

Practical Energy Management Systems for Isolated Microgrids

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Abstract—This paper presents practical Energy Management System (EMS) models which consider the operational constraints of Distributed Energy Resources (DERs), active-reactive power balance, unbalanced system configuration and loading, and voltage dependent loads. A novel linearization approach is proposed and validated based on the fact that, for isolated microgrids, due to the characteristics of feeders, network losses and voltage drops across feeders are relatively small. The proposed EMS models are Mixed Integer Quadratic Programming (MIQP) problems, requiring less computation time and thus suitable for online applications. The practical EMS models are compared with a typical decoupled Unit Commitment (UC) and Optimal Power Flow (OPF) based EMS with and without consideration of system unbalancing. The models, along with “standard” EMS models, are tested, validated, and compared using a CIGRE medium voltage benchmark system and the real isolated microgrid of Kasabonika Lake First Nation in Northern Ontario, Canada. The presented results demonstrate the effectiveness and practicability of the proposed models.

Index Terms—Energy Management System, microgrid, optimal power flow, renewable energy integration, unbalancing, unit commitment.

NOMENCLATURE

Indices and Superscripts

g	Generating units
i, j	Bus
k, t	Time steps
l, m	Phase
n	ESS units
$\rightarrow$	phasor
c	Commercial
PV	Photo-Voltaic (PV)
PW	Wind turbine
r	Residential
S	Energy Storage System

Sets

$\mathcal{G}$	Generator units
$\mathcal{I}$	Buses
$\mathcal{L}$	Phases (a, b, c)
$\mathcal{N}$	ESS units
$\mathcal{T}$	Time steps

Parameters

A, B, C, D	Three-phase ABCD parameter matrices [p.u.]
d	Quadratic term of cost function [\$/kWh ²]
e	Linear term of cost function [\$/kWh]
C^{LC}	Load curtailment cost [\$/kWh]
C^{sdn}	Shut-down cost of generating unit [\$]
C^{sup}	Start-up cost of generating unit [\$]
f	Constant term of cost function [\$/h]
$\bar{P}^S$	ESS charging/discharging power limit [p.u.]
P^{PV}	Photo-Voltaic (PV) plant output [p.u.]
P^{PW}	Wind turbine output [p.u.]
Pd	Active power demand [p.u.]
Qd	Reactive power demand [p.u.]
Y	Admittance [p.u.]
α	Voltage exponent for active load model
$\bar{\cdot}, \underline{\cdot}$	Maximum and minimum limits
β	Voltage exponent for reactive load model
Δt_k	Time interval between step k and step $k + 1$ [h]
η^{ch}, η^{dch}	Charging, discharging efficiency of ESS
$\gamma_{l,m}$	Angle difference between two balanced phases ($\pm 120^\circ$)
0	3-by-3 zero matrix
I	3-by-3 identity matrix
θ	Angle for line admittance [rad.]
$\wedge$	Variables considered as fixed parameters
Z	Impedance [p.u.]

Variables

E	Internal voltage of generator [p.u.]
I	Current [p.u.]
$\Delta P, \Delta Q$	Slack variables used for active and reactive power balance [p.u.]
P	Active power from generating units [p.u.]
P^{ch}, P^{dch}	ESS charging, discharging power [p.u.]
P^{LC}, Q^{LC}	Active and reactive Load curtailed [p.u.]
Q	Reactive power from generating units [p.u.]
S	Apparent power [p.u.]
D	Binary shut-down decision of generator (1 = OFF)
SoC	State of charge of ESS [p.u.]
U	Binary start-up decision of generator (1 = ON)
V	Voltage [p.u.]
W	ON/OFF decision (1 = ON, 0 = OFF)
δ	Voltage angle, [rad]
δ^E	Inter voltage angle of generator, [rad]

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I. INTRODUCTION

A Microgrid is defined as a cluster of small sources or Distributed Energy Resources (DERs) such as Distributed Generation (DG) units, Energy Storage Systems (ESS) and controllable and uncontrollable loads, presenting itself to the grid as a single controllable entity [1]. Microgrids can be permanently connected to the grid, permanently disconnected from the grid (referred to as isolated microgrids), or may transit between these two modes. There are many isolated microgrids worldwide, for example in Canada, there are around 280 remote communities supplying their electricity needs by locally available generation resources [2]. Most of these remote microgrids rely on diesel-based generation and have high electricity costs as compared to other parts of Canada, due to expensive fuel logistics. Also, in some of these microgrids, the electricity demand is reaching maximum generation capacity. Thus, to mitigate these issues, the integration of renewable energy sources (RES) is an option; however, this requires proper and efficient Energy Management Systems (EMS).

A microgrid EMS is responsible for its economic and reliable operation, and determines the optimal commitment and dispatch of the DERs [1]. The microgrid EMS needs to consider operational constraints, active and reactive power balance, and reserve constraints. Depending on the desired functionalities and characteristics of the microgrid, the EMS model can be developed considering different levels of detail. From practical measurements in actual remote microgrids, it can be observed that the voltage drops and feeder losses are relatively small because of the short length and large capacity of the feeders; hence, it is necessary to re-evaluate the network constraints for these microgrids in EMS models. Moreover, with the increasing share of RES in these systems and the tendency to deploy EMS based only on Unit Commitment (UC) models, reactive power dispatch needs to be integrated with active power dispatch in microgrid UC models [3]. Finally, loads in such isolated microgrids are sensitive to voltage variations, which need to be accounted for as well. Thus, there is a need for a practical EMS for such isolated microgrids that takes into account these specific issues, which have not been properly considered in previous EMS models.

As suggested in [1], achieving balance between the supply and demand is critical in isolated microgrids, while maintaining adequate reserve margins; this requires a high level of coordination among the DERs. A centralized EMS topology has been proposed in many of the works considering UC and/or Optimal Power Flow (OPF) models (e.g., [3]–[11]). UC-based EMS models include the ESS operational constraints in addition to the classical UC constraints for active power dispatch to minimize operating costs [4], [5], whereas OPF-based EMS models include network active and reactive power flow constraints, grid operational limits, and ESS constraints to determine the optimal active and reactive power dispatch decisions for DERs [6], [9]. Furthermore, in [7], [9], relevant unbalanced network conditions for microgrids are considered in the EMS models.

In [7], a decoupled UC and OPF based EMS is presented comprising a UC stage and a three-phase OPF dispatch stage.

The UC decisions are applied to the OPF subproblem, which obtains the optimal dispatch considering unbalanced network conditions. If a deficit in reactive power supply is observed, the UC subproblem is solved again by turning ON the next cheaper available DG unit; in this way, the approach in [7] arrives at a sub-optimal dispatch. Thus, the decoupled approach may not yield optimal active and reactive power dispatches, resulting in some cases of load shedding and overall higher operating costs, as demonstrated in [3] and [7]. To account for the UC and OPF constraints in a unified framework, an integrated UC-OPF based EMS is proposed in [3] to obtain the optimal dispatch of DERs; however, the proposed EMS model does not take into account the unbalanced system aspects of a microgrid, which should be considered, since this can lead to deviations from the optimal dispatch or inability to supply the reactive power demand, as demonstrated in [7].

The integrated UC and OPF problem is a Mixed Integer Non-Linear Programming (MINLP) optimization problem which is computationally challenging and not completely suitable for online applications, as shown in [3]. In [10], a two-stage EMS problem for grid-connected microgrids is proposed, where the first stage is a linearized UC-OPF problem, while the second stage is a three-phase OPF problem; the network equations are linearized using a Taylor series expansion. A linearized UC-OPF problem is presented in [11] for isolated microgrids, where the network equations are linearized using piece-wise functions; however, typical UC constraints such as ramp-up and ramp-down, and minimum-up and minimum-down time are ignored, which are necessary in microgrids. Both [10] and [11] do not consider the voltage dependency of the loads, which is required, given the load characteristics of isolated microgrids are voltage dependent. Furthermore, the results in [10] and [11] do not consider deviations in the forecast of RES and demand, which is important for isolated microgrids because of the need for demand-supply balance at all times.

From the review of the literature, there are only a few works that have considered microgrid EMS models that account for unbalanced network conditions, which increase the computational burden significantly. Since it is important to consider detailed network constraints for isolated microgrids, because of their specific feeder and loading characteristics, practical and realistic EMS models for such isolated microgrids need to be developed for online applications to provide economic, efficient, and realistic dispatch of microgrid DERs. Thus, this paper presents a practical EMS model which considers the operational constraints of DERs, simultaneous active-reactive power balance, unbalanced network conditions, and voltage dependent loads. The proposed EMS models are Mixed Integer Quadratic Programming (MIQP) problems, requiring less computational effort, and hence are suitable for online applications. This is accomplished by linearizing the network equations assuming that, for an isolated microgrid, because of the relative large capacity and short length of feeders, network losses and voltage drops across the feeder are relatively small. In order to validate the accuracy of the dispatch solution obtained from the proposed practical EMS models, a detailed power flow problem is utilized. Furthermore, in order to determine

the effectiveness of the proposed EMS model, a two stage decoupled UC-OPF EMS model is formulated both with and without unbalanced conditions for comparison purposes. An MPC approach is used to account for deviations in the forecast of renewables and electricity demand. Therefore, the main contributions of this work are two different proposed and studied mathematical models, one considering an approximate feeder representation and the other without modeling the feeders, with both including the UC operational constraints, a three-phase linearized generator model, voltage dependency of the loads, and unbalanced conditions; these models are referred here to as practical EMS for isolated microgrids. The effect of considering a detailed feeder representation is analyzed by comparing both practical EMS models with and without the inclusion of the grid. Furthermore, the dispatch results obtained from the proposed practical EMS models are compared with those obtained from decoupled UC-OPF based EMS models, considering balanced and unbalanced conditions. The proposed EMS models are tested and validated on two isolated microgrids representing practical extremes for these types of systems, i.e., the complex and large CIGRE medium voltage benchmark microgrid, and the real and simpler isolated system of the Kasabonika Lake First Nation (KLFN) community in Northern Ontario, Canada.

