Reverse Logistics Network Design for Additive Remanufacturing

by

Sheila Afros

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Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Statement of Contributions

I, Sheila Afros, am the sole author of Chapters 1 and 2, which were written under the supervision of Dr. Alumur Alev and are not intended for publication.

I, Sheila Afros, am the sole author for Chapter 3 (3.6), and 6 which was initially developed as a course project for MSCI 724 "Game Theory" taught by Dr. Dimitrov.

Chapters 3 (3.1-3.5), 4, 5, and 7 of this thesis have been incorporated into a manuscript that will be submitted for publication. The paper is co-authored by Dr. Alumur Alev and Atahan Bayir (PhD candidate).

I gathered and analyzed all the data. I completed the literature review. I developed all the mathematical models presented in the thesis. I developed the model code in Python for all four models (deterministic, stochastic, multi-period, and game theory). I completed the analysis of all results and insights based on the results of the code. The stochastic models were run by Atahan Bayir. I produced all the visualizations and maps in the thesis. I analyzed and provided a comparison of the results between the models for the sensitivity analysis. I completed the conclusions and managerial insights.

Abstract

The current push for businesses to adopt sustainable supply chain practices contributes to a circular economy, a systems-focused approach designed to allow resources to be used and reused for as long as possible. The Canadian Net-Zero Emissions Accountability Act and the 2030 Emissions Reduction Plan developed by the Canadian federal government aim to regulate the environmental footprint by reducing greenhouse gas (GHG) emissions. Some of these goals can be achieved by designing reverse logistics networks. Reverse logistics is the design of a network for the purpose of collecting end-of-life/end-of-use products and reusing, repairing, refurbishing, remanufacturing, and/or recycling.

This thesis proposes four models to design a reverse logistics network: deterministic, multi-period, stochastic, and a game theory model. The models are formulated as bi-objective mixed-integer linear programs. The design of the network is optimized with respect to economic and environmental objectives. The balancing of costs and environmental objectives during the design of the reverse logistics network provides the decision-maker with additional information on how environmental goals can be met. The models aim to determine the optimal locations for remanufacturing facilities and the optimal flow of parts to and from these facilities. The bi-objective models are solved using the weighted sum method, which allows for Pareto-optimal solutions to be analyzed. The models are solved using Gurobi in Python.

Deterministic, stochastic, and game theory models are applied to a case study for the remanufacturing of front lower control arms in the automotive industry in Ontario, Canada. The stochastic model is motivated by the uncertainty in the supply of end-of-life vehicles. The stochastic model is solved using the deterministic equivalent. The game theory model complements the aforementioned approach. It facilitates the triangulation of the results.

The models select the optimal locations for the remanufacturing facilities. Significantly, all three models applied to the case study produce similar results. The quantitative results demonstrate that the optimal solution based on the case study data is when multiple facilities are located, one facility should be opened in Ottawa and at least one other in the GTA. When only one facility is located, it should be placed in the GTA (either Mississauga or Brampton). Overall, the results reveal that small investments can lead to significant reductions in greenhouse emissions released during transportation.

The results can be scaled to design a reverse logistics network for Canada and inform

environmental policies. The results and findings of this study may be used to inform policies on the reduction of transportation emissions. The contributions that the thesis makes to the field are: (i) it incorporates greenhouse gas emissions into the models; (ii) it allows decision-makers to compare the results of the three models (deterministic, stochastic, and game theory) applied to the case study; and (iii) it applies the models to real-world data.

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Chapter 1

Introduction

The growing emphasis on sustainable supply chain practices encourages industry and policymakers to adopt the principles of the circular economy, a systems-oriented approach that aims to extend the lifecycle of resources through continuous use and regeneration (Geissdoerfer, Savaget, Bocken, & Hultink, 2017). A key component of this approach is the optimization of reverse logistics operations, which involves the collection of end-of-life or end-of-use products for reuse, repair, refurbishment, remanufacturing, or recycling purposes (Alumur & Bektaş, 2019). Another critical aspect of sustainability is minimizing the environmental footprint. The Canadian Net-Zero Emissions Accountability Act and the 2030 Emissions Reduction Plan developed by the Canadian federal government address these issues by regulating and reducing greenhouse gas (GHG) emissions (Government of Canada, 2022). Both of these goals — developing a circular economy and reducing GHG emissions — may be achieved through the design of effective reverse logistics networks.

This thesis examines remanufacturing in the automotive industry, where the benefits of remanufacturing have already been demonstrated for the environment and the economy. For example, in 2019 the estimated savings of emissions from remanufacturing in this industry were 269 kt CO_2e/year, which is approximately how much 58,478 typical passenger vehicles emit in a year (United States Environmental Protection Agency, 2025). The sales income generated in Canada from remanufacturing in the automotive industry is estimated to be approximately 1.8 billion dollars (Environment of Climate Change Canada, 2021).

The thesis focuses on how to design a reverse logistics network for end-of-life vehicles (ELVs). ELVs can be disassembled into components that may be recycled or remanufac-

tured. One of these components is the front lower control arm. The front lower control arm is a component of the suspension system that connects the front wheels to the chassis. It also houses the suspension spring that mitigates bouncing by keeping the tire in contact with the road. Front lower control arms are commonly made of aluminum, in cast or forged form. The used part can be remanufactured through additive remanufacturing and then sold to car manufacturers (reintroduced to the market). The additive remanufacturing process used is solid-state powder deposition technology, known as cold spray. Cold spray is a low-temperature, layer-by-layer additive manufacturing paradigm that uses micrometer-sized powder particles as feedstock. These particles are accelerated by a high-velocity, environmentally friendly carrier gas, typically compressed air or nitrogen and adhere to a substrate to form a dense, solid-state deposit, or rebuild the object (Jahed, 2023; Pathak & Saha, 2017). Additive remanufacturing facilities should be located to minimize the fixed cost, transportation cost, and GHG emissions released from transportation.

This thesis examines the benefits of remanufacturing to inform policies that support the development of a circular economy for the automotive industry in Canada. To achieve this goal, it designs a reverse logistics network for the remanufacturing of front lower control arms by determining the locations of remanufacturing facilities and the flow of parts. The design of the network is optimized with respect to economic and environmental objectives. The thesis proposes four bi-objective mixed-integer linear programs (deterministic, multi-period, stochastic, and a game theory model). These models are applied to real-world data. In addition, a sensitivity analysis is conducted to understand the effects of the facility cost, transportation cost, and supply of ELVs.

This thesis addresses the GHG emissions released from the transportation of parts from collection centers to remanufacturing facilities and from remanufacturing facilities to car manufacturers. The transportation emissions used are based on the MOtor Vehicle Emission Simulator (MOVES) (Wang, Saari, Bachmann, & Mukherjee, 2020). The models presented in the thesis are applied to a real-world case study. The case study provides a realistic design for a reverse logistics network in Ontario that can reduce emissions without significant costs. The thesis provides a comparison between deterministic, stochastic, and game theory models. A comparison between the deterministic and stochastic models improves the decision making process and identifies the differences that an uncertain supply of ELVs may lead to. The game theory model demonstrates how a different lens

provides a further understanding of the results and insights.

This thesis designs four mixed-integer linear programming models to determine the correlation between factors and results, identify common trends, and triangulate the results. Multiple models allow identifying which factors correspond to which changes in the solutions. In addition, multiple models allow determining common trends regardless of the theoretical lens. Furthermore, multiple models allow triangulation of the results, which boosts the confidence of decision-makers.

This thesis is structured as follows. Chapter 2 provides a literature review. Chapter 3 establishes the problem setting and presents mathematical formulations of the deterministic, stochastic, and game theory models. Chapter 3 also discusses the bi-objective nature of the models. Chapter 4 presents the data for the case study. Chapter 5 contains the results and insights for the stochastic and deterministic models as well as the sensitivity analysis for the stochastic and deterministic models. Chapter 6 provides the game theory model results. Chapter 6 contains the sensitivity analysis, conclusions, and next steps for the game theory model. Conclusions and managerial insights are offered in Chapter 7.

Chapter 2

Literature Review

The literature review is comprised of six subsections: facility location in reverse logistics networks, multi-objective models, multi-period models, stochastic models, game theory, and contributions compared to the literature. Section 2.1 provides a context of facility locations in reverse logistics. Section 2.2 reviews the common methodologies used to solve multi-objective models. Section 2.3 presents the publications on the design and purpose of multi-period models. Section 2.4 offers an overview of the literature discussing the context and methodology used to solve stochastic models. Section 2.5 offers an overview of facility location problems that use a game theory approach.

2.1 Facility Location in Reverse Logistics Networks

The facility location problem is a well-studied topic in the field of operations research (see literature reviews by Hachimi, Oubrich, and Souissi, 2018; Valenzuela, Alfaro, Fuertes, Vargas, and Sáez-Navarrete, 2021 for an overview of previous publications). The problem is generally developed as a mixed-integer linear programming model. The purpose of the facility location problem is to select a subset of facilities to open. The facility location problem may be applied in many industries, such as end-of-life vehicles (ELVs) (Karagoz, Aydin, & Simic, 2022; Rodriguez, Perez, & Boudaoud, 2023; Rosenberg, Huster, Rudi, & Schultmann, 2025; Xiao, Sun, Shu, & Wang, 2019), industrial waste (Trochu, Chaabane, & Ouhimmou, 2018, 2019), garments (Roudbari, Ghomi, & Eicker, 2024), healthcare waste management (Wang, Huang, & He, 2021), and non-hazardous recycling (Alarcon-Gerbier & Buscher, 2024; Alumur, Nickel, Saldanha-da Gama, & Verter, 2012; Liao, 2018;

Rossit & Bard, 2025; Santoso, Ahmed, Goetschalckx, & Shapiro, 2005; Tari & Alumur, 2014; Tosarkani & Amin, 2019; Vidovic, Ratkovic, Bjelic, & Popovic, 2016) (see Table 1).

Reverse logistics is a backward chain by which used products are re-introduced back into the system. The products are collected from consumers and sent to sorting/collection facilities where the products can be bought for remanufacturing and/or recycling and/or sent to a landfill. After the products are remanufactured, they can be sold to manufacturers and re-introduced into the market. The location of the remanufacturing/recycling facilities should be optimized for economic (profit) and/or green policy (e.g., carbon emissions), and/or equality (Alumur & Bektaş, 2019). These objective functions are subject to constraints such as budget, number of facilities, warehouse capacity, among others. Trochu et al. (2018) and Trochu et al. (2019) focus on a wood recycling reverse logistics network for recycled wood materials from the construction, renovation, and demolition industry. Alumur et al. (2012) and Yu and Solvang (2018) design reverse logistics models for household items. Rosenberg et al. (2025) design a reverse logistics network for end-of-life (EOL) electric vehicle battery systems. Rodriguez et al. (2023) design a reverse logistics network for EOL tires. Santoso et al. (2005) design a general reverse logistics model.

The model proposed in this thesis addresses remanufacturing of end-of-life vehicles (ELVs); that is, it uses the same setting as Karagoz et al. (2022) and Xiao et al. (2019) and provides optimal locations for remanufacturing facilities. Xiao et al. (2019) consider a four-echelon network design of ELVs in China; the model in the thesis, on the other hand, is a three-echelon system for ELVs in Ontario, Canada. The present model and the model in Xiao et al. (2019) consider the salvageable material based on percentages. Xiao et al. (2019) determine the locations of dismantlers/shredders, while this thesis locates remanufacturing facilities. Xiao et al. (2019) consider the costs of carbon emissions using a carbon tax, while this thesis uses the transportation emissions, which results in a multi-objective model. Xiao et al. (2019) only discuss a sensitivity analysis for supply increases, while this thesis considers facility cost, transportation cost, and supply increases/decreases. The model proposed by Karagoz et al. (2022) uses forecasting to predict the supply of ELVs; this thesis uses the number of registered vehicles in Ontario in 2022. Karagoz et al. (2022) use seven scenarios to account for the uncertainty in location and quantity of ELVs; this thesis uses ten scenarios to account for the quantity of ELVs.

2.2 Multi-Objective Models

The goal of facility locations in reverse logistics networks is to minimize/maximize an objective function(s). To consider multiple objective functions, multi-objective models can be used. For example, optimizing for one objective function may prevent at least one other objective function from reaching it's optimal value (for example, reducing carbon emissions should cause costs to increase and vice versa). Alumur and Bektaş (2019) suggest considering green policies as an additional objective to profit because remanufacturing facility operation, transportation, and production have a negative impact on the environment. The impacts are referred to as externalities and include the depletion of natural resources, noise, and pollution. This thesis considers GHG emissions from transportation as a secondary objective function.

Several methods are used to solve multi-objective models: the weighted sum method (which is a scalarization method where the sum of the objective function is weighted), the lexicographic method (in which subsequent optimization problems are solved without degrading previous optimal solution(s)), and the epsilon-constraint method (which converts one of the objectives into a constraint). The weighted sum method requires the modeler to understand and quantify the significance of each objective function. Rodriguez et al. (2023), Wang et al. (2021), and Yu and Solvang (2018) use the weighted sum method. The lexicographic method allows the decision maker to arrange the objectives in order of importance. Then each subsequent optimization is solved without degrading the previous optimal solution. The solution is very sensitive to the order of solving the objective functions. Tari and Alumur (2014) and Roudbari et al. (2024) use the lexicographic method. The constraint method allows for simultaneous balancing of the objectives where a maximum is set, and the model is optimized under the new constraint (Rodriguez et al., 2023; Rossit & Bard, 2025; Roudbari et al., 2024; Yu & Solvang, 2018).

Tari and Alumur (2014) propose three objective functions: the total cost of the reverse logistics network, an equity objective where the flow of products between planning horizons should be steady, and an objective where all collection centers receive equal numbers of reused parts over the time periods. This article uses the lexicographic method to solve the multi-objective optimization problem. This thesis uses the weighted sum method.

Yu and Solvang (2018) present a general bi-objective mixed-integer linear programming model to locate production facilities and distribution centers. The first objective

minimizes the costs and the second minimizes the carbon emissions from transportation. The article considers both the epsilon constraint method and the weighted sum method. Both this article and this thesis consider the same green objective function (minimizing GHG emissions from transportation) and both evaluate the bi-objective nature using the weighted sum method.

Wang et al. (2021) present a bi-objective mixed-integer non-linear programming model to locate the collection, transit centers, and processing facilities for a reverse logistics network for healthcare waste management. The first objective function minimizes building costs, operating costs, transportation costs, inventory costs, and closing costs of each type of center. The second objective function considers the environmental impact by measuring the amount of waste that did not end up in the landfill and the distance from a hospital (transportation emissions). This thesis also uses the weighted sum method.

Rodriguez et al. (2023) use a multi-objective mixed-integer linear programming model to determine network flow, locate collection facilities, and locate reprocessing plants. The article provides three objective functions: economic, social, and environmental. The economic objective function considers the revenue, the cost of operating the network, and the investment/reinvestment costs. The social objective function maximizes the social benefit based on a social vulnerability, which is region dependent. The environmental objective minimizes the ecological footprint of transportation, reprocessing, and landfill waste disposal. The multi-objective problem is solved using the epsilon constraint method and the weighted sum method. The model presented in this thesis considers economic and ecological objective functions. The economic objective function considers transportation and fixed cost. The carbon emissions objective function considers transportation costs (carbon emissions due to the operation of additive remanufacturing are zero, and the GHG emissions of building a remanufacturing facility are assumed to be identical across Ontario).

Roudbari et al. (2024) present a closed-loop supply chain network for garments on Montreal Island. A multi-objective two-stage stochastic model is solved using the deterministic equivalence model. The objective functions are: profit, carbon emissions, and social impact. Multiple clothing recovery options are considered: refurbishing, repairing, and remanufacturing. The multi-objective problem is solved using the lexicographical method. The model presented in this thesis designs a bi-objective two-stage stochastic model (cost and carbon emissions) that is solved using the deterministic equivalence

model and considers remanufacturing and recycling as recovery options.

Rossit and Bard (2025) locate waste bins for non-hazardous recycling. The model presented is a bi-objective robust optimization model. The first objective function minimizes the total cost. The second objective function maximizes the number of days that the bin can go uncollected without overflowing. This article uses the epsilon constraint method.

2.3 Multi-Period Models

Multi-period models are commonly used to have a finite planning horizon where the supply/demand of items is not constant. Multi-period models introduce inventory holding considerations into the model. Alumur et al. (2012) develop a multi-commodity multi-period mixed-integer linear programming model to determine the locations of inspection facilities and remanufacturing facilities for managing waste electrical and electronic equipment. The model provides the transportation, inventory, and operational facilities based on the time period. Tari and Alumur (2014) develop a multi-objective, multi-period reverse logistics design problem to locate collection centers for managing waste electrical and electronic equipment. Wang et al. (2021) design a bi-objective multi-period mixed-integer non-linear programming model for a reverse logistics network for healthcare waste management. The model is non-linear due to the constraints that capture the amount of healthcare waste transported. Karagoz et al. (2022) present a multi-period mixed-integer linear programming model for a reverse logistics network for end-of-life vehicles in Istanbul, Turkey, optimizing for profit under seven equally probable scenarios. Alarcon-Gerbier and Bucher (2024) present a multi-period location–production model to locate collection facilities, inventory, production, and transportation formulated as a mixed-integer linear optimization problem, in which a decentralized network with movable processing units is considered and their locations are determined for each time period. Rosenberg et al. (2025) design a multi-period two-stage stochastic model for a reverse logistics network for end-of-life electric vehicle battery systems, considering uncertainty in the quantity, quality, and revenue of end-of-life batteries from electric vehicles.

All the articles mentioned above propose a mixed-integer linear programming. The mixed-integer linear programming model proposed in this thesis extends the deterministic bi-objective model (Section 3.2) to use the multi-period setting to account for the potential for unequal demand and supply of ELVs. The present model considers transportation

of ELVs from collection centers to remanufacturing facilities, transportation of remanufactured parts from remanufacturing facilities to car manufacturers, inventory decisions, remanufacturing facility opening decisions, and remanufacturing facility placement.

2.4 Stochastic Models

Stochastic models are used to capture uncertainty of the location of collection points, supply of used materials in reverse logistics networks, and the price of selling recycled parts. Tosarkani and Amin (2019) consider the uncertainty in different rates of disposal. Karagoz et al. (2022) and Rossit and Bard (2025) consider uncertainty in the location and quantity of used parts. Trochu et al. (2019) design for uncertainty in quality of used parts, the quantity of used parts, and the locations of collection points. Shuang et al. (2019) and Keutchayan et al. (2023) discuss quantity of used parts under uncertainty. Rosenberg et al. (2025) consider the uncertainty in the quantity, quality, and revenue of EOL batteries from electric vehicles. Roudbari et al. (2024) consider the uncertainty in the rate of returned parts, the quality of end-of-life parts, and the rate of repairing the returned parts. The model in this thesis examines uncertainty in the supply of ELVs.

Researchers use a variety of methods to solve stochastic models; among them are: two-stage stochastic model (deterministic equivalent model), robust optimization, sample average approximation (SAA), and fuzzy logic. Karagoz et al. (2022), Roudbari et al. (2024), and Rosenberg et al. (2025) use two-stage stochastic model. Rossit and Bard (2025) use Bertsimas and Sim's robust methodology. Santoso et al. (2005) and Trochu et al. (2019) use SAA. Tosarkani and Amin (2019) use fully fuzzy logic. The model presented in this thesis solves the two-stage stochastic model using the deterministic equivalent. The ex-ante decision is the location of the remanufacturing facilities; and the ex-post decision is routing. This thesis utilizes a deterministic equivalent model to solve the stochastic model using ten scenarios.

To solve stochastic problems, the uncertainty is described by continuous distributed events, or a set of discrete scenarios is used to represent the uncertainties. Two approaches to the stochastic scenario reduction problem are distribution driven and problem driven. Distribution-driven methods generate scenarios based on the distribution of the original data. Monte Carlo, quasi-Monte Carlo methods, and sparse grid quadrature are some methods in this category. These methods aim to discretize the probability distribution

using a finite number of outcomes. Distribution-driven methods operate in the same way when applied to two different stochastic problems with the same underlying probability distribution. The problem-driven method extends the probability distribution method by including any relevant properties of the problem that can be drawn from its objective function and constraints. Two problems with the same underlying distribution but different objective functions are not formulated using the same set of scenarios Keutchayan et al. (2023).

The SAA method applied by Santoso et al. (2005) is a common methodology used to solve stochastic models. First, a stochastic version of the mixed-integer linear programming model is presented with the deterministic equivalent. Then a large number of scenarios for the supply of ELVs is generated to determine the optimal placement for the remanufacturing facilities. The supply of ELVs is assumed to be uncertain due to the high variability of deterioration of vehicles and the likelihood of a vehicle being brought to a collection center.

The distribution-driven approach, which is commonly used to solve stochastic problems, is the SAA. The SAA is a Monte Carlo method. SAA is employed because of the difficulty in evaluating the expectation in the objective function. In the SAA scheme, a random sample of scenario realizations are generated, and the expectation is approximated by the sample average function. As the sample size increases, the optimal value and optimal solution vector converge to true counterparts. The solution vector converges to an optimal solution of the true problem with probability approaching one exponentially fast suggesting that a fairly good approximate solution to the true problem can be obtained by solving an SAA problem with a modest sample size. Santoso et al. (2005) suggest that the required sample size is at most linear in the number of processing facilities. In practice, the SAA scheme involves repeated solutions of the SAA problem with independent samples. Statistical confidence intervals are then derived on the quality of the approximate solutions. The SAA algorithm generates independent samples. The corresponding SAA problem is solved for each sample. In each scenario, the problem becomes deterministic (deterministic equivalent model). A lower statistical bound for the optimal value of the true problem and an estimate of the variance of this estimator are computed. A feasible solution to the true problem is selected (from one of the computed solutions). The true objective function value is estimated, which is an estimate of an upper bound of the optimal value. The optimality gap of the solution is estimated using

the lower bound estimate and the objective function value estimate (Santoso et al., 2005). The SAA method is applied by Trochu et al. (2019) and Dell'Amico et al. (2018).

