

Predicting Energy Savings Associated with Falling-Film Drain Water Heat Recovery Systems in Residential Buildings

by

Ramin Manouchehri

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Examining Committee Membership

The following served on the Examining Committee for this thesis. The decision of the Examining Committee is by majority vote.

Supervisor:	Michael Collins, Professor Department of Mechanical and Mechatronics Engineering University of Waterloo
Internal Member:	Roydon Fraser, Professor Department of Mechanical and Mechatronics Engineering University of Waterloo
Internal Member:	Stewart McLachlin, Assistant Professor Department of Mechanical and Mechatronics Engineering University of Waterloo
Internal-External Member:	Costa Kapsis, Assistant Professor Department of Civil and Environmental Engineering University of Waterloo
External Member:	Alan Fung, Associate Professor Department of Mechanical, Industrial, and Mechatronics Engineering Toronto Metropolitan University

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Falling-film drain water heat recovery (DWHR) systems are heat exchangers utilized for recovering thermal energy from water travelling down shower drains in residential buildings. There are many commercially available DWHR heat exchangers on the market, and there exist a large variety of shower conditions that change from one house to another. For instance, the showerhead fixture and its associated flowrate, shower temperature and the plumbing system in a house are variables that directly impact energy savings, which can vary significantly between different houses and occupants. Clearly, reliable modeling procedures must be created to account for the large variety of operating conditions for DWHR heat exchangers.

The main goals for this project were to create and validate robust models to predict energy savings during steady-state and transient conditions, and to use these models in a building simulation software to identify the significance of plumbing configuration on energy savings. To this end, a thorough review of the literature concerning heat exchangers was done to identify the best procedure for modelling DWHR systems. For the steady-state analysis, a novel procedure was devised to derive correlations for the heat transfer coefficients in terms of heat transfer fundamentals. A semi-empirical correlation was also devised to allow faster calculations in cases where correlations for heat transfer coefficients are not readily available. Both methods were validated and were shown to have a mean absolute error smaller than 4%. Additionally, expressions for the transient behaviour of DWHR systems were derived based on heat exchanger theory and it was shown that these heat exchangers can be represented as first-order systems. Experimental data showed that the time constants associated with the typical operation of DWHR systems were generally on the order of a few seconds.

Next, the steady-state and transient models were programmed into TRNSYS (Transient System Simulation Tool) to perform monthly simulations to predict energy savings. For simulation purposes, five distinct plumbing configurations were considered, which covered all possible methods a DWHR heat exchanger can be incorporated into the plumbing system for a house. Identical draw schedules were devised and applied to all simulations, and the overall energy consumptions for all simulations were estimated and compared with a base case (i.e. a plumbing system without a heat exchanger). The results showed that small changes in plumbing configuration can have significant impact on the energy savings that can be achieved using DWHR heat exchangers. For instance, the results for one of the simulated heat exchangers showed that the energy savings can range from 25% to 35% depending on the plumbing configuration. These findings were then used as a basis to formulate how DWHR heat exchangers should be modeled, and perhaps more importantly, to highlight the urgent need to update the current building codes and standards to better reflect the actual performance for this particular heat exchanger technology.

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Finally, I want to thank my family for their continued support and encouragement. The past few years have been extremely difficult for me and there were a few times when I considered quitting my PhD program, but my parents always helped me get back on track. I would have never made it this far if not for them.

Dedication

This thesis is dedicated to my parents.

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List of Abbreviations

AMTD: Arithmetic-Mean Temperature Difference

CFD: Computational Fluid Dynamics

CSA: Canadian Standards Association

DWHR: Drain Water Heat Recovery

GUI: Graphical User Interface

HWT: Hot Water Tank

HX: Heat Exchanger

LMTD: Log-Mean Temperature Difference

LPM: Liters Per Minute

MAE: Mean Absolute Error

MAPE: Mean Absolute Percentage Error

MATLAB: Matrix Laboratory software

NTU: Number of Transfer Units

OMTD: Outlets-Mean Temperature Difference

RMS: Root Mean Square

RTD: Resistance Temperature Detector

TRNSYS: Transient System Simulation Tool

Nomenclature

A	Heat transfer area (m^2)
Bi	Biot number (dimensionless)
β	Heat Exchanger surface area density (m^2/m^3)
$\dot{C}_c, \dot{C}_h$	Heat capacity rates corresponding to the cold and hot sides, respectively ($\text{W}/^\circ\text{C}$)
$\dot{C}_{min}, \dot{C}_{max}$	Lesser and greater of $\dot{C}_c$ and $\dot{C}_h$, respectively ($\text{W}/^\circ\text{C}$)
C_p	Specific heat ($\text{kJ}/\text{kg } ^\circ\text{C}$)
d	Tube diameter (m)
De	Dean number (Dimensionless)
$\dot{E}$	Energy flow rate (kW)
h_{coils}	Convective heat transfer coefficient for the coils, ($\text{kW}/\text{m}^2 \text{ } ^\circ\text{C}$)
h_{film}	Convective heat transfer coefficient for the falling film, ($\text{kW}/\text{m}^2 \text{ } ^\circ\text{C}$)
k	Thermal conductivity ($\text{W}/\text{m } ^\circ\text{C}$)
M	Mass (kg)
$\dot{m}$	Mass flow rate (kg/s)
Nu	Nusselt number (Dimensionless)
Pr	Prandtl number (Dimensionless)
$\dot{q}$	Heat transfer rate (kW)
$\dot{q}_{HX}$	Heat exchanger duty (kW)
$\dot{q}_c$	Heat transfer rate based on the cold side of a DWHR system (kW)
$\dot{q}_{max}$	Maximum heat transfer rate (kW)
ρ	Density (kg/m^3)
P_x, P_y, P_{xy}	Regression coefficients for shortcut model (dimensionless)
R	Thermal resistance ($^\circ\text{C m}^2/\text{W}$)
Re	Reynolds number (Dimensionless)
t	Time (s)
t_{flush}	Flush time for a given heat exchanger (s)
τ	Time constant (s)
$T_{c,i}$	Temperature of mains-side, cold-water, inlet ($^\circ\text{C}$)
$T_{c,o}$	Temperature of mains-side, cold-water, outlet ($^\circ\text{C}$)
T_{drain}	Temperature of drain-side ($^\circ\text{C}$)
$T_{h,i}$	Temperature of drain-side, hot-water, inlet ($^\circ\text{C}$)
$T_{h,o}$	Temperature of drain-side, hot-water, outlet ($^\circ\text{C}$)
T_{mains}	Temperature of mains-side ($^\circ\text{C}$)
U	Overall heat transfer coefficient ($\text{kW}/\text{m}^2 \text{ } ^\circ\text{C}$)
$\dot{V}$	Volumetric flow rate (liter/min)
$\dot{V}_c$	Volumetric flow rate based on the cold side of a DWHR system (liter/min)

Chapter 1: Introduction

It is well understood that global energy consumption is on the rise and new measures are required to allow utilities to meet future demands. Climate change has also become a major concern, and many countries have agreed to implement measures to reduce their carbon emissions as per the Paris Agreement [1]. Energy recovery technologies can be used to counteract both aforementioned issues by reducing wasted energy, and thereby reducing the emissions associated with power generation. The residential sector accounts for about 13% of the total energy delivered around the world [2]. To put this into perspective, EIA's international energy outlook predicted that in year 2020, this would translate to 64.5 exajoules of energy, which is 5.87 times larger than the total predicted global nuclear energy generation in the year 2020 [3].

Water heating is a large contributor to the total energy consumed in residences. In Canada for example, surveys have shown that 23.6% of the total energy consumed in residences in 2014 was attributed to water heating [4]. This translates into 379.8 PJ, or 27.2 GJ per household. One of the main inefficiencies associated with water heating is that a significant portion of the energy that is put into heating of water goes down the drain at elevated temperatures. On average, hot water going down the drain retains 80-90% of its thermal energy relative to the mains cold water supply [5]. It is also worth noting that as buildings become more energy-efficient and move towards net-zero, the percentage of energy wasted through domestic water heating becomes more pronounced. According to a report by Schmid, 15 to 30% of the thermal energy provided to buildings is lost via the sewage system, with higher percentages attributed to the low-energy buildings [6]. Clearly, there is much potential in this area for energy recovery technologies, and one such technology is studied in this work.

Drain water heat recovery (DWHR) systems are heat exchangers designed to recover energy from greywater in buildings. There are many different designs available on the market, but in general, they can be divided into two main categories: near-horizontal and vertical. As these heat exchangers are mainly marketed towards small residential buildings, they are designed to easily adapt to the plumbing systems in houses. In other words, the drain-side of DWHR systems relies on gravity, and the mains-side relies on the mains water pressure. Given that no additional pumps are required to use these heat exchangers, and that they do not contain any electrical components, nor require any

external controls to operate, both DWHR categories are considered to be passive heat exchangers. Each of these categories is elaborated on in the following paragraphs.

Near-horizontal DWHR systems, or sometimes referred to as horizontal DWHR systems, refer to designs meant to be installed in a near-horizontal orientation, as the name suggests. See Figure 1-1 and Figure 1-2. This type of DWHR system is not commonly used, and it is generally only considered when vertical designs cannot be used. The main advantage of near-horizontal designs over vertical designs is that they can be installed in any house, whereas the vertical designs require a vertical drain stack. The disadvantages include lower savings in comparison with vertical designs, and higher likelihood and severity of fouling-related reduction in performance. There are no peer-reviewed papers available on the performance and fouling-related issues associated with near-horizontal DWHR systems; therefore, they are not discussed further in this work.



Figure 1-1: Horizontal DWHR system installed at the ground level [7].



Figure 1-2: Horizontal DWHR system installed at the ceiling level [7].

Vertical DWHR systems, also known as falling-film DWHR systems, are by far the most commonly used type of these heat exchangers. They are comprised of a large diameter copper drain pipe, wrapped tightly with a coil of smaller diameter copper tubes as shown in Figure 1-3. The drain water goes down the large inner pipe, which typically has diameters between 5.1 and 10.2 cm, to match the size of the drain stack they replace. The mains water is circulated within the smaller tubes to recover heat from the drain water. The ideal performance of DWHR systems relies on the formation a falling film of water within the drain pipe. That is to say the water sticks to the walls and falls as an annular film covering the entire inner surface of the pipe. An example of this is shown in Figure 1-4, where a camera was mounted vertically inside a DWHR system. The falling-film maximizes the area for heat transfer, while minimizing the thickness of water through which the heat must be transferred to the walls. Coiled falling-film DWHR systems depicted in Figure 1-3 are the subject of this study, and henceforth, will be referred to as DWHR systems.

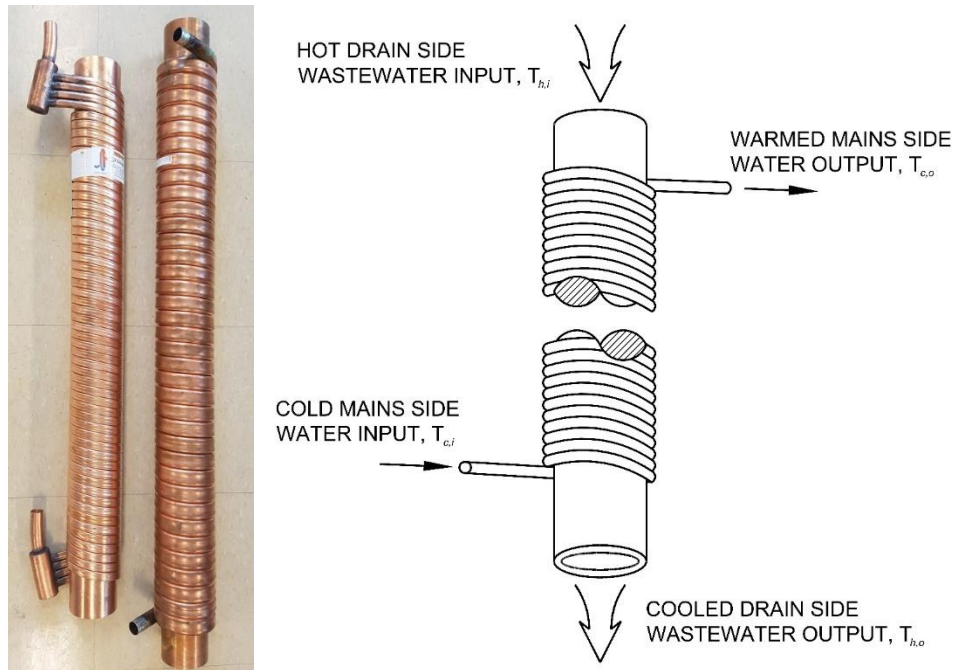


Figure 1-3: Common falling-film DWHR systems (left) and water flow directions (right).

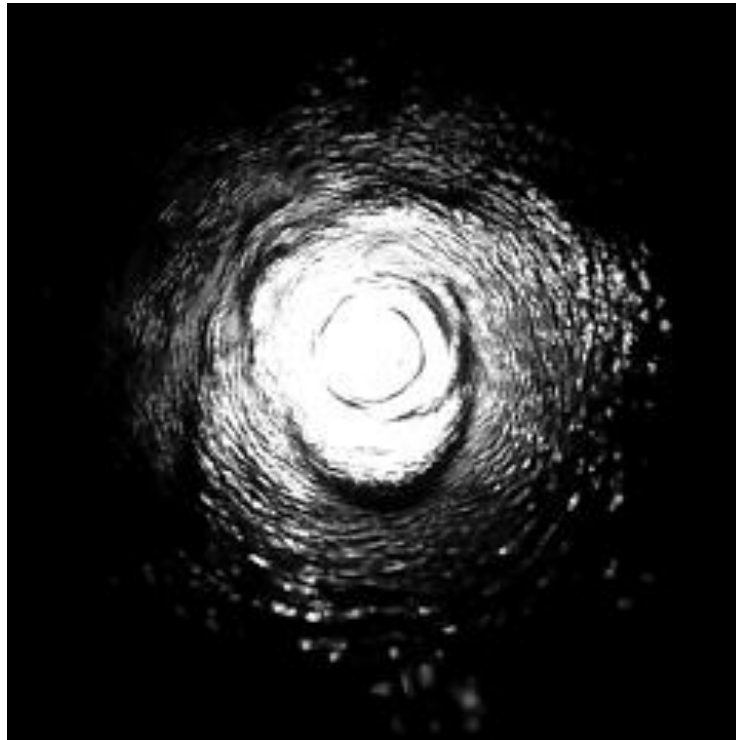


Figure 1-4: The falling-film in the drain-side pipe for a vertical installation.

DWHR systems have become more common in the residential buildings, and as such, building codes and incentive programs have been updated to include them. In Canada, the Governments of Ontario and Manitoba have updated their provincial building codes to promote the installation of DWHR systems in new residential buildings [8, 9]. The Canada Green Building Council has also incentivized the use of DWHR systems by making it possible to receive a point towards the Efficient Domestic Hot Water Equipment credit in the LEED Canada for Homes 2009 rating system [10]. Lastly, the Canadian Standards Association (CSA) has produced standard B55.1-15, which is used worldwide for rating the performance of DWHR systems [11].

The concept of using heat exchangers for heat recovery is not new, but DWHR systems have only been utilized for around a decade. They have slowly made their way into policies as listed in the previous paragraph, and the core design of these heat exchangers (i.e. a copper drainpipe wrapped with copper coils) has not changed appreciably since their commercialization. The majority of the studies available in the literature about DWHR systems have focused on discussing the feasibility of using them in residential buildings. In other words, most of the current literature can be considered as proof of concept, and so far, no complete model has been proposed for predicting accurate energy savings associated with DWHR systems. This presents a challenge to both designers and home owners alike, as it is not possible to predict accurate payback periods for any DWHR heat exchanger available on the market. A recent study by Kordana et al. highlighted this issue and concluded that one of the major threats to DWHR systems is the lack of acceptance from the users, which stems from insufficient knowledge about their operation and the related energy savings [12].

To further highlight the gaps in the literature, it is imperative to first cover fundamental heat exchanger terminology and the methods used to evaluate the performance of DWHR systems. These will be covered in detail in Chapter 2.

1.1 Research Problem and Objectives

The research problem is to develop a method to accurately estimate the energy savings associated with DWHR systems in residential buildings. This task can be broken down into three major objectives to support the overall goal:

- 1) Devise a method for predicting the steady-state performance of DWHR systems. This requires development of a new model, or a correction scheme to expand the existing model to encompass a much wider range of flow rates.
- 2) Experimentally investigate the transient response of DWHR systems and investigate its impact on the overall energy savings.
- 3) Combine the steady-state and transient models to create a TRNSYS (Transient System Simulation Tool) component for building simulation purposes. TRNSYS is a simulation software capable of accounting for transient conditions while calculating energy savings.

1.1.1 Steady State Model

A preliminary model was developed by the author in the early stages of this research, which will be described in Section 2.3. The model was purely based on experimental data and was not based on heat exchanger fundamentals. As such, a study in terms of the nondimensional variables such as the Reynolds number, Nusselt number, etc. cannot be performed to determine the applicability of the model at conditions outside the bounds for which it was developed. This led to two options for creating a more versatile steady-state model:

- 1) Determine whether or not the existing model applies to conditions outside the bounds for which it was developed, and devise correction factors if possible.
- 2) If the existing model is found to be inapplicable and correction schemes could not be developed, a new model would have to be created based on data collected over a larger range of operating conditions.

1.1.2 Transient Model

The energy savings associated with the steady-state performance of DWHR systems are expected to far outweigh the transient performance, and the conditions during shower events are expected to stay constant throughout the shower; therefore, more emphasis is placed upon the steady-state model.

However, it is imperative to have a means to determine the transient response of these heat exchangers for accurate performance modelling. To highlight this point, a DWHR system was tested and the plots of effectiveness and $\dot{q}$ as a function of time for the heat exchanger are shown in Figure 1-5, where the inlet temperatures were constant, and time 0 depicts the start of the pump. The flow rates were balanced and are shown in the plots using dotted lines.

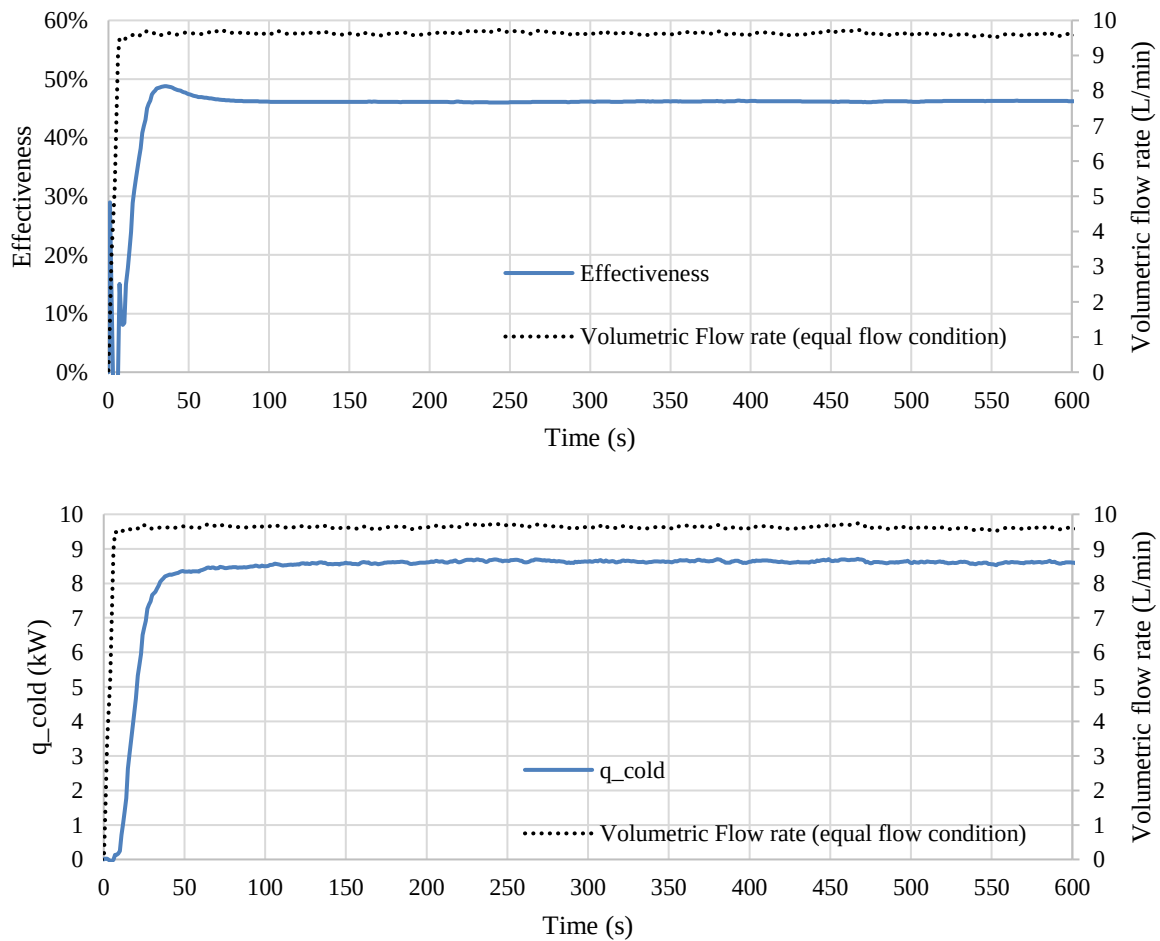


Figure 1-5: Effectiveness (top) and heat transfer rate (bottom) vs. time for a 7.6cm diameter, 122cm long DWHR system for a 10-minute test at a constant volumetric flow rate.

The duration of the test was 10 minutes, which was meant to simulate the duration of a typical shower event. As seen in the figure, the system took less 90 than seconds to reach steady-state and remained steady for the rest of the test. The magnitude of energy recovered by the DWHR system is the area under the $\dot{q}$ vs time curve, the majority of which can be attributed to the steady-state condition.

1.1.3 TRNSYS Model

Once both the steady-state and the transient models are complete, they must be combined into a building simulation software capable of simulating performance of DWHR systems under realistic conditions. Such conditions include variable inlet flow rates and temperatures into a DWHR system, subject to realistic water draws throughout the simulated days, in plumbing systems containing fixtures in addition to the showerhead. This will be done using the TRNSYS software to simulate the combined effect of all variables on the overall energy savings.

Chapter 2: Heat Exchanger Literature

This chapter provides an overview of heat exchanger fundamentals, followed by discussions pertaining to several publications focused on DWHR heat exchangers.

2.1 Heat Exchanger Fundamentals

This section is intended to provide a general background about heat exchanger theory and establish the nomenclature that will be used consistently throughout this thesis. It is the author's opinion that the terminology used in undergraduate textbooks is overly simplified and is only applicable to a limited number of heat exchanger designs. Furthermore, much of the discussions and fine details available in older prominent textbooks written by Shah and Kays/London [13, 14], have unfortunately not made it to modern textbooks. Perhaps this could be attributed to the fact that all fundamental methods for solving heat exchanger problems were devised long before the 1970s energy crisis, and the importance of very accurate heat exchanger models was not realized at the time. Regardless, this section contains heat exchanger theory and its connections to DWHR heat exchangers.

There are a multitude of equipment that can be constituted as heat exchangers, and it is best to start with a textbook definition. A heat exchanger is a device used to facilitate the transfer of thermal energy between two or more media (fluid or solid) that are in thermal contact while having different temperatures [14]. Due to this broad definition, the world of heat exchangers is extremely diverse and includes fundamentally different 'devices' such as human lungs and plate heat exchangers. As a result of this diversity, there is no universal technique that applies to all heat exchangers and appropriate solution procedures are design-based.

2.1.1 Heat Exchanger Classifications

To identify the best solution procedure, heat exchangers are classified based on different metrics, some of which have a direct impact on the thermal analysis. There are numerous methods to characterize heat exchangers, an extensive list of which is available in [14].

2.1.1.1 Classification by Size

Heat exchangers can be characterized by size, or more accurately surface area density β , which is the ratio of heat transfer surface area to volume occupied by a given heat exchanger. This leads to the concept of compact heat exchangers, where compactness is based on the area density. The exact β s

that would constitute compactness for a heat exchanger are arbitrarily selected in the literature and are notably based on heat transfer fluids. For gas-to-fluid heat exchangers, β must be greater than $700\text{m}^2/\text{m}^3$ ($213\text{ft}^2/\text{ft}^3$) for the heat exchanger to be considered compact. Alternatively, if one of the fluids is liquid, or is undergoing phase change, β must be greater than $400\text{m}^2/\text{m}^3$ ($122\text{ft}^2/\text{ft}^3$). Figure 2-1 depicts a few common heat exchangers and highlights their generalized position on the β spectrum [14, 15]. Different arrangements of fins or tube-bundles are often incorporated into compact heat exchangers to achieve a high area density ratio. DWHR heat exchangers are mostly hollow, given that the drainpipe is never flooded; hence, most of the volume occupied by the heat exchanger never participates in the heat transfer process. Therefore, falling-film DWHR heat exchangers cannot be considered compact.

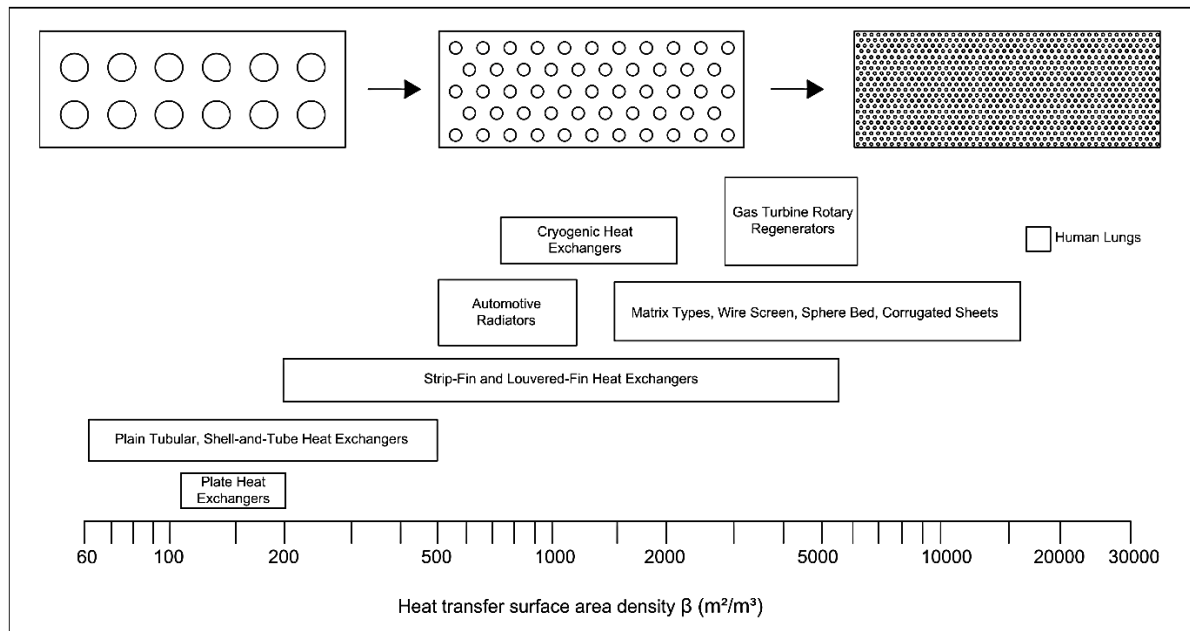


Figure 2-1: Selection of heat exchangers as a function of area density. Figure is adapted from [14, 15]

2.1.1.2 Classification by Flow Arrangement

As the name suggests, this classification is based on the relative direction of flow for the fluid media within the heat exchanger. The idealized flow arrangements are defined accordingly in heat exchanger literature [13] and are as follows:

Counter flow: A counter flow heat exchanger, also known as countercurrent or contraflow, is a heat exchanger in which the two fluids flow parallel to each other but in opposite directions. See Figure 2-2(a).

Parallel flow: A parallel flow heat exchanger is a heat exchanger in which the two fluids flow parallel to each other and in the same direction. See Figure 2-2(b).

Cross flow: A cross flow heat exchanger is defined as a heat exchanger in which the two fluids flow in directions normal to each other. See Figure 2-2(c).

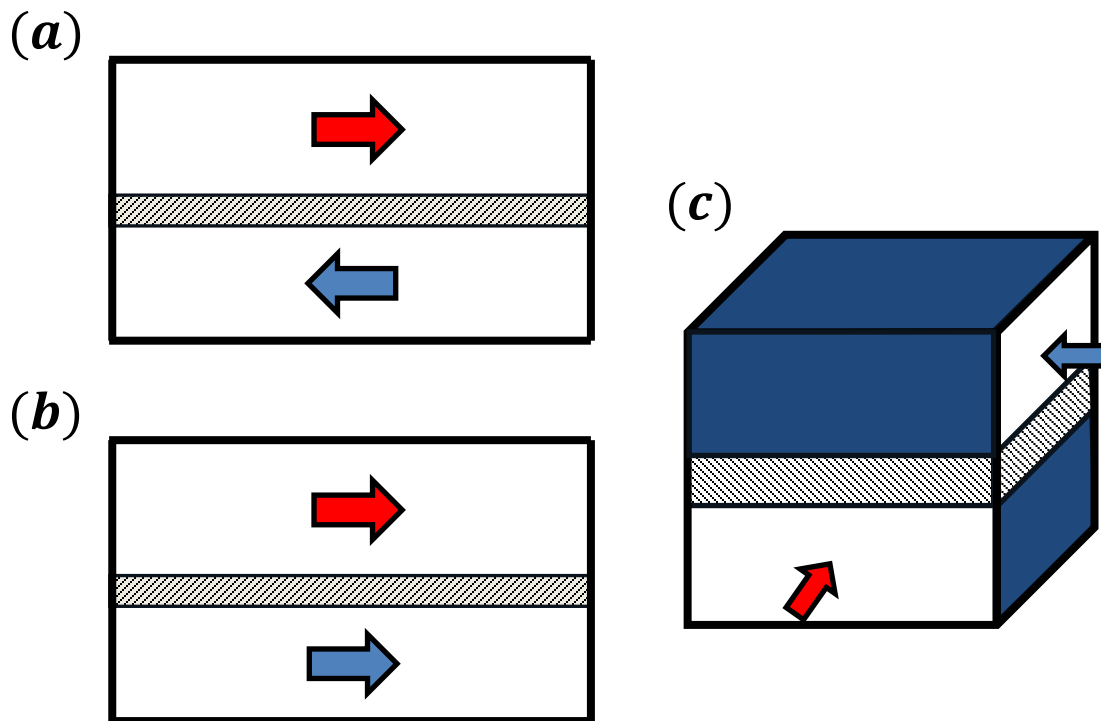


Figure 2-2: Flow arrangements for (a) counterflow, (b) parallel flow, and (c) cross flow heat exchangers.

Another flow-related distinction often applied to heat exchangers is based on the number of passes. A single pass is said to have occurred if the heat transfer fluid on one side of the heat exchanger

traverses through its entire length once. Likewise, a multi-pass heat exchanger is one where heat transfer fluid is circulated through the heat exchanger length more than once. This distinction becomes exceedingly important when dealing with shell-and-tube heat exchangers and signifies the need to use more complex solution methods.

There are numerous other classification methods available in the literature, including classification by construction features (such as fins), classification by construction type (plate, shell-and-tube, etc.) and classification by contact (direct vs. indirect); however, these classifications are not applicable to DWHR heat exchangers and as such, will not be used in this thesis. For a complete list of classifications, see [14].

2.1.2 Standard Presentation

It is customary to present heat exchanger concepts using graphs of fluid temperatures for each side as a function of heat exchanger length, like what is shown in Figure 2-3(a). This figure depicts a heat exchanger as a box, with the large red and blue arrows indicating hot and cold fluid directions respectively. Figure 2-3(b) is the complete form of the standard presentation, where the inlet and outlet temperatures are identified, along with the hypothetical temperature gradient as a function of length. Similar diagrams will be employed throughout this document to be consistent with the literature.

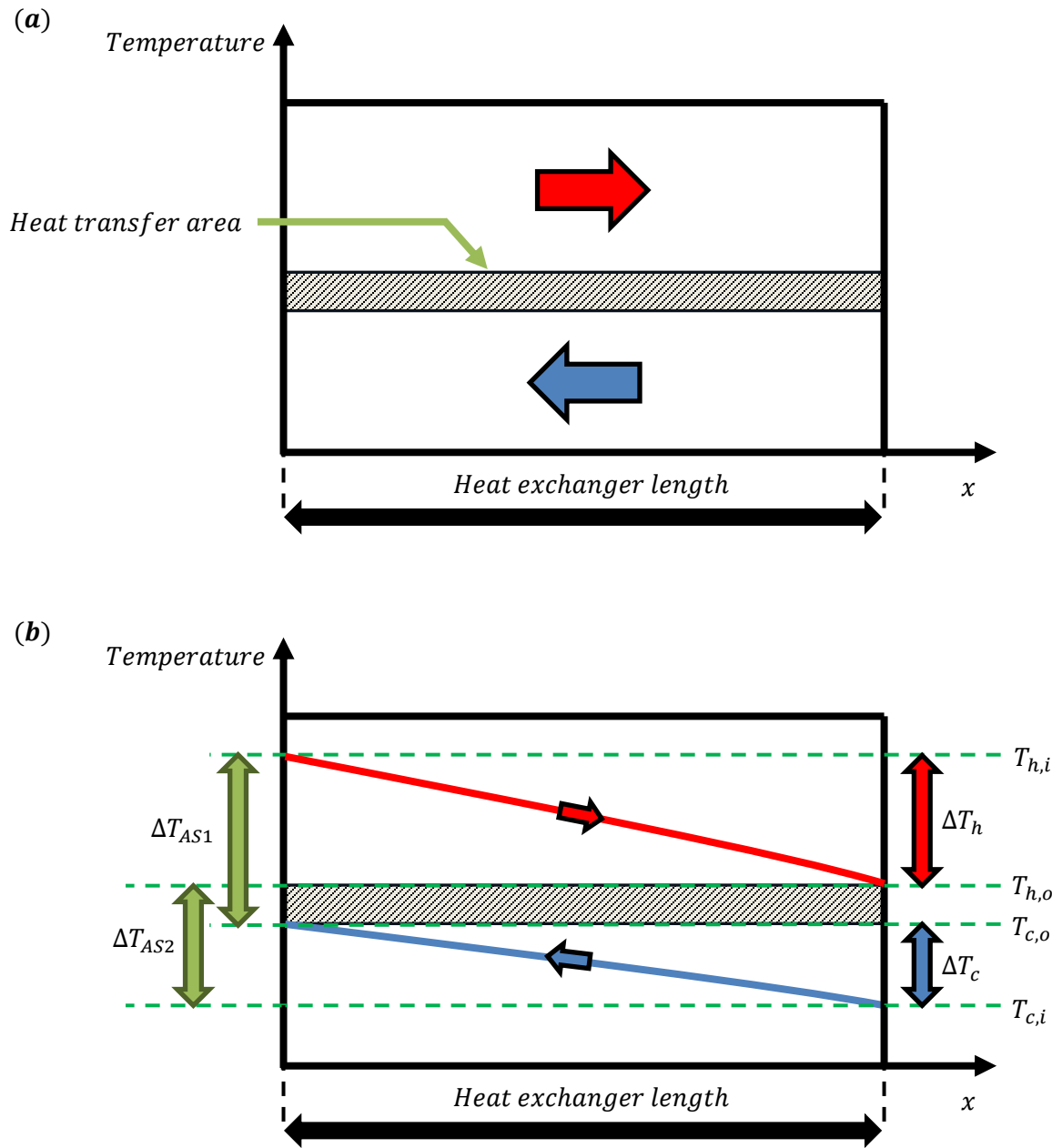


Figure 2-3: Standardized heat exchanger representations in terms of (a) flow arrangement and (b) temperature gradient as functions of heat exchanger length.

2.1.3 DWHR Heat Exchanger Classification

It is prudent to discuss and declare the exact classification of DWHR heat exchangers studied in this thesis, as the applicability of this work is limited to the class of heat exchangers involved in this research. In the context of this work, a DWHR heat exchanger is classified as a single-pass counter flow, water-to-water heat exchanger, installed vertically, where the cold fluid is fed through copper coils, and the warm fluid moves as a falling film. The lengthy description is necessary for classification of the heat exchanger and contains five distinct constraints that are repeated in bullet point form for clarity. The reason behind the repetition of these points is to emphasize that if any of the 5 constraints are not met, the heat exchanger in question does not belong to the classification of heat exchangers studied in this work, and the presented models are inapplicable without appropriate modifications.

- 1) Flow arrangement: single-pass counter-flow
- 2) Fluid media: single-phase water on both sides
- 3) Drain water (hot side) is designed to move as a falling film
- 4) Mains water (cold side) is fed through helical coils
- 5) Installation: Vertical

2.1.4 Theory and Solution Methods

As mentioned previously, there is no universal analysis technique that applies to all heat exchangers. This is due to the variety of devices that can be considered heat exchangers and the vast differences between them. Multiple techniques have been developed for heat exchanger analysis, such as ϵ -NTU and log-mean-temperature-difference (LMTD) methods, as well as lesser-known techniques such as P-NTU and theta methods [14, 13, 16, 17, 18]. Depending on the classification for the heat exchanger, different solution methods are employed in the literature, and that is partly the reason why DWHR heat exchangers were exhaustively classified in the previous section.

2.1.4.1 Energy Balance

This section will cover the fundamental approach to solving heat exchanger problems through performing energy balances.

The first energy balance is performed on a control volume drawn around the entire heat exchanger, as shown in Figure 2-4. This generic control volume is then subjected to relevant assumptions regarding heat exchangers to proceed with the algebra step-by-step.

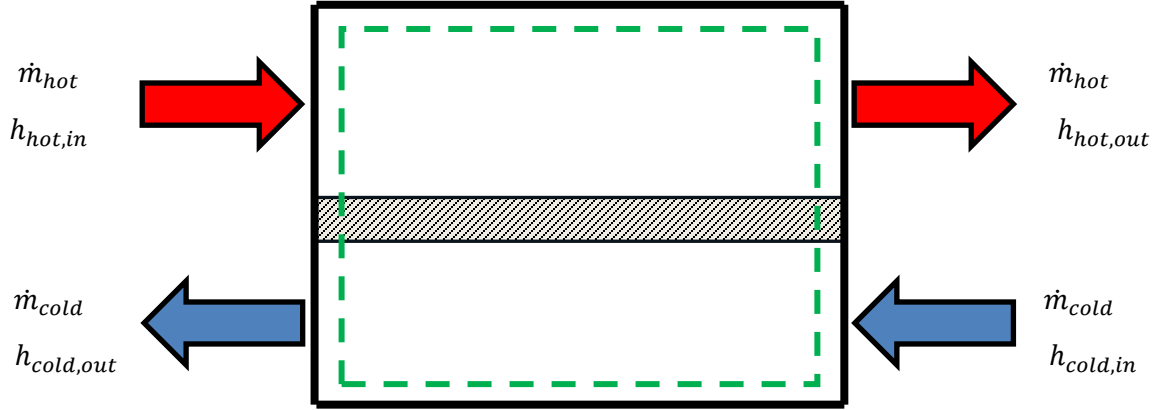


Figure 2-4: control volume drawn around a generic counter flow heat exchanger.

Firstly, the following assumptions have been implicitly applied to construct Figure 2-4:

- negligible thermal losses to the environment
- negligible axial conduction
- no energy sources or sinks

It bears repeating that due to the limited range of temperatures and flow rates experienced by DWHR heat exchangers during operation, the above assumptions are valid. Hence, the energy balance for the control volume from Figure 2-4 can be expressed in Eq. 2.1:

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{storage} \quad (2.1)$$

Heat exchangers are designed for operation under steady-state conditions; as a result, the change in thermal storage as a function of time (i.e. $\Delta \dot{E}_{storage}$) is often neglected, which results in the well-known expression in Eq. 2.2. Note that this does not imply that thermal storage term is zero; it should simply be interpreted as energy entering the control volume is equal to energy leaving the control volume under steady-state conditions.

$$\dot{E}_{in} = \dot{E}_{out} \quad (2.2)$$

where

$$\dot{E}_{in} = \dot{m}_{cold}h_{cold,in} + \dot{m}_{hot}h_{hot,in} \quad (2.3)$$

$$\dot{E}_{out} = \dot{m}_{cold}h_{cold,out} + \dot{m}_{hot}h_{hot,out} \quad (2.4)$$

Here, h denotes enthalpy. Hence, Eq. 2.2 is often presented in the form of Eq. 2.5 instead. The physical meaning behind Eq. 2.5 is that within the heat exchanger, the rate of energy removal from the hot stream is equal to the rate of energy addition to the cold stream.

$$\begin{aligned} \dot{m}_{cold}h_{cold,in} + \dot{m}_{hot}h_{hot,in} - \dot{m}_{cold}h_{cold,out} - \dot{m}_{hot}h_{hot,out} &= 0 \\ \rightarrow \dot{m}_{cold}(h_{cold,in} - h_{cold,out}) + \dot{m}_{hot}(h_{hot,in} - h_{hot,out}) &= 0 \\ \rightarrow \dot{m}_{cold}(h_{cold,out} - h_{cold,in}) &= \dot{m}_{hot}(h_{hot,in} - h_{hot,out}) \end{aligned} \quad (2.5)$$

The next set of simplifying assumptions routinely employed in the literature apply to the fluid media, and are as follows:

- Incompressible fluids
- Single-phase fluids
- Constant thermophysical properties for the fluids
- Linear relationship between changes in enthalpy and changes in temperature

Consequently, the energy balance is further simplified to Eq. 2.6. This equation is used predominantly to study heat exchangers that use water as the fluid media, where the above assumptions are extremely relevant. Due to the limited range of temperatures and flow rates in which DWHR heat exchangers operate, these assumptions are very close to the truth. This is also the equation often listed in heat transfer books to simplify problems dealing with heat exchangers [16].

$$\dot{m}_{cold}c_p(T_{cold,out} - T_{cold,in}) = \dot{m}_{hot}c_p(T_{hot,in} - T_{hot,out}) \quad (2.6)$$

As evident by the equation, the variables in Eq. 2.6 are based purely on the fluid media, and this equation does not contain any information specific to the heat exchanger depicted in the control volume. To establish an equation for the heat exchanger duty, $\dot{q}_{HX}$, energy balances are performed on each of the fluid streams in Figure 2-5(a) and (b), for the hot and cold streams respectively. Note that the dashed lines indicate where the control volume was drawn in the figure.

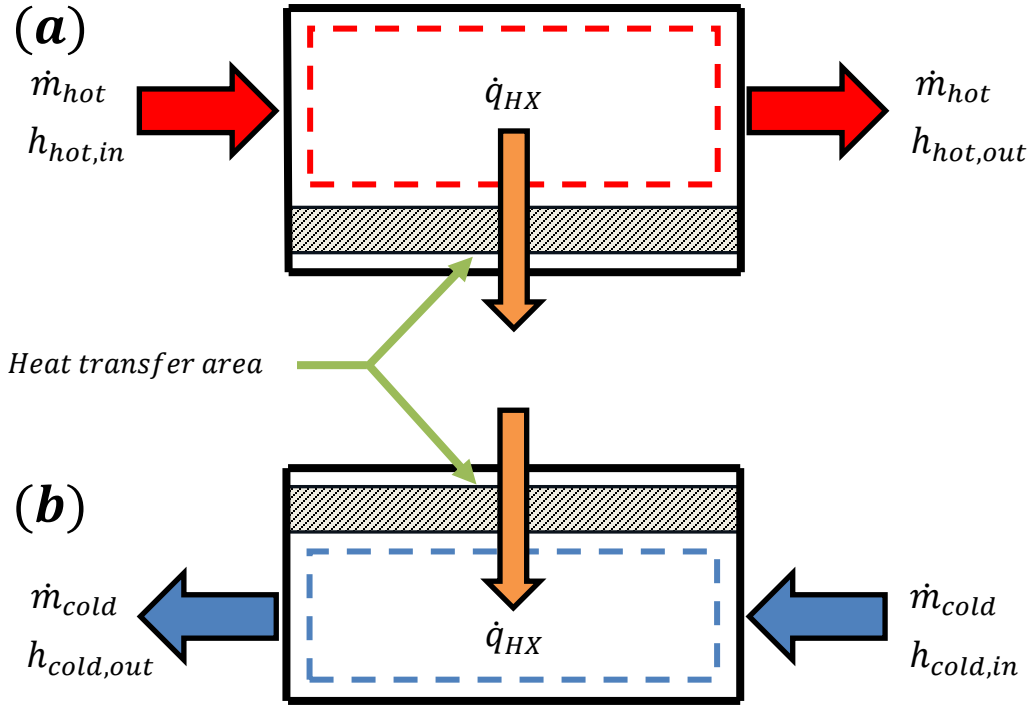


Figure 2-5: Control volumes drawn around (a) hot side and (b) cold side of a counter flow heat exchanger. Performing energy balances on control volumes depicted in Figure 2-5 results in the following equations:

$$\dot{q}_{HX} = \dot{m}_{cold} c_p (T_{cold,out} - T_{cold,in}) \quad (2.7)$$

$$\dot{q}_{HX} = \dot{m}_{hot} c_p (T_{hot,in} - T_{hot,out}) \quad (2.8)$$

Alternatively, the heat exchanger duty can be represented through a thermal resistor network, where the contributions from different components within the heat exchanger are combined to determine the value of overall heat transfer coefficient, U , based on the heat transfer area, A . See Eq. 2.9.

$$\dot{q}_{HX} = UA(T_h - T_c) = UA\Delta T_m \quad (2.9)$$

where T_h and T_c represent the mean temperatures on the hot and cold sides of the heat exchanger respectively, and ΔT_m is the mean temperature difference between the two sides. Eqs. 2.7, 2.8 and 2.9 are based on conservation of energy, and they are often equated and represented as follows:

$$\dot{q}_{HX} = \dot{m}_{cold}c_p(T_{cold,out} - T_{cold,in}) = \dot{m}_{hot}c_p(T_{hot,in} - T_{hot,out}) = UA\Delta T_m \quad (2.10)$$

Note that ΔT_m is an appropriate representation of the mean temperature difference for a specific heat exchanger, and it bears further discussions as it is an aspect of heat exchangers that is often overlooked. This will be addressed in Section 2.4.1.3.

2.1.4.2 Overall Heat Transfer Coefficient

U is the measure of total thermal transmittance (important note: at a given operating condition), and A is the area term associated with thermal transmittance. U is also called the overall heat transfer coefficient, or overall thermal conductance. Typically, what changes between operating points for a given heat exchanger is the value of total thermal transmittance, U , whereas the area term, A , is treated as a constant. It is customary to divide the UA product by a known area and provide the value of U by itself. This total transmittance is sometimes referred to as the “U-factor”. The U-factor is frequently encountered in building sciences and architectural designs, where thermal resistances for walls and windows are required to predict heating and cooling loads for HVAC systems. For example, wall insulations are compared using their “R-value” or “RSI-value” in imperial and metric systems accordingly. This R-value represents the total thermal resistance of an assembly of unidentical thermal resistors added in series or parallel, depending on the application. The U-factor is simply the reciprocal of the R-value, and instead of showing the total thermal resistance, it shows the effective thermal transmittance of the assembly, subject to certain boundary conditions. For example, windows and fenestrations are sold using the U-factor product measured under prescribed conditions.

Depending on the sensitivity of heat transfer coefficients to changes in temperature, flow rate, etc., the U-factor can stay relatively constant for the assembly’s intended usage. For example, In the context of a window in a wall assembly, the U-factor may be considered a constant because the boundary conditions for a building’s exterior wall are predictable. The temperatures and air velocities that a house’s window is exposed to are finite; hence, a range of possible convective heat transfer coefficients can be calculated.

Unlike window assemblies, heat transfer coefficients in heat exchangers are strong functions of mass flow rates on either side of the heat exchanger. Therefore, the assumption of constant U is invalid. Finding the overall thermal transmittance U requires knowledge of convective heat transfer coefficient h for both fluids traversing through the heat exchanger.

2.1.4.3 Mean Temperature Difference ΔT_m

As was listed in Eq. 2.9, ΔT_m is the mean temperature difference that gets multiplied by the overall heat transfer coefficient and the area to estimate the heat exchanger duty, $\dot{q}_{HX}$. There are different representations for ΔT_m , three of which are listed below:

$$\begin{aligned} 1) \Delta T_m &\cong \Delta T_{OMTD} = T_{hot,out} - T_{cold,out} \\ 2) \Delta T_m &\cong \Delta T_{AMTD} = \left(\frac{T_{hot,in} + T_{hot,out}}{2} \right) - \left(\frac{T_{cold,in} + T_{cold,out}}{2} \right) \\ 3) \Delta T_m &= \Delta T_{LMTD} = \frac{\Delta T_{AS1} - \Delta T_{AS2}}{\ln \left(\frac{\Delta T_{AS1}}{\Delta T_{AS2}} \right)} = \frac{\Delta T_{AS2} - \Delta T_{AS1}}{\ln \left(\frac{\Delta T_{AS2}}{\Delta T_{AS1}} \right)} \end{aligned}$$

In the above expressions, the subscripts mean the following:

- OMTD = outlets-mean temperature difference
- AMTD = arithmetic-mean temperature difference
- LMTD = log-mean temperature difference
- AS1 and AS2 = arbitrary sides 1 and 2. This is based on flow arrangement, and it must not be confused with heat exchanger sides (e.g. hot and cold)

For a counter flow heat exchanger, these temperature differences were previously depicted on Figure 2-3(b) and Eq. 2.11 represents the pair of mathematical expressions for AS1 and AS2 respectively. Note that it does not matter which side is arbitrarily designated as 1 or 2; the only requirement is to select one and be consistent in the algebra. Lastly, if a heat exchanger is being operated under parallel-flow conditions, the equations for ΔT_{AS1} and ΔT_{AS2} must be corrected accordingly. DWHR heat exchangers are counter flow heat exchangers by design; hence, detailed algebraic expressions for parallel-flow conditions are not listed nor will be discussed in this thesis.

$$\begin{cases} \Delta T_{AS1} = T_{hot,in} - T_{cold,out} \\ \Delta T_{AS2} = T_{hot,out} - T_{cold,in} \end{cases} , \text{ OR } , \begin{cases} \Delta T_{AS2} = T_{hot,in} - T_{cold,out} \\ \Delta T_{AS1} = T_{hot,out} - T_{cold,in} \end{cases} \quad (2.11)$$

The outlets-mean temperature difference, ΔT_{OMTD} , is the simplest and most erroneous representation of the true mean temperature amongst the three options listed above. Not only does this method ignore the inlet temperatures to the heat exchanger, but it also requires the difference between the outlets temperatures to be appreciably large or it is possible for ΔT_{OMTD} to approach zero, which does not lead to a solution. According to previous research by Jonsson and Holst [19] and Palsson

[20], using ΔT_{OMTD} results on temperature differences that are too small, which leads to significant steady-state errors [21]. Therefore, ΔT_{OMTD} has only been listed here for completeness and will not be employed anywhere in this thesis.

Next notable representation for ΔT_m is the arithmetic-mean temperature difference, ΔT_{AMTD} . The expression for ΔT_{AMTD} is only an estimate of the true mean temperature difference. In other words, the usefulness of this metric is problem dependent. For example, if the temperature gradients for the two sides of the heat exchanger are linear as a function of length, similar to Figure 2-3(b), then it is appropriate to use this approach. Furthermore, previous research has shown that if the ratio of $(\Delta T_{AS1}/\Delta T_{AS2})$ is between 1 and 2.2, the errors introduced into the calculations by using ΔT_{AMTD} instead of ΔT_{LMTD} are within 5% [14, 15]. In other words, $(\Delta T_{AMTD}/\Delta T_{LMTD}) < 1.05$, noting that ΔT_{LMTD} is always smaller in magnitude. In the next chapter, it will be shown that for DWHR heat exchangers, this condition applies, which leads to significant simplifications in the modelling process.

Finally, the mean temperature difference in a heat exchanger can be represented using the log-mean temperature difference, ΔT_{LMTD} . This is the only method that is based on algebraic expressions derived using conservation of energy. Hence, by definition, ΔT_{LMTD} is the “best” representation for the mean temperature difference in a heat exchanger. It is worth noting that methods covered in most undergraduate heat transfer textbooks mainly focus on solution procedures that rely on ΔT_{LMTD} [16].

Using the terminology discussed so far, it is now possible to introduce the most common techniques for solving heat exchanger problems, which form the theoretical basis for this thesis.

2.1.4.4 LMTD Method

As the name suggests, this method relies on the concept of ΔT_{LMTD} that was introduced previously. Unsurprisingly, the governing equation quantifying the rate of heat transfer is simply Eq. 2.12.

$$\dot{q}_{HX} = UA\Delta T_{LMTD} \quad (2.12)$$

The caveat for using this method is buried in the physical meaning of ΔT_{LMTD} , which bears highlighting here. ΔT_{LMTD} is the maximum temperature potential possible in a counter flow heat exchanger. Clearly, modifications are needed to extend this method to other flow arrangements such as cross flow. Consequently, the rate equation is modified as shown:

$$\dot{q}_{HX} = UAF\Delta T_{LMTD} \quad (2.13)$$

where F is the correction factor that is assigned based on the heat exchanger. For counterflow heat exchangers, the value of F is 1, and Eqs. 2.12 and 2.13 become identical. The mathematical expression for F is given in Eq. 2.14. Several expressions have been derived for calculating F in a variety of heat exchangers and an extensive list can be found in [14]. Noting that DWHR heat exchangers are counterflow heat exchangers, the correction factor F is always equal to 1; hence, this correction factor is not used nor discussed after this point.

$$F = \frac{(\Delta T_{mean})_{actual}}{(\Delta T_{LMTD})_{counterflow}} \quad (2.14)$$

There are also multiple alternative expressions for ΔT_{LMTD} , which are only available in older textbooks. Eq. 2.15 lists the different forms of ΔT_{LMTD} for completeness.

$$\Delta T_{LMTD} = \frac{\Delta T_{AS1} - \Delta T_{AS2}}{\ln \left(\frac{\Delta T_{AS1}}{\Delta T_{AS2}} \right)} = \frac{\Delta T_{AS2} - \Delta T_{AS1}}{N_h - N_c} = \frac{T_{cold,out} - T_{cold,in}}{N_c} = \frac{T_{hot,in} - T_{hot,out}}{N_h} \quad (2.15)$$

where

$$N_h = \frac{UA}{\dot{C}_h} = \frac{UA}{\dot{m}_h c_{p,h}} \quad (2.16)$$

$$N_c = \frac{UA}{\dot{C}_c} = \frac{UA}{\dot{m}_c c_{p,c}} \quad (2.17)$$

2.1.4.5 ϵ -NTU Method

This method has been widely used to characterize heat exchangers for over fifty years [13, 14]. It characterizes performance based on heat exchanger effectiveness, ϵ , with respect to the number of transfer units, NTU, under specific operating conditions. Eqs. 2.18 and 2.19 show the main mathematical representations for ϵ and NTU respectively,

$$\epsilon = \frac{\dot{q}}{\dot{q}_{max}} \quad (2.18)$$

$$NTU = \frac{UA}{\dot{C}_{min}} \quad (2.19)$$

In relation to heat exchanger terminology covered previously, either N_h or N_c is labeled as NTU depending on which side corresponds to the lower heat capacity rate, $\dot{C}_{min}$. Consequently, the

expression for the maximum heat transfer rate, $\dot{q}_{max}$, is based on $\dot{C}_{min}$ as per Eq. 2.20. Hence, the expression for heat exchanger effectiveness is often shown in the form of Eq. 2.21.

$$\dot{q}_{max} = \dot{C}_{min} \Delta T_{max} = (\dot{m} Cp)_{minimum} (T_{hot,in} - T_{cold,in}) \quad (2.20)$$

$$\varepsilon = \frac{\dot{q}}{\dot{q}_{max}} = \frac{(\dot{m} Cp)_{cold} (T_{cold,out} - T_{cold,in})}{(\dot{m} Cp)_{minimum} \Delta T_{max}} = \frac{(\dot{m} Cp)_{hot} (T_{hot,in} - T_{hot,out})}{(\dot{m} Cp)_{minimum} \Delta T_{max}} \quad (2.21)$$

In essence, heat exchanger effectiveness is simply the ratio between the actual performance of the heat exchanger operating at a given condition, to the maximum thermodynamic performance possible at that same given condition. Note that the ΔT_{max} term is implicitly accounting for the 2nd Law of thermodynamics, and through this term, temperature bounds are imposed on the outlet streams from the heat exchanger. In other words: $T_{cold,out}$ is always less than or equal to $T_{hot,in}$ and similarly, $T_{hot,out}$ is always greater than or equal to $T_{cold,in}$. As a result, heat exchanger effectiveness ε is a number between 0 and 1.