The rest of the paper is organized as follows: Section II presents an overview of existing dispatch models relevant to isolated microgrids. Section III introduces the proposed linearized UC-OPF models with and without inclusion of the network, considering an MPC implementation. Section IV presents and discusses two case studies, a CIGRE benchmark microgrid and the KLFN microgrid, validating and demonstrating the benefits of the proposed EMS models for the operation of isolated microgrids. Finally, Section V highlights the main conclusions and contributions of this paper.

II. BACKGROUND TO MICROGRID EMS MODELS

A centralized microgrid EMS includes operational constraints pertaining to DERs, active and reactive power balance, and reserve constraints. Depending on the desired functionalities and characteristics of the microgrid, the EMS model can be developed considering different levels of detail. A decoupled UC-OPF based EMS model has been presented in [3], [7], and [8] for microgrid operations, comprising two layers, a UC-based scheduling layer and an OPF-based dispatch layer; these layers are solved sequentially. In order to account for deviations in the forecast of renewables and demand, an MPC approach is used in most EMS formulations [3], [5], [7].

A. UC model

The UC subproblem seeks to minimize the operating cost of the microgrid, including generation costs, start-up and shut-down costs of diesel generators, and high costs associated with load curtailment [3], [7]. The model constraints include:

- Demand-supply balance, representing RES as uncontrollable injections, i.e., negative loads.
- Reserve constraints, which increase with RES.

- Generalized UC constraints, representing limits on active power generation, ramp-up and ramp-down, and minimum up-time and down-time limits of generators, plus coordination constraints.
- Energy Storage Systems (ESS), including the energy balance constraint and constraints to prevent simultaneous charging/discharging, limits on the State of Charge (SOC), and charging/discharging power [3].

B. Unbalanced OPF Model

The commitment decisions obtained from the UC subproblem are used in a three-phase OPF subproblem to obtain the dispatch [7], and [3], considering the minimization of the operating cost of the microgrid, as follows:

$$J = \sum_{g \in \mathcal{G}, k \in \mathcal{T}} [(d_g P_{g,k}^2 \Delta t_k + e_g P_{g,k} + f_g \widehat{W}_{g,k}) \Delta t_k + C_g^{sup} \widehat{U}_{g,k} + C_g^{sdn} \widehat{D}_{g,k}] + \sum_{i \in \mathcal{I}, k \in \mathcal{T}} [C^{LC} P_{i,k}^{LC} \Delta t_k] \quad (1)$$

where all variables, parameters, indices, and sets are defined in the Nomenclature section. The OPF constraints, as given in [7], [9], are discussed next.

1) *Feeders and Transformers*: Voltages and currents at buses i and j are related through the following rectangular phasor equations:

$$\begin{bmatrix} \vec{V}_{abc,i,k} \\ \vec{I}_{abc,i,k} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \vec{V}_{abc,j,k} \\ \vec{I}_{abc,j,k} \end{bmatrix} \quad \forall i, j \in \mathcal{I}_{i,j}, k \in \mathcal{T} \quad (2)$$

where $\vec{V}_{abc} = [\vec{V}_a \ \vec{V}_b \ \vec{V}_c]^T$, and $\vec{I}_{abc} = [\vec{I}_a \ \vec{I}_b \ \vec{I}_c]^T$.

Kirchhoff's current law at each node and phase is enforced, as follows:

$$\sum_{g \in \mathcal{G}_i} I_{l,g,k} + I_{l,i,k}^{PV} + I_{l,i,k}^{PW} + \sum_{n \in \mathcal{N}_i} I_{l,n,k}^S = \sum_{j \in \mathcal{I}_i} I_{l,i,j,k} + I_{l,k}^r + I_{l,k}^c \quad \forall i \in \mathcal{I}, l \in \mathcal{L}, k \in \mathcal{T} \quad (3)$$

And the following equation guarantees that voltage elements connected to the same node are equal, for each phase:

$$V_{l,g,k} = V_{l,i,k} = V_{l,n,k}^S \quad \forall i \in \mathcal{I}, l \in \mathcal{L}, k \in \mathcal{T} \quad (4)$$

2) *Generators*: Directly-connected synchronous generators are modeled as a special case of series element, as follows:

$$\begin{bmatrix} \vec{E}_{abc,g,k} \\ \vec{I}_{abc,g,k} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \vec{Z}_{abc,g,k} \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \vec{V}_{abc,g,k} \\ \vec{I}_{abc,g,k} \end{bmatrix} \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (5)$$

where $\vec{E}_{abc} = [\vec{E}_a \ \vec{E}_b \ \vec{E}_c]^T$, and Z_{abc} is the per-phase impedance matrix of the machine, which can be estimated from the sequence impedances of the generator. The internal synchronous machine positive sequence voltage is obtained as follows:

$$\sum_{l \in \mathcal{I}} \vec{E}_{l,g,k} = 0 \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (6)$$

$$E_{a,g,k} = E_{b,g,k} = E_{c,g,k} \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (7)$$

And the generator output powers are defined as follows:

$$P_{l,g,k} + jQ_{l,g,k} = \vec{V}_{l,g,k} \vec{I}_{l,g,k}^* \quad \forall g \in \mathcal{G}, l \in \mathcal{L}, k \in \mathcal{T} \quad (8)$$

$$P_g \widehat{W}_{g,k} \leq P_{g,k} \leq \bar{P}_g \widehat{W}_{g,k} \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (9)$$

$$Q_g \widehat{W}_{g,k} \leq Q_{g,k} \leq \bar{Q}_g \widehat{W}_{g,k} \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (10)$$

3) *Loads*: Residential and commercial loads in distribution systems can be considered to be a mix of constant impedance (Z), constant current (I) and constant power (P), i.e., ZIP loads, as discussed in [13] and [14], which can be modeled with voltage exponent power equations [15]. These voltage exponents are different for residential and commercial loads; hence, each are represented here as different exponential functions of the voltage as follows:

$$Pd_{l,i,k}^r V_{l,i,k}^{\alpha_k^r} + jQd_{l,i,k}^r V_{l,i,k}^{\beta_k^r} = \vec{V}_{l,i,k}(\vec{I}_{l,i,k}^r)^* \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T} \quad (11)$$

$$Pd_{l,i,k}^c V_{l,i,k}^{\alpha_k^c} + jQd_{l,i,k}^c V_{l,i,k}^{\beta_k^c} = \vec{V}_{l,i,k}(\vec{I}_{l,i,k}^c)^* \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T} \quad (12)$$

4) *Grid Operational Limits*: These include voltage limits:

$$\bar{V}_i \leq V_{l,i,k} \leq \bar{V}_i \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T} \quad (13)$$

and current limits:

$$\bar{I}_i \leq I_{l,i,k} \leq \bar{I}_i \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T} \quad (14)$$

5) *ESS*: These constraints include the following charging, discharging, and SOC equations and limits:

$$SoC_{n,k+1} - SoC_{n,k} = \left(P_{n,k}^{ch} \eta_n^{ch} - \frac{P_{n,k}^{dch}}{\eta_n^{dch}} \right) \Delta t_k \quad \forall n \in \mathcal{N}, k \in \mathcal{T} \quad (15)$$

$$P_{l,n,k}^{ch} + P_{l,n,k}^{dch} = \vec{V}_{l,n,k}^S \vec{I}_{l,n,k}^{S*} \quad \forall l \in \mathcal{L}, n \in \mathcal{N}, k \in \mathcal{T} \quad (16)$$

$$P_{n,k}^{ch} P_{n,k}^{dch} = 0 \quad \forall n \in \mathcal{N}, k \in \mathcal{T} \quad (17)$$

$$SoC_n \leq SoC_{n,k} \leq \bar{SoC}_n \quad \forall n \in \mathcal{N}, k \in \mathcal{T} \quad (18)$$

$$P_{n,k}^{ch} \leq \bar{P}_n^S \quad \forall n \in \mathcal{N}, k \in \mathcal{T} \quad (19)$$

$$P_{n,k}^{dch} \leq \bar{P}_n^S \quad \forall n \in \mathcal{N}, k \in \mathcal{T} \quad (20)$$

Battery degradation and operating costs are not considered in the aforementioned model, as is the case of most microgrid and practical EMSs (e.g., [3]–[5], [7], [8], [10], [11]), since this is typically considered a long-term issue that depends on the charging/discharging cycles, the rate of charging/discharging, ambient temperature, and depth of discharge [16]. For the daily operation EMS models proposed here in, degradation can be represented through the efficiency parameters in (15) and the SoC range in (18) as the battery degrades throughout the years, until it is eventually replaced. The degradation in these parameters for a typically battery lifetime are 90% to 86% in η_n^{ch} and η_n^{dch} , and 100% to 80% in $\bar{SoC}$ [16]. Furthermore, ESS operating costs are quite small compared to fuel costs, especially for remote microgrids, and hence are usually neglected in microgrid EMS cost functions. However,

battery degradation can be included in the EMS through appropriate inclusion of degradation and operation costs in the objective function, as discussed in [17].

