

Integrated Vehicle Stability Control and Power Distribution Using Model Predictive Control

by

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Abstract

There is a growing need for active safety systems to assist drivers in unfavorable driving conditions. In these conditions, the behavior of the vehicle is different than the linear response during everyday driving. Even experienced drivers usually lose control of the vehicle in such situations and that often results in a car accident.

Stability control systems have been developed over the past few decades to assist drivers in keeping the vehicle under control. Most of these control systems are comprised of separate modules, each responsible for one task such as yaw rate tracking, sideslip control, traction control or power distribution. These objectives may be in conflict in some driving situations. In such cases, individual controllers fight over priority and produce conflicting control commands, to the detriment of the vehicle performance.

In addition, in most stability control systems, transferring the controller from one vehicle to another with a different driveline and actuator configuration requires significant modifications in the controller and major re-tuning to obtain a similar performance. This is a major disadvantage for auto companies and increases the controller design and tuning costs.

In this thesis, an integrated control system has been designed to address vehicle stability, traction control and power distribution objectives at the same time. The proposed controller casts all of these objectives in a single objective function and chooses control actions to optimize this objective function. Therefore, the output of the integrated controller is not altered by another module and the optimality of the solution is not compromised.

Furthermore, the designed controller can be easily reconfigured to work with various driveline configurations such as all-wheel drive, front or rear-wheel drive. In addition, it can also work with various actuator configurations such as torque vectoring, differential braking or any combination of them on the front or rear axles. Moving from one configuration to another does not change the stability control performance and major re-tuning can be avoided.

The performance of the designed model predictive controller is evaluated in software simulations with a high fidelity model of an electric Equinox vehicle. The stability and

wheel slip control performance of the controller is evaluated in various driving and road conditions. In addition, the effect of integrated power distribution is studied.

Experimental tests with two different electric vehicles are also carried out to evaluate the real-time performance of the MPC controller. It is observed that the controller is able to maintain vehicle and wheel stability in all of the driving scenarios considered. The power distribution system is able to improve vehicle efficiency by approximately 1.5% and acts in cooperation with the stability control objectives.

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Dedication

This dissertation is dedicated to my brilliant and outrageously loving and supportive wife, Maryam and our exuberant, sweet, and kind-hearted little girl, Baran.

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Nomenclature

α_{ij}	Slip angle of tire ij
β	Sideslip angle of the vehicle
δ_i	Wheel steering angle of the axle i
η_T	Overall efficiency of electric vehicle in energy consumption
η_{ij}	Efficiency of electric motor attached to wheel ij
κ_{ij}	Slip ratio of tire ij
Q^{brake}	Driver's (hydraulic) brake torque request
Q^{max}	Maximum motor torque capacity at a given speed
Q^{min}	Minimum motor torque capacity at a given speed
$\mu_{x,ij}$	Friction coefficient of tire ij in longitudinal direction
$\mu_{y,ij}$	Friction coefficient of tire ij in lateral direction
ν_T	Reciprocal of the vehicle overall electrical efficiency
ν_{ij}	Reciprocal of electrical efficiency of motor ij
a_x	Longitudinal vehicle acceleration at C.G.

a_y	Lateral vehicle acceleration at C.G.
$e_{\omega_{ij}}$	Tracking error of the desired wheel speed ij
$F_{x_{ij}}$	Longitudinal force of tire ij
$F_{y_{ij}}$	Lateral force of tire ij
$F_{z_{ij}}$	Normal force of tire ij
g	Gravitational constant (9.81 m/s^2)
G_z	Total yaw moment acting on vehicle C.G.
I_w	Wheel moment of inertia about its roll axis
I_z	Yaw moment of inertia at the vehicle C.G.
k_{us}	Desired vehicle understeer gradient
L	Vehicle wheel base
L_i	Distance between vehicle C.G. and axle i
M_{F_x}	Yaw moment at the vehicle C.G. generated by longitudinal tire forces
M_{F_y}	Yaw moment at the vehicle C.G. generated by lateral tire forces
N_c	Number of points in the control horizon
N_p	Number of points in the prediction horizon
Q_{ij}	Total torque applied to wheel ij
r	Vehicle yaw rate
R_{eff}	Effective wheel radius
T_F	Total drive torque applied to the front axle

T_F^*	Optimal drive torque allocated to the front axle for maximum efficiency
T_R	Total drive torque applied to the rear axle
T_R^*	Optimal drive torque allocated to the rear axle for maximum efficiency
T_{demand}	Total drive torque demanded by driver
u	Longitudinal vehicle velocity at C.G.
u_{ij}	Longitudinal velocity at the center of wheel ij
v	Lateral vehicle velocity at C.G.
v_d	Desired vehicle lateral velocity
v_{ij}	Lateral velocity at the center of wheel ij
w_i	Trackwidth of axle i
4WD	Four-Wheel Drive
4WS	Four Wheel Steering
ABS	Anti-lock Brake System
AFS	Active Front Steering
ARS	Active Rear Steering
AWD	All-Wheel Drive
C.G.	Center of gravity of the vehicle
CAN	Controlled Area Network
CDC	Continuous Damping Control
DYC	Direct Yaw Control

EMU	Energy Management Unit
ESC	Electronic Stability Control
FWD	Front-Wheel Drive
GPS	Global Positioning System
HMPC	Hybrid Model Predictive Control
IMU	Inertial Measurement Unit
KKT	Karush-Kuhn-Tucker optimality conditions
LQR	Linear Quadratic Regulator
LTV-MPC	Linear Time Varying Model Predictive Control
MILP	Mixed Integer Linear Programming
MIQP	Mixed Integer Quadratic Programming
MPC	Model Predictive Control
NEDC	New European Driving Cycle
NLMPC	Nonlinear Model Predictive Control
PID	Proportional-Integral-Derivative (controller)
PWA	Piece-wise Affine
QFT	Quantitative Feedback Theory
QP	Quadratic Programming
RHC	Receding Horizon Control
RWD	Rear-Wheel Drive

SIL	Software-In-the-Loop
SOC	State Of Charge (for batteries)
SUV	Sport Utility Vehicle
TCS	Traction Control System
UCC	Unified Chassis Control

Chapter 1

Introduction

1.1 Motivation

Traffic accidents are one of the major causes for unnatural death throughout the world. In Canada, 1923 lost their lives in traffic accidents in 2013 [1]. These accidents typically happen in unfavorable driving conditions such as high vehicle speed, low surface friction, sudden change in road surface, etc. In these situations, the behavior of the vehicle is different from what drivers are used to in everyday driving. Therefore, it is difficult to respond correctly to these situations that cause loss of vehicle control.

This highlights the need for systems that can assist the driver in such situations. Vehicle safety features include ABS (Anti-lock Braking System), TCS (Traction Control System) and ESC (Electronic Stability Control). Although there are extensive studies that show effectiveness of control systems in vehicle stability (for example [2–4]), further development of stability control systems is still required to reduce road accidents.

Vehicles interact with the road by means of tires. Stability controllers use tires to affect the vehicle behavior. An over-spinning tire provides very little traction or lateral grip. Therefore, controlling wheel dynamics is an important part of any stability control system. When it comes to tire traction control, two approaches may be used: First is a

curative approach that is activated when the longitudinal slip ratio exceeds a certain limit. In this approach, corrective measures are then taken to reduce the slip ratio (usually by means of braking). The second approach is a preventive method. In this approach, when it is detected that a wheel is about to start over-spinning, the requested torque on the wheel is reduced to avoid over-spinning. In the latter case, because individual braking is not used (or is minimally used), less energy is wasted, which is an important achievement for an electric vehicle.

From the above discussion it is clear that the preventive approach is superior over the curative one. To adopt this approach, a control system must be able to anticipate the impending tire saturation and counteract it by reducing the wheel torque ahead of time. To this aim, the controller needs to be augmented with the ability to predict the behavior of the system for a finite window of time in the future. In control terminology, such control methods are known as Model Predictive Control (MPC). In model predictive control scheme, a model of the system is used to predict its behavior for a finite number of sample times in the future, and then find the optimal control actions that minimize a given cost function. Therefore, the future time points affect decision making at the present time.

In addition, the MPC technique allows considering the time delays that are present in all practical systems. These time delays consist of actuator or sensor lags as well as phase delays introduced by low pass filters that are used to remove high frequency noise of raw sensor data. Moreover, model predictive control in its general form can explicitly address inequality constraints in devising a control law. This means that saturation of actuators in the prediction window affects decision making at the current time. This is what makes MPC more suitable than other control algorithms for vehicle control systems.

The driver's input through the accelerator pedal determines the total tractive force that he/she demands at each instant of time. In an all-wheel drive vehicle, there is a degree of flexibility in distributing the drive torque between the front and the rear axles. In the case of electric vehicles, this degree of freedom is used to move the operating point of electric motors as close as possible to their optimal region of operation to improve the vehicle

range without adding extra battery cells and mass. This task is usually done by a separate Energy Management Unit (EMU). The EMU distributes the drive torques between the rear and front axles according to their efficiency map. The stability control unit then modifies this distribution for the purpose of stability control.

By separation of stability and power distribution units, their mutual effects are completely ignored. The basic assumption is that the torque vectoring done by a stability controller is so small that does not violate the optimal distribution of tractive torque between front and rear axles. Experimental data however shows that this is not always the case (see for example Figure 5.33). More importantly, transferring tractive torque from front to rear axle and vice-versa has considerable effect on vehicle stability, especially during cornering. Therefore distribution of tractive torque without considering its effect on vehicle stability can have detrimental effect on vehicle stability. In addition, in some driving situations the power distribution and stability control units fight each other.

Besides the flexibility in distributing the drive torque, there is another degree of flexibility in controller intervention. The controller determines the amount of yaw moment to adjust the vehicle response. This additional yaw moment is distributed among the existing actuators. In modern vehicles, there are usually more than a unique way to distribute the required control action. For example, in yaw stability and for an all-wheel drive vehicle, there are infinite choices for distributing the required yaw moment by torque vectoring between front and rear axle wheels. This degree of freedom provides another flexibility to consider other objectives besides vehicle stability. Therefore, when a vehicle system is over-actuated, an opportunity exists to choose a distribution of control action combined with efficiency map of electric motors to minimize the power consumption. This however, can only be done by integrating the vehicle stability and power distribution in one unit.

1.2 Objectives

The first objective of this thesis is to develop an integrated controller for the combined system of vehicle chassis and wheels. In this approach, there is no need for a separate slip

control unit and the output of the integrated controller maintains vehicle stability (small sideslip angle), steerability (yaw rate tracking) and wheel stability (small tire slip ratio) at the same time. The model predictive control technique will be used to augment the controller with the ability to anticipate impending wheel and vehicle slip. Therefore, the controller would be able to adjust individual wheel torques so that the wheel slip is avoided.

The second objective of this thesis is to integrate the stability control and power distribution systems. Integration of the stability controller and power distribution units provides an opportunity to exploit the interactions and the cross couplings that exist between the two systems. This integration will also eliminate situations in which vehicle stability and power distribution have conflict of interest.

The third objective is that the controller can be easily and quickly reconfigured to work with various drivetrain and actuation systems. The drivetrains of interest include front-wheel drive, rear-wheel drive and all-wheel drive configurations. The actuation methods include torque vectoring, differential braking and hybrid mode (e.g. torque vectoring on rear axle and differential braking on front axle).

In addition, the designed model predictive controller should be able to run in real-time for the purpose of experimental verification on the available electric vehicles.

1.3 Thesis Outline

In the second chapter of this thesis, the background of vehicle stability control is studied. The importance of vehicle directional control and its ability in stability enhancement, accident avoidance, particularly in adverse road conditions are discussed. The literature of vehicle stability control is reviewed with emphasis on the approaches that adopt the model predictive control technique.

In the third chapter of this thesis, the integrated vehicle stability and traction control system is developed. The double-track prediction model used for modeling the vehicle dynamics and the wheel dynamics equation is introduced. The combined prediction model

is expressed in state-space representation. Next, the objective function that indicates the yaw rate tracking, lateral stability and wheel slip control targets is introduced. It is shown that the objective function can be expressed in terms of the initial state and prospective control actions. Next, the tire capacity and motor torque limit constraints are developed. The problem is cast in form of a quadratic programming problem. By solving this QP problem, the optimum control action is obtained. In the rest of this chapter, integration of the power distribution unit with the stability controller is studied.

Chapter four is devoted to simulation of the closed-loop response of two representative electric vehicles in various driving and road conditions. These vehicles are different in their driveline configuration, tires and the actuation method. Software co-simulations are performed using MATLAB/Simulink and CarSim software packages. The simulations demonstrate the effectiveness of the proposed control strategy for both vehicles and in various driving scenarios.

Chapter five presents the experimental results of the vehicle performance. The developed controller is implemented on dSpace Autobox and tested in real-time on two different electric vehicles. The vehicle response with and without the controller is examined in a variety of driving maneuvers and on different road surfaces. The results show promising performance of the controller in all of the driving conditions tested and robustness with respect to the road condition.

In chapter six, the conclusions and the contributions of this thesis are presented. In addition, the possible future steps to continue the work done in this project are mentioned.

Chapter 2

Literature Review and Background

This chapter presents a review on the literature of the vehicle stability analysis and control. The focus is on application of model predictive control in vehicle stability control. In addition to stability control, the literature of power distribution of electric vehicles, with emphasize on integration with stability control is studied.

2.1 Vehicle Stability Analysis

Before addressing the problem of stability control, it is best that the stability of a vehicle itself is studied. The purpose of stability analysis is to find the conditions for stability of a vehicle, or to find regions inside which stability is inherently guaranteed. Many authors have studied stability analysis. Inagaki and Kshiro [5] did a precise stability analysis on vehicle dynamics. They suggested phase plane analysis for a two degree of freedom bicycle model combined with Pacejka tire model using sideslip angle (β) and sideslip angle rate ($\dot{\beta}$) as the state variables. These plots are very useful in studying system dynamic response and stability analysis. The effect of front wheel steering on the phase portraits was also investigated. The authors observed that as the vehicle is steered in one direction, the stability margin is reduced in the steered direction and increased in the opposite direction (Figure 2.1).

The problem with $\beta - \dot{\beta}$ phase plane is that neither state is measurable in current production vehicles. This makes it hard to use the results of the stability analysis. That is why many authors have used vehicle yaw rate as a replacement for the vehicle sideslip angle rate in phase plane analysis. Yaw rate can be easily measured using stock sensors available in most of the vehicles produced these days. In addition, yaw rate can easily be influenced by current actuation systems.

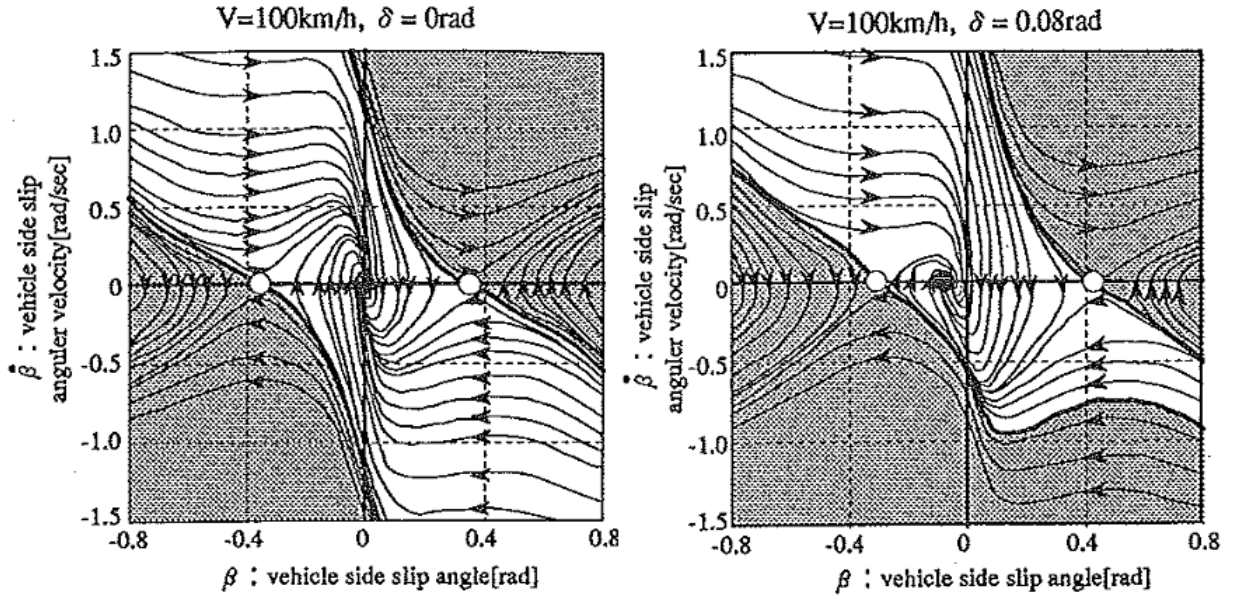


Figure 2.1: An example of movement of the stable equilibrium point and saddle points with change in steering angle (from [5]).

Hao, Xian-sheng et al [6] used a two degree of freedom bicycle model to analyze vehicle directional behavior in order to investigate the stability with different steering angles and forward velocities. They also used phase portrait method to graphically show the system response. They used Pacejka tire model to estimate tire lateral forces. They realized that as vehicle forward velocity is increased, the stable region of the phase portrait shrinks. Also, if the steering angle is introduced, the equilibrium points is moved along the vertical (yaw rate) axis until it vanishes, i.e. if the steering wheel angle is increased above a certain limit, the vehicle becomes unstable.

A similar study was done by Ono, Hosoe et al [7]. They also used a bicycle model to perform $\beta - r$ phase plane analysis. They also observed that an increase in the front wheel steering angle reduces the stable region within the phase portrait. Figure 2.2 shows a sample of such phase portraits. As it can be seen, when the vehicle is steered in one direction, the stable equilibrium point moves in the steered direction on the yaw rate axis. The stability margin reduces in the steered direction and increases in the opposite direction. If the steering angle is increased beyond a certain limit, the stable region vanishes.

Shibahata, Shimada et al [8] introduced a nonlinear analysis of the steady-state vehicle dynamics called the β -method. In this method, the stabilizing yaw moment and lateral force were synthesized at different vehicle speeds; sideslip angles and front wheel steering angles. They showed that at high sideslip angles, the front wheel steering has little effect on the yaw moment (reduced steerability). They primarily studied the effect of vehicle acceleration and deceleration during steady state cornering and developed a simple Direct Yaw Control (DYC) scheme to counteract the adverse effect of vehicle acceleration. They showed that torque vectoring based on the developed control algorithm can enlarge the limits of the vehicle maneuverability. However, the β -method is only valid in the steady state conditions and it ignores the dynamics during jumping from one state to another.

Similar stability analysis was done in [9–12]. In these studies, a nonlinear tire model is assumed and the open loop behavior of the vehicle is simulated with different forward velocities and front (and sometimes rear) steering angles. Afterwards, the stable and unstable regions of operation for each velocity and steering angle are obtained. The application of these stability regions are discussed in the next section.

2.2 Envelope Control

The concept of envelope control is widely used in aircraft control systems. It is based on the premise that as long as the uncontrolled dynamics of the aircraft are stable, there is no need for intervention from a controller and the pilot should be allowed to perform the maneuver that he/she intends. But if the aircraft enters the unstable region of operation,

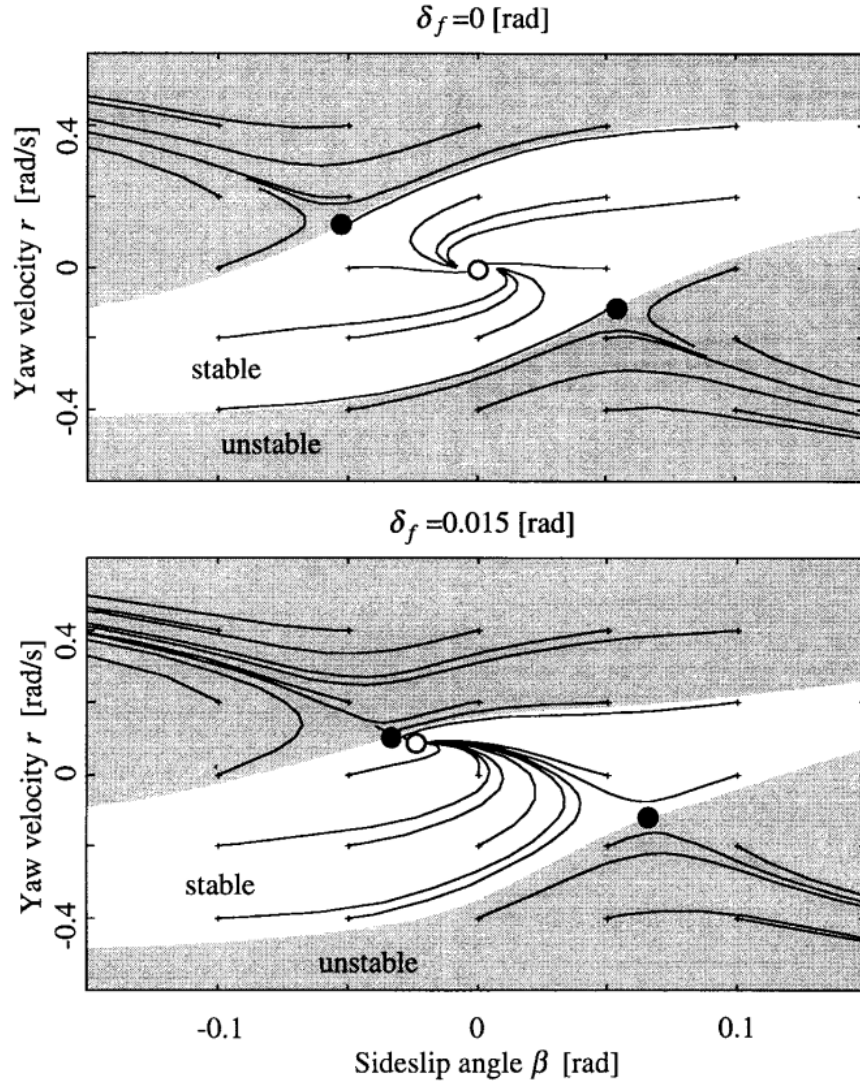


Figure 2.2: Sample β - r phase portraits in stability analysis, vehicle speed=20 m/s, $\bullet$ shows stable equilibrium point and $\circ$ show unstable equilibrium points (from [7]).

it should be returned the stable envelope by means of a controller. Such controllers impose constraints on the aircraft states such as speed, angle of attack, pitch and bank angle (see [13] as an example).

The notion of envelope control can be applied to a wide range of control applications. Vehicle stability is not an exception. In fact, using envelope control in vehicle systems has received considerable attention from researchers. Stability analysis, and in particular phase-plane analysis is extensively used in envelope control to determine the stable envelope when the controller is inactive.

Smakman [14,15] compared two control algorithms: defining reference value for the yaw velocity, thus a reference model, and defining a reference region for vehicle response. He argues that the second approach is superior since it minimizes controller intervention during regular driving. This stems from the common mindset that the driver should not notice the controller's intervention. He also compared effectiveness of two control algorithms: braking intervention system and integrated braking and wheel load control. He showed that design of stand-alone systems can have detrimental effects like cross-couplings and interferences. Besides, the opportunity for integration and exploiting the interaction between control systems is neglected.

Crolla et al [16] used a variable torque distribution controller to generate torque differential between left and right wheels to produce the yaw moment necessary to return the vehicle to the stable limits of vehicle as defined in [5]. The amount of the yaw moment is determined using a proportional controller and is based on the distance of the current operating point to the boundaries of the stable envelope. This control system is integrated with active front and rear steering that are used when the required yaw moment to track a reference yaw velocity is low.

Chung and Yi in [17] built upon Inagakis phase plane analysis. In their envelope control scheme, a closed region within the stability region in the phase portrait is defined. As the vehicle speed and steering angle is changed, the boundary of the safe region is updated to match the new vehicle state. They evaluated the controller performance during a lane change maneuver using a virtual test track. The method of actuation is differential braking

on front and rear wheels. They argued that the borders of the safe region should be flexible to be adjusted to the skill of the driver.

Bobier in his thesis [18] used phase plane analysis to define a closed safe region in the phase plane. He used sliding surface control to keep the vehicle state within the stable envelope in the phase plane. He showed the importance of precise estimation of the coefficient of friction for correct judgment of the safe region in the phase plane. Extra measures were taken to ensure that the safe region defined within the phase plane does not result in controller interference with vehicles natural stable transients. Sliding mode envelope control was used to move the vehicle states outside the safe area toward the borders. It turns out that the designed control scheme ensures the attractiveness of the envelope in large regions inside the phase plane.

Several other authors used similar concept in vehicle stability control. Beal [19] used phase plane analysis techniques to find the limits of vehicle stability and employed a model predictive control to keep the vehicle within the safe envelope. In [20], Yasui et al. designed a stability controller that would intervene by means of braking when the vehicle sideslip angle would exceed the stable limits. However, the main drawback of vehicle stability control using envelop control technique is that finding the stable region of the vehicle operation requires knowledge of the friction coefficient, a luxury that is unaffordable in practical control systems.

2.3 Yaw Rate and Sideslip Angle Tracking

As mentioned above, in the vehicle control literature, there are two competing views in controller design. One leads to envelope control which was reviewed briefly in Section 2.2, and the other one is reference tracking. In the latter, a reference model is designed that shows ideal vehicle behavior at all times, and the controller attempts to modify the response of the vehicle, so that it follows the desired behavior. The desired behavior of the vehicle is usually expressed in terms of the desired yaw rate and sometimes desired sideslip angle, which is often very small, if not zero. Manning and Crolla [21] wrote a review paper

on the state of the art of vehicle stability control. They summarized 68 prominent papers and categorized them into 3 categories based on the control goal: yaw rate control, sideslip angle control and both yaw rate and sideslip angle control. They argued that the papers focused on yaw rate tracking are mostly concerned with performance improvements in non-critical driving conditions. They also mention that as the vehicle approaches the limiting conditions, the actuation power required to push the nonlinear dynamics to the linear reference model increases beyond the capability of most actuation mechanisms. Regarding the sideslip control studies, they argue that even though Active Rear Steering (ARS) has been studied extensively in the literature, it is not been used much in production vehicles. The benefits of ARS for stability and safety are not significant compared to braking. Moreover, the deceleration disadvantage of braking is not a critical problem from a safety point of view. Regarding the integrated control papers, they criticize that most of the theoretical work lacked experimental verification and most of the practical works do not indicate the control algorithms used. They also noticed that many of the papers in this category use multiple actuators to track yaw rate and sideslip angle reference signals. Other papers use sliding surface control techniques to trade off multiple objectives when a single actuator is present.

Klomp [22] showed that the yaw moment generated by differential drive torques has a significant influence on the yaw stability of the vehicle, especially in limit cornering. He also uses a simple example to show that using a reference model requires an accurate measure of the coefficient of friction, thus confirming the claim in [14]. He used a simple proportional controller to minimize yaw tracking error with proper distribution of longitudinal forces.

Abe, Kano et al. [23] used sliding surface control to track reference values for vehicle side-slip angle and yaw rate. The reference values were obtained using a linearized model and were used to design the tracking controller. A simple sideslip angle estimation scheme was developed and experimentally verified. A rear-wheel steering system was used to track the desired value for the sideslip angle and DYC to track the yaw rate. They observed difficulty in precise sideslip tracking using 4WS and concluded that DYC is more effective in tracking sideslip angle.

Cho, Yoon et al. [24] used a linear bicycle model to generate desired values for yaw rate. They also included a first order filter in the model for the desired yaw rate to consider the transient conditions. A sliding mode control scheme was used to improve the robustness of the controller with respect to the uncertainty in cornering stiffness of the tires. Boundary layer technique was used to minimize chattering of the control signal. In this paper, a coordination of the chassis control systems was used to improve control effect on the lateral dynamics of the vehicle. The control systems included in this strategy were active front steering (AFS), electronic stability control (ESC) and continuous damping control (CDC) as well as four-wheel individual braking. Computer simulations showed improved performance of their Unified Chassis Control (UCC) over ESC.

Several other authors have used reference model for the purpose of vehicle control. Zhang et al. [25] used a sliding mode control to track a desired yaw rate by producing additional yaw moment by means of braking and used software simulation to show the effectiveness of the proposed stability controller. In [26], the authors study yaw rate tracking via active steering and differential braking, where braking intervention would only occur if the vehicle has reached the handling limits. The authors of [27] perform yaw rate tracking by means of controlling the drive and brake forces of all wheels in a four-wheel drive vehicle.

2.4 Stability Control Using Model Predictive Techniques

In the previous section, several papers were studied that have tackled the problem of vehicle stability control using a variety of techniques and control algorithms and using several types of actuators that all try to affect the behavior of the vehicle. Common to all the actuation methods is the fact that they have limited actuation capacity, which in the control theory, translates into a constraint. Very often, a controller is designed for an unconstrained system, and when the actuation signals reach the limits of capacity of the actuator, it is clipped. This approach however can lead to oscillatory system response and

also raises the question of the optimality of the adopted approach.

The Model Predictive Control (MPC) technique has the capability of explicitly considering the actuator and state constraints. In an MPC approach, not only the constraints are satisfied, their information is used to find an optimal solution. These properties put model predictive control in a unique position and very interesting for vehicle control systems. Consequently, it has received a lot of attention from the researchers in the past decade.

2.4.1 Accuracy and Complexity of Prediction Model

Choosing a proper prediction model in MPC is a challenging task. The prediction model needs to be descriptive enough to capture the important dynamics of the system, and at the same time, it has to be as simple as possible so that the resulting controller is simple and fast enough for practical applications. The performance of the resulting controller to a great extent depends on the selected prediction model. Some authors studied the importance of selecting a prediction model that is detailed enough to capture the nonlinear dynamics of the vehicle. Because it is in the nonlinear range that stability controllers are most needed.

For example, Falcone et al. [28] used model predictive control to perform path following via active front steering and differential braking on four wheels. They used two different models with different levels of complexity and accuracy. One was a full vehicle model of tenth order and the other one was a simple bicycle model. They used Pacejka's tire model to capture the nonlinear characteristics of tire. In the bicycle model, because it is a single track model of the vehicle, the braking torque is considered as an input variable and its optimal value is found during the in-loop optimization. Then an algorithm was used to distribute the braking torque between four wheels of the vehicle. They observed that the braking and steering outputs of the controller cooperate well to do the trajectory tracking. The simulation results also showed that the controller that is based on the simplified bicycle model is not able to stabilize the vehicle at high speeds.

Another study regarding the complexity of the prediction model was done by Plamieri et al. [29]. They studied the advantage of considering vehicle roll dynamics in the pre-

diction model for the purpose of vehicle stability control using active front steering. They compared the stabilization performance of two NL-MPC controllers: one based on a twelfth order nonlinear model including roll dynamics and the other one a tenth order nonlinear model that ignores the roll dynamics. Computer simulations showed that when roll dynamics are included, the controller can stabilize the vehicle with higher entry speeds. It is observed that considering roll dynamics is most helpful in high- μ and high speed maneuvers, where the roll motion of the vehicle is significant. Instability inevitably occurred for both controllers at higher speeds, because of the finite horizon of the controller.

2.4.2 Nonlinear Model Predictive Control

In addition to the modeling detail that matters in the prediction model, there is the question of using a linear or nonlinear model. Unlike linear models, nonlinear models are often accurate in a much broader range of vehicle operation. Therefore, they can provide a better description of the global dynamics of the system. A few authors have tried nonlinear model predictive control (NLMPC) in their work. For instance, Borrelli et al. [30] studied active steering of autonomous vehicle systems using model predictive control. They used a nonlinear bicycle model along with the Pacejka model for tire as the system model, and their method of actuation was active front steering. Using nonlinear MPC, they tried to find optimal control actions to perform path tracking. For NLMPC, the commercial code NPSOL ([31]) was used to solve the nonlinear programming problem. They showed effectiveness of the controller in a double lane change maneuver with increasing entry speed during vehicle coasting (i.e. no traction torque or brakes). They studied the required size of the prediction horizon and control horizon necessary to stabilize the double lane change maneuver with different entry speeds. A similar approach is used in [32, 33].

In spite of excellent performance of NLMPC controllers, their practical use is very limited. Using a nonlinear model as the prediction model leads to a nonlinear optimization problem that needs to be solved at each sampling time. Although there are good number of nonlinear programming solvers available (e.g. [31, 34]), due to practical implementation difficulties, using a nonlinear model is generally unfavorable.

2.4.3 Linear Time-Varying Model Predictive Control

As mentioned in the previous section, practical applications of NLMPC are limited due to difficulty in solving the resulting nonlinear programming problem. Therefore, many researchers start by a nonlinear model of the system and use successive linearization of that model to avoid nonlinear programming. This approach gives a sub-optimal controller. For example, Palmieri et al. [35] used a linear time-varying MPC method to stabilize a vehicle during harsh maneuvers such as high-speed double-lane change. The method of actuation was differential braking. They used a full vehicle model for prediction. The model was linearized at each time step, thus leading to a LTV-MPC problem which is much easier to tackle. They added a slip controller in series with the MPC controller to generate the desired braking force. Computer simulations were used to test the proposed control scheme.

Falcone et al. [36] used a linearized version of the nonlinear vehicle model for the purpose of design of a model predictive controller for path tracking of an autonomous vehicle. The controlled variables were the front steering angle and active braking/active differential. They investigated the tracking performance of the proposed controller in a double lane change maneuver on a slippery road with computer simulations and studied various actuation configurations (no braking, no traction/braking intervention). A similar technique is used in [37–42].

Canale et al. [43, 44] used a different technique to avoid nonlinear programming. They used a single track model with yaw rate and sideslip angle as the states for the purpose of prediction and started the controller design with developing a NLMPC controller. Afterward, they used an approximated control function with finite number of exact NLMPC solutions that were calculated offline to reduce the online computational task and make the real-time implementation feasible. They also proved the stability and constraint satisfaction of the proposed approximate NLMPC controller. They showed the effectiveness of the proposed method using software in the loop (SIL) test on an embedded device with limited computational capacity and a 14 DOF detailed vehicle model. The results showed the effectiveness of the proposed approximate NLMPC method and good stability in demanding

driving conditions.

2.4.4 Hybrid Model Predictive Control

One argument against LTV-MPC is that although linearization is performed at each sample time, it is only valid for small changes in the variables throughout the prediction window. If the changes are not small, then modelling inaccuracy can result in performance degradation. Another approach for having an accurate yet not so complex prediction model is using hybrid dynamic models (see [45–47] for more details). In this approach, the non-linearity of the model is approximated by piece-wise affine (PWA) functions. Based on the state of the system, one of the affine sections is active at each instant of time. The index of the active section(s) is one of the variables of the system, thus forming a hybrid system model or mixed integer dynamic systems. Using a hybrid prediction model leads to a mixed integer quadratic (or linear) programming (MIQP or MILP) that can be solved using available software (e.g. [48]).

Hybrid model predictive control (HMPC) has received some attention in recent years. Borrelli et al. [49] used a mixed logical dynamic model of the combined vehicle and tire system to capture the main behavior of the internal combustion engine and the wheel. The force developed in the tire contact patch is approximated with a piece-wise affine function in terms of the coefficient of friction and slip. The control problem is augmented with constraints on engine torque and engine torque gradient. The hybrid system consists of a continuous vehicle/wheel model and an auxiliary binary variable which indicates the region in the tire characteristic curve that is active (tire characteristic curve is divided into two regions). Their goal was to regulate the engine torque (by spark timing) so that the wheel slip remains in the target zone where the traction force is maximal.

Di Cairano et al. [50] studied the yaw stability problem for a non-autonomous vehicle equipped with active front steering and differential braking. They used tire brush model (see [51]) to approximate tire forces versus tire slip angles. They used a PWA system to describe the dynamics of the vehicle based on the current values of sideslip angles (hybrid MPC model). In formulating the MPC problem, limits on tire sideslip angles

were introduced. They also studied the sensitivity of the closed-loop system with respect to variation of parameters such as longitudinal velocity, road friction coefficient and also the slip angle corresponding to peak tire lateral force. It was observed that for large deviations between nominal and actual longitudinal velocity, loss of stability may occur. Similar observation can be made in case of road adhesion factor. Overall, the controller was judged reasonably robust with respect to parameter variations.

However, mixed integer programming is in general complex and requires expensive software and hardware. In [50], the authors use the assumption of no mode switches within the prediction horizon to convert the HMPC to a switching MPC, in order to reduce the computational effort and size of the problem.

