Computer Support for Environmental Multiple Criteria Decision Analysis Under Uncertainty

by

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Abstract

This thesis describes new techniques for Multiple–Criteria Decision Analysis (MCDA) under uncertainty with application to environmental problems. Decision support systems, such as the SEAL (Stochastic Environmental Analysis), REAL (Robust Environmental Analysis), and MEAL (Marginal Distributions for Environmental Analysis) systems, are developed to help decision makers improve their social, economic, and environmental decision making under uncertainty.

A major contribution of this thesis is the investigation of uncertainty approaches, including interval judgments (Saaty and Vargas, 1987), info-gap models (Ben-Haim, 1996), stochastic differential equations (Cox and Miller, 1965), and Bayesian techniques (Ludwig, 1996) in an MCDA context. For example, the proposed MCDA info-gap model approach is completely non-probabilistic; it captures a decision maker's preferences and attitude toward risk without resorting to "non-intuitive probabilistic concepts of gambling and indifference between lotteries" (Barzilai, 1997) and is the **first published info-gap model MCDA technique in the literature**. Convex modeling is particularly valuable since utility functions are not required (only value functions are necessary) and robust alternatives can be identified.

Significantly, **the stochastic water quality models used in the thesis investigates the use of Stratonovich calculus to model the classic interactions among biochemical oxygen demand (BOD), dissolved oxygen (DO), and other environmental variables**. In addition, the Streeter—Phelps equations are generalized to more realistically model hydrologic processes. Finally, a practical **colored noise approximation** is put forth and used to replace the abstract mathematical concept of 'white' (theoretical) noise. Replacing white noise with coloured noise is of great importance in water quality modeling since **in almost all cases the white noise assumption is not justified and is used only for mathematical convenience.**

Finally, the SEAL decision support system is applied to a wide range of

stochastic environmental problems, from water quality modeling to species extinction. **Here the 'First Passage Time' problem is considered in detail from an environmental perspective.** In the context of fisheries management, it is shown how regulating the 'fishing effort' can significantly reduce the risk of stock extinction.

Finally, it is described how the management of renewable resources, where it has been practiced at all, relies heavily on techniques from optimal control theory, cost-benefit analysis, and maximum sustainable yield (MSY). These approaches are critically reviewed and it is shown that formally modeling both the **risk of extinction** and the **'preservation value'** of a resource can improve the sustainable management of renewable resources.

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Chapter 1

Introduction

1.1 Uncertainty and Post-Modernism

For thousands of years events on earth were viewed as inherently uncontrollable and uncertain, governed by higher powers and forces. In the Middle Ages, spirits, miracles, and demons were held responsible for the unpredictability and disorder of life. The Greeks believed that fickle gods influenced their life course and destiny. More than twenty-five hundred years ago, at the very start of Western rationality, Epicurus and the ancient Greek 'physicists' proposed that the *chance* deviation of atoms (from their assumed parallel paths) produces the novelty associated with combinations of atoms, giving rise to all natural things: to this day the *Epicureanists* maintain that the world is a fortuitous combination of atoms[1].

In this context, the Newtonian revolution and the modern scientific worldview — with its emphasis on predictability, control, and regularity — can be viewed as a brief digression from the emphasis on chance and the supernatu-

[1] As written by Lucretius, the famous Roman poet and philsopher around 50 BC: "at uncertain times and places, the eternal, universal fall of atoms is disturbed by a very slight deviation: the 'clinamen' (Carus, 1947)". This spontaneous, unpredictable fluctuation in the direction of atoms forms the basis of Lucretian physics and attempts to explain events such as laminar flow.

ral that dominated previous millennia of Western thought. But in this brief window of certitude — the modern paradigm from Newton (1642-1727) until the late 20th century — science has become dominated by the philosophy of determinism: given a set of known initial conditions, the future can be completely predicted. The French scientist Pierre Simon Laplace (1749-1827), a leading expounder of determinism, proposed a set of scientific laws, "Celestial Mechanics", to permit the exact calculation of how the universe will unfold. Laplace even predicted that scientific laws would be found to explain human behaviour, similar to those governing the motion of planets (Hawking, 1988). Widespread acceptance of determinism coincided with the rise of a factory civilization and the Industrial Age: the world was perceived as a giant assembly line, in which all components of the universe came together like cogs in a cosmic machine, whose clockwork operations were subject to universal deterministic laws.

This fascination with determinism is perhaps the defining characteristic of Scientific Revolution: it can be found everywhere in Western scientific thought from the philosophy of Kant ('universal causal determinism') to the demon imagined by Laplace (capable of deducing the position and velocity of every mass in the universe, and inferring its course, both toward the past and future). Even the basic equation of quantum mechanics, Schrödinger's equation, is deterministic and time reversible. And many physicists argue that *the 'Book of Physics' will soon be closed*: it is only a matter of time until our knowledge of elementary particle physics and unified field theory becomes 'complete'. Hawking (1988) argues that we are close to this moment, the time when humanity shall "read the mind of God".

The first serious challenge to the dominance of determinism came from Karl Heisenberg's *Uncertainty Principle* (1927): it is not possible to know both the position and velocity of an object with absolute certainty. By the early twentieth century, Einstein had shown that our machine-universe was observer dependent. But it was still a deterministic machine: after all, God did not play dice.

Scientists have recently discovered that many physical systems are essentially deterministic, but extremely sensitive to initial conditions. Moreover, living and social systems are now recognized to be thermodynamically "open systems" that exhibit discontinuous change and self-organization (Kay, 1984). Increasingly, scholars are defining a new science based on novelty, choice, and spontaneous action. For example, Prigogine and Stengers (1997) argue that the we have reached "The End of Certainty":

> A new formulation of the laws of nature is required that is no longer based on certitudes, but rather possibilities. In accepting that the future is not determined, we come to the end of certainty.

What is now emerging is an "intermediate" description of nature that lies somewhere between the two alienating images of a deterministic world and pure chance. In this "new dialogue with nature" (Prigogine and Stengers, 1984), chance and necessity intertwine inextricably. For example, Prigogine and Stengers (1997) discuss how chance plays its role near the point of bifurcation after which a deterministic period ensues until the next bifurcation.

1.2 Uncertainty and Environmental Multiple Criteria Decision Analysis

Operational research (OR) consists of some general methodologies and many specific techniques for studying decision making problems (Hipel, 1992). Throughout World War II, the British employed OR in many of their military activities (Blackett, 1962). Until recently, OR focused on models which postulate the existence of a unique single-criterion function. However, practical decision situations are multidimensional and involve a wide range of criteria from economic considerations to qualitative notions of comfort, aesthetics, and sustainability (Bouyssou, 1993). The field of Multiple Criteria Decision Analysis (MCDA) is now widely used to solve environmental problems with multiple, and usually conflicting, objectives.

It appears that the spirit of MCDA was captured by decision makers more than 4,500 years ago in the Mesopotamian cities of Lagash and Umma to select among water resources alternatives (McDonald and Kay, 1988)[2]. Sustainable development is the epitome of MCDA since environmental problems are comprised of many interacting variables, conflicting objectives, and competing alternatives: explicitly modeling socio-cultural dimensions (International Union for Conservation of Nature and Natural Resources, 1980), biophysical sustainability (Munasinghe and Shearer, 1995), and the need for "lasting and secure livelihoods" (Barbier, 1987) will help to minimize resource depletion, cultural disruption, and social instability.

MCDA consists of a set of tools to help in the analysis of a decision problem by systematically *comparing, selecting, or ranking* a set of alternatives according to two or more criteria. The excitement generated by the first MCDA conference held at the University of South Carolina in 1973 contributed to the early growth of MCDA (Ignizio, 1983). Other reasons for the popularity of multiple criteria approaches in the 1970s include:

- dissatisfaction with conventional "single criterion" quantitative methods;
- recognition that multiple criteria approaches provide not only numbers but also an improved understanding of the decision problem; and
- the existence of software and algorithms for solving large scale multiple criteria problems.

In particular, this thesis addresses the difficult problem of environmental MCDA *under uncertainty*. For example, the evaluation of an alternative ("Rezone lands from Protected to Urban") on a given attribute ("Health of deer population") may be an uncertain quantity. With the exception

[2]The formal development of MCDA dates back to the late 19th century, when the concept of equilibrium in consumer economics was introduced by Edgeworth and Pareto (Stadler, 1988).

of Multi-Attribute Utility Theory (Keeney and Raiffa, 1976), commonly referred to as MAUT, there has been little formal treatment of uncertain outcomes in MCDA. However, the use of MAUT is restricted to problems involving probabilistic choice, in which case a cardinal von Neumann-Morgenstern utility function applies (von Neumann and Morgenstern, 1947; Luce and Raiffa, 1957).

1.3 Environmental Mental Models

The central problem in conservation ecology is that our expansive species of primate, *Homo sapiens* (so called "man the wise"), now enjoys a remarkable, although possibly temporary, dominance of its host ecosystem because of unprecedented neurological development. Christensen (1997) points out that resource problems are more "human problems" than environmental problems[3]. Over the past four decades the scope and scale of environmental problems has expanded considerably, from local pollution and resource depletion issues to regional and global problems including soil erosion, climate change, and ozone depletion (Havas et al., 1984; Prato, 1999). In light of the novelty, urgency, and complexity surrounding the environmental problems of modernity, decision makers are forced to learn more about their *ecological worldviews and beliefs.*

Consider the issue of global climate change. While scientists have developed complex computer models of the earth's atmosphere, there are significant uncertainties involved in predicting temperature, precipitation, and other variables. Estimates vary widely among the three to four well-known Global Climate Models (GCMs). It is uncertain how issues as diverse as energy use, food production, forest management, and transportation policies affect global warming. Meteorologists and other scientists may have the

[3]What we now call 'environmental problems' are by no means new: they probably contributed more to the collapse of earlier civilizations than did the typically cited military fortunes (Cronon, 1983; Weiskel, 1989).

much-awaited answer to these questions by 2050 (Karl and Trenberth, 1999); until then, whether humanity entertains a significant risk by continuing to release greenhouse gases into the atmosphere on a large scale (several billion tons of carbon per year) depends on one's "view of the world" or mental model (*Weltanschauung*).

These psychological models "predetermine how we perceive reality" (Mackay, 1994; Abel et al., 1998). They are constructed from past experiences (Abel, 1999) and help to simplify, structure, and 'make sense' of the deluge of information decision makers receive (Craik, 1952; Kelly, 1955; Johnson-Laird, 1983). Many authors have categorized these mental models: Harvey (1966) considers epistemological types; Maruyama (1980) explores various "mindscapes" or "causal metatypes"; Timmerman (1986) considers "myths and paradigms"; and Holling (1995) describes prevailing environmental "belief systems". *Mental models are the mechanisms through which we interpret reality and hence it is they, not reality, that guide our behaviour.* Accordingly, mental models hold supreme significance in political, economic, and ecological decision making.

Information that confirms our existing mental models is readily accepted, while information contradicting existing constructs is commonly ignored, reinterpreted, or even changed to suit the model better. Although less common, sometimes humans modify their mental models in order to accommodate new experiences and conflicting information. When mental models do change "they tend to do so rapidly, because the psychological re-structuring permits new kinds of information to enter" (Abel, 1999). When mental models change across an entire scientific discipline it is a "paradigm shift" (Kuhn, 1962; Young, 1991). Extending Kuhn's concept, one can consider mental models across an entire culture, or "cultural paradigms" (Kelly, 1955; Abel, 1999).

Often, mental models yield completely opposite conclusions, even among experts using the same data. For example, rangeland scientists disagree over whether rangelands are intrinsically stable (Stoddart et al., 1975), unstable (Ellis et al., 1993), fragile and extensively degraded (Dregne et al., 1991),

highly resilient (Scholes and Walker, 1993) or all of these, depending on rainfall variability (Coppock, 1993). An original categorization of human mental models is developed in Fig 1.1 to illustrate four mental models that have driven political debate, scientific research, and public concern about environmental issues: the Nature Constant, Nature Ephemeral, Nature Random, and Nature Resilient perspectives. For example, the 'Nature Resilient' mental model falls under the subdivision of models which incorporate 'biocentric ethics' and emphasize 'monitoring and adaptation'. Notice that human ethics can be either anthropocentric or biocentric.

Ethics is an important part of 'mental models' (Abel, 1999) since society must decide which system components are valued and should be preserved. Anthropocentric values tend to emphasize the use of natural resources for economic purposes and the well-being of humans; whereas, biocentric values deal with the intrinsic, non-utilitarian values of natural resources. The debate between John Muir and Gifford Pinchot serves to highlight this distinction: Pinchot (an early 'Conservationist/Utilist') argued that resources exist in part for human consumption and ecological resources should be managed to "reap maximum potential benefit for human life" (Lister, 1997). On the other hand, Muir, an 'Inherentist/Preservationist', emphasized the intrinsic value of living systems.

The anthropocentrism which separates humans from nature is quite specific to the Western world. In China and Japan, for example, nature means "what is by itself". Needham (1969) speaks of the irony with which the Chinese greeted the Jesuits' announcement of the triumphs of modern science. For them, the idea of 'managing' nature seemed a wonderful example of anthropocentric foolishness. According to Chinese tradition, nature is spontaneous harmony: "What can be controlled is never completely real; what is real can never be completely controlled" (Nabokov, 1974).

Managers and scientists in public policy hold competing perspectives on how organizations should respond to low probability, high consequence events, such as earthquakes, tornadoes, and hurricanes (Hart et al., 1993;

Sutphen and Bott, 1990). A tension exists between processes of command and control (Perrow, 1984; Sagan, 1993) and processes of innovation and discovery (Cohen and Levinthal, 1990). Both management approaches are illustrated in Fig 1.1. Theories of risk reduction through redundancy (Landau, 1991; Simon, 1969) (Holling, 1973) have been rejected in practice as too costly for low probability events, at least in certain situations (Rossi et al., 1982).

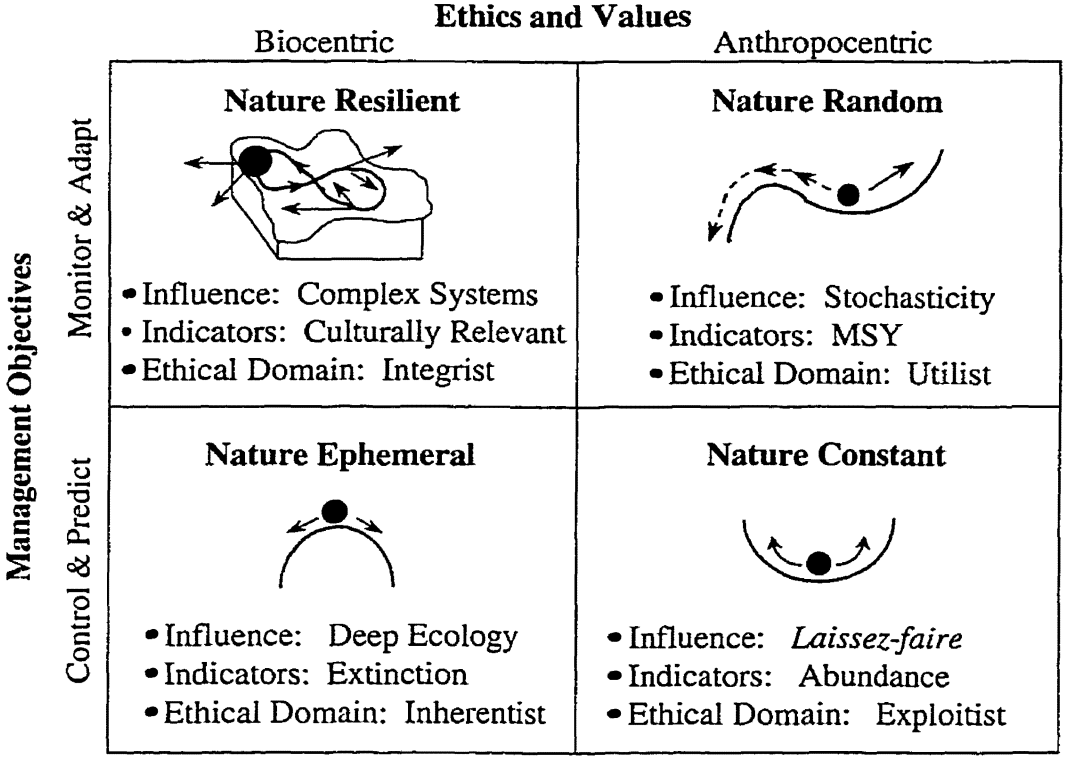


Figure 1.1: Schema of four human perceptions of natural systems (adapted from Holling (1995), Regier (1993), and Scheffer et al. (1993).

Note that each worldview in Fig 1.1 makes use of different socio-economic and ecological indicators — and is the result of different historical and cultural influences. The remainder of the introduction describes how the chapters of this thesis are organized according to Fig 1.1. To understand these

mental models better, it may be helpful to picture nature as a ball on a 'dynamical landscape'. Depending on the topography of the landscape, human activities will slightly oscillate the ball, or permanently dislodge it into the domain of another attractor. Here, the notion of ecological stability is crucial: do systems tend to a single unique stable state from all initial conditions and disturbances? Or do large perturbations carry these systems into a new region of state space. In the former case, historical accidents are unimportant; in the latter, chance events can be of "overriding significance" (May, 1977).

1.3.1 The Nature Constant View

A unique combination of events — the scientific-technological revolution, European domination of newly discovered lands, and seventeenth century laissez-faire — gave rise to the view that nature provides an endless supply of ecological resources for humans. The term 'frontier economics' was coined by Kenneth J. Boulding to describe this approach which prevailed in most countries until at least the late 1960s (Boulding, 1966): even if ecological limits are reached, they can be overcome by product substitution and other technological innovations. Examples include the 'developmental' paradigm that has dominated rangeland management (Walker, 1994). In an influential paper, Arrow et al. (1995) argue that technological optimism and cornucopian beliefs continue to dominate economic policy since national and international economic agreements "usually ignore the environment". In areas where the environment is beginning to impinge on economic policy, such as the General Agreement on Tariffs and Trade (GATT) and the North American Free Trade Agreement (NAFTA), it remains a "tangential concern".

The *Nature Constant* perspective focuses on global stability and the linear response of ecological systems to human disturbances. This concept is well represented by the "ecosystem linearization" models of Patten (1975) in which interactions between components are assumed to be linear. Proponents of the Nature Constant view point out that as per capita income

goes up, there is increasing environmental degradation up to a point, after which environmental quality ostensibly improves (Shafik and Bandyopadhay, 1992; Beckerman, 1992). This "inverted-U" relationship (Simon and Kahn, 1984) is provided as evidence that economic growth will lead to a healthier environment[4].

1.3.2 The Nature Ephemeral View

The *Nature Ephemeral* view argues that the environment cannot safely tolerate human activities (fundamental instability is the rule): survival is only deemed possible by applying safety factors (avoiding large scale irreversible damage); ensuring biological diversity (in structure and over space); curtailing human population growth (preserving future options and ecological possibilities); and abandoning technological innovation. Accordingly, decentralized governance with fine-scaled local autonomy is necessary. The Earth's resource base, it is argued, cannot support increased economic growth: draconian legislation is necessary to replace existing patterns of consumption and production with more environmentally sustainable ones.

In this worldview, all environmental phenomena (species, landscapes, *etc.*) have intrinsic value, independent of humans. As declared by Chief Seattle during the Treaty of Walla Walla negotiations: "every part of the earth is sacred." It follows that every organism has a right to exist and should be preserved. Early advocates of this "inherentist, preservationist" ethic include John Muir, a nineteenth century naturalist who successfully campaigned for forest preservation in the United States.

Ludwig et al. (1993) promote the *Nature Ephemeral* perspective in a provocative article on conservation policy: they argue that despite claims of sustainable environmental management, ecological resources are "inevitably overexploited, often to the point of collapse or extinction." At a minimum,

[4]On closer analysis, this "inverted-U" relationship may be limited to a select set of pollutants, with localized effects and short-term costs such as basic sanitation (fecal coliforms), sulfur, and particulates (Arrow et al., 1995).

the Nature Ephemeral perspective is a refreshing antidote to the anthropocentric, cornucopian perspective. Tropical rain forests are often held to fit this Ephemeral description (Gomez-Pompa et al., 1972). Perhaps the Nature Ephemeral perspective is the most accurate one: economic pressures are a reflection of human desires, and both human population growth and the per capita consumption of resources is increasing in an unsustainable fashion. The result of human greed, as we have seen too often, is the misuse, and subsequent destruction of resources.

In addition, even well meaning scientists often cannot detect initial signs of resource overexploitation until ecological damage is severe. Worse still, consensus among scientists is seldom achieved, even after the total collapse of a resource: humans seem unwilling to take prudent environmental measures, even when there is a good scientific understanding that certain practices are ultimately destructive.

An excellent example is the use of irrigation in arid regions. It is well known that in ancient Mesopotamia the once highly productive wheat crop had to be replaced by more salt-resistant plants. The increased soil salt was a result of irrigation (Vreede, 1977). While many scientists warned of similar consequences in California due to large scale planned irrigation, pleas from local biologists, some as early as 1899 (Hilgard, 1899) fell on deaf ears (Gard, 1988). Thus, 3,000 years of experience and sound scientific knowledge may not be sufficient to overcome shortsightedness and greed.

1.3.3 The Nature Resilient View

This view is frequently described as a synthesis of the *Nature Constant* and *Ephemeral* extremes: the environment is forgiving of most shocks, but large perturbations can knock variables into new regions of the landscape. In this view, the response of a living system to stress will be largely linear until a critical threshold is crossed, at which point a radical change (called a discontinuity or catastrophe) occurs (Thom, 1969; Kay, 1991). Here, severe environmental conditions (for example pest outbreaks, fires, and windstorms) are

used to test the survivability of system components or eliminate weak ones. In the Nature Resilient view, human culture is seen as embedded in nature, dependent on it, and capable to harm it (Regier, 1993). By emphasizing the resilience of systems (Gunderson et al., 1997), insights from the Nature Resilient view may help societies to overcome pathological behaviour including institutional rigidity, social dependencies, political hegemony, and ecological degradation.

1.3.4 The Nature Random View

Many environmental phenomena are highly variable. For example, in the savannas of Southern and East Africa, rainfall can vary from more than 750 mm per annum in wet years to nil at the driest extremes. To the extent that humans have made themselves dependent on non-extreme conditions (or have not prepared for these occurrences) such events can cause large scale destruction and even death.

None of these competing worldviews is correct to the exclusion of others. But there is merit in identifying different approaches to understanding, regulating, and managing natural systems. Since these four views have elements of truth within them, intelligent people have mobilized compelling examples to convincingly support these various views of the world.

1.4 Thesis Organization and Environmental Mental Models

Fig 1.1 is used to establish a meta-model for this thesis. Specifically, each MCDA technique described in this thesis can be considered in the context of the environmental worldviews of Fig 1.1. A more detailed integration of environmental mental models and MCDA topics is illustrated in Fig 1.2. For example, the discussion of deterministic MCDA is consistent with the Nature Constant (control Nature) perspective while info-gap models are consistent

with the Nature Resilient perspective. Note that issues related to species extinction are associated with the the Nature Ephemeral view.

In addition Fig 1.2 clearly shows the decision support programs associated with each MCDA technique. Computer programs developed by the author end in the acronym "EAL" because they are intended to facilitate environmental (E) analysis (A) for 'lokahi' (L), the Hawaiian word for sustainability (L). For example, the SEAL model facilitates stochastic environmental analysis; the REAL model identifies robust policy alternatives in the context of info-gap models; and the MEAL model is an extremely useful Gibbs sampler Monte-Carlo Markov-chain (Gelfand and Smith, 1990; Smith and Roberts, 1993) approach for use in Bayesian inferencing. Given a joint posterior probability density function, the MEAL decision support system can ascertain marginal probability density functions.

Note that Fig 1.2 provides a framework for the thesis by showing:

- The type of MCDA approach most suitable for each worldveiw;
- An explanation of the computer programs developed for the various MCDA techniques; and
- Each MCDA approach is illustrated using a case study from the field of water resources management[5].

In this way, Fig 1.2 helps to integrate and organize the thesis.

1.4.1 Optimal Control Theory: Nature Constant

Decision makers holding the Nature Constant view believe that Nature moves fast enough to be detected but slow enough to be controlled and managed.

[5]The use of multiple-criteria analysis for water resources planning is well established in the United States where it is required by law (Haimes and Hall, 1974; Prato, 1999). Relevant legislation includes the Clean Water Act (Adler et al., 1993) and the National Water Agenda for the 21st Century (Water Environment Foundation, 1992).

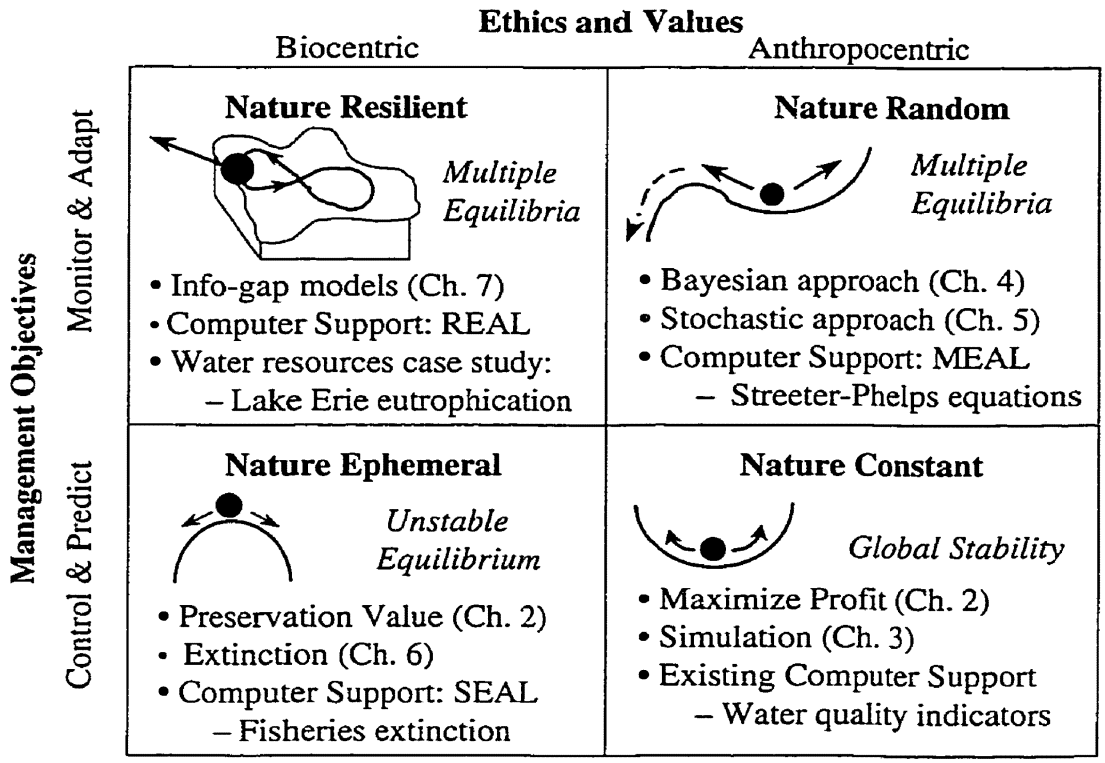


Figure 1.2: Schema of four human perceptions of natural systems (adapted from Holling (1995), Regier (1993), and Scheffer et al. (1993).

Much of **Chapter 2) is consistent with the Nature Constant perspective because it assumes that multiple criteria decision making occurs under certainty** (*i.e.* the performance profile of each alternative can be evaluated deterministically). In the Nature Constant perspective, the environment is perceived as a set of resources valued according to their economic worth. In Chapter 2 traditional deterministic optimal control theory is discussed in the spirit of the Nature Constant worldview. Specifically, the risk of species extinction is downplayed, the preservation value of a resource is not considered, and it is assumed that resource productivity can be controlled. As is well-known, **classic optimal control theory** is primarily concerned with the maximization of profits subject to economic and environmental constraints.

The development of sustainability indicators in Section (Chapter) also conforms to the Nature Constant view because a single stable equilibrium is assumed. For example, when managing agricultural systems from a Nature Constant perspective, sustainability often implies

- maintaining a constant (usually high) level of productivity; and
- quickly recovering from external disturbances.

Notions of productivity, constancy, and recovery from a Nature Constant perspective are shown in Fig 1.3.

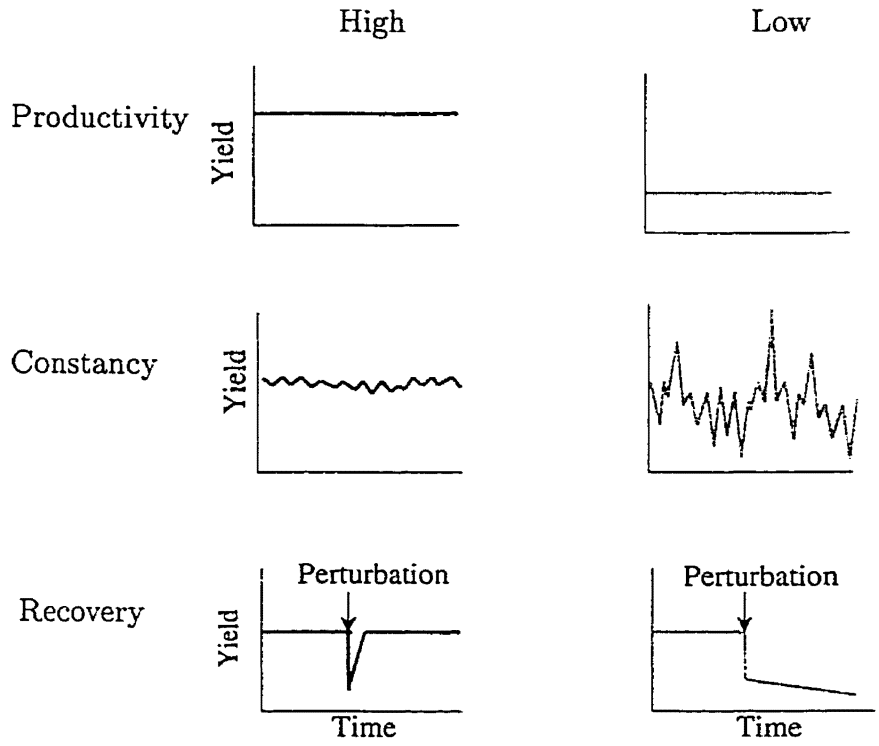


Figure 1.3: Various notions of agricultural sustainability.

Specifically, in Section the author illustrates the use of sustainability indicators in the context of forest management in New Brunswick, Canada (Clark

et al., 1979; Levy et al., 2000d). Existing decision support systems for the Nature Constant perspective include Web-HIPRE (HIerarchical PREference analysis software), a popular Java-applet for multiattribute decision making based on HIPRE 3+ Hämäläinen (1998).