SAA depends on generating a large number of scenarios to accurately represent uncertainty. The scenario reduction problem includes finding a smaller subset of scenarios to reduce the numerical complexity while keeping the error at an acceptable level. Keutchayan et al. (2023) develop a problem-driven scenario clustering method that produces a partition of the scenario set. Each cluster contains a representative scenario that reflects the optimal value of the objective function. Keutchayan et al. (2023) decompose the original problem into scenario-specific subproblems. The subproblems are solved (either approximations or exact), and a scenario tree is built based on the solutions (the tree is an mixed-integer linear programming model). Building a scenario tree is a method for discretizing the outcomes of the scenarios (Gülpınar, Rustem, & Settergren, 2004; Vázsonyi, 2006). To reduce the size of the scenario tree, Keutchayan et al. (2023) utilize scenario reduction through clustering. The clustering error is defined as the difference between the optimal value of the original problem and the value of the solution provided by solving the problem for the reduced scenario set. It corresponds to the error that would have been made by implementing the scenario tree decisions in the original problem. The output is a centroid scenario for each cluster; this centroid becomes the representative scenario of the cluster and is given a probability. The distance between points is measured by the value of the objective function. If two scenarios are very different but lead to the same value of the cost function, then these two scenarios are grouped together. The probability is doubled without changing the solution of the original problem, which results in the reduction of scenarios without loss of information (as far as the stochastic problem is concerned). The problem is formulated as a two-stage model. In the first stage, the decisions about the structure of the network are made (e.g., which arcs are opened), and the stochastic demands are revealed. In the second stage, the decisions for the commodity flows are made.

2.5 Game Theory Models

Cardinal and Hoefer (2010) present several different forms of the non-cooperative facility location problem. The article analyzes both uncapacitated and capacitated versions and proves the lower bound and upper bound to finding the Nash Equilibria. The article also

examines the existence of Nash Equilibrium and unique Nash Equilibrium. The model presented in this thesis is similar to the non-cooperative uncapacitated facility location problem model in Cardinal and Hoefer (2010). They differ in the facility location problem. This thesis uses a supply constraint (which makes the problem a mixed-integer problem instead of an integer problem). The cost-sharing structure is also modified. In Cardinal and Hoefer (2010) each player offers are based on their budget while in this thesis the cost sharing is equal between all players, or players can choose to not participate if their costs are more than the expected profit.

The Hotelling game is also another form of a facility location game that is based on placing a facility based on the customer demand. This type of game is discussed in Ben-Porat and Tennenholtz (2019) and Gur et al. (2018). The Hotelling problem evaluates the games as a function of the demand of the clients. In the Hotelling problem, the decision is up to the facility that chooses which clients it would like to serve. The clients then choose to go to the closest facility (if equidistant, then the clients split equally). In Hotelling games, the facilities compete for customers. In this thesis, the decision is based on the players (players want the facility as close as possible). The model presented in this thesis approaches the facility location game from a different perspective where players choose the facility location.

Krogmann et al. (2024) approach the facility location problem as a competition of customers to get the lowest waiting time (which depends on the location of the facility). The purpose of Krogmann et al. (2024) is to prove that Nash Equilibrium exists based on the shape of the network. Krogmann et al. (2024) considers the model where agents locate facilities sequentially, which leads to Stackelberg games. The players anticipate the moves of their opponents and play accordingly. The model in Krogmann et al. (2024) also uses the same assumption as in Ben-Porat and Tennenholtz (2019) and Gur et al. (2018), that is, if facilities are equidistant, then the clients are shared equally. Wexler et al. (2005) provides a general framework for location games. Wexler et al. (2005) has a similar purpose to Krogmann et al. (2024), that is, to prove Nash Equilibrium and uniqueness in facility location games. The model in this thesis has a similar goal to locate facilities (but does not discuss how the network shape affects Nash equilibrium).

Aziz et al. (2020) solve a capacitated facility location problem where the location of the facilities should minimize total cost. The paper proves that this problem is NP-hard (using dynamic programming) and introduces new mechanisms to solve the facility

location problem. Aziz et al. (2020) provide algorithmic approaches to the problem analyzed by Cardinal and Hoefer (2010). The model in this thesis is uncapacitated but in the real-world might need to be modified to be capacitated because warehouses are limited in the number of clients they can serve.

Vetta (2002) solves the non-cooperative facility location problem where every Nash equilibrium has social utility. The author proves that with certain conditions the Nash equilibrium solutions will be pure Nash. The version of the facility problem is competitive facility location problem. Consumers are in certain locations, and firms must choose where to put facilities (from a set of potential locations). This is analogous to the Hotelling game Ben-Porat and Tennenholtz (2019) and Gur et al. (2018). Both Aziz et al. (2020) and Vetta (2002) consider the social utility function (minimize the total cost). The paper also applies non-cooperative game theory to routing and class auctions. The model in this project uses Theorem 1 from Vetta (2002): "Any finite, k-person, non-cooperative game has at least one Nash equilibrium."

Chekuri et al. (2007) explores how flow can be split between facilities. This paper establishes that Nash equilibrium exists. The model in this thesis finds that splitting the flow will result in an equal or worse solution.

Anastasiadis and Deligkas (2020) approaches the k-facility location problem by allowing the agents to lie about their location. The goal is for the planner to locate facilities to maximize total utility and encourage all agents to tell the truth. The overall objective is the same as that of Aziz et al. (2020) and Vetta (2002) (whose objective is egalitarian). In the model presented in this thesis the goal is not egalitarian; it is individual (agents only minimize their own cost functions).

The model introduced in this thesis can be extended to include a carbon emissions cap. This model can also minimize greenhouse emissions produced through transportation instead of cost (see Section 6.3).

2.6 Contributions Compared to Literature

The models presented in this thesis are similar to Tosarkani and Amin (2019) and Roudbari et al. (2024) in that they provide a multi-objective stochastic location problem that is solved using an off-the-shelf solver and use Canadian data. Tosarkani and Amin (2019) use a fully fuzzy stochastic model, while this thesis uses a two-stage stochastic

model. Roudbari et al. (2024) consider multiple recovery options, while this thesis only considers remanufacturing and recycling. Roudbari et al. (2024) use a multi-objective, which is addressed using the lexicographic optimization method, while this thesis uses the weighted sum method. Roudbari et al. (2024) do not specify how transportation emissions were obtained, while this thesis uses the methodology developed by Wang et al. (2020) (MOVES).

To complement the research presented above, the present thesis develops three general bi-objective mixed-integer linear programming models (deterministic, multi-period, and stochastic) and applies two of them (deterministic and stochastic) to a case study of the automotive industry in Ontario, Canada. This thesis compares the deterministic and stochastic models to better understand the impact that uncertainty of supply has on the design of the reverse logistics network. Comparing these models allows stakeholders to make informed decisions about the design of the network. This thesis examines the Pareto-optimal solutions to explore the impact that incorporating an environmental objective function (transportation emissions) has on the design of the reverse logistics network.

The present thesis also presents a game theory model to compare the output of different approaches. The thesis provides insights on how the weight of the cost objective function can change the Nash equilibrium. The facility cost, transportation cost, supply variation, and the market price of a front lower control arm are perturbed to examine their effect on the Nash equilibrium.

Table 1: Summary of the Literature

Paper Citation	Number of Objective Functions		Type of Model		Solution Methodology		
	Single	**Multi**	**Deterministic**	**Stochastic**	**Exact (Solver)**	**Heuristic**	**Application**
Alumur, Nickel, Saldanha-da Gama, and Verter, 2012	✓		✓		✓		Electronics (general) WEEE
Tari and Alumur, 2014		✓	✓		✓		Electronics (general) WEEE
Vidovic, Ratkovic, Bjelic, and Popovic, 2016	✓		✓			✓	Non-Hazardous Recycling
Yu and Solvang, 2018		✓	✓		✓		General Reverse Logistics Model
Liao, 2018	✓		✓		✓	✓	Non-Hazardous Recycling
Trochu, Chaabane, and Ouhimmou, 2018	✓		✓		✓		Waste Management: wood
Trochu, Chaabane, and Ouhimmou, 2019	✓			✓	✓		Waste Management: wood
Xiao, Sun, Shu, and Wang, 2019	✓		✓			✓	ELVs
Tosarkani and Amin, 2019	✓	✓	✓	✓	✓		Acid Batteries
Wang, Huang, and He, 2021		✓	✓		✓		Healthcare Waste Management
Karagoz, Aydin, and Simic, 2022	✓		✓	✓	✓		ELVs
Rodriguez, Perez, and Boudaoud, 2023		✓	✓		✓		Tires
Alarcon-Gerbier and Buscher, 2024	✓		✓			✓	Non-Hazardous Recycling
Roudbari, Ghomi, and Eicker, 2024		✓	✓	✓	✓		Garments
Rosenberg, Huster, Rudi, and Schultmann, 2025	✓			✓	✓		EOL EV batteries
Rossit and Bard, 2025		✓	✓		✓		Non-Hazardous Recycling
Model in this thesis		✓	✓	✓	✓		ELVs

Chapter 3

Mathematical Formulations

3.1 Problem Description

Shifting towards remanufacturing and recycling of products reduces waste by keeping goods out of landfills, extends the life of products, and decreases anthropogenic GHG emissions. Moreover, recycling and remanufacturing of ELVs facilitate a closed-loop supply chain that reduces the need to mine raw materials, thus minimizing the environmental impact. Ontario Automotive Recycling Association (OARA) states that "[o]n average, 85 percent of a vehicle's weight can be recycled or reused" (Ontario Automotive Recyclers Association, 2022). Reverse logistics networks require collecting end-of-life parts, refurbishing them, and transporting them to manufacturers (see Figure 1). Developing a mathematical model that optimizes the locations of remanufacturing facilities is crucial to a circular economy.

This thesis develops four mixed-integer linear programming models that address the recycling of ELVs taking into account the carbon emissions and the cost of operating the network. The network is comprised of collection centers (which collect ELVs and sell the re-usable parts), remanufacturing facilities (which buy the used parts and refurbish them), and the manufacturing facilities (which buy the refurbished parts). As shown in Figure 1, ELVs are brought to collection centers.[1] Collection centers then send the EOL parts to remanufacturing facilities or other recovery options (recycling/landfill). Remanufacturing plants test the EOL parts from collection centers for suitability for remanufacturing. Parts

[1] In Canada, about 95 percent of vehicles are collected for recycling (Canadian Auto Recycler Magazine, 2019)

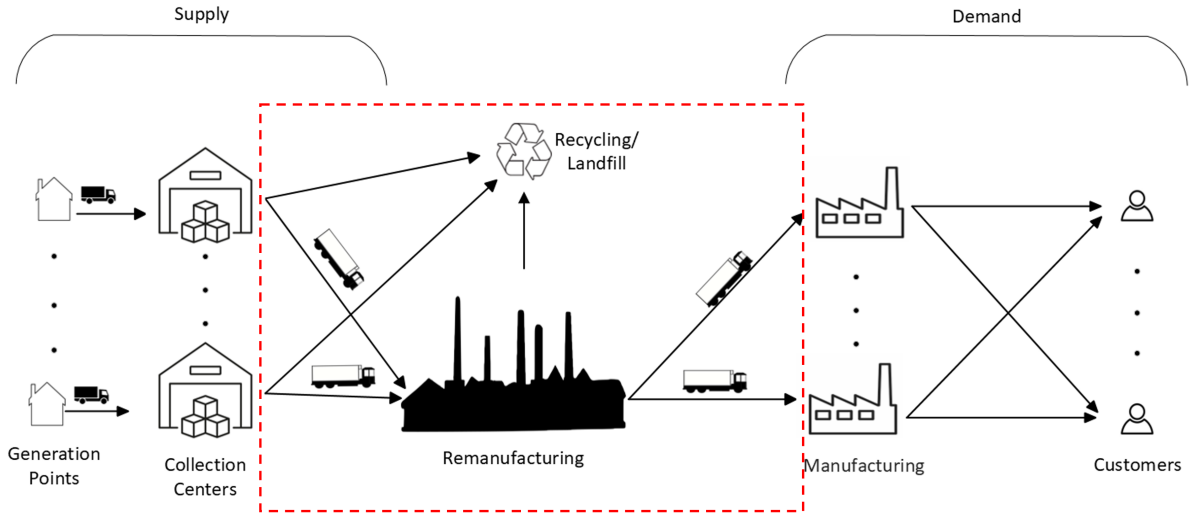


Figure 1: Network Flow for ELV Reverse Logistics

that are unsalvageable are sent to landfills or recycling facilities. The parts that are able to be remanufactured are sent to car manufacturers to be re-introduced them into the market.

The goal of the models presented in this thesis is to determine the best location(s) for the remanufacturing facilities to minimize the cost and the GHG emissions. The network is comprised of collection centers, remanufacturing facilities, city landfills, and car manufacturers (see Figure 1). This thesis considers the components inside of the red dashed boxed in Figure 1. The generation points and customers are considered out of scope because the locations of the collection centers and manufacturing plants are fixed points in the network and cannot be further optimized. The models solve for the optimal number of facilities, the locations of the facilities, and the optimal flow to and from the remanufacturing facilities. These models minimize the cost of operating the networks as well as the carbon emissions released by transportation. The models assume that the supply of ELVs is equal to the demand of the car manufacturing companies. This assumption implies that each car manufacturer has a demand and there is sufficient supply of EOL parts to meet the demand. The percentage of parts that are unsalvageable are assumed to be able to be reasonably approximated and is implemented as a parameter in the models.

3.2 Base Model

The parameters and decision variables required for the base model are as follows:

Sets

C	The set of collection centers
R	The set of potential remanufacturing facility locations
M	The set of manufacturing facilities
L	The set of landfills
A	The set of arcs that correspond to potential shipment connections defined as: $A = (i, j)$ where $(i \in C \wedge j \in R)$ or $(i \in R \wedge j \in L)$ or $(i \in R \wedge j \in M)$

Parameters

u_i	The amount of used parts generated at $i \in C$ suitable for remanufacturing
d_i	The demand of parts at manufacturing facility $i \in M$
f_i	The cost of locating a remanufacturing facility at $i \in R$
o_i	The operational cost of remanufacturing one unit at facility $i \in R$
Γ_i	The processing capacity of facility $i \in R$
c_{ij}	The cost of transporting one unit on arc $(i, j) \in A$
g_{ij}	The greenhouse gas emissions from transporting 1 unit on arc $(i, j) \in A$
β	The percentage of parts that need to be sent to the landfill (the percentage that is not suitable for remanufacturing)

Decision Variables

x_{ij}	The amount of parts to be transported on arc $(i, j) \in A$

$$y_i = \begin{cases} 1, \text{ if facility } i \in R \text{ is opened;} \\ 0, \text{otherwise.} \end{cases}$$

The mathematical model is as follows:

$$\text{Minimize} \quad \sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{\substack{i\in R \\ j\in M: \\ (i,j)\in A}} o_i x_{ij} + \sum_{i\in R} f_i y_i \tag{1}$$

$$\text{Minimize} \quad \sum_{(i,j)\in A} g_{ij}x_{ij} \tag{2}$$

$$\text{s.t.} \quad \sum_{\substack{j\in R:\\(i,j)\in A}} x_{ij} = u_i \qquad i \in C \tag{3}$$

$$\sum_{\substack{j\in R:\\(j,i)\in A}} x_{ji} \geq d_i \qquad i \in M \tag{4}$$

$$\sum_{\substack{j\in C:\\(j,i)\in A}} x_{ji} - \sum_{\substack{j\in M:\\(i,j)\in A}} (1-\beta)x_{ij} - \sum_{\substack{j\in L:\\(i,j)\in A}} \beta x_{ij} = 0 \qquad i \in R \tag{5}$$

$$\sum_{\substack{j\in C:\\(j,i)\in A}} x_{ji} + \sum_{\substack{j\in M:\\(i,j)\in A}} x_{ij} \leq \Gamma_i y_i \qquad i \in R \tag{6}$$

$$y_i \in \{0,1\} \qquad i \in R \tag{7}$$

$$x_{ij} \geq 0 \qquad (i,j) \in A \tag{8}$$

The problem is formulated as a bi-objective mixed-integer linear programming model, where first objective function (1) minimizes total cost, and the second objective function (2) minimizes the greenhouse gas emissions from transportation. The first additive term of the cost objective (1) calculates the transportation cost of shipping the parts to the possible network connections; the second additive term sums the operational cost at remanufacturing facilities; and the third additive term accounts for the fixed cost of opening a remanufacturing facility. The first additive term of the emission objective function (2) represents the greenhouse gas emissions generated from transporting the parts to the possible network connections.

Constraint (3) ensures that all parts generated at the collection centers and available for remanufacturing are shipped to a remanufacturing facility. Constraint (4) ensures that each manufacturer's demand is fully satisfied. Constraint (5) represents the flow balance at each remanufacturing facility, ensuring that the amount of parts received from the collection centers is equal to the total amount sent to landfills and manufacturing facilities. Constraint (6) is the processing capacity (including both inflows and outflows) constraint for remanufacturing. It ensures that collected items are shipped only to open facilities and that only open remanufacturing facilities send parts to manufacturers. Constraints (7) and (8) define the domains of the decision variables.

An additional constraint can be added to the base model in order to ensure that only

a certain number of facilities are opened. The additional constraint is as follows:

$$\sum_{i \in R} y_i = p \tag{9}$$

where p is the number of facilities to be opened.

3.3 Multi-period and Inventory Model

An extension of the base model implements time periods. The time periods are introduced to account for the instance when the supply and demand of ELVs are not equal [2]. In order to hold end-of-life parts that are not shipped for the next period(s), this model introduces inventory considerations. This model assumes that all EOL parts are bought by remanufacturers and all parts not bought by car manufacturers are kept in inventory. The model solves for the location and the time period to open remanufacturing facilities. The following additional parameters and decision variables are required:

Sets

T	The set of time periods where $\|T\|$ is the number of time periods

Parameters

u_{it}	The amount of used parts generated in node $i \in C$ available for remanufacturing in time period $t \in T$
$CapInventory_i$	the capacity of the inventory $i \in R$
d_{it}	The demand for parts from remanufacturing facility $i \in R$ in time period $t \in T$
k_i	The inventory cost of holding product in facility $i \in R$

Decision Variables

x_{ijt}	the amount of parts (e.g., in kg) to be transported on arc $(i,j) \in A$ in time period $t \in T$
I_{it}	how much product is in inventory at the end of the time period $t \in T$

$$y_{it} = \begin{cases} 1, \text{ if facility } i \in R \text{ is opened in time period } t \in T; \\ 0, \text{otherwise.} \end{cases}$$

[2] the multi-period and inventory model is not applied in the case study because the demand of ELVs is constant

The mathematical formulation is as follows:

$$\text{Minimize} \quad \sum_{(i,j)\in A}\sum_{t\in T} c_{ij}x_{ijt} + \sum_{i\in R}\sum_{\substack{j\in M:\\(i,j)\in A}}\sum_{t\in T} o_i x_{ijt} + \sum_{i\in R}\sum_{t\in T} f_i y_{it} + \sum_{i\in R}\sum_{t\in T} k_i I_{it} \tag{10}$$

$$\text{Minimize} \quad \sum_{(i,j)\in A}\sum_{t\in T} g_{ij}x_{ijt} \tag{11}$$

s.t.

$$\sum_{\substack{j\in R:\\(i,j)\in A}} x_{ijt} = u_{it} \qquad i \in C, t \in T \tag{12}$$

$$\sum_{\substack{j\in C:\\(j,i)\in A}} x_{jit} - \sum_{\substack{j\in M:\\(i,j)\in A}} (1-\beta)x_{ijt} - \sum_{\substack{j\in L:\\(i,j)\in A}} \beta x_{ijt} - I_{it} + I_{i,t-1} = 0 \qquad i \in R, t = 1, ..., |T| \tag{13}$$

$$\sum_{\substack{i\in R:\\(j,i)\in A}} x_{jit} \geq d_{it} \qquad i \in M, t \in T \tag{14}$$

$$\sum_{\substack{j\in C:\\(j,i)\in A}} x_{jit} + I_{it} \leq \Gamma_i y_{it} \qquad i \in R, t \in T \tag{15}$$

$$\sum_{\substack{j\in M:\\(i,j)\in A}} x_{ijt} \leq \Gamma_i y_{it} \qquad i \in R, t \in T \tag{16}$$

$$y_{it} \leq y_{i,t+1} \qquad i \in R, t = 0, ..., |T|-1 \tag{17}$$

$$I_{i,0} = 0 \qquad i \in R \tag{18}$$

$$y_{it} \in \{0,1\} \qquad i \in R, t \in T \tag{19}$$

$$x_{ijt} \geq 0 \qquad (i,j) \in A, t \in T \tag{20}$$

$$I_{it} \geq 0 \qquad i \in R, t \in T \tag{21}$$

$$I_{it} \leq CapInventory_i \qquad i \in R, t \in T \tag{22}$$

The objective function (10) minimizes total cost, whereas the second objective function (11) minimizes the greenhouse gas emissions from transportation. The first additive term of the cost objective (10) calculates the transportation cost of shipping parts on the possible shipment connections in each time period; the second additive term sums the operational cost at remanufacturing facilities in each time period; the third additive term

accounts for the fixed cost of opening a remanufacturing facility in each time period; and the fourth additive term accounts for the holding cost in each time period. The emission objective function (11) is the emissions produced from shipping parts to the possible shipment connections in each time period.