2.4.5 Explicit Model Predictive Control

Model predictive control has proved effective in many control applications. But the computational costs limit its use to systems with relatively fast hardware. In explicit MPC, the programming problem is solved offline in terms of the initial state of the system, using multiparametric programming (see [52]). In the final control law, at each instant of time, based on measurement of system state, the control law is looked up from memory and control action is calculated.

Using explicit MPC highly reduces the online computational effort, but to store the offline solution, it requires considerable memory space. Some authors have experimented with explicit version of MPC. For example, Tondel and Johansen [53] used multiparametric nonlinear programming (mp-NLP) to solve the control allocation problem. They assumed a high level controller is producing the required additional yaw moment based on the difference between desired yaw rate and actual yaw rate, and it is the role of the control allocation unit to generate the required torque by using brakes on individual wheels. In order to allow real-time implementation, they used mp-NLP techniques to find explicit solution for the optimization problem offline. The simulation results show good stabilization capacity of the proposed allocation scheme in maneuvers that open loop behavior would lead to instability. Although, as the number of parameters increase (for example when

using a more complex model or using fewer simplifying assumptions), multi-parametric programming techniques cannot help to solve the optimization problem offline.

Explicit MPC can also be used for hybrid problems which were discussed in the previous section. For example, Borelli et al. [49] obtained the explicit version of their HMPC controller using multi-parametric programming technique for mixed-integer linear programming problems, after it was tuned for desired performance. The resulting explicit controller was implemented on a low cost hardware and tested on a passenger vehicle. Results showed good traction control performed by controller.

2.5 Integration with Wheel Slip Control

Controlling the slip ratio of tires is an important part of the vehicle stability control. If the slip ratio of a tire exceeds a certain threshold, its force capacity in the lateral direction is severely reduced. This can lead to a significant understeer or oversteer during cornering especially on low friction surfaces, which can hardly be corrected by stability controllers. It is common practice to assume a separate tire slip control module exists that keeps the tire slip ratio within the permissible range (e.g. [54–59]). For instance, Feiqiang et al. [54] used a fuzzy logic controller to control directional stability of an all-wheel drive electric vehicle. In their control scheme, vehicle response is compared with the desired response and differential braking is used to modify vehicle response. However, tire slip ratios are controlled by a completely separate unit and any cross effect between slip control and stability control is ignored. Separate slip control and stability control modules means that the torque adjustments made by the stability controller are altered by a separate module; therefore, their optimality is compromised.

Few authors have attempted to design an integrated stability and traction control systems. Palmieri et al. [35] investigated integration of a model predictive stability control module with a slip control system. The method of actuation was differential braking. The desired braking forces are calculated in the stability controller and then passed to a slip control module to generate the brake force. However, even in this structure, the slip control

and stability control modules are separate entities and not fully integrated.

A better integration of the vehicle and wheel dynamics is done by Zhou et al. [60]. The state vector of their controller includes vehicle yaw rate, sideslip angle and tire slip ratios. Computer simulations are used to evaluate the performance of the controller. It is observed that careful tuning of controller parameters is required to achieve acceptable performance and avoid wheel lock.

Nonetheless, an integrated controller that controls vehicle and wheel stability and can be configured to work with various driveline and actuator configurations is unparalleled in the literature. The controller designed in this thesis allows integration of these controllers to obtain a highly optimal control actions that maintain vehicle and wheel stability at the same time.

2.6 Optimal Power Distribution Using MPC

In electric vehicles with independent front and rear motors, there is a degree of flexibility in torque distribution between front and rear motors. This provides opportunity for finding the most energy efficient torque allocation between front and rear motors. Several authors have studied the optimal torque distribution for improved energy efficiency of the vehicle. Yuan and Wang [61] studied the optimal torque distribution in an all-wheel drive electric vehicle over a range of speed and torques. It was observed that the maximum efficiency is generally achieved when the total torque is equally shared between identical motors. However, in the low-torque region, it is more efficient that just one of the motors is used to generate the torque demand. The torque redistribution was observed to improve the drive train efficiency by 4% over the New European Driving Cycle (NEDC).

Chen and Wang [62] studied the problem of energy efficient torque distribution in over-actuated systems such as electric vehicles with independent front/rear drives. They used a KKT¹-based numerical optimization techniques to find the globally optimal torque

¹Karush-Kuhn-Tucker (optimality conditions)

distribution between front and rear motors, based on the efficiency maps of the electric in-wheel motors. Computer simulations were used to evaluate the computational speed of the optimization algorithm proposed.

The above studies generally consist of solving a static optimization problem to find the optimal torque distribution. However, model predictive control technique is well suited for the purpose of power distribution in hybrid and/or full electric vehicles. In cases where driving cycle is known a priori, it can be easily used in the predictive approach of MPC. Some studies regarding optimal power distribution using MPC can be found in the literature. Borhan et al. [63,64] studied power distribution in a hybrid electric vehicle using model predictive control. A nonlinear model for the plant was developed that captured the essentials of the system behavior. This model was successively linearized at each sampling time about the working point. Constraints were considered on a couple of states such as state of the charge (SOC) of the battery, engine, motor and generator speed. The objective of the MPC controller is to keep the SOC at its desired level and minimize the fuel consumption rate and brake usage. Computer simulations show that the predictive controller has a better performance and results in better fuel economy.

Ripaccioli et al. [65] used hybrid modeling techniques to develop a hybrid dynamical model of different components in a hybrid electric vehicle. A hybrid model predictive controller was designed and tuned for the best performance. The resulting controller was used in closed-loop simulations using a high-fidelity nonlinear model.

2.7 Integration with Stability Control

Separation of the power distribution unit and the stability controller is commonly used in the literature (e.g. [66,67]). However, the power distribution unit can transfer significant amounts of torque between the front and rear axles. This is fully ignored in a separate design and in absence of cross-talk between power distribution and stability control units.

Some authors have attempted integration of the power distribution with stability control. For example, Chen and Wang [68] studied the electric vehicle motion control while

achieving optimal energy optimization. The high level controller finds the desired adjustments for vehicle motion control and the low-level controller allocates the control actions to each wheel while considering their energy efficiency. Similar approach is used in [69]. In these studies, the torque redistribution during normal longitudinal vehicle driving is not considered in the vehicle stability control system. In an integrated approach, the vehicle stability is the dominant objective of the controller and torque redistribution is only performed when vehicle and wheel stability objectives are attained.

2.8 Considering Control Loop Delays

Regardless of the control technique used, all controllers are subject to performance drop in the presence of delays in the control loop. A certain amount of delay can be found in any practical control system. This delay can have multiple sources such as communication, measurement or actuation. In the context of model predictive control, extensive computational demand at each sample time requires a non-trivial processing time and can contribute to the overall delay in the control loop.

Cortes, Rodriguez et al. [70] studied current control of a three-phase inverter using MPC. They studied the adverse effect of delay introduced because of the computational effort and proposed a method to compensate for this delay by estimating the current at the next sample time. Computer simulations and experimental tests showed that this method reduced the amount of current ripple. If the amount of delay is minimum, the controller is often designed by ignoring the delay. However, larger amount of delay can have a severe impact on the control loop. Sakai, Sado et al. [71] studied the wheel dynamics control in electric vehicles in presence of delays in the control loop. They noticed that in the presence of delay, even high values of feedback gains cannot prevent wheel skid. The experimental results also confirmed that with the delay, regardless of the tunings, the traction control system was not completely successful at preventing wheel skid.

Robust control techniques are commonly used to make the controller robust to time delays (see [72–75] for example). Chen and Ulsoy [75] studied design of a vehicle steering

controller considering the delay in the driver model. Using H_∞ and Quantitative Feedback Theory (QFT) techniques, they designed a robust Smith predictor controller to handle the delay in the driver model. The simulation results showed improved performance in critical situations. However, robust control techniques involve complicated design and tuning process and usually result in an overly conservative controller.

Another technique involves adding the delayed states to the state-space model of the system. For instance, Shuai, Zhang et al. [76] studied the lateral stability control of an electric vehicle considering the effect of time-varying CAN induced delays. They used active front steering and torque vectoring in order to enhance vehicle behavior. The communication delays were considered unknown but bounded. The system states were augmented with the delayed inputs, thus creating a larger state-space system. Using computer simulations, the controller was compared with a conventional controller and was observed to show improved performance. However, augmenting the system states with delayed states/inputs greatly increases the size of the system and computational cost, therefore is not suitable for real-time implementation.

Section 3.4 of this thesis proposes a method to handle a variety of dead-time and first order delays that can exist in a control loop. It is suggested that at each calculation step, the vehicle state at the end of the delay period is predicted and controlled. In the context of MPC, a prediction model is already available for the controller, as a result this method can be easily applied. Although this method is applied to a vehicle stability control problem, it is not specific to a certain application and can be used in a wide range of control problems.

2.9 Summary

In this chapter, the literature of vehicle stability control was studied, with special attention to the papers that used model predictive control. It was discussed that there are two different approaches for affecting the behavior of the vehicle: defining a reference region, or defining a reference model. In the first approach, the controller is only activated when

the vehicle is leaving the boundaries of a stable dynamics, and hence the controller tries to return the vehicle to its stable region of operation. In the second approach, the controller is active all the time and tries to push the dynamics of the vehicle towards a desired linear behavior. In this thesis, the second approach is used. Because switching the controller on and off is generally considered unfavorable. On the other hand, if the desired linear behavior is properly defined, in the linear and stable region it should more or less match the natural behavior of the vehicle and controller intervention will be minimal.

Several versions of model predictive control and their application in vehicle stability control were studied in Section 2.4. As discussed, each method has its own pros and cons. In this thesis, a nonlinear prediction model is used, however, in order to avoid dealing with a nonlinear programming problem, successive linearization is used to linearize the prediction model at each sample time. This results in a Quadratic Programming (QP) problem at each time step, which is much easier than a nonlinear programming problem.

In addition, nearly all of the stability control systems developed in the literature are designed separately from the wheel slip control module and are added on top of a individually designed traction control module. In this thesis, an integrated vehicle stability and traction control system is designed that controls wheel slip and vehicle stability at the same time.

As it was mentioned in Section 2.6, using model predictive control for the purpose of power distribution has not received much attention from the researchers and its integration with model predictive stability control has not been studied to the extent of this author's knowledge. The existing research mostly focuses on hybrid vehicles and limits the attention to the power distribution part only, without considering its effect on vehicle stability. That is why integrated stability control and power distribution using model predictive control is studied in this thesis.

Chapter 3

Design of an Integrated Model Predictive Controller

In this chapter, a model predictive control scheme for integrated control of vehicle and wheel stability is developed. To this aim, a double-track prediction model is used to predict the future vehicle states. This prediction model of the vehicle is augmented with the wheel dynamics to serve as the prediction model for the integrated vehicle and wheel system.

This chapter is structured as follows. First the desired vehicle responses which serve as reference values for the controller are defined. Next, the prediction model is presented and the governing equations are derived. The prediction model is then expressed in the discrete state-space format. Later, the objective function that drives the controller towards its tracking and regulation objectives is defined. Then, the constraints on the control actions are defined. In the last section, integration of the power distribution unit with the developed controller is discussed.

3.1 Introduction

The concept of model predictive control is illustrated in Figure 3.1. At each instant of time, the current system state is measured (or estimated) and using the prediction model, the behavior of the system at a finite number of points in the future (within a window called the prediction horizon) is predicted. System states (or outputs) at these points are expressed in terms of the current state and prospective control actions. Then, depending on the type of the control problem (regulation or tracking), the objective function is defined. In Figure 3.1, a tracking problem is used to illustrate the bases of model predictive control. In this case, the objective function can be defined as:

$$J(x(t), u_{t \rightarrow t+N|t}) = \sum_{k=1}^{N_p} q(y(t+k|t) - y_{ref}(t+k))^2 + u(t+k|t)^2 \quad (3.1)$$

In Equation (3.1), $y(t+k|t)$ stands for the output of the system at time $t+k$ predicted at the present time. Similarly, $u(t+k|t)$ denotes the input to the system at time $t+k$ calculated at the present time. $u_{t \rightarrow t+N|t}$ is a vector of inputs at times t to $t+k$ calculated at current time. $y_{ref}(t+k)$ is the desired value for the output of the system at time $t+k$ (if known, otherwise assumed constant) and the scalar q represents the trade-off between tracking error and control effort. The size of the prediction horizon is shown by N_p .

At each instant of time, the objective function J is minimized and a sequence of optimal control moves $u^*(t+k|t)$ is found. The first element in the sequence (i.e. $u^*(t+1|t)$) is then applied as the control action to the system and the rest of them are discarded. At the next sample time $t+1$, new measurements (estimations) are obtained and the whole process is repeated and a new series of optimal control actions is determined. The prediction window thus recedes towards future times, hence the alternative name receding horizon control (RHC).

In this thesis, a model predictive controller is designed to modify the vehicle response in order to improve yaw rate tracking, maintain small sideslip angle and tire slip ratios and also improve the overall vehicle efficiency. Figure 3.2 shows the block diagram of the control system. The controller receives estimates of vehicle longitudinal and lateral

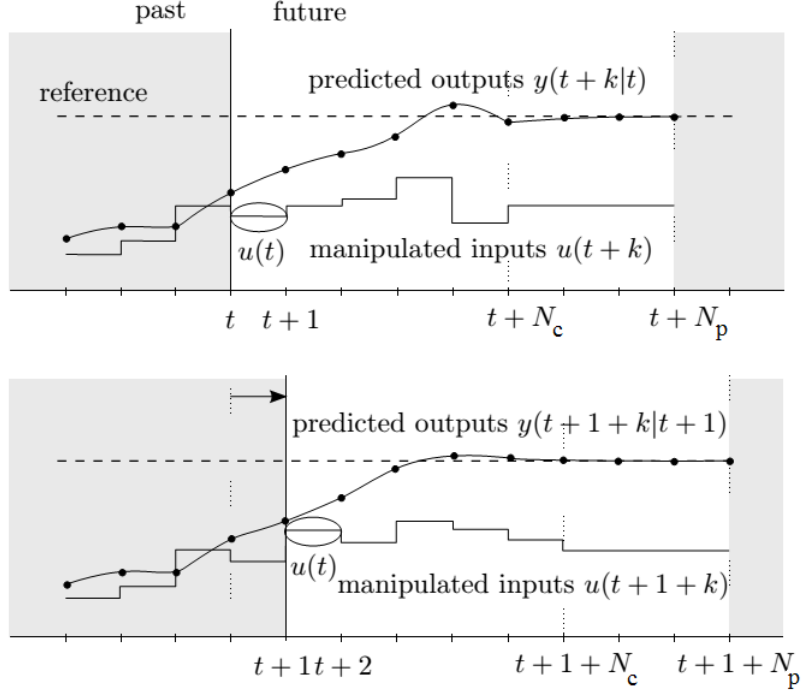


Figure 3.1: Illustration of model predictive control (MPC) concept ([47]).

velocities as well as tire forces from separate estimation schemes. In addition, the controller also receives feedback from the vehicle sensory system, including vehicle longitudinal and lateral accelerations, vehicle yaw rate as well as wheel rotational speeds. Using model predictive control technique, the controller calculates the optimal torque adjustment $\delta \mathbf{Q}$, that is added to the driver's requested torque and applied to the vehicle.

3.2 Desired Vehicle Response

In this section, the desired vehicle response is defined. The desired response consists of the desired yaw rate, desired lateral velocity and desired wheel speeds. These values serve as reference values when the control problem is cast in the form of a tracking problem.

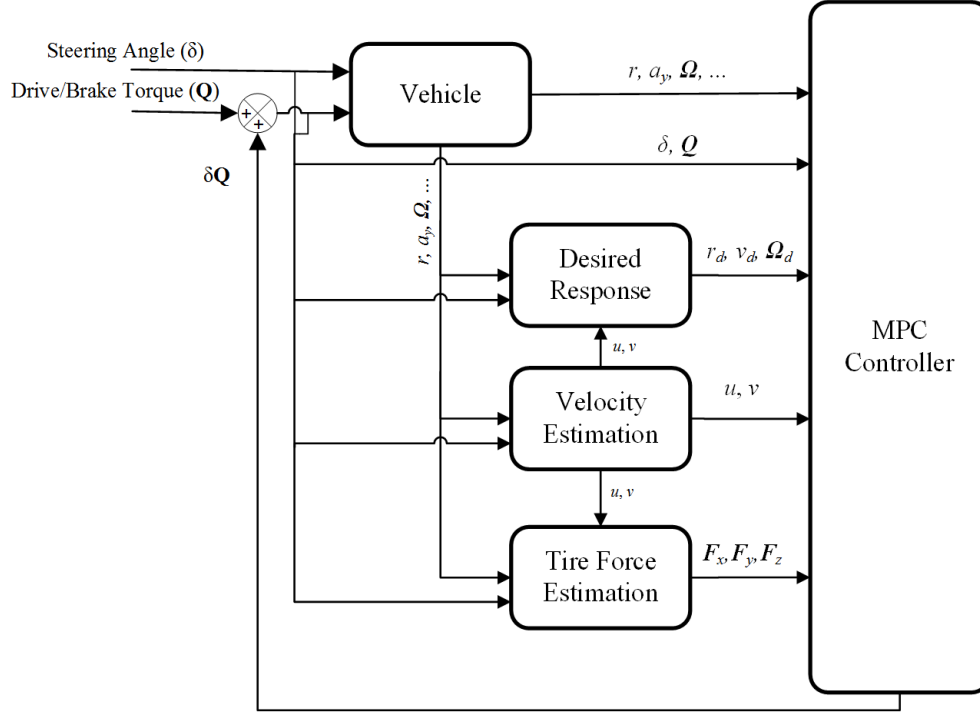


Figure 3.2: The structure of the model predictive control system.

3.2.1 Desired Yaw Rate

The desired vehicle yaw rate is defined in Equation (3.2) according to the steering angle, vehicle velocity and vehicle geometry ([77]):

$$r^* = \delta_F \frac{u}{L + k_{us} u^2 / g} \quad (3.2)$$

where δ_F is the steering angle of the front wheels, u is the vehicle longitudinal velocity, L is the vehicle wheel base, g is the gravitational constant and k_{us} is the desired understeer gradient. At a given speed, an increase in the understeer gradient results in a lower desired yaw rate, thus demanding a more understeer vehicle response.

The coefficient of friction between the tires and the road limit the maximum yaw rate that the vehicle can safely assume at a given speed. Any higher yaw rate can only be attained at the expense of a large vehicle sideslip angle, which is an unstable response.

Therefore, the yaw rate defined in Equation (3.2) has to be adjusted according to road conditions. During limit cornering, the vehicle lateral acceleration is directly related to the coefficient of friction between tires and the road. Therefore, the maximum vehicle lateral acceleration can be used as a measure of friction coefficient to limit the desired vehicle yaw rate ([50]):

$$r_d = \text{sign}(r^*) \min(|r^*|, |\frac{a_{y_{max}}}{u}|) \quad (3.3)$$

where $a_{y_{max}}$ is the maximum lateral acceleration at the C.G. of the vehicle.

3.2.2 Desired Lateral Velocity

In this thesis, two methods for controlling the lateral vehicle velocity are developed: the direct method and the indirect method. In the direct method, a desired lateral velocity is defined and the controller is configured to track this desired value. In the indirect method, the lateral velocity is controlled indirectly through adjusting the desired yaw rate based on the vehicle lateral velocity. In this approach, the controller does not directly track a value for the lateral velocity. Both methods are presented in this section.

Direct method

In the direct method, a desired vehicle lateral velocity is defined. In non-critical conditions that the vehicle sideslip angle (β) is small, the desired lateral velocity is defined as the actual lateral velocity. Therefore, yaw rate tracking and steerability are the primary objectives of the controller. On the contrary, if the sideslip angle increases beyond a certain threshold (β_{max}), the desired lateral velocity is assumed zero, so that the vehicle sideslip angle is controlled to the stable range. This is shown in Equation (3.4):

$$v_d = \begin{cases} 0 & |\beta| \geq \beta_{max} \\ v & \text{otherwise} \end{cases} \quad (3.4)$$

where v_d is the desired vehicle lateral velocity, v is the vehicle lateral velocity, β is the vehicle sideslip angle and β_{max} is a tunable threshold for controlling the lateral velocity.

Indirect method

In this method, the lateral vehicle velocity is controlled through adjusting the desired yaw rate that the controller is tracking. In order to maintain small sideslip angle, the desired vehicle yaw rate is adjusted according to the vehicle lateral velocity. The lateral vehicle acceleration is related to the vehicle velocities by:

$$a_y = \dot{v} + ru \quad (3.5)$$

Now, if the lateral velocity exceeds stable limits, the following stable dynamics is assumed for lateral velocity to bring it back to the stable range:

$$\dot{v} + \gamma v = 0 \quad (3.6)$$

where $\gamma > 0$ is a tuning parameter and determines the convergence speed of the lateral velocity. Combining Equations (3.5) and (3.6) gives:

$$ru = a_y + \gamma v \quad (3.7)$$

Now, the assumption of large lateral velocity results in tire saturation in the lateral direction. Therefore, the vehicle assumes its maximum lateral acceleration. In this case, Equation (3.7) becomes:

$$r = \frac{a_{y_{max}}}{u} + \gamma \frac{v}{u} \quad (3.8)$$

Comparing to Equation (3.3), it can be inferred that the second term in Equation (3.8) can be used to adjust the desired vehicle yaw rate such that the vehicle sideslip angle remains small. Therefore, the desired yaw rate for the vehicle is defined as:

$$r_{dv} = r_d + \gamma \beta \quad (3.9)$$

where r_{dv} is the desired yaw rate that indirectly controls the vehicle sideslip angle, r_d is defined in Equation (3.3) and γ serves as a tuning parameter. In the indirect method, the controller pushes the vehicle yaw rate towards the value defined in Equation (3.9) to maintain a small sideslip angle.

Figure 3.3 illustrates the desired yaw rates in various cases. Figure 3.3a shows an oversteer scenario. In this case, the vehicle has a large negative sideslip angle. According to Equation (3.9), this reduces the desired cornering yaw rate so that a negative yaw moment is generated by the controller at the vehicle C.G. to align the vehicle with the trajectory and reduce the sideslip angle. On the contrary, in Figure 3.3c the vehicle has a significant understeer behaviour. In this case, a positive sideslip angle increases the desired yaw rate in Equation (3.9) so that the controller applies a positive yaw moment at the vehicle C.G. to improve trajectory tracking of the vehicle. Figure 3.3b shows the stable cornering condition, where the sideslip angle is small the second term in Equation (3.9) is negligible.

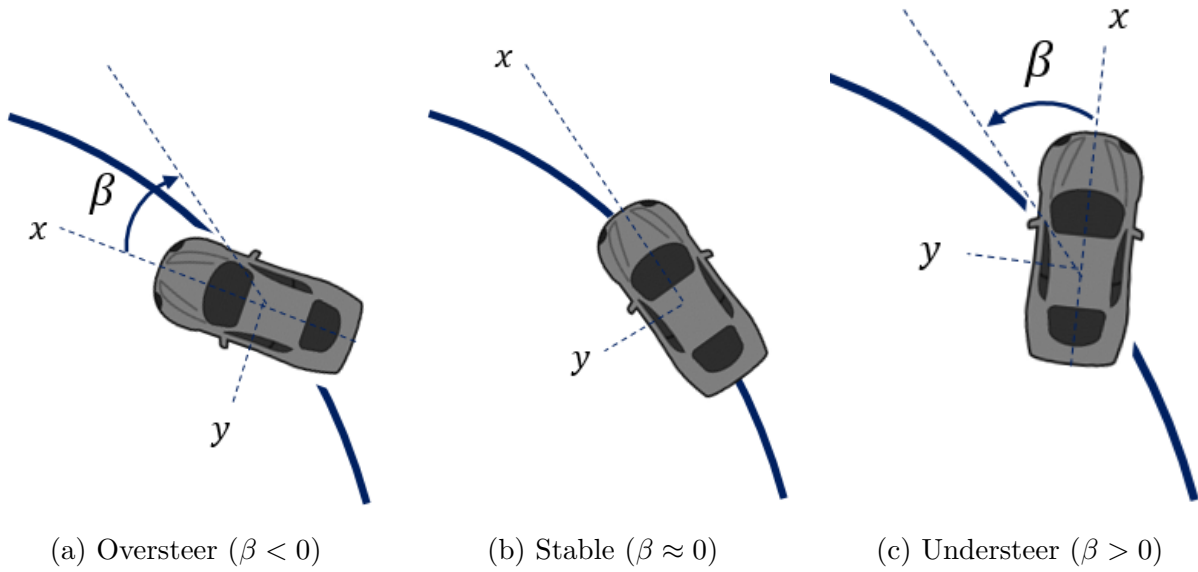


Figure 3.3: Comparison of the desired yaw rates in various driving situations using indirect method ($r_{dv} (a) < r_{dv} (b) < r_{dv} (c)$).

3.2.3 Desired Wheel Speeds

Controlling the wheel speeds is an essential part of vehicle stability control. If the tire slip ratio exceeds a certain limit, the longitudinal and lateral tire forces drop significantly. The tire slip ratio is defined as ([78]):

$$\kappa_{ij} = \frac{R_{eff}\omega_{ij} - u}{\max(u, R_{eff}\omega_{ij})} \quad (3.10)$$

where ω_{ij} is the measured speed of the wheel ij . Figure 3.4 shows the typical variation of longitudinal and lateral tire forces versus tire slip ratio. It can be seen that when the slip ratio exceeds a certain threshold, the lateral tire forces drop significantly. If this happens to the front tires, it results in poor steerability and severe understeer. If this occurs in the rear tires, it increases the risk of vehicle instability and oversteer. Therefore, maintaining the tire slip ratios in the stable range is crucial to vehicle stability control.

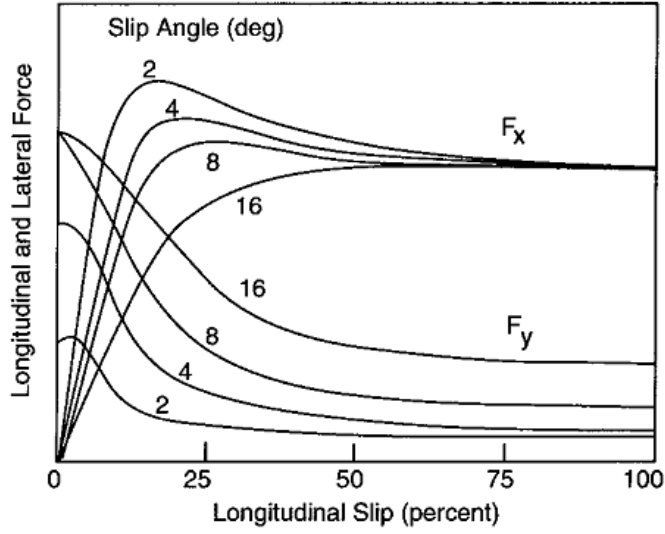


Figure 3.4: Typical variation of longitudinal and lateral tire forces versus slip ratio ([77]).

In this section, the desired wheel speeds are defined based on the longitudinal velocity

at the wheel center, divided by its effective radius:

$$\mathbf{\Omega}^* = \begin{bmatrix} \omega_{fl}^* \\ \omega_{fr}^* \\ \omega_{rl}^* \\ \omega_{rr}^* \end{bmatrix} = \frac{u}{R_{eff}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{r}{2R_{eff}} \begin{bmatrix} -w_F \\ +w_F \\ -w_R \\ +w_R \end{bmatrix} \quad (3.11)$$

where R_{eff} is the effective radius of the wheels, w_i is the trackwidth of axle i and r is the vehicle yaw rate. The wheel speeds defined in Equation (3.11) are used as the desired wheel speed when the tire slip ratio exceeds a certain threshold (κ_{max}):

$$\omega_{ij,d} = \begin{cases} \omega_{ij}^*, & |\omega_{ij}^* - \omega_{ij}| \geq \kappa_{max} \max(\omega_{ij}, \omega_{ij}^*) \\ \omega_{ij}, & \text{otherwise} \end{cases} \quad (3.12)$$

$$\mathbf{\Omega}_d = \begin{bmatrix} \omega_{fl,d} \\ \omega_{fr,d} \\ \omega_{rl,d} \\ \omega_{rr,d} \end{bmatrix} \quad (3.13)$$

The desired wheel accelerations are obtained calculating the time derivative of Equation (3.11):

$$\dot{\mathbf{\Omega}}_d = \frac{a_x}{R_{eff}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{\dot{r}}{2R_{eff}} \begin{bmatrix} -w_F \\ +w_F \\ -w_R \\ +w_R \end{bmatrix} \quad (3.14)$$

where a_x is the vehicle longitudinal acceleration and $\dot{r}$ is the vehicle yaw acceleration.

3.3 Prediction Model

The prediction model for the vehicle stability control is developed in this section. The accuracy and complexity of the prediction model has a severe impact on the closed-loop

performance of the controller. In this thesis, a double-track vehicle model is used as the prediction model. This model, captures the vehicle directional dynamics, yet, is simple enough for in-the-loop optimization of model predictive control.

The prediction model consists of two parts: vehicle dynamics and wheel dynamics. The vehicle dynamics part captures the directional response of the vehicle and predicts the vehicle yaw rate, yaw moment and sideslip angles within the prediction window. The wheel dynamics part describes the wheel response to the torque applied and is used to predict the wheel rotational speeds for the purpose of wheel slip control. In this section, each part is developed separately and combined at the end to form the integrated vehicle and wheel prediction model. The inputs to this integrated prediction model are the total torques applied at each wheel, and the outputs are the vehicle states that are required to be controlled, namely vehicle yaw rate, lateral velocity and wheel speeds.

3.3.1 Vehicle Dynamics

The vehicle part of the prediction model is based on a double-track vehicle model. To design the MPC controller, the prediction model has to be expressed in the state-space representation. The states of the prediction model are:

$$\mathbf{X}_1 = \begin{bmatrix} r & M_{F_y} & v \end{bmatrix}^T \quad (3.15)$$

where M_{F_y} is the yaw moment of lateral tire forces. The rest of this section is devoted to finding the update equation (i.e. $\dot{\mathbf{X}}_1$) for the state vector $\mathbf{X}_1$, according to the governing equations of the double-track vehicle model.

Figure 3.5 shows a double-track vehicle model. In order to predict the vehicle yaw rate and lateral velocity, the yaw moment acting at the vehicle center of gravity (C.G.) needs to be obtained. This, requires prediction of the tire forces within the prediction horizon. Assuming that the slip ratio of the tires are maintained within the linear range, the longitudinal tire forces may be approximated as:

$$F_{x_{ij}} = \frac{Q_{ij}}{R_{eff}} \quad i = F, R; \quad j = L, R \quad (3.16)$$

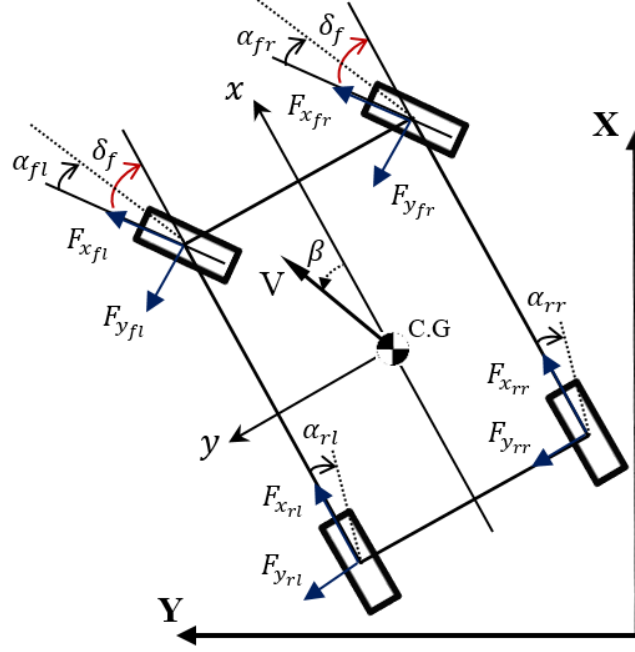


Figure 3.5: Double-track vehicle model used as prediction model.

where Q_{ij} is the total torque (drive and brake) applied to the wheel ij . The yaw moment at vehicle C.G. produced by these longitudinal forces can be expressed as:

$$M_{F_x} = \sum_{ij} \left(\zeta_j \frac{w_i}{2} \cos \delta_i + \xi_i L_i \sin \delta_i \right) F_{x_{ij}} \quad (3.17)$$

$$\text{where } \zeta_j = \begin{cases} -1 & j = L \\ +1 & j = R \end{cases}, \text{ and } \xi_i = \begin{cases} +1 & i = F \\ -1 & i = R \end{cases}$$

The steering angle of the rear wheels is considered zero ($\delta_R = 0$). Equation (3.17) can be expressed in the following compact form:

$$M_{F_x} = \mathbf{A}_x^T \mathbf{F}_x \quad (3.18)$$

where $\mathbf{F}_x$ and $\mathbf{A}_x$ are defined below:

$$\mathbf{F}_x = \begin{bmatrix} F_{x_{fl}} & F_{x_{fr}} & F_{x_{rl}} & F_{x_{rr}} \end{bmatrix}^T \quad (3.19)$$

$$\mathbf{A}_x = \begin{bmatrix} A_{x_{fl}} & A_{x_{fr}} & A_{x_{rl}} & A_{x_{rr}} \end{bmatrix}^T \quad (3.20)$$

$$\text{where } A_{x_{ij}} = \zeta_j \frac{w_i}{2} \cos \delta_i + \xi_i L_i \sin \delta_i$$

Next, the lateral tire forces and their evolution within the prediction horizon is investigated. The lateral tire forces are expressed in terms of the slip angle and cornering stiffness of the tires:

$$F_{y_{ij}} = C_{\alpha_{ij}} \alpha_{ij} \quad (3.21)$$

where $C_{\alpha_{ij}}$ and α_{ij} are respectively the cornering stiffness and the slip angle of the tire ij , illustrated in Figure 3.6. At each instant of time, tire cornering stiffness is obtained using look-up tables of the tire stiffness data. The slip angle α_{ij} is defined as:

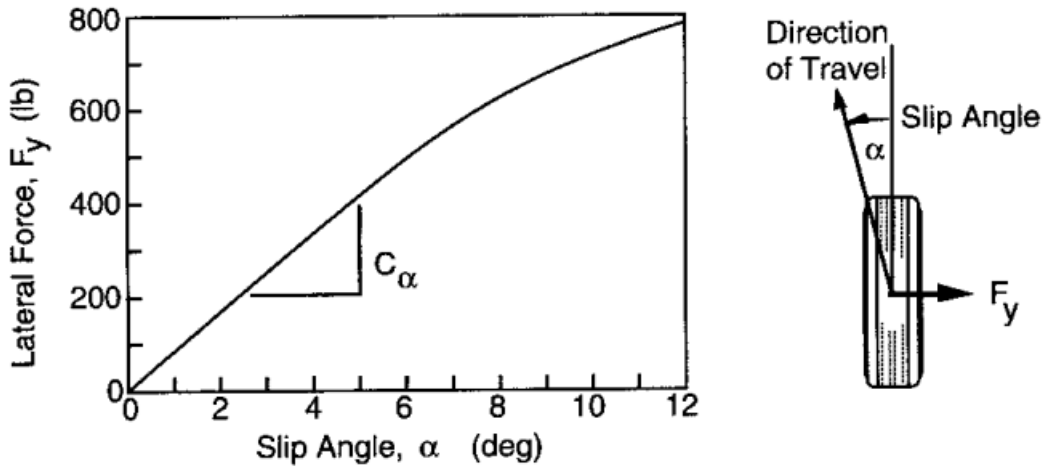


Figure 3.6: The cornering stiffness of the tire, defined as the variation of lateral tire force for a unit of change in tire slip angle ([77]).

$$\alpha_{ij} = \delta_i - \arctan \frac{v_{ij}}{u_{ij}} \quad (3.22)$$

where u_{ij} and v_{ij} are respectively the longitudinal and lateral velocities at the center of the wheel ij . These velocities can be expressed in terms of vehicle C.G. velocities and yaw

rate:

$$\begin{aligned} u_{ij} &= u + \frac{r}{2} \begin{bmatrix} -w_F & +w_F & -w_R & +w_R \end{bmatrix}^T \\ v_{ij} &= v + r \begin{bmatrix} +L_F & +L_F & -L_R & -L_R \end{bmatrix}^T \end{aligned} \quad (3.23)$$

where L_i is the distance between vehicle C.G. and the axle i . Since the vehicle yaw rate appears both in the numerator and denominator of Equation (3.22), it is nonlinear in terms of the vehicle yaw rate. A good approximation is to replace u_{ij} in the denominator by the C.G. velocity u . Since rw is often much smaller than u , this provides a good estimate of slip angles. Therefore, α_{ij} can be approximated as:

$$\alpha_{ij} = \delta_i - \arctan \frac{v + \xi_i r L_i}{u} \quad (3.24)$$

Inserting this in Equation (3.21) gives:

$$F_{y_{ij}} = C_{\alpha_{ij}} \left(\delta_i - \arctan \frac{v + \xi_i r L_i}{u} \right) \quad (3.25)$$

Equation (3.25) can be used to obtain the lateral tire forces. However, in this thesis, these forces are obtained from a separate estimation scheme, such as [79] or [80]. For the purpose of model predictive control, it is required to estimate the variation in these forces in the prediction horizon subject to the control actions. To this aim, a time derivative of Equation (3.21) gives:

$$\dot{F}_{y_{ij}} = C_{\alpha_{ij}} \dot{\alpha}_{ij} + \dot{C}_{\alpha_{ij}} \alpha_{ij} \quad (3.26)$$

The first term in the above equation is variation in tire lateral forces due to the change in tire slip angle and the second term accounts for the change in the tire cornering stiffness. Compared to the first term, variation of the tire cornering stiffness within the finite prediction window is small and is neglected. Therefore, Equation (3.26) is simplified as below:

$$\dot{F}_{y_{ij}} \approx C_{\alpha_{ij}} \dot{\alpha}_{ij} \quad (3.27)$$

Next, the time rate of tire slip angles needs to be found. Taking the time derivative of Equation (3.24) gives:

$$\dot{\alpha}_{ij} = \frac{\partial \alpha_{ij}}{\partial \delta_i} \dot{\delta}_i + \frac{\partial \alpha_{ij}}{\partial v} \dot{v} + \frac{\partial \alpha_{ij}}{\partial u} \dot{u} + \frac{\partial \alpha_{ij}}{\partial r} \dot{r} \quad (3.28)$$

In the context of model predictive control, it is assumed that the uncontrolled inputs such as the steering angle remain constant over the prediction horizon. Therefore, the first term in Equation (3.28) is ignored. The second term is the contribution of the change in the lateral vehicle velocity. In presence of the controller, the lateral velocity of the vehicle is controlled to remain in a small range. Furthermore, even in the case of large lateral velocities, the tires become saturated in this range and the lateral forces do not appreciably change. The third term is due to the change in vehicle longitudinal velocity. This term is also negligible because the u appears in the denominator of α_{ij} , therefore $\frac{\partial \alpha_{ij}}{\partial u}$ is smaller compared to the other three partial derivatives. Therefore, the last term is the dominant term in the time rate of tire slip angles in a controlled vehicle:

$$\dot{\alpha}_{ij} \approx \frac{\partial \alpha_{ij}}{\partial r} \dot{r} = \frac{\xi_i \frac{L_i}{u}}{1 + \left(\frac{v + \xi_i L_i r}{u} \right)^2} \dot{r} \quad (3.29)$$

Substituting Equation (3.29) in (3.27) gives:

$$\dot{F}_{y_{ij}} = C_{\alpha_{ij}} \frac{\xi_i \frac{L_i}{u}}{1 + \left(\frac{v + \xi_i L_i r}{u} \right)^2} \dot{r} \quad (3.30)$$

Equation (3.30) describes the evolution of the tire lateral forces within the prediction horizon in terms of the yaw acceleration. In this equation, a constant value for the lateral vehicle velocity v is used and its variation within the prediction horizon is neglected. In order to predict the vehicle yaw rate and yaw moment, it is not necessary to calculate the lateral force of each tire. It suffices to calculate and predict the yaw moment that these forces generate at the vehicle C.G. This moment is denoted by M_{F_y} and is defined as:

$$\begin{aligned} M_{F_y} = & \left(+\frac{w_F}{2} \sin \delta_F + L_F \cos \delta_F \right) F_{y_{FL}} - L_R F_{y_{RL}} \\ & + \left(-\frac{w_F}{2} \sin \delta_F + L_F \cos \delta_F \right) F_{y_{FR}} - L_R F_{y_{RR}} \end{aligned} \quad (3.31)$$

Using the time rate of lateral tire forces obtained in Equation (3.30), the time rate of M_{F_y} can be found¹:

$$\dot{M}_{F_y} = k_M \dot{r} \quad (3.32)$$

$$\text{where } k_M = \sum_{ij} C_{\alpha_{ij}} \frac{\xi_i \frac{L_i}{u}}{1 + \left(\frac{v + \xi_i L_i r}{u}\right)^2} \left(\xi_i L_i \cos \delta_i - \zeta_j \frac{w_i}{2} \sin \delta_i \right)$$

Now, the yaw moment G_z and yaw acceleration $\dot{r}$ can be written as:

$$G_z = M_{F_x} + M_{F_y} \quad (3.33)$$

$$\dot{r} = \frac{1}{I_z} G_z \quad (3.34)$$

Next, the time rate of the lateral vehicle velocity is obtained. The kinematic equation of the lateral vehicle acceleration can be written as:

$$a_y = \dot{v} + ru \quad (3.35)$$

where a_y is the lateral vehicle acceleration at the vehicle C.G. Re-arranging Equation (3.35) gives:

$$\dot{v} = a_y - ru \quad (3.36)$$

This completes the update equation for the state-space representation of the prediction model.