1.4.2 Bayesian and Stochastic Approaches: Nature Random

The inherent randomness and variability of environmental phenomena make it difficult to perform classic statistical inferencing: it is rarely possible to perform an infinite series of trials under identical conditions. **In circumstances where only limited data are available and uncertainty is large, the Bayesian/subjective interpretation of statistics (Chapter 4) can often provide more guidance to decision makers than the traditional, 'frequentist' approach.**

A Markov-Chain Monte-Carlo decision support system (known as MEAL) is developed based on Gibbs Sampling (Gelfand and Smith, 1990; Smith and Roberts, 1993) to ascertain the marginal density functions for all parameters in the mixed BOD-decay model. The MEAL computer system allows one to determine marginal distributions even when it is not possible to integrate the joint posterior over all nuisance parameters. For example, given a joint density $f(x, y_1, \ldots, y_p)$, to find the marginal distribution for x one would have to integrate over $y_1, \ldots, y_p$:

$$f(x) = \int \ldots \int f(x, y_1, \ldots, y_p)\, dy_1 \ldots dy_p \tag{1.1}$$

The MEAL system provides an alternative method for obtaining $f(x)$: rather than compute $f(x)$ directly, a sample

$$X_1, X_2 \ldots, X_m \sim f(x) \tag{1.2}$$

is generated *without requiring* $f(x)$. By simulating a large enough sample, the mean, variance, or any other characteristic of $f(x)$, even the density itself, can be calculated to the desired degree of accuracy.

Stochastic models are also consistent with the Nature Random view Stochastic models, developed in Chapter 5, allow for the order of occurrence of probabilistic events to be taken into account. The purely random ('white noise') process is frequently encountered in stochastic models. In a white noise process the future is independent of the past. **Accordingly, the white noise process is consistent with the Nature Random view that events are independent** (and that non-random structures are improbable and tend to decay).

Stochastic water quality models are used to capture the interactions among carbonaceous BOD (CBOD), nitrogenous BOD (NBOD) and dissolved oxygen (DO). In Chapter 5 temporal moment equations are derived for all state variables, the Streeter—Phelps equations are generalized, and a practical **coloured noise approximation** is put forth to replace the abstract mathematical concept of 'white' (theoretical) noise.

Finally, the SEAL model is developed to solve stochastic differential equations. The author programmed an explicit order 1.5 strong scheme to integrate Îto SDEs. The algorithm is found in Section 11.2 of Kloeden and Platen (1992). The SEAL model can help in ascertaining a probability density function for the minimum DO concentration (DO_{min})[6].

1.4.3 Species Extinction and Conservation: Nature Ephemeral

The Nature Ephemeral perspective is consistent with the thesis chapters dealing with species extinction (Chapters 2 and 6). In particular, the Allee effect (critical depensation) and the 'preservation value' of resources is formally modeled. In addition, the SEAL decision support system is applied to a number of species extinction problems. **Here the 'First Passage Time' problem is considered in detail from an environmental perspective.** The First Passage Time analysis is valuable because in real ecosystems it is known that if a population falls below a critical threshold

[6]If the concentration of DO falls below a critical threshold, fish and other organisms begin to die and floating sludges predominate.

(minimum viable level) the population may become extinct.

In the context of fisheries management, it is shown how regulating the 'fishing effort' can significantly reduce the risk of stock extinction. The precautionary principle provides additional incentive to model species extinction. Finally, it is discussed how increasing the conservation/preservation value of a fishery in an cost-benefit analysis framework can yield to more sustainable management. The objective of this research is to prevent tragedies such as the collapse of the Newfoundland northern cod fishery from being repeated.

1.4.4 Info-gap models: Nature Resilient

In recent years, Holling's concept of 'ecological resilience' (1973) has been applied in areas from water resources (the design of "safe-fail systems") to financial management (portfolio hedging and asset liquidity). The Nature Resilient perspective holds that natural and social systems are able to adapt and thrive under conditions of adversity. **Chapter 7 is consistent with the Nature Resilient view. Here, the concepts of system adaptability, flexibility, and robustness are emphasized. Specifically, an MCDA info-gap model is put forth to quantify the robustness of policy alternatives to uncertainty.**

The info-gap approach identifies policy alternatives that, while capable of coping with attribute variability, still achieve minimum socio-economic requirements. **The management of water quality (phosphorus and eutrophication concerns) in Lake Erie is used as an illustrative example of how info-gap models can be used to gain insights into environmental problems**: information about phosphorus levels is organized in terms of families of sets (or clusters).

The proposed multiple criteria info-gap model is entirely non-probabilistic and constitutes a viable medium for integrating environmental indicators, conflicting objectives, and ambient uncertainty in a complex decision context. Numerical results generated by the REAL decision support system

(Robust Environmental Analysis for Lokahi) show how the minimum required return and the available prior information determine which policy alternative can best cope with ambient uncertainty. Finally, the constructed robustness curves assess the global sensitivity of alternatives to uncertainty.

1.5 Additional Uncertainty Approaches

Uncertainty plays a significant role in how one perceives the external world. There are a variety of formal tools that can assist in the understanding, regulation, and optimization of uncertainty. For these reasons, uncertainty techniques are an "important contribution to our scientific understanding of complex phenomena" (Kapur and Kesevan, 1992). However, the field of uncertainty analysis is quite vast. Accordingly, this thesis is restricted to crisp (non-fuzzy) problems. While entropy methods are discussed briefly in the context of info-gap models, the field of entropy optimization is also outside the scope of this thesis. However, for completeness, both fuzzy approaches and entropy methods are briefly discussed below.

1.5.1 Entropy Methods

The concept of information-theoretic entropy plays a significant role in the modeling of uncertainty and the formulation of probabilistic systems. For example, there may be uncertainty as to whether a pair of dice will turn up two sixes or not. Similarly, there may be uncertainty about the impact of a chemical on a water body, or the future market price of a resource. There may be n possible discrete outcomes in each one of these situations, and their probabilities may be $p_1, p_2, \ldots, p_n$ where

$$p_1 \geq 0, p_2 \geq 0, \ldots, p_n \geq 0, \qquad \sum_{i=1}^{n} p_i = 1 \tag{1.3}$$

Different probability distributions are associated with different levels of entropy (probabilistic uncertainty). For example, the entropy (uncertainty)

in the probability distribution (0.5, 0.5) is more than the entropy of the probability distribution (0.01, 0.99). One frequently encounters probability distributions consistent with a set of given constraints. For example

$$\sum_{i=1}^{n} p_i = 1, \qquad \sum_{i=1}^{n} p_i g_{ri} = a_r, \qquad r = 1, 2, \cdots, m \tag{1.4}$$

where $m + 1 < n$ and $p_1 \geq 0, \cdots, p_n \geq 0$.

Scientific objectivity implies that one should select a probability distribution $\boldsymbol{p} = p_1, p_2, \ldots, p_n$ by using all information consistent with Eq 1.4 and carefully avoiding any information not given in Eq 1.4. The principle of scientific objectivity leads to the principle of maximum entropy (maximum uncertainty): "Out of all probability distributions consistent with a given set of constraints, choose the one that has maximum uncertainty" (Kapur and Kesevan, 1992).

In much of the probabilistic uncertainty literature, the term entropy is frequently used in place of uncertainty. The reason is primarily historical: in 1948 Claude Shannon, a communication engineer, developed the first measure of uncertainty of a probability distribution $\boldsymbol{p} = p_1, p_2, \ldots, p_n$. His uncertainty measure

$$S(\boldsymbol{p}) = -k \sum_{i=1}^{n} p_i \ln p_i \tag{1.5}$$

where k is an arbitrary positive constant, became known as entropy since Eq 1.5 had the same mathematical form as entropy in thermodynamics. The term entropy is advantageous because uncertainty, in all its forms, is too wide a concept to be encompassed in a single measure such as that of Shannon's. For example, the term entropy clarifies that one is not considering uncertainty due to fuzziness in information. Entropy also excludes situations in which uncertainty is partly probabilistic and partly non-probabilistic.

Fuzzy Sets and Linguistic Imprecision

While uncertainty is often associated with probabilistic phenomena (such as rolling a dice), uncertainty can also arise in the context of deterministic

phenomena, where we know for that the outcome is not a chance event but one is *fuzzy* about the meaning of this outcome. In both everyday discourse and professional writing one often uses imprecise language when referring to events or quantities. For example, the proposition that "the Grand River is highly polluted" is ill specified: there may be considerable variation in the way different people interpret this verbal phrase, and their interpretation is context dependent (Wallsten et al., 1986). Saying that a water body is "highly polluted" may have a different meaning depending on whether one lives in Canada or in a country where environmental standards are not as high. This suggests that simple mappings between verbal phrases and probabilities are likely to be problematic.

Fuzzy set theory (Zadeh, 1972) considers linguistic imprecision to be an unavoidable aspect of human conversation which should be explicitly handled by a formal axiomatic reasoning system. A conventional "crisp" set is defined by a membership function, which specifies for every object whether or not it is a member of the set. In contrast, a fuzzy set is defined by a fuzzy membership function, which allows degrees of membership intermediate between 0 and 1. For example, with a dissolved oxygen concentration level of 5 mg/L the Grand River might have degree of membership 0.6 in the fuzzy set of polluted rivers. Fuzzy set theory defines operations for the union, intersection, and complement of fuzzy sets, as generalizations of the corresponding crisp set operations.

Some uncertainty experts, such as Morgan and Henrion (1990) argue that linguistic imprecision can be "remedied" by providing a careful specification of all events and quantities. In this view, it is the role of the analyst to eliminate imprecise language. It may be possible to rephrase the statement, "the Grand River is highly polluted" more precisely: "In May, 2000, the Grand River has a BOD (biological oxygen demand) level exceeding 20 mg/L."

On the other hand, proponents of fuzzy set theory have developed sophisticated techniques for the formal representation of linguistic imprecision. There is a considerable body of research investigating the correspondence be-

tween verbal phrases such as "unlikely", "highly probable" *etc.* and numerical probabilities (Wallsten et al., 1986). While the controversy surrounding linguistic imprecision and fuzzy set theory raises important issues, it is beyond the scope of the thesis, which is concerned only crisp problems.

1.5.2 Which MCDA Approach to Use?

Every decision problem exists in a decision context (the set of circumstances and conditions that affect the decision making process). There are many ways to categorize decision problems from an operations research perspective. For example, Radford (1989) notes that important features of a decision problem include:

1. Whether or not uncertainty exists in the decision situation being considered.
2. Whether or not the benefits and costs can be completely assessed in quantitative terms.
3. Whether or not multiple criteria must be taken into account[7].

Rajabi et al. (1999) emphasize other important features of MCDA problems:

1. Whether economic, social and environmental impacts are considered.
2. Whether or not technical, legal, and operational issues are taken into account.
3. Whether or not the distribution of impacts is addressed.

[7]Most optimization techniques, including linear, dynamic, and nonlinear programming, are single criterion methods because they are often employed for maximizing monetary benefits (or minimizing costs) subject to various economic and physical constraints.

Because the type of information available (probabilistic/non-probabilistic, cardinal/ordinal, deterministic/stochastic) may vary from one problem to the next, different MCDA techniques have been developed to take advantage of the type of information available, as shown in the top row of Table 1.4. Which technique to use depends on the features of the decision problem. For example, ELECTRE methods (Roy, 1973) employ information in a fuzzy context, the Analytic Hierarchy Process (AHP) approach (Saaty, 1990) elicits ratio judgements, while Elimination Methods (MacCrimmon, 1973) require only ordinal rankings. Of course there other ways to view MCDA problems issues not addressed in Table 1.4. For example, MCDA methods can be classified according to whether the attributes are evaluated before (*a priori* preference articulation) or after the alternatives are presented (*posterior* preference articulation).

Techniques for decision making under certainty include Optimal Control Theory, CBA (Cost Benefit Analysis), the Elimination Method and MAVT (Multi-Attribute Value Theory). These four techniques are shown in the top row of Table 1.4 (from left to right). The remaining five approaches in Table 1.4 — Interval Methods, Entropy Techniques, MAUT (Multi-Attribute Utility Theory), and Info-gap methods — apply to decision making under uncertainty. This thesis begins with a discussion of MCDA under certainty and extends the discussion to new approaches for MCDA under uncertainty. Note that in Table 1.4 a checkmark is placed in a cell location, if a particular decision making technique is capable of handling the type of information available. While this type of evaluation matrix is highly subjective, it is extremely useful in illustrating the relative merits of different techniques.

Features of Problem	Section	Techniques: Optimal Control	CBA	Elimination Method	MAVT	Interval Methods	Entropy Measures	Bayesian	MAUT	Info-Gap	Relevant Approaches
Type of Impacts											
Economic Impacts	2.2.3	✓	✓	✓	✓	✓	✓	✓	✓	✓	9
Social Impacts	2.3.2			✓	✓	✓		✓	✓	✓	6
Environmental Impacts	2.2.2			✓	✓	✓		✓	✓	✓	6
Distribution of Impacts	2.2.4			✓	✓	✓			✓	✓	5
Feasibility											
Technical and Operational	6.4			✓	✓				✓	✓	4
Legal and Public Policy	6.4			✓	✓			✓	✓	✓	5
Type of Problem											
Physical Systems Model	5	✓		✓	✓			✓		✓	5
Societal Systems Model	2.2.2		✓		✓	✓		✓	✓	✓	6
Type of Tradeoffs											
Compensatory	2.1.1	✓	✓	✓	✓	✓	✓	✓	✓	✓	9
Non-compensatory	2.3.4			✓				✓			2
Type of Uncertainty											
Probabilistic Uncertainty	7.1.3	✓	✓			✓	✓	✓	✓		6
Non-probabilistic Uncertainty	7.3									✓	1
Type of Information											
Quantitative (Cardinal)	2.2.3	✓	✓	✓	✓	✓	✓	✓	✓	✓	9
Qualitative (Ordinal)	2.3.2			✓		✓					2
Type of Analysis											
Extinction Analysis	6.3	✓					✓	✓	✓	✓	5
Hierarchical Analysis	2.3.3	✓	✓	✓	✓	✓	✓	✓	✓	✓	9

Figure 1.4: Features of a Decision Problem and MCDA

1.5.3 Type of Impacts

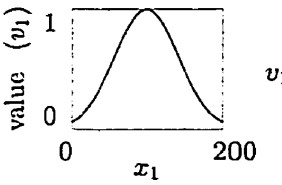
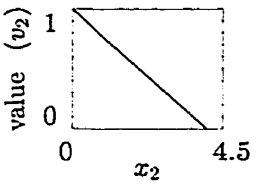
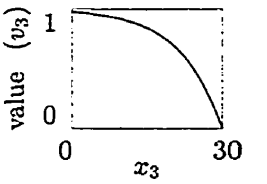
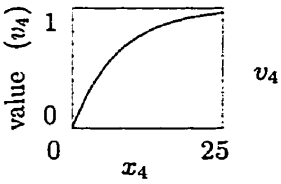
All nine of the techniques listed in Table 1.4 can be used to help make decisions when the criteria are expressed in dollars. However, the treatment of decision consequences in purely monetary terms has attracted considerable criticism, particularly for environmental applications (Sagoff, 1988), although it is certainly more convenient to assess projects and make decisions solely on the basis of monetary values. Faustmann (1849) designed one of the first harvesting models (a forest rotation system) in order to maximize long-term economic benefits[8].

Social and Environmental Impacts

Clearly, social and environmental criteria must be considered to ensure the integrity of life support systems. Multi-attribute Value Theory (MAVT) is a common approach that is able to handle social, environmental, and economic criteria. For example, in the spruce budworm analysis of Bell (1977) and Clark et al. (1979) value functions were elicited from an environmental official in the province of New Brunswick, Canada. A caricature of the results are shown in Table 1.1. Note that there are three generic value function shapes present in Table 1.1: Forest Volume possesses a *Desirable Range* value function; whereas, Recreational Quality is modelled with a *More is Better* value function; and Area Sprayed is of the *More is Worse* type.

[8]In the nearly two centuries since Faustmann's work, an overemphasis on economic consideration together with the introduction on non-native species and overharvesting has led to large-scale species extinction (Prato, 1999).

Table 1.1: Indicators, value functions and references.

Indicator			Value Function		Range	
x_i	Descriptor	Units	Graph	Equation: $v_i(x_i)$	Worst	Best
x_1	Forest Volume	m^3/ ha		$v_1 = e^{-(\frac{x_1-100}{60})^2}$	0 or 200	100
Ref.	Wymore (1988) Clark et al. (1979)				**Desirable Range**	
x_2	Spray Area	10^6 ha		$v_2 = 1 - \frac{1}{4.5}x_2$	4.5	0
Ref.	Lane et al. (1994) Baskerville (1995)				**More is Worse**	
x_3	Harvest Cost	\$ / m^3		$v_3 = 1 - e^{\frac{x_3-30}{8}}$	30	0
Ref.	Keeney (1980) Bell (1977)				**More is Worse**	
x_4	Rec. Quality	good sites		$v_4 = 1 - e^{-0.13x_4}$	0	25
Ref.	Bell (1977) Clark et al. (1979)				**More is Better**	

Distribution of Impacts

The types of environmental decisions for which fairness is likely to be a major concern range from the local and site-specific projects to national and policy-level endeavours. Intuitively, concerns of fairness and impact distributions relate to both substantive and procedural aspects of environmental decisions. In a substantive sense, adverse impacts ought not fall disproportionately on some groups, particularly on groups that have historically been economically and politically marginalized (or somehow disadvantaged). Those receiving most of the benefits should be paying most of the costs (or enduring most of the adverse impacts). 'Procedural fairness' refers to the procedures by which decisions are reached (Lind et al., 1990).

Joubert et al. (1997) argue that MAVT is a more appropriate tool than CBA with respect to procedural and substantive justice, particularly for evaluating environmental projects that generate significant social and ecological externalities. MAUT is a relevant technique to use because one may wish to include the risk attitudes of the stakeholders when considering the distribution of impacts. Due to the extreme uncertainty regarding future impacts, info-gap modeling may also be appropriate. Other relevant approaches are shown in Table 1.4.

1.5.4 Feasibility and Policy Considerations

Rajabi et al. (2000) emphasize that technical and operational feasibility, in addition to public policy issues and governance considerations should be considered in any MCDA. For instance, assessing the effect of a chemical on a population of fish prompts the questions: Are all species equally important? Are six unhealthy fish equivalent to one dead fish? What about the tradeoff between employment and ecosystem health? Intangible attributes (such as aesthetic considerations, pain and suffering, *etc.*) can be extremely difficult to include in any decision analysis. Finally, one must consider how consequences unfold over time (including issues of discounting and inter/intra–

generational equity). These significant uncertainties may be addressed in the context of info-gap modeling. MAVT may also be particularly relevant in light of the value judgements required to analyze these problems. The elimination method would be recommended if tradeoffs among alternatives is not permitted. Finally, the Bayesian approach may be useful to incorporate the opinions of experts.

1.5.5 Type of Problem

Social Systems Models

A typical societal systems problem involves deciding how much development to permit in a watershed. For example the potential impact of new development is of great concern for the community, government and environmental agencies in the Laurel Creek Watershed. A caricature of a social system problem is shown in Fig ??. Here, the relevant criteria are organized into a hierarchy, where the highest element of the hierarchy represents the overall goal, sustainable growth. Subgoals, such as housing and water quality are decomposed further until a sufficiently detailed representation of the decision problem is obtained. The decision alternatives, 'Business as Usual', 'Moderate Development' and 'Limit Development' are placed on the lowest level of the hierarchy.

A variety of approaches are suitable for social systems models. While CBA is frequently used, MAVT may be more appropriate since all impacts do not need to be converted into a dollar value. Moreover, Bayesian approaches may be useful to due a lack of data. In addition, interval approaches can be used when the relative criteria weights cannot be precisely specified. Finally, info-gap models and MAUT can be used to incorporate the risk attitudes of the stakeholders.

Physical Systems Models

Physical systems models include atmospheric and hydrological models. Stochastic differential equations are often useful for modeling physical systems. A stochastic process is a function of two variables: the parameter t and the probability parameter ω. A stochastic process is thus a mathematical model of a dynamic process whose dependence on a parameter t is governed by probabilistic laws.

This thesis considers physical systems models in hydrology. Sufficient quantities of organic material in a water body may lead to an increase in bacterial activity and a resulting decline in DO concentration. The removal of organic material by microorganisms (such as bacteria and algae), primarily through aerobic decomposition, gives rise to the classic "DO sag curve" of Fig 1.6. As the concentration of DO falls, the number of surviving life forms is reduced; in extreme cases, most forms of life are dead and odors, floating sludges, and fungal growth predominate. In addition to stochastic differential equations, time series analysis techniques and other statistical tools can also be used to model physical systems. Moreover, physical models have been used in the context of optimal control theory and MAVT. It is also possible to integrate physical models with Bayesian approaches, info-gap models and other techniques.

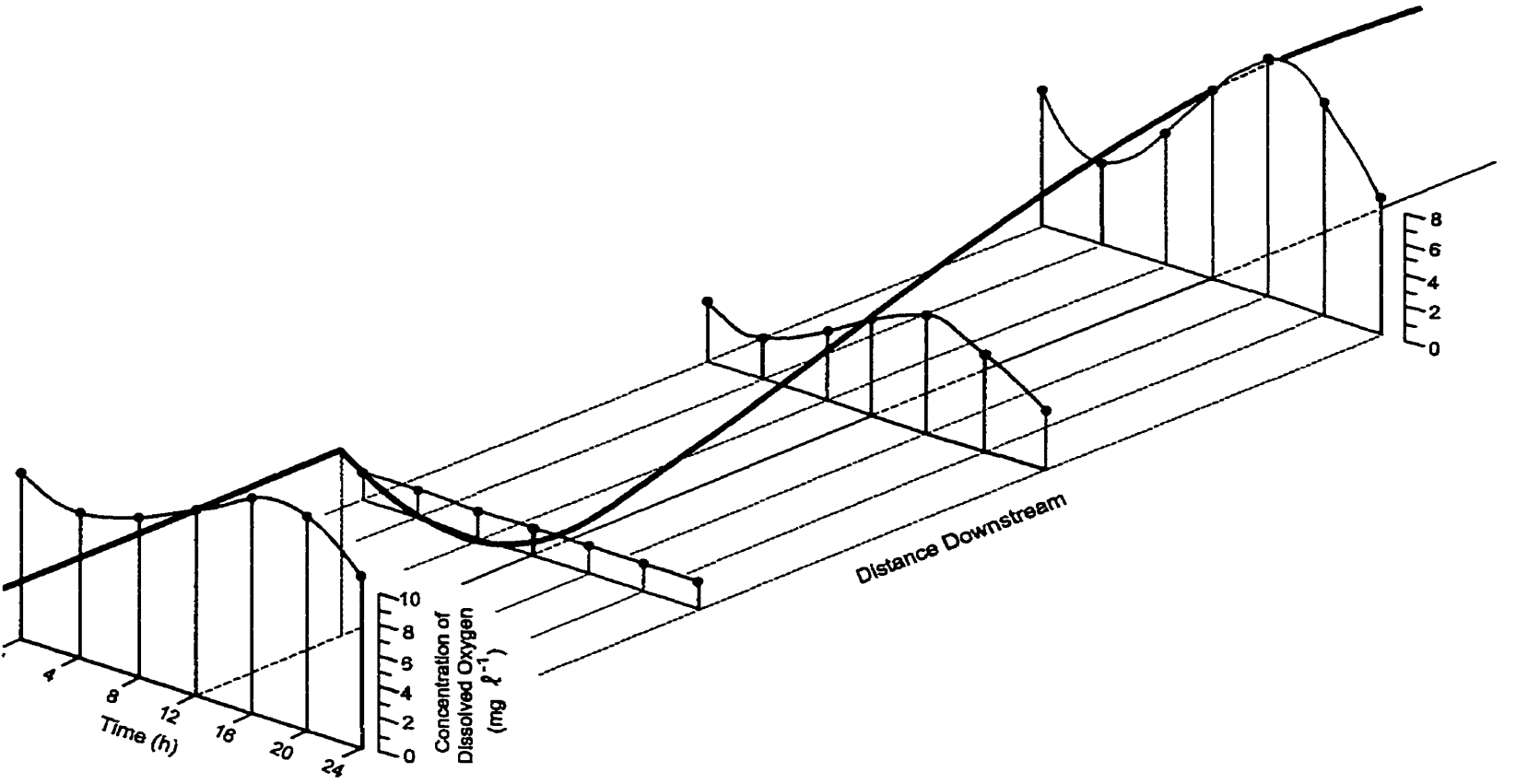


Figure 1.6: Dissolved oxygen (DO) vs time

1.5.6 Type of Tradeoffs

In compensatory methods, quantitative tradeoffs across objectives are allowed. Compensatory and noncompensatory approaches frequently coexist in decision problems. The choice ultimately depends on the characteristics of the problem: compensatory methods tend to be more demanding in terms of data and the amount of information to be elicited from experts. Munda (1993) and Opschoor and Hafkamp (1991) provide an overview of compensatory methods applied in the economic evaluations of environmental problems.

In many decision situations alternatives cannot be evaluated entirely in quantitative form. In this situation, noncompensatory models offer some capability of placing a number of alternatives in an order of preference. An important example of non-compensatory models is the lexicographic approach. In lexicographic preference, one attribute has overriding importance; decisions are made on the basis of it alone. If there are several options tied for performance on this attribute, the second and third most important attributes are used to break ties.

A popular non-compensatory technique is the *Elimination method* (MacCrimmon, 1973). The Elimination Method uses a stepwise process of screening alternatives: if one alternative performs better than another on the most important attribute, then it will be selected, however poorly it does on the remaining attributes. Alternatives not meeting a specified level of performance are eliminated until only one is left that has satisfied all the tests to that point. Ties are resolved by making the levels of performance or the criteria used more discriminating. Necessary conditions for use of the Elimination Method include ordinal or cardinal preferences for alternatives (for each objective) and an ordinal ranking of the criteria. The Elimination Method is illustrated in Fig 1.7 in the context of sixteen alternatives A through P.

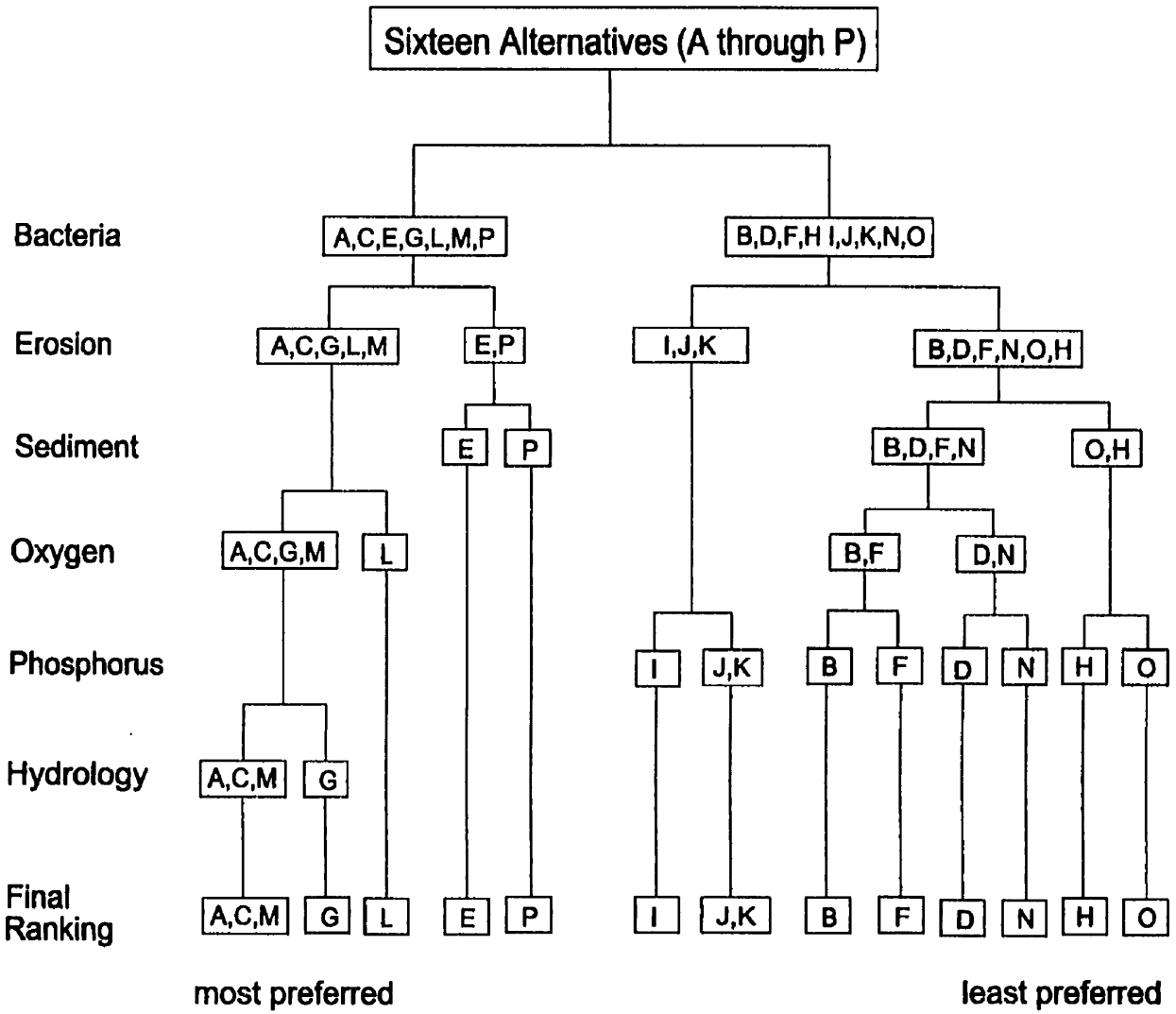


Figure 1.7: The Elimination Method and Stormwater Management

1.5.7 Type of Uncertainty

Probabilistic Uncertainty

Fig 1.8 shows an example of a response surface as a function of inputs x_1 and x_2: the surface displays how the output y changes with variations in the values of the inputs. Note that the inputs x_1 and x_2 are represented by the two horizontal dimensions, and the output y is represented by the vertical dimension. It is assumed that both uncertain inputs are empirical quantities (measurable in principle) and that uncertainty about them can be legitimately represented by probability distributions. A wide variety of techniques are available to handle probabilistic uncertainty, including MAUT and entropy methods. Info-gap methods are capable of handling both probabilistic and non-probabilistic models of uncertainty.