III. PROPOSED PRACTICAL EMS MODELS

In a decoupled UC-OPF problem, it is possible to account for the impact of reactive power supply in the UC decisions when solving the three-phase OPF problem, as proposed in [7]. Thus, if the OPF is infeasible, the commitment decisions of the UC subproblem can be modified by forcing the commitment of the next available cheaper generator. This process is repeated until a feasible OPF solution is obtained that satisfies the reactive power demand-supply balance and hence more expensive; however, the UC-OPF solution in this case is sub-optimal. Furthermore, a three-phase OPF problem is a complex Non-Linear Programming (NLP) problem that requires significant computation time and may not converge. Nevertheless, since isolated microgrids are generally radial networks with relatively short-length and high-capacity feeders, these networks do not usually play a significant role in microgrid energy management, as observed from measurements on actual remote microgrids [18] and demonstrated here. Therefore, there is a need to reevaluate the representation of the distribution network in EMS models, while considering the necessary voltage and reactive power management in the microgrid. Hence, practical EMS models are proposed next, which include linearization of detailed three-phase models of DG units, voltage dependent and unbalanced loads, reactive power balance constraints, and UC DER constraints, with and without linear approximations of the feeder equations.

A. Linearized UC-OPF Model (EMS-1)

The conventional UC-OPF is a complex MINLP optimization model [3], which may be transformed into an MILP problem by linearizing the non-linear network equations, based on the fact that isolated microgrids typically have short-length and high-capacity feeders, and thus bus voltage magnitudes and angles do not change significantly from node to node.

1) *Nodal Power Balance*: The demand-supply balance at each bus considers the output from dispatchable generators and non-dispatchable PV and wind generator units, plus the total power demand from commercial and residential customers, taking into account the generation from ESS units. In this case, feeders can be modeled as follows [19]:

$$\vec{V}_{abc,i} = \vec{V}_{abc,j} + \vec{I}_{abc,i,j} \vec{Z}_{abc,i,j} \quad \forall i, j \in \mathcal{I}_{i,j} \quad (21)$$

where

$$\vec{I}_{abc,i,j} = (\vec{V}_{abc,i} - \vec{V}_{abc,j}) \vec{Y}_{abc,i,j} \quad \forall i, j \in \mathcal{I}_{i,j}$$

$$\vec{Y}_{abc,i,j} = \vec{Z}_{abc,i,j}^{-1} \quad \forall i, j \in \mathcal{I}_{i,j}$$

$$[Y]_{i,j} = \begin{bmatrix} Y_{i,j}^{aa} & Y_{i,j}^{ab} & Y_{i,j}^{ac} \\ Y_{i,j}^{ba} & Y_{i,j}^{bb} & Y_{i,j}^{bc} \\ Y_{i,j}^{ca} & Y_{i,j}^{cb} & Y_{i,j}^{cc} \end{bmatrix} \quad \forall i, j \in \mathcal{I}_{i,j}$$

From these equations, the complex power injection at node i for phase l can be defined as follows:

$$\begin{aligned}
 \vec{S}_{l,i,k}^* &= \vec{V}_{l,i,k}^* \sum_{j \in \mathcal{I}_i} \vec{I}_{l,i,j,k} \\
 &= \vec{V}_{l,i,k}^* \sum_{j \in \mathcal{I}_i} \sum_{m \in \mathcal{L}} \vec{Y}_{i,j}^{l,m} (\vec{V}_{m,i,k} - \vec{V}_{m,j,k}) \\
 &= \sum_{j \in \mathcal{I}_i, m \in \mathcal{L}} Y_{i,j}^{l,m} [V_{l,i,k} V_{m,i,k} e^{j(\theta_{i,j}^{l,m} + \delta_{m,i,k} - \delta_{l,i,k})} \\
 &\quad - V_{l,i,k} V_{m,j,k} e^{j(\theta_{i,j}^{l,m} + \delta_{m,j,k} - \delta_{l,i,k})}] \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T}
 \end{aligned} \tag{22}$$

Thus, active and reactive power injection at node i for phase l can be obtained as follows:

$$\begin{aligned}
 &\sum_{g \in \mathcal{G}_i} P_{l,g,k} + P_{l,i,k}^{PV} + P_{l,i,k}^{PW} - P_{l,i,k}^{dc} V_{l,i,k}^{\alpha_k^c} \\
 &\quad - P_{l,i,k}^{dr} V_{l,i,k}^{\alpha_k^r} + \sum_{n \in \mathcal{N}_i} (P_{l,n,k}^{dch} - P_{l,n,k}^{ch}) \\
 &= \sum_{j \in \mathcal{I}_i, m \in \mathcal{L}} V_{l,i,k} Y_{i,j}^{l,m} [V_{m,i,k} \cos(\theta_{i,j}^{l,m} + \delta_{m,i,k} - \delta_{l,i,k}) \\
 &\quad - V_{m,j,k} \cos(\theta_{i,j}^{l,m} + \delta_{m,j,k} - \delta_{l,i,k})] \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T}
 \end{aligned} \tag{23}$$

$$\begin{aligned}
 &\sum_{g \in \mathcal{G}_i} Q_{l,g,k} - Q_{l,i,k}^{dc} V_{l,i,k}^{\beta_k^c} - Q_{l,i,k}^{dr} V_{l,i,k}^{\beta_k^r} \\
 &= \sum_{j \in \mathcal{I}_i, m \in \mathcal{L}} V_{l,i,k} Y_{i,j}^{l,m} [V_{m,i,k} \sin(\theta_{i,j}^{l,m} + \delta_{m,i,k} - \delta_{l,i,k}) \\
 &\quad - V_{m,j,k} \sin(\theta_{i,j}^{l,m} + \delta_{m,j,k} - \delta_{l,i,k})] \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T}
 \end{aligned} \tag{24}$$

Different from [11], where ac power flow equations are linearized by using piece-wise linearization and first-order Taylor series expansions, and UC constraints, internal generator and voltage dependent load models are not considered, (23) and (24) are linearized using the following assumptions:

- Since the bus voltages represented by $V = 1 + \Delta V$, lie between 0.95 p.u. and 1.05 p.u., the range for ΔV is ± 0.05 , which yields a negligible range for ΔV^2 of ± 0.0025 in the quadratic voltage products of the power flow equations.
- The bus voltage phasor angles do not vary much from node to node for typical feeder length and capacity in microgrids, as demonstrated in Section IV-C for different test systems; thus, the following approximations can be applied: $\sin(\Delta \delta_{i,j}) \approx \Delta \delta_{i,j}$, and $\cos(\Delta \delta_{i,j}) \approx 1$.
- Even though there can be significant unbalanced currents in the feeders due to load unbalances, the voltage angle difference between phases are close to 120° , due to relatively small feeder and internal generator impedances, as shown in Section IV-C for different test systems; thus, for $\delta_l - \delta_m = \Delta \delta_{l,m} + \gamma_{l,m}$, one can assume that $\Delta \delta_{l,m} \approx 0$, and hence $\sin(\Delta \delta_{l,m}) \approx \Delta \delta_{l,m}$ and $\cos(\Delta \delta_{l,m}) \approx 1$.
- The load power voltage functions can be approximated by $V^\alpha \approx 1 + \alpha \Delta V$ for active power loads, and $V^\beta \approx$

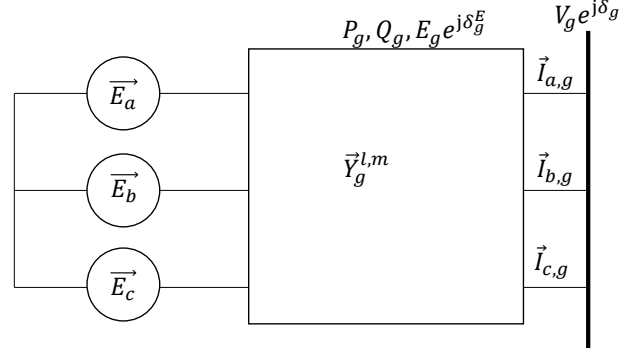


Fig. 1: Generator model.

$1 + \beta \Delta V$ for reactive power loads, based on a first-order Taylor series expansion, given the small ΔV range of ± 0.05 .