3.3.2 Wheel Dynamics

In this section, the wheel dynamics part of the prediction model is developed. The purpose of including the wheel dynamics in the prediction model is to control tire slip ratios by

¹As mentioned before, it is assumed that the steering angle does not change within the prediction horizon.

tracking a desired wheel speed, as defined in Section 3.2.3. Therefore, the following states are defined:

$$\mathbf{X}_2 = \mathbf{e}_\Omega = \begin{bmatrix} e_{\omega_{fl}} & e_{\omega_{fr}} & e_{\omega_{rl}} & e_{\omega_{rr}} \end{bmatrix}^T \quad (3.37)$$

where $e_{\omega_{ij}}$ is the tracking error of desired wheel speeds and is defined as:

$$e_{\omega_{ij}} = \omega_{ij,d} - \omega_{ij} \quad (3.38)$$

where $\omega_{ij,d}$ is the desired speed for wheel ij as defined in Equation (3.12). Figure 3.7 shows a simplified view of a wheel with relevant forces and moments acting on it. Writing the moment equation about the wheel axis gives:

$$I_w \dot{\omega}_{ij} = Q_{ij} - R_{eff} F_{x_{ij}} \quad (3.39)$$

where I_w is the wheel moment of inertia about its rolling axis and ω_{ij} is the rotational

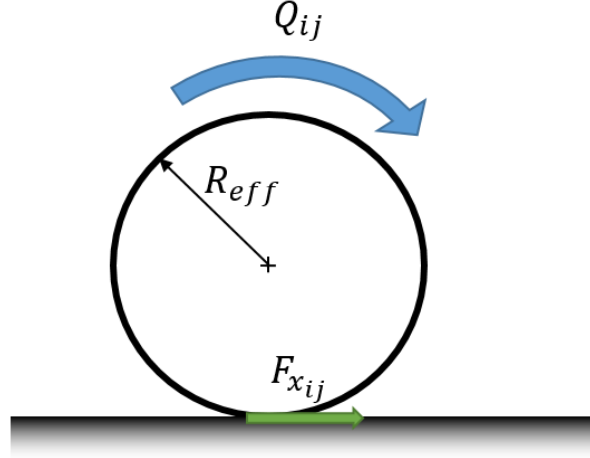


Figure 3.7: A sample wheel with torque, longitudinal force and radius shown.

speed of wheel ij . The rolling resistance torque is neglected in Equation (3.39), as it is much smaller than the drive or brake torque that can cause excessive tire slip ratios.

As it was shown in Figure 3.4, when the slip ratio of the tires exceeds a certain threshold, the tire cannot generate any more longitudinal force. In this case, Equation (3.39) can be

rewritten as:

$$\dot{\omega}_{ij} = \frac{1}{I_w} (Q_{ij} - G_{w_{ij}}) \quad (3.40)$$

where $G_{w_{ij}}$ is the opposing torque generated by the longitudinal tire forces, and is provided by separate estimation modules (such as [79] and [80]). Equation (3.40) combined with the desired wheel accelerations in Equation (3.14) provide the time derivative of the state vector $\mathbf{X}_2$:

$$\dot{\mathbf{X}}_2 = \dot{\mathbf{\Omega}}_d - \dot{\mathbf{\Omega}} \quad (3.41)$$

where

$$\dot{\mathbf{\Omega}} = \begin{bmatrix} \dot{\omega}_{fl} & \dot{\omega}_{fr} & \dot{\omega}_{rl} & \dot{\omega}_{rr} \end{bmatrix}^T \quad (3.42)$$

3.4 Handling Control Loop Delays

In this section, a method for handling delays of various sources is presented. Presence of delay in a control loop can severely degrade controller performance and even cause instability. The common approaches for handling delay are often complex in design and tuning or require an increase in the dimensions of the controller. The proposed method is easy to implement and does not entail a complex design or tuning process. Moreover, it does not increase the complexity of the controller, therefore the amount of online computations is not appreciably affected.

3.4.1 Pure Delay

In this section, the proposed method for treating system delays is presented. A certain amount of delay is present in any practical control system. There can be a multitude of sources contributing to the total delay in the control loop. For example, each actuator exhibits a certain delay between the time that the command is received until it is performed.

Transmission of signals within the communication network also exhibits a certain amount of delay (for instance see [81, 82]). Another source of delay is within the sensory system. Many sensors show a time lag for reporting reliable measurements. In addition, sometimes the output of the sensors carries a considerable amount of noise which needs to be filtered. However, filters additionally introduce a phase shift on the signal, which contributes to the overall delay in the control loop.

In the framework of model predictive control, it is easily possible to take delays into account and modify the MPC controller for the delayed system. Figure 3.8 depicts the proposed strategy for considering a pure delay of 200 ms in measurement. In this case, whenever the sensor reports data, it is 200 ms old. Therefore, any control action applied at this instant of time is only visible in the measured signals 200 ms later. Accordingly, instead of controlling the current state of the system, it makes sense to predict the system state 200 ms in the future and attempt to control that predicted state. For MPC, a prediction model is already required and it can be used to predict system state after the delay period.

In the proposed method, the total dead-time of the control loop is expressed as N_d , the number of delay steps. The history of the latest N_d inputs to the system is stored in memory. At each step, the sequence of the latest N_d inputs is applied to initial state $\mathbf{X}_0$ using the prediction model to find the system state $\mathbf{X}'_0$ at the end of the delay period. This procedure is illustrated in Figure 3.9. In this method, the prediction window starts at the end of the delay period (see Figure 3.10).

3.4.2 First Order Delay

The delay in the response of some elements in a control system is better described with a first order delay. The method proposed here can also consider first order delays besides the pure delay discussed in Section 3.4.1. This is shown in Figure 3.11, where the pure delay and the first order delay are considered in series. In this method, the first order delay is emulated by applying a first order filter to the input-output transfer function of the prediction model. Figure 3.12 shows the block diagram of the prediction model in the standard state-space form. The first order filter block can be considered in any of

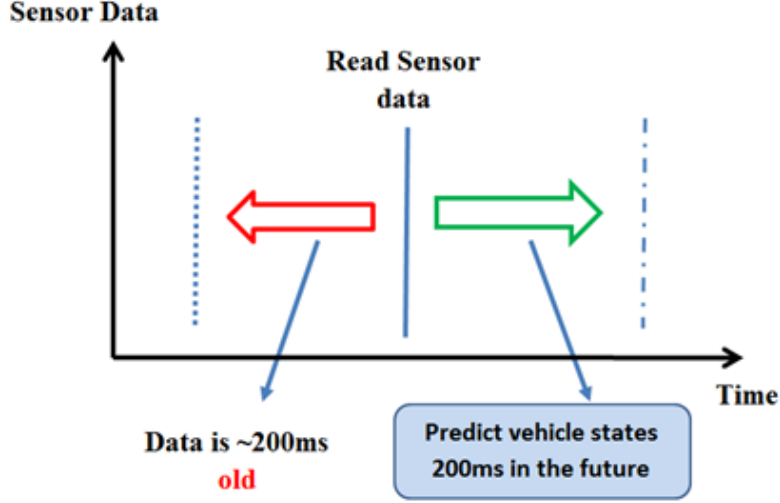


Figure 3.8: Proposed strategy for dealing with pure delays.

the positions 1 through 4, depending on the actual location of the delay in the system. Regardless of the position of the delay block, the input-output transfer function of the prediction model remains the same. In this thesis, the delay block is considered in position 2. Therefore, the first order filter (with the time constant τ_G is applied in Equation (3.33) where the yaw moment at the C.G. of the vehicle is related to the input torques. Therefore, a first order filter is added to Equation (3.18):

$$M_{F_x} = \frac{1}{1 + \tau_G p} \mathbf{A}_x^T \mathbf{F}_x \quad (3.43)$$

where τ_G is the first order delay constant and p is the time domain counterpart of the Laplace operator s , acting as the time derivative ($d(\dots)/dt$) operator. This requires adding M_{F_x} to the state vector $\mathbf{X}_1$.

3.5 State-space Representation

The state-space representation of the prediction model developed in Section 3.3 is presented here. The prediction model is expressed in the following standard format of continuous

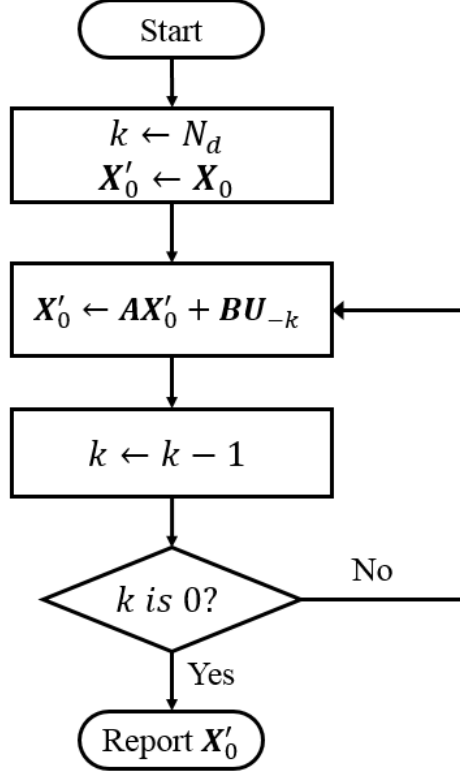


Figure 3.9: Procedure for finding system state at the end of pure delay period.

state-space model:

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{A}_c \mathbf{X} + \mathbf{B}_c \mathbf{U} + \mathbf{E}_c \mathbf{W} \\ \mathbf{Y} = \mathbf{C}_c \mathbf{X} \end{cases} \quad (3.44)$$

where the subscript c is short for continuous and is used to distinguish between the corresponding representation in the discrete form. The state vector ($\mathbf{X}$), controlled and uncontrolled inputs ($\mathbf{X}$ and $\mathbf{W}$ respectively) as well as the outputs ($\mathbf{Y}$) are defined as:

$$\mathbf{X} = \begin{bmatrix} r & M_{F_y} & M_{F_x} & v & e_{\omega_{fl}} & e_{\omega_{fr}} & e_{\omega_{rl}} & e_{\omega_{rr}} \end{bmatrix}_{8 \times 1}^T \quad (3.45)$$

$$\mathbf{U} = \begin{bmatrix} Q_{fl} & Q_{fr} & Q_{rl} & Q_{rr} \end{bmatrix}_{4 \times 1}^T \quad (3.46)$$

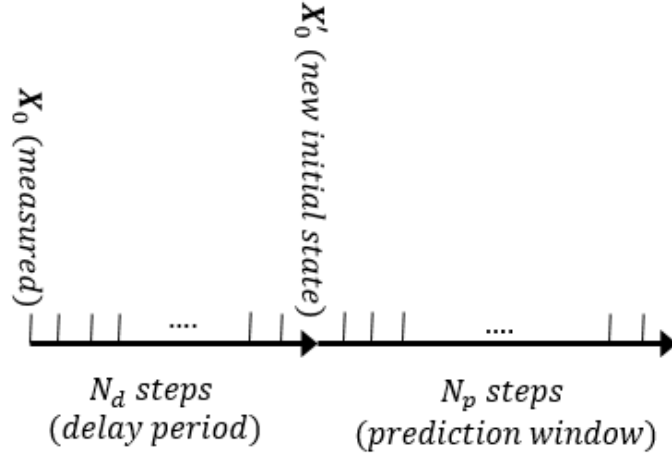


Figure 3.10: Delay period and prediction window.

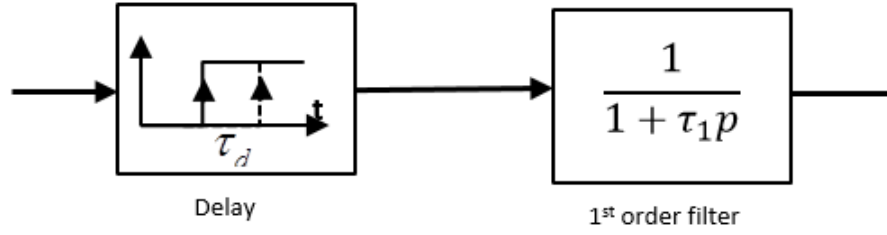


Figure 3.11: Overall delay is approximated by a pure delay and a first order delay in series.

$$\mathbf{W} = \begin{bmatrix} a_y & a_x & G_{w_{fl}} & G_{w_{fr}} & G_{w_{rl}} & G_{w_{rr}} \end{bmatrix}^T_{6 \times 1} \quad (3.47)$$

$$\mathbf{Y} = \begin{bmatrix} r & v & e_{\omega_{fl}} & e_{\omega_{fr}} & e_{\omega_{rl}} & e_{\omega_{rr}} \end{bmatrix}^T_{6 \times 1} \quad (3.48)$$

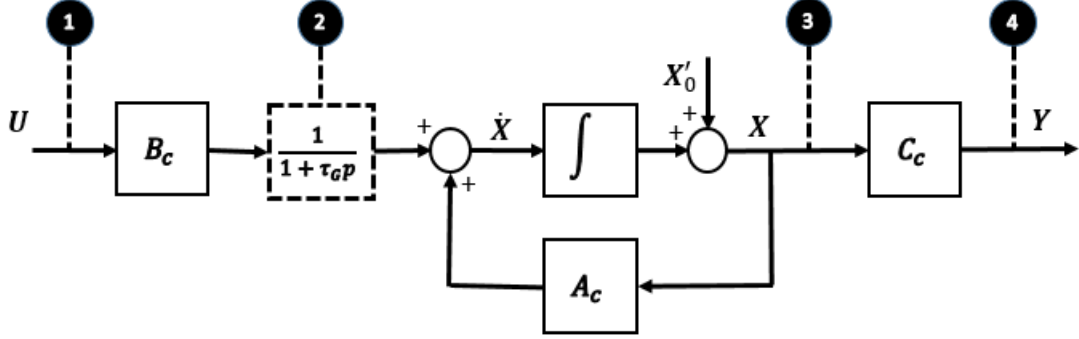


Figure 3.12: Adding first order delay in the prediction model. The filter block (currently in position 2) can take one of the positions 1 through 4.

Next, the matrices $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{E}_c$ and $\mathbf{C}_c$ are defined.

$$\mathbf{A}_c = \begin{bmatrix} 0 & \frac{1}{I_z} & \frac{1}{I_z} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{k_M}{I_z} & \frac{k_M}{I_z} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{\tau_G} & 0 & 0 & 0 & 0 & 0 \\ -u & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-w_F}{2I_z R_{eff}} & \frac{-w_F}{2I_z R_{eff}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{w_F}{2I_z R_{eff}} & \frac{w_F}{2I_z R_{eff}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-w_R}{2I_z R_{eff}} & \frac{-w_R}{2I_z R_{eff}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{w_R}{2I_z R_{eff}} & \frac{w_R}{2I_z R_{eff}} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{8 \times 8} \quad (3.49)$$

$$\mathbf{B}_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{A_{xfl}}{\tau_G R_{eff}} & \frac{A_{xfr}}{\tau_G R_{eff}} & \frac{A_{xrl}}{\tau_G R_{eff}} & \frac{A_{xrr}}{\tau_G R_{eff}} \\ \frac{-1}{I_w} & 0 & 0 & 0 \\ 0 & \frac{-1}{I_w} & 0 & 0 \\ 0 & 0 & \frac{-1}{I_w} & 0 \\ 0 & 0 & 0 & \frac{-1}{I_w} \end{bmatrix}_{8 \times 4} \quad (3.50)$$

$$\mathbf{E}_c = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_{eff}} & \frac{1}{I_w} & 0 & 0 & 0 \\ 0 & \frac{1}{R_{eff}} & 0 & \frac{1}{I_w} & 0 & 0 \\ 0 & \frac{1}{R_{eff}} & 0 & 0 & \frac{1}{I_w} & 0 \\ 0 & \frac{1}{R_{eff}} & 0 & 0 & 0 & \frac{1}{I_w} \end{bmatrix}_{8 \times 6} \quad (3.51)$$

$$\mathbf{C}_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6 \times 8} \quad (3.52)$$

The state-space representation of the prediction model provided in Equation (3.44) needs to be expressed in discrete format before it can be used for designing the MPC controller.

$$\begin{cases} \mathbf{X}_{k+1} &= \mathbf{A}\mathbf{X}_k + \mathbf{B}\mathbf{U}_k + \mathbf{E}\mathbf{W}_k \\ \mathbf{Y}_k &= \mathbf{C}\mathbf{X}_k \end{cases} \quad (3.53)$$

In this thesis, Euler approximation (see [83]) is used to perform the discretization:

$$\begin{cases} \mathbf{A} = \mathbf{I} + \mathbf{A}_c \Delta T_p \\ \mathbf{B} = \mathbf{B}_c \Delta T_p \\ \mathbf{E} = \mathbf{E}_c \Delta T_p \\ \mathbf{C} = \mathbf{C}_c \end{cases} \quad (3.54)$$

3.6 Performance Index

The desired responses of the vehicle and wheels were explained in Section 3.2. In this section, the performance index is defined to drive the responses of the system towards the desired responses²:

$$J = \frac{1}{2} \sum_{k=1}^{N_p} (\|\mathbf{Y}_d - \mathbf{Y}_k\|_{\mathbf{Q}}^2 + \|\mathbf{U}_k - \mathbf{V}\|_{\mathbf{R}}^2 + \|\mathbf{U}_k - \mathbf{U}_k^p\|_{\mathbf{T}}^2) \quad (3.55)$$

where N_p is the number of points considered in the prediction horizon, $\mathbf{U}_k$ are the prospective control actions and $\mathbf{V}$ is the driver's torque demand calculated according to the gas pedal position. The first term in Equation (3.55) is the tracking error of the desired responses. The second term is the prospective torque adjustments ($\delta\mathbf{Q}_k$) and minimizes the control effort. The third term is optional and enforces proximity to the previous optimal control actions ($\mathbf{U}_k^p$) to prevent chatter in control actions. The positive semi-definite $\mathbf{Q}$ and $\mathbf{T}$ and positive definite $\mathbf{R}$ are weighting matrices that reflect the relative importance of these terms in the performance index. These matrices serve as tuning parameters in the controller design.

In this section, using the prediction model introduced in Section 3.3, the performance index described in Equation (3.55) will be expressed in terms of the initial (current) system state and prospective control actions. Using the batch approach (see [47]), the future

² where the notation $\|\mathbf{X}\|_{\mathbf{Q}}^2 = \mathbf{X}^T \mathbf{Q} \mathbf{X}$ is used.

system states can be expressed as:

$$\begin{aligned}
\begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \vdots \\ \mathbf{X}_{N_p} \end{bmatrix} &= \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \vdots \\ \vdots \\ \mathbf{A}^{N_p} \end{bmatrix} \mathbf{X}_0 + \begin{bmatrix} \mathbf{B} & \mathbf{0} & \dots & \dots & \mathbf{0} \\ \mathbf{AB} & \mathbf{B} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{A}^{N_p-1}\mathbf{B} & \dots & \dots & \dots & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{U}_0 \\ \mathbf{U}_1 \\ \vdots \\ \vdots \\ \mathbf{U}_{N_p-1} \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{E} & \mathbf{0} & \dots & \dots & \mathbf{0} \\ \mathbf{AE} & \mathbf{E} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{A}^{N_p-1}\mathbf{E} & \dots & \dots & \dots & \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{W} \\ \mathbf{W} \\ \vdots \\ \vdots \\ \mathbf{W} \end{bmatrix}
\end{aligned} \tag{3.56}$$

where $\mathbf{X}_k$ is the predicted system state at (discrete) time k within the prediction horizon, $\mathbf{X}_0$ is the current system state (measured and/or estimated), $\mathbf{U}_k$ is the prospective control action at time k , $\mathbf{W}$ is the vector of uncontrolled inputs and is assumed constant throughout the prediction horizon. Matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$ matrices are the discrete state space representation of the prediction model as defined in Section 3.5. The outputs of the prediction model in the prediction window can be similarly expressed as:

$$\begin{aligned}
\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \vdots \\ \vdots \\ \mathbf{Y}_{N_p} \end{bmatrix} &= \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \vdots \\ \mathbf{CA}^{N_p} \end{bmatrix} \mathbf{X}_0 + \begin{bmatrix} \mathbf{CB} & \mathbf{0} & \dots & \dots & \mathbf{0} \\ \mathbf{CAB} & \mathbf{CB} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{CA}^{N_p-1}\mathbf{B} & \dots & \dots & \dots & \mathbf{CB} \end{bmatrix} \begin{bmatrix} \mathbf{U}_0 \\ \mathbf{U}_1 \\ \vdots \\ \vdots \\ \mathbf{U}_{N_p-1} \end{bmatrix} \\
&+ \begin{bmatrix} \mathbf{CE} & \mathbf{0} & \dots & \dots & \mathbf{0} \\ \mathbf{CAE} & \mathbf{CE} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{CA}^{N_p-1}\mathbf{E} & \dots & \dots & \dots & \mathbf{CE} \end{bmatrix} \begin{bmatrix} \mathbf{W} \\ \mathbf{W} \\ \vdots \\ \vdots \\ \mathbf{W} \end{bmatrix}
\end{aligned} \tag{3.57}$$

Equation (3.57) can be written in the following compact form:

$$\bar{\mathbf{Y}} = \mathbf{S}_x \mathbf{X}_0 + \mathbf{S}_u \bar{\mathbf{U}} + \mathbf{S}_w \bar{\mathbf{W}} \quad (3.58)$$

where the definition of $\bar{\mathbf{Y}}$, $\mathbf{S}_x$, $\mathbf{S}_u$, $\bar{\mathbf{U}}$, $\mathbf{S}_w$ and $\bar{\mathbf{W}}$ can be inferred by comparison with Equation (3.57). The vector of desired outputs over the prediction horizon can also be defined as:

$$\bar{\mathbf{Y}}_{ref} = \begin{bmatrix} \mathbf{Y}_{ref} & \mathbf{Y}_{ref} & \dots & \mathbf{Y}_{ref} \end{bmatrix}^T \quad (3.59)$$

This means that the desired values for system outputs are kept constant throughout the prediction horizon which is due to the assumption of constant (uncontrolled) inputs, a common practice in the literature of MPC. Similarly, the driver's torque demand is also assumed constant over the prediction horizon and is defined as:

$$\bar{\mathbf{V}} = \begin{bmatrix} \mathbf{V} & \mathbf{V} & \dots & \mathbf{V} \end{bmatrix}^T \quad (3.60)$$

The weighting matrices can also be merged to form the $\bar{\mathbf{Q}}$ and $\bar{\mathbf{R}}$ matrices:

$$\bar{\mathbf{Q}} = \text{blockdiag}(\mathbf{Q}, \mathbf{Q}, \dots, \mathbf{Q}) \quad (3.61)$$

$$\bar{\mathbf{T}} = \text{blockdiag}(\mathbf{T}, \mathbf{T}, \dots, \mathbf{T}) \quad (3.62)$$

$$\bar{\mathbf{R}} = \text{blockdiag}(\mathbf{R}, \mathbf{R}, \dots, \mathbf{R}) \quad (3.63)$$

Now, the performance index in Equation (3.55) can be expressed in a more compact form as:

$$\begin{aligned} 2J = & (\bar{\mathbf{Y}}_{ref} - \bar{\mathbf{Y}})^T \bar{\mathbf{Q}} (\bar{\mathbf{Y}}_{ref} - \bar{\mathbf{Y}}) + (\bar{\mathbf{U}} - \bar{\mathbf{V}})^T \bar{\mathbf{R}} (\bar{\mathbf{U}} - \bar{\mathbf{V}}) \\ & + (\bar{\mathbf{U}} - \bar{\mathbf{U}}_p)^T \bar{\mathbf{T}} (\bar{\mathbf{U}} - \bar{\mathbf{U}}_p) \end{aligned} \quad (3.64)$$

At this point, Equation (3.58) can be used to express the performance index fully in terms of the control input vector and initial condition:

$$J(\mathbf{X}_0, \bar{\mathbf{U}}) = \frac{1}{2} \bar{\mathbf{U}}^T \mathbf{H} \bar{\mathbf{U}} + \bar{\mathbf{U}}^T (\mathbf{F}_1 \mathbf{X}_0 + \mathbf{F}_2 \mathbf{Y}_{ref} + \mathbf{F}_3 \bar{\mathbf{W}}_0 - \bar{\mathbf{R}} \bar{\mathbf{V}} - \bar{\mathbf{T}} \bar{\mathbf{U}}_p) + \text{Const.} \quad (3.65)$$

where

$$\begin{aligned}
\mathbf{H} &= \mathbf{S}_u^T \bar{\mathbf{Q}} \mathbf{S}_u + \bar{\mathbf{R}} + \bar{\mathbf{T}} \\
\mathbf{F}_1 &= \mathbf{S}_u^T \bar{\mathbf{Q}} \mathbf{S}_x \\
\mathbf{F}_2 &= -\mathbf{S}_u^T \bar{\mathbf{Q}} \\
\mathbf{F}_3 &= -\mathbf{S}_u^T \bar{\mathbf{Q}} \mathbf{S}_w
\end{aligned} \tag{3.66}$$

In order to reduce the computational cost of the controller, it is customary to adopt a shorter control horizon. Therefore, it is assumed that:

$$\mathbf{U}_k = \mathbf{U}_{N_c} \quad \text{for } k > N_c \tag{3.67}$$

where N_c is the size of the control horizon.

3.7 Constraints

The performance index of the model predictive controller was introduced in Section 3.6. In this section, the constraints on the control actions are developed. Two sets of constraints are considered in this thesis: electric motor torque capacity and tire force capacity. Electric motors can deliver a limited amount of torque at each given speed. If the maximum and minimum torques that can be produced are denoted by $\mathbf{Q}^{max}$ and $\mathbf{Q}^{min}$, the lower and upper bounds on control actions $\mathbf{U}_k$ can be expressed as:

$$\begin{aligned}
\mathbf{lb}_1 &= \mathbf{Q}^{min} + \mathbf{Q}^{brake} \\
\mathbf{ub}_1 &= \mathbf{Q}^{max} + \mathbf{Q}^{brake}
\end{aligned} \tag{3.68}$$

where $\mathbf{Q}^{brake}$ is the driver's brake torque request that is provided by a hydraulic brake system. All of the prospective control actions $\mathbf{U}_1$ through $\mathbf{U}_{N_c}$ are subject to the constraints in Equation (3.68). Therefore, these constraints are augmented below to act as the constraint on the $\bar{\mathbf{U}}$ vector:

$$\begin{aligned}
\mathbf{LB}_1 &= \begin{bmatrix} \mathbf{lb}_1 & \mathbf{lb}_1 & \dots & \mathbf{lb}_1 \end{bmatrix}_{4N_c \times 1} \\
\mathbf{UB}_1 &= \begin{bmatrix} \mathbf{ub}_1 & \mathbf{ub}_1 & \dots & \mathbf{ub}_1 \end{bmatrix}_{4N_c \times 1}
\end{aligned} \tag{3.69}$$

The second set of constraints are developed due to the tire force capacity. In this thesis, an ideal ellipse tire model is used to obtain the tire capacity in the longitudinal direction (see [84]). In this model, an ellipse relates the capacity of the tire in the longitudinal and lateral directions. The maximum tire force capacity in the longitudinal and lateral direction is defined as:

$$\begin{aligned} F_{x,ij}^{max} &= \mu_{x,ij} F_{z,ij} \\ F_{y,ij}^{max} &= \mu_{y,ij} F_{z,ij} \end{aligned} \quad (3.70)$$

where $F_{z,ij}$ is the normal force of tire ij and μ_x, μ_y are the coefficient of the friction between the tires and the road in x and y directions respectively. If these estimates are not present, a default value of 1.0 is used instead. In this case, the tire slip control will have a more prominent role in keeping the wheels from overspinning. According to the ideal ellipse tire model, the available capacity in the longitudinal direction can be calculated as (see Figure (3.13)):

$$F_{x,ij}^{avail} = \sqrt{1 - \left(\frac{F_{y,ij}}{F_{y,ij}^{max}} \right)^2} \quad (3.71)$$

where $F_{y,ij}$ is the lateral force of tire ij which needs to be provided by a separate tire force estimation scheme such as [79]. The available tire force capacity in the longitudinal direction can be related to lower and upper bounds on control actions by the effective radius of the tire:

$$\begin{aligned} \mathbf{lb}_2 &= - \begin{bmatrix} F_{x,fl}^{avail} & F_{x,fr}^{avail} & F_{x,rl}^{avail} & F_{x,rr}^{avail} \end{bmatrix} \times R_{eff} \\ \mathbf{ub}_2 &= -\mathbf{lb}_2 \end{aligned} \quad (3.72)$$

Similar to Equation (3.69), the lower and upper bounds $\mathbf{LB}_2$ and $\mathbf{UB}_2$ can be defined. Combining the two sets of bounds developed in this section, the constraints to be imposed on the optimization problem are obtained:

$$\begin{aligned} \mathbf{LB} &= \max(\mathbf{LB}_1, \mathbf{LB}_2) \\ \mathbf{UB} &= \min(\mathbf{UB}_1, \mathbf{UB}_2) \end{aligned} \quad (3.73)$$

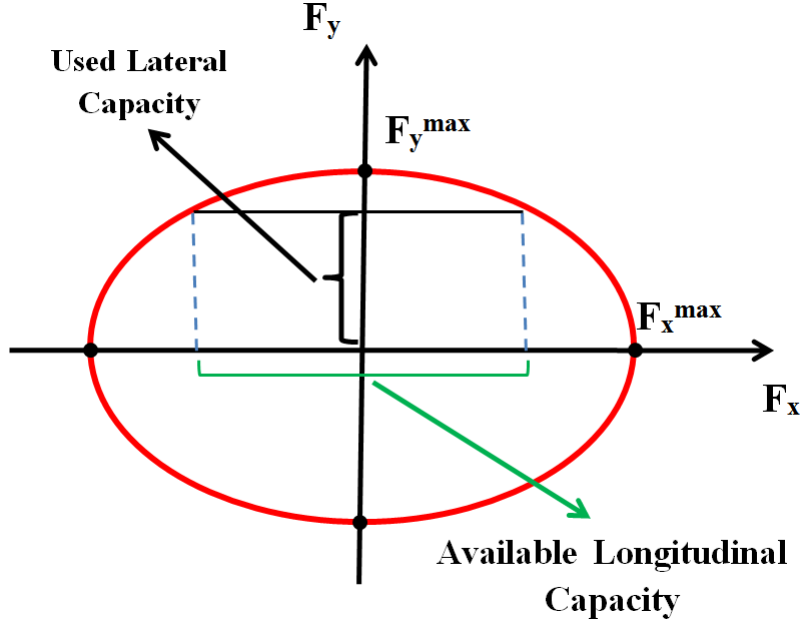


Figure 3.13: Tire capacity ellipse.

