# Development of a Risk Ranking System for Prioritizing Asset Maintenance Decisions

by

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## Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

**Abstract**

This thesis presents a modified approach to prioritize asset maintenance decisions by evaluating the overall risk rating of engineering systems. Traditional methods rely on subjective assessments by experts, which potentially lead to greater subjectivity and inconsistency in risk prioritization, thus requiring improvements. The thesis performs a comparative analysis of existing risk prioritization techniques to understand the challenges in ranking systems.

Analytical Hierarchical process (AHP) and fuzzy logic are proposed to develop a risk ranking system. AHP is employed to compute the weights of multiple criteria, providing a structured framework for decision-making and enabling systematic prioritization of system components. The Mamdani Fuzzy Inference System is integrated to manage the inherent uncertainty and imprecision in assigning ranks.

The proposed AHP-Fuzzy model is applied to a plant aging management problem in the nuclear industry with various maintenance tasks, demonstrating its effectiveness in decision-making. The risk rating distribution and sensitivity analyses are studied. The results indicate that integrating AHP and Fuzzy Logic improves decision-making by effective risk prioritization.

The thesis contributes to the field of engineering management by providing practical, actionable strategies for enhancing risk management practices to ensure the safety of engineering systems.

## Acknowledgments

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## Dedication

To all those whose presence in my life has left an indelible mark.

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# Chapter 1

# Introduction

## 1.1 Background and Motivation

Risk assessment and decision-making methods have become key components of infrastructure asset management, especially with the increasing demand for sustainable growth and the need to address aging infrastructure. This thesis explores the prioritization of risks based on their criticality to improve risk management by examining the factors influencing a system's performance beyond its immediate effects on production and safety.

research on risk assessment for various industries like gas pipelines [1], bridges and tunnels [2] [3], and railway transportation [4], among many others, is available. However, there are very few research papers on risk assessment and risk prioritization in the nuclear industry.

The Canadian Nuclear Industry is experiencing significant growth due to rising demands for power and clean energy. Due to the complex nature of the industry, there are engineering systems that run continuously, and these systems must be maintained without any downtime. The assets in each plant are assessed and ranked based on their criticality. Understanding the criticality of the system based on various criteria is crucial for forming a reliable ranking system. Criticality ranking systematically assigns a criticality rating to assets based on their potential risks and ranks them accordingly [5]. The Criticality Ranking (CR) process assesses the risk associated with each component and system within the plant based on the likelihood of failure and the severity of its potential consequences [6]. This strategy helps reduce operational disruptions by optimizing maintenance activities and resource planning. For example, assets critical for safety and production receive higher priority in maintenance schedules to reduce the risk of costly shutdowns and closures [7].

In most infrastructure risk management practices, ranking assets is predominantly subjective and relies heavily on insights from subject matter experts (SMEs). Traditional methods and practices in risk management for criticality ranking present opportunities for improvement in assigning ranks. Challenges include uncertainties in evaluations, intricate interdependencies among factors, biases in subjective assessments, and the dynamic evo-

lution of risk profiles due to technological advancements, regulatory shifts, and external factors.

The risk ranking process can be used not only to manage engineering systems but also to be adapted in various applications, as listed below.

**Site Selection for New Facilities:** These criteria are essential for evaluating potential locations for new nuclear facilities, such as reactors or waste disposal sites. Safety, environmental impact, and regulatory compliance are of paramount importance.

**Technology and Vendor Selection:** When choosing technologies or vendors for new installations or upgrades, considerations such as financial impact, regulatory compliance, safety, and security are critical.

**Work Order Approval:** Day-to-day work orders should be prioritized and assigned promptly with high reliability to reduce risks in the long run and effectively manage the asset.

**Investment Decisions:** Financial aspects, along with generation capacity, are essential for decisions regarding investments in the industry, whether for expansion, modernization, or research and development.

**Operational Strategies:** Decisions about day-to-day operations must consider safety, environmental impacts, regulatory frameworks, and the financial implications of operational choices.

**Decommissioning Plans:** Deciding when and how to decommission a plant involves safety, environmental, and regulatory considerations, alongside financial costs and security measures.

The above concept is taken as the basis for the development of an asset prioritization model.

## 1.2 Objective

The objectives of the thesis are outlined below:

- Investigate methods to develop a risk rating system for prioritizing maintenance decisions.
- Assess the efficacy of the model in its ability to prioritize risks on a daily basis by analyzing its distribution and sensitivity.

## 1.3 Thesis Outline

This thesis comprises six main chapters and three appendices. Chapter 1 discusses the background and importance of risk prioritization, along with the objectives and thesis

outline. In Chapter 2, the literature review, a summary of different risk prioritization methods is provided, along with the concepts required for the decision-making framework, and the proposed risk ranking system for nuclear power plants is introduced. Chapter 3 discusses the concept of AHP and its application to the nuclear risk ranking system for evaluating criterion weights. Chapter 4 discusses the coverage and sensitivity analysis of the nuclear risk ranking system using the AHP model and compares it with the Delphi risk ranking system. In Chapter 5, the AHP risk ranking system is integrated with fuzzy logic to finalize the proposed methodology. Finally, in Chapter 6, the thesis is concluded with a summary of the findings, limitations of the model, and its future scope. The categories and sub-categories used in the asset management problem are defined in Appendix A. Appendix B provides the types of fuzzy rules available for the Mamdani Fuzzy Inference System. Appendix C includes the developed MATLAB codes to perform the fuzzy modeling and validation.

# Chapter 2

# Literature Review

## 2.1 Risk Management

### 2.1.1 Background

The concept of risk and risk assessment has a long history, dating back 2400 years. The Institute of Risk Management defines risk as *"A combination of the probability of an event and its consequence, where the consequences can range from positive to negative"* [8]. There can be various types of risks like compliance risks, hazard risks, uncertainty risks, and opportunity risks. The impact of the risks and their effects on an organization must be studied to make informed decisions, which leads to risk management. The use of risk assessment as a scientific tool has flourished in the last 40 to 50 years. Risk management has evolved significantly over time due to exponential technological advancements and the increasing complexity of engineering systems. The Institute of Risk Management defines risk management as *"A set of activities within an organization that are undertaken to deliver the most favorable outcome and reduce the volatility or variability of that outcome"* [8].

In the early stages, asset assessment primarily focused on maximizing operational efficiency and safeguarding physical life. With such limited scope, the initial asset management practices focused on repairing faults and replacing failures rather than preventing unfavorable events [9]. With a steady evolution, the shift toward a more systematic approach to risk management became prominent with the introduction of preventive maintenance (PM), which paved the way to integrate risk management into asset management practices. The shift toward a systematic approach started the transition from a reactive to a proactive approach in asset management [10].

Risk management gained traction as organizations recognized the importance of understanding the possible risk profiles of any system and its components to operate a business effectively. The focus on the study of risks further paved the way for reliability-centered maintenance like Reliability, Availability, and Maintainability (RAM) analysis due to the increasing demands for system reliability and cost efficiency in various industries [11].

With the advent of the 21st century, various new methodologies are used to minimize risks through artificial intelligence by incorporating various algorithms that optimize the preferred metrics in an organization.

### 2.1.2 Asset Hierarchy and Criticality

For effective risk management, developing and maintaining an asset hierarchy and a master equipment list is essential to understand the workflow and overall workings of the system/industry. Asset Hierarchy is the process in which the organization's assets are categorized in a tree-like diagram to understand the interdependencies between assets at different levels. This process is essential for better risk management, maintenance planning, and asset tracking. Understanding just the structure is not sufficient; information on all the available assets of the organization, including their specifications, locations, conditions, maintenance history, and more, is essential. Thus, the master equipment list is used in the nuclear industry to streamline that process.

The Asset Hierarchy provides the basic knowledge about the assets. In addition, determining the importance of prioritizing assets to perform various operations is important through the process of Asset Criticality Ranking. Asset Criticality Ranking (ACR) is a formal process with specified criteria used to determine the importance of the asset to the overall function and process to generate the desired system's performance and output [12]. The same is true for risk prioritization and ranking various operations through criticality ranking.

However, recent trends show that the criteria factors include not only performance and output but also environmental impact, safety, regulations, and other factors. Thus, the factors are an ever-evolving list based on the system.

### 2.1.3 Risk Prioritization through Criticality Ranking

Risk prioritization through Criticality Ranking focuses on identifying, assessing, and arranging risks in a critical order from the perspective of engineers and operations. Various methods are used to rank the risks, with popular methods being factor-based and parameter-based statistical modeling [6].

#### Factor-based Ranking

The factor- and criteria-based approach applies importance factors to one or more criteria for every system based on a predefined definition agreed upon by subject matter experts and ranked based on an accepted mathematical calculation. This method is a qualitative-based risk analysis method.

#### System and Equipment Reliability Prioritization (SERP)

The SERP analysis ranks components or failure modes based on a set of criteria with equal importance. It calculates a criticality value using the square root of the sum of the squares of these criteria, as shown in equation 2.1, which reflects the overall severity of a component's failure potential [6].

$$\text{SERP} = \sqrt{\sum_{k=1}^{n}(CF_k^2)} \tag{2.1}$$

Where the terms are defined as:

1. $CF$: Weight of the criterion.
2. $n$: Total number of criteria.
3. $k$: Non-negative integer

### Prioritization Matrix (P-Matrix)

The P-Matrix extends the SERP by incorporating variable weights for each of the desired criteria to reflect its varying effect on the output function of the system. This method calculates a severity score by summing the products of each criterion's score and its respective weight, as shown in equation 2.2 [6].

$$\text{P-Matrix} = \sum_{k=1}^{n}(CF_k \times \text{Importance of } CF_k) \tag{2.2}$$

It is important to note that the factor-based approach should be handled properly to avoid double counting in the criteria factors. This method is highly relied upon for its flexibility to customize the criteria by combining and adding them. Additionally, this is the easiest and fastest method for criticality ranking and is thus widely followed.

### Parameter-based Ranking

Parameter-based ranking applies a system's or component's capacities with failure data in a statistical model to calculate and rank criticalities. This method is a quantitative risk analysis method. Various models such as reliability, availability, and maintainability (RAM) studies, discrete event simulators, and Failure Mode and Effect Analysis (FMEA) fall under this category. This approach requires technical expertise (usually external), which makes it more time-consuming and expensive.

The parameter-based approach is incorporated through various methods like the PageRank algorithm [13] to integrate heterogeneous information expression structures, the fuzzy VIKOR method [14] to handle uncertainty in expert judgments by finding a compromise solution that is close to ideal, and fault tree analysis (FTA) with fuzzy logic [15] to handle a lack of historical data and complex interactions between various risk factors.

## 2.2 Current Industry Practices

### 2.2.1 Delphi Method

The Delphi method is believed to have originated in the 1950s from the work of Helmer, Dalkey, and Rescher at the RAND Corporation to forecast the probability of an event [16]. However, other sources say that the first usage was in horse race predictions [17]. According to Turoff and Linstone (2002) [18], *"Delphi may be characterized as a method for structuring a group communication process so that the process is effective in allowing a group of individuals, as a whole, to deal with a complex problem"*.

This method depends on a group of experts who complete questionnaires in multiple rounds, making it a systematic and interactive approach to forecasting. After each round, a facilitator anonymously summarizes the experts' predictions and their rationales or can be programmed to be summarized based on predefined characteristics for the communications. The former process allows the experts to refine their subsequent responses, considering their peers' perspectives. The process of the Delphi method is explained as a flowchart in Figure 2.1 [19].

The application of the Delphi method in a large setting, such as the nuclear sector, is very useful as it facilitates an iterative process of evaluating the multitude of risks associated with nuclear power plant operations. The Electrical Power Research Institute (EPRI) developed a methodology for plant aging management that combines a range of impact factors with their likelihood of failure within the framework of the Delphi method.

Under this methodology, experts are first presented with the various impact factors: Nuclear Safety, Production Loss, Conventional Safety, External Environment, and Obsolescence. They are also given the corresponding likelihood of failure factors like the component's environment and equipment condition. These experts, typically drawn from diverse fields within the nuclear industry, independently score each factor based on their expertise and understanding of the operational context and provide a score for each criterion (usually from 1 to 9). The final score is obtained from the equation below.

$$\text{Risk Rating} = \text{Impact} + \text{Likelihood} \tag{2.3}$$

The Delphi method is useful in achieving consensus among experts, yet it comes with notable limitations.

A primary concern lies in the method's tendency to oversimplify risk ranking by merely adding the factors of impact and likelihood. This approach can skew the true importance of specific risks. For instance, a piece of equipment in poor condition with a low impact score might be ranked lower than it should be, especially when compared to high-impact equipment in better condition. This oversimplification can lead to flawed decision-making, as it underestimates the significance of more immediate risks.

Furthermore, while valuable, the Delphi method's reliance on expert input introduces subjectivity. While subject matter experts bring invaluable insights, their assessments

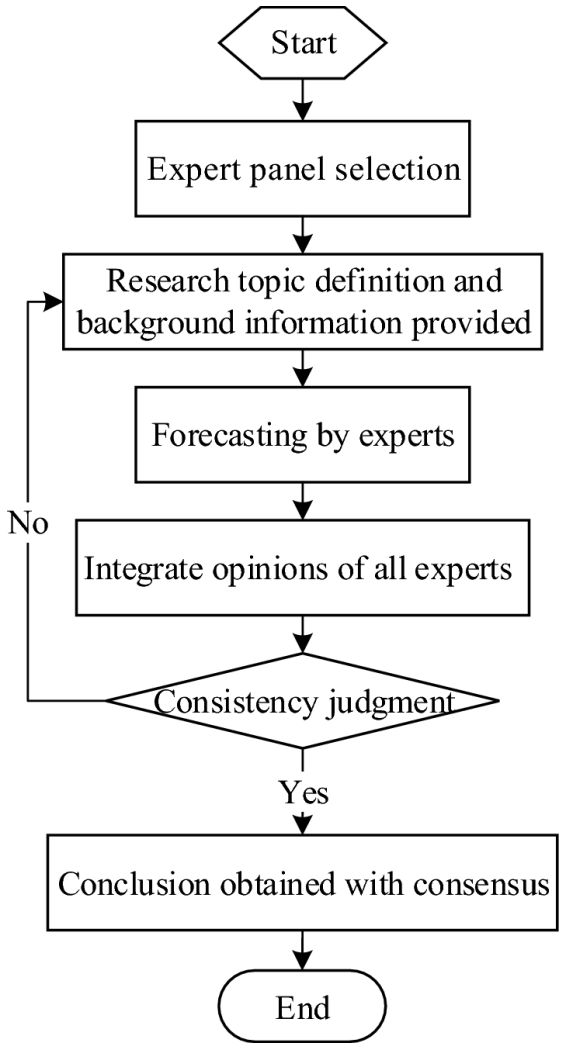



Figure 2.1: Procedure of the Delphi Technique

are not immune to personal biases [18], which can inadvertently influence the risk ranking process. The assumption that these evaluations are entirely objective overlooks the inherent subjectivity that any individual, including experts, might bring to the process. If not careful, this process inherently creates errors when ranking the risks.

### 2.2.2 Risk Matrix

The Risk Matrix, or Risk Assessment Matrix/Risk Ranking Matrix, is a crucial risk management tool used to evaluate and prioritize risks. It functions by assessing two main factors: the likelihood of a risk occurring and the severity of its impact. This method is widely adopted across various industries for identifying risks and devising appropriate response strategies.

According to Garvey and Lansdowne (1998) [20], the risk matrix was first devised by the

acquisition re-engineering team at the Air Force Electronics Systems Center (ESC) in 1995. However, the concept of the risk matrix, as used in modern risk management and decision-making, has various inventories and points of origin. It evolved as a tool for risk assessment in various fields, including engineering, project management, and military planning.

Academically, the matrix is recognized as a framework for categorizing and ranking risks by comparing their probability of occurrence against their impact severity. This approach is supported by standards from institutions like the Project Management Institute (PMI).

The process involves identifying potential risks and categorizing each based on its likelihood and impact, usually on predefined scales. These risks are plotted on the matrix, divided into zones (low, medium, high) to indicate overall risk levels, and followed by decision-making where strategies such as mitigation, transfer, acceptance, or avoidance are chosen based on the matrix's prioritization.

Applying a Risk Matrix is integral to maintaining safety and operational efficiency. Distinct from the Delphi method, this matrix provides a systematic framework to quantify and manage risks effectively.

The Risk Matrix approach begins in a nuclear setting by categorizing key impact areas like Nuclear Safety, Production, Environmental, and Regulatory compliance. Each category is critically evaluated, and the combined impact is measured on a 10-point scale. Subsequently, historical data analysis or subject matter expert (SME) opinion assesses the likelihood of these risks materializing.

The core of the Risk Matrix methodology, which differentiates it from the Delphi method, lies in its score calculation process, where the final risk score is determined by multiplying the impact score by the likelihood rating. This multiplication yields a quantifiable risk level, encapsulating the risk's severity and probability. This scoring system enables a more objective and measurable comparison of diverse risks, aiding in prioritizing mitigation strategies.

$$\text{Risk Rating} = \text{Impact} \times \text{Likelihood} \tag{2.4}$$

A key issue with the Risk Matrix is its approach to quantifying impact. The matrix uses a single 10-point scale to measure impact, combining factors such as nuclear safety, production, environmental impact, and regulatory compliance. This method of aggregating diverse impact types into a singular scale can lead to a loss of specificity. For instance, a high score in regulatory compliance might carry different implications than a high score in nuclear safety, yet these nuances can be lost in a uniform scoring system. The lack of granularity in scoring becomes particularly problematic when dealing with the extensive scope of the nuclear plant management portfolio.

The oversimplified scoring system might not provide enough resolution to distinguish between items of varying Criticality and urgency effectively.

### 2.2.3 Critical Impact Method

A new method that combines the advantages of the Delphi method and the risk matrix method was introduced to enhance the prioritization process of plant aging management. The proposed Critical Impact Method by OPG (Ontario Power Generation) also addresses some of the limitations of the above-discussed methods.

The complex nuclear systems in the nuclear power plant are analyzed by the nuclear experts, and each system and its components are assessed in terms of their impact, which consists of a comprehensive set of criteria such as Criticality Code (CC), Production Code (PC), Cost, Conventional Safety, Environmental (CCSE) Code, Integrated Safety Review (ISR), List of Safety Related Systems and Functions (SRSL), and Forced Loss Rate (FLR). Concurrently, the likelihood of failure based on the data and SME opinions is calculated regarding the system/component's Condition Assessment, Environment, and Duty Demand. The source of the above assessment is based on well-established databases and reviews, which ensures that the risk assessment is thorough and grounded on a solid background.

The risk score is calculated similarly to that of the risk matrix.

$$\text{Risk Rating} = \text{Impact} \times \text{Consequence} \tag{2.5}$$

This method provides a quantifiable metric to guide the decision-making process by ensuring a detailed analysis of impacts to reduce the loss of specificity. It also encompasses the severity and probability of the risk promptly.

This method highlights the need for assigning relative weights to these criteria. The method, as described, relies on aggregating individual scores to determine overall risk. However, this additive approach must reflect the varying importance levels of different criteria. For example, a high score in nuclear safety may be more critical than an equivalent score in production loss. However, if treated equally, the final risk calculation could misrepresent the actual risk. This lack of weighted scoring can lead to inconsistencies and variations in decision-making.

### 2.2.4 Limitations of Current Approaches

The qualitative analysis has various disadvantages, as listed below.

- **Subjectivity**

  Since the analysis depends on inputs from subject matter experts (SMEs), their perspectives and technical interpretations of a component/system can significantly influence the analysis, leading to biased results. It also paves the way for inconsistent results as every SME carries their interpretation of risk and risk factors.

  **So, what decision-making methods can we use to reduce subjectivity and obtain the importance factor? How can we deal with the uncertainty that affects the reliability, validity, and generalization of the findings?**

- **Coverage and Specificity of the Ranking System**

  Every objective and analysis should have a defined scope and level of detail required from the data. The level of detail (granularity) determines whether it is a fine-grained or coarse-grained analysis.

- **Time and Resource Sensitivity**

  Although qualitative analysis is less time-sensitive than quantitative analysis, it still faces significant obstacles in data collection and arriving at a consensus. Additionally, the number of subject matter experts and other project resources are limited at the site and must be optimized.

Several fundamental flaws were discussed by Cox and Louis (2008) [21] in the traditional model of determining the impact and consequence (threat, vulnerability, and consequence as noted in their paper). The paper highlighted the traditional model's inability to accommodate correlations between the factors and the simplifications, which could lead to inaccurate prioritization and misallocation of valuable resources. It also extends to the traditional model's inefficiency in handling the uncertainty and bias arising from subjective assessments. Traditional models fail to be adaptable and versatile to new data and thus might be rigid in some factors. Due to this, advanced decision-making models should be integrated into these practices for better accuracy in prioritization.

## 2.3 Methods to Improve Risk Prioritization

In risk management, various tools are available to understand the variables of complex systems and manage them based on various analyses and evaluations. One popular decision support tool to prioritize assets based on risk by integrating various technical information and stakeholder values to provide effective decisions is the Multi-Criteria Decision Making (MCDM) method. There are various methods and a wide range of applications for MCDM.

Understanding the system's basic structure and the risk calculation is important. Critical infrastructure is protected through a rank-ordered list based on the likelihood of occurrence of certain events, the component's vulnerability, and its importance to the system's performance [22]. The same can be broadly applied to the most significant critical infrastructure to enhance efficiency and increase the probability of a safe and sound structure/system. Other risk-based methodologies have been developed to facilitate the prioritization of risks. However, everything depends on understanding the possible threats/impacts on the system, its importance, and its likelihood of failure/occurrence to determine the risk. Determining the factors and the events that can cause the failures through an extensive survey combined with expert judgments is critical in making well-informed infrastructure management decisions.

Risk management in critical infrastructure is difficult due to the complex structure and the ever-evolving regulations and priorities. Prioritization of assets typically utilizes performance measures related to asset characteristics, operations, and output efficiency. Environmental criteria are paramount in asset criticality due to recent events in climatic change

and their adverse effects on various assets. A security index and rating as a function of likelihood and magnitude, resilience of the asset, and the resulting consequence in the event of a threat are helpful in investment evaluation involving multiple assets and multiple performance criteria [23]. Integrating various factors, including climate forecasts and asset criticality, to prioritize maintenance and investment decisions through a scenario-based multi-criteria decision framework is critical [24]. The importance of prioritizing assets through various criteria, including their vulnerability to climate change, is part of adaptive asset management and is widely followed now in various infrastructure management. J. Bödefeld and F. Marsili (2003) [25] also discuss how aging and neglected maintenance are critical challenges in any complex infrastructure system. It is crucial to integrate key performance indicators (KPIs) with the multi-criteria framework to effectively prioritize the needs of any infrastructure system by considering various factors such as condition, level of service, sustainability, and risk [26]. Strauss (2022) [27] also stresses the importance of using key performance indicators like reliability, safety, and environmental performance to optimize operation and maintenance strategies.

The optimization of decision-making is carried out by various methods like linear and quadratic models [28], fuzzy linear programming [29], Analytical Hierarchical Process (AHP) [30], Multi-Attribute Utility Method (MAUT), Simple Multi-Attribute Rating Technique (SMART), and Weighted Influence Non-Linear Gauge Systems (WINGS). These methods are used as strategic tools for prioritizing risks and deciding maintenance/work operations on various types of infrastructure [25].

A versatile decision-making method can prioritize the weights of criteria factors effectively and reduce the subjectivity associated with qualitative analysis. Of many decision-making methods, the AHP method is attractive due to its structured decision-making framework backed by mathematical representation to determine the importance of various criteria factors. The method uses relative importance between different criteria to calculate the weights rather than using absolute weights. The weights are then used to compare different alternatives and decide the best option among them based on the goals of the organization. The consistency of judgments from subject matter experts can be checked through the consistency ratio in AHP. It can also be used to combine both qualitative and quantitative analysis and is highly versatile as it can be integrated with various programming methods to optimize the process of decision-making. The integration of AHP with other methods is reviewed in detail in various research papers. Some are tabulated in Table 2.1.

The subjectivity and uncertainty in the judgments can also be addressed using the fuzzy logic concept. Although AHP can evaluate weights by comparing different criteria through relative importance, the initial evaluations from experts cannot be precise and often contain uncertainty. The expert judgments are also qualitative and imprecise. Fuzzy logic provides linguistic variables to describe risk factors, enabling a more intuitive and precise evaluation of the importance factors. The fuzzy logic approach also helps combine qualitative and quantitative data to perform risk assessment. Fuzzy logic provides a way to quantify and work with the vagueness associated with subjective assessments. It uses a "degree of truth" rather than a true or false approach. Various fuzzy models are listed in Table 2.2. It is used to narrow down the fuzzy model suitable for the ranking system.

Table 2.1: Integration of AHP with other methods to optimize the decision-making process

| **Methods** | **Description and Usage** |
|---|---|
| AHP-GP | AHP is used to evaluate and select alternate suppliers based on risk factors and decide accordingly. Goal Programming (GP) is used to optimize the objective functions based on the perceived goal of the workforce [31]. It is ideal for combining qualitative and quantitative analysis. |
| AHP-DP | The integrated AHP-DP approach is used for the supplier selection and order allocation problem where AHP was employed to evaluate potential suppliers and determine supplier ranking. The AHP-based supplier rankings were then incorporated into the Dynamic Programming (DP) model to determine the optimal order quantity to maximize a utility function and minimize the total supply chain costs [32]. Again, it is ideal for combining qualitative and quantitative analysis. |
| AHP-ANFIS | Adaptive neuro-fuzzy inference systems (ANFIS) and the analytic hierarchy process (AHP) are integrated and applied in FMEA to enhance decision-making in risk assessment, focusing on improving accuracy and effectiveness in complex systems, particularly in the oil and gas industry [33]. |
| FAHP | Fuzzy Analytic Hierarchy Process (FAHP) is used in project portfolio management to address the complexities and challenges in selecting and prioritizing project proposals within an organization by efficiently managing uncertainties in decision-making processes through a hierarchical model involving criteria and sub-criteria for practical project proposal evaluation and prioritization [34]. The regime-switching fuzzy Analytic Hierarchy Process (AHP) addresses the choice-varying priorities problem in multi-criteria decision-making and proposes a cubic fuzzy-priority matrix design for expert consistency prioritization [35]. FAHP delves into the complexities of decision-making processes where the relative importance of criteria varies across different choices, offering a novel approach to enhance the accuracy and consistency of decisions in uncertain environments [36]. |
| AHP-VIKOR | Analytical Hierarchy Process (AHP) and VIKOR methods are integrated within a fuzzy logic framework to form a hybrid method. This method considers various risk factors and their weights in determining the most significant contributors to workplace risk. It is beneficial in effectively identifying key risk areas and suggesting improvements for workplace safety [37]. |
| AHP-FST | An advanced risk management model for tunnel construction was developed by combining the Analytical Hierarchy Process (AHP) and Fuzzy Set Theory (FST) to prioritize hazardous zones. This model enhances the assessment of tunnel risks by considering the impact and probability of accidents through expert evaluations [38]. |

Table 2.2: Overview of Fuzzy Models and their Applications

| **Fuzzy Model** | **How It's Used** | **When to Use** |
|---|---|---|
| Mamdani [39] | Uses expert-derived fuzzy rules for input-output mapping. It involves fuzzification, inference, and defuzzification. | Ideal for control systems needing human-like decision-making using if-else rules. Suitable for complex systems controlled by humans. |
| Takagi-Sugeno-Kang (TSK) | Outputs are mathematical functions of input variables. The final output is a weighted average of each rule's output. | For systems requiring precise models like robotic control, pattern recognition, and predictive analyses. |
| Tsukamoto | Rules output fuzzy sets that are monotonic. Defuzzification yields a crisp value directly. | Applied when precise outcomes are necessary, and monotonic outputs effectively capture system behavior. |
| Neuro-Fuzzy Systems [33] | Combines neural networks with fuzzy logic to learn from data, using the structure of fuzzy systems for knowledge representation and neural networks for system adaptation. | Ideal for complex problem-solving and decision-making with plentiful data but hard-to-obtain explicit knowledge. |
| Genetic Fuzzy Systems | Uses genetic algorithms to optimize parameters of the fuzzy logic system, like membership functions and rule sets. | Suitable for dynamic environments needing optimized control strategies or where the optimal system structure is unknown. |
| Fuzzy Cognitive Maps [40] | Combines fuzzy logic with cognitive maps to model systems as networks of concepts with causal relationships. | Effective for modeling complex systems with interdependent variables with uncertain and qualitative relationships. It is useful in strategic planning and scenario analysis. |

Thus, AHP, along with fuzzy logic, can help address the importance of various factors and uncertainty issues in the subjective assessments of qualitative analysis.

