

Pressure-Velocity Coupling in Transpiration Cooling

by

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Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Statement of Contributions

The original modifications to the Hybrid code base to allow for simulation of transpiration cooling with uniform coolant injection were done by Nicholas Christopher for his Master's thesis at the University of Waterloo. The data for the reference cases used here (no blowing and uniform blowing) was provided by Nicholas Christopher. All other work was done by myself, with the supervision of Dr. Jean-Pierre Hickey.

Abstract

Transpiration cooling is an active thermal protection system of increasing interest in aerospace applications wherein a coolant is effused through a porous wall into a hot external flow. The wall is cooled due to convection within the porous structure and the formation of a coolant buffer layer between the wall and the main flow. In order to design an effective transpiration cooling system, it is necessary to understand the complex interaction between the high-temperature turbulent boundary layer and the pressure-driven coolant flow through the porous wall. However, high fidelity simulations of this interaction are rare and computationally limited to simple problems when accounting for both fluid domains. In the present work, two shallow convolutional neural networks (CNN) were trained on pore-network simulations of flow through a pressure-driven porous wall. CNN were used due to their ability to consider spatial correlations, meaning they can capture the influence of flow between neighbouring pores. The CNN are coupled with direct numerical simulations of a turbulent boundary layer over a massively-cooled flat plate. The coolant flow inside the porous medium is thus indirectly coupled to the near-wall pressure in the boundary layer, allowing the interaction between the two flow domains to be considered. Linear expressions that do not account for flow interactions between neighbouring pores were also coupled with direct numerical simulations in order to investigate the significance of this effect. With the incorporation of the pressure-coolant injection velocity coupling, the streamwise variation in mean pressure was found to have a significant impact on the local coolant injection. Blowing was reduced near the beginning of the transpiration region due to the high pressure region formed by the incoming boundary layer flow encountering the injected coolant. This effect reduced as the flow recovered and eventually reversed as the coolant film accumulated in the boundary layer, lifting it off of the wall. A corresponding trend was observed in the local cooling effectiveness, while the inverse was found in the local friction coefficient. The incorporation of lateral flow between neighbouring pores via the neural network was found to greatly attenuate these coupling effects. In all coupled cases, the turbulent kinetic energy was reduced at the beginning of the transpiration region due to the more gradual introduction of coolant. However, further downstream the rapid increase in coolant injection in the cases coupled using linear expressions resulted in increased turbulent production such that the turbulent kinetic energy was greater than in the uniform injection case at the end of the transpiration region. In the neural network cases, the increase in shear due to increased coolant injection was not significant enough to overcome the modulation of the turbulence due to the coupling. An analysis of the power spectral density of the pressure fluctuations at the wall within the transpiration region revealed that the implemented pressure-coolant velocity coupling only attenuated the largest scales of the turbulence, leaving the smaller scales relatively unaffected.

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Dedication

This thesis is dedicated to my family and friends, for all of their invaluable love and support, and for making sure I don't forget to eat.

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List of Abbreviations

AM	Additive Manufacturing
CFD	Computational Fluid Dynamics
CMC	Ceramic Matrix Composite
CNN	Convolutional Neural Network
DLR	German Aerospace Center
DNS	Direct Numerical Simulation
FFT	Fast Fourier Transform
LBM	Lattice Boltzmann Method
LES	Large Eddy Simulation
LPBF	Laser Powder Bed Fusion
PNM	Pore-Network Model(ling)
PSD	Power Spectral Density
RANS	Reynolds-Averaged Navier-Stokes
RDT	Rapid Distortion Theory [39]
ReLU	Rectified Linear Unit
RMS	Root Mean Square

SST	Shear Stress Transport
TKE	Turbulent Kinetic Energy
TPMS	Triply-Periodic Minimal Surface(s)
TPS	Thermal Protection System

Nomenclature

Γ	Gamma function
N_x, N_y, N_z	Number of grid points along the streamwise, wall-normal, and spanwise axes
Pr	Prandtl number
Re_τ	Tau Reynolds number: $\bar{\rho}_w u_\tau \delta_{99} / \bar{\mu}_w$
A	Cross-sectional area
F	Blowing ratio
c_p	Constant pressure heat capacity
h	Convection coefficient
β	Forchheimer coefficient
c_f	Local friction coefficient
η	Cooling effectiveness
φ_{pp}^{TM}	Wavenumber-based power spectral density of wall-pressure fluctuations due to the turbulence-mean shear interaction term
φ_{pp}	Frequency-based power spectral density of wall-pressure fluctuations
φ_{22}	Cross-spectral density of vertical velocity fluctuations
δ_{99}	Boundary layer thickness, determined as $U = 0.99U_\infty$
δ_{film}	Coolant film thickness
δ_i	Inlet boundary layer thickness
δ_{ij}	Kroenecker delta

ρ	Density
e	Total energy
ω	Angular frequency
γ	Exponential wall function constant ($1/2\Lambda$)
R	Gas constant
q	Heat flux
g	Hagen-Poiseuille hydraulic conductivity
i	Imaginary number
λ_2	Lambda2 vortex identification criterion
Λ	Longitudinal integral length scale of turbulence
M	Mach number
∇	Gradient operator
∇^2	Laplacian operator
σ_d	Pore diameter standard deviation
d_p	Pore diameter
Δx_i	Pore network grid spacing
ϕ	Porosity
$\boldsymbol{x}$	Position vector
ρ_0	Flow density in a quiescent medium
p	Pressure
R_i	Radius of influence
S_{ij}	Rate-of-strain tensor
σ_{ij}	Viscous stress tensor
T	Temperature
λ	Thermal conductivity
t	Time
τ	Total stress (sum of viscous and Reynolds stresses)

TKE	Turbulent kinetic energy
U_c	Streamwise convection velocity of turbulent eddies
u_τ	Turbulent friction velocity
v_D	Darcy velocity
U, V, W	Time-averaged streamwise, wall-normal, and spanwise velocity
u, v, w	Streamwise, wall-normal, and spanwise velocity
μ	Dynamic viscosity
ν	Generalised von Kármán spectrum constant
Ω_{ij}	Vorticity tensor
k	Planar wavenumber magnitude $\sqrt{k_1^2 + k_3^2}$
$\tilde{k}$	kl
$\mathbf{k}$	Planar wavenumber vector (k_1, k_3)
k_i	Angular wavenumber
x, y, z	Streamwise, wall-normal, and spanwise axes
$\hat{()}$	Fourier transform
$\overline{()}$	Average (in time and/or spanwise)
$()'$	Fluctuating component
$()^+$	Inner scaling
$()_{ad}$	Adiabatic
$()_c$	Coolant property
$()_\infty$	Property in the free-stream
$()_{ij}$	Einstein notation
$()_{rms}$	Root mean square
$()_w$	Property at the wall

Chapter 1

Introduction

The development of thermal protection systems (TPS) is a critical element in the advancement of aerospace technologies. In applications such as high speed external aerodynamics and propulsion, surfaces are exposed to significant thermal loads that severely limit both the type of materials which can be used and their operational lifetime [25, 43]. TPS are necessary to mitigate these limitations and are typically divided into two categories: passive and active.

Passive thermal protection systems do not require any additional power to operate, and therefore present an advantage due to their relative simplicity. However, this is offset by issues such as reusability (e.g., ablative coatings), cost, and maintenance (e.g., insulating tiles). Moreover, passive TPS are generally limited to brief exposure times, making them unsuitable for extended applications such as hypersonic flight [74].

Due to these issues, there has been an increasing push in the industry towards active TPS, which make use of a dedicated coolant to reduce thermal loads. These systems can potentially be used to build reusable thermal shields that require minimal maintenance compared to their passive counterparts, which typically suffer from ablative losses. Furthermore, their enhanced cooling efficiency is key to paving the way for the use of lower-cost and lighter materials as well as new manufacturing technologies such as additive manufacturing (AM).

The major types of active TPS are convective cooling, film cooling, and transpiration cooling. In the first type, the coolant and the external hot fluid are separated by the cooled wall and never come into contact. This is the principle behind regenerative cooling, which has been the popular choice for the cooling of rocket engines since the mid-20th century. In contrast, the coolant in film cooling is injected into the main hot flow via discrete channels

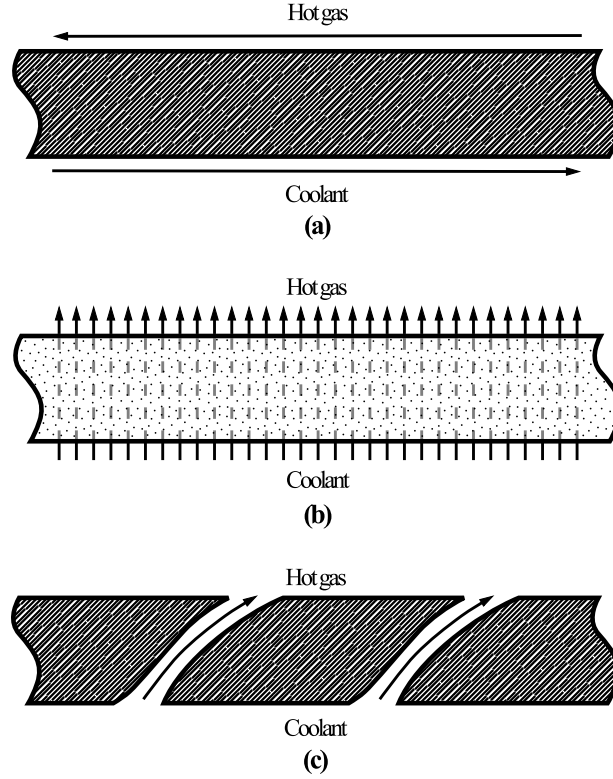


Figure 1.1: Illustration of convection (a), transpiration (b), and film (c) cooling.

in the cooled wall where it accumulates to form a barrier between the wall and the external fluid. This method is often used to cool modern gas turbine blades [22].

Finally, transpiration cooling bears some similarities to film cooling in that the coolant is injected into the main flow to form a thermal buffer layer. However, rather than moving through discrete channels, the coolant is instead effused through a porous wall into the hot boundary layer. As it moves through the wall, the surface area over which the coolant is exposed is maximised, leading to enhanced convective cooling. Therefore, there are two cooling mechanisms at play in transpiration cooling: convective cooling in the wall and a thermal buffer layer between the wall and the hot gas. As a result, transpiration cooling shows an increased cooling effectiveness compared to the other two methods [23]. Furthermore, transpiration cooling has also shown promise as a method of drag reduction due to the reduced skin friction corresponding to the formation of the cooling film [41], and as a means of controlling laminar-to-turbulent transition [11].

Despite improved cooling effectiveness, implementations of transpiration cooling have

remained largely limited to research applications. This is in part due to an incomplete understanding of the interaction between the main flow and that in the porous medium, which inhibits the development of predictive simulations. These are critical for the design of efficient and reliable transpiration cooling systems, which must control the pressure across the porous medium to minimise the momentum of the injected coolant and prevent turbulent mixing in the boundary layer (impeding coolant film formation) while also ensuring that none of the hot gas is driven backwards into the porous wall.

When considering transpiration cooling, the overall system can be subdivided into two fluid regions that are highly dependent on each other: the porous medium through which the coolant is effused and the external high-temperature flow over the cooled wall. The flows in these two regions have vastly different time and length scales, complicating efforts to build a resolved numerical model of such a system. In practice, high-fidelity simulations of transpiration cooling typically only model the turbulent boundary layer over the wall and assume a uniform injection of coolant, effectively decoupling any interactions with the porous medium flow [11, 17].

Although the assumption of uniform coolant injection is attractive in its simplicity, in reality it will vary both spatially and temporally. Even in macroscopically stable conditions, the presence of the turbulent boundary layer will result in small pressure fluctuations at the surface of the porous wall. While the magnitude of these fluctuations is small, they can still have a significant impact on the flow in the porous medium due to the minimal momentum of the coolant. The fluctuating pressure at the surface of the porous wall results in local variations in the pressure gradient normal to the wall surface across the porous medium, affecting the local injection mass flow rate. The result is a tight two-way coupling in which pressure fluctuations in the turbulent boundary layer impact the local rate of coolant injection and vice versa, as illustrated in Figure 1.2.

The vast majority of coupled simulations of transpiration cooling divide the problem into two numerical domains according to the two fluid regions. The flow through the porous medium is simulated by modelling the bulk flow using the Darcy-Forchheimer relationship. The transpiration cooling system is then modeled with two segregated solvers: one for the porous medium flow, solving a Darcy-type equation, and a separate one, solving the Navier-Stokes equations, for the flow above the wall [18, 51, 62]. These works have demonstrated the need to consider pore-scale interactions but necessarily use lower order flow models that are unable to do so in order to reduce the computational cost. Consequently, investigations using this method do not resolve the flow through individual pores and are therefore unable to capture pore-specific flow variations, as well as the impact of pore connectivity. Furthermore, most of the works that take this approach use a Reynolds-average Navier-Stokes (RANS) solver to model the main flow, which is unable to resolve the turbulent

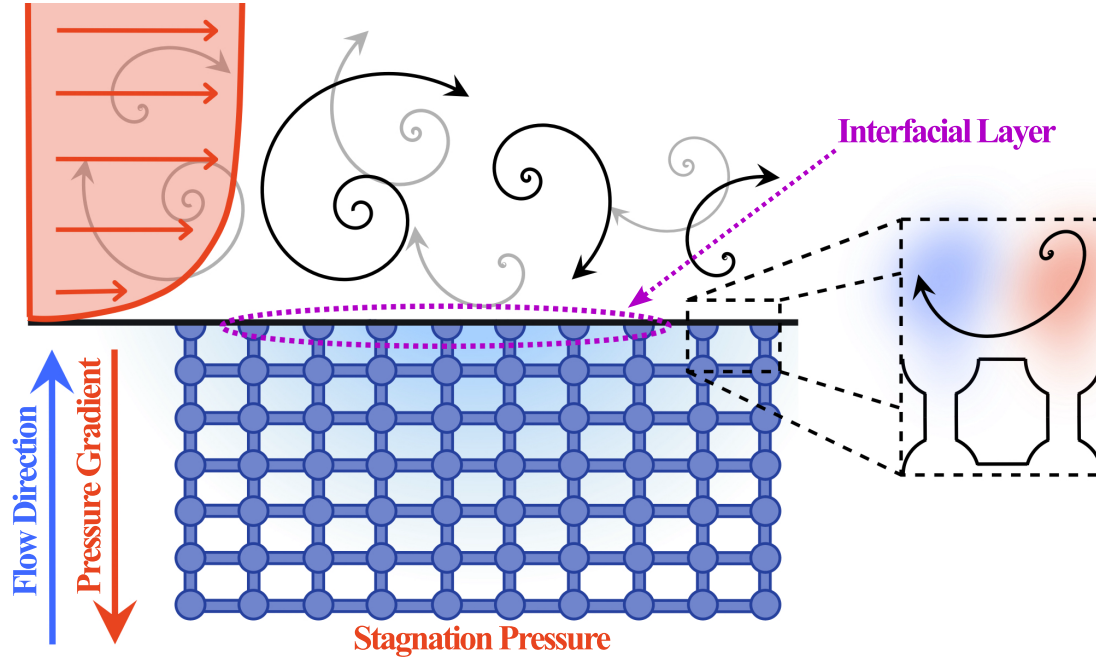


Figure 1.2: Schematic illustration of the coupling between the flow in the porous medium and the turbulent boundary layer in transpiration cooling.

structures formed in the boundary layer and the corresponding pressure fluctuations at the wall. **RANS** also requires the use of turbulence models which are not applicable to problems with blowing such as transpiration cooling without significant corrections [9, 14].

Ideally, a monolithic approach to modelling transpiration cooling would be used in which both flow domains are simulated simultaneously without the use of segregated solvers. This approach is very computationally expensive, and therefore its applications are limited to low Reynolds number flows and simplified geometries [12, 66, 79]. Although there are some examples of such coupled transpiration cooling simulations, they are very few and limited in the size, number, and geometry of pores considered in order to make the problem computationally tractable.

The purpose of the present work is to explore the significance of the coupling between the surface pressure fluctuations of the main flow and the corresponding injection velocity of the coolant effused through the porous wall. The impact of this coupling on the coolant injection distribution, the formation of the coolant buffer layer, the development of structures in the turbulent boundary layer, and the overall cooling effectiveness and skin friction reduction ability of the system will be considered. This information is required to

understand under what conditions this pressure-velocity coupling must be considered and will help to contextualise the results of previous research as well as inform new research on the topic.

In order to achieve this, a direct numerical simulation (DNS) of a subsonic turbulent boundary layer over a flatplate is indirectly coupled with a low-order pore-scale model of flow in a simplified porous medium via a variable boundary condition at the wall (interfacial layer). DNS solve the Navier-Stokes equations without any modelling and are therefore closest in accuracy to experimental results if constructed properly. In order to mitigate the computational cost and allow a large number of pores to be considered the porous medium solver is not coupled directly with the DNS but is instead used to train both a neural network and fit a simple algebraic expression via linear regression, each of which is then coupled to the main simulation.

This thesis is organised into multiple sections, beginning with a review of the published literature with respect to transpiration cooling, porous media flow modelling, and the application of machine learning to computational fluid dynamics (Chapter 2). Chapter 3 then provides an overview of the numerical setup including details regarding the solvers/models used while Chapter 4 outlines the efforts done to validate the simulations. Chapter 5 presents and analyzes the obtained results with respect to the previously stated goals of the work. Finally, the conclusions are presented in Chapter 6, including recommendations for possible avenues of further research.

The major contributions of the work presented in this thesis are as follows:

- Developed an algorithm to couple a DNS with a convolutional neural network to consider the coupling between two flow domains with a relatively small increase in computational cost.
- Used pore-network modelling to evaluate the importance of flow perpendicular to the driving pressure gradient between neighbouring pores when the surface pressure is allowed to vary due to turbulent pressure structures.
- Conducted high-fidelity DNS to evaluate the importance of the coupling between coolant injection and turbulent wall pressure fluctuations with respect to transpiration performance and the structure of the external flow features.

Chapter 2

Literature Review

2.1 Overview of transpiration cooling research

Interest in transpiration cooling first arose in the 1950s, when it was being considered in aerospace applications both for its potential as a [TPS](#) as well as for its ability to significantly reduce skin friction. In 1954, Eckert and Livingood [23] investigated the comparative cooling effectiveness of convection, film, and transpiration cooling. They found that transpiration cooling required significantly less coolant to achieve the same cooling performance as either of the other two methods, although the incorporation of radiative heat transfer to the wall considerably mitigated this advantage.

A few years later, Rubesin [64] conducted an experimental investigation of the efficacy of transpiration cooling compared to convection cooling. Rubesin concluded that the superiority of transpiration cooling became increasingly apparent as the Mach number of the external flow increased, but pointed out difficulties associated with the design, manufacturing, and structural integrity of porous plates for practical applications. Rubesin also identified the lack of well-established empirical models for transpiration cooling, thus forcing the designers to rely on a limited set of experimental data.

Due to these considerable challenges, research on transpiration cooling experienced a lull in subsequent years in favour of film cooling, which was viewed as a more practical alternative. This is because discrete coolant channels were much more feasible to manufacture than a porous medium with the requisite material properties [55]. However, the discrete nature of film cooling (typically composed of one or two rows of large channels) means that the coolant film does not completely cover the wall within the boundary layer.

Furthermore, the large amount of coolant required to ensure adequate film cooling is typically achieved by increasing the velocity in the coolant channels, resulting in the formation of jets that penetrate into the boundary layer. This causes the coolant film to detach from the wall, leading to a reduction in cooling effectiveness [49].

Full-coverage film cooling, also called effusion cooling, was developed to address these issues and represents a compromise between transpiration and film cooling wherein the cooled wall contains many rows of micro-channels [55]. In practice, the distinction between effusion and transpiration cooling is not clearly defined. Krewinkel [49] distinguishes the two by stating that in transpiration cooling, the coolant “reaches a thermal equilibrium with the solid it is moving through”. The key difference between the two methods is the contribution of convective cooling within the wall to the overall cooling performance of the system.