3.8 Controller Reconfigurability

The controller developed in this chapter can be easily adopted in vehicles with various driveline configurations (such as FWD, RWD and AWD), as well as different actuation methods. While the primary method of actuation in electric vehicles is torque vectoring using motors, the controller developed here is equally applicable to vehicles with an independent braking system. The constraints of the QP problem in Equation (3.73) need to be adjusted for each wheel that is intended for differential braking, so that the controller only applies braking (negative) torque on that wheel. To this aim, the upper bounds in Equation (3.68) need to be modified for each wheel ij that is intended for differential braking:

$$\mathbf{ub}_1 = Q_{drive}^{ij} \quad (3.74)$$

where the right hand side of the above equation is the drive torque demand from the motor on wheel ij , if any. Similarly, in case of driveline configurations, if a wheel is not used for

transmitting torque (such as rear wheels in a FWD configuration), both the lower and upper bounds of the QP problem are set to zero:

$$\mathbf{ub}_1 = \mathbf{lb}_1 = 0 \quad (3.75)$$

Using Equations (3.74) and (3.75), the controller can be adjusted to work in any combination of torque vectoring, differential braking and idle wheels.

3.9 Quadratic Programming Problem

The performance index required to achieve the control objectives was introduced in Equation (3.65). In order for the controller to perform yaw rate tracking and wheel slip control, the performance index needs to be minimized. At the same time, the constraints in Equation (3.73) need to be satisfied. Therefore, the objective function along with the constraints form a Quadratic Problem (QP) that needs to be solved (see [85] for details):

$$\begin{aligned} \bar{\mathbf{U}}^* &= \operatorname{argmin} \quad J \\ \text{subject to: } & \mathbf{LB} < \bar{\mathbf{U}} < \mathbf{UB} \end{aligned} \quad (3.76)$$

The solution of the above QP problem ($\bar{\mathbf{U}}^*$) contains a sequence of optimal control actions. In the context of MPC, only the first optimal control action is applied to the system, and the rest are discarded:

$$\begin{aligned} \bar{\mathbf{U}}^* &= \begin{bmatrix} \mathbf{U}_1^* & \mathbf{U}_2^* & \mathbf{U}_3^* & \dots & \mathbf{U}_{N_c}^* \end{bmatrix}^T \\ \delta \mathbf{Q}^* &= \mathbf{U}_1^* - \mathbf{V} \end{aligned} \quad (3.77)$$

3.10 Integration With Power Distribution

In this section, integration of the stability controller with power distribution is studied. It is common to consider separate power distribution and stability control units. In the

power distribution unit, the optimal distribution of torque between the front and rear axles are obtained according to the efficiency map of the electric motors. If the efficiency of the electric motors on front and rear axles are noticeably different, the optimal torque distribution involves most of the drive torque being applied to just one of the axles. However, this ignores the cross-effects between the two units, therefore the effect of the torque distribution on vehicle stability is ignored.

In this section, the power distribution unit is integrated with the stability control and an integrated controller is designed. The power distribution part of the controller will be added to the cost function of the MPC controller that yields the optimal torque adjustments. These control actions will be optimal in the sense of both stability control and power distribution. The new structure of the integrated controller is shown in Figure 3.14.

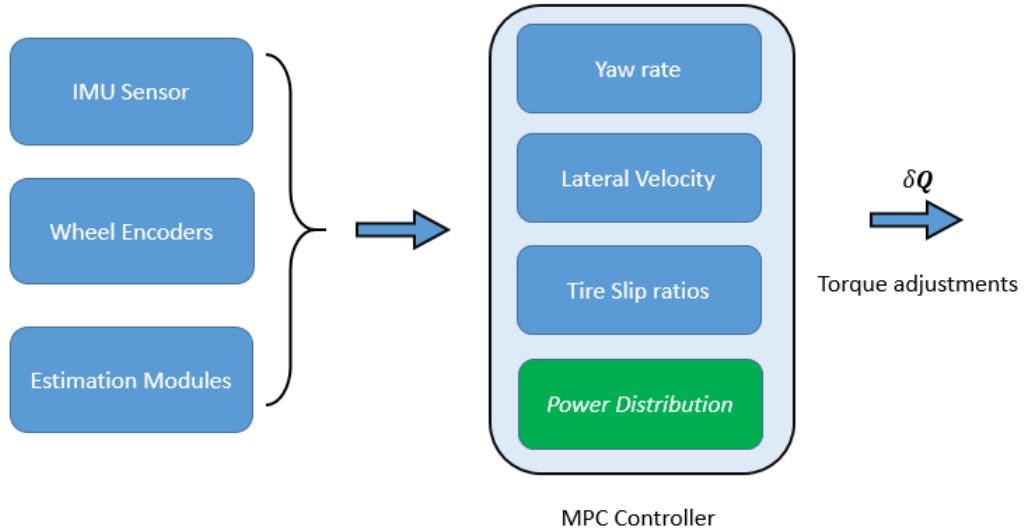


Figure 3.14: The desired structure of integrated power distribution and stability control.

3.10.1 Motor Efficiency

Efficiency of electric motors are defined as the ratio of the output mechanical power to the input electric power ([86]). This is shown in Equation (3.78).

$$\eta(T, \omega) = \frac{P_m}{P_e} = \frac{T\omega}{VI} \quad (3.78)$$

Efficiency is one of the characteristics of electric motors. A typical efficiency map of an electric motor is shown in Figure 3.15. It can be seen that efficiency typically varies according to the motor speed and the output torque.

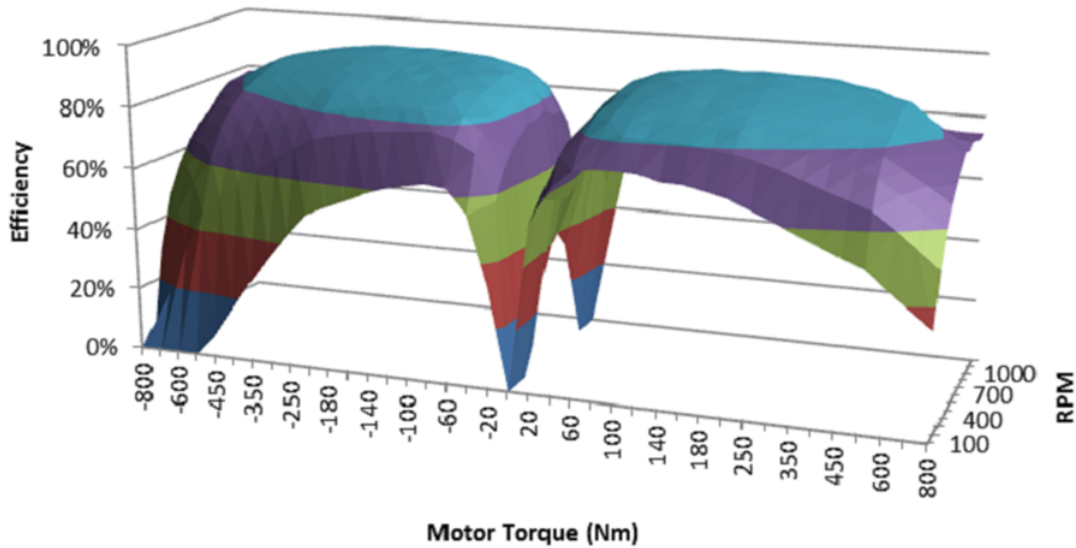


Figure 3.15: Typical efficiency map of electric motors.

3.10.2 Overall Vehicle Efficiency

Assuming an all-wheel drive configuration, where each wheel is independently driven by an electric motor, the overall efficiency of the vehicle can be expressed as:

$$\eta_T = \frac{\sum_{ij} P_{m_{ij}}}{\sum_{ij} P_{e_{ij}}} = \frac{\sum_{ij} T_{ij} \omega_{ij}}{\sum_{ij} \frac{T_{ij} \omega_{ij}}{\eta_{ij}(T_{ij}, \omega_{ij})}} \quad (3.79)$$

The above equation is nonlinear in terms of the torques T_{ij} . It is generally not easy to analytically find the torque distribution that maximizes the overall vehicle efficiency as shown in Equation (3.79). Therefore, a number of simplifying assumptions are made to arrive at a convex quadratic measure of the vehicle efficiency.

It is reasonable to assume that all four wheels have the same speed, different wheel speeds only occurs during wheel overspin, and since the wheel speeds are being controlled by the controller, it is safe to make this assumption. Therefore, Equation (3.79) can be simplified as:

$$\eta_T = \frac{\frac{T_{FL} + T_{FR} + T_{RL} + T_{RR}}{\eta_{FL}} + \frac{T_{FR} + T_{RL} + T_{RR}}{\eta_{FR}} + \frac{T_{RL} + T_{RR}}{\eta_{RL}} + \frac{T_{RR}}{\eta_{RR}}}{\frac{T_{FL} + T_{FR} + T_{RL} + T_{RR}}{\eta_{FL}} + \frac{T_{FR} + T_{RL} + T_{RR}}{\eta_{FR}} + \frac{T_{RL} + T_{RR}}{\eta_{RL}} + \frac{T_{RR}}{\eta_{RR}}} \quad (3.80)$$

Next, instead of entering the torque of each individual corner directly into the cost function, the total torque of the front and rear axles are considered and it is assumed that the left and right motors produce equal torque. The torque differential between left and right sides happens while torque vectoring (for example during cornering), but in this case, the torque is transferred from one side to another, and the total torque remains unchanged.

$$\eta_T = \frac{\frac{T_F + T_R}{\eta_F} + \frac{T_R}{\eta_R}}{\frac{T_F + T_R}{\eta_F} + \frac{T_R}{\eta_R}} \quad (3.81)$$

Showing the reciprocal of motor efficiency with ν , Equation (3.81) can be written as:

$$\nu_T = \frac{T_F \nu_F + T_R \nu_R}{T_F + T_R} \quad (3.82)$$

The denominator of Equation (3.82) is the total torque demand of the driver determined according to the accelerator pedal position. Therefore, it can be considered as constant.

$$\nu_T \propto T_F \nu_F + T_R \nu_R \quad (3.83)$$

The reciprocal of the motor efficiency (or inefficiency) curve at a given motor speed (Figure 3.16) can be approximated by the following parabolic curve:

$$\nu_i \propto k_i (T_i - T_i^*)^2 \quad (3.84)$$

Using Equation (3.84), Equation (3.83) can be rewritten as:

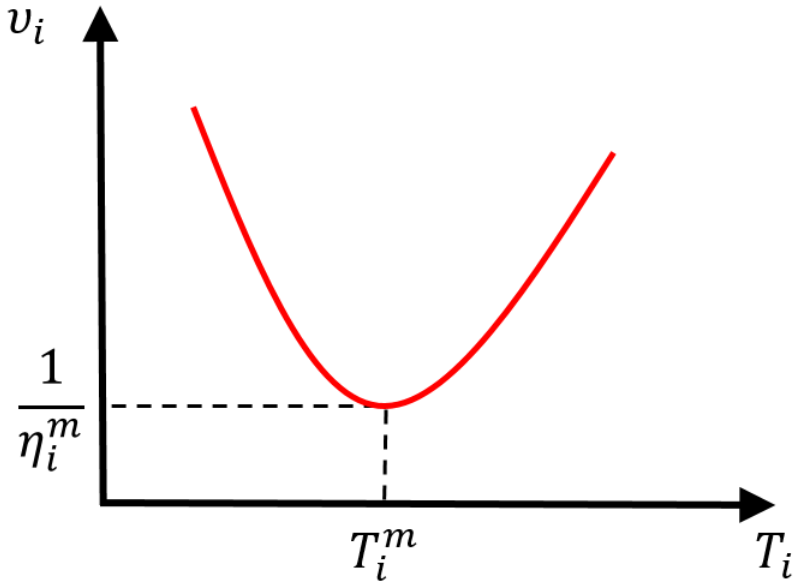


Figure 3.16: Typical shape of motor inefficiency curve (ν_i), at a given motor speed.

$$\nu_T \propto k_F T_F (T_F - T_F^*)^2 + k_R T_R (T_R - T_R^*)^2 \quad (3.85)$$

Equation (3.85) is obtained after a number of simplification steps and is still non-convex in terms of T_F and T_R , therefore cannot be added to the performance index in Equation (3.55) in its current form.

In this thesis, instead of entering the vehicle energy efficiency directly into the performance index, the optimal torque distribution between the front and rear axles is calculated separately and by direct search [87], and then the distance from this optimal distribution is entered into the cost function.

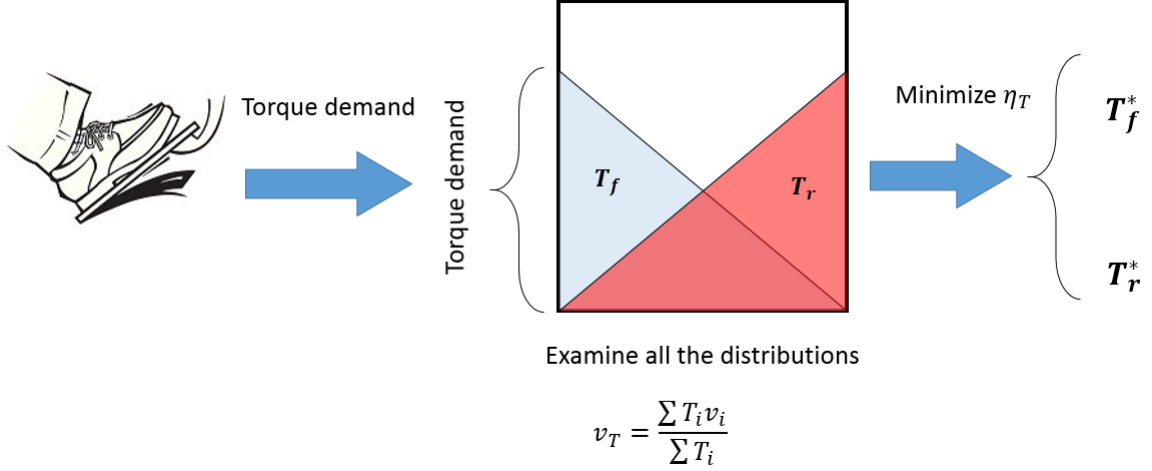


Figure 3.17: Direct search algorithm used in finding the optimal torque distribution between front and rear axles.

$$(T_F^*, T_R^*) = \underset{\text{subject to } T_F + T_R = T_{demand}}{\text{argmin}} \frac{T_F \nu_F + T_R \nu_R}{T_F + T_R} \quad (3.86)$$

The optimal torque distributions obtained by direct search are added to the performance index as an extra term. Therefore, the performance index in Equation (3.55) is modified as below:

$$J = \frac{1}{2} \sum_{k=1}^{N_p} (\|Y_d - Y_k\|_{\mathbf{Q}}^2 + \|U_k - V\|_{\mathbf{R}}^2 + \|U_k - U_k^p\|_{\mathbf{T}}^2 + \|U_k - T_{opt}\|_{\mathbf{S}}^2) \quad (3.87)$$

where

$$\mathbf{T}_{opt} = \begin{bmatrix} T_F & T_F & T_R & T_R \end{bmatrix}^T \quad (3.88)$$

$$\mathbf{S} = s \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad (3.89)$$

This term enforces proximity of the controller's solution to the optimal torque distribution. The tuning parameter s is chosen relatively smaller than the gains in the tracking term, so that the efficiency optimization has a lower weight than the stability and traction control terms.

With the addition of the extra term, minor changes in Equation (3.55) is required:

$$\begin{aligned} J(\mathbf{X}_0, \bar{\mathbf{U}}) = & \frac{1}{2} \bar{\mathbf{U}}^T \mathbf{H} \bar{\mathbf{U}} + \bar{\mathbf{U}}^T (\mathbf{F}_1 \mathbf{X}_0 + \mathbf{F}_2 \mathbf{Y}_{ref} + \mathbf{F}_3 \bar{\mathbf{W}}_0 \\ & - \bar{\mathbf{R}} \bar{\mathbf{V}} - \bar{\mathbf{T}} \bar{\mathbf{U}}_p - \bar{\mathbf{S}} \bar{\mathbf{T}}_{opt}) + \text{Const.} \end{aligned} \quad (3.90)$$

where

$$\bar{\mathbf{S}} = \text{blockdiag}(\mathbf{S}, \mathbf{S}, \dots, \mathbf{S}) \quad (3.91)$$

$$\bar{\mathbf{T}}_{opt} = \begin{bmatrix} \mathbf{T}_{opt} & \mathbf{T}_{opt} & \dots & \mathbf{T}_{opt} \end{bmatrix}^T \quad (3.92)$$

$$\mathbf{H} = \mathbf{S}_u^T \bar{\mathbf{Q}} \mathbf{S}_u + \bar{\mathbf{R}} + \bar{\mathbf{T}} + \bar{\mathbf{S}} \quad (3.93)$$

The rest of the parameters remain the same, as discussed in Section 3.6.

3.11 Summary

In this chapter, a model predictive control scheme was developed for stability and wheel slip control of electric vehicles. The prediction model comprises of two parts: vehicle model and

wheel dynamics. The vehicle model is based on a double-track model of vehicle dynamics. The desired vehicle yaw rate is defined according to the steering input, speed and the desired understeer gradient of the vehicle. The desired wheel speeds are also defined as the wheel center speed, allowing a certain slip percentage.

The stability and traction control objectives were described as quadratic terms in the objective function. The motor torque limit and tire force capacities are considered as constraints in the optimization problem. The required controller modifications for working with various driveline and actuator configurations were also studied.

In the last section, integration of the stability controller with optimal power distribution was studied. The optimal distribution is obtained by direct search and the controller is enforced to operate the front and rear motors in proximity of their respective maximum efficiency.

Chapter 4

Simulation Results

In this chapter, computer simulations are performed to evaluate the performance of the integrated model predictive controller that was developed in Chapter 3. The MPC controller is implemented in the MATLAB Simulink [88] environment. A high-fidelity CarSim [89] model, received from the manufacturer of the test vehicles, is used to represent the vehicle dynamics and its response to the driver's input as well as the controller's modifications. The accuracy of this CarSim model has been previously tested and found to be comparable to the actual vehicle response. In addition, CarSim also provides the feedback signals such as vehicle longitudinal and lateral accelerations, longitudinal and lateral speed and vehicle yaw rate as well as wheel speeds.

Two SUVs ¹ that are available for the experiments are also in the simulations. Table 4.1 shows the main properties of these two vehicles. Vehicles A and B are both electrified Chevrolet Equinox vehicles. Vehicle A is all-wheel drive and uses torque vectoring as method of actuation. Vehicle B is rear-wheel drive and is controlled via differential brakes.

Figure 4.1 shows the block diagram of the control loop used in Simulink/CarSim co-simulations. It can be seen that CarSim receives the steering input and drive/brake torques adjusted by the controller's differential torques. The Carsim solver provides the feedback

¹Sport Utility Vehicle

signals which are used by the predictive controller to calculate the optimal controller actions.

The parameters of the model predictive controller are summarized in Table 4.2. The sample time of the controller is set equal to 20 ms. The size of the prediction horizon is set equal to 8 and the size of the control horizon is 3. The size of the prediction horizon is limited by the computation time for real-time implementations. In addition, due to varying steering input, a very large prediction window results in poor transient performance of the controller. After evaluating several sizes for the prediction window and examining the steady state and transient response of the closed-loop system, a prediction horizon of 8 is selected. The control horizon is selected as 3 to reduce the online computation time while maintaining an acceptable closed-loop performance.

The elements of the tracking error weighting matrix (q_r , q_v and q_w) are tuned by trial and error. For instance, tuning for q_r starts by setting $q_r = 0$ in a sample steering maneuver, such as a flick maneuver (see Section 4.3). In this case, the controller makes no effort to track the desired yaw rate. The subsequent values for q_r are chosen so that the closed loop system shows satisfactory tracking speed and does not exhibit oscillatory response. A similar procedure is followed for q_v and q_w .

In this chapter, several maneuvers involving stability control, traction control or both are performed on a variety of road conditions to demonstrate the performance of the controller and robustness with respect to the road condition. In the end, the effectiveness of the proposed delay handling technique and integrated power distribution is also studied.

4.1 Launch on Snow

In this section, the launch maneuver on snowy road condition is performed with vehicle A. Due to simulation difficulties of a stationary car, the vehicle starts from a very low speed of 4 km/hr instead of zero. The steering wheel angle is kept straight and the acceleration pedal is given a step input from unpressed to fully pressed. The drive torque applied to the wheels is shown in Figure 4.2. The vehicle response is simulated with and without

Table 4.1: Inertial and geometric properties of the available vehicles used in simulations and experiments.

Parameter	Unit	Vehicle A	Vehicle B	Description
M	kg	2270	2043	Vehicle mass
I_z	kg.m ²	4600	4161	Yaw moment of inertia
I_w	kg.m ²	1.7	1.7	Wheel moment of inertia
L	m	2.858	2.858	Wheel base
W	m	1.589	1.585	Track width
R_{eff}	m	0.339	0.328	Wheel effective radius

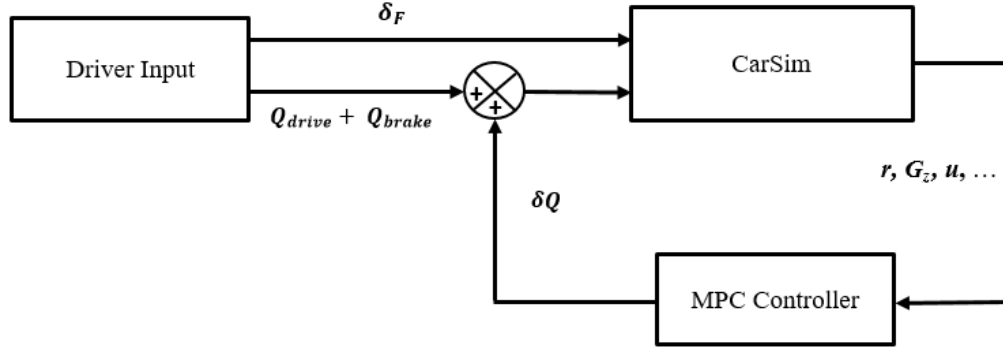


Figure 4.1: Simplified block diagram of the control loop of the controller and vehicle model in simulations.

the controller. Figure 4.4 shows the wheel speeds of the vehicle in the controlled and uncontrolled cases. It can be seen that when the controller is off, all four wheels are overspinning and marking speeds above 150 km/hr on the front and 100 km/hr on the rear. However, when the controller is active (Figure 4.4b), wheel speeds show only small overshoot and are quickly controlled and returned to the desired range. Wheel speeds show subsequent minor overshoots. These overshoots are desired and are important for detection of transitions to a surface with a higher friction coefficient. Figure 4.3 shows the torque adjustments of the controller for maintaining the traction of the tires. The controller is generating negative torques on all four wheels, which is the expected controller behaviour.

Table 4.2: Parameters of the model predictive controller.

Parameter	Description	Value
ΔT_p	Controller sample time (sec)	0.02
N_p	Size of the prediction horizon	8
N_c	Size of the control horizon	3
κ_{max}	Allowable longitudinal tire slip	8%
β_{max}	Allowable vehicle sideslip angle	4°
q_r	Weight of yaw rate tracking error	20
q_v	Weight of lateral velocity error	0.4
s	Weight on optimal torque tracking	10^{-5}
q_w	Weight of wheel speed tracking error	5×10^{-4}
$\mathbf{Q}$	Weight of reference tracking	$\text{diag}\{q_r, q_v, \dots$ $q_w, q_w, q_w, q_w\}$
$\mathbf{T}$	Weight of proximity to prev. solution	$10^{-5} \times \mathbf{I}_{4 \times 4}$
$\mathbf{R}$	Weight of control effort	$10^{-8} \times \mathbf{I}_{4 \times 4}$

The minor ripples in the torque adjustments are corresponding to the minor overshoots in the wheel speeds as discussed. Figure 4.5 shows the calculated tire slip ratios. It can be seen that when the controller is inactive, the tire slip ratios hit 90% slip. However, when the controller is active, after the initial overshoot, tire slip ratios are controlled and remain in the target range, as defined by κ_{max} in Table 4.2.

4.2 mu-Split Launch

In this section, a launch maneuver is performed when the right and left sides of the vehicle are on two different surface conditions. Vehicles that are equipped with an open differential have difficulty handling these situations, because the wheels that are positioned on a surface with a lower friction coefficient have less mechanical resistance and most of the engine torque passes through these wheels and the other wheel does not receive much drive torque.

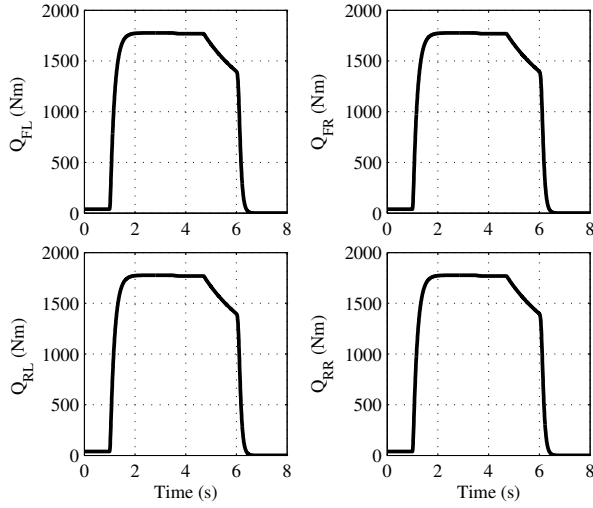


Figure 4.2: Drive torque of vehicle A in simulation of launch on snow with $\mu = 0.2$.

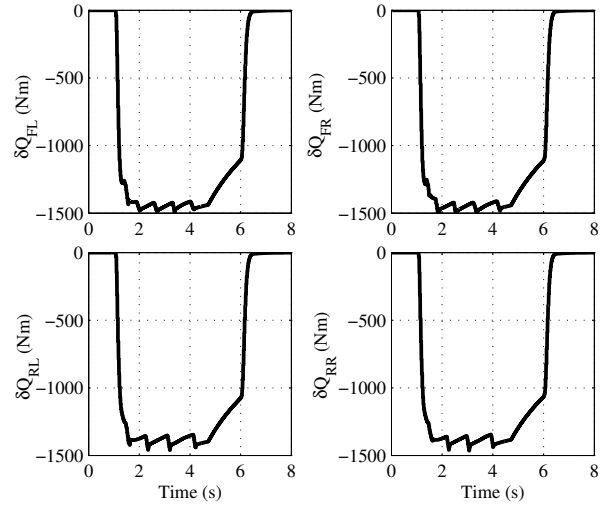


Figure 4.3: Controller torque adjustment in simulation of launch on snow with $\mu = 0.2$.

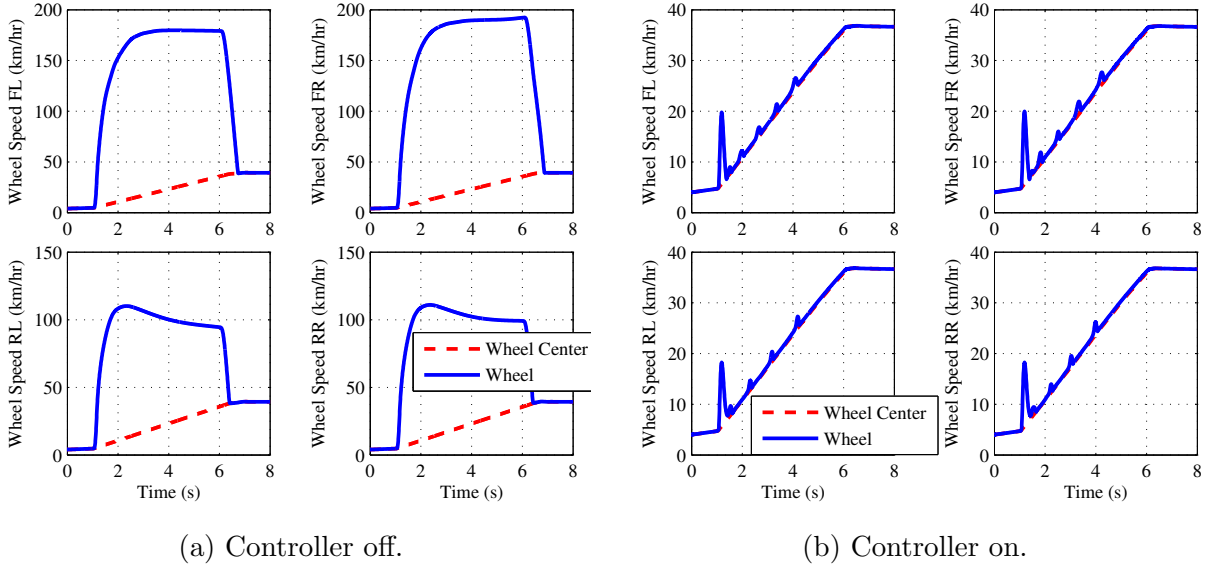


Figure 4.4: Wheel speeds of vehicle A in simulation of launch on snow with $\mu = 0.2$.

In this maneuver, the left wheels are on a surface with friction coefficient of $\mu = 0.2$ and the right wheels are on a surface with friction coefficient of $\mu = 0.9$. Figure 4.6 shows

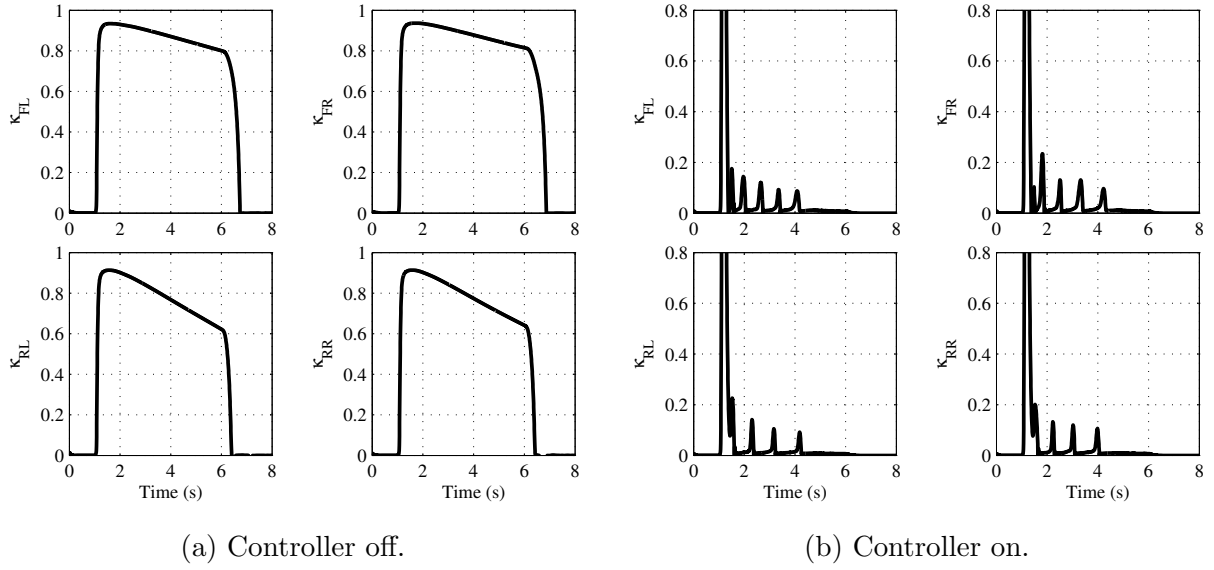


Figure 4.5: Tire slip ratios of vehicle A in simulation of launch on snow with $\mu = 0.2$.

the drive torques applied to all four wheels during the μ -split launch and Figure 4.7 shows the wheel speeds in each case. It can be seen that when the controller is off, the left wheels overspin and reach speeds above 150 km/hr. These wheels that overspin draw most of the battery current and reduce the torque applied to the right wheels. However, when the controller is active, the left wheels show only small overshoots at the beginning, which is quickly controlled and returned to the desired range. Controlled wheel speeds result in higher torque available for all four wheels as can be observed in Figure 4.6b.

Figure 4.8 compares the vehicle acceleration and speed in the controlled and uncontrolled cases. When the controller is active, the vehicle has a higher acceleration and reaches a higher speed at the end of the maneuver. However, absence of proper traction control results in a lower longitudinal acceleration and a smaller final speed.

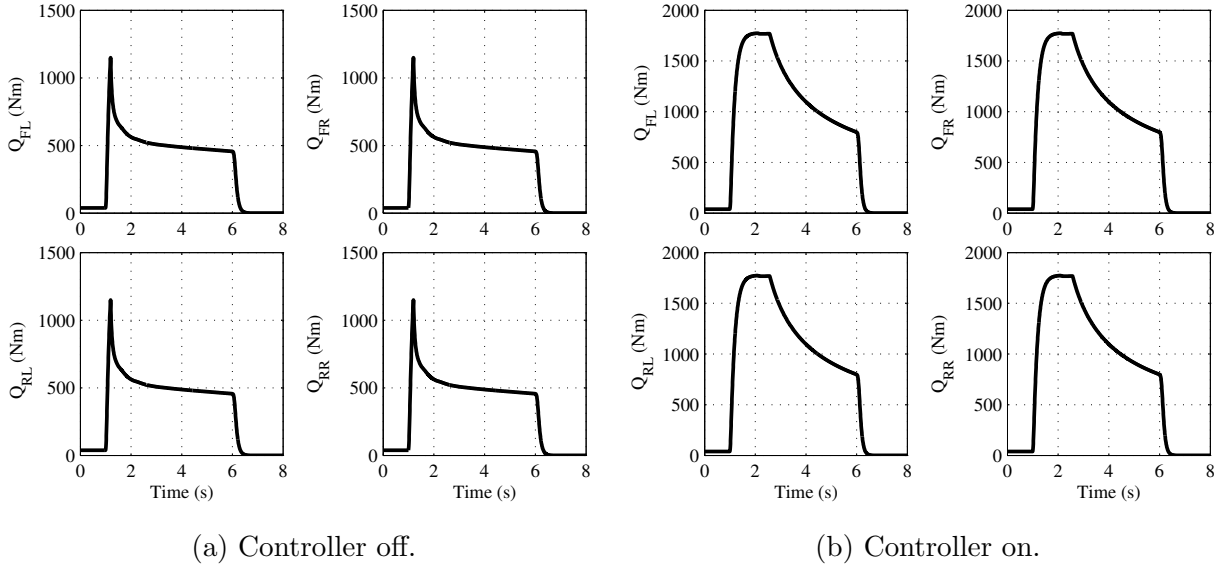


Figure 4.6: Drive torques of vehicle A in simulation of μ -split launch.

4.3 Flick Maneuver on Snow

This maneuver is widely used to induce vehicle drift without applying acceleration or braking. The road condition is assumed to be slippery with the friction coefficient of $\mu = 0.4$ (not made available to the controller). This maneuver is performed with vehicle A. The driving scenario involves cruising at the initial speed of 50 km/hr, steering to the right and then immediately counter steering and holding the steering wheel. The steering input and the resulting vehicle lateral acceleration are shown in Figure 4.9. As it can be seen, the lateral acceleration of the vehicle is close to the limit of the road condition.

Figure 4.11 shows the yaw rate tracking performance as well as the sideslip angle of the vehicle in the controlled and uncontrolled cases. In the uncontrolled maneuver, the vehicle is evidently unable to track the desired yaw rate and shows a steady-state offset. In addition, right after the counter steering the vehicle assumes large sideslip angle that exceeds 10 degrees and never returns to the stable bound. However, when the controller is in the loop, the vehicle can track the desired yaw rate very well and shows only minor overshoots. The sideslip angle of the vehicle also remains very small and never exceeds 5

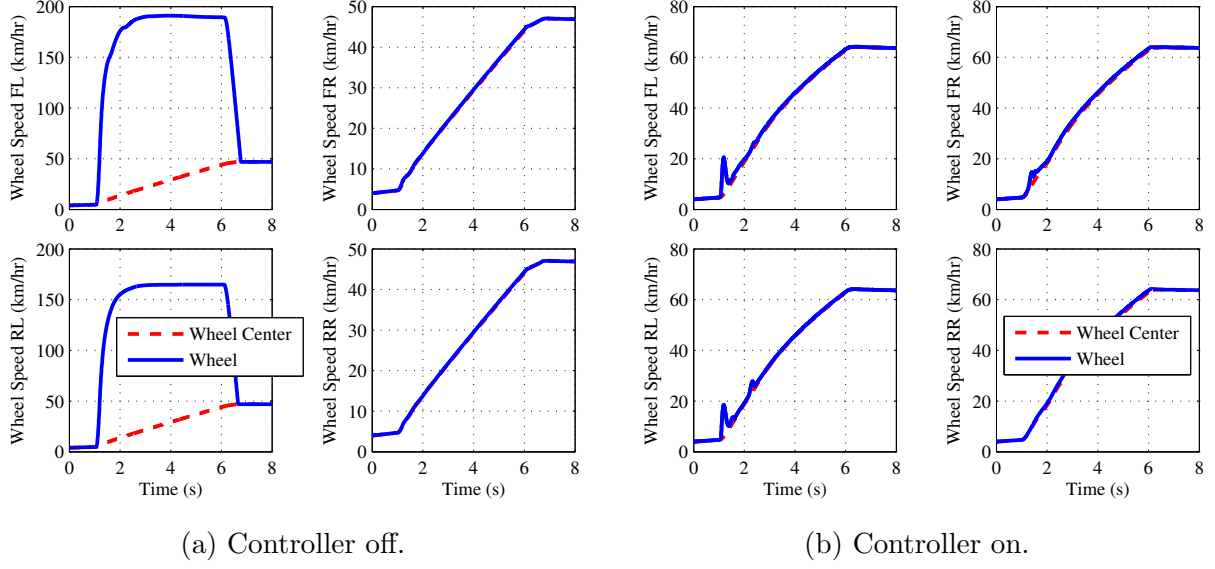


Figure 4.7: Wheel speeds of vehicle A in simulation of μ -split launch.