Non-probabilistic Uncertainty

In section 7.3, info-gap set models of uncertainty are proposed to handle non-probabilistic representations of uncertainty: here, the emphasis is on "cluster-thinking" (Ben-Haim, 1998) rather than on recurrence, probability, or likelihood. Given a particular piece of information an info-gap modeler might ask: what is the "cloud of possibilities" (Ben-Haim, 1999) consistent with this information? How does this cloud shrink, expand and shift as our information changes? What is the gap between what is known and what could be known? Info-gap modeling often takes place without recurrence information, and hence one can make no heuristic or lexical judgements of likelihood.

1.5.8 Type of Information

Quantitative Information

For the quantitatively oriented decision maker, there is great appeal in being able to establish some means of associating a numerical score with each de-

$$y = f(x_1, x_2)$$

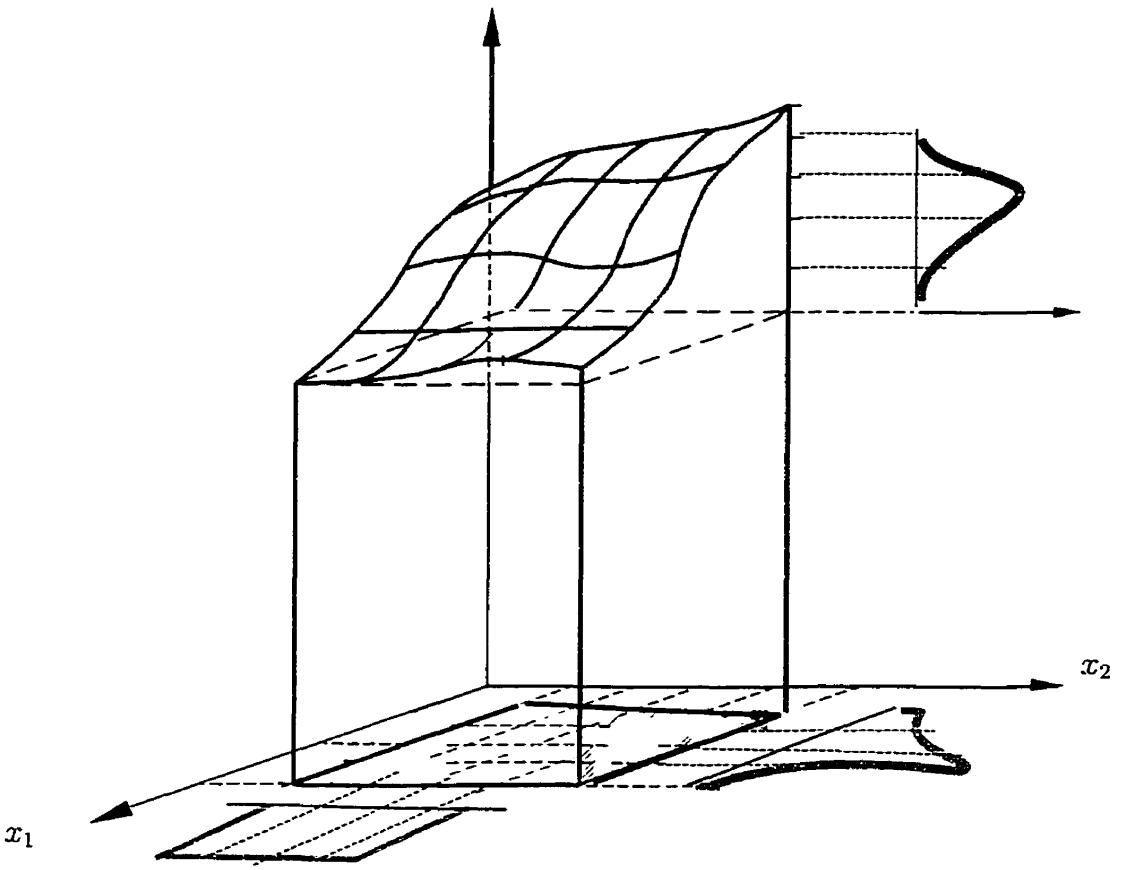


Figure 1.8: Propogation of the continuous probability distributions through a model

cision alternative, after which the choice of the optimal alternative becomes automatic. Accordingly, most optimization procedures are based on the assumption that one can assign a real number (such as a cost in dollars or the biomass of a fish stock in kg) to represent the consequences of an alternative according to a criterion.

Among cardinal decision analytic methods, a fundamental distinction exists between monetary and non-monetary evaluation approaches. This thesis

focuses on the latter; although popular examples of the former include cost-benefit analysis (CBA) and nonmarket valuation methods, such as contingent valuation (CV) (Mishan, 1988; Dasgupta and Pearce, 1972). Of course cardinal value functions require a more demanding set of assumptions than the ordinal case, but provide more information about the preference structure. A cardinal value function v preserves ordinal preference as well as an ordering on difference (under certainty).

Qualitative Information

In many environmental problems it is simply not possible to assign quantitative values to the consequences of each alternative. Often, the natural way to evaluate the performance of actions is by using qualitative, descriptive, or ordinal information to express concepts such as "degree of degradation" and "quality of life". Ordinal value functions attach numbers to objects in such as way that ordinal preference relationships are preserved. However, ordinal numbers are meaningful only in their ordinal content and differences between them are meaningless. Interval techniques and the Elimination method are well suited for ordinal information.

1.5.9 Type of Analysis

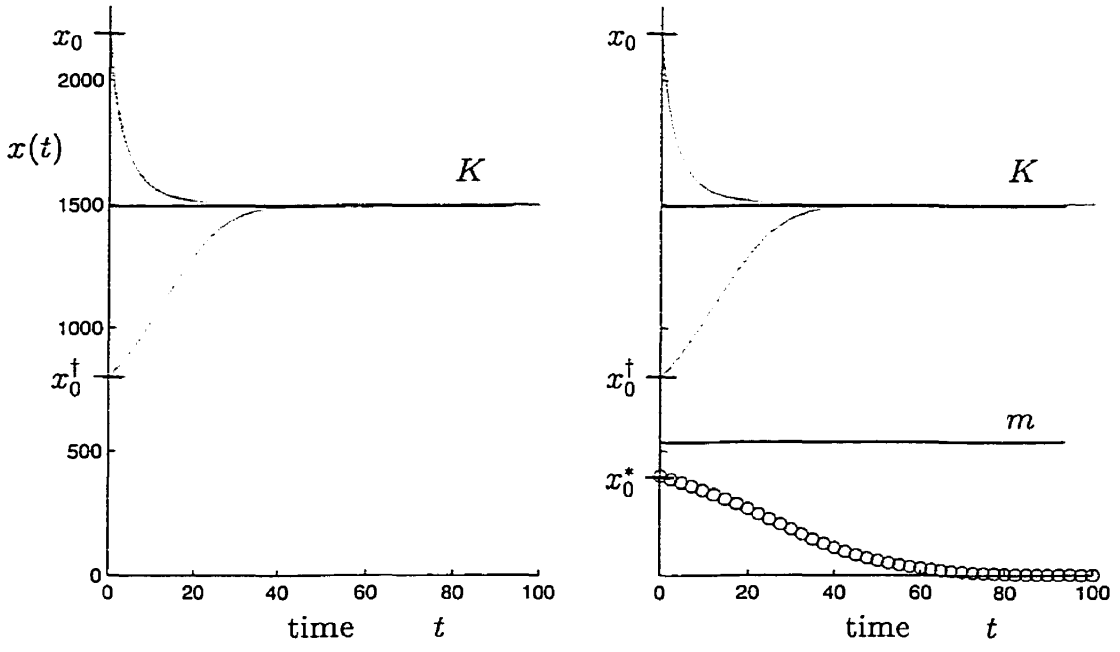
Extinction Analysis

Discussing the possibility of species extinction is particularly timely in light of the intense harvesting of biological resources currently taking place across the globe. Extinction analysis techniques allows one to model the expected time to extinction of a species. Appropriate techniques to model extinction issues include MAUT (to include risk profile of stakeholders), Bayesian approaches (to include subjective probabilities), and info-gap models (to handle extreme uncertainty). In this thesis the risk of species extinction was incorporated into an optimal control problem by using the following population growth

function $F(x)$ where $F(x) < 0$ for certain values of x near $x = 0$:

$$F(x) = rx\left(1 - \frac{x}{K}\right)\left(\frac{x}{m} - 1\right) \quad \text{where} \quad 0 < m < K \tag{1.6}$$

To model ecological criteria such as the risk of species extinction, it is instructive to compare and contrast the population dynamics of the pure compensation logistic growth model (Fig 1.9a) and its critical depensation analog (Fig 1.9b). Figure 1.9a illustrates two typical solution curves $x(t)$ approaching the equilibrium K from above and below. The lower curve is usually referred to as a logistic growth curve. Fig 1.9b) exhibits the phenomena of irreversibility. If the population is reduced to some level below the minimum viable population m, an irreversible extinction process begins: ultimate extinction of the population is ensured, regardless of what happens to future effort levels.



(a) Logistic Model

(b) Critical Depensation

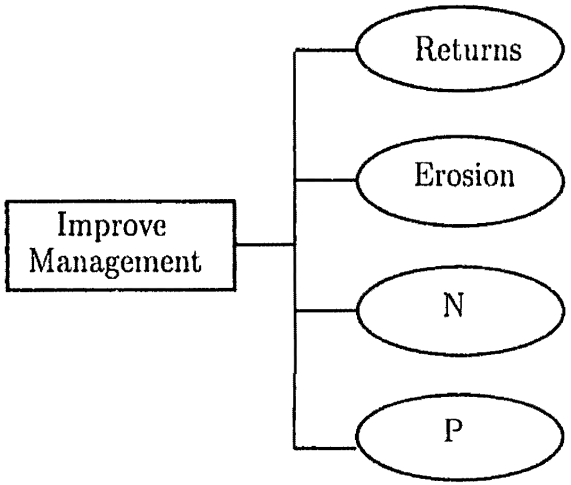
Figure 1.9: (a)Logistic growth model: $F(x) = rx\left(1 - \frac{x}{K}\right)$ (b) Critical Depensation: $F(x) = rx\left(1 - \frac{x}{K}\right)\left(\frac{x}{m} - 1\right)$

Hierarchical Analysis

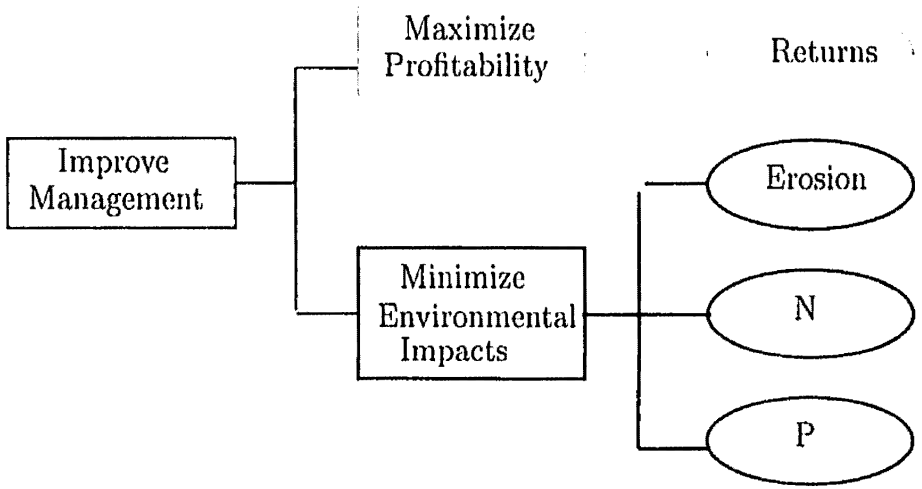
The Analytic Hierarchy Process (AHP) is a popular hierarchical technique to solve MADM problems. For a full exposition see Saaty (1980). The first stage in the AHP approach is problem structuring: breaking down the top level objective into subgoals until a sufficiently detailed representation of the decision problem is obtained. It is plausible that each of the techniques studied in this thesis could be applied to some part of a hierarchical problem.

Fig 1.10 illustrates a hierarchical value tree in the context of farm management. The highest system objective is often a broad statement about the overall goal, usually a universally acceptable statement. As one moves down

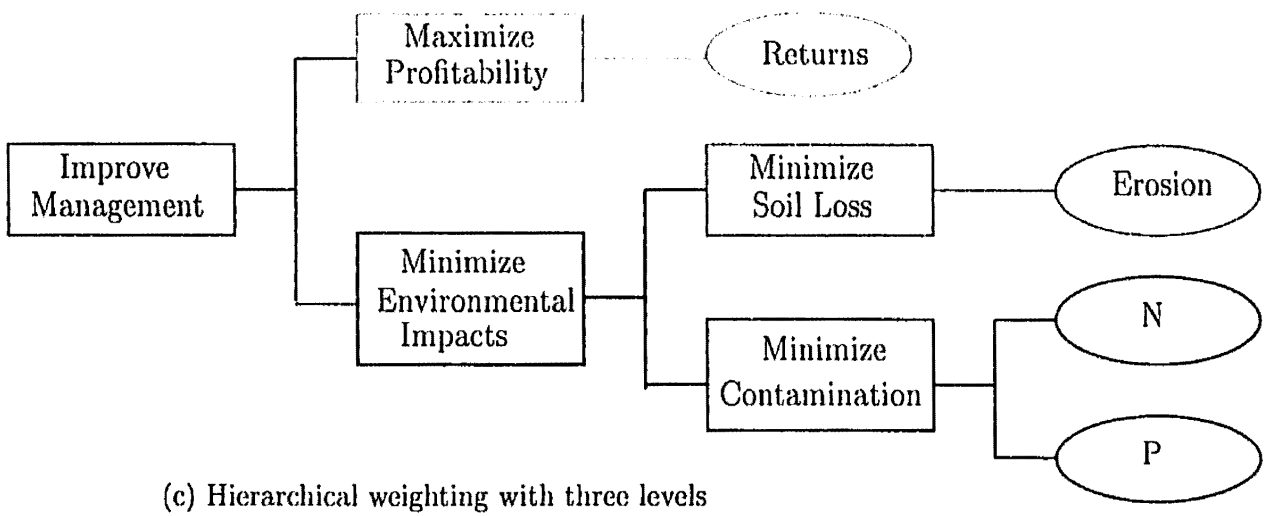
the 'value tree', the objectives become more specific, more operational, and at the same time conflict more with each other. For example, under conditions of water scarcity, will agricultural users have priority over industrial ones? What about allocation to residential users? How to meet domestic, industrial, and agricultural objectives simultaneously?



(a) Non-hierarchical weighting



(b) Hierarchical weighting with two levels



(c) Hierarchical weighting with three levels

Figure 1.10: Value Trees for Farm Management System

Chapter 2

Environmental Multiple Criteria Decision Analysis

In Section 2.1 both conservation and economic criteria are considered in the context of optimal control theory . Here, research has traditionally dealt with determining a harvest policy that maximizes the discounted net revenues derived from the exploitation of a renewable resource, subject to biological and economic constraints (Clark, 1985).

The MCDA framework is explained in Section 2.2. Here, important issues include alternative generation (Section 2.2.1), indicator selection (Section 2.2.2), the evaluation of alternatives (Section 2.2.3), and the fundamental problem of MCDA: comparing noncommensurate quantities, for example, comparing dollar costs with environmental quality. The traditional approach to this problem is cost-benefit analysis (CBA), whereby all considerations — economic, social and environmental — are converted to dollar values. Although this "compensation" permits tradeoffs among criteria, when the attributes of interest cannot meaningfully be reduced to a single measure such as cost, other methods of comparison must be found. In this context, the advantages of MCDA over CBA and non-market valuation procedures such as Willingness to Pay (WTP) and contingent valuation (CV) are discussed (Section 2.2.4).

Multiattribute decision making (MADM), the discrete version of MCDA, is the focus of Section 2.3. Here, discrete alternatives are evaluated against criteria ranging from cost (a quantitative criterion) to aesthetics (a qualitative criterion). Various MADM approaches are considered including outranking methods (Section 2.3.1), Multi-Attribute Value Theory (Section 2.3.2), the Analytic Hierarchy Process (Section 2.3.3), and lexicographic techniques (Section 2.3.4). Finally, the importance of performing a comprehensive sensitivity analysis is discussed (Section 3.3).

2.1 Optimal Control Theory

It is important to note that the 'environment' does not enter into either neoclassical or Marxist economic analysis. Consider the situation facing a private individual who owns a forest in a market economy. The entire forest can be cut immediately, generating many forest products but leaving only barren land, worth almost nothing. On the other hand, the forest can be run as a sustainable "timber factory", cutting trees slowly over time according to a well-selected pattern. However, the forest only grows at a limited rate, so the sustainable rate of timber production may be low. The owner of a forest (or any resource) tends to view her stock (in this case trees) as a *capital asset* (Clark, 1985). She expects the asset to earn dividends at the prevailing interest rate; otherwise, she will attempt to dispose of the asset. This result, which is often considered to be the fundamental theorem of resource economics, dates back to the pioneering work of Hotelling in the 1930s: if the return on other available investments (*i.e.* the interest rate) is greater than the growth rate of the forest, then the owner can maximize return by cutting the entire forest and investing the proceeds, rather than managing a sustainable forest.

As another example, consider how Antarctic blue whales (*Balaenoptera musculus*) might be managed by a single firm, or a consortium of countries such as the International Whaling Commission (IWC), that possess exploita-

tion rights. Until the 1950s the Antarctic whaling industry focused its efforts on the harvesting of blue whales (reducing the estimated original population of 150,000 to fewer than 1,000 by 1965, when their capture was prohibited by the IWC). Scientists estimate that a standing population of 75,000 blue whales is required to achieve a maximum sustainable yield (MSY) of approximately 2,000 whales per annum.

For the sake of discussion, assume that harvesting has reduced the blue whale population to 75,000 and that the market value of the average blue whale carcass is \$10,000. Here, industry has two options: sustainable harvesting or immediate extermination. In the former case, an MSY policy will result in an annual revenue of \$20 million; in the latter, immediately harvesting the remaining 75,000 whales yields a lump-sum revenue of \$750 million. Invested at a rate of 5% per annum, this sum would yield an annual return of \$37.5 million. From this rudimentary economic analysis, the extermination of blue whales appears considerably more profitable than pursuing an MSY policy (Clark, 1976).

Of course, this argument has been deliberately oversimplified. For example, how to deal with the problem of selling several million tons of whale products in a short time (demand inelasticity)? Moreover, the cost of catching blue whales becomes exorbitant as their population becomes depleted (Antarctic feeding grounds cover more than 20 million square kilometers)[1]. However, the conclusion may still be that government intervention is the only way to save the whales, the forests, and other natural resources. Intervention may be economically justified for the ecosystem services provided (soil formation, climate regulation, habitat for resident and transient species), in addition to recreational values and genetic diversity.

A traditional objective in the management of renewable resources is to

[1]In a personal communication, renowned bioeconomic modeler Dr. Colin Clark of the University of British Columbia noted that while the slow growth rate of the blue whale may cause industry to shun conservation (due to the "inferior asset" problem), the species may still survive because complete extermination is not economically feasible (due to what he tongue-and-cheek calls the "economic extinction" of blue whalers).

select a harvesting regime, $h(t)$, that maximizes total discounted net revenues from the resource in question. The goal of maximizing revenues derived from the exploitation of a resource may be expressed as maximizing:

$$\text{Present Value} = \int_0^\infty e^{-\delta t} [p - c(x)] \, h(t) \, dt \tag{2.1}$$

where the unit harvest cost, $c(x)$, price, p, and discount rate, δ, are important economic variables in the exploitation of renewable resources.

2.1.1 Optimal Fishery Management and MCDA

If $\delta > 0$ is a constant denoting the (continuous) rate of discount, p is a constant price per unit (eg \$ per kg), and $c[x(t)]$ equals the unit harvesting cost when the population level is x then the management objective of maximizing total discounted net revenues derived from the exploitation of a resource may be expressed as maximizing:

$$PV = \int_0^\infty e^{-\delta t} [p - c(x)] \, h(t) \, dt \tag{2.2}$$

where PV is present value. Traditional economic theory assumes that one attempts to utilize a harvest rate $h = h(t)$ that leads to the largest possible value for the expression in Eq 2.2. Note that Eq 2.2 also depends on the population level $x(t)$, which itself is related to the harvest rate according to

$$\frac{dx}{dt} = F(x) - h(t), \quad t \geq 0 \tag{2.3}$$

The variables $x(t)$ and $h(t)$ must also satisfy the constraints

$$x(t) \geq 0 \qquad \text{and} \qquad h(t) \geq 0 \tag{2.4}$$

Maximizing the expression in Eq 2.2 subject to these conditions is a problem in optimal control theory. Several mathematical techniques can be used to

determine the harvest policy $h(t)$ that maximizes Eq 2.2. Here we make use of the Euler equation as follows.

Substituting $h(t) = F(x) - \dot{x}$ from Eq 2.3 yields:

$$PV = \int_0^\infty e^{-\delta t} [p - c(x)] [F(x) - \dot{x}] \, dt \tag{2.5}$$

Since this integral has the form

$$\int \Delta(t, x, \dot{x}) \, dt \tag{2.6}$$

one may apply the classical Euler necessary condition for a maximum:

$$\frac{\partial \Delta}{\partial x} = \frac{d}{dt} \frac{\partial \Delta}{\partial \dot{x}} \tag{2.7}$$

In resource management, it is often assumed that resources should be exploited in such a way that the total discounted net revenues derived from the resources are maximized. Clearly this does not adequately consider the social, ecological, and recreational aspects of natural resource problems. Accordingly, Section 2.1.2 makes three significant and original contributions to the optimal control literature:

1. Formally modeling the 'preservation value' of the resource stock itself. It is important to capture the recreational value and ecological services provided by a resource system.

2. Allowing price and cost to vary as a function of time.

3. Including the possibility of species extinction (*i.e.* critical depensation).

The first two issues will be addressed with application to Schaefer's fisheries model. However, formally modeling species extinction requires the use of a growth function that exhibits critical depensation, sometimes referred to as the 'Allee effect'. Throughout this chapter, the effect of the discount rate, price, and other parameters on harvesting decisions is examined in detail.

2.1.2 Preservation Value and Critical Depensation

The value of the world's ecosystem services and natural capital should not be underestimated. For example, an international team of researchers recently approximated the monetary value of environmental processes, including the regulation of atmospheric gases, the treatment of wastes, and the cycling of nutrients to be "in the range of $16 - 54$ US trillion dollars per year" (Costanza et al., 1997). However, many authors express reservations about making such monetary based resource management decisions. For example, Goulder and Kennedy (1997) argue that such decisions will be biased in favor of economic-based considerations since the benefits of economic activities are better understood than ecological impacts. This was likely a factor in the development of hydropower systems in the U.S. Pacific Northwest, which drastically reduced salmon populations and their associated social benefits in the Columbia River Basin (Lee, 1995). In addition, the assignment of monetary values to ecological services is done, for the most part, independently of the environmental assessment and management process (Smith, 1992; Prato, 1999). Others reject the use of dollar estimates for ethical reasons (Mitchell and Carson, 1989). Moreover, since non-market values of ecological services are both site-specific and application dependent, the dollar amounts derived for one area are generally unsuitable for others (Bjornstad and Kahn, 1996). In fact, few valuation studies exist for some major biomes, including desert, tundra, ice/rock, and cropland (Costanza et al., 1997).

Others are more enthusiastic about the nonmarket valuation of natural resource systems. Postel and Carpenter (1997), for example, argue that better accounting for nonmarket values of natural systems can "help to ensure that land-use and water management decisions are both economically rational and environmentally sound". Economists commonly estimate the "nonmarket values" of ecosystem resources through surrogate market techniques, such as travel cost and hedonic pricing, or by nonmarket valuation methods, such as contingent valuation (CV). By so doing, a 'preservation value', $V(x)$, may be attached to the resource stock itself, so that the objective function

takes the following form:

$$J = \int_0^\infty e^{-\delta t} \{[p - c(x)]\, h(t) + V(x)\}\, dt \tag{2.8}$$

where J evaluates to the net discounted present value. Any expression for the preservation value $V(x)$ should satisfy $V'(x) > 0$. Here:

$$V(x) = \nu \ln(x) \tag{2.9}$$

Moreover, assume that the price per unit of harvested biomass $p(t)$ is a function of time. Possible models for $p(t)$ include damped trigonometric functions, such as

$$p(t) = a_1 + a_2 e^{-t} \sin t. \tag{2.10}$$

Finally, assume that unit harvest costs $\phi(t)c(x(t))$ are a function of time (Clark, 1985), where

$$\lim_{t \to \infty} \phi(t) = \overline{\phi} \tag{2.11}$$

Possible expressions for $\phi(t)$ satisfying 2.11 include:

$$\begin{aligned} \phi(t) &= \overline{\phi} + a_1 e^{-t} \quad \text{and} \\ \phi(t) &= \overline{\phi} + a_1 e^{-t} [a_2 + a_3 \sin t + a_4 \cos t] \end{aligned} \tag{2.12}$$

where a_1, a_2, a_3, and a_4 are constants.

Accordingly, Eq 2.8 can be re-written as:

$$J = \int_0^\infty e^{-\delta t} \{[p(t) - \phi(t)c(x)]\, [F(x) - \dot{x}] + \nu \ln(x)\}\, dt \tag{2.13}$$

Now, using Eq 2.6,

$$\Delta(t, x, \dot{x}) = e^{-\delta t} \{[p(t) - \phi(t)c(x)]\, [F(x) - \dot{x}] + V(x)\} \tag{2.14}$$

Applying the Euler necessary condition for a maximum in Eq 2.7 it follows that

$$\begin{aligned}\frac{\partial \Delta}{\partial x} &= \frac{\partial}{\partial x} e^{-\delta t} \{[p(t) - \phi(t)c(x)][F(x) - \dot{x}] + V(x)\} \\ &= e^{-\delta t} \{[p(t) - \phi(t)c(x)] F'(x)\} + \\ &\qquad e^{-\delta t} \{[F(x) - \dot{x}][-\phi(t)c'(x)]\} + e^{-\delta t} \{V'(x)\}\end{aligned} \tag{2.15}$$

$$\begin{aligned}\frac{d}{dt}\frac{\partial \Delta}{\partial \dot{x}} &= \frac{d}{dt}\frac{\partial}{\partial \dot{x}} e^{-\delta t} \{[p(t) - \phi(t)c(x)][F(x) - \dot{x}] + V(x)\} \\ &= e^{-\delta t} \left\{\delta p(t) - \dot{p}(t) + \phi(t)c'(x)\dot{x} + c(x)\dot{\phi}(t) - \delta\phi(t)c(x)\right\}\end{aligned} \tag{2.16}$$

Equating the expressions in Eq 2.15 and Eq 2.16 and simplifying yields:

$$\frac{V'(x)}{p(t) - \phi(t)c(x)} + F'(x) - \frac{\phi(t)c'(x)F(x)}{p(t) - \phi(t)c(x)} =$$

$$\delta - \frac{\dot{p}(t)}{p(t) - \phi(t)c(x)} + \frac{c(x)\dot{\phi}(t)}{p(t) - \phi(t)c(x)}$$

Further rearranging yields an important implicit formula for the population level x:

$$\begin{aligned}F'(x)[p(t) - \phi(t)c(x)] - \phi(t)c'(x)F(x) + \\ V'(x) - \dot{\phi}(t)c(x) = \delta[p(t) - \phi(t)c(x)] - \dot{p}(t)\end{aligned} \tag{2.17}$$

which can be re-written by use of the product rule of calculus:

$$\begin{aligned}\frac{d}{dx}\left[\{p(t) - \phi(t)c(x)\} F(x)\right] + V'(x) - \dot{\phi}(t)c(x) = \\ \delta[p(t) - \phi(t)c(x)] - \dot{p}(t)\end{aligned} \tag{2.18}$$

This is an equation for x that, according to optimal control theory, must hold when $x = x^*$, the optimal population level. For the Schaefer model the equations are

$$F(x) = rx\left(1 - \frac{x}{K}\right) \qquad \text{and} \qquad c(x) = \frac{c}{qx} \tag{2.19}$$

where $F(x)$ is the growth function and $c(x)$ represents the unit harvest costs. Substituting these expressions into the basic optimal control formula (Eq 2.18) one obtains:

$$\frac{d}{dx}\left[\left(p(t) - \frac{\phi(t)c}{qx}\right) rx\left(1 - \frac{x}{K}\right)\right] + \frac{\nu}{x} - \frac{\dot{\phi}(t)c}{qx} = \delta\left[p(t) - \frac{\phi(t)c}{qx}\right] - \dot{p}(t) \tag{2.20}$$

After differentiation this becomes a quadratic equation in x, the positive solution of which is given by

$$x^* = \frac{K}{4}\left(\frac{\phi(t)\,c}{p(t)qK} + 1 - \frac{\delta}{r} + \frac{\dot{p}(t)}{rp(t)}\right) + \frac{K}{4}\sqrt{\left(\frac{\phi(t)\,c}{p(t)qK} + 1 - \frac{\delta}{r} + \frac{\dot{p}(t)}{rp(t)}\right)^2 + 8\left[\frac{\phi(t)\,\delta c}{rp(t)qK} + \frac{\nu}{rp(t)K} - \frac{\dot{\phi}(t)\,c}{rp(t)qK}\right]} \tag{2.21}$$

To simplify this expression, the following dimensionless quantities are introduced:

$$\begin{aligned} z^*(t) &= \frac{x^*(t)}{K} \\ z_\infty &= \frac{x_\infty}{K} = \frac{c}{pqK} \\ \gamma &= \frac{\delta}{r} \end{aligned} \tag{2.22}$$

Thus z^* represents the biomass as a proportion of the environmental capacity K, and z_∞ is the corresponding open access, rent-dissipating biomass level. When q and K are given (as is usually the case), z_∞ is determined by the ratio c/p of fishing costs to price. γ is the ratio of the discount rate to the intrinsic growth rate of the population; this term is frequently referred to as the bionomic growth ratio.