Andrews et al. [2] conducted an experimental investigation comparing transpiration and effusion cooling. In both cases, a distinction is made between the overall cooling effectiveness and the adiabatic cooling effectiveness, which encapsulates only the portion due to the formation of a thermal coolant buffer in the boundary layer. It was found that for low coolant flow rates, effusion cooling was able to achieve similar adiabatic cooling effectiveness, with the overall superior cooling performance of the latter case stemming from enhanced convective heat transfer in the wall. However, as the coolant flow rate is increased, the adiabatic cooling effectiveness of transpiration cooling overtakes that of effusion cooling due to increased disruption of the cooling film in the effusion case.

Recently, there has been a resurgence of interest in transpiration cooling research which can be attributed to significant advances in the two key areas previously identified by Rubesin [64] as barriers to practical implementations of transpiration cooling systems. The first is the development of new manufacturing technologies that enable the manufacturing of porous walls with the required material properties and porous characteristics. The second is advancements in computing power and computational fluid dynamics (CFD), which have paved the way for numerical simulations of transpiration cooling that can be used to better understand the underlying flow mechanics and develop models for use in the design of transpiration systems.

2.1.1 Porous wall structures

Early implementations of transpiration cooling used sintered metal foams to construct the porous wall, which had lacklustre mechanical strength and a tendency to undergo complete

structural failure should they be exposed to local hot spots [50]. In order for practical implementation, there was a need for new types of porous materials, ideally ones with tuneable porous properties (e.g. porosity, permeability) to meet the needs of the specific application. Much of the recent work to develop materials that can satisfy these requirements has focused on ceramic matrix composites (CMC) and additively manufactured structures.

CMCs are a type of composite material wherein a ceramic matrix is reinforced by ceramic fibres, resulting in a material with both high temperature stability and mechanical strength. The manufacturing process typically involves a lay-up of woven ceramic fibres, which are then infiltrated by a matrix material before undergoing heat treatment to yield the final composite [76]. In the past few years, the German Aerospace Center (DLR) has spearheaded efforts to use these materials in active TPS, including transpiration cooling for rocket engine combustion chambers and hypersonic vehicles [10, 28, 34, 35, 63].

Peichl et al. [61] built an experimental setup consisting of four sequentially aligned carbon-fibre reinforced carbon (C/C) CMC plates of different permeability exposed to a hot gas flow. Using cold air as coolant, the thermal boundary layer was measured for a number of different injection mass flow rates. No coolant was injected through the third plate in order to investigate the impact of the cooled wake and its ability to recover over the final plate. A test case with no transpiration through any of the plates was also conducted to provide a comparison point. The cooling effect was clearly visible and increased with the coolant mass flow rate, demonstrating the suitability of the material for transpiration applications. Furthermore, while the temperature naturally increased on the plate without coolant injection, the surface temperature remained notably lower than in the uncooled case and rapidly recovered when injection resumed over the final plate.

Although the bulk porous properties of CMCs can be controlled, to some extent, during manufacturing, it remains a fundamentally stochastic process. Therefore, there has also been significant interest in using AM to manufacture particular porous structures as a deterministic alternative. This allows for careful tuning of not only the bulk properties of the porous material but also of optimal pore-scale flow paths. A common choice of AM technique is laser powder bed fusion (LPBF), also called direct metal laser sintering and selective laser melting, which applies a laser to a metal powder bed to selectively fuse the metal for each layer of the part.

Zhang [88] used LPBF to fabricate an architected porous material for transpiration cooling. The porous material consisted of an array of triply-periodic minimal surface (TPMS) unit cells. The fidelity of the resultant material to the original design was then evaluated to help understand the limitations and considerations associated with this technique. Although the structures of the printed part were macroscopically similar to those

of the original design, the irregular surface of the printed part resulted in an unexpected decrease in the bulk permeability.

Kim et al. [45] also used LPBF to fabricate a porous gas turbine blade to investigate the efficacy of transpiration cooling relative to effusion cooling for the same coolant flow rate. In addition to successfully using AM methods in the implementation of a transpiration cooling system, these results also demonstrated the superiority of transpiration cooling over other cooling methods predicted in the 1950s and observed in previous investigations (e.g. [2, 24, 78]), although at the cost of a significantly increased pressure drop across the porous wall.

2.1.2 Numerical simulations

Transpiration cooling systems present a significant challenge to simulate due to the large range of length and time scales that must be resolved to fully capture both the external flow and the flow in the porous medium. Doing so quickly becomes prohibitively computationally expensive. Generally speaking, there are three treatments of the porous medium and coolant injection that are used in the literature:

Decoupled approach

In the decoupled approach, the flow through the porous wall is not modeled, and transpiration is simulated by way of a constant velocity or mass flux boundary condition at the porous wall. This condition may be spatially varying, but importantly it is independent of the conditions in the main flow. The advantage of this technique is that the reduced complexity allows for highly resolved simulations of the main flow that can be used to infer an understanding of the influence of coolant injection on the structure of the turbulent boundary layer.

One example of this approach is the work by Christopher et al. [17], where multiple low Mach number DNS cases of the turbulent boundary layer over a flatplate were simulated with different values of coolant injection. In all cases, transpiration was simulated by imposing uniform wall-normal momentum at the porous wall, which was fixed at a temperature half that of the free-stream temperature. The turbulent transport of heat due to induced vortices from the interaction of the coolant and the main flow was observed to increase with increasing coolant injection.

Cerminara [11] used DNS of a Mach 5 flow over a flatplate with induced turbulent-to-laminar transition to explore the interactions between turbulence and coolant injection.

In contrast to the work of Christopher et al. [17], the wall-normal momentum boundary condition used to model transpiration varied in the streamwise and spanwise directions according to a trigonometric expression to simulate injection through discrete circular pores. Results showed that the diameter of the individual injection jets altered the flow patterns near the porous wall.

Dual solver approach

The dual solver approach incorporates the coupling of the main flow with the porous domain by using separate solvers for each flow domain which are run in alternation, exchanging information via boundary conditions at the interface. Typically, this is done without resolving the flow through individual pores by way of bulk porous properties such as Darcy permeability, porosity, and tortuosity. The result is a relatively computationally inexpensive simulation of coupled transpiration cooling, albeit with very limited resolution. Notably, nearly all instances using this method that were found by the author employed a **RANS** model to simulate the main flow domain and consequently are not able to resolve the turbulent structures formed in the boundary layer and the corresponding coupling between the pressure and velocity fluctuations at the wall interface. Furthermore, **RANS** simulations of transpiration cooling have been shown to over-predict near-wall turbulence production and inaccurately predict thermal mixing in transpiration cooling [9].

Dahmen et al. [18] implemented this approach using two separate solvers: a compressible **RANS** solver with the shear stress transport (**SST**) turbulence model to simulate the main flow and a finite-element solver based on the Darcy-Forchheimer relationship to model the flow in the porous wall. The two solvers were then run in alternation, exchanging information at the interface between the flow domains through updated boundary conditions. Originally, uniformity was imposed on the injection boundary condition, however this restriction was later eased with the injection distribution instead being prescribed based on empirical data [19]. König et al. [51] used the developed solver with injection distributions determined by trigonometric functions of either the spanwise or streamwise coordinates. It was determined that the injection pattern has a strong influence on the developed cooling film and the need to consider the micro-scale behaviour of injection through pores was recognised.

Prokein [62] built a similar model in the open-source **CFD** package OpenFOAM which is applicable to both sub- and supersonic hot gas flows. The flow in the porous medium was modeled using a three-dimensional expression of the Darcy-Forchheimer relationship and a **RANS** solver with the **SST** turbulence model was used to simulate the main flow, as in the work of Dahmen and König. As individual pores were not resolved, individual

injection jets were not modeled in favour of a local spatial averaging approach. This has the advantage of allowing for a non-uniform injection distribution over larger scales, however it also has the effect of smoothing out the individual higher-momentum jets corresponding to each pore which may impact the resultant fluid interactions.

One of the notable exceptions in the use of a [RANS](#) solver is the work of Xiao et al. [83] which coupled a Darcy-Forchheimer porous medium solver with a large eddy simulation ([LES](#)) of a hot turbulent channel flow. Their results indicated the presence of a Kelvin-Helmholtz instability due to the coolant injection as well as periodic vortex structures corresponding to periodic fluctuations of the mass flux within the porous medium.

Monolithic approach

As mentioned in chapter 1, a monolithic approach wherein both the porous medium flow and the main flow are solved simultaneously in the same solver is preferred, but examples of it are highly limited due to the often prohibitive computational demand. In order to make this approach feasible, the simulated porous domain must be kept small and the size of the pores relatively large.

Cerminara et al. [12] constructed a [DNS](#) of hypersonic flow over a flatplate with a porous section. The porous medium was simplified as a staggered arrangement of cylindrical or spherical elements using an upscaling process that preserves the Darcy-Forchheimer behaviour of a porous medium representative of those used in applications of transpiration cooling. Porous injection was found to promote the transition to turbulence compared to discrete injection (film cooling), corresponding to an increase in cooling effectiveness due to enhanced turbulent mixing.

Monolithic approaches have been used to model flow over porous media, although without the injection of coolant. While these are not examples of transpiration cooling, they nonetheless deserve mention as they provide insight into the interactions between the two flow domains, the outcomes of which may be extended to the transpiration case. Wang et al. [79] investigated information transfer between turbulent boundary layers and porous media using a pore-resolved [DNS](#) of turbulent channel flow over an array of cylinders. They observed that the interactions between the porous medium and the free-stream were limited to the near-wall region even in the highest porosity case. Furthermore, it was shown that the influence of larger scale turbulent structures began to dominate as the porosity was increased. Weishaupt et al. [80, 81] implemented a monolithic approach using a two-phase pore-network model ([PNM](#)) solved simultaneously with the incompressible Navier-Stokes

equations, however the main flow domain was limited to laminar flow. A model for pore-scale slip at the interface between the two flow domains was also incorporated.

2.2 Turbulent boundary layer pressure fluctuations

Small pressure fluctuations occur naturally at the bounding surface in a turbulent boundary layer as a result of momentum fluctuations. Classically, these pressure fluctuations are characterised by their power spectral density (PSD), however the mathematical characterisations of this spectrum have been limited to cases with a rigid wall. Very little information is available on the PSD above permeable walls. A summary of the treatment of these fluctuations in the literature is provided below, with the caveat that all available research on the topic is for a regular turbulent boundary layer over a solid wall without coolant injection. The applicability of these conclusions to the transpiration case is investigated as part of the present work.

The pressure fluctuations at the surface of a turbulent boundary layer can be estimated by applying the divergence operator to the complete incompressible momentum equation, resulting in a Poisson equation for pressure [32]:

$$\frac{1}{\rho} \nabla^2 p' = -2 \frac{\partial u'_j}{\partial x_i} \frac{\partial U_i}{\partial x_j} - \frac{\partial^2}{\partial x_i \partial x_j} (u'_i u'_j - \overline{u'_i u'_j}) \quad (2.1)$$

where ρ , p , and u_i are the density, pressure, and the i^{th} velocity component. ∇^2 is the Laplacian operator. Pressure and velocity are broken down by Reynolds decomposition into their mean and fluctuating components:

$$p = P + p' \quad (2.2)$$

$$u_i = U_i + u'_i \quad (2.3)$$

These fluctuations can be broken down into the contribution of two source terms, given on the right side of Equation (2.1), expressing the turbulence-mean shear and turbulence-turbulence interactions, respectively. By applying a rigid wall assumption and imposing a Dirichlet boundary condition stating that the pressure fluctuations disappear far from the wall, this equation can be manipulated to yield an expression in the space domain, given below [32]. However, obtaining the PSD of the fluctuations using Equation 2.4 is an arduous task, requiring the solving a six-dimensional integral which must then be Fourier-transformed. Efforts have therefore focused on developing a solution to the Poisson

equation directly in the wavenumber domain.

$$\frac{1}{\rho}p'(\mathbf{x}, t) = \int_{\mathbf{x}^*} - \left[2 \left(\frac{\partial U_i}{\partial x_j} u'_j \right) \frac{\partial G(\mathbf{x}, \mathbf{x}^*)}{\partial x_i} + (u'_i u'_j - \overline{u'_i u'_j}) \frac{\partial^2 G(\mathbf{x}, \mathbf{x}^*)}{\partial x_i \partial x_j} \right] d\mathbf{x}^* \quad (2.4)$$

where

$$G(\mathbf{x}, \mathbf{x}^*) = -\frac{1}{2\pi} \frac{1}{\|\mathbf{x} - \mathbf{x}^*\|} \quad (2.5)$$

$$\frac{\partial G(\mathbf{x}, \mathbf{x}^*)}{\partial x_i} = \frac{1}{2\pi} \frac{\|x_i - x_i^*\|}{\|\mathbf{x} - \mathbf{x}^*\|^3} \quad (2.6)$$

$$\frac{\partial^2 G(\mathbf{x}, \mathbf{x}^*)}{\partial x_i \partial x_j} = \frac{1}{2\pi} \left[\frac{\delta_{ij}}{\|\mathbf{x} - \mathbf{x}^*\|^3} - 3 \frac{\|(x_i - x_i^*)(x_j - x_j^*)\|}{\|\mathbf{x} - \mathbf{x}^*\|^5} \right]. \quad (2.7)$$

where δ_{ij} is the Kroenecker delta.

The Poisson equation for pressure was first characterised in the late 1940s, however the velocity fluctuations in the boundary layer could not be determined theoretically, meaning that attempts at a more complete solution were unable to progress [82]. In 1956, Kraichnan [48, 47], who was one of Albert Einstein's last assistants at Princeton, presented the first theoretical estimate of the wall pressure fluctuations with two particularly notable assumptions, the first of which stated that the turbulence is homogeneous in planes parallel to the wall. Kraichnan's second assumption required that the turbulence-mean shear interaction term in Equation (2.1) is dominant, allowing the turbulence-turbulence interaction term to be neglected.

This second assumption has proven to be contentious and, despite being a common simplification in later works, has yet to be unanimously validated. Slama et al. [69] developed an extended anisotropic model using Equation (2.4) which took into account both source terms and found that at low wavenumbers/frequencies, the turbulence-turbulence source term became dominant. Based on their results, Slama argued that even outside this range the turbulence-turbulence interaction term, while superseded by the turbulence-mean shear interaction term, was still too large to be readily ignored. Meanwhile, based on their results using a modified version of the model first developed by Hodgson [36], Grasso et al. [32] suggested instead that the turbulence-turbulence source term only becomes significant at high wavenumbers, although with the caveat that these results were obtained with the assumption of isotropic turbulence following a joint normal probability distribution.

Hu et al. [37] used a model for synthetic turbulence to determine the wall pressure

fluctuations and found that both source terms are on the same order of magnitude if anisotropic turbulence is considered, whereas in the case of isotropic turbulence (the other of Kraichnan’s assumptions), the turbulence-mean shear interaction term is dominant. Furthermore, the spectra of the turbulence-turbulence source term demonstrated a maximum plateau at lower frequencies, an effect which was similarly observed in the results of Grasso et al. [32] although deemed inconclusive due to several restrictive assumptions in their model derivation. As the turbulence-mean shear source term is observed to decay at lower frequencies at a rate of ω^2 , this supports Slama’s [69] conclusion that the turbulence-turbulence interaction term dominates at lower frequencies.

2.2.1 Models of the turbulence-mean shear interaction term

Of the source terms mentioned above, the contribution of the turbulence-mean shear interaction term to the PSD is often considered the most dominant of the two, in keeping with the precedent first set by Kraichnan [47]. As such, there are numerous models of this term’s contribution to the wall pressure fluctuation PSD. Grasso et al. [32] compared four of these analytical models to the empirical model formulated by Goody [29] in order to evaluate their strengths: the Gaussian, von Kármán [77], Liepmann [53], and rapid distortion theory (RDT) [39] models. The main steps in the derivation of these models are summarised in Appendix A for the reader’s reference.

It was found that at high and low frequencies the Liepmann and RDT models best replicated the empirically predicted PSD curve. However, none of the analytical models were able to reproduce the plateau at mid-range frequencies, which Grasso attributed to the unaccounted for increased influence of the physics of the log layer in this region.

Further comparison of the different analytical models with DNS data showed that the turbulence-mean shear interaction term alone was able to adequately predict wall pressure PSD across the range of frequencies considered, suggesting that the assumption first made by Kraichnan [47] about the dominance of the turbulence-mean shear interaction term was correct [32, 33]. However, as the turbulence-turbulence interaction term is only expected to be dominant at very low frequencies, it is possible that the range of observed frequencies was simply too small to capture this effect.

2.2.2 Models of the turbulence-turbulence interaction term

In contrast, the relevance of the turbulence-turbulence interaction term is a contentious topic among researchers and is often disregarded due to the additional complexities intro-

duced when solving the problem with the additional non-linear term added in.

Grasso et al. [32] suggest a modified version of that first proposed by Hodgson [36], which assumes a joint normal probability distribution of turbulence in the boundary layer as well as an isotropic distribution of turbulent kinetic energy, a Gaussian distribution of the cross-spectral density of velocity fluctuations in the wavenumber domain, and only minimal contribution of turbulence cross-spectral densities. This model exhibits a plateau in roughly the same region as that observed in the DNS data to which it was compared, however it also severely underestimates the power of low-frequency contributions to the pressure fluctuations, which is where this term is expected to dominate.

2.3 Pore-network modelling

Traditionally, modelling of transport in porous media has been separated into two approaches; a continuum approach using bulk properties of the medium, and a fully-resolved approach wherein the flow within the pores is fully modeled. The continuum model is based on Darcy’s law, which relates the flow through a homogeneous, isotropic porous medium to the applied pressure gradient:

$$v_D = -\frac{K_i}{\mu} \frac{\partial p}{\partial x_i} \quad (2.8)$$

K is the permeability and is found through experiments or empirical correlations. The Darcy velocity v_D , sometimes called the superficial velocity, is defined as the total volumetric flow rate through the porous medium divided by the total cross-sectional area ($v_D = Q/A_{tot}$). The actual mean coolant velocity is obtained by dividing the Darcy velocity by the porosity of the medium: $v = v_D/\phi$. Darcy’s law is well established, however it is only valid for steady-state, incompressible, viscous flow (Stokes flow). For higher flow rates where inertial effects become significant, the Forchheimer correction must be applied, resulting in the following relationship, where β is the Forchheimer or non-Darcy flow coefficient [87].

$$-\frac{\partial p}{\partial x_i} = \frac{\mu v_D}{K_i} + \beta_i \rho v_D^2 \quad (2.9)$$

Equation (2.9) is integrated into the momentum balance equation to yield the Darcy-Forchheimer equation below, which is the central expression used by continuum porous media solvers [18].

$$\rho \left(\frac{1}{\phi} \frac{\partial \vec{v}}{\partial t} + \frac{1}{\phi^2} (\vec{v} \cdot \nabla) v \right) = -\nabla p - \frac{\mu \vec{v}}{K} - \rho \vec{\beta} |\vec{v}| \vec{v} \quad (2.10)$$

The advantage of the continuum approach is that the structure of the porous medium does not need to be resolved; the flow space is completely described by the porosity, the permeability, and the Forchheimer coefficient. However, it requires that these properties be determined a priori via experiments or empirical correlations, which is not always possible or accurate. Furthermore, it is only applicable to macroscopically homogeneous porous media and is not able to resolve pore-scale flow phenomena. For this reason pore-resolved simulations using either Navier-Stokes or LBM solvers are preferred, however these require significant computational resources.