Due to the similarities between the concepts of effectiveness and efficiency, the two terms are often used interchangeably; however, they are fundamentally different concepts and should not be confused with each other. The term efficiency is used when one form of energy is being converted to another form of energy, or, when the performance of a system is being compared to its ideal performance under the same conditions. On the other hand, the concept of heat exchanger effectiveness is based purely on the transfer of thermal energy from one side of a heat exchanger to the other side; there is no energy conversion from one form to another. Additionally, heat exchanger effectiveness compares the actual performance under specific conditions, to the maximum performance under the same conditions. Note that the conditions that would result in $\dot{q}_{max}$ for a given heat exchanger are neither ideal nor are they necessarily achievable. Put simply, the idea behind $\dot{q}_{max}$ assumes infinite surface area for the heat exchanger sides, which is fictitious. Clearly, efficiency and heat exchanger effectiveness are conceptually different ideas that should not be confused.

Another common misconception about the term effectiveness is when heat exchanger effectiveness is confused with temperature effectiveness. Recall that heat exchanger effectiveness is the ratio of actual heat transfer to the maximum heat transfer, the simplified equation for which is repeated in Eq. 2.22. Note that both the numerator and the denominator contain a product of heat capacity rate and a temperature difference. This form represents heat exchanger effectiveness. Given this equation, it is now easier to introduce the concept of temperature effectiveness, as these terms are directly related.

Temperature effectiveness is simply the ratio of temperature differences, which is specific for each side of the heat exchanger. Thermal effectiveness for the cold and hot side of a single-pass heat exchanger are represented using Eqs. 2.23 and 2.24 respectively.

$$\varepsilon = \frac{q}{q_{max}} = \frac{\dot{C}_{cold} \Delta T_{cold}}{\dot{C}_{min} \Delta T_{max}} = \frac{\dot{C}_{hot} \Delta T_{hot}}{\dot{C}_{min} \Delta T_{max}} \quad (2.22)$$

$$\varepsilon_c = \frac{\Delta T_{cold}}{\Delta T_{max}} \quad (2.23)$$

$$\varepsilon_h = \frac{\Delta T_{hot}}{\Delta T_{max}} \quad (2.24)$$

Furthermore, temperature effectiveness terms are connected through energy balance; see Eq. 2.25.

$$\varepsilon_h \Delta T_{hot} = \varepsilon_c \Delta T_{cold} \quad (2.25)$$

The misconception likely stems from the fact that many heat exchangers are single-pass and are operated under balanced-flow conditions. Balanced flow implies that the heat capacity rates are equal on both sides of the heat exchanger (i.e. $\dot{C}_{min} = \dot{C}_{max}$). Therefore, for balanced flow conditions, heat exchanger effectiveness is equal to the temperature effectiveness for both the cold and hot side. In other words, if $\dot{C}_{min}$ equals $\dot{C}_{max}$, Eq. 2.26 is valid.

$$\varepsilon = \varepsilon_h = \varepsilon_c \quad (2.26)$$

The term effectiveness is used frequently in this thesis, and it should always be taken as heat exchanger effectiveness ε . The concept of temperature effectiveness is useful when dealing with heat exchangers with multiple passes, specifically shell-and-tube heat exchangers. The solution method that is based on this concept is referred to as P-NTU method in the literature, where P stands for the temperature effectiveness. The heat transfer rate equation in terms of P is shown in Eq. 2.27.

$$\dot{q} = P_1 \dot{C}_1 \Delta T_{max} = P_2 \dot{C}_2 \Delta T_{max} \quad (2.27)$$

where subscripts 1 and 2 correspond to the sides of the given heat exchanger. Allegedly, the P-NTU method was established first, and the ε -NTU method is a variation of it which was popularized in the 1940s [13, 14]. To reiterate, the ε -NTU method should not be confused with P-NTU method, as they are distinct methods which define the term effectiveness differently. DWHR heat exchangers are not multi-pass heat exchangers, nor can they be considered as shell-and-tube heat exchangers. Hence, the concept of temperature effectiveness is not used in this study.

2.1.5 DWHR Heat Exchangers vs. Industry Standards

The predominant method for marketing DWHR heat exchangers is ε -NTU. This is due to the simplicity of the method and the calculation procedure for calculating heat exchanger effectiveness. As a reminder, to calculate heat exchanger effectiveness, three temperature measurements and two flow rate measurements are required as per Eq. 2.21. This equation is even further simplified when used to measure performance of heat exchangers sold on the market.

The modified approach is developed by the Canadian Standards Association and mandated by the CSA standard B55.1-15 [11] for rating performance of DWHR systems. This approach is described in detail here, because the results generated through it are used directly in the steady-state model presented later in Section 2.3. In this approach, the properties of the fluid streams are assumed to be constant, and the flow rates through both sides of the heat exchanger are set to be equal. In other words:

$$(\dot{m}C_p)_{hot} = (\dot{m}C_p)_{cold} = (\dot{m}C_p)_{min} = (\dot{m}C_p)_{max} = \dot{m}C_p \rightarrow \frac{C_{min}}{C_{max}} = 1 \quad (2.28)$$

As a result, Eq. 2.21 is reduced to a simple ratio in terms of the fluid temperatures as follows:

$$\varepsilon = \frac{\dot{q}}{\dot{q}_{max}} = \frac{(\dot{m}C_p)_{\varepsilon}(T_{c,o} - T_{c,i})}{(\dot{m}C_p)_{min}(T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})} \quad (2.29)$$

The CSA standard mandates the use of Eq. 2.29 to measure the effectiveness of DWHR systems as a function of the volumetric flow rate, $\dot{V}$. The effectiveness is measured for a mains-side inlet temperature of $12 \pm 0.4^\circ\text{C}$, and a drain-side inlet temperature of $28 \pm 0.6^\circ\text{C}$ above the mains-side temperature, and at 6 volumetric flow rates: 5.5, 7, 9, 10, 12 and 14 L/min. A curve fit of the form shown in Eq. 2.30 is then fit to the data points. Here, a (min/L) and b (dimensionless) are fit coefficients, and $\dot{V}$ is the volumetric flow rate in L/min.

$$\varepsilon = \frac{1}{a\dot{V} + b} \quad (2.30)$$

The curve fit is then used to produce a single rated effectiveness value at 9.5 L/min, which is the metric used for selling DWHR systems on the market. This effectiveness value is what gets printed on the sales labels for these heat exchangers, a sample of which is shown in Figure 2-6. Unfortunately,

the rated conditions are not representative of what occurs during operation in a typical house and the reported number on the sales label is inapplicable for predicting savings.



Figure 2-6: A typical sales label attached to a DWHR system.

2.2 Literature Review

DWHR systems have only become popular over the past decade and as such, the studies covering their performance are few and far between. Many of the studies have focused on showing the reduction in water heating demands resulting from installing DWHR systems. A study by the Manitoba Advanced House in Winnipeg, estimated that 50% of a typical family's annual domestic hot water load could be recovered [22], and subsequent study by the Oak Ridge National Labs produced similar results [23]. Later research conducted by the Canadian Centre for Housing Technology examined DWHR systems and concluded that gas consumption for water heating could be reduced by 9 to 27% depending on system configuration [24]. Other studies conducted in Canada and the Netherlands support these results [25, 26].

Each of the previously mentioned studies focused only on proving the feasibility of using DWHR systems to reduce energy consumption. None attempted to produce performance models for DWHR systems. Theoretical and numerical approaches for characterization of DWHR systems are often not pursued due to the complexity of the falling-film. A recent joint graduate thesis by Bengtsson et al. attempted to develop a computational fluid dynamics (CFD) model for predicting the behaviour of the falling-film in a similar heat exchanger for a range of flow rates between 1 and 3 L/min [27]. The results for this study were presented in terms of the UA , which was experimentally shown to be a strong function of the flow rate. Unfortunately, the CFD model could not be validated as the UA values produced by the CFD model were less than half of the UA values measured experimentally, with the difference growing larger at increased flow rates. Clearly, further research is required before DWHR systems can be analysed in this way; hence, most of the studies in the literature are based on experimental approaches instead.

Due to the limited number of peer-reviewed research papers available on the subject, each of the major publications will be described in its own subsection, where its results and limitations are highlighted. This helps identify the gaps in the literature that need to be addressed. An extensive review of the literature pertaining to DWHR heat exchangers was published in the author's MASc thesis and is available in [28]. The studies presented in this section are focused on the most recent publications that attempt to model the performance of DWHR systems. Last but not least, significant efforts have been made to address this issue at the University of Waterloo through a series of studies, which will be described in Section 2.3.

2.2.1 Study #1 - Possibility of heat recovery from gray water in residential building [29]

This study focused on the financial benefits of installing DWHR systems in a detached house, as a function of the number of occupants and the duration for shower events. The authors investigated the financial benefits of DWHR systems in three different heating systems, referred to as variants 0 to 2. The description for each variant is as follows:

- Variant 0: traditional installation with condensing gas boiler with hot water storage tank
- Variant 1: installation of condensing gas boiler with hot water storage tank additionally equipped with vertical pipe heat exchanger DWHR1 (see Table 2-1)
- Variant 2: installation with condensing gas boiler with hot water storage tank additionally equipped with vertical pipe heat exchanger DWHR2 (see Table 2-1)

The authors performed lifecycle analyses for these variants based on the assumptions shown in Table 2-1, while accounting for the cost of energy, pipes, etc. at the time the work was published. The plumbing setup used for the analyses is shown in Figure 2-7, and the lifecycle cost analysis was done over a 20-year period. The results showed that the use of a DWHR system in a single-family house reduced the demand for domestic hot water by 23 to 31%, which translated to a cost savings of up to €2539 (3805 CAD) in a period of 20 years. The authors attributed the higher savings to families with larger number of occupants, who showered for longer durations.

Table 2-1: Simulation parameters in Study#1 [29].

Parameter	Parameter value
The number of occupants	3,4,5 persons
Daily hot water consumption in washbasins	6 L/person/day
Daily hot water consumption for cooking	2 L/person/day
Daily hot water consumption for cleaning	3 L/person/day
Daily hot water consumption for washing the dishes	5 L/person/day
Hot water temperature	55°C
Cold water temperature T_c	10°C
The temperature of the mixed water flowing out of the showerhead	40°C
Mixed water flow from the showerhead	9 L/min
Shower length	4,6,9 min/person/day
Performance of DWHR1 heat exchanger	52,7%
Performance of DWHR2 heat exchanger	60,1%
Water loss coefficient in the shower and tubes x	0,95
The efficiency of the hot water preparation system	68%

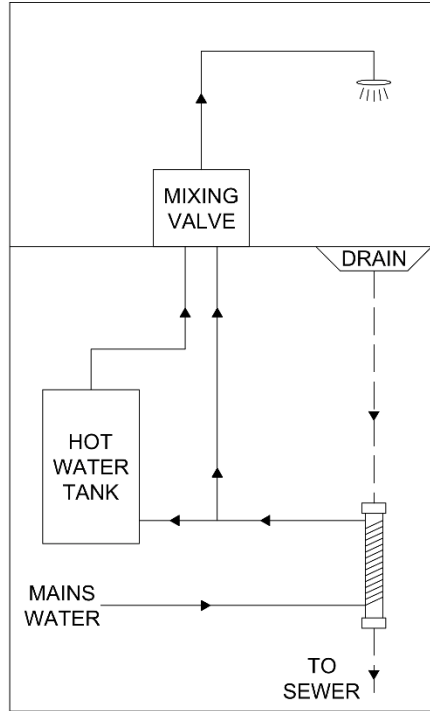


Figure 2-7: Plumbing configuration simulated in Study#1.

There are limitations and assumptions in this study that significantly affected the outcomes. The authors have neglected the transient effects, which have large impacts on the shorter shower durations in this study. Also, it is unclear whether or not the calculations accounted for concurrent events (e.g. the shower and kitchen sink being used simultaneously). Most importantly, some of the assumptions made in this study significantly limit the applicability of the results to a realistic installation. The plumbing system shown in Figure 2-7 is set up in such a way to force the DWHR system to always operate with equal flow rates through both sides of the heat exchanger, which is often not how these units are installed. The inlet temperatures were also fixed for a 20-year period, and the presence of any other plumbing fixture was omitted to avoid accounting for unequal flow conditions. Based on the reported simulation parameters, the authors should not have assumed constant effectiveness values of 52.7% and 60.1% for DWHR1 and DWHR2, respectively. The combination of the assumptions made by the authors allowed them to produce preliminary estimates for energy savings; however, the assumptions were unrealistic and results were not a good representation of the expected savings in real installations.

2.2.2 Study #2 - Characterization of greywater heat exchangers and the potential of implementation for energy savings [30]

This document is a master's thesis written by Jose Garcia in 2016. This work was published one year after Manouchehri's master's thesis [28], and due to the commonalities between the two theses, the limitations quickly became apparent. In this work, a theoretical approach was taken for estimating performance of DWHR systems, and experimental data from a study by Collins [31] were used to validate the theoretical approach. The theoretical approach was based on the thermal resistor network shown in Figure 2-8. Each resistor in the network was solved using relevant heat transfer correlations, and the results were compared to Collins' experimental results. Garcia has also developed a MATLAB GUI (Graphical User Interface), shown in Figure 2-9, where relevant information such as inlet temperatures, pipe diameter, energy price could be entered into the software to calculate savings. The author used this approach to show that higher savings were associated with larger households.

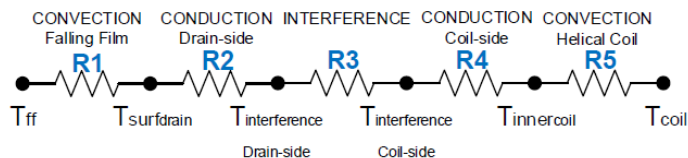


Figure 2-8: Thermal resistor network used to characterize DWHR systems in Study#2 [30].

Figure 2-9: MATLAB GUI for the energy calculator as appeared in Study#2 [30].

There are notable limitations and assumptions in this study that significantly affected the results. The most significant limitation of this work was the omission of unequal flow conditions. As described previously, DWHR systems often operate under unequal flow conditions, and it is not possible to produce realistic energy saving estimates while ignoring such conditions. Furthermore, the author assumed the effectiveness at each flow rate to be constant, regardless of inlet temperatures. This has been proven to be a false assumption [32] which could lead to large errors when constant effectiveness values are used to perform financial analyses over many years. Lastly, the MATLAB component of the model is not available as a part of the thesis package, and based on the interface from Figure 2-9, many of the required inputs for solving the thermal resistor network appear to be missing. A few of the missing variables include: the type of copper tubing (which determines its wall thickness), the hydraulic diameter of copper tubing, and the overall length of the drainpipe that is covered with the coils (always shorter than drainpipe). Overall, the impact of the assumptions made by the author, and the limitations of the work make it difficult to accept the results as a good representation of the expected energy savings in real installations.

2.2.3 Study #3 - Results of Vertical Drain Water Heat Recovery Phase Two Lab Testing and Data Analysis [33]

This study attempted to produce a method for predicting the effectiveness of DWHR systems under different flow conditions. Four different DWHR systems were tested in this study, with constant inlet temperatures of 10 and 38°C on the mains and drain sides respectively. The flow rates for the experiment were between 1 and 9 gallon/min (3.8 and 30.3 L/min).

For each DWHR system, a curve in the form of Eq. 2.31 was fit to the measured data, where the terms A, B, C, etc. are regression coefficients. These results were then used to develop a generic algorithm for predicting effectiveness of DWHR systems, though the details of this algorithm are not clearly defined in the study. The generic algorithm was used to estimate each system's performance under equal-flow conditions, as shown in Figure 2-10. Note that previous studies [34] have shown the falling-film breaks apart at low flow rates such as 1 GPM, and it unclear whether this was accounted for when the generic algorithm was devised.

$$\begin{aligned}\varepsilon &= (a\dot{V}_c^3 + b\dot{V}_c^2 + c\dot{V}_c + d)(e\dot{V}_h^3 + f\dot{V}_h^2 + g\dot{V}_h + h) \\ &= A\dot{V}_c^3\dot{V}_h^3 + B\dot{V}_c^3\dot{V}_h^2 + C\dot{V}_c^3\dot{V}_h + D\dot{V}_c^3 + E\dot{V}_c^2\dot{V}_h^3 + F\dot{V}_c^2\dot{V}_h^2 + G\dot{V}_c^2\dot{V}_h + H\dot{V}_c^2 + I\dot{V}_c\dot{V}_h^3 \\ &\quad + J\dot{V}_c\dot{V}_h^2 + K\dot{V}_c\dot{V}_h + L\dot{V}_c + M\dot{V}_h^3 + N\dot{V}_h^2 + O\dot{V}_h + P\end{aligned}\tag{2.31}$$

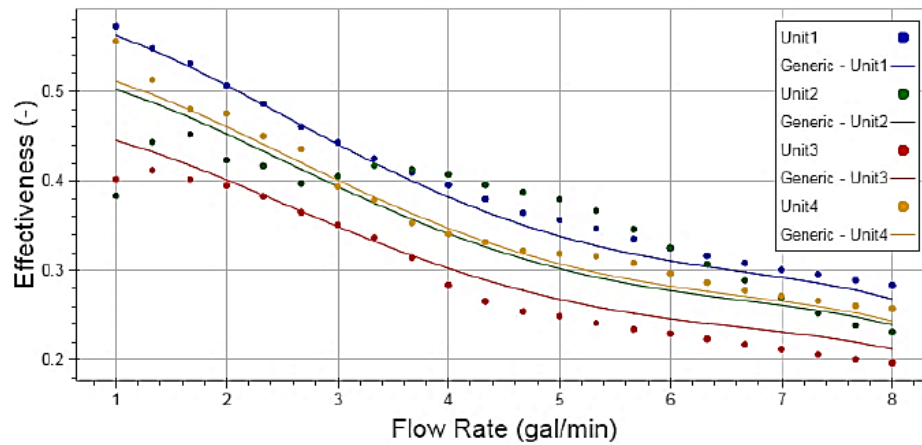


Figure 2-10: Comparison between measured and predicted effectiveness values in Study#3 [33].

Based on the figure, the generic algorithm is unable to consistently and accurately predict the effectiveness under equal-flow conditions for all DWHR systems. It is worth noting that the CSA rating standard discussed in Section 2.1.5 already provides the characteristic effectiveness vs. flow rate curve for DWHR systems under equal flow conditions, and it is unclear why the author chose to develop a new algorithm to regenerate that curve. Furthermore, it is uncertain if the algorithm is capable of estimating effectiveness under unequal flow rates, which was the intent of using Eq. 2.31, as no validations were done in this study. It is not possible to assess the validity of this predictive algorithm due to insufficient details and the absence of any experimental validation in the work. This study was included here because the results from this work were used to showcase the benefits of using DWHR systems to lawmakers in California, as these heat exchangers are new in the California Energy Code.

2.2.4 Study #4 - The impact of using of a DWHR heat exchanger on operating costs for a hot water preparation system and the amount of carbon dioxide emissions entering the atmosphere [35]

This article analyzed the impact of DWHR systems on the operating costs and CO₂ emissions associated with water heating. Six different water heating methods were investigated which included an electric heater, a gas boiler, a biomass boiler, a heating oil boiler, a coal boiler and an air-to-water heat pump. The effectiveness of the DWHR system was assumed to be constant at 57.6%, and simulations were performed for different number of occupants, with different shower durations. Figure 2-11 shows the annual simulation results for the reductions in CO₂ emissions due to

installation a DWHR system. Counterintuitively, the emissions associated with heating systems powered by electricity were much higher than burning fuels on site, which the author attributed to the relatively high emission rates from electricity generation in power plants in that location. Lastly, the simulations showed that implementation of a DWHR system resulted in annual savings of 20-30%.

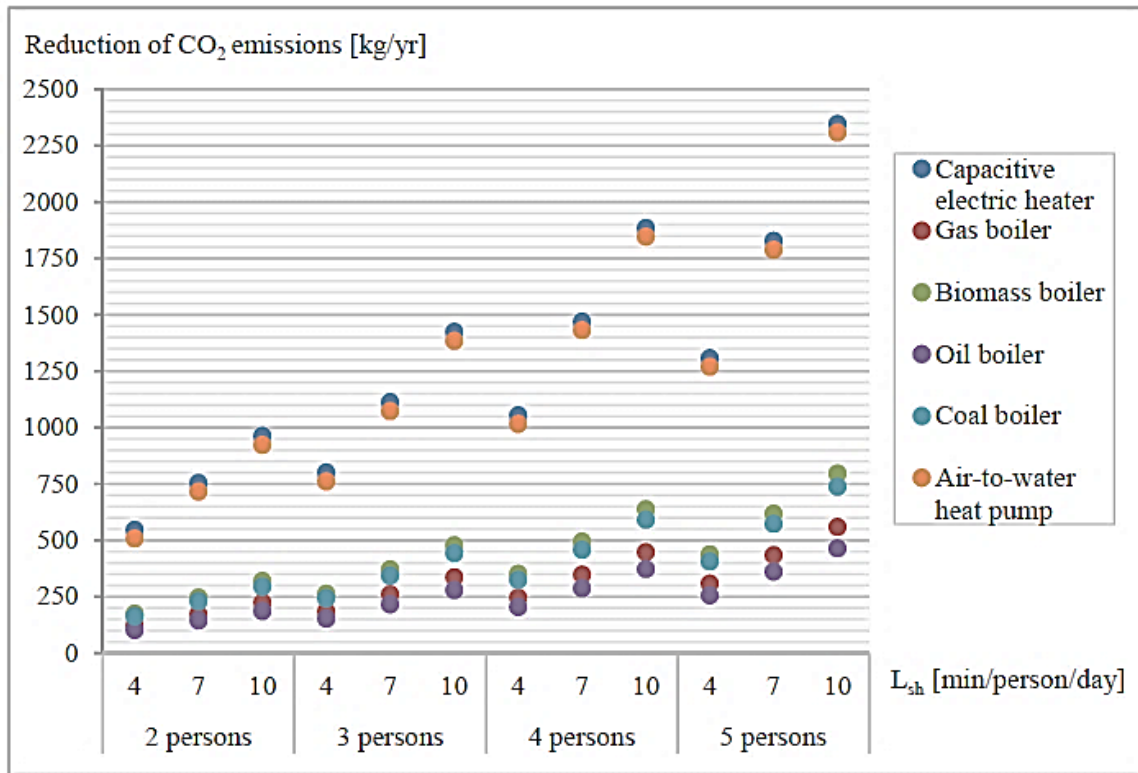


Figure 2-11: Annual reduction in CO₂ emissions associated with DWHR systems for six different water heating methods as predicted in Study#4 [35].

This study has done well in showing the potential positive environmental impacts of using DWHR heat exchangers; however, the assumptions made for simulating the performance of the DWHR system were unrealistic. As discussed earlier, unequal-flow conditions are very common, and assuming a constant effectiveness for all conditions throughout a year would not generate realistic results.

2.3 University of Waterloo's Contributions

As discussed, the most recent publications that attempted to model the performance of DWHR systems have all made assumptions that significantly limit their results, rendering them unrealistic. The lack of a reliable model capable of predicting the performance of DWHR systems has hindered progress in this field, and consequently, accurate payback periods cannot be estimated to better market these products.

Significant efforts have been made to address this issue at the University of Waterloo through a series of studies. The first study aimed to develop the characteristic effectiveness vs. flow rate curves for multiple DWHR systems [31], while others investigated the impacts of drain-side wetting [34], off-vertical system installation [36], variable inlet temperatures [32] and unequal flow rates [37]. The results from these studies were combined into a semi-empirical model capable of predicting the steady-state performance of DWHR systems using data obtained from the CSA's rating process [28, 38]. This model was developed during the author's master's research, and the essential parts of it will be briefly summarized in the next section. There are significant limitations associated with the said model that were never addressed in the past, which will be highlighted as well.

The experimental work for this project was done at the Solar Thermal Research Laboratory, where an apparatus for testing the performance of DWHR systems was built. The apparatus is capable of measuring and recording the inlet and outlet water temperatures to within $\pm 0.1^\circ\text{C}$ on both sides of the heat exchanger. Tests can be performed for volumetric flow rates of up to 20 L/min, and each flow meter has an accuracy of $\pm 1\%$ of the reading. The system is set up to allow testing both under equal flow, and unequal flow conditions. A detailed description of this apparatus was published in the author's master's thesis [28] and is available in Appendix A. It is noted that the majority of the CSA testing performed in Canada has been done on this apparatus.

2.3.1 Model Development

Based on the author's experiences in testing DWHR systems, a multistep strategy was laid for using data obtained from the rating process to estimate steady-state performance. In short, one must first estimate the equal flow effectiveness for the DWHR system, correct it for the inlet fluid temperatures, and if needed, use this effectiveness to predict the unequal flow performance. Each step is briefly described in the following sections. Full details of the experiments and model validations can be found in Manouchehri's MAsC thesis [28].

2.3.2 Estimating the Rated Equal-Flow Effectiveness Curve

To estimate the equal flow effectiveness curve, some data related to the DWHR system performance is needed. Two likely data sources are presented:

- 1) The user may have access to CSA test data from a system manufacturer, or from CSA reports. If available, this would contain the equal flow effectiveness curve-fit in the form of Eq. 2.30 (*i.e.* $\varepsilon = 1/(a\dot{V} + b)$).
- 2) The user may have equal flow effectiveness measurements taken at the CSA test flow rates of 5.5, 7, 9, 10, 12 and 14 L/min, or similar data produced from other sources. The equal flow effectiveness curve can be developed by fitting Eq. 2.30 to this data. Care must be taken when doing this. At low flow rates, the falling-film on the drain-side becomes unstable, and as a result, the steady-state performance varies between DWHR systems. Therefore, when generating the curve fit, the user is advised not to use any data obtained at flow rates below 5.5 L/min, or below 7 L/min for DWHR systems with diameters of 10.2 cm or larger.

Option (2) is the process specified in the CSA standard to generate the equal flow effectiveness curve mentioned in Option (1). Figure 2-12 shows an example of the curve fit for a 7.6 cm diameter, 153 cm long DWHR system.

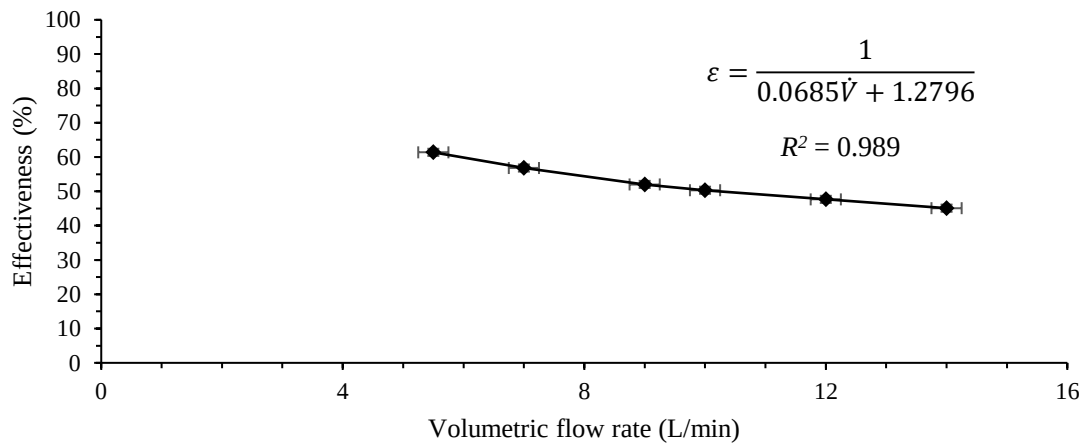


Figure 2-12: Equal flow effectiveness vs. volumetric flow rate for a 7.6 cm diameter, 153 cm long DWHR system.

2.3.3 Temperature Adjustment of the Equal Flow Effectiveness Curve

During previous experimentation, it was observed that varying inlet temperatures had an impact on the measured effectiveness for DWHR systems. At the time when the effects of inlet temperatures

were investigated, CSA standard B55.1-12 [39] was used for the rating process, which allowed a wider range of inlet temperatures compared to the new version of the CSA standard [11]. The old standard allowed mains-side temperature of $12 \pm 5^\circ\text{C}$, and a drain-side at $28 \pm 1^\circ\text{C}$ above the mains-side temperature. Within the bounds of the old standard, if one were to test the effectiveness of a DWHR system at a mains-side inlet temperature of 7°C and drain-side inlet temperature of 35°C , and then again at 17°C and 45°C , the effectiveness would vary by as much as 2%. To investigate this, a theoretical analysis of heat transfer in a DWHR system was conducted and successfully used to predict the impact of inlet temperatures on system effectiveness [28]. This work showed that the change in effectiveness is primarily due to changes in dynamic viscosity with temperature, and the resulting impact of this on heat transfer coefficients. It was evident that the inlet temperatures used during the production of the equal flow effectiveness curve were important, which necessitated a correction of the curve to suit the conditions being studied by the user.

Knowledge of the inlet temperatures used to produce the equal flow effectiveness curve is likely not an issue. For CSA rated data, the actual temperatures measured during testing are required as part of the CSA test report, and those should be available to the user. For user-produced data, the temperatures would be known.

A temperature correction could be developed based on the aforementioned theoretical analysis, but it was deemed to be overly complex. Instead, it was decided that a simpler correlation could be developed based on experimental data. To do so, four DWHR systems were studied: two 7.6 cm diameter systems with lengths of 122 and 153 cm, and two 10.2 cm diameter systems with lengths of 122 and 153 cm. Performance tests were done under equal flow conditions at flow rates of 5.5, 9.5 and 14 L/min, and inlet temperatures were selected to be 25, 30, 35, 40 and 45°C on the drain-side, and 5, 10, 15 and 20°C on the mains-side. A minimum temperature difference of 10°C was maintained between the two inlets to minimize experimental uncertainties. A sample of the results is shown in Figure 2-13 for the 7.6 cm diameter, 122 cm long system tested at a flow rate of 9.5 L/min. More details about these tests can be found in a previous publication by Manouchehri and Collins [32], as well as in Manouchehri's MASc thesis [28].

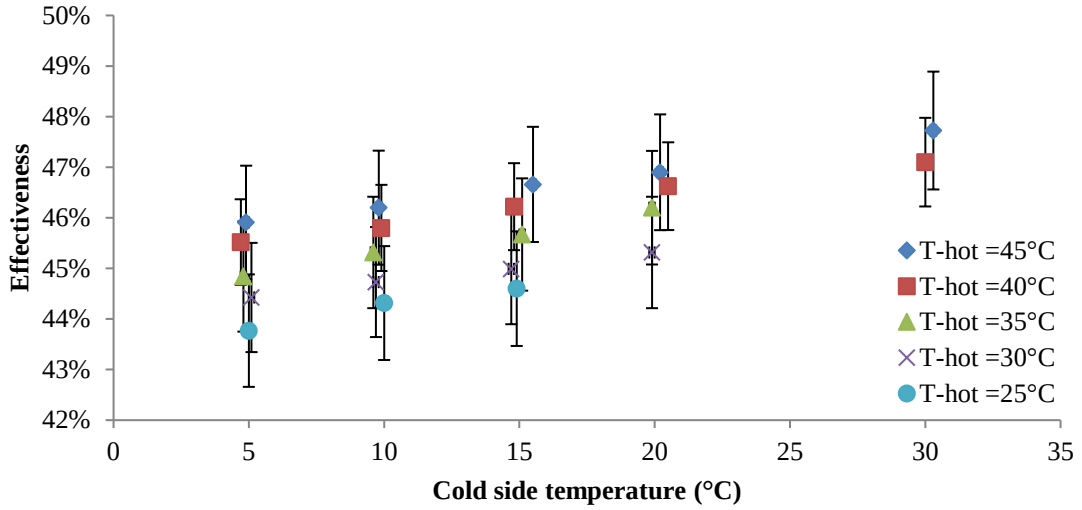


Figure 2-13: Measured effectiveness values for the 7.6 cm diameter, 122 cm long DWHR system, at 9.5 L/min [28].

In all cases, the measured system effectiveness changed with inlet temperature in a predictable way. At a constant drain-side temperature, the effectiveness increased linearly with increasing mains-side temperature. The same behaviour was observed when drain-side temperature was increased linearly at a constant mains-side temperature. It was decided, therefore, to develop a linear correction factor with interaction. The correction factor was to take the form shown in Eq. 2.32.

$$F_C = \frac{\varepsilon}{\varepsilon_{ref}} = A \cdot T_{h,i} \cdot T_{c,i} + B \cdot T_{h,i} + C \cdot T_{c,i} + D \quad (2.32)$$

In this equation, ε is the equal flow effectiveness at a set of known inlet temperatures, ε_{ref} is the equal flow effectiveness determined at a set of inlet reference temperatures, and F_C is the correction factor. A ($1/^\circ\text{C}^2$), B ($1/^\circ\text{C}$), C ($1/^\circ\text{C}$), and D (dimensionless) are fit coefficients. The reference temperatures were chosen to be at $T_{h,i} = 40^\circ\text{C}$ and $T_{c,i} = 10^\circ\text{C}$. Further examination of the data revealed that when applied to each data set, nearly identical correlations were produced. Therefore, a single temperature correction was produced using all of the data for the four DWHR systems tested. See Eq. 2.33.

$$F_C = \frac{\varepsilon}{\varepsilon_{ref}} = \left(\frac{2.37 \times 10^{-6}}{^\circ\text{C}^2} \right) \cdot T_{h,i} \cdot T_{c,i} + \left(\frac{1.75 \times 10^{-3}}{^\circ\text{C}} \right) \cdot T_{h,i} + \left(\frac{1.24 \times 10^{-3}}{^\circ\text{C}} \right) \cdot T_{c,i} + 0.917 \quad (2.33)$$

Here, all T_s are in °C. It is important to note that fit coefficients in Eq. 2.33 are specific to the previously specified reference temperatures of $T_{h,i} = 40^\circ\text{C}$ and $T_{c,i} = 10^\circ\text{C}$. This equation is used within the model to account for different inlet temperatures that a DWHR system may experience during operation.

2.3.4 Performance at Unequal Flow Conditions

The final step needed for determining performance was to examine conditions of unequal flow. To do this, 5 DWHR systems were studied: one 5.1 cm diameter system with a length of 122 cm, two 7.6 cm diameter systems with lengths of 122 and 153 cm, one 10.2 cm diameter system with a length of 122 cm, and one 7.6 cm diameter system with a length of 102 cm from a different manufacturer than the other 4 systems. Tests were performed at inlet temperatures of $38 \pm 1^\circ\text{C}$ and $10 \pm 1^\circ\text{C}$ on the drain-side and the mains-side, respectively. The test flow rates through the two sides of the heat exchanger were combination pairs based on the CSA standard flow rates of 5.5, 7, 9, 10, 12 and 14 L/min. Except for the 10.2 cm diameter system, 36 flow combinations were tested on each system. For the 10.2 cm diameter system, a flow rate of 5.5 L/min was not considered on either side of the heat exchanger, which reduced the flow combinations to 25. A sample of the results is shown in Figure 2-14 for a 7.6 cm diameter and 122 cm long system. More details about these tests can be found in a publication by Manouchehri and Collins [37], as well as in Manouchehri's MASc thesis [28].

At this point, it was decided to move from an examination in terms of effectiveness, to an examination in terms of heat transfer rate, $\dot{q}$. This was done for three reasons. The first was that this conversion would be needed by the user at some point in order to calculate energy savings. The second was to simplify the development of a correlation. Examination of Figure 2-14 (top) shows that an inflection point occurs in the effectiveness curve when the flow rates for the drain-side and mains-side are equal. By switching to heat transfer rate, Figure 2-14 (bottom), the inflection point is removed. The third reason was that higher effectiveness does not necessarily represent higher savings. Hence, it was concluded that presenting discussion in terms of effectiveness could be highly misleading. Note that the calculation of heat transfer rate assumed constant density ($\rho = 1000 \text{ kg/m}^3$) and specific heat ($C_p = 4.18 \text{ kJ/kg}^\circ\text{C}$) for water.

Several attempts were made to reduce the data shown in Figure 2-14 to a correlation. Eventually, it was found that the data was best represented by:

$$\dot{q}_{(\dot{V}_c=F1 \ \& \ \dot{V}_h=F2)} = \dot{q}_{(\dot{V}_c=F1 \ \& \ \dot{V}_h=F1)} \times \left[M \times \ln \left(\frac{\dot{V}_h = F2}{\dot{V}_c = F1} \right) + N \right] \quad (2.34)$$

Here $\dot{q}_{(\dot{V}_c=F1 \ \& \ \dot{V}_h=F2)}$ is the heat transfer rate measured at the unequal flow condition,

$\dot{q}_{(\dot{V}_c=F1 \ \& \ \dot{V}_h=F1)}$ is the heat transfer rate measured at the equal flow condition, $F1$ is the flow rate through the coils, $F2$ is the flow rate of the falling film, and M and N are dimensionless constants determined experimentally through regression. Figure 2-15 shows the application of Eq. 2.34 to the data from Figure 2-14.

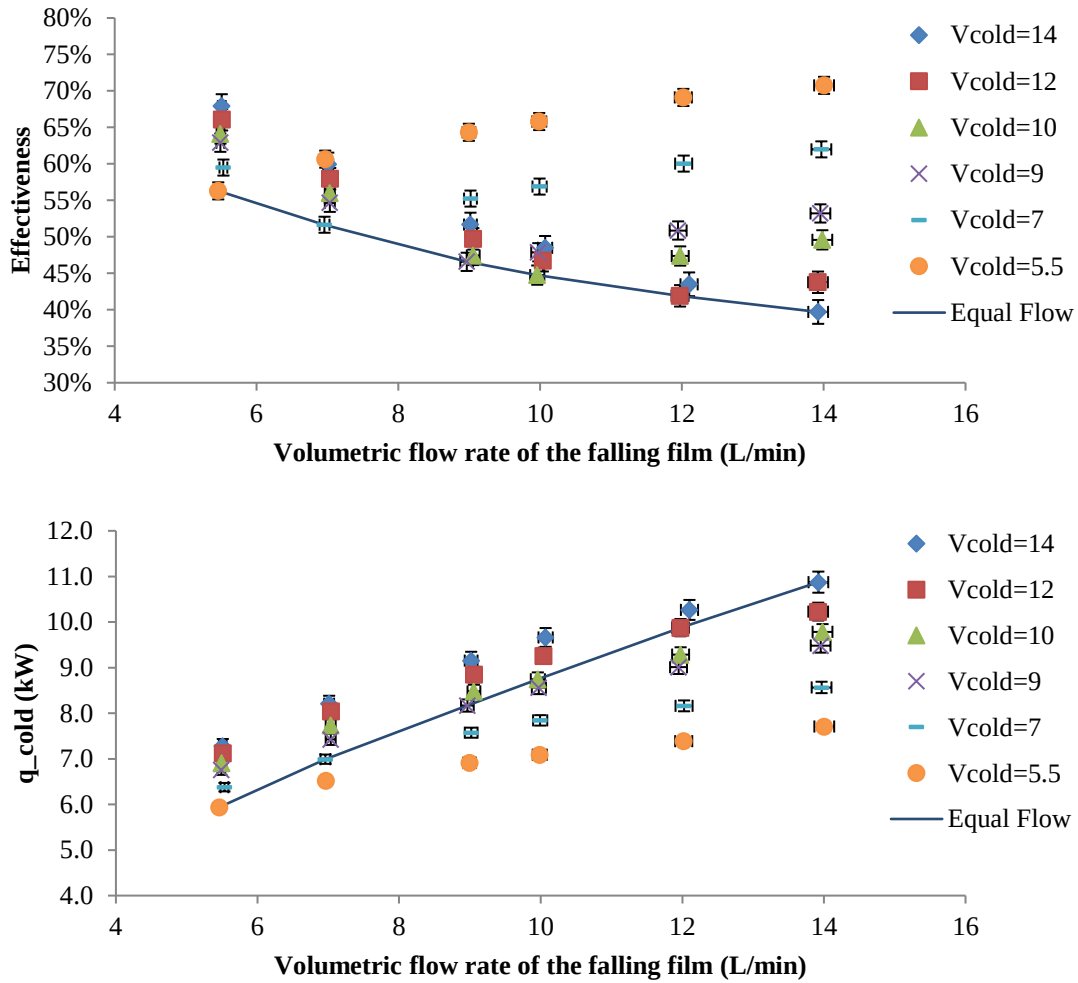


Figure 2-14: Mains-side effectiveness (top) and heat transfer rate (bottom) vs. volumetric flow rates for the 7.6 cm diameter, 122 cm long DWHR system [28].

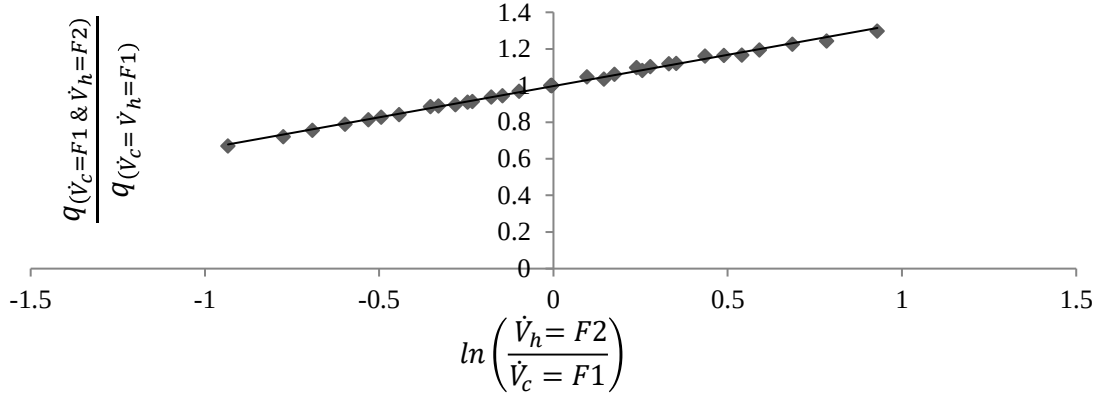


Figure 2-15: Normalized heat transfer data for the 7.6 cm diameter, 122 cm long system [28].

The constant N would ideally be 1 so that the equal flow heat transfer rate would be obtained when the drain-side and mains-side flow rates are equal. Similar to the temperature-correction scheme discussed previously, this correlation turned out to be independent of DWHR system length, diameter, or coil design. Therefore, the data from all 5 DWHR systems were combined to develop the final correlation shown in Eq. 2.35.

$$\dot{q}_{(\dot{V}_c=F1 \ \& \ \dot{V}_h=F2)} = \dot{q}_{(\dot{V}_c=F1 \ \& \ \dot{V}_h=F1)} \times \left[0.3452 \times \ln\left(\frac{\dot{V}_h = F2}{\dot{V}_c = F1}\right) + 1 \right] \quad (2.35)$$

As discussed previously, it is not recommended to present the final results in terms of effectiveness because it could be misleading in the context of unequal flow performance for DWHR systems. However, if necessary, the heat transfer rate calculated in Eq. 2.35 can easily be converted into an effectiveness value using Eq. 2.29.

2.3.5 Overall Model

Using the methods presented so far, it is possible to predict the steady-state performance of a DWHR system based on equal flow data. The overall model combines these methods in the order presented in this section. A complete example will be presented later in Section 3.1. The steps for applying the model are as follows:

- 1) Obtain the equal flow effectiveness curve. The curve may be directly available from CSA reports. If not, fit a curve of best fit in the form of Eq. 2.30 to the equal flow data from CSA or an equivalent rating process. The user is advised not to use any data obtained at flow rates below 7 L/min for DWHR systems with diameters of 10.2 cm or larger, or flow

rates below 5.5 L/min for DWHR systems with smaller diameters. Use this curve fit to calculate the equal flow effectiveness at the desired mains-side flow rate.

- 2) Adjust the calculated effectiveness to represent the input temperatures being considered using Eq. 2.33. This is a 2-stage process. If the effectiveness curves determined in step (1) are not taken from the reference condition of $T_{h,i} = 40^\circ\text{C}$ and $T_{c,i} = 10^\circ\text{C}$, then Eq. 2.33 must first be used to determine ε_{ref} . Following this, Eq. 2.33 is used again to calculate ε at the inlet temperatures of interest.
- 3) A conversion from effectiveness to heat transfer rate is required. Convert using:

$$\begin{aligned}\dot{q} &= \varepsilon \times \dot{q}_{max} = \varepsilon C_{min}(T_{h,i} - T_{c,i}) \\ &= \varepsilon(\rho \dot{V} C_p)_{min}(T_{h,i} - T_{c,i}) = \frac{4180 \times \dot{V} \times \varepsilon \times (T_{h,i} - T_{c,i})}{60000}\end{aligned}\quad (2.36)$$

Where $\dot{V}$ is the flow rate in L/min, the heat transfer rate, $\dot{q}$, is in kW, and T s are in $^\circ\text{C}$. The numbers in Eq. 2.36 assume constant fluid density ($\rho = 1000 \text{ kg/m}^3$) and specific heat ($C_p = 4.18 \text{ kJ/kg}^\circ\text{C}$) for water. The 60000 in the denominator is to convert $\dot{V}$ from L/min into m^3/s .

- 4) If required, find the unequal flow heat transfer rate using Eq. 2.35.

This model was developed for DWHR systems with mains-side inlet temperatures between 5°C and 20°C , drain-side inlet temperatures between 25°C and 45°C , and flow rates between 5.5 L/min and 14 L/min except for systems with diameters of 10.2 cm or larger, where the valid flow rates are between 7 and 14 L/min. The preliminary validation for the model in this range of flow rates was presented at the Canadian eSim conference in 2016 [38].

2.3.6 Existing Model's Limitations

As summarized in the previous paragraph, the model was developed for a very limited range of inlet flow rates and temperatures. The performance of DWHR systems is a strong function of inlet flow rates, and the range of flow rates chosen for model development were adapted from those covered in the CSA standard [11].

The range of flow rates for which the model was validated is insufficient to cover what occurs in many plumbing systems where DWHR systems would be installed. To clarify this statement, Figure

2-16 was created to show three distinct plumbing systems containing a DWHR heat exchanger. The three variations shown in the figure are simplified to only contain a showerhead for water-draw purposes. To better represent a realistic case scenario, each showerhead is equipped with a thermostatic mixing valve. In the figure, T is the temperature, $\dot{V}$ is the volumetric flow rate, and the fluid properties are assumed to be constant. Figure 2-16(A) depicts a system where all preheated water from the DWHR system goes to the water heater, Figure 2-16(B) depicts a system where all preheated water goes to the mixing valve at the showerhead, and Figure 2-16(C) shows a combination of the two previous cases.

For all three cases shown in Figure 2-16, and considering that a typical showerhead has a flow rate of 9.5 L/min (2.5 gallons/min), if two or more shower events occur simultaneously, the total flow rate exceeds the maximum flow rate for which the model was validated. This situation could apply to low-rise and high-rise residential buildings, college dormitories, gymnasiums, and any other application where the flow rate of drain water exceeds 14 L/min. Clearly, further investigation is required to determine if the model would work for flow rates higher than 14 L/min.

It is also possible for the flow rate through a DWHR system's coils to fall below the lower limit of the model, even for a typical showerhead flow rate of 9.5 L/min. Consider the following case for plumbing setup depicted in Figure 2-16(A): the showerhead flow rate is fixed at 9.5 L/min, the mains temperature is 10°C, the user chooses shower temperature of 35°C, and the hot water tank's temperature is the typical 60°C. Ignoring all losses, and by performing mass and energy balances on the mixing valve, the volumetric flow rate of water going through the DWHR system's coils can be calculated. In this case, the flow rate was determined to be 4.75 L/min, which is below the lower limit for which the model was validated (i.e. 5.5 L/min). This exercise was repeated for the same showerhead flow rate of 9.5 L/min, for constant mains water temperatures of 5, 10 and 15°C, and shower temperatures varying from 15 to 45°C. The results are shown in Figure 2-17, where the solid lines represent constant mains temperatures as labelled, and the shaded area enclosed by dashed lines represents the region in which the model was validated.

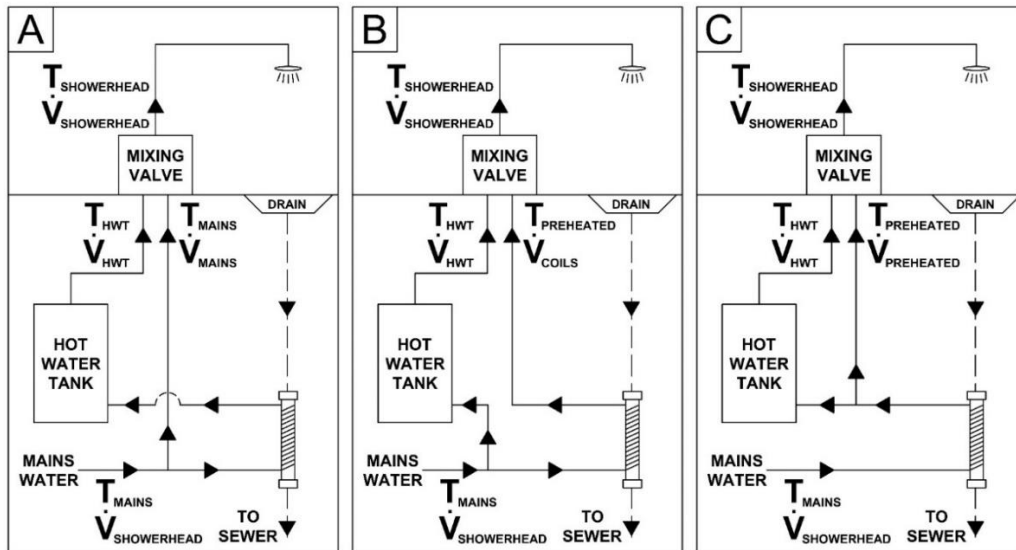


Figure 2-16: Possible plumbing setups for DWHR systems in a residential building. In the figure, (A) depicts a system where all preheated water from the DWHR's outlet is fed to the water heater, (B) depicts a system where all preheated water is fed to the mixing valve.

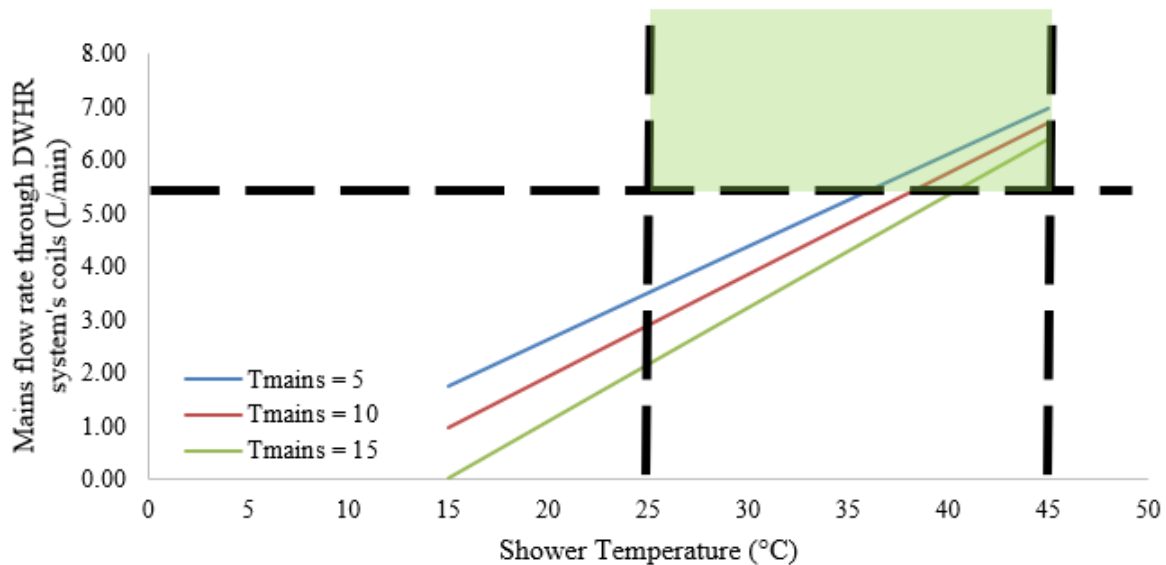


Figure 2-17: Mains water flow rate through a DWHR system's coils as a function of shower temperature for a 9.5 L/min showerhead, for constant mains water temperatures of 5, 10 and 15°C.

It is clear from the figure that the existing model is only applicable to a small region of operating conditions for the plumbing configuration depicted in this example. This exercise was repeated to generate similar plots for showerhead flow rates of 14 and 7.5 L/min as shown in Figure 2-18. Note that for the 7.5 L/min showerhead, the model does not apply at all.