Based on these assumptions, the linearized active and reactive power balance equations can be formulated as follows:

$$\begin{aligned}
 &\sum_{g \in \mathcal{G}_i} P_{l,g,k} + P_{l,i,k}^{PV} + P_{l,i,k}^{PW} + P_{l,i,k}^{LC} \\
 &\quad - P_{l,i,k}^{dc} (1 + \alpha_k^c \Delta V_{l,i,k}) - P_{l,i,k}^{dr} (1 + \alpha_k^r \Delta V_{l,i,k}) \\
 &\quad + \sum_{n \in \mathcal{N}_i} (P_{l,n,k}^{dch} - P_{l,n,k}^{ch}) \\
 &= \sum_{j \in \mathcal{I}_i, m \in \mathcal{L}} Y_{i,j}^{l,m} [(\Delta V_{m,i,k} - \Delta V_{m,j,k}) \cos(\theta_{i,j}^{l,m} + \gamma_{l,m}) \\
 &\quad + (\delta_{m,j,k} - \delta_{m,i,k}) \sin(\theta_{i,j}^{l,m} + \gamma_{l,m})] \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T}
 \end{aligned} \tag{25}$$

$$\begin{aligned}
 &\sum_{g \in \mathcal{G}_i} Q_{l,g,k} - Q_{l,i,k}^{LC} - Q_{l,i,k}^{dc} (1 + \beta_k^c \Delta V_{l,i,k}) \\
 &\quad - Q_{l,i,k}^{dr} (1 + \beta_k^r \Delta V_{l,i,k}) \\
 &= - \sum_{j \in \mathcal{I}_i, m \in \mathcal{L}} Y_{i,j}^{l,m} [(\Delta V_{m,i,k} - \Delta V_{m,j,k}) \sin(\theta_{i,j}^{l,m} + \gamma_{l,m}) \\
 &\quad - (\delta_{m,j,k} - \delta_{m,i,k}) \cos(\theta_{i,j}^{l,m} + \gamma_{l,m})] \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T}
 \end{aligned} \tag{26}$$

2) *Generator Constraints:* As per Fig. 1, at the generator terminal, the currents in each phase can be determined from (5). Hence, the per-phase complex power output of the generator can be determined as follows:

$$\begin{aligned}
 \vec{S}_{l,g,k}^* &= \vec{V}_{l,g,k}^* \vec{I}_{l,g,k} \\
 &= \vec{V}_{l,g,k}^* \sum_{m \in \mathcal{L}} \vec{Y}_g^{l,m} (\vec{E}_{m,g,k} - \vec{V}_{m,g,k}) \\
 &= \sum_{m \in \mathcal{L}} Y_{l,m,k}^g [V_{l,g,k} E_{m,g,k} e^{j(\theta_{l,g}^{l,m} + \delta_{m,g,k}^E - \delta_{l,g,k})} \\
 &\quad - V_{l,g,k} V_{m,g,k} e^{j(\theta_{l,g}^{l,m} + \delta_{m,g,k} - \delta_{l,g,k})}] \quad \forall l \in \mathcal{L}, g \in \mathcal{G}, k \in \mathcal{T}
 \end{aligned} \tag{27}$$

The active and reactive power outputs can hence be expressed as:

$$P_{l,g,k} = \sum_{m \in \mathcal{L}} V_{l,g,k} Y_g^{l,m} [E_{m,g,k} \cos(\theta_g^{l,m} + \delta_{m,g,k}^E - \delta_{l,g,k}) - V_{m,g,k} \cos(\theta_{l,m}^g + \delta_{m,g,k} - \delta_{l,g,k})] \quad \forall l \in \mathcal{L}, g \in \mathcal{G}, k \in \mathcal{T} \quad (28)$$

$$Q_{l,g,k} = \sum_{m \in \mathcal{L}} V_{l,g,k} Y_g^{l,m} [E_{m,g,k} \sin(\theta_g^{l,m} + \delta_{m,g,k}^E - \delta_{l,g,k}) - V_{m,g,k} \sin(\theta_{l,m}^g + \delta_{m,g,k} - \delta_{l,g,k})] \quad \forall l \in \mathcal{L}, g \in \mathcal{G}, k \in \mathcal{T} \quad (29)$$

In order to linearize (28)-(29), one can assume that $V_{l,g,k} = 0.95$ p.u., because in isolated microrids with voltage dependent loads, the UC-OPF procedure will try to minimize the network voltage to reduce demand, thus reducing costs, and hence voltages will be close to their minimum value. Furthermore:

$$\delta_{m,g,k}^E - \delta_{l,g,k} = \delta_{m,g,k}^E - \delta_{l,g,k}^E + \delta_{l,g,k}^E - \delta_{l,g,k} \quad (30)$$

where $\delta_{m,g,k}^E - \delta_{l,g,k}^E = \gamma_{l,m}$, and thus:

$$\delta_{m,g,k}^E - \delta_{l,g,k} \approx \gamma_{l,m} + \delta_{l,g,k}^E - \delta_{l,g,k} \approx \gamma_{l,m} + \Delta\delta_{l,g,k}^E \quad (31)$$

On the other hand, defining the variables $EX_{l,g,k} = E_{l,g,k} \cos(\Delta\delta_{l,g,k}^E)$, and $EY_{l,g,k} = E_{l,g,k} \sin(\Delta\delta_{l,g,k}^E)$, it follows that:

$$E_{a,g,k} = E_{b,g,k} = E_{c,g,k} \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (32)$$

$$\Delta\delta_{a,g,k}^E = \Delta\delta_{b,g,k}^E = \Delta\delta_{c,g,k}^E \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (33)$$

$$EX_{g,k} = EX_{a,g,k} = EX_{b,g,k} = EX_{c,g,k} \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (34)$$

$$EY_{g,k} = EY_{a,g,k} = EY_{b,g,k} = EY_{c,g,k} \quad \forall g \in \mathcal{G}, k \in \mathcal{T} \quad (35)$$

Hence, (28) and (29) can be expressed in linear form as follows:

$$P_{l,g,k} = \sum_{m \in \mathcal{L}} Y_g^{l,m} [0.95EX_{g,k} \cos(\theta_g^{l,m} + \gamma_{l,m}) - 0.95EY_{g,k} \sin(\theta_g^{l,m} + \gamma_{l,m}) - (1 + \Delta V_{l,g,k} + \Delta V_{m,g,k}) \cos(\theta_g^{l,m} + \gamma_{l,m}) + (\delta_{m,g,k} - \delta_{l,g,k}) \sin(\theta_g^{l,m} + \gamma_{l,m})] \quad \forall l \in \mathcal{L}, g \in \mathcal{G}, k \in \mathcal{T} \quad (36)$$

$$Q_{l,g,k} = - \sum_{m \in \mathcal{L}} Y_g^{l,m} [0.95EX_{g,k} \sin(\theta_g^{l,m} + \gamma_{l,m}) + 0.95EY_{g,k} \cos(\theta_g^{l,m} + \gamma_{l,m}) - (1 + \Delta V_{l,g,k} + \Delta V_{m,g,k}) \sin(\theta_g^{l,m} + \gamma_{l,m}) - (\delta_{m,g,k} - \delta_{l,g,k}) \cos(\theta_g^{l,m} + \gamma_{l,m})] \quad \forall l \in \mathcal{L}, g \in \mathcal{G}, k \in \mathcal{T} \quad (37)$$

3) *Objective Function*: The objective function representing the cost of the microgrid, including generation costs, start-up and shut-down costs of diesel generators, and high costs for

load curtailment can be defined as:

$$J = \sum_{g \in \mathcal{G}, k \in \mathcal{T}} [(d_g P_{g,k}^2 \Delta t_k + e_g P_{g,k} + f_g W_{g,k}) \Delta t_k + C_g^{sup} U_{g,k} + C_g^{sdn} D_{g,k}] + \sum_{i,k} [C^{LC} P_{i,k}^{LC} \Delta t_k] \quad (38)$$

4) *Reserve Constraint*: The following constraint guarantees that proper spinning reserves for the microgrid are provided by the committed generators:

$$\sum_{g \in \mathcal{G}} (\bar{P}_g W_{g,k} - P_{g,k}) \geq R^{sv} \sum_{l \in \mathcal{L}, i \in \mathcal{I}} [P_{l,i,k}^{PV} + P_{l,i,k}^{PW} + Pd_{l,i,k}^c (1 + \Delta V_{l,i,k} \alpha_k^c) + (Pd_{l,i,k}^r) (1 + \Delta V_{l,i,k} \alpha_k^r) - P_{l,i,k}^{LC}] \quad \forall k \in \mathcal{T} \quad (39)$$

5) *Generalized UC Constraints*: These include limits on active power generation, ramp-up and ramp-down, minimum up-time and down-time constraints, and coordination constraints [3].

6) *ESS*: These constraints are the same as (16) to (20).

7) *Grid Operational Constraints*: These include the bus voltage constraints as follows:

$$\underline{\Delta V} \leq \Delta V_{l,i,k} \leq \overline{\Delta V} \quad \forall l \in \mathcal{L}, i \in \mathcal{I}, k \in \mathcal{T} \quad (40)$$

Current limits are not considered in this model, because the short-length and high-capacity of feeders in isolated microgrids make these limits unnecessary, as demonstrated by the results discussed in Section IV. However, these limits could be readily included if required without affecting the linearity of the model.