Pore-network modelling (PNM) is an alternative to the above modelling approaches, in which the empty (void) space in a porous medium is represented by an equivalent resistive pipe network. The void region of the material is broken down into disparate pores which are connected via conduits referred to as throats [30]. The resulting model preserves information regarding the flow paths through the material and can be used to obtain the mean of properties such as pressure, temperature, and flow rate at each pore, however, the distribution of these properties within the pores and conduits themselves is not resolved. This method is described as pore-scale to distinguish it from fully-resolved simulations where the sub-pore-scale flow is captured.

PNM can be used to model many different types of problems regarding transport in porous media, including multi-phase transport in fuel cell components [72] and gas diffusivity in soil [46]. With respect to Darcy-type single-phase fluid mass transport, the conductivity (inverse of resistivity) of the individual conduits in the network is determined using the Hagen-Poiseuille equation for one-dimensional pipe flow, where L is the length of pore-throat-pore conduit and A_{eff} is its effective cross-sectional area and depends on the cross-sectional areas of pores i and j and that of the throat connecting them

$$q_{i \rightarrow j} = -\frac{A_{eff}^2}{8\pi\mu L} \Delta p_{i \rightarrow j} = -g_{i \rightarrow j} \Delta p_{i \rightarrow j}. \quad (2.11)$$

Applying conservation of mass yields an equation for each pore in the network, where N is the total number of pores connected to pore i and x_{ij} is the property corresponding to the conduit connecting pores i and j :

$$\sum_{j=1}^N g_{i \rightarrow j} \Delta p_{i \rightarrow j} = q_i = 0 \quad (2.12)$$

By applying Equation (2.12) to each pore in the network a linear system of equations is formed that can be solved very efficiently using matrix inversion techniques. The particular problem to be solved can be defined by imposing Dirichlet (pressure) or Neumann (volumetric flow rate) boundary conditions at the pores in question.

2.3.1 Network generation methods

In order to use the above method, a network representation of the porous medium onto which the model is to be applied must be generated. This is a non-trivial task and can have a significant impact on the results output by the model [8]. While there have been some successful efforts to generate the porous network stochastically, such as the work of Jiang et al. [40], it is preferred and more common to generate a network using high-resolution images of the internal porous structure.

There are two primary methods of pore network extraction from images of the internal structure (typically obtained using computed tomography), both based on the concept of applying successive morphological operations on the images. The first of these, called the maximal ball algorithm, was first developed by Silin et al. [68] in 2006 and defines spheres based on the maximum sphere that can be inscribed in the void space. An extension of this model by Dong et al. [21] was applied to internal images of different rock types to obtain permeability values that were found to be in good agreement with the experimental values. More recent efforts have improved on this method by incorporating a medial axis method to improve the accuracy of the constructed network [5, 86].

The second technique is the watershed algorithm, first introduced by Thompson et al. [71] and Sheppard et al. [67]. This algorithm segments the void space of the image into distinct regions according to maxima in the distance transform of the image. The pores are then taken as the largest sphere that can be inscribed in each region. It has been applied successfully to a number of different porous media, including fibrous mats and sandstone [15, 30].

Baychev et al. [6] compared the performance of these two algorithms by applying both to three porous mediums with known properties: an idealised medium, a sample of carbonate, and a sample of sandstone. Both methods displayed reasonable accuracy in predicting permeability values despite differing significantly in their topology.

2.3.2 Transient effects

While [PNM](#) can be a powerful tool, its simplicity imposes some critical limitations that must be considered. Namely, the linear network model on which it is based is predicated on a mass balance approach rather than a momentum balance. Consequently, it cannot model transient effects in flow through porous media.

In the case of compressible porous flow, transient effects may become significant and should not be neglected [\[38\]](#). Furthermore, the bulk properties of porous media such as permeability and tortuosity have previously demonstrated a strong frequency dependence, indicating the possible significance of incorporating transient effects [\[13\]](#).

It is possible to integrate these transient effects into a pore-network approach by extending the circuit analogy to include inductive and capacitive elements. This has been explored with respect to acoustics in the equivalent circuit model proposed by Miki [\[57, 56\]](#) as well as the transfer-matrix formulation [\[20, 54\]](#). However, the resulting systems no longer possess the mathematical simplicity of the basic resistive network which made [PNM](#) advantageous relative to higher-order methods in the first place.

In the present work, the coolant within the porous medium was assumed incompressible such that the response time is much smaller than the time scale of the surface turbulent fluctuations in the boundary layer. This steady-state approximation allows us to neglect the transient effects in the present case so that [PNM](#) can be used to model the porous medium flow.

2.4 Convolutional neural networks

Artificial neural networks are a machine learning method inspired by the structure of the human brain and consist of a large network of nodes, or neurons, organised into different layers and connected by weighted edges. The network learns by systematically modifying the values of these weights to reduce the error of its output in a process known as training. Neural networks are increasingly popular as they allow the characterisation of complex processes and relationships using very simple mathematical operations, including the incorporation of spatial correlations within the data.

Convolutional neural networks ([CNNs](#)) are a type of neural network in which convolution operations are performed on the dataset in successive layers. This translates to a more strategic method of connecting adjacent neurons than the traditional fully-connected layers of the first artificial neural networks. Since their inception, [CNNs](#) have experienced

wide-spread adoption, particularly with respect to image processing applications such as image classification and segmentation [52].

CNNs have been applied to a multitude of problems in CFD, including force estimation on airfoils and turbulence modelling [27, 58]. With respect to flow in porous media, a fair amount of work has been done to predict bulk properties of the medium by applying a CNN to images of internal porous structures [1, 4, 31]. Santos et al. [65] developed a 3D CNN that can predict the flow field within a porous medium on which a small pressure gradient is applied with reasonable accuracy given a reconstructed image of the pore space.

With respect to transpiration cooling, Yang et al. [84] successfully used a simplified CNN with only one convolution layer and no fully-connected layers to model the anticipated cooling effectiveness for an arbitrary injection hole distribution. However, the simulations used to train the network were run using a RANS solver and represented the porous media as a collection of straight channels. This work might therefore be better considered as a case of effusion cooling as the pores do not have internal connectivity. The limitations imposed by the choice of solver and porous media representation prevent any characterisation of the pressure-coolant velocity coupling that is the subject of this thesis; however, it does set a compelling precedent for the use of convolutional networks to encapsulate the influence of porous media geometry on transpiration cooling performance.

The works to integrate CNNs with DNS have mainly focused on numerical methods that use the former to accelerate the latter and relieve some of the computational burden [59, 60]. This contrasts with the present work, where a CNN is used instead of a porous medium solver to incorporate an external coupling into a DNS.

Chapter 3

Numerical Details

The numerical details of the simulations, discussed below, are divided into three sections. The first section outlines the details of the [DNS](#) of a turbulent boundary layer over the transpirative wall. The second section discusses the pore-network model used to simulate the flow through individual pores in the porous medium. Finally, the construction of the coupling functions (neural network or linear expression) based on the [PNM](#) results that are coupled with the [DNS](#) are outlined. Figure [3.1](#) illustrates the interrelation between these three components.

3.1 Direct numerical simulation

The [DNS](#) solver Hybrid was used for this work, developed by Bermejo-Moreno et al. [\[7\]](#), which solves the compressible Navier-Stokes equations on a structured grid. In conservative form using Einstein notation, these equations are as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0 \quad (3.1)$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_i u_j + p \delta_{ij}) = \frac{\partial \sigma_{ij}}{\partial x_j} \quad (3.2)$$

$$\frac{\partial(\rho e)}{\partial t} + \frac{\partial[(\rho e + p)u_j]}{\partial x_j} = \frac{\partial(\sigma_{ij}u_i)}{\partial x_j} - \frac{\partial q_j}{\partial x_j} \quad (3.3)$$

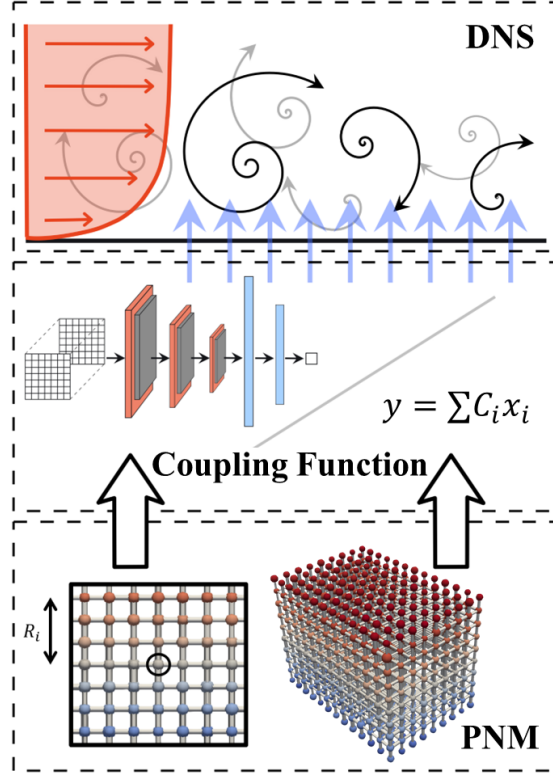


Figure 3.1: Sketch of the interrelation between the three major numerical components of the considered simulations.

Here, δ_{ij} is the Kronecker delta, e is the total energy, and q_{ij} is the heat flux vector. σ_{ij} is the viscous stress tensor:

$$\sigma_{ij} = 2\mu \left(S_{ij} - \frac{\delta_{ij}}{3} \frac{\partial u_k}{\partial x_k} \right) \quad (3.4)$$

The above equations are used in conjunction with the ideal gas equation $p = \rho RT$ to solve for the five conserved variables: ρ , ρu , ρv , ρw , and ρe . Aliasing errors are reduced by using a split form of the convective terms and the spatial derivatives are approximated using a sixth-order central finite-difference scheme with a higher-order spatial filtering to stabilise the numerics. Time integration is done using a fourth-order Runge-Kutta scheme.

In addition to its accuracy, speed, and parallelisability, Hybrid was chosen because it was previously modified by Christopher et al. [17] for the transpiration cooling case with

uniform coolant injection. With the groundwork already laid, the work required to build the [DNS](#) cases with indirectly coupled coolant injection was significantly reduced.

A single fluid is used for both the hot gas and coolant, and the temperature dependence of the fluid viscosity is modeled using a power-law function with similar accuracy to Sutherland’s law at reduced computational cost,

$$\mu = \mu_0 \left(\frac{T}{T_0} \right)^{0.75}. \quad (3.5)$$

Both μ_0 and T_0 are constant reference values set by the user. Non-dimensional units are used such that the inlet boundary layer thickness δ_i and the free-stream density, streamwise velocity, and temperature $(\rho_\infty, U_\infty, T_\infty)$ are all unity. The Prandtl number is fixed at a constant value, and the Mach number is set to 0.3 to minimise the effects of compressibility.

The flow domain is shown in [Figure 3.2](#) and has a size of $60 \times 6 \times 6$ with respect to the inlet boundary layer thickness. The mean flow profile and Reynolds stresses at the inlet are taken from those of an established [DNS](#) of an incompressible turbulent boundary layer with $Re_\tau = 450$ [\[42\]](#). The boundary condition at the inlet plane is set by superimposing the mean flow profile with randomly generated turbulence corresponding to the Reynolds stress profiles. The resulting signal is digitally filtered in both space and time to ensure a realistic structure of the generated turbulence fluctuations. In/outflow boundary conditions are assigned to both the top and outlet of the flow domain using a summation-by-parts scheme with a simultaneous approximation term penalty, with sponge layers to dampen any numerical reflections. Periodic boundary conditions are applied at the front and back faces.

The wall at the bottom face of the domain is set to a uniform temperature of $T_w = 0.5T_\infty$ and is split into two segments. At $0 \leq x < 30$ and $x \geq 50$ it corresponds to a regular non-permeable flatplate, represented by the typical no-slip boundary condition. However, for $30 \leq x < 50$ a modified no-slip boundary condition is used to represent transpiration wherein a non-zero wall-normal velocity is permitted. Depending on the case, the magnitude of this velocity is either a prescribed constant (uniform injection) or is set by the coupled neural network at each time step based on the surface pressure distribution. When the neural network is used, hydraulic conductivity values are randomly generated according to the same normal distribution as those of the [PNM](#) simulation the network was trained on. These are then assigned to each grid element at the wall within the transpiration region prior to the initial start of the simulation.

Whether uniform or coupled injection is used, it is assumed that the injected coolant is

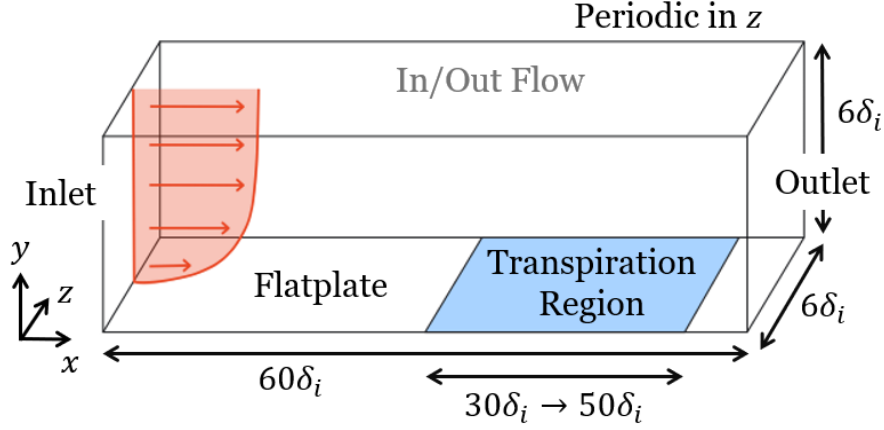


Figure 3.2: Overview of the computational domain for all [DNS](#) cases.

laminar. Given the small pore sizes and low injection velocities typically used in transpiration cooling, this is considered a reasonable assumption. This assumption is consistent with the pore-network model approach used to train the coupled neural network; as discussed in Section 2.3, the Hagen-Poiseuille equation used to obtain hydraulic conductivity is only applicable for Darcy-type flow.

The degree of transpiration is quantified using the blowing ratio $F = \rho_c v_c / \rho_\infty u_\infty$ where v_c is the Darcy velocity of the coolant (volume of fluid per unit time through a unit cross-section of the porous medium). In all instances, a mean blowing ratio of $\bar{F} = 0.6\%$ was targeted over the transpiration region, although in the coupled cases there is some variation in the local blowing ratio due to changes in pressure across the transpiration region.

In all cases, the same fully-structured mesh with a wall-normal grid refinement was used for the [DNS](#), containing roughly 118 million grid points. The number of grid points along each axis is given in Table 3.1 along with a summary of the main flow conditions.

Pr	M	μ_0	T_0	$Re_{\tau, x=0}$	$\bar{F}$	N_x	N_y	N_z
0.7	0.3	$1.47 \cdot 10^{-4}$	0.5	450	0.6%	2560	180	256

Table 3.1: Specifications of the [DNS](#) for all considered cases

3.2 Pore-network model

The Python package OpenPNM [30] was used to simulate flow through a simple porous medium based on a cubic lattice structure in order to obtain the data necessary to construct the different coupling functions. The PNM uses Equation (2.11) as discussed in Section 2.3 to build a sparse system of linear equations which can then be solved using matrix inversion techniques.

Each network is composed of 2.2 million spherical pores ($200 \times 43 \times 256$) connected by cylindrical throats, a section of which is shown in Figure 3.3. The diameter of the pores is randomised according to a normal distribution in order to obtain the desired porosity.

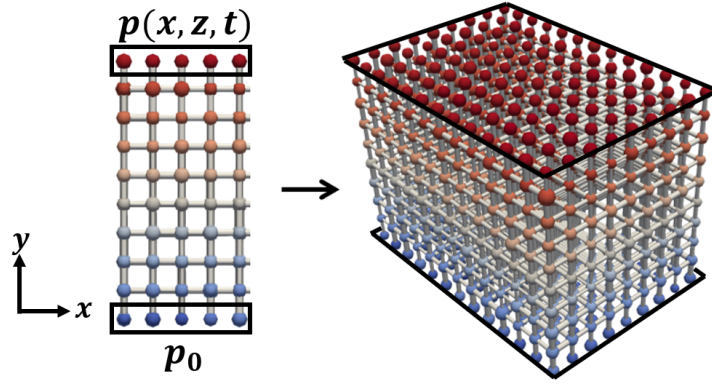


Figure 3.3: Straight-on and isometric views of subset of the pores from one of the pore networks, showing the basic structure and relevant boundary conditions.

Two different pore-networks were constructed according to the details described in Table 3.2 and indirectly coupled to the DNS to explore the impact of the pore geometry on the coupling. A third network with uniform pore sizes corresponding to the mean pore diameter of the 30.7% porosity case was also constructed as part of an exploration of sources of error in the neural networks; however, it was not coupled with a DNS due to time limitations.

ϕ	$\overline{d_p}$	σ_d	Δx_i
24.6%	0.01150	0.0010	0.0234375
30.7%	0.01295	0.0011	

Table 3.2: Details of the constructed pore networks coupled with a DNS.

It is important to note that all cases have the same centre-to-centre distance between neighbouring pores along all three axes (Δx_i), which corresponds to the streamwise and spanwise grid spacing of the [DNS](#) grid at the transpirative wall. This was required for the network to be compatible with the coupling method. Similarly, viscosity and temperature were taken from the conditions set in the [DNS](#) at the transpirative wall and all variables are solved for in the same non-dimensional units.

To set the boundary conditions of the [PNM](#) simulations, the time-and-space-dependent pressure signal at the wall of the transpiration region was extracted from the uniform injection [DNS](#) case. Using this signal, two simulations were run for each network for over 1000 time units. Due to the steady-state limitations of the utilised Hagen-Poiseuille equation, time convergence was not a concern in these simulations and running the simulations for longer has the sole purpose of generating more data to inform the construction of the coupling functions.

Of these two simulations, one applied the extracted [DNS](#) pressure signal to the boundary of the pore network representing the interface between the wall and the main flow while the other applied a random signal with the same mean and standard deviation as the extracted pressure signal. The latter was used to build the coupling functions without biasing them towards correlations in the extracted signal, while the former was then used to test the coupling functions ability to deal with coherent pressure structures. This will be discussed in more detail in the following section. The pressure at the opposite face was set to a uniform value such that the mean blowing ratio is set to 0.6%, corresponding with the [DNS](#) cases.

3.3 Coupling functions

Coupling functions were used as an intermediary between the pore-network simulations and the [DNS](#) to make the latter computationally feasible while still incorporating the coupling with the former. They act as transfer functions, taking as input the local pore conductivity and pressure at the porous wall and outputting the predicted local flow rate without having to fully resolve the porous medium with a separate solver. Two forms of coupling function are considered: linear expressions and neural networks.

3.3.1 Linear expressions

Linear expressions are the simplest possible form of the coupling function and express the coolant injection velocity at a surface pore as a linear function of the local pressure drop across the wall at the pore in question (Δp_{ij}) and the hydraulic conductivity of the conduit connecting this pore to the bulk porous network (g_{ij}). Importantly, this method does not consider the influence of neighbouring pores in the network via the redirection of the internal flow.

The use of this linear expression approach is motivated by the well-established Darcy law, which establishes a linear relationship between the bulk Darcy velocity through a porous medium and the pressure gradient across it (see Section 2.3). Being a bulk medium approach, it is not suitable for capturing local velocity variations; however, it provides a good benchmark against which to compare the performance of the more sophisticated approach using a CNN.

In an effort to express the coupling function as a linear expression without reducing the problem to the effusion cooling case, as with the Hagen-Poiseuille expression (2.11), an alternative expression was found by fitting PNM results with the random pressure signal boundary condition using a multi-variable linear regression.

Initial attempts sought to perform the regression using only the local pressure gradient as a variable, as it is expected that the flow velocity exiting a surface pore will have a linear relationship with the local pressure gradient across the porous wall following Darcy's law. This relationship was observed in the porous flow simulation with uniformly sized pores; however, in the simulations with a normal distribution of pore sizes this relationship is obfuscated by the corresponding variation in pore hydraulic conductivity values through the network. It was therefore necessary to include the pore hydraulic conductivity to obtain a decent fit.