An enhanced methodology using AHP and a fuzzy inference system is developed to address the challenges discussed earlier for effective risk management.

## 2.4 Proposed Methodology

In the field of risk management, the priority lies in establishing a robust and dynamic system that meticulously balances the crucial aspects of safety, operational efficiency, and strategic resource management.

In the current scenario, experts in the Canadian Nuclear Industry use a Plant Aging Management (PAM) Risk Ranking system to prioritize tasks based on various criteria, understand the order of maintenance and operations, and ensure a safe environment with high productivity. This approach combines the likelihood of component failures with the potential consequences, effectively aligning maintenance tasks with plant safety and production goals. The Delphi system discussed in the literature review is used to calculate the weights by comparing the criteria in an absolute way. There are various criteria like criticality code, internal safety review, forced loss rate, reactor safety, and equipment obsolescence. However, when exploring the hierarchy and the definitions in the existing internal reports, it becomes evident that the system double counts different criteria by defining production loss and safety in multiple variables. Additionally, the hierarchy does not equally support all systems, as some categories are not relevant to certain systems, making the ranking process less robust.

Based on these limitations, a revised methodology is proposed, which uses the analytical hierarchical process (AHP) to update the method for evaluating the weights of the categories/criteria efficiently. This method also incorporates fuzzy logic to reduce the uncertainties in the process. Thus, it ensures a more reliable, accurate, and effective system for the critical asset management task.

The proposed risk ranking system is explicitly tailored for plant aging management in a Nuclear Power Plant. The model incorporates AHP and Fuzzy Logic to prioritize risks and rank them based on a qualitative and quantitative approach. The model starts by understanding various criteria for defining the model's hierarchy as its bare skeleton. The hierarchy structure helps gather inputs from subject matter experts for the AHP and fuzzy models to calculate the weights. The weights are combined using a defined aggregation method to facilitate decision-making. Figure 2.2 illustrates the proposed process as a flowchart for better visualization.

Each step of the above process is discussed in the upcoming chapters for better understanding. The proposed risk ranking methodology is carried out in two parts. First, a risk ranking system solely dependent on AHP is developed, and then Fuzzy Logic is integrated to obtain the final risk ranking system.

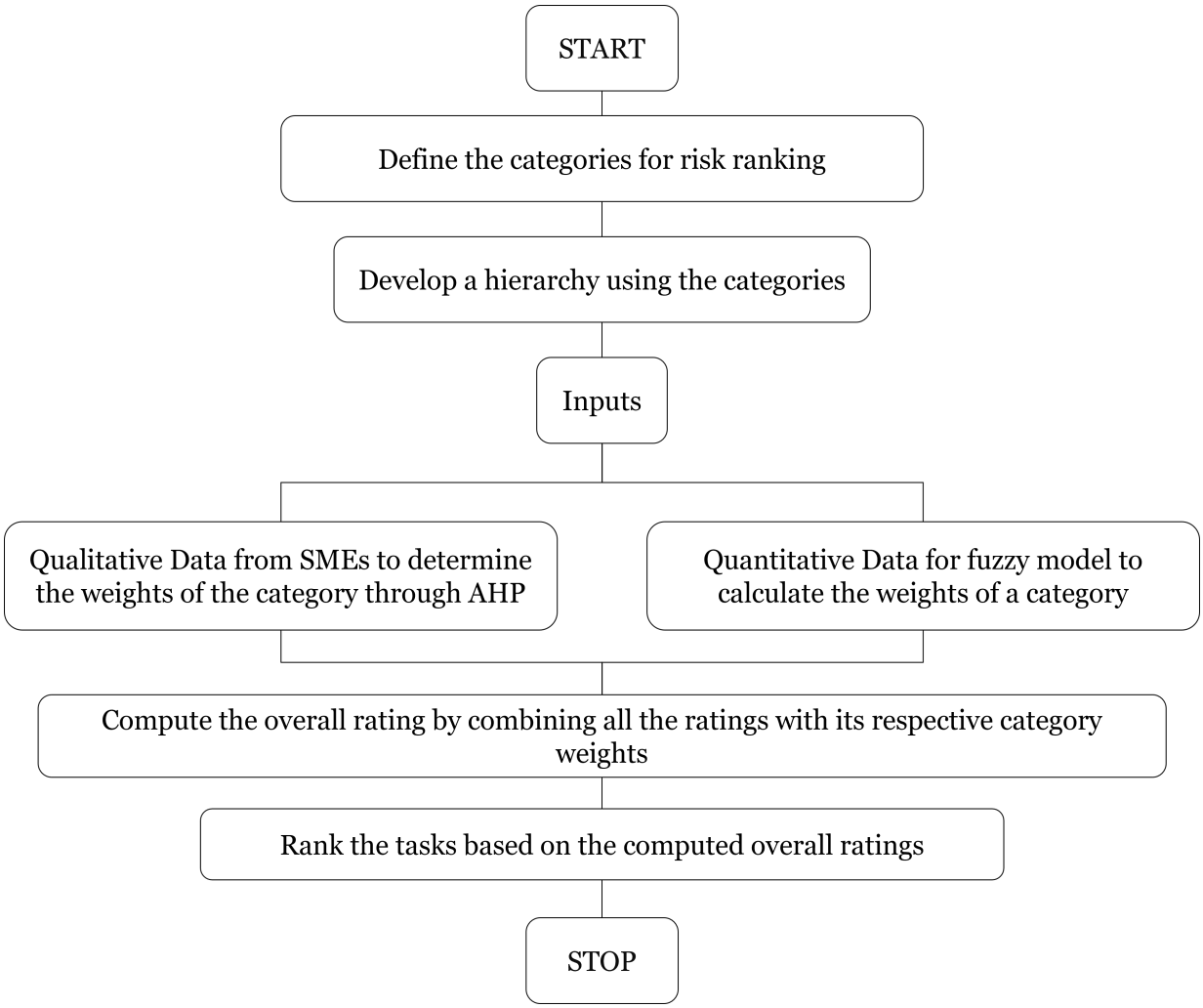



Figure 2.2: Flowchart of the proposed model

## 2.5 Concluding Remarks

The literature review highlights the evolution of risk management, particularly in the nuclear industry, from reactive to proactive approaches. It examines various risk prioritization methods, including the Delphi method and Risk Matrix, noting their strengths and limitations. Advanced models like AHP and fuzzy logic are recommended for their ability to combine qualitative and quantitative data, reduce subjectivity, and handle uncertainty, offering a robust framework for effective risk prioritization and decision-making in complex engineering systems. Based on the literature review, a modified risk ranking methodology is proposed for prioritizing risks in the nuclear power plant.

# Chapter 3

# Analytical Hierarchical Process

## 3.1 Introduction

The Analytic Hierarchy Process (AHP) is a pivotal tool in the contemporary landscape of decision-making methodologies. This approach is remarkably effective when the objective is to rank alternatives and deal with attributes that defy straightforward quantification, as discussed in the literature review.

AHP's position is firmly established at the intersection of two critical fields: decision analysis and operations research [41]. In decision analysis, AHP aligns seamlessly with its goal of selecting choices among predefined alternatives. It provides a structured approach for decision-makers to evaluate and prioritize criteria-based options. Simultaneously, AHP resonates with operations research's core principles, which involve applying scientific and mathematical methods to solve complex problems within intricate systems. Its flexibility allows it to adapt to various problem scenarios, making it a valuable asset in both domains. However, AHP can be problematic when there are many criteria and alternatives, leading to a scaling problem due to increased comparisons.

## 3.2 Evaluating Weights using AHP

The process begins by understanding the criteria and metrics behind the engineering system in question. Follow the steps below to apply AHP successfully to the ranking system. The AHP process is also provided as a flow chart in figure 3.1 [42].

- Define a hierarchy that clearly explains the goals, categories, and sub-categories of criteria/factors under Impact and its Likelihood of failure.
- Prepare a Questionnaire to get inputs from Subject Matter Expert(SME) to prioritize each layers within the hierarchical system

- Tabulate Pairwise Comparison Matrix and calculate Priority Weight Vector to find the weights of each layer within the hierarchy.
- Check if Consistency is within the predefined bound. i.e., Consistency Ratio (CR) is less than or equal to 0.1.

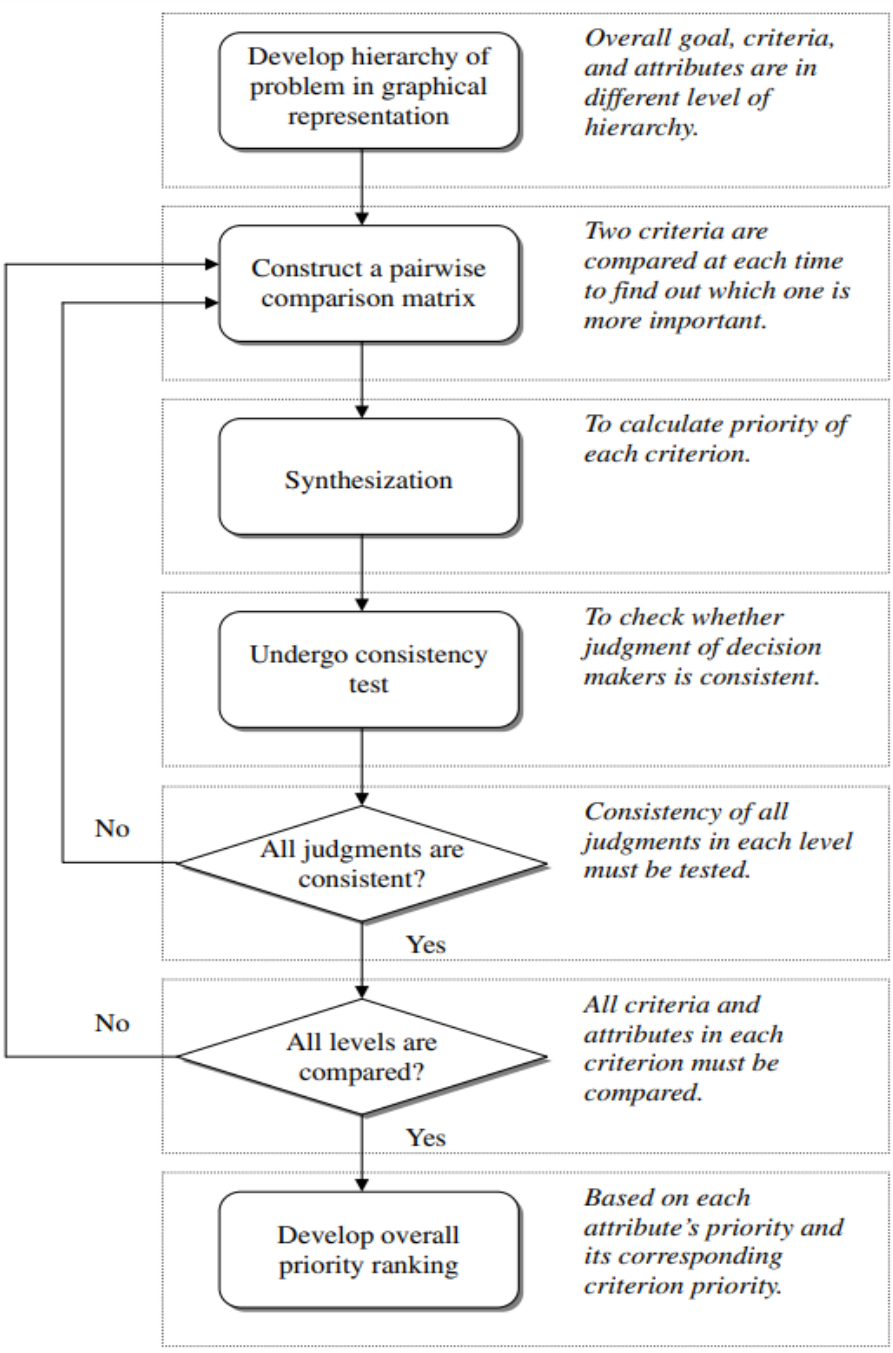



Figure 3.1: Analytic Hierarchy Process (AHP) flowchart

### 3.2.1 Hierarchy

As implied by its name, AHP involves the construction of a hierarchical structure to address complex decision-making scenarios. This hierarchical framework comprises multiple layers and sub-layers, organized based on the intricacy of the problem or system under consideration. At the apex of this hierarchy resides the ultimate goal of the decision-making process, with subsequent layers comprising criteria and their corresponding sub-criteria as shown in Figure 3.2. The AHP approach adheres to a 'bottom-up' methodology during its application, commencing the analysis from the lowermost layer and progressively working towards the solution. This methodological approach is instrumental in structuring intricate problems, offering clarity by breaking them into more manageable constituents.

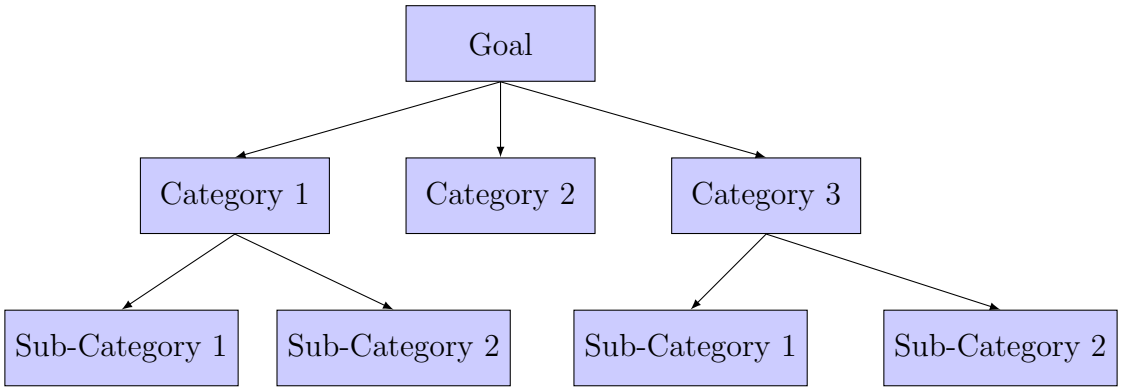



Figure 3.2: AHP Hierarchy (General)

### 3.2.2 Questionnaire for assessing relative weights

A questionnaire comparing criteria at the same level based on the relative scale is used to evaluate the weights. For example, the input of 2 as a relative scale between a and b means that a is twice as important as b. The qualitative data from the questionnaire is used for further computation.

### 3.2.3 Pairwise Comparison Matrix (A)

In scenarios where decision-makers evaluate numerous options through a questionnaire, understanding the relative weights on long tables and interpreting the meaning of the same is complex, and thus, pairwise comparison is used. This technique simplifies the decision-making process by breaking down a complex problem into smaller, more manageable comparisons between pairs of options. This approach allows the decision-maker to concentrate on only two options simultaneously, thereby converting the original complex problem into a collection of simpler sub-problems.

The results of these pairwise comparisons are arranged in a pairwise comparison matrix, denoted as $A$ (3.1), where each element $a_{ij}$ quantifies the preference of criterion $x_i$ over criterion $x_j$.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \tag{3.1}$$

Ideally, each matrix entry aims to reflect the weight ratio between the criteria, where each element $w_{ij}$ quantifies the ratio between the weight $w_i$ of criterion $x_i$ over the weight $w_j$ of criterion $x_j$. More precisely, according to **Saaty's theory**, each entry is supposed to approximate the ratio between two weights of the criterion.

$$a_{ij} \approx \frac{w_i}{w_j} \quad \forall i, j. \tag{3.2}$$

By substituting (3.2) in (3.1), the updated pairwise comparison matrix (3.3) quantitatively reflects the criterion deemed to be of greater preference.

$$A = \left(\frac{w_i}{w_j}\right)_{n\times n} = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{pmatrix}. \tag{3.3}$$

Under perfect conditions where matrix entries accurately represent weight ratios, the matrix would exhibit a property of multiplicative reciprocity. Consequently, the matrix adopts a simplified structure in which each entry $a_{ij}$ is the reciprocal of $a_{ji}$.

$$a_{ij} \approx \frac{1}{a_{ji}} \quad \forall i, j. \tag{3.4}$$

By substituting (3.4) in (3.1), the updated pairwise comparison matrix (3.5) reflects the transpose nature of a well-constructed matrix.

$$A = \begin{pmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1 \end{pmatrix} \tag{3.5}$$

The main objective is to find the priority weight vector($\mathbf{w}$) from $\mathbf{A}$ to facilitate the decision-making process.

### 3.2.4 Priority Weight Vector (w)

In the pairwise comparison matrix, if an element $a_{ij}$ of the matrix precisely mirrors the weight ratio $w_i/w_j$, all columns in the matrix $A$ are proportional. Thus, the priority vector aligns with any normalized column of $A$. Realistically, achieving such precision in decision-making is uncommon. AHP is typically applied to less exact pairwise comparisons, necessitating the estimation of an effective priority vector. Various methods for this have been proposed. The most notable is the eigenvector method.

The eigenvector method, proposed by Saaty, is an approach to estimate the priority vector in the Analytic Hierarchy Process (AHP). This method identifies the priority vector as the principal eigenvector of the matrix $A$, often referred to in linear algebra as the Perron-Frobenius eigenvector[41]. This approach arises from the observation that, for a matrix $A$ with entries as exact ratios of weights, multiplying it by a weight vector $w$ yields:

$$Aw = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \dots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \dots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \dots & \frac{w_n}{w_n} \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} = \begin{pmatrix} nw_1 \\ \vdots \\ nwn \end{pmatrix} = nw. \tag{3.6}$$

In linear algebra, if $Aw = nw$, then $n$ and $w$ are an eigenvalue and eigenvector of $A$, respectively. Given that $A$'s other eigenvalue is 0 with multiplicity $n-1$, $n$ is its largest eigenvalue. If $A$'s entries are ratios between weights, its weight vector is the eigenvector $(w)$ associated with the eigenvalue $n$. Saaty suggested generalizing the use of maximum eigenvalue,$\lambda_{\max}$, for all pairwise comparison matrices by replacing the previously used $n$.

Thus, the vector $w$ can be derived from any pairwise comparison matrix $A$ as a solution to the system:

$$Aw = \lambda_{\max} w, \quad w^T 1 = 1$$

where $\lambda_{\max}$ is the maximum eigenvalue of $A$, and 1 is a vector of ones. Although this problem can be addressed with mathematical software and spreadsheets, its practical interpretation can be challenging for practitioners.

### 3.2.5 Consistency

In AHP, consistency in decision-making is critical. Ideally, a rational decision maker's pairwise preferences should satisfy $a_{ij} = \frac{w_i}{w_j} \forall i, j$ as discussed in previous sections. This leads to the condition $a_{ik} = a_{ij}a_{jk} \forall i, j, k$, indicating a consistent matrix.

$$a_{ij}a_{jk} = \frac{w_i}{w_j}\frac{w_j}{w_k} = \frac{w_i}{w_k} = a_{ik}. \tag{3.7}$$

However, as the number of criteria increases, the number of pairwise comparisons increases, and it is hard to maintain the property of the pairwise comparison matrix. This leads

to a recognition of a diverse array of inconsistency indices. The consistency condition, in addition to the (3.2), can generate from multiple equivalent formulations, which are prominently identified as follows:

1. The product of the pairwise comparisons, such that $a_{ik} = a_{ij}a_{jk}$ for all $i, j, k$.
2. The existence of a weight vector $(w_1, \ldots, w_n)^T$ where each pairwise comparison $a_{ij}$ is represented by the ratio $\frac{w_i}{w_j}$ for all $i, j$.
3. The columns of the comparison matrix $A$ are proportional, indicating that $A$ possesses a rank of one.
4. The pairwise comparison matrix $A$ has its maximum eigenvalue $\lambda_{\max}$ equal to its order $n$.

Consistency is quantified using various indices, such as the eigenvector method, which relates to the matrix's eigenvalues and eigenvectors. For a consistent matrix $A$, the largest eigenvalue corresponds to the number of elements in $A$. Other methods to assess inconsistency involve analyzing the determinants and characteristic polynomials of the matrix.

### Consistency Index (CI)

In the Analytic Hierarchy Process, the Consistency Index (CI) is used to measure the reliability of pairwise comparisons. It is calculated as:

$$\mathrm{CI}(A) = \frac{\lambda_{\max} - n}{n - 1} \tag{3.8}$$

where $\lambda_{\max}$ is the matrix $A$'s maximum eigenvalue, and $n$ is the size of the matrix. A larger CI indicates greater inconsistency and vice versa.

However, based on numerical research [43], the expected Consistency Index (CI) value for a random matrix of size $n+1$ exceeds that of a random matrix of size $n$. This trend suggests that directly comparing CI values across matrices of varying sizes may not provide equitable assessments. As a result, adjusting the CI to ensure a fair comparison between matrices of different orders by normalization becomes necessary.

### Consistency Ratio (CR)

The Consistency Ratio (CR) addresses the limitation in the Consistency Index(CI) by scaling CI with a Random Index (RI), resulting in the below equation.

$$\mathrm{CR}(A) = \frac{\mathrm{CI}(A)}{\mathrm{RI}_n} \tag{3.9}$$

where $RI_n$ are estimated in the numerical studies [43] and tabulated in Table 3.1. A large number of random matrices of the same order are generated to determine the RI for a matrix of a given size. For each of these random matrices, the principal eigenvalue,$\lambda_{\max}$, is calculated, and the Consistency Index (CI) is determined using the equation 3.9. The average CI of all these random matrices gives the RI. CR value below 0.1 is typically considered acceptable. The CR value of less than 0.1 means that the inconsistency in the pairwise comparison matrix should not exceed the cap of 10 %.

Table 3.1: Values of $RI_n$

| $n$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|
| $RI_n$ | 0.5247 | 0.8816 | 1.1086 | 1.2479 | 1.3417 | 1.4057 | 1.4499 | 1.4854 |

Thus, all the optimization techniques and methods used to find the priority vector (w) use consistency as one of the objective functions by minimizing it.

### Index of Determinants

Peláez and Lamata [44] proposed an inconsistency index based on the determinant properties of a matrix. The determinant of a matrix $A$ can be expressed as:

$$\det(A) = \frac{a_{13}}{a_{12}a_{23}} + \frac{a_{12}a_{23}}{a_{13}} - 2. \tag{3.10}$$

The above comparison in equation (3.10) holds to be 0 for a consistent matrix as $a_{13} = a_{12}a_{23}$ by the properties of elements in an ideal pairwise comparison matrix.

The inconsistency for a 3x3 real matrix, can be discerned if $a_{13} \neq a_{12}a_{23}$ and the determinant, given by $\det(A)$ is positive. This is because generally

$$ab + \frac{1}{ab} - 2 > 0 \quad \forall a \neq b, a, b > 0.$$

This principle is extendable to larger matrices by averaging the determinants of all possible 3x3 sub matrices $T_{ijk}$ of a given pairwise comparison matrix of size $n$, which is constructed as shown below.

$$T_{ijk} = \begin{pmatrix} 1 & a_{ij} & a_{ik} \\ a_{ji} & 1 & a_{jk} \\ a_{ki} & a_{kj} & 1 \end{pmatrix}, \quad \forall i < j < k.$$

The total count of such submatrices is $\binom{n}{3} = \frac{n!}{3!(n-3)!}$. The inconsistency index $CI^*$ for a pairwise comparison matrix $A$ is then calculated as:

$$CI^*(A) = \frac{n-2}{\binom{n}{3}} \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} \left( \frac{a_{ik}}{a_{ij}a_{jk}} + \frac{a_{ij}a_{jk}}{a_{ik}} - 2 \right) \det(T_{ijk}). \tag{3.11}$$

The above consistency index for a general $n$ x $n$ pairwise comparison matrix is the average inconsistency computed for all the possible submatrices $T_{ijk}$, where $i < j < k$.

The inconsistency index in (3.11) is also proportional to another inconsistency index known as the coefficient $c_3$ [45]. Shiraishi and Obata [46] and Shirashi*et al.* [47, 48] proposed to use the third coefficient of the characteristic polynomial of $A$, $c_3$, as an inconsistency index.The characteristic polynomial $P_A(\lambda)$ of a matrix $A$ is given by:

$$P_A(\lambda) = \lambda^n + c_1\lambda^{n-1} + +c_2\lambda^{n-2} + c_3\lambda^{n-3} + \cdots + c_{n-1}\lambda + c_n, \tag{3.12}$$

where $\lambda$ is an eigenvalue of $A$, and $c_1, \ldots, c_n$ are coefficients determined by the entries of $A$. The above expression can be derived from the method to calculate eigenvalues by calculating the determinant of $Aw - \lambda w$.

A negative $c_3$ indicates inconsistency as proved by Shirashi *et al.*[47] . The proportionality between $CI^*$ and $c_3$ is also highlighted in Shirashi *et al.*[47] and the corresponsing analytical expression for $c_3$ is given below.

$$c_3 = \frac{2}{\binom{n}{3}} \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} \left(2 - \frac{a_{ik}}{a_{ij}a_{jk}} - \frac{a_{ij}a_{jk}}{a_{ik}}\right) \tag{3.13}$$

Since $c_3 < 0$, all the optimization techniques and methods used to find the priority vector (w) use consistency as one of the objective functions by maximizing it. The same can be applied to find missing elements in the pairwise comparison matrix.

### Missing Comparisons in Pairwise Comparison Matrix

As the quantity of criteria in a decision-making problem increases, fully completing pairwise comparison matrices becomes increasingly challenging. This escalation in criteria often leads to the emergence of incomplete matrices, posing difficulties in their resolution. However, there are several methodologies available to address these incomplete matrices. These include the optimization of the $c_3$ coefficient, applying the revised geometric mean method, and adhering to the fundamental principles of matrix consistency as outlined in Equation 3.7 and 3.4.

Notably, the consistency of a matrix can be effectively leveraged to populate all its entries, provided that a subject matter expert accurately fills out the initial row of the matrix. This approach ensures a systematic and coherent completion of the matrix, even in the face of increasing complexity due to the growing number of criteria.