B. Linearized UC-OPF Model (EMS-2)

As discussed before, in isolated microgrids, the voltage drop across the feeder and losses in the feeders are not significant, and hence one may assume that all the DERs and loads are basically connected to a single bus, as shown in Fig. 2. However, it is necessary to consider unbalanced system loading, voltage dependent loads, and reactive power regulation in general. In this case, the objective function is the same as (38), and the model constraints change as discussed next.

1) *Demand-Supply Balance*: This constraint ensures that the total generation is equal to the total demand, for each phase, at every time interval as follows:

$$\sum_{g \in \mathcal{G}} P_{l,g,k} + P_{l,k}^{PV} + P_{l,k}^{PW} + \sum_{n \in \mathcal{N}} (P_{l,n,k}^{dch} - P_{l,n,k}^{ch}) = Pd_{l,k}^c V_{l,k}^{\alpha_k^c} + Pd_{l,k}^r V_{l,k}^{\alpha_k^r} - P_{l,k}^{LC} \quad \forall l \in \mathcal{L}, k \in \mathcal{T} \quad (41)$$

Hence, assuming that $V = 1 + \Delta V$, where ΔV is small, so that it can be approximated by $V^\alpha \approx 1 + \alpha \Delta V$, (41) can be

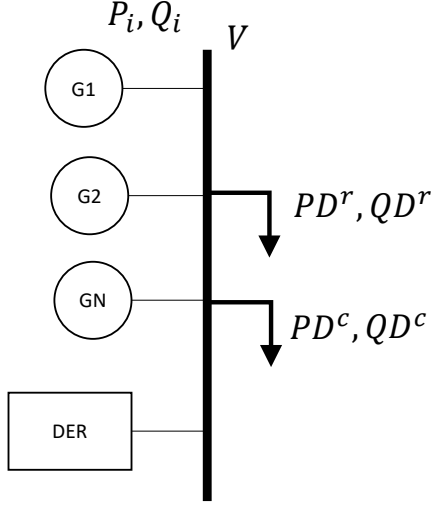


Fig. 2: Single-node representation of an isolated microgrid.

represented by:

$$\begin{aligned} \sum_{g \in \mathcal{G}} P_{l,g,k} + P_{l,k}^{PV} + P_{l,k}^{PW} + \sum_{n \in \mathcal{N}} (P_{l,n,k}^{dch} - P_{l,n,k}^{ch}) \\ = Pd_{l,k}^c (1 + \alpha_k^c \Delta V_{l,k}) \\ + Pd_{l,k}^r (1 + \alpha_k^r \Delta V_{l,k}) - P_{l,k}^{LC} \quad \forall l \in \mathcal{L}, k \in \mathcal{T} \end{aligned} \quad (42)$$

Similarly for reactive power balance constraints:

$$\begin{aligned} \sum_{g \in \mathcal{G}} Q_{l,g,k} = Qd_{l,k}^c (1 + \beta_k^c \Delta V_{l,k}) \\ + Qd_{l,k}^r (1 + \beta_k^r \Delta V_{l,k}) - Q_{l,k}^{LC} \quad \forall l \in \mathcal{L}, k \in \mathcal{T} \end{aligned} \quad (43)$$

2) *Reserve Constraint*: The following constraint guarantees that spinning reserve requirements for the microgrid are provided by the committed generators:

$$\begin{aligned} \sum_{g \in \mathcal{G}} \left(\bar{P}_g W_{g,k} - \sum_{l \in \mathcal{L}} P_{l,g,k} \right) \geq R^{sv} \sum_{l \in \mathcal{L}} [Pd_{l,k}^c (1 + \alpha_k^c \Delta V_{l,k}) \\ + Pd_{l,k}^r (1 + \alpha_k^r \Delta V_{l,k}) - P_{l,k}^{LC} + P_{l,k}^{PV} + P_{p,k}^{PW}] \\ \forall k \in \mathcal{T} \end{aligned} \quad (44)$$

3) *Other Constraints*: In addition to the above, the generator model (36)-(37), generalized UC constraints, ESS constraints, and voltage limit constraints defined for the EMS-1 model are also considered in this model. Furthermore, in order to consider the deviations in forecast of renewables and demand, an MPC technique is adopted here, where the forecasts of renewables and demand are updated at time t for the time horizon $\mathcal{T}$, and the EMS models are then solved for the 42 time intervals depicted in Fig. 3. The obtained solution defines DERs' set points at time $t+5$ min only. The time horizon is then shifted forward by one 5 min time interval, the forecasts of renewables and demand are updated for the time horizon $\mathcal{T}$, and the EMS models are solved again, repeating the process every 5 min, as illustrated in Fig. 4.

For computational efficiency, a scheduling horizon with non-uniform time resolution is used with the first few time steps of higher resolution and the latter time steps of low resolution, as discussed in [3] and [7], and depicted in Fig. 3.

The practical EMS models proposed here are MIQP problems, and are solved using the CPLEX solver in GAMS [20], on an Intel® Xeon® CPU L7555, 1.87 GHz 4-processor server. The UC subproblem of the decoupled UC-OPF model is also an MIQP problem solved with CPLEX, while the OPF subproblem is an NLP problem solved using the SNOPT solver [20].

IV. RESULTS AND DISCUSSIONS

In order to evaluate the accuracy of the proposed practical EMS models, a power flow problem is solved using the generator dispatch obtained from these models and including slack active power ΔP and reactive power ΔQ variables at each node; the sums of these slack variables, for each EMS model, yield a total measure of the power flow accuracy of the linearized model. The forecasting errors are considered using the same approach as in [3], where the forecast errors of renewables and demand are considered via probability density functions (pdfs) for 1-day and 1-hour ahead to obtain the wind, PV, and demand power profiles, using linear approximations of the difference between the 24-hour- and 1-hour-ahead forecast errors with respect to time to get forecasts for time intervals in between. To simulate the actual implementation of the proposed models and examine the applicability of the dispatch results, the real-time operation of the microgrid is also evaluated in an open loop for the dispatch schedule obtained, solving a three-phase power flow problem using the actual demand and renewable generation, with the internal voltage angle and magnitude being kept fixed for the generator controlling the system frequency (slack bus), and fixed active and reactive power outputs for the rest of the DERs. Based on the change of power in the slack generator for this power flow problem, the actual microgrid operating costs can be calculated and compared for all EMS models.

The developed dispatch models are tested and validated on a modified CIGRE benchmark system [3], [7], and on the real Northern Ontario-Canada isolated microgrid of KLFN [12], to evaluate their practical application. It should be mentioned that these systems represent two extreme cases of practical/real isolated microgrids, i.e., the large and practical CIGRE benchmark system, with multiple DERs, nodes, and relatively long feeders, and the simpler but real KLFN microgrid, with just a few DERs with a simple dispatch strategy, less nodes, and short feeders. This allows demonstrating the applicability and feasibility of the proposed EMSs for practical isolated microgrids, considering that there are no more complex isolated systems in practice than the CIGRE benchmark microgrid, thus illustrating the scalability of the proposed approach for actual applications.

A. CIGRE Benchmark System

The modified CIGRE benchmark system with 25% more ESS capacity, shown in Fig. 5 and Appendix, is used to test

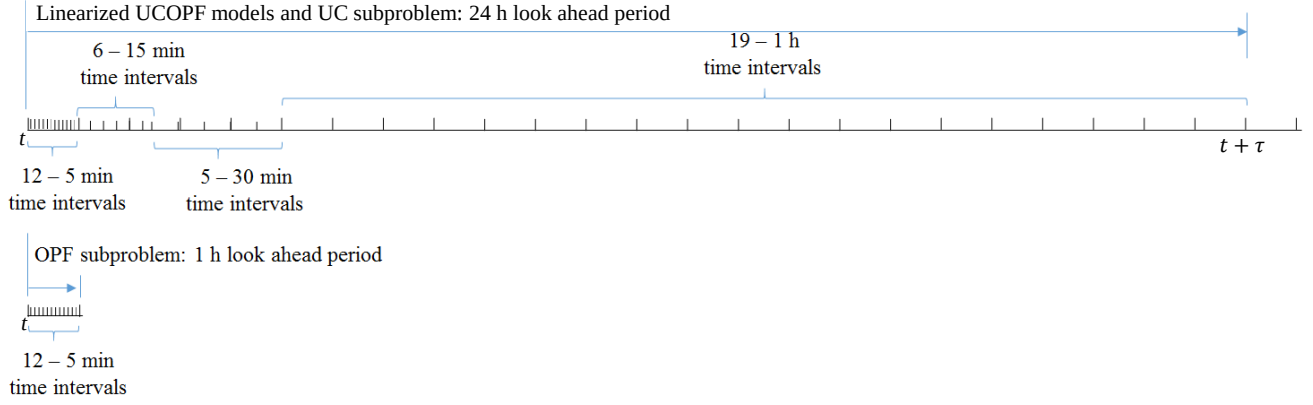


Fig. 3: MPC time intervals and horizon.