The resulting expression takes the following form, where C_1, C_2, C_3, C_4 are constants obtained from the linear regression and are unique to each of the pore networks considered:

$$v_c(g_{ij}, \Delta p_{ij}) = C_1 + C_2 g_{ij} + C_3 \Delta p_{ij} + C_4 g_{ij} \Delta p_{ij} \quad (3.6)$$

Here Δp_{ij} and g_{ij} are the local wall-normal pressure gradient and connection hydraulic conductivity, respectively, corresponding to the surface pore at location (i, j) within the interfacial layer (as defined in Figure 1.2).

3.3.2 Neural network

In order to try and capture the spatial dependence of the relationship between the local pressure gradients and the coolant injection velocity, a shallow CNN was developed using PyTorch [3]. This neural network was designed to be as small as possible in order to mitigate the overhead increase when incorporated in the DNS while still effectively capturing the behaviour of the flow within the porous medium.

In essence, the neural network takes as input two “images” consisting of the pressure gradient across the porous medium and the surface pore hydraulic conductivity in the local field surrounding the point/pore of interest. This data is then passed through a sequence of convolution and rectified linear unit (ReLU) activation layers followed by some fully-connected linear layers. The final output of the network is a single scalar value representing the wall-normal coolant injection velocity at the central pore.

To keep the network small and thereby reduce the overhead associated with coupling it to the DNS, it is necessary to limit the size of the field used as input. Therefore, a radius of influence R_i is defined that dictates the maximum distance in terms of the number of pores that must be included in the input field of the neural network (see Figure 3.4).

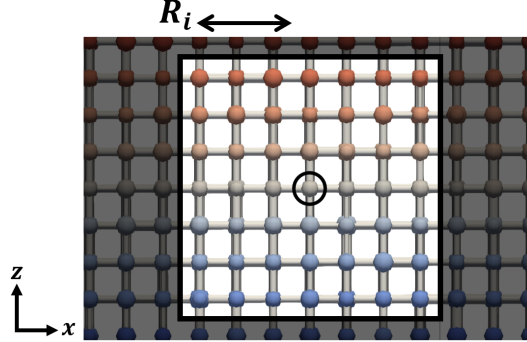


Figure 3.4: Illustration (top view) of the region captured by the radius of influence R_i around the circled central pore.

Upon inspection of the cross-correlation between the coolant injection velocity and surface pressure on the wall a radius of influence of three pores was selected for the present cases. The impact of this selection is discussed further in Section 4.1.

A sketch of the final structure of the neural network, which was then trained and coupled with the DNS, is given in Figure 3.5. It takes as input two 7×7 images containing the pressure drop across the wall and the surface pore hydraulic conductivity in the area

surrounding the central pore of interest. These are passed through 3 2D convolution layers, the first two with a kernel size of 3×3 and the last with a kernel size of 2×2 . The number of channels is 10, 32, and 64, respectively. In all cases a unit stride is used, no padding is applied, and a [ReLU](#) activation layer is applied following each convolution. The subsequent output is flattened and passed through two fully-connected layers of size 256 and 100, respectively, to yield the coolant injection velocity at the central pore of interest.

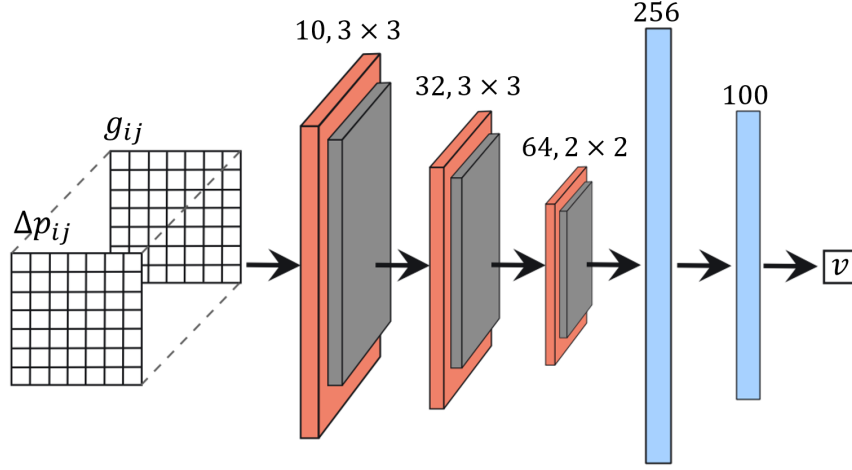


Figure 3.5: Sketch of the basic structure of the neural network which was coupled with the [DNS](#). The red represents convolution layers of varying channel and kernel sizes, the grey [ReLU](#) activation layers, and the blue indicates fully-connected linear layers.

Separate versions of the neural network were trained on the [PNM](#) data with the random fluctuation boundary condition applied for each of the network porosities. It was necessary to train the networks using data with a randomised boundary condition to avoid biasing the network according to the length correlations that were present in the uniform injection [DNS](#) case but may not be present once the pressure fluctuation and injection velocity coupling have been incorporated. In all cases, the Adam optimiser was used for training in conjunction with a root mean square ([RMS](#)) error loss function. In each case, the network was allowed to train until the training loss function plateaued, corresponding to roughly 50 epochs. The learning rate was initially set at 10^{-3} and was reduced up to 10^{-6} as the training progressed. Section 4.1 presents the errors associated with the neural network and compares its performance with that of the expression obtained via linear regression.

3.4 Simulated cases

A total of 4 cases were simulated for the present work: 2 cases using linear expressions obtained via linear regression for the injection coupling and 2 cases using neural networks for the coupling. Additional cases from the work of Christopher et al. [16] were used for the DNS validation and comparison of the results with cases with no and uniform coolant injection. These have the same details as described in Section 3.1; given the data was already available, it was deemed unnecessary to run these simulations a second time for the present work. A summary of the relevant cases is given in Table 3.3.

Coupling	Case	Notes
None	No Injection	From Christopher et al. [17]
	Uniform Injection	From Christopher et al. [17]
Linear Expression	LR-24	$\phi = 24.6\%$
	LR-30	$\phi = 30.7\%$
Neural Network	NN-24	$\phi = 24.6\%$
	NN-30	$\phi = 30.7\%$

Table 3.3: Summary of the DNS cases.

The porosities of the pore networks are in the lower range of those typically chosen for transpiration systems (between 20% and 40%). Pore network simulations for an additional case with a porosity of 40% were run, however there was insufficient time to implement and run the associated DNS cases. As will be shown in Chapter 5, even with the relatively small differences in porosity of the cases in Table 3.3, the influence of this parameter is clearly visible.

Chapter 4

Validation

The present chapter discusses the validity of the constructed simulations and investigates the uncertainty in the results. First, the validity of the coupling functions (neural network and linear expression) is investigated in section 4.1, where it is found that the neural network is able to replicate the PNM results with acceptable accuracy whereas the linear expression yields significant errors when a coherent pressure signal is applied. The impact of the selected radius of influence is also investigated, where it was found that the use of a larger radius of influence would allow for better resolution of the smaller scale features, however this advantage is deemed minimal compared to the associated increase in computational cost.

Section 4.2 focuses on a validation of the DNS itself. This is done by confirming the extensibility of the validation previously done by Christopher [16] to the new coupled cases that were conducted for the present work. In particular, grid convergence and time convergence are confirmed by comparing the near-wall mesh resolution and mean eddy turnover time of the new cases with that of the no injection and uniform injection cases, respectively.

4.1 Coupling functions

As discussed previously, due to computational limitations, the PNM used to simulate flow through the porous medium is not directly coupled to the DNS but is instead encapsulated within a coupling function, using either an expression obtained via multi-variable linear regression or a CNN. This process naturally introduces some errors, and the purpose of this

section is to explore the degree of these errors and determine the suitability of these coupling functions in representing the coupled-domain behaviour in transpiration cooling. To this end, the ability of the neural network and the linear regression expression to capture the [PNM](#) results is evaluated for the network with 30.7% porosity. Both coupling functions had greater difficulty reproducing the local coolant velocity values from the [PNM](#) simulations for this pore network due to its increased sensitivity to changes in the wall-normal pressure gradient.

In order to quantify the ability of the trained [CNN](#) and the expression obtained via linear regression to capture the flow in the porous medium, the error relative to the [PNM](#) results was obtained. This error is defined as the difference between the local injection velocity predicted by the coupling function and the corresponding value extracted from the [PNM](#) simulations, which is taken as the true value. Figure 4.1 displays the error histogram when both coupling functions were applied to the data with random pressure fluctuations (4.1a, no coherent structures) as well as the data with the wall pressure signal extracted from the uniform injection [DNS](#) case (4.1b). It should be noted that the reported error values correspond to the discrepancy between the injection velocity predicted by the coupling functions and that given by the [PNM](#) simulations. The injection velocity v_c is the value actually used to set the transpiration boundary condition in the [DNS](#) cases. A mean blowing ratio of $\bar{F} = 0.6\%$, which was used in the present cases, corresponds to a mean injection velocity of $\bar{v}_c = 0.003$.

Even when comparing the [PNM](#) results with a random pressure signal applied at the wall surface, the neural network significantly outperforms the linear expression. The network had a [RMS](#) error of roughly 12% of the mean injection velocity, while the error in the predictions of the linear expression was approximately 25%. As both coupling functions were obtained using data from the same [PNM](#) simulation on which they were tested, this reflects the optimal performance of either function.

When the same neural network and linear regression expression are applied to the [PNM](#) simulation results using the wall pressure signal extracted from the uniform injection case, the difference in performance becomes even more apparent. The [RMS](#) error corresponding to the neural network increases to 20.5%, while that of the linear regression increased to 111.5% of the mean injection velocity.

On the basis of these results, it is clear that the linear regression is unable to predict the flow exiting individual surface pores. As mentioned in section 3.3.1, this is an expected result, as the basis for using a linear expression was a Darcy-type bulk medium approach. The pressure signal extracted from the uniform injection case has natural spatial correlations due to the turbulent structures at the wall. Consequently, there are coherent

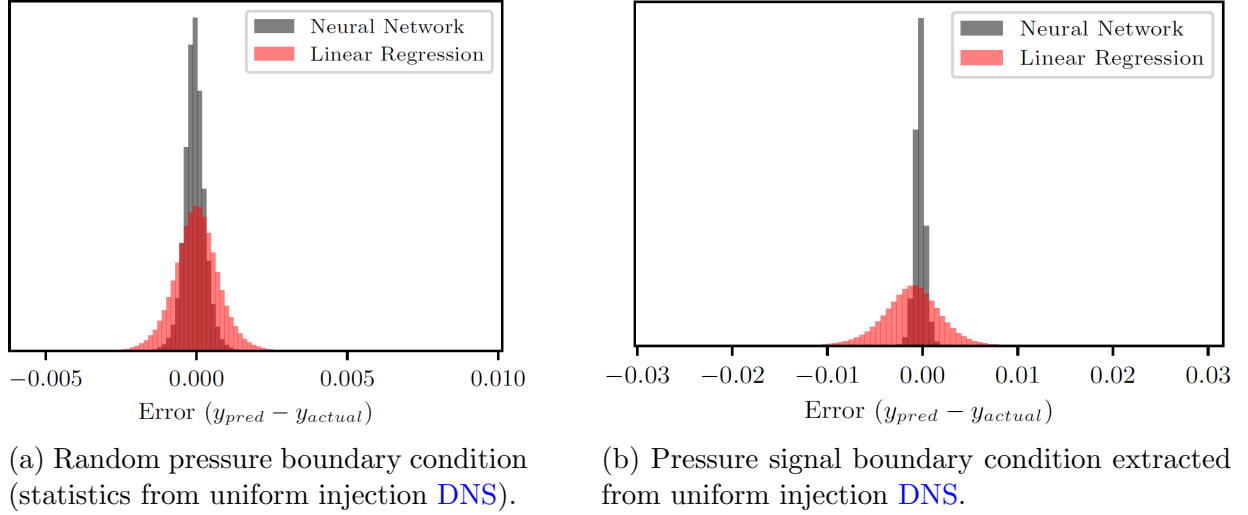


Figure 4.1: Error histograms of both the [CNN](#) (grey) and linear regression expression (red) when applied to the [PNM](#) results for the $\phi = 30.7\%$ network with different imposed wall interface boundary conditions.

pressure “footprints” at the wall which cover multiple surface pores and are able to strongly influence the flow inside the porous medium.

The influence of the flow between neighbouring pores is expected to become most significant in this case, explaining why the linear expression, which does not consider spatial correlations, performs so much worse in this case. It is therefore not suitable for implementing the coupling effects between the two flow domains in transpiration cooling, being more representative of an effusion cooling-type scenario where there is no lateral flow of coolant prior to injection. [DNS](#) cases coupled using the linear expressions were nonetheless conducted to compare the results with those of the neural network cases. This allows for insight on the impact of the lateral flow in the porous medium on the observed coupling effects in the main flow.

In contrast, the increase in the error of the predictions made by the [CNN](#) is much more moderate due to its ability to incorporate the influence the neighbouring pores. This highlights the importance of considering interactions between nearby pores when modelling the coupling of the porous medium domain with the main turbulent flow.

It should be noted that, despite being an improvement over those of the linear expression, the errors in the [CNN](#) predictions are not insignificant. One major source of the observed errors is the selected radius of influence R_i which limits the size of the field sur-

rounding the central pore that is considered. Figure 4.2 shows instantaneous snapshots of the injection velocity field from the **PNM** simulation with the **DNS** pressure signal applied, as well as those predicted by the linear regression and two **CNN**s with different values of R_i .

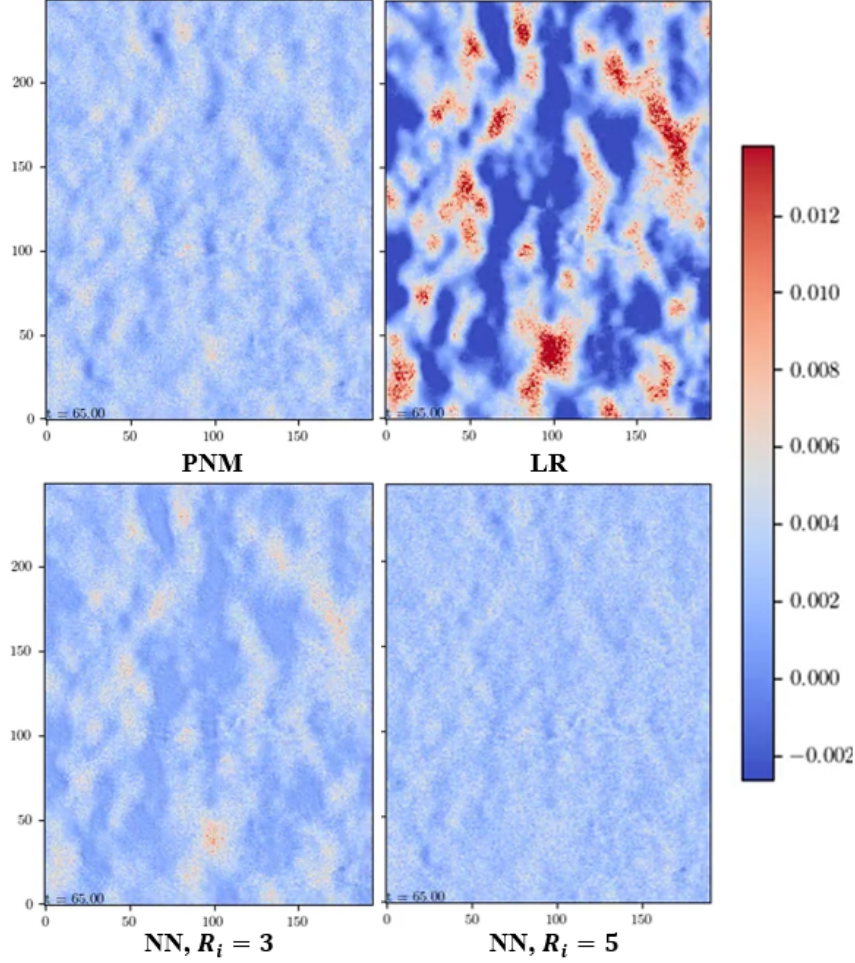


Figure 4.2: Instantaneous wall-normal injection velocity field for the network with 30.7% porosity as given by the **PNM** (top left) and predicted by the linear regression expression (top right), **CNN** with $R_i = 3$ (bottom left), and **CNN** with $R_i = 5$ (bottom right).

As previously established, the predictions obtained using linear regression significantly deviate from the **PNM** results. It is also apparent that these errors affect the overall structure of the signal, further reinforcing the previous conclusion that it is unable to

capture the interactions of neighbouring pores. What is of greater interest, however, is the difference between the predictions of the two neural networks, which demonstrates the impact of the selected radius of influence.

The [CNN](#) with the smaller radius of influence of three is the same as the one used to generate the error histograms in Figure 4.1. Although it is able to recreate the general structures and trends from the [PNM](#) case, it also has the effect of smoothing out the smaller visible structures. Meanwhile, the neural network with an increased radius of influence of five is able to capture these small structures, which is reflected in a corresponding reduction in the [RMS](#) error. However, it underpredicts the magnitude of the deviations in the injection velocity from the mean.

While the neural network with the larger radius of influence does perform better overall, it was nonetheless decided to use the one with a radius of influence of three. This is because increasing the radius of influence is associated with a significant increase in the computational expense of the [CNN](#). As discussed in Section 2.3, pore-network modelling itself comes with important limitations that make it not strictly applicable for the present work. In light of this, it was decided to sacrifice some accuracy in order to have a neural network that is lightweight enough to couple with a [DNS](#). This network is still able to reproduce the general reaction of the local coolant injection to changes in the surface pressure, and is therefore sufficient for the present purpose.

4.2 Direct numerical simulation

The present section deals with the validation of the [DNS](#) to which the coupling functions are applied. As the simulations conducted in this work are extensions of some of the cases from the work of Christopher [16], it was deemed unnecessary to conduct new wall scaling, grid resolution, and time convergence studies.

To confirm that the validation work done by Christopher can reasonably be extended to the new [DNS](#) cases with the implemented pressure-coolant velocity coupling, the scaled wall-normal mesh resolution as well as the mean eddy turnover times are extracted for each case.

4.2.1 Grid convergence

The grid convergence study conducted by Christopher [17] involved three simulations of the no blowing case: one with the baseline mesh, one with double the number of grid points in

the spanwise direction ($2560 \times 180 \times 512$), and one with the number of grid points doubled in all directions ($5120 \times 360 \times 512$). It was found that the deviation of the mean heat flux distribution between all cases within the designated transpiration region ($30 \leq x < 50$) was found to be within 1%, which is sufficient for the present work.

Table 4.1 compares the wall-normal mesh resolution of the no injection case with that of the coupled cases to determine if this finding can be extended to the latter. The inner scaling developed by Trettel and Larsson [73] is applied, which is a correction to the commonly used Van Driest scaling [75] to account for compressibility and heat transfer effects:

$$y^+ = \frac{\bar{\rho} \sqrt{\bar{\tau}_w / \bar{\rho}}}{\bar{\mu}} y \quad (4.1)$$

Case	No Injection	LR-24	NN-24	LR-30	NN-30
y_w^+	1.35	1.23	1.18	1.26	1.22

Table 4.1: Wall-normal mesh resolution at $x = 30$ for the studied case and all coupled cases.

In all of the coupled cases the y^+ value at the wall is less than that of the no injection case on which the grid refinement study was done, corresponding to a slightly improved wall-normal resolution. The grid convergence study is therefore applicable to the coupled cases, as the blowing reduces the velocity gradient at the wall. Therefore, the selected mesh resolution is considered sufficient for all of the cases considered.