The following symbols will also help to simplify the notation:

$$\begin{aligned} H(t) &= \frac{\dot{p}(t)}{rp(t)} \\ \Omega(t) &= \frac{\nu}{rp(t)K} \end{aligned} \tag{2.23}$$

where $H(t)$ is the ratio of the rate of change of price divided by price times the intrinsic growth rate. Finally, $\Omega(t)$ is the ratio of the 'preservation coefficient' ν divided by $Krp(t)$. Note that it is the ratio of ν to $p(t)$ that is important (K and r are usually given). This model shows that regardless how much society values a resource (represented by a high value for ν) one must also consider the price per unit biomass of the resource (since $p(t)$ may offset ν). With the above substitutions, Eq 2.21 now becomes:

$$z^* = \frac{1}{4}\left(\phi(t)z_\infty(t) + 1 - \gamma + H(t)\right) + \frac{1}{4}\sqrt{\left[\phi(t)z_\infty(t) + 1 - \gamma + H(t)\right]^2 + 8\left[\phi(t)z_\infty(t)\gamma + \Omega(t) - \frac{\dot{\phi}(t)z_\infty(t)}{r}\right]} \tag{2.24}$$

Note that from Eq 2.24, zero discounting ($\gamma = 0$) produces the highest

optimal population level given by[2]

$$z^* = \frac{1}{4}(\phi(t)z_\infty(t) + 1 + H(t)) + \frac{1}{4}\sqrt{(\phi(t)z_\infty(t) + 1 + H(t))^2 + 8\left[\Omega(t) - \frac{\dot{\phi}(t)z_\infty(t)}{r}\right]} \tag{2.25}$$

Critical Depensation

This analysis will now be extended to the situation of critical depensation (the "Allee effect"). As before, optimal population levels will be calculated. However, the pure compensation logistic Schaefer model of population growth is now replaced by the following growth curve:

$$F(x) = rx\left(1 - \frac{x}{K}\right)\left(\frac{x}{m} - 1\right) \tag{2.26}$$

After substituting $F(x)$ into the optimal control formula (Eq 2.18), then differentiating, simplifying, and introducing the dimensionless quantities of Eq 2.22, one obtains a cubic equation in x:

$$\begin{aligned} & z^3 - \frac{2}{3}\left[\phi(t)\, z_\infty(t) + \theta + 1\right] z^2 + \\ & \frac{\theta}{3}\left[1 + \gamma + \left(\frac{1+\theta}{\theta}\right)\phi(t)\, z_\infty(t) - \frac{\dot{p}(t)}{rp(t)}\right] z \\ & - \frac{\theta}{3}\left[\frac{\nu}{Kp(t)r} + \gamma\phi(t)\, z_\infty(t) - \frac{\dot{\phi}(t)z_\infty(t)}{r}\right] = 0 \end{aligned} \tag{2.27}$$

Here, the dimensionless quantity θ is introduced to represent the ratio of the extinction threshold (m) to the carrying capacity (K):

$$\theta = \frac{m}{K}$$

[2]It is also of interest whether z^* is higher than z_{MSY} in the case of a zero-discount rate.

Note that a higher value of θ corresponds to a higher risk of species extinction. Using the quantities defined in Eq 2.23, the notation can be further simplified:

$$z^3 - \frac{2}{3}\left[\phi(t)\, z_\infty(t) + \theta + 1\right] z^2 + \frac{\theta}{3}\left[1 + \gamma + \left(\frac{1+\theta}{\theta}\right)\phi(t)\, z_\infty(t) - H(t)\right] z - \frac{\theta}{3}\left[\Omega(t) + \gamma\,\phi(t)\, z_\infty(t) - \frac{\dot{\phi}(t) z_\infty(t)}{r}\right] = 0 \tag{2.28}$$

While a closed form analytic solution is possible for the cubic equation of Eq 2.28, mathematical software (MAPLE) was used to highlight the relationship between the optimal population level (z^*) and key parameters, with $\theta = 0.05$. Specifically, the relationship between z^* (normalized) and the bionomic growth ratio γ for various values of z_∞ is illustrated in Fig 2.1. Note that z^* is relatively insensitive to γ for moderate to high values of the rent dissipating biomass level ($z_\infty > 0.45$). However for lower values of z_∞, z^* drops off markedly for $\gamma > 2$. Fig 2.2 illustrates the normalized optimal population level z^* as a function of the bionomic growth ratio γ for various normalized values of the preservation ratio Ω. Note that increasing Ω can have a significant role in increasing z^*.

Note that price $p(t)$ and the coefficient of the unit harvesting cost $\phi(t)$ are modeled as functions of time (*i.e.* price and cost changes are not exogenous to the system). This satisfies a long-standing void in the environmental optimal control literature (Clark, 1976; 1985; Lohmander, 1990) and yields penetrating insights as shown in Fig 2.3. Note that as the fishing costs $\phi(t)$ increase at time T_1, the optimal population level (z^*) initially decreases as fisherman rush to harvest more fish (before the costs rise even further). However, in the long term, the higher costs lead to a higher z^*. Similarly, the price drop at time T_3 leads initially to a lower value of z^*, but ultimately a higher one (less fish are harvested in the long term since they yield less profit).

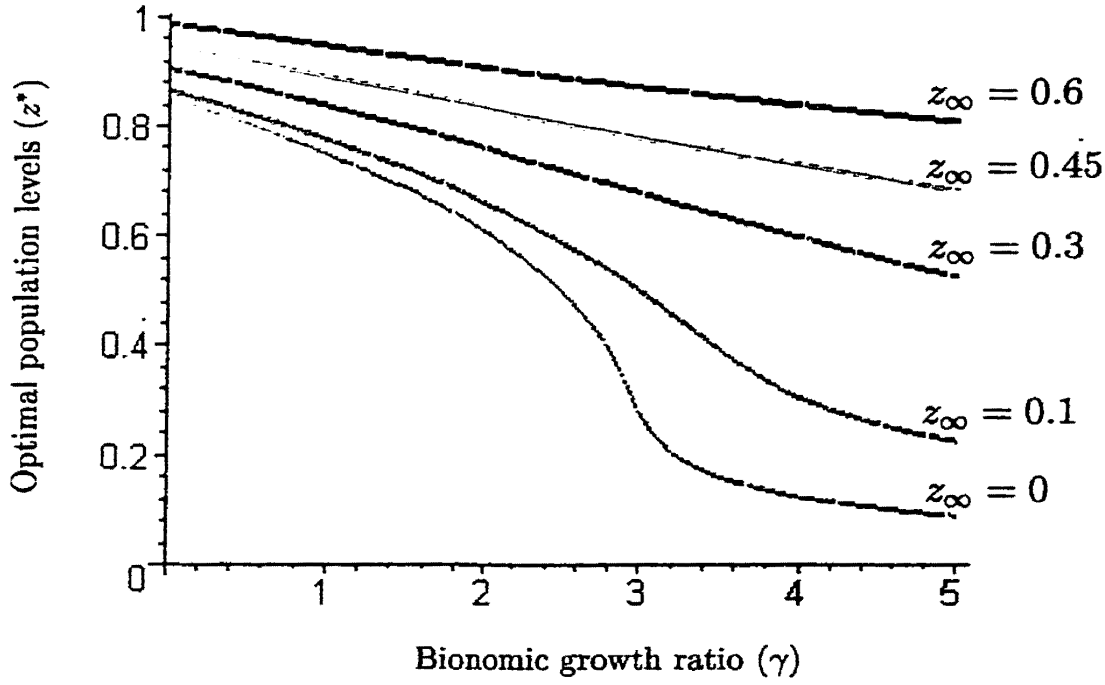


Figure 2.1: Optimal population levels (normalized) z^* as a function of the bionomic growth ratio γ for various values of z_∞.

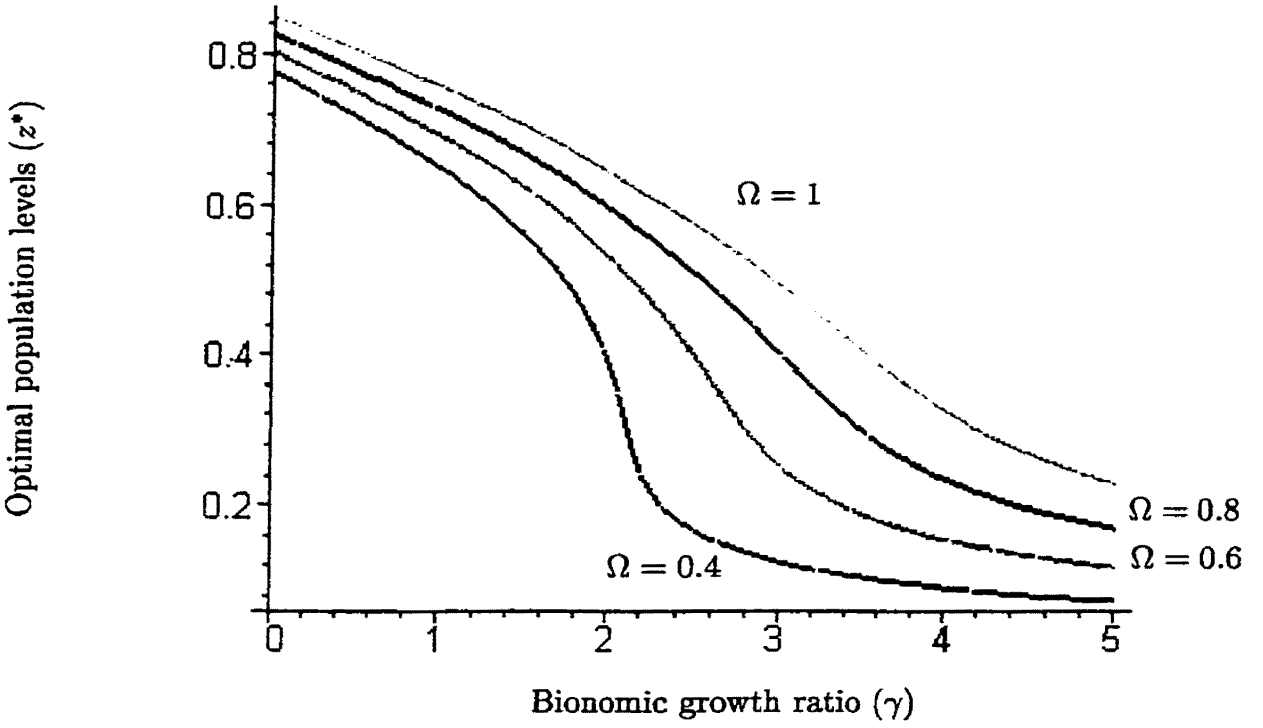


Figure 2.2: Optimal population levels (normalized) z^* as a function of the bionomic growth ratio γ for various values (normalized) of preservation ratio, Ω.

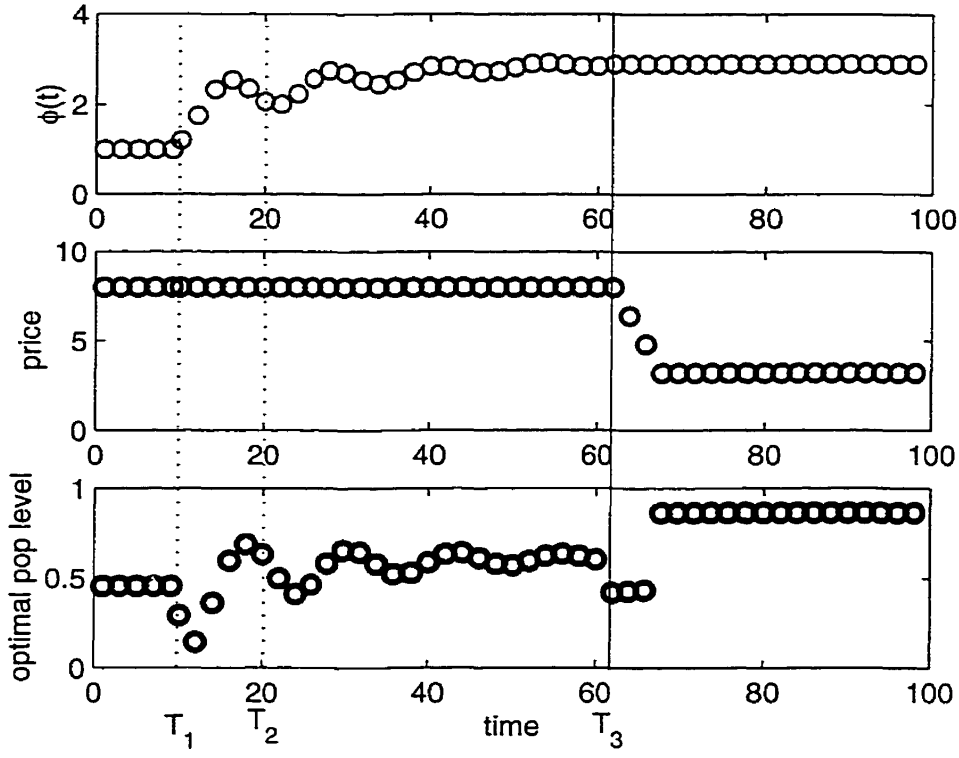


Figure 2.3: Relationship among price, $p(t)$, cost coefficient, $\phi(t)$, and optimal population levels, z^*, vs time

2.2 Multiple Criteria Decision Analysis (MCDA)

Multiple Criteria Decision Analysis (MCDA) is a collection of methodologies to select among alternatives that involve incommensurate attributes. MCDA can be classified into two main branches, Multiple Attribute Decision Making (MADM) and Multiple Objective Mathematical Programming (MOMP). The former applies to decision problems with a discrete set of alternatives (*i.e.* when the set of alternatives can be explicitly defined by listing its finite, and usually small, members); the latter when the set of alternatives is implicitly defined by a set of constraints to be satisfied (resulting in a large or infinite number of alternatives). A good discussion of MOMP can be found in Steuer (1986) and Appendix A. While MOMP represents a useful generalization of continuous SPSC problems, it is beyond the scope of this thesis.

Discrete multiattribute problems can be found in almost all types of private and public decisions, such as the evaluation of projects, plans, and policies. In many cases, the discrete alternatives take on a specific "identity" which captures the public's imagination, such as the locations where nuclear wastes can be "buried" (Yucca mountain, deep space, Pacific ocean, *etc.*). In other cases, the alternatives are more naturally linked to various *actors* in the decision process (for example, the nuclear industry or conservation groups) or specific value systems (conservative solutions, draconian solutions, and so on). Finally, continuous and discrete problems are not mutually exclusive. For example, the siting of a nuclear power plant (an inherently discrete problem) can be supported by continuous programming techniques (Nijkamp, 1990).

2.2.1 Generation of Alternatives

Alternatives (also referred to as decision options, courses of action, strategies, or means) are potential solutions to a decision problem. It is important that no alternative is excluded *a priori* at the early stages because of some

particular criterion such as high cost; a more expensive option may become acceptable when mitigatory costs and environmental impacts are taken into account (Munasinghe, 1993). For some criteria, for example, cost of machinery, alternatives are readily evaluated.

The definition and generation of alternatives is an important, but frequently overlooked, step in the process of MCDA (Keeney, 1992; Stewart, 1992; Vincke, 1992). For most environmental problems there is no pre-existing set of well-defined alternatives. Most often, before any formal decision analysis can proceed, some preliminary work to define, expand, or reduce the set of feasible alternatives is necessary.

In many real world problems, the decision maker is interested in selecting a combination of alternatives rather than an individual alternative. For example, a government which is responsible for developing the long term water supply for a region may employ a variety of sources, such as underground aquifers, treated river water, and imported lake water to satisfy future demand. The set of feasible alternatives can be reduced by removing "inferior" alternatives, identifying those that do not meet performance standards on key environmental indicators (Ulungu and Teghem, 1994; Rajabi, 1997).

2.2.2 Selection of Criteria (Indicators)

The highest objective in an environmental problem is usually a broad, unmeasurable goal, often to minimize adverse environmental effects. The overall objective may be broken down into a hierarchy of goals, where lower levels become more detailed and measurable, but also more conflicting. The degree to which objectives are achieved is measured through a set of performance indicators (also referred to as criteria or attributes). The criteria are usually in conflict with each other, especially if each criterion represents the interests of a specific group of decision makers. For example, closing down a nuclear power plant may reduce the risk of radiation leaks, while laying off workers, and increasing energy costs. Thus, it is rare to find an action that is best according to all criteria, and one must search for a *compromise* solution (rather

than an optimal one) that appropriately reconciles the different criteria.

Eckel, Levy, Hipel, and Kilgour (1998a) describe the role of environmental indicators (including output, input, and process indicators) for planning and management in the context of the Pressure—State—Response (PSR) model (see Fig 2.4). The PSR model was developed by the Organization for Economic Cooperation and Development (OECD) in the early 1990s and is described in detail by Hammond et al. (1995) and OECD (1993). Pressure refers to human activities directly affecting the environment (such as methane emissions); state refers to observable changes of the environment (*e.g.* rising global temperatures) while response deals with the ways in which society chooses to address environmental problems, ranging from expenditures for environmental protection to solar energy research, environmental education, and energy taxes.

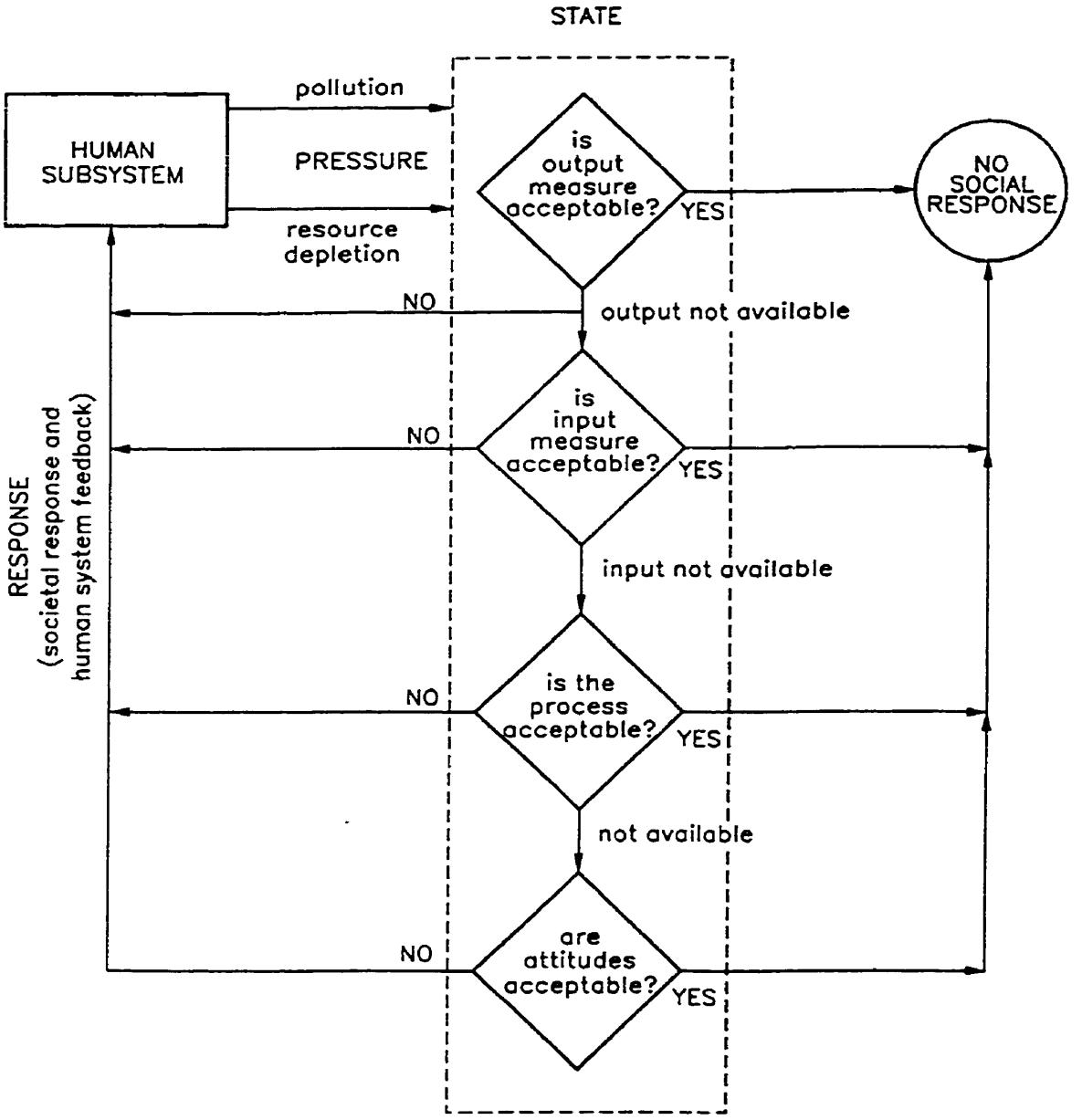


Figure 2.4: Indicator use in the Pressure-State-Response model as described by Eckel, Levy, Hipel, and Kilgour (1998a). This figure is modified from the work of OECD (1993) and Hammond et al. (1995).

After long debate among scientists and indicator experts, the *Driving forces—Pressure— State—Impact—Response* model (DPSIR) was recently adopted as the most appropriate way to structure environmental information by most countries of the European Union and by international environmental organizations, such as the European Environment Agency (EEA) and the OECD (Eurostat, 1999). Significant emphasis is now placed on the two new additions to the PSR model: driving forces (environmentally relevant sectoral trends, such as energy generation, transportation, industry, agriculture, and tourism) and impacts (effects of a changed environment, *e.g.* a decrease in agricultural production, increased flood risks *etc.*)

Ideally, the state of a nation's environment should be measured using output measures: when output measures are unavailable, Eckel, Levy, Hipel and Kilgour (1998a) propose using inputs, and sometimes the process itself. Output and input measures are essentially different: output refers to the end result of the process; whereas, inputs measure the resources put into the process, with no attempt to measure their effects. The output of a wastewater treatment facility is improved water quality which could be measured using a single chemical that acts as a *leading indicator*, giving early warning of the presence of other chemicals. Output indicators are often described in relative terms, such as "percent of the maximum acceptable level".

Where output measures are unavailable or inappropriate, input measures are capable of quantifying the *effort* being made by a country to improve the state of the environment, but not the *effectiveness* of such actions: a given amount of input may have little or no impact on output. Examples of environmental inputs include spending on greenhouse gas reduction and the resources devoted to environmental enforcement. In order to be a valid indicator, the chosen input measure must demonstrably affect the output. When neither output nor input measures are available governmental attitudes can provide clues as to the state of the environment. For example, many governments lack the resources and commitment to enforce illegal activities that ultimately impact the environment, such as the black market for CFCs.

2.2.3 Evaluation of Alternatives

For the quantitatively oriented decision maker, there is great appeal in being able to establish some means of associating a numerical score with each decision alternative, after which the choice of the optimal alternative becomes automatic. Accordingly, most optimization procedures are based on the assumption that one can assign a real number (such as cost in dollars or the biomass of a fish stock in kg) to represent the consequences of an alternative according to a criterion. However, in many environmental problems this is simply not possible. Often, the natural way to express the consequences of actions is by using qualitative (descriptive) or ordinal information.

For this reason, MCDA techniques have been developed to take advantage of the type of information available. For example, the ELECTRE methods (Roy, 1973) employ information in a fuzzy context, the Analytic Hierarchy Process (AHP) approach (Saaty, 1980; Saaty and Vargas, 1987; Saaty, 1990; Belton, 1986) elicits ratio judgements, while Elimination Methods (MacCrimmon, 1973) require only ordinal rankings. Moreover, MCDA methods can be classified according to whether the attributes are evaluated before (*a priori* preference articulation) or after the alternatives are presented (*posterior* preference articulation).

This thesis addresses primarily the prior articulation of preferences. This means that the value system of the stakeholder is analyzed before the evaluation of the actual decision alternatives. This separation is never completely possible as the decision context affects the elicitation process. Nevertheless, attempting to separate values from alternatives has two key advantages. First, it can help to systematically explore the stakeholder's value functions. Second, the results can be used to explore new alternatives in the same decision problem (or even to address a new decision problem).

Finally, compensatory methods are the focus of this thesis: quantitative tradeoffs across objectives are allowed. Although compensatory and noncompensatory approaches frequently coexist in decision problems. The choice ultimately depends on the characteristics of the problem: compen-

satory methods tend to be more demanding in terms of data and the amount of information to be elicited from experts. Munda (1993) and Opschoor and Hafkamp (1991) provide an overview of compensatory methods applied in the economic evaluations of environmental problems.

Among cardinal decision analytic methods, a fundamental distinction exists between monetary and non-monetary evaluation approaches. This thesis focuses on the latter; although popular examples of the former include cost-benefit analysis (CBA) and nonmarket valuation methods, such as contingent valuation (CV) (Mishan, 1988; Dasgupta and Pearce, 1972). The treatment of decision consequences in purely monetary terms has attracted considerable criticism, particularly for environmental applications (Nijkamp, 1980; Sagoff, 1988; Prato, 1999), although it is certainly more convenient to assess projects and make decisions solely on the basis of monetary values.

2.2.4 MCDA and Cost-Benefit Analysis

MCDA is a conceptual framework for evaluating environmental projects that alleviates some of the ethical, theoretical, and practical shortcomings of CBA (Prato, 1999; Bishop, 1993; Perrings, 1994): both economists and ecologists note that basing environmental investment decisions on CBA will not ensure the sustainability of essential natural services, such as flood protection, water purification, and biodiversity. There are several reasons why MCDA is preferred to CBA (Naiman et al., 1997; Cameron, 1997). First, CBA compromises the "authenticity, richness, and quality" (Prato, 1999) of decision making since an inherently multiple criteria problem (with socio-cultural dimensions) must be analyzed with a single monetary criterion (net present value).

Second, Willingness to Pay (WTP) estimates of ecological services elicited with Contingent Valuation (CV) methods are routinely incorporated into the CBA of environmental projects (Feather et al., 1995; Cameron, 1997). In summarizing the weakness of the CV method Kahn (1996) argues that "contingent valuation is associated with controversy and is far from universally ac-

cepted, even among environmental economists." Moreover, Cameron (1997) notes that respondents have trouble stating their WTP for improved water quality[3]. Finally, estimating dollar values for goods and services is particularly impractical in developing countries where most business transactions and social activities occur outside of any formal market setting (Bjornstad and Kahn, 1996).

2.3 Multi-Attribute Decision Making (MADM)

MADM, a subset of MCDA, is a class of problems in which there are a *discrete* set of alternatives. MADM situations are sometimes termed 'selection problems'. This thesis focuses on cardinal MADM approaches although most of the results could be reformulated to accommodate ordinal data (Keeney, 1992). In this section a variety of MADM approaches are highlighted and contrasted, including Outranking methods (Section 2.3.1), Multi-Attribute Value Theory, MAVT (Section 2.3.2), the Analytic Hierarchy Process, AHP (Section 2.3.3), and lexicographic techniques, such as the 'Elimination Method' (Section 2.3.4).

2.3.1 Outranking methods

The ELECTRE technique was developed by Benayoun et al. (1966) and Roy (1973); the name ELECTRE is an acronym for ELimination Et (and) Choice TRanslating algorithm. Three versions of ELECTRE have been presented by Roy: ELECTRE I seeks to reduce the number of alternatives under consideration; ELECTRE II ranks nondominated alternatives; while ELECTRE III discusses the notion of "pseudo-criteria". Proponents of outranking methods argue that their lack of an axiomatic foundation is compensated by their descriptive reality (Bouyssou, 1993). In ELECTRE the decision maker

[3]Sagoff (1988) introduced another criticism of CV: survey respondents tend to express their WTP for a good or service from the viewpoint of a concerned citizen rather than as a consumer or user of that good or service.

must provide both a set of weights reflecting the relative importance of the objectives and numerical scores evaluating the alternatives. Information pertaining to the 'discordance' and 'concordance' indices is also required.

Outranking methods are based on the concept of an *outranking relation* (S). Given two discrete alternatives a_i and a_j it is said that

- a_i outranks a_j ($a_i S a_j$) if there is enough evidence to suggest that 'a_i is at least as good as a_j'
- a_i does not outrank a_j (not($a_i S a_j$)) if the arguments in favor of the proposition 'a_i is at least as good as a_j' are considered insufficient

Several common criticisms of ELECTRE are:

- there are often no alternatives selected by ELECTRE;
- the methods lack a strong axiomatic basis;
- many input parameters are required which may have little intuitive meaning (such as the discordance and concordance thresholds);
- a consultant often adjusts the thresholds and the weights in order to obtain the desired solution;
- the method is quite complicated;
- if one possesses the information necessary for building a linear utility function, the use of ELECTRE may be gratuitous.

Responding to these criticisms Brans and Vincke (1985) developed PROMETHEE, an offshoot of ELECTRE. The ELECTRE approach to MADM problems continues to be a popular technique in the European school of MCDA (although much of the literature is only available in French).

2.3.2 Multi-attribute Value Theory (MAVT)

The need to clarify values has long been recognized as a critical component of decision analysis. For example over 200 years ago, the American inventor and statesman Benjamin Franklin (Franklin, 1772) emphasized the importance of personal values. Even today, the management gurus Peters and Waterman Jr (1982) propose one all-purpose bit of advice for managerial excellence: "figure out your value system". Similarly, Keeney (1994) refers to *value-focused thinking* as a way to facilitate creative thinking and to improve communication among stakeholders. In fact, values pervade the entire field of operations research; they are used to build a quantitative objective function, which provides the basis for evaluating alternatives. Most importantly, values are essential in defining the goals we strive to meet (and the indicators we select to measure our progress toward these goals).

Cardinal Value Functions

Value functions are a mathematical representation of human judgments. They attempt to analytically describe the value system of the individuals involved in a decision and realistically capture aspects of human judgment. A cardinal value function v is often referred to as a "measurable value function", a "cardinal value function", or a "value difference function". Cardinal value functions require a more demanding set of assumptions than the ordinal case, but provide more information about the preference structure. A cardinal value function v preserves ordinal preference as well as an ordering on difference (under certainty). Thus $v(x_1) > v(x_2)$ implies that x_1 is preferred to x_2 and

$$v(x_1) - v(x_2) \; > \; v(x_3) - v(x_4) \tag{2.29}$$

implies that the value difference between x_1 and x_2 is greater than that between x_3 and x_4 (where x_3 is preferred to x_4). Hence, cardinal value functions can measure 'strength of preference' under certainty. Debreu (1959), Scott

and Suppes (1958), Frisch (1964), Alt (1971), and others have proposed various axiomatizations of such 'strength of preference' measures. The formal properties of this so-called "positive-difference" structure are reviewed by Krantz et al. (1971).

A decision alternative can typically be described by a vector of attributes (x_1, x_2, x_3) where, for example x_1 may denote profit, x_2 the flooding risk, and x_3 the recreational benefits of a particular alternative. An attribute y is preferentially independent of z if preferences for values of y do not depend on the value of z. If also z is preferentially independent of y, then y and z are mutually preferentially independent. In general, preferential independence states that the preferences for some subset of the attributes do not depend on the level fixed for the other attributes. Price and quantity in many commodities are naturally preferentially independent: people always prefer affordable to expensive items given a fixed quality level. This holds for any fixed quality level. And people always prefer higher quality to inferior quality for any fixed cost.