Constraint (12) ensures that the amount of parts generated at collection centers in each time period and available for remanufacturing are shipped to a remanufacturing facility. Constraint (13) is the flow balance for remanufacturing facilities that ensures that the amount of parts shipped from collection centers is equal to the amount sent to the landfill in addition to the manufacturing facilities in each time period. Constraint (14) ensures that the demand from the manufacturer is being met in every time period. Constraint (15) is the processing capacity constraint for remanufacturing; it also ensures that the collected items can only be shipped to open facilities in each time period. Constraint (16) ensures that only open facilities can send parts to manufacturers in each time period. Constraint (17) ensures that once a remanufacturing facility is opened in a time period, it remains open. Constraint (18) ensures that the inventory in the first time period is zero. Constraints (19), (20), and (21) define the domains of the decision variables. Constraint (22) ensures that the inventory capacity is not surpassed.

3.4 Two-stage Stochastic Model

This section presents a stochastic model. The stochastic model is introduced in order to account for uncertainty in the supply of parts. The deterioration of vehicles and the likelihood of a vehicle being brought to a collection center create uncertainty in the supply of ELVs to the system. Therefore, multiple scenarios to capture a variety of supply originating from all collection centers are examined. The first-stage decisions are the locations of the remanufacturing facilities, while the second-stage decisions determine the flow of parts. It is assumed that the remanufacturing facility location decisions are made before the uncertainty in supply is revealed. Once the supply is known, the second-stage decisions specify how parts are routed to and from the remanufacturing facilities.

Let $u_i(\psi)$ denote the random vector representing the supply of parts from collection center $i \in C$, associated with realization $\psi \in \Psi$, where Ψ is the known joint distribution of the supply. The first-stage decision variables are the locations of the remanufacturing facilities, denoted by y_i, and the second-stage decisions are the flows, denoted by $x_{ij}(\psi)$

for realization $\psi \in \Psi$. The two-stage stochastic program is then modeled as:

$$\text{Minimize} \quad \mathbb{E}_\psi(Q(\mathbf{x}, \psi)) + \sum_{i \in R} f_i y_i \tag{23}$$

$$\text{Minimize} \quad \mathbb{E}_\psi(G(\mathbf{x}, \psi)) \tag{24}$$

$$\text{s.t.} \quad (7).$$

The objective function (23) minimizes the total expected cost and the second objective function (24) minimizes the expected total GHG emissions produced by the transportation of parts, where $\mathbb{E}_\psi$ denotes the expectation with respect to ψ. Constraint (9) can be included in the above formulation if the number of facilities to be opened should be restricted to p. $Q(\mathbf{x}, \psi)$ represents the optimal value of the following second-stage problem for each realization of $\psi \in \Psi$:

$$\mathrm{Q}(\mathbf{x}, \psi) = \min \quad \sum_{(i,j) \in A} c_{ij} x_{ij}(\psi) + \sum_{\substack{i \in R, j \in M: \\ (i,j) \in A}} o_i x_{ij}(\psi) \tag{25}$$

$$\text{s.t.} \quad \sum_{\substack{j \in R: \\ (i,j) \in A}} x_{ij}(\psi) = u_i(\psi) \qquad i \in C, \psi \in \Psi \tag{26}$$

$$\sum_{\substack{j \in R: \\ (j,i) \in A}} x_{ji}(\psi) \geq d_i \qquad i \in M, \psi \in \Psi \tag{27}$$

$$\sum_{\substack{j \in C: \\ (j,i) \in A}} x_{ji}(\psi) - \sum_{\substack{j \in M: \\ (i,j) \in A}} (1 - \beta) x_{ij}(\psi) - \sum_{\substack{j \in L: \\ (i,j) \in A}} \beta x_{ij}(\psi) = 0 \qquad i \in R, \psi \in \Psi \tag{28}$$

$$\sum_{\substack{j \in C: \\ (j,i) \in A}} x_{ji}(\psi) + \sum_{\substack{j \in M: \\ (i,j) \in A}} x_{ij}(\psi) \leq \Gamma_i y_i \qquad i \in R, \psi \in \Psi \tag{29}$$

$$x_{ij}(\psi) \geq 0 \qquad (i,j) \in A, \psi \in \Psi \tag{30}$$

Similarly, $G(\mathbf{x}, \psi)$ represents the optimal value of the following second-stage problem for

each realization of $\psi \in \Psi$:

$$\mathrm{G}(\mathbf{x}, \psi) = \min \quad \sum_{(i,j)\in A} g_{ij} x_{ij}(\psi) \tag{31}$$
$$\text{s.t.} \quad (26) - (30).$$

Objective function (25) minimizes the transportation cost and the operating cost for each realization of $\psi \in \Psi$. Objective function (31) minimizes the GHG emissions for each realization of $\psi \in \Psi$. Constraint (26) ensures all the supply is sent. Constraint (27) ensures demand is met. Constraint (28) is the flow balance constraint, which ensures that the supply is equal to the amount sent to the landfill and remanufacturing facilities. Constraint (29) ensures the parts can only be sent to/from open facilities and that the amount never surpasses the capacity of a facility. Constraint (30) defines the domain of the decision variable.

Under the assumption that the uncertainty in the supply can be described by a finite set of scenarios, we can formulate a deterministic equivalent of the stochastic problem. Given the finite set of scenarios for the uncertain supply S, let p^s denote the probability of scenario $s \in S$, where $\sum_{s\in S} p^s = 1$. The deterministic equivalent of the stochastic problem can then be written as follows:

$$\text{Minimize} \quad \sum_{s\in S} p^s \left(\sum_{(i,j)\in A} c_{ij} x_{ij}^s + \sum_{\substack{i\in R, j\in M: \\ (i,j)\in A}} o_i x_{ij}^s \right) + \sum_{i\in R} f_i y_i \tag{32}$$

$$\text{Minimize} \quad \sum_{s\in S} p^s \sum_{(i,j)\in A} g_{ij} x_{ij}^s \tag{33}$$

s.t.

$$\sum_{\substack{j\in R: \\ (i,j)\in A}} x_{ij}^s = u_i^s \qquad i \in C, s \in S \tag{34}$$

$$\sum_{\substack{j\in R: \\ (j,i)\in A}} x_{ji}^s \geq d_i^s \qquad i \in M, s \in S \tag{35}$$

$$\sum_{\substack{j\in C: \\ (j,i)\in A}} x_{ji}^s - \sum_{\substack{j\in M: \\ (i,j)\in A}} (1-\beta) x_{ij}^s - \sum_{\substack{j\in L: \\ (i,j)\in A}} \beta x_{ij}^s = 0 \qquad i \in R, s \in S \tag{36}$$

$$\sum_{\substack{j\in C:\\(j,i)\in A}} x_{ji}^s + \sum_{\substack{j\in M:\\(i,j)\in A}} x_{ij}^s \leq \Gamma_i y_i \qquad i \in R, s \in S \qquad (37)$$

$$y_i \in \{0,1\} \qquad i \in R \qquad (38)$$

$$x_{ij}^s \geq 0 \qquad (i,j) \in A, s \in S \qquad (39)$$

The first objective function (32) minimizes the cost (transportation and operation) multiplied by the probability that this scenario will occur plus the facility cost. The second objective (33) minimizes the emissions released through transportation, which is multiplied by the probability that this scenario will occur. Constraint (34) ensures all the supply is sent for each scenario. Constraint (35) ensures demand is met for each scenario. Constraint (36) is the flow balance ensuring that the supply is equal to the amount sent to the landfill and remanufacturing facilities for each scenario. Constraint (37) ensures that parts can only be sent to/from open facilities and that the amount never exceeds the capacity of a facility for each scenario. Constraints (38) and (39) define the domains of the decision variables.

Note that the above model cannot be decomposed for each scenario $s \in S$, since the location decisions are independent of the scenario index and must be optimized considering all scenarios simultaneously. Hence, this is not an easy problem to solve, especially when the number of scenarios is large.

Constraint (9) can be added to the deterministic equivalent of the stochastic model to ensure that a certain number of facilities are opened.

3.5 Addressing the Bi-Objective Nature

All the models presented are bi-objective. All models contain one objective function for cost and a second objective function for GHG emissions released during transportation. The objective functions must be formulated separately because they are in different units of measurement (the cost in dollars and the emissions in kg of CO_2e). The objective functions are in tension because improving the cost function objective may lead to a decrease in the emissions objective. For example, if building a facility is expensive, minimizing the cost objective function would require lowering the number of facilities selected. On the other hand, minimizing the emissions objective function would require increasing the number of facilities selected in order to reduce the transportation emissions.

The weighted sum method is used to obtain Pareto-optimal solutions for these bi-objective models. The weighted sum method is a common approach used in multi-objective optimization to combine multiple objectives into a single objective function. It allows the decision-maker to express preferences among the objectives by assigning weights to them. Rodriguez et al. (2023), Xiao et al.(2019), Yu and Solvang (2018) discuss implementations of the weighted sum method used to identify Pareto-optimal solutions.

For a given scalar $\alpha \in [0, 1]$, representing the weighting of the cost and emissions objectives, the objective function for the deterministic (base) model can be written as follows:

$$\text{Minimize} \quad \alpha \left[\sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{\substack{i\in R, j\in M: \\ (i,j)\in A}} o_i x_{ij} + \sum_{i\in R} f_i y_i \right] + (1-\alpha) \left[\sum_{(i,j)\in A} g_{ij}x_{ij} \right] \tag{40}$$

Similarly, the weighted sum objective function for the deterministic equivalent of the stochastic problem is:

$$\text{Minimize} \quad \alpha \left[\sum_{s\in S} \sum_{(i,j)\in A} p^s c_{ij} x^s_{ij} + \sum_{s\in S} \sum_{\substack{i\in R, j\in M: \\ (i,j)\in A}} p^s o_i x^s_{ij} + \sum_{i\in R} f_i y_i \right] + (1-\alpha) \left[\sum_{s\in S} \sum_{(i,j)\in A} p^s g_{ij} x^s_{ij} \right] \tag{41}$$

Note that varying α shifts the emphasis between the cost and emissions objectives. As α approaches 1, the emphasis will be on the cost objective, whereas as α approaches 0, the emphasis will be on the emissions objective. The decision-maker can obtain Pareto-optimal solutions by systematically changing the values of α.

Since the objectives considered are total cost and total GHG emissions, their units are different and may differ by orders of magnitude. Therefore, it is recommended to scale the objectives when using the weighted sum method. Without scaling, the objective with the larger numerical range may dominate the optimization, regardless of the assigned weights.

3.6 Game Theory Model

The game theory model is a non-cooperative facility location problem. The game developed in this thesis is an n-player game. There are two types of players: collection centers and car manufacturers. They select locations for remanufacturing facilities. All players aim to lower their costs by having a remanufacturing facility located close to themselves (to lower transportation costs) or lowering the facility cost (through picking a cheaper location or sharing the opening cost with other players). Players' locations and supply/demands are fixed and known. The known locations and supply allow the game to be a one-shot game where all players simultaneously solve their mixed-integer programming models.

Sets

C	The set of collection centers
M	The set of car manufacturers
R	The set of potential remanufacturing facility locations
A	The set of arcs that correspond to potential shipment connections defined as: $A = (i, j)$ where $(i \in C \wedge j \in R)$ or $(i \in R \wedge j \in L)$ or $(i \in R \wedge j \in M)$

Parameters

u_i	The amount of used parts generated at $i \in C$ suitable for remanufacturing
d_i	The demand of parts at manufacturing facility $i \in M$
f_i	The cost of locating a remanufacturing facility at $i \in R$
Γ_i	The processing capacity of facility $i \in R$
c_{ij}	The cost of transporting one unit on arc $(i, j) \in A$
g_{ij}	The greenhouse gas emissions from transporting one unit on arc $(i, j) \in A$
p_i	The price of facility $i \in R$ divided by the number of players that agree to open facility $i \in R$
w	The wholesale price of a front lower control arm
h	The marginal price from second-hand to new parts

Decision Variables

x_{ij} The amount of parts to be transported on arc $(i, j) \in A$

$$y_i = \begin{cases} 1, \text{ if facility } i \in R \text{ is opened;} \\ 0, \text{ otherwise.} \end{cases}$$

The costs of a collection center opening a remanufacturing facility is the supply multi-

plied by the cost of transportation and the cost of opening a facility. The payoff function is:

$$P_{collectioncenter_i} = -\left(\sum_{j \in R} c_{ij} x_{ij} + p_i\right) \tag{42}$$

If the collection center chooses not to open a facility, the payoff is:

$$L_{collectioncenter_i} = -u_i w \tag{43}$$

This penalty is the profit that would be lost by not selling end-of-life parts.

In addition to their greenhouse gas emissions:

$$G_{collectioncenter_i} = \sum_{j \in R} g_{ij} x_{ij} \tag{44}$$

The bi-objective nature of the players' model uses the weighted sum method (see Section 3.5). This results in an objective function of:

$$S_{collectioncenter_i} = (\alpha)\left(\sum_{j \in R} c_{ij} x_{ij} + p_i\right) + (1-\alpha)\sum_{j \in R} g_{ij} x_{ij} \tag{45}$$

The costs of a car manufacturer opening a remanufacturing facility is the supply multiplied by the cost of transportation and the cost of opening a facility. The payoff function is:

$$P_{manufacturerer_i} = -\left(\sum_{j \in R} c_{ij} x_{ij} + p_i\right) \tag{46}$$

If the car manufacturer chooses not to open a facility, the payoff is:

$$L_{manufacturer_i} = -d_i h \tag{47}$$

This penalty is the profit that would be lost if remanufactured parts are not purchased.

In addition to their greenhouse gas emissions:

$$G_{manufacturerer_i} = \sum_{j \in R} g_{ij} x_{ij} \tag{48}$$

The bi-objective nature of the players' model uses the weighted sum method (see Section

3.5). This results in an objective function of:

$$S_{manufacturerer_i} = (\alpha)\left(\sum_{j\in R} c_{ij}x_{ij} + p_i\right) + (1-\alpha)\sum_{j\in R} g_{ij}x_{ij} \tag{49}$$

With equations (45) and (49) the players wish to minimize the amount of cost incurred (in terms of money and GHG emissions). Therefore, each player solves their own minimization problem.

Each collection center solves the following facility location problem:

$$\text{Minimize} \quad \alpha\left(\sum_{j\in R} c_{ij}x_{ij} + \sum_{j\in R} f_j y_j\right) + (1-\alpha)\sum_{j\in R} g_{ij}x_{ij} \tag{50}$$

where | other players actions given and fixed

$$\text{s.t.} \quad \sum_{j\in R} x_{ij} = u_i \tag{51}$$

$$x_{ij} \leq \Gamma_j y_j \qquad j \in R \tag{52}$$

$$y_j \in \{0,1\} \qquad j \in R \tag{53}$$

$$x_{ij} \geq 0 \qquad j \in R \tag{54}$$

where i is the collection facility $i \in C$.

Each manufacturer solves the following facility location problem:

$$\text{Minimize} \quad \alpha\left(\sum_{j\in R} c_{ij}x_{ij} + \sum_{j\in R} f_j y_j\right) + (1-\alpha)\sum_{j\in R} g_{ij}x_{ij} \tag{55}$$

where | other players actions are given and fixed

$$\text{s.t.} \quad \sum_{j\in R} x_{ij} = d_i \tag{56}$$

(52), (53), and (54)

where i is the manufacturing facility $i \in M$.

The objective functions (50) and (55) minimize the total cost of transportation, opening a facility, and the emissions generated during transportation given that the other players' decisions are given and fixed. The constraints (51) and (56) ensure that the number of parts transported from the facility is the number of parts the facility has (sup-

ply) or the amount the facility demands. Constraint (52) ensures that parts can only be bought/sold to a facility that is opened. Constraints (53) and (54) define the domain of the decision variables.

If players decide to open a remanufacturing facility together, the fixed cost of a facility is evenly split between them (cost sharing). Since all players choose the facility to open simultaneously, the problem can be solved using one linear programming model. This model combines all players' minimization objective functions and all their constraints. This linear programming model is as follows:

Sets

P The set of players, where $|P|$ is the number of players

R The set of potential remanufacturing facility locations

Parameters

D_i The supply/demand for a player $i \in P$

M A very big number

Decision Variables

x_{ij} The amount of parts to be transported on arc $(i,j) \in A$ where $i \in P, j \in R$

w_j The number of times a facility $j \in R$ is selected

$$y_{ij} = \begin{cases} 1, \text{ if player } i \in P \text{ is assigned to facility } j \in R; \\ 0, \text{ otherwise.} \end{cases}$$

$$a_j = \begin{cases} 1, \text{ if facility } j \in R \text{ is selected;} \\ 0, \text{ otherwise.} \end{cases}$$

$$\text{Minimize} \quad \alpha \left(\sum_{i \in P} \sum_{j \in R} c_{ij} x_{ij} + \sum_{j \in R} \frac{a_j f_j}{w_j} \right) + (1-\alpha) \sum_{j \in R} g_{ij} x_{ij} \tag{57}$$

$$\text{s.t.} \quad \sum_{j \in R} x_{ij} = D_i \qquad i \in P \tag{58}$$

$$\sum_{i \in P} x_{ij} \leq \Gamma_j a_j \qquad j \in R \tag{59}$$

$$\sum_{j \in R} y_{ij} = 1 \qquad i \in P \tag{60}$$

$$\sum_{i \in P} y_{ij} = w_j \qquad j \in R \tag{61}$$

$$w_j \leq Ma_j \qquad j \in R \tag{62}$$

$$y_{ij} \in \{0,1\} \qquad i \in P, j \in R \tag{63}$$

$$x_{ij} \geq 0 \qquad i \in P, j \in R \tag{64}$$

$$w_j \geq 0 \qquad j \in R \tag{65}$$

$$a_j \in \{0,1\} \qquad j \in R \tag{66}$$

Objective function (57) minimizes the transportation cost of every player plus the facility cost plus the emissions generated by transportation. Constraint (58) ensures that the players transport all their supply/demand to their chosen facility. Constraint (59) ensures that the capacity of the facility is not surpassed and that no parts can be sent to a facility if it is not opened. Constraint (60) ensures that every player is assigned to exactly one facility. Constraint (61) obtains the number of players that have been assigned to each facility. Constraint (62) ensures a facility is opened if players have been assigned to the facility. Constraints (63)-(66) are the variable domains.

An additional constraint can be added to the model in order to ensure that only a certain number of facilities are opened. The additional constraint is as follows:

$$\sum_{j \in R} a_j = p \tag{67}$$

where p is the number of facilities to be opened.

The formulation above is a non-linear programming model. To linearize it, additional variables will be used. First,

$$z_j = \frac{a_j}{w_j}$$

replaces the non-linear term in the objective function (57). This requires the following constraints:

$$z_j \leq Ma_j \forall j \in R \tag{68}$$

$$z_j \geq \frac{1}{w_j + 1 - a_j} - (1 - a_j)M \forall j \in R \tag{69}$$

Constraint (68) ensures that when a facility is closed, then no facility cost is added. Constraint (69) ensures that if the facility is not opened, then the z_j variable will be 0; if the facility is opened, then the z_j variable will be one divided by the number of facilities opened. Equation (69) is not a linear function. Therefore, a new variable δ_j should be

added:

$$\delta_j(w_j + 1 - a_j) = \delta_j w_j + \delta_j - a_j\delta_j = 1$$

This new constraint must also be linearized because it is a polynomial. Let:

$$U_j = \delta_j w_j$$

$$T_j = \delta_j a_j$$

The domains of $w_j, \delta_j,$ and a_j allow the variables U_j and T_j to be linearized[3]. The domains of the variables w_j, δ_j, a_j, U_j and T_j are as follows:

$$w_j \in [0, |P|]$$

$$\delta_j \in \left[\frac{1}{|P|}, 1\right]$$

$$a_j \in \{0, 1\}$$

$$U_j \in [0, |P|]$$

$$T_j \in [0, 1]$$

The fully linearized mixed-integer linear programming model is as follows:

Variables for Linearization

z_j $\quad z_j = \frac{a_j}{w_j}$ where $j \in R$

δ_j $\quad \delta_j = \frac{1}{w_j+1-a_j}$ where $j \in R$

U_j $\quad U_j = \delta_j w_j$ where $j \in R$

T_j $\quad T_j = \delta_j a_j$ where $j \in R$

$$\text{Minimize} \quad \alpha\left(\sum_{i\in P}\sum_{j\in R} c_{ij}x_{ij} + \sum_{j\in R} z_j f_j\right) + (1-\alpha)\sum_{j\in R} g_{ij}x_{ij} \tag{70}$$

$$\text{s.t.} \quad z_j \leq M a_j \qquad j \in R \tag{71}$$

$$z_j \geq \delta_j - (1 - a_j)M \qquad j \in R \tag{72}$$

$$U_j \geq w_j \qquad j \in R \tag{73}$$

[3] U_j is linearized using McCormick envelopes (Urban, 2021).

$$U_j \geq \delta_j \times |P| + w_j - |P| \qquad j \in R \qquad (74)$$

$$U_j \leq \delta_j \times |P| + \frac{1}{|P|} \times w_j - 1 \qquad j \in R \qquad (75)$$

$$U_j \leq w_j \qquad j \in R \qquad (76)$$

$$T_j \leq a_j \qquad j \in R \qquad (77)$$

$$T_j \leq \delta_j \qquad j \in R \qquad (78)$$

$$T_j \geq 2 * a_j - 1 \qquad j \in R \qquad (79)$$

$$z_j \geq 0 \qquad j \in R \qquad (80)$$

$$0 \leq U_j \leq |P| \qquad j \in R \qquad (81)$$

$$0 \leq T_j \leq 1 \qquad j \in R \qquad (82)$$

$$\frac{1}{|P|} \leq \delta_j \leq 1 \qquad j \in R \qquad (83)$$

(58)-(66)

Constraints (71)-(79) linearize the nonlinear term in objective function (57). Constraints (80)-(83) define the domains of the linearization variables.