4.2.2 Time convergence

Turbulent statistics are central to understanding how the implemented coupling affects the structure of the turbulent boundary layer. It is therefore important to ensure that the DNS results are statistically stationary. All of the coupled cases were started from the uniform injection case, which had been run for over 10 mean eddy turnover times. They were then allowed to run for a minimum of four additional mean eddy turnover times with the implemented coupling before averages started being collected.

For the time convergence study, Christopher [16] collected averages for the uniform injection case (called Uniform-0.6% in the referred to work) after three, six, and seven mean

eddy turnover times. It was found that within the transpiration region the standard deviation of the average values was 2%, which is considered an acceptable level of uncertainty. Table 4.2 shows that the the mean eddy turnover time of all of the coupled cases is less than that of the uniform injection case, suggesting that this estimation of uncertainty can be extended to the cases conducted for the present work.

Case	Uniform Injection	LR-24	NN-24	LR-30	NN-30
$\overline{\delta_{99}/u_\tau}$	47.93	47.18	47.14	46.41	47.25

Table 4.2: Mean eddy turnover time for the studied case and all coupled cases.

In addition, Christopher evaluated the uncertainty associated with the averaging window and found that an average over between one and two eddy turnover times was sufficient to keep the standard deviation of the mean properties within 2%. The averages for all coupled case were therefore collected over a minimum of two eddy turnover times (correspond to over 4.5 average flow through times over the porous region) to keep the associated uncertainty within this threshold. Due to limitations on time and computational resources, it was not possible to extend the averaging window further.

Chapter 5

Results and Discussion

The results from the [DNS](#) are presented in this chapter and discussed to infer the impact of the implemented coupling on transpiration cooling. First, a general discussion of the influence of the coupling on the distribution of the injected coolant and coolant film formation is presented in [Section 5.1](#), including the influence of the pore-network porosity. [Sections 5.2](#) and [5.3](#) explore changes in system performance with respect to cooling effectiveness and reduction of skin friction. Changes in the nature of the turbulence are then investigated in [Section 5.4](#), including quantitative results based on turbulent statistics and qualitative results from the visualisation of vortical structures. Finally, [Section 5.5](#) investigates the influence of coolant injection on the resultant pressure fluctuations at the cooled wall by comparing the pressure [PSD](#) obtained from the analytical Liepmann model with that from the different simulated cases.

5.1 Injection distribution and film formation

One of the simplest metrics used to evaluate the influence of the imposed coupling is the streamwise distribution of coolant injection. Changes in the local rate at which coolant is effused through the porous wall reflect changes to the local flow structure and transpiration performance parameters. This is quantified by the streamwise blowing ratio, which is shown alongside the corresponding mean pressure at the wall in [Figure 5.1](#).

It is immediately apparent that both the linear expression and neural network couplings have a significant impact on the injection of cooling, with a reduced blowing ratio at the beginning of the transpiration region relative to the uniform injection case. The blowing ratio increases further downstream, eventually exceeding that of the uniform injection case.

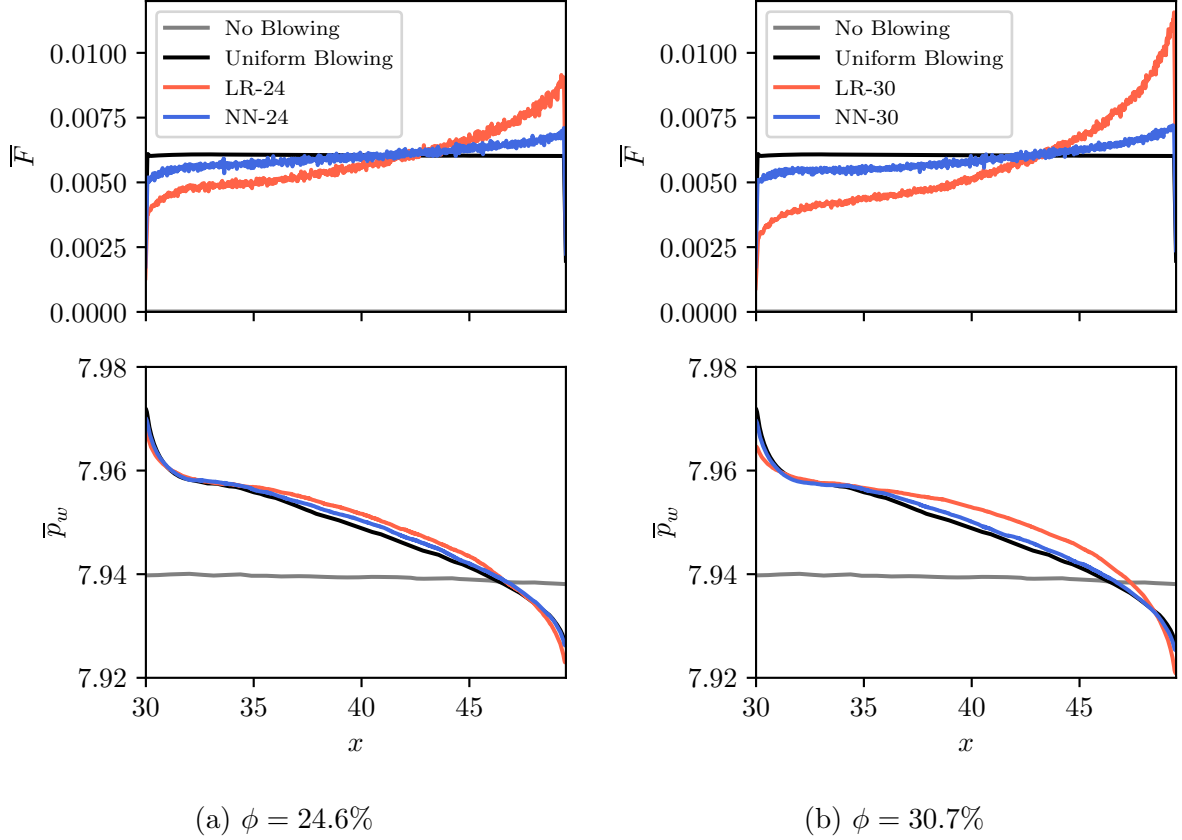


Figure 5.1: Streamwise variation in blowing ratio (top row) and mean wall pressure (bottom row) for all cases. The blowing ratio $\bar{F}$ for the no blowing case is present but not visible as it is directly on the x-axis.

The streamwise increase in the blowing ratio corresponds directly to a decrease in the mean pressure at the wall through the transpiration region. This is expected. A larger pressure at the wall results in a reduced wall-normal pressure gradient across the porous medium, which locally decreases the coolant injection velocity. The mean pressure over the transpiration region, in all three cases with coolant injection, is notably larger than the no injection case. This is due to the boundary layer flow suddenly encountering the wall-normal coolant injection at the beginning of the transpiration region. The pressure eventually decreases as the flow adapts to the change, and continues to decrease as the accumulation of the coolant film lifts the turbulent flow off of the wall.

With respect to the two different coupling functions employed, the cases using a linear

expression obtained via linear regression (LR-24 and LR-30) demonstrate a significantly increased sensitivity to pressure variations relative to the simulations coupled with the neural networks, suggesting that incorporating the influence of flow between neighbouring pores within the porous medium reduces the impact of the coupling. It is also observed that the higher porosity ($\phi = 30.7\%$) cases are more sensitive than their lower porosity ($\phi = 24.6\%$) counterparts, which is anticipated as it has a correspondingly higher permeability and therefore requires a smaller change in pressure drop to yield the same change in the velocity of the flow through the porous medium.

Due to differences in coolant injection, the development of the coolant film is also influenced by the introduced coupling. This is accentuated by the change in streamwise pressure development, which takes longer to decrease in the coupled cases due to the increased distance required for the coolant film to lift the flow off of the wall.

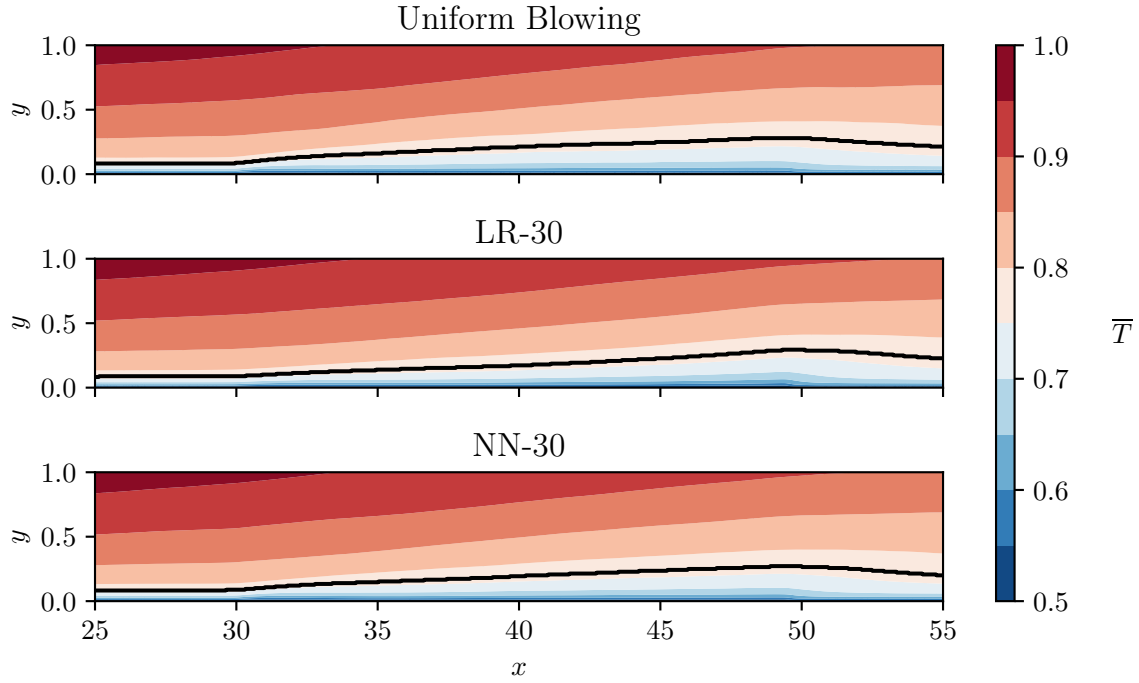


Figure 5.2: Identified coolant film interface for the uniform blowing case compared with cases coupled to the $\phi = 30.7\%$ pore network. The solid black line indicates the location of the interface, while the contours correspond to the time and spanwise-averaged temperature field.

To better visualise this, the interface between the coolant film and the hot main flow was obtained by applying an automatic interface identification algorithm based on a Fuzzy C-means clustering technique to the mean temperature field [26]. Three clusters (regions) were identified, corresponding to the free-stream flow, the temperature boundary layer, and the coolant film. The interface between the latter two of these is shown in Figure 5.2 for the uniform injection and $\phi = 30.7\%$ cases. The LR-30 case has a larger peak film thickness while the NN-30 case has a reduced peak film thickness. In both cases the film thickness in the first half of the domain is less than that of the uniform blowing case, corresponding to the observed trends in the blowing ratio. A similar trend was observed in the lower porosity cases.

5.2 Cooling effectiveness

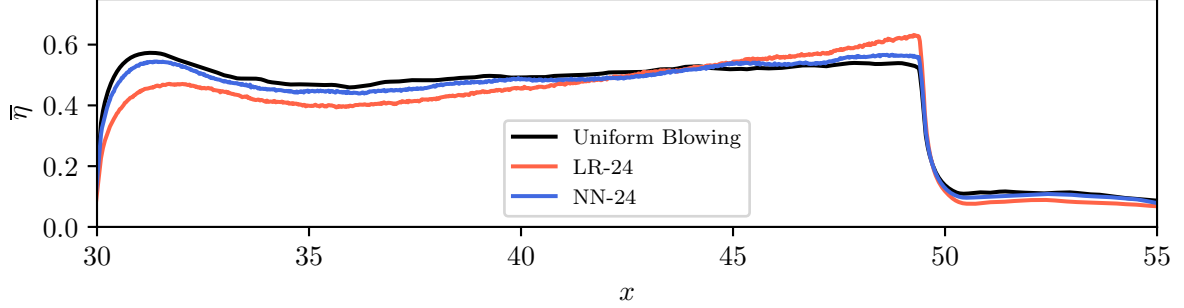
The cooling effectiveness is one of the most important parameters to consider when it comes to transpiration cooling. It is defined as the ratio of the difference in temperature between the free-stream and adiabatic wall to that between the free-stream and the injected coolant:

$$\eta = (T_\infty - T_{w,ad}) / (T_\infty - T_c) = 1 - (q_w / q_{w,ref}) \quad (5.1)$$

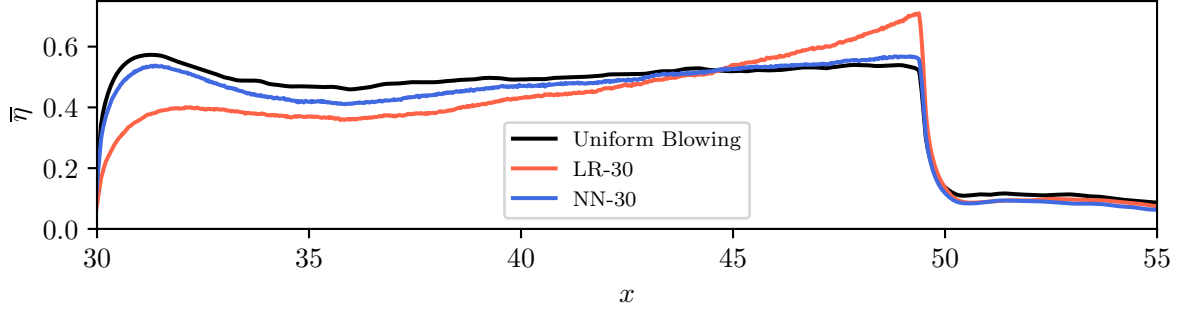
where $\eta = 1$ corresponds to an adiabatic wall temperature equal to that of the coolant (the wall is completely insulated from the main flow), while $\eta = 0$ occurs when the adiabatic wall temperature is equal to the temperature of the free-stream, in which case there is no cooling. However, in the present cases a constant temperature boundary condition was applied at the wall which also corresponded to the injected coolant temperature ($T_w = T_c$). Therefore, the above equation cannot be used directly but must instead be substituted into the convective heat transfer equation to relate the cooling effectiveness to the heat flux and thereby the temperature gradient at the wall [16]:

$$q_w = h(1 - \eta)(T_\infty - T_w) = \lambda \left. \frac{\partial T}{\partial y} \right|_w \quad (5.2)$$

Using this equation, the time and spanwise-averaged cooling effectiveness along the streamwise direction was calculated for each of the DNS cases, as shown in Figure 5.3. Overall, the same trends are visible as were observed previously when looking at the streamwise variation in blowing ratio, which makes sense as the two are closely tied.



(a) $\phi = 24.6\%$



(b) $\phi = 30.7\%$

Figure 5.3: Mean cooling effectiveness along the spanwise direction for all cases.

5.2.1 Turbulent transport of heat

One notable difference incurred by the coupling is a smoothing out of the peak in cooling effectiveness at the beginning of the transpiration region. This peak is explained by considering the turbulent transport of heat to the wall. The difference in velocity between the coolant and the main flow introduces shear, resulting in the development of strong vortices which transport heat to the wall. These vortices require a finite amount of time to develop, however. Consequently the turbulent transport of heat at the beginning of the transpiration region is primarily due to the vortices that were already present in the turbulent boundary layer. Further downstream, the strong vortices due to transpiration develop and begin to participate significantly in the transport of heat. The result is an initial peak in cooling effectiveness until the vortices develop sufficiently, at which point the turbulent transport of heat to the wall increases and the cooling efficiency is reduced.

Christopher [16] observed that for higher blowing ratios, the magnitude of the peak in cooling effectiveness increased and was compressed closer to the beginning of the transpiration region, which is in accordance with the mechanism described above. Greater blowing ratios further reduce the vorticity and thereby turbulent transport of heat at the beginning of the transpiration region, however the greater difference in velocity between the coolant and the main flow results in stronger and more rapid vortex production, increasing the turbulent heat transport just downstream.

Figure 5.4 depicts the spanwise vorticity for each DNS case with isocontours. In all coupled cases the mean vorticity at the wall near the beginning of the transpiration region is greater than in the uniform injection case. The locations of the minimum near-wall vorticity coincide with the locations of the observed peaks in the cooling effectiveness.

The vorticity is greatest at the wall in the LR-30 case (which also had the lowest blowing rate at this location), whose cooling effectiveness curve had a minimal, elongated peak. The injected coolant decreases the vorticity by reducing the near-wall velocity gradient, corresponding to a reduction in the turbulent transport of heat. However, in the LR-30 case, the blowing rate is small at the beginning of the transpiration region due to the higher pressure in that area, limiting its ability to reduce the vorticity.

Figure 5.5 shows the mean turbulent transport of heat to the wall for all DNS cases coupled with the higher porosity pore network. The transport is initially reduced in the coupled cases as compared to the uniform blowing case due to the more gradual introduction of coolant (reduced blowing ratio at the beginning of the transpiration region). The increased shear associated with the increasing coolant injection rate in the coupled cases, however, results in increasing turbulent transport of heat such that all three cases are on par near the end of the transpiration region.

5.2.2 Heat advection and film accumulation

The cooling effect in the boundary layer can be attributed to two effects: the decreasing temperature gradient at the wall due to the accumulation of a coolant film in the boundary layer, and the advection of heat away from the wall due to the non-zero wall-normal coolant injection velocity. It is of interest to see how both of these components are influenced by the introduced coupling.

The first mechanism considered is the coolant film accumulation in the boundary layer, which modifies the boundary layer temperature profile, lowering the amount of heat flux entering the wall. This is demonstrated in Figure 5.6, where it can be seen that the average temperature of the boundary layer decreases as the fluid moves further downstream.

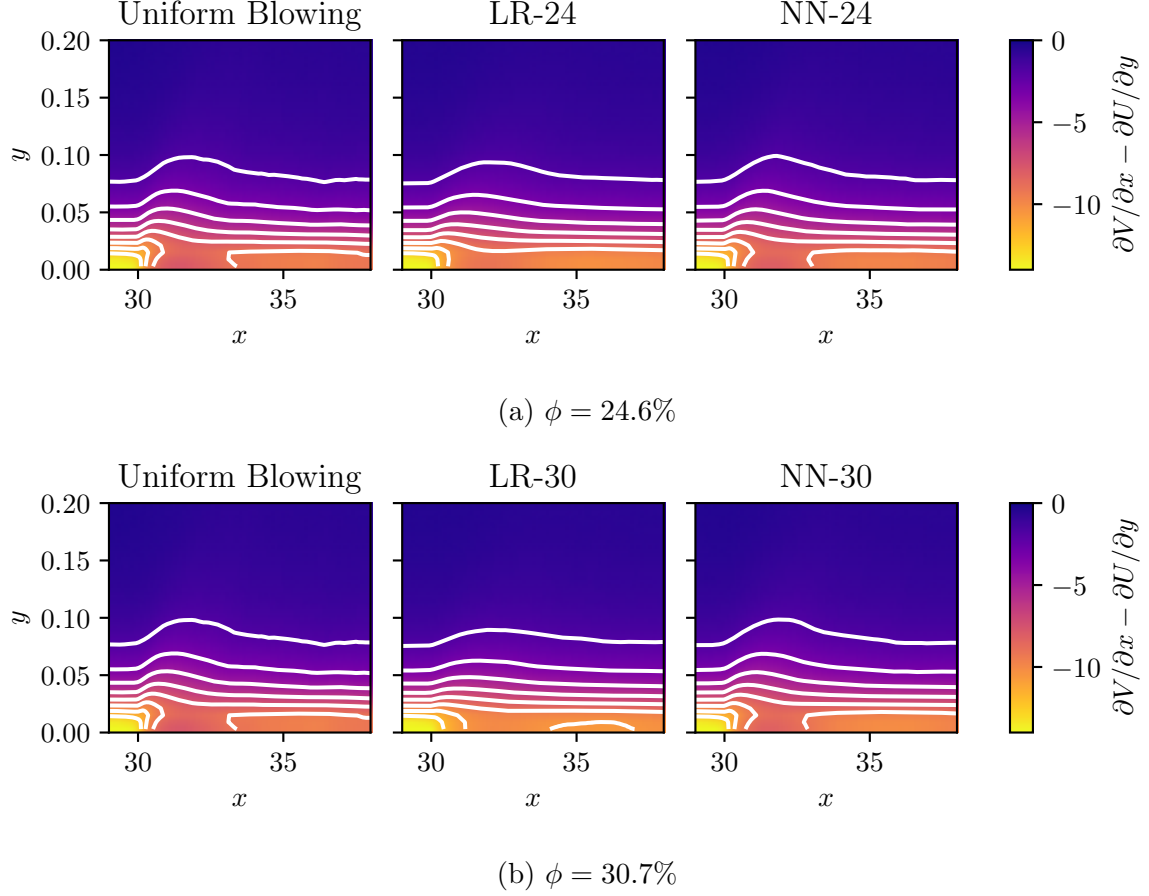


Figure 5.4: Mean spanwise vorticity around the beginning of the transpiration region.