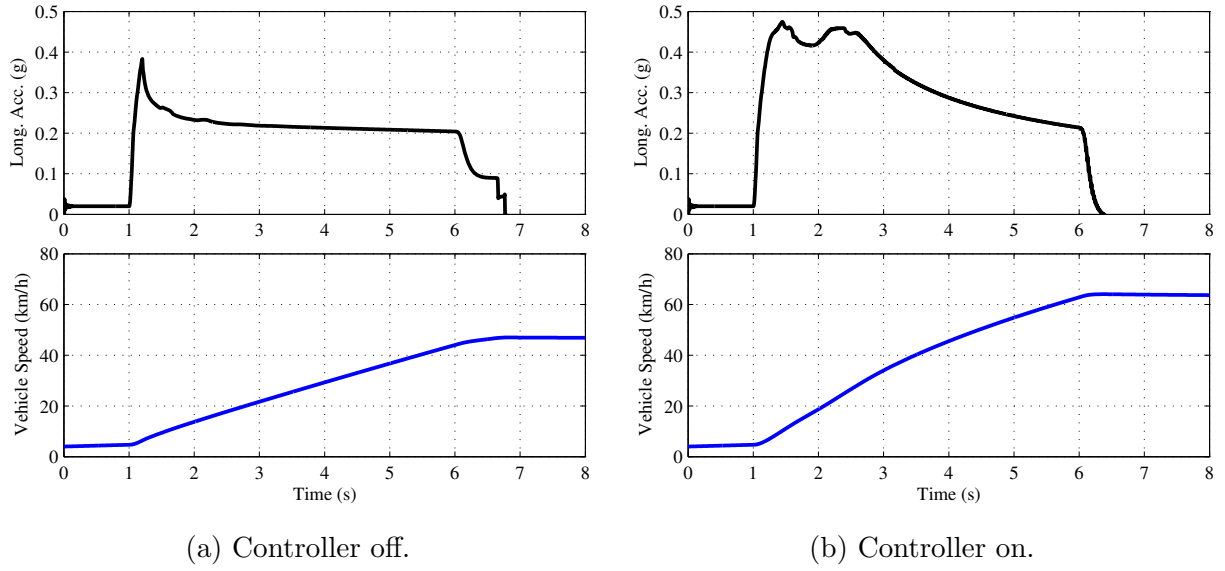


Figure 4.8: Longitudinal acceleration and speed of vehicle A in simulation of μ -split launch.

degrees. The torque adjustments of the controller are shown in Figure 4.10. It can be seen that the torque differential required to maintain vehicle stability are quite small, smooth and symmetric and do not show any oscillations.

4.4 Acceleration in Turn, RWD mode

Vehicle B is used to performed the acceleration in turn (AIT) maneuver. Since vehicle B uses differential brakes, the constraints in Section 3.7 are adjusted according to the reconfigurability discussion in Section 3.8. The road condition is the same as the previous maneuver with $\mu = 0.4$. The initial vehicle speed is 40 km/hr. The driving scenario involves steering and applying acceleration while cornering. The steering input and the lateral acceleration of the vehicle is shown in Figure 4.12. It is observed that the steering action results in significant lateral acceleration given the slippery road condition. The torque applied to the rear wheels is shown in Figure 4.13. It can be seen that the drive torque is requested as the vehicle is cornering. Since the vehicle is rear-wheel drive, if the slip of the rear tires is not controlled, the cornering force of the rear axle will be significantly reduced and the vehicle will oversteer.

Figure 4.14 shows the vehicle yaw rate and sideslip angle in the controlled and uncontrolled cases. In the uncontrolled case, the vehicle becomes unstable as soon as the driver applies acceleration. The vehicle yaw rate overshoots the desired yaw rate for an extended period of time. Moreover, the vehicle sideslip angle increases unboundedly and exceeds 10 degrees and indicates a full vehicle spin. On the other hand, when the controller is in the loop, the vehicle can track the desired yaw rate very well and the vehicle sideslip angle also remains small and in the stable range of vehicle dynamics.

The slip ratio of the rear tires is shown in Figure 4.15. In the uncontrolled case, the slip ratio of the tires quickly exceed 20%, while with the controller, the slip ratios are kept with the desired range (determined by κ_{max} in Equation (3.12) and according to the driving condition). The slip ratio of the rear right tire shortly exceeds the desired range, but is quickly controlled.

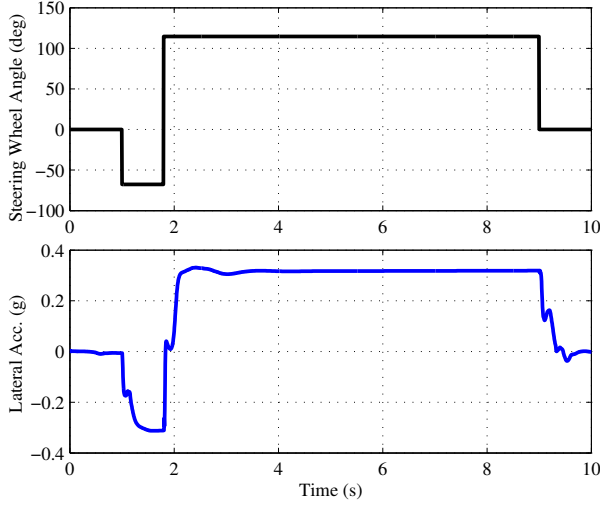


Figure 4.9: Steering wheel angle and lateral acceleration of vehicle A in simulation of flick maneuver on snow with $\mu = 0.4$.

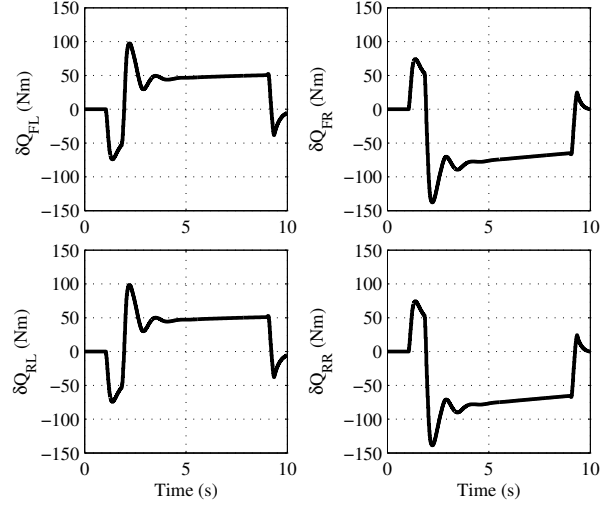
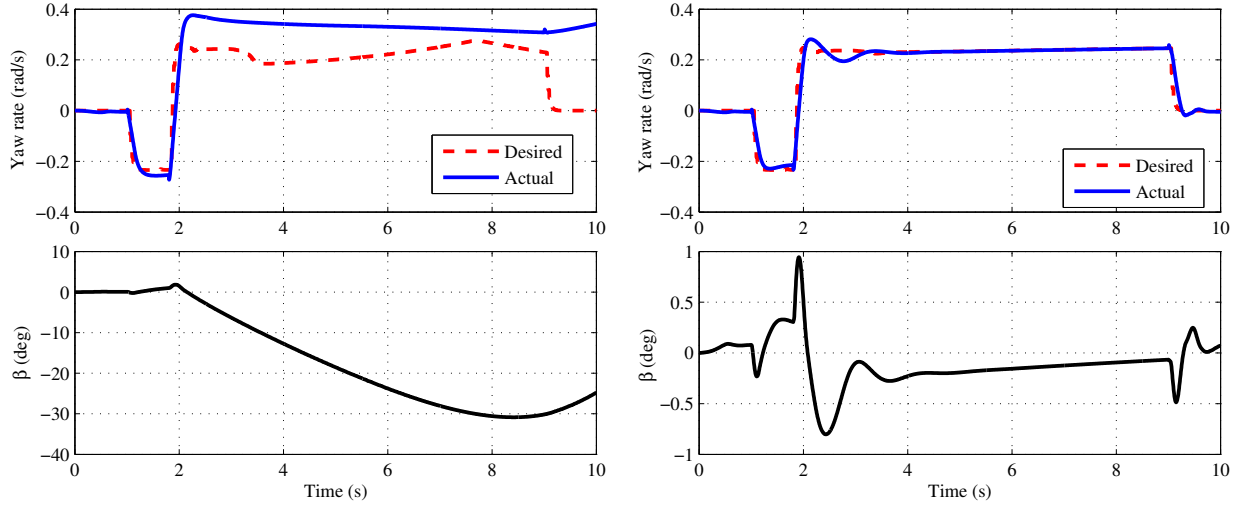


Figure 4.10: Torque adjustments of the controller in simulation of flick maneuver on snow with $\mu = 0.4$.



(a) Controller off.

(b) Controller on.

Figure 4.11: Yaw rate and sideslip angle of vehicle A in simulation of flick maneuver on snow with $\mu = 0.4$.

The control actions are shown in Figure 4.16. As it can be seen, only braking torques are generated by the controller since the vehicle is assumed to be equipped with differential brakes. The negative torque on the left side of the vehicle is applied mostly to prevent the vehicle from oversteering, it also helps to prevent overspinning of the rear left wheel. The negative torque applied to the rear right wheel is a result of tire slip control.

Figure 4.17 shows the vehicle longitudinal velocity and acceleration in the. As it can be seen, as the driver demands more drive torque, the longitudinal acceleration of the vehicle is increased by about 0.1g and the vehicle speed is gradually increased.

4.5 Delay Handling Technique

In this section, the performance of the proposed delay handling technique is studied using vehicle B. In the simulations, the delay is artificially inserted at the output of the controller. The delay consists of a pure (transport) delay of $\tau_d = 200$ ms and a first order delay of $\tau_1 = 100$ ms in series as shown in Figure 3.11. The performance of two similar MPC controllers will be compared: controller A and controller B. In controller A, direct sensor readings (i.e. $\mathbf{X}_0$) are used in the control law and the first order delay is not taken into account (i.e. $\tau_G = 0$ in Equation (3.43)). On the contrary, controller B is using the proposed delay handling method. It is designed based on the predicted system states at the end of the delay period (i.e. $\mathbf{X}'_0$). This means that $N_d = 10$ and $\tau_G = 0.1$ s. Besides the delay handling feature, the two controllers have identical structure and tunings as listed in Table 4.2.

A sample driving scenario is used to compare the performance of controllers A and B. Figure 4.18 shows the steering input. The driving scenario consists of steering and then immediately counter-steering, without applying any acceleration or brake. A slippery road surface with the friction coefficient of $\mu = 0.4$ is considered. The vehicle entry speed is 50 km/h.

Figure 4.19 compares the yaw rate tracking performance of the two controllers. In the top graph, it can be seen that after counter-steering, controller A is having trouble sta-

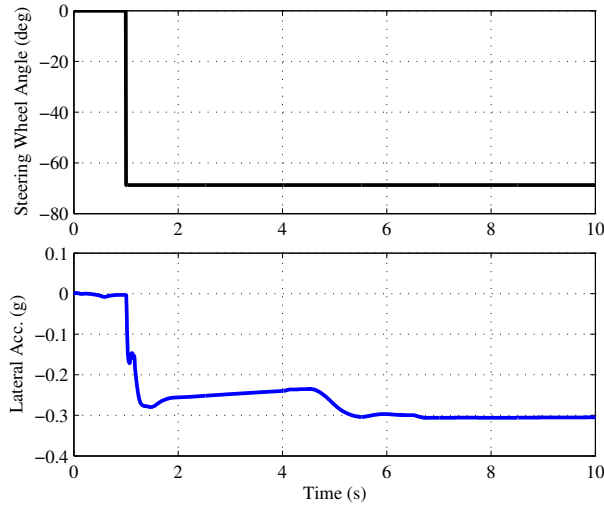


Figure 4.12: Steering wheel angle and lateral acceleration of vehicle B in simulation of AIT maneuver in RWD mode on snow with $\mu = 0.4$.

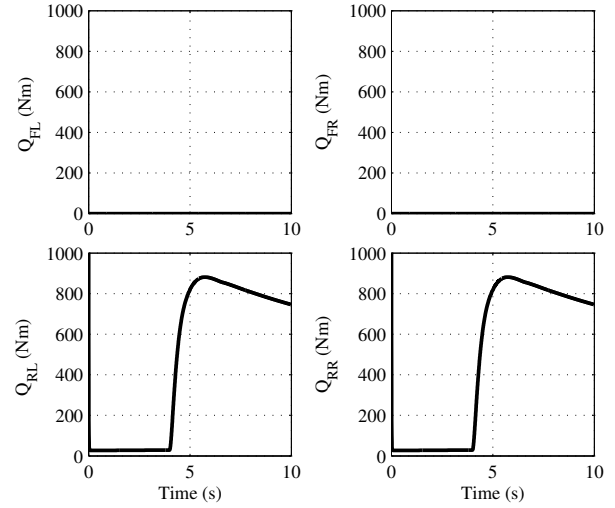
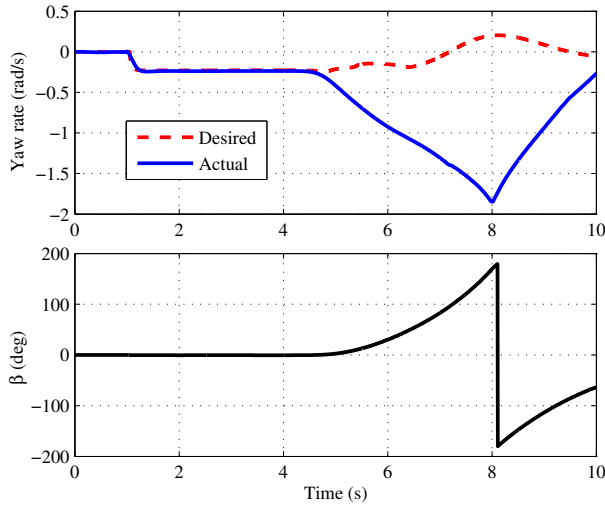
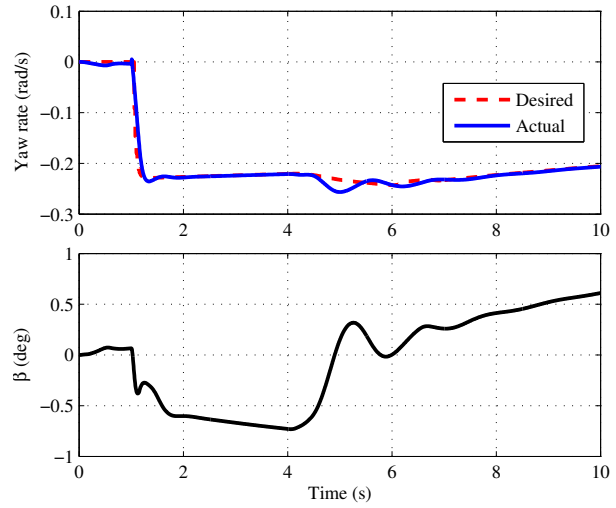


Figure 4.13: Drive torques applied to vehicle B in simulation of AIT maneuver in RWD mode on snow with $\mu = 0.4$.



(a) Controller off.



(b) Controller on.

Figure 4.14: Yaw rate and sideslip angle of vehicle B in simulation of AIT in RWD mode on snow with $\mu = 0.4$.

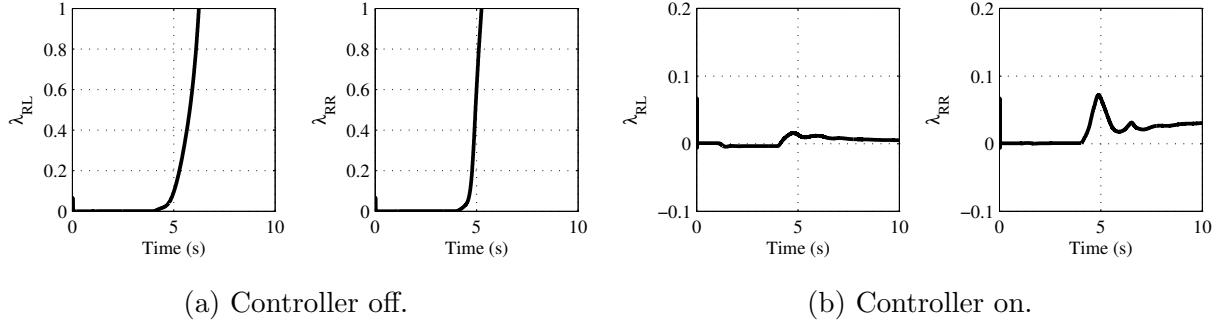


Figure 4.15: Slip ratio of the rear tires of vehicle B in simulation of AIT in RWD mode on snow with $\mu = 0.4$.

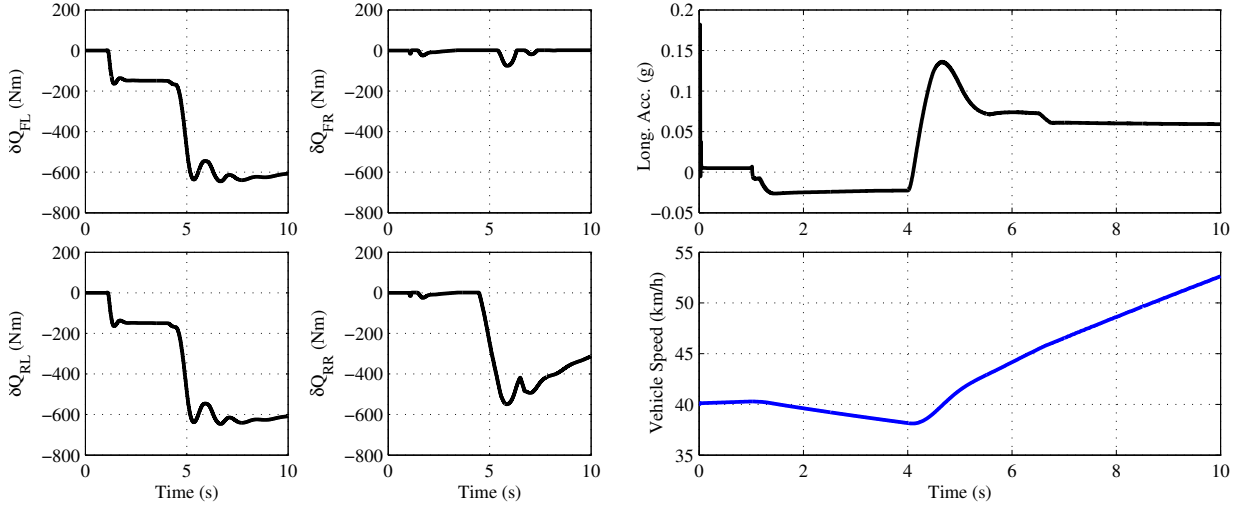


Figure 4.16: Torque adjustments of the controller of vehicle B in simulation of AIT in RWD mode on snow with $\mu = 0.4$.

Figure 4.17: Longitudinal acceleration and speed of vehicle B in simulation of AIT maneuver in RWD mode on snow with $\mu = 0.4$.

bilizing the vehicle and it is constantly oscillating about the reference yaw rate with progressively increasing amplitudes. In the bottom graph, controller B that uses the proposed delay handling method, shows only minor overshoot and undershoot and then converges to the desired yaw rate.

The vehicle sideslip angle is compared in Figure 4.20. As expected, the large oscillations in the yaw rate tracking of controller A result in large sideslip angles that progressively increase in peak values. Over the 12 seconds of simulation, the sideslip angle of the vehicle using controller A exceeds 15 degrees which is clearly past the stable limits of the vehicle dynamics. However, when using controller B, the sideslip angle remains below 3 degrees which is well within the stable limits of vehicle dynamics.

Figure 4.21 shows the torque adjustments made by the two controllers. The same trend can be observed here. The control commands of controller A are oscillatory and the amplitude of their oscillations increase over time. While, the torque adjustments of controller B do not exhibit such behaviour and converge to a steady state value over time.

From the above discussion, it can be inferred that controller B shows a much better performance and can maintain the stability of the vehicle in the situations where controller A becomes unstable.

4.6 Evaluation of Optimal Power Distribution

In this section, the effect of the power distribution part of the controller is studied. As developed in Section 3.10, the optimal torque distribution between front and rear axles is determined by a direct search. This distribution enters the objective function of the controller and the controller tries to operate the front and rear motors in proximity of their respective optimal distribution, if stability control objectives permit.

This section consists of two parts: In the first part, the potential energy savings resulting from torque redistribution in a standard FTP-75 driving cycle is investigated. In the second part, the cooperation of the power distribution and stability control objectives in a maneuver that involves both objectives is studied.

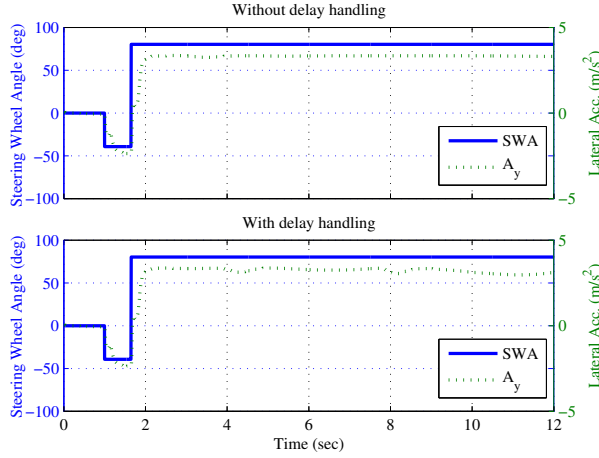


Figure 4.18: Steering wheel angle and lateral acceleration of vehicle with controllers A & B in evaluation of delay handling.

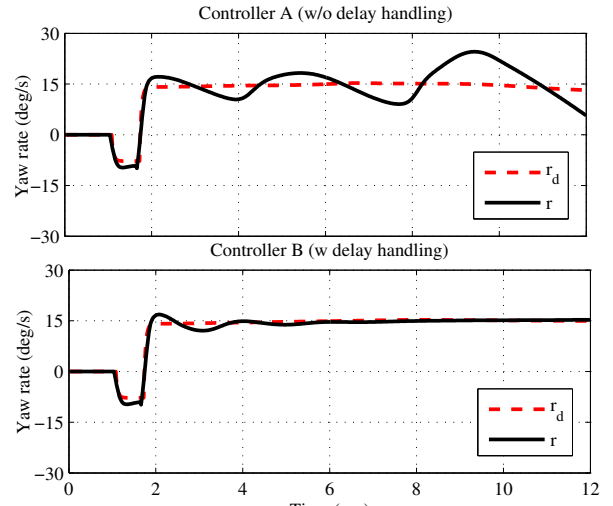


Figure 4.19: Yaw rate tracking performance of controllers A & B in evaluation of delay handling.

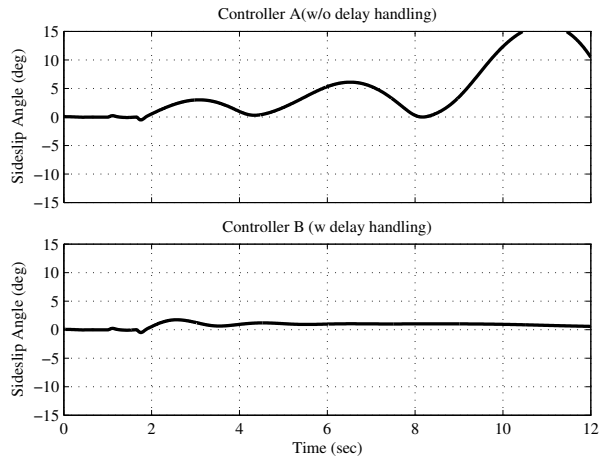


Figure 4.20: Sideslip angle of the vehicle with controllers A & B in evaluation of delay handling.

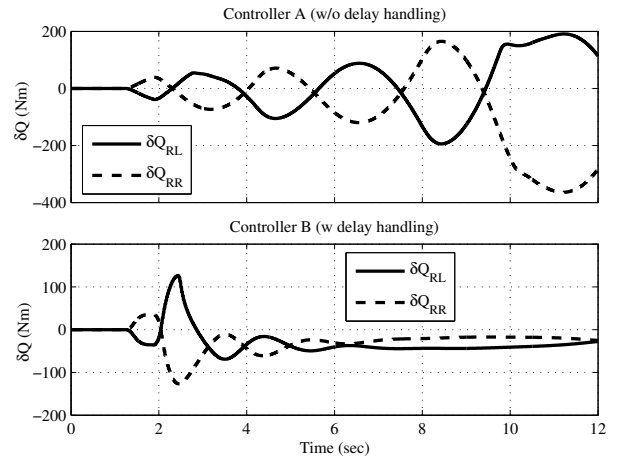


Figure 4.21: Torque adjustments of the controllers A & B in evaluation of delay handling.

4.6.1 Efficiency Improvement

In this section, the prospective energy savings resulting from optimal torque distribution between front and rear axles is studied. The efficiency maps of the electric motors of vehicle A is used in the simulations. Figure 4.22 shows the efficiency map of the electric motors used in vehicle A. It can be seen that contrary to the internal combustion engines, electric motors have a significantly higher efficiency ($\eta > 90\%$) in most of the operating regions. It should be noted that a reduction gearbox with gear ratio of 8:1 is used between motors and wheels.

In order to investigate the effect of drive torque redistribution based on the motor efficiency map, FTP driving cycle [90] is simulated (Figure 4.23). The driving cycle is performed with and without ² optimal power distribution and the overall vehicle efficiency in the whole cycle is calculated according to the efficiency map. The electrical energy consumed by the electric motors is calculated using the following equation:

$$E_{elec} = \int \sum_{ij} V_{ij} I_{ij} dt \quad (4.1)$$

where V_{ij} and I_{ij} are respectively the input voltage and current of the motor attached to wheel ij . Similarly, the mechanical energy produced by the electrical motors are calculated as:

$$E_{mech} = \int \sum_{ij} T_{ij} \omega_{ij} dt \quad (4.2)$$

where T_{ij} and ω_{ij} are respectively the delivered torque and speed of motor ij . Figure 4.24 shows a portion of the FTP driving cycle and the controller torque adjustments. It can be seen that during vehicle acceleration, the controller transfers all of the drive torque to one axle, in this case the rear axle. This is to ensure that motors are operating closer to their optimum efficiency point.

²In absence of optimal power distribution, the drive torque is simply split between front and rear axles at all times.

The overall vehicle efficiency is measured throughout the cycle and reported in Table 4.3. It can be seen that the vehicle consumes the same amount of mechanical energy in both cases. This is expected, since the mechanical energy mostly depends on the vehicle mass and speed versus time profile. However, when the optimal power distribution unit is active, the vehicle consumes less electrical energy (therefore less battery drain). The overall vehicle efficiency increases by about 2.3% with torque redistribution.

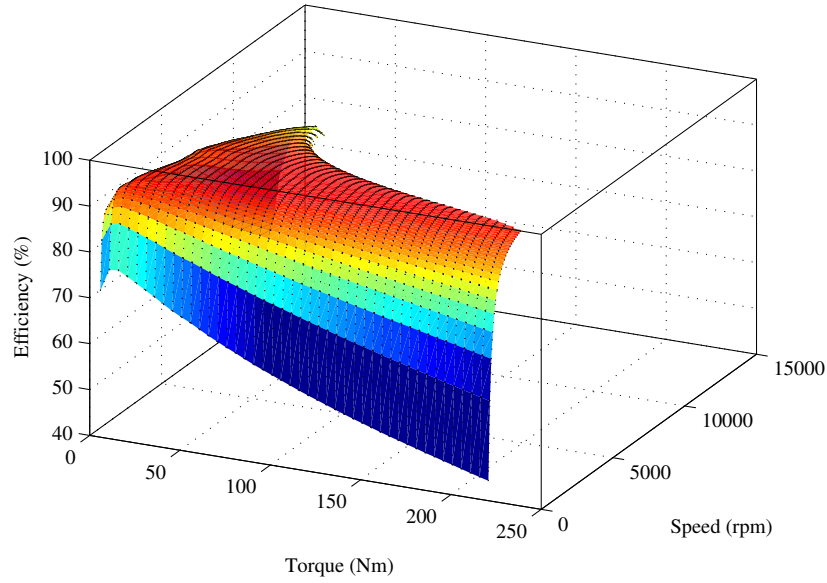


Figure 4.22: Efficiency map of electric motors of vehicle A.

Table 4.3: Energy consumption in FTP-75 driving cycle.

Test Number	Torque redistribution	E_{mech} (MJ)	E_{elec} (MJ)	η
1	on	1.736	1.886	0.9205
2	off	1.736	1.935	0.8971

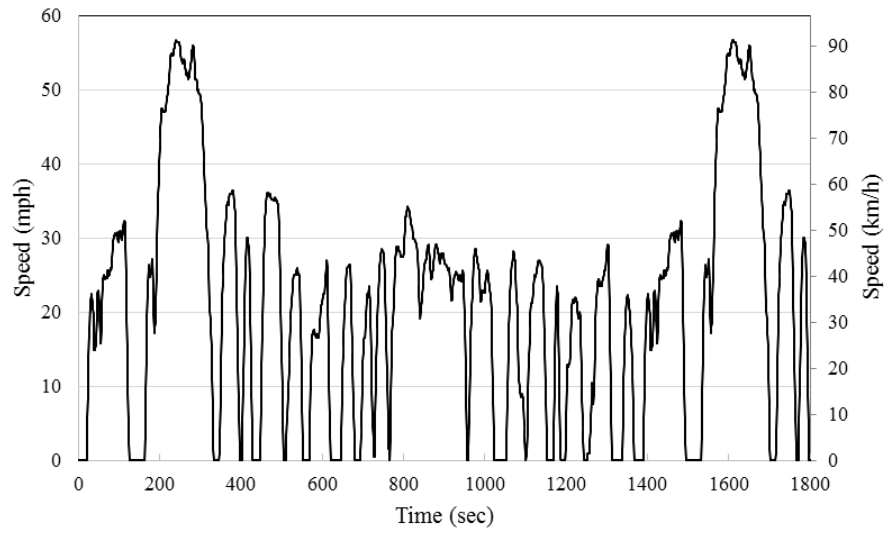


Figure 4.23: FTP-75 federal test driving cycle.

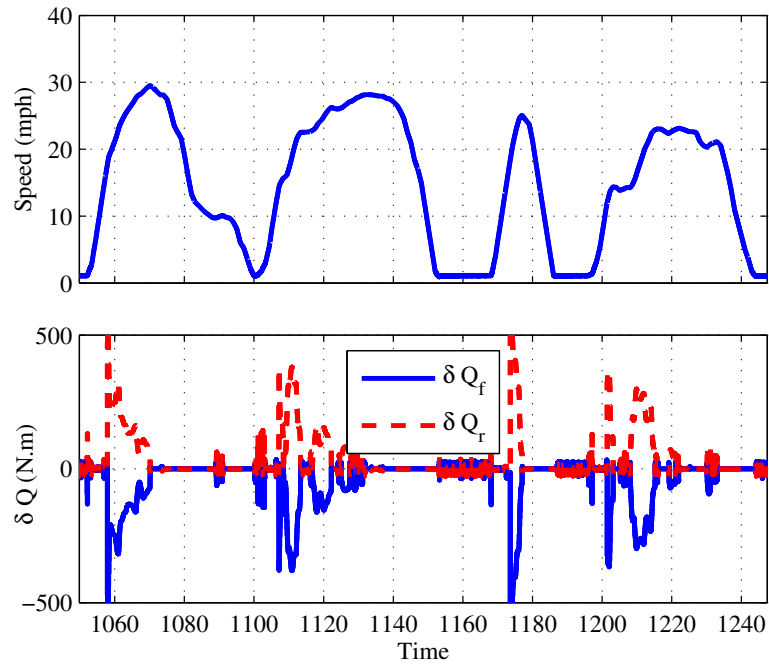


Figure 4.24: Vehicle speed and controller torque adjustments in a portion of the FTP driving cycle.

4.6.2 Cooperation with Stability Control

In this section, cooperation of the power distribution unit with the stability control system is studied. In order to further amplify the effect of torque redistribution, the efficiency map of the electric motors is changed to the map shown in Figure 4.25. In addition, the efficiency of the front motors is slightly reduced (by 2%), so that the controller favors the rear motors.

The driving scenario consists of flick maneuver on wet road with a friction coefficient of $\mu = 0.40$. The initial vehicle speed is 40 km/hr. The acceleration pedal is slightly pressed to generate some drive torque (Figure 4.26). The steering wheel angle and the lateral vehicle acceleration are shown in Figure 4.27. In the second steering action, the lateral vehicle acceleration is about $0.4g$, which is the limit on the road surface. Figure 4.28 shows the yaw rate tracking performance of the controller and the vehicle sideslip angle. It can be seen that the controller is tracking the desired yaw rate very well while maintaining a small sideslip angle. In addition, at the end of the maneuver, when the steering angle is returned to zero, the vehicle follows the desired yaw rate, showing great steerability after an extended period of critical cornering.

Figure 4.29 shows the torque adjustments of the controller. It is observed that the controller is producing negative torques on the front axle and positive torques of the same magnitude on the rear axle, thus transferring the drive torque to the rear motors that supposedly have a higher efficiency. In addition, the torque differential between left and right motors can be noticed that is generated to modify the vehicle response and track the desired yaw rate. It can be seen that the controller is satisfying power distribution and stability control objectives at the same time.

Figure 4.30 shows the adjusted torques ($Q_{ij} + \delta Q_{ij}$). It can be seen that the controller has transferred most of the drive torque to the rear motors that have a higher efficiency. In addition, the controller can still perform torque vectoring without interference with optimal power distribution.

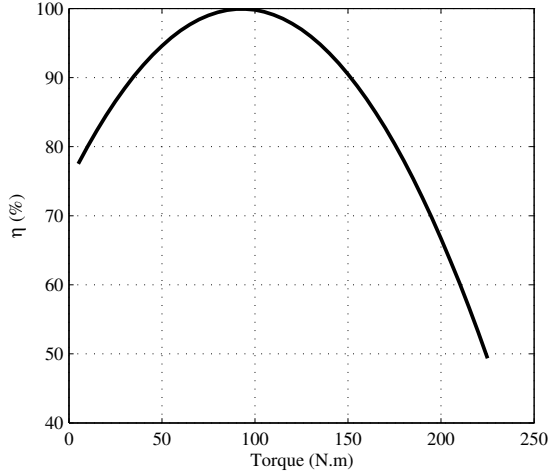


Figure 4.25: The alternative electric motor efficiency map used to amplify the front/rear torque transfer.

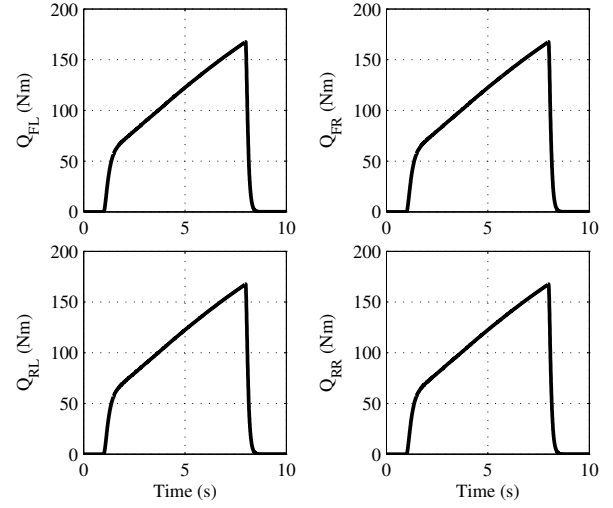


Figure 4.26: Drive torques in simulation of flick maneuver on wet road with optimal power distribution using vehicle A.