Nash equilibrium should always be possible in this game (Vetta, 2002). In the scenarios solved in this project, there is a Nash equilibrium (all players choose a strategy and have one strategy preferable to all other possible strategies). The strategies available to players in this game are to participate in opening a facility or not to participate in the market. The players do not participate in the market if equation (42) is smaller than equation (43) or equation (46) is smaller than equation (47). See Appendix A, for an example of a payoff matrix.

This model is applied to the case study as described in Section 4.[4] The average price for a second-hand front lower control arm is $37.50 (Parts 4 less U pull, 2025) (w in the mathematical model). The average wholesale price for a front lower control arm is $78.50 (Auto Shack, 2024). This makes the margin (h in the mathematical model) $41.00.

[4]In Ontario there are 77 collection centers and 18 manufacturing facilities. The model uses 101 potential locations (based on the most populated cities in Ontario) for potential locations for a remanufacturing facility.The base price of opening a facility is $2 million plus a land factor adjustment cost that is calculated based on the price of a plot of land. The cost of shipping one part is $0.00005696/kg-km. The distance to potential locations is the shortest path (in km) obtained using ArcGIS. The supply of collection centers is based on vehicle registration in Ontario based on the county. The demand of manufacturers is 80% of the supply (percentage of salvageable parts) that is evenly split between them.

Chapter 4

Case Study Data

The models are implemented to a case study in Ontario. All maps in this thesis are visualized using Folium (Story, 2020). The following section describes the data that is used (for a summary of the data used in this thesis, see Appendix B). The case study focuses on one EOL part — the front lower control arm. This vehicle component is part of the suspension system. It is typically made of aluminum due to its favorable strength-to-weight ratio, corrosion resistance, and manufacturability (Aldan American, 2025). In this case study, the control arm is composed of an aluminum alloy—commonly AA 356 or AA 6061—which is widely used in automotive applications for its high strength and fatigue resistance (Aldan American, 2025). In this case study, the remanufacturing process is additive manufacturing. The additive manufacturing process is cold spray. Cold spray is a solid-state additive manufacturing technique in which metal powders are accelerated to supersonic speeds and deposited onto a substrate without melting (Pathak & Saha, 2017; Titomic, 2024). The case study in this thesis designs a reverse logistics network that locates additive manufacturing facilities (referred to as *remanufacturing facilities*) in Ontario to manage the reverse flow of front lower control arms.

The models described above use four sets of data: collection centers, potential remanufacturing locations, the facilities of current car manufacturers, and landfills. The locations of the collection centers in Ontario (set C in the mathematical models) are based on OARA members (Ontario Automotive Recyclers Association, 2024). There are 77 cities that have an OARA member (see Figure 2a). The locations of the top 101 most populated cities in Ontario (World Population by Country 2024, 2024) are used to provide possible locations for remanufacturers (set R in the mathematical models) (see Figure

2b). The locations of the five car manufacturing companies in Ontario (Government of Canada, 2021) that serve as the car manufacturing locations (set M in the mathematical models) are: Stellantis, Ford, GM, Honda, and Toyota. These companies have 18 factories in Ontario (see Figure 2c). The costs and emissions of trucking to the landfill are assumed to be negligible because each city has a landfill (set L in the mathematical models) (Government of Ontario, 2022b).

The annual supply of used front lower control arms (u_i in the mathematical models) is based on the number of registered vehicles by county in 2022 (Government of Ontario, 2023). The supply for each collection center is calculated based on the number of registered vehicles in the county where the collection center is located. If there are multiple collection centers in a county, then the number of registered vehicles is split evenly between them. In order to determine the kilograms of parts that are produced by each collection center, the number of registered vehicles is multiplied by two (two front lower control arms per vehicle) and then multiplied by 25 kg (the average weight of a control arm according to Stellantis Canada, 2024).

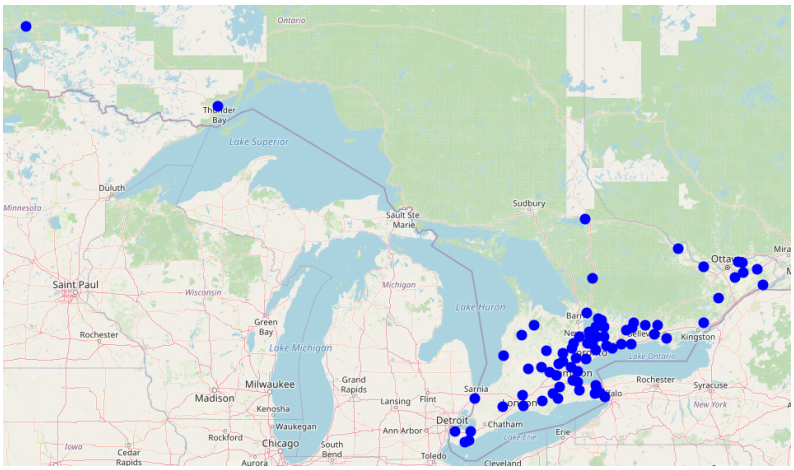
The total yearly demand for parts (d_i in the mathematical models) is assumed to be equal to 80% of the supply of parts (the salvageable portion of the parts). This ensures that all possible parts that are collected are re-introduced into the market (remanufactured). This case study assumes that the demand is evenly split between all car manufacturers. This assumption is necessary because the production volumes at each manufacturer are unknown.

The cost of the facility (f_i in the mathematical models) is calculated using the following formula:

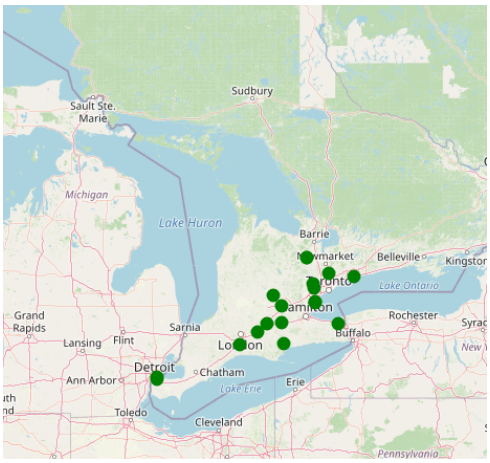
$$f_i = BaseCost \left(1 + \frac{LandCost}{\sum_{i \in R} landCost}\right) \forall i \in R \tag{84}$$

where f_i is the cost of the facility, R is the set of remanufacturing facilities, $LandCost$ is the estimated value of an acre of farm land in 2021 based on the county (Government of Canada, 2024b), and $BaseCost$ is the base cost for the facility assumed to be two million as a baseline. This formula is used to incorporate a correlation between the cost of a facility and its location into the model. For example, the formula shows that facilities in the Greater Toronto Area (GTA) region are more expensive than those in remote areas.

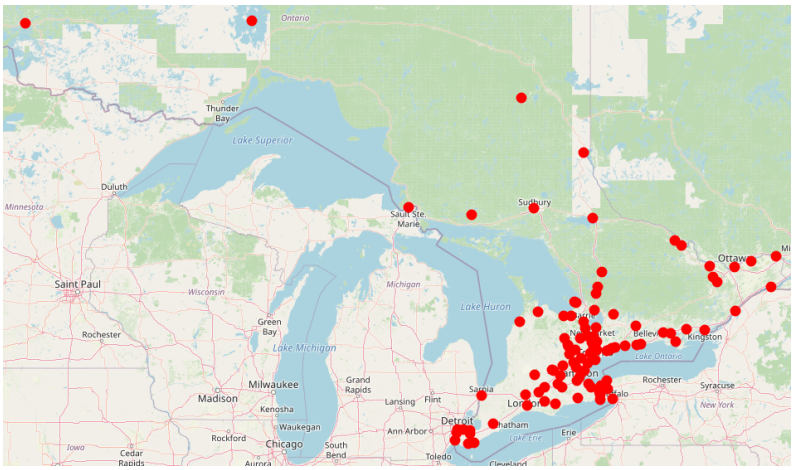
The operating cost of a facility (o_i in the mathematical models) is assumed to be identical for all potential locations for a remanufacturing facility in Ontario. Therefore,



(a) Locations of the 77 Collection Centers (Ontario Automotive Recyclers Association, 2024)



(b) Locations of the 18 Car Manufacturing Plants in Ontario Operated by Ford, GM, Honda, Stellantis, and Toyota



(c) Locations of the 101 most Populated Cities in Ontario Considered as Potential Sites for Remanufacturing Facilities

Figure 2: Map of the sets of Locations Across Ontario

the operating cost has no impact on the solutions produced by the models.

The facilities for this case study are considered uncapacitated (Γ_i in the mathematical models). All facilities are assumed to be able to accommodate the supply of end-of-life parts.

To calculate the transportation costs on each network connection, \$1.65/mile is used for a 18,000kg truck (Li, Taherkhani, Alumur, & Hewitt, 2025). This results in \$0.000091667 / mile-kg which is converted to \$0.00005696/kg-km. The trucking rate is multiplied by the shortest trucking distance (obtained using ArcGIS) (c_{ij} in the mathematical models). This trucking rate is measured based on weight and distance of a 18,000 kg long-haul combination truck (Garner, 2024).

The emissions released from transporting one long-haul combination truck on arc $(i, j) \in A$ (g_{ij} in the mathematical models) is calculated based on the census divisions of Ontario (Wang et al., 2020). Each census division in Ontario is associated with a distinct emissions factor. The shortest path between nodes in the network is generated (using ArcGIS). The emissions are computed by identifying the census divisions with which the path intersects and applying the corresponding emissions factors to the distance traveled within each county (in kilometers). Long-haul combination trucks have a capacity of approximately 19,000 kg (Garner, 2024). In this thesis, the capacity of a truck is considered to be 18,000 kg due to the irregular shape of the part and the possibility of not filling the truck to its capacity. This case study does not account for empty return trips of trucks to the facility or another pick-up location. In addition, emissions from collection centers and remanufacturing facilities to recycling centers or landfills are assumed to be negligible since a landfill is available at all potential locations within the set.

The amount of product sent to a landfill (β in the mathematical models) (because it is not salvageable) is assumed to be 20%.

The stochastic model is introduced to capture the uncertainty in the quantity and quality of the front lower control arms. The uncertainty can arise from the unknown condition of the parts that are brought to collection centers and the unknown number of cars that are brought to collection centers. Therefore, supply is assumed to follow a normal distribution. Each instance of the deterministic equivalent of the stochastic model is solved using a total of ten scenarios generated from this normal distribution, with each scenario representing different supply levels occurring with equal probability. The case study assumes a supply variation of 10% around the mean as a baseline.

The sensitivity analysis examines the parameters of fixed cost, transportation cost, and supply variation. The fixed cost (f_i in the mathematical models) is varied by setting the $baseCost$ of a facility from one to five million. The transportation cost is varied by fluctuating the trucking rate. The supply (u_i in the mathematical models) is varied by percentage decreases/increases in the deterministic version and variation in the stochastic model. Fluctuation of supply also accounts for the variation of the amount of product that is not salvageable (β in the mathematical models) because the parameter only affects the amount of outflow from remanufacturing facilities.

Chapter 5

Results and Insights

This section first discusses the base model and then proceeds to the stochastic model. The base models were run on a Dell with an Intel Core i5 7th Gen CPU, with 3.5 GHz and 8.00 GB of RAM, running Windows 11 operating system. The stochastic models were run on Mac with an Apple M2 Pro chip, 16 GB of RAM, running macOS Sequoia 15.5 by Atahan Bayir. Gurobi was used as a mixed-integer linear programming solver. All instances were solved to optimality using the default settings of Gurobi (the default value of this parameter is 0.01%), coded in Python.

5.1 Base Model

The objective function equation (40) from Section 3.5 is used. The objective function used to solve this model incorporates weights assigned to the cost and emissions objective functions (denoted by α and $1 - \alpha$) to observe the locations selected based on how important cost-effectiveness or environmental sustainability are for the decision maker. As the value of α increases, a greater weight is placed on the cost objective function. When $\alpha = 1$, only the cost is minimized. When $\alpha = 0$, only the emissions are minimized. For this initial analysis, Constraint (9) is included to restrict the number of facilities to be located to p.

Table 2 presents the results of the Base model (deterministic). Table 2 contains the p value, which represents the number of facilities that should be located. The objective weights are reported. The total cost of the solution (made up of the transportation cost and the fixed cost for a facility) and the total emissions released (transportation emissions)

by the solution are indicated. The locations of the facilities are indicated. The rightmost column contains Gurobi's solution time (in seconds).

Table 2: Deterministic Results with the Base Model

p	**Objective Weights** $(\alpha, 1-\alpha)$	**Total Cost ($)**	**Transportation Emissions (kg of CO_2e)**	**Optimal Locations**	**Gurobi Time (sec)**
	1,0	6,664,747	13,789,622	Mississauga	0.62
	0.9,0.1	6,664,747	13,789,622	Mississauga	0.58
	0.8,0.2	6,664,747	13,789,622	Mississauga	0.56
	0.7,0.3	6,686,167	13,457,394	Brampton	0.58
	0.6,0.4	6,686,167	13,457,394	Brampton	0.62
1	0.5,0.5	6,686,167	13,457,394	Brampton	0.58
	0.4,0.6	6,686,167	13,457,394	Brampton	0.53
	0.3,0.7	6,686,167	13,457,394	Brampton	0.58
	0.2,0.8	6,686,167	13,457,394	Brampton	0.59
	0.1,0.9	6,686,167	13,457,394	Brampton	0.58
	0,1	6,686,167	13,457,394	Brampton	0.56
	1,0	7,569,496	10,463,233	Ottawa, Mississauga	10.48
	0.9,0.1	7,569,496	10,463,233	Ottawa, Mississauga	11.11
	0.8,0.2	7,569,496	10,463,233	Ottawa, Mississauga	10.23
	0.7,0.3	7,571,189	10,435,033	Ottawa, Mississauga	19.67
	0.6,0.4	7,571,189	10,435,033	Ottawa, Mississauga	12.23
2	0.5,0.5	7,617,576	10,122,450	Ottawa, Brampton	12.48
	0.4,0.6	7,617,576	10,122,450	Ottawa, Brampton	13.37
	0.3,0.7	7,617,576	10,122,450	Ottawa, Brampton	13.95
	0.2,0.8	7,617,576	10,122,450	Ottawa, Brampton	13.53
	0.1,0.9	7,617,576	10,122,450	Ottawa, Brampton	11.65
	0,1	7,623,159	10,120,578	Ottawa, Brampton	14.64
	1,0	8,724,390	7,722,761	Ottawa, Vaughan, Woodstock	6.36
	0.9,0.1	8,724,390	7,722,761	Ottawa, Vaughan, Woodstock	6.76
	0.8,0.2	8,726,160	7,668,744	Ottawa, Vaughan, Woodstock	5.06
	0.7,0.3	8,726,160	7,668,744	Ottawa, Vaughan, Woodstock	5.48
	0.6,0.4	8,726,160	7,668,744	Ottawa, Vaughan, Woodstock	6.86
3	0.5,0.5	8,726,160	7,668,744	Ottawa, Vaughan, Woodstock	6.31
	0.4,0.6	8,774,484	7,442,406	Ottawa, Mississauga, Windsor	7.84
	0.3,0.7	8,838,735	7,288,989	Ottawa, Brampton, Windsor	7.26
	0.2,0.8	8,838,735	7,288,989	Ottawa, Brampton, Windsor	8.42
	0.1,0.9	8,949,811	7,180,639	Ottawa, Brampton, Sarnia	6.69
	0,1	10,544,208	7,163,250	Ottawa, Brampton, Sault Ste. Marie	5.97
	1,0	10,208,394	5,656,944	Ottawa, Vaughan, Windsor, Cambridge	3.26
	0.9,0.1	10,208,394	5,656,944	Ottawa, Vaughan, Windsor, Cambridge	3.14
	0.8,0.2	10,208,394	5,656,944	Ottawa, Vaughan, Windsor, Cambridge	3.17
	0.7,0.3	10,208,498	5,655,389	Ottawa, Vaughan, Windsor, Cambridge	3.06
	0.6,0.4	10,208,498	5,655,389	Ottawa, Vaughan, Windsor, Cambridge	3.20
4	0.5,0.5	10,209,301	5,650,033	Ottawa, Vaughan, Windsor, Brant	3.09
	0.4,0.6	10,209,301	5,650,033	Ottawa, Vaughan, Windsor, Brant	2.66
	0.3,0.7	10,214,550	5,632,072	Ottawa, Vaughan, Windsor, Brant	2.97
	0.2,0.8	10,214,550	5,632,072	Ottawa, Vaughan, Windsor, Brant	2.97
	0.1,0.9	10,221,115	5,627,883	Ottawa, Vaughan, Windsor, Brant	2.95
	0,1	11,528,612	5,627,761	Ottawa, Vaughan, Brant, Sault Ste. Marie	3.73

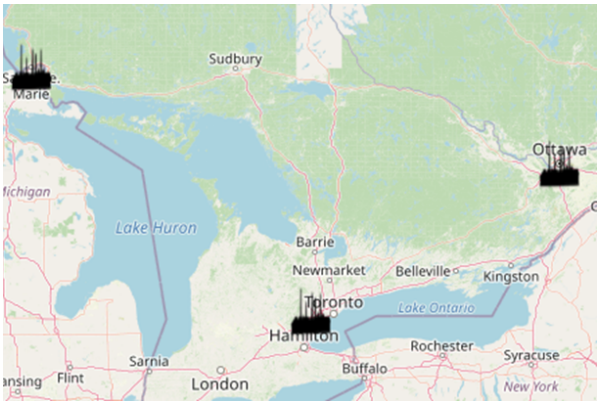
Table 2 shows that when one or two facilities are located, the difference in total costs for all α values are small. For example, when $p = 1$, the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is \$21,420. When $p = 2$, the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is \$53,663. There is a sharp increase in the difference in total cost as α decreases when three or four facilities are built. For example, when $p = 3$, the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is about \$1,819,818. When $p = 4$, the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is about \$1,320,218.

Table 2 shows that the lowest total cost is achieved when one facility is built. Building multiple facilities increases the total cost because the fixed cost of building a facility (2 million) is substantially higher than the transportation cost (\$0.00005696 kg/km). Table 2 shows that there is an inverse relationship between the number of facilities and the amount of released emissions. As the number of facilities increases, emissions decrease because the transportation distance traveled decreases.

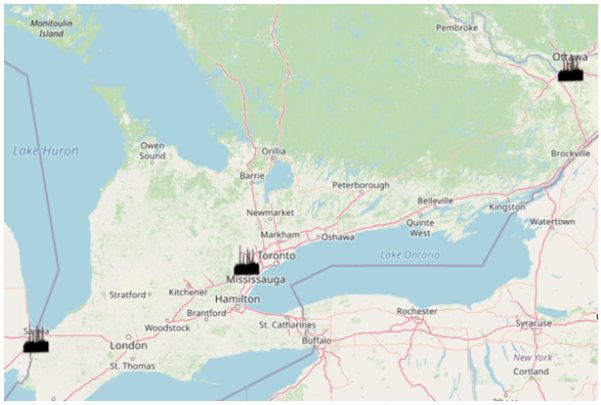
Table 2 shows that the model locates the remanufacturing facilities in the GTA area because of the clustering of collection centers (see Figure 2). If the company only plans to build one facility in Ontario, it should be located in either Mississauga or Brampton (both located in the GTA). By investing approximately \$21,420 and locating the facility in Brampton the company can save approximately 332,228 kg of CO_2e yearly. If building more than one facility in the long run is anticipated, then the company should first place a facility in Ottawa, followed by another facility in the GTA. Table 2 shows that Ottawa is always selected when two or more facilities are located.

Figures 3 and 4 (based on Table 2) illustrate the optimal locations produced by solving the deterministic model when three or four facilities are located. Figure 3 shows that the selected locations do not change significantly if some weight is placed on the cost objective. The trend of Ottawa, a location in the GTA, and a location near the US border are recurrent. Figure 4 shows that the selected locations do not change significantly. The trend of Ottawa, a location in the GTA, a location between London and the GTA, and a location near the US border are recurrent.