Let us assume a 5 x 5 matrix with the first row of pairwise comparisons filled and others empty($\emptyset$) as in eq. 3.14.

$$\mathbf{A} = \begin{pmatrix} 1 & a_{12} & a_{13} & a_{14} & a_{15} \\ \emptyset & 1 & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & 1 & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & 1 & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & 1 \end{pmatrix} \tag{3.14}$$

The missing entries like $a_{23}$ and other entries can be calculated using the equation mentioned earlier. For instance $a_{23} = a_{21}.\ a_{13}$ and $a_{21} = \frac{1}{a_{12}}$. Thus, $a_{23} = \frac{a_{13}}{a_{12}}$ [41]. Similarly, we can find all the missing variables based on the entries in the first row of the pairwise comparison matrix.

$$\mathbf{A} = \begin{pmatrix} 1 & a_{12} & a_{13} & a_{14} & a_{15} \\ \frac{1}{a_{12}} & 1 & \frac{a_{13}}{a_{12}} & \frac{a_{14}}{a_{12}} & \frac{a_{15}}{a_{12}} \\ \frac{1}{a_{13}} & \frac{a_{12}}{a_{13}} & 1 & \frac{a_{14}}{a_{13}} & \frac{a_{15}}{a_{13}} \\ \frac{1}{a_{14}} & \frac{a_{12}}{a_{14}} & \frac{a_{13}}{a_{14}} & 1 & \frac{a_{15}}{a_{14}} \\ \frac{1}{a_{15}} & \frac{a_{12}}{a_{15}} & \frac{a_{13}}{a_{15}} & \frac{a_{14}}{a_{15}} & 1 \end{pmatrix} \tag{3.15}$$

The missing comparisons, if any, from the questionnaire are calculated using the above-mentioned concept.

### 3.2.6 Aggregation Method

The evaluated weights are used to calculate the overall risk rating using any defined aggregation method.

## 3.3 AHP Risk Ranking Model

For the Nuclear Risk Ranking System, define the goals, categories/sub-categories, and their respective ratings. The hierarchy is inspired by the current hierarchy used in the nuclear industry and developed in consultation with a nuclear plant expert. The categories are chosen to minimize any double counting between them.

There are two goals as per the proposed ranking system.

- Consequence
- Likelihood of Failure

The risk effect decreases as the index of the ratings increases in the respective categories/sub-categories as shown in table 3.2 and 3.3.

The entire hierarchy of the goal "Consequence" and "Likelihood of Failure" is also shown in Figure 3.3 and 3.4 for better visualization.

The definition of all the ratings within categories and sub-categories is provided in the Appendix A.

Table 3.2: Categories and Ratings within the goal "Consequence"

| **Category** | **Sub-Category** | **Possible Ratings** |
|---|---|---|
| Nuclear Safety (S) | N/A | S1: Reactor Safety 1 |
| | | S2: Reactor Safety 2 |
| | | S3: Reactor Safety 3 |
| | | S4: Reactor Safety 4 |
| Production Loss (PL) | N/A | PL1: Production Loss 1 |
| | | PL2: Production Loss 2 |
| | | PL3: Production Loss 3 |
| | | PL4: Production Loss 4 |
| | | PL5: Production Loss 5 |
| | | PL6: Production Loss 6 |
| | | PL7: Production Loss 7 |
| | | PL8: Production Loss 8 |
| | | PL9: Production Loss 9 |
| Cost, Conventional Safety, and Environment (C) | N/A | C1: Cost, Conventional Safety, and Environment 1 |
| | | C2: Cost, Conventional Safety, and Environment 2 |
| | | C3: Cost, Conventional Safety, and Environment 3 |
| | | C4: Cost, Conventional Safety, and Environment 4 |
| | | C5: Cost, Conventional Safety, and Environment 5 |
| | | C6: Cost, Conventional Safety, and Environment 6 |
| | | C7: Cost, Conventional Safety, and Environment 7 |
| Ease of Repairability (ER) | Equipment Obsolescence (EO) | EO1: Very High |
| | | EO2: High |
| | | EO3: Medium/Normal |
| | | EO4: Low |
| | | EO5: Negligible |
| | Spare Unavailability (SU) | SU1: Very High |
| | | SU2: High |
| | | SU3: Medium/Normal |
| | | SU4: Low |
| | | SU5: Negligible |

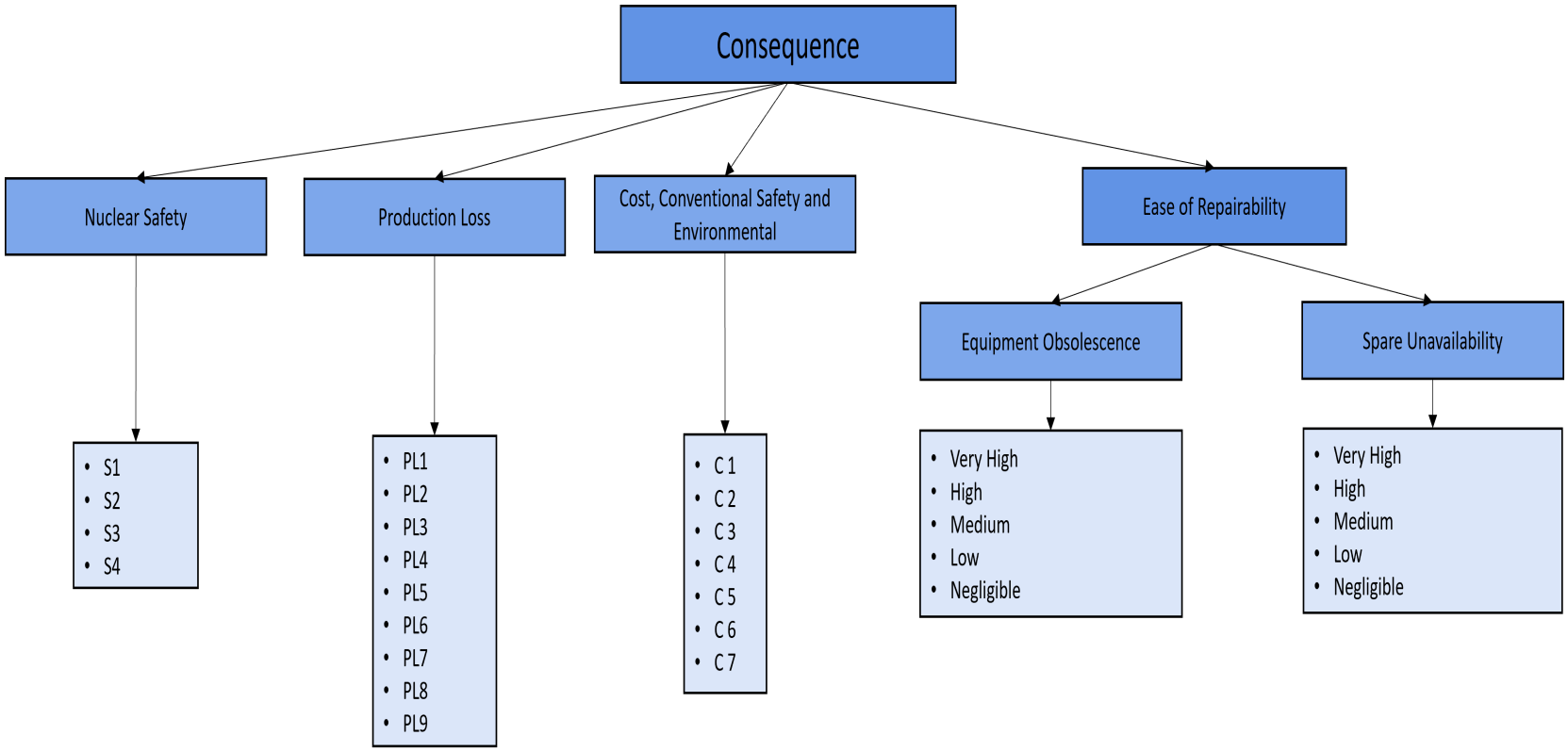



Figure 3.3: Hierarchy of Consequence

Table 3.3: Categories and Ratings of goal "Likelihood of Failure"

| **Category** | **Sub-Category** | **Possible Ratings** |
|---|---|---|
| Equipment Condition (EC) | N/A | VP: Very Poor |
| | | P: Poor |
| | | S: Satisfactory |
| | | G: Good |
| | | VG: Very Good |
| Equipment Environment (EE) | N/A | Severe |
| | | Mild |
| Equipment Demand (ED) | N/A | High Duty Cycle |
| | | Low Duty Cycle |

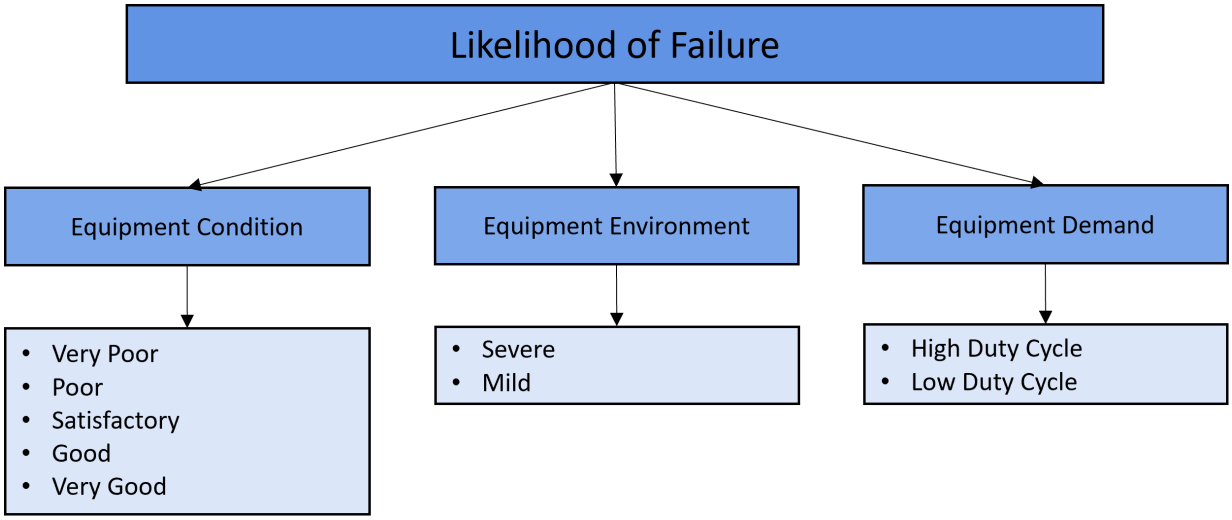



Figure 3.4: Hierarchy of Likelihood of Failure

The questionnaire that compares the relative scale between various categories and possible ratings is completed by a single respondent, a nuclear power plant expert. These missing comparisons in the questionnaire were calculated using the subsection 3.2.5.

Tabulated below are the Questionnaire and its relative weights for the goal "Consequence".

Table 3.4: Questionnaire for Categories in Consequence

| **Question** | **Relative Scale** |
|---|---|
| How critical is Nuclear Safety when compared with Production Loss? | 1.50 |
| How critical is Nuclear Safety when compared with Cost, Conventional Safety, and Environment? | 2.00 |
| How critical is Nuclear Safety when compared with Ease of Repairability? | 4.00 |
| How critical is Production Loss when compared with Cost, Conventional Safety, and Environment? | 1.50 |
| How critical is Production Loss when compared with Ease of Repairability? | 3.00 |
| How critical are Cost, Conventional Safety, and Environment when compared with Ease of Repairability? | 2.00 |

Table 3.5: Questionnaire for Sub-Categories within Ease of Repairability

| **Question** | **Relative Scale** |
|---|---|
| How critical is Equipment Obsolescence when compared with spare unavailability? | 1.50 |

Table 3.6: questionnaire for ratings within Nuclear Safety

| **Question** | **Relative Scale** |
|---|---|
| How critical is S1 when compared with S2? | 1.50 |
| How critical is S1 when compared with S3? | 3.00 |
| How critical is S1 when compared with S4? | 4.50 |
| How critical is S2 when compared with S3? | 2.00 |
| How critical is S2 when compared with S4? | 4.00 |
| How critical is S3 when compared with S4? | 1.50 |

Table 3.7: questionnaire for ratings within Production Loss

| **Question** | **Relative Scale** |
|---|---|
| How critical is PL1 when compared with PL2? | 1.50 |
| How critical is PL1 when compared with PL3? | 2.50 |
| How critical is PL1 when compared with PL4? | 3.50 |
| How critical is PL1 when compared with PL5? | 4.50 |
| How critical is PL1 when compared with PL6? | 6.00 |
| How critical is PL1 when compared with PL7? | 7.00 |
| How critical is PL1 when compared with PL8? | 8.50 |
| How critical is PL1 when compared with PL9? | 10.00 |
| How critical is PL2 when compared with PL3? | 1.50 |
| How critical is PL2 when compared with PL4? | 2.50 |
| How critical is PL2 when compared with PL5? | 3.00 |
| How critical is PL2 when compared with PL6? | 4.00 |
| How critical is PL2 when compared with PL7? | 5.00 |
| How critical is PL2 when compared with PL8? | 6.00 |
| How critical is PL2 when compared with PL9? | 7.00 |
| How critical is PL3 when compared with PL4? | 1.50 |
| How critical is PL3 when compared with PL5? | 2.00 |
| How critical is PL3 when compared with PL6? | 2.50 |
| How critical is PL3 when compared with PL7? | 3.00 |
| How critical is PL3 when compared with PL8? | 3.50 |
| How critical is PL3 when compared with PL9? | 4.00 |
| How critical is PL4 when compared with PL5? | 1.50 |
| How critical is PL4 when compared with PL6? | 2.00 |
| How critical is PL4 when compared with PL7? | 2.25 |
| How critical is PL4 when compared with PL8? | 2.50 |
| How critical is PL4 when compared with PL9? | 3.00 |
| How critical is PL5 when compared with PL6? | 1.50 |
| How critical is PL5 when compared with PL7? | 1.75 |
| How critical is PL5 when compared with PL8? | 2.00 |
| How critical is PL5 when compared with PL9? | 2.25 |
| How critical is PL6 when compared with PL7? | 1.50 |
| How critical is PL6 when compared with PL8? | 1.75 |
| How critical is PL6 when compared with PL9? | 2.00 |
| How critical is PL7 when compared with PL8? | 1.25 |
| How critical is PL7 when compared with PL9? | 1.50 |
| How critical is PL8 when compared with PL9? | 1.50 |

Table 3.8: questionnaire for ratings within Cost, Conventional Safety and Environment

| **Question** | **Relative Scale** |
|---|---|
| How critical is C1 when compared with C2? | 1.50 |
| How critical is C1 when compared with C3? | 2.00 |
| How critical is C1 when compared with C4? | 2.50 |
| How critical is C1 when compared with C5? | 3.00 |
| How critical is C1 when compared with C6? | 4.00 |
| How critical is C1 when compared with C7? | 5.00 |
| How critical is C2 when compared with C3? | 1.50 |
| How critical is C2 when compared with C4? | 1.75 |
| How critical is C2 when compared with C5? | 2.25 |
| How critical is C2 when compared with C6? | 3.00 |
| How critical is C2 when compared with C7? | 3.50 |
| How critical is C3 when compared with C4? | 1.25 |
| How critical is C3 when compared with C5? | 1.50 |
| How critical is C3 when compared with C6? | 2.00 |
| How critical is C3 when compared with C7? | 2.50 |
| How critical is C4 when compared with C5? | 1.50 |
| How critical is C4 when compared with C6? | 2.00 |
| How critical is C4 when compared with C7? | 2.50 |
| How critical is C5 when compared with C6? | 1.50 |
| How critical is C5 when compared with C7? | 2.00 |
| How critical is C6 when compared with C7? | 2.00 |

Table 3.9: questionnaire for ratings within Equipment Obsolescence

| **Question** | **Relative Scale** |
|---|---|
| How critical is Very High when compared with High? | 2.00 |
| How critical is Very High when compared with Medium? | 2.50 |
| How critical is Very High when compared with Low? | 3.50 |
| How critical is Very High when compared with Negligible? | 5.00 |
| How critical is High when compared with Medium? | 1.50 |
| How critical is High when compared with Low? | 2.00 |
| How critical is High when compared with Negligible? | 3.00 |
| How critical is Medium when compared with Low? | 1.50 |
| How critical is Medium when compared with Negligible? | 2.00 |
| How critical is Low when compared with Negligible? | 2.00 |

Table 3.10: questionnaire for ratings within Spares Unavailability

| **Question** | **Relative Scale** |
|---|---|
| How critical is Very High when compared with High? | 1.50 |
| How critical is Very High when compared with Medium? | 2.00 |
| How critical is Very High when compared with Low? | 3.00 |
| How critical is Very High when compared with Negligible? | 4.00 |
| How critical is High when compared with Medium? | 1.50 |
| How critical is High when compared with Low? | 2.00 |
| How critical is High when compared with Negligible? | 3.00 |
| How critical is Medium when compared with Low? | 1.50 |
| How critical is Medium when compared with Negligible? | 2.00 |
| How critical is Low when compared with Negligible? | 1.50 |

Tabulated below are the questionnaire and its relative weights for the goal - "Likelihood of Failure."

Table 3.11: Questionnaire for Categories of Likelihood of Failure

| **Question** | **Relative Scale** |
|---|---|
| How critical is Equipment Condition when compared with Equipment Environment? | 2.00 |
| How critical is Equipment Condition when compared with Equipment Demand? | 3.00 |
| How critical is Equipment Environment when compared with Equipment Demand? | 1.50 |

Table 3.12: Questionnaire for Sub-Categories of Equipment Environment

| **Question** | **Relative Scale** |
|---|---|
| How critical is Severe when compared with Mild? | 1.25 |

Table 3.13: Questionnaire for Sub-Categories of Equipment Demand

| **Question** | **Relative Scale** |
|---|---|
| How critical is the High Duty Cycle when compared with the Low Duty Cycle? | 1.10 |

Table 3.14: Questionnaire for Sub-Categories of Equipment Condition

| **Question** | **Relative Scale** |
|---|---|
| How critical is Very High when compared with High? | 1.50 |
| How critical is Very High when compared with Medium? | 2.00 |
| How critical is Very High when compared with Low? | 2.50 |
| How critical is Very High when compared with Negligible? | 4.00 |
| How critical is High when compared with Medium? | 1.50 |
| How critical is High when compared with Low? | 2.00 |
| How critical is High when compared with Negligible? | 2.50 |
| How critical is Medium when compared with Low? | 2.00 |
| How critical is Medium when compared with Negligible? | 3.00 |
| How critical is Low when compared with Negligible? | 2.50 |

The above-discussed properties are used to convert the inputs received in consultation with the Nuclear Power Plant expert to form pairwise comparison matrices.

The pairwise comparison matrices of goal"Consequence" are provided below.

Table 3.15: Pairwise comparison matrix for categories of Consequence

| **Categories of Consequence** | **S** | **PL** | **C** | **ER** |
|---|---|---|---|---|
| S | 1.000 | 1.500 | 2.000 | 4.000 |
| PL | 0.667 | 1.000 | 1.500 | 3.000 |
| C | 0.500 | 0.667 | 1.000 | 2.000 |
| ER | 0.250 | 0.333 | 0.500 | 1.000 |

Table 3.16: Pairwise comparison matrix for the Ratings of Nuclear Safety

| **Ratings of S** | **S 1** | **S 2** | **S 3** | **S4** |
|---|---|---|---|---|
| S 1 | 1.000 | 1.500 | 3.000 | 4.500 |
| S 2 | 0.667 | 1.000 | 2.000 | 4.000 |
| S 3 | 0.333 | 0.500 | 1.000 | 1.500 |
| S 4 | 0.222 | 0.250 | 0.667 | 1.000 |

Table 3.17: Pairwise comparison matrix for sub-categories of Loss of Generation (P)

| Ratings of PL | PL1 | PL2 | PL3 | PL4 | PL5 | PL6 | PL7 | PL8 | PL9 |
|---|---|---|---|---|---|---|---|---|---|
| PL1 | 1.00 | 1.50 | 2.50 | 3.50 | 4.50 | 6.00 | 7.00 | 8.50 | 10.00 |
| PL2 | 0.67 | 1.00 | 1.50 | 2.50 | 3.00 | 4.00 | 5.00 | 6.00 | 7.00 |
| PL3 | 0.40 | 0.67 | 1.00 | 1.50 | 2.00 | 2.50 | 3.00 | 3.50 | 4.00 |
| PL4 | 0.29 | 0.40 | 0.67 | 1.00 | 1.50 | 2.00 | 2.25 | 2.50 | 3.00 |
| PL5 | 0.22 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 1.75 | 2.00 | 2.25 |
| PL6 | 0.17 | 0.25 | 0.40 | 0.50 | 0.67 | 1.00 | 1.50 | 1.75 | 2.00 |
| PL7 | 0.14 | 0.20 | 0.33 | 0.44 | 0.57 | 0.67 | 1.00 | 1.25 | 1.50 |
| PL8 | 0.12 | 0.17 | 0.29 | 0.40 | 0.50 | 0.57 | 0.80 | 1.00 | 1.50 |
| PL9 | 0.10 | 0.14 | 0.25 | 0.33 | 0.44 | 0.50 | 0.67 | 0.67 | 1.00 |

Table 3.18: Pairwise comparison matrix for Ratings of Cost, Conventional Safety and Environment

| Ratings of C | C 1 | C 2 | C 3 | C 4 | C 5 | C 6 | C 7 |
|---|---|---|---|---|---|---|---|
| C 1 | 1.00 | 1.50 | 2.00 | 2.50 | 3.00 | 4.00 | 5.00 |
| C 2 | 0.67 | 1.00 | 1.50 | 1.75 | 2.25 | 3.00 | 3.50 |
| C 3 | 0.50 | 0.67 | 1.00 | 1.25 | 1.50 | 2.00 | 2.50 |
| C 4 | 0.40 | 0.57 | 0.80 | 1.00 | 1.50 | 2.00 | 2.50 |
| C 5 | 0.33 | 0.44 | 0.67 | 0.67 | 1.00 | 1.50 | 2.00 |
| C 6 | 0.25 | 0.33 | 0.50 | 0.50 | 0.67 | 1.00 | 2.00 |
| C 7 | 0.20 | 0.29 | 0.40 | 0.40 | 0.50 | 0.50 | 1.00 |

Table 3.19: Pairwise comparison matrix for sub-categories of Equipment Obsolescence

| Ratings of EO | Very High | High | Medium | Low | Negligible |
|---|---|---|---|---|---|
| Very High | 1.000 | 2.000 | 2.500 | 3.500 | 5.000 |
| High | 0.500 | 1.000 | 1.500 | 2.000 | 3.000 |
| Medium | 0.400 | 0.667 | 1.000 | 1.500 | 2.000 |
| Low | 0.286 | 0.500 | 0.667 | 1.000 | 2.000 |
| Negligible | 0.200 | 0.333 | 0.500 | 0.500 | 1.000 |

Table 3.20: Pairwise comparison matrix for Ratings of Spares Unavailability

| Ratings of SU | Very High | High | Medium | Low | Negligible |
|---|---|---|---|---|---|
| Very Poor | 1.000 | 1.500 | 2.000 | 3.000 | 4.000 |
| High | 0.667 | 1.000 | 1.500 | 2.000 | 3.000 |
| Medium | 0.500 | 0.667 | 1.000 | 1.500 | 2.000 |
| Low | 0.333 | 0.500 | 0.667 | 1.000 | 1.500 |
| Negligible | 0.250 | 0.333 | 0.500 | 0.667 | 1.000 |

The pairwise comparison matrices of goal"Likelihood of Failure" are provided below.

Table 3.21: Pairwise comparison matrix for Categories of Likelihood of Failure

| **Categories of Likelihood of Failure** | **EC** | **EE** | **ED** |
|---|---|---|---|
| EC | 1.000 | 2.000 | 3.000 |
| EE | 0.500 | 1.000 | 1.500 |
| ED | 0.333 | 0.667 | 1.000 |

Table 3.22: Pairwise comparison matrix for Sub-Categories of Equipment Condition

| **Ratings of EC** | **Very Poor** | **Poor** | **Satisfactory** | **Good** | **Very Good** |
|---|---|---|---|---|---|
| Very Poor | 1.000 | 1.500 | 2.000 | 2.500 | 4.000 |
| Poor | 0.667 | 1.000 | 1.500 | 2.000 | 2.500 |
| Satisfactory | 0.500 | 0.667 | 1.000 | 2.000 | 2.500 |
| Good | 0.400 | 0.500 | 0.500 | 1.000 | 2.500 |
| Very Good | 0.250 | 0.400 | 0.333 | 0.400 | 1.000 |

Based on the pairwise comparison matrix derived from the relative comparison between the categories and ratings, its priority weight vectors for the goal 'Consequence' are tabulated below. The AHP model applies to only levels with criteria greater than 3. If it is just 2, we can find the weights directly using the AHP Properties.

Table 3.23: Priority Weight Vector for categories of Consequence

| **Categories of Consequence** | **S** | **PL** | **C** | **ER** | **Weights** |
|---|---|---|---|---|---|
| S | 0.414 | 0.429 | 0.400 | 0.400 | 0.411 |
| PL | 0.276 | 0.286 | 0.300 | 0.300 | 0.290 |
| C | 0.207 | 0.190 | 0.200 | 0.200 | 0.199 |
| ER | 0.103 | 0.095 | 0.100 | 0.100 | 0.100 |

Table 3.24: Priority Weight Vector for the Ratings of Nuclear Safety

| **Ratings of S** | **S 1** | **S 2** | **S 3** | **S4** | **Weights** |
|---|---|---|---|---|---|
| S 1 | 0.450 | 0.462 | 0.450 | 0.409 | 0.443 |
| S 2 | 0.300 | 0.308 | 0.300 | 0.364 | 0.318 |
| S 3 | 0.150 | 0.154 | 0.150 | 0.136 | 0.148 |
| S 4 | 0.100 | 0.077 | 0.100 | 0.091 | 0.092 |

Table 3.25: Priority Weight Vector for Ratings of Production Loss

| Ratings of PL | PL1 | PL2 | PL3 | PL4 | PL5 | PL6 | PL7 | PL8 | PL9 | Weights |
|---|---|---|---|---|---|---|---|---|---|---|
| PL1 | 0.32 | 0.32 | 0.34 | 0.32 | 0.32 | 0.32 | 0.30 | 0.31 | 0.31 | 0.32 |
| PL2 | 0.21 | 0.21 | 0.20 | 0.23 | 0.21 | 0.21 | 0.22 | 0.22 | 0.22 | 0.22 |
| PL3 | 0.13 | 0.14 | 0.13 | 0.14 | 0.14 | 0.14 | 0.13 | 0.13 | 0.13 | 0.12 |
| PL4 | 0.09 | 0.09 | 0.09 | 0.09 | 0.11 | 0.11 | 0.10 | 0.09 | 0.09 | 0.10 |
| PL5 | 0.07 | 0.07 | 0.07 | 0.06 | 0.07 | 0.08 | 0.08 | 0.07 | 0.07 | 0.07 |
| PL6 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.07 | 0.06 | 0.06 | 0.06 |
| PL7 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.05 | 0.05 | 0.04 |
| PL8 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.03 | 0.04 | 0.05 | 0.04 |
| PL9 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.03 | 0.03 |

Table 3.26: Priority Weight Vector for Ratings of Cost, Conventional Safety and Environment

| Ratings of C | C 1 | C 2 | C 3 | C 4 | C 5 | C 6 | C 7 | Weights |
|---|---|---|---|---|---|---|---|---|
| C 1 | 0.30 | 0.31 | 0.29 | 0.31 | 0.29 | 0.29 | 0.27 | 0.294 |
| C 2 | 0.20 | 0.21 | 0.22 | 0.22 | 0.22 | 0.21 | 0.19 | 0.209 |
| C 3 | 0.15 | 0.14 | 0.15 | 0.15 | 0.14 | 0.14 | 0.14 | 0.144 |
| C 4 | 0.12 | 0.12 | 0.12 | 0.12 | 0.14 | 0.14 | 0.14 | 0.129 |
| C 5 | 0.10 | 0.09 | 0.10 | 0.08 | 0.10 | 0.11 | 0.11 | 0.098 |
| C 6 | 0.07 | 0.07 | 0.07 | 0.06 | 0.06 | 0.07 | 0.11 | 0.075 |
| C 7 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.04 | 0.05 | 0.052 |

Table 3.27: Priority Weight Vector for Ratings of Equipment Obsolescence

| Ratings of EO | Very High | High | Medium | Low | Negligible | Weights |
|---|---|---|---|---|---|---|
| Very High | 0.419 | 0.444 | 0.405 | 0.412 | 0.385 | 0.413 |
| High | 0.210 | 0.222 | 0.243 | 0.235 | 0.231 | 0.228 |
| Medium | 0.168 | 0.148 | 0.162 | 0.176 | 0.154 | 0.162 |
| Low | 0.120 | 0.111 | 0.108 | 0.118 | 0.154 | 0.122 |
| Negligible | 0.084 | 0.074 | 0.081 | 0.059 | 0.077 | 0.075 |

Table 3.28: Priority Weight Vector for Ratings of Spares Unavailability

| Ratings of SU | Very High | High | Medium | Low | Negligible | Weights |
|---|---|---|---|---|---|---|
| Very Poor | 0.364 | 0.375 | 0.353 | 0.367 | 0.348 | 0.361 |
| High | 0.242 | 0.250 | 0.265 | 0.245 | 0.261 | 0.253 |
| Medium | 0.182 | 0.167 | 0.176 | 0.184 | 0.174 | 0.177 |
| Low | 0.121 | 0.125 | 0.118 | 0.122 | 0.130 | 0.123 |
| Negligible | 0.091 | 0.083 | 0.088 | 0.082 | 0.087 | 0.086 |

Based on the pairwise comparison matrix derived from the relative comparison between the categories and ratings, its priority weight vectors for the goal "Likelihood of Failure" are tabulated below. The AHP model applies to only levels in which the category is greater than 3. If it is just 2, we can find the weights directly using the AHP Properties.