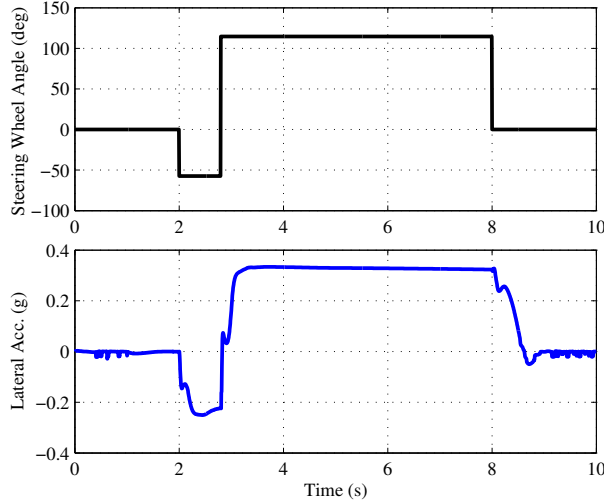


Figure 4.27: Steering wheel angle and lateral acceleration in simulation of flick maneuver on wet road with optimal power distribution using vehicle A.

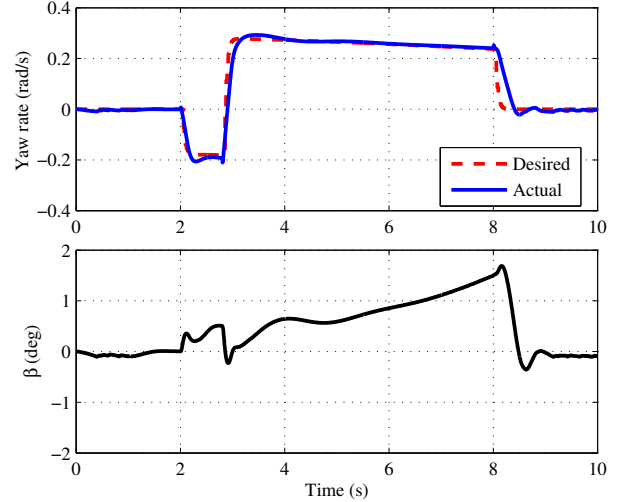


Figure 4.28: Yaw rate and vehicle sideslip angle in simulation of flick maneuver on wet road with optimal power distribution using vehicle A.

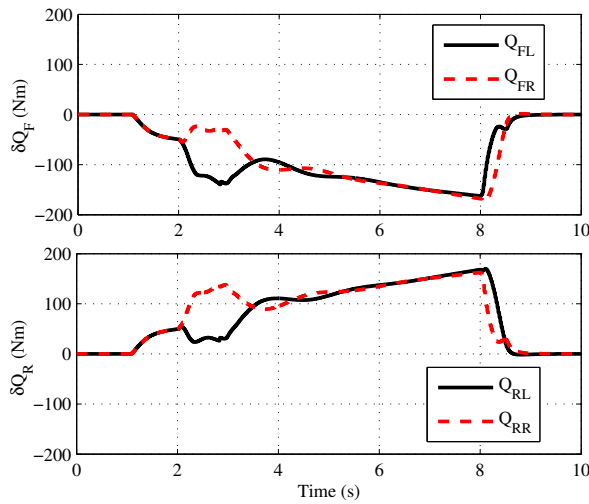


Figure 4.29: Torque adjustments of the controller in simulation of flick maneuver on wet road with optimal power distribution using vehicle A.

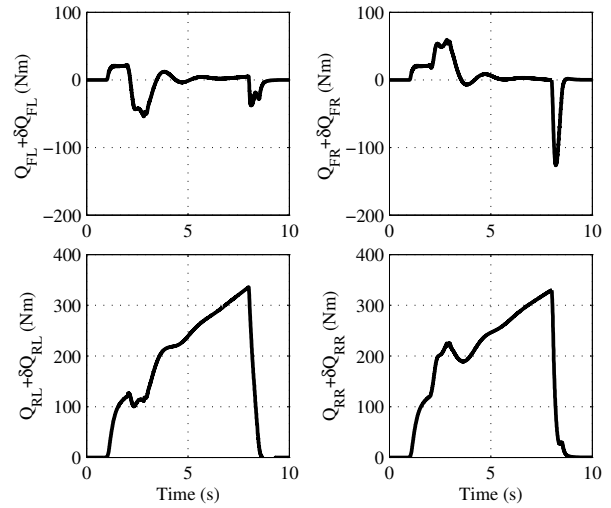


Figure 4.30: Adjusted total torque applied to each wheel in simulation of flick maneuver on wet road with optimal power distribution using vehicle A.

4.7 Summary

In this chapter, the performance of the model predictive controller developed in Chapter 3 was evaluated in several computer simulations. A high-fidelity CarSim model was used to model the vehicle response to the driver inputs and controller adjustments. The traction control performance of the controller was examined in a launch and μ -split maneuver. In both maneuvers, the controller was successful in quickly controlling wheel speeds and providing maximum acceleration on slippery surfaces.

A flick maneuver was performed without any acceleration or brake to examine the performance of the stability control part of the controller, without any adverse effect from excessive tire slip ratios. The controller was able to maintain vehicle stability with small amounts of torque vectoring. The combined stability and traction control performance of the controller was studied in an acceleration in turn maneuver where the uncontrolled vehicle response leads to a major vehicle spin. However, the controller was able to maintain

small slip ratios on the rear tires and maintain the directional stability of the vehicle.

The energy savings of the controller by redistribution of the drive torque between the front and rear motors was evaluated in an FTP-75 driving cycle. This test revealed a 2.3% improvement in the overall vehicle efficiency. Furthermore, the controller response when vehicle stability and vehicle efficiency objectives are both non-trivial was studied and it was observed that these objectives cooperate well and do not show any fight over priority.

Chapter 5

Experimental Results

In this chapter, the experimental verification of the performance of the controller developed in the previous chapters is presented. The same controller that was used in the simulations in chapter 4, is used in the experiments. The controller is developed in MATLAB Simulink ([88]) environment and is compiled and implemented on the dSpace micro-Autobox. The micro-Autobox communicates with the ABS ¹ encoders, electric motors (Figure 5.1), GPS unit (Figure 5.2) and vehicle IMU sensor through the CAN ² network. The experimental setup is shown in Figure 5.3. The stock vehicle IMU ³ sensor measures the vehicle longitudinal and lateral accelerations and the vehicle yaw rate. The RT2500 inertial and GPS navigation systems from the OxTS company is installed on the vehicle to provide accurate measures of the vehicle longitudinal and lateral velocities. Estimates of tire forces are provided by a separate code ([79]), also implemented in Simulink environment. The wheel encoders in the ABS module provide the wheel angular speeds and the hydraulic brake system can regulate the brake pressure and brake torque according to the received command signal. Similarly, the electric motors receive the torque command through the CAN bus and delivers the requested drive torque.

¹Anti-Block System

²Controlled Area Network

³ Inertial Measurement Unit

Two electric Chevrolet Equinox vehicles have been used in the experiments (Figure 5.4). These test vehicles are maintained, modified and driven by the technicians of "Mechatronic Vehicle Systems Laboratory". Vehicle A is an all-wheel drive (AWD) vehicle with four electric motors installed, one on each corner of the vehicle. The method of actuation is torque vectoring across both front and rear axles. Sport tires are installed on this vehicle. Vehicle B is rear-wheel drive (RWD) thus has only two electric motors installed on the rear axle. It uses differential braking as the method of actuation and has all season tires. Table 5.1 summarizes the main differences between the two vehicles. Figure 5.4 shows both test vehicles used in experimental verification. The main geometric and inertial properties of the two vehicles are listed in Table 4.1.

Several critical maneuvers are performed with the test vehicles A and B. The performance of the model predictive controller developed in Chapter 3 is evaluated in these maneuvers and compared to the uncontrolled vehicle response where possible. These maneuvers are performed on various road surfaces such as dry pavement, wet sealer and snow. The tuning parameters of the controller are the same as those listed in Table 4.2, except for some fine tunings.

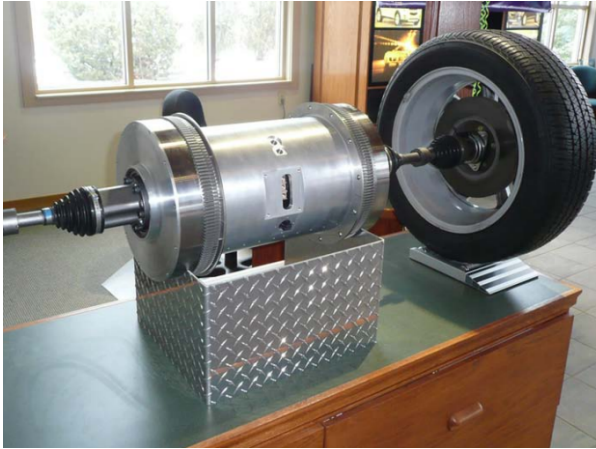


Figure 5.1: Electric motors modules [91].



Figure 5.2: RT2500 GPS module.

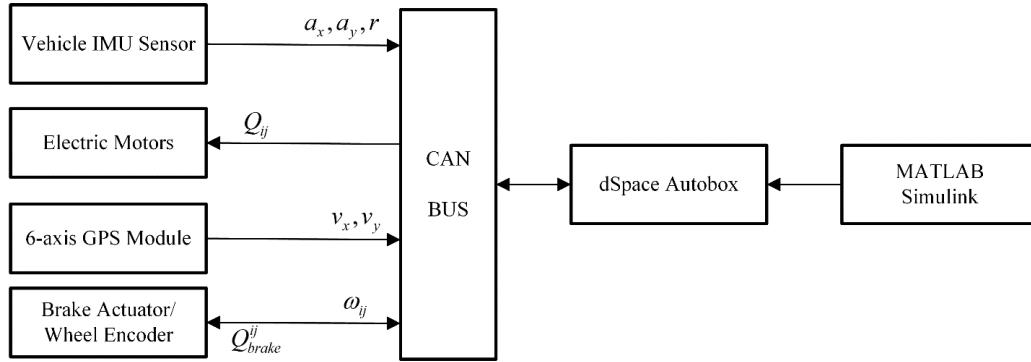


Figure 5.3: Experimental setup for measurement and control of vehicle.



(a) Vehicle A



(b) Vehicle B

Figure 5.4: Test vehicles used in experimental verifications.

Table 5.1: Properties of the two electric vehicles used in experiments.

Vehicle Name	Vehicle A	Vehicle B
Appearance	Black Equinox	White Equinox
Driveline	All Wheel Drive	Rear Wheel Drive
Actuation	Torque Vectoring	Differential Brakes
Tires	Sport Tires	All-season Tires

5.1 Slalom on Dry Pavement

This maneuver is performed with vehicle A. The vehicle is initially at rest on dry pavement. A sine steering input with full throttle ⁴ acceleration is applied. The test is performed with and without the controller in the loop, for the purpose of comparison.

The drive torque requested by the driver is shown in Figure 5.5. It can be seen that the drive torque is reduced over time, even though the acceleration pedal is fully pressed. This is because of the increased vehicle (and motor) speed and is a normal characteristic of electric motors. The steering wheel input and the vehicle lateral acceleration are shown in Figure 5.6. It can be seen that as the vehicle speed is increased, the magnitude of the vehicle lateral acceleration also increases until it reaches the 1.0g acceleration limit on dry pavement. Figure 5.7a shows the vehicle yaw rate and the vehicle sideslip angle in the uncontrolled maneuver. It is observed that the vehicle yaw rate deviates from the reference yaw rate and shows significant overshoot. This overshoot is accompanied by large vehicle sideslip angle as shown in the bottom graph. In the end, the driver has to react and apply brakes to stop the vehicle and prevent a major spin.

This maneuver is repeated with the controller. Figure 5.7b shows the yaw rate tracking performance of the controller and the vehicle sideslip angle. It can be seen that when the controller is active, the vehicle closely tracks the desired yaw rate. In addition, the vehicle sideslip angle remains small ($|\beta| < 3$ deg) and within the stable range of vehicle dynamics. Figure 5.8 shows the torque adjustments made by the controller to modify the vehicle response and track the reference yaw rate. Since the maneuver is performed with full throttle, the torque adjustments are all negative. In addition, for $t > 8$ s the torque adjustments δQ_{FL} and δQ_{FR} remain around -600 N.m, which almost cancel the total torque on the front axle. This is due to the combined effect of high longitudinal and lateral vehicle acceleration. The high longitudinal acceleration results in load transfer towards the rear axle and during high lateral acceleration, the tire capacity is used in the lateral direction.

⁴Although there is no physical "throttle" in electric vehicles, the terms "full throttle" and "partial throttle" are used several times in this thesis to indicate the driver's torque demand based on the acceleration pedal position.

Therefore, the combined effect of the high longitudinal and lateral acceleration is significant reduction in the longitudinal tire capacity on the front tires.

5.2 Slalom on Wet Sealer

In this maneuver, the test vehicle B is used to perform a full-throttle slalom maneuver on wet sealer with friction coefficient of $\mu \approx 0.4$. The controller is configured to work with the differential brakes according to Equation (3.74) in Section 3.8. This maneuver evaluates the combined performance of the traction and stability control of the MPC controller as well as its performance with the differential brakes. The maneuver is performed with and without the controller for comparison. The initial vehicle speed is 35 km/hr in both maneuvers.

The steering wheel input and the rear axle drive torque are shown in Figure 5.9. Despite small discrepancies, the controlled and uncontrolled maneuvers are conducted in the same way, with the acceleration applied during the steering action. Figure 5.10 compares the yaw rate tracking and sideslip angle of the vehicle in the controlled and uncontrolled maneuvers. It can be seen that when the controller is inactive, the vehicle yaw rate deviates from the reference values between $5 < t < 7$ seconds. In the same time interval, the vehicle sideslip angle increases above 20 degrees which is clearly past the stable range of the vehicle dynamics. At this point, the driver reacts and counter-steers and brakes to prevent a major vehicle spin. On the other hand, when the controller is active, the vehicle tracks the desired yaw rate very well without any significant overshoot or undershoot. The vehicle sideslip angle also remains small ($|\beta| < 2^\circ$) and in the allowable range.

The wheel speeds are shown in Figure 5.11. When the controller is off, it is observed that the rear wheels have lost grip between $4 < t < 6$ seconds. This results in reduced tire capacity in the lateral direction which is the reason for the vehicle oversteer that is observed in Figure 5.10a. However, when the controller is active, the wheel speeds remain close to the desired speeds and do not exhibit any overspin or lock-up.

The differential brake torque requests of the controller are shown in Figure 5.12. It

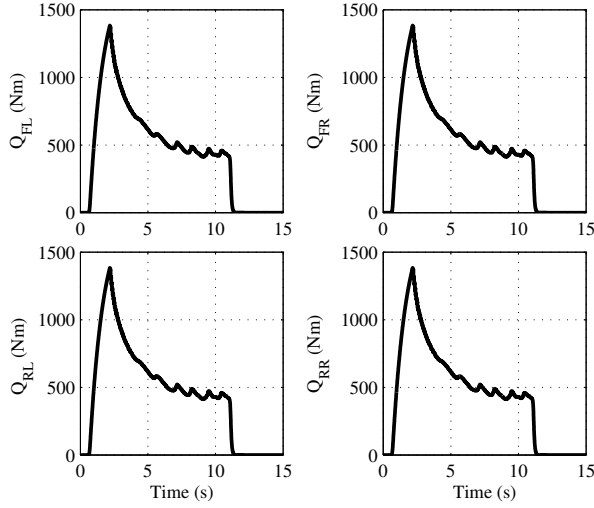


Figure 5.5: Drive torques requested by driver in the slalom maneuver on dry pavement with vehicle A.

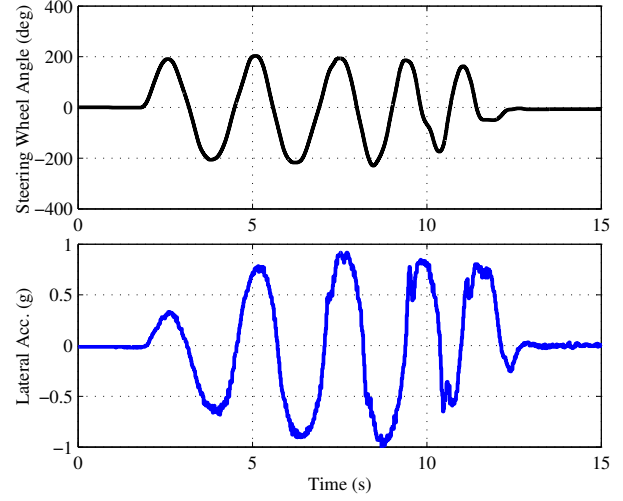
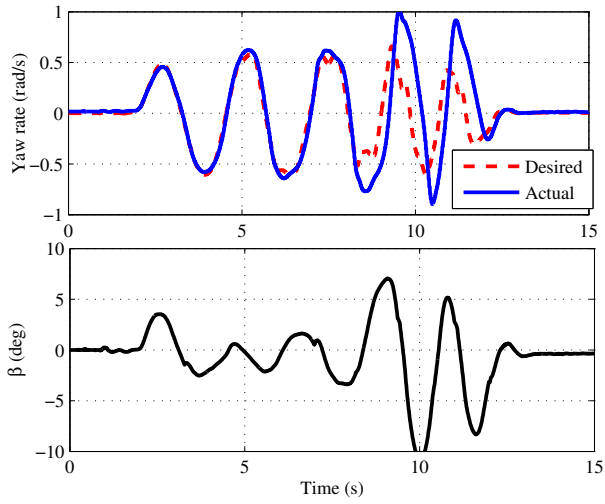
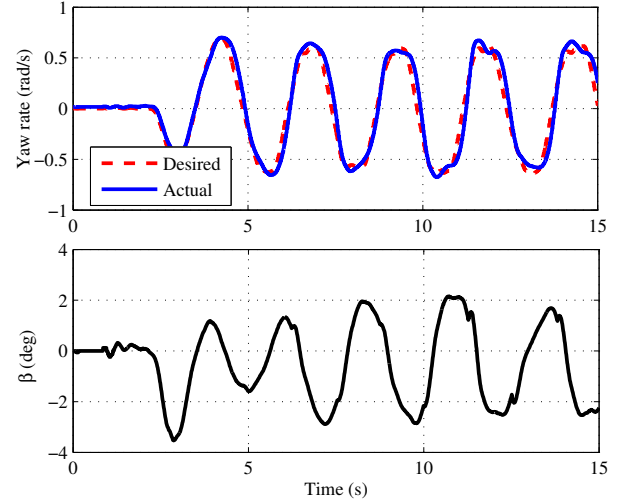


Figure 5.6: Steering wheel input and lateral acceleration in slalom maneuver on dry pavement with vehicle A.



(a) Controller off.



(b) Controller on.

Figure 5.7: Yaw rate tracking and vehicle sideslip angle in slalom maneuver on dry pavement with vehicle A.

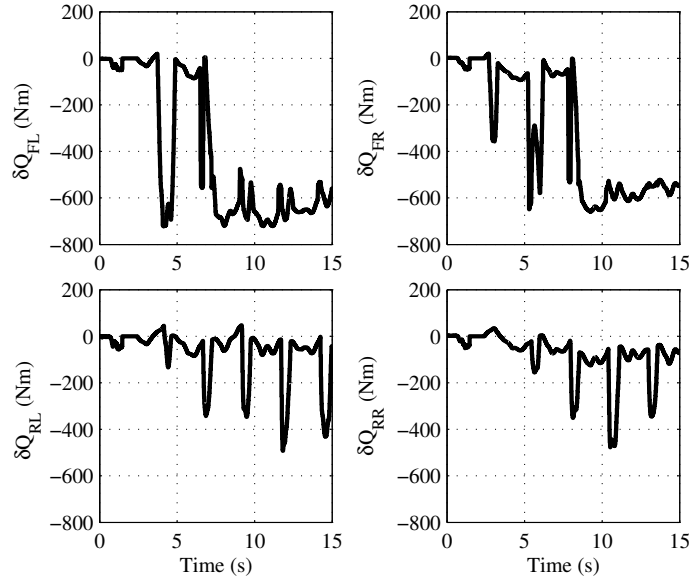


Figure 5.8: Torque adjustments performed by the controller in the slalom maneuver on dry pavement with vehicle A.

is observed that the controller is only demanding negative (brake) torques, and transition of braking torque request from the left wheels to the right wheels is smooth and without chatter. In addition, the brake torques requested on the front wheels are within the tire capacity and do not cause any wheel lock-up.

5.3 Launch on Wet Sealer

In this maneuver, vehicle A is positioned on a wet sealer with coefficient of friction of approximately $\mu \approx 0.40$. The steering wheel angle is kept straight and the accelerator pedal is fully pressed at some instant of time. This maneuver is performed with or without the controller to verify the effectiveness of the traction control part of the controller. Figure 5.13 shows the drive torque requested by the driver on all four wheels. The torque request peaks to about 1400 N.m. and then reduces as the motor speeds increase. Figure 5.15a shows the wheel speeds during this maneuver. It can be seen that the front right wheel

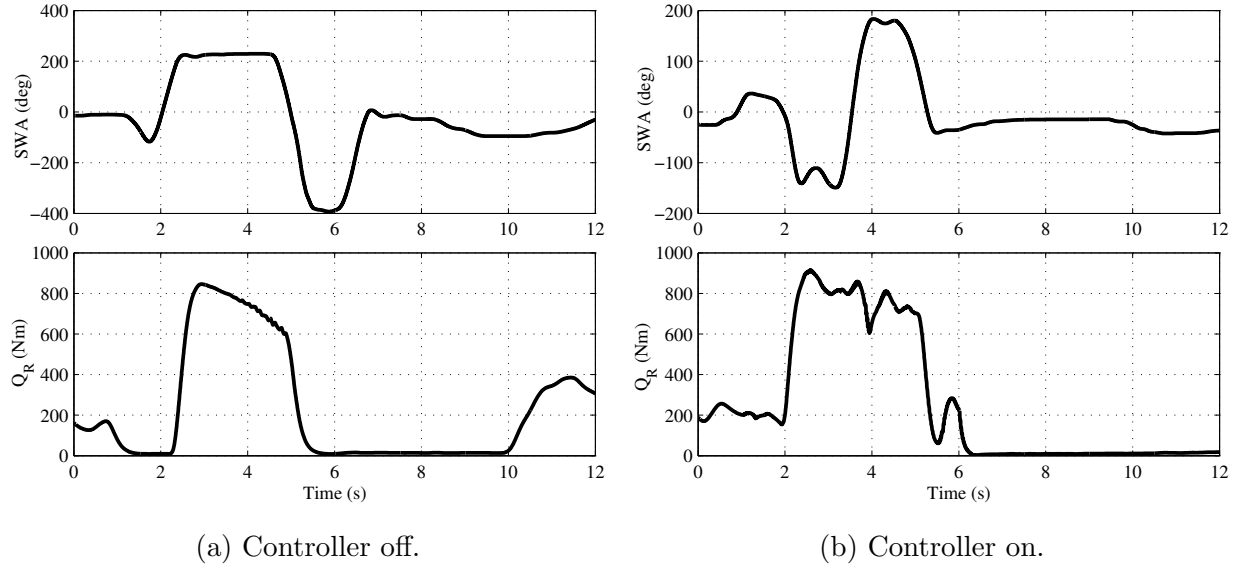


Figure 5.9: Steering wheel angle and rear axle drive torque in full-throttle slalom maneuver on wet sealer with vehicle B.

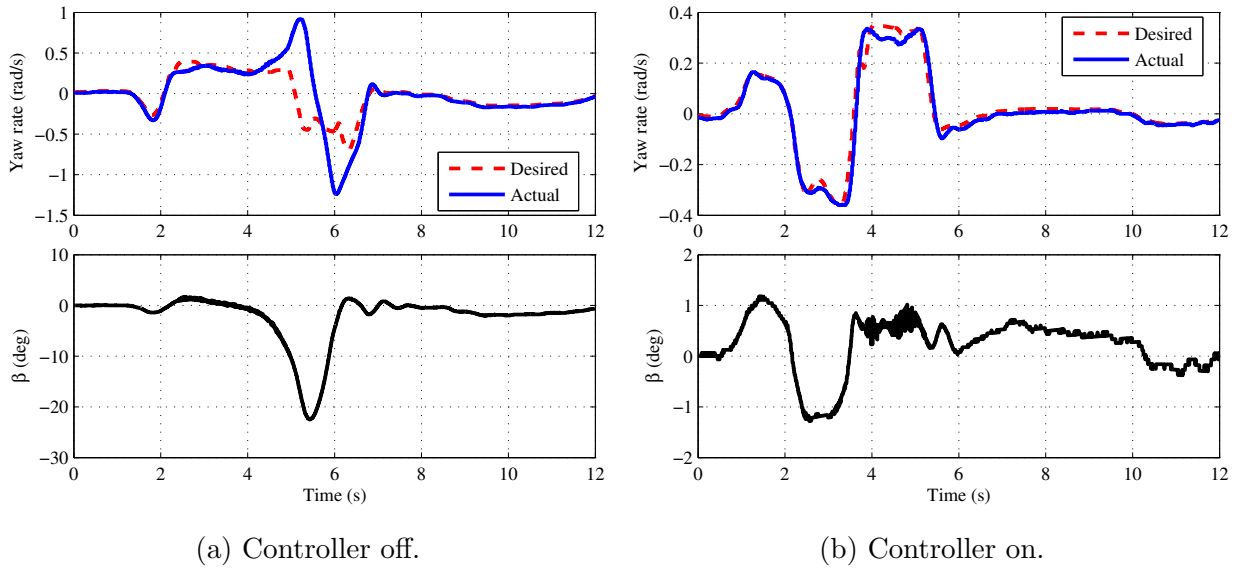


Figure 5.10: Yaw rate tracking and sideslip angle in full-throttle slalom maneuver on wet sealer with vehicle B.

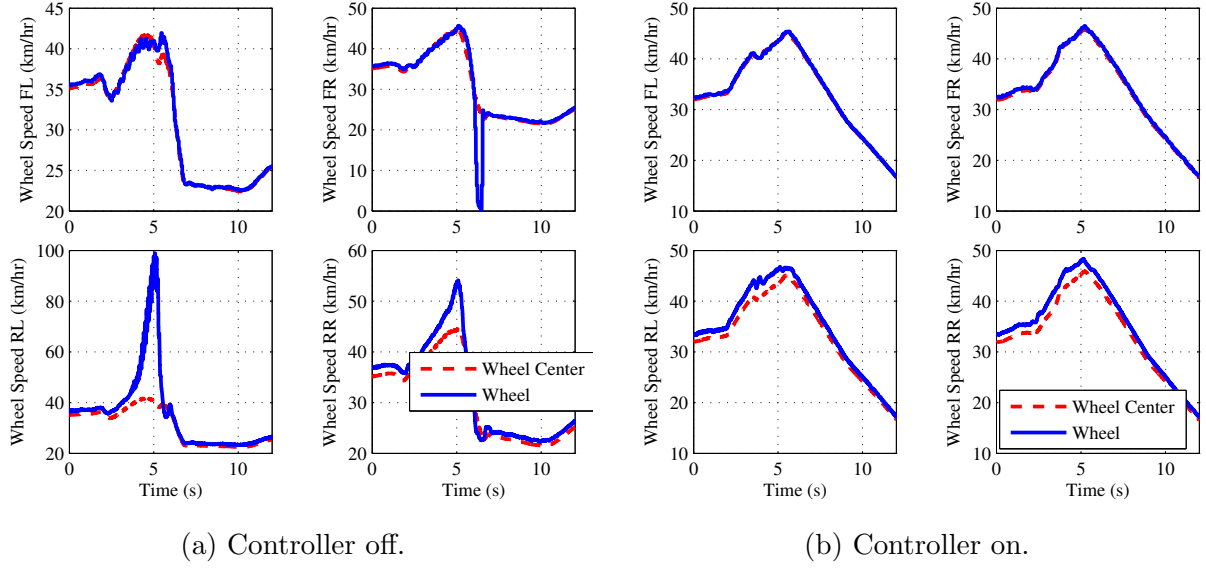


Figure 5.11: Wheel speeds in full-throttle slalom maneuver on wet sealer with vehicle B.

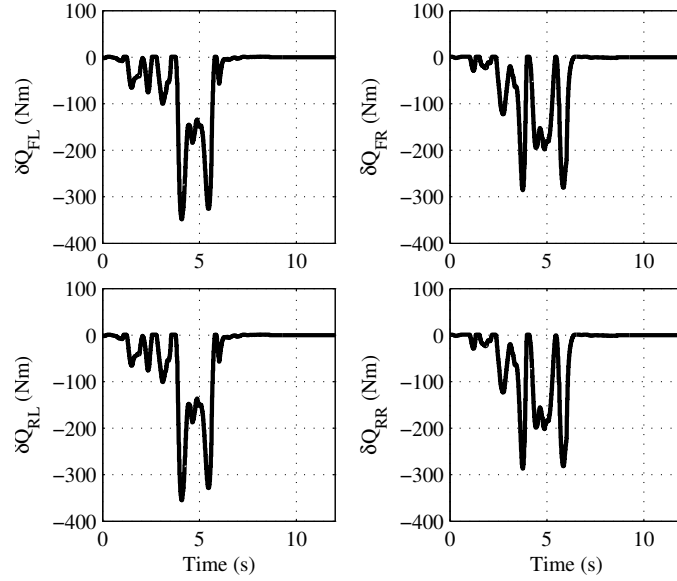


Figure 5.12: Torque adjustments performed by the controller in the slalom maneuver on wet sealer with vehicle B.

loses its grip and overspins to speed of 110 km/h. It can be seen that the rest of the wheels do not show significant overshoots. The reason for this is that the high speed of the front right wheel draws most of the current flowing from the battery pack and the power limiter module reduces the current passing to the other three wheels.

Figure 5.15b shows the wheel speeds in the controlled maneuver. At $t \approx 2s$, the front wheels start to lose grip with the ground, but the controller quickly controls the wheel speeds and returns them to the desired wheel speeds by adjusting the applied torque. The control actions are shown in Figure 5.14. The controller is generating negative torque adjustments to maintain tire grip on low- μ surface. Furthermore, the negative torque on the front wheels are much more than the rear wheels. This is expected, since the vehicle acceleration reduces the vertical tire forces on the front axle, making them more likely to lose grip.

The longitudinal vehicle acceleration and speed are compared in Figure 5.16. In the uncontrolled maneuver, the vehicle acceleration increases in accordance with the applied torque and peaks at $t = 2s$. After that, the vehicle acceleration is reduced. Comparing with the wheel speeds in Figure 5.15a, it can be seen that $t = 3s$ is the moment that the front wheels lose grip and the front right wheel overspins. For $4 \leq t \leq 6s$, the vehicle acceleration is constant and about $0.2g$.

Figure 5.16b shows the vehicle longitudinal acceleration and velocity when the controller is active. Compared to Figure 5.16a, the vehicle acceleration peaks at the same value (about $0.40g$ in agreement to the coefficient of friction). However, when the controller is active, the average vehicle acceleration is higher and it does drop as fast as in Figure 5.16a. In addition, the vehicle achieves a higher speed when the controller is active.

5.4 Launch on Packed Snow

In this maneuver, vehicle A is used in a launch test on packed snow. The drive torque applied to the wheels are shown in Figure 5.17. Figure 5.18 shows the individual wheel speeds compared to the wheel center speed. It can be seen that upon applying the wheel

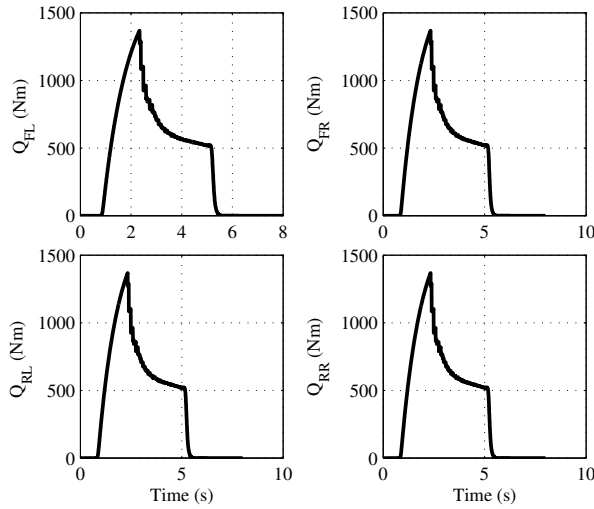


Figure 5.13: Drive torques requested by driver in the launch maneuver on wet sealer with vehicle A.

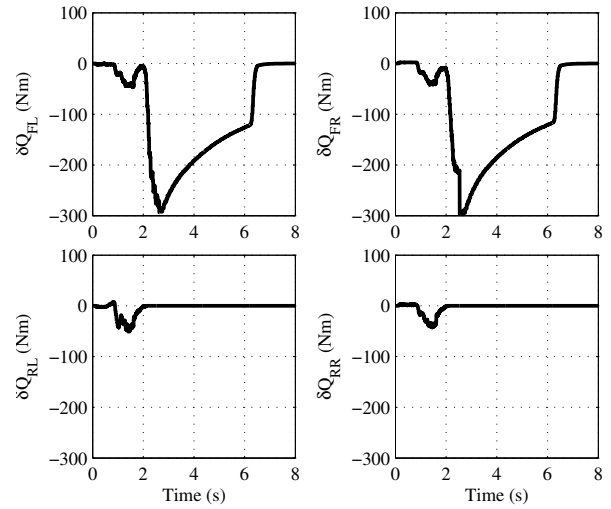


Figure 5.14: The torque adjustments on four wheels in the launch maneuver on wet sealer with vehicle A, controller on.

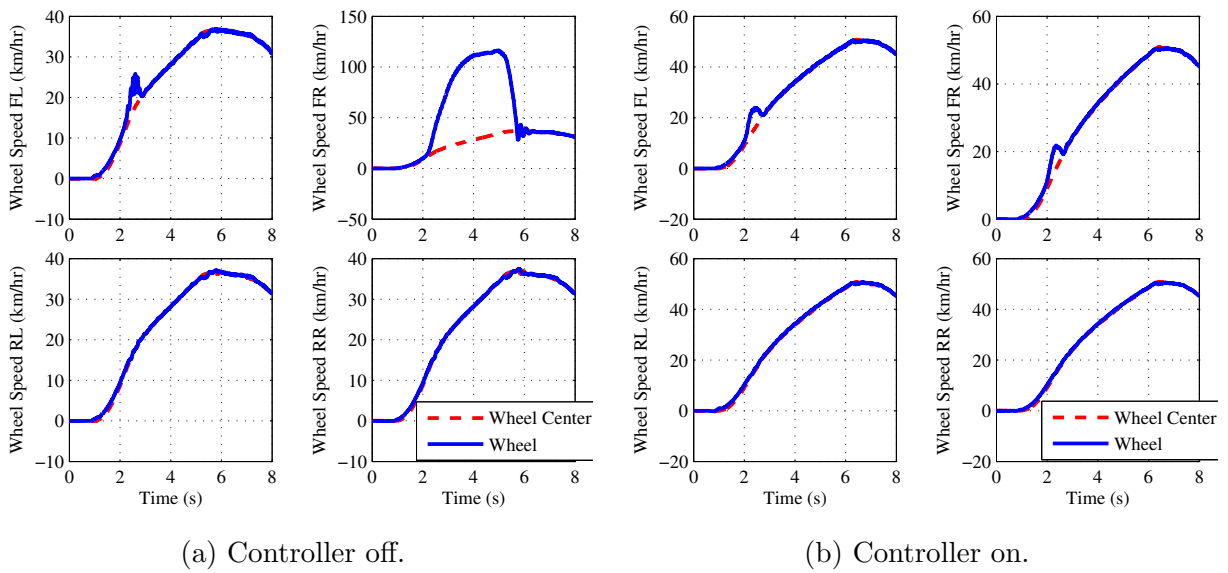


Figure 5.15: Wheel speeds in launch maneuver on wet sealer with vehicle A.