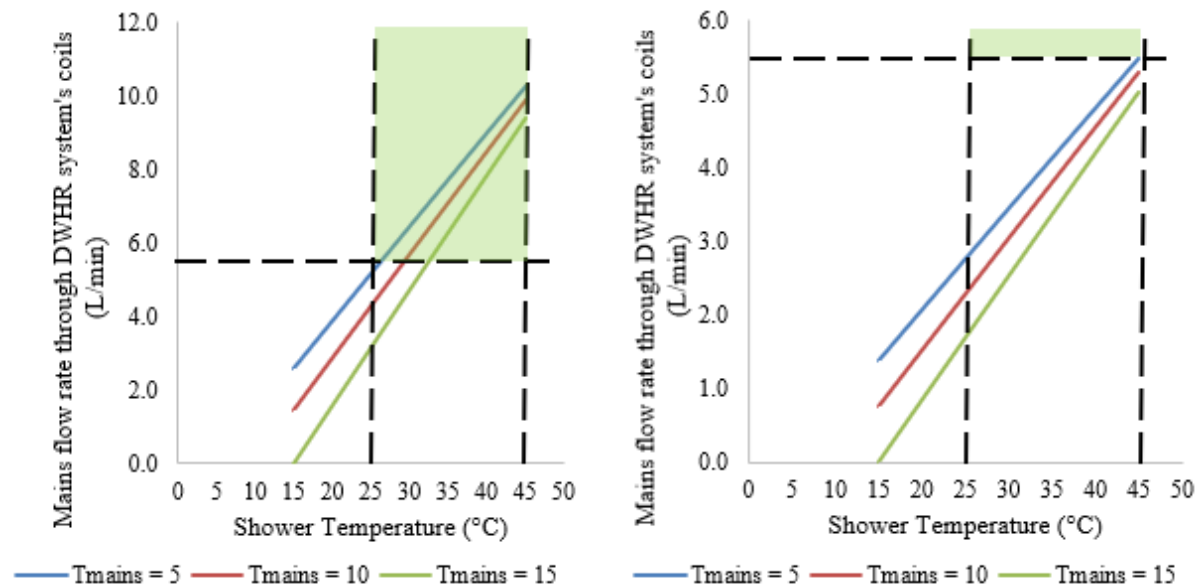


Figure 2-18: Mains water flow rate through a DWHR system's coils as a function of shower temperature for a 14 L/min showerhead (left) and a 7.5 L/min showerhead (right), for constant mains water temperatures of 5, 10 and 15°C.

2.4 Summary

Numerous studies have shown that DWHR systems can lead to substantial energy savings; however, the existing literature is insufficient for modelling their performance. Most of the current studies assume a constant heat exchanger effectiveness, and use that value to perform simulations. This leads to unrealistic results, as the effectiveness of DWHR heat exchangers changes significantly in the range of flow rates they operate. The overall model produced at the University of Waterloo is the only experimentally-validated model capable of predicting accurate heat transfer rates. However, it was developed for the range of flow rates prescribed by the CSA standard; thus, it cannot be assumed to work for other conditions. In short, the preliminary validation presented in Manouchehri [28] did not account for the impact of plumbing configuration on system performance.

Chapter 3 – Steady State Model

This chapter covers the research and modeling of DWHR heat exchangers during their steady state operation. It establishes different approaches for modeling DWHR heat exchangers. Each model is validated using the dataset produced as part of the following section.

3.1 Existing Model Validation

As mentioned in Section 2.3.5, the steady-state model developed during the author's MASc studies was limited. An investigation was needed to determine whether or not the existing model would apply to conditions outside the bounds for which it was developed, and devise correction factors if necessary. In the worst-case scenario, the existing model would be deemed inapplicable, and a new model would have to be created.

The existing model was developed using experimental results for a limited range of conditions, and no assumptions could be made regarding its validity outside of that range. Prior to assuming the worst case, it was decided to experimentally investigate the validity of the existing model for a broad set of inlet temperatures and flow rates. To do so, eight different DWHR systems representing three different diameters and two different manufacturers were selected for testing. Table 3-1 lists the systems examined.

Table 3-1: DWHR systems tested to generate data for validation purposes.

System #	Diameter cm	Length cm
1	5.1	91
2	5.1	152
3	7.6	91
4	7.6	107
5	7.6	152
6	7.6	244
7	10.2	122
8	10.2	152

To produce performance data equivalent to those from the CSA rating [11], all heat exchangers were tested at equal flow rates of 5.5, 7, 9, 10, 12 and 14 L/min, and at inlet temperatures of $40\pm0.5^{\circ}\text{C}$ and $12\pm0.5^{\circ}\text{C}$ on the drain-side and the mains-side, respectively.

Each system was then tested under many different inlet conditions to produce data for the validation process. Mains inlet temperatures were randomly chosen to be between 4 and 22°C , while the drain inlet temperatures were between 25 and 50°C . These temperature ranges cover most (if not all) temperatures expected to occur in residential buildings. Note that some of the inlet temperatures used for these tests were outside of the range for which the model was developed; this was done to determine whether the model remains accurate for such temperatures. The drain side flow rates were between 7 and 20 L/min for DWHR systems with a diameter of 10.2 cm, or between 5.5 and 20 L/min for systems with smaller diameters. Validation of the model for drain side flow rates lower than the ones listed above was not attempted in this work for two reasons. Firstly, in typical installations, this flow rate is dictated by the flow rate through the showerhead, which is usually above 5.5 L/min; secondly, the falling-film becomes unstable and tends to break apart at very low flow rates [34]. As for the mains side flow rates, the range of flow rates in this study were between 4 and 20 L/min. The lower bound of 4 L/min was picked mainly because the test apparatus is unable to hold steady flow rates below 4 L/min.

In total, over one hundred validation cases were considered, and for each case, the predicted performance was compared to the measured result. Prior to presenting the final results, it was deemed necessary to first show an example for how the model is applied to the validation cases. The calculation procedure is demonstrated here for one of these cases. The 5.1cm diameter, 91cm long DWHR system (System #1) was first tested to produce the CSA-equivalent data listed in Table 3-2. The data from Table 3-2 were then used to generate a curve fit in the form of Eq. 2.30 to characterize effectiveness as a function of flow rate, as shown in Eq. 3.1.

Table 3-2: CSA-equivalent data for System #1.

$\dot{V}$ (L/min)	ε	q_c (kW)
5.49	39.2%	4.06
7.00	35.2%	4.80
8.94	31.5%	5.36
9.99	30.0%	5.88
12.11	27.6%	6.57
14.11	25.7%	6.94

$$\varepsilon = \frac{1}{0.1548\dot{V} + 1.7513} \quad (3.1)$$

This heat exchanger's performance was then evaluated at random conditions, one of which had $T_{c,in} = 4.7^\circ\text{C}$, $T_{h,in} = 47.3^\circ\text{C}$, a mains-side flow rate of 3.97 L/min, and a drain-side flow rate of 7.97 L/min. The measured steady-state heat transfer rate for this test was 6.25 kW.

Following the procedure described at the beginning of Section 2.3.5, the first step is to obtain the equal flow effectiveness curve for this DWHR system, shown in Eq. 3.1, and use it to predict the equal flow effectiveness at the mains-side flow rate as shown in Eq. 3.2.

$$\varepsilon = \frac{1}{0.1548(3.97) + 1.7513} = 0.4226 \quad (3.2)$$

This effectiveness must then be adjusted for the inlet temperature during the test in a 2-stage process. First, the reference effectiveness is calculated using Eq. 2.33. The CSA-equivalent tests were performed at $T_{c,in} = 12^\circ\text{C}$, $T_{h,in} = 40^\circ\text{C}$; therefore, the reference effectiveness is calculated as shown in Eq. 3.3.

$$F_C = (2.37 \times 10^{-6}) \cdot 40 \cdot 12 + (1.75 \times 10^{-3}) \cdot 40 + (1.24 \times 10^{-3}) \cdot 12 + 0.917 = \frac{0.4226}{\varepsilon_{ref}} \quad (3.3)$$

Solving gives a value of $\varepsilon_{ref} = 0.4213$. The next stage is to use ε_{ref} in Eq. 2.33 to determine the effectiveness at the inlet temperatures of $T_{c,in} = 4.7^\circ\text{C}$, $T_{h,in} = 47.3^\circ\text{C}$ as shown in Eq. 3.4.

$$F_C = (2.37 \times 10^{-6}) \cdot 47.3 \cdot 4.7 + (1.75 \times 10^{-3}) \cdot 47.3 + (1.24 \times 10^{-3}) \cdot 4.7 + 0.917 = \frac{\varepsilon}{0.4213} \quad (3.4)$$

Solving gives a temperature corrected equal flow effectiveness value of $\varepsilon = 0.4238$. Using Eq. 2.36, this effectiveness is then converted to a heat transfer rate as shown in Eq. 3.5.

$$\dot{q} = \frac{4180 \times 3.97 \times 0.4238 \times (47.3 - 4.7)}{60000} = 4.99 \text{ kW} \quad (3.5)$$

This heat transfer rate corresponds to the equal flow condition, and since the flow rates in the validation case were not equal, Eq. 2.35 must be used. The unequal flow calculation is shown in Eq. 3.6.

$$\dot{q}_{(\dot{V}_c=3.97 \text{ \& } \dot{V}_h=7.97)} = 4.99 \times \left[0.3452 \times \ln\left(\frac{7.97}{3.97}\right) + 1 \right] \quad (3.6)$$

This gives a value of $\dot{q}_{(\dot{V}_c=3.97 \text{ \& } \dot{V}_h=7.97)} = 6.19 \text{ kW}$ as the predicted heat transfer rate for the validation case. This is 0.06 kW lower than the measured value of 6.25 kW, with a prediction error smaller than 1% for this example.

A total of 135 validation tests were performed, 93 of which had at least one of the inlet flow rates outside of the CSA's range of flow rates. The predicted heat transfer rates for all DWHR units were plotted against the measured values, as shown in Figure 3-1 for all 135 validation cases. The identity line of $y=x$, which represents a perfect model, was also added to the figure to visually highlight the model's inaccuracies. As shown in the figure, the test results lie in close proximity to the $y=x$ line, even though the majority of the validation cases were outside the bounds in which the model was developed. The validation data are available in Appendix D.

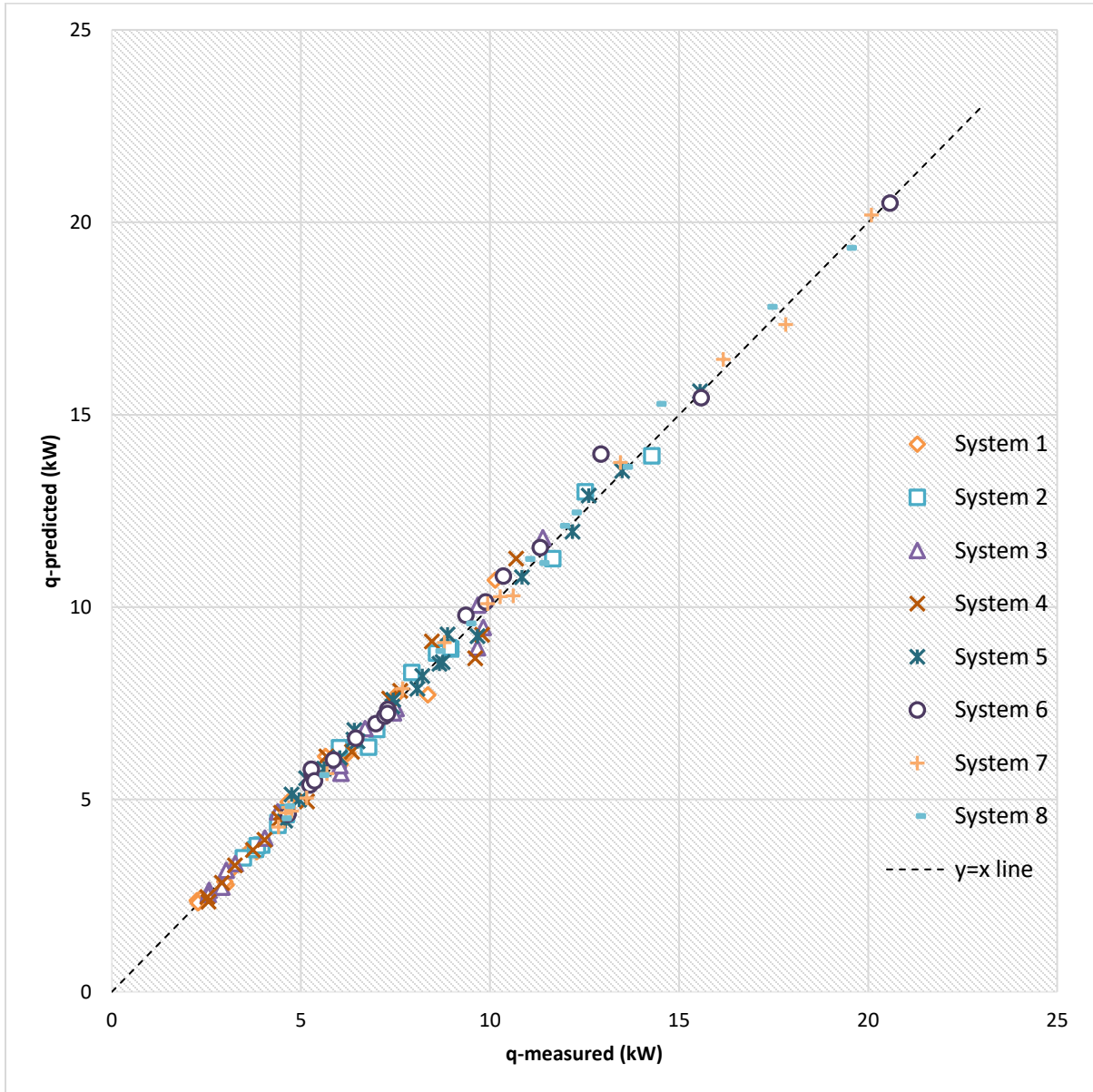


Figure 3-1: Predicted vs. measured heat transfer rates for the 135 test cases, including the 93 test cases with at least one flow rate outside of the CSA range.

To provide an estimate for the prediction errors associated with the model, the Mean Absolute Error (MAE) associated with the results was calculated using Eq. 3.7. MAE represents the average absolute vertical (or horizontal) distance between each point in the scatter plot to the $y=x$ line. The MAE was calculated to be 0.271kW for the case with 93 non-CSA flow rates, and the MAE for the entire data set was 0.214kW.

$$MAE = \frac{\sum_1^n |Actual - Predicted|}{n} \quad (3.7)$$

The problem with MAE is its scale-dependency, which became very apparent since the magnitude of $\dot{q}$ in the data set ranges between 2 and 21kW. The calculated MAE of 0.271kW is highly misleading for quantifying the errors associated with the model, as it is overpredicting the errors for small values of $\dot{q}$, and underpredicting the errors for large values of $\dot{q}$. Hence, the Mean Absolute Percentage Error (MAPE) was employed to represent the error as a percentage instead, and to eliminate the scale-dependency of the outcome. Eq. 3.8 was used to calculate MAPE. The MAPE was calculated to be 3.47% for the case with 96 non-CSA flow rates, and the MAPE for the entire data set was 2.96%. Table 3-3 summarizes the statistical results, as well as the maximum prediction errors between the measured and predicted heat transfer rates.

$$MAPE = \frac{\sum_1^n \left| \frac{Actual - Predicted}{Actual} \right|}{n} \times 100 \quad (3.8)$$

Table 3-3: Comparison of model's accuracy for different flow regions.

Data Set	Number of Tests	Mean Absolute Error (kW)	Mean Absolute Percentage Error	Maximum Prediction Error
CSA flow rates	42	0.088	1.8%	7.9%
Non-CSA flow rates	93	0.271	3.5%	9.7%
All test Points	135	0.214	3.0%	

The results listed in Table 3-3 showed that the model, on average, was almost twice as erroneous when it was used to predict heat transfer rates outside of the CSA's range of flow rates. The increased errors in model predictions were not large enough to require the development of a new model from scratch; however, there are aspects of the model that can be improved to reduce errors associated with operating conditions outside of the CSA flow rates. These improvements are discussed below.

The main input to the model, which specifies the DWHR system, is the characteristic effectiveness vs. flow rate curve developed during the CSA rating procedure. The curve fit used in the model is based on the range of flow rates prescribed by the CSA standard, which then gets extrapolated to predict performance for much different flow rates. The model cannot account for errors caused by extrapolating the curve-fits; instead, it stands to reason to expand the CSA standard's range of flow

rates to include additional data points in order to generate better curve-fits, and eliminate the need for extrapolation.

To investigate this, each of the eight DWHR systems from Table 3-1 was tested at an additional high flow rate of 18 L/min under an equal flow condition to measure effectiveness. This additional point was added to the points gathered during the CSA rating procedure, and a new characteristic curve was generated for each heat exchanger. Table 3-4 contains the curve fits generated using CSA data, along with the newly generated curve fits that have an additional test point at 18 L/min.

Table 3-4: Characteristic equations with and without the 18 L/min measurement point.

	Curve-fit generated using CSA flow rates	Curve-fit generated using CSA flow rates + 18 L/min
System 1	$\varepsilon = \frac{1}{0.1548\dot{V} + 1.7513}$	$\varepsilon = \frac{1}{0.1479\dot{V} + 1.8101}$
System 2	$\varepsilon = \frac{1}{0.0859\dot{V} + 1.4603}$	$\varepsilon = \frac{1}{0.0827\dot{V} + 1.4880}$
System 3	$\varepsilon = \frac{1}{0.1142\dot{V} + 1.4203}$	$\varepsilon = \frac{1}{0.1034\dot{V} + 1.5152}$
System 4	$\varepsilon = \frac{1}{0.1209\dot{V} + 1.4731}$	$\varepsilon = \frac{1}{0.1126\dot{V} + 1.5447}$
System 5	$\varepsilon = \frac{1}{0.0681\dot{V} + 1.2315}$	$\varepsilon = \frac{1}{0.0607\dot{V} + 1.2950}$
System 6	$\varepsilon = \frac{1}{0.0409\dot{V} + 1.1576}$	$\varepsilon = \frac{1}{0.0363\dot{V} + 1.1973}$
System 7 Note: neither curve fit includes 5.5 L/min.	$\varepsilon = \frac{1}{0.0641\dot{V} + 1.3276}$	$\varepsilon = \frac{1}{0.0610\dot{V} + 1.3578}$
System 8 Note: neither curve fit includes 5.5 L/min.	$\varepsilon = \frac{1}{0.0529\dot{V} + 1.2544}$	$\varepsilon = \frac{1}{0.0477\dot{V} + 1.3045}$

The new equations from Table 3-4 were used to repeat the entire exercise to predict heat transfer rates for all 135 test conditions. The main goal here was to determine if the mean absolute percentage error would be improved for the entire range. It was also of interest to know whether or not the errors would become significantly larger within the CSA's range of flow rates. The final results are listed in Table 3-5 along with the results previously listed in Table 3-3, to enable comparison of model outputs between the two curve-fits. For data within the CSA's range of flow rates, the MAPE did not change between the two curve fits, and the maximum prediction error was reduced; hence, it was concluded that inclusion of an additional test point at 18L/min would not, on average, negatively affect the

model's outputs. On the other hand, for data sets containing data outside of the CSA's range of flow rates, the MAPE's were reduced by 0.3%. Hence, it was concluded that adding a test point at 18L/min to the CSA standard would result in less erroneous predictions by this model.

Table 3-5: Comparison of model's accuracy for different flow regions, calculated using different characteristic curves.

Data Set	Number of Tests	Curve-fit Used for Analysis	Mean Absolute Percentage Error	Maximum Prediction Error
CSA flow rates	42	CSA	1.8%	7.9%
		CSA+18L/min	1.8%	7.0%
Non-CSA flow rates	93	CSA	3.5%	9.7%
		CSA+18L/min	3.2%	11.2%
All test Points	135	CSA	3.0%	9.7%
		CSA+18L/min	2.7%	11.2%

It should be noted that the 0.3% improvement is only associated with the current data set, which only contains flow rates of up to 20L/min. The improvements are expected to be much more pronounced for flow rates higher than 20L/min, which cannot be tested using the test apparatus. To support this point, Figure 3-2 was generated using the curve-fit equations for System 6 listed in Table 3-4. As shown in the figure, the curves drift farther apart as flow rate is increased; thus, it was concluded that the CSA standard must be expanded to include additional test points for DWHR systems designed to operate at very high flow rates.

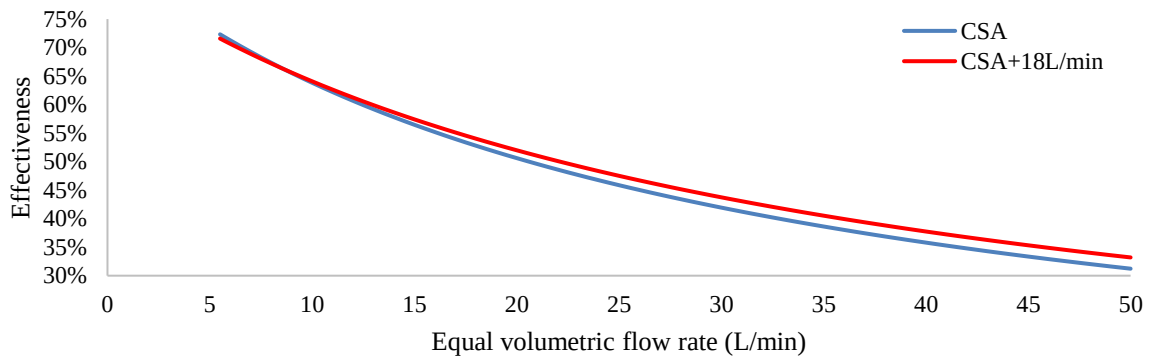


Figure 3-2: Characteristic curves for System 6, plotted between 5.5 and 50L/min.

Once the investigation into the impact of using improved curve-fits was finished, further attempts were made to determine other potential sources of error within the model. Consequently, it was

observed that the model tended to overpredict heat transfer rates as the ratio of $(\dot{V}_h/\dot{V}_c)$ became larger, as shown in Figure 3-3. This ratio was used in the development of Eq. 3.9, which is used to predict heat transfer rates under unequal flow conditions in the model. The value of 0.3452 in this equation is a regression coefficient based on data within the CSA's range of flow rates, as was discussed previously. Based on the plot, for $(\dot{V}_h/\dot{V}_c)$ ratios larger than 2, the model overpredicts the heat transfer rates, as shown in Figure 3-4. A portion of the error can be attributed to curve-fit extrapolation discussed previously. Using better curve-fits did reduce the overpredicted values, on average; however, the overprediction issue prevailed even when improved curve-fits were used.

$$q_{(\dot{V}_c=F1 \ \& \ \dot{V}_h=F2)} = q_{(\dot{V}_c=F1 \ \& \ \dot{V}_h=F1)} \times \left[0.3452 \times \ln\left(\frac{\dot{V}_h}{\dot{V}_c}\right) + 1 \right] \quad (3.9)$$

One method to address this issue is using a different coefficient instead of 0.3452 for $(\dot{V}_h/\dot{V}_c)$ ratios larger than 2. For example, if a value of 0.28 is used as the new coefficient, the upward trend in model prediction is minimized, and the errors become much smaller in this range, as shown in Figure 3-5, which can be directly compared to Figure 3-4 where a constant value of 0.3452 was used as the coefficient.

At this point, it was clear that the existing CSA curve fit (Eq. 2.30), should not be used for flow rates above 14L/min, which is the upper bound of flow rates included in the CSA testing procedure. The test results showed that extrapolating the existing curve-fits will consistently underpredict performance. Consequently, if a correction factor is to be developed, it would be a quick-fix at best, and would not address the underlying issues with the curve-fit. In other words, without delving into heat exchanger theory, it is not possible to discern the issues with the existing effectiveness curve-fit, and any correction schemes developed at this point would be specific to individual products.

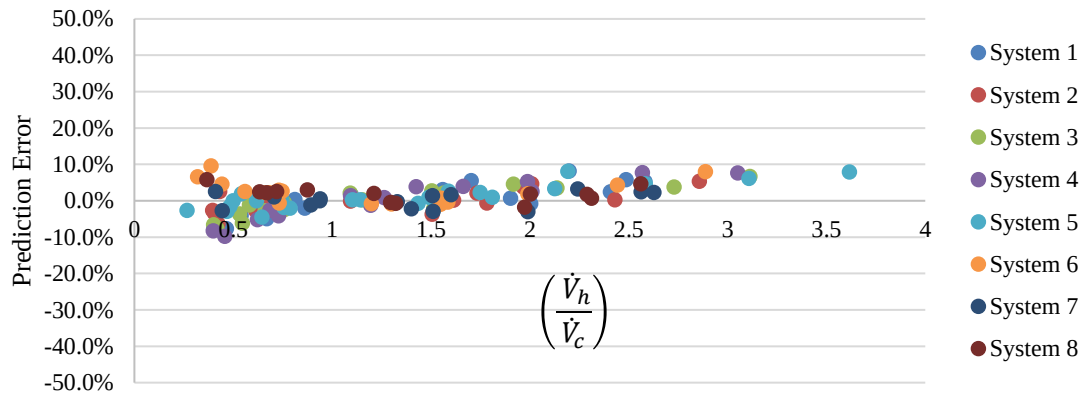


Figure 3-3: Prediction errors plotted as a function of the ratio of volumetric flow rates for all 135 test points.

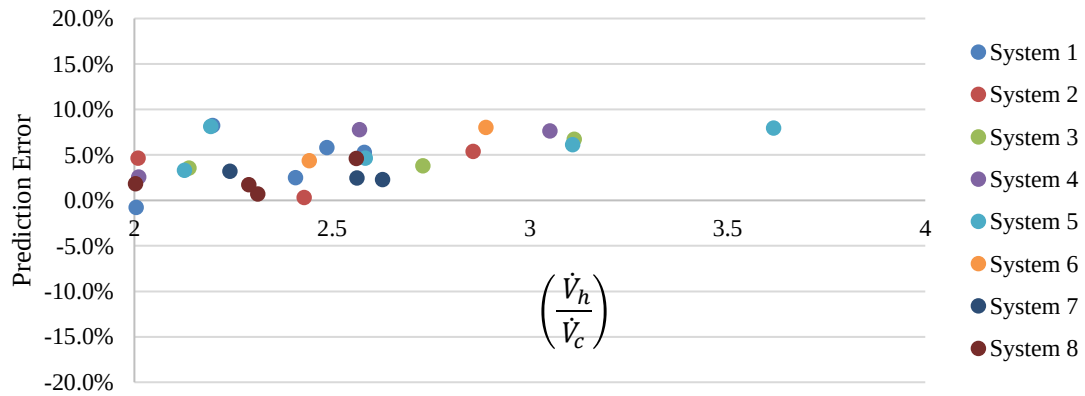


Figure 3-4: Prediction errors vs. the ratio of volumetric flow rates between 2 and 4, using 0.3452 as the constant in Eq. 3.9.

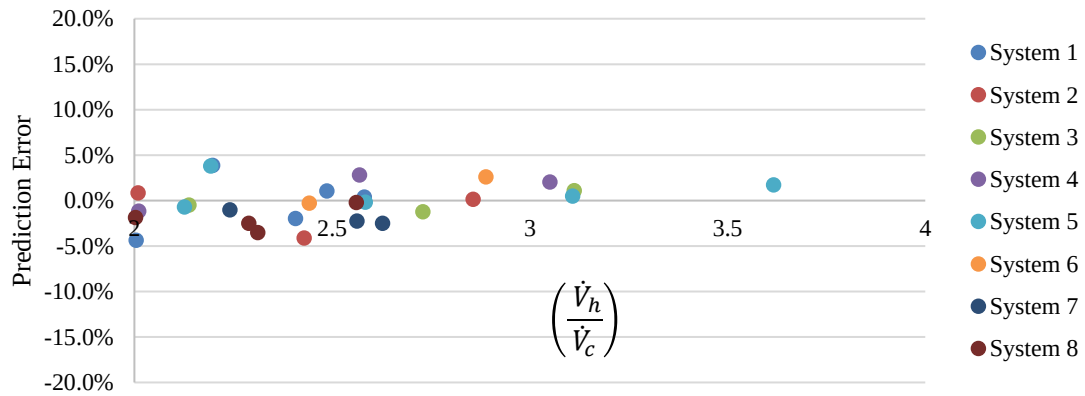


Figure 3-5: Prediction errors vs. the ratio of volumetric flow rates between 2 and 4, using 0.28 as the constant in Eq. 3.9.

3.2 Theoretical Model (Thermal Resistor Network)

The goal for this section is to establish a means to model the performance of DWHR heat exchangers in terms of fundamental heat transfer concepts such as thermal transmittance, resistance, and heat transfer coefficients. If such a model were to exist, it would have numerous applications including the means to detect fouling as well as a mathematical equation that would allow for optimizations. Chapter 2 covered the fundamental methods for solving heat exchanger problems, and these fundamentals will be used as the basis for the new model that will be covered in this section.

One of the broadly accepted methods for estimating heat exchanger performance is to experimentally test the given heat exchanger under conditions that mimic its typical operating conditions. Often, this is done using the previously discussed ϵ -NTU method, whereby the heat exchanger is tested at known inlet temperatures and flow rates, and this data is used to calculate heat exchanger effectiveness. Another method to model heat exchangers is to follow the textbook approach of representing the given heat exchanger as a thermal resistor network. This resistor network is shown for a counterflow heat exchanger in Figure 3-6.

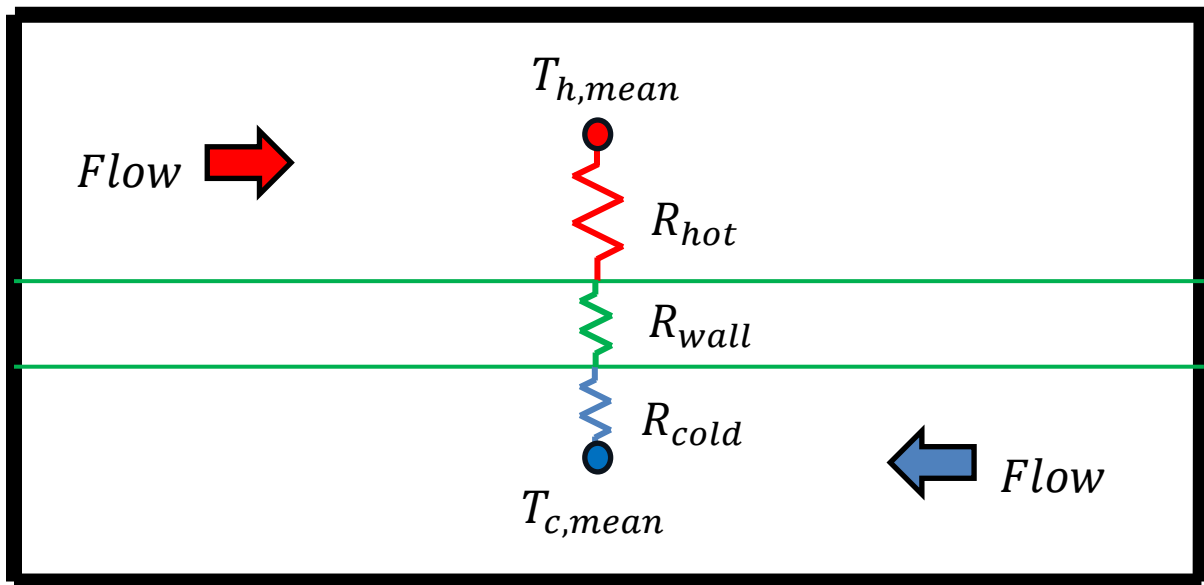


Figure 3-6: General representation of a counterflow heat exchanger using thermal resistors.

The classical approach to solving heat transfer problems containing resistors is to use appropriate Nusselt number correlations that are also found in textbooks. Nusselt number represents the ratio of convective to conductive heat transfer across a boundary. There are many correlations available in the

literature in terms of Nusselt number for different geometries; however, using such correlations to approximate the heat transfer coefficient does not necessarily lead to accurate results. This is well known, and some authors even advise against trying to find highly accurate heat transfer coefficients in design guidelines for heat exchangers. The quotation below is taken directly from a textbook on heat exchanger design engineering [18], which is a good summary of the classical approach to heat exchanger design problems.

“Guideline 23: It is counterproductive to strive for accuracies of the order of 1-2% for heat transfer coefficients and friction factor correlations, in view of unavoidable manufacturing tolerances, test inaccuracies, the approximate nature of some key transport properties, and such uncertain effects as fouling resistance. Accuracies of the order of 10-20% are adequate and more realistic in real world forced-convection heat exchangers.” -Soumerai [18]

Another shortcoming of this approach is that Nusselt number correlations for many complex geometries do not exist. As a result, designers often have no choice but to select a correlation that is deemed “close enough” to the problem at hand. This further exacerbates the problem of finding relatively accurate heat transfer coefficients for heat exchangers.

Finally, there are conditions and limitations associated with every Nusselt number correlation, which do not necessarily apply to heat exchangers. Nusselt number correlations are derived using the assumption of constant wall temperature, or constant heat flux at the wall, neither of which are accurate representations of what occurs in a heat exchanger. The wall separating the two fluids, more specifically its temperature, is a function of what occurs around it, and one should not expect the Nusselt number correlation that was derived based on the above assumptions to fully account for the physics of heat exchangers. In essence, if a Nusselt number correlation from a heat transfer textbook is applied to for the cold side of a hypothetical heat exchanger, this correlation would be completely blind to the fluid motion on the hot side of this hypothetical heat exchanger. In other words, the fluid on the hot side is essentially treated as motionless, and the impacts of fluid motion and heat capacity on the wall temperature are not properly addressed.

The problem with Nusselt number correlations is not specific to heat exchangers and it merits more discussion. Named after Wilhelm Nusselt, Nusselt number represents the ratio of convective to conductive heat transfer at a fluid boundary as depicted in Eq. 3.10.

$$Nu_L = \frac{h \cdot L}{k} \quad (3.10)$$

where h is the convective heat transfer coefficient, L is a characteristic length (e.g. diameter), and k is the thermal conductivity of the fluid. Like h , Nusselt number varies as a function of distance along the heat transfer area; this is because every point along the surface has its own local heat flux as a result of the heat transfer process occurring at the surface. However, the Nusselt number is often represented as an average quantity along the entire surface, which provides a better understanding of the overall heat transfer process. The average Nusselt number equation for forced convection is often listed in heat transfer textbooks as follows [16]:

$$\overline{Nu}_L = \frac{\bar{h}L}{k_{fluid}} = f(Re_L, Pr) \quad (3.11)$$

where Re is the Reynolds number and Pr is the Prandtl number.

Likewise, if the Nusselt number is developed for internal flow, the characteristic dimension is often a diameter or a hydraulic diameter, and the equation is written as follows:

$$\overline{Nu}_D = \frac{\bar{h}D}{k_{fluid}} = f(Re_D, Pr) \quad (3.12)$$

It is possible to derive Nusselt number correlations analytically for simple geometries, but usually, analytical solutions are not available for specific geometries of interest. That is why engineers are encouraged to use well-established correlations listed in heat transfer textbooks, such as the one shown in Eq. 3.13:

$$Nu_D = X \cdot Re_D^m \cdot Pr^n \quad (3.13)$$

where X is a numerical constant, m is the power of the Reynolds number, and n is the power for Prandtl number. This form of representation for the Nusselt number has become known as the Dittus-Boelter equation [16, 15]. However, the actual Dittus-Boelter equations are as follows:

$$Nu_D = 0.0243 Re_D^{4/5} \cdot Pr^{0.4} \quad (\text{for heating; i.e. } T_{surface} > T_{mean}) \quad (3.14)$$

$$Nu_D = 0.0265 Re_D^{4/5} \cdot Pr^{0.3} \quad (\text{for cooling; i.e. } T_{surface} < T_{mean}) \quad (3.15)$$

Note that these correlations are not universal, and each correlation is accompanied with additional information about its applicability to similar geometries, often listed as ranges for Reynolds and Prandtl numbers. For example, the Dittus-Boelter equation is only valid for:

- Turbulent flow
- Fully developed
- $0.6 \leq Pr \leq 160$
- $Re_D \geq 10000$
- $\left(\frac{L}{d}\right) \geq 10$

In order to properly use Eqs. 3.14 and 3.15, one needs to determine applicable thermophysical properties such as the density, viscosity, and the specific heat. For Dittus-Boelter type equations, the fluid properties are taken at the mean fluid temperature T_m , also referred to as bulk fluid temperature in some textbooks. Therefore, the underlying assumption is that the properties do not change significantly as a function of temperature, and the accuracy of what results from this equation is directly tied to the validity of that assumption.

Eqs. 3.14 and 3.15 show good agreement with experimental findings; however, as the temperature difference between the wall and the fluid becomes larger, the assumption of constant properties breaks down. More specifically, the dynamic viscosity of water tends to change rapidly as a function of temperature, and depending on the end-use temperatures and the fluid properties, Dittus-Boelter type equations are often corrected. Figure 3-7 shows the dynamic viscosity of water as a function of temperature to highlight the significant changes that can be expected for simple fluids such as water. Note that many heat exchangers use water as heat transfer media, including DWHR heat exchangers, so this correction is vital to many heat exchanger calculations.

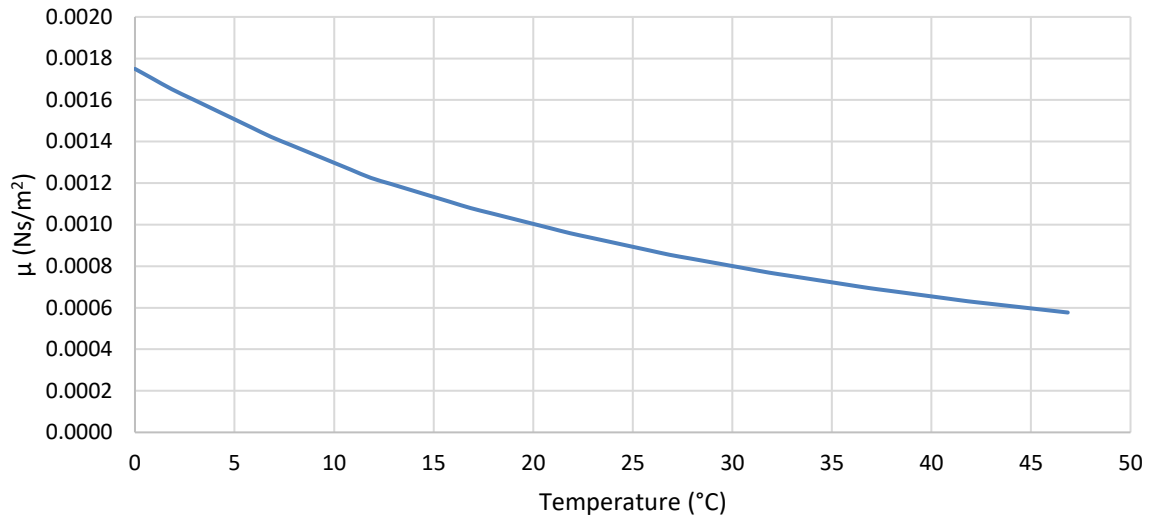


Figure 3-7: Viscosity of water as a function of temperature. Data taken from Incropera et al [16].

As a result of varying properties, Nusselt number correlations are often corrected using a property ratio. This was first introduced by Sieder and Tate [40], where the suggested correction was to multiply the Dittus-Boelter type equation by a ratio of properties taken at the wall and the mean fluid temperature. See Eq. 3.16:

$$Nu = 0.027 Re_D^{4/5} \cdot Pr^{1/3} \cdot \left(\frac{\mu_{mean}}{\mu_{wall}} \right)^{0.14} \quad (3.16)$$

This correction factor has come to be known as the Sieder and Tate viscosity ratio, and it is often included in Nusselt number correlations for internal flow, where the wall temperature is expected to be very different than the mean fluid temperature. To solve this equation, the wall temperature has to be known, which is usually not possible. Hence, iterative solutions are often employed to find the appropriate wall temperature in the problem of interest. This is time consuming, difficult to instrument, and often requires iterative solutions to find the wall temperature, which is not ideal for modelling heat exchangers where wall temperature is expected to be different under different operating conditions. In other words, for a given heat exchanger, the wall temperature gradient and surface temperatures are directly influenced by the fluids passing around it. Therefore, using Sieder and Tate viscosity ratio becomes problematic as iterative solutions may be required for every combination of flows and temperatures. It is also worth noting that, by definition, Nusselt number correlations correspond to the surface being kept at a constant temperature, or the surface being

exposed to a constant heat flux. Neither of these assumptions can be assumed to hold true for heat exchangers. Given the known shortcomings of Nusselt number correlations discussed thus far, it is now appropriate to examine other methods with which heat exchangers are characterized.

3.2.1 ϵ -NTU revisited

This method was briefly discussed in the previous chapters, as it is another well-known and widely used method for measuring thermal performance of heat exchangers. Rather than looking at correlations, resistors, etc., this method is simply based on comparing the actual performance of the heat exchanger operating at a given condition, to the maximum thermodynamic performance possible at that same given condition. The metric that was created for this purpose was dubbed “effectiveness”, which is the ratio of actual heat transfer to the total heat transfer, as shown in Eq. 3.17. Effectiveness implicitly accounts for the 2nd Law of thermodynamics, which is absent in the resistor network analysis.

$$\epsilon = \frac{q}{q_{max}} = \frac{(\dot{m} Cp)_{cold}(T_{cold,out} - T_{cold,in})}{(\dot{m} Cp)_{minimum} \Delta T_{max}} = \frac{(\dot{m} Cp)_{hot}(T_{hot,in} - T_{hot,out})}{(\dot{m} Cp)_{minimum} \Delta T_{max}} \quad (3.17)$$

$$where \Delta T_{max} = (T_{hot,in} - T_{cold,in})$$

Note that effectiveness is a result of the condition under which the heat exchanger is being operated, and by no means is it supposed to, nor should be expected to stay constant when the conditions change. Additionally, by definition, the maximum temperature is included in the derivation; in other words, the maximum drive for heat transfer (i.e. ΔT) is fixed. This is ideal, because this method allows for direct comparison of different heat exchangers subjected to identical inlet conditions. The experimental nature of this method also accounts for any manufacturing defects or other irregularities that are present in the heat exchanger design, as the effectiveness is calculated based on the real temperature differences as opposed to the ones predicted using the resistor network approach. What is lacking in the ϵ -NTU method is the means to use an experimentally determined effectiveness value at a certain condition to predict performance for the same heat exchanger operating under different conditions. More specifically, the UA product for the heat exchanger varies based on inlet conditions, and without the means to predict what happens to UA as a function of inlet conditions, it is not possible to accurately model any heat exchanger. To reiterate, the effectiveness values are great for allowing point-comparisons at specific and identical inlet conditions for different

heat exchangers. Without understanding the physics behind the UA product, knowing the effectiveness at a point is insufficient for performance predictions.

As was explained in Section 2.1.4.2., the assumption of constant U does not apply to heat exchangers because heat transfer coefficients are strong functions of mass flow rates on either side of the heat exchanger. Finding the overall thermal transmittance U requires knowledge of convective heat transfer coefficient h for both fluids traversing through the heat exchanger. As already discussed, using Nusselt number correlations for finding these convective heat transfer coefficients can also lead to inaccurate results. In essence, the problem in solving heat exchanger problems boils down to U , and the challenge is how to find expressions for heat transfer coefficients. It would be ideal if the heat exchanger itself could be used as the means to create expressions for convective heat transfer coefficient h .

One method for finding expressions for convective heat transfer coefficients in heat exchangers is referred to as Wilson's plot. This method was first used by Wilson [41] to experimentally determine one of the convective heat transfer coefficients for a shell-and-tube heat exchanger. This is done by treating the heat exchanger as a thermal resistor network. See below.

$$\begin{aligned}
 R_{total} &= R_{convection,cold} + \sum R + R_{convection,hot} \\
 R_{convection,cold} &= \frac{1}{h_{convection,cold} A_{convection,cold}} \\
 R_{convection,hot} &= \frac{1}{h_{convection,hot} A_{convection,hot}} \\
 \sum R &= R_{wall,conduction} + R_{fouling} + R_{manufacturing\ defects} \\
 \sum R &\approx R_{wall,conduction} = \frac{Length_{conduction}}{k_{wall} A_{wall}} \\
 R_{total} &= \frac{1}{h_{convection,cold} A_{convection,cold}} + \sum R + \frac{1}{h_{convection,hot} A_{convection,hot}} \quad (3.18)
 \end{aligned}$$

The definition of Nusselt number in the form of a Dittus-Boelter type equation is then used to derive an expression for h in terms of non-dimensional variables that can be determined experimentally. See Eq. 3.19.

$$Nu_D = X \cdot Re_D^m \cdot Pr^n = \frac{h \cdot d}{k}$$

$$h = \left(\frac{k}{d}\right) \cdot (X \cdot Re_D^m \cdot Pr^n) \quad (3.19)$$

The nondimensional Re and Pr numbers can also be expressed in terms of dimensional variables; this is done using the fundamental equations representing each of the nondimensional terms shown in Eq. 3.20 and 3.21. Note that when the velocity term in Reynolds number is converted into $\dot{m}$, it is assumed that the density is constant. This is a good assumption for DWHR heat exchangers since the density of water does not change significantly in the range of temperatures associated with shower events.

$$Re_D = \left(\frac{u \cdot d \cdot \rho}{\mu}\right) = \left(\frac{\dot{m} \cdot d}{A_{cross-section} \cdot \mu}\right) \quad (3.20)$$

$$Pr = \left(\frac{c_p \cdot \mu}{k}\right) \quad (3.21)$$

Next, the expression for h is further simplified as follows:

$$h = \left(\frac{k}{d}\right) \cdot X \cdot \left(\frac{\dot{m} \cdot d}{A_{cross-section} \cdot \mu}\right)^m \cdot \left(\frac{c_p \cdot \mu}{k}\right)^n$$

$$h = \underbrace{\left(\frac{X \cdot d^{m-1}}{A_{cross-section}^m}\right)}_{\text{Constant}} \cdot \underbrace{(c_p^n \cdot k^{1-n} \cdot \mu^{n-m})}_{\text{Function of temperature}} \cdot \underbrace{(\dot{m}^m)}_{\text{Function of flowrate}} \quad (3.22)$$

In Eq. 3.22, the terms in the first bracket do not change with flow rate or temperature and are constants for a given heat exchanger. The second bracket contains the thermophysical properties that are expected to change as a function of temperature. The third bracket contains the mass flow rate. Note that the third term implicitly assumes constant fluid density to turn fluid velocity into a mass flow rate (based on Reynolds number), and the validity of this assumption is problem dependent. Eq. 3.22 can be generalized as follows:

$$h = (Constant) \cdot (c_p^n \cdot k^{1-n} \cdot \mu^{n-m})_{\text{evaluated at } T} \cdot (\dot{m}^m) \quad (3.23)$$

For heat exchangers operating in a limited range of temperatures where the properties are expected to stay relatively constant, the dependence of h on temperature can be dropped, which simplifies the equation to only be a function of the mass flow rate.

$$h = (Constant) \cdot (\dot{m}^m)$$

$$\rightarrow h = K \cdot \dot{m}^m \quad (3.24)$$

This expression for h is incredibly useful, yet there are limitations associated with it that are worth mentioning and are as follows:

- The heat transfer fluid does not change phase and its thermophysical properties can be approximated as constants in the range of temperatures between the inlet and outlet.
- Similar to Nusselt number correlations, the expression for h is dependent on the flow regime. The underlying assumption in this approach is that the flow regime does not change for the range of flowrates where this expression is applied. This is analogous to the range of Reynolds numbers where Nusselt number correlations are valid.
- Other typical simplifications often applied to heat exchangers including: no fouling, negligible axial conduction, and negligible losses to the environment.

Using the expression for h , the convective resistors from Eq. 3.18 can be represented as follows:

$$R_{convection,cold} = \frac{1}{h_{convection,cold} \cdot A_{convection,cold}} = \frac{1}{K_{cold} \cdot \dot{m}_{cold}^m \cdot A_{convection,cold}}$$

$$R_{convection,hot} = \frac{1}{h_{convection,hot} \cdot A_{convection,hot}} = \frac{1}{K_{hot} \cdot \dot{m}_{hot}^m \cdot A_{convection,hot}}$$

$$R_{total} = \frac{1}{UA} = \frac{1}{K_{cold} \cdot \dot{m}_{cold}^m \cdot A_{convection,cold}} + \sum R + \frac{1}{K_{hot} \cdot \dot{m}_{hot}^m \cdot A_{convection,hot}} \quad (3.25)$$

Finally, Wilson's plot method can be explained given the above simplifications. Wilson plot was first used when the magnitudes and expressions for one of the convective resistors as well as the conduction resistor were known. For example, if the heat transfer coefficient for the hot fluid in Eq. 3.25 is known, and if the magnitude of this resistor is kept constant in experiments (i.e. constant mass flow rate), the following simplification can be made:

$$K_1 = \sum R + \frac{1}{K_{hot} \cdot \dot{m}_{hot}^m A_{convection,hot}} = constant \quad (3.26)$$

Hence, the expression for total thermal resistance is reduced to:

$$R_{total} = \frac{1}{K_{cold} \cdot \dot{m}^{m_{cold}} A_{convection,cold}} + K_1 \quad (3.27)$$

As per Wilson plot's procedure, Eq. 3.27 is then turned into the form of an equation for a line. Note that the log-mean temperature difference is kept constant during experiments related to Wilson's plot for this linear relationship to be valid. The visual representation of this is shown in Figure 3-8.

$$y = m \cdot x + b \quad (3.28)$$

where

$$m = \frac{1}{K_{cold} \cdot A_{convection,cold}} = \text{constant}$$

$$x = \frac{1}{\dot{m}^{m_{cold}}}$$

$$b = K_1 = \text{constant}$$

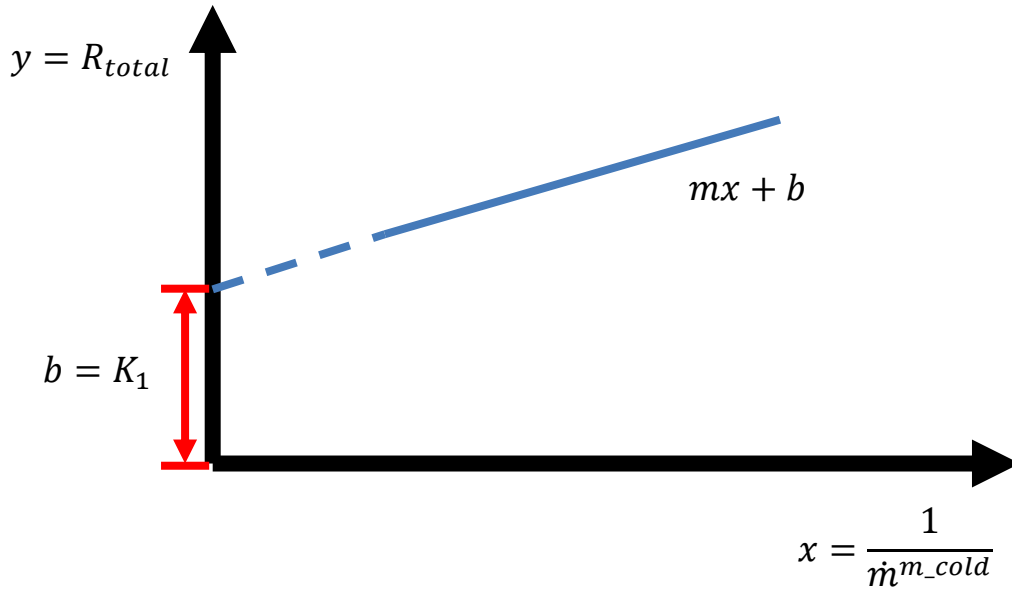


Figure 3-8: Wilson plot's graphical representation.

Based on the above, experiments are performed, and the measured overall thermal resistance values are plotted like what is shown in Figure 3-8. The values for constant K_{cold} and power m_c can be estimated from experimental data using regression. This general method for experimental estimation

of heat transfer coefficients has become known as the Wilson's plot method. Note that Wilson's work is over a century old (published in 1915) and unknown terms are now found using computers.

Despite the usefulness of Wilson's plot technique, it is extremely sensitive to temperature measurements. Simulations have shown that RMS errors of 0.1°C can lead to errors as large as 20% in the magnitude of the heat transfer coefficient [42]. Other researchers have also claimed that the use of Wilson's plot can underpredict heat transfer coefficients by up to 25% [43]. As a result, pursuing the formulation of heat transfer coefficients for DWHR heat exchangers using Wilson's plot method in its current form would not necessarily provide any improvements over the erroneous Nusselt number correlations.

The literature on DWHR heat exchangers has mostly been in terms of the ϵ -NTU method, which is based on experimental data, just like Wilson's plot. ϵ -NTU is based on the laws of thermodynamics whereas Wilson's plot is based on a thermal resistor network. There must exist a way to combine the two approaches, and perhaps address the shortcomings of both. This will be covered in the next section.

3.2.2 Model Derivation

This section will cover the derivation of a theoretical model that is constructed for solving problems involving DWHR heat exchangers. The derivation and solution methods are based on heat exchanger fundamentals covered in Chapter 2, as well as known information about DWHR heat exchanges covered thus far. Hence, it is assumed that the readers are familiar with heat transfer terminology.

The working fluid for DWHR heat exchangers is water on both sides, which is assumed to have the same specific heat capacity. The heat transfer area can also be assumed to stay constant. Therefore, the temperature profile for the heat exchanger would have to look linear, as depicted in Figure 3-9. Note that the typical assumptions of zero losses and negligible axial conduction are applied here implicitly. This allows for one of the common simplifications applied to simple heat exchangers, which is that the mean fluid temperature is the arithmetic mean of the inlet and outlet temperatures for each side of the heat exchanger. This is mathematically shown in Eqs. 3.29, 3.30 and 3.31.

$$T_{h,mean} = \frac{(T_{h,in} + T_{h,out})}{2} \quad (3.29)$$

$$T_{c,mean} = \frac{(T_{c,in} + T_{c,out})}{2} \quad (3.30)$$

$$\Delta T_{mean} = \Delta T_{AMTD} = T_{h,mean} - T_{c,mean} = \frac{(T_{h,in} + T_{h,out})}{2} - \frac{(T_{c,in} + T_{c,out})}{2} \quad (3.31)$$

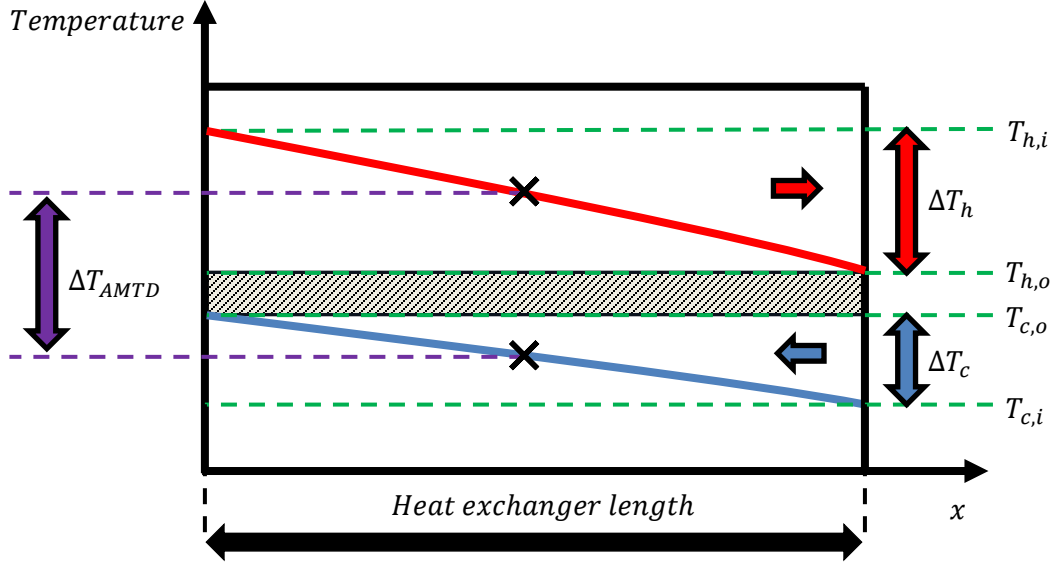


Figure 3-9: Hypothetical temperature profiles as a function of length in a counterflow DWHR heat exchanger.