The streamwise evolution of the coolant film was briefly discussed in Section 5.1, where it was determined that the peak coolant film thickness is increased in the coupled cases compared to the uniform injection case. This is in spite of the fact that slightly less coolant is injected over the transpiration region in the coupled cases. In order to get a better idea of the streamwise development of the coolant film, its thickness is plotted in Figure 5.7 without the additional temperature information from Figure 5.2.

As anticipated, based on the blowing ratio distribution, the coolant film thickness in the coupled cases is lower than in the uniform injection case at the beginning of the transpiration region. In the cases corresponding to the lower porosity network (LR-24 and NN-24), the film thickness eventually reaches that of the uniform blowing case at the

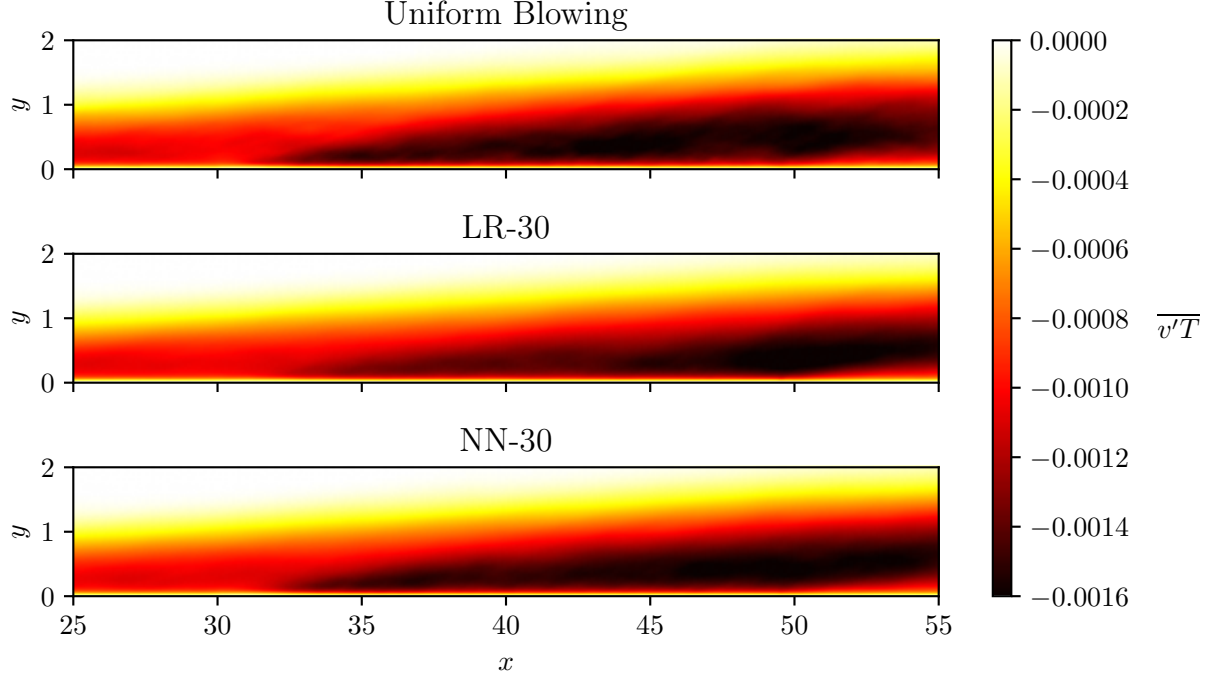


Figure 5.5: Mean turbulent transport of heat to the wall.

end of the transpiration region. Meanwhile, the cases associated with the higher porosity network do not converge to the uniform blowing film thickness. In the LR-30 case, the film thickness eventually exceeds that of the reference case, whereas in the NN-30 case the film thickness is less than that of the uniform blowing case throughout the entire transpiration region. This is particularly notable as all coupled cases had a cooling effectiveness greater than in the uniform blowing case at the end of the transpiration region, in contrast to the observed variation in film thickness. This means that the cooling effectiveness recovers faster from the coupling effects than the coolant film itself.

In order to gain a better understanding of this, it is necessary to look at the effect of advection on heat transfer throughout the transpiration region. To this end, the advection term $\bar{\rho}Vc_p(\partial\bar{T}/\partial y)$ for all cases at different streamwise locations is plotted in Figure 5.8. This term is only significant very close to the wall where the turbulence is small; further out, turbulence dominates the heat transfer. In the uniform injection case the advection term is strongest at the beginning of the transpiration region, decreasing further downstream due to the change in temperature gradient as was illustrated in Figure 5.6. However, in

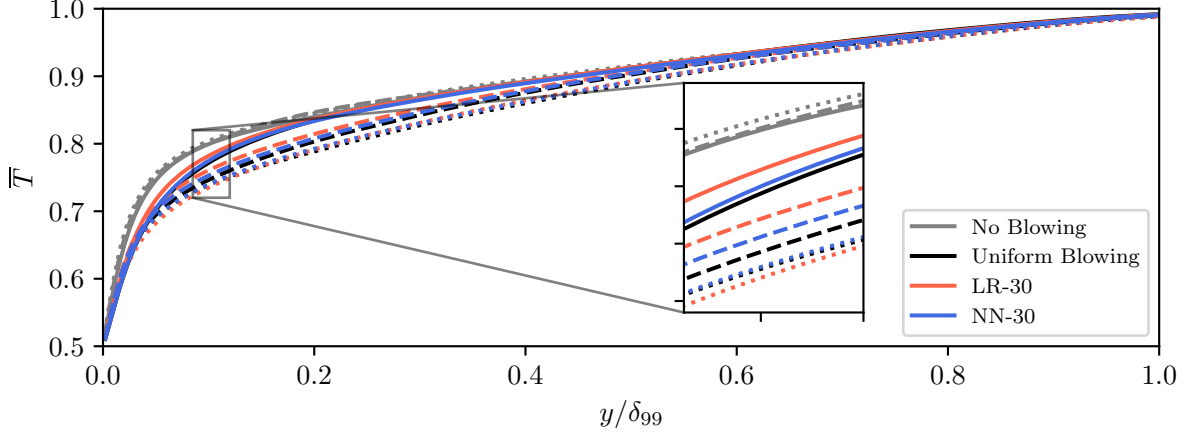


Figure 5.6: Mean temperature profile of $\phi = 30.7\%$ cases at different streamwise locations: $x = 32.5$ (—), $x = 40$ (---), $x = 47.5$ (····).

the coupled cases the lower blowing ratio near $x = 30$ inhibits film formation, meaning the temperature gradient is larger than at the same location in the uniform injection case. However, the blowing ratio and concomitantly the coolant injection velocity increases further downstream such that the advection term in the latter half of the transpiration region is greater for the coupled cases than for the uniform injection case, which explains the disproportional increase in cooling effectiveness relative to the coolant film thickness. This illustrates the importance of near-wall advective heat transfer on the downstream recovery of the cooling effectiveness in the coupled cases.

5.3 Skin Friction

In addition to wall cooling, transpiration cooling is of interest due to its ability to reduce skin friction, which is tied to the formation of the coolant film between the porous wall and the main flow. To consider the influence of the implemented pressure-injection velocity coupling, the local friction coefficient is computed along the streamwise direction:

$$\overline{c_f} = \frac{2\overline{\tau_w}}{\overline{\rho_\infty}U_\infty} = \frac{2\mu_w}{\overline{\rho_\infty}U_\infty} \left. \frac{\partial U}{\partial y} \right|_w \quad (5.3)$$

The resulting friction coefficient distribution is plotted in Figure 5.9. As anticipated,

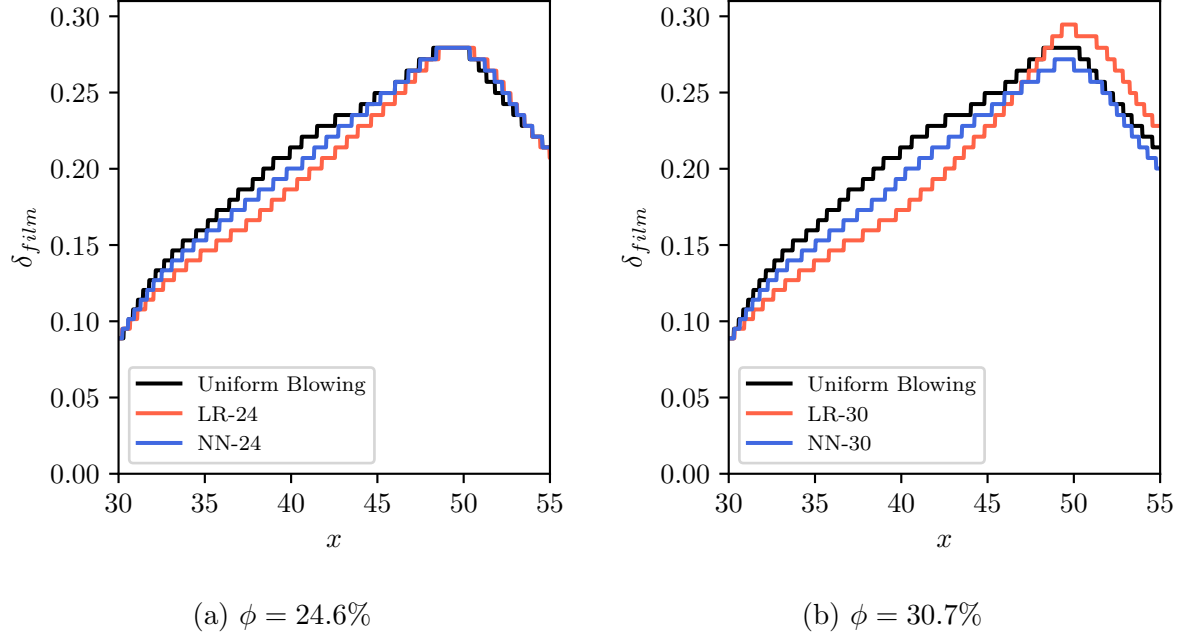


Figure 5.7: Coolant film thickness along the streamwise axis.

the trends bear similarities to the streamwise distribution of the blowing ratio and the coolant film thickness previously discussed, however, there is one particular difference to take note of, which is the local minimum of the friction coefficient at the beginning of the transpiration region.

Intuitively, it seems likely that this is due to the reduction in vorticity observed at the beginning of the transpiration region, as was discussed in Section 5.2.1. This is reinforced by the near lack of a local minimum in the LR-30 case, which is reminiscent of the lack of a peak in the cooling effectiveness for this case as observed in Figure 5.3.

A similar local minimum at the beginning of the blowing region was observed in an investigation done by Kametani and Fukagata [44]. In this work Kametani and Fukagata identified the Reynolds normal and shear stresses as sources of drag augmentation. Meanwhile, the mean convection ($-UV$) term, representing the convection of the free stream away from the wall due to blowing, was found to be a source of drag reduction. These two effects are analogous to the competing influences of vorticity generation and blowing ratio on cooling effectiveness as discussed earlier.

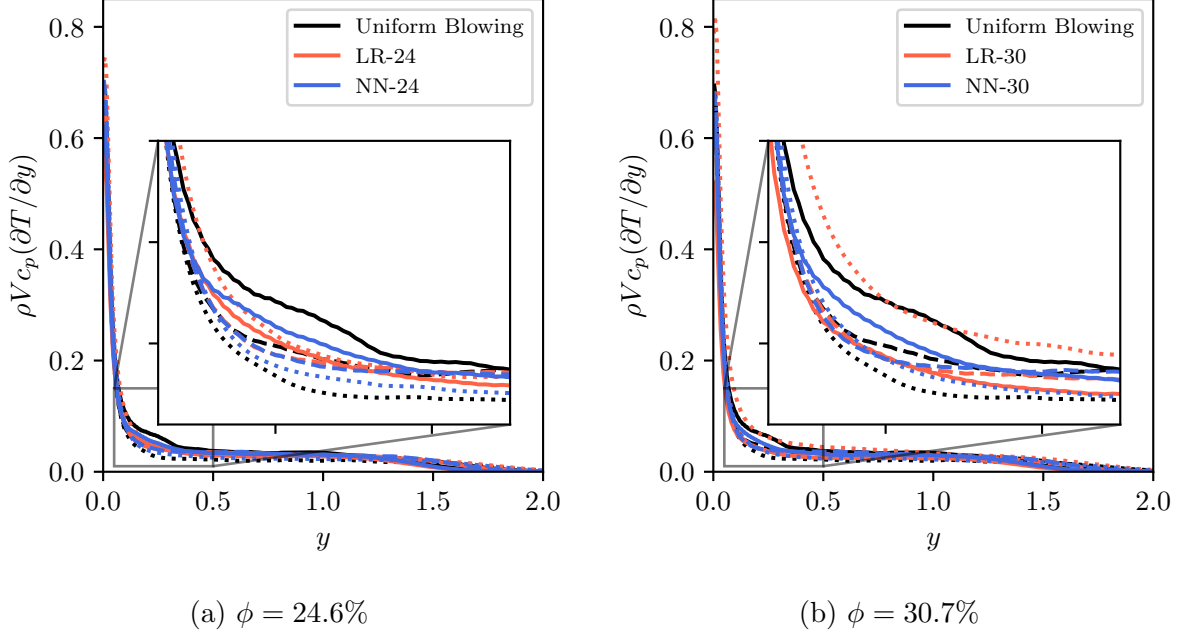


Figure 5.8: Effect of advection on heat transfer at different streamwise locations: $x = 32.5$ (—), $x = 40$ (---), $x = 47.5$ (.....).

5.4 Impact on Turbulence

In the following Section the changes in the turbulence of the flow over the wall due to the implemented coupling between the local near-wall pressure and coolant injection velocity will be investigated. This is first done quantitatively by comparing turbulent statistics over the different DNS cases. A qualitative analysis via a visualisation of the large-scale vortical structures present in the flow is then conducted to confirm the observations.

5.4.1 Turbulent statistics

In order to compare the nature of the turbulence in the different DNS cases, the turbulent kinetic energy (TKE) is used:

$$TKE = \frac{1}{2} (\overline{u'u'} + \overline{v'v'} + \overline{w'w'}) \quad (5.4)$$

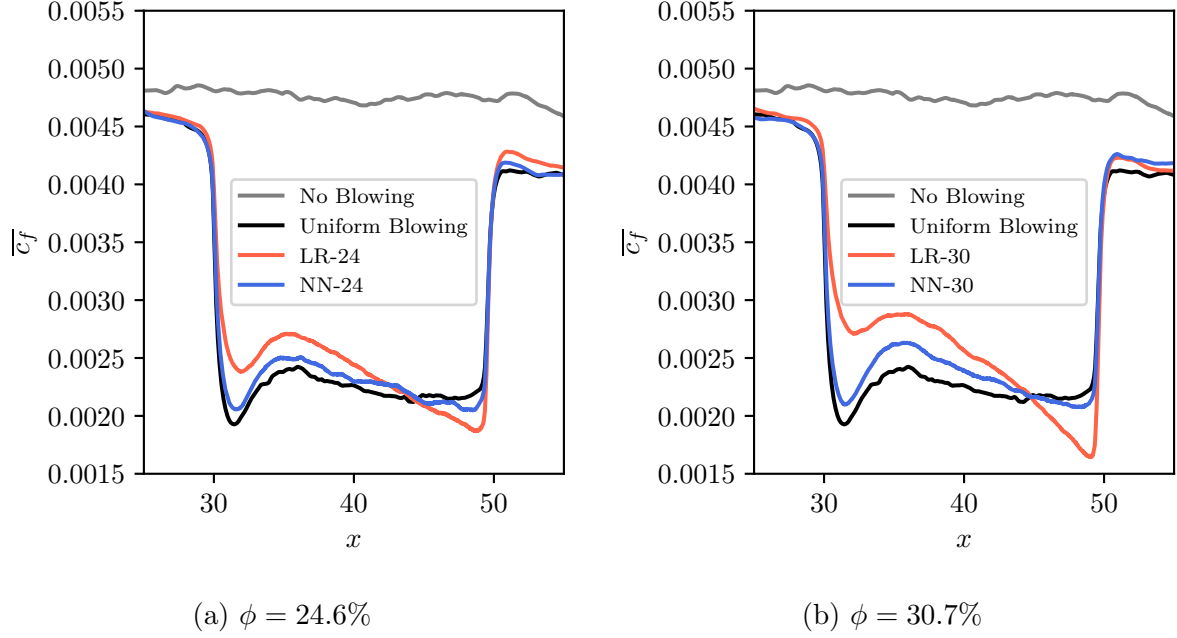


Figure 5.9: Local friction coefficient along the streamwise axis.

The time and spanwise-averaged [TKE](#) corresponding to the reference cases and those coupled with the higher porosity pore network (LR-30 and NN-30) is shown in Figure [5.10](#). Figure [5.11](#) displays the streamwise distribution in [TKE](#) at three different wall-normal distances such that the discussed trends are more clearly visible. As expected, blowing significantly increases the turbulent kinetic energy near the wall relative to the no injection case. Close to the beginning of the transpiration region both coupled cases have less [TKE](#) near the wall than the uniform injection case, however, as the flow progresses through the first half of the transpiration region the LR-24 and NN-24 cases maintain the large amount of turbulent energy at the wall, while the [TKE](#) in the uniform blowing case decreases slightly past $x \approx 38$ before reaching a plateau.

This initially seems to be an unexpected result; both coupled cases have a lower blowing ratio than the uniform injection case throughout the entire first half of the transpiration region and therefore were anticipated to have a correspondingly lower [TKE](#). An explanation for this is that the blowing ratio is slowly increasing in this region for the coupled cases, resulting in additional turbulent production, whereas the blowing ratio is fixed in the uniform injection case.