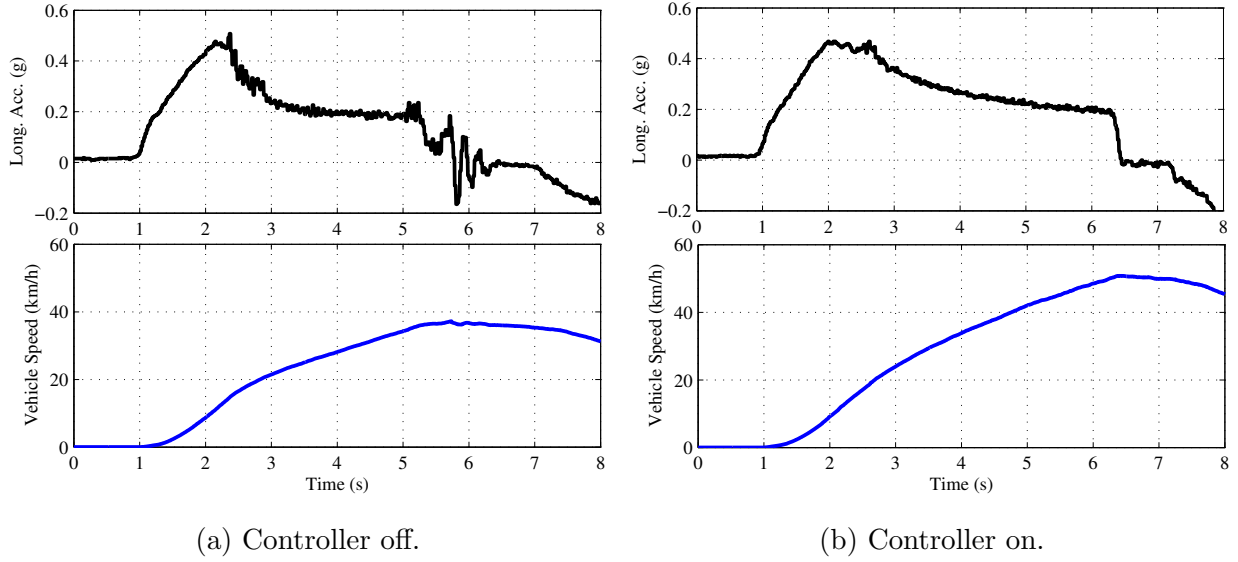


Figure 5.16: Longitudinal vehicle acceleration and vehicle speed in the launch maneuver on wet sealer with vehicle A.

torques, all wheels show slight overshoot, but it is quickly controlled and returned to the allowable slip range. The overshoot is particularly noticeable in the front wheels, since the longitudinal acceleration results in reduced vertical tire forces. The calculated tire slip ratios are shown in Figure 5.19. There are peaks in the slip ratios at the same times that overshoot occurs in wheel speeds. However, the slip ratios are quickly controlled and returned to the desired range.

The torque adjustments of the MPC controller is shown in Figure 5.20. It can be seen that as soon as wheel speeds start to overshoot, the controller requests negative torque so that the overall torque on the wheels is reduced and that the wheel speeds are returned to the desired range. The longitudinal acceleration and vehicle speed are shown in Figure 5.21. The vehicle acceleration between 2 and 3 seconds into the test is close to $0.3g$ which is very close to the friction coefficient of the packed snow, thus confirming the effectiveness of the wheel slip control of the controller.

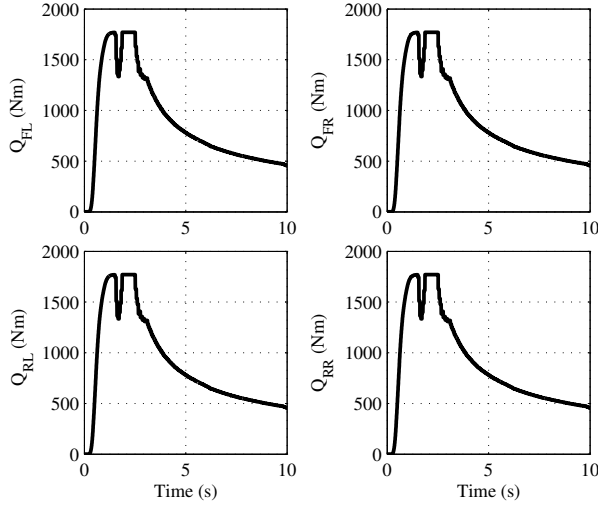


Figure 5.17: The driver's torque demand on four wheels in the launch test on snow with vehicle A.

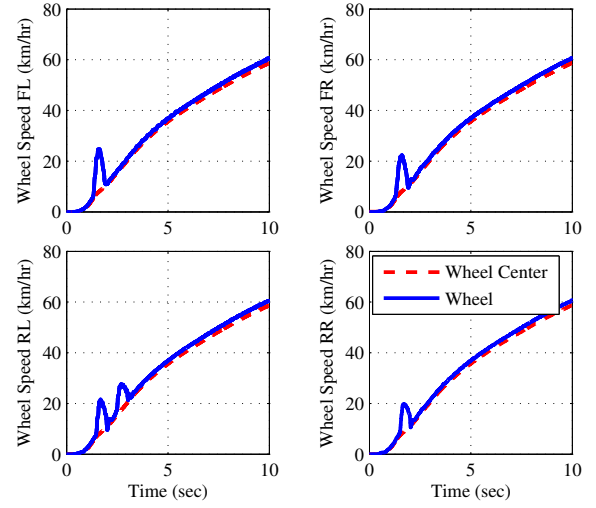


Figure 5.18: Wheel speeds and wheel center speeds in the launch test on snow with vehicle A.

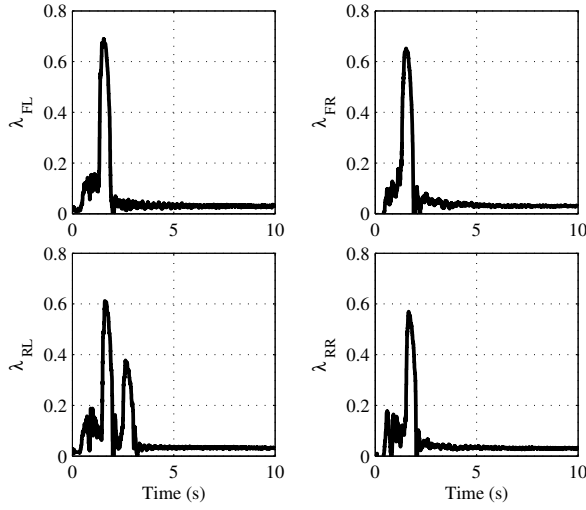


Figure 5.19: Slip ratios in the launch test on snow with vehicle A.

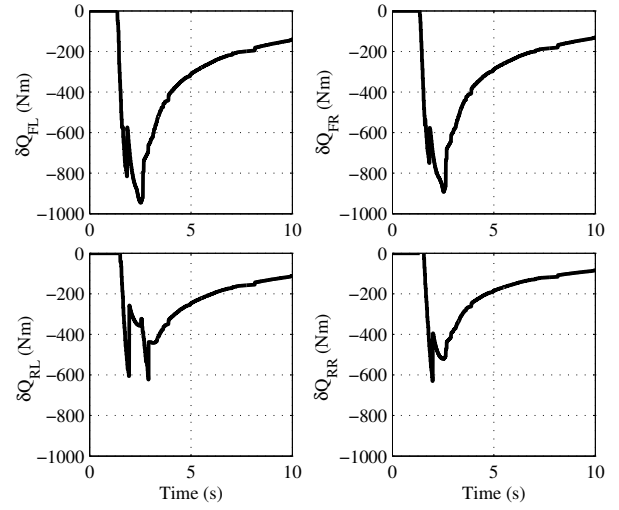


Figure 5.20: Torque adjustments of the controller in the launch test on snow with vehicle A.

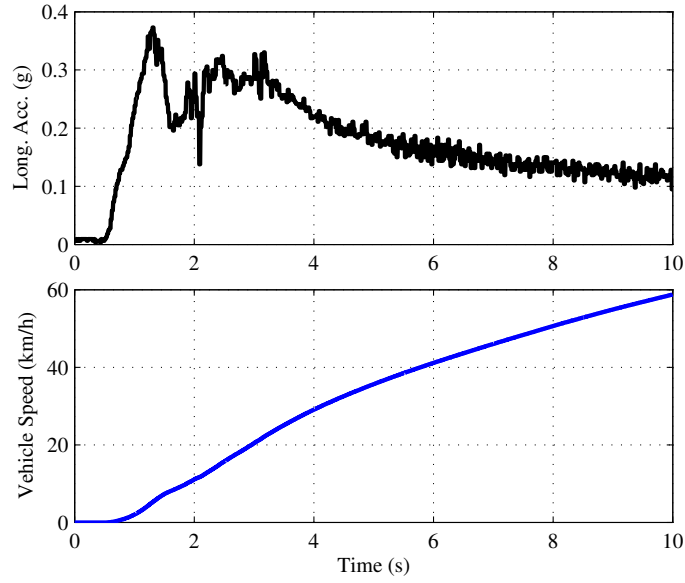


Figure 5.21: Vehicle longitudinal acceleration and velocity in the launch test on snow with vehicle A.

5.5 Acceleration in Turn on Wet Sealer, RWD mode

In this experiment, vehicle A is configured to run in rear-wheel drive (RWD) mode, i.e. the front motors are disconnected from power and are not used for driving or torque vectoring. The initial vehicle speed is 40 km/hr and at some instant of time, a step steering of 200 degrees is applied followed immediately by fully pressing the accelerator pedal. This maneuver is performed with and without the controller and the results are compared. Figure 5.22a and 5.22b compare the applied steering wheel angle and the drive torque applied to the rear wheels.

Figure 5.23a shows the yaw rate and the vehicle sideslip angle in the uncontrolled maneuver. It can be seen that the vehicle severely overshoots the reference yaw rate. In addition, the vehicle sideslip angle has increased to 80 degrees, which shows a massive oversteer. Looking at Figure 5.24a, it is noted that the rear wheel speeds have significantly exceeded their reference values. This results in significant drop in the lateral force capacity

of the tires. Therefore, vehicle oversteer is inevitable.

Figure 5.23b shows the vehicle yaw rate and sideslip angle when the controller is active. It can be seen that the controller maintains the vehicle stability and tracks the desired yaw rate relatively well. The vehicle sideslip angle is quite small and the vehicle maintains the steerability after the steering angle is returned to zero degrees. The wheel speeds are shown in Figure 5.24b. It is observed that the wheel speeds closely track the desired values and do not show any significant deviation. The torque adjustments made by the controller are shown in Figure 5.25. The controller is trying to reduce the applied torque to the rear wheels and prevent them from overspinning.

5.6 Acceleration in Turn on Wet Sealer, FWD mode

In this experiment, vehicle A is configured to operate in the front-wheel drive mode. The rear wheels are disconnected from power and are not used for drive or torque vectoring. The vehicle is given an initial velocity of 25 km/hr. A step steering input of about 200 degrees is applied and at the same time the accelerator pedal is fully pressed. The maneuver is performed with and without the controller for the purpose of comparison. The steering wheel angle and the vehicle lateral acceleration are shown in Figure 5.26. The lateral vehicle acceleration is about 0.4g, which is consistent with the friction coefficient of the wet sealer. Comparing Figures 5.26a and 5.26b, it can be seen that when the controller is active, the vehicle assumes a larger lateral acceleration.

Figure 5.27 compares the yaw rate tracking and the sideslip angle of the vehicle in the controlled and uncontrolled maneuver. It is observed that the vehicle sideslip angle remains small ($|\beta| < 4^\circ$) with or without the controller. However, when the controller is active, the vehicle assumes a larger yaw rate and can more closely track the desired yaw rate. Therefore, the controller can severely reduce the understeer behaviour of the car in the acceleration in turn maneuver in the front-wheel drive mode.

Figure 5.28 shows the wheel speeds in this maneuver. When the controller is inactive, the front wheels overshoot the reference speeds and the front right wheel reaches speed of

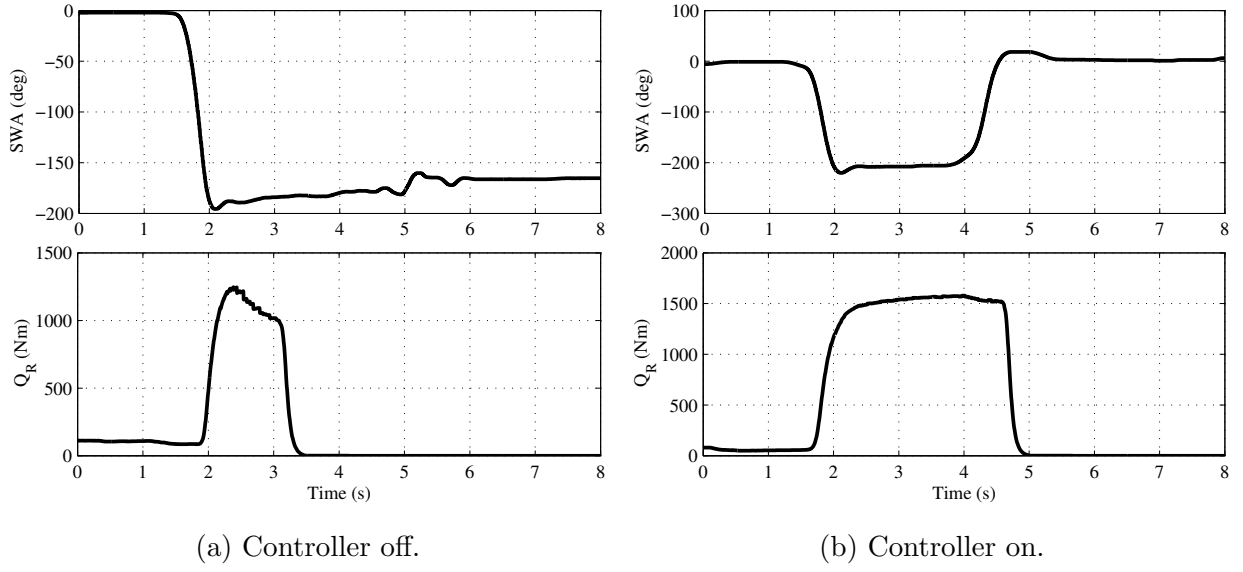


Figure 5.22: The steering wheel angle and the drive torque on rear wheels during the RWD AIT maneuver on wet sealer with vehicle A.

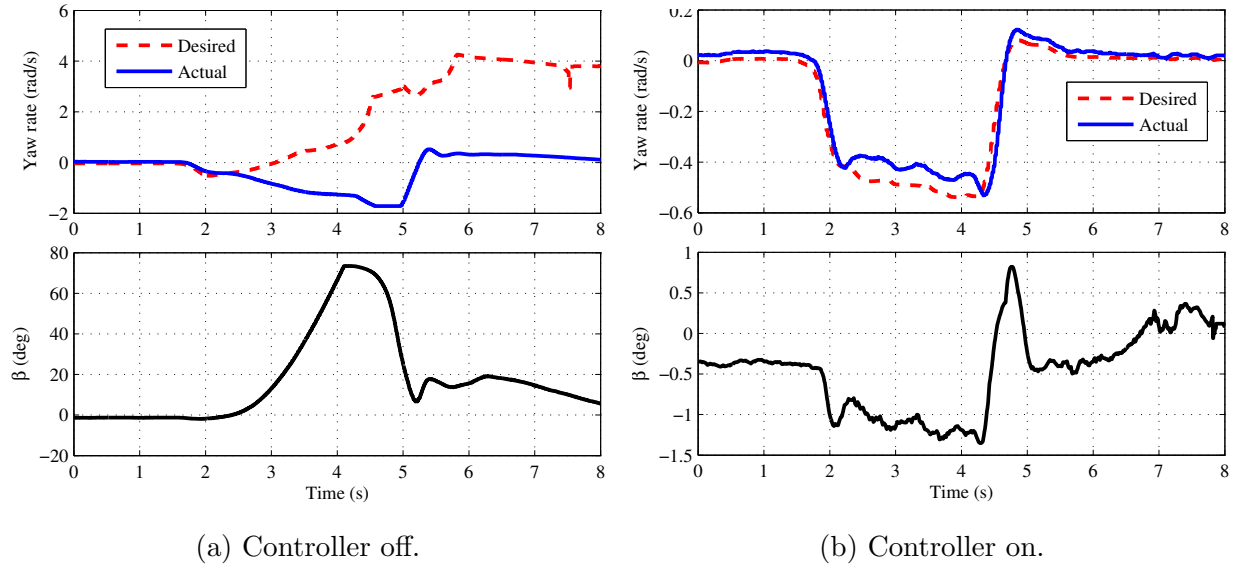


Figure 5.23: Yaw rate tracking and vehicle sideslip angle in the RWD AIT maneuver on wet sealer with vehicle A.

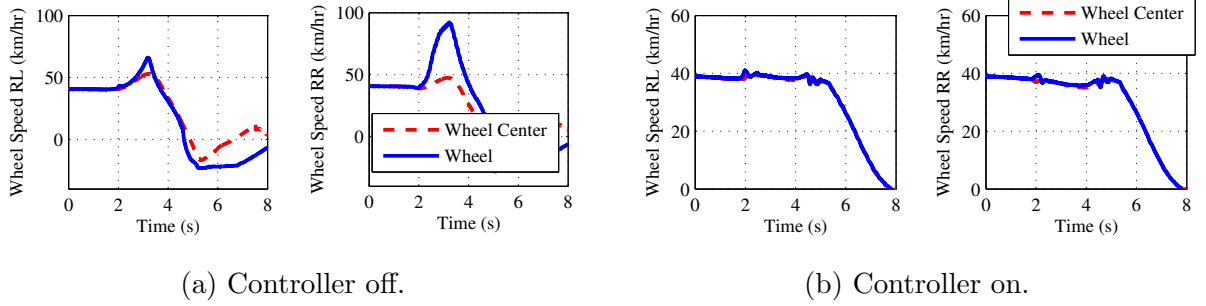


Figure 5.24: Wheel speeds in the RWD AIT maneuver on wet sealer with vehicle A.

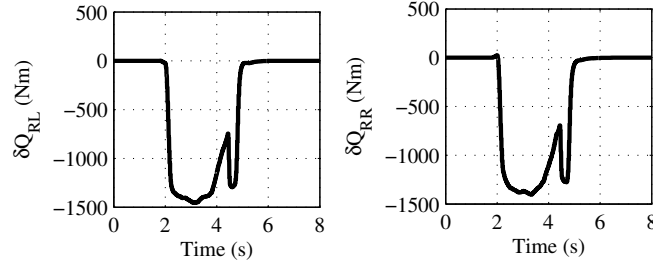
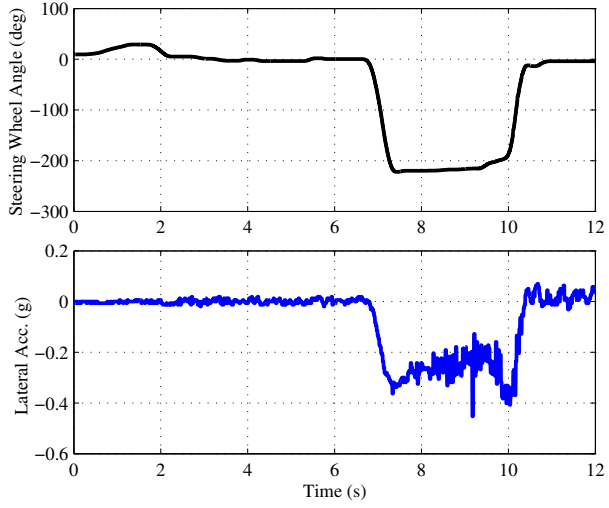
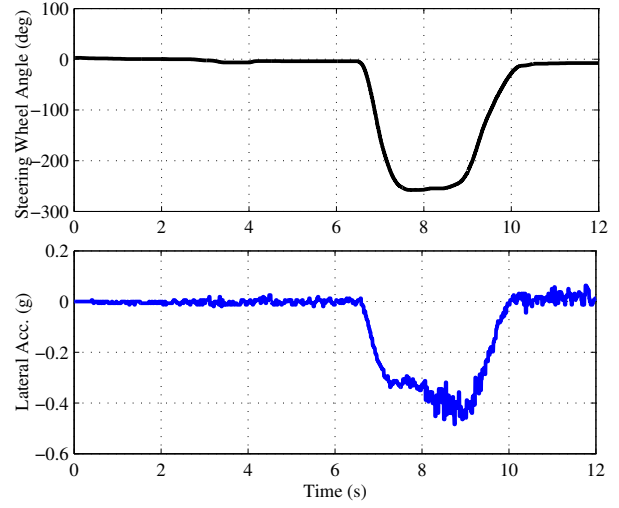


Figure 5.25: Torque adjustments made by controller in the RWD AIT maneuver on wet sealer with vehicle A.

about 120 km/hr. Since the front wheels have exceeded the permissible slip ratios, their lateral forces drop significantly and result in reduced vehicle steerability. This reflects as reduced vehicle yaw rate and lateral acceleration as discussed above. On the other hand, when the controller is active, wheel speeds are controlled and track the desired speeds quite well. Therefore, the vehicle shows improved steerability (Figure 5.27b) and has a higher lateral acceleration (Figure 5.26b). The torque adjustments of the controller are shown in Figure 5.29. The controller generates negative torques on the front wheels to maintain the wheel speeds in the permissible range and tracks the desired yaw rate.

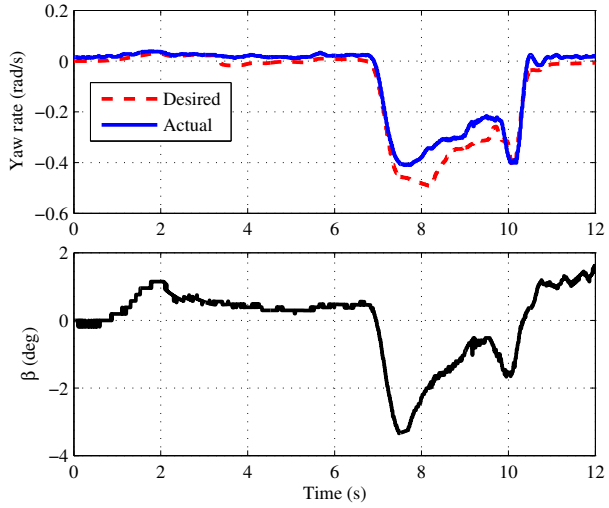


(a) Controller off.

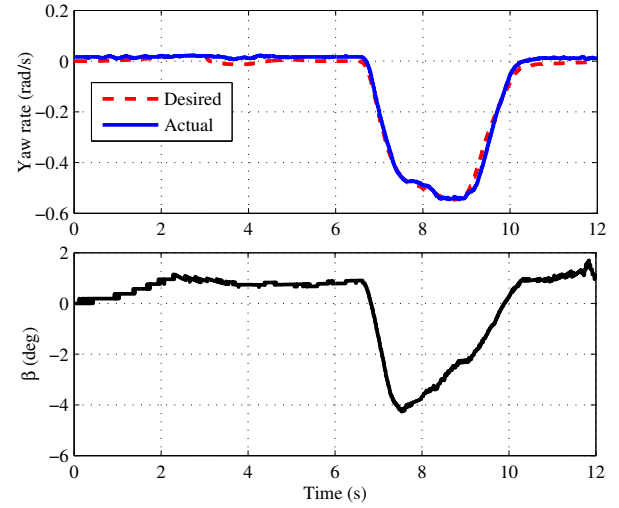


(b) Controller on.

Figure 5.26: Steering wheel angle and the lateral acceleration in FWD AIT maneuver on wet sealer with vehicle A.



(a) Controller off.



(b) Controller on.

Figure 5.27: Yaw rate tracking and sideslip angle in FWD AIT maneuver on wet sealer with vehicle A.

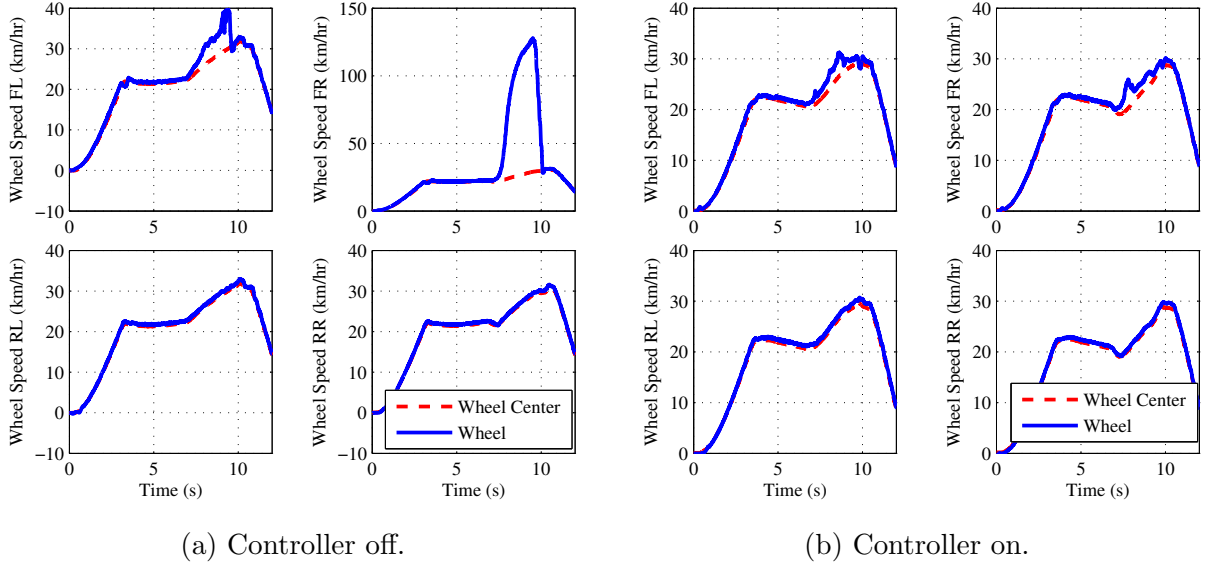


Figure 5.28: Wheel speeds in FWD AIT maneuver on wet sealer with vehicle A.

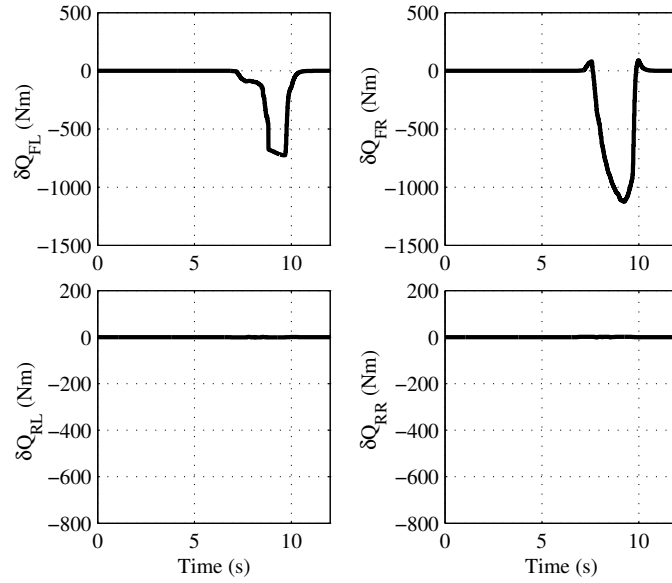


Figure 5.29: Torque adjustments made by controller in the FWD AIT maneuver on wet sealer with vehicle A.

5.7 Verification of Delay Handling Technique

Vehicle B is used for experimental verification of the delay handling technique developed in Section 3.4. Similar to the computer simulations, an artificial pure delay of $\tau_d = 200$ ms is added at the output of the controller. First order delay is not considered in this experiment, therefore $N_d = 10$ and $\tau_G = 0$ is assumed. The driving scenario consists of a slalom maneuver on dry pavement with an entry speed of 55 km/hr. During the maneuver, no acceleration or braking is applied, thus the vehicle is coasting. The maneuver is performed twice, once with the MPC controller without the delay handling technique (Controller A) and once with the same controller, but equipped with the delay handling technique as discussed in Section 3.4 (Controller B).

Figure 5.30 shows the steering input and the resulting lateral acceleration of the vehicle. It can be observed that the steering is very similar for driving with controllers A and B. The last steering action with controller B at about $t=8$ sec is at low speed and insignificant in this discussion. The lateral acceleration of the vehicle in both runs peaks at about $g=9.8$ m/s², which is the limit of the road condition, therefore the maneuver is an example of a critical driving situation.

Figure 5.31 shows the yaw rate tracking performance of controllers A and B. The vehicle response at around $t = 4$ sec is of particular interest, because it is right after the counter steering and the vehicle has not had enough time to settle yet. It is observed that when using controller A, the vehicle yaw rate shows more overshoot in comparison with controller B, particularly around time $t = 4$ sec. The consequence of this is more evident in Figure 5.32 where the vehicle sideslip angle is compared for controllers A and B. With controller A, the maximum sideslip angle of the vehicle is about 9 degrees whereas when controller B is used, the maximum sideslip angle is only 5 degrees, about half of that of controller A. This is the result of having a control action that is more in phase with the vehicle state. This is better observed in Figure 5.33 where the control actions of the two controllers are shown. In the highlighted 200 ms window, when the vehicle yaw rate is overshooting the reference values, controller A is just about to produce torque vectoring to correct the overshoot while the controller B has a significant torque vectoring already developed. This

confirms that controller B is generating control actions that are more in phase with the vehicle state, whereas controller A that is not equipped with the proposed delay handling method is late in producing the corrective control actions.

5.8 Evaluation of Optimal Power Distribution

In this section, the optimal power distribution objective of the integrated controller is assessed. Vehicle A that is an all-wheel drive vehicle with four identical electric motors, one on each corner is used in this experiment. This assessment is divided into two parts: In the first part, the energy consumption of the vehicle with and without optimal power distribution control is evaluated. In the second part, the cooperation of the optimal power distribution and the stability control tasks of the integrated MPC controller is studied.

5.8.1 Effect on Energy Consumption

In this section, the long-term effect on energy consumption of electric vehicles is studied. The efficiency maps of the electric motors as provided by the manufacturer is used in determining the optimal operating point of the vehicle as discussed in Section 3.10.1. In order to calculate the electrical energy consumed by the electric motors, Equation (4.1) is used. The mechanical energy developed by the motors is also calculated using Equation (4.2).

For this experiment, the vehicle is driven in laps around the test track facility. Full range of vehicle acceleration is attempted in each lap with as much consistency across the laps as humanly possible. The Google Earth photograph of the test track and some details of this experiment are shown in Figure 5.34. The length of each lap is 1.13 km as measured using Google Maps. The vehicle is driven 6 laps around the facility, 3 laps with power distribution and 3 laps without it. The results are presented in Table 5.2.

According to the results, in the first three laps the optimal power distribution unit is active and around 5.5 MJ mechanical energy is used in the 3.4 km length of travel. The

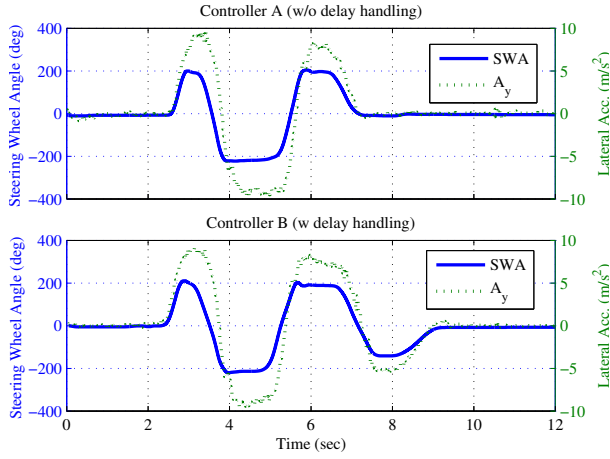


Figure 5.30: Steering wheel angle and vehicle lateral acceleration with controller A & B.

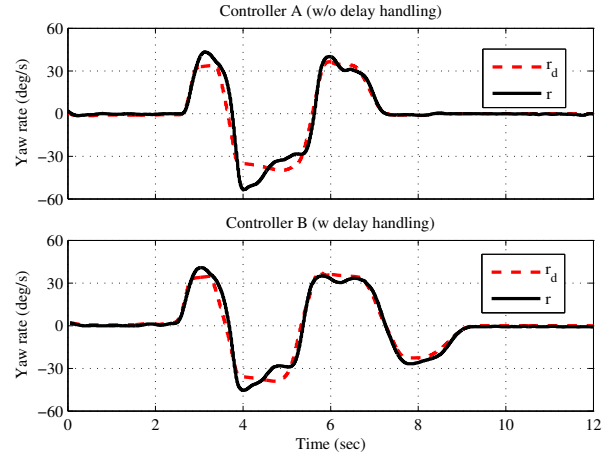


Figure 5.31: Yaw rate tracking performance with controllers A & B.

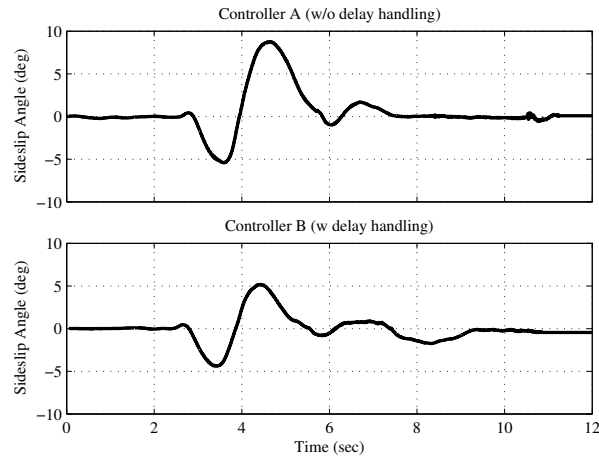


Figure 5.32: Vehicle sideslip angle with controller A & B.

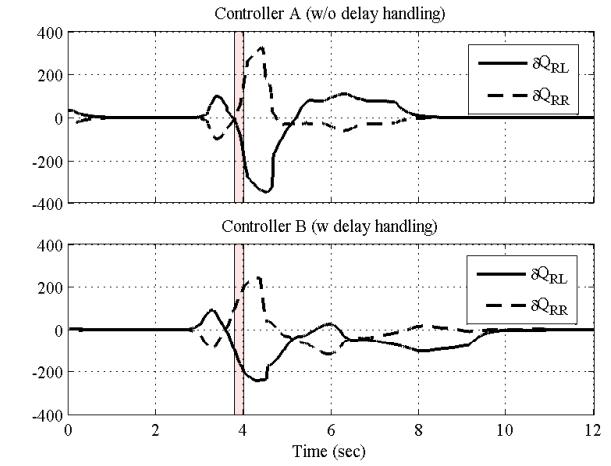


Figure 5.33: Torque adjustments made by controllers A & B.

measured electrical energy is 5.4 MJ, which is slightly less than the mechanical energy. This is obviously a measurement error and can be traced back to the measurement and filtering error in voltage, current, motor speed and specially the produced torque that are all reported by the electric motor drivers. The overall efficiency of the vehicle in this case is 1.021. In the next three laps, the optimal power distribution unit is inactive and the maneuver is repeated. The mechanical energy consumed is 5.2 MJ. The difference in the consumed mechanical energy compared to the first three laps is about 8% and is the result of unavoidable differences in repeating the laps with a human driver. The electrical energy used in this maneuver is 5.1 MJ. Similar to the first three laps, the measured mechanical energy exceeds the spent electrical energy due to the same measurement errors. The overall vehicle efficiency amounts to 1.015. In comparison, this is 1.5% less than the vehicle efficiency with the optimal power distribution.

Overall, this experiment is deemed inconclusive in demonstration of vehicle efficiency improvement. Since identical electric motors are installed on all four corners, the expected efficiency improvement is around 1 to 2%. This requires an accurate measure of motor voltage, current, speed and torque for experimental verification.

Table 5.2: Energy consumption in laps around the test track facility.