This linear gradient was extensively researched by Dhillon [44], the previous graduate student working on DWHR heat exchangers at University of Waterloo. In his work, the temperatures along the length of heat exchanger were measured using thermocouples as well as thermography. Figure 3-10 is taken from his work, which shows temperatures as a function of length. The data correspond to a 3inch diameter heat exchanger having a length of 60inches; the red markers correspond to values measured at steady-state and those points are of interest in this discussion. As shown in the figure, the temperature profile at steady state looks linear as a function of length. Many similar plots for different DWHR heat exchangers can be found in that work [44]. Therefore, the assumption of having a linear temperature profile for DWHR heat exchangers is justified and as a result, ΔT_{AMTD} will be used instead of ΔT_{LMTD} to express the mean temperature difference between the fluids.

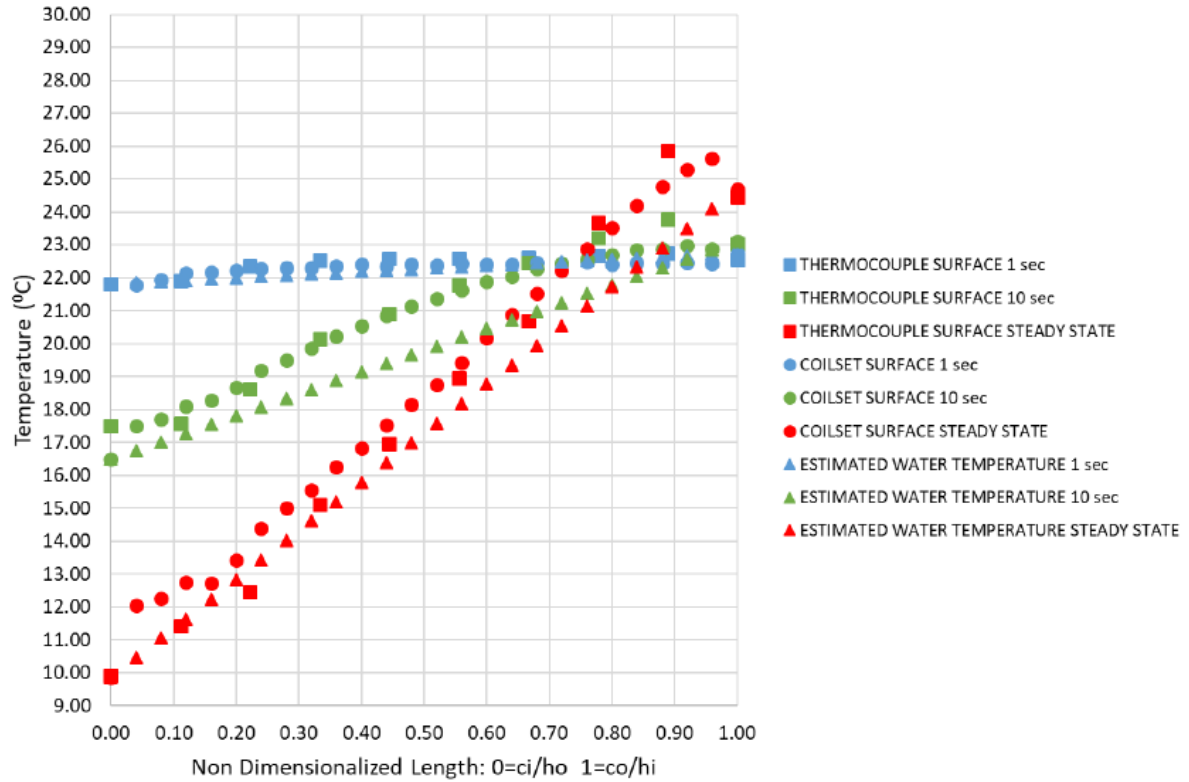


Figure 3-10: Temperature gradient for the coils on a DWHR (R360) heat exchanger, as a function of length [44].

To further support this approach, the entire dataset from a previous publication [37] was used to compare ΔT_{LMTD} to ΔT_{AMTD} for realistic conditions. The experimental data pertains to 4 DWHR heat exchangers having different lengths, diameters and designs. There are a total of 144 experiments in this data set, each of which having its own ΔT_{LMTD} and ΔT_{AMTD} based on the inlet and outlet temperatures. These data are plotted in Figure 3-11, which show that for the majority of data points, the two temperature differences are almost identical.

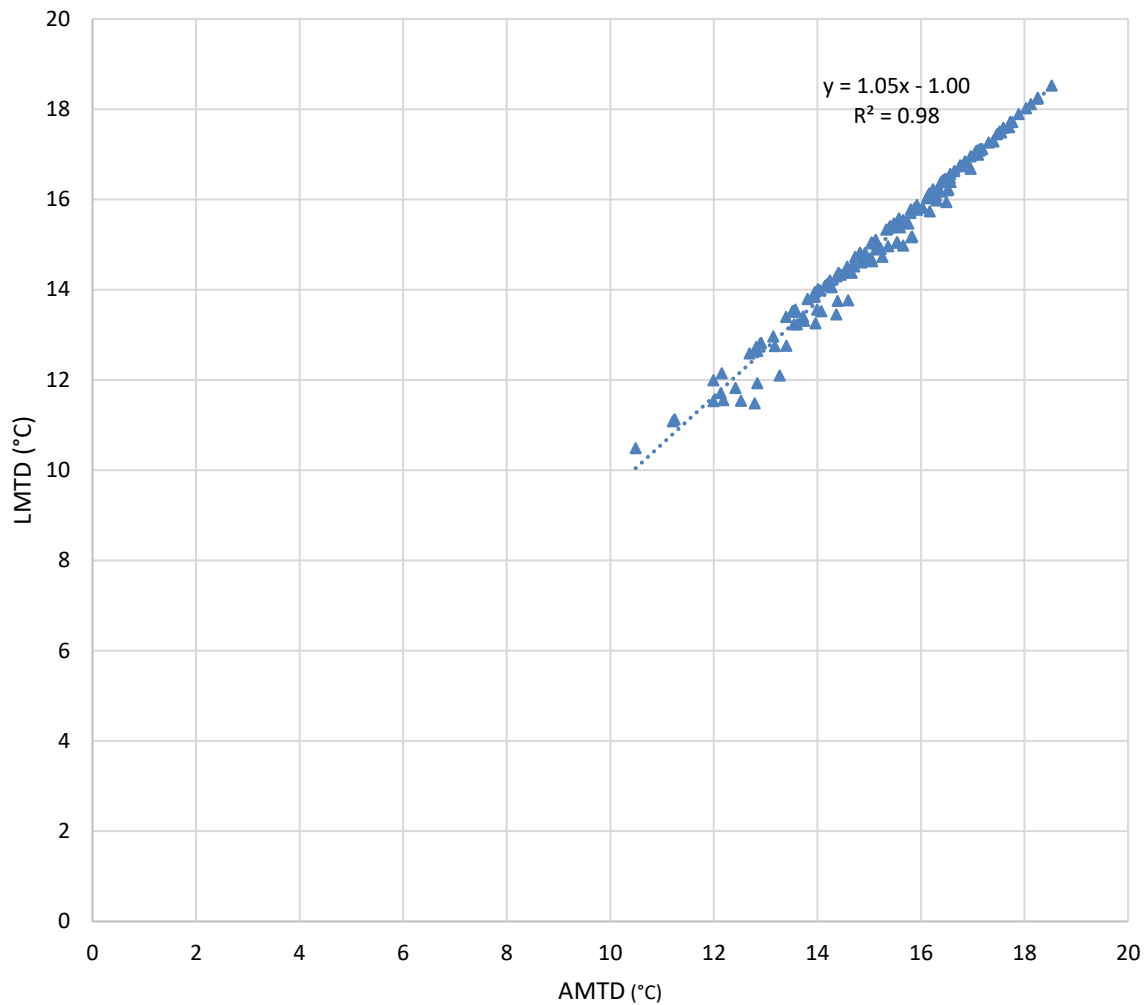


Figure 3-11: Log-mean temperature difference vs. arithmetic mean temperature difference for all DWHR heat exchangers tested.

The differences between the two can be attributed to two separate causes. Firstly, as the magnitude of the mean temperature difference gets smaller, the difference between ΔT_{LMTD} and ΔT_{AMTD} becomes larger. This can be partly attributed to the fact that as temperature differences become smaller, the uncertainty in measurements becomes larger and better instruments are required; otherwise, the error will naturally become larger. Note that a small mean temperature difference across the heat exchanger can also correspond to a poor operating condition for the heat exchanger with minimal energy savings. However, upon closer inspection of experimental data, the main source of error was revealed to be C_r . As a reminder, this metric is the ratio between the minimum and

maximum heat capacity rate for a heat exchanger at a given operating point. This can be thought of as the ratio of the smaller mass flow rate, to the larger mass flow rate for a DWHR heat exchanger. As C_r gets farther from 1, the heat exchanger gets farther from its intended design condition which has balanced flow rates (i.e. $C_r = 1$). To justify this point, a closer inspection of data is required. To this end, Figure 3-12 was constructed to highlight the difference between ΔT_{LMTD} and ΔT_{AMTD} for all DWHR units tested. As shown, the error increases as C_r gets farther from unity. To clarify, error in this context refers to the difference between ΔT_{LMTD} and ΔT_{AMTD} .

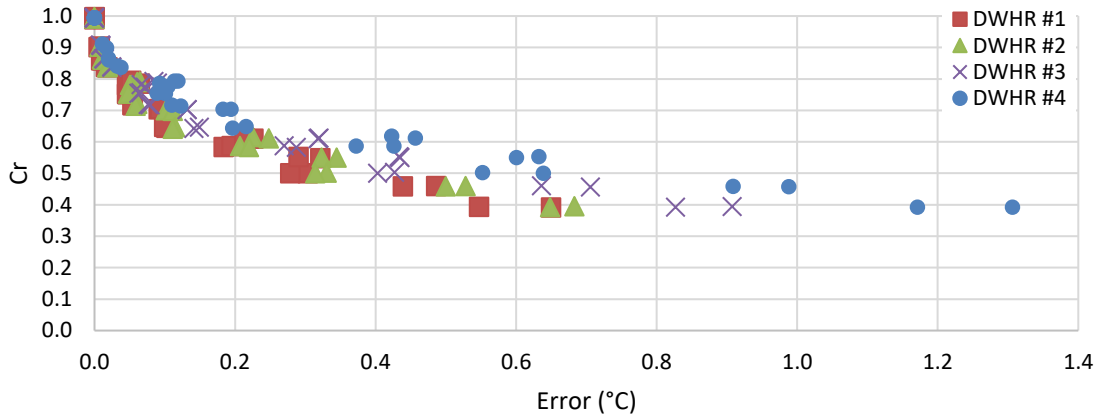


Figure 3-12: Influence of C_r on temperature-related error.

Perhaps it is easier to explain this trend if one were to think of C_r as a bypass factor for the heat exchanger. In other words, as the mass flow rates become more out of phase with each other, a larger volume of water would traverse through one side of the heat exchanger without contributing to the heat transfer process. As a reminder, ΔT_{LMTD} is by definition the maximum temperature difference that is achievable in a counterflow heat exchanger, and since the heat exchanger is underperforming in cases where C_r is lower than 1, it makes sense that ΔT_{LMTD} and ΔT_{AMTD} would differ in magnitude.

Considering that DWHR heat exchangers are designed for operation under balanced flow rates (i.e. $C_r = 1$), and that the difference between ΔT_{LMTD} and ΔT_{AMTD} is smaller than 0.5°C for all experiments where C_r is between 0.5 and 1, it can be concluded that ΔT_{AMTD} is sufficiently accurate for modeling purposes. This will allow for significant simplifications in the derivations that will follow. Hence, Eq. 3.31 will be used in the derivations.

In the expression for the ΔT_{AMTD} (Eq. 3.31), the inlet temperatures are inputs (known), and the outlet temperatures are outputs (unknown). Using conservation of energy, the outlet temperatures can be written in terms of known inlet temperatures and flow rates instead:

$$\dot{q} = \dot{m}_c \cdot cp \cdot (T_{c,out} - T_{c,in}) = \dot{m}_h \cdot cp \cdot (T_{h,in} - T_{h,out}) \quad (3.32)$$

$$T_{c,out} = T_{c,in} + \left(\frac{\dot{q}}{\dot{m}_c \cdot cp} \right) \quad (3.33)$$

$$T_{h,out} = T_{h,in} - \left(\frac{\dot{q}}{\dot{m}_h \cdot cp} \right) \quad (3.34)$$

Next, these expressions for the outlet temperatures are substituted into the expression for ΔT_{AMTD} :

$$\begin{aligned} \Delta T_{AMTD} &= \left(\frac{T_{h,in} + T_{h,out}}{2} \right) - \left(\frac{T_{c,in} + T_{c,out}}{2} \right) \\ \rightarrow \Delta T_{AMTD} &= \left[\frac{T_{h,in} - T_{c,in} + T_{h,in} - \left(\frac{\dot{q}}{\dot{m}_h \cdot cp} \right) - T_{c,in} - \left(\frac{\dot{q}}{\dot{m}_c \cdot cp} \right)}{2} \right] \\ \rightarrow \Delta T_{AMTD} &= T_{h,in} - T_{c,in} - \left(\frac{\dot{q}}{2 \cdot cp} \right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h} \right) \\ \therefore \Delta T_{AMTD} &= \Delta T_{max} - \left(\frac{\dot{q}}{2 \cdot cp} \right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h} \right) \end{aligned} \quad (3.35)$$

At this stage, it is appropriate to highlight the significance of Eq. 3.35. Firstly, the need to instrument and measure the two outlet temperatures has been eliminated, which reduces the number of variables and their associated measurement errors. Secondly, the outlet temperatures were replaced with known inlet variables based on the conservation of energy; therefore, the approach is rooted into the first law of thermodynamics. Thirdly, eq 3.35 contains the term ΔT_{max} , which acknowledges that the second law of thermodynamics is indeed involved in heat exchangers. Last but not least, unlike the expressions for ΔT_{LMTD} , the expression for ΔT_{AMTD} does not contain any exponential terms, which eliminate the discontinuities that result from using exponentials and logarithmic functions [19].

Given the newly derived expression for ΔT_{AMTD} , it is time to revisit the energy equation for the heat exchangers which is shown in Eq. 3.36:

$$\dot{q} = UA \cdot \Delta T_{LMTD} \cong UA \cdot \Delta T_{AMTD} \quad (3.36)$$

The expression for UA comes from the thermal resistor network analysis discussed previously.

$$R_{total} = \frac{1}{UA} = \frac{1}{K_{cold} * \dot{m}^{m_{cold}} A_{convection,cold}} + \sum R + \frac{1}{K_{hot} * \dot{m}^{m_{hot}} A_{convection,hot}} \quad (3.37)$$

For clarity, Eq. 3.37 is re-written as

$$\frac{1}{UA} = \frac{1}{a \cdot \dot{m}_c^c} + R_{wall} + \frac{1}{b \cdot \dot{m}_h^h} \quad (3.38)$$

where

$$a = K_{cold} * A_{convection,cold} = constant$$

$$b = K_{hot} * A_{convection,hot} = constant$$

$$R_{wall} = constant$$

$$\dot{m}_c^c = \dot{m}^{m_{cold}}$$

$$\dot{m}_h^h = \dot{m}^{m_{hot}}$$

Consequently, Eq. 3.36 can be expressed as:

$$\Delta T_{AMTD} = \frac{\dot{q}}{UA} = \dot{q} \times \left(\frac{1}{a \cdot \dot{m}_c^c} + R_{wall} + \frac{1}{b \cdot \dot{m}_h^h} \right) \quad (3.39)$$

At this stage, there are two expressions for ΔT_{AMTD} ; one comes from energy balance and the definition of ΔT_{AMTD} , and the other comes from thermal resistors. They can be equated and combined as follows, and the final expression is listed in Eq. 3.40.

$$\left[T_{h,in} - T_{c,in} - \left(\frac{\dot{q}}{2 \cdot cp} \right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h} \right) \right] = \frac{\dot{q}}{a \cdot \dot{m}_c^c} + \dot{q} R_w + \frac{\dot{q}}{b \cdot \dot{m}_h^h}$$

$$\Delta T_{max} - \left(\frac{\dot{q}}{2 \cdot cp} \right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h} \right) = \frac{\dot{q}}{a \cdot \dot{m}_c^c} + \dot{q} R_w + \frac{\dot{q}}{b \cdot \dot{m}_h^h}$$

$$\Delta T_{max} = \frac{\dot{q}}{a \cdot \dot{m}_c^c} + \dot{q} R_w + \frac{\dot{q}}{b \cdot \dot{m}_h^h} + \left(\frac{\dot{q}}{2 \cdot cp} \right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h} \right)$$

$$\frac{\Delta T_{max}}{\dot{q}} = \frac{1}{a \cdot \dot{m}_c^c} + R_w + \frac{1}{b \cdot \dot{m}_h^h} + \left(\frac{1}{2 \cdot cp} \right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h} \right)$$

$$\frac{\dot{q}}{\Delta T_{max}} = \frac{1}{\frac{1}{a \cdot \dot{m}_c^c} + R_w + \frac{1}{b \cdot \dot{m}_h^h} + \left(\frac{1}{2 \cdot cp}\right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h}\right)}$$

$$\dot{q} = \frac{\Delta T_{max}}{\frac{1}{a \cdot \dot{m}_c^c} + R_w + \frac{1}{b \cdot \dot{m}_h^h} + \left(\frac{1}{2 \cdot cp}\right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h}\right)} \quad (3.40)$$

Given that there is a ΔT_{max} term in this derivation, it can be directly linked to the ε -NTU method. See below.

$$\varepsilon = \frac{\dot{q}}{\dot{q}_{max}} = \frac{\dot{q}}{\dot{C}_{min} \Delta T_{max}} = \frac{1}{\dot{C}_{min}} \left(\frac{\dot{q}}{\Delta T_{max}} \right)$$

$$\varepsilon = \frac{1}{\dot{C}_{min}} \left[\frac{1}{\frac{1}{a \cdot \dot{m}_c^c} + R_w + \frac{1}{b \cdot \dot{m}_h^h} + \left(\frac{1}{2 \cdot cp}\right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h}\right)} \right]$$

$$\varepsilon = \frac{1}{\dot{C}_{min}} \left[\frac{1}{\left(\frac{1}{UA}\right) + \left(\frac{1}{2 \cdot cp}\right) \left(\frac{1}{\dot{m}_c} + \frac{1}{\dot{m}_h}\right)} \right]$$

$$\varepsilon = \left[\frac{1}{\left(\frac{1}{NTU}\right) + \left(\frac{1}{2}\right) \left(\frac{\dot{C}_{min}}{\dot{C}_c} + \frac{\dot{C}_{min}}{\dot{C}_h}\right)} \right] \quad (3.41)$$

As a check, assume $Cr=1$ (i.e., $\dot{C}_c = \dot{C}_h = \dot{C}_{min} = \dot{C}_{max}$) to compare this equation to the one available in textbooks:

$$\varepsilon = \left[\frac{1}{\left(\frac{1}{NTU}\right) + \left(\frac{1}{2}\right) \left(\frac{\dot{C}_{min}}{\dot{C}_c} + \frac{\dot{C}_{min}}{\dot{C}_h}\right)} \right] = \frac{1}{\left(\frac{1}{NTU}\right) + 1}$$

$$\varepsilon = \frac{NTU}{1 + NTU} \quad (3.42)$$

Eq. 3.42 is identical to the equation found in heat transfer textbooks, which supports the fact that the approach is mathematically sound. It is worth taking a step back to discuss the terms in Eq. 3.40. One

key observation is that the term $\dot{q}_{max}$ from ε -NTU method would occur when R_{total} or its reciprocal, $(1/UA)$, goes to 0. This can be mathematically written as follows:

$$\dot{q}_{max} = \frac{\Delta T_{max}}{\left(\frac{1}{UA}\right) + \left(\frac{1}{2 \cdot cp \cdot \dot{m}_c} + \frac{1}{2 \cdot cp \cdot \dot{m}_h}\right)} = \frac{\Delta T_{max}}{\left(\frac{(\dot{m}_c + \dot{m}_h)}{2 \cdot cp \cdot \dot{m}_c \cdot \dot{m}_h}\right)} = \left(\frac{2 \cdot cp \cdot \dot{m}_c \cdot \dot{m}_h}{\dot{m}_c + \dot{m}_h}\right) \Delta T_{max} \quad (3.43)$$

For the case where $Cr=1$, this reduces to Eq. 3.44, which is also found in textbooks.

$$\dot{q}_{max} = \dot{C}_{min} \Delta T_{max} \quad (3.44)$$

It must be emphasized that the presented form for Eq. 3.40 is due to the assumption made about the log mean temperature difference based on Eq. 3.31. As a result, logarithmic averaging using natural log terms was avoided. Consequently, the form of Eq. 3.41 differs from the one published in heat transfer textbooks (see Eq. 3.45). In summary, due to the assumptions made along the way, this method is only valid for heat exchangers with linear temperature profiles. If this method is to be applied to other heat exchangers, the derivation will have to be changed accordingly. As an aside, the following equations can be used as guidelines for developing an expression in terms of inlet temperatures for heat exchangers with non-linear temperature profiles [17]:

$$\varepsilon = \left[\frac{1 - \exp[-NTU(1 - Cr)]}{1 - Cr \cdot \exp[-NTU(1 - Cr)]} \right] \quad (3.45)$$

$$T_{c,o} = \frac{a(b - 1)T_{h,i} - b(a - 1)T_{c,i}}{(b - 1) - (a - 1)} \quad (3.46)$$

$$T_{h,o} = \frac{(a - 1)T_{h,i} - (b - 1)T_{c,i}}{(a - 1) - (b - 1)} \quad (3.47)$$

$$a = \frac{N_c}{N_h} = \frac{\dot{m}_c cp_c}{\dot{m}_h cp_h} = \frac{\dot{C}_c}{\dot{C}_h} = \frac{\Delta T_c}{\Delta T_h} \quad (3.48)$$

$$b = \frac{\Delta \theta_1}{\Delta \theta_2} = \exp(N_h - N_c) = \exp \left[\frac{UA(\dot{C}_c - \dot{C}_h)}{\dot{C}_c \dot{C}_h} \right] \quad (3.49)$$

With the connection established between the two experimental methods, it is now possible to use experimental data to solve for the unknown terms in the resistor network.

3.2.3 Connection to resistor network

The conventional thermal resistor network for a simple counterflow heat exchanger was shown in Figure 3-6. The total resistance is calculated using Eq. 3.50. Analogous to electrical resistor networks, one can think of the total heat transfer rate $\dot{q}$ as the current running through the circuit, and the potential (voltage) across the circuit is the temperature difference ΔT_{LMTD} , which is an unknown prior to testing. If everything was known, $\dot{q}$ can be calculated from Eq. 3.40.

$$R_{total} = \frac{1}{UA} = R_c + R_w + R_h \quad (3.50)$$

The new approach taken here was to transform the conventional resistor network to remove its dependency on the unknown potential ΔT_{LMTD} , and instead rely on the controlled inlet temperatures to the heat exchanger, combined with the conservation of energy to solve the circuit. In this case, the classic expression for R_{total} remains the same, as it represents the thermal resistance imposed by the heat exchanger and its combined convective and conductive heat transfer rates. However, due to the removal of ΔT_{LMTD} from the expression for $\dot{q}$, Eq. 3.40 must be used. This introduces two additional terms to the circuit that have the same units and act similar to thermal resistors in the system, referred to as $R_{c,AMTD}$ and $R_{h,AMTD}$. It is imperative to reiterate that these additional resistors are separate from the components that attribute to the UA term; in other words, they are independent of heat transfer rates, geometry, fouling, etc. and are based solely on conservation of energy. The absence of these resistor-like components in Wilson's plot approach is likely the reason why the method was found to be problematic.

$$R_{c,AMTD} = \frac{1}{2 \cdot \dot{m}_c \cdot cp} \quad (3.51)$$

$$R_{h,AMTD} = \frac{1}{2 \cdot \dot{m}_h \cdot cp} \quad (3.52)$$

$$\dot{q} = \frac{\Delta T_{max}}{\frac{1}{a \cdot \dot{m}_c^c} + R_w + \frac{1}{b \cdot \dot{m}_h^h} + \frac{1}{2 \cdot \dot{m}_c \cdot cp} + \frac{1}{2 \cdot \dot{m}_h \cdot cp}}$$

$$\dot{q} = \frac{\Delta T_{max}}{(R_{c,resistor} + R_w + R_{h,resistor}) + (R_{c,AMTD} + R_{h,AMTD})}$$

$$\dot{q} = \frac{\Delta T_{max}}{\left(\frac{1}{UA}\right) + (R_{c,AMTD} + R_{h,AMTD})} \quad (3.53)$$

These additional resistor-like terms are the reason behind the drop in performance in counterflow heat exchangers as the operating points deviate from $Cr=1$. A visual representation of this is given in Figure 3-13, where the effectiveness is plotted as a function of NTU, and each curve represents a different Cr . This plot should be interpreted as follows: A counterflow heat exchanger can provide X number of NTUs at $Cr=1$. If the same heat exchanger is to provide the same number of NTUs at a Cr that is lower than 1, it will have to have a higher effectiveness to do so. In other words, ε (measured at $Cr=1$) is smaller than ε (measured at $Cr<1$) for the same NTU. Therefore, reducing Cr from 1 should be seen as a penalty towards heat exchanger performance. This plot is often misinterpreted; the heat exchanger appears to become more effective (i.e. has higher effectiveness) as Cr is reduced. But this is due to the definition of effectiveness, in which the denominator contains $\dot{C}_{min}$; hence the magnitude of effectiveness changes as $\dot{C}_{min}$ changes. In summary, a higher effectiveness at $Cr=0.5$ vs. $Cr=1$ does not mean a higher heat transfer rate, and effectiveness values measured under different Cr values are not directly comparable. This is another reason why effectiveness shall not be confused with efficiency.

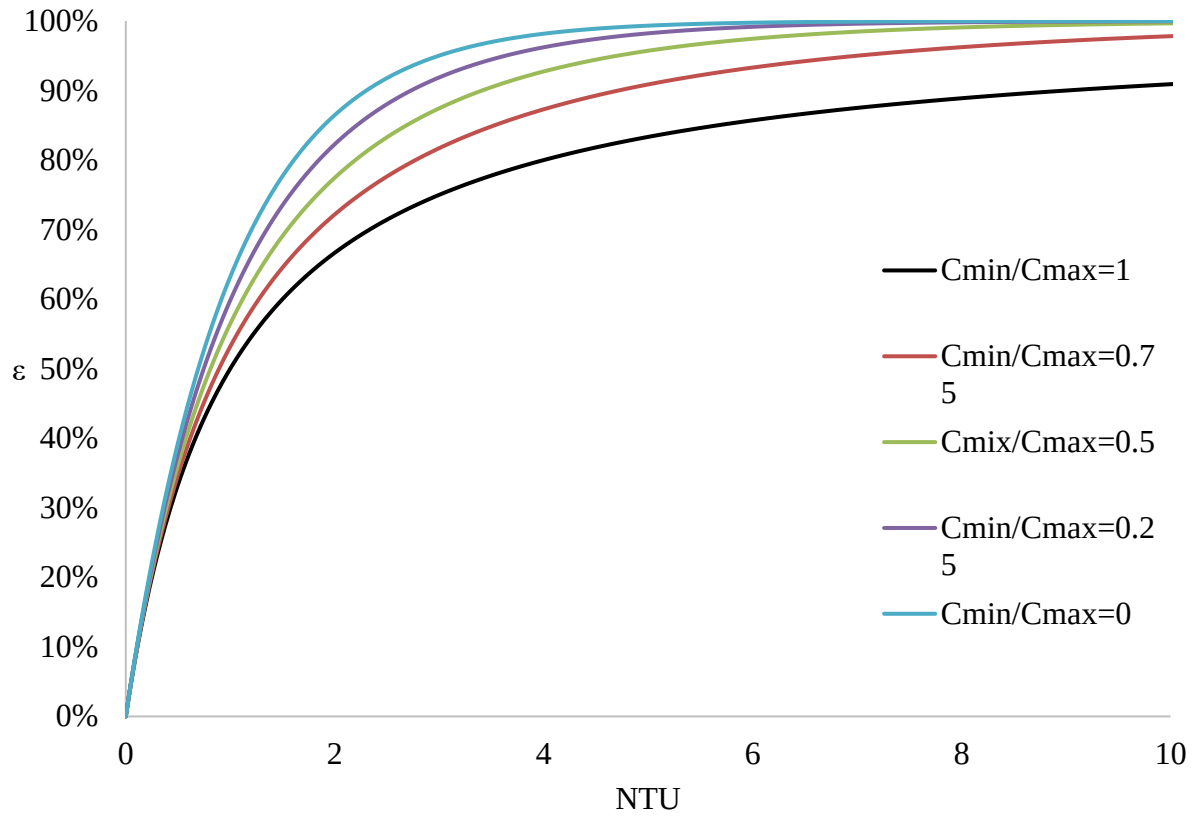


Figure 3-13: Effectiveness as a function of NTU for counterflow heat exchangers.

3.2.4 Application

This method can be used to estimate the unknown resistor terms for a heat exchanger. To do so, Eq. 3.54 is used:

$$\frac{\dot{q}}{\Delta T_{max}} = \frac{1}{\left(\frac{1}{UA}\right) + \left(\frac{1}{2 \cdot \dot{m}_c \cdot cp} + \frac{1}{2 \cdot \dot{m}_h \cdot cp}\right)} \quad (3.54)$$

There are 5 unknowns in this equation, which assumes nothing is known about the heat transfer characteristics of the given heat exchanger. If correlations are known, appropriate priors can be applied to help convergence. It is important to emphasize that this method is blind to fluid flow regimes in the heat exchanger; it is data-driven, and it has no way of predicting $\dot{m}$ at which the flow regime changes from laminar to turbulent or vice-versa. In terms of convective resistors, this would mean that the coefficients a and b , as well as powers c and h , are based on fluid flow regimes. Depending on the experimental data available for finding the unknown terms, it is possible to find

two distinct solutions through curve fitting. Therefore, additional care must be taken when applying this method to heat exchangers where the Reynolds number for the transition from laminar to turbulent flow is not known.

3.2.5 Experimental conditions for data collection

The experimental setup discussed in Manouchehri [28] was used to collect performance data for multiple DWHR heat exchangers. To produce data, heat exchangers were tested under all flowrate conditions prescribed by the CSA standard (i.e. 5.5, 7, 9, 10, 12, 14 L/min on each side) while measuring $\dot{q}$ based on the cold side of the heat exchanger. Finally, the maximum potential ΔT_{max} was kept constant at 28°C. Note that $\Delta T_{max}=28^\circ\text{C}$ used here was based on the application of DWHR heat exchangers and comes directly from CSA standard [11].

$$\Delta T_{max} = T_{h,in} - T_{c,in} = 38^\circ\text{C} - 10^\circ\text{C} = 28^\circ\text{C} \quad (3.55)$$

Fortunately, the author has collected a great deal of experimental data over the past few years based on ε -NTU method, and it is possible to revisit the data for multiple DWHR heat exchangers to determine the applicability of this method for finding the unknown resistors.

To illustrate the solution procedure, the performance data for a 2-inch diameter, 48-inch long DWHR heat exchanger is to be studied closely. The first step in doing so is to divide the experimentally calculated values for $\dot{q}$ by the corresponding ΔT_{max} during the experiment when the given $\dot{q}$ was measured. This represents the left-hand side of Eq. 3.54, which can then be plotted as a function of $\dot{m}_c$ and $\dot{m}_h$ as shown in Figure 3-14. It bears repeating that the maximum potential, ΔT_{max} , must be kept constant for generating the dataset.

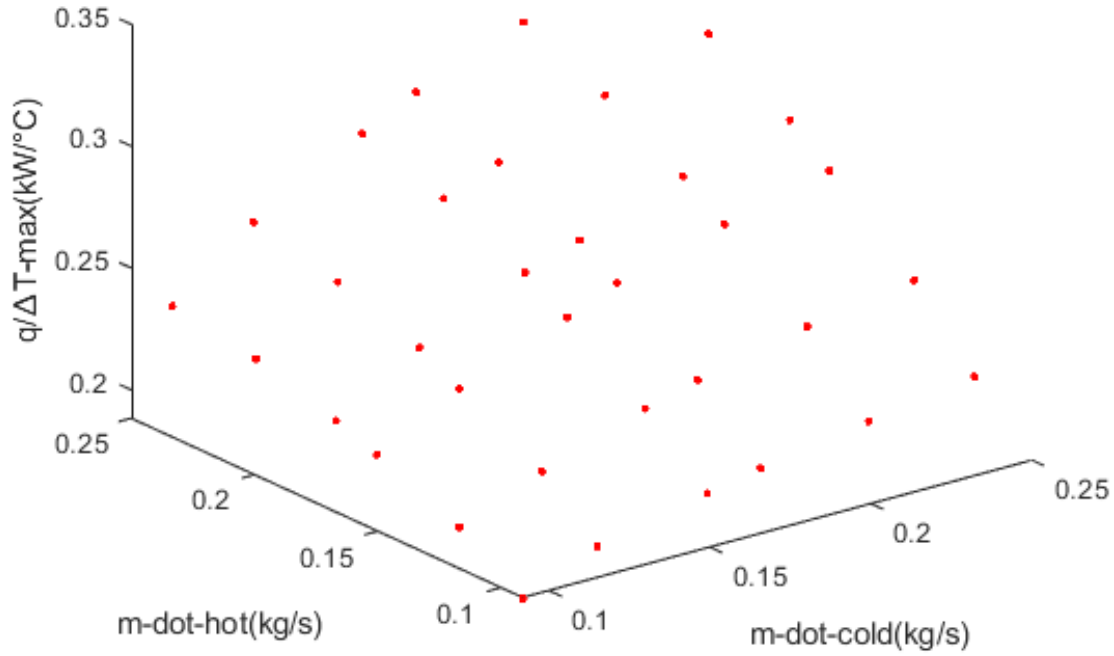


Figure 3-14: Experimental results for R248 (2inch diameter, 48inch long unit) plotted as a function of mass flowrates.

The next step is to invert both sides of the equation to simplify the math and remove the unknown terms from the denominator. Then, all the unknowns can be moved to the right-hand side of the equation as shown in Eq. 3.56.

$$\frac{\Delta T_{max}}{\dot{q}} - \left(\frac{1}{2 \cdot \dot{m}_c \cdot cp} + \frac{1}{2 \cdot \dot{m}_h \cdot cp} \right) = \frac{1}{a \cdot \dot{m}_c^c} + R_w + \frac{1}{b \cdot \dot{m}_h^h} \quad (3.56)$$

Wilson's plot is not being employed here and this approach is closer to ϵ -NTU than it is to Wilson's work. The data shown in Figure 3-14 were then fed to the curve fitting tool in MATLAB to find the unknown terms using a custom equation in form of Eq. 3.54. If any prior knowledge exists about the unknown resistors, these can be input into the curve fitting tool. The following logical priori were used: the constants a , b , and R_w are greater than 0 and the powers c and h are between 0 and 1. The calculated unknowns are listed in Table 3-6, including the priori used to help find them. In terms of goodness of fit, an R^2 value of 0.9961 for the fit was provided by MATLAB. Clearly, this equation fits the data well; whether the coefficients are accurately calculated or not is a result of data collection and uncertainty analysis. However, this level of accuracy was deemed sufficient for modelling purposes, as will be shown in the validation section.

Table 3-6: Results of curve fitting for R248 (2inch diameter, 48inch long unit).

Unknown Constants	Experimental Values	Applied Priori
a	2.265	Greater than 0
b	1.234	Greater than 0
R_w (°C/kW)	0.01	Greater than 0
c (dimensionless)	0.29	Between 0 and 1
h (dimensionless)	0.34	Between 0 and 1

The units for coefficients a and b can only be assigned after coefficients c and h have been determined. The unit for each thermal resistor is known (°C/kW) and once the powers for mass flowrates are known, the appropriate units can be determined using the following equations such that the unit on the left-hand side matches the right-hand side. Therefore, the units for a and b are written in terms of c and h ; a is measured in $\frac{kW \cdot s^c}{^\circ C \cdot kg^c}$ and b is measured in $\frac{kW \cdot s^h}{^\circ C \cdot kg^h}$.

$$R_{convection} = \frac{1}{h_{convection} Area} = \frac{1}{(Constant) * (\dot{m}^m)} \quad (3.57)$$

$$R_{coils} = \frac{1}{h_{coils} \cdot Area} = \frac{1}{a \cdot \dot{m}_c^c} \quad (3.58)$$

$$R_{film} = \frac{1}{h_{film} \cdot Area} = \frac{1}{b \cdot \dot{m}_h^h} \quad (3.59)$$

The heat transfer coefficients in these equations are based on experimental data, and are dimensional; hence, these should not be confused with the nondimensional Nusselt number derivations in terms of Re and Pr . It is now possible to write the governing equation for this heat exchanger as follows.

$$\frac{q}{\Delta T_{max}} = \frac{1}{\frac{1}{2.265 \cdot \dot{m}_c^{0.29}} + 0.01 + \frac{1}{1.234 \cdot \dot{m}_h^{0.34}} + \frac{1}{2 \cdot \dot{m}_c \cdot (4.18)} + \frac{1}{2 \cdot \dot{m}_h \cdot (4.18)}} \quad (3.60)$$

It is helpful to graphically illustrate the shape of the surface that was fit to the data. This shape is difficult to show on paper, and is consequently shown from multiple views in Figure 3-15 through Figure 3-18, and the mathematical name for this surface is a hyperbolic paraboloid.

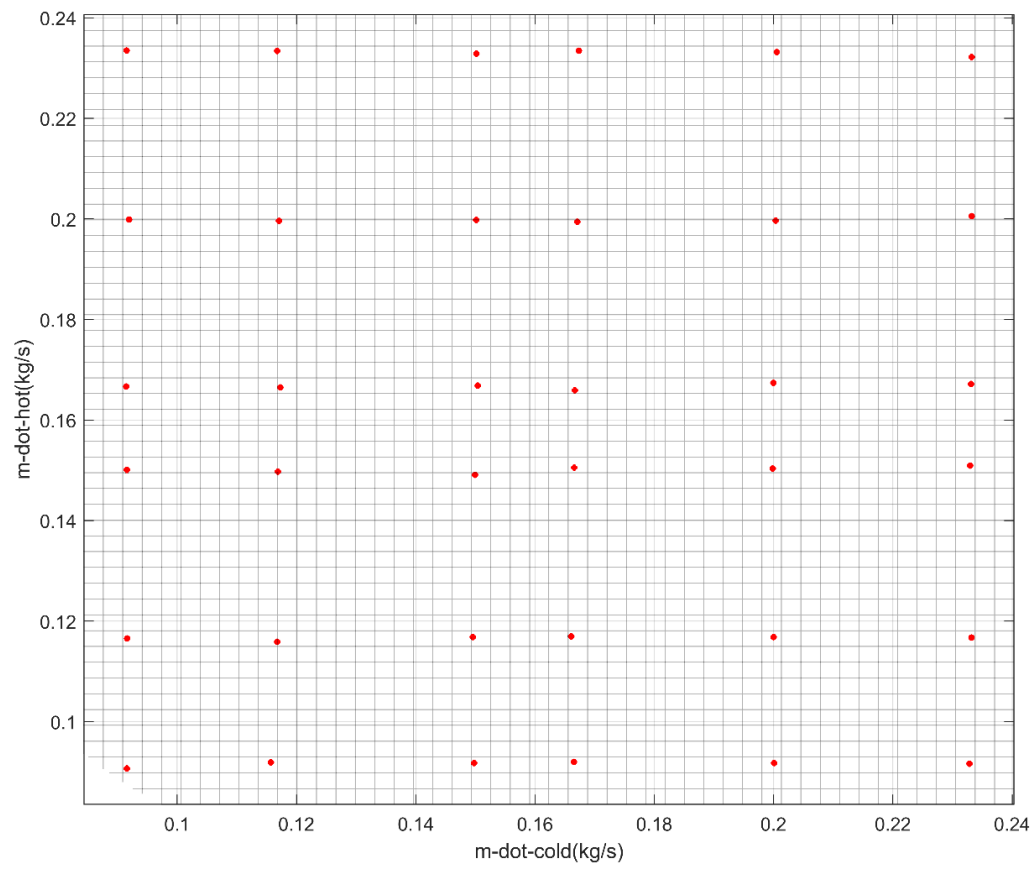


Figure 3-15: Top view of the surface, showing grid discretization in terms of mass flowrates.

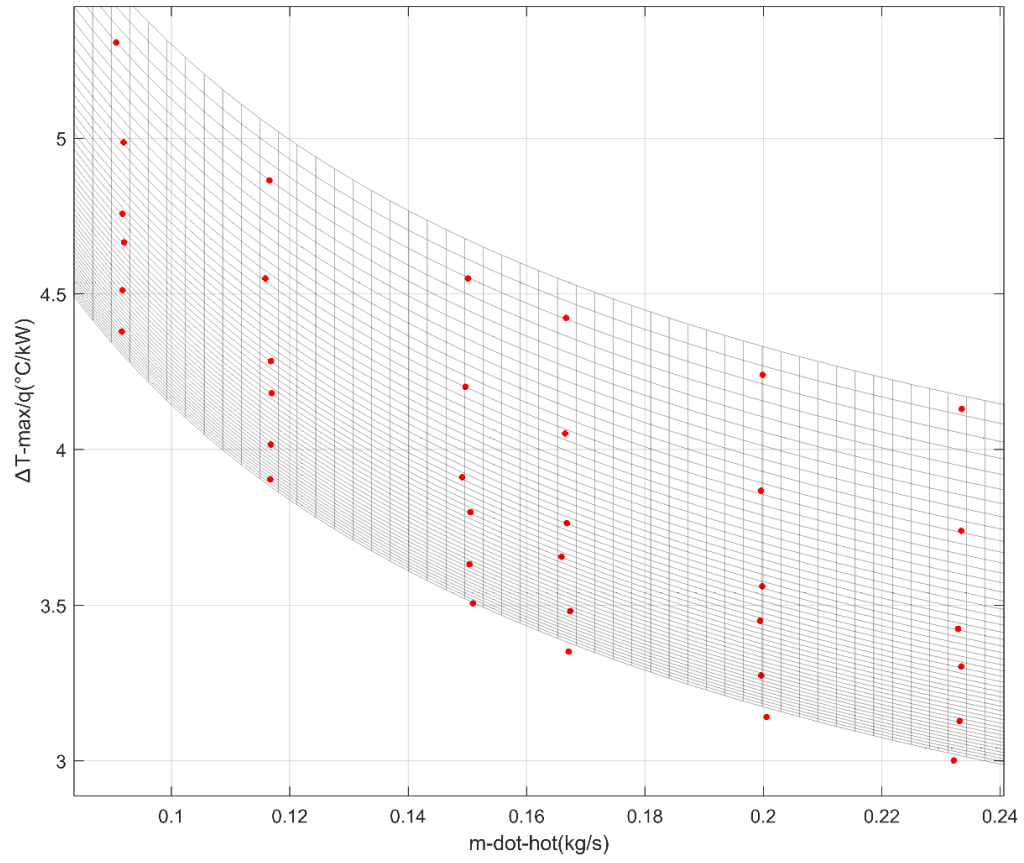


Figure 3-16: $(\Delta T_{max}/\dot{q})$ plotted as a function of mass flowrate for the falling-film in a R248 heat exchanger. The red dots represent experimental results.

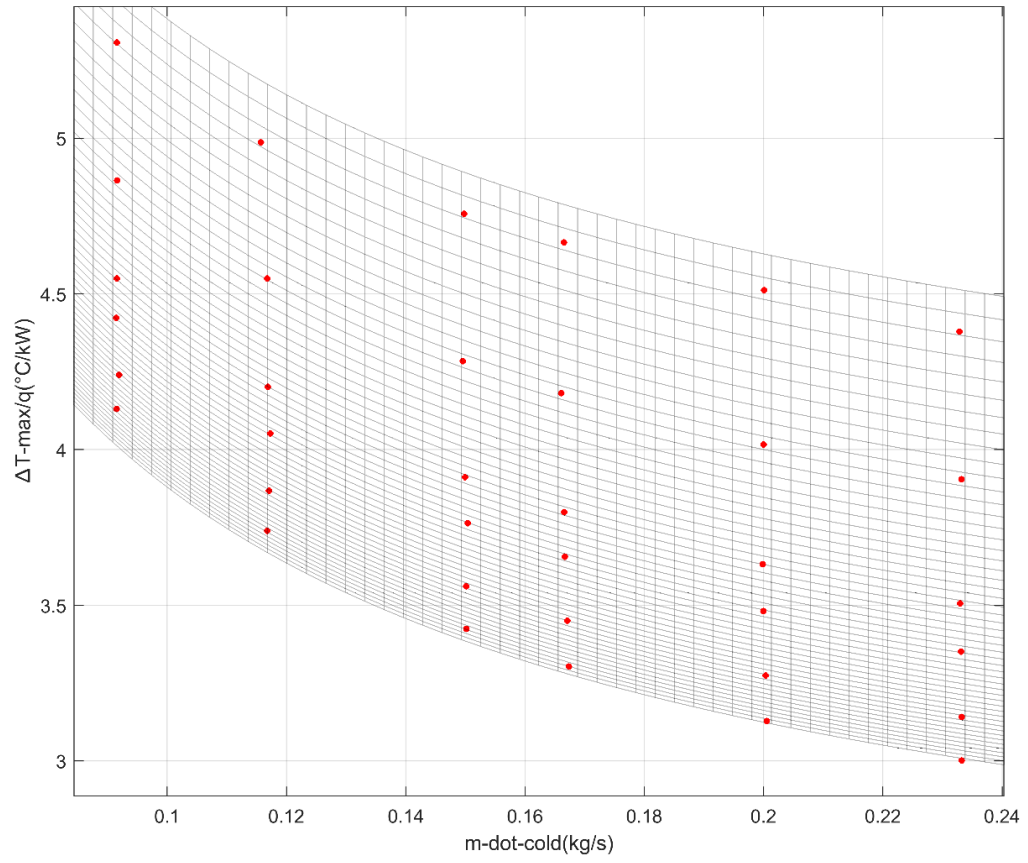


Figure 3-17: $(\Delta T_{max}/\dot{q})$ plotted as a function of mass flowrate for the coils in a R248 heat exchanger. The red dots represent experimental results.

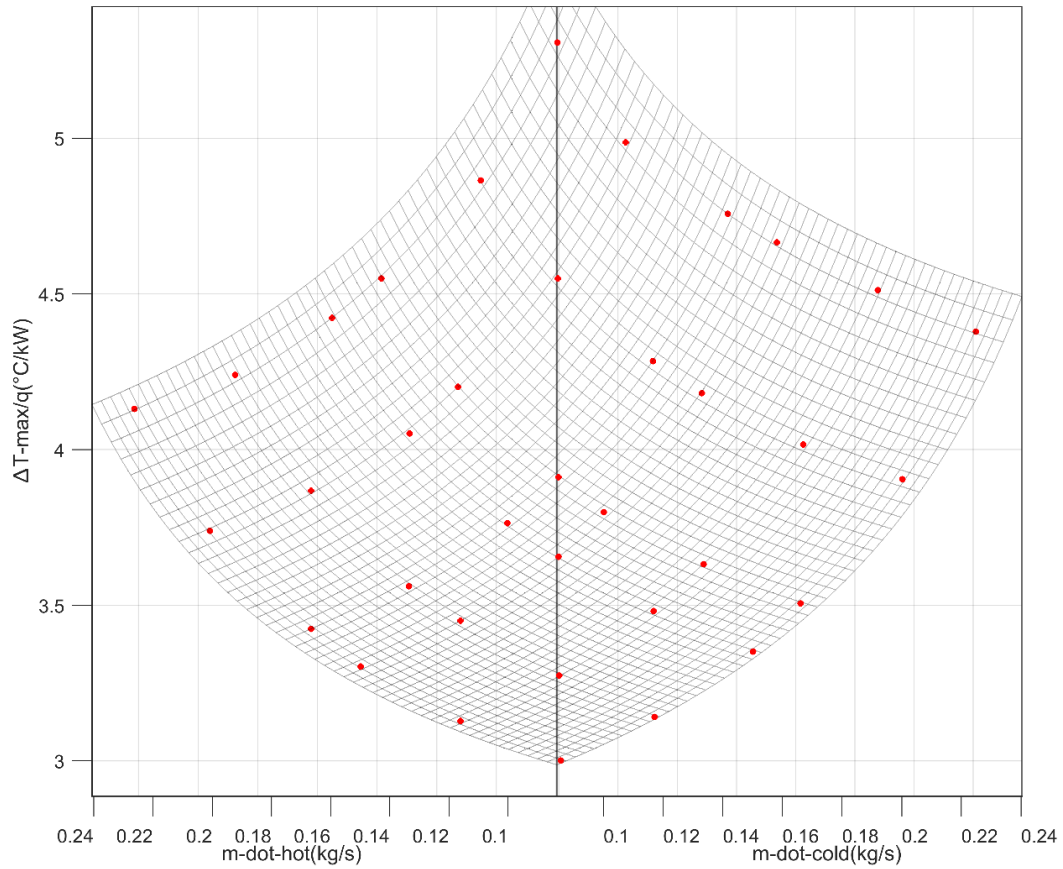


Figure 3-18: Isometric view of the surface generated using Eq. 3.54 for R248. The red dots represent experimental results.

This method was applied to four other DWHR heat exchangers to find the unknown constants in the resistor network. The results are presented graphically in Figure 3-19 and Figure 3-20 for two of these heat exchangers to illustrate the similarity between the plots for different heat exchangers. The results for all heat exchangers are presented in tabular format in Table 3-7.

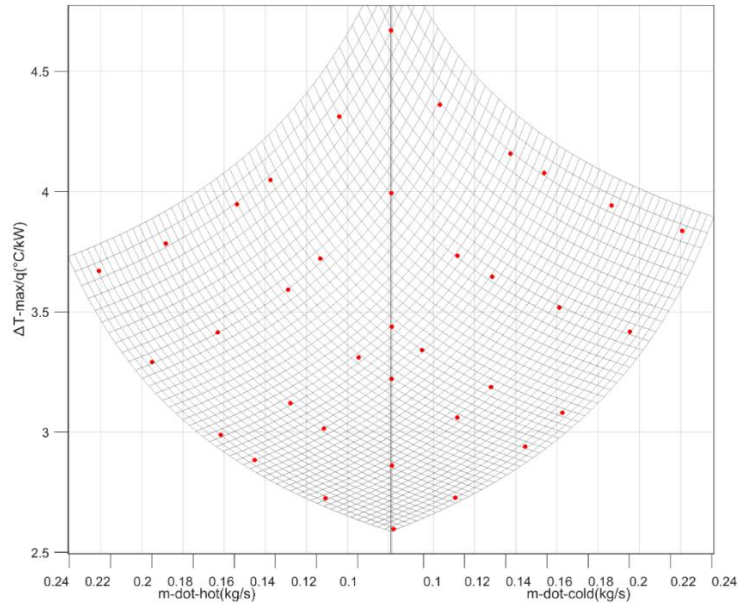


Figure 3-19: Isometric view of the surface generated using Eq. 3.54 for R348. The red dots represent experimental results.

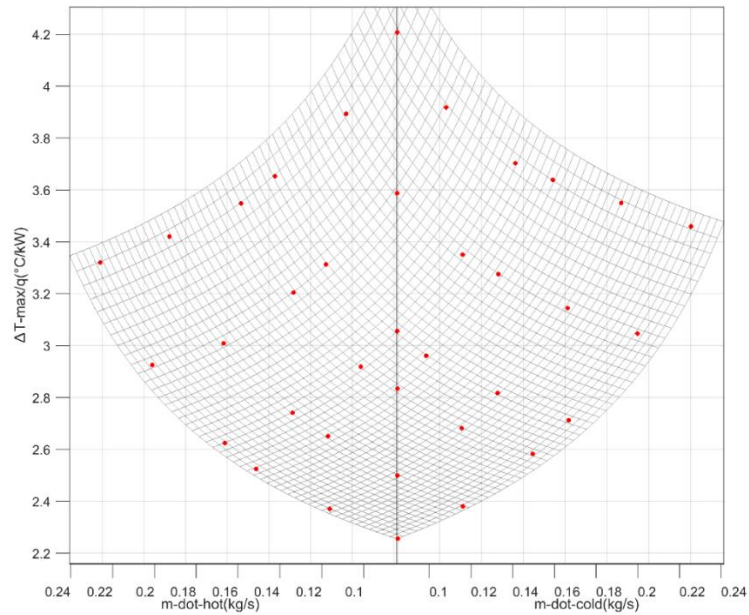


Figure 3-20: Isometric view of the surface generated using Eq. 3.54 for R360. The red dots represent experimental results.

Table 3-7: Regression coefficients. Note that for each product (e.g. R248), the first digit denotes the diameter in inches, and the following two digits denote the overall length of the heat exchanger in inches.