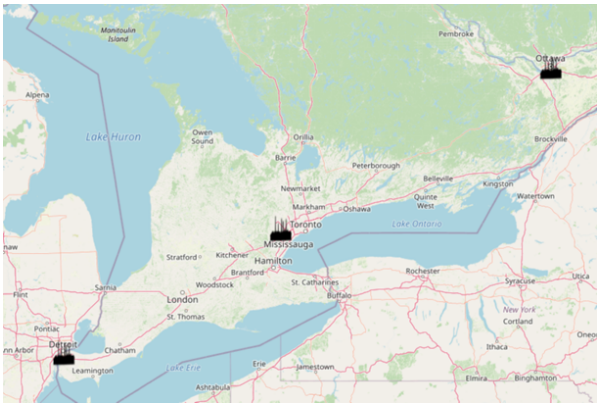
Figure 5 shows four trade-off curves based on Table 2. Each subfigure displays non-dominated solutions representing the trade-off between cost and emissions, where the x-axis shows the total cost (in millions of \$) and the y-axis shows the total emissions (in millions of kg of CO_2e). Figure 5a displays the two non-dominated solutions: Mississauga



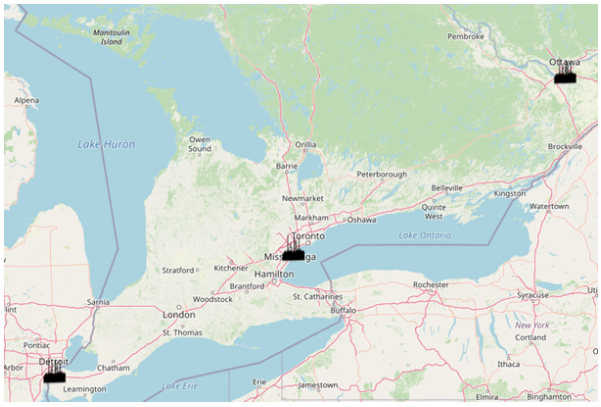
(a) Ottawa, Brampton, and Sault Ste. Marie ($\alpha = 0$)



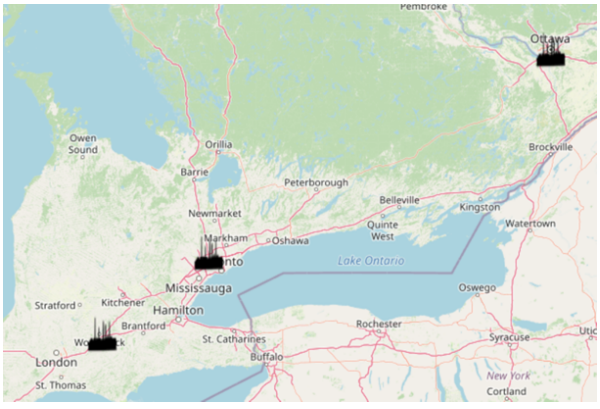
(b) Ottawa, Brampton, and Sarnia ($\alpha = 0.1$)



(c) Ottawa, Brampton, and Windsor ($\alpha = 0.2, 0.3$)

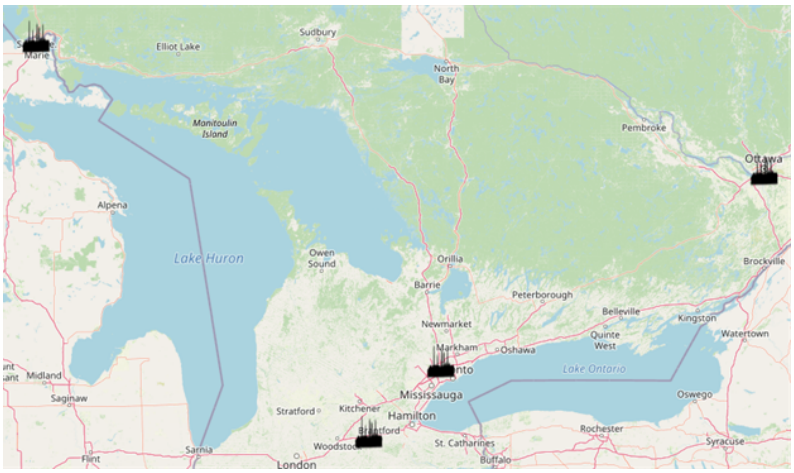


(d) Ottawa, Mississauga, and Windsor ($\alpha = 0.4$)

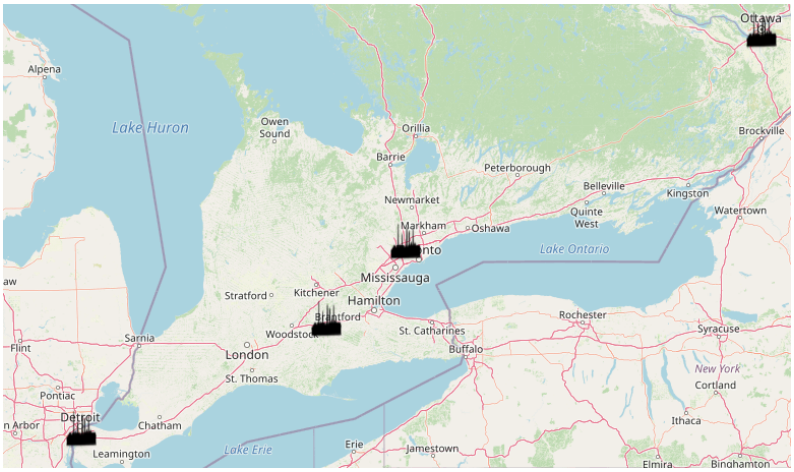


(e) Ottawa, Vaughan, and Woodstock ($\alpha = 0.5 - 1$)

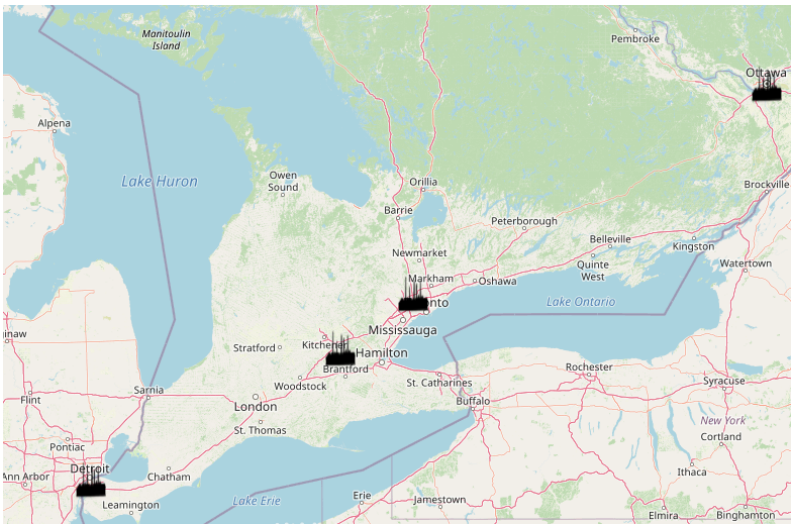
Figure 3: Optimal Facility Locations Under Different Values of α with $p = 3$ Using the Base Model



(a) Ottawa, Vaughan, Sault Ste. Marie, and Brant ($\alpha = 0$)

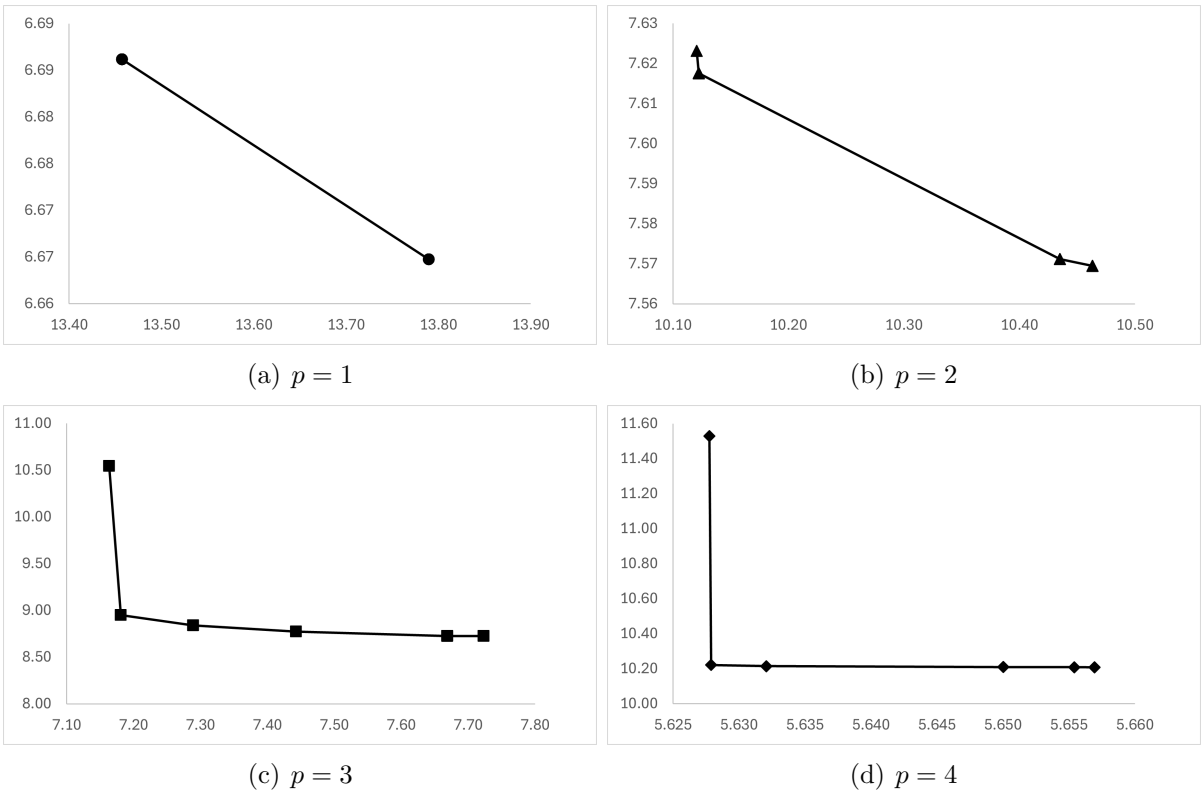


(b) Ottawa, Vaughan, Windsor, and Brant ($\alpha = 0.1 - 0.5$)



(c) Ottawa, Vaughan, Windsor, and Cambridge ($\alpha = 0.6 - 1$)

Figure 4: Optimal Facility Locations Under Different Values of α with $p = 4$ Using the Base Model



(a) $p = 1$ (b) $p = 2$

(c) $p = 3$ (d) $p = 4$

Figure 5: Trade-off Curves Based on the Deterministic Base Model: x-axis total cost (in millions of \$) and y-axis total emissions (in millions of kg of CO_2e).

and Brampton. Figure 5b depicts the two non-dominated solutions: Ottawa and Mississauga and Ottawa and Brampton as four points because changing the flow of parts does not change the locations but changes the emissions and transportation costs. The same trend may be observed with $p = 3$ and $p = 4$ in Figures 5c and 5d.

As can be seen in Figure 5, building one facility ($p = 1$) has the lowest total cost because the fixed cost of building a facility (2 million plus the land cost adjustment) outweighs the savings from the transportation cost. The plateau in the trade-off curves in Figures 5c and 5d demonstrates that an increase in spending does not correlate linearly with a decrease in emissions. Therefore, an α value between 0.6 and 0.4 provides a solution where emissions are reduced without extreme expenses.

When three facilities are opened ($p = 3$), the Pareto-optimal solutions that significantly improve emissions without significant costs correspond to the following locations: Ottawa, Brampton, and Sarnia; Ottawa, Brampton, and Windsor; and Ottawa, Mississauga, and Windsor (see Figure 3b, c, and d). For four facilities ($p = 4$), the Pareto-optimal solutions that substantially reduce emissions while maintaining economic viability

are Ottawa, Vaughan, Windsor, and Cambridge, as well as Ottawa, Vaughan, Windsor, and Brant (see Figure 4b and c).

5.2 Stochastic Model

The objective function equation (41) from Section 3.5 is used. This objective function incorporates weights to observe the locations selected based on how important cost-effectiveness or environmental sustainability is to the decision maker.[5] For this initial analysis, Constraint (9) is included to restrict the number of facilities to be located to p.

Table 3 presents the results of the Base model (deterministic). Table 3 contains the p value, which represents the number of facilities that should be located. The objective weights are reported. The total cost of the solution and the total emissions released by the solution are indicated. The locations of the facilities are indicated. The runtime of the model is indicated in the rightmost column.

As Table 2 and Table 3 demonstrate, for each value of p, there is no significant increase in cost as the cost objective weight (α) increases. Table 3 differs from Table 2 when $p = 4$ and $\alpha = 0$. Ottawa, Vaughan, Brant, Sault Ste. Marie is the solution in the base model versus Ottawa, Vaughan, Windsor, Brant is the solution in the stochastic model. When $p = 4$ the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is about \$20,250 (which is significantly smaller than the difference in Table 2).

Table 3 shows that the maximum emissions difference occurs when three facilities ($p = 3$) are built (approximately 0.56 kg of CO_2e). If one, two, or four facilities are built, the difference is approximately 0.332, 0.342, and 0.03 kilotons of CO_2e, respectively. A slight change in cost can reduce emissions. However, in order to minimize emissions, a much greater investment may be required.

Table 3 shows that when α decreases and when $p = 1$, $p = 2$, and $p = 3$, the model replaces Mississauga with Brampton. Although Brampton and Mississauga have the same facility cost, the emissions for the routes to and from Mississauga are higher. When $p = 3$, it can be observed that Vaughan is replaced by Mississauga as α decreases. Vaughan has a lower facility cost than Mississauga but has higher emissions to and from collection centers and manufacturers. When $p = 4$, as α decreases, Cambridge is replaced with

[5]Supply is assumed to follow a normal distribution with a 10% variation around the mean, and the deterministic equivalent of the stochastic model is solved using ten scenarios, each occurring with equal probability.

Table 3: Results with the Stochastic Model

p	Objective Weights $(\alpha, 1-\alpha)$	Total Cost ($)	Transportation Emissions (kg of CO_2e)	Optimal Location(s)	Gurobi Solution Time (sec)
	1,0	6,664,775	13,789,707	Mississauga	1.87
	0.9,0.1	6,664,775	13,789,707	Mississauga	1.59
	0.8,0.2	6,664,775	13,789,707	Mississauga	1.50
	0.7,0.3	6,686,190	13,457,468	Brampton	1.54
	0.6,0.4	6,686,190	13,457,468	Brampton	1.71
1	0.5,0.5	6,686,190	13,457,468	Brampton	1.94
	0.4,0.6	6,686,190	13,457,468	Brampton	1.85
	0.3,0.7	6,686,190	13,457,468	Brampton	1.83
	0.2,0.8	6,686,190	13,457,468	Brampton	1.88
	0.1,0.9	6,686,190	13,457,468	Brampton	1.68
	0,1	6,686,190	13,457,468	Brampton	1.74
	1,0	7,569,508	10,463,268	Ottawa, Mississauga	728.55
	0.9,0.1	7,569,508	10,463,268	Ottawa, Mississauga	1,247.69
	0.8,0.2	7,569,508	10,463,268	Ottawa, Mississauga	803.16
	0.7,0.3	7,571,200	10,435,073	Ottawa, Mississauga	606.72
	0.6,0.4	7,571,200	10,435,073	Ottawa, Mississauga	1,030.46
2	0.5,0.5	7,617,583	10,122,478	Ottawa, Brampton	1,041.32
	0.4,0.6	7,617,583	10,122,478	Ottawa, Brampton	1,209.41
	0.3,0.7	7,617,583	10,122,478	Ottawa, Brampton	205.87
	0.2,0.8	7,617,583	10,122,478	Ottawa, Brampton	214.08
	0.1,0.9	7,617,583	10,122,478	Ottawa, Brampton	202.29
	0,1	7,623,166	10,120,603	Ottawa, Brampton	105.49
	1,0	8,724,399	7,722,788	Ottawa, Vaughan, Woodstock	1,119.30
	0.9,0.1	8,724,399	7,722,788	Ottawa, Vaughan, Woodstock	921.41
	0.8,0.2	8,726,169	7,668,768	Ottawa, Vaughan, Woodstock	980.46
	0.7,0.3	8,726,169	7,668,768	Ottawa, Vaughan, Woodstock	1,270.07
	0.6,0.4	8,726,169	7,668,768	Ottawa, Vaughan, Woodstock	1,278.39
3	0.5,0.5	8,726,169	7,668,768	Ottawa, Vaughan, Woodstock	1,497.28
	0.4,0.6	8,774,487	7,442,405	Ottawa, Mississauga, Windsor	1,546.25
	0.3,0.7	8,838,734	7,288,978	Ottawa, Brampton, Windsor	1,486.99
	0.2,0.8	8,838,734	7,288,978	Ottawa, Brampton, Windsor	1,250.01
	0.1,0.9	8,949,811	7,180,638	Ottawa, Brampton, Sarnia	2,345.95
	0,1	10,544,228	7,163,239	Ottawa, Brampton, Sault Ste. Marie	1,134.60
	1,0	10,208,393	5,656,931	Ottawa, Vaughan, Windsor, Cambridge	474.82
	0.9,0.1	10,208,393	5,656,931	Ottawa, Vaughan, Windsor, Cambridge	323.94
	0.8,0.2	10,208,393	5,656,931	Ottawa, Vaughan, Windsor, Cambridge	321.60
	0.7,0.3	10,208,497	5,655,381	Ottawa, Vaughan, Windsor, Cambridge	306.37
	0.6,0.4	10,208,497	5,655,381	Ottawa, Vaughan, Windsor, Cambridge	281.93
4	0.5,0.5	10,209,301	5,650,033	Ottawa, Vaughan, Windsor, Brant	323.81
	0.4,0.6	10,209,301	5,650,033	Ottawa, Vaughan, Windsor, Brant	320.59
	0.3,0.7	10,214,550	5,632,072	Ottawa, Vaughan, Windsor, Brant	328.24
	0.2,0.8	10,214,550	5,632,072	Ottawa, Vaughan, Windsor, Brant	240.96
	0.1,0.9	10,221,117	5,627,880	Ottawa, Vaughan, Windsor, Brant	390.11
	0,1	10,228,643	5,627,757	Ottawa, Vaughan, Windsor, Brant	415.97

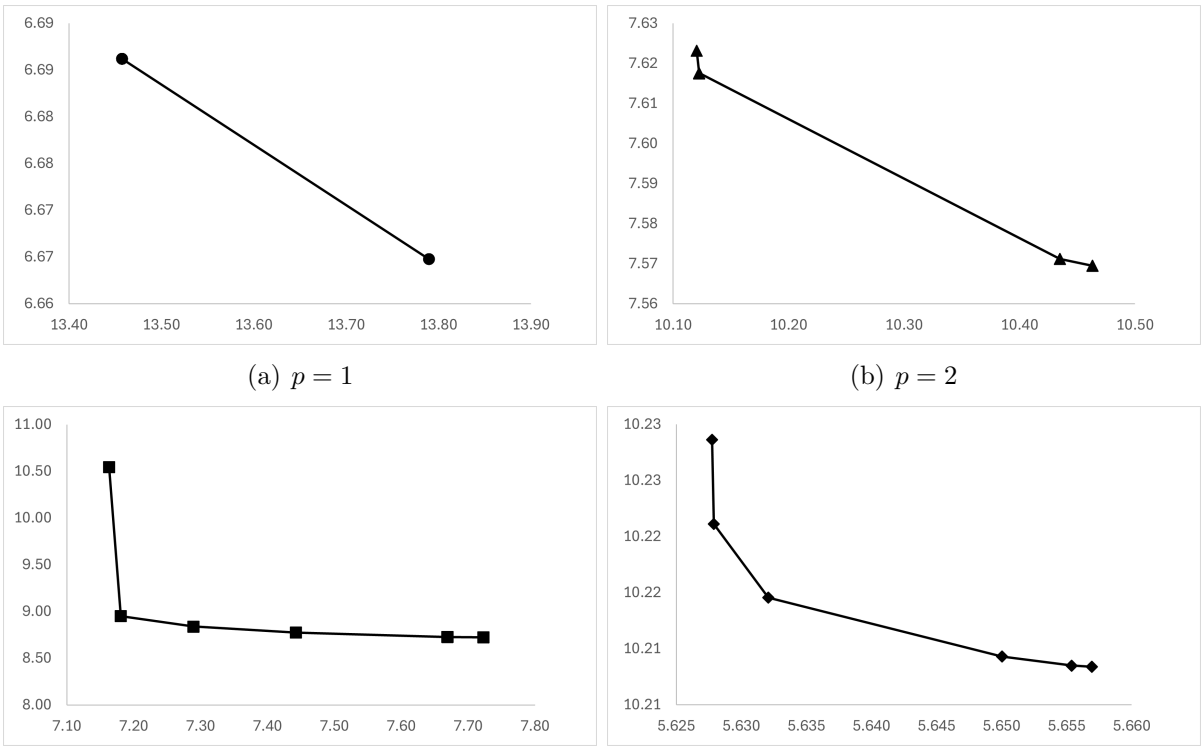
Brant because the emissions for the routes to and from Cambridge are higher. As shown above (see Table 2), if a company chooses to open one facility, it should be in the GTA, and if a company chooses to open multiple facilities, one should be in Ottawa and another in the GTA.

There is a considerable variation in the solution times for the stochastic model. The minimum solution time observed was approximately 1.5 seconds. The maximum solution time was 39.1 minutes. The average solution time was around 9.8 minutes. The variation reflects the computational complexity. The long solution times may be due to the number of decision variables. Despite the variation, these times remain reasonable given the strategic nature of the problem.

The stochastic and base models yield the same optimal locations for the majority of instances. This demonstrates that the base model (deterministic) is a good approximation for the stochastic model when there is a small supply variation (10%). The only difference between Table 2 and Table 3 occurs when $p = 4, \alpha = 0$, where Ottawa, Vaughan, Brant, Sault Ste. Marie is the solution in the base model versus Ottawa, Vaughan, Windsor, Brant is the solution in the stochastic model. When $p = 4$ and $\alpha = 0$, the base model solution total cost is worse than the stochastic model by \$1.3 million, but the emissions for the solutions are identical. This is likely due to the random scenarios taken in the stochastic model.