This explanation is sufficient to explain the high [TKE](#) trend throughout the entirety of

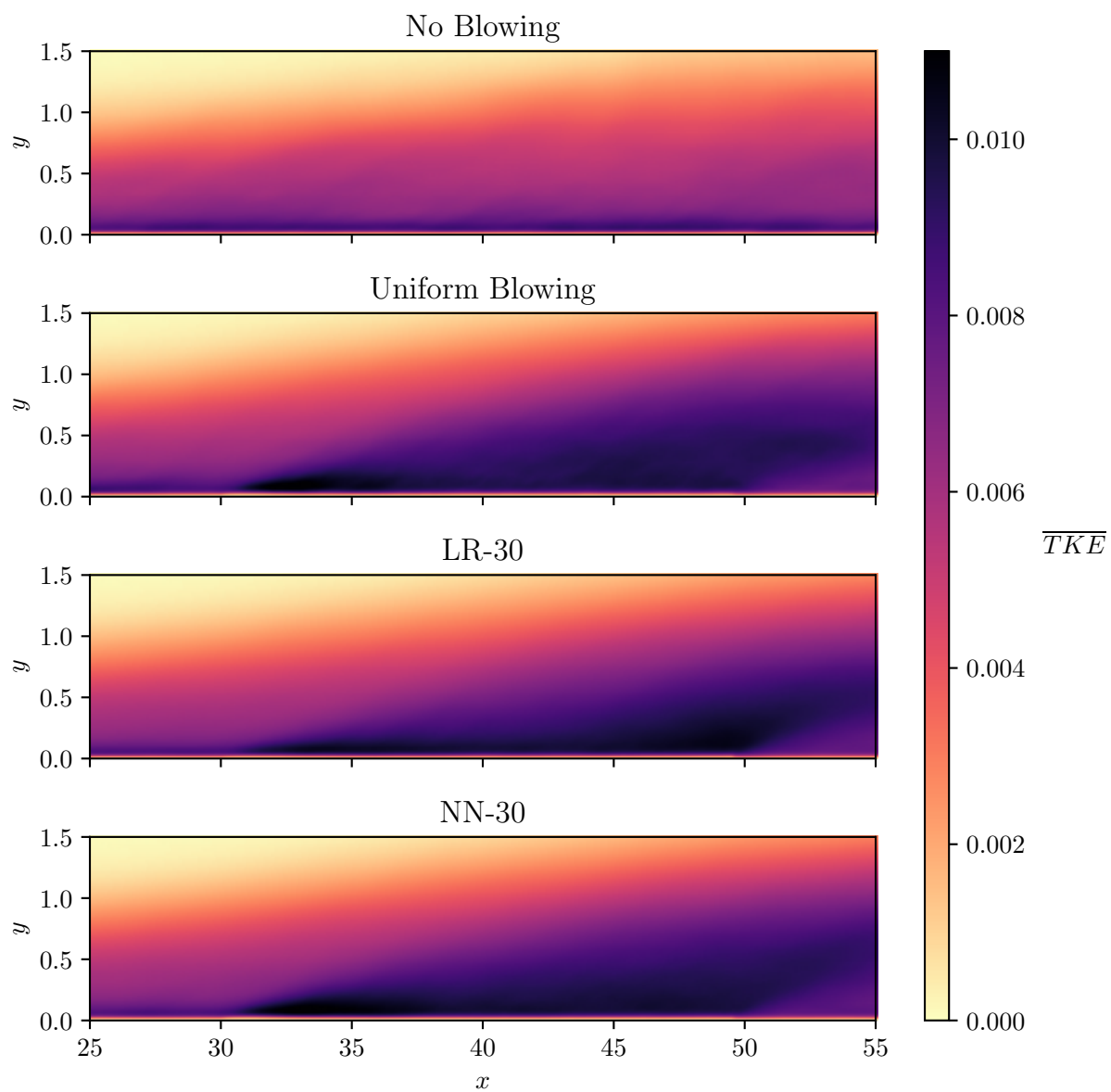


Figure 5.10: Time and spanwise-averaged TKE field.

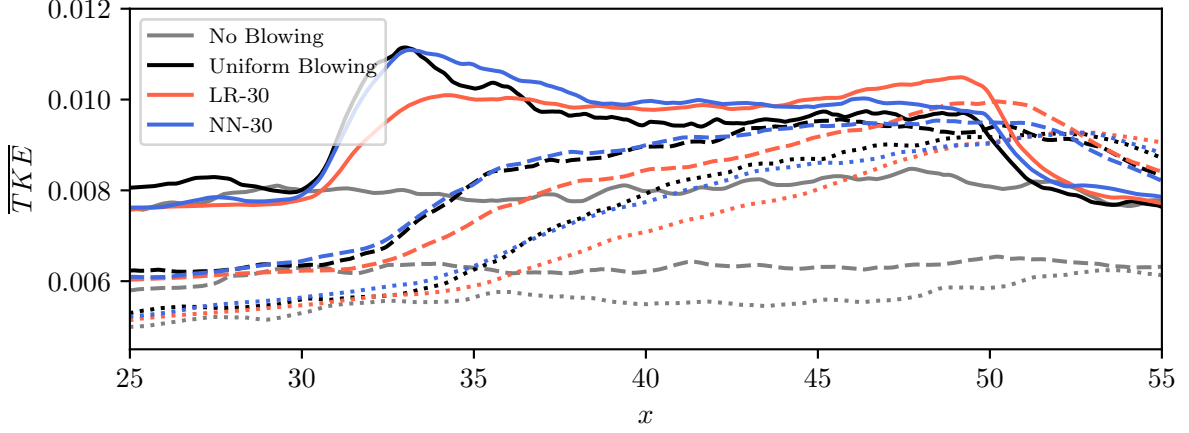


Figure 5.11: Time and spanwise-averaged TKE along the streamwise direction. Different linestyles correspond to different distances from the wall: $y = 0.1$ (—), $y = 0.3$ (---), $y = 0.5$ (.....).

the transpiration region in the LR-30 case, corresponding to the rapidly increasing blowing ratio. However, it fails to account for the gradual decrease in TKE observed in the NN-30 case in spite of the increasing blowing ratio. This is best seen by looking at the solid lines (TKE profile sampled closest to the wall) in Figure 5.11 and suggests the presence of another mechanism that counters the increased shear production due to the increase in the blowing ratio.

In light of this, it is hypothesised that the influence of the imposed coupling on the TKE distribution is the product of two competing factors: a reduction due to the response of the local blowing ratio to turbulent pressure fluctuations modulating the turbulence, and an augmentation due to the increasing blowing ratio along the streamwise direction. In the NN-30 case the blowing ratio increases slowly and, therefore, as the flow adjusts the attenuation of the pressure fluctuations due to the coupling is able to dominate, resulting in a decrease in TKE. In contrast, the blowing ratio in the LR-30 case increases extremely rapidly downstream such that the increased shear and therefore turbulent production dominates and the turbulent energy increases downstream. This modulation of the near-wall turbulent structures due to the implemented coupling is explored further in Section 5.5.

In addition to the TKE plots, the normal (diagonal component) Reynolds stresses are shown for all DNS cases in Figure 5.12. As the coupling is expected to impact the turbulence near the wall, a log scaling is applied to the x-axis to make differences in this

region more easily visible. Furthermore, an inner wall scaling was not used for the wall-normal distance due to the fact that the shear stress at the wall varies significantly between the different cases, as shown in Figure 5.9. This would therefore result in a shift of the apparent stresses relative to each other, complicating comparisons between the different cases.

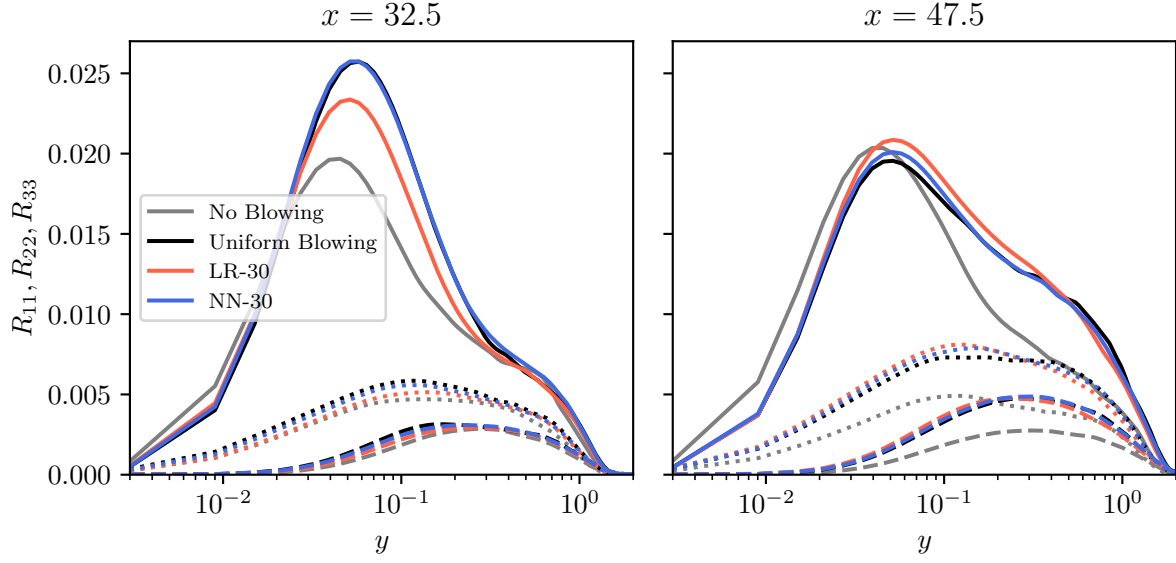


Figure 5.12: Reynolds normal stresses: R_{11} (—), R_{22} (---), R_{33} (.....).

The observed trends in the TKE fields discussed above are also present in all of the wall-normal profiles of the diagonal components of the Reynolds stresses, suggesting that the mechanism resulting in these changes to the turbulence of the flow is isotropic in nature, corroborating the theory that the increase in TKE is due to the shear produced by interactions between the increasing coolant injection and the main hot gas flow. The shear and resultant vortices due to this interaction are expected to produce velocity fluctuations in all directions, as observed.

5.4.2 Vortical structures

An alternative method of analyzing the impact of the implemented coupling on the turbulence is via visualisation of the large scale vortical structure. This was done using the λ_2 criterion, which is based on the decomposition of the velocity gradient tensor into its

symmetric and anti-symmetric components and can be used to identify vortex cores in compressible flows [85].

The symmetric component corresponds to the rate-of-strain tensor $S_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$, while the anti-symmetric components is the vorticity tensor $\Omega_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i})$. The sum of the squares of these two terms is related to the local pressure:

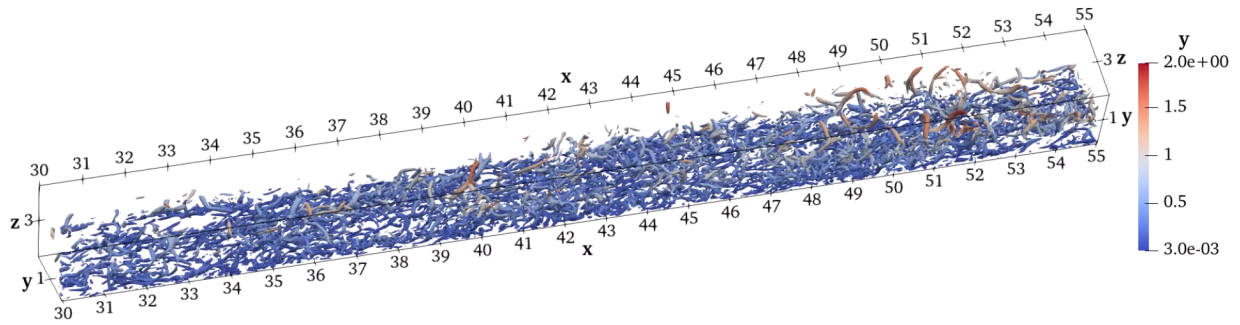
$$S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj} = -\frac{1}{\rho}p_{ij} \quad (5.5)$$

A vortex core is defined as the pressure minimum due to vortical motion, in which case at least two of the eigenvalues of the left-hand side expression in the above equation must be negative. If the eigenvalues are ordered such that $\lambda_1 \geq \lambda_2 \geq \lambda_3$, the regions of the flow where this is satisfied are easily determined by identifying the locations in the flow where $\lambda_2 < 0$. Using this method, the isosurfaces corresponding to $\lambda_2 = -2$ for the reference and $\phi = 24.6\%$ cases are shown in Figure 5.13 in order to identify the location of strong vortices. Only half of the domain along the spanwise axis is shown, and the isosurfaces have been coloured according to the normal distance from the wall.

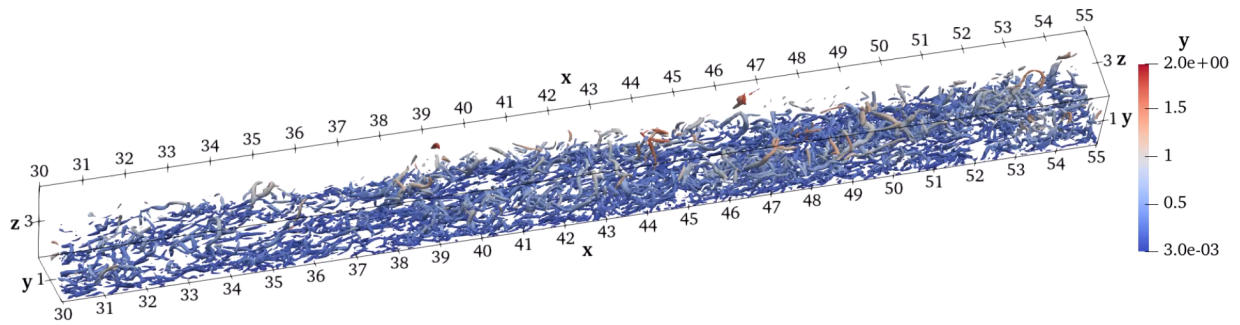
The observed trends, which were present in all extracted snapshots of each case, complement those found in the statistical analysis of the turbulence. At the beginning of the transpiration region there are fewer vortices in the LR-30 case than in the uniform blowing and NN-30 cases; however, the rapidly increasing blowing rate in the former case introduces further turbulence such that the vortex density appears roughly similar for all three cases in the middle of the transpiration region and is greatest in the LR-30 case at the end of the transpiration region. In contrast to this increase in TKE, however, the LR-30 case appears to have less large-scale vortices than the other two cases in the latter half of the transpiration region. This observation and its cause are explored further in the following section.

5.5 Surface pressure spectrum

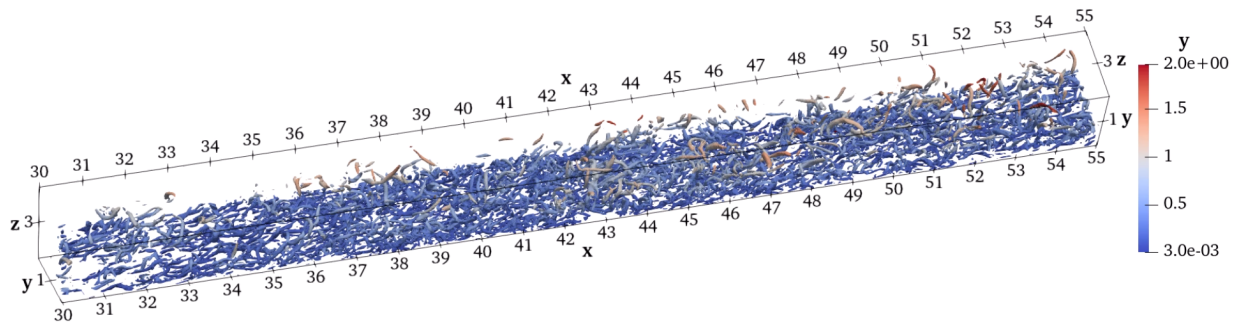
A driving consideration of the present work was the turbulent pressure fluctuations that naturally occur at the wall in a turbulent boundary layer. Figure 5.14 shows several instantaneous snapshots of the large scale pressure structures at the wall corresponding to the different cases considered. It is evident that coolant injection significantly modifies the near-wall pressure fluctuations.



(a) Uniform blowing



(b) LR-30



(c) NN-30

Figure 5.13: Visualisation of large scale vortices using isosurfaces of $\lambda_2 = -2$.

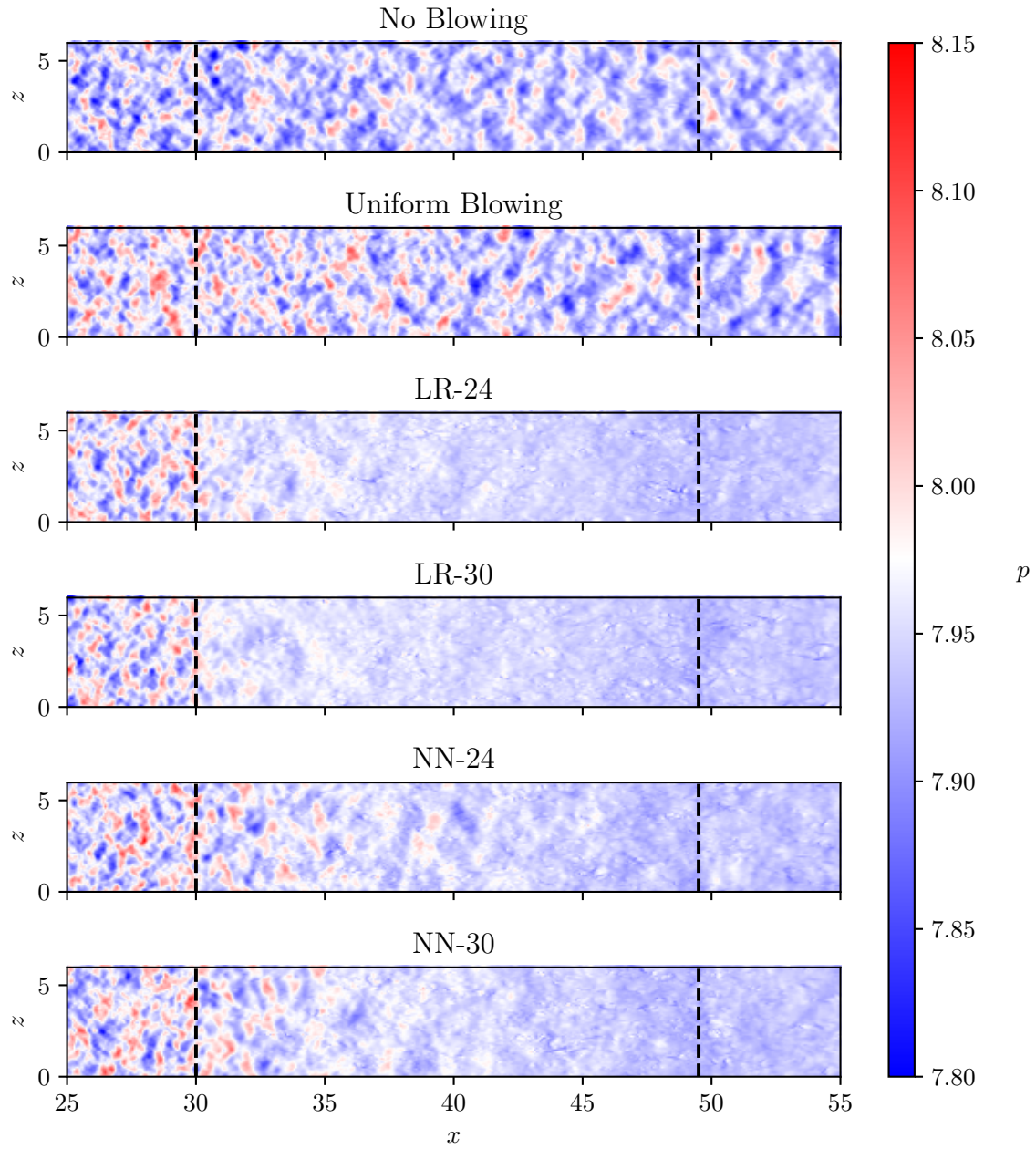


Figure 5.14: Instantaneous snapshots of large-scale pressure structures at the wall for all cases. The beginning and end of the transpiration region are demarcated by black dashed lines.

In the uniform blowing case, the introduction of coolant injection clearly increases the pressure fluctuations both inside and upstream of the transpiration region due to the increase in produced shear and turbulence. The pressure structures near the beginning of the transpiration region are also noticeably compressed as a result of the impingement of the incoming main flow on the wall-normal coolant flow. Further downstream these structures stretch back out and are similar in size to their counterparts in the no blowing case, albeit with increased magnitude due to the additional turbulence.