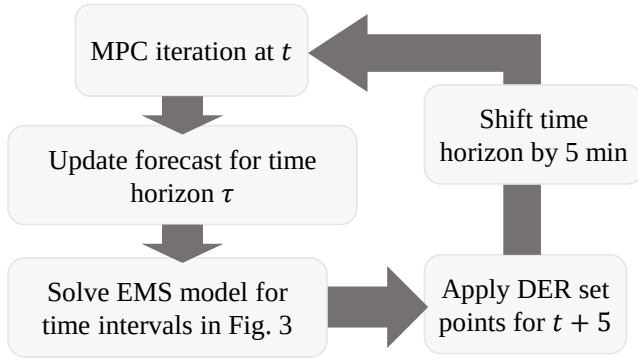


Fig. 4: MPC process.

and validate the proposed EMS models; the total installed capacity is 9,216 kW including all DERs. The line, load and generator parameters given in [7] are used, and the full system data can be found in the Appendix. The dispatch models are solved for both meshed (all switches closed) and radial (all switches open) versions of the system for 24 h of operation, with an MPC recalculation time of 5 min, thus yielding 288 MPC iterations. For comparison purposes, four different EMS models are considered: EMS-1 (Case I), EMS-2 (Case II), unbalanced decoupled UC-OPF (Case III), and balanced decoupled UC-OPF (Case IV). It is assumed that generator G1 is responsible for frequency control because of its large capacity.

1) *Meshed Microgrid*: Table I presents a summary of results obtained for all the EMS models for meshed CIGRE system, where all the switches are considered in closed condition. The values of ΔP and ΔQ are presented in % of the total demand, and the maximum observed errors over the 24 hours period are shown. Observe that the EMS-1 model (Case I) yields the least operating cost dispatch, with maximum errors in active and reactive power injection of 0.6% and 3%, respectively. The EMS-2 model (Case II) yields a solution with larger errors as compared to Case I, at lower computational costs, as expected; various computational parameters for an

MPC iteration are also provided for comparison purposes. It is important to note that the dispatch solutions obtained from the decoupled UC-OPF with and without the unbalanced condition, i.e., Case III and Case IV, respectively, resulted in higher estimated and actual operating costs as compared to the proposed practical EMS models, at much higher computational costs, as expected. The power flow errors in Case IV are due to the fact that a balanced power flow model was used in the dispatch solution, but the system is in fact unbalanced.

Fig. 6 presents the plots of dispatch for all DERs. Note that G1 and G13 are committed for the entire 24 hour operating horizon, in all cases. However, during hours 15-17, G9 is committed in Cases I and II in order to meet the reactive power requirements, since it has a low minimum generation limit, but has a high operating cost. On the other hand, in Cases III and IV, G2 or G3 replaces G9 during hours 15-17, because these are the next available cheaper units; however, this results in higher operating costs overall, thus showing that the proposed practical EMS methods yield more optimal solutions than the decoupled UC-OPF approaches. None of the feeders reached their current capacity limits.

2) *Radial Microgrid*: Table II presents a summary of results obtained for all the EMS models for the radial CIGRE system, with all switches open. Note that the EMS-2 model (Case II) yields the least cost dispatch, with maximum errors of 0.47% in active power and 2.94% in reactive power injections. The EMS-1 model (Case I) yields an operating cost very close to the one obtained with the EMS-2 model, with maximum errors of 0.34% in active power and 2.24% in reactive power injections. Observe that, the errors in active and reactive power injections are smaller for Cases I and II; also, the total computation times for all the models decrease, compared to the meshed CIGRE system; various computational parameters for an MPC iteration are also provided for comparison purposes. However, no major change is observed in operating costs in Case I and Case II. As in the case of the meshed system, the dispatch solutions obtained from the decoupled UC-OPF with and without unbalanced conditions, i.e., Cases III and IV, respectively, resulted in higher estimated and actual operating costs compared to the proposed practical EMS models, at

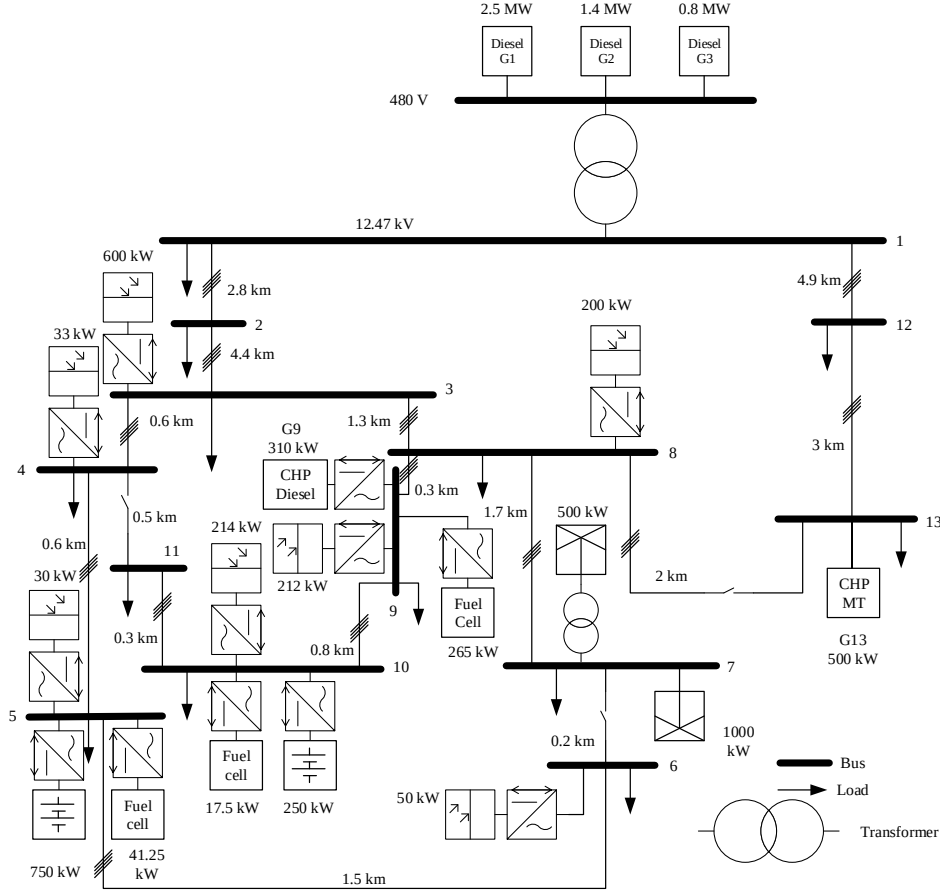


Fig. 5: Modified CIGRE benchmark system [3].

TABLE I: Summary of Results for meshed CIGRE system

Case	EMS Operating Cost [\$]	Actual Operating Cost [\$]	Max. ΔP [%]	Max. ΔQ [%]	Total Computation Time [s]	MPC Iteration		
						Number of Optimization Iterations	Average Number of Variables	Number of Equations
I	12,775	12,750	0.6	3	10,800	29,600	23,800	8,011
II	12,828	12,705	1.1	6	2,760	16,700	21,300	4,913
III	13,037	12,975	0	0	18,600	10,560	24,700	5,802
IV	12,970	12,907	3.6	4.12	9,960	9,860	13,537	5,300

TABLE II: Summary of Results for radial CIGRE system

Case	EMS Operating Cost [\$]	Actual Operating Cost [\$]	Max. ΔP [%]	Max. ΔQ [%]	Total Computation Time [s]	MPC Iteration		
						Number of Optimization Iterations	Average Number of Variables	Number of Equations
I	12,839	12,830	0.34	2.24	8,460	10,082	23,800	8,011
II	12,830	12,704	0.47	2.94	2,520	10,081	21,000	4,913
III	13,044	12,982	0	0	16,600	7,795	23,889	5,802
IV	15,634	15,565	3.6	4.2	6,660	10,044	13,537	5,300

higher computational costs, as expected.

B. Kasabonika Lake First Nation System

The KLFN system model used in this paper, shown in Fig. 7, allows to evaluate the performance of the proposed EMS models in a real existing Northern Canada community

microgrid. The measured generation data from existing 12.4 kW roof-top PV panels and 30 kW wind turbines are used in these simulations. There are 3 diesel units, operated one at a time as per the utility's dispatch rules, with the largest unit being rated at 1.5 MW. The commercial demand is represented by the data measured from the store (STR), the school (SCL),