Intuitively, preferential independence suggests that each alternative contributes independently to the overall score, or, in other words, that some additive form of the value function $v(x_1, x_2, \ldots, x_n)$ may be appropriate. In fact, Keeney and Raiffa (1976) prove that if the set of attributes x_1, x_2, $\ldots, x_n$ is mutually preferentially independent, a decision maker's preferences can be represented by an additive value function:

$$v(x_1, x_2 \ldots, x_n) \;=\; \sum_{i=1}^{n} k_i \, v_i(x_i) \tag{2.30}$$

The additive representation explicitly introduces the component (marginal) value functions v_i for attribute x_i. A common technique for specifying component value functions, is the bisection technique (Keeney and Raiffa, 1976). On the other hand, the shape of marginal value functions can be directly selected. Two commonly used functional forms for decreasing, convex value

functions are:

$$\begin{aligned} v(x) &= ax^2 + bx + c \\ v(x) &= -e^{-ax} - be^{-cx} \end{aligned}$$

After imposing $v(x^0) = 0$ and $v(x^*) = 1$, a value function has one degree of freedom which can be used to modify its shape. Often the assessor tries to elicit the score $x^\dagger$ corresponding to $v(x^\dagger) = 0.5$ from the decision maker. This approach is particularly appealing when the decision maker can interactively modify the shape of the curve using graphical software (Hämäläinen, 1998). Finally, "sophisticated and formally trained" assessors should be consulted before attempting to elicit value functions (von Winterfeldt and Edwards, 1986; Huber, 1974).

2.3.3 Value Hierarchy: AHP approach

The highest system objective is often a broad statement about the overall goal, usually a universally acceptable statement. In the context of water resources, the highest objective for a nation may be 'the ability to supply all present and future water needs for economic development and the welfare of its inhabitants'. The lower level, or subordinate, objectives are introduced as answers to the question of how the higher level objectives will be achieved. After moving down the hierarchy several levels, the answers become "implementation activities" rather than objectives *per se* (Jousma et al., 1987).

The Analytic Hierarchy Process (AHP) is a popular hierarchical technique to solve MADM problems. For a full exposition see Saaty (1980). The AHP decision support process consists of three phases: problem structuring, preference elicitation, and synthesis. Problem structuring involves breaking down the top level objective into subgoals until a sufficiently detailed representation of the decision problem is obtained. Preference elicitation consists of a series of pairwise comparisons where the decision maker considers the relative importance of two attributes at a time, such as housing and water quality. For each pair, one must decide which attribute is more important,

and by how much? To answer this question, the decision maker provides a verbal statement to represent the intensity of her preferences, such as "I perceive water quality to be demonstrably more important than additional housing". This preference statement is then cast into a numeric scale.

To illustrate the use of AHP, consider the issue of stormwater management in Subwatershed 314, Laurel Creek Watershed, Ontario, Canada. As specified in the Laurel Creek Watershed Study (Grand River Conservation Authority, 1993) each subwatershed in the Laurel Creek Watershed must meet minimum standards for the eight environmental criteria listed in Table 2.1. Weights for the eight environmental criteria in Table 2.1 were ascertained by eliciting pairwise comparisons from local residents.

The pairwise comparison matrix (PCM) shown in Fig 2.5 was developed by aggregating individual scores using the geometric mean technique. The resulting eigenvector E of this PCM is:

$$E = \begin{pmatrix} 0.74 & 0.47 & 0.29 & 0.25 & 0.19 & 0.15 & 0.13 & 0.11 \end{pmatrix} \qquad (2.31)$$

After normalizing E the weightings are bacteria = 0.74/2.33 = 0.32, phosphorus = 0.20, sediment = 0.12, DO = 0.11, hydrology = 0.08, hydrogeology = 0.06, erosion = 0.06, and temperature = 0.05.

	Bacteria	Phosphorus	Sediment	Dissolved Oxygen	Hydrology	Hydrogeology	Erosion Control	Temperature
Bacteria	1.00	1.59	2.29	3.16	3.91	4.97	5.83	6.28
Phosphorus	0.63	1.00	1.82	2.19	2.29	2.97	3.53	3.80
Sediment	0.44	0.55	1.00	1.21	1.59	1.80	2.22	2.64
Dissolved Oxygen	0.32	0.46	0.83	1.00	1.51	1.71	1.85	2.19
Hydrology	0.26	0.44	0.63	0.66	1.00	0.43	1.54	1.66
Hydrogeology	0.20	0.34	0.56	0.58	0.70	1.00	1.08	1.46
Erosion Control	0.17	0.28	0.45	0.54	0.65	0.93	1.00	1.36
Temperature	0.16	0.26	0.38	0.46	0.60	0.68	0.74	1.00

Figure 2.5: Pairwise Comparison Matrix for Stormwater Management

1. Hydrogeology	Infiltration is to be maintained on an average annual basis for the entire subwatershed
2. Hydrology	• **peak flows** must be controlled to existing levels • **peak flow timing** must be maintained to within 75 % of existing conditions • **runoff volume** is to be matched to existing levels, exclusive of extended detention volumes
3. Erosion Control	future distribution of impulse is to be maintained at existing levels
4. Temperature	stormwater discharged to the receiving stream is not to exceed: 26°C from June 1 to August 1 29°C from August 1 to October 31
5. Dissolved Oxygen	stormwater discharged to the receiving stream should not have less than 5 mg/l D.O.
6. Phosphorous	90% removal of Phosphorous from urban development is required and in stream levels are to be less than 0.05 to 0.08 mg/l
7. Sediment	stormwater discharged to the stream (both during and after construction) is to have less than 25 mg/l of suspended solids
8. Bacteria	in stream levels are to be less than 200 counts/100 ml E.coli

Table 2.1: Subwatershed 314 Targets

2.3.4 Lexicographic Approach

In many decision situations alternatives cannot be evaluated entirely in quantitative form. In addition, numeric weighting factors are often unavailable to express the priorities of the objectives. In this situation, noncompensatory models offer some capability of placing a number of alternatives in an order of preference. An important example of non-compensatory models is the lexicographic approach. In lexicographic preference, one attribute has overriding importance; decisions are made on the basis of it alone. If there are several options tied for performance on this attribute, the second and third most important attributes are used to break ties. Some researchers, such as French (1986) eschew lexicographic preference because there is no trade-off at all between alternatives.

A popular lexicographic technique is the *Elimination method* (MacCrimmon, 1973). Since significant emphasis is placed on the primary attribute, to the exclusion of other issues, the Elimination Method and other lexicographic methods should be used judiciously to ensure that all possible information is used in the analysis. Necessary conditions for use of the Elimination Method include ordinal or cardinal preferences for alternatives (for each objective) and an ordinal ranking of the criteria. In summary, the Elimination Method uses a stepwise process of screening alternatives: if one alternative performs better than another on the most important attribute, then it will be selected, however poorly it does on the remaining attributes. Alternatives not meeting a specified level of performance are eliminated until only one is left that has satisfied all the tests to that point. Ties are resolved by making the levels of performance or the criteria used more discriminating.

Consider the evaluation of sixteen stormwater management alternatives (A through P) in Subwatershed 314. The evaluation of these sixteen alternatives based on the eight environmental criteria listed in the Laurel Creek Watershed document is illustrated in Table 2.1. More details can be found in the technical report by Dorfman (1996). Of the eight criteria shown in Fig 2.6 and Table 2.1 many local residents feel that 'bacteria' is the most

important criterion (citing the effects of high bacteria levels on fish, dissolved oxygen, swimmers, and the water supply).

BMP(brief definition)	BMP Number	Targets: Hydrogeologic target	Runoff volume	Erosion control	Temperature	Dissolved Oxygen	Phosphorous	Sediment	Bacteria	Targets addressed	Comments
At Source											
Lot Level Infiltration	*A*	√	√	√	√	√	√	√	√	8	easily implemented
Vegetated Filter Strips	*B*					√	√	√		3	best suited for commercial
Conveyance (to stream)											
Pervious Pipes	*C*	√	√	√	√	√	√	√	√	8	groundwater contamination
Grassed Swales	*D*						√	√		2	inexpensive
Sand Filters	*E*				√		√	√	√	4	high maintenance
Cooling Trench	*F*				√	√		√		3	well suited to temperature
Conveyance (in stream)											
Erosion Protection	*G*			√		√	√	√	√	5	improves stream erosion
Restore Canopy	*H*				√		√			2	not preferred
Buffer Strips	*I*			√			√	√		3	required as part of study
End of Pipe											
Detention Wet Ponds	*J*		√	√				√		3	effective sediment removal
Detention Dry Ponds	*K*		√	√				√		3	not preferred; ineffective
Detention Wetlands	*L*		√	√			√	√	√	5	effective for bacteria
Infiltration Basins/Trenches	*M*	√	√	√	√	√	√	√	√	8	high failure rate
Oil/Grit Separators	*N*							√		1	not preferred
Quantity Detention Basins	*O*		√							1	control large storms
Detention Wetlands	*P*								√	1	preferred as contingency

Figure 2.6: Sixteen stormwater management BMPs

Using the criteria rankings derived in the previous section (*i.e.* bacteria is the most important criteria, followed by phosphorous, sediment, DO, hydrology, hydrogeology, erosion, and temperature), the Elimination Method shows that the ordering of alternatives (from most to least preferred) is

$$\text{most preferred} \quad \underbrace{A \sim C \sim M}_{\text{tie}} \gg G \gg L \gg E \gg P \gg B \gg D \gg I \gg H \gg F \gg \underbrace{J \sim K}_{\text{tie}} \gg N \gg O \quad \text{least preferred} \tag{2.32}$$

Here, 'Lot Level Infiltration' (A), 'Pervious Pipes' (C), and 'Infiltration Basins' (M) are the most attractive alternatives. Deciding among these three may require additional information. For example it is known that 'Pervious Pipes' are susceptible to clogging and 'Infiltration Basins' possess a "very high failure rate" (Dorfman, 1996). Accordingly, the most suitable alternative may be 'Lot Level Infiltration'. Chipman (1971) and Fishburn (1970) discuss lexicographic preference in more detail.

2.4 Conclusions

Values of z^* (normalized optimal population levels) are plotted as a function of the ratio of the discount rate to the intrinsic growth (the bionomic growth rate, γ) and the cost to price ratio (q and K are usually given) z_∞. One can conclude that as the harvest costs increase (or resource prices decrease) the optimal population level z_∞ increases in a non-linear fashion. An extreme case arises when $z_\infty = 0$, that is, when the costs of fishing are zero: here the optimal population level is quite low (but not zero due to the structure of the cost function). In addition, increasing the discount rate (making the future worth less relative to the present) leads to progressively decreasing optimal population levels z^* that approach z_∞ as $\gamma \to +\infty$. In addition, increasing the preservation value of the stock (and decreasing the price per unit of the harvested resource) leads to higher optimal population levels. Consequently,

explicitly including conservation concerns in a cost-benefit analysis may lead to increased protection of resources, provided that the price of the resource remains low.

The dynamic relationships among price, cost, and optimal population levels are also modeled. It is shown that as the costs of harvesting increase, the optimal population level initially decreases (as harvesters rush to extract more resources). However, in the long term, as harvesting costs continue to increase, eventually harvesting is reduced. To address this temporary increase in harvesting, a government might announce a tax in advance and implement the full tax immediately. A reduction in the price per unit of harvested biomass was shown to increase the optimal population level.

This chapter illustrates the use of the Elimination Method and Multi-Attribute Value Theory to select among sixteen stormwater management alternatives in subwatershed 314, in the Laurel Creek Watershed. It was shown that 'Lot Level Infiltration' (A), 'Pervious Pipes' (C), and 'Infiltration Basins' (M) are the most attractive alternatives. However, since 'Pervious Pipes' are susceptible to clogging and 'Infiltration Basins' possess a "very high failure rate" (Dorfman, 1996) the most suitable alternative may be 'Lot Level Infiltration'.

Chapter 3

MCDA under Uncertainty

A plethora of techniques is currently available to describe and model environmental systems under uncertainty — soft systems methodologies, bootstrapping, qualitative simulation, fuzzy logic, scenario analysis, Monte Carlo simulation, what-if conjectures, risk analysis, perturbation theory, spectral analysis, and the statistical design of experiments, to name a few. For a detailed discussion of these uncertainty methods, see, for example, Morgan and Henrion (1990), Zadeh (1972), Restrepo et al. (1993), and references therein. Which uncertainty technique to select depends on the purpose of the uncertainty analysis, the nature of the uncertainties (local or global), and whether model inputs are endogenous or exogenous to the model.

Section 3.1 deals with the use of *interval approaches* to formally model the uncertainty in a decision maker's preferences. When a multiple criteria situation is characterized by severe uncertainty, a decision maker may be unable to provide precise estimates for her preferences. In fact, a decision maker might feel comfortable specifying only ordinal information. Or the decision maker may feel inclined to make interval 'strength of preference' statements consistent with 'natural' verbal expressions such as "the i-th attribute is two to three times more important than the j-th attribute". Accordingly, methodologies for processing ordinal information and interval judgments are provided.

Section 3.2 uses simulation models and sustainability indicators to address the uncertainty inherent in multi-attributed resource management decisions. Here, several sustainability indicators are put forth, including measures to capture 'failure intensity' and 'system resistance'. These indicators are considered in the context of Web-HIPRE, a Java-applet based on Hämäläinen's HIPRE 3+ (1998), in order to promote interactive web-based MCDA under uncertainty (Section 3.2.1). Finally, these concepts are illustrated using a forest management case study in New Brunswick, Canada (Section 3.2.2).

3.1 Interval Approaches to Uncertainty

Preference judgments are notoriously tentative, imprecise, approximate, and incomplete. Accordingly, when multiple attribute problems are characterized by severe uncertainty a decision maker may be unable to provide precise estimates for her preferences, such as the relative importance of criteria. In fact, a decision maker might feel comfortable only specifying an ordinal ranking of the criteria weights. It follows that a decision maker should be allowed to specify a range of value judgments (approximate preference statements).

Consider a decision problem with three attributes, c_1, c_2, and c_3, and corresponding 'relative importance' scores of w_1, w_2, and w_3 (w_i's are commonly referred to as 'weights', although more technically they are 'scaling factors'). Assume that the only preference information available to the analyst is the following ordinal information:

$$w_3 \geq w_1 \text{ and } w_1 \geq w_2 \quad \text{with} \quad w_1 + w_2 + w_3 = 1 \tag{3.1}$$

Consider the question of estimating w_1, the relative importance of attribute c_1. It is assumed that each w_i is non-negative and less than or equal to 1. It follows that

$$0 \leq w_1 \leq 1 \text{ and } 0 \leq w_2 \leq 1 \text{ and } 0 \leq w_3 \leq 1 \tag{3.2}$$

To determine the average value of w_1 consistent with $w_3 \geq w_1 \geq w_2$, an

intuitive approach pursued by the author is to take the average value of w_1 over all points in the feasible region (the pink plane shown in Fig 3.1).

From elementary calculus, the average value of x weighted according to a function $\lambda(x)$ over the interval $[a, b]$ is

$$\text{Average value of } x \;=\; \frac{\int_a^b x\,\lambda(x)dx}{\int_a^b \lambda(x)\,dx} \tag{3.3}$$

By convention, the weighting function $\lambda(x)$ is non-negative with a positive integral over the range $[a, b]$. For the problem at hand, $x = w_1$ since we are trying to find the average value of w_1. As shown in Fig 3.1 there will be two linear weighting functions: $\lambda_1(w_1)$, the solid green rectangle, applies when $0 \leq w_1 \leq \frac{1}{3}$; and $\lambda_2(w_1)$, the open blue rectangle, applies when $\frac{1}{3} \leq w_1 \leq \frac{1}{2}$. Geometrically, for any particular value of w_1, where $0 \leq w_1 \leq \frac{1}{3}$, the set of points that is consistent with this value of w_1 is proportional to the length of the green rectangle in Fig 3.1. Similarly, for any value of w_1, with $\frac{1}{3} \leq w_1 \leq \frac{1}{2}$, the set of points that share this value of w_1 is proportional to the length of the open blue rectangle in Fig 3.1.

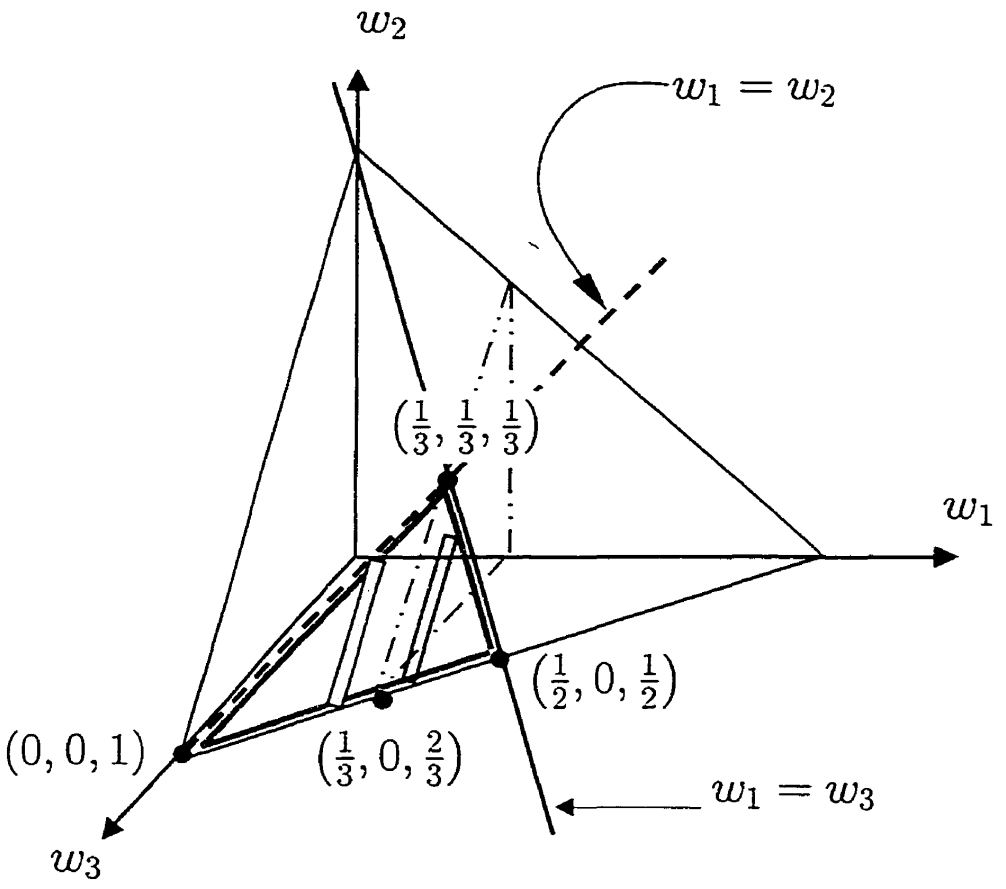


Figure 3.1: Calculation of average value of w_1

Accordingly, Eq 3.3 becomes:

$$\text{Average value of } w_1 \;=\; \frac{\int_0^{1/3} w_1\lambda_1(w_1)\,dw_1 \;+\; \int_{1/3}^{1/2} w_1\lambda_2(w_1)\,dw_1}{\int_0^{1/3} \lambda_1(w_1)\,dw_1 \;+\; \int_{1/3}^{1/2} \lambda_2(w_1)\,dw_1} \tag{3.4}$$

Since the weighting functions $\lambda_1(w_1)$ and $\lambda_2(w_1)$ are both linear, elementary algebra and Fig 3.1 yield:

$$\lambda_1(w_1) = \sqrt{2}\,w_1 \quad \text{and} \quad \lambda_2(w_1) = 2\sqrt{2}\left(\frac{1}{2} - w_1\right) \tag{3.5}$$

Substituting this into Eq 3.4 yields

$$\begin{aligned}\text{Average value of } w_1 \;&=\; \frac{\int_0^{1/3} w_1\sqrt{2}w_1\,dw_1 \;+\; \int_{1/3}^{1/2} w_1 2\sqrt{2}(\tfrac{1}{2} - w_1)\,dw_1}{\int_0^{1/3} \sqrt{2}w_1\,dw_1 \;+\; \int_{1/3}^{1/2} 2\sqrt{2}(\tfrac{1}{2} - w_1)\,dw_1} \\ &= \frac{5}{18}\end{aligned} \tag{3.6}$$

Using this approach one can derive the average value of the weights (w_1, w_2, and w_3) to be (5/18, 2/18, and 11/18).

3.2 Simulation tools for MCDA

Advances in computer and information technologies have revolutionized the daily lives of many humans: everything from electronic shopping to political activism has been transformed. Environmental managers have been quick to take advantage of the Internet's ability to disseminate environmental information quickly, cheaply, and efficiently; web-based decision support systems have become a popular means to help institutions and nations achieve their environmental objectives. For example, Bhargava and Tettelbach (1997)

present an Internet-based system for efficient waste disposal and recycling, while Heilman et al. (1999) use the Extensible Markup Language (XML) for the design of web-accessible databases to support rangeland conservation. Decision support systems not only capitalize on the Internet's ability to efficiently incorporate new knowledge (and make it instantly available to end-users) but also assist decision makers in evaluating alternatives across physical, biological, and social dimensions (Gunderson et al., 1995).

3.2.1 Sustainability Indicators

The author developed several sustainability indicators to evaluate soil tillage practices (Levy et al., 2000c) and forest management approaches (Levy et al., 2000d) in the context of Web-HIPRE (Hämäläinen, 1998). Specifically, Web-HIPRE, a Java-applet based on Hämäläinen's HIPRE 3+ (1998) is an interactive web-based tool for decision analysis which integrates a number of MCDA techniques including AHP (Saaty, 1980), SMART (Edwards and Barron, 1994), and SWING (von Winterfeldt and Edwards, 1986). The meaning of sustainability remains unclear: some interpret sustainable development as a lofty philosophical goal such as "the pursuit of happiness" and "justice" (Manning, 1990), while others dismiss the concept as an oxymoron, or a political shibboleth (Livingston, 1994). To others, sustainability implies a condition in which the frequency and severity of societal risks are decreasing over time.

The interpretation of sustainability is also context dependent. For example, when managing agricultural systems, sustainability might imply productivity, constancy, and recovery (the ability to bounce back from a perturbation). In order to formally evaluate system sustainability using Web-HIPRE, several new sustainability concepts are defined, extending the work of Pimm (1984) and Loucks (1997).

Let the status of the system at time t, $t = 1, 2, 3...n$, be represented by the variable X_t, where the possible values of X_t are divided into two sets: S, the set of satisfactory values ($X_t \in S$) and F, the set of unsatisfactory

values ($X_t \in F$). Satisfactory and unsatisfactory ranges of criterion values are subjective. They are based on human judgment or human goals. In some cases, they may be based on well-defined health standards; however, most criteria ranges will not have predefined or published standards.

Fig 3.2(a) shows an illustrative time series of values of a typical system performance indicator, along with associated satisfactory and unsatisfactory ranges. Each criterion will have its own range of satisfactory values. An episode is defined as the maximal set of *consecutive* years in which an indicator fails in the same direction; it is possible, of course, for the system to immediately switch from failing 'low' to failing 'high', but this 'Scylla to Charybdus' scenario will not be considered further. Note that in Fig 3.2(a) there are four episodes occurring in years $\{3,4\}$, $\{6,7\}$, $\{9,10,11,12\}$, and $\{17,18\}$. Note that in Fig 3.2(a) the deviation from satisfactory values are also given.

The degree to which the system tends to maintain satisfactory values is its *resistance*, γ, measured by the probability that the system remains satisfactory immediately following a satisfactory value (i.e. that good follows good):

$$\gamma = P(X_{t+1} \in S \mid X_t \in S). \tag{3.7}$$

This result is closely linked to the mean inter-episode time, Γ, which is defined as follows:

$$\Gamma = \frac{1}{\#\text{ inter-episodes}} \sum_{j \in \text{inter-episodes}} \mathcal{L}_j \tag{3.8}$$

where there are i episodes, j inter-episodes (with $j = i - 1$) and the length of the jth inter-episode is $\mathcal{L}_j$. The mean episode time, μ, is measured as the average length of an episode:

$$\mu = \frac{1}{\#\text{ episodes}} \sum_{i \in \text{episodes}} \mathcal{L}_i \tag{3.9}$$

where $\mathcal{L}_i$ is the length of episode i.

Extreme events typically bring substantial economic damages. Thus, the prevention, management, and control of extreme events may have a high priority. When a system fails, a measure of its vulnerability is the extent of failure, *i.e.* the expected magnitude of failure. As an estimator, ν, the mean maximum failure is used. This represents the average maximum failure extent over all episodes in a time series:

$$\nu = \frac{1}{\#\text{ episodes}} \sum_{i \in \text{episodes}} \max\{\text{ fail}_i \} \tag{3.10}$$

where "$\max\{\text{ fail}_i\}$" represents the maximum failure during episode i and "# episodes" stands for the number of episodes in the time series.

One could combine aspects of μ and ν to capture information about how effectively the system is buffering shocks. To this end, ψ, the mean failure sharpness is defined as:

$$\psi = \frac{1}{\#\text{ episodes}} \sum_{i \in \text{episodes}} \frac{\max\{\text{ fail}_i \}}{\mathcal{L}_i} \tag{3.11}$$

ψ would be more meaningful if it could consider the time the system requires to return from its peak failure to the satisfactory region; we shall call this $\mathcal{L}_i^*$. This concept is included in the mean recovery time, Ψ, which is defined as:

$$\Psi = \frac{1}{\#\text{ episodes}} \sum_{i \in \text{episodes}} \frac{\max\{\text{ fail}_i \}}{\mathcal{L}_i^*} \tag{3.12}$$

All of the aforementioned sustainability indicators are now computed for the time series in Fig 3.2(a) as follows:

$$\begin{aligned}
\text{Resistance:} \quad \gamma &= \frac{5}{10} = 0.5 \\
\text{Mean Inter-Episode Time:} \quad \Gamma &= \frac{1+2+3}{3} = 2.0 \\
\text{Mean Episode Time:} \quad \mu &= \frac{2+2+4+2}{4} = 2.5 \\
\text{Mean Maximum Failure:} \quad \nu &= \frac{4+7+8+9}{4} = 7.0 \\
\text{Mean Failure Sharpness:} \quad \psi &= \frac{1}{4}\left[\frac{4}{2}+\frac{7}{2}+\frac{8}{4}+\frac{9}{2}\right] = 3.0 \\
\text{Mean Recovery Time:} \quad \Psi &= \frac{1}{4}\left[\frac{4}{1}+\frac{7}{1}+\frac{8}{3}+\frac{9}{1}\right] = 5.7
\end{aligned}$$

These indicators should be combined with those already existing in the literature. For example, Hashimoto et al. (1982) defined system resilience to be the average probability of a recovery from the failure set in a single time step.

3.2.2 Sustainability Indicators and Forest Management

To illustrate the use of sustainability indicators in the context of environmental management, consider the following forest management problem in New Brunswick, Canada (Clark et al., 1979). At the heart of this issue is the spruce budworm, a lepidopteran defoliator of conifers (*Chorisoneuma fumiferana*). This insect oscillates from low to high population levels every 30-60 years. During an outbreak, typically lasting about eight years, the budworm may kill up to 90% of trees in a stand; balsam fir (*Abies balsamea*) and spruce are particularly vulnerable. In an attempt to minimize disruption of pulp and paper production, industry and government in New Brunswick began the aerial application of pesticide in 1952. While initially 'successful' at suppressing budworm outbreaks, chemical spraying gradually became less and less effective (Baskerville, 1995). When an unprecedented outbreak

Table 3.1: Subset of indicators developed from a simulation model built at the University of British Columbia and International Institute for Applied Systems Analysis (Clark et al., 1979).

Resource Indicators
Forest volume in m^3 of merchantable timber / ha of fully stocked forest
Proportion of total volume harvested
Environmental Indicators
Insecticide impact in terms of fraction of province sprayed
Age class diversity of the forest
Economic Indicators
Cost per unit volume of harvested wood
Cost of insecticide spraying
Social Indicators
Recreational quality (number of subregions meeting a predefined rating)
Employment rate reflecting proportion of mill capacity utilized

erupted in the mid 1970s, the Canadian government recognized the need for indicators to determine the health of the forests and related socio-economic variables (see Table 3.1 for a subset of the indicators considered).

During the mid-1970s, the government of Canada reviewed possible forest management alternatives: how best to schedule the harvesting and spraying of trees so as to maintain a viable lumber industry, promote ecological integrity, and preserve recreational opportunities? As part of the attempt to answer this question, a detailed simulation model was built (by researchers at the Institute for Resource Ecology, University of British Columbia, Canada and at the International Institute for Applied Systems Analysis, Laxenberg, Austria) to examine the impacts of different harvesting alternatives on crit-

ical socio-economic and resource variables. Three of the key alternatives investigated by Clark et al. (1979) were:

- "Historical Management" (policies used in New Brunswick in the 1960s and 1970s);
- "Winkler Dantzig" Management (the use of dynamic programming to determine the 'optimal' schedule for spraying and harvesting);
- "Branch Density Hybrid Management" (judicious forest harvesting to avoid triggering a budworm outbreak).

Fig 3.2(c) illustrates the hierarchical arrangement of attributes in this forest management problem: resource (x_1), economy (x_2), ecology (x_3), recreation (x_4), and employment (x_5). Note that there are four 'level one' objectives, upon which the three alternatives (Historical Management, Winkler-Dantzig Management, and Branch-Density Hybrid Management) are evaluated. The overall value of each alternative is determined using the formula

$$V(x_j) = k_1 x_{1j} + k_2 x_{2j} + k_3 x_{3j} + k_6(k_4 x_{4j} + k_5 x_{5j}) \tag{3.13}$$

For simplicity, assume that the level one criteria are of equal importance: *i.e.* $k_1 = 0.25$, $k_2 = 0.25$, $k_3 = 0.25$, and $k_6 = 0.25$; and that the level two criteria, namely recreational quality and mill employment, have weights of $k_4 = 1/3$ and $k_5 = 2/3$ respectively.

Using the sustainability indicators of the previous section, the value function of Eq 3.13, and the time series provided in the work of Clark et al. (1979), overall sustainability scores can be determined for each of the three forest management alternatives. As previously mentioned, determining the 'acceptable region' for each indicator is highly subjective. For example, in the case of harvest costs, the 'acceptable region' was assumed to be

$$x_3 < \$20/m^3 \tag{3.14}$$

Additional details can be found in Levy et al. (2000d) where the overall sustainability scores for the Historical Management, Winkler-Dantzig, and

Branch-Density Hybrid alternatives are found to be 0.05, 0.50, and 0.65 respectively. This is illustrated graphically in Fig 3.2(b). Note that Branch-Density Hybrid is non-dominated on all criteria except x_1 (the resource indicator), on which Winkler-Dantzig is best.