Unknowns	R248	R348	R360	WC340	R448
R (°C/kW)	0.01	0.1	0.1	0.05	0.054
a ($kW \cdot s^c / ^\circ C \cdot kg^c$)	2.265	2.452	3.203	1.891	2.498
b ($kW \cdot s^h / ^\circ C \cdot kg^h$)	1.234	1.85	2.35	1.576	2.55
c (dimensionless)	0.29	0.29	0.29	0.29	0.29
h (dimensionless)	0.34	0.34	0.34	0.34	0.34
Fit information	R248	R348	R360	WC340	R448
Sample Size	36 points	36 points	36 points	36 points	22 points
R²	0.996	0.995	0.993	0.997	0.991
SSE	0.045	0.044	0.063	0.029	0.026
RMSE	0.036	0.036	0.043	0.029	0.035

Solely judging by the information provided by MATLAB regarding the goodness of fit as listed in Table 3-7, it can be concluded that Eq. 3.54 is a good fit for characterizing DWHR heat exchangers. However, it is prudent to discuss every single one of the constants found in this manner and determine if they make physical sense.

3.2.5.1 Regression Constant **R**:

The constant R represents the sum of terms listed in Eq. 3.61. The data sets correspond to heat exchangers without any fouling present, and the manufacturing defects are not known, so to approximate this term, the conduction resistance will be estimated as per Eq. 3.62. Here, the conduction length can be calculated based on the wall thicknesses for the copper walls in terms of their radii. k_{wall} denotes the conductivity of the wall, which is made of copper and has a value of $401 W/m^\circ C$. A_{wall} is the heat transfer area.

$$\sum R = R_{wall,conduction} + R_{fouling} + R_{manufacturing\ defects} \quad (3.61)$$

$$\sum R \approx R_{wall,conduction} = \frac{Length_{conduction}}{k_{wall}A_{wall}} \quad (3.62)$$

Using the physical dimensions of the heat exchangers and the conductivity of copper, the conduction term was calculated for R348 and R448 from Table 3-7. The results were 0.022 (°C/kW) and 0.017 (°C/kW) respectively. Both results are within the same order of magnitude as the ones recovered using MATLAB; they are smaller in magnitude as well. This is reasonable because in reality, manufacturing quality/defects and contact resistances contribute to $\sum R$, and the experimentally measured values should always be higher than the theoretical conduction resistances.

3.2.5.2 Regression Constant a :

This constant is part of the expression for heat transfer coefficient and can be expressed as follows:

$$R_{coils} = \frac{1}{h_{coils} \cdot Area} = \frac{1}{a \cdot \dot{m}_c^c}$$

$$\rightarrow a = \frac{h_{coils} \cdot Area}{\dot{m}_c^c} \quad (3.63)$$

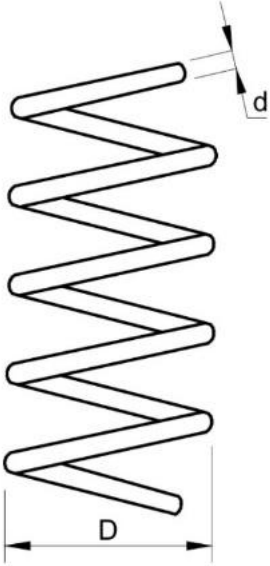
To compare the values of a for different heat exchangers listed in Table 3-7, they must be first nondimensionalized with respect to an area. The known area for DWHR heat exchangers is the drain area, which is a simple circular drainpipe. The results are shown in Table 3-8.

Table 3-8: Constant a per unit area for each heat exchanger.

Unknown per unit area of drain	R248	R348	R360	WC340	R448
a'' ($kW \cdot s^c / ^\circ C \cdot kg^c \cdot m^2$)	5.820	4.201	4.390	3.887	3.210

It is encouraging that the recovered constants for the heat exchangers are close and within the same order of magnitude. However, they are not equal and the magnitude of constant a seems to decrease as diameter of the drainpipe is increased. This can be explained through the physics of DWHR heat exchangers, or more specifically the copper coils. Helical coils are known to be impacted by the non-dimensional number known as the Dean number. See Eq. 3.64. It is the product of Reynolds number and the ratio of certain diameters pointed out in Figure 3-21. Based on the equation, the Dean number would get smaller as the drainpipe diameter D is increased. This suggests that the constant a includes the contribution from the Dean number, and if Nusselt number correlations were to be developed for helical coils, they should account for this effect. This falls outside the scope of this thesis. Instead, it

is recommended that constant a'' be found and listed for DWHR heat exchangers as a function of Drainpipe diameter, noting that these constants cannot readily be scaled to model different diameters.

$$De = Re_D \sqrt{\frac{d}{D}} \quad (3.64)$$


The diagram shows a vertical helical coil. A dimension line labeled 'd' indicates the diameter of the coil's cross-section. Another dimension line labeled 'D' indicates the overall diameter of the coil, measured from the outer edge of the helix to the outer edge of the helix.

Figure 3-21: Helical coils' dimensions involved in the calculation of Dean number.

3.2.5.3 Regression Constant c :

The method for assessing c is to compare it with the power for Reynolds number in existing Nusselt correlations for similar geometries. The problem is that there are no correlations that are specific for DWHR heat exchangers and their rectangular coil geometry. The only option is to look at similar geometries, or in this case, helical coils with circular cross section, as listed in Eq. 3.65. This correlation was developed by Dravid et al. for Laminar flow inside helical coils [45]. The two observations that can be made here are that: 1) the Dean number is present in the correlation, and 2) the power for Reynolds number is $\frac{1}{4}$. Constant c was approximated by MATLAB to be 0.29 through the regression process, which is close enough to the value of 0.25 from the Nusselt number correlation derived for a similar geometry.

$$Nu = (0.76 + 0.65\sqrt{De})Pr^{0.175} \quad (3.65)$$

3.2.5.4 Regression Constant b :

This constant is part of the expression for heat transfer coefficient for the falling film and it can be expressed as follows:

$$R_{film} = \frac{1}{h_{film} \cdot Area} = \frac{1}{b \cdot \dot{m}_h^h}$$

$$b = \frac{h_{film} \cdot Area}{\dot{m}_h^h} \quad (3.66)$$

Following the same procedure as Section 3.2.5.2, this constant is nondimensionalized with respect to the area of the drainpipe for each heat exchanger and the results are in Table 3-9. It is evident that the magnitude of this constant is scalable between different heat exchanger sizes. The implication here is that b does not contain other dimensionless constants such as the Dean number for the coils.

Table 3-9: Constant b per unit area for each heat exchanger.

Unknown per unit area of drain	R248	R348	R360	WC340	R448
$b'' (kW \cdot s^h / ^\circ C \cdot kg^h \cdot m^2)$	3.171	3.169	3.221	3.240	3.276

3.2.5.5 Regression Constant h :

The procedure for determining the validity of constant h is similar to that of constant c , and that is to find a correlation in the literature that is close enough to the falling film within DWHR heat exchangers. The correlation in Eq. 3.67 was developed by McAdams et al. for falling film within small tubes [46]. Based on this equation, 1) there are no dimensionless terms present except for Reynolds and Prandtl numbers, which were accounted for in the derivation for constant b , and 2) the power for the Reynolds number is $\frac{1}{3}$. The power of Reynolds number was approximated by MATLAB to be 0.34 through regression, as was listed in Table 3-7.

$$h_{film} = 0.01(Re_D)^{\frac{1}{3}} \left((Pr)^{\frac{1}{3}} \left(\frac{k^3 g \rho^2}{\mu^2} \right)^{\frac{1}{3}} \right) \quad (3.67)$$

3.2.6 Potential Applications for the Resistor Model:

Now that there is enough information to model each of the resistors in the previously shown thermal resistor network, and there is a procedure for finding them for new and novel DWHR heat exchangers, the following generalizations can be made:

- 1) Fouling can be detected. This will both affect $\sum R$ and the general shape that was shown in Figure 3-18.
- 2) Designs and sizing can be tailored towards realistic operating conditions.
- 3) Predicting performance at different flow conditions can easily be done with Eq. 3.54.

One indirect application of the results is to revisit the effectiveness curve prescribed by the CSA standard, and further assess its validity in terms of the findings. As part of the validation results that were presented during the comprehensive examination (Section 3.1), it was shown that extrapolating the CSA curve was problematic, and that it would always underpredict performance at increased flowrates. The solution that was recommended at the time was to add an additional point to the standard and expand the curve fit to include a test point at 18 L/min. The impact of doing so was previously shown on Figure 3-2. Using the resistor network and Eq. 3.41, the effectiveness curve can be generated and compared with the CSA curve, as shown in Figure 3-22. As evident by the figure, CSA curve matches the results of the resistor network closely for the range of flowrates included in the standard. It is also clear that extrapolating the curve will result in errors, and as the flowrate is increased, the existing CSA curve underpredicts performance in comparison to those predicted by the resistor network. This is inline with the previous finding that this curve should not be extrapolated unless additional points are added (i.e., 18 L/min).

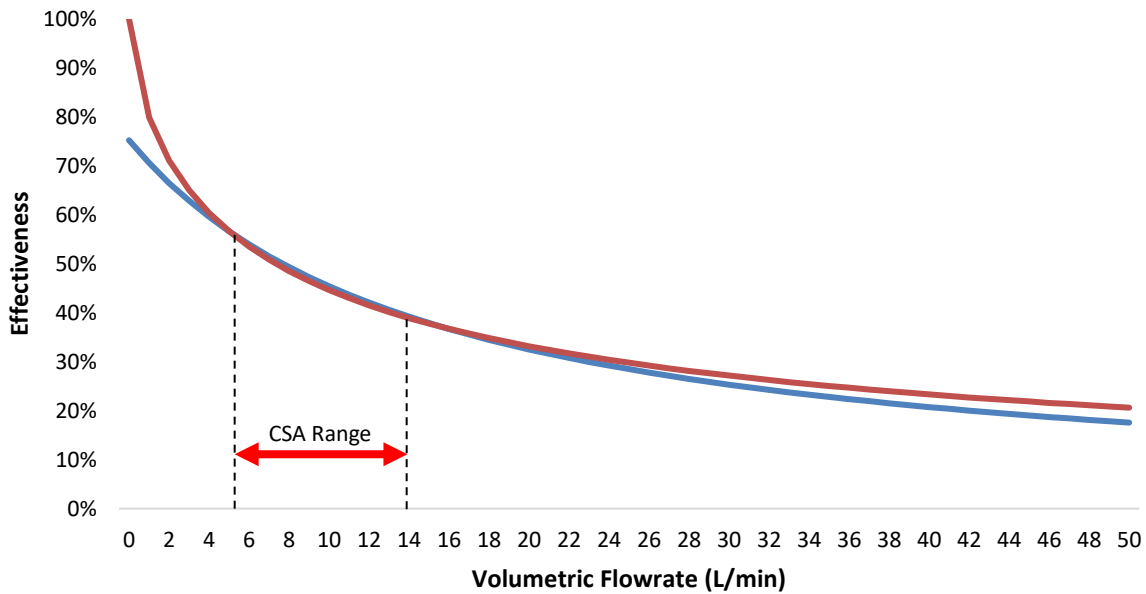


Figure 3-22: Comparison between the existing CSA curve and the results from the model. The blue curve represents the effectiveness curve from CSA, and the red curve represents the calculated effectiveness.

3.2.6.1 Validation of Resistor Network Model

This section uses the same dataset generated for Section 3.1 for validation, to allow direct comparison between the models. Due to the complexity of the model, an example will be provided to show how the model can be applied to DWHR heat exchangers. This example will use the same data point as the example provided in Section 3.1. The inputs used in the example were $T_{c,in} = 4.7^{\circ}\text{C}$, $T_{h,in} = 47.3^{\circ}\text{C}$, a mains-side flow rate of 3.97 L/min, and a drain-side flow rate of 7.97 L/min. The measured steady-state heat transfer rate for this test was 6.25 kW.

The first step is to select appropriate constants from Table 3-7 based on diameter. Noting that the data in this example corresponds to a 2" diameter, the constants under column R2-48 were selected. The next step is to scale constants a , b , and R using the ratio of heat exchanger lengths between the one in this example, and the one from Table 3-7. This is necessary step because these constants include heat transfer areas, and these areas are a function of length. Therefore, the constants will be scaled by the ratio of lengths, or rather $\left(\frac{32}{48}\right)$ for this example. See Table 3-10 for a list of constants.

Table 3-10: Summary of data used in the example for resistor network.

Variables of Interest	R248 (from Table 3-7)	Heat exchanger in this example
Diameter (inches)	2	2
Length (inches)	48	32
R (°C/kW)	0.01	$0.01 \times \left(\frac{32}{48}\right) = 0.0067$
a (kW · s ^c / °C · kg ^c)	2.265	$2.265 \times \left(\frac{32}{48}\right) = 1.51$
b (kW · s ^h / °C · kg ^h)	1.234	$1.234 \times \left(\frac{32}{48}\right) = 0.8227$
c (dimensionless)	0.29	0.29
h (dimensionless)	0.34	0.34

The next step is to write the governing equation for this heat exchanger using the newly calculated constants from Table 3-10. See Eq. 3.68.

$$\dot{q} = \frac{\Delta T_{max}}{\frac{1}{a \cdot \dot{m}_c^c} + R_w + \frac{1}{b \cdot \dot{m}_h^h} + \frac{1}{2 \cdot \dot{m}_c \cdot cp} + \frac{1}{2 \cdot \dot{m}_h \cdot cp}}$$

$$\rightarrow \dot{q} = \frac{\Delta T_{max}}{\frac{1}{1.51 \cdot \dot{m}_c^{0.29}} + 0.0067 + \frac{1}{0.8227 \cdot \dot{m}_h^{0.34}} + \frac{1}{2 \cdot \dot{m}_c \cdot (4.18)} + \frac{1}{2 \cdot \dot{m}_h \cdot (4.18)}} \quad (3.68)$$

It is now possible to insert the input conditions into the governing equation to estimate the heat transfer rate for this example. As shown in Eq. 3.69, the heat transfer rate was predicted to be 6.47 kW using the governing equation, which is 0.23 kW higher than the measured value of 6.25 kW. The prediction error in this case was 3.6%.

$$\dot{q} = \frac{(47.3^\circ\text{C} - 4.7^\circ\text{C})}{\frac{1}{1.51 \times \left(\frac{3.97}{60}\right)^{0.29}} + 0.0067 + \frac{1}{0.8227 \times \left(\frac{7.97}{60}\right)^{0.34}} + \frac{1}{2 \times \left(\frac{3.97}{60}\right) \times (4.18)} + \frac{1}{2 \times \left(\frac{7.97}{60}\right) \times (4.18)}}$$

$$\rightarrow \dot{q} = \frac{42.6^\circ\text{C}}{1.4554 + 0.0067 + 2.4151 + 1.8065 + 0.901} = \frac{42.6^\circ\text{C}}{6.5847^\circ\text{C}/\text{kW}} = 6.47 \text{ kW} \quad (3.69)$$

This solution method was applied to the rest of the experimental data from the dataset containing a total of 135 tests. Figure 3-23 shows a plot of the predicted heat transfer rates using this method versus the experimental data. The identity line represents a perfect model and was also added to the figure to visually highlight the new model's inaccuracies. The Mean Absolute Percentage Error between the measured and predicted values for the entire dataset was 2.44%. This is relatively close to, but smaller than the error associated with the existing model presented in Section 3.1.

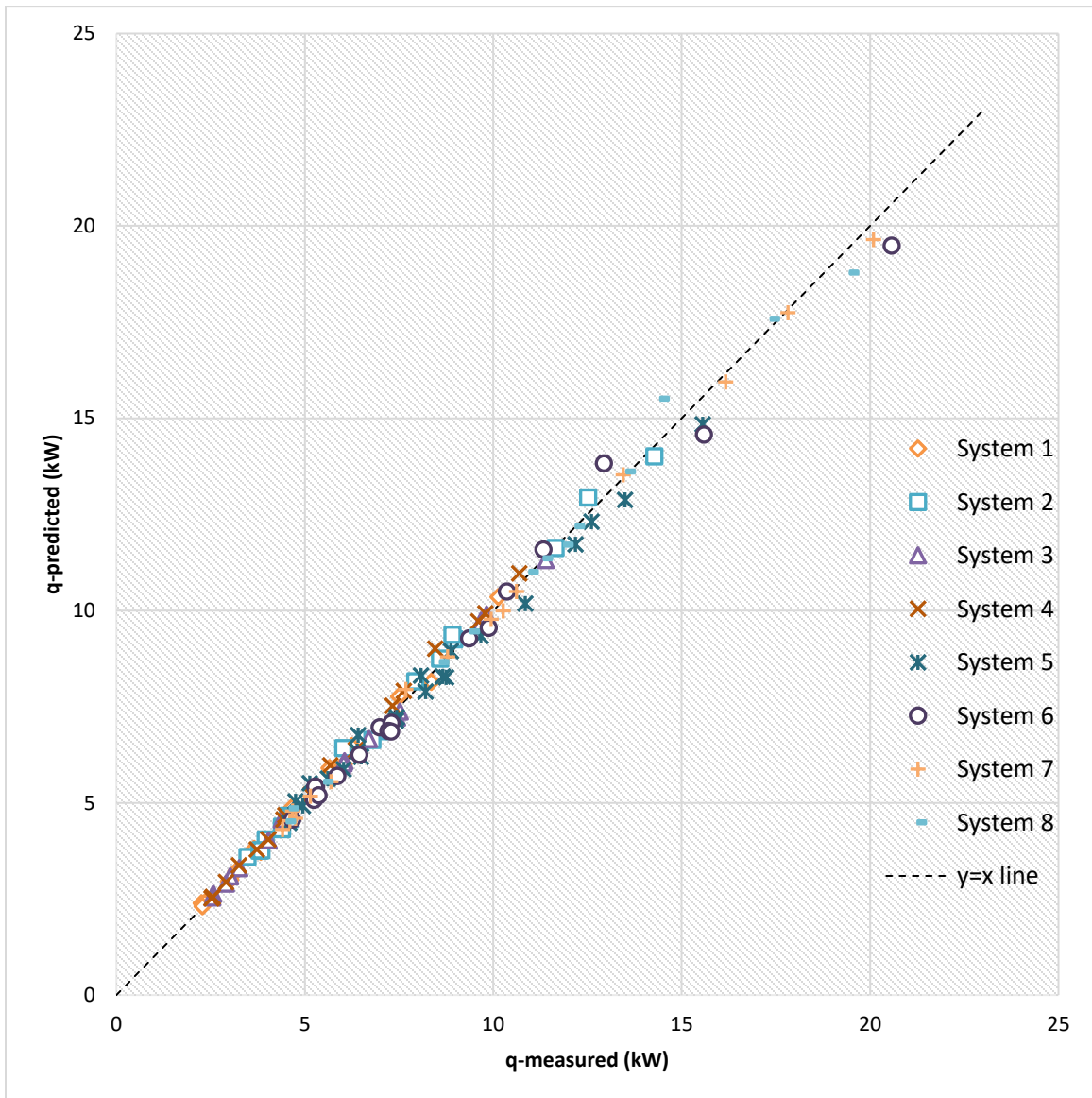


Figure 3-23: Predicted vs. measured heat transfer rates when the resistor model was applied to the dataset from Section 3.1.

3.3 Shortcut model:

Given the high degree of similarity between these heat exchangers, it was of interest to develop a shortcut to allow approximation of a heat exchanger's performance relative to an experimentally measured point that lies on the performance curve. A preliminary form of this approximation method was developed in the author's MASc thesis [28], where the heat transfer $\dot{q}$ at an unbalanced flow condition (i.e. $Cr < 1$) could be approximated using the performance of the same heat exchanger under balanced flow.

$$\dot{q}_{(\dot{V}_{cold}=F1 \ \& \ \dot{V}_{hot}=F2)} = \dot{q}_{(\dot{V}_{cold}=\dot{V}_{hot}=F1)} \times \left[0.3452 \times \ln \left(\frac{\dot{V}_{hot} = F2}{\dot{V}_{cold} = F1} \right) + 1 \right] \quad (3.70)$$

A visual representation of this method is shown in Figure 3-24, where the equal-flow performance is the curve along the dashed line and is known from the characteristic effectiveness curve. Eq. 3.70 allows taking a point along the dashed line and estimating performance at another point on the surface with equal $\dot{m}_c$ while varying $\dot{m}_h$. Graphically, the reference point is on the dashed line and the equation allows translating the performance along the y-axis (varying $\dot{m}_h$). In summary, to use this approach, the following must be known:

- Characteristic effectiveness curve; used to predict performance for a point on the dashed line
- Eq. 3.70, and the regression constant that allows movement along $\dot{m}_h$ -axis.

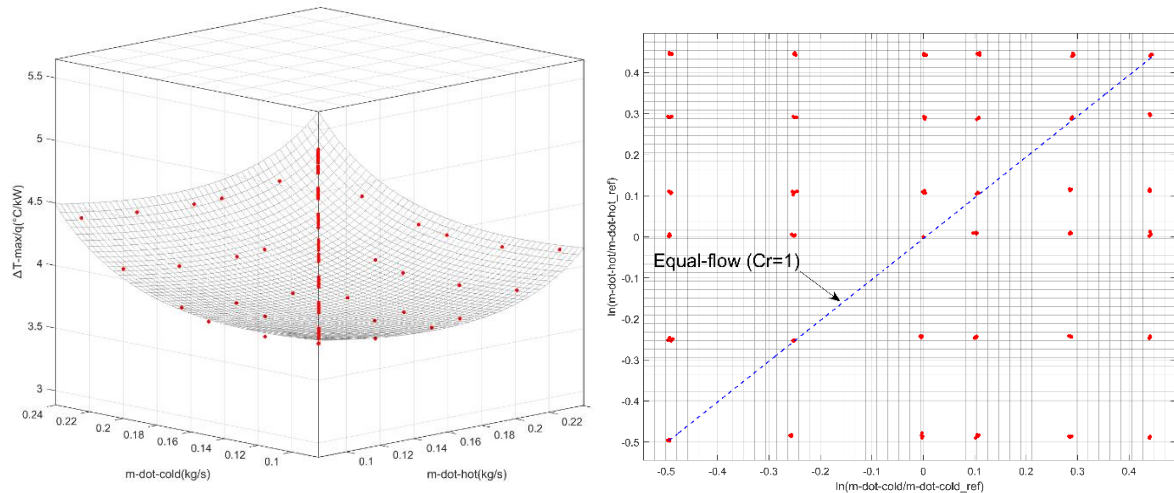


Figure 3-24: Graphical representation of the location for the CSA curve on the surface plot for $(\Delta T_{max}/\dot{q})$.

Now that more information is known about the CSA effectiveness curve and its limitations, it stands to reason to expand this approach so that it relies on individual performance points, rather than an approximated performance point based on the CSA effectiveness curve. As a reminder, this is an experimental fit and it does not necessarily apply to other heat exchangers.

The following equation can be used to estimate performance of a DWHR heat exchanger based on its performance at a reference point.

$$\frac{\left(\frac{\Delta T_{max,ref}}{q_{ref}}\right)}{\left(\frac{\Delta T_{max}}{q}\right)} = \left(\frac{q}{q_{ref}}\right) \left(\frac{\Delta T_{max,ref}}{\Delta T_{max}}\right)$$

$$= 1 + P_x * \ln\left(\frac{\dot{m}_c}{\dot{m}_{c,ref}}\right) + P_y * \ln\left(\frac{\dot{m}_h}{\dot{m}_{h,ref}}\right) + P_x * \ln\left(\frac{\dot{m}_c}{\dot{m}_{c,ref}}\right) * P_y * \ln\left(\frac{\dot{m}_h}{\dot{m}_{h,ref}}\right) * P_{xy} \quad (3.71)$$

where P_s are dimensionless regression constants to be found experimentally. The reference point was chosen to be at 9L/min, equal-flow condition at ΔT_{max} of 28°C. This is a performance point that should be available for every DWHR heat exchanger that is sold on the market in North America, as it is a part of the certification process. This reference point is shown on Figure 3-25.

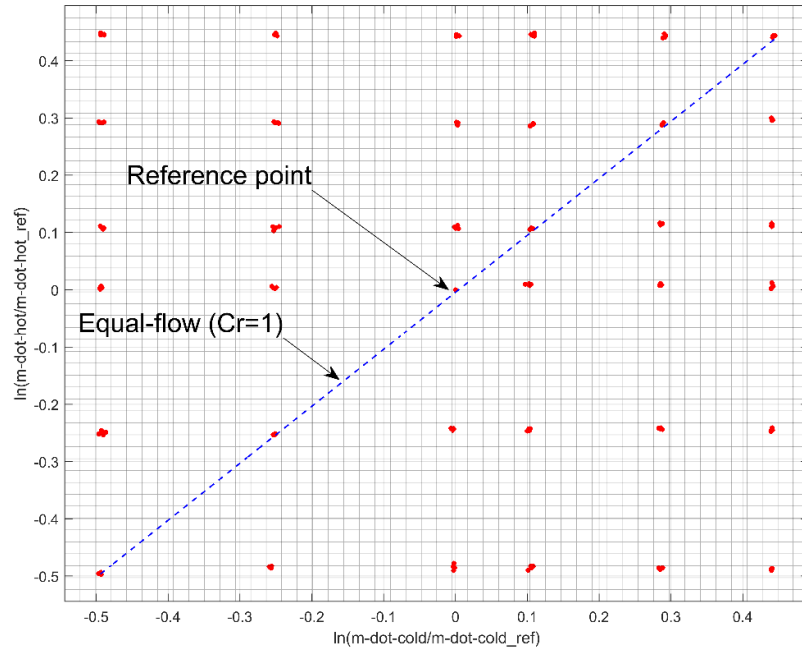


Figure 3-25: Graphical representation of the location for the new reference point.

To find regression constants, data from multiple DWHR units were nondimensionalized according to Eq. 3.71 and the result is shown in Figure 3-26 and in Table 3-11. The findings suggest that regardless of diameter, length, coil design or manufacturer, performance for these heat exchangers can be approximated using Eq. 3.72. Note that this equation is based on ratios; in other words, the units cancel out and this can be thought of as a way of approximating distances on the surface plot, rather than a method that is based on energy balances. Hence, it is an approximate method, albeit a really good one, and should be used with utmost care. In summary, to use this method, the only required info are constants P_x , P_y , P_{xy} (which are provided here) and a reference point for the performance of the heat exchanger, which is printed on the product label.

$$\left(\frac{q}{q_{ref}}\right)\left(\frac{\Delta T_{max,ref}}{\Delta T_{max}}\right) = 1 + 0.2896 \cdot \ln\left(\frac{\dot{m}_c}{\dot{m}_{c,ref}}\right) + 0.3429 \cdot \ln\left(\frac{\dot{m}_h}{\dot{m}_{h,ref}}\right) + 0.2896 \cdot \ln\left(\frac{\dot{m}_c}{\dot{m}_{c,ref}}\right) \cdot 0.3429 \cdot \ln\left(\frac{\dot{m}_h}{\dot{m}_{h,ref}}\right) \cdot 2.67 \quad (3.72)$$

Table 3-11: Fit constants corresponding to Eq. 3.71 based on data for all heat exchangers.

Unknowns	Value
P_x	0.2896
P_y	0.3429
P_{xy}	2.67
Sample Size	142 points
R²	0.9945
SSE	0.01706
RMSE	0.01096

The advantage of using this method over the classical approach of using thermal resistors is that it requires fewer number of constants to be calculated and allows generalizations between heat exchangers having different sizes. However, due to its semi-empirical nature, it does not necessarily apply to other heat exchangers, as is the nature of any experimental fit.

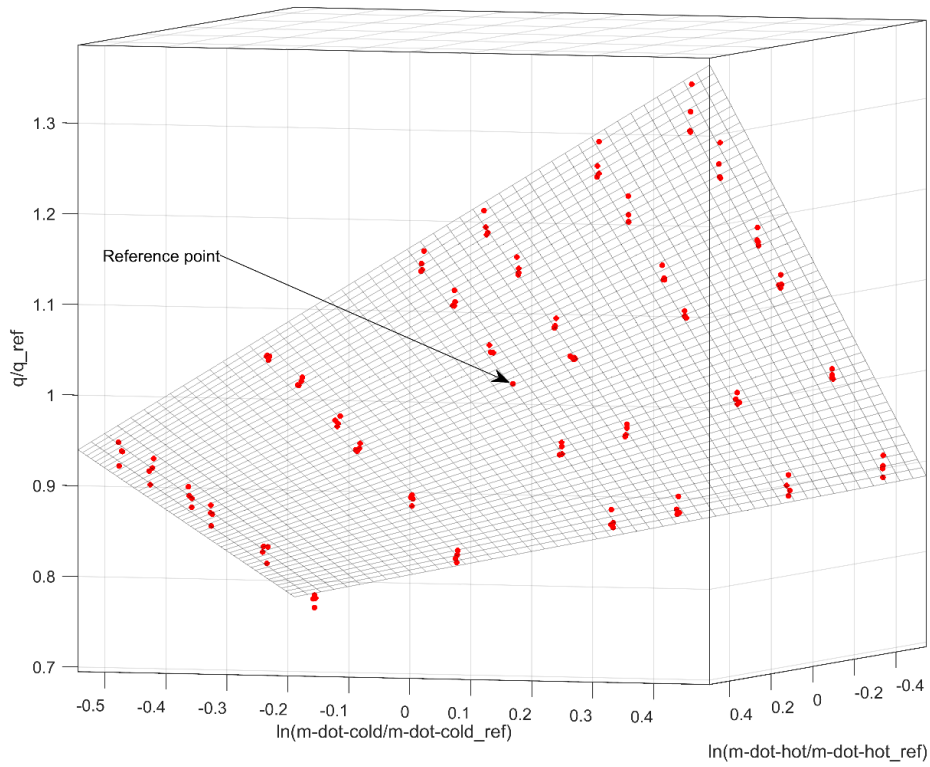


Figure 3-26: Graphical representation of Eq. 3.72. The red dots are experimental points for all heat exchangers in the dataset.

3.3.1.1 Validation of Shortcut Model

The application of the shortcut will first be demonstrated through an example. This will then be applied to the rest of the dataset from section 3.1 to demonstrate the validity of the model and its prediction errors.

This example will use the same data as the example provided in Section 3.1. The inputs were $T_{c,in} = 4.7^{\circ}\text{C}$, $T_{h,in} = 47.3^{\circ}\text{C}$, a mains-side flow rate of 3.97 L/min, and a drain-side flow rate of 7.97 L/min. The measured steady-state heat transfer rate for this test was 6.25 kW. Based on the inlet temperatures, ΔT_{max} can be calculated, and is equal to 42.6°C .

The first step in using this model is to find a reference point for which data exists. Table 3-12 contains information about the reference point that will be used for this heat exchanger. This data was partially presented in Table 3-2, which is further expanded here to include the inlet temperatures.

Table 3-12: Reference point for system 1.

$\dot{V}$ (L/min)	$T_{cold,in}$ (°C)	$T_{hot,in}$ (°C)	ΔT_{max} (°C)	ε	$\dot{q}_c$ (kW)
8.94	12.42	39.79	27.37	31.5%	5.36

Sufficient information is now available to use the shortcut presented in Eq. 3.72. The second step is to rearrange Eq. 3.72 for the variable of interest, namely the heat transfer rate $\dot{q}$. This is presented in Eq. 3.73.

$$\dot{q} = \dot{q}_{ref} \left(\frac{\Delta T_{max}}{\Delta T_{max,ref}} \right) \left[1 + 0.2896 \times \ln \left(\frac{\dot{m}_c}{\dot{m}_{c,ref}} \right) + 0.3429 \times \ln \left(\frac{\dot{m}_h}{\dot{m}_{h,ref}} \right) + 0.2896 \times \ln \left(\frac{\dot{m}_c}{\dot{m}_{c,ref}} \right) \times 0.3429 \times \ln \left(\frac{\dot{m}_h}{\dot{m}_{h,ref}} \right) \times 2.67 \right] \quad (3.73)$$

Next, simply insert numerical values for the inlet temperatures and flowrates into the equation and carry out the math. See below. Note that this model is based on the assumption of constant properties and as such, there is no need to convert volumetric flowrates to mass flowrates. As shown in Eq. 3.74, the predicted heat transfer rate using the shortcut is 6.27 kW, which is very close to the measured value of 6.25 kW.

$$\begin{aligned} \dot{q} &= 5.36 \times \left(\frac{42.6}{27.37} \right) \times \left[1 + 0.2896 \times \ln \left(\frac{3.97}{8.94} \right) + 0.3429 \times \ln \left(\frac{7.97}{8.94} \right) + 0.2896 \times \ln \left(\frac{3.97}{8.94} \right) \times 0.3429 \times \ln \left(\frac{7.97}{8.94} \right) \times 2.67 \right] \\ \dot{q} &= 5.36 \times (1.56) \times [1 + 0.2896 \times \ln(0.444) + 0.3429 \times \ln(0.891) + 0.2896 \times \ln(0.444) \times 0.3429 \times \ln(0.891) \times 2.67] \\ \dot{q} &= 5.36 \times 1.56 \times 0.75 = 6.27 kW \end{aligned} \quad (3.74)$$

Similar to the previous sections, this method was applied to the validation data from Section 3.1 to determine the validity of the shortcut across different heat exchangers. Figure 3-27 shows a plot of the predicted heat transfer rates using this method versus the experimental data. The Mean Absolute Percentage Error between the measured and predicted values for the entire dataset was 3.57%, which is slightly worse than the other two methods.

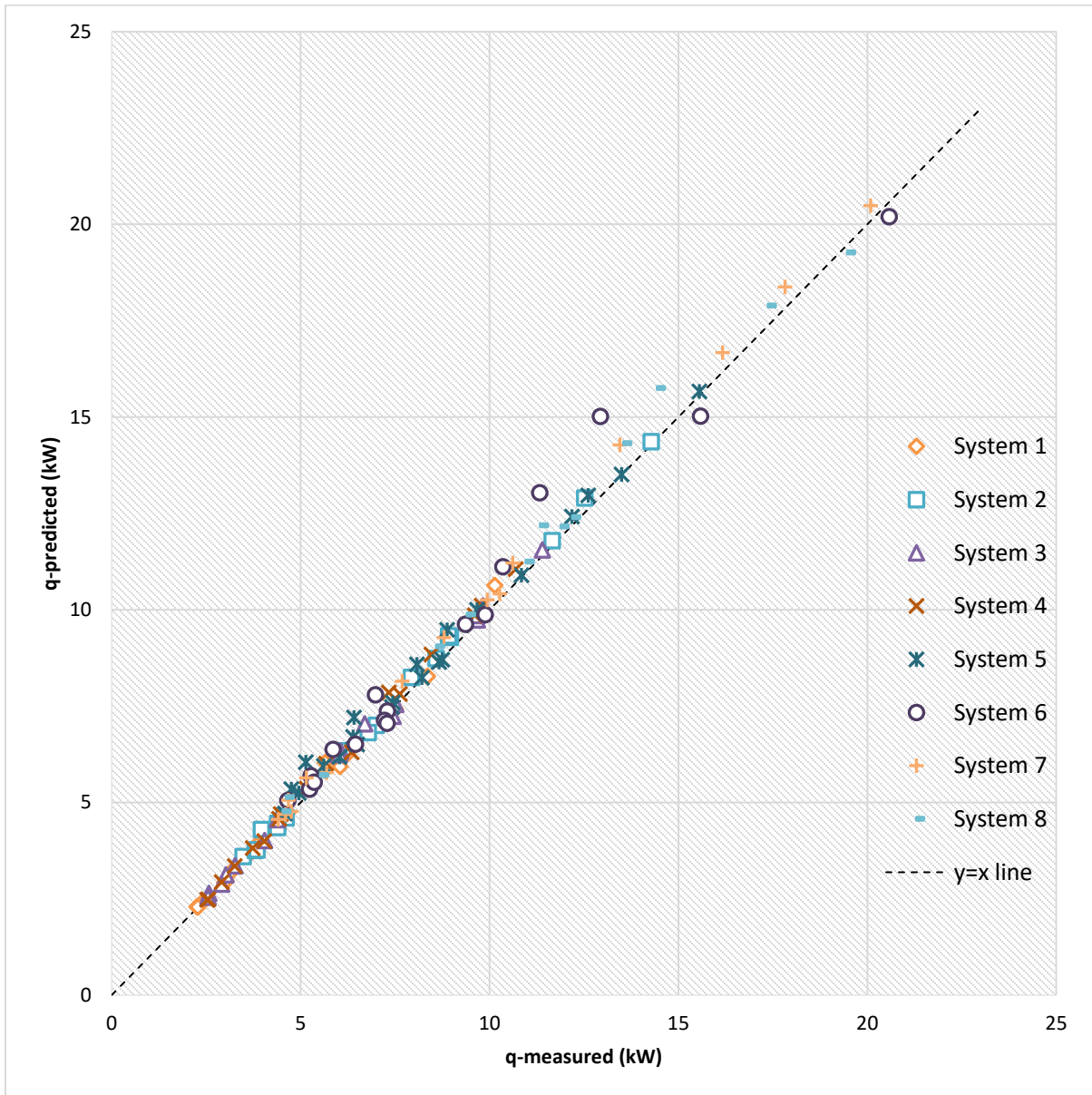


Figure 3-27: Predicted vs. measured heat transfer rates when the shortcut was applied to the data from Section 3.1.

3.4 Discussion

Three independent models for approximating performance of DWHR heat exchangers were presented in this chapter. Each model has its advantages and disadvantages, some of which will be discussed in this section to provide a generic guideline for those reading this document.

It is prudent to start with comparison of the models. Table 3-13 shows a statistical comparison between the three models and provides the magnitude of error for each approach. On average, the errors associated with all three models were smaller than 4%. The numbers also indicate that the resistor network is the most accurate, and the shortcut is the least accurate. This is inline with the complexity of the models and their development.

Table 3-13: Statistical comparison between the three models when applied to the validation data from Section 3.1.

Model	Mean Absolute Error (kW)	Mean Absolute Percentage Error	Maximum Prediction Error
Classical Model (Section 2.3)	0.214	2.96%	9.7%
Resistor Network Model (Section 3.2)	0.194	2.44%	7.4%
Shortcut Model (Section 3.3)	0.274	3.57%	17.7%

The first model was discussed and validated in Section 3.1. Despite its accuracy in predicting heat transfer rate, it is a model that can be considered purely experimental, as it relies entirely on the existence of sufficient experimental data for each individual heat exchanger. Furthermore, this model was shown to be problematic when applied to flow rates exceeding 14 L/min. This was deemed to be due to the equation used in the CSA standard to represent effectiveness as a function of flowrate. Lastly, several new product lines have become available on the market over the course of the author's graduate degree, and there is very little data available for these heat exchangers. Hence, the existing model would only apply to a select few heat exchangers on the market and may not apply to newer products. This would provide an unfair advantage to certain manufacturers who have been in business for longer and have had their products rated based on the CSA standard. This was one of the main motivations behind developing a theoretical model, which can be used by anyone.

The second model presented in this chapter was based on heat exchanger fundamentals, resistor networks and conservation of energy. Through this novel method, expressions were derived for the convective heat transfer coefficients for several DWHR heat exchangers. These expressions were then

used to validate performance for 8 other DWHR heat exchangers that had various diameters and lengths. The results showed good agreement with the experimental measurements. Note that once heat transfer coefficients are derived for a family of similar heat exchangers, this model is no longer semi-empirical, and only requires inlet temperatures and flowrates; hence, it is both applicable to heat exchangers currently sold on the market, as well as any newer products that rely on the same physics. It is the author's hope that the manufacturers of DWHR heat exchangers will start employing this model to derive heat transfer coefficients for their products as part of their R&D process, which would pave the way for innovation and improvement in their design.

This model can also be used to detect faults during operation. For example, it is well known that the falling film becomes unstable at low flowrates in heat exchangers with drainpipe diameters larger than 3 inches. This phenomenon can be observed in the data as well. Figure 3-28 shows the dataset for a 4-inch diameter heat exchanger, where the surface plot at the top represents the governing equation, the individual points on the plot are measured data, and the residual plot is provided directly below. A close inspection of this figure reveals that the measurement points for tests having a falling film flow rate of 0.09 kg/s (i.e. 5.5 L/min) do not lie on the surface, but below it. The residual plot also concurs with this observation. However, this discrepancy is not due to the model, but the physics of the falling film. As shown in Figure 3-29, the falling film did not fully cover the inner surface area of the drainpipe at these low flow rates, and as such, the heat exchanger underperformed as was shown in the plot. This is only one simple application of how this model can be used to detect fault during performance. Another common issue that can be detected in this way is heat exchanger fouling, which would cause similar drops in performance. Due to the absence of in-situ data, this application for the model cannot be shown here and is left as part of the future work for this project.

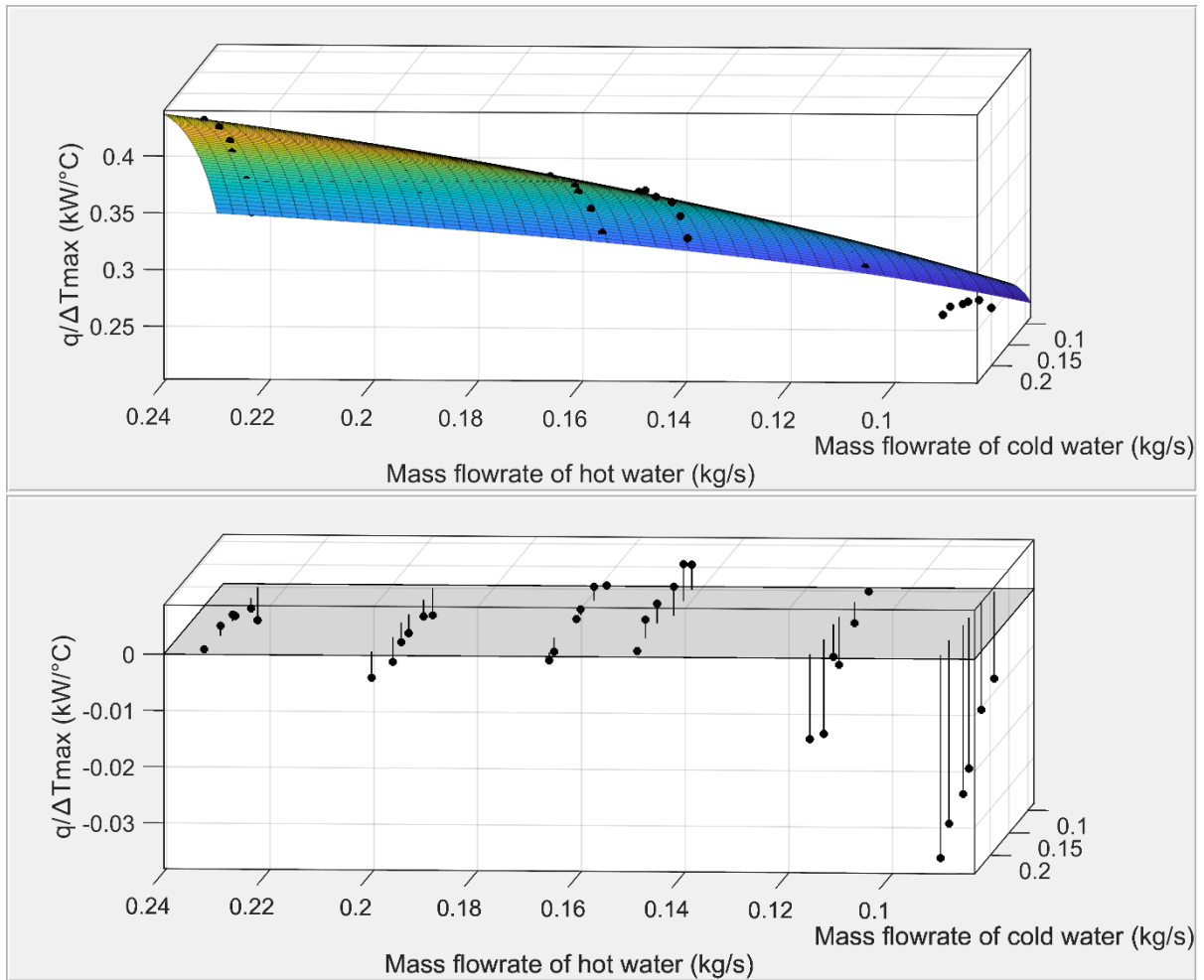


Figure 3-28: Visual depiction of how the governing equation (depicted by the surface plot) can be used to identify underperformance/faults. Data pertains to a 4" diameter heat exchanger.



Figure 3-29: Partial falling film; the root cause behind underperformance identified in Figure 3-28.

The third and final model presented in this chapter is meant to act as a shortcut for quick and relatively accurate calculations. This model is semi-empirical and uses measured data from one performance point on the surface plot, to predict performance for other points for the same heat exchanger.

This model is by far the easiest to use, as it only relies on one simple equation, which is independent of length and diameter. The shortcut was created to address a consistent problem in the literature concerning DWHR heat exchanger, which is the wide usage of constant-effectiveness models by other researchers. The point-measurement that is currently used in constant-effectiveness models can be replaced by this shortcut, and the known effectiveness can be used as the reference condition in the shortcut.

Chapter 4– Transient Model

This chapter covers the study and modelling of DWHR heat exchangers under transient conditions. Although there are great similarities between different chapters in this thesis, the transients are unique, and as such, this chapter can be considered standalone.

4.1 Introduction into transients

As suggested by the name, transient operation refers to conditions under which the performance varies as a function of time, due to changes in inlets to the heat exchanger. Heat exchanger transients are often modeled as first order systems [21, 44, 16]. Preliminary data gathered during transient operation of DWHR heat exchangers suggested that this was likely the case for these heat exchangers as well. The next section uses the ideas and terminology discussed in Chapters 2 and 3 to derive expressions for transient performance in DWHR heat exchangers.

4.2 Derivations

Transient performance of heat exchangers can be modeled through energy balances and proper accounting for movement of energy through the heat exchanger as a function of time. Figure 4-1 depicts a counterflow heat exchanger along with its inlets and outlets as a function of time; note that each side of the heat exchanger is represented as a separate control volume, with arrows indicating the direction for energy flow into and out of each control volume. The energy balance for each side can be qualitatively written as per Eq. 4.1. Note that the change in energy storage term is equal to 0 for steady-state conditions.

$$Energy_{in} - Energy_{out} = \Delta Energy_{stored} \quad (4.1)$$

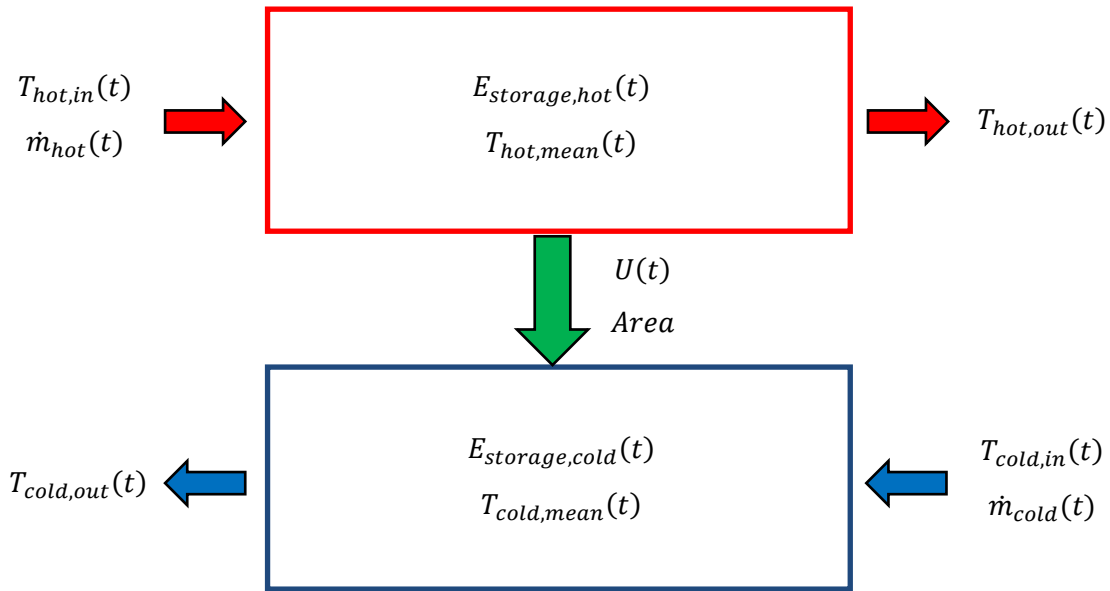


Figure 4-1: Control volumes in a generic counterflow heat exchanger during transients.

Assuming negligible losses to the environment, the quantitative form of the energy balance for the cold side of the heat exchanger is written as follows:

$$[(\dot{m} \cdot h)_{cold,in} - (\dot{m} \cdot h)_{cold,out}] + UA \cdot \Delta T_{HX_{sides}} = \Delta Energy_{stored} \quad (4.2)$$

Assuming that the fluid is incompressible and has a constant specific heat, the following simplification is often made to heat exchangers:

$$(\dot{m} \cdot h)_{cold,in} - (\dot{m} \cdot h)_{cold,out} = \dot{m}_{cold} \cdot cp \cdot (T_{cold,in} - T_{cold,out}) \quad (4.3)$$

Using this relationship, the steady-state energy balance for the cold side of a heat exchanger is reduced to Eq. 4.4, which is found in textbooks. Similarly, the energy balance for the hot side of the heat exchanger can be developed, subject to the same assumptions. See Eq. 4.5.

$$\dot{m}_{cold} \cdot cp \cdot (T_{cold,out} - T_{cold,in}) = \dot{q} = UA \cdot \Delta T_{LMTD} \quad (4.4)$$

$$\dot{m}_{hot} \cdot cp \cdot (T_{hot,in} - T_{hot,out}) = \dot{q} = UA \cdot \Delta T_{LMTD} \quad (4.5)$$

The equations shown so far in this section have been in terms of steady-state conditions, where the time-dependency was dropped for all terms inside equations. To solve transient heat exchanger problems, the derivations must be revisited to account for the physics as a function of time. This

means that unlike steady-state analysis, the energy storage term can no longer be assumed to be zero, and that appropriate differential equations must be used.

DWHR heat exchangers are counterflow heat exchangers; therefore, the derivations shown in this chapter do not apply to crossflow nor parallel-flow heat exchangers without modifications. The typical assumptions applied to heat exchangers are still applied, including negligible axial conduction, negligible losses to the environment and single-phase fluids with constant thermophysical properties. The differential form for the energy equation is formulated in terms of variables shown in Figure 4-1. Eq. 4.6 shows the energy balance for the cold side of the heat exchanger, and Eq. 4.7 corresponds to the hot side.

$$M_{cold_side} c_{p,cold_side} \frac{dT_{cold_side}(t)}{dt} = \dot{m}_{cold_water}(t) c_{p,cold_water} [T_{cold_water,in}(t) - T_{cold_water,out}(t)] + AU(t) \Delta T_{HX_sides}(t) \quad (4.6)$$

$$M_{hot_side} c_{p,hot_side} \frac{dT_{hot_side}(t)}{dt} = \dot{m}_{hot_water}(t) c_{p,hot_water} [T_{hot_water,in}(t) - T_{hot_water,out}(t)] - AU(t) \Delta T_{HX_sides}(t) \quad (4.7)$$

where M represents mass, t represents time, and ΔT_{HX_sides} refers to the temperature difference between the two sides of the heat exchanger, which is often approximated as the ΔT_{AMTD} . Note that the definition for ΔT_{LMTD} assumes steady-state and this assumption breaks down during transient conditions. Hence, log-mean temperature differences are not used in this derivation. Instead, Eq. 4.8 is used.

$$\begin{aligned} \Delta T_{HX_sides}(t) &\cong \Delta T_{LMTD}(t) = \left(\frac{T_{h,out}(t) + T_{h,in}(t)}{2} \right) - \left(\frac{T_{c,out}(t) + T_{c,in}(t)}{2} \right) \\ &= \left(\frac{T_{h,out}(t) + T_{h,in}(t) - T_{c,out}(t) - T_{c,in}(t)}{2} \right) \end{aligned} \quad (4.8)$$

Another significant difference between transient and steady-state analysis is that the steady-state approach completely disregards mass and heat capacity of metal separating the fluid media. During transient operation, the temperature(s) and/or the flowrate(s) within the heat exchanger are changing as a function of time; therefore, the change in the energy content of each side, denoted by $\Delta E_{storage}$, varies as a function of time until steady state is reached. These storage terms are mathematically represented in Eqs. 4.9 and 4.10 for the cold and hot side of the heat exchanger respectively.

$$\Delta E_{storage,cold}(t) = M_{cold_{side}} c_{p,cold_{side}} \frac{dT_{cold_{side}}(t)}{dt} \quad (4.9)$$

$$\Delta E_{storage,hot}(t) = M_{hot_{side}} c_{p,hot_{side}} \frac{dT_{hot_{side}}(t)}{dt} \quad (4.10)$$

where

$$M_{cold_{side}} c_{p,cold_{side}} = (M_{cold_{water}} c_{p,cold_{water}} + M_{metal,cold_{side}} c_{p,metal,cold_{side}}) \quad (4.11)$$

$$M_{hot_{side}} c_{p,hot_{side}} = (M_{hot_{water}} c_{p,hot_{water}} + M_{metal,hot_{side}} c_{p,metal,hot_{side}}) \quad (4.12)$$

In the case of DWHR heat exchangers, water is the operating fluid for both sides of the heat exchanger, and the metal is copper; therefore, the corresponding specific heats can readily be found in textbooks. The problem is finding appropriate values to assign to variables representing mass in the above equations.

In terms of fluids, the total mass of water on the cold side of the heat exchanger can be estimated by measuring the volume of water contained in the coils; however, no accurate methods exist for estimating the mass of the falling film. The film velocity and film thickness are both unknowns, and due to the gravity-driven nature of the falling film, water does not linger inside the heat exchanger if the flow is stopped. This contrasts with the coils which are always flooded with water.

Evaluating the mass of metal attributed to each side of the heat exchanger is another hurdle. One school of thought suggests that if the heat exchanger is deconstructed, and the coils are separated from the drainpipe, they could be weighed separately to measure the total mass of metal on each side. At first glance, this may appear as a good method for estimating masses, but it contradicts with what is known about the metal interface. Copper is used as the metal in construction of DWHR heat exchangers because it has a very high thermal conductivity. Since the entire heat exchanger is made up of copper, it stands to reason that thermal energy is stored in the entire body of the heat exchanger. Therefore, it is futile to deconstruct the heat exchanger and weigh its components individually to estimate the mass of metal on each side, because the entire mass of metal is involved in the heat transfer process, and arbitrary divisions cannot be drawn accurately.

There are different schools of thoughts regarding how the mass of metal should be dealt with during transient operation of heat exchangers. Some researchers tend to neglect the effect of metal

altogether; this is reportedly done because introducing the mass of metal into the model will overcomplicate it and introduce additional uncertainties, while providing no appreciable improvements over models that do not account for the metal [21]. Conversely, some researchers claim that the mass of metal should be accounted for, either as part of the heat exchanger side where the savings are being realized (i.e., mains water being preheated) or as part of the side of the heat exchanger that corresponds to $\dot{C}_{min}$.