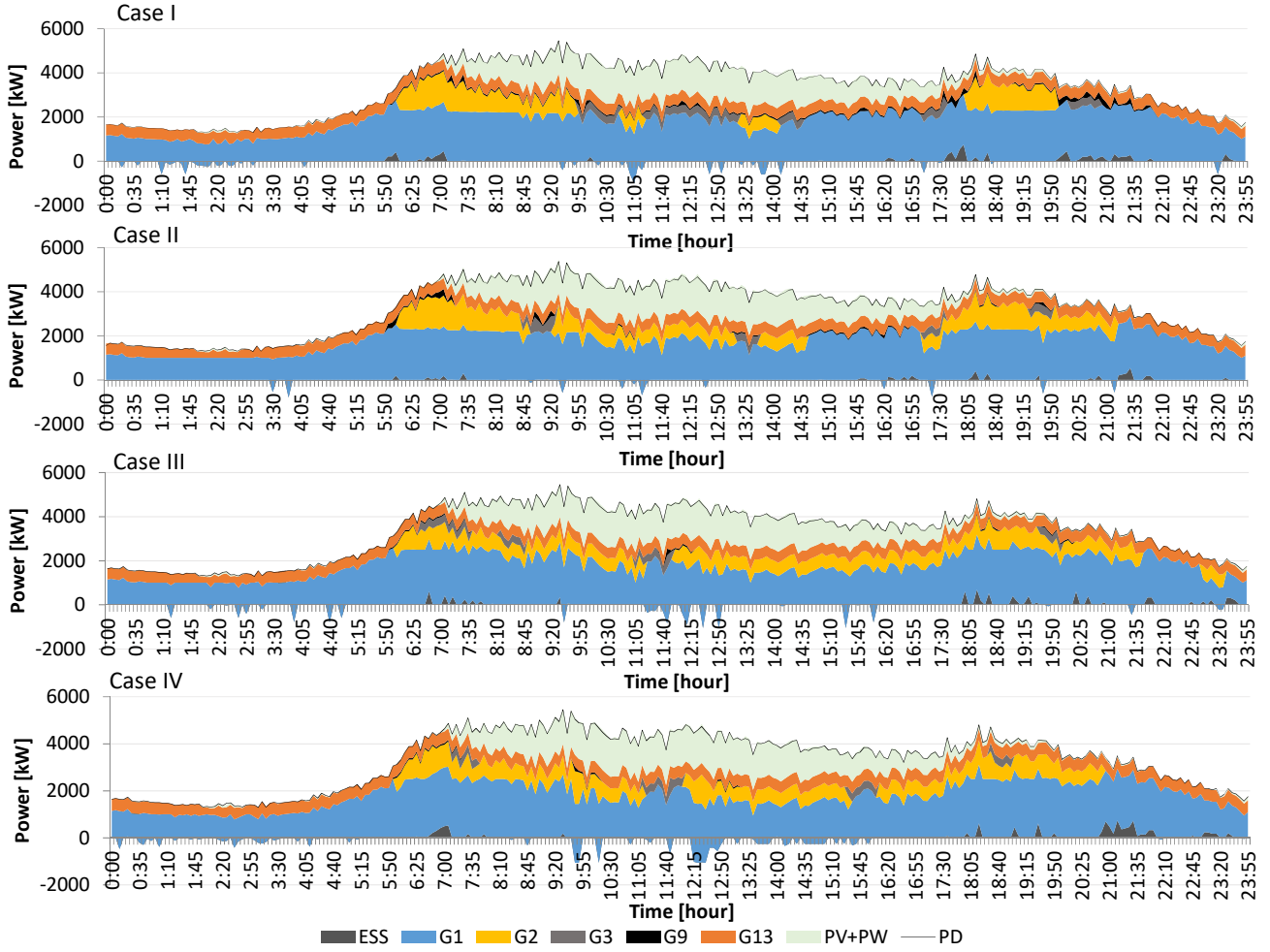


Fig. 6: Dispatch for all four EMS cases for the meshed CIGRE test system.

the police station (PLC), the nursery station (NRS), and the water treatment plant (WTP). The total residential demand at an hour is calculated from the measured total generation minus the total commercial demand, assuming that losses are not really significant in this microgrid, given the length and capacity of the feeders. Note that a 100% unbalance condition is considered, where load on one phase is twice the load in the other two phases, as is the actual case during the summer months. All data for this microgrid is provided in the Appendix.

Table III presents the summary of results obtained from the EMS models for the KLFN system. Observe that, similarly to the CIGRE test system, the EMS-1 model provides the least cost dispatch, with the EMS-2 following closely, with negligible values of ΔP and ΔQ errors in both cases. The latter can be attributed to the relatively smaller system size of KLFN microgrid as compared to the CIGRE microgrid, and is also the reason for the low computational burden of Cases III and IV; various computational parameters for an MPC iteration are also provided for comparison purposes. Note that there is a significant difference of around 10% in the operating costs of the practical EMS models compared to the decoupled UC-OPF models. Observe also that in spite of the 100%

unbalanced condition, there is no significant difference in operating costs between Case III and Case IV, since only one generator is dispatched at a time, with negligible differences in the generator active power output between the two cases.

Fig. 8 depicts the stacked area plot of the dispatch for all DERs. There were only two main generators participating in the dispatch, G1 and G2, with G1 operating during the peak hours only. For the practical EMS models (Cases I and II), G1 was dispatched only during hours 19-22, while for the decoupled UC-OPF models (Cases III and IV), G1 was dispatched during hours 14, 16, and 18-22, since the UC subproblems in the decomposed EMS approach (Cases III and IV) do not consider voltage dependency of the loads, thus committing more capacity. Generator G1 mostly operates near its minimum generation limit, which is the least efficient operating point of the generator; therefore, there is significant difference in the operating cost in Cases I and II compared to Cases III and IV.

Note in Tables I to III that the active and reactive power dispatch errors vary with feeder characteristics and network topology and size, but these errors are not significant nor negatively influence the dispatch results, which is the main purpose of an EMS. Furthermore, the impact on EMS cal-

TABLE III: Summary of Results for KLFN system

Case	EMS Operating Cost [\$]	Actual Operating Cost [\$]	Max. ΔP [%]	Max. ΔQ [%]	Total Computation Time [s]	MPC Iteration		
						Number of Average Optimization Iterations	Number of Variables	Number of Equations
I	12,928	12,863	0.24	0	13,000	47,070	14,100	8,035
II	12,979	12,958	0.25	0	1,980	6,520	11,218	4,913
III	14,457	14,415	0	0	5,640	3,300	14,100	8,148
IV	14,452	14,573	17	25	5,660	3,285	7,300	2,668

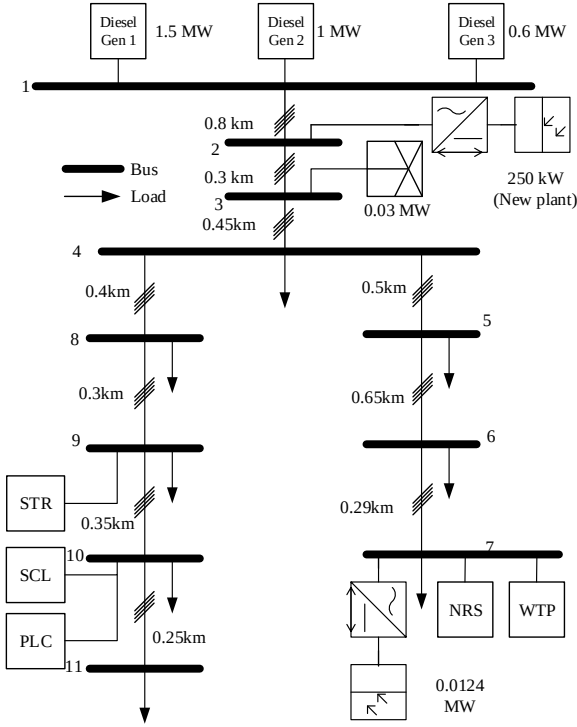


Fig. 7: Kasabonika Lake First Nation microgrid.

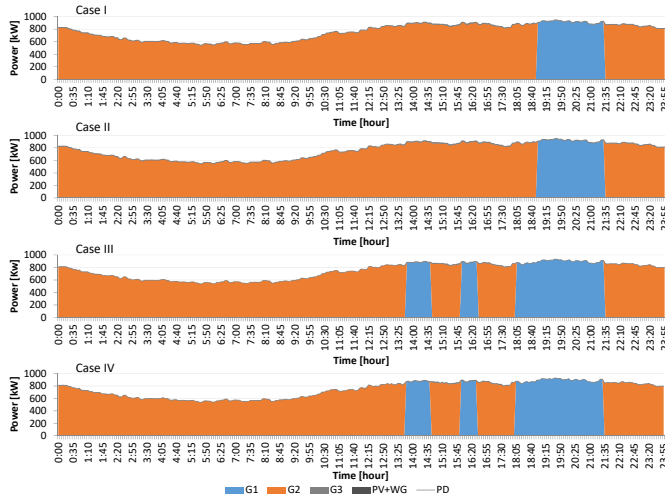


Fig. 8: Dispatch for all cases for the KLFN system.

ulation times of topology and size of the system as well as optimization model complexity can be evaluated from the computational statics depicted in these tables. Thus, observe that the computation times are directly affected by the number of optimization variables and iterations, as well as topology and system size, although the former impacts more these values than the latter, as expected.

C. Assumption Validation

The proposed practical EMS models are for isolated microgrids, i.e., grids with short high capacity rural feeders, where higher feeder voltage levels are used compared to urban primary distribution feeders, to reduce the number of poles and wire sizes. Thus, typically, the feeders in isolated microgrids are short in length and of large capacity. For example, as per the data given in [21], the medium voltage line losses for rural feeders are in the range of 1.2 to 2.5%, whereas the results obtained for both CIGRE and the actual KLFN system, shows that the losses range from 0.44 to 1.52%.