Figure 6 shows four trade-off curves. Figure 6a displays two non-dominated solutions: Mississauga and Brampton. Figure 6b shows four non-dominated solutions corresponding to two facility locations: Ottawa and Mississauga, and Ottawa and Brampton. Although the facility locations remain unchanged in some of the non-dominated solutions, variations in transportation routes impact both cost and emissions as placing greater weight on cost results in the use of less environmentally friendly routes.

Figure 6 shows that locating one facility ($p = 1$) is optimal in terms of cost because the transportation cost is substantially lower than the fixed cost of building a facility. Increasing the number of facilities decreases the emissions (locating four facilities is optimal for minimum emissions values). The sharp decreases show that small investments in emissions reductions may be beneficial, but the plateau shows that the trend is not linear (more investment does not result in an equal reduction in emissions). Figure 6 shows that the objective cost weights (α) of approximately 0.6 to 0.4 should be considered because small investment results in significant improvements in reducing GHG emissions.



(a) $p = 1$ (b) $p = 2$

(c) $p = 3$ (d) $p = 4$

Figure 6: Trade-off Curves with the Stochastic Model: x-axis total cost (in millions of \$) and y-axis total emissions (in millions of kg of CO_2e).

When three facilities are located ($p = 3$), the Pareto front solutions that significantly improve emissions without significant costs coincide with those of the deterministic model (see Figure 3). When four facilities are located ($p = 4$), these solutions correspond to the locations Ottawa, Vaughan, Windsor, and Cambridge as well as Ottawa, Vaughan, Windsor, and Brant (see Figures 4a and 4b).

5.3 Sensitivity Analysis

This section provides a sensitivity analysis. The analysis consists of perturbing the base facility cost, transportation cost, and supply variation. This section presents the results for the stochastic model. It offers an assessment of the differences between the results of the base and stochastic models. Constraint (9) is removed to determine the optimal number of facilities that should be located.

The sensitivity analysis is conducted with the following cost objective weight values (α): 1, 0.75, 0.5, and 0.25. Cost objective weight values (α) below 0.25 are not considered because lower values of α do not result in significant additional emissions savings and when only the emissions objective is considered in the model ($\alpha = 0$) with no maximum number of facilities, the optimal solution opens all potential remanufacturing facilities, which is unrealistic.

5.3.1 Effect of Facility Cost

The cost of the facility is adjusted, from one to five million dollars, in order to observe the effect that the base cost of operating a facility has on the locations of the remanufacturing facilities and the overall cost of operating the network. The rows with two million as the base facility cost in Table 4 correspond to the results provided in Table 3. The sensitivity analysis on the fluctuation of the facility cost with the base and stochastic models produces the same solutions and similar total cost and transportation emissions (which can be accounted for through the generation of scenarios for the stochastic solution). The leftmost column contains the objective weights. The base facility cost is recorded. The total cost and total emissions are included. The optimal locations for the facilities are listed. The rightmost column shows the runtime.

Table 4 shows that as the fixed cost to locate a facility increases, the number of facilities located by the model decreases. As the weight of the cost objective function decreases (decreasing α), the total cost increases. The difference in total cost increases by approximately \$0.63, \$2.17, \$0.03, \$0.04, and \$0.05 million when comparing $\alpha = 1$ and $\alpha = 0.25$ when fixed cost is one, two, three, four, and five million respectively. The difference in base fixed cost of a facility causes a larger fluctuation in the total cost of the solution when the cost objective is weighted more. The difference in total cost increases by approximately \$4.15, \$3.99, \$4.04, and \$3.56 million when comparing fixed cost one

Table 4: Effect of Facility Cost on the Resulting Solutions

Objective Weights $(\alpha, 1-\alpha)$	**Base Cost (\$ in mil)**	**Total Cost (\$)**	**Transportation Emissions (kg of CO_2e)**	**Optimal Locations**	**Gurobi time (sec)**
1,0	**1**	5,545,302	10,463,268	Ottawa, Mississauga	795.64
	2	6,664,775	13,789,707	Mississauga	1,619.36
	3	7,676,590	13,821,396	Vaughan	1,415.73
	4	8,684,797	13,821,396	Vaughan	55.96
	5	9,693,005	13,821,396	Vaughan	38.75
0.75,0.25	**1**	5,698,059	7,668,768	Ottawa, Vaughan, Woodstock	3,093.85
	2	6,664,775	13,789,707	Mississauga	1,586.36
	3	7,676,590	13,821,396	Vaughan	1,273.10
	4	8,684,797	13,821,396	Vaughan	76.26
	5	9,693,005	13,821,396	Vaughan	50.56
0.5,0.5	**1**	5,698,059	7,668,768	Ottawa, Vaughan, Woodstock	1,823.43
	2	6,686,190	13,457,468	Brampton	949.15
	3	7,704,125	13,457,468	Brampton	1,160.77
	4	8,722,060	13,457,468	Brampton	1,614.38
	5	9,739,994	13,457,468	Brampton	88.39
0.25, 0.75	**1**	6,175,688	5,637,420	Ottawa, Vaughan, Windsor, Cambridge	621.86
	2	8,838,734	7,288,978	Ottawa, Brampton, Windsor	688.18
	3	7,704,125	13,457,468	Brampton	822.41
	4	8,722,060	13,457,468	Brampton	3,088.03
	5	9,739,994	13,457,468	Brampton	758.57

million and five million when the weight of the objective function is $\alpha = 1$, $\alpha = 0.75$, $\alpha = 0.5$, and $\alpha = 0.25$.

Table 4 shows that as the number of facilities decreases, the emissions increase. The solution with the lowest emissions has four opened facilities ($\alpha = 0.25$ and the base cost is one million dollars). All solutions that produce the same number of facilities opened yield similar emissions. For example, all solutions with three facilities opened (when $\alpha = 0.75$ and fixed cost is one million, $\alpha = 0.5$ and fixed cost is one million, $\alpha = 0.25$ and fixed cost is two million) result in emissions of approximately 7 kilotons of CO_2e.

Table 4 confirms that when only one facility is opened, it is located in the GTA due to the proximity to the majority of collection centers and manufacturers (see Figure 2). Table 4 displays the same trend that opening multiple facilities results in one facility in Ottawa and another in the GTA.

5.3.2 Effect of Transportation Cost

The transportation cost is perturbed in order to observe the effect that the cost of using an arc in the network has on the location(s) of the remanufacturing facilities and the overall cost of operating the network. The transportation cost may fluctuate due to gas

prices, tariffs, and workers' wages. The transportation cost of \$0.00005696 per kg-km is increased/decreased by up to 50%. The rows with 0% in Table 5 correspond to the results provided in Table 3. The sensitivity analysis on the fluctuation of the facility cost with the base and stochastic models produces the same solutions and similar total cost and transportation emissions (which can be accounted for by the generation of scenarios for the stochastic solution). The leftmost column of Table 5 contains the objective weights. The percentage of the increase/decrease of the transportation cost is recorded. The total cost and total emissions are recorded. The optimal locations of the facilities are listed. The rightmost column shows the runtime.

Table 5: Effect of Transportation Cost Variations on the Solutions

Objective Weights $(\alpha, 1-\alpha)$	**Transportation Cost Perturbation**	**Total Cost (\$)**	**Transportation Emissions (kg of CO_2e)**	**Optimal Locations**	**Gurobi time (sec)**
	50% dec	4,342,398	13,821,396	Vaughan	506.85
	25 % dec	5,505,390	13,821,396	Vaughan	1,324.88
	10% dec	6,201,885	13,789,707	Mississauga	2,267.04
1,0	**0%**	6,664,775	13,789,707	Mississauga	1,622.03
	10% inc	7,127,666	13,789,707	Mississauga	5,119.18
	25% inc	7,822,002	13,789,707	Mississauga	2,949.09
	50% inc	8,979,228	13,789,707	Mississauga	299.32
	50% dec	4,361,030	13,457,468	Brampton	105.75
	25 % dec	5,523,610	13,457,468	Brampton	3,413.17
	10% dec	6,221,158	13,457,468	Brampton	3,996.78
0.75,0.25	**0%**	6,664,775	13,789,707	Mississauga	1,594.42
	10% inc	7,127,666	13,789,707	Mississauga	2,273.69
	25% inc	7,822,002	13,789,707	Mississauga	1,158.47
	50% inc	8,979,228	13,789,707	Mississauga	722.81
	50% dec	4,361,030	13,457,468	Brampton	2,359.28
	25 % dec	5,523,610	13,457,468	Brampton	2,572.14
	10% dec	6,221,158	13,457,468	Brampton	496.39
0.5,0.5	**0%**	6,686,190	13,457,468	Brampton	945.57
	10% inc	7,151,222	13,457,468	Brampton	567.05
	25% inc	7,848,770	13,457,468	Brampton	816.21
	50% inc	9,332,594	10,435,073	Ottawa, Mississauga	903.16
	50% dec	5,832,998	10,122,478	Ottawa, Brampton	437.77
	25 % dec	8,146,624	7,288,978	Ottawa, Brampton, Windsor	669.45
	10% dec	8,561,890	7,288,978	Ottawa, Brampton, Windsor	4,285.50
0.25,0.75	**0%**	8,838,734	7,288,978	Ottawa, Brampton, Windsor	686.79
	10% inc	9,115,578	7,288,978	Ottawa, Brampton, Windsor	1,283.10
	25% inc	9,530,844	7,288,978	Ottawa, Brampton, Windsor	588.02
	50% inc	10,126,584	7,442,405	Ottawa, Mississauga, Windsor	1,781.53

Table 5 demonstrates that the fluctuation in transportation cost has a significant effect on the location and number of facilities and, subsequently, the total cost of the solution. When the weight on the cost objective is higher ($\alpha = 1$), decreasing the transportation

cost significantly lowers the total cost (by $2.32 million). Increasing the transportation cost significantly increases the total cost (by $2.31 million). When the weight of the emissions objective is higher ($\alpha = 0.25$), decreasing the transportation cost significantly lowers the total cost (by $3.01 million). Increasing the transportation cost significantly increases the total cost (by $1.29 million). When the weight of the cost objective is equal to the weight of the emissions objective ($\alpha = 0.5$), decreasing the transportation cost lowers the total cost (by $2.33 million). Increasing the transportation cost increases the total cost (by $2.65 million).

Table 5 demonstrates that when the weight of the cost objective is higher ($\alpha = 1$), decreasing the transportation cost increases the emissions (by 0.031 kiloton of CO_2e compared to 0% perturbation). Increasing the transportation cost does not change the emissions. When the emissions objective function is weighted higher ($\alpha = 0.25$), decreasing the transportation cost increases the emissions significantly (by 2.83 kiloton of CO_2e compared to 0% perturbation) because the model locates one less facility. Increasing the transportation cost increases the emissions (by 0.153 kiloton of CO_2e compared to 0% perturbation) because the cost of the model increases substantially which causes less emissions efficient paths to be taken (Mississauga). When the weight of the cost objective is equal to the weight of the emissions objective ($\alpha = 0.5$), decreasing the transportation cost does not impact the emissions. Increasing the transportation cost significantly decreases the emissions (by 3.02 kiloton of CO_2e compared to 0% perturbation).

Table 5 demonstrates that locating one facility is the most beneficial. The common locations selected by the model are Brampton (15 out of 28 solutions) and Mississauga (11 out of 28 solutions). If an increase in transportation costs is likely, then Mississauga should be selected. It can again be observed that if multiple facilities are located, then the facility locations that are selected are typically the first facility is in Ottawa, the second in the GTA region, and the third near the US border.

5.3.3 Effect of Supply Variation

The variation (distribution) of the supply of parts is perturbed to simulate the effects that uncertainty of the number of ELVs has on the location(s) of the remanufacturing facilities and the overall cost of operating the network. This section first presents the base model and then the stochastic results because the supply variation perturbation is

expressed differently.

The effect of supply variation on the base model is observed by increasing and decreasing the supply of parts in kg by up to 50% in increments of 10%. The first column of Table 6 is the objective weights. The percentage increase/decrease of the supply in kilograms of parts is indicated. The rows with "Mean" in Table 6 correspond to the results provided in Table 2. The total cost and total emissions are indicated. The optimal locations of the facilities are listed. The rightmost column shows the runtime.

Table 6 shows that the fluctuation on supply has a large effect on the total cost of the solution. If the cost objective function is weighted more ($\alpha = 1, 0.75$), then the difference between the mean and 50% increase and the mean and 50% decrease is \$2.31 million and \$2.32 million respectively. If the cost objective function and emissions objective are weighted equally ($\alpha = 0.5$), then the difference between the mean and 50% increase and the mean and 50% decrease is \$3.37 million and \$2.33 million respectively. If the emissions objective function is weighted more ($\alpha = 0.25$), then the difference between the mean and 50% increase and the mean and 50% decrease is \$2.44 million and \$4.48 million respectively. This shows that fluctuations in supply has a significant effect on cost within the same objective weight due to transportation costs. The difference in total cost increases by approximately \$2.3, \$2.17, and \$0.02 million when comparing $\alpha = 1$ and $\alpha = 0.25$ for a supply change of 50% increase, no change (0%), and 50% decrease. This shows that an increase in the supply has a significant effect on the total cost due to the number of facilities built.

Table 6 shows that the supply has a significant effect on emissions because the number of trucks that are needed increases as the supply increases, which significantly increases the GHG emissions released by transportation. If the cost objective function is weighted more ($\alpha = 1$), then the difference between the mean and 50% increase and the mean and 50% decrease is 6.89 and 6.88 kiloton of CO_2e respectively. If the cost objective function and emissions objective are weighted equally ($\alpha = 0.5$), then the difference between the mean and 50% increase and the mean and 50% decrease is 1.95 and 6.73 kiloton of CO_2e respectively. If the emissions objective are is weighted more ($\alpha = 0.25$), then the difference between the mean and 50% increase and the mean and 50% decrease is 1.17 and 0.56 kiloton of CO_2e respectively. The difference in emissions decreases by 12.23, 6.5, and 0.18 kiloton of CO_2e when comparing $\alpha = 1$ and $\alpha = 0.25$ for a supply change of 50% increase, no change (mean), and 50% decrease respectively. This shows that the number

Table 6: Effect of the Variation in Supply with the Deterministic Model

Objective Weights $(\alpha, 1-\alpha)$	**Supply Variation**	**Total Cost ($)**	**Transportation Emissions (kg of CO_2e)**	**Optimal Location(s)**	**Gurobi Solution Time (sec)**
	50% inc	8,979,185	20,684,433	Mississauga	7.33
	40% inc	8,516,297	19,305,467	Mississauga	9.60
	30 % inc	8,053,410	17,926,506	Mississauga	43.96
	20 % inc	7,590,522	16,547,544	Mississauga	2.13
	10% inc	7,127,634	15,168,583	Mississauga	19.98
1,0	**Mean**	6,664,747	13,789,622	Mississauga	4.32
	10% dec	6,201,859	12,410,661	Mississauga	16.58
	20% dec	5,737,967	11,057,050	Vaughan	8.93
	30% dec	5,272,773	9,674,922	Vaughan	1.80
	40% dec	4,807,579	8,292,789	Vaughan	15.93
	50% dec	4,342,385	6,910,656	Vaughan	10.41
	50% inc	8,979,185	20,684,433	Mississauga	6.82
	40% inc	8,516,297	19,305,467	Mississauga	8.73
	30 % inc	8,053,410	17,926,506	Mississauga	13.66
	20 % inc	7,590,522	16,547,544	Mississauga	1.59
	10% inc	7,127,634	15,168,583	Mississauga	7.90
0.75,0.25	**Mean**	6,664,747	13,789,622	Mississauga	16.41
	10% dec	6,201,859	12,410,661	Mississauga	16.73
	20% dec	5,738,971	11,031,694	Mississauga	14.52
	30% dec	5,272,773	9,674,922	Vaughan	10.10
	40% dec	4,807,579	8,292,789	Vaughan	10.36
	50% dec	4,342,385	6,910,656	Vaughan	18.51
	50% inc	10,061,130	11,503,111	Ottawa, Vaughan, Woodstock	14.15
	40% inc	9,045,240	14,171,433	Ottawa, Brampton	15.82
	30 % inc	8,688,324	13,159,189	Ottawa, Brampton	18.04
	20 % inc	8,331,408	12,146,944	Ottawa, Brampton	11.67
	10% inc	7,151,196	14,803,133	Brampton	7.03
0.5,0.5	**Mean**	6,686,167	13,457,394	Brampton	22.53
	10% dec	6,221,137	12,111,656	Brampton	11.65
	20% dec	5,756,107	10,765,917	Brampton	12.61
	30% dec	5,291,077	9,420,178	Brampton	15.77
	40% dec	4,826,048	8,074,433	Brampton	9.53
	50% dec	4,361,018	6,728,694	Brampton	2.06
	50% inc	11,282,563	8,456,139	Ottawa, Vaughan, Windsor, Cambridge	10.26
	40% inc	11,072,461	7,884,900	Ottawa, Vaughan, Windsor, Brant	11.05
	30 % inc	9,669,268	9,475,683	Ottawa, Brampton, Windsor	10.45
	20 % inc	9,392,424	8,746,783	Ottawa, Brampton, Windsor	11.38
	10% inc	9,115,579	8,017,883	Ottawa, Brampton, Windsor	14.29
0.25,0.75	**Mean**	8,838,735	7,288,989	Ottawa, Brampton, Windsor	14.91
	10% dec	8,561,891	6,560,089	Ottawa, Brampton, Windsor	21.25
	20% dec	6,903,743	8,097,961	Ottawa, Brampton	21.71
	30% dec	6,546,827	7,085,717	Ottawa, Brampton	20.97
	40% dec	4,826,048	8,074,433	Brampton	20.14
	50% dec	4,361,018	6,728,694	Brampton	15.68

of facilities placed also has a significant effect on the emissions released.

Table 6 shows that when the cost objective function is weighted higher ($\alpha = 1, 0.75$), the number of facilities opened is one (Mississauga or Vaughan). If only the cost objective ($\alpha = 1$) is considered, then the same location (Mississauga) is valid for fluctuations ranging from 10% decrease to 50% increases. If the cost objective is equally important to the emissions objective ($\alpha = 0.5$), then the same location (Brampton) is valid for fluctuations ranging from 50% decrease to 10% increase. If the cost objective is weighted less than the emissions objective ($\alpha = 0.25$), then the same locations (Ottawa, Brampton, Windsor) is valid for fluctuations ranging from 10% decrease to 30% increases. Locating one facility in Mississauga is a common solution when only one facility is desired (this follows the same trends discussed above). When locating multiple facilities, the trend of locations demonstrates the same trends as discussed above (Ottawa, GTA, US border, and then a location between London and the GTA).

The effect of supply variation on the stochastic model is observed by increasing the supply variation around the mean from 10% to 50% in increments of 10%. The rows with 10% in Table 7 correspond to the results provided in Table 3. Table 7 indicates the objective weights for the objective function. The percentage of variation from the mean used to produce the scenarios is indicated. The total cost and transportation emissions are indicated. The optimal locations of the remanufacturing facilities are listed. The runtime for the model is in the rightmost column.

Table 7 shows that the solutions remain robust to fluctuations in supply regardless of the objective function weights.[6] The total cost does not change substantially due to fluctuations in supply within objective values. As the cost objective function weight decreases (from $\alpha = 1$ to $\alpha = 0.25$), the cost difference is \$2.17 million for all the supply variations of 10, 20, 30, 40, and 50 respectively.

Table 7 shows that the emissions are generally stable for all fluctuations in supply. The emissions are highly dependent on the number of facilities (the greater the number of opened facilities the lower the transportation emissions). When $\alpha = 0.25$, the solution obtained has three opened facilities, which results in the lowest transportation emissions.

Table 7 shows that as the weight assigned to the cost objective decreases, Mississauga is replaced by Brampton due to the lower emissions associated with routes to and from

[6]Note that the facilities are assumed to be uncapacitated in this analysis, which allows only one facility to be located regardless of the quantity of parts. The capacity of the additive manufacturing facility depends on the processing rate and quantity of machines.

Table 7: Effect of the Variation in Supply with the Stochastic Model

Objective Weights $(\alpha, 1-\alpha)$	**Supply variation (%)**	**Total Cost ($)**	**Transportation Emissions (kg of CO_2e)**	**Optimal Locations**	**Gurobi Solution time (sec)**
1,0	**10**	6,664,775	13,789,707	Mississauga	1,614.79
	20	6,664,688	13,789,431	Mississauga	4,233.43
	30	6,664,787	13,789,718	Mississauga	1,660.80
	40	6,664,761	13,789,665	Mississauga	1,603.90
	50	6,664,628	13,789,313	Mississauga	4,579.09
0.75,0.25	**10**	6,664,775	13,789,707	Mississauga	1,581.62
	20	6,664,688	13,789,431	Mississauga	1,249.18
	30	6,664,787	13,789,718	Mississauga	1,525.19
	40	6,664,761	13,789,665	Mississauga	2,851.06
	50	6,664,628	13,789,313	Mississauga	1,680.17
0.5,0.5	**10**	6,686,190	13,457,468	Brampton	1,050.23
	20	6,686,107	13,457,199	Brampton	940.59
	30	6,686,203	13,457,469	Brampton	1,330.75
	40	6,686,186	13,457,436	Brampton	1,338.47
	50	6,686,055	13,457,109	Brampton	719.69
0.25,0.75	**10**	8,838,734	7,288,978	Ottawa, Brampton, Windsor	714.63
	20	8,838,693	7,288,902	Ottawa, Brampton, Windsor	698.32
	30	8,838,784	7,289,083	Ottawa, Brampton, Windsor	554.03
	40	8,838,768	7,289,041	Ottawa, Brampton, Windsor	1,571.42
	50	8,838,680	7,288,853	Ottawa, Brampton, Windsor	850.36

Brampton. When the emissions objective is weighted more heavily than the cost objective, the number of open facilities increases to three. All solutions follow the same trend of choosing locations as observed in the previous analyses.