In the context of DWHR heat exchangers, the savings correspond to the heat recovered by the cold side of the heat exchanger, where the mains water is heated while traversing the coils. Also, $\dot{C}_{min}$ is predominantly attributed to the cold side for these heat exchangers. To put this into perspective: if the flow rate for a showerhead (i.e. hot side of the heat exchanger) is 10 liters per minute, a portion of that flow rate is coming from the hot water tank while the rest of it is coming from mains water (i.e. cold side of the heat exchanger). In other words, for shower events, $\dot{C}_{min}$ usually refers to the cold side, unless the occupant is showering at mains temperature, in which case the heat exchanger provides no savings. Therefore, in this derivation, the entire mass of the heat exchanger will be attributed to the cold side with the coils. Eqs. 4.13 and 4.14 represent heat capacity for the cold and hot side of the heat exchanger respectively. Note that in writing Eq. 4.13 in its current form, the assumption of lumped thermal capacitance has been implicitly applied. This assumption was rigorously investigated by Dhillon [44], and the results showed that the Biot number for the fluid in the coils is much smaller than 0.1. This is the metric used to determine the validity of the lumped sum capacitance model in the literature [16, 44].

$$C_{cold_side} = M_{cold_side} c_{p,cold_side} = (M_{cold_water} c_{p,cold_water} + M_{copper} c_{p,copper}) \quad (4.13)$$

$$C_{hot_side} = M_{hot_side} c_{p,hot_side} = (M_{hot_water} c_{p,hot_water}) \quad (4.14)$$

Given the expressions for heat capacities for the two sides of a heat exchanger, the energy storage terms in the governing differential equations can be written in terms of Cs as shown:

$$C_{cold_side} \frac{dT_{cold_side}(t)}{dt} = \dot{m}_{cold_water}(t) c_{p,cold_water} [T_{cold_water,in}(t) - T_{cold_water,out}(t)] + AU(t) \Delta T_{HX_sides}(t) \quad (4.15)$$

$$C_{hot_side} \frac{dT_{hot_side}(t)}{dt} = \dot{m}_{hot_water}(t) c_{p,hot_water} [T_{hot_water,in}(t) - T_{hot_water,out}(t)] - AU(t) \Delta T_{HX_sides}(t) \quad (4.16)$$

Given the governing Eqs. 4.15 and 4.16, expressions can be derived to represent the transient response of DWHR heat exchangers to changes in input variables, namely temperatures and/or flow rates. In the literature, this response is often characterized for step changes in one of the input variables [21]. As step changes to flow rates are the most impactful and realistic changes that DWHR heat exchangers can experience during operation, the first derivation in this section will correspond to step changes in flow rates. Next, an expression will be derived for step changes in temperatures, which will be followed with a derivation for step change in both flowrates and temperatures.

Prior to showing the mathematical derivations, it is important to reiterate that savings are realized on the cold side of the heat exchanger; therefore, expressions for transients are based on the cold side of the heat exchanger. It is only unnecessary to attempt to derive expressions for the hot side of the heat exchanger, as the two sides are connected via conservation of energy, and the second expression is redundant.

4.2.1 Step change in flowrate

A step change in flow rate implies that $\dot{m}_{cold_water}$, or $\dot{m}_{hot_water}$, or both are changed as a function of time, and this change happens immediately in the time domain. This can be thought of as a shower tap that is closed (i.e., 0 flowrate) being suddenly opened.

To solve this problem as a function of time, Eq. 4.15 is used to derive expressions for the heat exchanger corresponding to conditions prior to the step change (i.e. $t \leq 0$) and conditions after the step change (i.e. $t > 0$). To improve the presentation of the math, the following substitutions are utilized:

$$c_{p,w} = c_{p,cold_water} = c_{p,hot_water}$$

$$C_{cold_side} = M_{cold_side} c_{p,cold_side}$$

$$\dot{C}_{cold} = \dot{m}_{cold_water}(t) c_{p,cold_water}$$

$$\dot{C}_{hot} = \dot{m}_{hot_water}(t) c_{p,hot_water}$$

$$T_{c,out}(t) = T_{cold_water,out}(t)$$

$$T_{c,in}(t) = T_{cold_water,in}(t)$$

$$T_{h,out}(t) = T_{hot_water,out}(t)$$

$$T_{h,in}(t) = T_{hot_water,in}(t)$$

$$T_{c,out,0} = T_{c,out}(t = 0)$$

$$T_{c,in,0} = T_{c,in}(t = 0)$$

$$T_{h,out,0} = T_{h,out}(t = 0)$$

$$T_{h,in,0} = T_{h,in}(t = 0)$$

$$\dot{C}_{cold,0} = \dot{C}_{cold}(t = 0)$$

$$\dot{C}_{hot,0} = \dot{C}_{hot}(t = 0)$$

$$AU_0 = AU(t = 0)$$

4.2.1.1 Algebraic Derivation

The first step is to rewrite the governing differential equation for the cold side of the heat exchanger in terms of the substitutions listed above, combined with the assumption of a linear temperature profile from Eq. 4.8. Other notable assumptions worth reiterating include: single phase incompressible fluids with constant properties; negligible losses to the environment; negligible axial conduction.

$$C_{cold_side} \frac{dT_{cold_side}(t)}{dt} = \dot{C}_{cold} [T_{c,in}(t) - T_{c,out}(t)] + AU(t) \left(\frac{T_{h,out}(t) + T_{h,in}(t) - T_{c,out}(t) - T_{c,in}(t)}{2} \right) \quad (4.17)$$

The next step is to write the corresponding energy equation prior to the application of a step change in flow rate (i.e., $t \leq 0$), noting that the storage term is 0.

$$0 = \dot{C}_{cold,0} [T_{c,in,0} - T_{c,out,0}] + AU_0 \left(\frac{T_{h,out,0} + T_{h,in,0} - T_{c,out,0} - T_{c,in,0}}{2} \right) \quad (4.18)$$

Subtracting Eq. 4.18 from Eq. 4.17 results in Eq. 4.19 which can eventually be solved for $T_{c,out}(t)$.

$$C_{cold_side} \frac{dT_{cold_side}(t)}{dt} = \dot{C}_{cold} [T_{c,in}(t) - T_{c,out}(t)] - \dot{C}_{cold,0} [T_{c,in,0} - T_{c,out,0}] + AU(t) \left(\frac{T_{h,out}(t) + T_{h,in}(t) - T_{c,out}(t) - T_{c,in}(t)}{2} \right) - AU_0 \left(\frac{T_{h,out,0} + T_{h,in,0} - T_{c,out,0} - T_{c,in,0}}{2} \right) \quad (4.19)$$

Given that the problem involves a step change in one or both flow rates, the time dependence for the input temperatures can be dropped in the equation. See Eqs. 4.20 and 4.21. Furthermore, since the assumption of a linear temperature profile was employed in the derivations, the derivative of $T_{cold_side}(t)$ can be written in terms of $T_{c,out}(t)$ using chain rule. See Eq. 4.22. Consequently, the governing differential equation is simplified to the expression shown in Eq. 4.23.

$$T_{c,in}(t) = T_{c,in,0} = T_{c,in} \quad (4.20)$$

$$T_{h,in}(t) = T_{h,in,0} = T_{h,in} \quad (4.21)$$

$$\frac{dT_{cold_side}(t)}{dt} = \frac{d}{dt} \left(\frac{T_{c,out}(t) + T_{c,in}(t)}{2} \right) = \frac{1}{2} \left(\frac{dT_{c,out}(t)}{dt} \right) \quad (4.22)$$

$$\begin{aligned} C_{cold_side} \left[\frac{1}{2} \left(\frac{dT_{c,out}(t)}{dt} \right) \right] &= \dot{C}_{cold} [T_{c,in} - T_{c,out}(t)] - \dot{C}_{cold,0} [T_{c,in} - T_{c,out,0}] \\ &\quad + \left(\frac{AU(t)}{2} \right) [T_{h,out}(t) + T_{h,in} - T_{c,out}(t) - T_{c,in}] \\ &\quad - \left(\frac{AU_0}{2} \right) [T_{h,out,0} + T_{h,in} - T_{c,out,0} - T_{c,in}] \\ &= \left[\dot{C}_{cold} - \dot{C}_{cold,0} - \frac{AU}{2} + \frac{AU_0}{2} \right] (T_{c,in}) + \left[-\dot{C}_{cold} - \frac{AU}{2} \right] (T_{c,out}(t)) + \left[\dot{C}_{cold,0} + \frac{AU_0}{2} \right] (T_{c,out,0}) \\ &\quad + \left[\frac{AU}{2} - \frac{AU_0}{2} \right] (T_{h,in}) + \left[\frac{AU}{2} \right] (T_{h,out}(t)) + \left[-\frac{AU_0}{2} \right] (T_{h,out,0}) \end{aligned} \quad (4.23)$$

Before applying further simplifications, some of the terms in this equation bear discussing. Note that both AU_0 and AU are known quantities; the first comes from initial conditions, and the latter comes from the predicted steady-state condition after the step change in flow rate. Hence, neither term is a function of time in this equation. Furthermore, this expression explicitly accounts for the change in flow rate on the cold side because it contains the term $\dot{C}_{cold}$; however, $\dot{m}_{hot_water}$ is absent from the equation and is only implicitly accounted for through the predicted AU term. Another problematic term in this equation is the outlet temperature on the hot side, which also changes as a function of time. Consequently, the equation cannot readily be integrated and solved for the variable of interest, namely $T_{c,out}(t)$.

Next, the two sides of the heat exchanger must be connected through energy balances. This means that the differential Eqs. 4.15 and 4.16 are to be solved simultaneously, which overcomplicates the problem and requires additional information which are unknown. For example, the mass of water contained in the falling film is unknown; approximating this mass requires having accurate expressions to predict the thickness of the falling film, which can then be used to approximate its volume and mass. Furthermore, the entire mass of copper was attributed to the cold side in the derivations. Hence, the transient storage term C_{hot_side} cannot be accurately predicted as a function of time without applying gross simplifications to the problem.

Instead, it was decided to incorporate information about the hot side of the heat exchanger into Eq. 4.15, to eliminate the need to solve two differential equations simultaneously. Therefore, the goal was to identify the unknown terms in Eq. 4.15 that are based on the hot side of the heat exchanger, namely $T_{h,out}(t)$ and $T_{h,out,0}(t)$, and replace them with known variables. $T_{h,out,0}(t)$ refers to the outlet temperature of the falling film prior to the step change in flow rate. This is a known variable as it is an initial condition; however, because the storage term was equal to 0 prior to the step change, this temperature can be written in terms of other known variables. This can be done through performing an energy balance on the flow streams and solving for $T_{h,out,0}(t)$ as shown in Eq. 4.24.

$$\begin{aligned}\dot{C}_{cold,0}[T_{c,out,0} - T_{c,in,0}] &= \dot{C}_{hot,0}[T_{h,in,0} - T_{h,out,0}] \\ \rightarrow T_{h,out,0} &= T_{h,in,0} - \left[\frac{\dot{C}_{cold,0}}{\dot{C}_{hot,0}} \right] (T_{c,out,0} - T_{c,in,0})\end{aligned}\quad (4.24)$$

The only problematic term left in the equation is $T_{h,out}(t)$. The correct expression to represent this outlet temperature during transient conditions comes from solving Eq. 4.16, which relies on knowing accurate information about the falling film, which are unavailable. Instead of making assumptions about the film thickness and volume, the entire falling film was assumed to be in quasi-steady conditions; in other words, $T_{h,out}(t)$ was approximated using Eq. 4.25. This may seem like an egregious assumption, but it is shown to work well for DWHR heat exchangers and its validity will be demonstrated through experiments.

$$T_{h,out}(t) = T_{h,in} - \left[\frac{\dot{C}_{cold}}{\dot{C}_{hot}} \right] (T_{c,out}(t) - T_{c,in}) \quad (4.25)$$

Using the expressions for the outlet temperatures from Eqs. 4.24 and 4.25, the differential equation is written in terms of known temperatures. The only unknown temperature is $T_{c,out}(t)$ on the right-hand side of the equation, and its derivative with respect to time on the left-hand side of the equation. See Eq. 4.26.

$$\begin{aligned} C_{cold,side} \left[\frac{1}{2} \left(\frac{dT_{c,out}(t)}{dt} \right) \right] &= \left[\dot{C}_{cold} - \dot{C}_{cold,0} - \frac{AU}{2} + \frac{AU_0}{2} + \frac{AU\dot{C}_{cold}}{2\dot{C}_{hot}} - \frac{AU_0\dot{C}_{cold,0}}{2\dot{C}_{hot,0}} \right] (T_{c,in}) \\ &+ \left[-\dot{C}_{cold} - \frac{AU}{2} - \frac{AU\dot{C}_{cold}}{2\dot{C}_{hot}} \right] (T_{c,out}) + \left[\dot{C}_{cold,0} + \frac{AU_0}{2} + \frac{AU_0\dot{C}_{cold,0}}{2\dot{C}_{hot,0}} \right] (T_{c,out,0}) \\ &+ \left[\frac{AU}{2} - \frac{AU_0}{2} + \frac{AU}{2} - \frac{AU_0}{2} \right] (T_{h,in}) \end{aligned} \quad (4.26)$$

Combining the terms and cleaning up the equation results in:

$$\begin{aligned} &C_{cold,side} \left[\frac{1}{2} \left(\frac{dT_{c,out}(t)}{dt} \right) \right] \\ &= \left[\frac{AU\dot{C}_{hot,0}(\dot{C}_{cold} - \dot{C}_{hot}) - AU_0\dot{C}_{hot}(\dot{C}_{cold,0} - \dot{C}_{hot,0}) + 2\dot{C}_{cold}\dot{C}_{hot}\dot{C}_{hot,0}}{2\dot{C}_{hot}\dot{C}_{hot,0}} \right] (T_{c,in}) \\ &+ \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} - AU(\dot{C}_{cold} + \dot{C}_{hot})}{2\dot{C}_{hot}} \right] (T_{c,out}) + \left[\frac{2\dot{C}_{cold,0}\dot{C}_{hot,0} + AU_0(\dot{C}_{cold,0} + \dot{C}_{hot,0})}{2\dot{C}_{hot,0}} \right] (T_{c,out,0}) \\ &+ [AU - AU_0](T_{h,in}) \end{aligned} \quad (4.27)$$

Eq. 4.27 is a differential equation that can be solved in its current form; however, it is customary to present and solve transient problems in their non-dimensional form. The intent is to provide a solution that applies to similar heat exchangers undergoing similar operating conditions.

Nondimensionalization is an extremely useful tool that is frequently applied in fluid mechanics

analysis using Buckingham- π theorem. A similar approach is taken here to select appropriate nondimensional terms that would lead to a solution while allowing a means to directly compare different heat exchangers through familiar terms such as time constants.

4.2.1.2 Nondimensionalization

The first and perhaps the most important variable to nondimensionalize is time. This is often done using the concept of flush time for heat exchangers [19]. Flush time is defined as the time it takes to empty the volume of fluid contained in the heat exchanger based on the flow rate of fluid that is being supplied to it. For example, if a DWHR heat exchanger can contain 1 liter of water in the coils, and the volumetric flow rate through the coils is 4 liters per minute, the flush time is 15 seconds. See Eq. 4.28 for the definition of flush time for DWHR heat exchangers with respect to the cold side. Note that the assumption of an incompressible fluid has been implicitly applied to the water traversing the coils.

$$t_{flush,water} = \frac{C_{cold}}{\dot{C}_{cold}} \quad (4.28)$$

It stands to reason that every heat exchanger would contain a different volume of water based on its size; hence, this nondimensional group would allow direct comparison between DWHR heat exchangers on a common time domain. It is imperative to note that this flush time is based only on mass and properties of water in the coils; it does not account for the mass and specific heat of copper. Therefore, the equation for flush time will be revisited later to account for the heat capacity of metal.

The next variable to be nondimensionalized is the heat capacity of the cold side, C_{cold_side} , which is the lumped sum of the total heat capacity of water in the coils and the heat capacity of metal in the heat exchanger. The nondimensional heat capacity, C_{HX} , is the ratio of the lumped heat capacity to the heat capacity of water on the same side. This is shown in Eq. 4.29. The main reason behind selecting this nondimensional group will reveal itself shortly; it was to cancel out $\dot{C}_{cold}$. Furthermore, C_{HX} is a nondimensional number; therefore, it can be tuned based on experimental results, which will be illustrated in the validation section.

$$C_{HX} = \frac{(M_{cold_water} c_{p,cold_water} + M_{copper_coils} c_{p,copper_coils})}{M_{cold_water} c_{p,cold_water}} = \frac{(C_{cold} + C_{metal})}{C_{cold}} \quad (4.29)$$

Using C_{HX} , the equation for flush time is updated to account for the heat capacity of metal. Consequently, the nondimensional time, t^* , is defined in Eq. 4.30. In essence, this equation replaces the metal and its heat capacity, with an equivalent amount of water. In other words, it takes the mass-based flush time and turns it into an energy-based flush time to account for the energy stored in the metal.

$$t^* = C_{HX} \left(\frac{t}{t_{flush,water}} \right) \quad (4.30)$$

Next, using the nondimensional terms shown so far, the left-hand side of the governing differential Eq. 4.27 can be put in terms of nondimensional variables. See Eq. 4.31.

$$\begin{aligned} C_{cold_side} \left[\frac{1}{2} \left(\frac{dT_{c,out}}{dt} \right) \right] &= \left(\frac{C_{cold_side}}{2} \right) \left(\frac{dT_{c,out}}{dt} \right) = \left(\frac{C_{cold} + C_{metal}}{2} \right) \left(\frac{dT_{c,out}}{dt} \cdot \frac{dt}{dt^*} \right) \\ &= \left(\frac{C_{cold} + C_{metal}}{2} \right) \left(\frac{dT_{c,out}}{dt^*} \cdot \frac{dt^*}{dt} \right) = \left(\frac{C_{cold} + C_{metal}}{2} \right) \left(\frac{dT_{c,out}}{dt^*} \cdot \frac{1}{t_{flush}} \right) \\ \therefore C_{cold_side} \left[\frac{1}{2} \left(\frac{dT_{c,out}}{dt} \right) \right] &= \left(\frac{C_{HX} \dot{C}_{cold}}{2} \right) \frac{dT_{c,out}}{dt^*} \end{aligned} \quad (4.31)$$

Next, both sides of the differential equation are divided by coefficient of the derivative term:

$$\begin{aligned} \frac{dT_{c,out}}{dt^*} &= \left[\frac{AU \dot{C}_{hot,0} (\dot{C}_{cold} - \dot{C}_{hot}) - AU_0 \dot{C}_{hot} (\dot{C}_{cold,0} - \dot{C}_{hot,0}) + 2 \dot{C}_{cold} \dot{C}_{hot} \dot{C}_{hot,0}}{C_{HX} \dot{C}_{cold} \dot{C}_{hot} \dot{C}_{hot,0}} \right] (T_{c,in}) \\ &+ \left[\frac{-2 \dot{C}_{cold} \dot{C}_{hot} - AU (\dot{C}_{cold} + \dot{C}_{hot})}{C_{HX} \dot{C}_{cold} \dot{C}_{hot}} \right] (T_{c,out}) + \left[\frac{2 \dot{C}_{cold,0} \dot{C}_{hot,0} + AU_0 (\dot{C}_{cold,0} + \dot{C}_{hot,0})}{C_{HX} \dot{C}_{cold} \dot{C}_{hot,0}} \right] (T_{c,out,0}) \\ &+ \left[\frac{2(AU - AU_0)}{C_{HX} \dot{C}_{cold}} \right] (T_{h,in}) \end{aligned} \quad (4.32)$$

At this point, this differential equation should look familiar. All terms within square brackets on the right-hand side of the equations are constants with respect to time (in the context of a step change). The inlet temperatures are known; hence the only unknowns here is $T_{c,out}(t^*)$. The constant terms are combined together and substituted with variables a and b shown in Eqs. 4.33 and 4.34 respectively.

$$a = \left[\frac{-2 \dot{C}_{cold} \dot{C}_{hot} - AU (\dot{C}_{cold} + \dot{C}_{hot})}{C_{HX} \dot{C}_{cold} \dot{C}_{hot}} \right] \quad (4.33)$$

$$b = \left[\frac{AU\dot{C}_{hot,0}(\dot{C}_{cold} - \dot{C}_{hot}) - AU_0\dot{C}_{hot}(\dot{C}_{cold,0} - \dot{C}_{hot,0}) + 2\dot{C}_{cold}\dot{C}_{hot}\dot{C}_{hot,0}}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}\dot{C}_{hot,0}} \right] (T_{c,in}) \\ + \left[\frac{2\dot{C}_{cold,0}\dot{C}_{hot,0} + AU_0(\dot{C}_{cold,0} + \dot{C}_{hot,0})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot,0}} \right] (T_{c,out,0}) + \left[\frac{2(AU - AU_0)}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in}) \quad (4.34)$$

So, the differential equation becomes:

$$\frac{dT_{c,out}}{dt^*} = a \cdot T_{c,out}(t^*) + b \quad (4.35)$$

The solution to this ordinary differential equation is:

$$T_{c,out}(t^*) = W \cdot e^{at^*} - \frac{b}{a} \quad (4.36)$$

where W is a constant of integration to be found using initial conditions to the problem. Solving for W using initial condition at $t = t^* = 0$ leads to Eq. 4.37.

$$T_{c,out}(t^* = 0) = T_{c,out,0} = W \cdot e^0 - \frac{b}{a} \\ \therefore W = T_{c,out,0} + \frac{b}{a} \quad (4.37)$$

Using the expression for W, the differential equation can be written as follows:

$$T_{c,out}(t^*) = \left(T_{c,out,0} + \frac{b}{a} \right) e^{at^*} - \frac{b}{a} \\ \frac{T_{c,out}(t^*) + \left(\frac{b}{a} \right)}{T_{c,out,0} + \left(\frac{b}{a} \right)} = e^{at^*} \quad (4.38)$$

The expression for (b/a) may seem intimidating, but it is in terms of known variables that are easy to program into computer software. Additionally, if one were to carry out the math, this term will reveal itself to be equal to the negative of the outlet temperature that corresponds to the value of AU plugged into the equation. In other words, it represents the negative of the final steady-state temperature. This is used in Eq. 4.39 to nondimensionalize the outlet temperature in terms of θ .

$$\left(\frac{b}{a} \right) = -T_{c,out}(t^* = \infty) = -T_{c,out,\infty}$$

$$\frac{T_{c,out}(t^*) - T_{c,out,\infty}}{T_{c,out,0} - T_{c,out,\infty}} = \theta_{c,out}(t^*) = e^{at^*} = e^{\frac{-t^*}{\tau^*}} \quad (4.39)$$

This expression represents the nondimensional outlet temperature in terms of nondimensional time and nondimensional constant a . Conventionally, this constant is presented in form of the well-known term called the time constant. This time constant is denoted in literature using τ . The time taken to reach steady state is often listed in terms of multiples of τ , and the heat exchanger is assumed to have reached steady-state when time equal to 5τ has passed since application a step change in input(s). The time constant for DWHR heat exchangers is listed in Eq. 4.40.

$$\tau^* = -\frac{1}{a} = \left[\frac{C_{HX} \dot{C}_{cold} \dot{C}_{hot}}{2\dot{C}_{cold} \dot{C}_{hot} + AU(\dot{C}_{cold} + \dot{C}_{hot})} \right] \quad (4.40)$$

It is possible to simplify the expression for τ^* using heat exchanger terminology. This is demonstrated in Eq. 4.41.

$$\tau^* = \left[\frac{C_{HX}}{2 + AU \left(\frac{\dot{C}_{cold} + \dot{C}_{hot}}{\dot{C}_{cold} \dot{C}_{hot}} \right)} \right] = \left[\frac{C_{HX}}{2 + N_c + N_h} \right] \\ \approx \frac{\text{Heat Exchanger Lumped Thermal Capacitance}}{\text{Heat Exchanger Thermal Resistance}} \quad (4.41)$$

where

$$N_c = \frac{UA}{\dot{C}_c} = \frac{UA}{\dot{m}_c c_{p,c}} \quad (4.42)$$

$$N_h = \frac{UA}{\dot{C}_h} = \frac{UA}{\dot{m}_h c_{p,h}} \quad (4.43)$$

Finally, the nondimensional form of the governing equation is simplified to Eq. 4.44. This is a well-known form to represent transient response for a first-order system.

$$\theta_{c,out}(t^*) = e^{-\left(\frac{t^*}{\tau^*}\right)} \quad (4.44)$$

4.2.2 Step change in one or both inlet temperatures

The solution procedure is almost identical to the one shown for step changes in flow rates. What differentiates the procedures is the parameters that are treated as constants as a function of time. Hence, the procedure will be discussed here, while skipping the trivial steps that are identical to the previous section.

The first step is to return to the dimensional form of the governing differential equation, Eq. 4.6. Subject to similar assumption regarding a linear temperature profile, negligible losses, etc., the differential form is simplified to Eq. 4.45, which is the same as the one derived in the previous section.

$$C_{cold_side} \frac{dT_{cold_side}}{dt} = \dot{C}_{cold} [T_{c,in}(t) - T_{c,out}(t)] + AU \left(\frac{T_{h,out}(t) + T_{h,in}(t) - T_{c,out}(t) - T_{c,in}(t)}{2} \right) \quad (4.45)$$

Next, initial conditions are inserted into the governing differential equation to produce Eq. 4.46. This equation is then subtracted from Eq. 4.45 to produce Eq. 4.47, which can eventually be solved for $T_{cold_side}(t)$.

$$0 = \dot{C}_{cold} [T_{c,in,0} - T_{c,out,0}] + AU_0 \left(\frac{T_{h,out,0} + T_{h,in,0} - T_{c,out,0} - T_{c,in,0}}{2} \right) \quad (4.46)$$

$$C_{cold_side} \frac{dT_{cold_side}}{dt} = \dot{C}_{cold} [T_{c,in} - T_{c,out}] - \dot{C}_{cold} [T_{c,in,0} - T_{c,out,0}] + AU \left(\frac{T_{h,out} + T_{h,in} - T_{c,out} - T_{c,in}}{2} \right) - AU_0 \left(\frac{T_{h,out,0} + T_{h,in,0} - T_{c,out,0} - T_{c,in,0}}{2} \right) \quad (4.47)$$

At this point, the differences between the assumption that apply here vs. those applied to the previous section become clear. Notably, the inlet temperatures prior to the step change are no longer equivalent to the inlet temperatures after the step change. In other words, Eqs. 4.20 and 4.21 are no longer valid. Therefore, there will be additional (constant) temperature terms in the derivation.

$$C_{cold_side} \frac{dT_{cold_side}}{dt} = \left[\dot{C}_{cold} - \frac{AU}{2} \right] (T_{c,in}) + \left[-\dot{C}_{cold} + \frac{AU_0}{2} \right] (T_{c,in,0}) + \left[-\dot{C}_{cold} - \frac{AU}{2} \right] (T_{c,out}) + \left[\dot{C}_{cold} + \frac{AU_0}{2} \right] (T_{c,out,0}) + \left[\frac{AU}{2} \right] (T_{h,in}) + \left[-\frac{AU_0}{2} \right] (T_{h,in,0}) + \left[\frac{AU}{2} \right] (T_{h,out}) + \left[-\frac{AU_0}{2} \right] (T_{h,out,0}) \quad (4.48)$$

Next, the outlet temperature for the hot stream is written in terms of other variables using the conservation of energy, similar to what was shown in the previous section. Hence, the differential

equation reduces to Eq. 4.49, which is a function of six temperatures, five of which are known constants in this problem.

$$\begin{aligned}
C_{cold_side} \frac{dT_{cold_side}}{dt} = & \left[\frac{2\dot{C}_{cold}\dot{C}_{hot} - AU\dot{C}_{hot} + AU\dot{C}_{cold}}{2\dot{C}_{hot}} \right] (T_{c,in}) \\
& + \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} + AU_0\dot{C}_{hot} - AU_0\dot{C}_{cold}}{2\dot{C}_{hot}} \right] (T_{c,in,0}) \\
& + \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} - AU\dot{C}_{hot} - AU\dot{C}_{cold}}{2\dot{C}_{hot}} \right] (T_{c,out}) \\
& + \left[\frac{2\dot{C}_{cold}\dot{C}_{hot} + AU_0\dot{C}_{hot} + AU_0\dot{C}_{cold}}{2\dot{C}_{hot}} \right] (T_{c,out,0}) \\
& + [AU](T_{h,in}) + [-AU_0](T_{h,in,0})
\end{aligned} \tag{4.49}$$

Next step is to nondimensionalize the equation. To do so, identical nondimensional groups as the previous section are employed, which result in Eq. 4.50. Note that the terms contained in square brackets on the right-hand side of the equation are constants with respect to time.

$$\begin{aligned}
\frac{dT_{c,out}}{dt^*} = & \left[\frac{2\dot{C}_{cold}\dot{C}_{hot} + AU(\dot{C}_{cold} - \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,in}) + \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} - AU_0(\dot{C}_{cold} - \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,in,0}) \\
& + \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} - AU(\dot{C}_{cold} + \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,out}) + \left[\frac{2\dot{C}_{cold}\dot{C}_{hot} + AU_0(\dot{C}_{cold} + \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,out,0}) \\
& + \left[\frac{2AU}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in}) + \left[\frac{-2AU_0}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in,0})
\end{aligned} \tag{4.50}$$

Next, the constants in the equation are combined, and the simplified version is shown in Eq. 4.51.

$$\frac{dT_{c,out}}{dt^*} = a \cdot T_{c,out}(t^*) + b \tag{4.51}$$

where

$$a = \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} - AU(\dot{C}_{cold} + \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] \tag{4.52}$$

$$\begin{aligned}
b = & \left[\frac{2\dot{C}_{cold}\dot{C}_{hot} + AU(\dot{C}_{cold} - \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,in}) + \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} - AU_0(\dot{C}_{cold} - \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,in,0}) \\
& + \left[\frac{2\dot{C}_{cold}\dot{C}_{hot} + AU_0(\dot{C}_{cold} + \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,out,0}) + \left[\frac{2AU}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in}) + \left[\frac{-2AU_0}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in,0})
\end{aligned} \tag{4.53}$$

The rest of the solution procedure is identical to the previous section and is not repeated here. Carrying out the math will lead to Eq. 4.54, and the time constant is denoted by Eq. 4.55.

$$\frac{T_{c,out}(t^*) - T_{c,out,\infty}}{T_{c,out,0} - T_{c,out,\infty}} = \theta_{c,out}(t^*) = e^{at^*} = e^{-\left(\frac{t^*}{\tau}\right)} \quad (4.54)$$

$$\tau^* = \left[\frac{C_{HX}}{2 + AU \left(\frac{\dot{C}_{cold} + \dot{C}_{hot}}{\dot{C}_{cold}\dot{C}_{hot}} \right)} \right] = \left[\frac{C_{HX}}{2 + N_c + N_h} \right] \quad (4.55)$$

4.2.3 Simultaneous step change in temperature and flowrate

The procedure to solve for $T_{c,out}(t)$ is almost identical to the previous sections. Therefore, only the key parts of the solution will be discussed here. Starting from the governing differential equation and carrying out the math will result in Eq. 4.56.

$$\begin{aligned} \frac{dT_{c,out}}{dt^*} = & \left[\frac{2\dot{C}_{cold}\dot{C}_{hot} - AU\dot{C}_{hot} + AU\dot{C}_{cold}}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,in}) \\ & + \left[\frac{-2\dot{C}_{cold,0}\dot{C}_{hot,0} + AU_0\dot{C}_{hot,0} - AU_0\dot{C}_{cold,0}}{C_{HX}\dot{C}_{cold}\dot{C}_{hot,0}} \right] (T_{c,in,0}) \\ & + \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} - AU\dot{C}_{hot} - AU\dot{C}_{cold}}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,out}) \\ & + \left[\frac{2\dot{C}_{cold,0}\dot{C}_{hot,0} + AU_0\dot{C}_{hot,0} + AU_0\dot{C}_{cold,0}}{C_{HX}\dot{C}_{cold}\dot{C}_{hot,0}} \right] (T_{c,out,0}) \\ & + \left[\frac{2AU}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in}) + \left[\frac{-2AU_0}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in,0}) \end{aligned} \quad (4.56)$$

Combining the constant terms leads to Eq. 4.57, and the solution to it is shown in Eq. 4.60, which is identical to those from the previous sections.

$$\frac{dT_{c,out}}{dt^*} = a \cdot T_{c,out}(t^*) + b \quad (4.57)$$

where

$$a = \left[\frac{-2\dot{C}_{cold}\dot{C}_{hot} - AU(\dot{C}_{cold} + \dot{C}_{hot})}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] \quad (4.58)$$

$$\begin{aligned}
b = & \left[\frac{2\dot{C}_{cold}\dot{C}_{hot} - AU\dot{C}_{hot} + AU\dot{C}_{cold}}{C_{HX}\dot{C}_{cold}\dot{C}_{hot}} \right] (T_{c,in}) \\
& + \left[\frac{-2\dot{C}_{cold,0}\dot{C}_{hot,0} + AU_0\dot{C}_{hot,0} - AU_0\dot{C}_{cold,0}}{C_{HX}\dot{C}_{cold}\dot{C}_{hot,0}} \right] (T_{c,in,0}) \\
& + \left[\frac{2\dot{C}_{cold,0}\dot{C}_{hot,0} + AU_0\dot{C}_{hot,0} + AU_0\dot{C}_{cold,0}}{C_{HX}\dot{C}_{cold}\dot{C}_{hot,0}} \right] (T_{c,out,0}) \\
& + \left[\frac{2AU}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in}) + \left[\frac{-2AU_0}{C_{HX}\dot{C}_{cold}} \right] (T_{h,in,0})
\end{aligned} \tag{4.59}$$

$$\frac{T_{c,out}(t^*) - T_{c,out,\infty}}{T_{c,out,0} - T_{c,out,\infty}} = \theta_{c,out}(t^*) = e^{at^*} = e^{-\left(\frac{t^*}{\tau^*}\right)} \tag{4.60}$$

4.3 Discussion

This chapter covered the derivation of expressions for the design parameter of interest, $T_{c,out}$, as a function of time in a DWHR heat exchanger subjected to a step change in input variables. As was shown, the non-dimensional form of the solution is identical, regardless of the input that gets modified through a step change. What does change is the expression for b , while the expression for a remains the same for each solution. Hence, the expression for the time constant τ for DWHR heat exchangers subjected to a step change in inputs is identical, and is repeated here again:

$$\tau^* = -\frac{1}{a} = \left[\frac{C_{HX}\dot{C}_{cold}\dot{C}_{hot}}{2\dot{C}_{cold}\dot{C}_{hot} + AU(\dot{C}_{cold} + \dot{C}_{hot})} \right] = \left[\frac{C_{HX}}{2 + AU\left(\frac{\dot{C}_{cold} + \dot{C}_{hot}}{\dot{C}_{cold}\dot{C}_{hot}}\right)} \right] = \left[\frac{C_{HX}}{2 + N_c + N_h} \right] \tag{4.61}$$

This expression is in terms of known heat capacity rates $\dot{C}_{cold}$ and $\dot{C}_{hot}$, nondimensional lumped mass C_{HX} , and AU that is a result of heat exchanger size and inlet conditions. The inlet flowrates can be measured with accuracy; hence, the main sources of error for calculating the time constant are C_{HX} and AU . The term C_{HX} requires knowing the mass of water in the coils, as well as mass of copper in the heat exchanger. Note that in the derivation, the entire mass of copper was attributed to the cold side of the heat exchanger; this is a number that can be tuned if needed. As for AU , it determines the final outlet temperature when steady state is reached; in other words, without a robust model that can predict U under various steady state conditions, the transient approach will be inaccurate.

4.4 Experimental Validation

To validate the transient expressions derived in this chapter, experimental data were required. The test apparatus described in Manouchehri [28] was used to generate data during transient operation of DWHR heat exchangers.

To produce data, a 3inch diameter, 96inch long DWHR heat exchanger was installed in the test apparatus. In each test, the heat exchanger is supplied with water at temperatures of 10 and 38°C on the mains side and drain side respectively. The inlet flowrates are kept constant until all inlet variables reach a steady state. The data collection commences afterwards. It is imperative that in each test, the inlet conditions are first allowed to reach a steady condition; otherwise, the transients would include impacts from the test apparatus as well, which are independent from the transients associated with DWHR heat exchangers.

An example of a transient test is provided in Figure 4-2. The primary y-axis shows the outlet temperature, the secondary y-axis shows the (balanced) flowrate supplied to the heat exchanger, and the x-axis denotes time measured in seconds. Note that time zero in the tests refers to when the inlet conditions were subjected to a change; in this case, the flowrate was lowered at time 0, from 12 L/min to 7 L/min. Also note that the outlet temperature does not react immediately, and the time-delays associated with instruments must be accounted for during modeling. The plots that will follow this figure will be constructed similarly to allow visual comparison of the transient responses.

Before applying the model, a few comments can be made about the transient behaviour depicted in Figure 4-2. Firstly, the measured outlet temperature closely follows the expected behaviour for a first order system. Secondly, the time constant τ can be estimated graphically to check the validity of transient models. Based on the figure, it takes about 20 seconds for the system to reach steady state; knowing that it takes 5 times the magnitude of τ to reach 99.3% of the steady state value, the time constant is approximated to be 4 seconds.

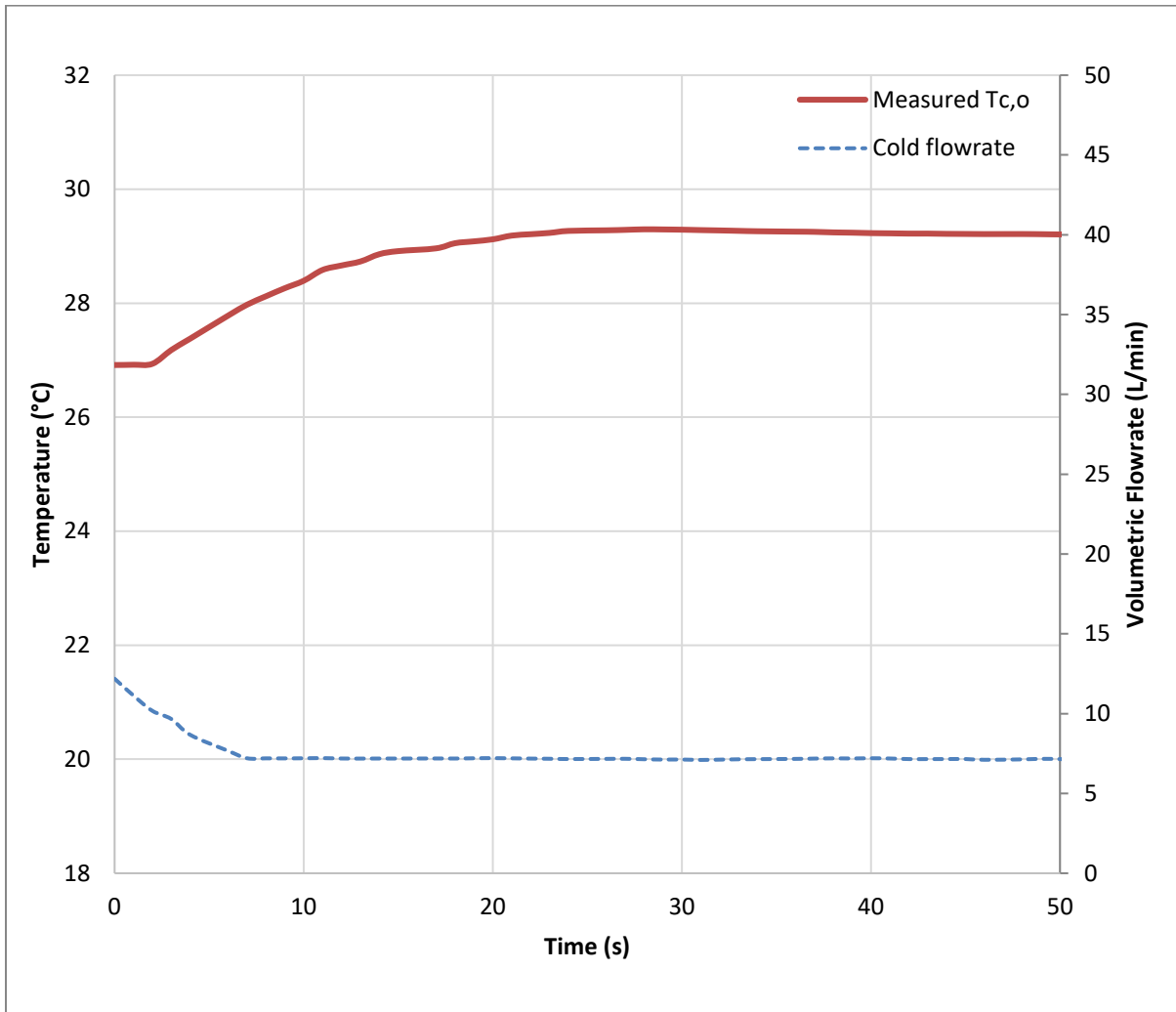


Figure 4-2: Transient response of a 3" diameter, 96" long DWHR heat exchanger to a change in flowrate.

The next step is to apply the equations derived in this chapter and determine how well they apply to the experimental data. This application will be shown through an example, using the data associated with Figure 4-2. Table 4-1 lists the relevant inputs to the model. Note that the flush time and the time constant are condition-dependent and are calculated based on the inlet flowrate. In other words, the time constant cannot be assumed to stay constant for other conditions.

Table 4-1: Required information for modeling the transient response shown in Figure 4-2.

Variables of interest	Measured values	Units
$\dot{C}_{cold,0}$	0.849	kW/°C
$\dot{C}_{hot,0}$	0.845	kW/°C
$T_{c,in,0}$	9.7	°C
$T_{c,out,0}$	26.9	°C
$T_{h,in,0}$	38.2	°C
$\dot{C}_{cold}$	0.502	kW/°C
$\dot{C}_{hot}$	0.497	kW/°C
$T_{c,in}$	9.7	°C
$T_{h,in}$	38.2	°C
Mass of Water	2	kg
Mass of Copper	20	kg
C_{HX}	1.921	Nondimensional
$t_{flush,water}$	31.98	seconds
Time constant τ	3.01	seconds

Using the values from Table 4-1, it is now possible to plot the predicted cold outlet temperature as a function of time. This is shown in Figure 4-3. There is a small difference between the predicted and measured outlet temperatures shown in the plot, but the magnitude of energy associated with the difference is not significant compared to the overall savings achieved during a shower event. The error can be attributed to simplifications applied in the derivation process, as well as the inability to perfectly replicate a step-change in flowrate in lab setting. This can readily be observed on the plots; it is more accurate to refer to flowrate adjustments as ramp-inputs instead of step-change inputs. This is the likely reason why the measured temperatures seem to overshoot what was predicted using the first order model.

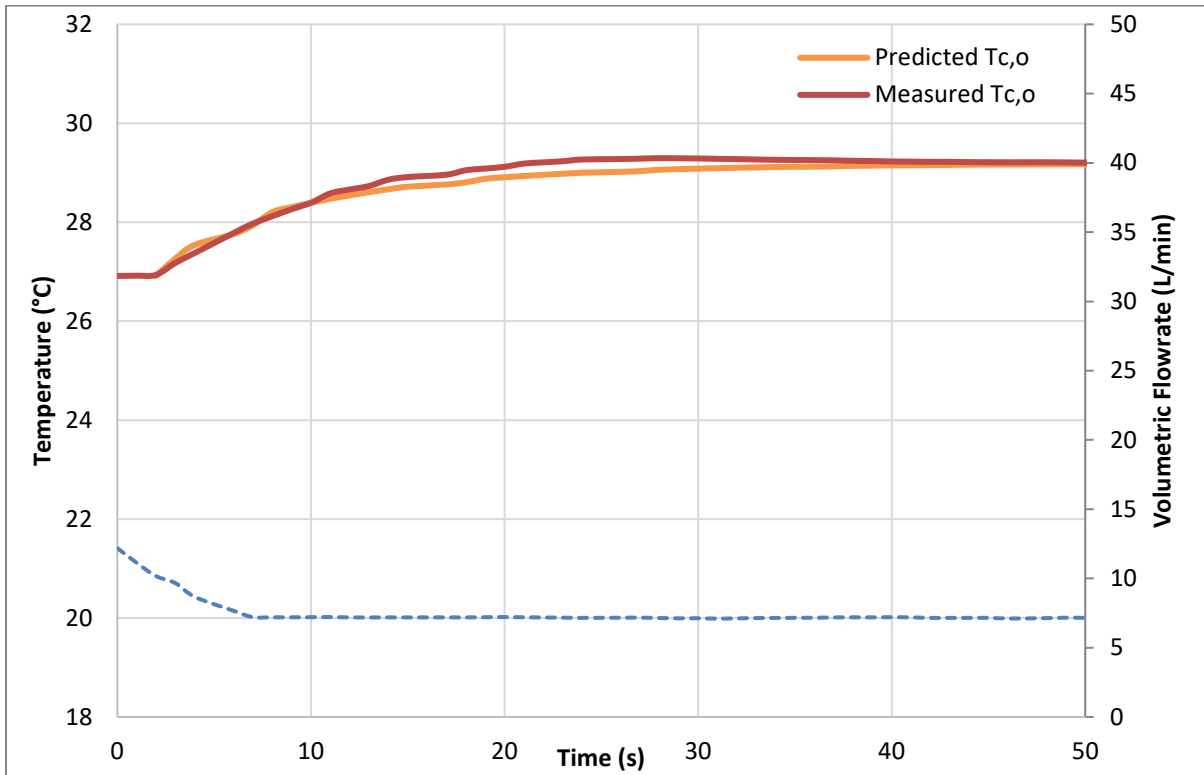


Figure 4-3: Measured vs. predicted transient response of a 3" diameter, 96" long DWHR heat exchanger to a change in flowrate.

To further illustrate the application of the transient model, additional data were gathered using the same heat exchanger. The data that was shown in Figure 4-2 and Figure 4-3 were limited to the first 50 seconds of the experiment, where the flowrate was only adjusted once. This experiment was in fact continued for much longer, and the flowrate was adjusted multiple times after reaching steady state. Figure 4-4 depicts the application of the model to the entire dataset. The data indicate that the time to reach steady state is always shorter than a minute, and the time constants are on the order of seconds. Also, the discrepancies between the measured and predicted outlet temperatures are smaller than 1°C, and last on the order of seconds; hence, the quantity of energy associated with transient errors is expected to be minimal when compared to the quantity of energy saved during steady-state operation.

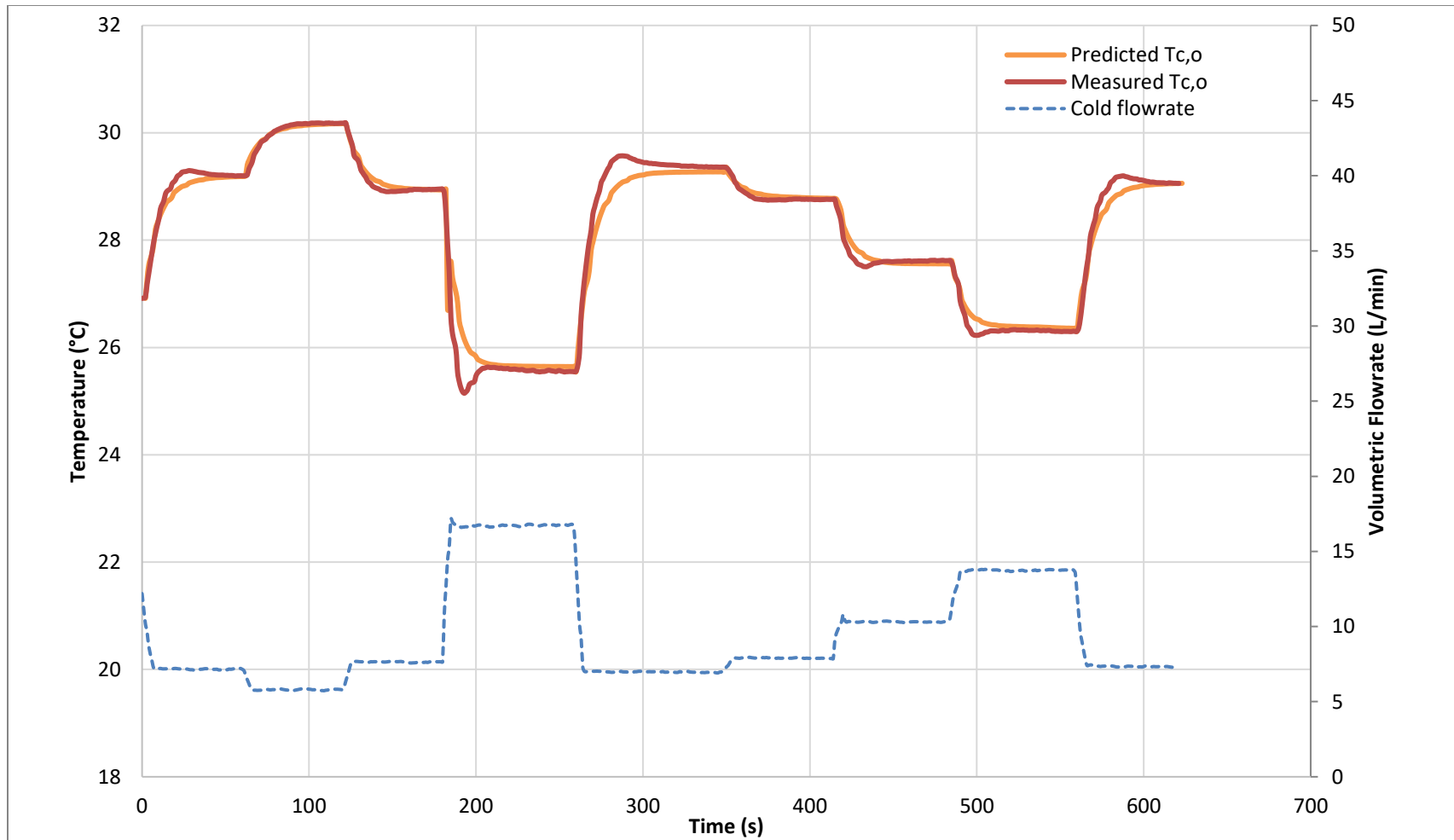


Figure 4-4: Measured vs. predicted transient responses for a 3" diameter, 96" long DWHR heat exchanger subjected to multiple consecutive changes in flowrate.

Given the small magnitude for time constants associated with the previous figures, it was of interest to determine the applicability of the model in cases where the heat exchanger is not allowed to reach steady state. To this end, the same DWHR heat exchanger was tested under balanced flow conditions, where the flowrate was changed at approximately every 20 seconds. Figure 4-5 shows the application of the model to the data gathered for this test. As shown, the first order model developed to represent transients in DWHR heat exchangers is clearly a good approximation of what actually occurs during transient testing. It is worth noting that the numerous flowrate adjustments observed in Figure 4-5 are not expected to occur during a shower, and this experiment was designed to illustrate the applicability of the model to high frequency changes, which may be of interest to future researchers.

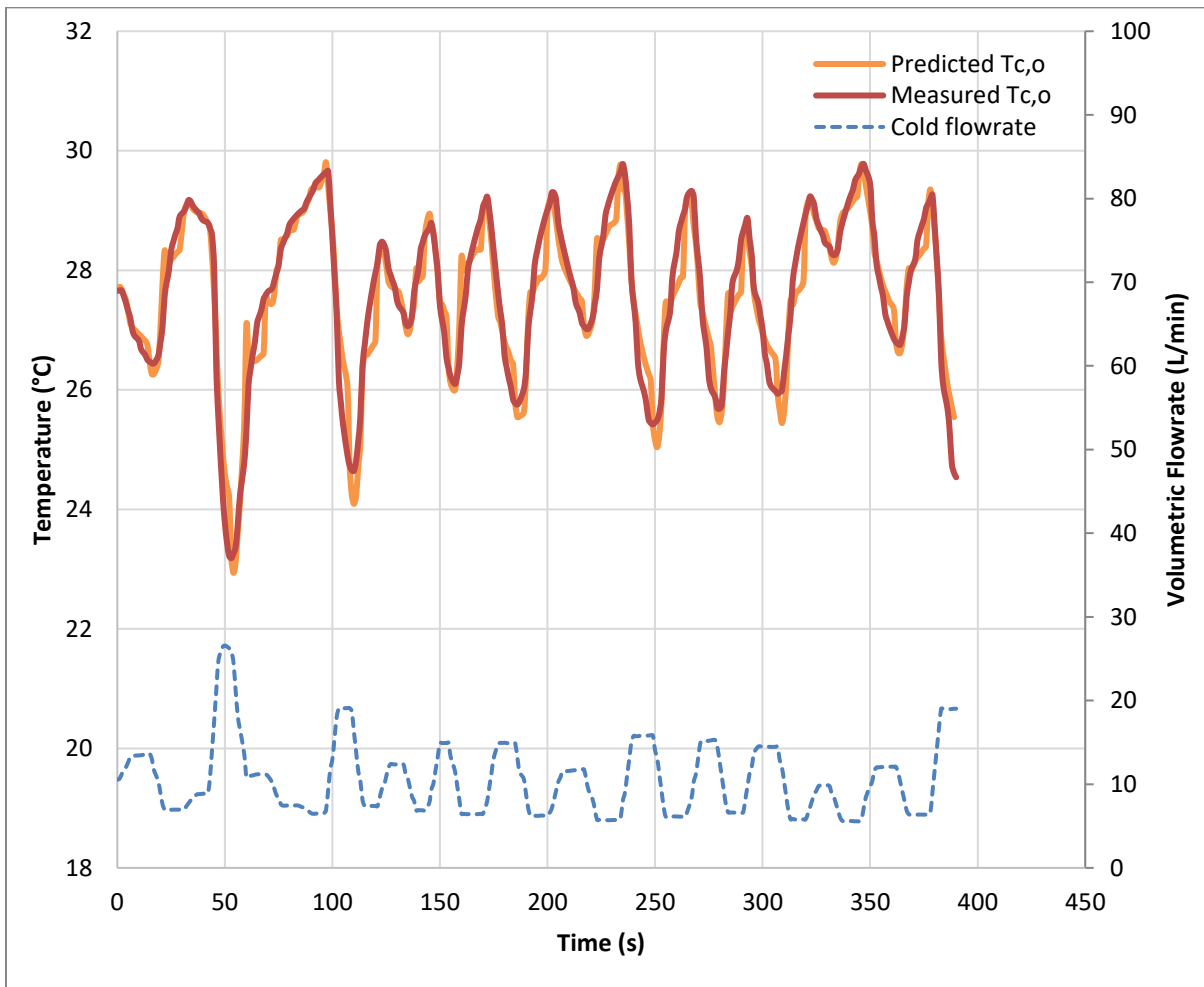


Figure 4-5: Measured vs. predicted transient responses for a 3" diameter, 96" long DWHR heat exchanger subjected to multiple consecutive changes in flowrate over 400 seconds, noting that a steady state was never reached.