Lap numbers	Torque redistribution	E_{elec} (MJ)	E_{mech} (MJ)	η
1-3	on	5.426	5.539	1.021
4-6	off	5.080	5.157	1.015

5.8.2 Cooperation with Stability Control

In this section, cooperation of the optimal power distribution unit with the stability control objectives of the controller is studied. To this aim, an uphill double lane change maneuver on dry pavement is selected with partial throttle. In order to better see the role of the optimal power distribution unit, an alternative efficiency map (Figure 4.25) is used and the efficiency of the front motors is artificially reduced by 2%. Consequently, the controller

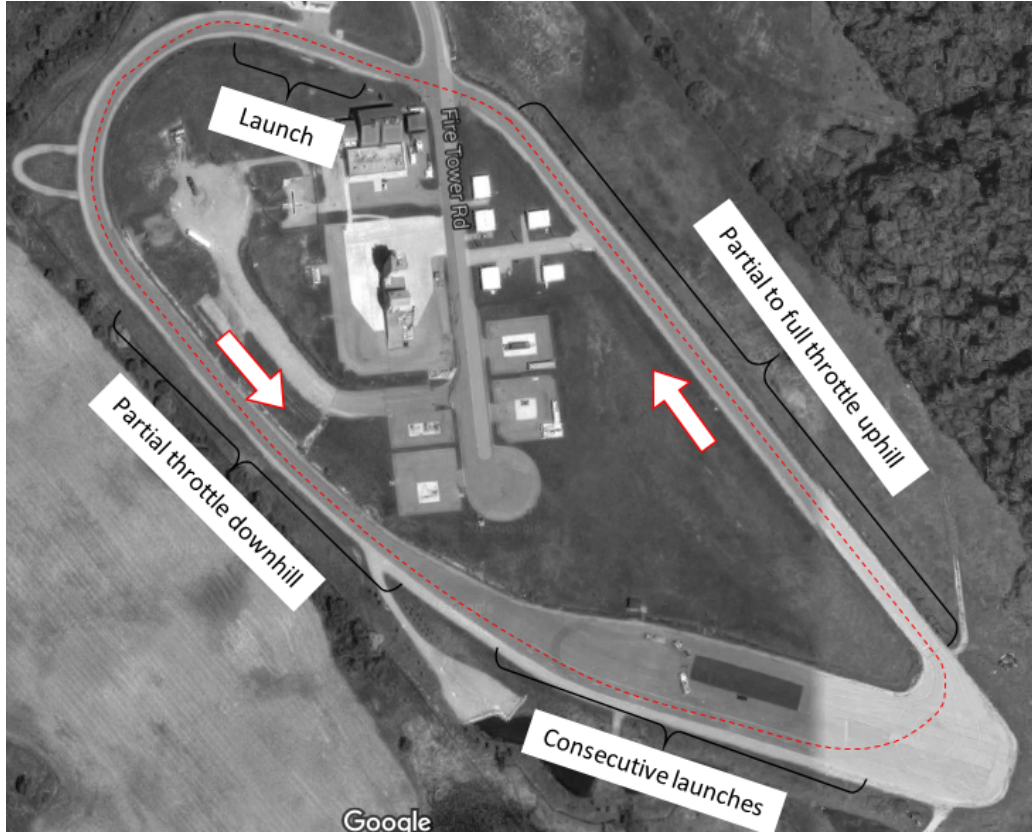


Figure 5.34: Specifics of a lap around the test track facility.

favors using only rear motors when the torque request can be met (i.e. partial throttle).

The steering input and the resulting lateral acceleration are shown in Figure 5.35. The two consecutive lane changes are visible in the steering input and the lateral acceleration peaks at about 0.6 g which shows sufficient lateral excitation. Figure 5.36 shows the driver's torque request. The vehicle starts from rest and accelerates to speed of 40 km/hr. Then a partial throttle is applied throughout the double lane change maneuver. Figure 5.37 shows the yaw rate tracking and the vehicle sideslip angle. It can be seen that the vehicle tracks the desired yaw rate pretty well and the sideslip angle is pretty small ($|\beta| < 7^\circ$).

The torque adjustments of the controller are shown in Figure 5.38. When $t < 2$ sec, and as the torque request is increasing (Figure 5.36), the controller starts to transfer the

drive torque towards the rear motors. But as the request passes the torque capacity of the rear tires, the controller uses both axles to match the total torque request of the driver. During the partial throttle section of the maneuver ($5 < t < 10$ sec), the controller mostly transfers the torque towards the rear motors to increase the overall vehicle efficiency and then torque vectors around that to achieve the yaw rate tracking and stability objectives.

5.9 Summary

In this chapter, the real-time performance of the model predictive controller developed in Chapter 3 was studied. Several critical maneuvers such as launch, acceleration in turn and slalom were performed to examine the stability and wheel slip control performance of the controller. In most cases, the closed-loop vehicle performance was compared with the uncontrolled vehicle response. The results show excellent stability control and quick traction control in all of the attempted maneuvers.

Two different test vehicles were used in the experiments to demonstrate reconfigurability of the MPC controller and ability to work with various driveline and actuation systems. Moving from the all-wheel drive test vehicle A that uses torque vectoring to the rear-wheel drive test vehicle B that is equipped with differential brakes is very quick and requires minimum change in the controller parameters.

The effectiveness of the proposed delay handling technique was experimentally investigated in presence of a known delay in the control loop. It was observed that it can improve the controller performance by providing control actions that are more in phase with the vehicle response.

The effect of the optimal power distribution objective of the controller was also studied. It was observed that the optimal torque distribution can improve the overall vehicle efficiency by 1.5%. Moreover, it was noted that the optimal torque redistribution and the stability control objectives of the integrated controller work well with each other without fighting over priority.

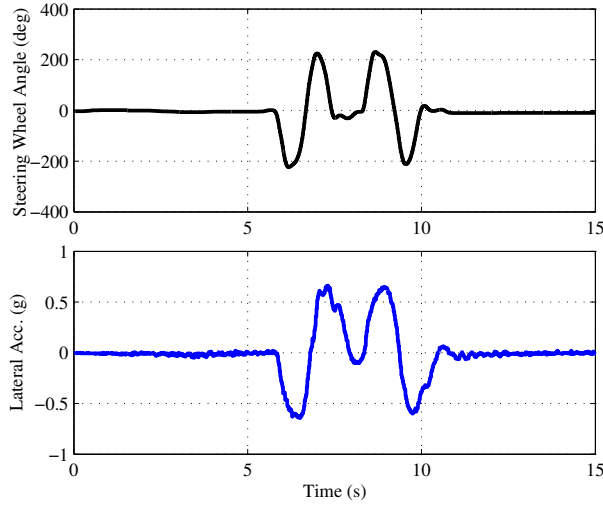


Figure 5.35: Steering wheel angle and lateral acceleration in uphill double lane change using partial throttle with vehicle A.

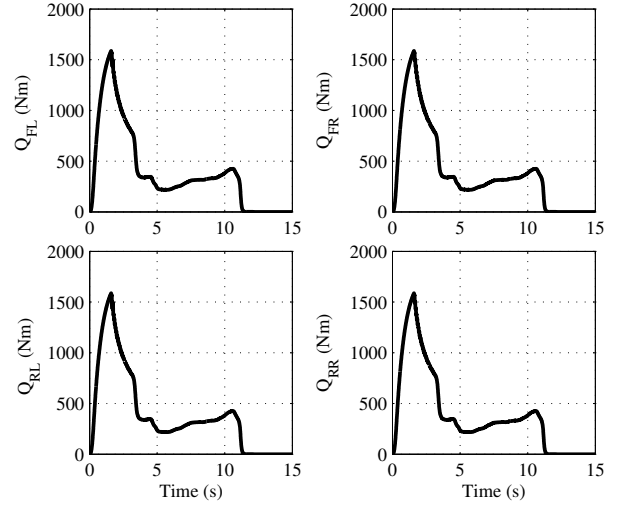


Figure 5.36: Drive torque request in uphill double lane change using partial throttle with vehicle A.

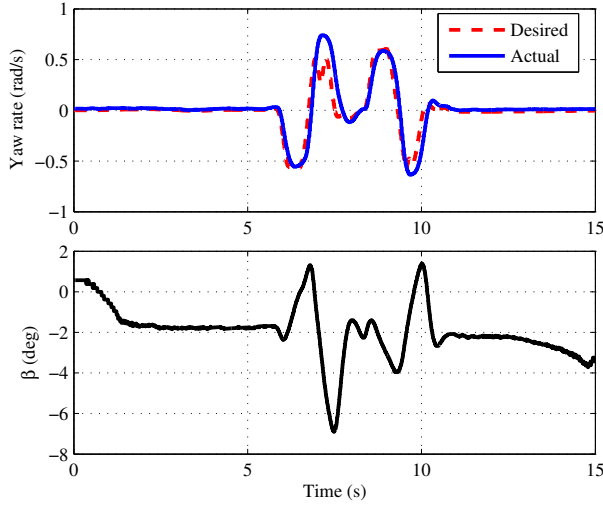


Figure 5.37: Yaw rate tracking and sideslip angle in uphill double lane change using partial throttle with vehicle A.

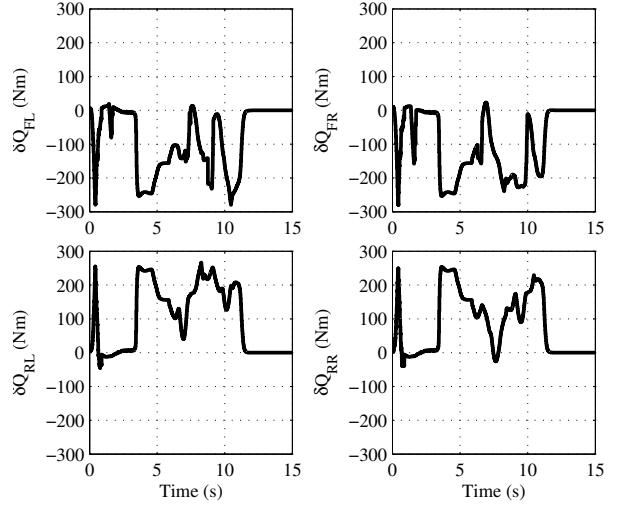


Figure 5.38: Torque adjustments in uphill double lane change using partial throttle with vehicle A.

Chapter 6

Conclusions and Future Work

6.1 Conclusions and Summary

In this thesis, an integrated stability control and power distribution system was designed using the model predictive control technique. The integrated controller maintains vehicle and wheel stability and maximizes the overall energy efficiency of the vehicle by redistribution of torque between the front and rear axles. Furthermore, it can be easily reconfigured to work with various driveline and actuator configurations without re-tuning the controller parameters. The model predictive control approach allows the system constraints to be explicitly considered at the controller design stage and results in control actions that are highly optimal. However, this increases the computational cost of the controller compared to conventionally used PID or LQR controllers. The closed-loop vehicle performance was evaluated using software simulations as well as experimental tests. The major findings and contributions of this thesis are listed below.

Vehicle stability and wheel slip controllers were integrated in this thesis. This results in a highly optimal control actions. The customary approach is that wheel slip control is a separate unit and adjusts the drive/brake torque as it goes to the tires. The separation of these two modules ignores the cross effects between them and allows occasional conflicts

of interest. Integration minimizes conflicting objectives between separate controllers and prevents possible fights over priority.

Stability control and optimal power distribution units were also integrated. It is common to separate the power distribution unit from the stability controller to obtain simpler controllers. However, the power distribution unit transfers significant amounts of torque between the front and rear axles, especially if the efficiency maps of the front and rear motors are significantly different. This can severely impact the vehicle stability in slippery road conditions and during cornering. Separate power distribution and stability control units ignore the cross effects and compromise vehicle stability. However, in an integrated approach, the vehicle stability is the dominant objective of the controller and torque redistribution is only performed when vehicle and wheel stability is in check.

A delay handling method was proposed to consider pure and first order delays in model predictive stability control of electric vehicles. The proposed method can be easily implemented and does not require a complicated tuning process. In addition, since it does not increase the size of the system model, the amount of online computation is not appreciably affected and hence, can be used in real-time implementations. The proposed method is not specific to applications in vehicle control and can be used in a wide variety of problems. To demonstrate the performance of the proposed technique, computer simulations and experimental tests were presented. The results confirm that the proposed delay handling method can improve the controller performance in the presence of delays.

The proposed integrated controller can be easily reconfigured to work with various driveline and actuator configurations. Moving from one driveline configuration to another does not incur controller redesign. The controller can be easily configured to work with front-wheel drive, rear-wheel drive or all-wheel drive vehicles. In addition, various actuation methods can be used: torque vectoring, differential braking as well as a hybrid combination of these two are supported. Changing the actuation method does not require re-tuning of the controller as the controller tracking and regulation performance does not change with the change in the actuation method, as long as actuator dynamics are the same.

The performance of the proposed controller was evaluated using computer simulations

in MATLAB/Simulink and CarSim. A high-fidelity model of the available electric SUVs in CarSim was used to simulate the vehicle response to the given inputs. The controller was implemented in the Simulink environment. The performance of the controller was evaluated in several driving scenarios such as straight-line launch, split- μ launch, acceleration in turn, etc. Various road conditions such as wet, snowy and dry were used to examine the robustness of the controller with respect to the road condition. The vehicle response was compared with the uncontrolled vehicle response. The simulations showed excellent performance of the controller. The controller was able to maintain vehicle stability and control tire slip ratios in all of the driving scenarios.

Furthermore, the energy savings potential of the controller through torque redistribution between the front and rear axles in an FTP-75 driving cycle was studied and it was observed that torque redistribution could reduce the overall energy consumption by 2.3%. In addition, it was observed that the optimal power distribution and stability control objectives of the controller cooperate in maneuvers where both objectives were non-trivial.

The proposed control scheme was implemented in real-time on dSpace Autobox and tested on two electric Equinox vehicles with different driveline configurations and actuation mechanisms. The real-time performance of the controller was evaluated in a number of driving conditions and on a variety of available road conditions. The controller was observed to improve the vehicle response and stability in all of the driving conditions. The energy savings of the controller was also tested. In the test scenario, the controller was observed to reduce the energy consumption of the vehicle by 1.5% just by redistributing the drive torques between the front and rear axles. In addition, similar to the simulations, the optimal power distribution objective of the controller was observed to concede to the stability control objective whenever stability became a concern.

6.2 Future Work

To continue the work done in this thesis and improve the performance of the controller, a few suggestions are made in this section. Since the closed-loop performance of model

predictive controllers to a great extent depend on accuracy of the prediction model, these suggestions are mostly centered on improving the accuracy of the prediction model.

- Considering longitudinal vehicle dynamics: Adding the vehicle longitudinal speed to the prediction model can improve the accuracy of the prediction of the vehicle directional dynamics. When differential brakes are used for vehicle control, the vehicle speed reduces during controller intervention. This allows for more vehicle yaw rate on the same road condition, not foreseen by a prediction model that neglects longitudinal deceleration. Therefore, considering vehicle speed in the prediction model can improve the accuracy of the prediction model.
- Considering roll dynamics: As literature of vehicle dynamics suggest, adding the roll dynamics to the prediction model can improve the accuracy of the prediction model in high-speed maneuvers on surfaces with high friction coefficient. In these driving situations, the vehicle roll rate and roll angle is considerable, resulting in significant lateral load transfer. Therefore, by including the vehicle roll dynamics, the controller can foresee the lateral load transfer and can better predict tire capacity.
- Using combined slip tire models: The tire model used in this thesis are pure slip, where the tire longitudinal force depends only on the tire slip ratios and the lateral force depends only on the tire slip angles. However, variation on the tire lateral force with the tire longitudinal slip is quite significant and can greatly affect vehicle response. For example, if a combined slip tire model is used, the controller can foresee impending vehicle oversteer when rear tires are about to overspin (or lock). Similarly, it can increase (or decrease) the slip ratio of front tires to reduce their lateral force and prevent a major oversteer before it happens. Therefore, considering a combined slip tire model presents the controller with a new set of optimal solutions not previously available and can improve the prediction ability of the controller in high-slip situations.
- Designing an explicit MPC controller: the linear prediction model can be converted to an explicit MPC controller using multi-parametric programming. This requires

the size of the prediction window and the tuning parameters to be finalized. The explicit controller has much less computational time; therefore, the controller can be implemented on production vehicles with less expensive computational hardware.

- Using nonlinear prediction model: In this thesis, the tire model is linearized about the operating point at each sample time, in order to arrive at a linear prediction model. Using a nonlinear tire model greatly increases the accuracy of prediction model as well as its accuracy range and enables the controller to find a globally optimal solution. However, an accurate tire model requires an estimate of road condition, which limits the use of this approach.
- Extension to hybrid vehicles: In this thesis, optimal power distribution was studied for fully electric vehicles. This method can be extended to various configurations of hybrid vehicles. Internal combustion engines and electric motors have a significantly different efficiency maps. Therefore, proper distribution of the torque demand between these sources is essential. In this case, the state of charge of the battery should be included in the controller design.
- Considering the effect of road angles: In this thesis, the focus was on flat roads where no significant road grade or bank angles are present. However, if the road angles are not negligible, the acceleration measurements of the IMU sensor are affected. This must be corrected either by an estimation scheme that provides estimates of the road angles, or by making the controller robust with respect to the uncertainty in the acceleration measurements.

References

- [1] Transport Canada. Canadian motor vehicle traffic collision statistics. https://www.tc.gc.ca/media/documents/roadsafety/cmvtcs2013_eng.pdf, 2013. Accessed: 2016-06-27.
- [2] Alena Høyen. The effects of electronic stability control (ESC) on crashes-an update. *Accident Analysis & Prevention*, 43(3):1148–1159, 2011.
- [3] Allan Lyckegaard, Tove Hels, and Inger Marie Bernhoft. Effectiveness of electronic stability control on single-vehicle accidents. *Traffic injury prevention*, 16(4):380–386, 2015.
- [4] M Starnes. Estimating lives saved by electronic stability control, 2008–2012. <http://www-nrd.nhtsa.dot.gov/Pubs/812042.pdf>, 2014. Accessed: 2016-06-27.
- [5] Shoji Inagaki, Ikuo Kshiro, and Masaki Yamamoto. Analysis on vehicle stability in critical cornering using phase-plane method. In *Proceedings of the International Symposium on Advanced Vehicle Control*, pages 287–292, 1994.
- [6] Zhang Hao, Li Xian-sheng, Shi Shu-ming, Liu Hong-fei, Guan Rachel, and Liu Li. Phase plane analysis for vehicle handling and stability. *International Journal of Computational Intelligence Systems*, 4(6):1179–1186, 2011.
- [7] E. Ono, S. Hosoe, H. D. Tuan, and S. Doi. Bifurcation in vehicle dynamics and robust front wheel steering control. *Control Systems Technology, IEEE Transactions on*, 6(3):412–420, 1998.

- [8] Y Shibahata, K Shimada, and T Tomari. Improvement of vehicle maneuverability by direct yaw moment control. *Vehicle System Dynamics*, 22(5-6):465–481, 1993.
- [9] Shuiwen Shen, Jun Wang, Peng Shi, and Giuliano Premier. Nonlinear dynamics and stability analysis of vehicle plane motions. *Vehicle System Dynamics*, 45(1):15–35, 2007.
- [10] Young Eun Ko and Jang Moo Lee. Estimation of the stability region of a vehicle in plane motion using a topological approach. *International Journal of Vehicle Design*, 30(3):181–192, 2002.
- [11] John Samsundar and Jeffrey C. Huston. Estimating lateral stability region of a non-linear 2 degree-of-freedom vehicle. *SAE Technical Paper*, 1998.
- [12] A. Stotsky and X. Hu. Stability analysis of robustly decoupled car steering system with nonlinear tire model. In *Decision and Control., Proceedings of the 36th IEEE Conference on*, volume 5, pages 4750–4755 vol.5, 1997.
- [13] Klaus H. Well. Aircraft control laws for envelope protection. In *AIAA Guidance, Navigation and Control Conference*, pages 1–10, 2006.
- [14] Henk Smakman. Functional integration of active suspension with slip control for improved lateral vehicle dynamics. In *Proceedings of AVEC00*, Ann Arbor, Michigan, USA, 2000.
- [15] Henk Smakman. *Functional integration of slip control with active suspension for improved lateral vehicle dynamics*. PhD thesis, Technische Universiteit Delft, 2000.
- [16] Junjie He, David A. Crolla, Martin C. Levesley, and Warren J. Manning. Integrated active steering and variable torque distribution control for improving vehicle handling and stability. *SAE Technical Paper*, 113(6):638–647, 2004.
- [17] Taeyoung Chung and Kyongsu Yi. Design and evaluation of side slip angle-based vehicle stability control scheme on a virtual test track. *Control Systems Technology, IEEE Transactions on*, 14(2):224–234, 2006.

- [18] Carrie Gwendolene Bobier. *A Phase Portrait Approach to Vehicle Stabilization and Envelope Control*. PhD thesis, Stanford University, 2012.
- [19] Craig Earl Beal. *Applications of model predictive control to vehicle dynamics for active safety and stability*. PhD thesis, Stanford University, 2011.
- [20] Yoshiyuki Yasui, Kenji Tozu, Noriaki Hattori, and Masakazu Sugisawa. Improvement of vehicle directional stability for transient steering maneuvers using active brake control. *SAE Technical Paper*, 1996.
- [21] WJ Manning and DA Crolla. A review of yaw rate and sideslip controllers for passenger vehicles. *Transactions of the Institute of Measurement and Control*, 29(2):117–135, 2007.
- [22] Matthijs Klomp. *Longitudinal force distribution and road vehicle handling*. PhD thesis, Chalmers University of Technology, 2010.
- [23] Masato Abe, Yoshio Kano, Kazuasa Suzuki, Yasuji Shibahata, and Yoshimi Furukawa. Side-slip control to stabilize vehicle lateral motion by direct yaw moment. *JSAE Review*, 22(4):413–419, 10 2001.
- [24] Wanki Cho, Jangyeol Yoon, Jeongtae Kim, Jaewoong Hur, and Kyongsu Yi. An investigation into unified chassis control scheme for optimised vehicle stability and manoeuvrability. *Vehicle System Dynamics*, 46:87–105, 2008.
- [25] Siqi Zhang, Shuwen Zhou, and Jun Sun. Vehicle dynamics control based on sliding mode control technology. In *Control and Decision Conference*, pages 2435–2439. IEEE, 2009.
- [26] Moustapha Doumiati, Olivier Sename, John Jairo Martinez Molina, Luc Dugard, Peter Gaspar, Zoltan Szabo, and Jozsef Bokor. Vehicle yaw control via coordinated use of steering/braking systems. In *18th IFAC World Congress*, pages 644–649, 2011.
- [27] Hui Lu, Jing Li, Bing Zhu, Yuelei Gao, and Bohua Sun. Traction and yaw-rate control of 4WD vehicle with drive force distribution. In *Electronic and Mechanical Engineering*

- and Information Technology (EMEIT), *International Conference on*, volume 2, pages 1062–1065, 2011.
- [28] Paolo Falcone, H. Eric Tseng, Francesco Borrelli, Jahan Asgari, and Davor Hrovat. MPC-based yaw and lateral stabilisation via active front steering and braking. *Vehicle System Dynamics*, 46(S1):611–628, 2008.
 - [29] Giovanni Palmieri, Paolo Falcone, H. Eric Tseng, and Luigi Glielmo. A preliminary study on the effects of roll dynamics in predictive vehicle stability control. In *Decision and Control. 47th IEEE Conference on*, pages 5354–5359. IEEE, 2008.
 - [30] Francesco Borrelli, Paolo Falcone, Tamas Keviczky, and Jahan Asgari. MPC-based approach to active steering for autonomous vehicle systems. *International journal of vehicle autonomous systems*, 3(2):265–291, 2005.
 - [31] Philip E Gill, Walter Murray, Michael A Saunders, and Margaret H Wright. User’s guide for NPSOL (version 4.0): A fortran package for nonlinear programming. Technical report, DTIC Document, 1986.
 - [32] Bingtao Ren, Hong Chen, Haiyan Zhao, and Lei Yuan. MPC-based yaw stability control in in-wheel-motored EV via active front steering and motor torque distribution. *Mechatronics*, 2015.
 - [33] Paolo Falcone, Francesco Borrelli, H Eric Tseng, Jahan Asgari, and Davor Hrovat. Linear time-varying model predictive control and its application to active steering systems: Stability analysis and experimental validation. *International journal of robust and nonlinear control*, 18(8):862–875, 2008.
 - [34] Philip E. Gill, Walter Murray, and Michael A. Saunders. SNOPT: An SQP algorithm for large-scale constrained optimization. *SIAM journal on optimization*, 12(4):979–1006, 2002.
 - [35] Giovanni Palmieri, Osvaldo Barbarisi, Stefano Scala, and Luigi Glielmo. A preliminary study to integrate LTV-MPC lateral vehicle dynamics control with a slip control. In

- Decision and Control. Proceedings of the 48th IEEE Conference on*, pages 4625–4630. IEEE, 2009.
- [36] P. Falcone, M. Tufo, F. Borrelli, J. Asgari, and H. E. Tsengz. A linear time varying model predictive control approach to the integrated vehicle dynamics control problem in autonomous systems. In *Decision and Control, 46th IEEE Conference on*, pages 2980–2985, 2007.
 - [37] Milad Jalali, Amir Khajepour, Shih-Ken Chen, and Bakhtiar Litkouhi. Integrated stability and traction control for electric vehicles using model predictive control. *Control Engineering Practice*, pages 256–266, 2016.
 - [38] Valerio Turri, Ashwin Carvalho, Hongtei Tseng, Karl Henrik Johansson, and Francesco Borrelli. Linear model predictive control for lane keeping and obstacle avoidance on low curvature roads. In *Intelligent Transportation Systems-(ITSC), 16th International IEEE Conference on*, pages 378–383, 2013.
 - [39] Osvaldo Barbarisi, Giovanni Palmieri, Stefano Scala, and Luigi Glielmo. LTV-MPC for yaw rate control and side slip control with dynamically constrained differential braking. *European Journal of Control*, 15(3):468–479, 2009.
 - [40] Paolo Falcone, Francesco Borrelli, H. Eric Tseng, Jahan Asgari, and Davor Hrovat. Linear time-varying model predictive control and its application to active steering systems: Stability analysis and experimental validation. *International journal of robust and nonlinear control*, 18(8):862–875, 2008.
 - [41] Alexander Katriniok and Dirk Abel. LTV-MPC approach for lateral vehicle guidance by front steering at the limits of vehicle dynamics. In *Decision and Control and European Control Conference (CDC-ECC), 50th IEEE Conference on*, pages 6828–6833. IEEE, 2011.
 - [42] Sohail Anwar. Generalized predictive control of yaw dynamics of a hybrid brake-by-wire equipped vehicle. *Mechatronics*, 15(9):1089–1108, 2005.

- [43] Massimo Canale, Lorenzo Fagiano, and Valentino Razza. Approximate NMPC for vehicle stability: Design, implementation and SIL testing. *Control Engineering Practice*, 18(6):630–639, 2010.
- [44] Massimo Canale, Lorenzo Fagiano, and Valentino Razza. Vehicle lateral stability control via approximated NMPC: real-time implementation and software-in-the-loop test. In *Decision and Control, Proceedings of the 48th IEEE Conference on*, pages 4596–4601. IEEE, 2009.
- [45] Eduardo F. Camacho and Carlos Bordons Alba. *Model Predictive Control*. Springer, London, 2nd edition, 2013.
- [46] Manfred Morari, Mato Baotic, and Francesco Borrelli. Hybrid systems modeling and control. *European Journal of Control*, 9(2):177–189, 2003.
- [47] F. Borrelli, A. Bemporad, and M. Morari. Predictive control for linear and hybrid systems. http://www.mpc.berkeley.edu/mpc-course-material/MPC_Book.pdf. Accessed: 2016-02-20.
- [48] ILOG Cplex. 11.0 users manual. *ILOG SA, Gentilly, France*, 2007.
- [49] Francesco Borrelli, Alberto Bemporad, Michael Fodor, and Davor Hrovat. An MPC/hybrid system approach to traction control. *Control Systems Technology, IEEE Transactions on*, 14(3):541–552, 2006.
- [50] Stefano Di Cairano, Hongtei Eric Tseng, Daniele Bernardini, and Alberto Bemporad. Vehicle yaw stability control by coordinated active front steering and differential braking in the tire sideslip angles domain. *Control Systems Technology, IEEE Transactions on*, 21(4):1236–1248, 2013.
- [51] Aldo Sorniotti and Mauro Velardocchia. Enhanced tire brush model for vehicle dynamics simulation. *SAE Technical paper*, 2008.
- [52] E. N. Pistikopoulos, M. C. Georgiadis, and V. Dua. *Multi-parametric programming: theory, algorithms, and applications*. Wiley-VCH, 2007.

- [53] P. Tondel and TA Johansen. Control allocation for yaw stabilization in automotive vehicles using multiparametric nonlinear programming. In *Proceedings of the American Control Conference*, volume 1, page 453. Citeseer, 2005.
- [54] Paolo Falcone, Francesco Borrelli, J. Asgari, HE Tseng, and Davor Hrovat. Low complexity MPC schemes for integrated vehicle dynamics control problems. In *9th International Symposium on Advanced Vehicle Control*, 2008.
- [55] Kyongsu Yi, Taeyoung Chung, Jeontae Kim, and Seungjong Yi. An investigation into differential braking strategies for vehicle stability control. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 217(12):1081–1093, 2003.
- [56] BL Boada, MJL Boada, and V Diaz. Fuzzy-logic applied to yaw moment control for vehicle stability. *Vehicle System Dynamics*, 43(10):753–770, 2005.
- [57] Daegun Hong, Inyong Hwang, Paljoo Yoon, and Kunsoo Huh. Development of a vehicle stability control system using brake-by-wire actuators. *Journal of dynamic systems, measurement, and control*, 130(1):011008, 2008.
- [58] J-S Jo, S-H You, Ji Yoel Joeng, Kyo Il Lee, and Kyongsu Yi. Vehicle stability control system for enhancing steerability, lateral stability, and roll stability. *International Journal of Automotive Technology*, 9(5):571–576, 2008.
- [59] Li Feiqiang, Wang Jun, and Liu Zhaodu. Motor torque based vehicle stability control for four-wheel-drive electric vehicle. In *Vehicle Power and Propulsion Conference. VPPC’09. IEEE*, pages 1596–1601. IEEE, 2009.
- [60] HongLiang Zhou and ZhiYuan Liu. Design of vehicle yaw stability controller based on model predictive control. In *Intelligent Vehicles Symposium, 2009 IEEE*, pages 802–807. IEEE, 2009.
- [61] Xibo Yuan and Jiabin Wang. Torque distribution strategy for a front- and rear-wheel-driven electric vehicle. *Vehicular Technology, IEEE Transactions on*, 61(8):3365–3374, 2012.

- [62] Yan Chen and Junmin Wang. Fast and global optimal energy-efficient control allocation with applications to over-actuated electric ground vehicles. *Control Systems Technology, IEEE Transactions on*, 20(5):1202–1211, 2012.
- [63] H. Ali Borhan, Ardalan Vahidi, Anthony M. Phillips, Ming L. Kuang, and Ilya V. Kolmanovsky. Predictive energy management of a power-split hybrid electric vehicle. In *American Control Conference.*, pages 3970–3976. IEEE, 2009.
- [64] Hoseinali Borhan, Ardalan Vahidi, Anthony M. Phillips, Ming L. Kuang, Ilya V. Kolmanovsky, and Stefano Di Cairano. MPC-based energy management of a power-split hybrid electric vehicle. *Control Systems Technology, IEEE Transactions on*, 20(3):593–603, 2012.
- [65] Giulio Ripaccioli, Alberto Bemporad, Francis Assadian, Clement Dextreit, Stefano Di Cairano, and Ilya V. Kolmanovsky. *Hybrid modeling, identification, and predictive control: An application to hybrid electric vehicle energy management*, pages 321–335. Hybrid Systems: Computation and Control. Springer, 2009.
- [66] Yan Chen and Junmin Wang. Design and experimental evaluations on energy efficient control allocation methods for overactuated electric vehicles: Longitudinal motion case. *Mechatronics, IEEE/ASME Transactions on*, 19(2):538–548, 2014.
- [67] Jong-Seob Won and Reza Langari. Fuzzy torque distribution control for a parallel hybrid vehicle. *Expert Systems*, 19(1):4–10, 2002.
- [68] Yan Chen and Junmin Wang. Energy-efficient control allocation with applications on planar motion control of electric ground vehicles. In *American Control Conference (ACC)*, pages 2719–2724. IEEE, 2011.
- [69] Avesta Goodarzi and Masoud Mohammadi. Stability enhancement and fuel economy of the 4-wheel-drive hybrid electric vehicles by optimal tyre force distribution. *Vehicle System Dynamics*, 52(4):539–561, 2014.

- [70] Patricio Cortes, Jose Rodriguez, Cesar Silva, and Alexis Flores. Delay compensation in model predictive current control of a three-phase inverter. *Industrial Electronics, IEEE Transactions on*, 59(2):1323–1325, 2012.
- [71] Shin ichiro Sakai, Hideo Sado, and Yoichi Hori. Anti-skid control with motor in electric vehicle. In *Advanced Motion Control. 6th International Workshop on*, pages 317–322. IEEE, 2000.
- [72] S. Khosravani, A. Kasaiezadeh, A. Khajepour, B. Fidan, S. Chen, and B. Litkouhi. Torque vectoring based vehicle control robust to driver uncertainties. *Vehicular Technology, IEEE Transactions on*, PP(99):1–1, 2014.
- [73] M. Canale and L. Fagiano. Stability control of 4WS vehicles using robust IMC techniques. *Vehicle System Dynamics*, 46(11):991–1011, 2008.
- [74] Qing-Chang Zhong. *Robust control of time-delay systems*. Springer Science & Business Media, 2006.
- [75] Liang-Kuang Chen and A. Galip Ulsoy. Design of a vehicle steering assist controller using driver model uncertainty. *International journal of vehicle autonomous systems*, 1(1):111–132, 2002.
- [76] Zhibin Shuai, Hui Zhang, Junmin Wang, Jianqiu Li, and Minggao Ouyang. Combined AFS and DYC control of four-wheel-independent-drive electric vehicles over CAN network with time-varying delays. *Vehicular Technology, IEEE Transactions on*, 63(2):591–602, 2014.
- [77] D Gillespie Thomas. Fundamentals of vehicle dynamics. *Society of Automotive Engineering Inc*, pages 168–193, 1992.
- [78] Rajesh Rajamani. *Vehicle dynamics and control*. Springer Science & Business Media, 2011.
- [79] A. Rezaeian, R. Zarringhalam, S. Fallah, W. Melek, A. Khajepour, S. K Chen, N. Moshchuk, and B. Litkouhi. Novel tire force estimation strategy for real-time

- implementation on vehicle applications. *IEEE Transactions on Vehicular Technology*, 64(6):2231–2241, 2014.
- [80] Ayyoub Rezaeian. *Development of an Integrated Estimation Method for Vehicle States, Parameters and Tire Forces*. PhD thesis, University of Waterloo, 2015.
 - [81] Ulrich Klehmet, Thomas Herpel, Kai-Steffen Hielscher, and Reinhard German. Delay bounds for CAN communication in automotive applications. In *Measuring, Modelling and Evaluation of Computer and Communication Systems (MMB), 14th GI/ITG Conference*, pages 1–15. VDE, 2008.
 - [82] Poi Loon Tang and Clarence W. de Silva. Compensation for transmission delays in an ethernet-based control network using variable-horizon predictive control. *Control Systems Technology, IEEE Transactions on*, 14(4):707–718, 2006.
 - [83] Hans-Georg Matuttis and Jian Chen. *Understanding the Discrete Element Method: Simulation of Non-Spherical Particles for Granular and Multi-body Systems*. John Wiley, 2014.
 - [84] Raymond Brach and Matthew Brach. The tire-force ellipse (friction ellipse) and tire characteristics. *SAE Technical Paper 2011-01-0094*, 2011.
 - [85] Hans Joachim Ferreau, Christian Kirches, Andreas Potschka, Hans Georg Bock, and Moritz Diehl. qpOASES: a parametric active-set algorithm for quadratic programming. *Mathematical Programming Computation*, 6(4):327–363, 2014.
 - [86] Hamid A Toliyat and Gerald B Kliman. *Handbook of electric motors*, volume 120. CRC press, 2004.
 - [87] Tamara G. Kolda, Robert Michael Lewis, and Virginia Torczon. Optimization by direct search: New perspectives on some classical and modern methods. *SIAM Review*, 45(3):385–482, 2003.
 - [88] The Mathworks, Inc., Natick, Massachusetts. *MATLAB version 7.13.0.564 (R2011b)*, 2011.

- [89] CarSim User Manual. Mechanical simulation corporation. *Ann Arbor, MI*, 48013, 2002.
- [90] United States Environmental Protection Agency. Federal test procedure revisions. <https://www3.epa.gov/otaq/sftp.htm#cycles>, 2016. Accessed: 2016-06-24.
- [91] Green Car Congress. Amp unveils electric version of GM equinox; secret sauce is the drivetrain, 2010.