Chapter 6

Game Theory Model Results

The game theory model was run on a Dell with an Intel Core i5 7th Gen CPU, with 3.5 GHz and 8.00 GB of RAM, running Windows 11 operating system. Gurobi was used as a mixed-integer linear programming solver. All instances were solved to optimality using the default settings of Gurobi (the default value of this parameter is 0.01%), coded in Python.

6.1 Game Theory Model

The Nash equilibrium (a strategy where no players have incentive to deviate) is calculated based on the mixed-integer linear modeling problem presented in 3.6. The players strategy set for this case study is {open facility 0, ..., open facility 100, or stay out of the market}. The players calculate their payoff based on equation (42) for collection centers or equation (46) for manufacturers. In this case study the game outcomes are {open facility 0, ... , open facility 100, open facilities 0 & 1, ... , open all facilities, and do not open any facilities}.

Constraint (67) is included to restrict the number of facilities to be located to p.

Table 8 presents the results of the game theory model (deterministic). Table 8 contains the p value, which represents the number of facilities that should be located. The objective weights are reported. The total cost of the solution (made up of the transportation cost and the fixed cost for a facility) and the total emissions released (transportation emissions) by the solution are indicated. The locations of the facilities are indicated. The rightmost column contains Gurobi's solution time (in seconds).

Table 8: Results with Game Theory Model

p	Objective Weights $(\alpha, 1-\alpha)$	Total Cost ($)	Emissions (kg of CO_2e)	Optimal Location(s)	Gurobi Solution Time (sec)
1	**1,0**	6,664,746	13,789,622	Mississauga	1.23
	0.75,0.25	6,664,746	13,789,622	Mississauga	1.12
	0.5,0.5	6,686,166	13,457,394	Brampton	1.14
	0.25,0.75	6,686,166	13,457,394	Brampton	0.99
	0,1	6,686,166	13,457,394	Brampton	2.96
2	**1,0**	7,569,496	10,463,233	Ottawa, Mississauga	17.92
	0.75,0.25	7,569,496	10,463,233	Ottawa, Mississauga	18.43
	0.5,0.5	7,617,575	10,122,450	Ottawa, Brampton	20.63
	0.25,0.75	7,617,575	10,122,450	Ottawa, Brampton	20.03
	0,1	7,623,159	10,120,578	Ottawa, Brampton	17.89
3	**1,0**	8,635,007	7,444,950	Ottawa, Vaughan, Woodstock	11.43
	0.75,0.25	8,635,007	7,444,950	Ottawa, Vaughan, Woodstock	9.01
	0.5,0.5	8,635,007	7,444,950	Ottawa, Vaughan, Woodstock	9.7
	0.25,0.75	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	12.28
	0,1	10,731,532	7,091,817	Ottawa, Brampton, Sault Ste. Marie	11.67
4	**1,0**	10,128,021	5,415,007	Ottawa, Vaughan, Windsor, Brant	7.67
	0.75,0.25	10,128,021	5,415,007	Ottawa, Vaughan, Windsor, Brant	6.45
	0.5,0.5	10,128,021	5,415,007	Ottawa, Vaughan, Windsor, Brant	6.25
	0.25,0.75	10,133,269	5,397,044	Ottawa, Vaughan, Windsor, Brant	5.53
	0,1	10,149,347	5,393,596	Ottawa, Vaughan, Windsor, Brant	6.15

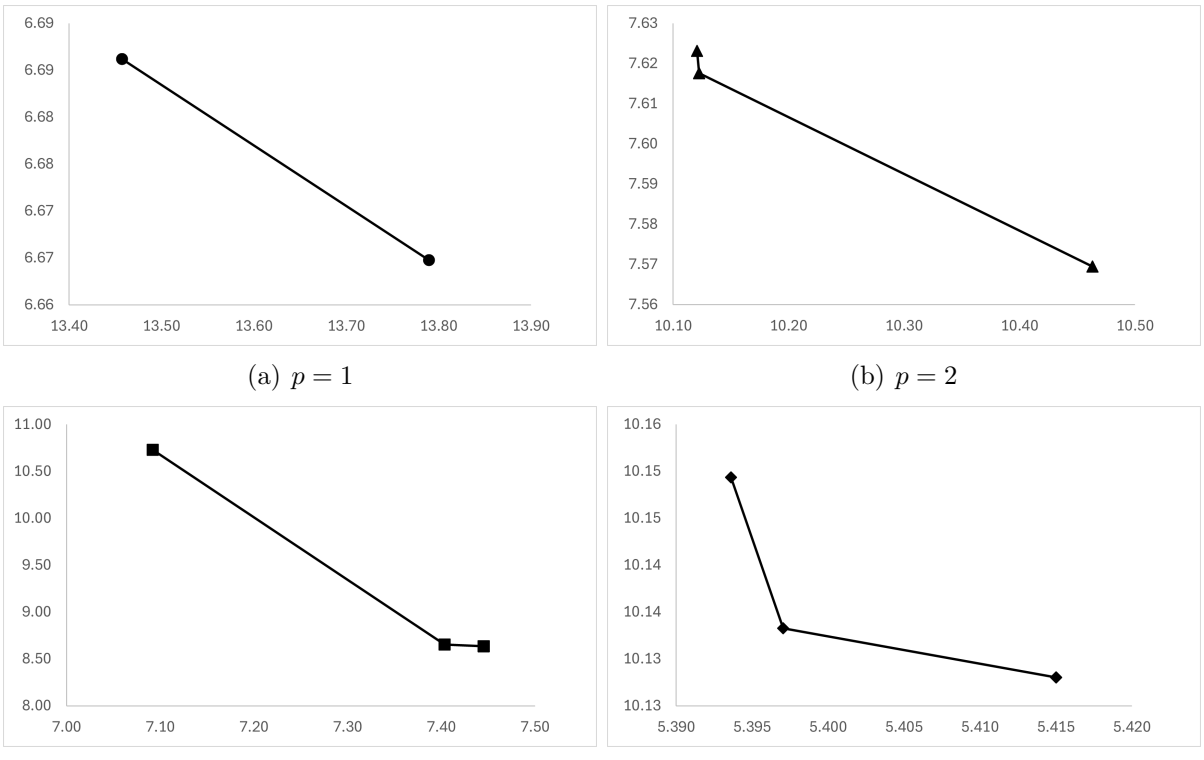
Table 8 shows that when one or two facilities are located, the difference in total costs for all α values are small. For example, when $p = 1$, the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is \$21,420. When $p = 2$, the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is \$53,663. There is a sharp increase in the difference in total cost as α decreases when three facilities are located. When $p = 1$, the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is approximately \$2.1 million. When $p = 4$, the location does not change but the selection of players who are assigned to the facility change, therefore, the difference in total cost between the solutions with $\alpha = 1$ and $\alpha = 0$ is \$21,326.

Table 8 shows that the emissions are highly dependent on the number of facilities placed. There is no significant drop in emissions between $\alpha = 1$ and $\alpha = 0$ when the number of facilities is constant (the differences are 0.33, 0.34, 0.35, and 0.021 kilotons of CO_2e for $p = 1$, $p = 2$, $p = 3$, and $p = 4$ respectively). There is a significant drop between opening one facility and four facilities (by approximately 8 kilotons of CO_2e).

The locations selected by the model follow the trends that were shown in the determin-

istic and stochastic models. When one or two facilities are opened the trend is identical to the trends in the deterministic and stochastic models (see Tables 2 and 3). When four facilities are selected ($p = 4$) the set of facilities opened is the same regardless of the objective weights.

When $p = 3$ and $p = 4$ and $\alpha = 1, 0.75, 0.5, 0.25, 0$, there are players who do not wish to participate in the market because their costs exceed the selling price multiplied by the supply that they would anticipate selling. The other players choose to remain in the market and the location indicated in Table 8 is still the Nash equilibrium solution.



(a) $p = 1$ (b) $p = 2$

(c) $p = 3$ (d) $p = 4$

Figure 7: Trade-off Curves with the Game Theory Model: x-axis total cost (in millions of \$) and y-axis total emissions (in millions of kg of CO_2e).

Figure 7 shows four trade-off curves based on Table 8. Each subfigure displays non-dominated solutions representing the trade-off between cost and emissions. Figure 7a displays the two non-dominated solutions: Mississauga and Brampton. Figure 7b depicts the two non-dominated solutions: Ottawa and Mississauga and Ottawa and Brampton as three points because changing the flow of parts does not change the locations but changes the emissions and transportation costs. The same trend may be observed with $p = 3$ and $p = 4$ in Figures 7c and 7d.

6.2 Sensitivity Analysis

The model is very sensitive to parameters. The factors affecting the equilibrium of the game are: facility cost, transportation cost, supply/demand, and cost of parts on the market. Each of the factors listed above is permutated while keeping the other factors stable to determine at which point the Nash equilibrium changes. Constraint (67) is removed in order to determine the optimal number of facilities to be located.

6.2.1 Effect of Facility Cost

The base facility cost is perturbed from one to five million dollars. The rows with 0% in Table 9 correspond to the results provided in Table 8. Table 9 contains the base facility cost, the location(s) selected, the total cost for all players, the emissions released, and the solution time for the model (in seconds).

Table 9: Effect of Facility Cost with the Game Theory Model

Objective Weights $(\alpha, 1-\alpha)$	**Fixed Cost (\$ in mil)**	**Total Cost (\$)**	**Emissions (kg of CO_2e)**	**Optimal Location(s)**	**Gurobi Solution Time (sec)**
1,0	**1**	5,545,290	10,463,233	Ottawa, Mississauga	36.09
	2	6,664,746	13,789,622	Mississauga	30.76
	3	7,676,563	13,821,317	Vaughan	25.3
	4	8,684,770	13,821,317	Vaughan	23.06
	5	9,692,978	13,821,317	Vaughan	11.97
0.75,0.25	**1**	5,606,897	7,444,950	Ottawa, Vaughan, Woodstock	20.48
	2	6,664,746	13,789,622	Mississauga	27.29
	3	7,676,563	13,821,317	Vaughan	48.14
	4	8,684,770	13,821,317	Vaughan	46.57
	5	9,692,978	13,821,317	Vaughan	37.47
0.5,0.5	**1**	5,606,897	7,444,950	Ottawa, Vaughan, Woodstock	32.1
	2	6,686,166	13,457,394	Brampton	62.56
	3	7,704,101	13,457,394	Brampton	65.97
	4	8,722,036	13,457,394	Brampton	55.46
	5	9,739,970	13,457,394	Brampton	47.08
0.25, 0.75	**1**	6,098,383	5,397,044	Ottawa, Vaughan, Windsor, Brant	19.68
	2	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	38.08
	3	7,704,101	13,457,394	Brampton	80.7
	4	8,722,036	13,457,394	Brampton	69.41
	5	9,739,970	13,457,394	Brampton	66.95

Table 9 shows that as the fixed cost to locate a facility increases, the number of facilities located by the model decreases. As the weight of the cost objective function

decreases (decreasing α), the total cost increases. The difference in total cost increases by approximately \$0.55, \$1.99, \$0.03, \$0.04, and \$0.05 million when comparing $\alpha = 1$ and $\alpha = 0.25$ when fixed cost is one, two, three, four, and five million respectively. The difference in base fixed cost of a facility causes a larger fluctuation in the total cost of the solution when the cost objective is weighted more. The difference in total cost increases by approximately \$4.15, \$4.09, \$4.13, and \$3.64 million when comparing fixed cost one million and five million when the weight of the objective function is $\alpha = 1$, $\alpha = 0.75$, $\alpha = 0.5$, and $\alpha = 0.25$.

Table 9 shows that as the number of facilities decreases, the emissions increase. The solution with the lowest emissions has four opened facilities ($\alpha = 0.25$ and the base cost is one million dollars). As the facility cost rises (fixed cost of facilities 3 million dollars and up) all solutions result in placing one facility.

Table 9 confirms that when only one facility is opened, it is located in the GTA due to the proximity to the majority of collection centers and manufacturers (see Figure 2). Table 9 displays the same trend that opening multiple facilities results in one facility in Ottawa and the other in the GTA.

6.2.2 Effect of Transportation Cost

The transportation cost of \$0.00005696 per kg-km is increased/decreased by up to 50%. The rows with 0% in Table 10 correspond to the results provided in Table 8. Table 10 contains the transportation variation, the location(s) selected, the total cost for all players, the emissions released, and the solution time for the model (in seconds).

Table 10 shows that the transportation costs have a large effect on the total cost of the solution. When the weight on the cost objective is higher ($\alpha = 1$), decreasing the transportation cost significantly lowers the total cost (by \$1.16 million). Increasing the transportation cost significantly increases the total cost (by \$2.31 million). When the weight of the emissions objective is higher ($\alpha = 0.25$), decreasing the transportation cost significantly lowers the total cost (by \$2.65 million). Increasing the transportation cost significantly increases the total cost (by \$1.16 million). When the weight of the cost objective is equal to the weight of the emissions objective ($\alpha = 0.5$), decreasing the transportation cost significantly lowers the total cost (by \$1.28 million). Increasing the transportation cost increases the total cost (by \$0.65 million).

Table 10: Effect of Transportation Cost with the Game Theory Model

Objective Weights $(\alpha, 1-\alpha)$	**Transportation Cost Perturbation**	**Total Cost ($)**	**Emissions (kg of CO_2e)**	**Optimal Location(s)**	**Gurobi Solution Time (sec)**
1,0	**50% inc**	8,979,185	13,789,622	Mississauga	30.52
	25% inc	7,821,965	13,789,622	Mississauga	30.35
	0%	6,664,746	13,789,622	Mississauga	30.63
	25% dec	5,505,370	13,821,317	Vaughan	25.81
	50% dec	5,505,370	13,821,317	Vaughan	26.54
0.75,0.25	**50% inc**	8,979,185	13,789,622	Mississauga	30.16
	25% inc	7,821,965	13,789,622	Mississauga	30.76
	0%	6,664,746	13,789,622	Mississauga	26.54
	25% dec	5,523,592	13,457,394	Brampton	33.1
	50% dec	5,523,592	13,457,394	Brampton	37.88
0.5,0.5	**50% inc**	9,332,576	10,435,033	Ottawa, Mississauga	25.4
	25% inc	7,848,740	13,457,394	Brampton	29.43
	0%	6,686,166	13,457,394	Brampton	30.633
	25% dec	5,523,592	13,457,394	Brampton	33.91
	50% dec	5,523,592	13,457,394	Brampton	33.95
0.25, 0.75	**50% inc**	9,932,273	7,426,989	Ottawa, Vaughan, Woodstock	15.67
	25% inc	9,286,265	7,426,989	Ottawa, Vaughan, Woodstock	15.39
	0%	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	17.79
	25% dec	8,003,530	7,403,367	Ottawa, Vaughan, Woodstock	17.61
	50% dec	5,832,994	10,122,450	Ottawa, Brampton	20.83

Table 10 demonstrates that when the weight of the cost objective is higher ($\alpha = 0.75, 1$), decreasing the transportation cost slightly increases the emissions (by 0.031 kiloton of CO_2e compared to 0% perturbation). Increasing the transportation cost does not change the emissions. When the emissions objective function is weighted higher ($\alpha = 0.25$), decreasing the transportation cost increases the emissions significantly (by 2.72 kiloton of CO_2e compared to 0% perturbation) because the model locates one less facility. Increasing the transportation cost does not significantly impact the emissions (increases by 0.02 kiloton of CO_2e compared to 0% perturbation). When the weight of the cost objective is equal to the weight of the emissions objective ($\alpha = 0.5$), decreasing the transportation cost does not impact the emissions. Increasing the transportation cost significantly decreases the emissions (by 3.02 kiloton of CO_2e compared to 0% perturbation).

Table 10 demonstrates that when the cost objective function is weighted more than the emissions objective function ($\alpha = 0.75, 1$), the selected location, Mississauga, is resilient to increases in transportation cost. When the emissions objective function is weighted more than the cost objective function ($\alpha = 0.25$), the selected location remains the same

for fluctuations in transportation costs between 25% decrease to 50% increase. When the cost and emissions objective function are weighted equally ($\alpha = 0.5$), the selected location, Brampton, is valid for fluctuations ranging from 50% decrease to 25% increases.

6.2.3 Effect of Supply Variation

The supply is increased/decreased by up to 50%. The rows with 0% in Table 11 correspond to the results provided in Table 8. Table 11 contains the supply variation, the location(s) selected, the total cost for all players, the emissions released, and the solution time for the model (in seconds).

Table 11: Effect of Supply Variation with the Game Theory Model

Objective Weights $(\alpha, 1-\alpha)$	**Supply Variation**	**Total Cost ($)**	**Emissions (kg of CO_2e)**	**Optimal Location(s)**	**Gurobi Solution Time (sec)**
1,0	**50% inc**	8,979,185	20,684,433	Mississauga	29.88
	25% inc	7,821,965	17,237,028	Mississauga	30.46
	0%	6,664,746	13,789,622	Mississauga	30.73
	25% dec	5,505,370	10,365,983	Vaughan	25.66
	50% dec	4,342,385	6,910,656	Vaughan	24.63
0.75,0.25	**50% inc**	8,979,185	20,684,433	Mississauga	48.02
	25% inc	7,821,965	17,237,028	Mississauga	38.6
	0%	6,664,746	13,789,622	Mississauga	26.6
	25% dec	5,505,370	10,365,983	Vaughan	26.95
	50% dec	4,342,385	6,910,656	Vaughan	21.49
0.5,0.5	**50% inc**	9,924,400	11,167,422	Ottawa, Vaughan, Woodstock	17.42
	25% inc	8,509,865	12,653,067	Ottawa, Brampton	26.1
	0%	6,686,166	13,457,394	Brampton	31.25
	25% dec	5,523,592	10,093,044	Brampton	27.48
	50% dec	4,361,018	6,728,694	Brampton	23.26
0.25, 0.75	**50% inc**	11,165,018	8,095,567	Ottawa, Vaughan, Windsor, Brant	11.84
	25% inc	10,649,144	6,746,306	Ottawa, Vaughan, Windsor, Brant	14.12
	0%	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	17.49
	25% dec	6,725,284	7,591,839	Ottawa, Brampton	25.09
	50% dec	4,361,018	6,728,694	Brampton	31.93

Table 11 shows that fluctuations in supply have a significant effect on cost within the same objective weight due to transportation costs. The difference in total cost increases by approximately \$2.19, \$2.00, and \$0.02 million when comparing $\alpha = 1$ and $\alpha = 0.25$ for a supply change of 50% increase, no change (0%), and 50% decrease. This shows that an increase in the supply has a significant effect on the total cost due to the number of facilities built.

Table 11 shows that the supply has a significant effect on emissions because the number of trucks that are needed increases as the supply increases, which significantly increases the GHG emissions released by transportation. The difference in emissions decreases by 12.59, 6.39, and 0.18 kiloton of CO_2e when comparing $\alpha = 1$ and $\alpha = 0.25$ for a supply change of 50% increase, no change (mean), and 50% decrease respectively. This shows that the number of facilities placed also has a significant effect on the emissions released.

Table 11 shows that when the cost objective function is weighted higher ($\alpha = 1, 0.75$), the number of facilities opened is one in Mississauga or Vaughan. If only the cost objective ($\alpha = 1$) is considered, then the same location (Mississauga) is valid for increases in supply. Locating one facility in Mississauga is a common solution when only one facility is desired (this follows the same trends discussed above). When locating multiple facilities, the trend of locations demonstrates the same trends as discussed above (Ottawa, GTA, US border, and then a location between London and the GTA).

6.2.4 Effect of Market Price

The price of front lower control arms (w and h in the mathematical model) only affects the number of players who choose to stay in the market. The market price is not in the objective function of the mathematical model (only the costs of participating in the market); therefore, the market price only determines if a player makes more profit (supply multiplied by price) than the cost (transportation and facility). If players cannot make enough money to cover the costs, then players choose not to participate in the market. Table 12 contains the variation in wholesale and marginal prices, the location(s) selected, the total cost for all players, the emissions released, the solution time for the model (in seconds).

Table 12 demonstrates that the fluctuation of the price of the model between 50% increase and 50% decreases does not effect the number of players who participate in the market. No players drop out of the market until the price drops approximately 90% (in this case study, the prices become \$3.75 for the wholesale price and \$4.10 for the marginal price). This shows that players have monetary incentive to participate in the remanufacturing market.