Table 3.29: Priority Weight Vector for Categories of Likelihood of Failure

| **Categories of Likelihood of Failure** | **EC** | **EE** | **ED** | **Weights** |
|---|---|---|---|---|
| EC | 0.545 | 0.545 | 0.545 | 0.545 |
| EE | 0.273 | 0.273 | 0.273 | 0.273 |
| ED | 0.182 | 0.182 | 0.182 | 0.182 |

Table 3.30: Priority Weight Vector for Ratings of Equipment Condition

| **Ratings of EC** | **Very Poor** | **Poor** | **Satisfactory** | **Good** | **Very Good** | **Weights** |
|---|---|---|---|---|---|---|
| Very Poor | 0.355 | 0.369 | 0.375 | 0.316 | 0.308 | 0.345 |
| Poor | 0.237 | 0.246 | 0.281 | 0.253 | 0.192 | 0.242 |
| Satisfactory | 0.178 | 0.164 | 0.188 | 0.253 | 0.231 | 0.203 |
| Good | 0.142 | 0.123 | 0.094 | 0.127 | 0.192 | 0.136 |
| Very Good | 0.089 | 0.098 | 0.063 | 0.051 | 0.077 | 0.075 |

Based on the pairwise comparison matrix and priority vector, the eigenvalue is calculated to compute the consistency ratio from the evaluated questionnaire, as discussed in Section 3.2.5. The consistency ratio is verified to be below 0.1, indicating acceptable consistency. Consequently, the weights for the goals "Consequence" and "Likelihood of Failure" are tabulated and presented in Table 3.31 and 3.32, respectively.

The consistency of the nuclear ranking system is checked for all the pairwise comparison matrices, and the final set of weights for the goals are given in table 3.31 and 3.32.

Table 3.31: Weight Summary - Consequence

| S.No | Categories/Sub-Categories | Weights |
|---|---|---|
| | **CONSEQUENCE** | |
| **1** | **Nuclear Safety** | 0.41 |
| 1.1 | S1 | 0.44 |
| 1.2 | S2 | 0.32 |
| 1.3 | S3 | 0.15 |
| 1.4 | S4 | 0.09 |
| **2** | **Production Loss** | 0.29 |
| 2.1 | PL 1 | 0.32 |
| 2.2 | PL 2 | 0.22 |
| 2.3 | PL 3 | 0.13 |
| 2.4 | PL 4 | 0.10 |
| 2.5 | PL 5 | 0.07 |
| 2.6 | PL 6 | 0.06 |
| 2.7 | PL 7 | 0.04 |
| 2.8 | PL 8 | 0.04 |
| 2.9 | PL 9 | 0.03 |
| **3** | **Cost, Conventional Safety and Environment** | 0.20 |
| 3.1 | C 1 | 0.29 |
| 3.2 | C 2 | 0.21 |
| 3.3 | C 3 | 0.14 |
| 3.4 | C 4 | 0.13 |
| 3.5 | C 5 | 0.10 |
| 3.6 | C 6 | 0.08 |
| 3.7 | C 7 | 0.05 |
| **4** | **Ease of Repairability** | 0.10 |
| 4.1 | **Equipment Obsolescence** | 0.60 |
| 4.1.1 | Very High | 0.41 |
| 4.1.2 | High | 0.23 |
| 4.1.3 | Medium | 0.16 |
| 4.1.4 | Low | 0.12 |
| 4.1.5 | Negligible | 0.08 |
| 4.2 | **Spares Unavailability** | 0.40 |
| 4.2.1 | Very High | 0.36 |
| 4.2.2 | High | 0.25 |
| 4.2.3 | Medium | 0.18 |
| 4.2.4 | Low | 0.12 |
| 4.2.5 | Negligible | 0.09 |

Table 3.32: Weight Summary - Likelihood of Failure

| S.No | Categories | Weights |
|---|---|---|
| **LIKELIHOOD OF FAILURE** | | |
| **1** | **Equipment Condition** | 0.55 |
| 1.1 | Very Poor | 0.35 |
| 1.2 | Poor | 0.24 |
| 1.3 | Satisfactory | 0.20 |
| 1.4 | Good | 0.14 |
| 1.5 | Very Good | 0.08 |
| **2** | **Equipment Environment** | 0.27 |
| 2.1 | Severe | 0.56 |
| 2.2 | Mild | 0.44 |
| **3** | **Equipment Demand** | 0.18 |
| 3.1 | High Duty Cycle | 0.52 |
| 3.2 | Low Duty Cycle | 0.48 |

The weights are then used for calculating the overall risk rating. The following formula represents the overall risk rating based on the category weights and ratings from the AHP Model.

$$OverallRiskRating\ (original) = \left(\sum_{i=1}^{N}\left(W_i \cdot \sum_{j=1}^{M_i}(w_{ij} \cdot R_{ij} \cdot 100)\right)\right) \times \left(\prod_{k=1}^{P}(S_k \cdot R_k)\right) \tag{3.16}$$

**Normalized Overall Risk Rating**

$$R = \left(\frac{\left(\sum_{i=1}^{N}\left(W_i \cdot \sum_{j=1}^{M_i}(w_{ij} \cdot R_{ij} \cdot 100)\right)\right) \times \left(\prod_{k=1}^{P}(L_k \cdot R_k)\right) - a}{b - a}\right) \times 100 \tag{3.17}$$

**Note:** If the normalized overall risk rating is high, then it means that the task or alternate is of higher importance and ranked accordingly.

Where the terms are defined as:

1. $W_i$: Weight of the $i$-th category in Consequence.
2. $w_{ij}$: Weight of the $j$-th sub-category within the $i$-th category of Consequence. If there are no sub-categories within the category, $w_{ij} = 1$.

3. $R_{ij}$: Rating of the $j$-th sub-category within the $i$-th category of Consequence. If there are no sub-categories within the category, $R_{ij}$ represents the rating of the category.

4. $S_k$: Weight of the $k$-th Likelihood of Failure category.

5. $R_k$: Rating of the $k$-th Likelihood of Failure.

6. $N$: Total number of categories (Consequence).

7. $M_i$: Number of sub-categories within the $i$-th category. If there are no sub-categories, $M_i = 1$.

8. $P$: Total number of categories (Likelihood of Failure).

9. $a$: Normalization factor - the minimum possible value of eq. 3.16.

10. $b$: Normalization factor - the maximum possible value of eq. 3.17.

## 3.4 Demonstration of the AHP Risk Ranking System

To understand the application of the weights and the aggregation method to rank tasks within the nuclear power plant, the following five different tasks with defined category and sub-category ratings per our hierarchy are used. The tasks used below as examples are obtained from internal reports of the nuclear industry to compare the efficiency of the model.

**Tasks considered for the risk ranking system:**

1. Replace level transmitters with a new technology. (Tritium Removal Facility)

2. Procure and replace Heat Exchanges. (High-Temperature Circuit)

3. Replace ball screw and crane limit switches. (Regulated Asset Base and Carriage system)

4. Replace reverse rotation devices. (Main Boiler Feed Pump)

5. Internal inspection of the circulatory water system. (Water Circulation System)

Table 3.33: Evaluated Ratings of different systems in a given task

| Tasks | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **Consequence** | | | | | |
| Nuclear Safety | S2 | S2 | S1 | S3 | S2 |
| Production Loss | PL2 | PL2 | PL3 | PL6 | PL2 |
| Cost, Conventional Safety and Environment | C3 | C6 | C1 | C3 | C3 |
| Equipment Obsolescence | High | High | Very High | Medium | High |
| Spares Unavailability | High | High | Very High | Medium | High |
| **Likelihood of Failure** | | | | | |
| Equipment Condition | VP | P | S | VP | S |
| Equipment Environment | Mild | Mild | Mild | Mild | Severe |
| Equipment Demand | L | L | H | H | L |

The formulations discussed in this chapter are used to evaluate the above tasks, and the results are discussed below.

The rating of goal "Consequence" is multiplied by 100 throughout to ensure better range and representation. The weight of the category "Ease of Repairability" is included within its sub-categories and denoted directly for easier understanding. The updated ratings and category weights for each task are shown in Table 3.34. The overall risk rating is computed using Equation 3.17 and ranked from 1 to 5 based on this rating, as shown in Table 3.35. Rank 1 indicates that the item is critical and should be prioritized. This process effectively prioritizes various factors, enhancing decision-making accuracy for effective asset management. In spite of the ranks being of importance, the decision makers can dig deep by taking the tasks with high overall risk ratings and understand the reason behind such high scores to make a more informed decision.

Table 3.34: Weights of Category and its ratings of the systems based on the given tasks

| **Tasks** | | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|
| **Category / Sub-Category** | **Weights** | **Ratings of Category / Sub-Category** | | | | |
| Nuclear Safety | 0.41 | 31.78 | 31.78 | 44.27 | 14.76 | 31.78 |
| Production Loss | 0.29 | 21.58 | 21.58 | 13.36 | 5.55 | 21.58 |
| Cost, Conventional Safety and Environment | 0.20 | 14.44 | 7.46 | 29.37 | 14.44 | 14.44 |
| Equipment Obsolescence | 0.06 | 22.82 | 22.82 | 41.31 | 16.17 | 22.82 |
| Spares Unavailability | 0.04 | 25.26 | 25.26 | 36.14 | 17.65 | 25.26 |
| Component Condition | 0.55 | 0.34 | 0.24 | 0.20 | 0.34 | 0.20 |
| Component Environment | 0.27 | 0.44 | 0.44 | 0.44 | 0.44 | 0.56 |
| Component Demand | 0.18 | 0.48 | 0.48 | 0.52 | 0.52 | 0.48 |

Table 3.35: Overall Risk Rating based on the AHP Ranking Method

| **Tasks** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| **Consequence** | | | | | |
| Nuclear Safety | 13.05 | 13.05 | 18.18 | 6.06 | 13.05 |
| Production Loss | 6.27 | 6.27 | 3.88 | 1.61 | 6.27 |
| Cost, Conventional Safety and Environment | 2.88 | 1.48 | 5.86 | 2.88 | 2.88 |
| Equipment Obsolescence | 1.37 | 1.37 | 2.47 | 0.97 | 1.37 |
| Spares Unavailability | 1.01 | 1.01 | 1.44 | 0.70 | 1.01 |
| **Likelihood of Failure** | | | | | |
| Component Condition | 0.19 | 0.13 | 0.11 | 0.19 | 0.11 |
| Component Environment | 0.12 | 0.12 | 0.12 | 0.12 | 0.15 |
| Component Demand | 0.09 | 0.09 | 0.10 | 0.10 | 0.09 |
| **Overall Risk Rating (Normalized)** | | | | | |
| Overall Risk Ratings | 46.55 | 29.86 | 38.52 | 24.17 | 33.45 |
| Rank | 1 | 4 | 2 | 5 | 3 |

## 3.5 Concluding Remarks:

In the proposed approach, the concepts and formulations to calculate weights using qualitative data are discussed in this chapter. The Nuclear Risk Ranking system's hierarchy is introduced, and the weights of the categories are evaluated using AHP, and the application of the same is showcased using an example.

# Chapter 4

# Coverage and Sensitivity Analysis

coverage of the model should be assessed by examining the cardinality (count) of data in a specific bin. The more data points contained within the bounds, the better. Empty bins indicate inefficiency. High cardinality results in higher coverage, indicating that the aggregation method used for the ranking system is legitimate and effective for the management process.

The sensitivity of a ranking system helps in understanding the impact of various inputs on the output variable. It can be used to predict the outcome based on specific ranges of the input variable. If the output variable is constant for various input ratings, the output variable is less sensitive to the input ratings and vice versa.

The distribution a ranking system defines the extent at which the model captures and represents the full spectrum of possible ratings and the same is used to visualize the sensitivity of the various categories.

## 4.1 Simulation to analyze the sensitivity and coverage of the Ranking System

Evaluate the distribution of the overall risk ratings for the proposed AHP model by following the sequence of operations shown in Figure 4.1. Each step in the simulation process is critical. The iteration process within the simulation is explained below, along with the assumptions made for the analysis is given below.

**Steps in every iteration:**

1. Index of the category weights and ratings of both Consequence and Likelihood of Failure is generated through a discrete random sampling technique, where each rating and weight is equally likely to be selected. (Assumption: All the ratings are equally likely to occur)

2. Select the corresponding weights and rating of the category with the simulated indices.
3. Compute the Consequence by summing the weighted ratings and multiplying the sum by 100 as given in eq. 3.16.
4. Compute the Likelihood of Failure by evaluating the product of its weighted ratings as given in eq. 3.16 .
5. Normalized overall risk rating is calculated using the eq. 3.17.
6. Store all the values in the predefined variables for further visualization and analysis.

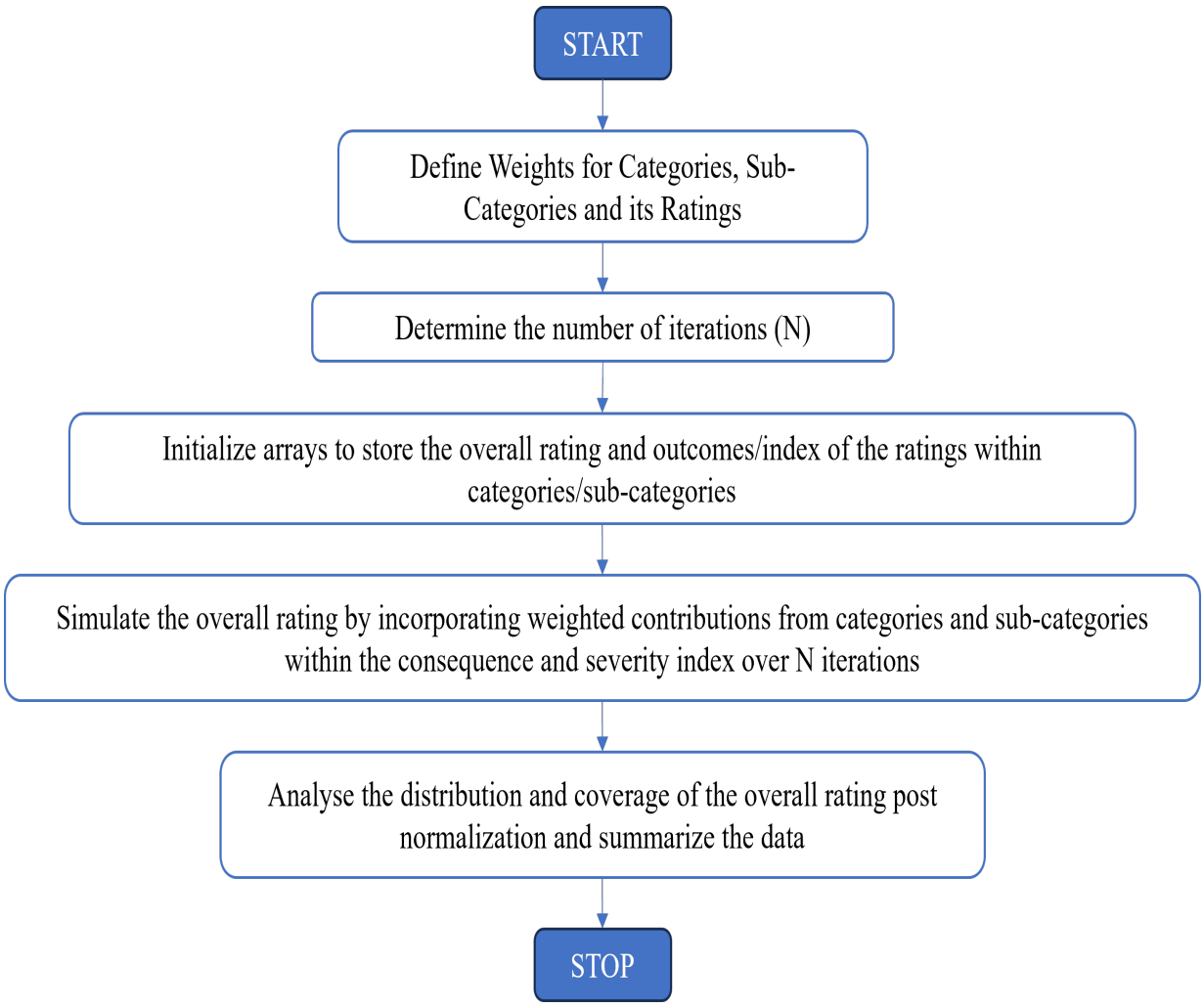



Figure 4.1: Simulation Model to analyze the distribution of the overall risk rating

The variable that stored the possible risk rating of 'N' various simulated risks is used to plot the distribution of the overall risk rating.

Example: In the simulation, during each iteration, the indices of all the ratings are generated using a discrete random sampling technique. For instance, in the first iteration, the ratings might be as follows: 1 for nuclear safety, 2 for production loss, 1 for cost, conventional safety and environment, 2 for equipment obsolescence, 3 for spares unavailability, 2 for equipment condition, 1 for equipment demand, and 2 for equipment environment. This means that the inputs for the simulated tasks are S1, P2, C1, EO2, SU3, Poor, Heavy Duty Cycle, and Mild. Based on these inputs, the aggregation method calculates the overall risk rating, which is then stored in a variable. This process is repeated for N iterations to plot the distribution of the overall risk rating.

## 4.2 Comparison of the Results: AHP Method and Delphi Method

The AHP model must be validated based on its coverage and specificity [49]. The overall risk rating of the ranking system using the Delphi method is compared with the AHP method.

Though the current hierarchy and practices used in nuclear plant aging management differ, the proposed hierarchy is utilized to obtain the absolute weights in consultation with a nuclear plant expert. The weights obtained heavily depend on the existing ranking scheme evaluated using the Delphi method, as discussed in subsection 2.2.1. These weights are accepted and used for comparison with the AHP Model. Table 4.1 presents the current industry ratings for the defined hierarchy.

Table 4.1: Ratings of the categories (Delphi Method)

| **Category** | **Ratings** |
|---|---|
| Nuclear Safety | $\{30, 20, 10, 0\}$ |
| Production Loss | $\{30, 26, 22, 18, 14, 10, 6, 3, 0\}$ |
| Cost, Conventional Safety and Environment | $\{40, 35, 25, 20, 15, 10, 0\}$ |
| Equipment Obsolescence | $\{40, 30, 20, 10, 0\}$ |
| Spare Unavailability | $\{20, 15, 10, 5, 0\}$ |
| Equipment Condition | $\{5, 4, 3, 2, 1\}$ |
| Equipment Environment | $\{1.25, 1\}$ |
| Equipment Demand | $\{1.1, 1\}$ |

The simulation to calculate the overall distribution is carried out for both the Delphi and AHP Methods. The results are shown in Figure 4.2 and Figure 4.3. Summarize the data to provide the distribution's mean, median, mode, and skewness.

**Observations of the Distributions**

**Delphi Model**

Mean: 25.79 ; Median: 23.18 ; Mode: 30.00 ; Skewness: 0.77 ;

- The distribution shows a positive skew with a long tail extending towards higher ratings.
- Both the mean and median are lower, indicating that many ratings are clustered at a lower value to drag the parameters to the lower percentile.
- The mode is higher than the mean and median, which might be due to the multi-modal distribution nature of the distribution.
- The mean and median are relatively close, indicating a somewhat symmetric distribution around the lower ratings but with a skew towards higher values.

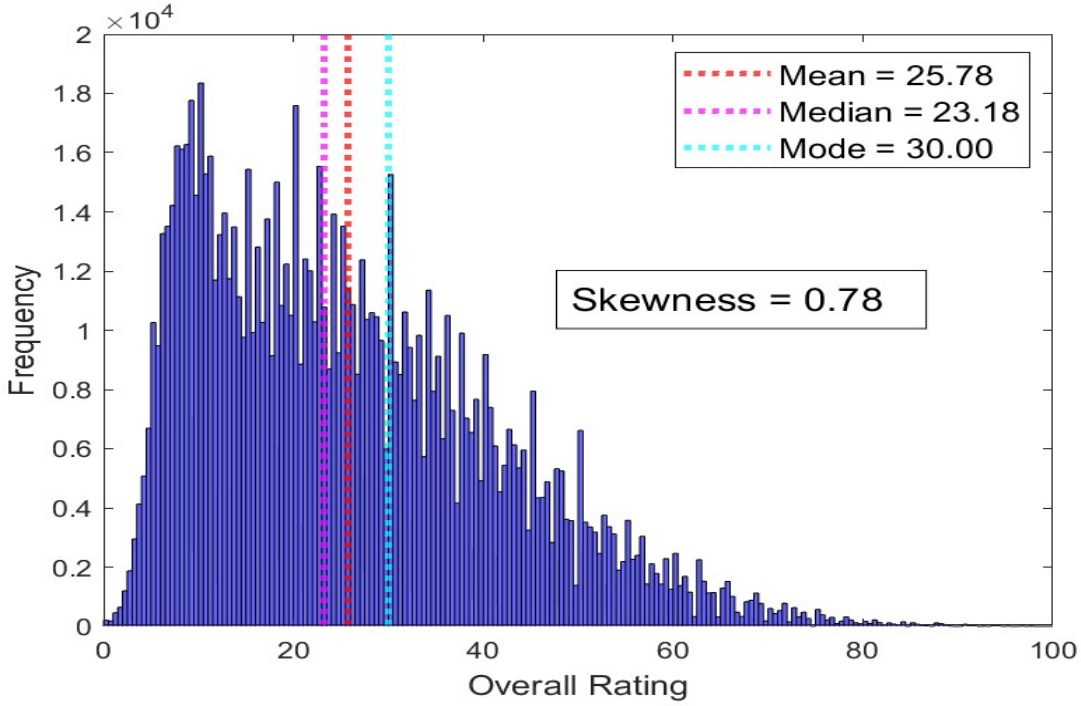



Figure 4.2: Distribution of Overall Risk Rating (Delphi Method)

**AHP Model**

Mean: 22.40 ; Median: 18.84 ; Mode: 38.20 ; Skewness: 1.06

- The distribution also shows a positive skew with a long tail extending towards higher ratings.
- The mode is significantly higher than both the mean and median, indicating that many ratings are clustered at a lower value, but there are notable occurrences of higher ratings.

- The mean and median are relatively close, indicating a somewhat symmetric distribution around the lower ratings but with a skew towards higher values.

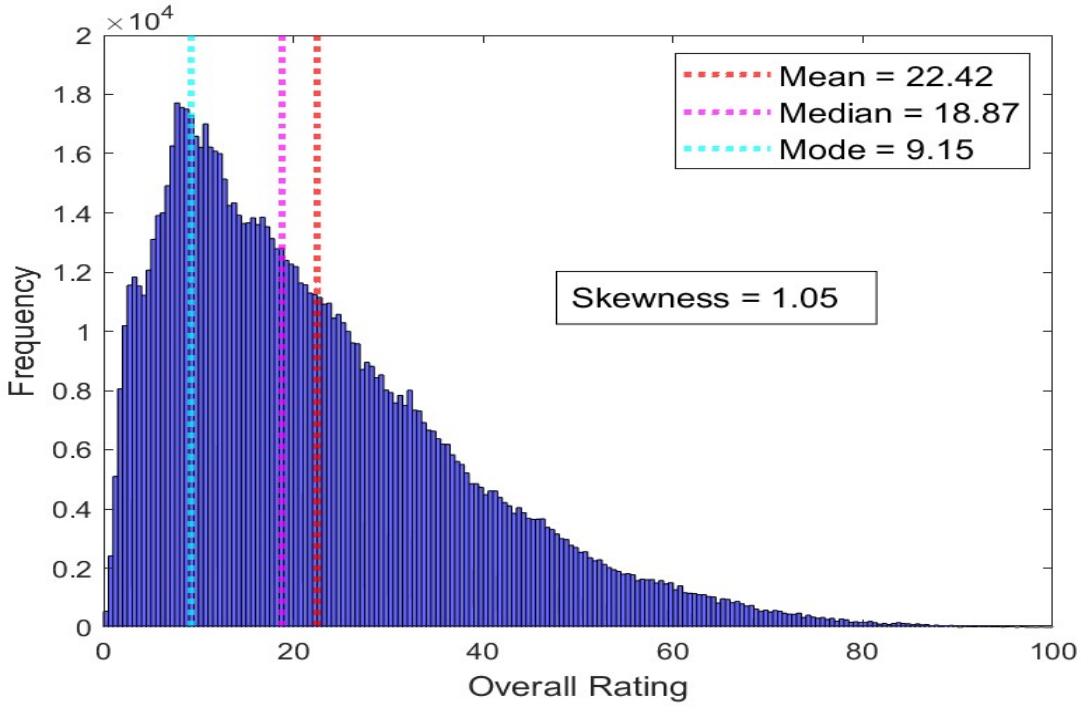



Figure 4.3: Distribution of Overall Risk Rating (AHP Method)

**Interpretations of the Distributions**

- The finer granularity of the AHP model is evident when compared to the Delphi model, particularly in the lower ratings due to the higher number of elements within each bin of the histograms. Additionally, the AHP model has fewer empty bins in the higher ratings compared to the Delphi method, indicating its high coverage in these higher ratings.
- Higher right skewness in the AHP model denotes that a significant concentration of iterations falls in the lower ratings, allowing it to differentiate more effectively between items in the lower rating range. This is also supported by the separation of central tendency measures.
- The higher mode values pull the average to the higher end.
- The Delphi method is relatively more balanced, as its skewness is less when compared to the AHP Model.