3.3 Sensitivity Analysis

Practicing decision analysts employ a variety of sensitivity techniques to model uncertainties ranging from unknown weights to uncertain preference information. However, sensitivity analyses are often *ad hoc* and inadequate for a number of reasons. First, traditional sensitivity methods occupy a separate phase of the MCDA cycle, usually at the "back end" (Chávez and Shachter, 1998). They inform the decision maker which uncertainties are important, but do not provide the necessary feedback to the decision maker ("at the front end"). Second, while many sensitivity methods provide clues as to how changes in model inputs will impact the recommended action, they do not represent a comprehensive basis for measuring the relative robustness of competing decisions to uncertainty. Third, the term "sensitivity analysis" is sometimes used loosely and many authors have introduced their own specific definitions. For example Kleijnen (1994) explicitly defines "sensitivity analysis" as the response of model outputs to *extreme* values of the model inputs and drastic changes of the model structure. In contrast, Morgan and Henrion (1990) defines "uncertainty analysis" as the process of sampling model inputs from probability distributions to quantify the consequences of uncertainty on the model output.

In a typical sensitivity analysis involving unknown weights, one might modify the relative importance of a particular criteria weight and observe the effect on the overall result. For instance, in the Stormwater Management problem, consider the effect of placing more importance on "Erosion" and less on "Phosphorus". The resulting ordering of the sixteen alternatives (from most to least preferred) is:

$$\text{most preferred} \quad \underbrace{A \sim C \sim M}_{\text{tie}} \succ G \succ L \succ E$$
$$\overbrace{\succ P \succ \underbrace{I \succ J \sim K}_{\text{tie}} \succ H \succ B \succ F \succ D}^{\text{new positions}} \succ N \succ O \quad \text{least preferred} \tag{3.15}$$

3.4 Sensitivity Analysis Techniques

Consider a model represented as a function f, with two uncertain inputs x_1 and x_2, and one output y:

$$y = f(x_1, x_2) \tag{3.16}$$

Further, assume that various uncertainties in the inputs have been identified.

A scenario is a particular situation, specified by a single value for each input variable which defines a single point on the response surface. Accordingly, a scenario can be defined as a vector of values for the inputs, $\boldsymbol{x}$:

$$\boldsymbol{x} = (x_1, x_2) \tag{3.17}$$

A nominal, or "base-case" scenario, consists of a single *nominal value* for each input and represents the initial "best guess" values for the inputs (perhaps the mean, median, or mode values of the full probability distribution). These nominal input values are denoted $\overline{x}_1$ and $\overline{x}_2$. Together these two input values specify the nominal scenario:

$$\overline{\boldsymbol{x}} = (\overline{x}_1, \overline{x}_2) \tag{3.18}$$

The corresponding nominal output value is defined as:

$$\overline{y} = (\overline{x}_1, \overline{x}_2) \tag{3.19}$$

The analysis of uncertainty involves measuring the degree to which each input x_i (here x_1 and x_2 are the only inputs) contributes to uncertainty in the output y. Perhaps the simplest method to quantify this uncertainty is a measure called simply *sensitivity*, U_S. It is the rate of change of the output y with respect to variation in an input x_i (Morgan et al., 1984). In this case, the two sensitivities are the partial derivatives of output y with respect to each input, evaluated at values of the nominal scenario. In general U_S can be defined as follows:

$$U_S(\boldsymbol{x}, y) = \left.\frac{\partial y}{\partial x_i}\right|_{\overline{\boldsymbol{x}}} \tag{3.20}$$

Hence, these sensitivities are the slopes of the two tangents to the response surface at the nominal scenario. One obvious problem with U_S for comparing the uncertainty of different inputs is that it depends on the scale, or units of measurement of x_i and y. Of course sensitivity to an input measured in centimeters will be a hundred times greater than an input measured in meters. To ensure that measures of uncertainty are unaffected by the unit of measurement, U_S should be normalized, defining changes in x_i and y in relative terms as a fraction of their nominal values. This measure of uncertainty is sometimes known as *elasticity*, U_E.

$$U_E(\boldsymbol{x}, y) = \left.\frac{\partial y}{\partial x_i}\right|_{\overline{\boldsymbol{x}}} \times \frac{\overline{x}_i}{\overline{y}} \tag{3.21}$$

A drawback of both U_S and U_E is that they ignore the degree of variation in each input. An input that has a small sensitivity, but a large variation about its nominal value may be just as important as an input with a larger sensitivity but smaller variation. The simplest approach that considers both sensitivity and variation is generally known as the first order approximation or *Gaussian approximation* after the German mathematician Karl Friedrich Gauss (the "Prince of Mathematics"), who is credited with developing this approach in the early nineteenth century. Here, the variance of the output $\mathrm{Var}[y] \equiv \sigma_y^2$ is estimated as the sum of squares of the contributions from each

input. Denote the variance of each input as $\text{Var}[x_1] \equiv \sigma_1^2$ and $\text{Var}[x_2] \equiv \sigma_2^2$. Then the variance of the output is given by the Gaussian approximation as:

$$Var[y] \approx \left[\frac{\partial y}{\partial x_1}\right]^2\Bigg|_{\overline{x}} Var[x_1] \quad + \quad \left[\frac{\partial y}{\partial x_2}\right]^2\Bigg|_{\overline{x}} Var[x_2] \tag{3.22}$$

Here, the total uncertainty in the output, expressed as variance, is explicitly decomposed as the sum of the uncertainty contributions from each input (the product of its partial derivative times its standard deviation). This is the basis for many uncertainty techniques in the physical sciences and environmental engineering.

The Gaussian approach is a *local* approach in that it considers the behavior of the function only in the vicinity of the nominal scenario. This may be fairly accurate when functions are smooth and inputs are near nominal values, but is likely to produce misleading results for more complicated functions and large deviations from nominal values. In such cases, a *global* approach to uncertainty is called for that explicitly evaluates the function for large uncertainties (scenarios distant from the nominal scenario).

Suppose that a low and high value is selected for each input, chosen to bound its range of plausible variation (it is not necessary for the bounds to be symmetrically placed around each input's nominal value). Assume that the ranges for our two inputs x_1 and x_2 are denoted as $[x_1^0, x_1^*]$ and $[x_2^0, x_2^*]$ respectively. The nominal range sensitivity method, U_R, computes the effect on the output of varying each input from its low to high value, while keeping the other inputs at their nominal values (Morgan et al., 1984). For example:

$$\begin{aligned} U_R(x_1, y) &= f\left(x_1^*, \overline{x}_2\right) - f\left(x_1^0, \overline{x}_2\right) \\ U_R(x_2, y) &= f\left(\overline{x}_1, x_2^*\right) - f\left(\overline{x}_1, x_2^0\right) \end{aligned} \tag{3.23}$$

3.4.1 Analytic Approaches: Taylor Series Approximation

For all except the simplest cases, such as linear combinations of normal variables, Springer (1979) argues that exact analytic methods for the propagation of uncertainty are intractable (or require sophisticated numerical integration techniques). However, there are a variety of well-known approximate analytic techniques based on Taylor series expansions (Cheney, 1966). These techniques are sometimes called *Method of Moments* because they analyze uncertainty using the mean, variance, and sometimes higher order moments of a random (output) variable which is itself a function of one or more random (input) variables. In many engineering fields, the term 'First Order Uncertainty Analysis' is used (Burges and Lettenmaier, 1975; Benjamin and Cornell, 1970), because only the first order term in the Taylor series expansion is considered.

Consider a vector of n uncertain inputs

$$\boldsymbol{x} = (x_1, x_2, \ldots x_n) \tag{3.24}$$

so $y = f(\boldsymbol{x})$ It is assumed that the nominal value for each input is equal to its expectation, so that for $i = 1$ to n, $E[x_i] = \overline{x}_i$. Accordingly, the nominal scenario is equal to the mean scenario, or the expectation of $\boldsymbol{x}$:

$$E[\boldsymbol{x}] = (\overline{x}_1, \overline{x}_2, \ldots \overline{x}_n) \tag{3.25}$$

The Taylor series expansion provides a way to express deviations of output, y, from its nominal value $y - \overline{y}$ in terms of deviations of inputs from their nominal values, $x_i - \overline{x}_i$. Successive terms contain higher order powers of deviations and higher order derivatives of the function with respect to each input (Korn and Korn, 1968). The expansion around the nominal scenario with the first three terms is given by

$$
\begin{aligned}
y - \overline{y} \;\; &= \sum_{i=1}^{n} (x_i - \overline{x}_i) \frac{\partial y}{\partial x_i}\bigg|_{\overline{\boldsymbol{x}}} \; + \\
&\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - \overline{x}_i)(x_j - \overline{x}_j) \frac{\partial^2 y}{\partial x_i \partial x_j}\bigg|_{\overline{\boldsymbol{x}}} \; + \\
&\frac{1}{3!} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} (x_i - \overline{x}_i)(x_j - \overline{x}_j)(x_k - \overline{x}_k) \frac{\partial^3 y}{\partial x_i \partial x_j \partial x_k}\bigg|_{\overline{\boldsymbol{x}}} \; + \; \cdots
\end{aligned}
\tag{3.26}
$$

Note that all derivatives are evaluated at the nominal (*i.e.* mean) scenario $\overline{\boldsymbol{x}}$ where

$$\overline{\boldsymbol{x}} \; = \; (\overline{x}_1, \overline{x}_2, \; \ldots \; \overline{x}_n) \tag{3.27}$$

If the deviations $x_i - \overline{x}_i$ are relatively small, the higher powers will become very small. And if the function is relatively smooth in the region of $\overline{\boldsymbol{x}}$, the higher derivatives will be small too. Under these conditions, the Taylor series produces a good approximation when the higher order terms are ignored.

For example, consider an approximation for the mean of the output deviation given in Eq. 3.26 using only terms up to the second order:

$$
\begin{aligned}
E\left[y - \overline{y}\right] \;\; &\approx \sum_{i=1}^{n} E\left[x_i - \overline{x}_i\right] \frac{\partial y}{\partial x_i}\bigg|_{\overline{\boldsymbol{x}}} \; + \\
&\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} E\left[(x_i - \overline{x}_i)(x_j - \overline{x}_j)\right] \; \frac{\partial^2 y}{\partial x_i \partial x_j}\bigg|_{\overline{\boldsymbol{x}}}
\end{aligned}
\tag{3.28}
$$

Since the nominal value of each x_i is equal to its mean, we know

$$E[x_i - \overline{x}_i] = 0,$$

so the first term disappears. The covariance between x_i and x_j is given by

$$Cov[x_i, x_j] \; \equiv \; E[(x_i - \overline{x}_i)(x_j - \overline{x}_j)] \tag{3.29}$$

Substituting this into Eq 3.28 yields

$$E\left[y - \overline{y}\right] \approx \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} Cov[x_i, x_j] \frac{\partial^2 y}{\partial x_i \partial x_j}\bigg|_{\overline{\boldsymbol{x}}} \tag{3.30}$$

Note that as long as the function y is non-linear (at least one of the second order or higher derivative terms are nonzero) the mean output value $E[y]$ cannot be computed simply by evaluating the model with all inputs set to their expected values.

First Order Approximation

To make things simpler, it is common to take only the first order term. To first order, the expected value of the deviation in y is zero:

$$E[y - \overline{y}] \equiv 0 \tag{3.31}$$

and so the expected value of y can be approximated simply by the nominal value $\overline{y}$:

$$E(y) \equiv \overline{y} \tag{3.32}$$

From the definition of variance we have

$$\begin{aligned} Var(y) &= E\left[(y - E[y])^2\right] \\ &\equiv E\left[(y - \overline{y})^2\right] \end{aligned} \tag{3.33}$$

It follows that the first order approximation for the variance of the output can be obtained using only the first order term from Eq 3.26.

$$\begin{aligned} Var[y] &\approx E\left[\left(\sum_{i=1}^{n}(x_i - \overline{x}_i)\frac{\partial y}{\partial x_i}\bigg|_{\overline{\boldsymbol{x}}}\right)^2\right] \\ &\approx \sum_{i=1}^{n}\sum_{j=1}^{n} E\left[(x_i - \overline{x}_i)(x_j - \overline{x}_j)\right]\frac{\partial y}{\partial x_i}\bigg|_{\overline{\boldsymbol{x}}}\frac{\partial y}{\partial x_j}\bigg|_{\overline{\boldsymbol{x}}} \end{aligned} \tag{3.34}$$

Using the covariance formula in Eq 3.29, $Var[y]$ can be expressed as:

$$Var[y] \approx \sum_{i=1}^{n}\sum_{j=1}^{n} Cov\left[x_i, x_j\right]\frac{\partial y}{\partial x_i}\bigg|_{\overline{\boldsymbol{x}}}\frac{\partial y}{\partial x_j}\bigg|_{\overline{\boldsymbol{x}}} \tag{3.35}$$

More insights can be gleaned by separating the variance terms,

$$Var[x_i] \equiv Cov[x_i, x_i],$$

from the covariance terms. Moreover, since

$$Cov[x_i, x_j] = Cov[x_j, x_i]$$

it can be shown (Morgan et al., 1984) that

$$Var[y] \approx \sum_{i=1}^{n} Var\,[x_i] \left[\left. \frac{\partial y}{\partial x_i} \right|_{\overline{x}} \right]^2 + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} Cov\,[x_i, x_j] \left. \frac{\partial y}{\partial x_i} \right|_{\overline{x}} \left. \frac{\partial y}{\partial x_j} \right|_{\overline{x}} \tag{3.36}$$

If the inputs are independent, the second term containing the covariances is zero, and this collapses into the simple Gaussian approximation discussed in Eq 3.22 for the two input case. Assuming independence of the x_i and x_j terms the variance of the output is approximately the sum of the squares of the products of the standard deviation $\sigma[x_i]$ and sensitivity $\partial y/\partial x_i$ of each input:

$$Var[y] \approx \sum_{i=1}^{n} Var\,[x_i] \left[\left. \frac{\partial y}{\partial x_i} \right|_{\overline{x}} \right]^2 \tag{3.37}$$

Both the first order approximation (Gaussian) and higher order approximations (method of moments) have been applied quite widely to the analysis of complex problems in engineering and the physical sciences. These analytic methods have a number of advantages: numerical calculations are relatively simple (once the algebraic analysis has been completed); the contribution of each input toward the variance in the output is clearly illustrated, and the entire probability distribution of the input parameters does not require specification (only the first few moments, typically the mean and variance). However, these analytic methods suffer from a number of disadvantages:

1. If the model is complex and higher order terms are necessary, the algebra can become intractable.

2. It is difficult to obtain estimates for the tails of the output distribution (usually only the mean and variance are conveniently calculated).

3. Large uncertainties in the input variables will cause significant inaccuracies since the Method of Moments is a "local approach".

4. First order approximations replace the actual function by a linear one (a hyperplane tangent to the response surface at the nominal scenario).

5. The method breaks down if the response surface has discontinuities or important covariance terms are omitted.

3.5 Conclusions

In order to evaluate the sustainability of ecological systems over time, several new sustainability indicators are developed, extending the work of Pimm (1984) and Loucks (1997). Using the time series provided in the classic spruce budworm paper of of Clark et al. (1979), overall sustainability scores were determined for three forest management alternatives (Historical Management, Winkler-Dantzig, and Branch-Density Hybrid). The 'acceptable region' for each indicator was subjectively determined. For example, in the case of harvest costs, the 'acceptable region' was assumed to be less than $\$20/m^3$. The overall sustainability scores for the Historical Management, Winkler-Dantzig, and Branch-Density Hybrid alternatives are found to be 0.05, 0.50, and 0.65 respectively. The Branch-Density Hybrid alternative was found to be non-dominated on all criteria except x_1 (the resource indicator), on which Winkler-Dantzig is best.

Since preference judgments are notoriously tentative, imprecise, approximate, and incomplete an intuitive interval approach to estimating criterion weights is developed that requires only an ordinal ranking of the weights.

This approach determines the average value of each criterion weight over all values in the feasible region (*i.e.* consistent with the constraints). The results are useful although often surprising without the aid of a diagram. Finally, practicing decision analysts employ a variety of uncertainty/sensitivity techniques, including analytic approaches such as the Taylor Series Approximation, to model uncertainties ranging from unknown weights to uncertain preference information.

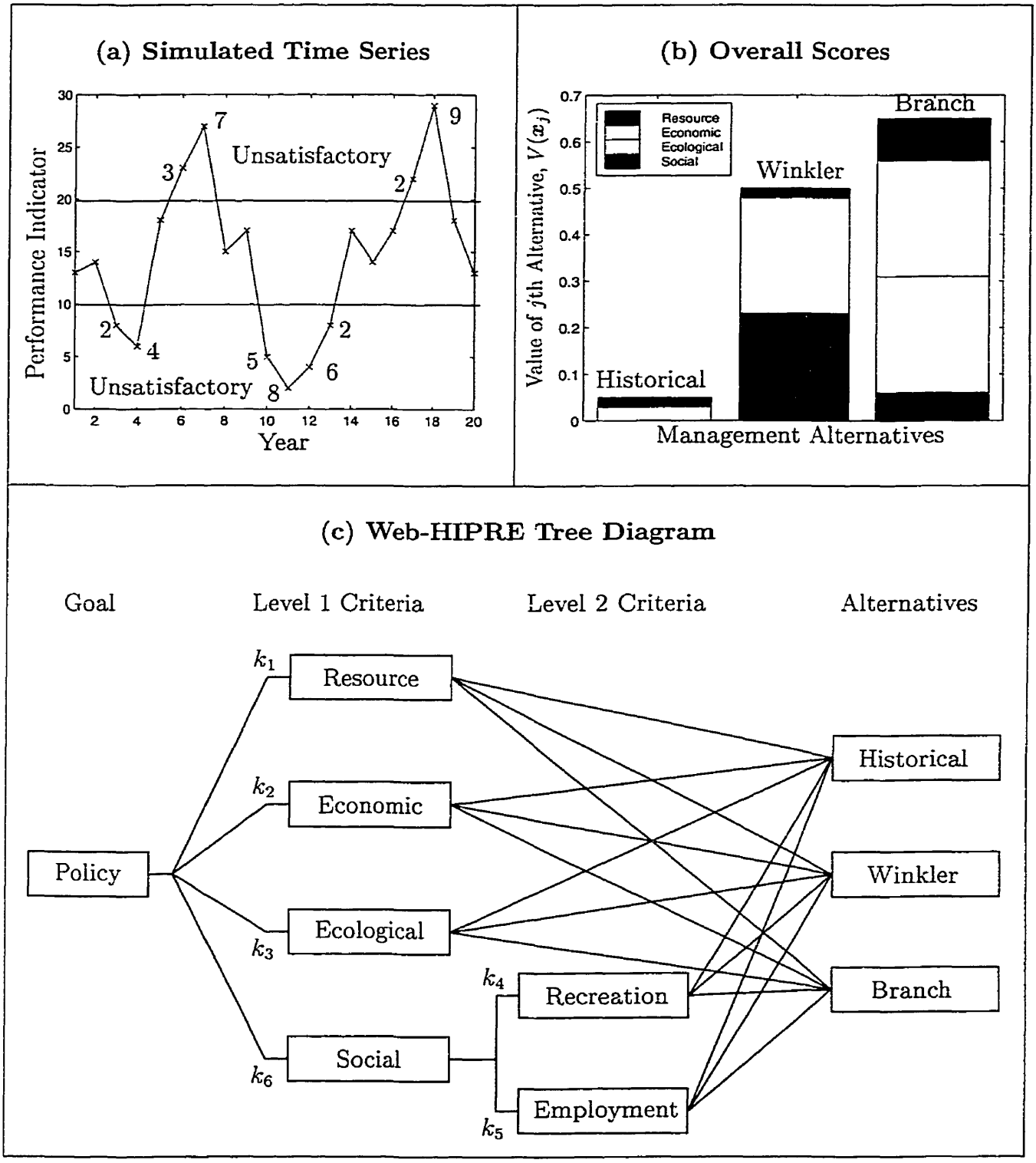


Figure 3.2: Web-HIPRE Framework and Results

Chapter 4

Bayesian Uncertainty Analysis

By the end of the 1970s, under the influence of powerful computing machines, there was an explosion of immensely complex, often arbitrarily precise, and predominantly deterministic environmental models[1]. Despite the enthusiasm of the time, most of the large scale systems and policy oriented models built in the 1970s fell significantly short of their original expectations. A few shortcomings figured prominently in the limited utility of these efforts:

1. Failure to carefully examine the implications of uncertainty in the input variables.

2. Inability to deal with exogenous events and stochastic phenomena, particularly in physical systems such as climate models.

3. Inadequate and incomplete understanding of the system being modeled.

Clearly, environmental decision making under uncertainty remains a difficult research problem, particularly in complex, multi-attribute situations. Yet, without a thorough analysis of the uncertainty in a policy problem "we

[1]A number of other global modeling projects were undertaken in the decade that followed publication of *The Limits to Growth* (Meadows et al., 1974). Many of them, such as the Global 2000 Report to the President (U.S. Council on Environmental Quality, 1980) and the Forrester-Meadows model are succinctly summarized in Meadows et al. (1982).

cannot be sure that the results of a model, especially a very large and complex one, mean anything at all." (Morgan et al., 1984). To improve the analysis and modeling of uncertainty in environmental systems, this chapter considers uncertainty at all stages in the environmental modeling cycle. The Bayesian approach to uncertainty analysis is emphasized throughout. As shown in Fig 4.1, uncertainty in environmental modeling can be categorized into the stages of model building (Section 4.1), parameter estimation (Section 4.2), and prediction. Of course, models can be used for purposes other than prediction, such as simulations, forecasting, and analysis,

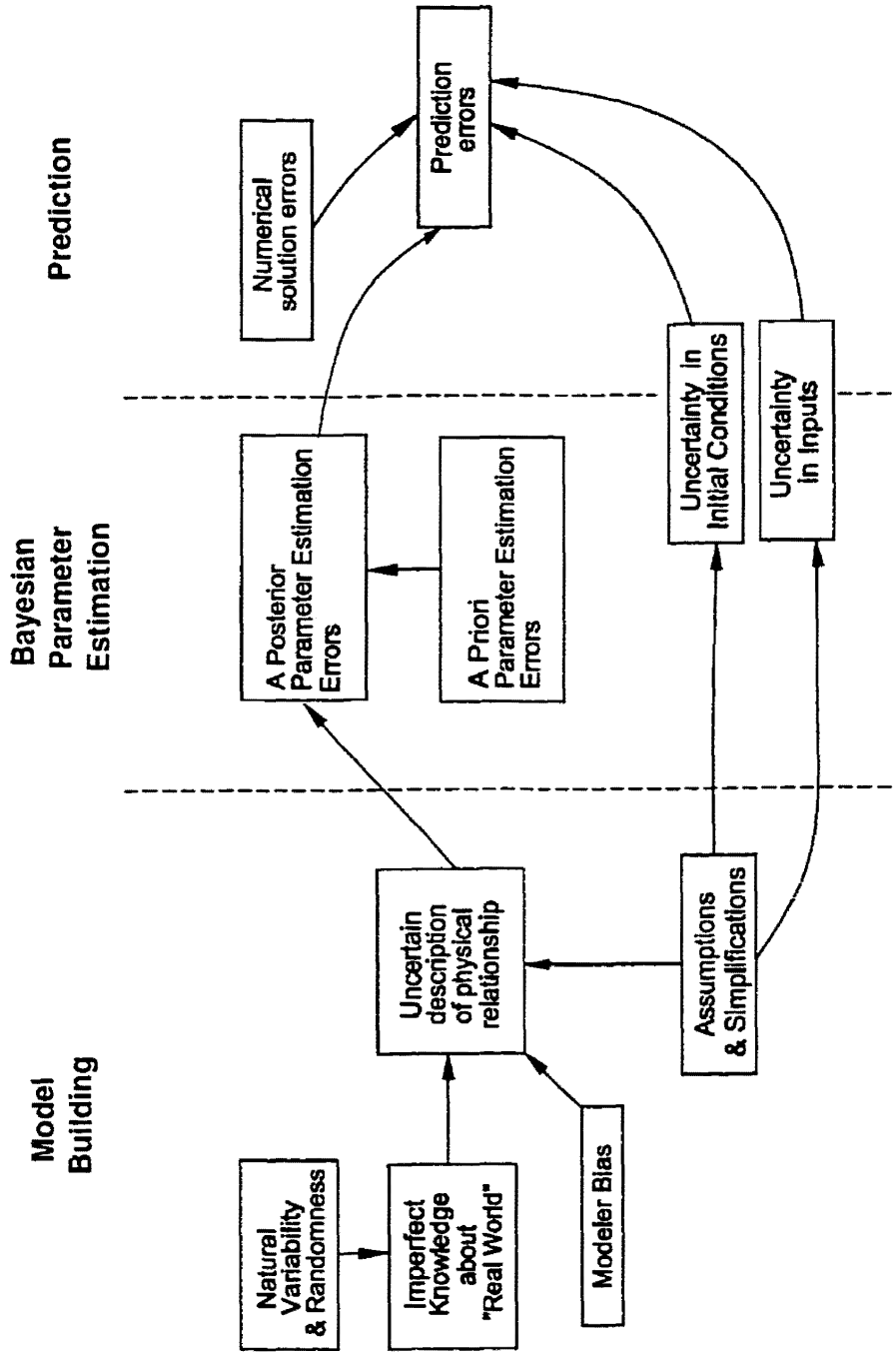


Figure 4.1: Uncertainty analysis in environmental modelling (Beck, 1987)

4.1 Model Building

Rosen (1985) addresses some of the nuances that arise in the modelling of physical systems. He argues that modeling a "Natural System" N (some aspect of the world gleaned through interaction) in terms of a syntactic "Formal System" F (which manipulates symbols according to explicit rules, as in mathematics) requires a sequence of three steps:

1. System identification: *encoding* linkages and environmental phenomena observed in N as propositions in F.

2. The *syntactic entailment* of F: the application of its rules of inferences.

3. *Decoding* theorems back into causal phenomena by way of prediction.

The distinction between parameter and model uncertainty is often ambiguous and different model structures can be assimilated into a single *meta-model*, which contains separate models as special cases, according to one or more parameters. For example, Howard and Matheson (1984) and Howard (1988) consider a dose-response function with uncertain form: it may be linear or exponential. It is straightforward to define a dose-response function with an exponent parameter which will reproduce linear models if the exponent is 1 (Henrion and Fischoff, 1986). Similarly a dose-response function with a threshold parameter can be defined that will characterize non-threshold models, if the threshold parameter is zero (Morgan et al., 1984). In this way, uncertainty about the model form can be converted into uncertainty about the parameter values (Genest and Zidek, 1986).

4.1.1 Assumptions and Simplifications

Any model should be as simple as possible, yet still provide a reasonable explanation of what is happening according to the principle of Occam's Razor (Hipel and McLeod, 1994). Even the most carefully constructed and sophisticated model is only an approximation to reality: inaccuracies and

uncertainties will arise when modelling real-world systems (Wynne, 1992). Furthermore, data is often lacking, measurements may be imprecise, and environmental phenomena are inherently random.

The use of a first order equation for the decay of organic wastes dates back to the pioneering work of Phelps (1909) and Streeter and Phelps (1925). A majority of modelers continue to assume that environmental processes occur at rates proportional to the concentration of the substance of interest. Specifically, environmental processes are often described mathematically as:

$$\frac{dC}{dt} = -kC \tag{4.1}$$

where C is a concentration (mass/volume), t is time, and k is a rate constant (1/time). The integrated form of this expression is the familiar first-order or exponential decay formula:

$$C(t) = C_0 \exp(-kt) \tag{4.2}$$

where $C(t)$ is the concentration of C at time t, and C_0 is the initial concentration of C. This approach has intuitive appeal, and the parameters C_0 and k are easy to estimate given only a few measurements of $C(t)$. Environmental processes that have been approximated in this manner include the oxidation of carbonaceous biochemical oxygen demand (BOD) and nitrogenous BOD by oxygen demanding organisms (O'Connor and Dobbins, 1958; Metcalf and Eddy, 1991).

The decay process in Eq 4.1 is referred to as a 'first-order' reaction. The term first-order arises from an implicit exponent of a '1' on the C in this equation. This assumption is so common and subtle that many researchers are unaware that a subjective decision has been made by choosing a first-order model (Berger and Berry, 1988). For fundamental processes such as nuclear decay and chemical reactions, the first order assumption has "sound theoretical and empirical support" (Bates and Watts, 1988). However, environmental processes occur at a very different scale from chemical reactions (Swamee and Ojha, 1991) and are an aggregation of numerous underlying,

often unknown, individual processes (Adrian and Sanders, 1992). To address this situation, the first-order decay model of Eq 4.1 can be more generally expressed as:

$$\frac{dC}{dt} = -kC^{\theta} \tag{4.3}$$

where the exponent θ is a free parameter to be estimated from the observations. The integrated form of Eq 4.3 is

$$C(t) = \left[C_0^{1-\theta} - k(1-\theta)t\right]^{\frac{1}{1-\theta}} \tag{4.4}$$

Note that when $\theta = 0$, the concentration decreases linearly to zero. On the other hand, when θ is greater than zero, the rate of concentration decrease slows with time, with a zero asymptote. When $\theta = 1$, the rate of decrease is proportional to C and the solution is given by Eq 4.2. Large values of θ indicate faster initial concentration decreases, followed by increasingly slower decreases. This is illustrated in Fig 4.2. Finally, θ is not restricted to integers (fractional values are also possible). For example, Adrian and Sanders (1992) consider the oxygen sag equation for half order BOD kinetics ($\theta = 0.5$ in Eq 4.3).

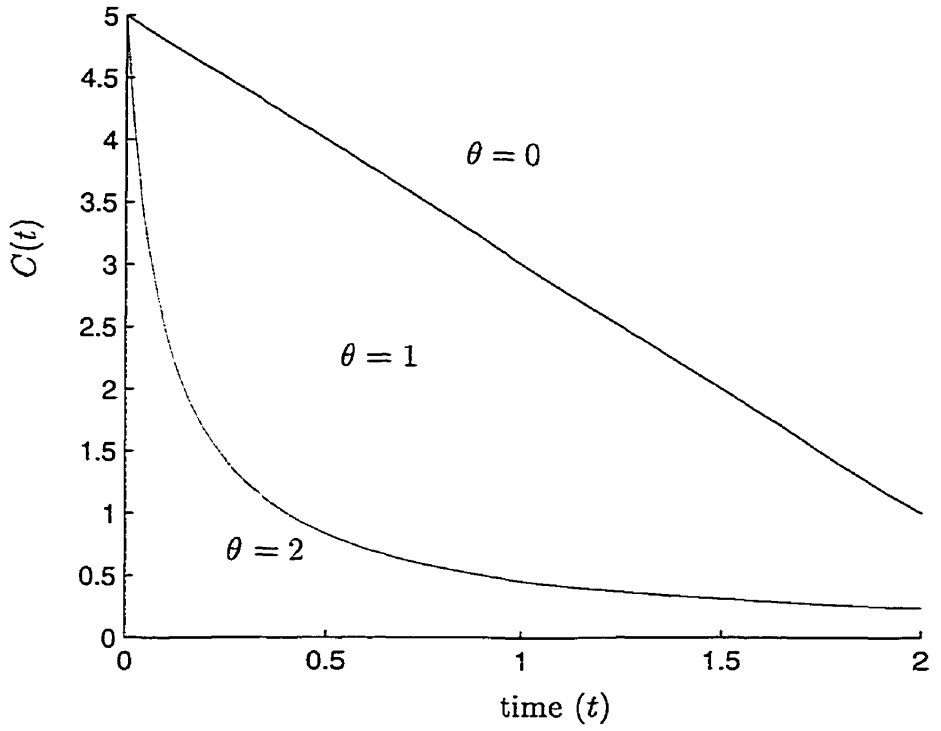


Figure 4.2: Decay of $C(t)$ for various values of θ

4.2 Parameter Estimation: the Bayesian approach

Assessing the relative plausibility of a variety of parameter values, given the available data, is an important component of ecological modeling. Such an assessment is awkward using classical statistics, since the frequentist approach assumes that the value of parameters are fixed (known by God), not random: each parameter has a single, true, though often unknown value (at least to mortals). However, in Bayesian statistics parameters are not fixed: they are themselves random variables from a given probability distribution.