Next, the same heat exchanger was subjected to test condition where the flowrates were no longer balanced. To generate data, the heat exchanger was subjected to inlet flowrates of 6.5 and 15 L/min on the mains and drain sides respectively. Once steady state was reached, the flowrates in the system were suddenly brought to a balanced condition, which corresponds to time = 0 in the dataset. This process was repeated 3 times in a row to investigate the repeatability of transients, as well as the applicability of the model.

Figure 4-6 shows the measured data corresponding to this test, where the outlet temperature for the coils as well as the flowrates for the mains and the drain side are shown as a function of time. Before applying the model, a few observations can be made. First, the transient behaviour is very repeatable. Second, the time to reach steady state is longer relative to balanced flow conditions tested previously. This is inline with what is known about heat exchangers; as a heat exchanger deviates from a balanced flow condition (i.e. $C_r = 1$), the heat exchanger becomes less effective, both in steady state and transient operating conditions. Further evidence of this phenomenon is visually present in the figure: the time to reach steady state is longer when going from balanced flow conditions to unbalanced flow conditions relative to the time it takes to move in reverse.

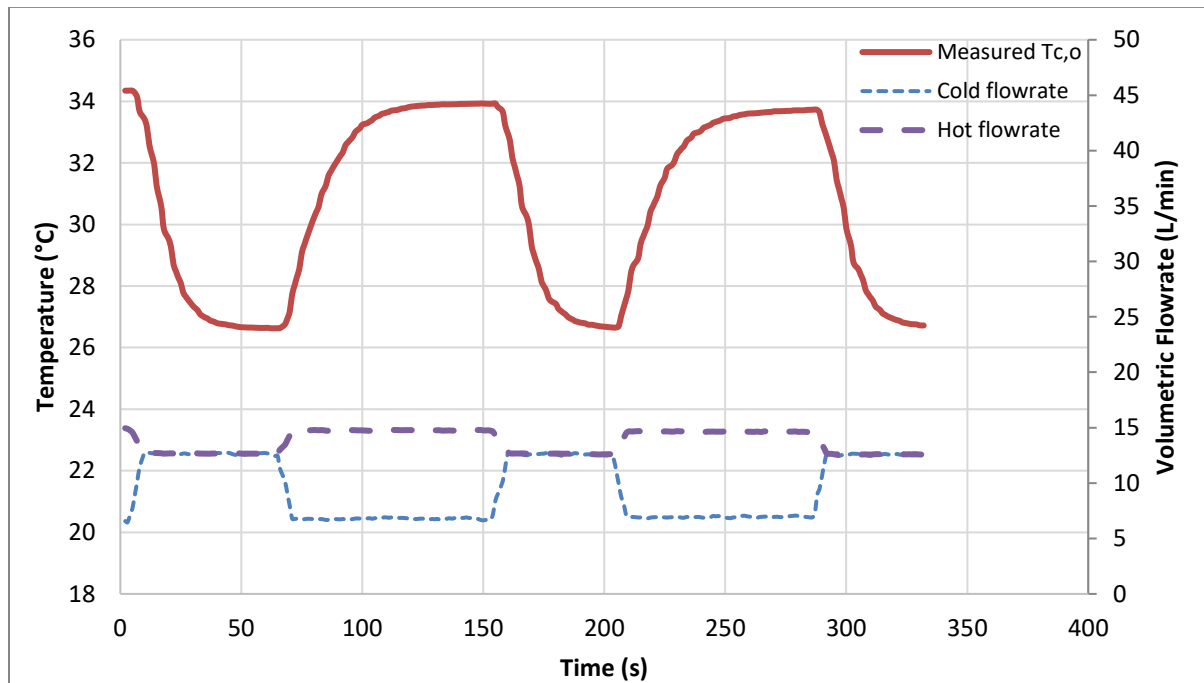


Figure 4-6: Measured transient responses for a 3" diameter, 96" long DWHR heat exchanger subjected to multiple consecutive changes in both flowrates.

Next, the model is applied to the measured data from Figure 4-6 to determine its validity. Figure 4-7 shows the results. Once again, there is a small discrepancy between the measured and predicted outlet temperatures, which only lasts a few seconds. Otherwise, the first order behaviour of transients in the heat exchanger, as well as time to reach steady state are predicted well. It is possible to tune the model to better match the experimental data, by slightly modifying the value of C_{HX} , noting that this variable was created based on the lumped sum assumption, which is far from a perfect assumption for transients. This is illustrated in Figure 4-8, which uses the same data as the previous figure, but instead uses a value of C_{HX} that is higher by a factor of 1.15. That is not to say that a correction factor of 1.15 should be applied to all data produced with this approach; the numerous assumptions that have gone into the derivation of the transient model will inevitably cause error, but this error can be minimized by tuning C_{HX} , if deemed necessary.

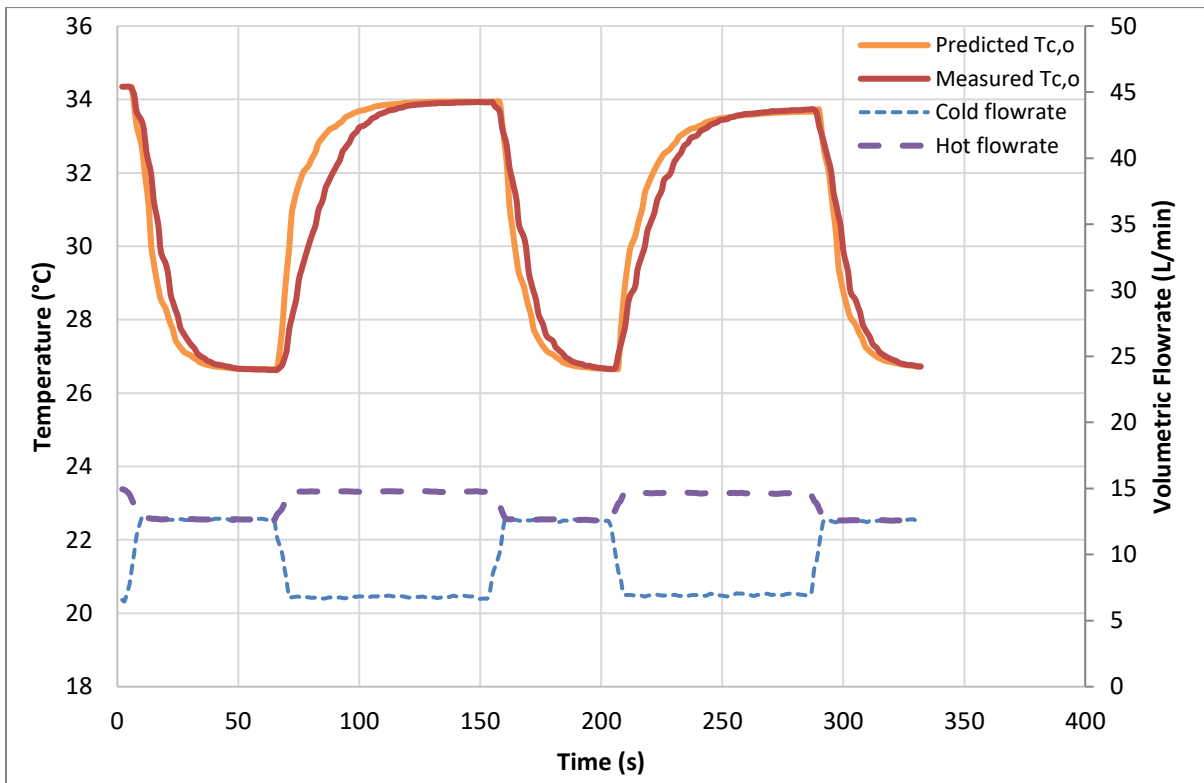


Figure 4-7: Measured vs. predicted transient responses for a 3" diameter, 96" long DWHR heat exchanger subjected to multiple consecutive changes in both flowrates. The prediction relies on the value of C_{HX} calculated originally.

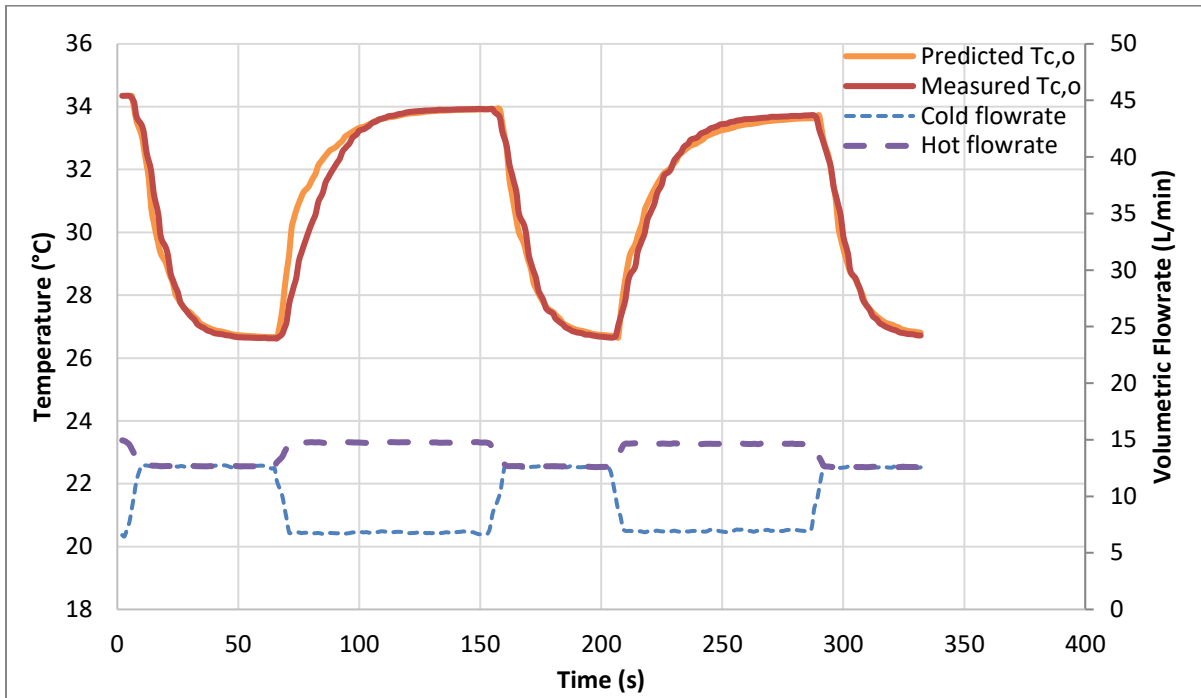


Figure 4-8: Measured vs. predicted transient responses for a 3" diameter, 96" long DWHR heat exchanger subjected to multiple consecutive changes in both flowrates. The prediction relies on the tuned value for C_{HX} .

At this point, there was sufficient evidence that the transient model applies to the 3inch diameter, 96inch long DWHR heat exchanger. To further test the validity of the transient model, it must be applied to other DWHR heat exchangers. To this end, similar transient experiments were performed on two heat exchangers having different lengths and diameters. The first heat exchanger had a diameter of 2 inches and a length of 60 inches, whereas the second heat exchanger had a diameter of 4 inches and a length of 48 inches. The relevant physical information about each unit is listed in Table 4-2. The experimental results for the two heat exchangers are presented in Figure 4-9 and Figure 4-10.

Table 4-2: Inputs required for modeling transient behaviour for DWHR heat exchangers tested as part of the validation.

Variables of interest	2" diameter, 60" long unit	4" diameter, 48" long unit
Mass of Water (kg)	1.3	2.5
Mass of Copper (kg)	11	14.5

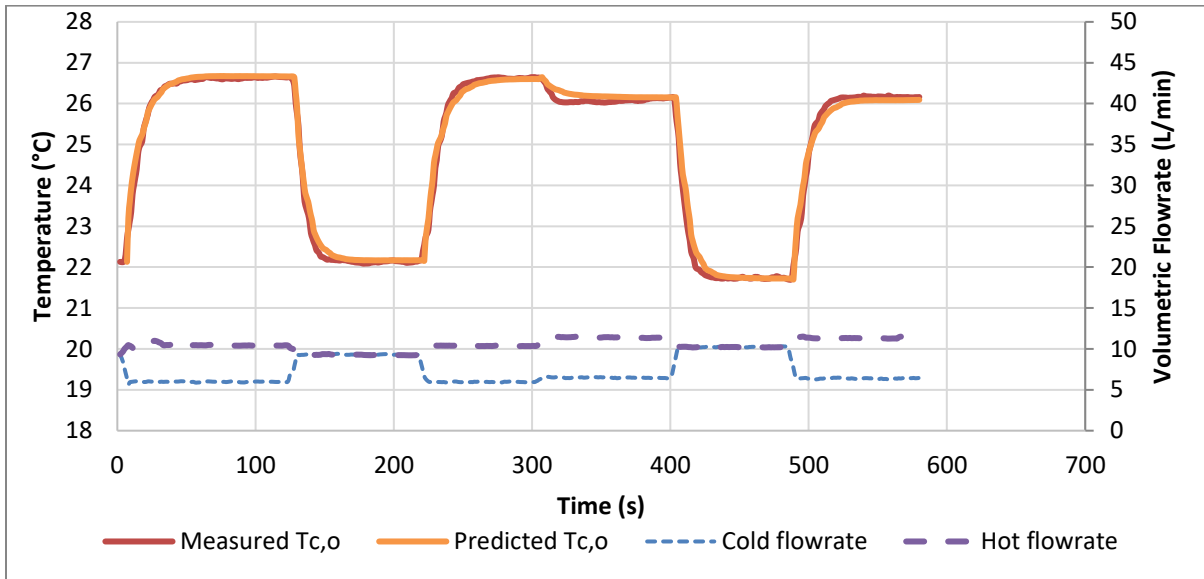


Figure 4-9: Measured vs. predicted transient responses for a 2" diameter, 60" long DWHR heat exchanger subjected to multiple consecutive changes in both flowrates.

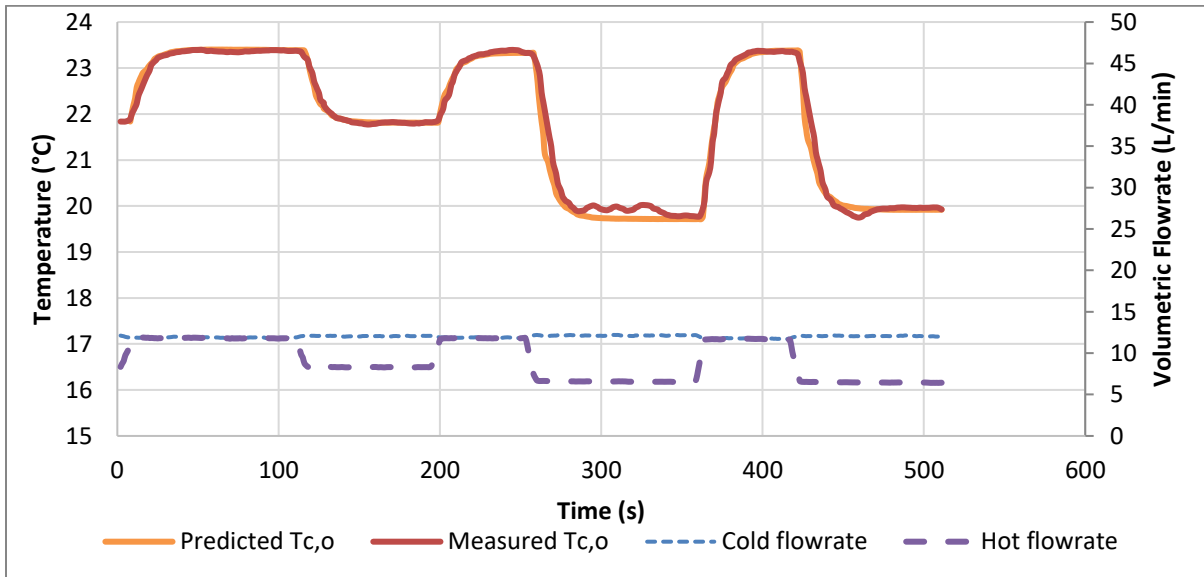


Figure 4-10: Measured vs. predicted transient responses for a 4" diameter, 48" long DWHR heat exchanger subjected to multiple consecutive changes in both flowrates.

Based on the experimental results the transient model was deemed sufficiently accurate for representing DWHR heat exchangers. Additional testing performed on other heat exchangers yielded similar results and are not presented here.

Chapter 5– TRNSYS Model

This chapter focuses on the application of the steady state and transient models developed in this thesis in a simulation software. The majority of current publications on DWHR heat exchangers focus on the estimating or simulating performance based on steady state data collected during balanced flow operation. The goal for this chapter is to provide guidelines about how DWHR should be simulated in computer software, and to illustrate the significant impact of plumbing configurations on the overall energy savings. This will be done using the TRNSYS software to simulate the combined effect of all variables on the overall energy savings. A part of this chapter has already been published in the journal of Energies [47].

5.1 TRNSYS Software

TRNSYS (Transient System Simulation Tool) is a software package used to simulate performance of transient systems. Chapters 3 and 4 established how an individual DWHR heat exchanger should be modelled during transient and steady state conditions, and TRNSYS will be used to determine performance in a realistic residential setting.

There are many ways to install plumbing in a house, which are influenced by building codes and plumbing codes. DWHR heat exchangers are intended to be installed as part of the plumbing system and as such, it is imperative to investigate their performance in realistic installations. Prior to any modeling, it is helpful to conceptualize all possible plumbing configurations in a simple residential dwelling.

To illustrate the differences, Figure 5-1 was constructed. Figure 5-1(A1) depicts a system where all preheated water from the DWHR system goes to the water heater, Figure 5-1(B1) depicts a system where all preheated water goes to the mixing valve at the showerhead, and Figure 5-1(C1) shows a combination of the two previous cases. Lastly, Figure 5-1(D1) is provided to show the same plumbing system without a DWHR heat exchanger installed. For the configurations B1 and C1, it is possible to make a slight modification to supply the toilet using mains water instead of preheated water. These configurations are shown in Figure 5-2. Henceforth, the configurations in Figure 5-1 will be referred to as A, B1, C1 and D; similarly, the modified configurations in Figure 5-2 will be referred to as B2 and C2. The final results will be presented in terms of these configurations to allow direct comparison amongst them.

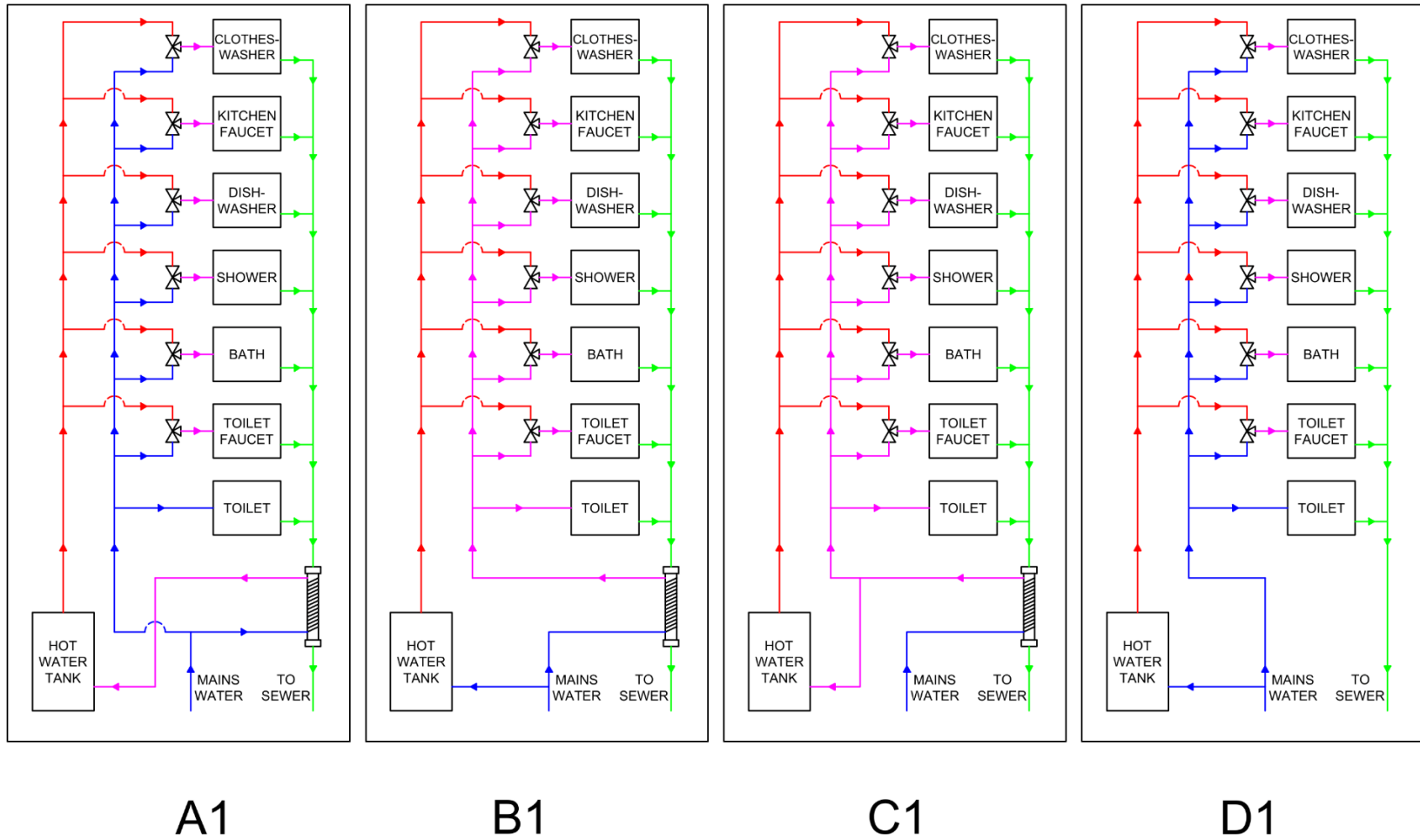


Figure 5-1: Schematic of plumbing configurations used in the simulations. (A1) depicts a plumbing configuration where all the preheated water is sent to the water heater, (B1) depicts a configuration where all the preheated water is sent to the fixtures, and (C1) depicts a combination of the two. (D1) represents a setup with no heat exchanger.

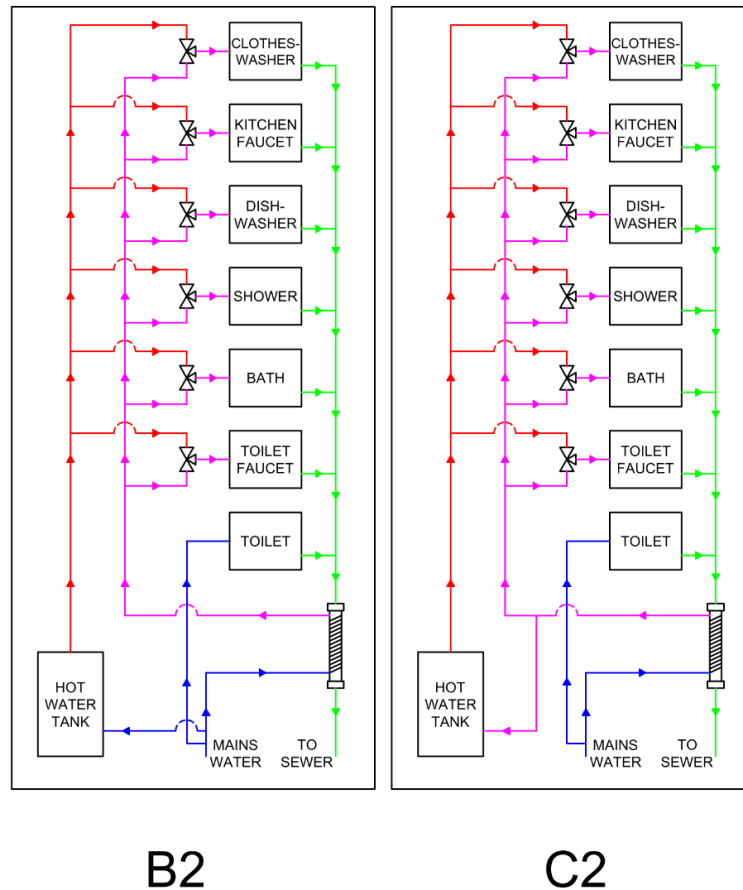


Figure 5-2: Alternative configurations to (B1) and (C1), where the preheated water supplied to the toilet fixture is replaced with mains water.

Configuration C depicts equal flowrates of water through both sides of the heat exchanger, whereas configurations A and B depict unequal-flow conditions. The performance of DWHR systems is a strong function of flowrates and based on the configurations shown, the flowrate of water through the heat exchanger coils is expected to vary based on plumbing configuration to provide the same temperature at the showerhead.

Researchers have been focused on simulating the performance of DWHR heat exchangers under equal-flow arrangements (i.e. config C). This is because the data available on the performance of DWHR heat exchangers is limited to the effectiveness data gathered during the CSA rating process [11]. The rating system is meant to provide a means for comparing different DWHR heat exchangers on an equal basis; it does not and cannot provide any accurate estimates for different configurations in which a heat exchanger could be installed.

It is worth noting that despite being the only configuration covered in standards, configuration C is not always an option, especially in retrofit settings where the plumbing has already been installed and additional modifications are costly. Furthermore, local building codes can impose limitations on which configurations are allowed. For example, configurations B and C are not permitted in Québec, Canada [48]. This restriction is due to the possibility of bacteria growth, such as *Legionella*, in preheated water that bypasses the water heater. In other words, only plumbing setups in which all preheated water is directed to the water heater are allowed, and that limits the allowable configurations to what was shown in Figure 5-1(A1). Clearly, assuming that the DWHR heat exchangers are always installed and operated under equal-flow conditions is incorrect, and potentially against local codes. Thus, building simulation models must be created to predict performance of such heat exchangers under unequal-flow conditions.

The steady-state and transient models developed in chapters 3 and 4 can be used to simulate the performance of DWHR heat exchangers under realistic conditions, including the configurations depicted previously in Figure 5-1 and Figure 5-2. The rigorous details regarding internals of the model are not repeated here; instead, this study aims to use the aforementioned models to perform energy simulations for DWHR systems installed in different plumbing configurations. To this end, the model was implemented into TRNSYS software as a component and used in simulations described in the following sections.

5.2 TRNSYS Component

TRNSYS was selected as a suitable software for simulations because it allows performing simulations while accounting for transient behaviour for components such as water heaters. A TRNSYS simulation works by allowing interaction between different ‘components’ on a timestep-by-timestep basis, and this time step is selected by the user at the beginning of each simulation. Each component consists of ‘parameters’ and ‘inputs’ which are used to generate ‘outputs’ for the components for each time step.

Parameters are relevant values that are used by TRNSYS to simulate a component throughout the simulation period. Parameters are independent of time and remain the same throughout the modelling process. In the context of the DWHR component in TRNSYS, the size of the heat exchanger is a parameter in the simulations. On the other hand, inputs to the simulation are variables that can change as a function of time. The inputs to the DWHR component include the inlet water temperatures and

flowrates to the heat exchanger. The simulation then uses the inputs and parameters to generate outputs for the component for each time step. The outputs for the DWHR component include the heat recovery rate, heat exchanger effectiveness, and the outlet temperatures for both streams. A summary of the parameters, inputs, and outputs for the DWHR component are shown in Table 5-1. It is worth noting that Chapter 3 covered three methods for modelling steady-state performance of DWHR conditions, and only the model which relies on regression constants a and b is used in simulations. The rationale behind this choice is that this method allows manufacturers to use pre-existing data generated during the CSA rating procedure.

Table 5-1: Parameters, inputs and outputs associated with the DWHR component created in TRNSYS.

Type	Variable Name	Unit	Notes
Parameter	a	min/L	Regression coefficients taken from the characteristic effectiveness curve for the DWHR unit being simulated.
	b	Dimensionless	
Input	$T_{c,i}$	°C	Inlet water temperature to the coils
	$T_{h,i}$	°C	Inlet water temperature to the drainpipe
	$\dot{m}_{coils}$	kg/s	Mass flowrate of water through the coils
	$\dot{m}_{drain}$	kg/s	Mass flowrate of water through the drainpipe
Output	$T_{c,o}$	°C	Outlet water temperature from the coils
	$T_{h,o}$	°C	Outlet water temperature from the drainpipe
	q	kW	Heat recovery rate
	ε	Dimensionless	Heat exchanger effectiveness

Parameters a and b are regression coefficients taken from the characteristic effectiveness vs. flowrate curve that is generated during the rating process under CSA B55.15 standard [11]. Figure 5-3 shows an example of this characteristic curve, where parameters a and b are determined to be 0.0685 min/L and 1.2796 respectively, and $\dot{V}$ denotes the volumetric flow rate in L/min. The TRNSYS component uses these parameters and the inputs assigned by the user to calculate outputs for each time step.

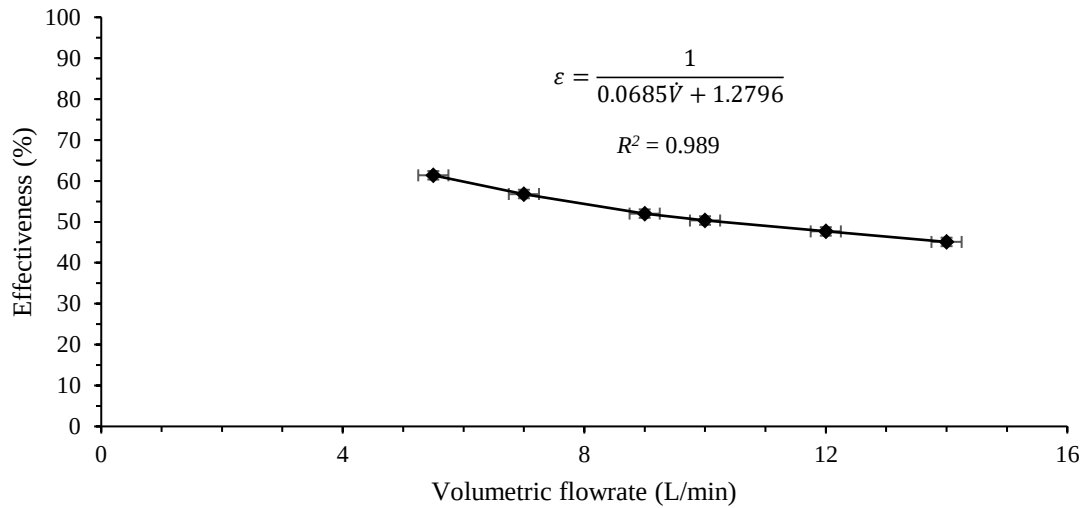


Figure 5-3: Equal-flow effectiveness vs. volumetric flowrate for a 7.6 cm diameter, 153 cm long DWHR heat exchanger.

5.3 Method

To compare the energy savings associated with DWHR systems in different plumbing configurations, important simulation components such as the water heater, water-draw schedule and time step size must be selected. The following sections describe the key constraints used in the simulations.

5.3.1 Water Draw Schedule

The operation of DWHR heat exchangers relies on simultaneous draw and drainage of water; hence, the water draw schedule has a significant influence on any savings achieved these heat exchangers. Water usage in homes has been a subject of study since the 1940s [49], and this interest exponentially grew in the 1990s due to the demand for low-flow fixtures as mandated by Energy Policy Act 1992 and other standards. Data on water demand is also very useful for water utility companies, as it guides their decisions on metering, forecasting and other planning and management strategies. As a result, many statistical studies exist that have attempted to collect data from end-users to come up with typical draw schedules. One such study is the Residential End Uses of Water [49], which is a report put together as a joint effort by many organizations spanning from utility companies, regional utilities as well as researchers. The dataset used in this study also includes the Region of Waterloo; hence, it was deemed sufficiently accurate to guide the end-uses for water in simulations. Additionally, the

report surveyed over 4000 households, 762 of which had detailed end-uses for water; hence the sample size was deemed large enough to be a good statistical representation of reality.

The statistical findings from report were used to construct a water-draw schedule for a house to be simulated. The average flowrates and durations for significant events such as showers were also taken directly from the report. The constructed draw schedule for residence is shown in Table 5-2.

Additional information about the draw events is found in Appendix B. Note that due to the statistical nature of the draw profile, the draw events shown in Table 5-2 correspond to only one week worth of draws, and if the simulation period is longer than one-week, identical draw profiles are used. This draw profile is used for all TRNSYS simulations presented in this chapter.

Table 5-2: Detailed water draw schedule used in the simulations.

Day of the week	Start	End	Event	Flowrate (liter/s)
M, T, W, Th, F, Sa, Su	6:00:00 AM	6:08:00 AM	Shower	0.1325
M, T, W, Th, F, Sa, Su	6:10:00 AM	6:10:05 AM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	6:10:05 AM	6:10:25 AM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	6:10:10 AM	6:10:40 AM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	6:30:00 AM	6:40:00 AM	Misc faucet usage	0.0631
M, T, W, Th, F, Sa, Su	6:30:00 AM	6:30:05 AM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	6:30:05 AM	6:30:25 AM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	6:30:10 AM	6:30:40 AM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	6:50:00 AM	6:50:05 AM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	6:50:05 AM	6:50:25 AM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	6:50:10 AM	6:50:40 AM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	7:00:00 AM	7:08:00 AM	Shower	0.1325
M, T, W, Th, F, Sa, Su	7:10:00 AM	7:10:05 AM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	7:10:05 AM	7:10:25 AM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	7:10:10 AM	7:10:40 AM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	7:30:00 AM	7:30:05 AM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	7:30:05 AM	7:30:25 AM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	7:30:10 AM	7:30:40 AM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	7:50:00 AM	7:50:05 AM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	7:50:05 AM	7:50:25 AM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	7:50:10 AM	7:50:40 AM	Handwash	0.0631

Sa	9:00:00 AM	9:02:00 AM	Washing machine draw (load1)	0.4889
Sa	9:10:00 AM	9:12:00 AM	Washing machine drain (load1)	0.4889
Sa	9:12:00 AM	9:14:00 AM	Washing machine draw (load1)	0.4889
Sa	9:22:00 AM	9:24:00 AM	Washing machine drain (load1)	0.4889
Sa	9:30:00 AM	9:32:00 AM	Washing machine draw (load2)	0.4889
Sa	9:40:00 AM	9:42:00 AM	Washing machine drain (load2)	0.4889
Sa	9:42:00 AM	9:44:00 AM	Washing machine draw (load2)	0.4889
Sa	9:52:00 AM	9:54:00 AM	Washing machine drain (load2)	0.4889
Sa	10:00:00 AM	10:02:00 AM	Washing machine draw (load3)	0.4889
Sa	10:10:00 AM	10:12:00 AM	Washing machine drain (load3)	0.4889
Sa	10:12:00 AM	10:14:00 AM	Washing machine draw (load3)	0.4889
Sa	10:22:00 AM	10:24:00 AM	Washing machine drain (load3)	0.4889
Sa	10:30:00 AM	10:32:00 AM	Washing machine draw (load4)	0.4889
Sa	10:40:00 AM	10:42:00 AM	Washing machine drain (load4)	0.4889
Sa	10:42:00 AM	10:44:00 AM	Washing machine draw (load4)	0.4889

Sa	10:52:00 AM	10:54:00 AM	Washing machine drain (load4)	0.4889
Sa, Su	11:00:00 AM	11:10:00 AM	Misc faucet usage	0.0631
Sa, Su	12:00:00 PM	12:00:05 PM	Flush drain	1.9684
Sa, Su	12:00:05 PM	12:00:25 PM	Flush draw	0.4921
Sa, Su	12:00:10 PM	12:00:40 PM	Handwash	0.0631
Sa, Su	12:10:00 PM	12:10:05 PM	Flush drain	1.9684
Sa, Su	12:10:05 PM	12:10:25 PM	Flush draw	0.4921
Sa, Su	12:10:10 PM	12:10:40 PM	Handwash	0.0631
Sa, Su	12:20:00 PM	12:20:05 PM	Flush drain	1.9684
Sa, Su	12:20:05 PM	12:20:25 PM	Flush draw	0.4921
Sa, Su	12:20:10 PM	12:20:40 PM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	5:00:00 PM	5:00:05 PM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	5:00:05 PM	5:00:25 PM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	5:00:10 PM	5:00:40 PM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	5:10:00 PM	5:10:05 PM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	5:10:05 PM	5:10:25 PM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	5:10:10 PM	5:10:40 PM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	5:20:00 PM	5:20:05 PM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	5:20:05 PM	5:20:25 PM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	5:20:10 PM	5:20:40 PM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	6:00:00 PM	6:10:00 PM	Misc faucet usage	0.0631
M, W, F, Su	8:00:00 PM	8:10:25 PM	bath-draw	0.1325
W, Su	8:00:00 PM	8:02:00 PM	Dishwasher draw	0.1924
M, W, F, Su	8:25:00 PM	8:30:00 PM	bath-drain	0.2763
W, Su	8:28:00 PM	8:30:00 PM	Dishwasher drain	0.1924
W, Su	8:30:00 PM	8:32:00 PM	Dishwasher draw	0.1924
M, T, W, Th, F, Sa, Su	9:00:00 PM	9:00:05 PM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	9:00:05 PM	9:00:25 PM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	9:00:10 PM	9:00:40 PM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	9:10:00 PM	9:10:05 PM	Flush drain	1.9684
M, T, W, Th, F, Sa, Su	9:10:05 PM	9:10:25 PM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	9:10:10 PM	9:10:40 PM	Handwash	0.0631
M, T, W, Th, F, Sa, Su	9:20:00 PM	9:20:05 PM	Flush drain	1.9684

M, T, W, Th, F, Sa, Su	9:20:05 PM	9:20:25 PM	Flush draw	0.4921
M, T, W, Th, F, Sa, Su	9:20:10 PM	9:20:40 PM	Handwash	0.0631
W, Su	9:58:00 PM	10:00:00 PM	Dishwasher drain	0.1924
W, Su	10:00:00 PM	10:02:00 PM	Dishwasher draw	0.1924
W, Su	11:28:00 PM	11:30:00 PM	Dishwasher drain	0.1924

5.3.2 Water Heater

A typical 40-gallon electric water heater was selected for simulations. The tank volume was found to be sufficient to accommodate all shower durations at all flowrates without depleting the tank. In other words, the shower temperatures do not drop below 38°C during shower events.

There are multiple water heater types available for simulation in TRNSYS, and it was crucial to select one that has been validated experimentally. An in-depth study by Allard et al. focused on validating different electric water heaters used in TRNSYS, and the results showed that TRNSYS ‘Type 534’ accurately depicts the variables of interest, namely the energy consumption and supply temperature during simulations [50]. Type 534 simulates the tank using a nodal approach while accounting for stratification effects. Hence, Type 534 was selected for simulation purposes. This tank was divided into 50 nodes for the simulations, which exceeds the minimum number of nodes recommended by Kleinbach et al. per Eq. 5.1 [51]. Here, N_{FIXED} denotes the minimum number of nodes in the tank and $Tank_{Turnover}$ is the ratio of daily water draw and total tank volume. $Tank_{Turnover}$ varies based on the showerhead flowrate used in the simulation; hence, N_{FIXED} was calculated for all flowrates. As a result, the tank was divided into 50 nodes, as this number of nodes satisfies the minimum number of nodes required for all flowrates and having a consistent number of nodes in the tank for all simulations allows fair comparison of the results. Table 5-3 provides a summary of relevant parameters for the water heater used in all simulations in this study.

$$N_{FIXED} = 45.8 \times (Tank_{Turnover})^{-1.218} \quad (5.1)$$

Table 5-3: Information used to model the water heater in TRNSYS.

Parameter	Value used in simulations	Unit
Number of Nodes	50	Dimensionless
Tank Volume	40	Gallons
Tank Height	41.4	Inches
Environment Temperature	20	°C
Upper Element	3000	W
Lower Element	3000	W
Initial Tank Temperature	10 (at all nodes)	°C

All other TRNSYS components used in the simulations, such as flow mixers, diverters, etc., were default components available in TRNSYS.

5.3.3 Shower Conditions

A shower temperature of 38°C was selected for all shower events. This temperature is based on the temperature used for rating all DWHR systems according to the CSA standard. The mains temperature was also taken from the CSA standard and was simulated as 10°C for all water draws.

The flowrate of water through the showerhead is expected to have a significant impact on simulation results, as it would affect both the energy consumption by the water heater, and the energy recovery by the heat exchanger. It is difficult to select one showerhead flowrate to represent all households, as there are many fixtures available on the market, each having its own rated flowrate. In a typical dwelling, this flowrate is also affected by the available water pressure in the plumbing system. Hence, to simplify the matter, a showerhead flowrate of 7.95 L/min was selected based on the statistical average gathered from the residential end-use report [49].

5.3.4 DWHR Heat Exchangers Simulated

There are many commercially available DWHR heat exchangers on the market, each of which has its own thermal performance. The simulations only focus on a couple of DWHR heat exchangers to illustrate potential savings. See Table 5-4 for additional information about these heat exchangers. Note that heat exchanger 2 is larger, and is expected to provide higher energy savings, which may be difficult to discern in Table 5-4.

Table 5-4: DWHR heat exchangers used in the simulations.

	Curve-fit generated using CSA flow rates	Parameter <i>a</i>	Parameter <i>b</i>
Heat Exchanger 1	$\varepsilon = \frac{1}{0.0681\dot{V} + 1.2315}$	0.0681	1.2315
Heat Exchanger 2	$\varepsilon = \frac{1}{0.0553\dot{V} + 1.2601}$	0.0553	1.2601

5.3.5 Simulation Time Step

The next step was to determine appropriate time steps for performing simulations. Selecting appropriate time steps is crucial for accurate simulations. Smaller time steps often provide higher accuracy at the expense of CPU-time. However, the time domain must be discretized such that all shower events can be appropriately captured by the chosen time step size. To investigate discretization errors, simulations were run for several time step sizes, and for all cases, the energy consumption by the water heater was calculated for a period of a month. A time step size of 1 second was used as the reference, and the results from other time steps were compared to the reference case to estimate the discretization error. See Table 5-5. Based on the results, a time step size of 1 second was found to be suitable for simulation purposes. The simulation period used in this chapter was one month.

Table 5-5: Discrepancies in the overall energy consumption for various time step sizes, relative to simulations having a time step size of 1 second.

Time step size	Discretization Error
1s	0% (base)
15s	0.20%
30s	0.41%
1m	0.86%
5m	16.62%
30m	200.05%
1h	500.09%

5.3.6 Metric for Comparison

The energy savings associated with DWHR systems can be looked at from different perspectives, and it is crucial to clarify how savings are presented in this study. It is insufficient to simply compare the magnitude of energy that was recovered by the DWHR system in each simulation, as there are other components present in domestic water heating. Most importantly, the rate of heat recovery is expected to impact how often the heating elements within the water heater tank must activate to provide auxiliary heat. Thus, it would be more appropriate to compare the total energy consumed by the water heater instead. For this purpose, the total energy consumed by the water heater in configurations A, B and C are compared to the energy consumption in configuration D under identical simulation parameters. This allows presentation of the savings in terms of percentages relative to configuration D.

Eq. 5.2 is used to calculate the energy savings as a percentage of total energy consumed in the reference case (configuration D). E_D denotes the total auxiliary energy consumed by the water heater in configuration D, and E_{sim} represents the total auxiliary energy consumed by the water heater in configurations A, B or C.

$$Energy\ Savings = \frac{E_D - E_{sim}}{E_D} \times 100 \quad (5.2)$$

5.4 Simulation Results

The information regarding water draws, equipment, time step size, etc. from Section 5.3 were used as inputs to TRNSYS to perform a month-long simulation for all plumbing configurations shown previously. Table 5-6 shows a summary of the results for heat exchanger 1 from Table 5-4. The energy balance for all simulation cases was smaller than 2%. Note that the energy balance is defined as the absolute percent-difference between the total energy added to the system, and the total energy extracted from the system.

Table 5-6: Simulation results for heat exchanger 1.

Configuration	Energy consumed by the heating elements (kJ)	Energy delivered from the HWT (kJ)	Change in thermal energy storage within HWT (kJ)	Energy recovered by DWHR (kJ)	Energy savings based on Eq. 5.2
A	1011443.97	955911.17	55532.97	389673.50	27.7%
B1	1049369.97	994371.97	54998.11	398070.90	25.0%
B2	985886.97	930861.90	55025.19	416927.77	29.5%
C1	922640.98	867265.12	55375.97	467230.89	34.0%
C2	911651.98	856262.12	55389.97	489501.18	34.8%
D	1398929.96	1343972.10	54957.95	0.00	0.0%

Note that there is a large discrepancy between the total energy consumption by the heating elements, resulting in energy savings ranging from 25% to 35%. Also, note that the magnitude of heat recovery by the heat exchanger is not necessarily aligned with %savings. For example, the magnitude of energy recovered in configuration A is smaller than that of configuration B1, while having higher energy savings as a percentage. This further illustrates why the plumbing configuration must be accounted for when it comes to simulating performance of DWHR heat exchangers.

Table 5-7 shows a summary of the results for heat exchanger 2 from Table 5-4. The results concur with those from heat exchanger 1 and show that the range in energy savings occurs regardless of the heat exchanger size. The magnitude of energy consumption for heat exchanger 2 was higher than heat exchanger 1 in all cases, which is reasonable due to its larger size.

Table 5-7: Simulation results for heat exchanger 2.

Configuration	Energy consumed by the heating elements (kJ)	Energy delivered from the HWT (kJ)	Change in thermal energy storage within HWT (kJ)	Energy recovered by DWHR (kJ)	Energy savings based on Eq. 5.2
A	1006061.97	950529.14	55532.99	395097.54	28.1%
B1	1041044.97	986045.52	54999.55	407029.82	25.6%
B2	977153.97	922125.75	55028.33	425718.50	30.1%
C1	908465.98	853083.01	55383.07	481624.97	35.1%
C2	897299.98	841903.93	55396.16	503950.94	35.9%
D	1398929.96	1343972.10	54957.95	0.00	0.0%

5.5 Discussion

Several useful conclusions can be drawn based on the simulation results presented in the previous section. Firstly, without a clear specification for the plumbing configuration, the savings associated with DWHR heat exchangers should be presented as a range (e.g. 25% to 35%). This also implies that if in-situ data were to be collected, it must be accompanied by complete information about the plumbing configuration. Secondly, the rated performances determined during CSA rating procedure, which get printed on the labels for DWHR heat exchanger, are in fact misleading and potentially inapplicable to many installations. As a reminder, the rated performance is measured during equal-flow conditions (i.e. config C), and the savings associated in this case could be vastly different than those from other configurations. The configuration is not always a choice and can be influenced by the local codes and standards. For example, only configuration A is allowed in the province of Quebec [48], while the heat exchangers sold on the market are rated for conditions that never occur in the province. Therefore, the models developed in this thesis are not only useful for performing energy calculations, but are also very useful for guiding codes and standards to be aligned with realistic installations.

Chapter 6 – Conclusions

Falling-film drain water heat recovery (DWHR) heat exchangers were extensively studied in the course of the author's graduate studies, and this thesis contains detailed information with regards to how these heat exchangers can be modelled. Three distinct methods were presented in Chapter 3 for estimating the steady-state performance of DWHR heat exchanger, which accounts for the majority of energy savings associated with them. All three models had a MAPE smaller than 4%, with maximum prediction errors smaller than 10%. The most accurate steady-state model is based on resistor networks, and had a MAPE of 2.44%, whereas the most erroneous model (shortcut-model) had a MAPE of 3.57%. The maximum prediction error was also associated with the shortcut model, and the relatively larger errors were attributed to the simplifying assumptions used in model development.

The transient performance was modelled and validated in Chapter 4. An algebraic expression was derived based on heat exchanger fundamentals to express the time constant for DWHR heat exchangers, and the validation experiments showed good agreements between the predicted time constants and those found experimentally. Note that time constants for all heat exchangers tested were smaller than 20 seconds for balanced conditions.

Lastly, the models were programmed into TRNSYS to perform month-long simulations for various plumbing configurations having identical heat exchangers and water-draw profiles. The simulation results showed that plumbing configuration has a direct impact on the savings associated with DWHR heat exchangers, and accurate energy models cannot be created without having information about the plumbing system. Maximum energy savings were attributed to the configuration having balanced flow rates (i.e. Configuration C), and it was shown that alternative plumbing configurations could reduce savings by more than 10%, subjected to identical input conditions.

6.1 Future work

There are many direct and indirect applications for the material contained in this thesis. This section aims to list the ones that were deemed the most important by the author.

- 1) The hyperbolic-paraboloid method of modelling heat exchangers at a constant ΔT_{max} , can be expanded and applied to similar heat exchangers. Although not shown in the thesis, this approach was applied to experimental data for a horizontal DWHR heat exchanger and the results were promising. However, the experimentation was not continued as horizontal

DWHR heat exchangers fall outside the scope of this thesis. Perhaps, this approach can replace Wilson plot procedure, or at the very least, augment it.

- 2) Chapter 3 covered the procedure for estimating convective heat transfer coefficients as a function of the mass flow rate. Although useful, this is not aligned with the conventional approach of finding correlations in terms of Re and Pr numbers. It is possible to perform a series of experiments to gather sufficient experimental data to allow probabilistic determination of the unknown coefficients through inverse heat transfer analysis.
- 3) Preliminary experimental data showed that the transient modelling approach covered in Chapter 4 applies to horizontal DWHR heat exchangers as well.
- 4) All models developed in this thesis were validated under laboratory conditions. It is prudent to collect in-situ data to compare, validate and fine-tune the models to account for factors that exist in real installations, which cannot be accounted for in lab settings. These factors include fouling, off-vertical installation, inappropriate installation, etc.
- 5) The models can be used to update codes and standards to better reflect reality. Currently, the rating procedure generates a point that is only applicable to configuration C in Chapter 5. The rating procedure must be updated to include test points that correspond to other plumbing configurations, which allows designers and customers to select more appropriate heat exchangers for their applications.
- 6) Although not pursued in this thesis, it is possible to convert energy savings associated with DWHR heat exchangers to reductions in GHG emissions.

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Appendix A-Test Rig and instrumentation details

This section contains the description of the test apparatus used for analysing DWHR systems, which published in the author's MASc thesis [28]. Several minor changes have been made to the test apparatus since this section was originally written; however, none of the changes affect the data acquisition method or the sensors. As such, they are deemed irrelevant to the performance of DWHR systems and are not described here as a result. It is important to note this appendix has already received credit as part of the author's MASc thesis [28], and the text is only repeated here for completeness.

Figures A1 through A4 depict the schematic design of the system under different modes of operation, where the greyed out portions are the sections of the system that are not involved in a given operation. A photograph of the system is presented in Figure A5. Specific aspects of the design will be described in the sections which follow.

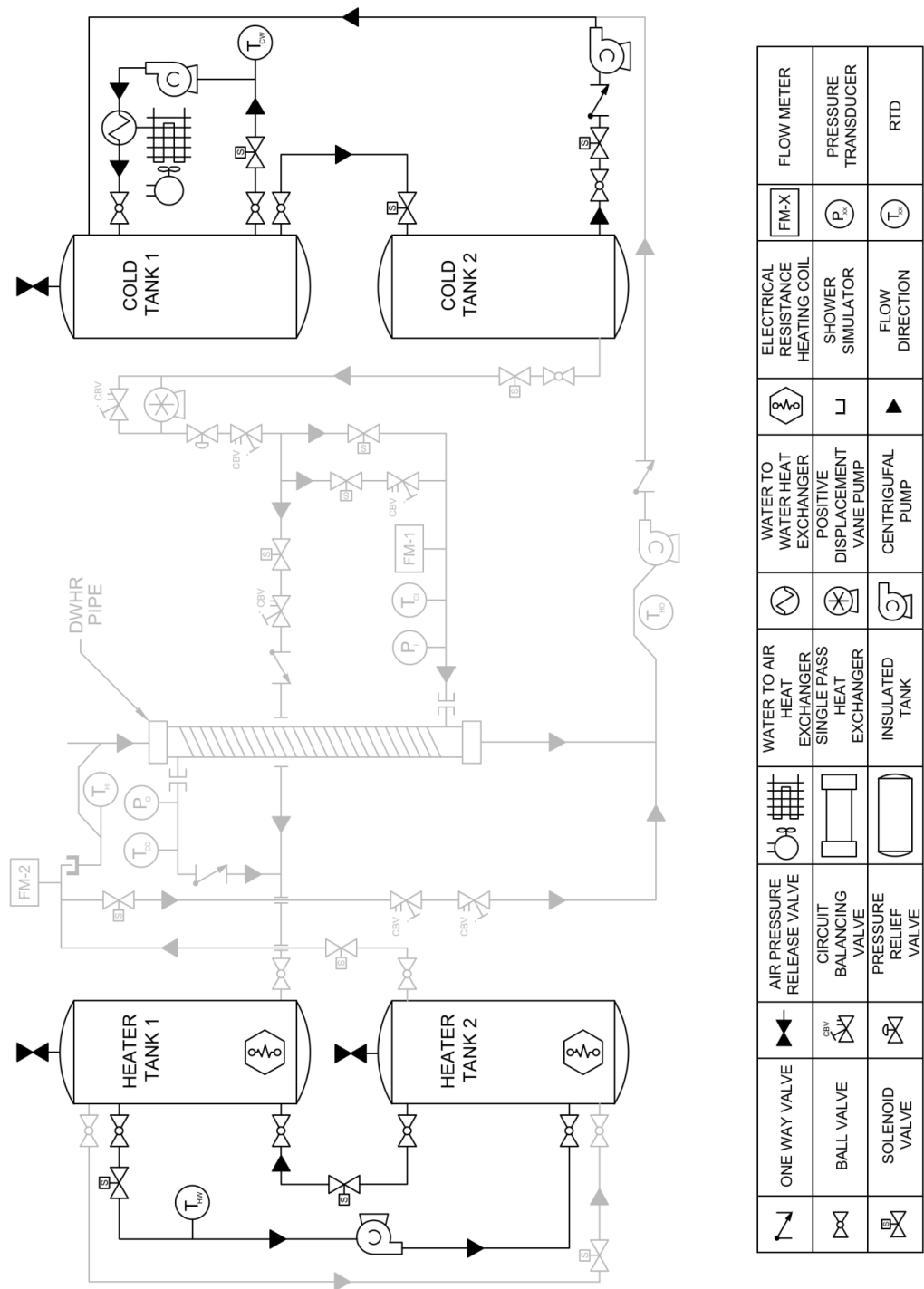


Figure A1- Main diagram of the plumbing configuration for DWHR apparatus operating under 'conditioning mode'. The greyed out portions are the sections of the system that are not included in a given operation.