## 4.3 Sensitivity Analysis

Perform sensitivity analysis by observing changes in the overall risk rating when varying the rating of a specific category or sub-category. Filter the outcomes of a particular rating from the simulation results and study how the overall risk rating responds to understand the sensitivity of that specific input.

Example: To understand the sensitivity of the overall risk rating due to S1, the N simulated outcomes from the simulation should be filtered such that the index of the nuclear safety category in that iteration is 1. The distribution of these filtered outcomes is then analyzed to understand how the chosen variable affects the overall risk rating.

Figure 4.4 to 4.11 gives the visualization of the sensitivity analysis on the overall risk rating based on every single rating under each category and sub-category.

**Key Insights of the Sensitivity Analysis:**

- Nuclear Safety remains a top priority, with the highest overall risk ratings requiring immediate attention among the categories of the goal "Consequence" and Equipment Condition for the goal "Likelihood of Failure".
- Equipment Demand and Environment also play a crucial role in determining the overall risk ranking.
- Production Loss, Cost, Safety, Environment, Equipment Obsolescence, and Spares Unavailability provide critical insights, as their sensitivity is subtle yet significant. These factors can help prioritize resource allocation and mitigation strategies in the lower risk rating scale due to the high specificity in that region.

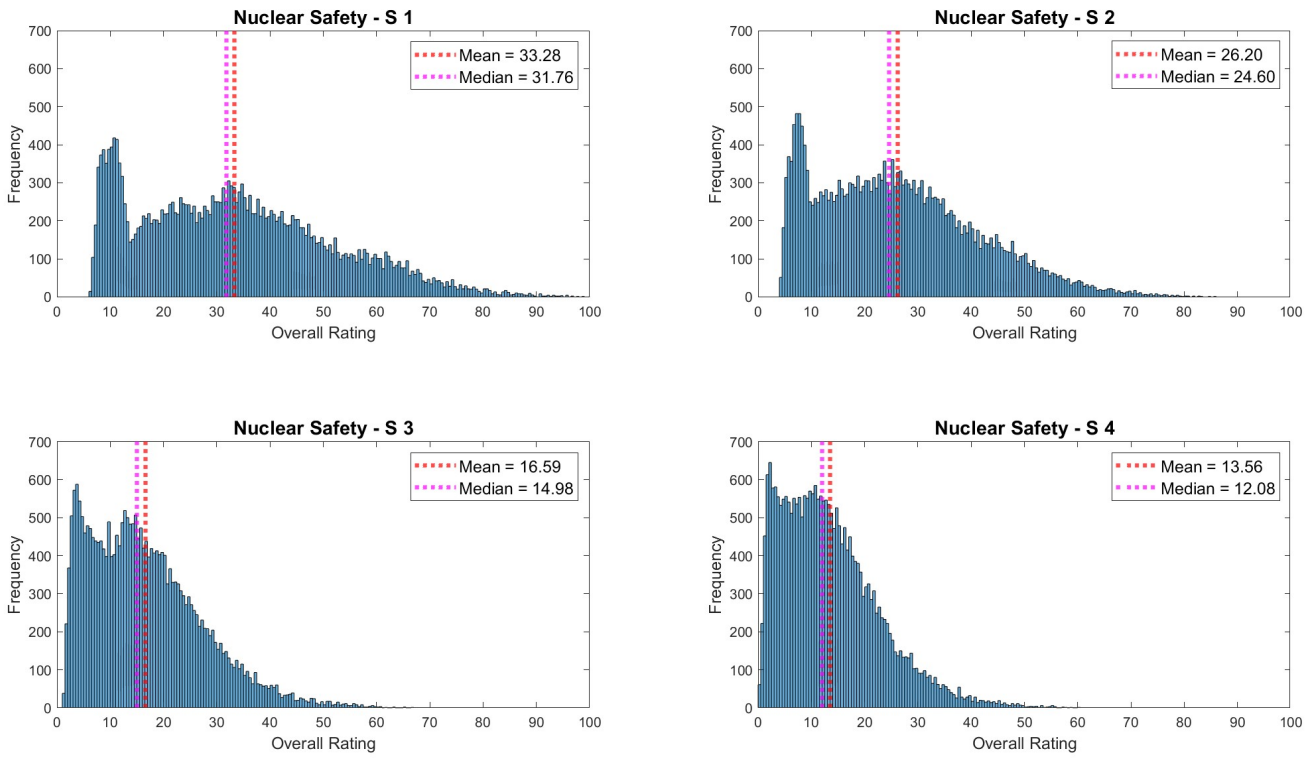



Figure 4.4: Sensitivity Analysis for the category - Nuclear Safety

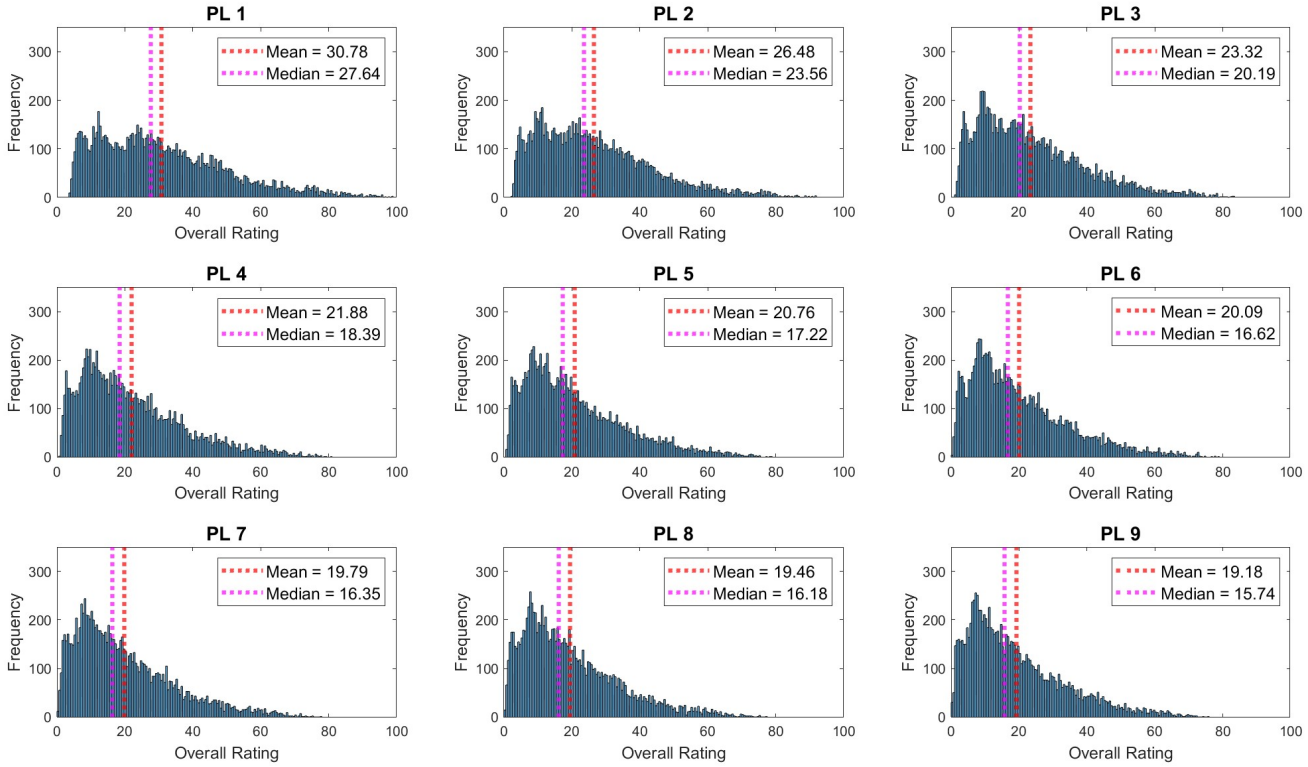



Figure 4.5: Sensitivity Analysis for the category - Production Loss

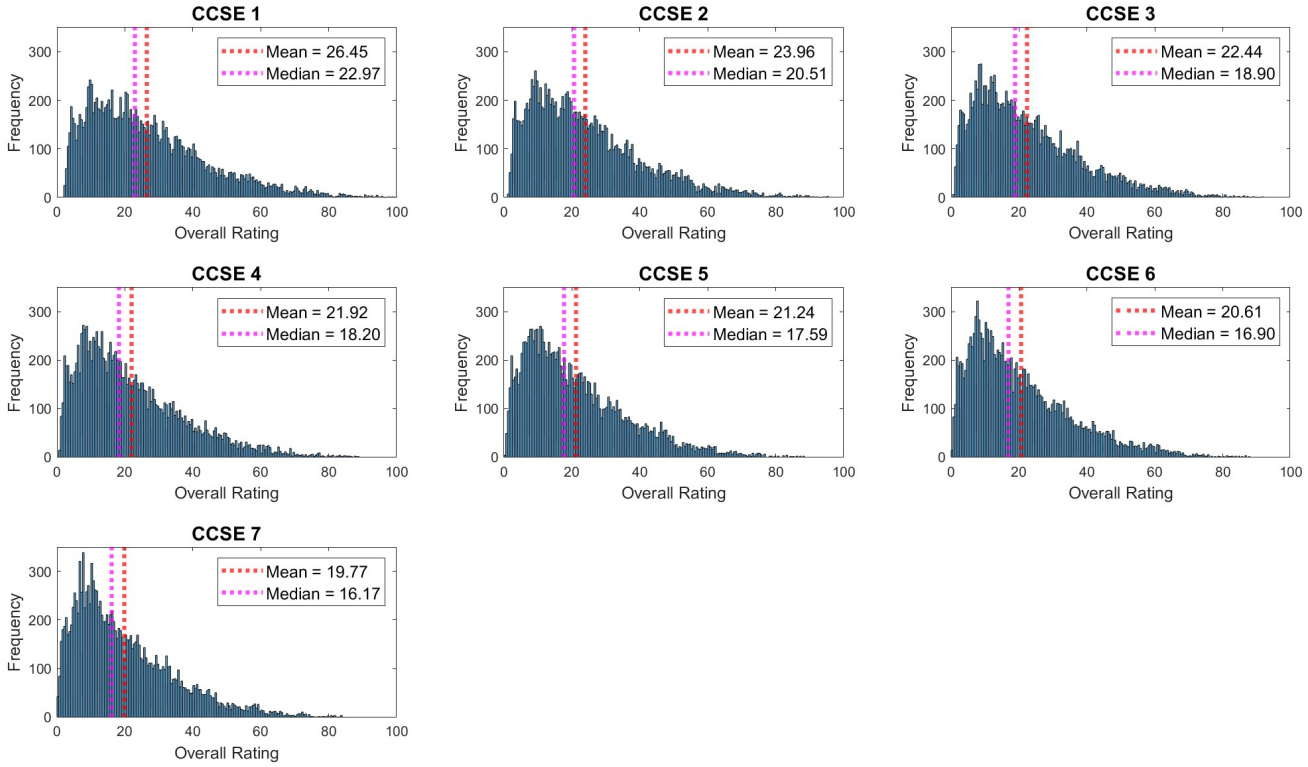



Figure 4.6: Sensitivity Analysis for the category - Cost, Conventional Safety and Environment

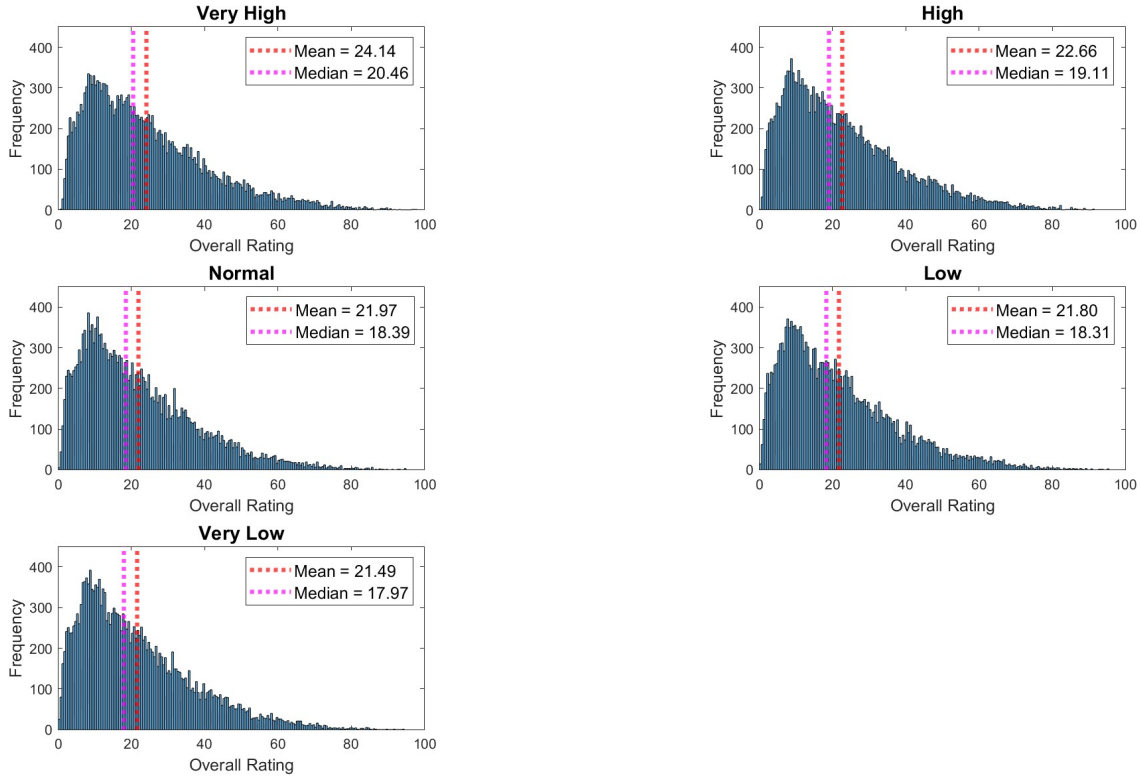



Figure 4.7: Sensitivity Analysis for the sub-category - Equipment Obsolescence

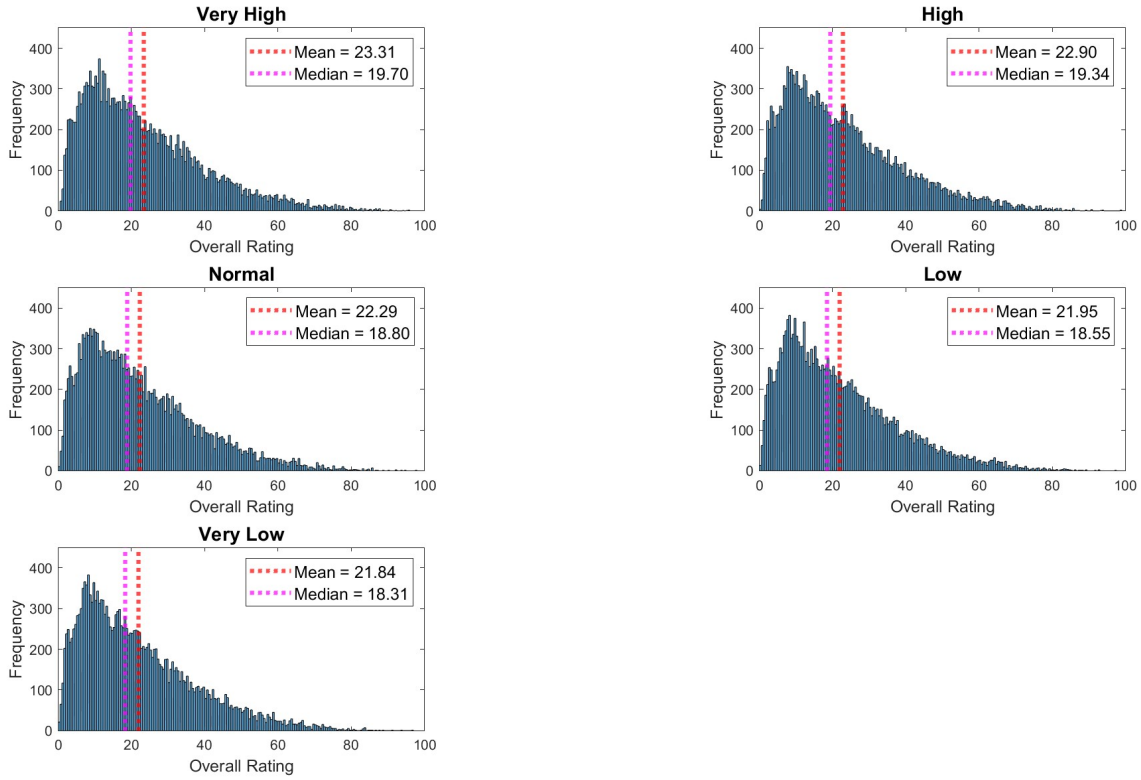



Figure 4.8: Sensitivity Analysis for the sub-category - Spares Unavailability

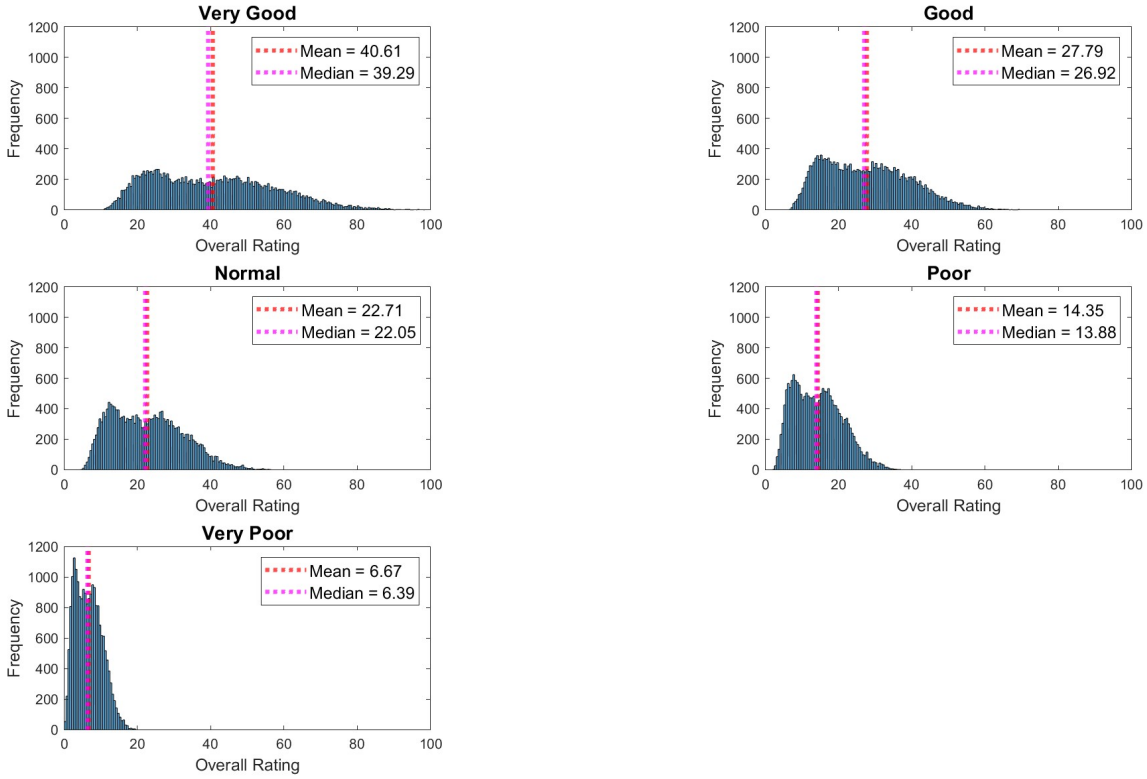



Figure 4.9: Sensitivity Analysis for the category - Equipment Condition

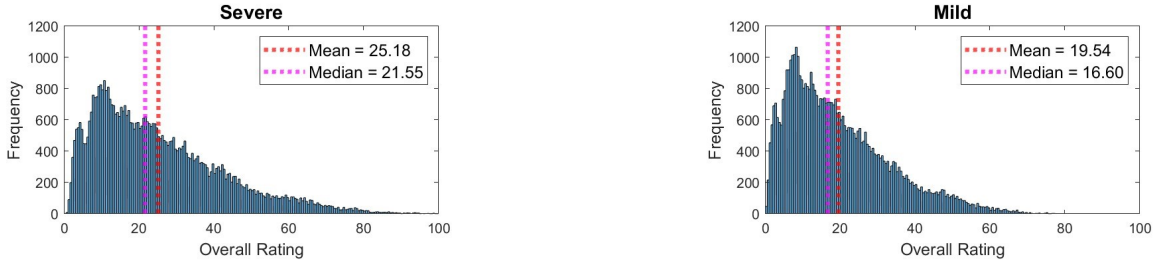



Figure 4.10: Sensitivity Analysis for the category - Equipment Environment

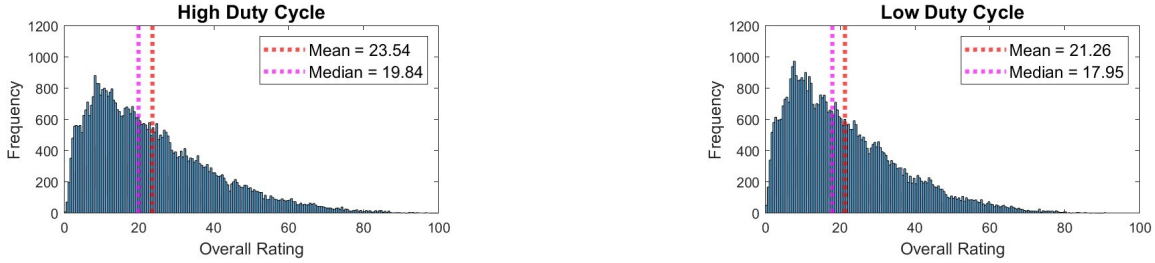



Figure 4.11: Sensitivity Analysis for the category - Equipment Demand

## 4.4 Concluding Remarks

This chapter analyzes the sensitivity of the AHP Risk ranking system on its own. A comparison between the existing method and the proposed method is presented to highlight the advantages of the new model. Additionally, the chapter underscores the importance of evaluating the equipment's condition and its crucial role in assessing the overall risk rating of a task. To reduce bias and uncertainty, equipment condition is assessed using fuzzy logic, and the weights of the ratings are calculated accordingly.

# Chapter 5

# Fuzzy Logic

## 5.1 Introduction

Fuzzy Logic is a logical system that aims to formalize approximate reasoning in a narrow sense [50]. Lotfi Zadeh first developed it in the 1960s and extended the traditional Boolean Logic, which operates on binary values like true or false, to handle the concept of partial truth. However, it is quite different from traditional multivalued logical concepts. In particular, such key concepts in Fuzzy Logic as the concept of a linguistic variable, canonical form, fuzzy if-then rule, fuzzy quantification and defuzzification, predicate modification, truth qualification, the extension principle, the compositional rule of inference and interpolative reasoning, among others, are not addressed in traditional systems [51]. It is instrumental in systems where the input might be imprecisely or ambiguously defined, making it ideal for dealing with uncertain or qualitative information. These are the reasons for the wide range of applications of Fuzzy Logic compared to any other traditional system.

## 5.2 Advantages of Fuzzy Logic

1. **Tolerance:** Fuzzy logic is a robust methodology that functions effectively without precise and noise-free inputs. This attribute renders it highly applicable in various real-world scenarios, particularly those characterized by significant uncertainty and data irregularities.

2. **Flexibility:** Fuzzy Logic can model the qualitative dimensions of human knowledge and logical reasoning without relying on precise quantitative analyses. This capability makes it highly adaptable to diverse and novel inputs, enhancing its applicability in dynamic environments.

3. **Simplicity of Design:** The design of fuzzy logic systems is usually simpler and more intuitive because it replicates human reasoning and decision-making, which are often

comprised of linguistic(verbal) and imprecise inputs. Consequently, these systems are typically easier to develop and maintain.

4. **Suitability for Real-Time Operations:** Intensive numerical computations are not required by fuzzy logic algorithms, inherently enhancing their efficiency. As a result, they are well-suited for real-time operations in applications such as control systems and decision-making processes.

5. **Integration with Other Methods:** Fuzzy Logic can be seamlessly integrated with various decision-making and predictive models, demonstrating its versatility across various applications.

Fuzzy Logic has been widely applied in control systems, automotive systems, consumer electronics, and more, demonstrating its versatility and effectiveness in various applications.

## 5.3 Fuzzy Sets

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function that assigns to each object a grade of membership ranging between zero and one [50]. There are various definitions. It can also be defined as:

**Definition.**

A fuzzy set $A$ within a universe of discourse $X$ is defined by a membership function $\mu_A(x)$ which assigns to each element $x$ in $X$ a degree of membership ranging from 0 to 1. This function quantifies how much $x$ is considered part of the set $A$.

Mathematically, the fuzzy set $A$ can be represented as:

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

Here:

1. $x$ is an element of the universe $X$,

2. $\mu_A(x)$ (also represented as $f_A(x)$) is the membership function of the fuzzy set $A$, which maps $x$ to a value between 0 and 1.

The fuzzy sets are broadly classified into two types, and they are described below.

### Type-1 Fuzzy Sets

Type-1 fuzzy sets are the primary form of fuzzy sets where each element has a crisp degree of membership between 0 and 1. These sets are widely used for their simplicity and effectiveness in various applications.

A Type-1 fuzzy set, denoted by $A$ and represented by a Type-1 fuzzy set membership function $\mu_A(x)$, where $x \in X$ and $X$ is the universe of discourse, is defined as:

$$A = \{(x, \mu_A(x)) \mid x \in X\},$$

in which $0 \leq \mu_A(x) \leq 1$. $A$ can be expressed as:

$$A = \bigcup_{x \in X} \{(x, \mu_A(x))\},$$

where $\bigcup$ denotes union over all admissible $x$.

For Example:

Consider a fuzzy set $T$ representing "High Temperature." We define $T$ with a triangular membership function:

$$\mu_T(x) = \max\left(0, \min\left(\frac{x-70}{30}, 1, \frac{110-x}{20}\right)\right)$$

Here, $x$ represents the temperature in degrees Celsius. This function assigns a membership degree between 0 and 1 to temperatures, where values close to 90°C have higher degrees of membership.

### Type-2 Fuzzy Sets

Type-2 fuzzy sets allow the degree of membership to be fuzzy, providing an additional layer of uncertainty. This makes them suitable for more complex applications where a simple fuzzy degree does not accurately capture uncertainty.

A type-2 fuzzy set, denoted by $\tilde{A}$ and represented by a type-2 fuzzy set membership function $\mu_{\tilde{A}}(x, u)$ where $x \in X$ and $u \in J_x \subseteq [0, 1]$, is defined as

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) \mid \forall u \in J_x \subseteq [0, 1]\},$$

in which $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$. $\tilde{A}$ can be expressed as

$$\tilde{A} = \bigcup_{x \in X} \bigcup_{u \in J_x} \{(x, u), \mu_{\tilde{A}}(x, u)\},$$

where $\bigcup$ denotes union over all admissible $x$ and $u$.