Even with copious volumes of data, it is often not possible to recover a uniquely best set of parameter estimates allowing a match between the model

and the observations (Sorooshian and Gupta, 1985; Johnston and Pilgrim, 1976). Moreover, data of ecological processes are always produced with a certain error (Jorgensen, 1979). In the modelling process these errors will be propagated to the model parameters. Moreover, model parameters must also compensate for the shortcomings of over-simplified model constructs.

Any statistic used to estimate the value of an unknown parameter θ is called an estimator of θ. The observed value of the estimator is called the estimate. For instance, the usual estimator of the mean of a normal population, based on a sample $x_1, \ldots, x_n$ from that population, is the sample average

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{4.5}$$

If a sample of size 3 yields the data $x_1 = 5$, $x_2 = 5$ and $x_3 = 2$, then the estimate of the population mean, resulting from the estimator $\bar{x}$ is the value 4. Hence, an estimator is a function of the observed sample values that provides an estimate of a parameter of the parent distribution (such as a moment). The method of moments, maximum likelihood method, and method of least squares are three general approaches for obtaining point estimates of unknown parameters (estimators). Bayesian approaches can help in the estimation of unknown parameters when prior information available.

The earliest general method for determining an estimator of an unknown parameter is the method of moments (introduced by Karl Pearson in 1894). It works as follows: the parameters of a population distribution are selected to match the estimate of the sample data. It follows that the method of moments' estimate of a population mean is always the sample mean. This procedure is usually quite straightforward to implement. The method of maximum likelihood is more widely used in modern statistics and involves the selection of parameter values most likely to yield the observed data set. Polacheck et al. (1993), Punt and Butterworth (1993), and Punt et al. (1994) use the method of maximum likelihood estimation in the context of fisheries. The evaluation of point estimators is described in more detail in Appendix

B.

Bayesian Regression Analysis

Many engineering and scientific problems are concerned with determining a relationship between a (random) response variable Y and one or more explanatory or predictor variables $x_1, \ldots, x_p$:

$$Y = f(x_1, \ldots, x_p) + \epsilon \tag{4.6}$$

where the function f is unknown and ϵ is random error. For instance, in a chemical process, the relationship between the amount of catalyst employed and the output of the process might be of interest. The simplest type of relationship between the dependent variable Y and the p predictor variables $x_1, \ldots, x_p$ is a linear relationship:

$$Y = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p \tag{4.7}$$

where $\beta_0, \beta_1, \ldots, \beta_p$ are unknown coefficients, usually estimated from a set of data.

Linear Regression

If the relationship between Y and the x_i's, $i = 1, \ldots, p$, is given by Eq 4.7 then it would be possible (once β_i were learned) to exactly predict the response for any set of input values. However, in practice, such precision is almost never attainable, and the most that one can expect is that Eq 4.7 would be valid subject to random error. This means that the explicit relationship is:

$$Y = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p + \epsilon \tag{4.8}$$

where ϵ, the random error, is assumed to be a random variable having mean 0. Hence, another way of expressing Eq 4.8 is

$$E[Y|\boldsymbol{x}] = \beta_0 + \beta_1 x_1 + \cdots + \beta_p x_p \tag{4.9}$$

where $\boldsymbol{x} = (x_1, x_2, \ldots, x_p)$ is the set of independent variables, and $E[Y|\boldsymbol{x}]$ is the expected response given the inputs $\boldsymbol{x}$. Equation 4.9 is called a regression equation because it describes the 'regression' of Y on the set of independent variables $x_1, x_2, \ldots x_p$. A regression equation containing a single independent variable is called a *simple regression*, whereas one containing many independent variables is called a *multiple regression* equation.

If there are n observations $(x_1, Y_1), \ldots, (x_n, Y_n)$ on a single independent variable x and output Y, the common model is:

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon \tag{4.10}$$

so that $\beta_0 + \beta_1 x_i$ represents the systematic relationship and ϵ is random error. Y is clearly a random variable as it depends on ϵ. In this thesis x is always regarded as non-random.

A number of assumptions about the random errors $\epsilon_1, \ldots, \epsilon_n$ are now made formally. Not all of these assumptions are needed for some results; in rough order of importance they are:

1. $\mathrm{E}(\epsilon_i) = 0$.
2. $\epsilon_1, \ldots, \epsilon_n$ are statistically independent.
3. $\mathrm{Var}(\epsilon_i) = \sigma^2$, *i.e.* constant for all observations $i = 1, \ldots, n$.
4. ϵ_i is normally distributed.

These four assumptions are often summarized as saying that $\epsilon_1, \ldots, \epsilon_n$ are independent and identically distributed $N(0, \sigma^2)$. In particular, Assumption 1 rules out data in which the errors have a positive expectation (mean) in parts of the x range, and negative expectation in others. As previously discussed, if $E(\epsilon_i) = 0$ for $i = 1, \ldots, n$ then

$$E(Y_i) = \beta_0 + \beta_1 x_i \tag{4.11}$$

i.e. we have a linear relationship between $E(Y)$ and x. Similarly, Assumptions 2, 3, and 4 also translate immediately into assumptions about $Y_1, \ldots, Y_n$.

Assumption 2 implies that $Y_1, \ldots, Y_n$ are statistically independent, Assumption 3 implies that $Var(Y_i) = \sigma^2$, *i.e.* a constant over observations, and Assumption 4 implies that Y_i is normally distributed. Thus the four assumptions about $\epsilon_1, \ldots, \epsilon_n$ may be summarized by saying that $Y_1, \ldots, Y_n$ are independent, and that Y_i has a $N(\beta_0 + \beta_1 x_i, \sigma^2)$ distribution, as illustrated in Fig 4.3.

Least squares estimation

The usual procedure to estimate β_0 and β_1 is to select them such that

$$\sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 \tag{4.12}$$

is a minimum. This is the method of least squares. Each $[y_i - (\beta_0 + \beta_1 x_i)]^2$ is the square of the vertical distance from the line one is drawing; the "best" line is determined by the condition that the sum of squares of the vertical distances between observations and the line be a minimum.

Using the popular linear model in Eq 4.10, the likelihood of the data, $\boldsymbol{x}$ and $\boldsymbol{y}$ given the model parameters, β_0, β_1, and σ is a normal distribution expressed as:

$$p(\boldsymbol{x}, \boldsymbol{y} | a, b, \sigma) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 \right) \tag{4.13}$$

Note that the sum of the squares

$$\sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 \tag{4.14}$$

becomes part of the exponent in the likelihood probability density function. It follows that

$$p(\boldsymbol{x}, \boldsymbol{y} | a, b, \sigma) \propto \frac{1}{\sigma^n} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 \right) \tag{4.15}$$

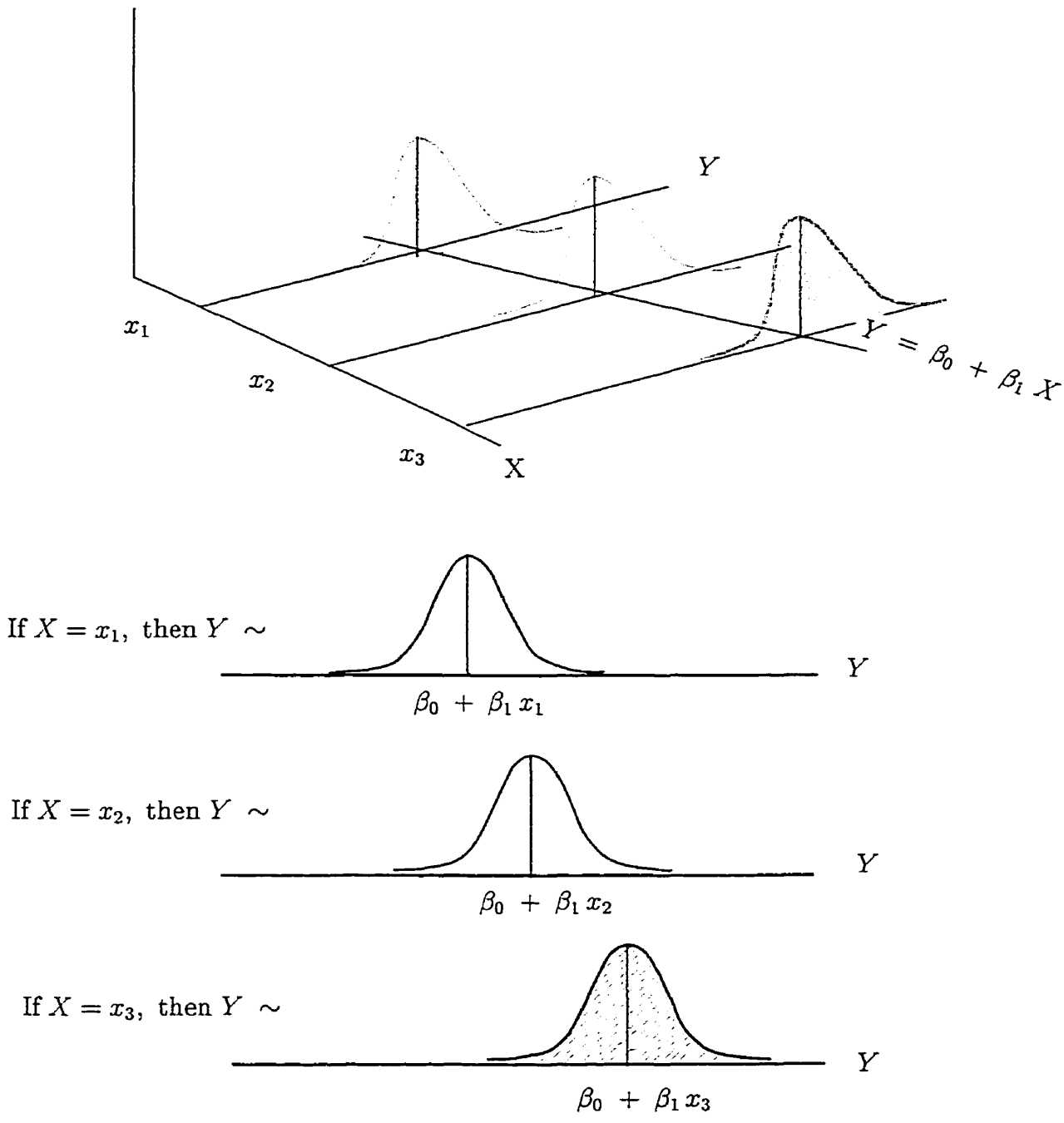


Figure 4.3: Y_i has a $N(\beta_0 + \beta_1 x_i, \ \sigma^2)$ distribution

In a Bayesian context, a prior distribution is needed. When no prior knowledge (initial ignorance) of model parameter values is assumed, a Jeffrey's prior distribution on σ (variance of the model residuals) is appropriate. This results in a joint prior distribution of the form:

$$p(\beta_0, \beta_1, \sigma) = \frac{1}{\sigma} \tag{4.16}$$

According to Bayes' theorem, the prior in Eq 4.16 is then combined with the likelihood function given in Eq 4.15 to yield the joint posterior probability density of the form:

$$p(a, b, \sigma | \boldsymbol{x}, \boldsymbol{y}) \propto \frac{1}{\sigma^{n+1}} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^{n} [y_i - (\beta_0 + \beta_1 x_i)]^2 \right) \tag{4.17}$$

Note from Eq 4.17 that minimizing the sum of squares implies finding the maximum of the posterior density.

4.2.1 Mixed Order Model of BOD Decay

A compelling reason for using Bayesian parameter estimation over maximum likelihood methods, or other estimation procedures, is that much information is lost when model parameters are represented by a single value rather than by a full distribution. In many modeling applications, one is interested in estimating the value of an unknown parameter, θ, or a vector of n parameters

$$\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_n) \tag{4.18}$$

about which there may be some prior beliefs. These prior beliefs may be interpreted as the assigned probability before the collection of new data. This prior distribution is often determined by using either previously existing data, subjective scientific judgment, or by employing a statement of ignorance. The latter, called a noninformative prior, is often a uniform distribution in which all possible parameter values are equally likely. After obtaining m new observations

$$\boldsymbol{x} = (x_1, x_2, \cdots, x_m) \tag{4.19}$$

which have a probability distribution that is a function of the n unknown parameters, the dependence of $\boldsymbol{x}$ on $\boldsymbol{\theta}$ can be expressed as the probability density function $p(\boldsymbol{x}|\boldsymbol{\theta})$. This pdf is often referred to as the *likelihood function.*

The next step in the Bayesian approach is to update the prior beliefs on $\boldsymbol{\theta}$ to account for the new data, $\boldsymbol{x}$. This is done using Bayes' theorem:

$$p(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\boldsymbol{\theta}) \times p(\boldsymbol{\theta})}{p(\boldsymbol{x})} \tag{4.20}$$

In the above expression $p(\boldsymbol{\theta}|\boldsymbol{x})$ is called the Bayesian *posterior distribution* and expresses the probability of the parameter values given the observed data. The denominator, $p(\boldsymbol{x})$, is the expected value of the likelihood function over the parameter distribution and it acts as a normalizing constant. Because the denominator in Eq 4.20 is a constant, Bayes' theorem is often expressed in words as:

$$\text{posterior} \propto \text{likelihood} \times \text{prior} \tag{4.21}$$

indicating that the prior expectations are modified by the likelihood function to yield the posterior beliefs. Once the normalized posterior distribution is derived, it can be used for inference in a number of ways. Marginal distributions can be obtained for each model parameter θ_i by integrating the joint posterior over all the other parameters in $\boldsymbol{\theta}$.

The presence of dissolved oxygen is essential for maintaining the biological integrity of the aquatic environment. For this reason, agencies responsible for water quality management seek to quantify the impact of municipal and industrial wastes on the dissolved oxygen concentration of receiving waters. The amount of oxygen required by aerobic microorganisms (bacteria) to stabilize the organic matter of waste water (from metabolism of organic waste compounds) is termed the biochemical oxygen demand (BOD)[2].

Conventionally, BOD exertion is modeled as a first-order decay process (Phelps, 1909), in which oxygen consumption is proportional to the concen-

[2]This indicator has been extensively used to measure the "rate and extent of bioavailability of the organic material present in waste water" (Constable and McBean, 1977).

tration of the BOD remaining (the BOD 'yet to be satisfied') at time t, $L(t)$, where L is measured in mg/L:

$$\frac{dL}{dt} = -k_1 L \tag{4.22}$$

Here, k_1 is the first order BOD rate constant in units of (1/day) and L_0 is the initial concentration of L (the amount of BOD remaining at $t = 0$, or ultimate BOD). This model was used in the pioneering work of Streeter and Phelps (1925), who developed the relationship between the stabilization of an organic waste measured by the BOD and the dissolved oxygen (DO) levels of a river. Theriault (1927) and Fair (1939) did additional early work in estimating the parameters of the first-order BOD decay model. The first order BOD decay has been widely used for nearly a century to describe the deoxygenation rate of municipal and industrial organic wastes. However, this model is often chosen more on the basis of mathematical convenience, rather than as a description of the complex transformations that occur as BOD decays; a number of authors have cautioned against assuming that all BOD data are described by a first order model (Orford and Ingram, 1953; Adrian and Sanders, 1992; 1998).

Specifically, many authors have pointed out that second order reactions frequently describe the stabilization of wastewaters (Thomas, 1957; Young and Clark, 1965; Tebbutt and Berkun, 1976; Nemerow, 1974). After extensive examination of municipal sewage Tebbutt and Berkun (1976) note that "... the oxygen uptake relationship could be satisfactorily modeled by both first and second order formulations." Data taken from the Waterloo Pollution Control Plant (Constable and McBean, 1977) is used to compare the first and second order BOD decay models in Appendix C using the root mean squared error (RMSE) statistic

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[y_p(t_i) - y_m(t_i)]^2} \tag{4.23}$$

where $y_p(t_i)$ is the predicted value of y on day t_i; $y_m(t_i)$ is the measured value

of y on day t_i; and N is the number of measurements of $y_m(t_i)$. Appendix C shows that the RMSE value is lower for the second order model.

Many studies have reported varying degrees of success modelling BOD decay with first, second, and half order models. An alternative to the fixed exponent approach is to model the parameter on L in Eq 4.22 as a free parameter. Rather than assuming a first-order (or any fixed order) decay process *a priori*, this formulation acknowledges that BOD decay is a mixture of decay processes and allows the data to determine the reaction order, which need not be constrained to integer values. With the exponent as a free parameter, the first-order decay model of Equation 4.22 can be more generally expressed as:

$$\frac{dL}{dt} = -k_n L^n \tag{4.24}$$

where the exponent n is a free parameter to be estimated from the observations. The integrated form of Eq 4.24 is

$$L(t) = \left[L_0^{1-n} - k_n(1-n)t\right]^{\frac{1}{1-n}} \tag{4.25}$$

where n is a 'pseudo-order' parameter, k_n is a mixed-order reaction rate constant and the other variables have been previously defined. This "mixed order" model was first proposed for application to BOD by Hewitt et al. (1979) who fit Eq 4.25 to oxygen uptake curves obtained from stream samples in New Jersey.

Bayesian parameter estimation is now applied to the mixed-order BOD decay model. Bayes' theorem is used to develop a joint posterior distribution for the parameters in $\boldsymbol{\theta}$ conditional on observed data. By implementing a sampling based approach known as the Gibbs sampler (Gelfand and Smith, 1990) marginal parameter distributions are then derived, allowing competing parameter values to be compared quantitatively to assess which are most plausible for the fitted data set. Bayesian computation via the Gibbs sampler and related Markov Chain Monte Carlo Methods is described in Smith and Roberts (1993).

The mixed-order BOD decay model can be represented in vector form as the nonlinear model

$$y = f(\boldsymbol{\theta}, t) + \epsilon \tag{4.26}$$

where $\epsilon \sim N(0, \sigma^2)$ and y and $f(\boldsymbol{\theta}, t)$ are the left and right-hand sides of Eq 4.25 respectively. Note that the vector of parameters, $\boldsymbol{\theta}$, represents the three parameters of the BOD model, L_0, k_n, and n, *i.e.* :

$$\boldsymbol{\theta} = [L_0, k_n, n] \tag{4.27}$$

Using this model, the likelihood of the data given the model parameters is a normal distribution expressed as

$$p(\boldsymbol{x}|\boldsymbol{\theta}, \sigma) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left[y_i - \left(L_0^{1-n} - k_n(1-n)t_i\right)^{\frac{1}{1-n}}\right]^2\right) \tag{4.28}$$

where the data is given by

$$\boldsymbol{x} = [(t_1, y_1); \ldots (t_i, y_i); \ldots (t_n, y_n)] \tag{4.29}$$

Assuming initial ignorance of model parameter values, Jeffreys' non-informative prior chosen (Jeffreys, 1961). This results in a joint prior distribution of the form:

$$p(\boldsymbol{\theta}, \sigma) = \frac{1}{\sigma} \tag{4.30}$$

According to Bayes' theorem, this prior is then combined with the likelihood function given in Eq 4.28 to yield the posterior density function:

$$p(\boldsymbol{\theta}|\boldsymbol{x}, \sigma) \propto \frac{1}{\sigma^{n+1}} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left[y_i - \left(L_0^{1-n} - k_n(1-n)t_i\right)^{\frac{1}{1-n}}\right]^2\right) \tag{4.31}$$

Determining Marginal Distributions

Now that we have determined the joint posterior distribution for the parameters in $\boldsymbol{\theta}$, how do we obtain the marginal density functions for n, L_0, or

k_n. In general, given a joint density $f(x, y_1, \ldots, y_p)$ one could obtain the marginal distribution for each parameter by integrating the joint posterior over all the other parameters. For example

$$f(x) = \int \ldots \int f(x, y_1, \ldots, y_p)\, dy_1 \ldots dy_p \tag{4.32}$$

However, there are many cases where the integrations shown in Eq 4.44 are extremely difficult to perform, either analytically or numerically. In such cases, the Gibbs sampler provides an alternative method for obtaining $f(x)$: rather than compute or approximate $f(x)$ directly, the Gibbs generates a sample

$$X_1, X_2 \ldots, X_m \sim f(x) \tag{4.33}$$

without requiring $f(x)$.

A Markov-Chain Monte-Carlo decision support system (or MEAL for marginal distributions, environmental analysis, and 'lokahi') is developed based on Gibbs Sampling (Gelfand and Smith, 1990; Smith and Roberts, 1993) to ascertain marginal density functions of interest. By simulating a large enough sample, the mean, variance, or any other characteristic of $f(x)$, even the density itself, can be calculated to the desired degree of accuracy. To understand the Gibbs sampler better, consider the two variable case. Starting with a pair of random variables (X,Y), the Gibbs sampler generates a sample from $f(x)$ by sampling instead from the conditional distributions $f(x|y)$ and $f(y|x)$, distributions that are often known in statistical models. This is done by generating a "Gibbs sequence" of random variables:

$$Y'_0,\ X'_0,\ \ Y'_1,\ X'_1,\ \ Y'_2,\ X'_2,\ \ \ldots,\ \ Y'_K,\ X'_K \tag{4.34}$$

The initial value $Y_0 = y_0$ is specified, and the rest of Eq 4.34 is obtained iteratively by alternately generating values from

$$\begin{aligned} X'_j &\sim f(x \mid Y'_j = y'_j) \\ Y'_{j+1} &\sim f(y \mid X'_j = x'_j) \end{aligned} \tag{4.35}$$

The generation of Eq 4.34 is known as Gibbs sampling. It turns out that under reasonably general conditions, the distribution of X'_k converges (in distribution) to $f(x)$, the true marginal distribution of X, as $k \to \infty$. Thus, for k large enough, the final observation in Eq 4.34 is effectively a sample point from $f(x)$.

To test the accuracy of the Gibbs sampler program written by the author in MATLAB, consider the following joint distribution of X and Y:

$$f(x,y) \propto \binom{n}{x} y^{x+\alpha-1}(1-y)^{n-x+\beta-1}, \quad x = 0,1,\ldots,n \quad \text{and} \quad 0 \le y \le 1 \tag{4.36}$$

Suppose that the marginal distribution $f(x)$ of X is of interest. The Gibbs sampler allows us to generate a sample from $f(x)$ by sampling instead from the conditional distributions:

$$\begin{aligned} &f(x \mid y) \text{ is Binomial } (n,\ y) \\ &f(y \mid x) \text{ is Beta } (x+\alpha,\ n-x+\beta) \end{aligned} \tag{4.37}$$

Applying the iterative scheme of Eq 4.35 to the conditional distributions in Eq 4.37 one can generate a sample $X_1, X_2, \ldots, X_m$ from $f(x)$ and use this sample to estimate any desired characteristic of the marginal distribution $f(x)$. In this example, Gibbs sampling is not needed since $f(x)$ can be analytically obtained from Eq 4.36 as

$$f(x) = \binom{n}{x} \frac{\Gamma(\alpha+\beta)\,\Gamma(n-x+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)\,\Gamma(\alpha+\beta+n)} \qquad x = 0,1,\ldots,n \tag{4.38}$$

the beta-binomial distribution. Hence, characteristics of $f(x)$ can be obtained either

- directly from Eq 4.38 (either analytically or by generating a sample directly from the beta-binomial distribution) or by
- using Gibbs sampling from the conditional distributions in Eq 4.37

Fig 4.4 compares the Gibbs sample obtained from the conditional distributions in Eq 4.37 with $n = 16$, $\alpha = 2$, and $\beta = 4$ (green histogram) and the analytic Beta-Binomial distribution of Eq 4.38 (grey line). Note that the two histograms are very similar, giving credence to the claim that the Gibbs scheme for random variable generation is indeed generating variables from the marginal distribution.

Of course Gibbs sampling is not essential in any bivariate situation where the joint distribution $f(x, y)$ can be calculated, since

$$f(x) = \frac{f(x, y)}{f(y \mid x)} \tag{4.39}$$

On the other hand, Gibbs sampling may be indispensable in situations where there are more than two variables, and when $f(x, y)$, $f(x)$, or $f(y)$ cannot be calculated.

For example, in the BOD decay model we have an entire vector of parameters (random variables), $\boldsymbol{\theta}$. The Gibbs sampling approach is used to determine the marginal distributions for the parameters L_0, k_n, and n in $\boldsymbol{\theta}$. Specifically, the marginal distribution for L_0 (the ultimate BOD, defined as the amount of BOD remaining at $t = 0$) is given in Fig 4.5.

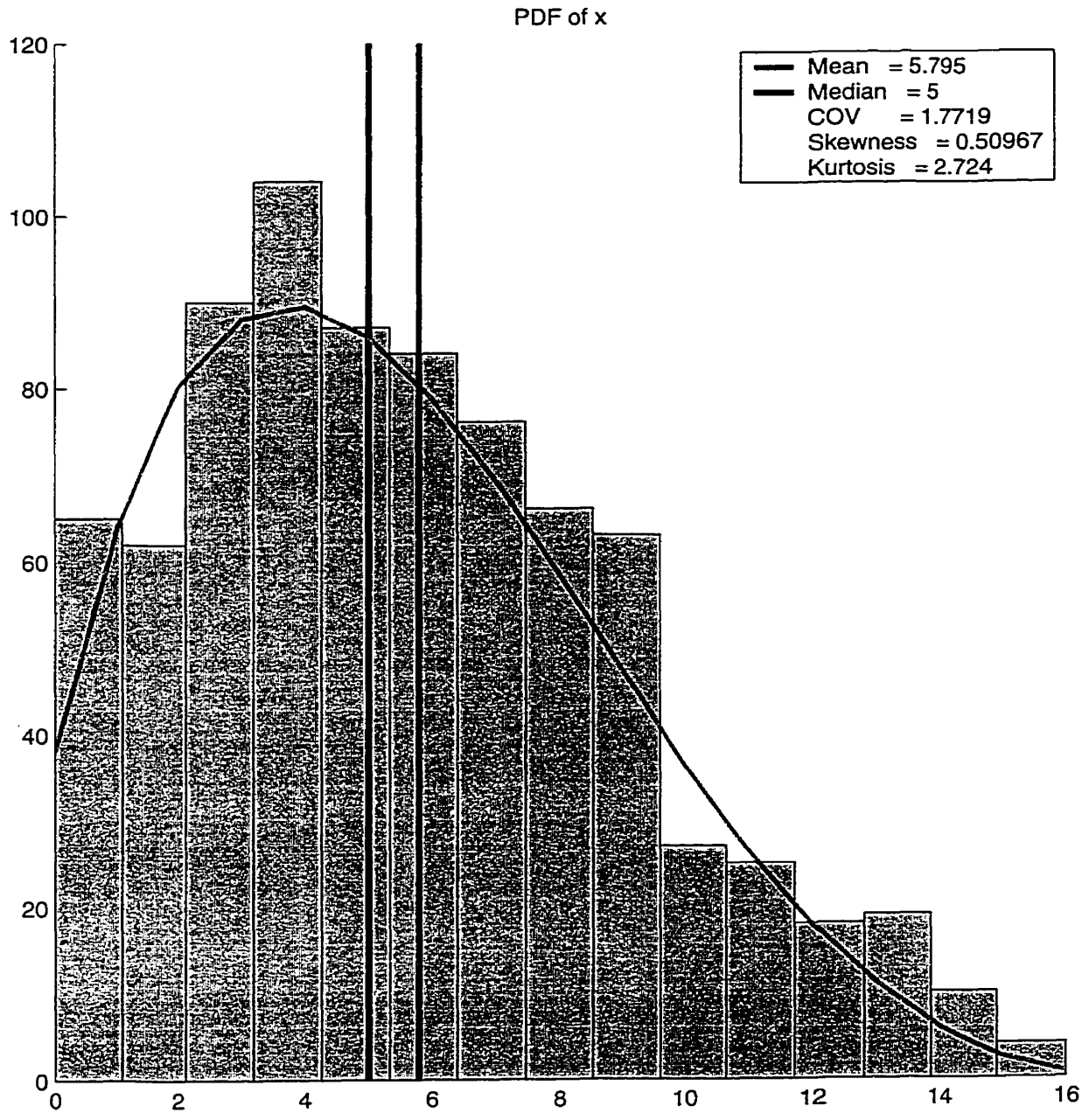


Figure 4.4: Comparison of analytical Beta-Binomial distribution (grey line) with the green histogram sample obtained using Gibbs Sampling with $n = 16$, $\alpha = 2$, and $\beta = 4$

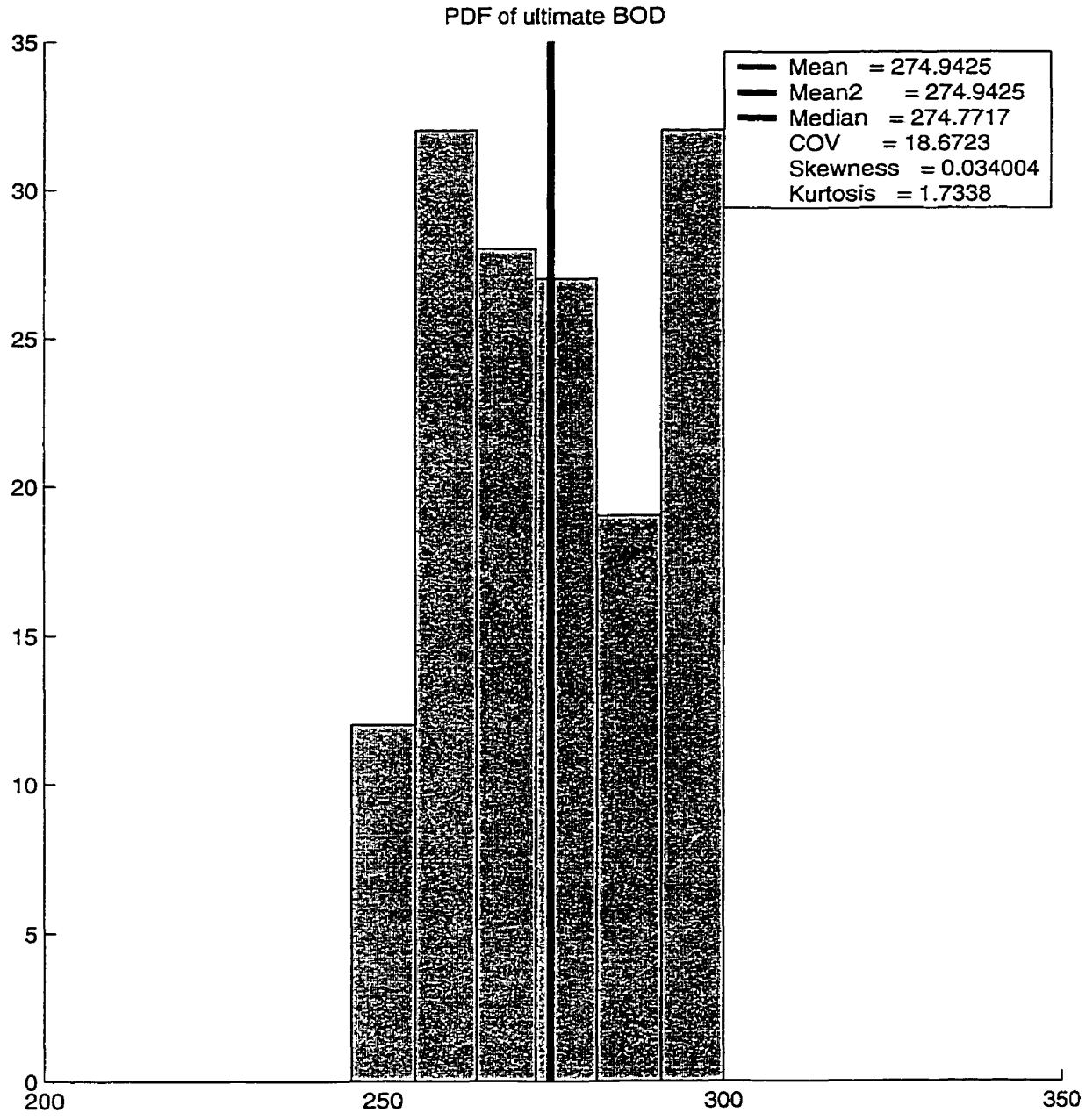


Figure 4.5: Marginal probability density function for L_0 (ultimate BOD)

4.3 Conclusions

Data taken from the Waterloo Pollution Control Plant (Constable and McBean, 1977) is used to compare the first and second order BOD decay models in using the root mean squared error (RMSE) statistic

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[y_p(t_i) - y_m(t_i)]^2} \tag{4.40}$$

where $y_p(t_i)$ is the predicted value of y on day t_i; $y_m(t_i)$ is the measured value of y on day t_i; and N is the number of measurements of $y_m(t_i)$. It is shown that the RMSE value is lower for the second order model than the first order model.