The same structure compression and increase in fluctuations seen in the uniform blowing case is also observed upstream of the transpiration region in the coupled cases, however once transpiration begins these large pressure structures are rapidly attenuated due to the coupling between the injection velocity and the pressure fluctuations. This attenuation occurs more rapidly in the cases coupled to the pore network with the higher porosity value ($\phi = 30.7\%$) as compared to their lower porosity counterparts. This is a consequence of the increased sensitivity of the porous medium with higher porosity and permeability to small changes in the wall-normal pressure gradient, as previously observed in Section 5.1.

Similarly, the LR-24 and LR-30 cases disrupt the large scale structures very rapidly due to the previously observed higher sensitivity to changes in the pressure signal at the wall as a consequence of not considering the lateral flow of fluid between neighbouring pores. In the cases coupled with the neural network, which do consider the influence of neighbouring pores, it takes longer for the large scale pressure structures to be attenuated after coolant injection begins.

It is important to note that the visible structures in Figure 5.14 are only the largest, high energy-containing structures. In order to gain an understanding of how the implemented injection coupling interacts with the smallest pressure scales, the PSD within the transpiration region was plotted for the reference cases as well as LR-30 and NN-30 in Figure 5.15.

The data output by the DNS cases was not sufficiently time-resolved to be able to directly calculate the frequency-based wall-pressure PSD within the frequency range of interest. Instead, a 2D Fast Fourier Transform (FFT) in the wavenumber domain was applied to the pressure signal at the wall for each case. This was done over several snapshots in order to obtain the time-averaged PSD of the pressure signal in the wavenumber domain at the plane of the wall. This was then converted to the frequency domain using Taylor’s hypothesis of frozen convection [70], which assumes a constant convection speed of the turbulent eddies in the flow direction (A.13). A mean convective speed of $U_c = 0.8U_\infty$ was used here, as suggested by Grasso et al. [32].

The analytical PSD was also calculated using the form of the Liepmann model [53]

presented by Grasso et al. (see Appendix A for the details including a derivation):

$$\varphi(\mathbf{k})_{pp}^{TM} = \frac{\rho_0^2 \tau_w^2 \overline{u'^2} \Lambda^5}{\pi \mu^2} \frac{k_1^2}{k + \gamma} \frac{8(1 + k^2 \Lambda^2) + 9(k + \gamma) \Lambda \sqrt{1 + k^2 \Lambda^2} + 3(k + \gamma)^2 \Lambda^2}{(1 + k^2 \Lambda^2)^{(5/2)} ((k + \gamma) \Lambda + \sqrt{1 + k^2 \Lambda^2})^3} \quad (5.6)$$

This model includes some highly limiting assumptions (including isotropic turbulence and an isothermal fluid) that mean it is not formally applicable to the DNS cases considered here, however it provides an interesting point of comparison to see how the observed PSD structures compare to the current understanding of wall pressure fluctuations over a rigid wall. The fluid density in a quiescent medium ρ_0 was set to unity (corresponding to the free-stream density), while the dynamic viscosity and wall shear stress were extracted from the no injection DNS case, averaged in time as well as over the span- and streamwise axes within the transpiration region. The integral length was selected as the mean boundary layer thickness in the transpiration region in the no blowing case, and the mean square velocity fluctuation was chosen as the mean of the that along each axis.

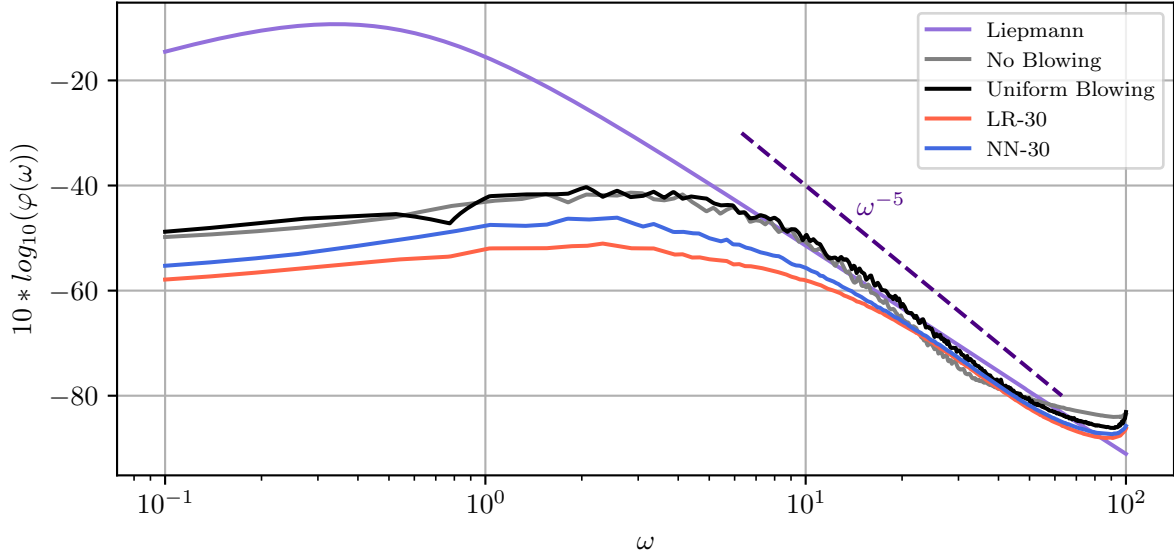


Figure 5.15: Power spectral density of the reference and $\phi = 30.7\%$ simulated cases compared to that predicted by the analytical Liepmann model.

The Liepmann model performed surprisingly well compared to the no blowing case in the high-frequency region in spite of its assumptions that make it non-applicable, suggesting

that the effects of the anisotropy of the turbulence and the flow temperature gradient can be adequately captured in this region by careful selection of the analytical model’s input flow parameters. As the frequency decreases the Liepmann PSD curve deviates from those obtained from the DNS cases, which begin to plateau and then decrease, however this is a known limitation of the model attributed to an overlap between the inner and outer scaling parameters in this region [32].

While the limited size and resolution of the DNS means that the PSD cannot be obtained for very low or high frequencies, two of the major regions identified by Goody [29] are clearly present in both the no blowing and uniform blowing cases, these being the ω^{-5} trend at high frequencies, and the near plateau at middling frequencies (corresponding to a slope of $\omega^{-0.7}$). The increased near-wall turbulence in the uniform injection case results in a slight increase of the associated pressure fluctuation PSD as anticipated, though due to the low blowing ratio the difference is relatively small. The impact is also somewhat offset by the formation of the coolant film, which pushes some of the structures responsible for the pressure fluctuations off of the wall.

By contrast, the PSD curves corresponding to the coupled cases differ significantly both from the analytical Liepmann model as well as the reference cases. The previously discussed attenuation of the large-scale pressure fluctuations, corresponding to lower frequencies, is clearly visible, with the effect being more significant in the LR-24 case than in the NN-24 case. These large structures account for a lot of the turbulent energy within the boundary layer and are expected to be particularly susceptible to decorrelation due to the variation in local injection velocity, causing them to be broken down and dissipated.

Meanwhile, small-scale pressure fluctuations contain very little energy and cover very few pores, making them more robust against decorrelation. This is observed in Figure 5.5, where the PSD of the coupled cases is very close to that of the uniform injection case at higher frequencies. While this too is an anticipated result, it is also important to keep in mind the inability of the neural network to capture and respond to the smaller-scale fluctuations, as discussed in Section 4.1. It is possible that some of the lack of attenuation of the high frequency structures in the neural network case is due to its difficulty responding to these structures, however the general trends discussed here are still expected to be relevant.

Chapter 6

Conclusions

The present work sought to explore the coupling between coolant injection and turbulent pressure fluctuations in the boundary layer in transpiration cooling systems. This was motivated by a gap in the existing literature on the significance of this coupling and its implications with respect to the nature of the turbulence over the porous wall. To this end, an algorithm was developed to indirectly couple high-fidelity direct numerical simulations of flow in a turbulent boundary layer over a flatplate with a pore-network model through the use of a shallow convolutional neural network.

Two versions of the neural network were trained to predict the flow velocity at individual surface pores of two simplified pore networks with different porosity values. The networks were able to successfully reproduce the major components of the coolant velocity distributions at the surface of the porous wall using images of the local wall-pressure distribution within a specified radius. Linear expressions were also obtained by performing a linear regression on the pore-network modelling data and coupled with the [DNS](#) to gain insight into the importance of the spatial correlations of the flow within the porous medium, which is accounted for by the neural network but not by the linear expression.

Overall, the cases coupled using the linear expressions were much more sensitive to pressure variations within the turbulent boundary layer, whereas the internal flow interactions between neighbouring pores that were accounted for in the neural network coupling significantly reduced the coupling effects. It is therefore important to consider the influence of neighbouring pores in the porous medium on each other when considering the coupling between the external turbulent boundary layer and coolant injection.

The coupling of the coolant injection with the near-wall pressure resulted in an inconsistent blowing ratio, being much smaller than in the uniform injection case at the beginning

of the transpiration region but surpassing it further downstream. A corresponding reduction in cooling effectiveness and coolant film thickness was observed near the beginning of the transpiration region, as well as a higher local skin friction coefficient compared to the constant blowing case. All of these parameters eventually recovered as the blowing ratio increased. The importance of the advection of heat away from the wall in the recovery of the cooling effectiveness was highlighted.

The reduced blowing ratio at the beginning of the transpiration region resulted in reduced shear and, therefore, turbulent transport of heat to the wall, mitigating the initial peak and then decline in cooling effectiveness observed in the uniform blowing case. The streamwise variation in blowing ratio, however, resulted in increased turbulent production further downstream. In the neural network coupled cases, this effect was offset by an attenuation of the turbulence due to interactions of the resultant pressure fluctuations with the local coolant injection. Consequently, in these cases the turbulent kinetic energy and Reynolds stresses decreased over the transpiration region in spite of the increasing blowing ratio. However, the cases coupled using linear expressions were unable to sufficiently attenuate the pressure fluctuations to offset the effect of the increasing blowing ratio, resulting in increasing turbulent kinetic energy downstream.

Finally, the influence of the imposed coupling on the pressure fluctuations at the porous wall was investigated. It was shown that the introduced coupling disrupts the high energy, large scale pressure structures while leaving the higher frequency fluctuations relatively untouched. In keeping with the trends observed in the other results, the cases coupled using the linear expressions attenuated the large scale fluctuations nearly instantaneously due to the high sensitivity of the coupling to small changes in pressure, while the fluctuations were able to persist slightly further downstream in the neural network cases.

While the work presented in this thesis demonstrates the importance of considering the coupling between the porous medium and external flows in transpiration cooling, it leaves many unanswered questions regarding this coupling. Possible future avenues of research to this end are as follows:

- An exploration of how pore connectivity and geometry within the porous medium influences the observed coupling behaviour, particularly the attenuation of the pressure fluctuations.
- An extension of the simulations to the supersonic/hypersonic regime, where most proposed use-cases for transpiration cooling are located. The large pressure gradients associated with shocks are likely to significantly impact the local coolant injection.

- The inclusion of transient/compressible effects in the modelling of the flow through the porous medium. The bulk properties of porous media have been shown to have a frequency dependence, which would likely change how the coupling interacts with the different scales in the turbulent flow.
- Improvements to the neural network coupled with the direct numerical simulations. This is particularly important to contextualise the current result in light of the inability of the present neural networks to interact with the smaller pressure structures.
- A revision of the current coupling implementation to allow for pore sizes and spacings different from the grid geometry of the direct numerical simulations, allowing for the consideration of high momentum injection jets.
- A revision of the no-slip condition at the wall within the transpiration region to account for porosity.
- Introducing a functional gradation of the porous wall properties to try to offset some of the observed coupling effects by controlling the streamwise blowing ratio distribution.
- Simulations of two-phase transpiration cooling, which is desirable due to the increased cooling potential when taking advantage of the latent heat of vaporisation but has been shown to be prone to instabilities.
- The use of different species for the coolant and hot gas, which is often the case in implementations of transpiration cooling systems. It may be interesting to see how the coupling effects change when the two fluids have different properties, as well as the impact of the changes to the coolant film structure and the near-wall turbulence on the distribution of coolant in the boundary layer.

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Appendices

Appendix A

Derivation of Analytical Models for the Turbulence Mean Shear Interaction Term

The following is a summary of the main steps in the derivation of the main turbulence mean shear interaction term analytical models discussed in section 2.2.1 as given by Grasso et al [32]. While this derivation is not the author's work, it is nonetheless outlined in this thesis as it provides important context as to the limiting assumptions of these models and why they cannot be indiscriminately applied to the transpiration cooling case. Herein, the follow Fourier transform convention is used:

$$\hat{f}(k) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx \quad (\text{A.1})$$

Equation (2.1) is taken and, assuming homogeneous turbulence in planes parallel to the wall, Fourier transformed in space over the streamwise and transverse directions, (x_1, x_3) . Assuming that the only mean velocity gradient that does not vanish is that of the stream-wise mean velocity component in the wall-normal direction then yields a modified variation of the Helmholtz equation:

$$\frac{\partial^2 \hat{p}'(\mathbf{k}, x_2)}{\partial x_2^2} - k^2 \hat{p}'(\mathbf{k}, x_2) = -2\rho i k_1 \frac{\partial U_1}{\partial x_2} \hat{u}'_2(\mathbf{k}, x_2). \quad (\text{A.2})$$

The PSD of the wall pressure fluctuations can then be expressed as the product of

Equation (A.2) by its complex conjugate which, in conjunction with taking the ensemble average, results in the following equation:

$$\varphi_{pp}^{TM}(\mathbf{k}) = 4\rho_0^2 \iint_0^\infty \frac{k_1^2}{k^2} e^{-(X_2+X_2^*)k} \frac{\partial U_1(X_2)}{\partial x_2} \frac{\partial U_1(X_2^*)}{\partial x_2} \varphi_{22} dX_2 dX_2^* \quad (\text{A.3})$$

The different analytical models examined by Grasso et al. [32], namely the Gaussian, von Kármán, Liepmann, and RDT models, can all be expressed using Equation (A.3) and vary in their expression of the vertical velocity cross-spectral density, φ_{22} , according to their corresponding longitudinal correlation function.

The Gaussian model is based on the corresponding Gaussian longitudinal correlation function:

$$F(r) = \exp\left(\frac{-r^2}{L^2}\right) \quad (\text{A.4})$$

$$L = \frac{2}{\sqrt{\pi}} \Lambda \quad (\text{A.5})$$

wherein Λ is the longitudinal integral length scale of turbulence. This results in the simplest expression for the cross-spectral density of the vertical velocity fluctuations of all the considered analytical models, given in equation A.6. It should be noted, however, that the accuracy of this model has been put in doubt by multiple works.

$$\varphi_{22}(\mathbf{k}, x_2, x_2^*) = \frac{\sqrt{u'^2(x_2)u'^2(x_2^*)}L^4}{16\pi} k^2 \exp\left(-\frac{L^2 k^2}{4} - \frac{(x_2 - x_2^*)^2}{L^2}\right) \quad (\text{A.6})$$

Meanwhile, the latter three models are all based on the generalised von Kármán correlation function:

$$F(r) = \frac{1}{2^{\nu-1}\Gamma(\nu)} \left(\frac{r}{l}\right)^\nu K_\nu\left(\frac{r}{l}\right) \quad (\text{A.7})$$

where the characteristic length

$$l = \frac{\Gamma(\nu)}{\sqrt{\pi}\Gamma(\nu + 1/2)} \Lambda. \quad (\text{A.8})$$

The resulting cross-spectral density of vertical velocity fluctuations is therefore

$$\varphi_{22}(\mathbf{k}, x_2, x_2^*) = \frac{\sqrt{u_2'^2(x_2)u_2'^2(x_2^*)l^2\tilde{k}^2}\zeta^{\nu+2}}{\Gamma(\nu)\pi 2^{\nu+1}(1+\tilde{k}^2)^{\nu+2}}K_{\nu+2}(\zeta) \quad (\text{A.9})$$

where

$$\zeta = \frac{||x_2 - x_2^*||}{l}\sqrt{1 + l^2k^2}. \quad (\text{A.10})$$

Equation (A.9) is then integrated for all values of x_2 and x_2^* , including the case where $x_2 = x_2^*$ wherein the associated Bessel function $K_{\nu+2}(\zeta)$ is undefined. It is therefore important to make note of the generalised auto-spectrum as well:

$$\varphi_{22}^a(\mathbf{k}, x_2) = \lim_{x_2 \rightarrow x_2^*} \varphi_{22}(\mathbf{k}, x_2, x_2^*) = \frac{\nu(\nu+1)\overline{u_2'^2(x_2)}l^4k^2}{\pi(1+l^2k^2)^{\nu+2}} \quad (\text{A.11})$$

The von Kármán, Liepmann, and RDT models are constructed from Equation (A.9) using different values for the parameter ν . The Liepmann model uses $\nu = 1/2$, while the von Kármán and RDT models use $\nu = 1/3$ and $\nu = 7/6$, respectively.

The vertical velocity cross-spectral density assumes isotropic turbulence. In order to account for anisotropic turbulence φ_{22} can optionally be multiplied by a value α given as the ratio of streamwise to transverse integral length scales:

$$\varphi_{22}^{NI}(k_1, k_3, x_2, x_2^*) = \alpha\varphi_{22}(\alpha k_1, k_3, x_2, x_2^*) \quad (\text{A.12})$$

α is expected to be a function of both the streamwise wavenumber k_1 and distance from the wall x_2 . Grasso et al. [32] suggested that as the chosen expression will be subject to a double integration over the distance from the wall, it is only with the variation of α with respect to this distance that is of importance. Following further examination by Grasso et al. in the same work it was furthermore determined that at the mid-range frequencies considered α had no significant effect on the resulting PSD, which was attributed to the fact that at this range it is determined by the isotropic or near-wall portion of the boundary layer. Consequently the turbulence will be assumed isotropic and α will be omitted (set to unity) in subsequent equations.

The resulting wavenumber spectrum given by Equation (A.3) can then be converted to the frequency domain by assuming that the convection speed of the wall pressure fluctuations is constant, in which case only the streamwise wavenumber contributes to a given

frequency and is given by $k_c = \omega/U_c$. This allows a final expression for the wall pressure fluctuations in the wavenumber domain to be given as follows:

$$\varphi_{pp}(\omega) = \frac{\int_{-\infty}^{+\infty} \varphi_{pp}^{TM}(k_c, k_3) dk_3}{U_c} \quad (\text{A.13})$$

A.1 Liepmann model

The above was a general derivation valid for all of the analytical models considered by Grasso et al. [32]. In order to obtain a final expression for the Liepmann model, which was used as part of the original work of this thesis, some additional work must be done. Firstly, it is assumed that the turbulent kinetic energy doesn't vary with vertical distance from the wall and is isotropic, such that

$$\overline{u'^2}(x_2) = \overline{u'^2}. \quad (\text{A.14})$$

Secondly, the streamwise velocity gradient is expressed as a simple exponential function with respect to the vertical distance from the wall:

$$\frac{\partial U_1}{\partial x_2} = \frac{\tau_w}{\mu} e^{-\gamma x_2} \quad (\text{A.15})$$

wherein γ is required to be within the same order of magnitude as $1/\Lambda$ such that Equation (A.15) is a decent approximation of the law of the wall.

The substitution of assumptions (A.14) and (A.15) into (A.9) with parameter $\nu = 1/2$ results in a function that when subsequently substituted into Equation (A.3) yields the PSD curve of wall pressure fluctuations in the wavenumber domain:

$$\varphi(\mathbf{k})_{pp}^{TM} = \frac{\rho_0^2 \tau_w^2 \overline{u'^2} \Lambda^5}{\pi \mu^2} \frac{k_1^2}{k + \gamma} \frac{8(1 + k^2 \Lambda^2) + 9(k + \gamma) \Lambda \sqrt{1 + k^2 \Lambda^2} + 3(k + \gamma)^2 \Lambda^2}{(1 + k^2 \Lambda^2)^{(5/2)} ((k + \gamma) \Lambda + \sqrt{1 + k^2 \Lambda^2})^3} \quad (\text{A.16})$$