For Example:

Define $U$ as a type-2 fuzzy set for "Very High Temperature," where the membership degree for each temperature $x$ is a fuzzy set:

$$\mu_U(x) = \left[\exp\left(-\frac{(x-90)^2}{2 \cdot 15^2}\right), \exp\left(-\frac{(x-90)^2}{2 \cdot 10^2}\right)\right]$$

In this example, the lower and upper bounds of the membership function are defined using Gaussian functions centered at 90°C with different spreads. This interval represents a range of possible membership values, encapsulating the uncertainty in determining how "very high" a temperature is.

## 5.4 Fuzzy Variables

Fuzzy variables are variables whose values are fuzzy sets. They are used in fuzzy logic systems to represent linguistic terms and qualitative descriptors. By using fuzzy variables, the systems can process input and output data that are inherently imprecise or vague.

**Definition:** A linguistic variable $V$ is characterized by its name $x$, an universe $U$, a term set $T(x)$, a syntactic rule $G$ for generating names of values of $x$, and a set of semantic rule $M$ for associating with each value its meaning.

For example, if the temperature of a room is interpreted as a linguistic variable, then its term set could be $T(x) = \{\text{cold, cool, warm, hot}\}$ where each term is characterized by a fuzzy set in a universe of discourse $U = [0, 40]$ (degrees Celsius). We might interpret: *cold* as "a temperature below about 10°C", *cool* as "a temperature close to 20°C", *warm* as "a temperature close to 30°C", *hot* as "a temperature above 35°C".

## 5.5 Fuzzy Number

Zadeh introduced Fuzzy Numbers to deal with imprecise numerical quantities. A fuzzy number refers to a set of possible values where each possible value has its weight between 0 and 1 [52]. This weight is called the membership function.

**Definition:** A fuzzy number $F$ in a continuous universe $U$, e.g., a real line, is a fuzzy set $F$ in $U$ which is normal and convex, i.e.,

**Convexity:**

The membership function of a fuzzy number must produce a convex set. This means that for any two elements $u_1$ and $u_2$ in the fuzzy set and any $\lambda$ between 0 and 1, the membership value at $\lambda u_1 + (1-\lambda)u_2$ must be at least as large as the minimum of the membership values at $u_1$ and $u_2$. Mathematically, this is expressed as:

$$\mu_F(\lambda u_1 + (1-\lambda)u_2) \geq \min(\mu_F(u_1), \mu_F(u_2)), \quad u_1, u_2 \in U, \lambda \in [0,1] \quad \text{(convex)}$$

where $\mu$ denotes the membership function.

**Normality:**

The membership function must reach a value of 1 at least at one point. This property ensures that at least one element in the universe entirely belongs to the fuzzy set, representing the peak of the fuzzy number.

$$\max_{u \in U} \mu_F(u) = 1 \quad \text{(normal)}$$

A membership function is a more general concept used in Fuzzy Logic to define how each element in the universe of discourse (which can be numbers, terms, or objects) belongs to a fuzzy set. The shape of the membership function can vary depending on the context,

with common shapes being triangular, trapezoidal, and bell-shaped. These functions are crucial for defining how fuzzy sets are interpreted in the calculus of fuzzy rules applied in fuzzy logic systems. The popular membership functions are defined below.

A triangular membership function is defined by three parameters $a$, $b$, and $c$, representing the left foot, peak, and right foot of the triangle, respectively.

$$\mu_{\text{triangular}}(x; a, b, c) = \begin{cases} \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } x = b \\ \frac{c-x}{c-b} & \text{if } b < x \leq c \\ 0 & \text{otherwise} \end{cases}$$

1. When $x$ is less than or equal to $a$, the membership grade is 0.
2. As $x$ increases from $a$ to $b$, the membership grade increases linearly from 0 to 1.
3. At $x = b$, the membership grade is 1.
4. As $x$ increases from $b$ to $c$, the membership grade decreases linearly from 1 to 0.
5. When $x$ is greater than $c$, the membership grade is 0.

A trapezoidal membership function is defined by four parameters $a$, $b$, $c$, and $d$, representing the left foot, left shoulder, right shoulder, and right foot of the trapezoid, respectively.

$$\mu_{\text{trapezoidal}}(x; a, b, c, d) = \begin{cases} \frac{x-a}{b-a} & \text{if } a \leq x < b \\ 1 & \text{if } b \leq x \leq c \\ \frac{d-x}{d-c} & \text{if } c < x \leq d \\ 0 & \text{otherwise} \end{cases}$$

1. When $x$ is less than or equal to $a$, the membership grade is 0.
2. As $x$ increases from $a$ to $b$, the membership grade increases linearly from 0 to 1.
3. At $x = b$, the membership grade is 1.
4. When $x$ is between $b$ and $c$, the membership grade is 1.
5. As $x$ increases from $c$ to $d$, the membership grade decreases linearly from 1 to 0.
6. When $x$ is greater than $d$, the membership grade is 0.

.

## 5.6 Fuzzy Inference Engine

A Fuzzy Inference System can simulate human decision-making based on fuzzy concepts and inferring fuzzy control action employing fuzzy implication and the rules of inference in fuzzy logic [39]. There are various fuzzy inference models, as discussed in the literature review. Mamdani Fuzzy Inference Model is chosen for the asset prioritization and ranking using linguistic variables. The general process of the Mamdani model is shown in figure 5.1. The Mamdani fuzzy inference model is a widely used method for fuzzy logic control. It consists of the following components: Fuzzification, Rule Evaluation, Aggregation, and Defuzzification.

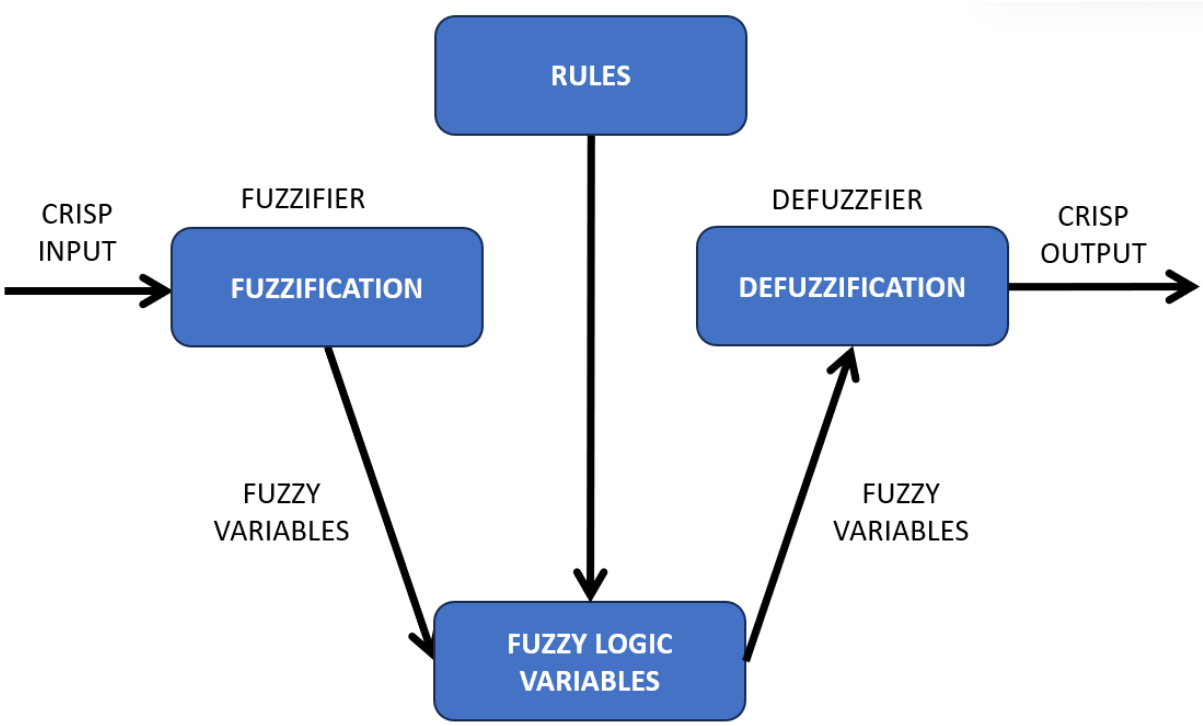



Figure 5.1: Mamdani Model flowchart

### 5.6.1 Fuzzification

Fuzzification is the process of converting a crisp input into fuzzy sets using a fuzzification operator. So, the crisp inputs are evaluated, and the degree to which they belong to each of the appropriate fuzzy sets is determined.

- **Defining Fuzzy Sets:** The fuzzy sets describe each input's degree of truth or membership levels. For example, in the case of temperature, the fuzzy set can be cold, warm, and hot.

- **Input Scaling:** The crisp inputs are scaled according to the universe of discourse for the fuzzy variable, and the scaling aligns with the range considered in the fuzzy logic system.
- **Application of Membership Function:** The scaled input (typically between 0 to 1 or any other values) is processed using a membership function that defines the degree to which the input belongs to a fuzzy set.
- **Fuzzy Output:** The result from the membership functions is a set of fuzzy values, which are the degree of membership of the input associated with each predefined fuzzy set. For example, a temperature of 70°F might yield membership values like 0.1 in Cold, 0.8 in Warm, and 0.0 in Hot.

### 5.6.2 Rule Evaluation

Fuzzy rules in Fuzzy Logic are a way to impose the subject matter expert's knowledge on the system when the decision-making environment is highly uncertain or ambiguous. The rules are similar to traditional rules. However, the elements can partially belong to multiple sets. Fuzzy rules are formulated in the form of "If-Then"[53]. The "IF" part of the rule is called the antecedent, and the "THEN" part is called the consequent. Different types of fuzzy rules are based on semantics, such as certainty rules, possibility rules, gradual rules, anti-gradual rules, and exception rules (unless rules). The definitions for the same are added in appendix B.

### 5.6.3 Aggregation

In most fuzzy inference models, the inputs might call multiple rules at an instance. Thus, aggregation is the process of combining all the fuzzy outputs into a single fuzzy set. Aggregate the fuzzy outputs of all rules to obtain a single fuzzy output set. This process entails selecting the highest membership value from the output fuzzy sets of each rule for all possible outputs. The outcome is an aggregate fuzzy set that encapsulates the total output of the system prior to converting it into a precise, actionable decision.

### 5.6.4 Defuzzification

Convert the fuzzy output set into a crisp output value.

- **Center-of-Gravity:**

  The center of gravity method is one of the most common defuzzification techniques used in Mamdani fuzzy inference systems. It calculates the crisp output value by finding the centroid (center of gravity) of the area under the fuzzy output set curve. The process is explained in figure 5.2 [54].

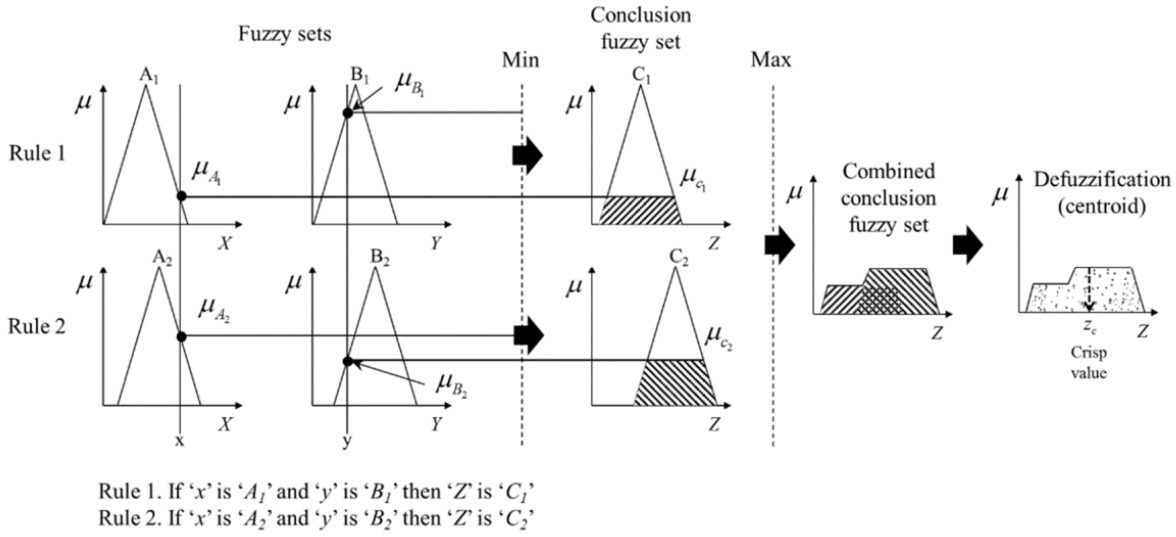



Figure 5.2: Defuzzification using Center-of-Gravity Method

**Procedure:**

1. **Calculate the Area:** Determine the area under the fuzzy output set curve.
2. **Calculate the Centroid:** Divide the area into small rectangles and find the centroid of each rectangle. Then, find the weighted average of the centroids using the area of each rectangle as the weight.
3. **Crisp Output:** The crisp output value is the x-coordinate of the centroid.

The Center-of-Gravity method computes the defuzzified output, $z_0$, as the weighted average of the universe of discourse elements. The membership function $\mu_C(z_j)$ represents the membership degree of the element $z_j$ [55]. The formula is given by:

$$z_0 = \frac{\sum_{j=1}^{N} \mu_C(z_j) z_j}{\sum_{j=1}^{N} \mu_C(z_j)}$$

- **Center of Sums (COS) Method:** This method combines the effects of multiple fuzzy sets, counting overlapping areas multiple times to capture their combined impact.

$$z^* = \frac{\sum_{k=1}^{n} \mu_k \cdot Z_k}{\sum_{k=1}^{n} \mu_k}$$

Where $\mu_k$ is the membership value for the $k$-th fuzzy set and $Z_k$ is the center of area for the $k$-th fuzzy set.

- **Center of Area / Bisector of Area Method (BOA):** Determines the defuzzified output by finding a vertical line that bisects the area under the curve of the

membership function into two halves.

$$\int_a^{z^*} \mu_C(x)\,dx = \int_{z^*}^{b} \mu_C(x)\,dx$$

Where $\mu_C(x)$ is the membership function, $a$ and $b$ are the limits of the function, and $z^*$ is the defuzzified output [55].

- **Weighted Average Method:** Computes the defuzzified value by taking the weighted average of the outputs, where weights are the maximum membership values.

$$z^* = \frac{\sum_{i=1}^{N} \mu_i \cdot x_i}{\sum_{i=1}^{N} \mu_i}$$

Where $\mu_i$ are the maximum membership values and $x_i$ are the corresponding output values.

- **Maxima Methods:** These methods focus on the maximum values of the membership functions.
  - **First of Maxima (FOM)**: Chooses the smallest domain value where the membership function reaches its maximum.

$$z^* = \min\{x \mid \mu_C(x) = \max(\mu_C)\}$$

  - **Last of Maxima (LOM)**: Chooses the most significant domain value where the membership function reaches its maximum.

$$z^* = \max\{x \mid \mu_C(x) = \max(\mu_C)\}$$

  - **Mean of Maxima (MOM)**: Calculates the average of all domain values where the membership function is at its maximum.

$$z^* = \frac{1}{N} \sum_{i=1}^{N} x_i$$

where $x_i$ are values where $\mu_C(x_i) = \max(\mu_C)$

The center-of-gravity method from the above defuzzification methods is preferred to develop the risk ranking method with the fuzzy integrated system.

General Example of Fuzzy Mamdani Model: A general fuzzy Mamdani system for temperature control involves mapping inputs (temperature and humidity) to outputs (fan speed) using fuzzy logic. In this example, the system defines input variables like temperature (with categories: low, medium, high) and humidity (low, medium, high), along with an output variable for fan speed (low, medium, high). Membership functions for these variables are defined using triangular shapes, where specific ranges represent each category.

The system employs a set of fuzzy rules, such as "If temperature is low and humidity is low, then fan speed is low," to determine the appropriate fan speed based on the input conditions. For instance, if the temperature is high and humidity is high, the rule would set the fan speed to high. The process begins with fuzzification, where crisp input values are converted to fuzzy values, enabling the system to apply the fuzzy rules and determine the necessary adjustments to fan speed, ensuring optimal room temperature control.

## 5.7 Fuzzy Integration into AHP Risk Ranking Model

The practice of subjective analysis for asset management decisions is still widely followed in the industry due to its ease and quick implementation. However, calculating the overall risk rating using equation 2.4 based on likelihood factors resembling the probability of failure is inherently flawed as they are just the subjective weights assigned to them. These methods must be flexible and incorporate quantitative data to improve their validity. Therefore, as discussed earlier, the fuzzy Mamdani model defines and evaluates equipment conditions based on various factors. This approach helps mitigate the uncertainty inherent in the category, resulting in a more refined and reliable model.

While the equipment condition is determined using various criteria, I have selected critical factors such as a component/system's age, maintenance frequency, operational performance, and material condition. These factors were considered the quantitative data for the same and are readily available in every plant for asset management. All these are defined as type 1 fuzzy sets with fuzzy variables. The fuzzy variables are defined by triangular, trapezoidal, and Gaussian membership functions.

Figure 5.8 illustrates the fuzzy sets and the respective types of membership functions used in the model.

Based on the above definition, the visualization of the fuzzy sets is given below in figure 5.3, 5.4, 5.5 and 5.6.

Below are the interpretations for a clearer understanding of the input of the fuzzy variables.

- Age increases with an increase in the input value.
- Maintenance Frequency decreases with an increase in the input value.
- Operational Performance decreases with an increase in the input value.
- Material Condition deteriorates with an increase in the input value.
- Equipment Condition worsens with the increase in the output value.

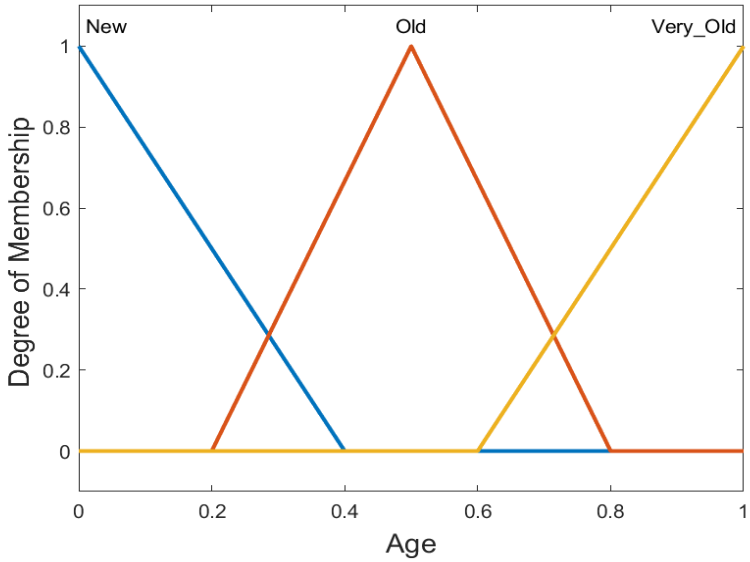



Figure 5.3: Membership Function - Age

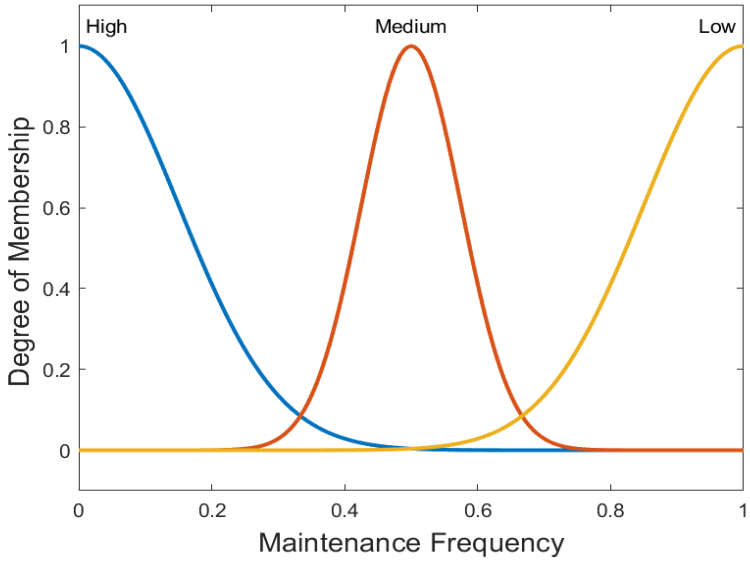



Figure 5.4: Membership Function - Maintenance Frequency

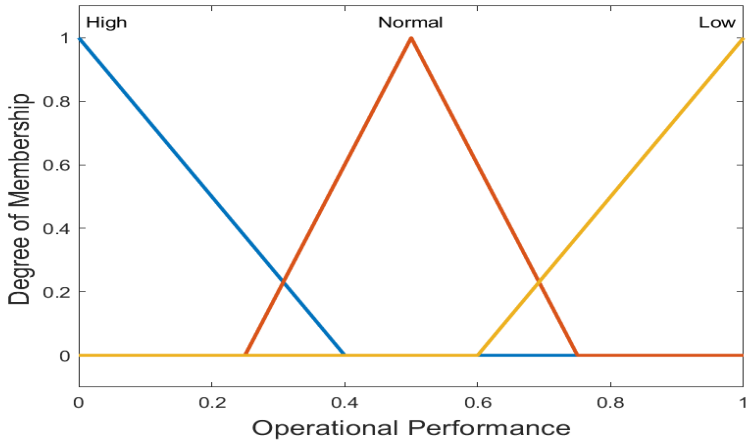



Figure 5.5: Membership Function - Operational Performance

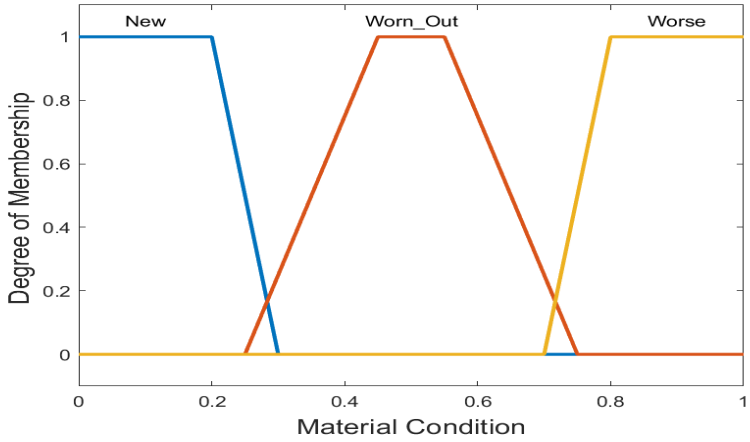



Figure 5.6: Membership Function - Material Condition

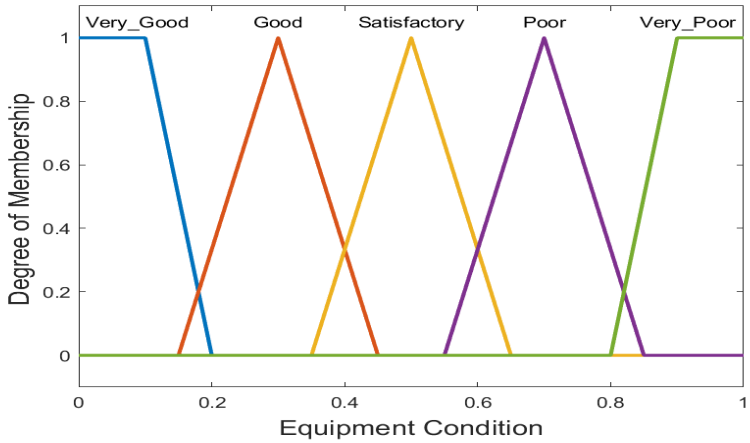



Figure 5.7: Membership Functions - Equipment Condition

**Input Fuzzy Variables and its Membership Functions**

**Age**

| | | |
|---|---|---|
| [0 0 0.1 0.4] | - | New |
| [0.2 0.5 0.8] | - | Old |
| [0.6 0.9 1 1 ] | - | Very Old |

**Maintenance Frequency**

| | | |
|---|---|---|
| [0.15 0] | - | High |
| [0.075 0.5] | - | Medium |
| [0.15 1] | - | High |

**Operational Performance**

| | | |
|---|---|---|
| [0 0 0.4] | - | High |
| [0.25 0.45 0.55 0.75] | - | Normal |
| [0.6 1 1] | - | Low |

**Material Condition**

| | | |
|---|---|---|
| [0 0 0.2 0.3] | - | New |
| [0.25 0.45 0.55 0.75] | - | Worn Out |
| [0.7 0.8 1 1] | - | Worse |

**Output Fuzzy Variable and its Membership Function**

**Equipment Condition**

| | | |
|---|---|---|
| [0 0 0.1 0.2] | - | Very Good |
| [0.15 0.3 0.45] | - | Good |
| [0.35 0.5 0.65] | - | Satisfactory |
| [0.55 0.7 0.85] | - | Poor |
| [0.8 0.9 1 1] | - | Very Poor |

Figure 5.8: Fuzzy Input and Output Variables and its respective Membership functions.

The membership functions and their boundaries are determined by referring to previously evaluated tasks according to various risks. The membership function is manually tuned by adjusting its shape and boundaries to align with the expected outputs described in internal reports. For example, the "poor" fuzzy variable should be within the range of 0.55 to 0.85 to achieve a mean of approximately 0.7. The inputs corresponding to a poor condition should, in a heuristic manner, match that outcome after applying the model. In other words, the boundaries and parameters are adjusted to reflect the data in the internal reports better.

Once the input variables are fuzzified, the fuzzy rules have to be applied to the input to create a fuzzy output as discussed in figure 5.1.

The fuzzy rules are defined as shown below for the fuzzy inference system. The rules mentioned are determined based on the possible combination of inputs in the industrial sector's daily workings. A total of 135 rules are defined based on a heuristic approach by understanding the ratings and the possible outcomes.

1. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Very Good with a weight of 1.0.

2. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the

Equipment Condition is Satisfactory with a weight of 1.0.

3. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Very Poor with a weight of 1.0.

4. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Very Poor with a weight of 0.75.

5. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Very Poor with a weight of 0.75.

6. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Very Poor with a weight of 0.75.

7. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Very Poor with a weight of 0.75.

8. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Poor with a weight of 0.25.

9. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.25.

10. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.25.

11. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.25.

12. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Poor with a weight of 0.5.

13. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.5.

14. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Poor with a weight of 0.5.

15. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Poor with a weight of 0.5.

16. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.5.

17. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.5.

18. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

19. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

20. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

21. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

22. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

23. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

24. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.25.

25. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Poor with a weight of 0.25.

26. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Poor with a weight of 0.25.

27. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Poor with a weight of 0.25.

28. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

29. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

30. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

31. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.75.

32. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Very Poor with a weight of 0.9.

33. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Very Poor with a weight of 1.0.

34. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Very Poor with a weight of 0.1.

35. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Very Poor with a weight of 0.25.

36. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.1.

37. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.9.

38. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Poor with a weight of 0.75.

39. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.1.

40. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

41. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.1.

42. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

43. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.9.

44. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Poor with a weight of 0.5.

45. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.9.

46. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Poor with a weight of 0.5.

47. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.9.

48. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

49. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Good with a weight of 0.1.

50. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

51. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

52. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.9.

53. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.1.

54. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

55. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.75.

56. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

57. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

58. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Good with a weight of 0.1.

59. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.25.

60. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.9.

61. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.25.

62. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Good with a weight of 0.25.

63. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.1.

64. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.9.

65. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.5.

66. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.25.

67. If the Age is Very Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.5.

68. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.25.

69. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

70. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

71. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

72. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

73. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.25.

74. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.25.

75. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.25.

76. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.25.

77. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Good with a weight of 0.25.

78. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Good with a weight of 0.5.

79. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Good with a weight of 0.5.

80. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.25.

81. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.5.

82. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.75.

83. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.25.

84. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.25.

85. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.25.

86. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.75.

87. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

88. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

89. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

90. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

91. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.5.

92. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Satisfactory with a weight of 0.25.

93. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

94. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

95. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

96. If the Age is Old AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.25.

97. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 1.

98. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.6.

99. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Poor with a weight of 0.25.

100. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Poor with a weight of 0.75.

101. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Poor with a weight of 0.75.

102. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Poor with a weight of 0.1.

103. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.4.

104. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.75.

105. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.25.

106. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Satisfactory with a weight of 0.25.

107. If the Age is Very Old AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Satisfactory with a weight of 0.9.

108. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Very Good with a weight of 0.5.

109. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Very Good with a weight of 0.5.

110. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Very Good with a weight of 0.5.

111. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Very Good with a weight of 0.5.

112. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Very Good with a weight of 0.5.

113. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Very Good with a weight of 0.5.

114. If the Age is Old AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.5.

115. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.5.

116. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.5.

117. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.5.

118. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.5.

119. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.5.

120. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Good with a weight of 0.75.

121. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.75.

122. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.5.

123. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.5.

124. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worse, THEN the Equipment Condition is Very Good with a weight of 0.25.

125. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Low AND the Material Condition is New, THEN the Equipment Condition is Very Good with a weight of 0.25.

126. If the Age is New AND the Maintenance Frequency is Low AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Very Good with a weight of 0.5.

127. If the Age is Very Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Very Good with a weight of 0.5.

128. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.75.

129. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.75.

130. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.75.

131. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.75.

132. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is Worn Out, THEN the Equipment Condition is Good with a weight of 0.25.

133. If the Age is New AND the Maintenance Frequency is High AND the Operational Performance is Normal AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.25.

134. If the Age is New AND the Maintenance Frequency is Medium AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.25.

135. If the Age is Old AND the Maintenance Frequency is High AND the Operational Performance is High AND the Material Condition is New, THEN the Equipment Condition is Good with a weight of 0.25.

Executing the fuzzy interference system with the fuzzy rules provides the equipment condition rating as a fuzzified output variable, which is defuzzified using the center-of-gravity defuzzification method. The relationship between the input variables and output variables is studied by comparing the input data with the output (equipment condition rating). This involves fixing all input variables except the one under study to understand how this single input variable independently affects the output variable. The study is also repeated for pairs of variables to examine their combined effects on the output.

**Interpretation of the relationship between each Fuzzy Input and the Output**

The linguistic fuzzy model is also used to understand the dependency between every possible input factor and the Output (equipment condition) as shown in the figure 5.9.

- Age - It can affect the equipment condition rating independently by around 0.1 in value when all other criteria are fixed.

  The equipment condition worsens (rating increases) as age increases gradually from 0 to 0.4.

  - This is true as the equipment condition decreases with the increase in age due to wear and tear.

  The equipment condition stabilizes as age increases gradually from 0.4 to 0.6.

  - This is true as the conditional rating can be managed by focusing on other input criteria as they are more dominant in the mid-zone. Example: The maintenance frequency is on par with the aging process to stabilize the conditional rating.

  The equipment condition improves (rating decreases) as age gradually increases from 0.6 to 0.8.

  - This is true as the conditional rating improves as the increases in age are over-compensated with the maintenance and performance criteria, which enables the system to work at its peak performance.

  The equipment condition decreases (rating increases) as age increases from 0.8 to 1.

  - This is true as the conditional rating increases with the increase in age, which might have regulatory effects to change the components past a certain age, and it can potentially lead to high maintenance frequency and many more.

- Maintenance Frequency - It can affect the equipment condition rating independently by around 0.15 in value when all other criteria are fixed.

  The equipment condition worsens (rating increases) as maintenance frequency reduces (rating increases). It is predominant for the entire input range.

  - This is true as the equipment condition worsens (rating increases) with decreases in the frequency of maintenance (rating increases) as frequent maintenance makes the system function as intended and increases its efficiency and lifespan.
  - The plateau denotes the optimized rate for maintenance frequency.

- Operational Performance - It can affect the equipment condition rating independently by around 0.15 in value when all other criteria are fixed.

  The equipment condition worsens (rating increases) as operational performance reduces (rating increases). It is predominant for the entire input range.

  - This is true as the equipment condition worsens (rating increases) with a decrease in the operating performance of the component (rating increases), as a decrease in operating performance would mean a decrease in production, efficiency, profits, and many more.

- Material Condition - It can independently affect the equipment condition rating by around 0.15 in value when all other criteria are fixed.

  The equipment condition worsens (rating increases) as the material condition deteriorates (rating increases). It is predominant for the entire input range.

  - This is true as the equipment condition worsens (rating increases) with decreases in the material condition of the component (rating increases) as material deterioration may cause negative effects on performance, efficiency, safety, and many more.

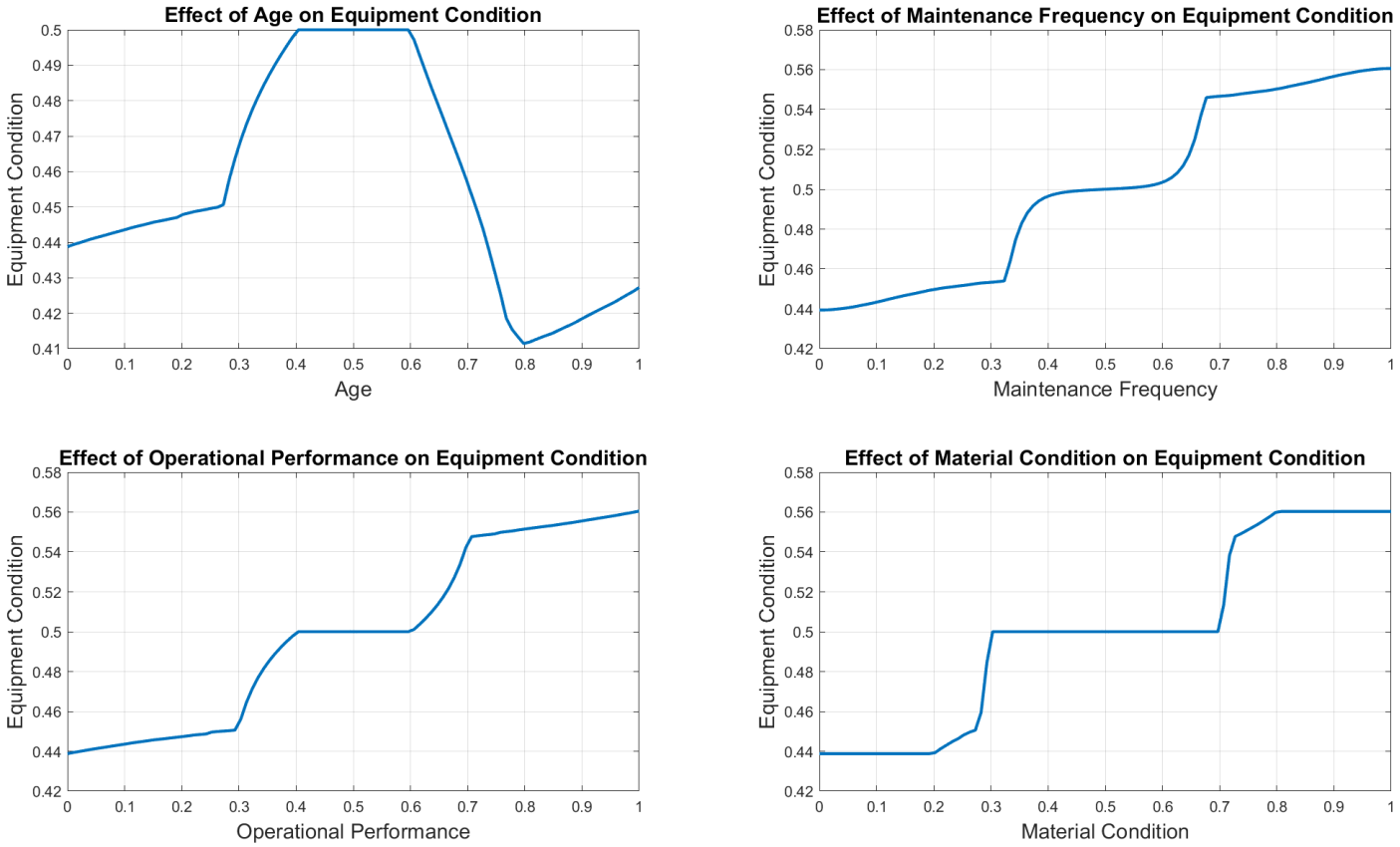



Figure 5.9: Relation between each fuzzy input variable with the output variable

**Interpretation of the relationship between pair of Fuzzy Inputs and the Output**

The linguistic fuzzy model is also used to understand the dependency between every possible pair of input factors with the Output (equipment condition) as shown in the figure 5.10.

**Age and Maintenance Frequency**

- **X-axis**: Age
- **Y-axis**: Maintenance Frequency
- **Z-axis**: Equipment Condition
- **Observations**:
  - Equipment condition is very good at low age and high maintenance frequency.
  - Condition deteriorates sharply with increasing age and decreasing maintenance frequency.
  - A plateau in the middle age range indicates a temporary improvement in condition.
- **Possible Reasons**:
  - High maintenance frequency helps maintain good condition in younger equipment.
  - The plateau could be due to major overhauls or effective maintenance practices during middle age.

**Age and Operational Performance**

- **X-axis**: Age
- **Y-axis**: Operational Performance
- **Z-axis**: Equipment Condition
- **Observations**:
  - Equipment condition is very good at low age and high operational performance.
  - Equipment condition declines as age increases and operational performance decreases.
- **Possible Reasons**:
  - Optimal operational performance minimizes wear and tear, especially in younger equipment.

  - Declining performance and increasing age contribute to accelerated deterioration.

**Age and Material Condition**

- **X-axis**: Age
- **Y-axis**: Material Condition
- **Z-axis**: Equipment Condition
- **Observations**:
  - Equipment condition is excellent when both material condition and age are new.
  - Condition declines with increasing age and worsening material condition.
  - A peak in the middle age range indicates temporary improvement in condition.
- **Possible Reasons**:
  - New materials ensure good initial condition, which deteriorates over time.
  - The peak could be due to material upgrades or refurbishments during middle age.

**Maintenance Frequency and Operational Performance**

- **X-axis**: Maintenance Frequency
- **Y-axis**: Operational Performance
- **Z-axis**: Equipment Condition
- **Observations**:
  - Equipment condition is perfect, with high maintenance frequency and high operational performance.
  - Condition declines with decreasing maintenance frequency and operational performance.
  - The highest condition is achieved when both factors are optimized.
- **Possible Reasons**:
  - Regular maintenance and optimal performance contribute to prolonged equipment lifespan and better condition.
  - Lack of maintenance and poor performance lead to accelerated wear and tear, negatively affecting the management's goals.

**Maintenance Frequency and Material Condition**

- **X-axis**: Maintenance Frequency
- **Y-axis**: Material Condition
- **Z-axis**: Equipment Condition
- **Observations**:
  - Equipment condition is very good with high maintenance frequency and good material condition.
  - Condition declines with decreasing maintenance frequency and worsening material condition.
- **Possible Reasons**:
  - High-quality materials and regular maintenance ensure good equipment condition.
  - Poor materials and infrequent maintenance lead to rapid deterioration.

**Operational Performance and Material Condition**

- **X-axis**: Operational Performance
- **Y-axis**: Material Condition
- **Z-axis**: Equipment Condition
- **Observations**:
  - Equipment condition is very good with high operational performance and good material condition.
  - Condition declines with decreasing operational performance and worsening material condition.
- **Possible Reasons**:
  - Optimal performance and good materials contribute to prolonged equipment lifespan and better condition.
  - Poor performance and materials lead to rapid wear and tear.

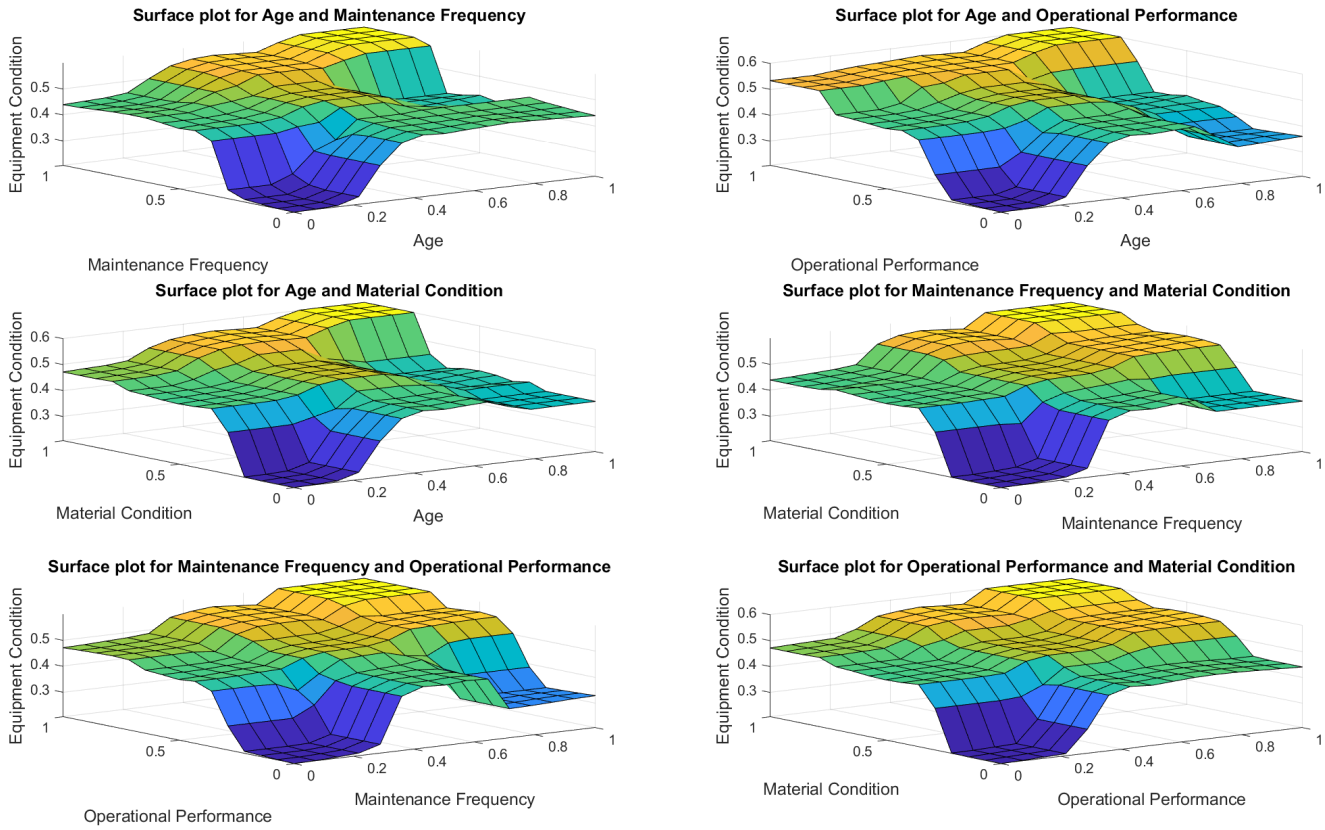



Figure 5.10: Relation between every pair of input variables with the output variable

**Weight of the Equipment Condition category from the fuzzy logic model**

The equipment/component condition ratings from the fuzzy model are scaled to match the maximum equipment condition rating obtained from the AHP. For instance, if the fuzzy model gives an equipment condition rating of 0.5, it is normalized to align with the maximum component condition rating from AHP (e.g., 0.345) by multiplying by 0.345 and dividing by the maximum possible value from the fuzzy model. This normalization is necessary because the fuzzy system operates within a range of 0 to 1 due to its convexity and normality. Without scaling, using the same range would lead to varied results, making comparative studies challenging.

## 5.8 Demonstration of the Fuzzy Integrated AHP Risk Ranking System

The same example used in the Nuclear Risk ranking system discussed in the AHP Chapter is used here with one alteration. The equipment condition has more inputs for the fuzzy model. The updated inputs for the fuzzy integrated model are given in table 5.1, and the same is used to calculate the overall risk rating.

Table 5.1: Inputs of Criteria and Ratings of the systems for the given tasks

| **Tasks** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| **Consequence** | | | | | |
| Nuclear Safety | S2 | S2 | S1 | S3 | S2 |
| Production Loss | PL2 | PL2 | PL3 | PL6 | PL2 |
| Cost, Conventional Safety and Environment | C3 | C6 | C1 | C3 | C3 |
| Equipment Obsolescence | High | High | Very High | Medium | High |
| Spares Unavailability | High | High | Very High | Medium | High |
| **Likelihood of Failure** | | | | | |
| Equipment Condition | | | | | |
| Age | 0.90 | 0.70 | 0.45 | 0.80 | 0.55 |
| Maintenance Frequency | 0.90 | 0.75 | 0.35 | 0.70 | 0.50 |
| Operational Performance | 0.85 | 0.75 | 0.50 | 0.75 | 0.50 |
| Material Condition | 0.95 | 0.70 | 0.40 | 0.85 | 0.60 |
| Equipment Environment | Mild | Mild | Mild | Mild | Severe |
| Equipment Demand | L | L | H | H | L |

The evaluation process is similar to the AHP model discussed in Chapter 3. The weights from the AHP and fuzzy logic are combined to get the overall risk rating, which is normalized between 0 and 100 using eq. 3.17.

Table 5.2 represents the weights and ratings of each task with respect to the categories/sub-categories, and Table 5.3 represents the normalized overall risk rating for the tasks selected from the internal reports of Canadian Nuclear Power Plant.

Table 5.2: Nuclear Risk Ranking System: Weights and Ratings

| **Tasks** | | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|
| **Category / Sub-Category** | **Weights** | **Ratings of Category / Sub-Category** | | | | |
| Nuclear Safety | 0.41 | 31.78 | 31.78 | 44.27 | 14.76 | 31.78 |
| Production Loss | 0.29 | 21.58 | 21.58 | 13.36 | 5.55 | 21.58 |
| Cost, Conventional Safety and Environment | 0.20 | 14.44 | 7.46 | 29.37 | 14.44 | 14.44 |
| Equipment Obsolescence | 0.06 | 22.82 | 22.82 | 41.31 | 16.17 | 22.82 |
| Spares Unavailability | 0.04 | 25.26 | 25.26 | 36.14 | 17.65 | 25.26 |
| Component Condition | 0.55 | 0.34 | 0.27 | 0.18 | 0.33 | 0.19 |
| Component Environment | 0.27 | 0.44 | 0.44 | 0.44 | 0.44 | 0.56 |
| Component Demand | 0.18 | 0.48 | 0.48 | 0.52 | 0.52 | 0.48 |

Table 5.3: Nuclear Risk Ranking System: Normalized Overall Risk Rating and its Rankings

| **Tasks** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| **Consequence** | | | | | |
| Nuclear Safety | 13.05 | 13.05 | 18.18 | 6.06 | 13.05 |
| Production Loss | 6.27 | 6.27 | 3.88 | 1.61 | 6.27 |
| Cost, Conventional Safety and Environment | 2.88 | 1.48 | 5.86 | 2.88 | 2.88 |
| Equipment Obsolescence | 1.37 | 1.37 | 2.47 | 0.97 | 1.37 |
| Spares Unavailability | 1.01 | 1.01 | 1.44 | 0.70 | 1.01 |
| **Likelihood of Failure** | | | | | |
| Component Condition | 0.19 | 0.14 | 0.10 | 0.18 | 0.10 |
| Component Environment | 0.12 | 0.12 | 0.12 | 0.12 | 0.15 |
| Component Demand | 0.09 | 0.09 | 0.10 | 0.10 | 0.09 |
| **Overall Risk Rating (Normalized)** | | | | | |
| Overall Risk Ratings | 47.09 | 34.18 | 34.88 | 24.57 | 31.75 |
| Rank | 1 | 3 | 2 | 5 | 4 |

## 5.9 Concluding Remarks

This Chapter discusses the fuzzy logic and how the category "Equipment Condition" is defined using the fuzzy sets to obtain weights for different ratings within it. It also discusses the aggregation and normalization of the weights to compare the results from the AHP model and the final Fuzzy Integrated AHP Ranking system. The obtained ranks are similar for tasks with high overall risk ratings, while the ranks of the tasks in the low percentile

region of overall risk rankings have changed. Therefore, it can be said that both methods are efficient.

# Chapter 6

# Conclusion

## 6.1 Summary

An integrated Analytical Hierarchy Process (AHP) and Fuzzy Logic model were developed for risk prioritization within infrastructure asset management by evaluating the overall risk rating of the system.

- A comparative study of various risk ranking systems in the current nuclear industry was conducted, highlighting their advantages and limitations.
- The literature review discussed approaches to overcoming the limitations of the current risk ranking system.
- An updated approach for risk ranking is proposed using AHP and fuzzy logic to address the shortcomings of existing methods.
- The proposed model demonstrates good coverage and high specificity, offering a more detailed spread of overall risk ratings and finer distinctions between different rating levels.
- High ratings correspond to complete system failure, which is unlikely in day-to-day activities, highlighting the importance of effectively segregating lower criticality tasks, making this method more practical. This is evident from the ranking order, where changes occur only in the lower tiers.
- The relative pairwise comparisons simplify complex problems and are more user-friendly due to their transparent and adaptive nature compared to the absolute rating systems prevalent in the industry.
- The manual tuning of the fuzzy inference system's membership functions with the available dataset is reliable, as the updated weights from the fuzzy Mamdani model produced similar rankings to those of the AHP model in the high percentile region.

- The findings show that the integrated AHP-Fuzzy model offers a robust and systematic way to handle the complexities and uncertainties in expert judgment and qualitative data.

The proposed method facilitates a nuanced quantification and analysis of various maintenance tasks, thereby supporting more informed and effective decision-making. Ultimately, this research contributes substantially to engineering management and risk assessment by providing a scalable and adaptable tool that can be incorporated into standard practices to improve critical infrastructure systems' safety and longevity.

## 6.2 Limitations of the Model

The model primarily focuses on a select set of criteria—safety, production, finance, environment, and maintenance. This focus may omit critical aspects that influence risk assessments in specific contexts. Including a broader range of factors could enhance the model's comprehensiveness. The accuracy and consistency of the relative comparisons can be improved by getting inputs from many experts by conducting a survey, as it may be skewed and highly subjective. Currently, the model employs Type 1 Fuzzy Sets, which are somewhat limited in handling high degrees of uncertainty. Exploring Type 2 Fuzzy Sets could provide a more robust framework for managing uncertainties, enhancing the model's reliability. The fuzzy rules are designed primarily for general scenarios and may not adequately address exceptions or specific cases. Developing precise rules that cater to these unique situations could significantly improve the model's applicability and accuracy. Data unavailability of the work order and its rating to train the model limits its efficiency.

## 6.3 Scope for Future Research

The research conducted offers a strong foundation for future studies and several avenues for expansion:

1. Implementing longitudinal studies to track the model's effectiveness over extended periods and under varying operational conditions would provide deeper insights into its adaptability and long-term benefits.

2. Developing user-friendly software tools that embody the AHP-Fuzzy model would facilitate its adoption in industry practices. These tools can help simplify the process for decision-makers unfamiliar with the underlying mathematical theories.

Through these future research directions, the utility and impact of the AHP-Fuzzy model can be significantly expanded, further enhancing its contribution to safe and effective infrastructure asset management.

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# Appendices

# Appendix A

# Categorization and Description of Ratings within the Consequence goal

## Rating Definitions

**Nuclear Safety (S):**

- **S1:** Concerned with a single component's failure that could lead to immediate serious consequences, such as reactor trip, turbine trip, or manual shutdown within a specified duration (e.g. 24 hours), or cause a significant performance reduction.
- **S2:** Concerned with component failures that could impair the reactor or turbine within certain duration (e.g. 7 days) if not resolved, potentially leading to station shutdown or significant performance de-rating. Excludes S1.
- **S3:** Concerned with component failures that would result in a reduction in station capacity (e.g., $< 5\%$), or where maintenance is more cost-effective than replacement (e.g., savings exceeding \$250,000), or could lead to a station shutdown for safety reasons after 7 days. Excludes S1 and S2.
- **S4:** Concerned with those component failures and subsequent unavailable which will not trigger a reactor trip, scram, or de-rating of the station. Excludes S1-S3.

**Production Loss (PL):**

- **PL1**: Concerned with a component failure that could lead to a shutdown of the reactor, turbine, or the entire unit within 24 hours if the issue is not resolved.
- **PL2**: Concerned with component failures would prevent the system from restarting after an outage, or whose failure results in an outage after 24 hours for a duration less than 7 days.

- **PL3**: Applies to auxiliary components that are essential for maintaining operations of major components (e.g., those rated as PL1 or PL4), ensuring that they function at capacity $\geq 10\%$.
- **PL4**: Concerned with component failures that results in a long-term outage of a major operational unit, defined in the operational documentation.
- **PL5**: Concerned with redundant equipment where a secondary failure in parallel equipment could trigger a significant system disruption, such as a reactor or turbine trip.
- **PL6**: Concerned with redundant equipment where a secondary failure could lead to a reduction in operational capacity by more than 10%.
- **PL7**: Pertains to auxiliary equipment necessary for the ongoing operation of systems classified under P2.
- **PL8**: Involves equipment (including support or auxiliary units) whose failure leads to a loss of redundancy. However, this does not result in an outage because either additional redundancy is available or the system design accommodates such failures without leading to an outage within a week.
- **PL9**: Involves situations where equipment failures result in a loss of some system redundancy but have no substantial impact on overall system operations or safety.

**Cost, Conventional Safety, and Environment (C):**

- **C1:** Failure of a component that leads to non-compliance with federal or provincial safety or environmental regulations (e.g., OHSA, MOL, MOE, MISA, emissions standards).
- **C2:** Equipment failure results in negative environmental consequences.
- **C3:** Failure involves very costly equipment, such as cases where maintenance or workaround costs exceed $250,000, making it more economical to maintain than replace.
- **C4:** A failure that imposes a heavy maintenance workload, requiring significant operator intervention or making it more cost-effective to maintain rather than replace the equipment.
- **C5:** Concerned with components that require frequent maintenance. For example, a component that received two corrective maintenance tasks (or work orders) within the past five years.
- **C6:** Concerned with a failure that could lead to failures in other critical components identified as S3, PL5, PL6, PL8, C3, C4, or C5.

- **C7:** Concerned with failures that are tolerable, In such cases condition monitoring strategies can be used to manage the situation.

**Equipment Obsolescence:**

- **Very High:** Component failure resulting in consequences described in risk ranks, S1,P1,P2 and P5.
- **High:** Component failure resulting in consequences described in S2,P3 and P8.
- **Medium/Normal:** Component failure resulting in consequences described in S3,P4,P6,P7,P9, C1, C2, C3, and C4.
- **Low:** Component failure resulting in consequences described in S4,P9 and C7.
- **Negligible:** Component failure resulting in negligible effects and does not fall under any risk rink discussed previously.