The voltage phase shifts are all close to 120° , in spite of the significant load unbalancing, as supported by actual measurements on the real KLFN remote microgrid, due to the small impact of the feeders and generator internal impedances, plus the fact that all generator internal voltages are balanced. The unbalancing is observed in the system currents, where there are significant magnitude differences among the 3 phases, while the phase angle differences depend on the load power factors, which in the test systems studied here are all the same, as expected. Thus, for the CIGRE system, the voltage angle deviations between phases vary by less than 1° from 120° , and similarly for the KLFN system, where the voltage angle deviations between phases vary by less than 5° from the ideal 120° . For the CIGRE system, the main generator phase currents at peak hour are $2.917\angle-26.8^\circ$, $2.977\angle214.1^\circ$, and $2.79\angle93.7^\circ$ in p.u., which are out of phase by less than 1° with respect to the ideal 120° , and the magnitudes vary by about 6% between the phases. For the KLFN system, due to a 100% unbalanced load, the main generator phase currents are $0.64\angle-13.35^\circ$, $1.28\angle225.79^\circ$, and $0.64\angle106.36^\circ$, showing significant magnitude difference, while the phase angle deviate by less than 1° from the ideal 120° .

The bus voltage phasor angles do not vary significantly from source to node. Thus, for the CIGRE system, these angles vary by at most 5° , while for the KLFN system, these vary by less than 1° . Furthermore, the voltage drops across the feeders are relatively small; thus for the CIGRE system, the maximum voltage drop from source to end node is 0.0139 p.u., while

for the KLFN system, the maximum voltage drop is 0.001305 p.u.. These observations justify the single-node approximation assumption made in the EMS-2 model.

Note in Tables I, II, and III, that for the EMS-2 model (Case II single-bus approximation), the active power injection errors are quite low for both the CIGRE and KLFN systems (max. 1.1%). The reactive power injection errors, on the other hand, are zero for the KLFN network, and although these are more significant for the CIGRE test system (max. 6%), they are still reasonable, especially considering that these would represent the worst possible case for an isolated microgrid. Thus, it can be inferred that the proposed models are applicable and scalable to any isolated microgrid.

V. CONCLUSIONS

Novel and practical EMS models were proposed by linearizing UC-OPF models with and without the grid, and considering DER operational constraints, active-reactive power balance, voltage-dependent loads, and unbalanced system loading. The proposed practical EMS models were compared with previously proposed EMS models being used in practice, for a CIGRE benchmark system and the real KLFN microgrid, based on operating costs, errors, computational time, and actual generator dispatches, showing that the proposed models yield better results at significantly reduced computational costs, thus demonstrating their feasibility for practical applications.

The results show that neglecting the network is a valid approach for isolated microgrids, as the power flow errors are relatively low. Thus, the proposed practical EMS model with a single node approximation of the system is shown to yield reasonable results, when considering voltage-dependent loads, active-reactive power management, and unbalanced system conditions, and hence should be the preferred approach for EMS applications in practice. Note that in practice, a simple linear UC approach is used for generators dispatch, which yields significant networks error and high-cost dispatches; therefore, the proposed EMS models should yield a better DER dispatch results for isolated microgrids in practice.

APPENDIX

TABLE IV: Load model exponents for both systems

Load Exponent	Residential		Commercial	
	α	β	α	β
Day	0.72	2.96	1.25	3.5
Night	0.92	4.04	0.99	3.95

TABLE V: CIGRE system ESS parameters

ESS Unit No	$\bar{P}$ [kW]	η^{ch} [%]	η^{dch} [%]
1	750	86	86
2	41.25	55	55
3	265	55	55
4	250	86	86
5	17.5	55	55

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TABLE VI: CIGRE system line parameters

Node from-to	R_{ph} [Ω]	X_{ph} [Ω]	B_{ph} [μS]	R_0 [Ω]	X_0 [Ω]	B_0 [μS]
1-2	0.208	0.518	4.596	0.421	2.160	1.884
2-3	0.173	0.432	3.830	0.351	1.800	1.570
3-4	0.106	0.264	2.336	0.214	1.098	0.958
4-5	0.097	0.242	2.145	0.197	1.008	0.879
5-6	0.266	0.665	5.898	0.541	2.772	2.418
6-7	0.042	0.104	0.919	0.084	0.432	0.377
7-8	0.289	0.721	6.396	0.586	3.006	2.622
8-9	0.055	0.138	1.226	0.112	0.576	0.502
9-10	0.133	0.333	2.949	0.270	1.386	1.209
10-11	0.057	0.143	1.264	0.116	0.594	0.518
11-4	0.085	0.212	1.877	0.172	0.882	0.769
3-8	0.225	0.562	4.979	0.456	2.340	2.041
1-12	0.846	2.112	18.729	1.716	8.802	7.677
12-13	0.517	1.292	11.452	1.049	5.382	4.694
13-8	0.346	0.864	7.660	0.702	3.600	3.140

TABLE VII: CIGRE system load parameters

Node	Apparent Power [kVA]						Power Factor	
	Phase A		Phase B		Phase C		Res	Com
	Res	Com	Res	Com	Res	Com		
1	344	80	172	180	200	180	0.90	0.80
2	100	200	50	200	0	200	0.95	0.85
3	0	80	200	80	50	80	0.90	0.80
4	200	0	100	0	100	0	0.90	1
5	200	50	172	200	0	50	0.95	0.85
6	50	0	100	0	172	0	0.95	1
7	0	100	100	100	0	100	0.95	0.95
8	100	0	150	0	0	200	0.90	0.90
9	100	0	150	0	100	0	0.95	1
10	150	0	100	0	250	0	0.90	1
11	50	150	50	150	0	150	0.95	0.85
12	0	145	0	145	0	145	0.95	0.85
13	0	90	0	90	172	90	0	0

TABLE VIII: CIGRE system synchronous generator parameters

Unit No	S_{base} [kVA]	V_{base} [kV]	$\bar{P}$ [kW]	$\bar{P}$ [kW]	$\bar{Z}_g^{l,l}$ [p.u.]	$\bar{Z}_g^{l,m}$ [p.u.]	$\bar{Z}_g^{m,l}$ [p.u.]
1	3125	0.48	2500	1000	1.0815j	-0.8390-0.5153j	0.8390-0.5153j
2	1750	0.48	1400	600	1.0815j	-0.8390-0.5153j	0.8390-0.5153j
3	1000	0.48	800	350	1.0815j	-0.8390-0.5153j	0.8390-0.5153j



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TABLE IX: CIGRE system generator parameters

DG Unit No	d [US\$/kWh ²]	e [US\$/kWh]	f [US\$]	C^{sup} [US\$]	C^{sdn} [US\$]	R [kW/min]	T^{up} [min]	T^{dn} [min]
1	0.0000088	0.197	40.04	83.6	13.464	250	60	60
2	0	0.226	22.44	39.6	7.304	140	60	60
3	0	0.253	6.6	13.2	4.664	100	30	30
4	0	0.253	0	6.468	1.2672	60	30	30
5	0.0000976	0.00552	0.018414	0.829	0	100	30	30

TABLE X: KLFN system line parameters

Node from-to	R_{ph} [Ω]	X_{ph} [Ω]	B_{ph} [μS]	R_0 [Ω]	X_0 [Ω]	B_0 [μS]
1-2	0.2688	0.3984	3.1576	0.5744	1.2416	1.368
2-3	0.1008	0.1494	1.1841	0.2154	0.4656	0.513
3-4	0.1512	0.2241	1.77615	0.3231	0.6984	0.7695
4-5	0.168	0.249	1.9735	0.359	0.776	0.855
5-6	0.2184	0.3237	2.56555	0.4667	1.0088	1.1115
6-7	0.09744	0.14442	1.14463	0.20822	0.45008	0.4959
4-8	0.1344	0.1992	1.5788	0.2872	0.6208	0.684
8-9	0.1008	0.1494	1.1841	0.2154	0.4656	0.513
9-10	0.1176	0.1743	1.38145	0.2513	0.5432	0.5985
10-11	0.084	0.1245	0.98675	0.1795	0.388	0.4275

TABLE XI: KLFN system synchronous generator parameters

Unit No	S_{base} [kVA]	V_{base} [kV]	$\bar{P}$ [kW]	$\underline{P}$ [kW]	$\bar{Z}_g^{l,l}$ [p.u.]	$\bar{Z}_g^{l,m}$ [p.u.]	$\bar{Z}_g^{m,l}$ [p.u.]
1	2750	0.48	1500	600	1.0331j	-0.7419-0.5j	0.7419-0.5j
2	1875	0.48	1000	400	1.0331j	-0.7419-0.5j	0.7419-0.5j
3	1000	0.48	600	240	1.0755j	-0.7967-0.5227j	0.7967-0.5227j

TABLE XII: KLFN system generator parameters

DG Unit No	d [US\$/kWh ²]	e [US\$/kWh]	f [US\$]	C^{sup} [US\$]	C^{sdn} [US\$]	R [kW/min]	T^{up} [min]	T^{dn} [min]
1	0.000123	1.0027	107.3	83.6	13.464	150	30	30
2	0	0.5	88.06	39.6	7.304	100	30	30
3	0.0000187	0.171	37.69	13.2	4.664	60	30	30

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