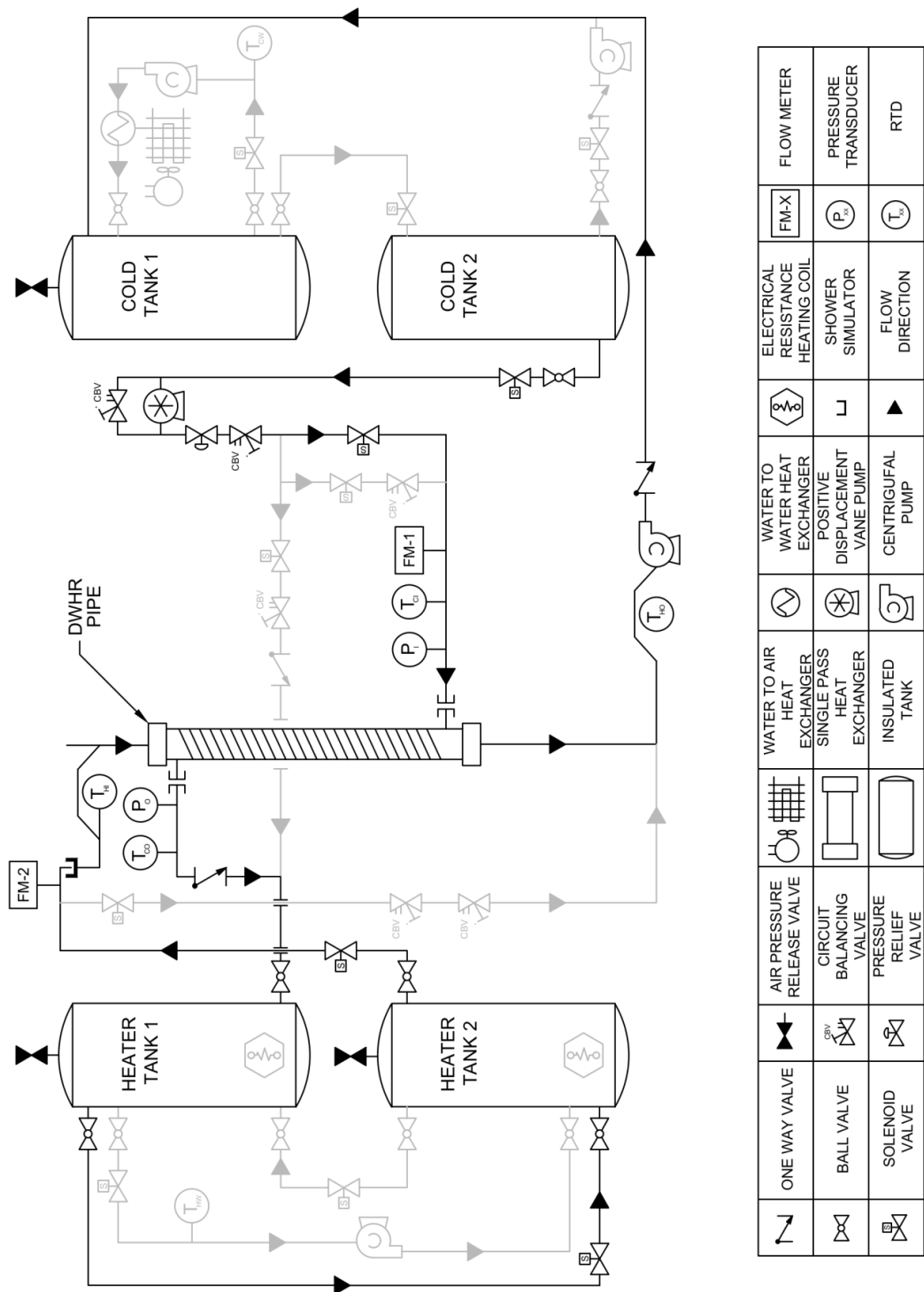


Figure A2- Main diagram of the plumbing configuration for DWHR apparatus operating under 'equal flow mode'. The greyed out portions are the sections of the system that are not included in a given operation.

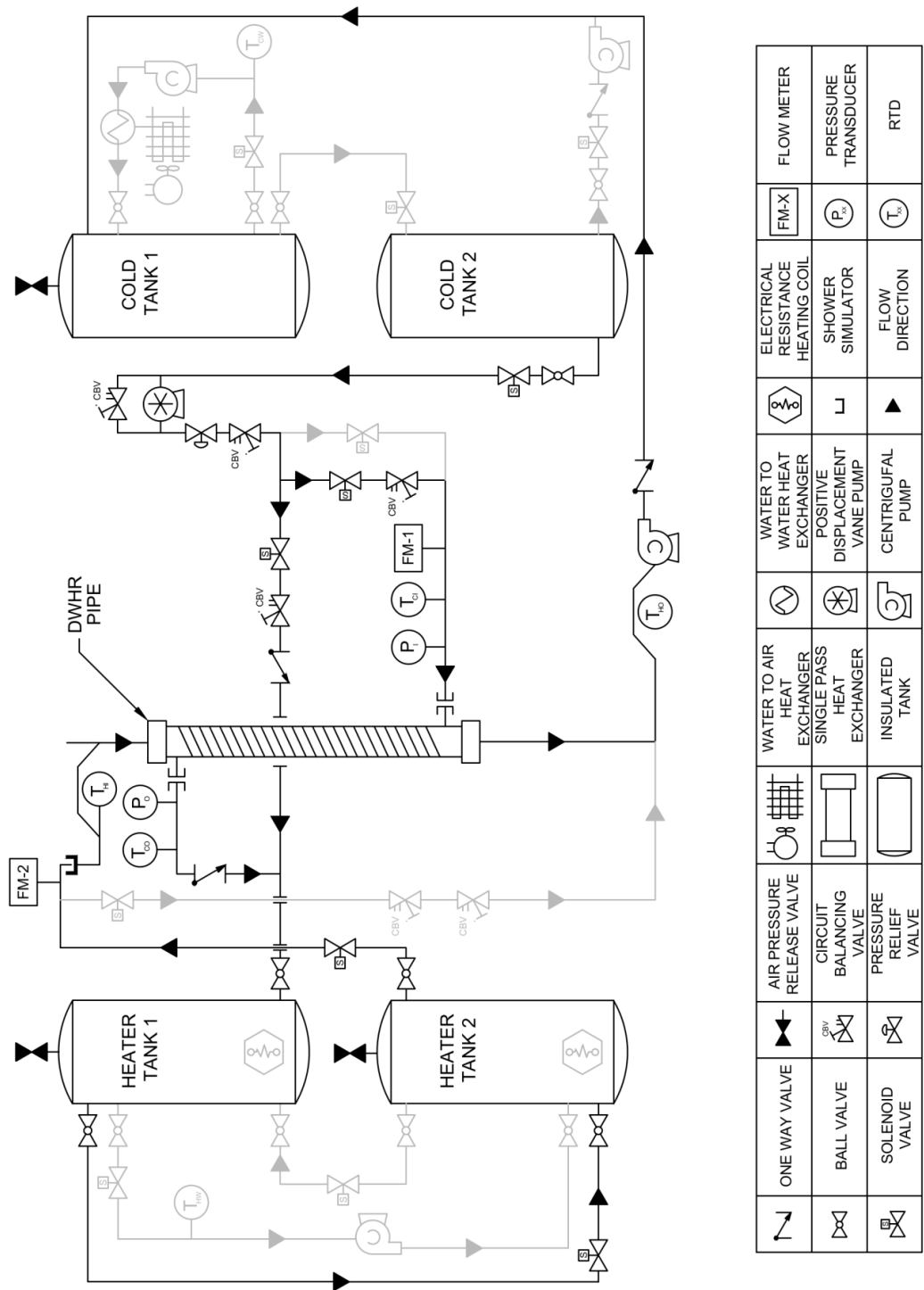


Figure A3- Main diagram of the plumbing configuration for DWHR apparatus operating under 'variable cold flow rate mode'. The greyed out portions are the sections of the system that are not included in a given operation.

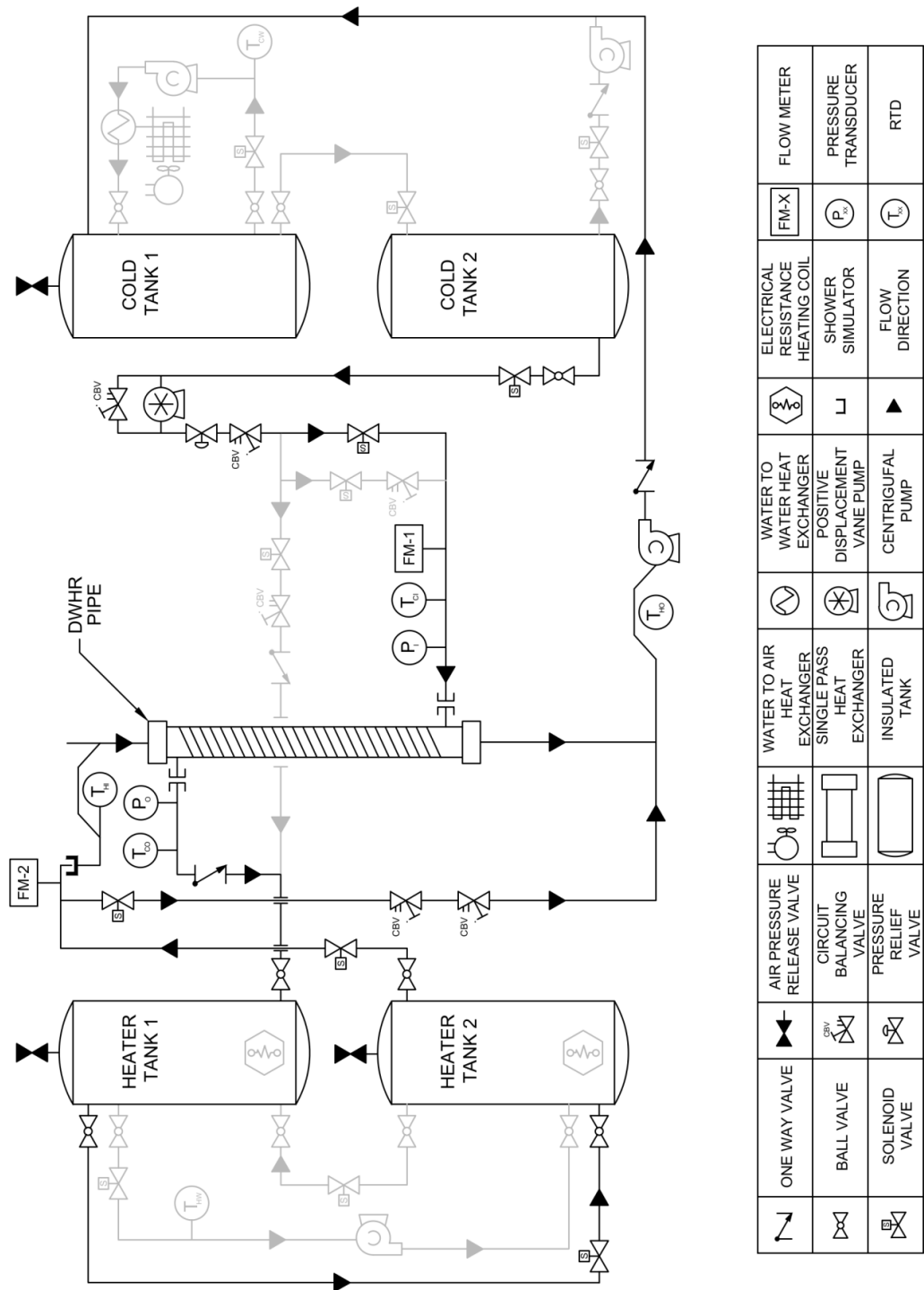


Figure A4- Main diagram of the plumbing configuration for DWHR apparatus operating under 'variable hot flow rate mode'. The greyed out portions are the sections of the system that are not included in a given operation.

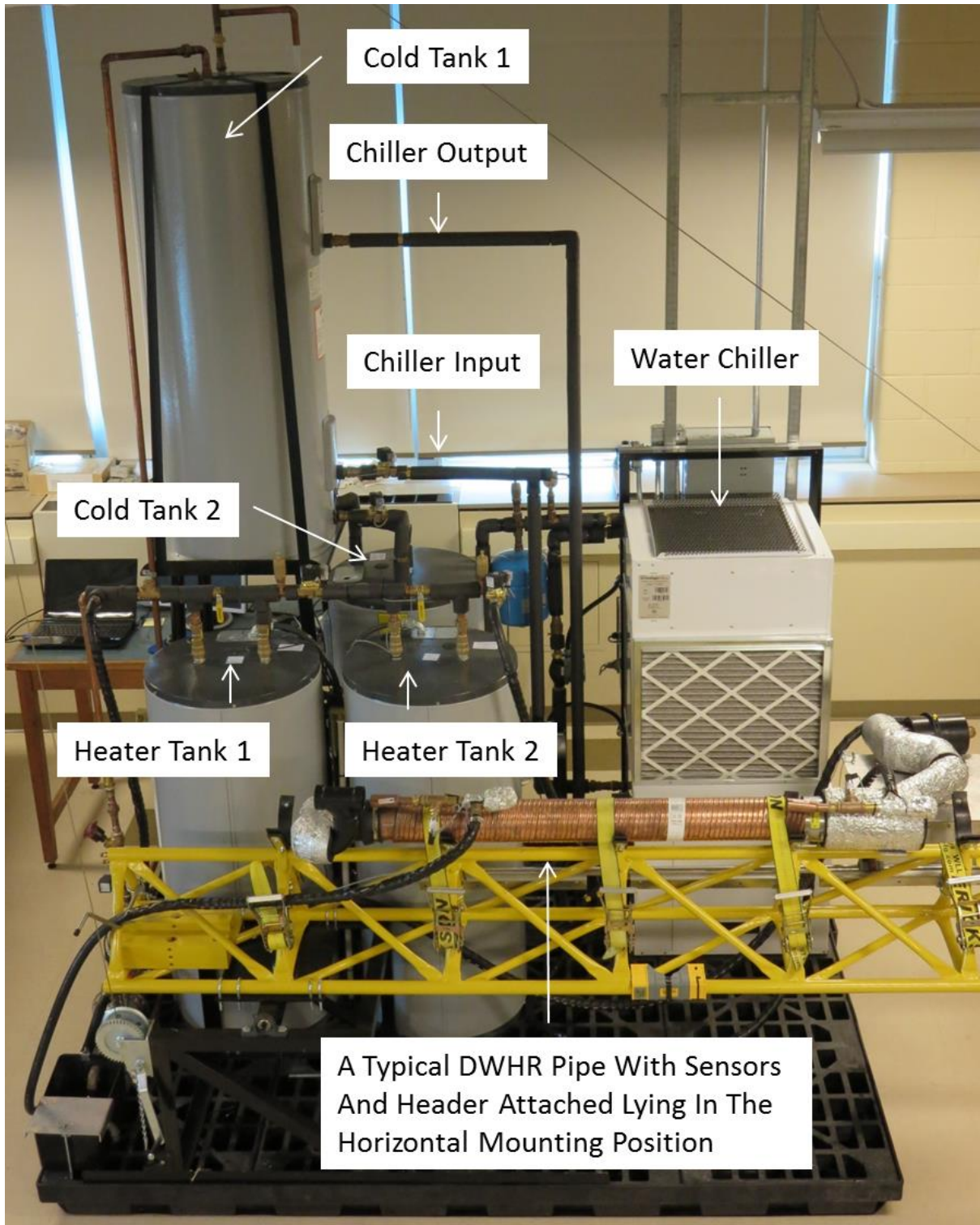


Figure A5- The DWHR apparatus with the main pivoting arm and DWHR pipe in the horizontal mounting position.

This description of components uses the schematic from Figure A1 to sequentially move through the components. To perform a test, the water entering either side of the DWHR system must be chilled and heated to predetermined temperatures. Cold water for the test is chilled by a 10.5 kW (36000 Btu/h) water-to-water heat exchanger, and then the extracted heat is expelled by a 10 kW (34000 Btu/h) water-to-air heat exchanger. The chilled water circulates through Cold Water Tank 1, a 284-liter insulated tank, gradually dropping the temperature to the pre-set cold-side temperature. The chilling system can drop the water to its freezing point. Freshly chilled water is pumped into the very top of Cold Water Tank 1, through a modified 90° angled pipe, and then extracted from the bottom of the tank (see Figure A6). This configuration utilizes the tendency of the cold water to sink to maximize the overall stirring effect. Much attention was given to achieving a stable temperature throughout the tank, due to the effect of temperature measurements on ε . Once the desired temperature is reached, the chilling system shuts off, and a volume of water sufficient in amount to run a test is released into the second 184-liter cold-water tank, giving the cold water a further mixing (see Cold Water Tank 2 in Figure A1).

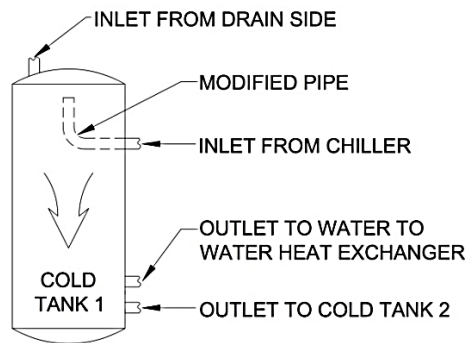


Figure A6- Diagram of the water flow in Cold Tank 1, and the modified pipe used to direct the cold water from the chiller to the very top of the tank.

On the hot side, the water is heated in two hot-water tanks by 4.5 kW electrical resistance elements at the bottom of each tank. To equalize the temperature between the two tanks, a circulator pump draws hot water from the top of one heater tank and inputs it into the bottom of the second tank. An ‘open’ line equalizes the pressure difference between the two tanks using the same principle. This mixing action is amplified by the warmer water rising to the top of the tank in conjunction with the currents of hot water rising from the heating elements (see Figure A7).

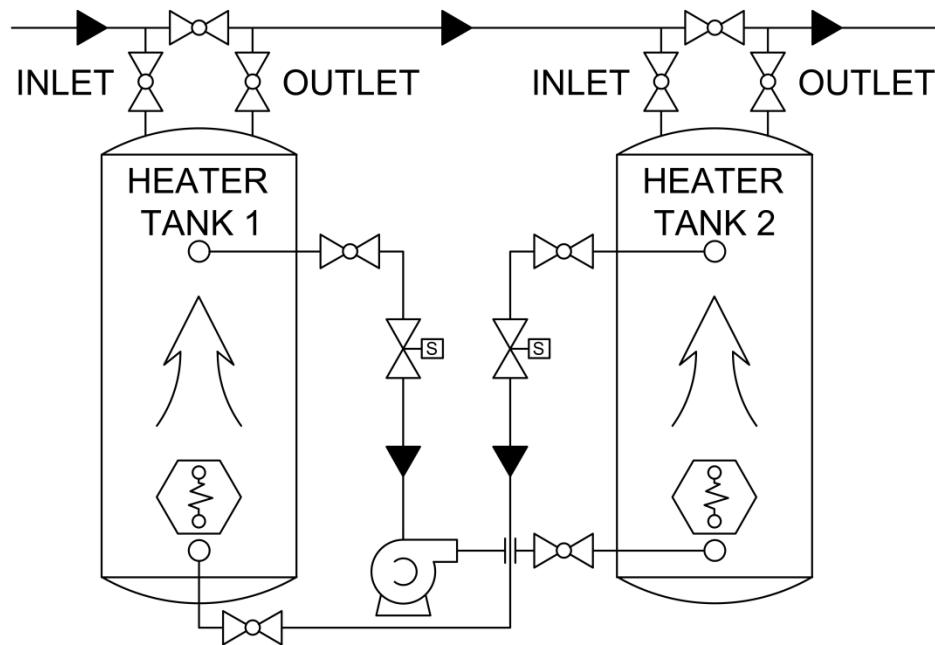


Figure A7- Diagram of the water flow in Heater Tanks 1 and 2.

To achieve equal flow, a single vane pump drives the water through the entire system. This positive displacement pump stabilizes the water flow despite the range of lengths of DWHR systems. The conditioned cold water, after being drawn by the vane pump, from Cold Tank 2, passes through the mains-side of the DWHR system. After exiting the top of the DWHR system, the water enters the bottom of Heater Tank 1, and leaves from the top. The same configuration applies to Heater Tank 2. Upon exiting Heater Tank 2, it drops down through the DWHR system's drain-side. This configuration prevents significant mixing of the conditioned hot water, and the water entering the hot water tanks from the DWHR system, and allows for a constant temperature supply of hot water to the drain-side of the DWHR system. The water is finally collected below the DWHR system and sent to Cold Tank 1 for conditioning, completing the circuit of the system.

The CSA standard requires testing at multiple flow rates; however, since the positive displacement pump cannot be regulated to change its output, a bypass was built into the system. The given bypass simply feeds some of the pump's output flow back into its input. Minor adjustment of a circuit balancing valve is needed during each test to achieve desired flow rates. Metering of the hot and cold water flows is done by two turbine flow meters with an accuracy of $\pm 1\%$ of reading [52].

For tests conducted under unequal flow conditions, the same positive displacement pump is used to achieve different flow rates on both sides without the assistance of any additional pumps. This is done using different bypasses to allow different volumes of water to flow through the drain-side and the mains-side, as was shown in the schematics in Figures A6 and A7. These flow rates are adjusted using circuit balancing valves.

The system's cold mains side temperature and pressure sensors are mounted right at the inlet connection to a given DWHR system (see Figure A8). Water temperatures are measured throughout the system by an ISO 19025 calibrated/certified RTD whose accuracy is ± 0.03 to 0.08°C [53]. A conservative worst case accuracy of $\pm 0.1^\circ\text{C}$ is used when measuring temperature. Water pressure is measured at the cold mains water input and output on a DWHR system. They are measured by ISO 19025 calibrated/certified pressure transducers which have an accuracy of 0.25% full scale [54].

Table A1-The DWHR apparatus' parameters and sensors.

Parameter	Operating Range	Accuracy
RTD Probes	0-100 °C	± 0.1 °C
Temperature of Cold-side	0-20 °C	± 0.1 °C
Temperature of Hot-side	20-50 °C	± 0.1 °C
Cold Water Flow Meter	2.8-28 L/min	$\pm 1\%$ Reading
Hot Water Flow Meter	2.8-28 L/min	$\pm 1\%$ Reading
Pressure Transducer	0-100 psig	$\pm 0.25\%$ FS

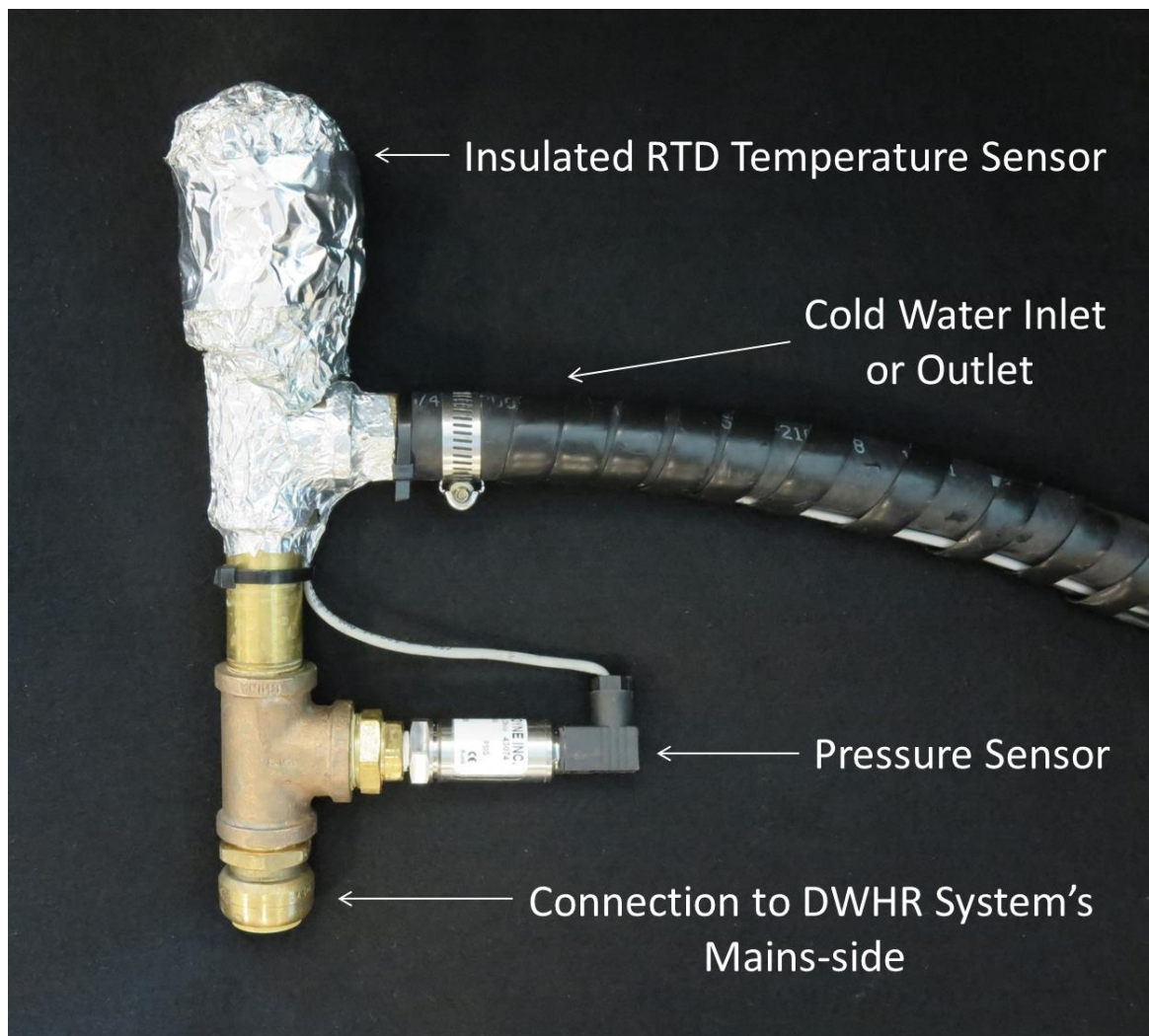


Figure A8- One of the two identical sensor bearing units mounted on a DWHR system's mains-side input and output.

The ABS pipe that connects to the DWHR system's hot water drain-side input is called the *header*. An open to atmosphere water line (simulating a drain) channels the hot water from the heater tanks into an insulated trap where the temperature is measured. The water exits the trap into a Tee connected to the DWHR system. This header was built to the specifications given by the CSA standard which stipulates "*The vertical straight distance from the bottom of the ...DWV Tee... to the DWHR shall be $750\pm50\text{mm}$.*" [55]. Three headers were built with diameters of 5.1, 7.6, and 10.2cm to allow testing of all DWHR systems available. All headers were insulated similar to what is shown in Figure A9 to minimize thermal losses.



Figure A9- The 10.2cm diameter ABS header covered in insulation to minimize thermal losses.

Hot water exiting the DWHR pipe is collected and its temperature measured in another insulated trap. Finally, the water enters a catch basin from which it is returned to the main cold-water tank.

Realizing water spills would occur during operation of the DWHR apparatus, a series of chemical catchment platforms were connected together to form a single base capable of holding 551L. The DWHR apparatus rests on top of this foundation as shown in Figure A10.



Figure A10- Containment platforms installed beneath the test apparatus to prevent water spillage onto the floors.

Operation by a single technician, even with systems of 366cm in length weighing up to 40 kg was achieved by constructing a pivoting arm on which a DWHR system could be placed while lying in the horizontal mounting position (see Figure A5). Once a DWHR system was secured in place, the arm could then be winched into the vertical testing position. Other amenities such an adjustable board on which the sensors and DWHR system header are attached were affixed to the arm. To reduce the risk of accidents, counter weights, safety locks and a safety factor of ten, for cables and bolts, were incorporated into the arm's manufacture.

Customized software controls the entire system and records the data. A variety of safeguards including automatic pressure, temperature, time and pump cut-offs were incorporated to protect the test apparatus. The user selects a test from a drop down menu and then watches the test running to determine when sufficient data on the system's performance,

after reaching steady state, has been collected. Over this period of time, the flow rates and temperatures for the cold and hot water are recorded. The inlet and outlet pressures for the cold water are also measured to determine the total pressure drop across the DWHR system.

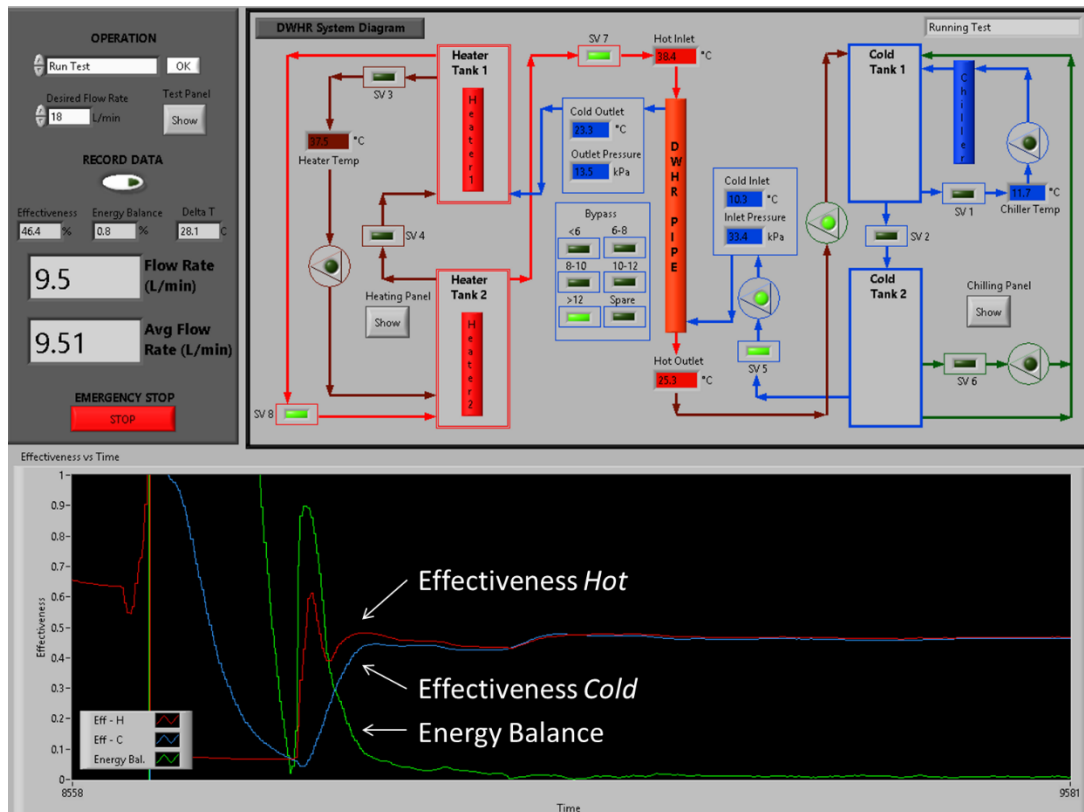


Figure A11- The LabView interface used by the operator to run a test at a chosen flow rate. This screen shot is the plot of a 9.5 L/min test on a 10.2cm diameter DWHR system.

In Figure A11, the ‘effectiveness *hot*’ represents the effectiveness, ϵ , calculated using the hot water side of the DWHR system. The ‘effectiveness *cold*’ represents the effectiveness based on the cold water side. The ‘Energy balance’ is the expression for the difference between the two effectiveness values, with zero being the ideal. The displayed effectiveness’ show an initial period of adjustment while a given test comes to steady state. The last third of the graph displays what is considered steady state. Once this steady state has been reached, the operator

can start recording the data by pressing the button called “Record Data” on the user interface. By doing so, the temperatures, pressures and flow rates for the duration of the test will be recorded in an Excel file for data processing.

Appendix B- Detailed water-draw schedule

A portion of this data was presented in Chapter 5. This table contains complete information, along with the temperatures corresponding to each fixture's draw and drain.

Table A 2: Detailed draw schedule for water usage in the simulations.

Day of the week	Start	End	Event	Flowrate (liter/s)	Fixture Temperature (°C)	Drainage Temperature (°C)
M, T, W, Th, F, Sa, Su	6:00:00 AM	6:08:00 AM	Shower	0.1325	38	35
M, T, W, Th, F, Sa, Su	6:10:00 AM	6:10:05 AM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	6:10:05 AM	6:10:25 AM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	6:10:10 AM	6:10:40 AM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	6:30:00 AM	6:40:00 AM	Misc faucet usage	0.0631	35	25
M, T, W, Th, F, Sa, Su	6:30:00 AM	6:30:05 AM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	6:30:05 AM	6:30:25 AM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	6:30:10 AM	6:30:40 AM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	6:50:00 AM	6:50:05 AM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	6:50:05 AM	6:50:25 AM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	6:50:10 AM	6:50:40 AM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	7:00:00 AM	7:08:00 AM	Shower	0.1325	38	35
M, T, W, Th, F, Sa, Su	7:10:00 AM	7:10:05 AM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	7:10:05 AM	7:10:25 AM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	7:10:10 AM	7:10:40 AM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	7:30:00 AM	7:30:05 AM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	7:30:05 AM	7:30:25 AM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	7:30:10 AM	7:30:40 AM	Handwash	0.0631	40	38

M, T, W, Th, F, Sa, Su	7:50:00 AM	7:50:05 AM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	7:50:05 AM	7:50:25 AM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	7:50:10 AM	7:50:40 AM	Handwash	0.0631	40	38
Sa	9:00:00 AM	9:02:00 AM	Washing machine draw (load1)	0.4889	50	45
Sa	9:10:00 AM	9:12:00 AM	Washing machine draw (load1)	0.4889	50	45
Sa	9:12:00 AM	9:14:00 AM	Washing machine draw (load1)	0.4889	50	45
Sa	9:22:00 AM	9:24:00 AM	Washing machine draw (load1)	0.4889	50	45
Sa	9:30:00 AM	9:32:00 AM	Washing machine draw (load2)	0.4889	30	25
Sa	9:40:00 AM	9:42:00 AM	Washing machine draw (load2)	0.4889	30	25
Sa	9:42:00 AM	9:44:00 AM	Washing machine draw (load2)	0.4889	30	25
Sa	9:52:00 AM	9:54:00 AM	Washing machine draw (load2)	0.4889	30	25
Sa	10:00:00 AM	10:02:00 AM	Washing machine draw (load3)	0.4889	30	25
Sa	10:10:00 AM	10:12:00 AM	Washing machine draw (load3)	0.4889	30	25
Sa	10:12:00 AM	10:14:00 AM	Washing machine draw (load3)	0.4889	30	25
Sa	10:22:00 AM	10:24:00 AM	Washing machine draw (load3)	0.4889	30	25
Sa	10:30:00 AM	10:32:00 AM	Washing machine draw (load4)	0.4889	30	25
Sa	10:40:00 AM	10:42:00 AM	Washing machine draw (load4)	0.4889	30	25

Sa	10:42:00 AM	10:44:00 AM	Washing machine draw (load4)	0.4889	30	25
Sa	10:52:00 AM	10:54:00 AM	Washing machine drain (load4)	0.4889	30	25
Sa, Su	11:00:00 AM	11:10:00 AM	Misc faucet usage	0.0631	30	25
Sa, Su	12:00:00 PM	12:00:05 PM	Flush drain	1.9684	T_in (ANY)	23
Sa, Su	12:00:05 PM	12:00:25 PM	Flush draw	0.4921	T_in (ANY)	23
Sa, Su	12:00:10 PM	12:00:40 PM	Handwash	0.0631	40	38
Sa, Su	12:10:00 PM	12:10:05 PM	Flush drain	1.9684	T_in (ANY)	23
Sa, Su	12:10:05 PM	12:10:25 PM	Flush draw	0.4921	T_in (ANY)	23
Sa, Su	12:10:10 PM	12:10:40 PM	Handwash	0.0631	40	38
Sa, Su	12:20:00 PM	12:20:05 PM	Flush drain	1.9684	T_in (ANY)	23
Sa, Su	12:20:05 PM	12:20:25 PM	Flush draw	0.4921	T_in (ANY)	23
Sa, Su	12:20:10 PM	12:20:40 PM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	5:00:00 PM	5:00:05 PM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	5:00:05 PM	5:00:25 PM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	5:00:10 PM	5:00:40 PM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	5:10:00 PM	5:10:05 PM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	5:10:05 PM	5:10:25 PM	Flush draw	0.4921	T_in (ANY)	23
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M, T, W, Th, F, Sa, Su	5:20:00 PM	5:20:05 PM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	5:20:05 PM	5:20:25 PM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	5:20:10 PM	5:20:40 PM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	6:00:00 PM	6:10:00 PM	Misc faucet usage	0.0631	30	25
M, W, F, Su	8:00:00 PM	8:10:25 PM	bath-draw	0.1325	38	30

W, Su	8:00:00 PM	8:02:00 PM	Dishwasher draw	0.1924	55	50
M, W, F, Su	8:25:00 PM	8:30:00 PM	bath-drain	0.2763	38	30
W, Su	8:28:00 PM	8:30:00 PM	Dishwasher drain	0.1924	55	50
W, Su	8:30:00 PM	8:32:00 PM	Dishwasher draw	0.1924	55	50
M, T, W, Th, F, Sa, Su	9:00:00 PM	9:00:05 PM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	9:00:05 PM	9:00:25 PM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	9:00:10 PM	9:00:40 PM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	9:10:00 PM	9:10:05 PM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	9:10:05 PM	9:10:25 PM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	9:10:10 PM	9:10:40 PM	Handwash	0.0631	40	38
M, T, W, Th, F, Sa, Su	9:20:00 PM	9:20:05 PM	Flush drain	1.9684	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	9:20:05 PM	9:20:25 PM	Flush draw	0.4921	T_in (ANY)	23
M, T, W, Th, F, Sa, Su	9:20:10 PM	9:20:40 PM	Handwash	0.0631	40	38
W, Su	9:58:00 PM	10:00:00 PM	Dishwasher drain	0.1924	55	50
W, Su	10:00:00 PM	10:02:00 PM	Dishwasher draw	0.1924	55	50
W, Su	11:28:00 PM	11:30:00 PM	Dishwasher drain	0.1924	55	50

Appendix C- Uncertainty Calculations

This appendix focuses on explaining how the uncertainty calculations were done for the experimental results. The test instruments used to take measurements in every test are summarized in Table A-, along with their respective accuracies. All uncertainty calculations performed in this thesis are based on Moffat's constant odds method [56]. Note that the procedure provided in this appendix has already received credit as part of the author's MASc thesis [28], and due to the identical nature of uncertainty calculations, the content are repeated here.

Table A-3 The measurement accuracies for all instruments used in this thesis.

Parameter	Accuracy
RTD Temperature Probes	$\pm 0.1\text{ }^{\circ}\text{C}$
Cold Water Flow Meter	$\pm 1\%$ Reading
Hot Water Flow Meter	$\pm 1\%$ Reading
Pressure Transducers	$\pm 862.5\text{ Pa}$

Uncertainty Associated With the Effectiveness Calculations

The effectiveness of a DWHR system operating under equal flow conditions is calculated using Eq. A.1 below:

$$\varepsilon = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{\Delta T_{cold}}{\Delta T_{max}} \quad (A.1)$$

The first step is to calculate the uncertainty for the ΔT_{cold} and ΔT_{max} :

$$\delta \Delta T_{cold} = \sqrt{(\delta T_{c,o})^2 + (\delta T_{c,i})^2} = \sqrt{(0.1^{\circ}\text{C})^2 + (0.1^{\circ}\text{C})^2} = \pm 0.14^{\circ}\text{C} \quad (A.2)$$

$$\delta \Delta T_{max} = \delta \Delta T_{cold} = \pm 0.14^{\circ}\text{C} \quad (A.3)$$

The uncertainty in the effectiveness calculation can now be expressed as:

$$\delta \varepsilon = \varepsilon \times \sqrt{\left(\frac{\delta \Delta T_{cold}}{\Delta T_{cold}}\right)^2 + \left(\frac{\delta \Delta T_{max}}{\Delta T_{max}}\right)^2} = \varepsilon \times \sqrt{\left(\frac{0.14^{\circ}\text{C}}{\Delta T_{cold}}\right)^2 + \left(\frac{0.14^{\circ}\text{C}}{\Delta T_{max}}\right)^2} \quad (A.4)$$

As is clear from Eq. A.4, the actual value of the uncertainty in effectiveness calculation varies from test to test and requires substitution of values for ε , ΔT_{cold} and ΔT_{max} .

For example, for a test data with $\varepsilon = 0.4453$, $\Delta T_{cold} = 12.51^\circ\text{C}$ and $\Delta T_{max} = 28.1^\circ\text{C}$, the uncertainty is calculated to be:

$$\delta\varepsilon = 0.4453 \times \sqrt{\left(\frac{0.14^\circ\text{C}}{12.51^\circ\text{C}}\right)^2 + \left(\frac{0.14^\circ\text{C}}{28.1^\circ\text{C}}\right)^2} = \pm 0.55\% (F.S.) \quad (A.5)$$

The effectiveness of a DWHR system operating under unequal flow conditions is calculated using Eq. A.6 below:

$$\varepsilon = \frac{\dot{V}_{cold}(T_{c,o} - T_{c,i})}{\dot{V}_{min}(T_{h,i} - T_{c,i})} = \frac{\dot{V}_{cold}\Delta T_{cold}}{\dot{V}_{min}\Delta T_{max}} \quad (A.6)$$

The first step is to calculate the uncertainty for the numerator and the denominator as follows:

$$\frac{\delta(\dot{V}_{cold}\Delta T_{cold})}{\dot{V}_{cold}\Delta T_{cold}} = \sqrt{\left(\frac{\delta\dot{V}_{cold}}{\dot{V}_{cold}}\right)^2 + \left(\frac{\delta\Delta T_{cold}}{\Delta T_{cold}}\right)^2} = \sqrt{(1\%)^2 + \left(\frac{0.14^\circ\text{C}}{\Delta T_{cold}}\right)^2} \quad (A.7)$$

$$\frac{\delta(\dot{V}_{min}\Delta T_{max})}{\dot{V}_{min}\Delta T_{max}} = \sqrt{\left(\frac{\delta\dot{V}_{min}}{\dot{V}_{min}}\right)^2 + \left(\frac{\delta\Delta T_{max}}{\Delta T_{max}}\right)^2} = \sqrt{(1\%)^2 + \left(\frac{0.14^\circ\text{C}}{\Delta T_{max}}\right)^2} \quad (A.8)$$

Now the uncertainty in the effectiveness calculation can now be expressed as:

$$\delta\varepsilon = \varepsilon \times \sqrt{(1\%)^2 + (1\%)^2 + \left(\frac{0.14^\circ\text{C}}{\Delta T_{cold}}\right)^2 + \left(\frac{0.14^\circ\text{C}}{\Delta T_{max}}\right)^2} \quad (A.9)$$

Similar to the case for the equal flow conditions, calculating the actual uncertainty in the effectiveness value requires substitution of values for ε , ΔT_{cold} and ΔT_{max} .

For example, for a test data with $\varepsilon = 0.5598$, $\Delta T_{cold} = 11.1^\circ\text{C}$, $\Delta T_{max} = 28.2^\circ\text{C}$, $\dot{V}_{cold} = 10 \text{ LPM}$ and $\dot{V}_{min} = 7 \text{ LPM}$, the uncertainty is calculated to be:

$$\delta\varepsilon = 0.5598 \times \sqrt{(1\%)^2 + (1\%)^2 + \left(\frac{0.14^\circ\text{C}}{11.1}\right)^2 + \left(\frac{0.14^\circ\text{C}}{28.2}\right)^2} = \pm 1.1\% (F.S.) \quad (A.10)$$

Uncertainty Associated With the Heat Transfer Rate Calculations

The uncertainty calculation procedure for the heat transfer rate is the same for both equal and unequal flow conditions. Eq. A.11 is used to calculate the heat transfer rate based on the cold side of the heat exchanger:

$$q = \frac{4180 \times \dot{V}_{cold} \times \Delta T_{cold}}{60000} \quad (A.11)$$

The uncertainty for the heat transfer calculations can be expressed as:

$$\delta q = q \times \sqrt{\left(\frac{\delta \dot{V}_{cold}}{\dot{V}_{cold}}\right)^2 + \left(\frac{\delta \Delta T_{cold}}{\Delta T_{cold}}\right)^2} = q \times \sqrt{(1\%)^2 + \left(\frac{0.14^\circ\text{C}}{\Delta T_{cold}}\right)^2} \quad (A.12)$$

Again, the actual value of this uncertainty depends on the measured values for q and ΔT_{cold} .

For example, for a test data with for $q = 6.99 \text{ kW}$ and $\Delta T_{cold} = 14.34^\circ\text{C}$, the uncertainty is calculated to be:

$$\delta q = 6.99 \text{ kW} \times \sqrt{(1\%)^2 + \left(\frac{0.14^\circ\text{C}}{14.34^\circ\text{C}}\right)^2} = \pm 0.097 \text{ kW} \quad (A.13)$$

Appendix D- Validation Data

This appendix contains the experimental data used in Chapter 3 for validation of the three steady-state models.

Test #	System #	$T_{c,in}$ °C	$T_{h,in}$ °C	$\dot{V}_c$ L/min	$\dot{V}_c$ L/min	ϵ	$\dot{q}$ kW
1	1	4.5	47.4	9.94	16.94	34.2%	10.14
2	1	4.5	47.4	4.96	11.94	50.6%	7.50
3	1	4.6	47.3	3.97	7.97	53.0%	6.25
4	1	14.2	38.2	9.10	20.01	37.2%	5.65
5	1	14.2	38.2	6.05	15.03	46.2%	4.66
6	1	14.3	38.0	4.46	8.48	49.7%	3.65
7	1	19.6	31.9	5.44	14.05	48.4%	2.26
8	1	19.6	31.8	7.02	10.94	39.2%	2.35
9	1	19.6	31.6	8.12	8.91	33.6%	2.28
10	1	4.1	45.0	15.03	7.01	41.9%	8.36
11	1	4.4	44.9	5.92	5.10	42.1%	6.04
12	1	18.7	35.2	14.98	10.02	33.3%	3.83
13	1	14.8	31.3	13.92	5.59	46.9%	3.02
14	1	14.8	31.4	10.99	5.93	43.5%	2.98
15	1	14.8	31.3	9.94	8.06	34.2%	3.18
16	2	5.0	45.1	5.03	8.97	63.7%	8.97
17	2	4.9	45.1	4.52	10.97	70.4%	8.91
18	2	4.8	45.1	8.08	15.89	55.1%	12.52
19	2	15.3	37.6	9.98	20.05	51.2%	7.93
20	2	15.4	37.5	5.98	17.08	65.5%	6.02
21	2	15.6	37.2	4.01	6.03	65.7%	3.96
22	2	18.3	33.3	5.56	8.98	59.8%	3.48
23	2	18.3	33.3	8.05	13.95	53.9%	4.56
24	2	18.3	33.3	10.01	10.94	44.3%	4.62
25	2	5.1	45.3	17.04	6.91	60.1%	11.66
26	2	5.0	45.6	16.07	11.04	45.7%	14.28
27	2	5.1	45.3	10.47	4.51	68.1%	8.59
28	2	16.2	37.4	15.52	9.54	49.8%	7.01
29	2	16.2	37.3	19.10	8.08	57.3%	6.79
30	2	16.4	37.0	7.44	5.01	60.9%	4.40
31	2	19.4	34.2	14.04	5.56	66.2%	3.80

32	2	19.4	34.2	12.98	7.96	53.4%	4.39
33	2	19.4	34.2	10.47	6.49	57.8%	3.85
34	3	5.5	44.7	4.59	6.94	60.0%	7.52
35	3	5.0	44.9	5.50	15.01	63.3%	9.68
36	3	4.9	44.5	7.88	16.85	52.4%	11.40
37	3	15.0	34.5	12.48	19.53	39.5%	6.70
38	3	15.1	34.1	5.12	15.94	64.6%	4.39
39	3	15.1	33.5	13.09	15.09	35.6%	5.99
40	3	17.2	29.3	5.81	8.74	52.6%	2.57
41	3	17.2	29.2	10.05	10.97	38.8%	3.26
42	3	17.2	29.1	7.05	13.50	52.0%	3.02
43	3	4.6	41.3	16.96	7.06	53.6%	9.67
44	3	4.6	41.2	14.97	7.97	48.4%	9.83
45	3	4.9	40.7	7.95	5.26	56.7%	7.45
46	3	12.3	32.6	17.57	9.59	44.5%	6.06
47	3	12.3	32.3	14.54	11.00	39.2%	6.03
48	3	12.4	31.9	8.97	5.22	57.0%	4.04
49	3	16.2	28.7	14.05	5.64	59.5%	2.91
50	3	16.4	28.4	7.96	6.04	50.7%	2.55
51	4	5.4	45.0	8.08	16.06	47.9%	10.69
52	4	5.4	45.0	4.93	15.05	62.1%	8.46
53	4	5.5	44.7	4.71	9.47	59.2%	7.62
54	4	12.7	35.6	12.03	20.01	38.3%	7.33
55	4	12.8	34.9	7.01	18.01	52.6%	5.68
56	4	13.0	33.9	4.99	5.96	51.3%	3.73
57	4	12.4	29.5	9.12	12.98	41.2%	4.48
58	4	12.4	29.5	10.03	10.96	37.1%	4.43
59	4	12.5	29.3	5.72	7.23	48.6%	3.26
60	4	4.5	39.1	18.43	8.43	47.2%	9.61
61	4	4.5	39.0	15.78	9.82	41.4%	9.79
62	4	4.9	38.6	6.98	5.09	53.0%	6.35
63	4	15.9	33.1	15.79	11.51	37.4%	5.16
64	4	16.0	32.9	10.53	7.49	45.8%	4.04
65	4	19.2	30.4	14.04	5.60	58.4%	2.56
66	4	19.2	30.5	12.47	8.54	43.4%	2.91
67	4	19.2	30.4	9.48	6.77	48.1%	2.53
68	5	14.2	35.8	13.95	20.01	51.6%	10.85
69	5	6.5	31.6	6.97	18.01	72.7%	8.88

70	5	6.6	31.2	4.50	13.99	83.3%	6.41
71	5	9.6	26.1	4.98	18.02	82.8%	4.75
72	5	9.5	25.5	10.06	15.98	57.2%	6.39
73	5	12.3	30.0	5.49	9.95	73.0%	4.95
74	5	12.2	29.8	6.39	13.58	71.9%	5.62
75	5	12.3	29.6	8.02	11.95	62.3%	6.03
76	5	5.2	41.4	9.97	11.00	53.6%	13.50
77	5	5.0	41.6	7.42	12.98	66.7%	12.61
78	5	5.0	41.5	12.15	13.90	50.3%	15.56
79	5	13.9	37.1	4.08	8.94	77.7%	5.13
80	5	9.4	30.1	20.01	12.90	52.0%	9.67
81	5	14.0	26.4	17.04	7.98	66.4%	4.59
82	5	6.8	35.0	18.95	5.04	81.8%	8.08
83	5	6.9	34.9	15.95	12.07	51.7%	12.18
84	5	9.6	36.2	10.08	5.46	73.7%	7.44
85	5	9.6	36.2	12.04	6.01	73.5%	8.21
86	5	9.5	35.8	13.54	6.54	72.3%	8.67
87	5	13.6	34.6	14.08	11.09	53.9%	8.75
88	5	13.6	34.6	12.98	8.01	63.3%	7.42
89	5	13.6	34.4	9.46	7.05	63.6%	6.49
90	6	5.4	45.2	4.51	8.98	90.5%	11.33
91	6	5.2	45.2	5.01	14.47	92.8%	12.93
92	6	5.1	44.7	10.07	15.96	74.0%	20.58
93	6	12.4	37.5	13.00	20.12	68.5%	15.59
94	6	12.4	37.1	6.96	16.99	86.5%	10.36
95	6	12.5	36.1	5.03	7.97	84.6%	6.98
96	6	16.1	31.8	9.03	14.02	74.2%	7.29
97	6	16.2	31.5	9.98	11.97	67.7%	7.22
98	6	16.3	31.2	5.73	7.45	78.2%	4.66
99	6	14.4	35.1	18.04	7.99	81.5%	9.36
100	6	14.4	35.1	16.05	8.98	76.4%	9.88
101	6	14.6	34.9	6.93	5.06	81.8%	5.86
102	6	17.4	31.2	19.01	6.08	91.1%	5.30
103	6	17.5	31.3	15.10	11.06	68.5%	7.29
104	6	17.5	31.1	9.94	7.43	74.3%	5.24
105	6	13.8	29.2	14.03	5.43	90.0%	5.28
106	6	13.8	29.1	12.03	8.00	75.3%	6.45
107	6	13.8	28.8	9.53	6.50	79.1%	5.36

108	7	5.0	45.0	9.99	16.00	58.1%	16.17
109	7	5.0	45.0	6.46	16.97	74.8%	13.45
110	7	5.2	44.7	4.99	9.93	77.4%	10.62
111	7	14.2	36.7	8.49	19.04	66.0%	8.79
112	7	14.2	36.5	11.95	18.01	53.6%	9.94
113	7	14.4	35.9	4.97	6.96	69.0%	5.15
114	7	19.5	35.4	5.47	14.01	77.2%	4.67
115	7	19.5	35.2	8.97	11.92	58.0%	5.69
116	7	19.6	35.1	5.92	8.95	69.0%	4.41
117	7	5.0	49.8	19.97	8.88	64.4%	17.82
118	7	5.0	49.9	14.97	14.07	45.7%	20.09
119	7	17.7	40.7	17.07	6.99	68.6%	7.69
120	7	17.8	40.6	14.94	14.01	46.0%	10.27
121	7	21.5	36.3	9.97	7.05	63.0%	4.60
122	7	21.5	36.3	8.99	8.03	57.5%	4.76
123	8	5.2	44.9	12.08	15.96	58.3%	19.48
124	8	5.3	44.8	6.06	14.01	81.2%	13.55
125	8	5.4	44.6	5.04	9.94	82.6%	11.35
126	8	13.2	38.2	10.04	20.10	68.1%	11.89
127	8	13.2	38.1	6.99	16.00	77.7%	9.41
128	8	18.5	33.1	6.51	8.44	68.2%	4.53
129	8	18.4	33.2	5.51	14.10	81.6%	4.61
130	8	18.5	33.1	9.04	10.95	60.1%	5.52
131	8	4.7	44.7	18.98	6.96	74.6%	14.44
132	8	4.8	44.7	15.98	10.12	61.7%	17.37
133	8	13.0	41.4	14.91	9.97	61.9%	12.20
134	8	13.2	41.1	7.96	6.97	63.3%	8.60
135	8	9.5	39.8	11.08	7.97	65.0%	10.97