**Spares Unavailability:**

- **Very High:** Component failure resulting in consequences described in S1,PL1,PL2 and PL5.
- **High:** Component failure resulting in consequences described in S2,PL3 and PL8.
- **Medium/Normal:** Component failure resulting in consequences described in S3, PL4, PL6, PL7, PL9, C1, C2, C3, and C4.
- **Low:** Component failure resulting in consequences described in of S4,PL9 and C7.
- **Negligible:** Component failure resulting in negligible effects and does not fall under any risk rink discussed previously.

# Appendix B

# Types of Fuzzy Rules

## Types of Fuzzy Rules [53]

### 1. Certainty Rules

Defined as: These imply that the more certain the condition, the more certain the outcome. For example, "IF the temperature is definitely high, THEN the fan will definitely be on high."

$$\mu_{\text{output}}(y) = \min(\mu_{\text{antecedent}}(x), \mu_{\text{consequent}}(y))$$

This implies the consequent is as true as the antecedent.

### 2. Possibility Rules

Defined as: These suggest that the condition makes the outcome possible to a certain degree. "IF the temperature is somewhat high, THEN the fan might be on high."

$$\pi_{\text{output}}(y) = \max(\pi_{\text{antecedent}}(x), \mu_{\text{consequent}}(y))$$

This states that the possibility of the consequent is at least as great as the most supportive evidence from the antecedent or the consequent itself.

### 3. Gradual Rules

Defined as: These indicate that the degree to which the condition holds affects the outcome in a proportional manner. "The higher the temperature, the higher the fan speed."

$$\mu_{\text{output}}(y) = \min(1, 1 - \mu_{\text{antecedent}}(x) + \mu_{\text{consequent}}(y))$$

Gradual rules express a direct proportionality between the truth of the antecedent and the truth of the consequent.

## 4. Exception Rules (Unless Rules)

Defined as: These incorporate exceptions into the fuzzy logic. "IF the temperature is high THEN the fan should be high UNLESS it is winter."

$$\mu_{\text{output}}(y) = \min(\mu_{\text{antecedent}}(x) \cdot (1 - \mu_{\text{exception}}(z)), \mu_{\text{consequent}}(y))$$

This rule type introduces an exception that can override the usual consequent if the exception condition is met.

## 5. Anti-Gradual Rules

Defined as: These indicate that the degree to which the condition holds affects the outcome in a inversely proportional manner. "The higher the temperature, the lower the fan speed."

$$\mu_{\text{output}}(y) = \max(0, \mu_{\text{antecedent}}(x) + \mu_{\text{consequent}}(y) - 1)$$

Anti-gradual rules imply an inverse relationship between the antecedent and the consequent, where increasing truth in the antecedent decreases the truth in the consequent.

# Appendix C

# Matlab Code

Given below is the code for simulating various ratings of the proposed model through random sampling technique.

## C.1 Overall Risk Rating Distribution

```
% Define the weights for Impact and its sub-categories
impact_weights = [0.411, 0.290, 0.199, 0.100];

nuclear_safety_weights = [0.443, 0.318, 0.148, 0.092];
loss_of_generation_weights = [0.319, 0.216, 0.134, 0.095, 0.071, 0.055,
    0.043, 0.037, 0.030];
ccse_weights = [0.294, 0.209, 0.144, 0.129, 0.098, 0.075, 0.052];
ease_of_repairability_weights = [0.60, 0.40];

equipment_obsolescence_weights = [0.413, 0.228, 0.162, 0.122, 0.075];
spare_unavailability_weights = [0.361, 0.253, 0.177, 0.123, 0.086];

% Define weights for Likelihood of Failure
likelihood_weights = [0.545, 0.273, 0.182];

equipment_condition_weights = [0.345, 0.242, 0.203, 0.136, 0.075];
equipment_environment_weights = [0.556, 0.444];
equipment_demand_weights = [0.524, 0.476];

% Number of simulations
n_simulations = 1000000;

% Generate random outcomes for each category and sub-category
nuclear_safety_subcategories = 1:4;
loss_of_generation_subcategories = 1:9;
ccse_subcategories = 1:7;
ease_of_repairability_subcategories = 1:2;
equipment_obsolescence_subcategories = 1:5;
spare_unavailability_subcategories = 1:5;
```

```
equipment_condition_subcategories = 1:5;
equipment_environment_subcategories = 1:2;
equipment_demand_subcategories = 1:2;

% Initialize final scores and outcomes arrays
final_scores = zeros(n_simulations, 1);
x = zeros(n_simulations, 1);
combined_impact_scores = zeros(n_simulations, 1);
combined_likelihood_scores = zeros(n_simulations, 1);

outcomes_ns = zeros(n_simulations, 1);
outcomes_lg = zeros(n_simulations, 1);
outcomes_ccse = zeros(n_simulations, 1);
outcomes_eo = zeros(n_simulations, 1);
outcomes_su = zeros(n_simulations, 1);
outcomes_ec = zeros(n_simulations, 1);
outcomes_ee = zeros(n_simulations, 1);
outcomes_ed = zeros(n_simulations, 1);

nuclear_safety_score = zeros(n_simulations, 1);
loss_of_generation_score = zeros(n_simulations, 1);
ccse_score = zeros(n_simulations, 1);
equipment_obsolescence_score = zeros(n_simulations, 1);
spare_unavailability_score = zeros(n_simulations, 1);

equipment_condition_score = zeros(n_simulations, 1);
equipment_environment_score = zeros(n_simulations, 1);
equipment_demand_score = zeros(n_simulations, 1);

% Simulation loop
for i = 1:n_simulations
    % Select random sub-category weights for each category within Impact
    idx_ns = randsample(length(nuclear_safety_weights), 1, true);
    idx_lg = randsample(length(loss_of_generation_weights), 1, true);
    idx_ccse = randsample(length(ccse_weights), 1, true);
    idx_eo = randsample(length(equipment_obsolescence_weights), 1, true);
    idx_su = randsample(length(spare_unavailability_weights), 1, true);

    nuclear_safety_score(i) = nuclear_safety_weights(idx_ns) *
       impact_weights(1);
    loss_of_generation_score(i) = loss_of_generation_weights(idx_lg) *
       impact_weights(2);
    ccse_score(i) = ccse_weights(idx_ccse) * impact_weights(3);
    equipment_obsolescence_score(i) =
       equipment_obsolescence_weights(idx_eo) *
       ease_of_repairability_weights(1) * impact_weights(4);
    spare_unavailability_score(i) = spare_unavailability_weights(idx_su)
       * ease_of_repairability_weights(2) * impact_weights(4);

    % Select random likelihood weights for each category within
       Likelihood of Failure
    idx_ec = randsample(length(equipment_condition_weights), 1, true);
    idx_ee = randsample(length(equipment_environment_weights), 1, true);
```

```
    idx_ed = randsample(length(equipment_demand_weights), 1, true);

    equipment_condition_score(i) = equipment_condition_weights(idx_ec) *
        likelihood_weights(1);
    equipment_environment_score(i) =
        equipment_environment_weights(idx_ee) * likelihood_weights(2);
    equipment_demand_score(i) = equipment_demand_weights(idx_ed) *
        likelihood_weights(3);

    % Combine impact and likelihood scores
    combined_impact_scores(i) = (nuclear_safety_score(i) +
        loss_of_generation_score(i) + ccse_score(i) +
        equipment_obsolescence_score(i) + spare_unavailability_score(i))
        * 100;
    combined_likelihood_scores(i) = equipment_condition_score(i) *
        equipment_environment_score(i) * equipment_demand_score(i);

    x(i)=combined_impact_scores(i)*combined_likelihood_scores(i);

    final_scores(i) = ((combined_impact_scores(i) *
        combined_likelihood_scores(i)) - 0.002781356956822080) * 100 /
        (0.101334947234032000 - 0.002781356956822080);

    % Store outcomes for cumulative distribution plot
    outcomes_ns(i) = idx_ns;
    outcomes_lg(i) = idx_lg;
    outcomes_ccse(i) = idx_ccse;
    outcomes_eo(i) = idx_eo;
    outcomes_su(i) = idx_su;
    outcomes_ec(i) = idx_ec;
    outcomes_ee(i) = idx_ee;
    outcomes_ed(i) = idx_ed;
end

% Display the average final score
average_final_score = mean(final_scores);
disp(['Average Final Score: ', num2str(average_final_score)]);

% Calculate mean, median, mode, and skewness
mean_score = mean(final_scores);
median_score = median(final_scores);
mode_score = mode(final_scores);
skewness_score = skewness(final_scores);

% Plot the histogram of final scores
x1_limits = [0 100];
y1_limits = [0 20000];
figure;
histogram(final_scores,'BinWidth', 0.5, 'FaceColor', 'blue',
    'EdgeColor', 'black');
saveas(gcf, 'histogram_overall rating.png');
xlabel('Overall Rating', 'FontSize', 12);
ylabel('Frequency', 'FontSize', 12);
```

```
xlim(x1_limits);
ylim(y1_limits);

% Add mean, median, mode, and skewness lines
hold on;
mean_line = xline(mean_score,'r:','Color', 'r', 'LineWidth', 2.5);
median_line = xline(median_score,'r:','Color', 'm', 'LineWidth', 2.5);
mode_line = xline(mode_score,'r:','Color', 'c', 'LineWidth', 2.5);

% Add a legend to show the values
legend_handle = legend([mean_line, median_line, mode_line], ...
    {['Mean = ' num2str(mean_score, '%.2f')], ...
     ['Median = ' num2str(median_score, '%.2f')], ...
     ['Mode = ' num2str(mode_score, '%.2f')]}, ...
    'Location', 'northeast');
set(legend_handle, 'FontSize', 12);

% Display skewness as text in the plot

dim = [.5 .3 .3 .3];
str = ['Skewness = ' num2str(skewness_score, '%.2f')];
annotation_handle = annotation('textbox', dim, 'String', str,
    'FitBoxToText', 'on', 'BackgroundColor', 'white');
set(annotation_handle, 'FontSize', 12);

hold off;

saveas(gcf, 'Overall Rating (AHP)-updated bin size.jpeg');
```

Listing C.1: MATLAB Code for Overall Condition Rating Distribution

## C.2 Fuzzy Inference System

```
% Create a new fuzzy inference system
fis = mamfis('Name', 'EquipmentConditionAssessment');

% Define input ranges
ageRange = [0 1];
maintenanceFrequencyRange = [0 1];
operationalPerformanceRange = [0 1];
materialConditionRange = [0 1];

% Add input variables with specified ranges
fis = addInput(fis, ageRange, 'Name', 'Age');
fis = addInput(fis, maintenanceFrequencyRange, 'Name',
    'MaintenanceFrequency');
fis = addInput(fis, operationalPerformanceRange, 'Name',
    'OperationalPerformance');
fis = addInput(fis, materialConditionRange, 'Name', 'MaterialCondition');

```

```
% Define the output range
equipmentConditionRange = [0 1];

% Add output variable with specified range
fis = addOutput(fis, equipmentConditionRange, 'Name',
    'EquipmentCondition');

% Add membership functions for Age
fis = addMF(fis, 'Age', 'trimf', [0 0 0.4], 'Name', 'New');
fis = addMF(fis, 'Age', 'trimf', [0.2 0.5 0.8], 'Name', 'Old');
fis = addMF(fis, 'Age', 'trimf', [0.6 1 1 ], 'Name', 'Very_Old');

% Add membership functions for MaintenanceFrequency
fis = addMF(fis, 'MaintenanceFrequency', 'gaussmf', [0.15 0], 'Name',
    'High');
fis = addMF(fis, 'MaintenanceFrequency', 'gaussmf', [0.075 0.5], 'Name',
    'Medium');
fis = addMF(fis, 'MaintenanceFrequency', 'gaussmf', [0.15 1], 'Name',
    'Low');

% Add membership functions for OperationalPerformance
fis = addMF(fis, 'OperationalPerformance', 'trimf', [0 0 0.4], 'Name',
    'High');
fis = addMF(fis, 'OperationalPerformance', 'trimf', [0.25 0.5 0.75],
    'Name', 'Normal');
fis = addMF(fis, 'OperationalPerformance', 'trimf', [0.6 1 1], 'Name',
    'Low');

% Add membership functions for MaterialCondition
fis = addMF(fis, 'MaterialCondition', 'trapmf', [0 0 0.2 0.3], 'Name',
    'New');
fis = addMF(fis, 'MaterialCondition', 'trapmf', [0.25 0.45 0.55 0.75],
    'Name', 'Worn_Out');
fis = addMF(fis, 'MaterialCondition', 'trapmf', [0.7 0.8 1 1], 'Name',
    'Worse');

% Add membership functions for EquipmentCondition
fis = addMF(fis, 'EquipmentCondition', 'trapmf', [0 0 0.1 0.2], 'Name',
    'Very_Good');
fis = addMF(fis, 'EquipmentCondition', 'trimf', [0.15 0.3 0.45], 'Name',
    'Good');
fis = addMF(fis, 'EquipmentCondition', 'trimf', [0.35 0.5 0.65], 'Name',
    'Satisfactory');
fis = addMF(fis, 'EquipmentCondition', 'trimf', [0.55 0.7 0.85], 'Name',
    'Poor');
fis = addMF(fis, 'EquipmentCondition', 'trapmf', [0.8 0.9 1 1], 'Name',
    'Very_Poor');

% Define the rules
rules = [
1 1 1 1 1 1 1;
2 2 2 2 3 1 1;
3 3 3 3 5 1 1;
```

```
54
55 3 3 2 3 5 0.75 1;
56 3 2 3 3 5 0.75 1;
57 3 3 3 2 5 0.75 1;
58 2 3 3 3 5 0.75 1;
59 3 3 3 2 4 0.25 1;
60 2 3 3 3 4 0.25 1;
61 3 3 2 3 4 0.25 1;
62 3 2 3 3 4 0.25 1;
63
64 2 3 3 2 4 0.5 1;
65 2 2 3 3 4 0.5 1;
66 3 3 2 2 4 0.5 1;
67 3 2 3 2 4 0.5 1;
68 3 2 2 3 4 0.5 1;
69 2 3 2 3 4 0.5 1;
70 2 3 3 2 3 0.5 1;
71 2 2 3 3 3 0.5 1;
72 3 3 2 2 3 0.5 1;
73 3 2 3 2 3 0.5 1;
74 3 2 2 3 3 0.5 1;
75 2 3 2 3 3 0.5 1;
76
77 2 2 2 3 4 0.25 1;
78 2 2 3 2 4 0.25 1;
79 2 3 2 2 4 0.25 1;
80 3 2 2 2 4 0.25 1;
81 2 2 2 3 3 0.75 1;
82 2 2 3 2 3 0.75 1;
83 2 3 2 2 3 0.75 1;
84 3 2 2 2 2 0.75 1;
85
86 1 3 3 3 5 0.9 1;
87 3 1 3 3 5 1 1;
88 3 3 1 3 5 0.1 1;
89 3 3 3 1 5 0.25 1;
90 1 3 3 3 4 0.1 1;
91 3 3 1 3 4 0.9 1;
92 3 3 3 1 4 0.75 1;
93
94 1 2 3 3 3 0.1 1;
95 1 3 3 2 3 0.5 1;
96 2 1 3 3 3 0.1 1;
97 2 3 3 1 3 0.5 1;
98 1 2 3 3 4 0.9 1;
99 1 3 3 2 4 0.5 1;
100 2 1 3 3 4 0.9 1;
101 2 3 3 1 4 0.5 1;
102
103 1 2 2 3 3 0.9 1;
104 1 3 2 2 3 0.75 1;
105 2 2 1 3 2 0.1 1;
106 2 2 3 1 3 0.75 1;
```

```
2 1 2 3 3 0.75 1;
2 1 3 2 3 0.9 1;
2 3 1 2 3 0.1 1;
2 3 2 1 3 0.5 1;
3 2 1 2 2 0.75 1;
3 2 2 1 3 0.5 1;
3 1 2 2 3 0.75 1;
1 2 2 3 2 0.1 1;
1 3 2 2 2 0.25 1;
2 2 1 3 3 0.9 1;
2 2 3 1 2 0.25 1;
2 1 2 3 2 0.25 1;
2 1 3 2 2 0.1 1;
2 3 1 2 2 0.9 1;
2 3 2 1 2 0.5 1;
3 2 1 2 3 0.25 1;
3 2 2 1 2 0.5 1;
3 1 2 2 2 0.25 1;

1 2 2 2 3 0.75 1;
2 2 2 1 3 0.75 1;
2 2 1 2 3 0.75 1;
2 1 2 2 3 0.75 1;
1 2 2 2 2 0.25 1;
2 2 2 1 2 0.25 1;
2 2 1 2 2 0.25 1;
2 1 2 2 2 0.25 1;

1 1 2 3 2 0.25 1;
1 2 1 3 2 0.5 1;
2 1 1 3 2 0.5 1;
1 1 3 2 2 0.25 1;
1 3 1 2 2 0.5 1;
3 1 1 2 2 0.75 1;
1 2 3 1 2 0.25 1;
1 3 2 1 2 0.25 1;
2 1 3 1 2 0.25 1;
2 3 1 1 2 0.75 1;
1 1 2 3 3 0.75 1;
1 2 1 3 3 0.5 1;
2 1 1 3 3 0.5 1;
1 1 3 2 3 0.75 1;
1 3 1 2 3 0.5 1;
3 1 1 2 3 0.25 1;
1 2 3 1 3 0.75 1;
1 3 2 1 3 0.75 1;
2 1 3 1 3 0.75 1;
2 3 1 1 3 0.25 1;

1 1 3 3 4 1 1;
1 3 1 3 4 0.6 1;
1 3 3 1 4 0.25 1;
3 1 3 1 4 0.75 1;
```

```
3 1 1 3 4 0.75 1;
3 3 1 1 4 0.1 1;
1 3 1 3 3 0.4 1;
1 3 3 1 3 0.75 1;
3 1 3 1 3 0.25 1;
3 1 1 3 3 0.25 1;
3 3 1 1 3 0.9 1;


2 2 1 1 1 0.5 1;
2 1 2 1 1 0.5 1;
2 1 1 2 1 0.5 1;
1 2 2 1 1 0.5 1;
1 1 2 2 1 0.5 1;
1 2 1 2 1 0.5 1;
2 2 1 1 2 0.5 1;
2 1 2 1 2 0.5 1;
2 1 1 2 2 0.5 1;
1 2 2 1 2 0.5 1;
1 1 2 2 2 0.5 1;
1 2 1 2 2 0.5 1;

1 1 1 3 2 0.75 1;
1 1 3 1 2 0.75 1;
1 3 1 1 2 0.5 1;
3 1 1 1 2 0.5 1;
1 1 1 3 1 0.25 1;
1 1 3 1 1 0.25 1;
1 3 1 1 1 0.5 1;
3 1 1 1 1 0.5 1;

1 1 1 2 1 0.75 1;
1 1 2 1 1 0.75 1;
1 2 1 1 1 0.75 1;
2 1 1 1 1 0.75 1;
1 1 1 2 2 0.25 1;
1 1 2 1 2 0.25 1;
1 2 1 1 2 0.25 1;
2 1 1 1 2 0.25 1;


];

% Add rules to the system
for i = 1:size(rules,1)
    fis = addRule(fis, rules(i,:));
end

% Plot membership functions for all input and output variables
figure;

% Plot for 'Age'

```

```
plotmf(fis, 'input', 1);
set(findobj(gca, 'Type', 'line'), 'LineWidth', 2);

xlabel('Age', 'FontSize', 14);
ylabel('Degree of Membership ', 'FontSize', 14);

% Plot for 'MaintenanceFrequency'
figure;
plotmf(fis, 'input', 2);
set(findobj(gca, 'Type', 'line'), 'LineWidth', 2);
xlabel('Maintenance Frequency', 'FontSize', 14);
ylabel('Degree of Membership ', 'FontSize', 14);


% Plot for 'OperationalPerformance'
figure;
plotmf(fis, 'input', 3);
set(findobj(gca, 'Type', 'line'), 'LineWidth', 2);

xlabel('Operational Performance', 'FontSize', 14);
ylabel('Degree of Membership ', 'FontSize', 14);

% Plot for 'MaterialCondition'
figure;
plotmf(fis, 'input', 4);
set(findobj(gca, 'Type', 'line'), 'LineWidth', 2);
xlabel('Material Condition', 'FontSize', 14);
ylabel('Degree of Membership ', 'FontSize', 14);


% Plot for 'EquipmentCondition'
figure;
plotmf(fis, 'output', 1);
set(findobj(gca, 'Type', 'line'), 'LineWidth', 2);
xlabel('Equipment Condition', 'FontSize', 14);
ylabel('Degree of Membership ', 'FontSize', 14);


% Define your input values
inputValues = [0.45, 0.35, 0.5, 0.4];

% Evaluate the equipment condition using the fuzzy inference system
equipmentCondition = evalfis(fis, inputValues);

% Display the evaluated equipment condition
disp(['The evaluated equipment condition is: ',
    num2str(equipmentCondition)]);

% Assuming fis is your fuzzy inference system with a single output or
    you're focusing on the first output
% Select two inputs to vary in the plot
inputIndex1 = 1; % Index of the first input variable you want to plot
inputIndex2 = 2; % Index of the second input variable you want to plot
```

```
inputIndex3 = 3; % Index of the first input variable you want to plot
inputIndex4 = 4; % Index of the second input variable you want to plot

% Set up the figure window
figure;

% Number of subplots needed (depends on the number of graphs)
rows = 3; % Adjust this if needed
cols = 2; % Adjust this if needed

% First surface plot
subplot(rows, cols, 1);
gensurf(fis, [inputIndex1 inputIndex2]);
title('Surface plot for Age and Maintenance Frequency', FontSize=12);
xlabel('Age', FontSize=12);
ylabel('Maintenance Frequency', FontSize=12);
zlabel('Equipment Condition', FontSize=12);

% Second surface plot
subplot(rows, cols, 2);
gensurf(fis, [inputIndex1 inputIndex3]);
title('Surface plot for Age and Operational Performance', FontSize=12);
xlabel('Age', FontSize=12);
ylabel('Operational Performance', FontSize=12);
zlabel('Equipment Condition', FontSize=12);

% Third surface plot
subplot(rows, cols, 3);
gensurf(fis, [inputIndex1 inputIndex4]);
title('Surface plot for Age and Material Condition', FontSize=12);
xlabel('Age', FontSize=12);
ylabel('Material Condition', FontSize=12);
zlabel('Equipment Condition', FontSize=12);

% Fourth surface plot
subplot(rows, cols, 4);
gensurf(fis, [inputIndex2 inputIndex4]);
title('Surface plot for Maintenance Frequency and Material Condition',
    FontSize=12);
xlabel('Maintenance Frequency', FontSize=12);
ylabel('Material Condition', FontSize=12);
zlabel('Equipment Condition', FontSize=12);

% Fifth surface plot
subplot(rows, cols, 5);
gensurf(fis, [inputIndex2 inputIndex3]);
title('Surface plot for Maintenance Frequency and Operational
    Performance', FontSize=12);
xlabel('Maintenance Frequency', FontSize=12);
ylabel('Operational Performance', FontSize=12);
zlabel('Equipment Condition', FontSize=12);

% Sixth surface plot
```

```
subplot(rows, cols, 6);
gensurf(fis, [inputIndex3 inputIndex4]);
title('Surface plot for Operational Performance and Material Condition',
    FontSize=12);
xlabel('Operational Performance', FontSize=12);
ylabel('Material Condition', FontSize=12);
zlabel('Equipment Condition', FontSize=12);

% Adjust layout to prevent subplot overlap
set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0, 1, 1]);

% Define the fixed values for the other three input variables
fixedValues = [0.5, 0.5, 0.5];

% Define a range of values for each input variable
ageRange = linspace(0, 1, 100);
maintenanceFrequencyRange = linspace(0, 1, 100);
operationalPerformanceRange = linspace(0, 1, 100);
materialConditionRange = linspace(0, 1, 100);

% Preallocate the output arrays
outputValuesAge = zeros(size(ageRange));
outputValuesMaintenanceFrequency =
    zeros(size(maintenanceFrequencyRange));
outputValuesOperationalPerformance =
    zeros(size(operationalPerformanceRange));
outputValuesMaterialCondition = zeros(size(materialConditionRange));

% Compute the output for each value of the single input variables
for i = 1:length(ageRange)
    age = ageRange(i);
    outputValuesAge(i) = evalfis(fis, [age, fixedValues(1),
        fixedValues(2), fixedValues(3)]);
end

for i = 1:length(maintenanceFrequencyRange)
    maintenanceFrequency = maintenanceFrequencyRange(i);
    outputValuesMaintenanceFrequency(i) = evalfis(fis, [fixedValues(1),
        maintenanceFrequency, fixedValues(2), fixedValues(3)]);
end

for i = 1:length(operationalPerformanceRange)
    operationalPerformance = operationalPerformanceRange(i);
    outputValuesOperationalPerformance(i) = evalfis(fis,
        [fixedValues(1), fixedValues(2), operationalPerformance,
        fixedValues(3)]);
end

for i = 1:length(materialConditionRange)
    materialCondition = materialConditionRange(i);
    outputValuesMaterialCondition(i) = evalfis(fis, [fixedValues(1),
        fixedValues(2), fixedValues(3), materialCondition]);
end
```

```
360
361 % Plot the results for each input variable
362 figure;
363
364 subplot(2, 2, 1);
365 plot(ageRange, outputValuesAge, 'LineWidth', 2);
366 xlabel('Age', FontSize=14);
367 ylabel('Equipment Condition', FontSize=14);
368 title('Effect of Age on Equipment Condition', FontSize=14);
369 grid on;
370
371 subplot(2, 2, 2);
372 plot(maintenanceFrequencyRange, outputValuesMaintenanceFrequency,
    'LineWidth', 2);
373 xlabel('Maintenance Frequency', FontSize=14);
374 ylabel('Equipment Condition', FontSize=14);
375 title('Effect of Maintenance Frequency on Equipment Condition',
    FontSize=14);
376 grid on;
377
378 subplot(2, 2, 3);
379 plot(operationalPerformanceRange, outputValuesOperationalPerformance,
    'LineWidth', 2);
380 xlabel('Operational Performance', FontSize=14);
381 ylabel('Equipment Condition', FontSize=14);
382 title('Effect of Operational Performance on Equipment Condition',
    FontSize=14);
383 grid on;
384
385 subplot(2, 2, 4);
386 plot(materialConditionRange, outputValuesMaterialCondition, 'LineWidth',
    2);
387 xlabel('Material Condition', FontSize=14);
388 ylabel('Equipment Condition', FontSize=14);
389 title('Effect of Material Condition on Equipment Condition',
    FontSize=14);
390 grid on;
391
392 % Adjust layout to prevent subplot overlap
393 set(gcf, 'Units', 'Normalized', 'OuterPosition', [0, 0.04, 1, 0.96]);
394
395 saveAllFigures();
396
397 function saveAllFigures()
398     % Get a list of all open figure handles
399     figHandles = findall(0, 'Type', 'figure');
400
401     for i = 1:length(figHandles)
402         % Get the current figure handle
403         fig = figHandles(i);
404
405         % Set the current figure to the one being processed
406         set(0, 'CurrentFigure', fig);
```

```
407
408         % Generate a filename based on the figure number
409         filename = sprintf('figure_%d.png', fig.Number);
410
411         % Save the figure as a PNG file
412         saveas(fig, filename);
413
414         % Alternatively, use exportgraphics for better control over
                output
415         % exportgraphics(fig, filename, 'Resolution', 300);
416     end
417 end
```

Listing C.2: MATLAB Code for Fuzzy Inference System