Table 12: Effect of Part Price with the Game Theory Model

Objective Weights $(\alpha, 1-\alpha)$	Price Fluctuation	Total Cost ($)	Emissions (kg of CO_2)	Optimal Location(s)	Gurobi Solution Time (sec)	Number of players who drop out of the market
1,0	**50% inc**	6,664,746	13,789,622	Mississauga	37.02	0
	25% inc	6,664,746	13,789,622	Mississauga	35.73	0
	0%	6,664,746	13,789,622	Mississauga	37.22	0
	25% dec	6,664,746	13,789,622	Mississauga	37.86	0
	50% dec	6,664,746	13,789,622	Mississauga	38.38	0
0.75,0.25	**50% inc**	6,664,746	13,789,622	Mississauga	30.61	0
	25% inc	6,664,746	13,789,622	Mississauga	32.74	0
	0%	6,664,746	13,789,622	Mississauga	30.32	0
	25% dec	6,664,746	13,789,622	Mississauga	30.1	0
	50% dec	6,664,746	13,789,622	Mississauga	31.14	0
0.5,0.5	**50% inc**	6,686,166	13,457,394	Brampton	35.38	0
	25% inc	6,686,166	13,457,394	Brampton	34.88	0
	0%	6,686,166	13,457,394	Brampton	35.03	0
	25% dec	6,686,166	13,457,394	Brampton	36.21	0
	50% dec	6,686,166	13,457,394	Brampton	35.14	0
0.25, 0.75	**50% inc**	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	19.99	0
	25% inc	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	20.4	0
	0%	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	20.65	0
	25% dec	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	20.66	0
	50% dec	8,652,633	7,403,367	Ottawa, Vaughan, Woodstock	20.54	0

6.3 Conclusions and Next Steps

The model shows that the locations for facilities follows a trend. When opening one facility it is always located in the GTA (Mississauga, Vaughan, or Brampton should be considered). When opening two facilities one should be located in Ottawa and the other in the GTA. The opening of three facilities only occurs when the emissions objective is considered. The placement of three facilities follows locating the first in Ottawa, the second in the GTA, and the third between London and the GTA or near the US border. Opening four facilities is not a good solution because it forces a large number of players to not participate in the market. The model demonstrates that opening one facility results in the lowest costs. The opening of one facility also results in the largest release of emissions. The results demonstrate that monetary investment is not proportional to the amount of emissions reduced.

Fluctuations in the parameters: facility costs, transportation costs, supply variation, and market price of a front lever control arm are examined. As the facility increases in price, placing one facility becomes beneficial. This is due to the ratio of fixed facility cost and transportation costs. The model demonstrates that transportation costs contribute heavily to the locations of the remanufacturing facility. The results show that Mississauga

is resilient to increases in transportation costs and Vaughan is resilient to decreases in transportation costs. The results show that the supply has a significant effect on costs and emissions. The increase in supply causes more emissions and results in higher total costs. The results show that the market price of front lower control arms can drop significantly before players choose to leave the market. This demonstrates that there is a monetary incentive for collection centers and manufacturers to participate in selling and buying remanufactured products.

Further extensions to the model may include modifying the facility location model that each player minimizes. The new model may be as follows:

$$\text{Minimize} \quad \sum_{j \in R} c_{ij} x_{ij} + \sum_{j \in R} f_j y_j \tag{85}$$

$$|\text{where other players actions are given and fixed}$$

$$\text{s.t.} \quad \sum_{j \in R} x_{ij} = D_i \tag{86}$$

$$x_{ij} \leq \Gamma_j y_j \qquad j \in R \tag{87}$$

$$\sum_{j \in R} g_{ij} x_{ij} \leq CarbonCap \tag{88}$$

$$y_j \in \{0, 1\} \qquad j \in R \tag{89}$$

$$x_{ij} \geq 0 \qquad j \in R \tag{90}$$

where $i \in P$ (collection center or manufacturing facility). $CarbonCap$ is the maximum amount of carbon emissions allowed by government policies. Adjusting the carbon capacity may affect the location of the remanufacturing facility. Using the carbon capacity may discourage trucking materials or placing facilities in high-emissions routing areas (i.e., Mississauga and Vaughan). This model (equations (85)-(90)) employs the constraint method instead of the weighted sum method for modeling the bi-objective nature of the problem.

Chapter 7

Conclusion

The present thesis offers four mixed-integer linear programming models that facilitate the design of a reverse logistics network for the recovery of parts in the automotive industry in Ontario, Canada. The models determine the optimal locations of remanufacturing facilities for processing end-of-use or end-of-life parts using additive manufacturing technologies, as well as the optimal flow of shipments from collection centers that are responsible for gathering used parts and transporting them to remanufacturing facilities and from the remanufacturing facilities to manufacturing plants.

The deterministic, stochastic, and game theory models are formulated as bi-objective mixed-integer linear programming models. The stochastic model is introduced to account for the uncertainty in the supply of parts for remanufacturing. The game theory model is introduced to account for the possibility of cost sharing of facilities. The objectives of these models are to minimize the total cost of building and operating the reverse logistics network and to minimize the GHG emissions generated by transportation. Pareto-optimal solutions are obtained using the weighted sum method.

The models are applied to case study data from Ontario, Canada, to optimize the design of a reverse logistics network for the recovery of front lower control arm car parts using cold spray additive manufacturing technology. The deterministic, stochastic, and game theory models are run under varying weights assigned to the two objective functions. Trade-off curves are generated to analyze the relationship between total cost and emissions. Sensitivity analyses are performed to examine how changes in facility costs, transportation costs, and variation in supply uncertainty affect the resulting solutions.

This thesis shows that the results are robust across different modeling approaches. The

quantitative results (regardless of the objective weights, value of α) demonstrate that the optimal solution based on the case study data is when multiple facilities are located, one facility should be opened in Ottawa and at least one other in the GTA. When only one facility is located, it should be placed in the GTA (either Mississauga or Brampton). This trend can be explained by the clustering of most collection centers and car manufacturers around the GTA. The results of all the models demonstrate that there is a diminishing return on investment in decreasing emissions. There is no linear relationship between investment and lowering emissions. Therefore, investing small sums can reduce emissions significantly, and considering some weighting for an emissions objective can lead to better solutions without significant costs. The results of all the models show that the main factor behind emissions is the number of facilities located. The results demonstrate that building one facility is preferable (Mississauga). Locating one facility is a good solution because it is able to absorb transportation cost increases, facility cost increases, and supply variation.

One of the main limitations of this case study is the accuracy of the data. For example, the supply of automotive parts for this thesis is based on the number of registered vehicles in Ontario and is a rough estimate of the number of ELVs per year. The number of ELVs arriving at each collection center in Ontario cannot be accessed due to the private nature of the information. In future research, it would be beneficial to acquire more accurate data in order to better understand the optimal location for remanufacturing facilities. In addition, the assumption that all car manufacturers evenly split the supply of parts may not be accurate. In further analyses the models can be re-run with different shares to understand how the locations may differ. The present study mitigates the data limitations through sensitivity analysis.[7]

The insights offered by this thesis pave the way for further analysis. The present study can be scaled up to design a reverse logistics network for Canada or for individual provinces and territories. The results of the present study may serve as a basis for comparison with the current network to better understand the potential GHG emissions savings achieved by the implementation of remanufacturing. The results and findings of this study may be used to inform policies on the reduction of transportation emissions.

[7]This thesis varies the supply in the sensitivity analysis. Further research may vary β as well.

Bibliography

Alarcon-Gerbier, E., & Buscher, U. (2024). The dynamic facility location problem with mobile production units: A waste recycling application. *Computers & Operations Research*, *166*, 106609. doi:https://doi.org/10.1016/j.cor.2024.106609

Aldan American. (2025). Aluminum vs steel control arms: Which one is right for you? Retrieved from https://aldanamerican.com/blog/steel-vs-aluminum-control-arms/?srsltid=AfmBOoohJfXUrG-tUk-EYfydyrEnhpmbVD9WOr7npBzY8gXP4WGgQWGd

Alumur, S. A., & Bektaş, T. (2019). Green location problems. In *Location science* (Chap. 20, pp. 591–610). Cham: Springer International Publishing.

Alumur, S. A., Nickel, S., Saldanha-da Gama, F., & Verter, V. (2012). Multi-period reverse logistics network design. *European Journal of Operational Research*, *220*(1), 67–78.

Amanda Garner. (2024). How many pallets fit in a 53-foot dry van trailer? Retrieved from https://www.atsinc.com/blog/how-many-pallets-fit-in-trailer-explained

Anastasiadis, E., & Deligkas, A. (2020). Heterogeneous facility location games. *CoRR*, *abs/2005.03095*. doi:https://doi.org/10.48550/arXiv.2005.03095. arXiv: 2005.03095

Auto Shack. (2024). Control arms. Retrieved from https://autoshack.ca/en/suspension/control-arms

AZ Group. (2025). Retrieved from https://oxfordconnection.ca/target-sectors/automotive

Aziz, H., Chan, H., Lee, B., Li, B., & Walsh, T. (2020). Facility location problem with capacity constraints: Algorithmic and mechanism design perspectives. *Proceedings of the AAAI Conference on Artificial Intelligence*, *34*(2), 1806–1813.

Ben-Porat, O., & Tennenholtz, M. (2019). Multiunit facility location games. *Mathematics of Operations Research*, *44*(3), 865–889.

Canadian Auto Recycler Magazine. (2019). The low-down on canada's automotive recycling industry. Retrieved from https://canadianrecycler.ca/the-low-down-on-canadas-automotive-recycling-industry/

Canadian Tire. (2025). Pro-series oe+ control arm. Retrieved from https://www.canadiantire.ca/en/pdp/pro-series-oe-control-arm-0227405p.html

Cardinal, J., & Hoefer, M. (2010). Non-cooperative facility location and covering games. *Theoretical Computer Science*, *411*(16), 1855–1876.

Chekuri, C., Chuzhoy, J., Lewin-Eytan, L., Naor, J., & Orda, A. (2007). Non-cooperative multicast and facility location games. *IEEE Journal on Selected Areas in Communications*, *25*(6), 1193–1206.

Choksey, J. S. (2021). What is a control arm on a car? Retrieved from https://www.jdpower.com/cars/shopping-guides/what-is-a-control-arm-on-a-car

Dell'Amico, M., Iori, M., Novellani, S., & Subramanian, A. (2018). The bike sharing rebalancing problem with stochastic demands. *Transportation Research. Part B: Methodological*, *118*, 362–380.

Environment of Climate Change Canada. (2021). Socio-economic and environmental study of the Canadian remanufacturing sector and other value-retention processes in the context of a circular economy / prepared for Environment and Climate Change Canada by Oakdene Hollins and Dillon. Retrieved from https://publications.gc.ca/site/eng/9.900569/publication.html

Geissdoerfer, M., Savaget, P., Bocken, N. M., & Hultink, E. J. (2017). The circular economy – a new sustainability paradigm? *Journal of Cleaner Production*, *143*, 757–768.

Government of Canada. (2021). Canadian automotive industry. Retrieved from https://ised-isde.canada.ca/site/canadian-automotive-industry/en

Government of Canada. (2022). 2030 emissions reduction plan – sector-by-sector overview. Retrieved from https://www.canada.ca/en/services/environment/weather/climatechange.html

Government of Canada. (2024a). Climate change: Strategies and initiatives. Retrieved from https://www.canada.ca/en/services/environment/weather/climatechange.html

Government of Canada. (2024b). Estimated value and rental rate of farmland by county and township - estimated value and rental rate of farmland by county and township. Retrieved from https://data.ontario.ca/dataset/estimated-value-and-rental-rate-of-farmland-by-county-and-township

Government of Ontario. (2022a). Executive summary of the socio-economic and environmental study of the canadian remanufacturing sector and other value-retention processes in the context of a circular economy. Retrieved from https://www.canada.ca/en/services/environment/conservation/sustainability/circular-economy/summary-study-remanufacturing-sector-value-retention-processes.html#toc8

Government of Ontario. (2022b). Ontario landfills. Retrieved from https://data.ontario.ca/dataset/ontario-landfills/resource/b7c52570-83bd-40a7-939a-8eb3742c4140

Government of Ontario. (2023). Vehicle population data. Retrieved from https://data.ontario.ca/dataset/vehicle-population-data

Gur, Y., Saban, D., & Stier-Moses, N. E. (2018). Technical note—the competitive facility location problem in a duopoly: Advances beyond trees. *Operations Research*, *66*(4), 1058–1067.

Gülpınar, N., Rustem, B., & Settergren, R. (2004). Simulation and optimization approaches to scenario tree generation. *Journal of Economic Dynamics & Control*, *28*(7), 1291–1315.

Hachimi, H. E., Oubrich, M., & Souissi, O. (2018). The optimization of reverse logistics activities: A literature review and future directions. *2018 IEEE International Conference on Technology Management*, 18–24.

Jahed, V. (2023). Remanufacturing - a manufacturing paradigm shift for deep decarbonization in a sustainable economy.

Karagoz, S., Aydin, N., & Simic, V. (2022). A Novel Stochastic Optimization Model for Reverse Logistics Network Design of End-of-Life Vehicles: A Case Study of Istanbul. *Environmental Modeling & Assessment*, *27*(4), 599–619.

Keutchayan, J., Ortmann, J., & Rei, W. (2023). Problem-driven scenario clustering in stochastic optimization. *Computational Management Science*, *20*(1), 13.

Krogmann, S., Lenzner, P., Molitor, L., & Skopalik, A. (2021). Two-stage facility location games with strategic clients and facilities. *CoRR*, *abs/2105.01425*. doi:https://doi.org/10.48550/arXiv.2105.01425. arXiv: 2105.01425

Lei, J., Che, A., & Van Woensel, T. (2024). Collection-disassembly-delivery problem of disassembly centers in a reverse logistics network. *European Journal of Operational Research*, *313*(2), 478–493. doi:https://doi.org/10.1016/j.ejor.2023.07.008

Li, H., Taherkhani, G., Alumur, S. A., & Hewitt, M. (2025). Strategic expansion of freight transportation hub networks under demand uncertainty. *Omega*, *131*.

Liao, T.-Y. (2018). Reverse logistics network design for product recovery and remanufacturing. *Applied Mathematical Modelling*, *60*, 145–163.

Ontario Automotive Recyclers Association. (2022). Where do old cars go? Retrieved from https://oara.com/where-do-old-cars-go/

Ontario Automotive Recyclers Association. (2023). The Environmental Benefits of Green Recycled Parts in Ontario. https://oara.com/wp-content/uploads/2024/01/OARA-Final-GreenRecycledParts-report.pdf.

Ontario Automotive Recyclers Association. (2024). Active members. Retrieved from https://oara.com/members/

Parts 4 less U pull. (2025). U - pull parts price list - 2025. Retrieved from https://parts4lessupull.ca/price_list.asp

Pathak, S., & Saha, G. (2017). Development of sustainable cold spray coatings and 3d additive manufacturing components for repair/manufacturing applications: A critical review. *Coatings*, *7*(8), 122.

Revolution Parts. (2024). What is a lower control arm and why is it important? Retrieved from https://www.nissanpartsplus.com/blog/lower-control-arms-explained

Rodriguez, I. C., Perez, A. E., & Boudaoud, H. (2023). A multi-objective model for sustainable reverse logistics design for end-of-life tires under a regulatory framework: A Chilean case study. In *2023 3rd international conference on electrical, computer, communications and mechatronics engineering (iceccme)* (pp. 1–6). IEEE.

Rosenberg, S., Huster, S., Rudi, A., & Schultmann, F. (2025). Assessing economic uncertainty in dynamic reverse logistics networks – a stochastic modeling approach for planning circular battery treatment. *Computers & Industrial Engineering*, *201*, 110900. doi:https://doi.org/10.1016/j.cie.2025.110900

Rossit, D., & Bard, J. (2025). Solving the waste bin location problem with uncertain waste generation rate: A bi-objective robust optimization approach. *Waste Management & Research*, *43*(3), 421–437.

Roudbari, E. S., Ghomi, F., & Eicker, U. (2024). Designing a multi-objective closed-loop supply chain: a two-stage stochastic programming, method applied to the garment industry in Montréal, Canada: Designing a multi-objective closed-loop supply chain: a two-stage stochastic programming, method applied to the garment industry in Montréal, Canada. *Environment, Development and Sustainability*, *26*(3), 6131–6162.

Santoso, T., Ahmed, S., Goetschalckx, M., & Shapiro, A. (2005). A stochastic programming approach for supply chain network design under uncertainty. *European Journal of Operational Research*, *167*(1), 96–115.

Scrap Monster. (2025). Scrap metal recycling companies in onatrio. Retrieved from https://www.scrapmonster.com/companies/scrap-metal-recycling/region/canada/ontario/397/60

Shuang, Y., Diabat, A., & Liao, Y. (2019). A stochastic reverse logistics production routing model with emissions control policy selection. *International Journal of Production Economics*, *213*, 201–216.

Stellantis Canada. (2024). Personal communication, [email]. Estimate of Front LCA Usage & Production.

Story, R. (2020). Folium (Version 0.15.1).

Tari, I., & Alumur, S. A. (2014). Collection center location with equity considerations in reverse logistics networks. *INFOR. Information Systems and Operational Research*, *52*(4), 157–173.

Titomic. (2024). What is cold spray additive manufacturing? Retrieved from https://titomic.com/articles/what-is-cold-spray-additive-manufacturing/

Tosarkani, B. M., & Amin, S. H. (2019). An environmental optimization model to configure a hybrid forward and reverse supply chain network under uncertainty. *Computers & Chemical Engineering*, *121*, 540–555.

Trochu, J., Chaabane, A., & Ouhimmou, M. (2018). Reverse logistics network redesign under uncertainty for wood waste in the CRD industry. *Resources, Conservation and Recycling*, *128*, 32–47.

Trochu, J., Chaabane, A., & Ouhimmou, M. (2019). A two-stage stochastic optimization model for reverse logistics network design under dynamic suppliers' locations. *Waste Management*, *95*, 569–583.

United States Environmental Protection Agency. (2025). Greenhouse gas emissions from a typical passenger vehicle. Retrieved from https://www.epa.gov/greenvehicles/greenhouse-gas-emissions-typical-passenger-vehicle

Urban, S. (2021). Mccormick envelopes. Retrieved from https://optimization.cbe.cornell.edu/index.php?title=McCormick_envelopes

Valenzuela, J., Alfaro, M., Fuertes, G., Vargas, M., & Sáez-Navarrete, C. (2021). Reverse logistics models for the collection of plastic waste: A literature review. *Waste Management & Research*, *39*, 1116–1134.

Vetta, A. (2002). Nash equilibria in competitive societies, with applications to facility location, traffic routing and auctions. In *43rd Symposium on Foundations of Computer Science (FOCS 2002)* (pp. 416–425). Los Alamitos, CA: IEEE.

Vidovic, M., Ratkovic, B., Bjelic, N., & Popovic, D. (2016). A two-echelon location-routing model for designing recycling logistics networks with profit: MILP and heuristic approach. *Expert Systems with Applications*, *51*, 34–48.

Vázsonyi, M. (2006). Overview of scenario tree generation methods, applied in financial and economic decision making. *Periodica Polytechnica. Social and Management Sciences*, *14*(1), 29.

Wang, W., Saari, R. K., Bachmann, C., & Mukherjee, U. (2020). Estimating transboundary economic damages from climate change and air pollution for subnational incentives for green on-road freight. *Transportation Research. Part D, Transport and Environment*, *82*, 102325.

Wang, Z., Huang, L., & He, C.-X. (2021). A multi-objective and multi-period optimization model for urban healthcare waste's reverse logistics network design. *Journal of Combinatorial Optimization*, *42*(4), 785–812.

Wexler, Thomas and Tardos, Eva. (2005). A facility location game. Retrieved from https://www.cs.cornell.edu/courses/cs684/2005fa/9-28.pdf

World Population by Country 2024. (2024). World population review. Retrieved from https://worldpopulationreview.com/cities/province/ontario

Xiao, Z., Sun, J., Shu, W., & Wang, T. (2019). Location-allocation problem of reverse logistics for end-of-life vehicles based on the measurement of carbon emissions. *Computers & Industrial Engineering*, *127*, 169–181.

Yu, H., & Solvang, W. D. (2018). A trade-off model for green supply chain design: An efficiency-versus-emission analysis. In *2018 7th International Conference on Industrial Technology and Management (ICITM)* (pp. 136–142). IEEE.

Appendices

Appendix A

Game Theory Diagram

To illustrate the game dynamics, an example with three players A, B, and C and two potential remanufacturing locations (facility 0 and facility 1) is presented in Table 13. The lower case letters are used to indicate the payoff of the players. A similar matrix can be completed for player B given the decisions of players A and C and player C given the decisions of players A and B. All players make their decisions simultaneously; therefore, there is no order of game play.

Table 13: Payoff Matrix

		B and C				
		Given B and C open facility 0	Given B and C open facility 1	Given B opens facility 0 and C opens facility 1	Given B opens facility 1 and C opens facility 0	Given B and C stay out of the market
A	Open facility 0	a,b,c	d,e,f	g,h,i	j,k,l	m,n,o
	Open facility 1	p,q,r	s,t,u	v,w,x	y,z,aa	ab,ac,ad
	Stay out of the market	ae,af,ag	ah,ai,aj	ak,al,am	an,ao,ap	aq,ar,as

Appendix B

Data Table

Table 14 summarizes the information presented in Chapter 4.

Table 14: Summary of Data Used for the Case Study

Parameter	Description	Reference
Sets		
C	Set of automotive collection centers in Ontario	(Ontario Automotive Recyclers Association, 2024)
R	101 most populated cities in Ontario	(World Population by Country 2024, 2024)
M	Plants of Ford, GM, Honda, Stellantis, and Toyota	(AZ Group, 2025)
L	Landfills and metal recyclers in Ontario	(Government of Ontario, 2022b), (Scrap Monster, 2025)
Parameters		
u_i	Supply from each collection center	(Government of Ontario, 2023)
f_i	Cost of a facility	(Government of Canada, 2024b)
c_{ij}	Cost of transportation on an arc	(Li, Taherkhani, Alumur, & Hewitt, 2025)
g_{ij}	GHG emissions released on an arc	(Wang, Saari, Bachmann, & Mukherjee, 2020)
w	The wholesale price of a front lower control arm	(Parts 4 less U pull, 2025)
h	The marginal price from second-hand to new parts	(Auto Shack, 2024)