Next, the BOD decay exponent n is allowed to take on any real value (giving rise to a mixed-order BOD decay model), *i.e.* n is not restricted to simply 1 or 2. According to Bayes' theorem, the following joint posterior density function for the parameters in $\boldsymbol{\theta}$ is:

$$p(\boldsymbol{\theta}|\boldsymbol{x},\sigma) \propto \frac{1}{\sigma^{n+1}}\exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left[y_i - \left(L_0^{1-n} - k_n(1-n)t_i\right)^{\frac{1}{1-n}}\right]^2\right) \tag{4.41}$$

where the vector of parameters, $\boldsymbol{\theta}$, represents the three parameters of the BOD model, L_0, k_n, and n, *i.e.* :

$$\boldsymbol{\theta} = [L_0, k_n, n] \tag{4.42}$$

and the data are given by

$$\boldsymbol{x} = [(t_1, y_1); \ldots (t_i, y_i); \ldots (t_n, y_n)] \tag{4.43}$$

A Markov-Chain Monte-Carlo decision support system (MEAL) is developed based on Gibbs Sampling (Gelfand and Smith, 1990; Smith and Roberts, 1993) to ascertain the marginal density functions for n, L_0, or k_n. This technique is valuable because integrating the joint posterior over all

nuisance parameters is often intractable. For example, given a joint density $f(x, y_1, \ldots, y_p)$, to find the marginal distribution for x one would have to integrate over $y_1, \ldots, y_p$:

$$f(x) = \int \ldots \int f(x, y_1, \ldots, y_p)\, dy_1 \ldots dy_p \tag{4.44}$$

The MEAL system provides an alternative method for obtaining $f(x)$: rather than compute $f(x)$ directly, the MEAL system generates a sample

$$X_1, X_2 \ldots, X_m \sim f(x) \tag{4.45}$$

without requiring $f(x)$. By simulating a large enough sample, the mean, variance, or any other characteristic of $f(x)$, even the density itself, can be calculated to the desired degree of accuracy. The Gibbs sampling approach is used to determine the marginal distributions for the parameters L_0, k_n, and n in $\boldsymbol{\theta}$. Some scientists have difficulty accepting Bayesian methods and interpretations in view of their apparent "arbitrariness and subjectivity" (Berger, 1985). This chapter addresses these objections through the systematic use of prior density functions for unknown parameters.

Chapter 5

Dynamic Environmental Modeling under Uncertainty: Stochastic Differential Equations

5.1 Stochastic Processes

There are many examples of stochastic (random) processes in physical situations: ground acceleration due to an earthquake, windload on structures, *etc.* A stochastic process is a function of two variables: the parameter t and the probability parameter ω. A stochastic process is thus a mathematical model of a dynamic process whose dependence on a parameter t is governed by probabilistic laws[1]. It follows that a complete notation of a stochastic process is hence $x(t,\omega)$, $t \in T$ and $\omega \in \Omega$. For a fixed t, $x(t,\omega)$ is a function on the probability space Ω and thus a random variable. On the other hand, for fixed ω, $x(t,\omega)$ defines a function of t and is a realization or a sample function of the stochastic process. For notational convenience, the

[1] In this thesis, t will refer only to time, however, it can also denote a spatial coordinate.

dependence of a stochastic process on ω is often not explicitly shown.

		STATE SPACE	
		Discrete	**Continuous**
TIME	**Discrete**	Markov Chain Models	Time Series Models
	Continuous	Point Processes	Stochastic Differential Equations

Table 5.1: Classifications of Stochastic Models (Cox and Miller, 1965)

A common categorization of stochastic models was proposed by Cox and Miller (1965) and is summarized in Table 5.1. Notice that time can be either discrete or continuous and the state space of the variables describing the system can also be divided into discrete and continuous values. Using this distinction, four kinds of stochastic models are grouped in Table 5.1. Markov chains, for instance, fall under the subdivision of stochastic models which incorporate discrete time and discrete values of the state space in their mathematical structure. This chapter is primarily concerned with continuous time, continuous real-valued (continuous values of the state space) stochastic processes. In particular, stochastic differential equations (SDEs), first addressed by Langevin (1908) in the study of the Brownian motion are investigated. Stochastic models falling in all categories in Table 5.1 have been employed for addressing problems arising in stochastic hydrology and water quality modeling (Hipel, 1994). For example, when deciding upon the design of a multipurpose reservoir, a time series model fitted to the historical river flows can be used for simulating other possible flow sequences.

5.1.1 Wiener Process, $W(t)$

The Scottish botanist Robert Brown noted that individual pollen grains in water moved about irregularly (due to the random bombardment of the pollen grain by molecules of water). This type of erratic motion of tiny objects in a fluid or gas can be described by Brownian motion, commonly referred to as the *Wiener Process* $W(t)$ and formally defined as the limiting position of a random walk process[2]. The Wiener process is one of the most useful stochastic processes in applied probability theory and has been applied for purposes such as analyzing price levels on the stock market and quantum mechanics. In this thesis, only the 'unit' or 'standard' Wiener process, $W(t)$, is dealt with (the variance of the unit Wiener process is simply t, rather than the more general $\sigma^2 t$).

Elementary properties of $W(t)$

The *Wiener process* $W(t)$ can be formally defined as follows:

1. for every $t > 0, W(t)$ is normally distributed with mean 0 and variance t
2. $W(0) = 0$
3. The process W has independent increments, *i.e.* if $r < s \leq t < u$ then $W(u) - W(t)$ and $W(s) - W(r)$ *etc.* are independent stochastic variables.
4. For $s < t$ the stochastic variable $W(t) - W(s)$ has the Gaussian distribution $N\left(0, \sqrt{t-s}\right)$.

[2]The random walk process can be envisaged as the sequence of plays of a game: if you win any play you receive 1 unit; if you lose you lose 1 unit where successive plays are assumed to be independent. Gamow, one of the pioneers of nuclear physics 1947 modeled the progress of a drunk staggering away from a lamp post (with equal-sized steps) as a random walk process.

5. The sample functions of $W(t)$ are continuous but almost surely (with probability one) nondifferentiable functions at every point.

The continuity of $W(t)$ in mean square is easy to prove since

$$E\left[|W(s) - W(t)|^2\right] = 0 \text{ as } s \to t \tag{5.1}$$

It can be also be shown that $W(t)$ is not differentiable in mean square:

$$E\left[\left|\frac{W(t+h) - W(t)}{h}\right|^2\right] = \frac{1}{|h|} \to \infty \text{ as } |h| \to 0 \tag{5.2}$$

Similarly it can be shown that $W(t)$ is also not differentiable with probability 1. Thus a typical Weiner trajectory consists entirely of "corners".

An additional important characteristic of the Wiener Process is that:

$$\begin{aligned} E\left[W(t)\,W(s)\right] &= \min(t,s) \\ &= \begin{cases} t & : \quad t < s \\ s & : \quad t > s \\ t = s & : \quad t = s \end{cases} \end{aligned} \tag{5.3}$$

This property implies that

1. $E\left[W^2(t)\right] = t$
2. $E\left[|W(t) - W(s)|^2\right] = t - s, \quad \text{for } t \geq s$
3. $E\left[|dW(t)|^2\right] = dt \quad \text{where } dW(t) = W(t+dt) - W(t)$
4. $W(t)$ has the The Lévy oscillation property (described below)

Let us fix two points in time, s and t with $s < t$ and use the convenient notation

$$\begin{aligned} \Delta t &= t - s \\ \Delta W(t) &= W(t) - W(s) \end{aligned}$$

5.2 River Water Quality Modeling

A water quality model typically describes the chemical, physical, and biological processes that occur in a water body, such as the reaction of chemical constituents and the uptake of nutrients by living organisms. In this section, four important contributions are made to the field of stochastic water quality modeling. First, the classic deterministic Streeter-Phelps equations are modelled in a stochastic context. Second, the use of white noise processes in water quality models is questioned. It is proposed that colored noise (the Ornstein-Uhlenbeck process) replace the standard white noise assumption. Approximation schemes are put forth so that practitioners can use colored noise as a viable replacement for white noise. Third, generalized CBOD and NOD decay models are used (so that decay parameters can be estimated from data). Fourth, a decision support system, SEAL (Stochastic Environmental Analysis for "Lokahi") is designed to help environmental managers improve water quality modeling in a multiple-criteria, stochastic context. In the context of decision analysis, Yakowitz and Hipel (1997) refer to the importance of "Lokahi" in their interesting paper: "Multiple objective decision making for *Lokahi* in environmental management".

Since the survival of aquatic organisms depends principally upon the amount of available oxygen, dissolved oxygen (DO) is an important concern in water quality management. Other important factors include water temperature since biochemical processes and organism growth rates are regulated to a large extent by temperature (Culberson and Piedrahita, 1996). An increase of atmospheric carbon dioxide and/or other greenhouse gases is projected to cause climate warming which could significantly alter DO characteristics in water bodies. These changes are in turn expected to have a profound effect on indigenous fish populations (Fang et al., 1999).

The earliest models of water quality involved two linear deterministic differential equations of biochemical oxygen demand (BOD) and dissolved oxygen (DO) based on the pioneering work of Phelps (1909) and Streeter and Phelps (1925). While working for the U.S. Public Health Service, these

researchers and their colleagues investigated the water quality of American rivers. The seminal Streeter—Phelps equations form the foundation for many of today's sophisticated water quality models which are able to capture phenomena such as the phosphorus cycle, carnivores, phytoplankton, and contaminants in a "nonlinear compartmental approach" (Thompson, 1982).

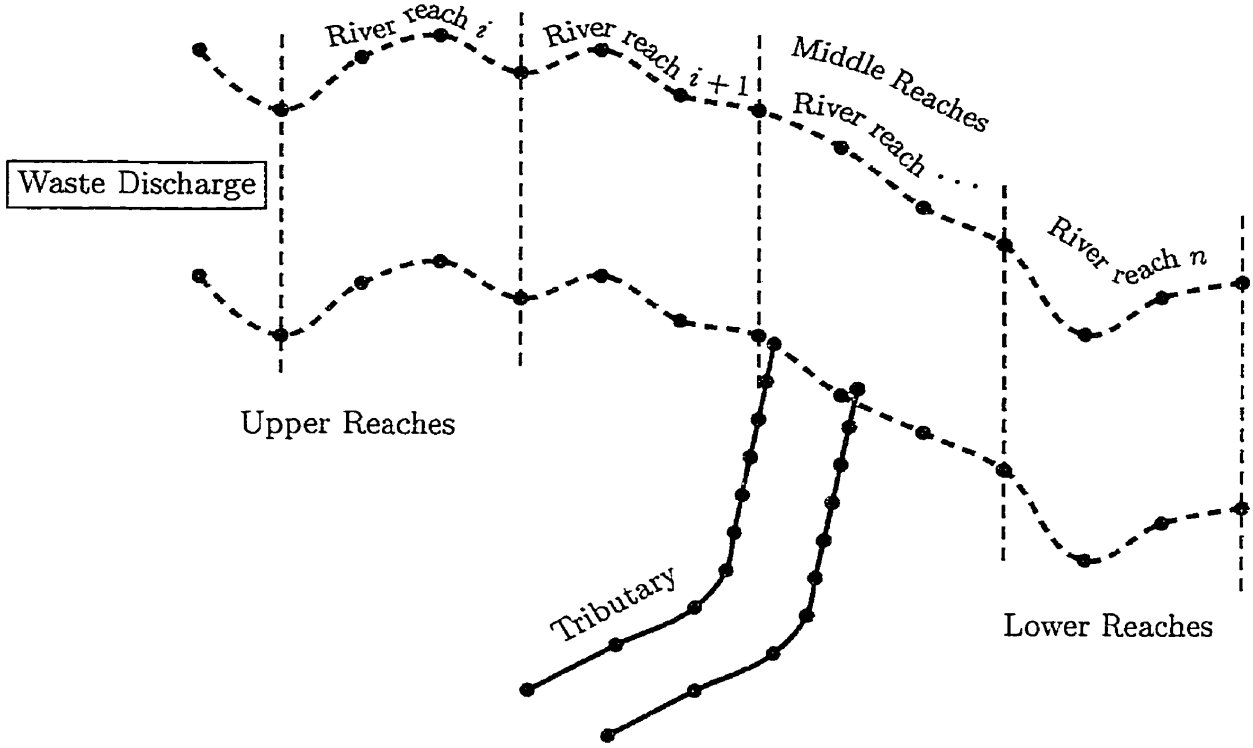


Figure 5.1: The reaches of a stream

Consider a river with multiple reaches and a treatment plant discharging at the head of the reach, as illustrated in Fig 5.1. An environmental agency may be interested in monitoring the discharge of contaminants into the river.

Mass or energy balance equations are often used to describe the dynamics of constituent concentrations of natural water bodies. The health of aquatic systems (algae, fish, micro-organisms, *etc.*), aesthetics (such as odor and color), potability, taste, and so on depend upon the resulting concentrations of dissolved oxygen. DO levels naturally cycle over the course of a day (and throughout the year). In the steady state conditions resulting from the natural balance of various chemical and biological processes, the DO concentration fluctuates about a saturation concentration (O_S). Whenever untreated waste waters are discharged into the stream, the concentration of DO may be adversely affected.

In this chapter, the steady-state stochastic DO models of Bowles et al. (1977), Finney et al. (1982), Dewey (1984), Zielinsky (1989), and Curi et al. (1995) are considered. These models address three water quality constituents: DO, carbonaceous biochemical oxygen demanding substances (CBOD) and nitrogenous oxygen demanding substances (NOD). Coupled CBOD-NOD-DO reactions are an important component of water quality modeling and data for the concentration of these constituents are more readily available than for other substances. It is known that CBOD is increased by nonpoint load sources of carbon (S_C) and decreased by oxidation (k_1), sedimentation, and adsorption ($\mathcal{L}$). NOD is also increased by nonpoint load sources (S_N) and decreased by oxidation (k_2). Finally, DO is supplied by reaeration (k_3) and photosynthesis (P) and decreased by respiration (R), CBOD (k_1), and NOD (k_2). The following three deterministic differential equations have been used for describing the water quality of a river (Zielinsky, 1989; Curi et al., 1995):

$$\begin{aligned}
\frac{dC}{dt} &= -(k_1 + \mathcal{L})\, C \;+\; S_C \\
\frac{dN}{dt} &= -k_2 N \;+\; S_N \\
\frac{dO}{dt} &= k_3(O_s - O) \;-\; k_1 C \;-k_2 N \;+\; P \;-\; R
\end{aligned} \tag{5.4}$$

where the photosynthetic term, P, in Eq 5.4 is represented by

$$P_m \sin[\nu(t+\theta)] \tag{5.5}$$

These equations describe how a spike input of CBOD, NOD (or other organic material) generates the classic transient DO "sag curve" shown in Fig 1.6. In Eq 5.5 P_m is the maximum rate of photosynthetic DO production in mg/L/day. A detailed modeling of photosynthesis is beyond the scope of this thesis[3]. However, the photosynthetic term used in Eq 5.5 is general enough to encompass the comprehensive photosynthesis models of O'Connor and Toro (1978) and Curi (1992).

The units of the variables in Eq 5.4 are well-known and reproduced here. First the three state variables are defined: C is the carbonaceous biochemical oxygen demand (CBOD) in mg/L; N is the nitrogenous oxygen demand (NOD) in mg/L; and O is the dissolved oxygen concentration (DO) in mg/L. Next, the four decay constants are defined: k_1 is the CBOD decay rate per day; $\mathcal{L}$ is the sedimentary and adsorption loss rate for CBOD per day; k_2 is the decay rate of NOD per day; and k_3 is the reaeration rate per day. In addition, O_S is the saturation concentration of oxygen in mg/L while R is the loss rate of DO due to respiration in mg/L/day. Finally, S_C and S_N are the nonpoint source loads of carbon and nitrogen respectively in mg/L/day.

Replacing the state variables C, N, and O with x_1, x_2, and x_3 respectively, equation 5.4 can be re-written in matrix form:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{A}\,\boldsymbol{x} + \boldsymbol{b} \tag{5.6}$$

where the 3×1 column vector $\boldsymbol{x}$ is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{5.7}$$

[3] ν is often taken to be a function of the fraction of the day with sunshine, while θ is a value chosen to ensure that the photosynthesis cycle at any point of its periodic function coincides with the origin of the river (Curi et al., 1995).

The 3×3 matrix $\boldsymbol{A}$ is given by

$$\begin{bmatrix} -(k_1 + \mathcal{L}) & 0 & 0 \\ 0 & -k_2 & 0 \\ -k_1 & -k_2 & -k_3 \end{bmatrix} \tag{5.8}$$

while the 3×1 column vector $\boldsymbol{b}$ is

$$\begin{bmatrix} S_C \\ S_N \\ P_m \sin[\nu(t+\theta)] - R + k_3 O_S \end{bmatrix} \tag{5.9}$$

Accordingly, Eq. 5.6 becomes

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} -(k_1 + \mathcal{L}) & 0 & 0 \\ 0 & -k_2 & 0 \\ -k_1 & -k_2 & -k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ &+ \begin{bmatrix} S_C \\ S_N \\ P_m \sin[\nu(t+\theta)] - R + k_3 O_S \end{bmatrix} \end{aligned} \tag{5.10}$$

5.2.1 Random Processes and Water Quality Modeling

There is a great need for modeling and understanding the "imposing number of uncertainties" (Tung and Hathhorn, 1988) associated with biological and physical processes occurring within the stream or river environment. More generally, uncertainty pervades all aspects of the hydrological cycle which leads to significant complexities in the modelling and prediction of water quality (Loucks and Lynn, 1966; Padgett and Rao, 1979; Chadderton et al., 1982). For example, random water quality processes are due in part to the variability and randomness of atmospheric conditions (Hobbie and Tiwari, 1978). Curi et al. (1995) argue that the parameters and coefficients in Eq 5.4, namely k_1, $\mathcal{L}$, k_2, k_3, S_C, S_N, R, and P_m "vary significantly" at different locations of the stream.

In addition, obtaining reliable estimates for the reaeration coefficient (k_3) and other parameters continues to be a significant challenge since satisfactory empirical or theoretical expressions have yet to be devised. For example, Wilcock (1988) and references therein describe how seven different predictive formulas for the reaeration coefficient vary with discharge rate for a given river. While the formulas provide reasonable agreement for low flows, they diverge by more than an order of magnitude at discharge rates higher than $3\,m^3/s$.

Other factors which contribute to system noise (and hence uncertainty) include measurement errors, unreliable estimates for initial conditions, non-point source loading, uncertainty in respiratory and photosynthetic activities, and random fluctuations in parameters that define decay and reaeration. To account for this random behavior the quantities k_1, $\mathcal{L}$, k_2, k_3, S_C, S_N, R, and P_m are considered as random variables given as a superposition of their mean values and a Gaussian white noise process $\xi_t(\cdot)$ where $(\cdot)$ is the parameter or input coefficient to be specified:

$$\begin{aligned} k_1 &= \overline{k}_1 + \sigma_1 \xi_1(t) & P_m &= \overline{P}_m + \sigma_4 \xi_4(t) \\ \mathcal{L} &= \overline{\mathcal{L}} + \sigma_L \xi_L(t) & R &= \overline{R} + \sigma_5 \xi_5(t) \\ k_2 &= \overline{k}_2 + \sigma_2 \xi_2(t) & S_C &= \overline{S}_C + \sigma_6 \xi_6(t) \\ k_3 &= \overline{k}_3 + \sigma_3 \xi_3(t) & S_N &= \overline{S}_N + \sigma_7 \xi_7(t) \end{aligned} \tag{5.11}$$

A physical continuous process, such as BOD and DO must be interpreted in the Stratonovich SDE sense. This was formally proven by Wong and Zakai (1965). For related discussions see Stratonovich (1967a), Stratonovich (1966), Gray and Caughey (1965), and Mortensen (1968). Unfortunately, many environmental researchers have failed to include this fact in their analyses. For example, Curi et al. (1995) mentioned neither Wong and Zakai (1965) nor Stratonovich calculus in their stochastic modeling of the the Thames river, Ontario, Canada. Several researchers, such as Ponnambalam et al. (1997) correctly describe the stochastic calculus necessary to model en-

vironmental processes. Ponnambalam et al. (1997) note that the reason they treated the BOD decay stochastic differential equation in the Stratonovich sense is that if they had taken it in the Ito sense, "with higher and higher variance value for the [reaction rate coefficient], the probability of exceedance calculated becomes close to zero which would be hard to understand for practioners although is correct theoretically".

To correctly model this water quality situation, the set of Stratonovich stochastic differential equations corresponding to Eq 5.10 must be derived:

$$\frac{dx_1^*}{dt} = -(\overline{k}_1 + \overline{\mathcal{L}})\, x_1 + \overline{S}_C - [\sigma_1 \xi_1(t) + \sigma_L \xi_L(t)]\, x_1 + \sigma_6 \xi_6(t)$$

$$\frac{dx_2^*}{dt} = -\overline{k}_2 x_2 + \overline{S}_N - \sigma_2 x_2 \xi_2(t) + \sigma_7 \xi_7(t)$$

$$\frac{dx_3^*}{dt} = \overline{k}_3(O_S - x_3) - \overline{k}_1 x_1 - \overline{k}_2 x_2 + \overline{P}_m \sin[\nu(t+\theta)] - \overline{R} - \sigma_1 x_1 \xi_1(t)$$
$$- \sigma_2 x_2 \xi_2(t) + \sigma_3 (O_S - x_3)\, \xi_3(t) + \sigma_4 \overline{P}_m \sin[\nu(t+\theta)]\, \xi_4(t) - \sigma_5 \xi_5(t) \tag{5.12}$$

These Stratonovich SDEs can be re-written in differential form:

$$dx_1^* = \left[-(\overline{k}_1 + \overline{\mathcal{L}})\, x_1 + \overline{S}_C\right] dt - \sigma_1 x_1 \circ dW_1(t) - \sigma_L x_1 \circ dW_L(t) + \sigma_6 \circ dW_6(t)$$

$$dx_2^* = \left[-\overline{k}_2\, x_2 + \overline{S}_N\right] dt - \sigma_2 x_2 \circ dW_2(t) + \sigma_7 \circ dW_7(t)$$

$$dx_3^* = \left[\overline{k}_3(O_S - x_3) - \overline{k}_1 x_1 - \overline{k}_2 x_2 + \overline{P}_m \sin[\nu(t+\theta)] - \overline{R}\right] dt$$
$$- \sigma_1 x_1 \circ dW_1(t) - \sigma_2 x_2 \circ dW_2(t) + \sigma_3 (O_S - x_3) \circ dW_3(t)$$
$$+ \sigma_4 \overline{P}_m \sin[\nu(t+\theta)] \circ dW_4(t) - \sigma_5 \circ dW_5(t)$$

The solution to the coupled CBOD-NOD-DO equations is properly obtained by transforming the above Stratonovich SDE (Eq 5.12) to its equivalent Îto form through the Stratonovich–Îto conversion relations (using Eq

??):

$$\frac{dx_1}{dt} = -\alpha_1 x_1 + \overline{S}_C - [\sigma_1 \xi_1(t) + \sigma_L \xi_L(t)]\, x_1 + \sigma_6 \xi_6(t)$$

$$\frac{dx_2}{dt} = -\alpha_2 x_2 + \overline{S}_N - \sigma_2 x_2 \xi_2(t) + \sigma_7 \xi_7(t)$$

$$\frac{dx_3}{dt} = \alpha_3 x_3 - \beta_1 x_1 - \beta_2 x_2 + \overline{P}_m \sin[\nu(t+\theta)] - \gamma - \sigma_1 x_1 \xi_1(t)$$

$$- \sigma_2 x_2 \xi_2(t) + \sigma_3 (O_S - x_3)\, \xi_3(t) + \sigma_4 \overline{P}_m \sin[\nu(t+\theta)]\, \xi_4(t) - \sigma_5 \xi_5(t) \tag{5.13}$$

The Îto SDEs in differential form are hence as follows:

$$dx_1 = \left[-\alpha_1 x_1 + \overline{S}_C\right] dt - \sigma_1 x_1 dW_1(t) - \sigma_L x_1 dW_L(t) + \sigma_6 dW_6(t)$$

$$dx_2 = \left[-\alpha_2 x_2 + \overline{S}_N\right] dt - \sigma_2 x_2 dW_2(t) + \sigma_7 dW_7(t)$$

$$dx_3 = \left[-\alpha_3 x_3 - \beta_1 x_1 - \beta_2 x_2 + \overline{P}_m \sin[\nu(t+\theta)] - \gamma\right] dt$$

$$- \sigma_1 x_1 dW_1(t) - \sigma_2 x_2 dW_2(t) + \sigma_3 (O_S - x_3)\, dW_3(t)$$

$$+ \sigma_4 \overline{P}_m \sin[\nu(t+\theta)]\, dW_4(t) - \sigma_5 dW_5(t) \tag{5.14}$$

where

$$\alpha_1 = \overline{k}_1 + \overline{\mathcal{L}} - \frac{\sigma_1^2 + \sigma_2^2}{2} \qquad \beta_1 = \overline{k}_1 - \frac{\sigma_1^2}{2}$$

$$\alpha_2 = \overline{k}_2 - \frac{\sigma_2^2}{2} \qquad \beta_2 = \overline{k}_2 - \frac{\sigma_2^2}{2} \tag{5.15}$$

$$\alpha_3 = \overline{k}_3 - \frac{\sigma_3^2}{2} \qquad \gamma = R - \overline{k}_3 O_S$$

5.2.2 Formulation of Moment Equations

Consider the steps required to calculate the first moment (expected value) for x_1 (CBOD) as a function of time, *i.e.* $< x_1 >$. First take expected values

of both sides of the first equation in Eq 5.14.

$$\begin{aligned} d < x_1 > = \left[-\alpha_1 < x_1 > + \overline{S}_C\right] dt - \sigma_1 < x_1 dW_1(t) > \\ - \sigma_L < x_1 dW_L(t) > + \sigma_6 < dW_6(t) > \end{aligned} \tag{5.16}$$

Since this is an Îto SDE it follows that

$$d < x_1 > = \left[-\alpha_1 < x_1 > + \overline{S}_C\right] dt \tag{5.17}$$

Therefore we have a linear deterministic differential equation of first order:

$$\frac{d < x_1 >}{dt} + \alpha_1 < x_1 > = \overline{S}_C \tag{5.18}$$

Using the obvious integrating factor $e^{\alpha_1 t}$ it follows that

$$< x_1 > = A_1 \exp(-\alpha_1 t) + \frac{\overline{S}_C}{\alpha_1} \tag{5.19}$$

where

$$A_1 = x_1(0) - \frac{\overline{S}_C}{\alpha_1} \tag{5.20}$$

Similarly, the first moment (expected value) for x_2 (NOD) as a function of time, *i.e.* $< x_2 >$ is found to be

$$< x_2 > = A_2 \exp(-\alpha_2 t) + \frac{\overline{S}_N}{\alpha_2} \tag{5.21}$$

where

$$A_2 = x_2(0) - \frac{\overline{S}_N}{\alpha_2} \tag{5.22}$$

Finally, the first moment (expected value) for x_3 (DO) as a function of time, *i.e.* $< x_3(t) >$ is the solution of the following differential equation

$$\begin{aligned} \frac{d < x_3 >}{dt} + \alpha_3 < x_3 > = - \beta_1 < x_1 > - \beta_2 < x_2 > \\ + \overline{P}_m \sin\left[\nu(t+\theta)\right] - \gamma \end{aligned} \tag{5.23}$$

The solution can be found by trying the particular solution:

$$< x_3 >_p = G_1 \exp(-\alpha_1 t) + G_2 \exp(-\alpha_2 t) + G_3 \sin \nu t + G_4 \cos \nu t + G_5$$

Using elementary techniques from differential equations the general solution can be found as:

$$\begin{aligned} < x_3 > = A_3 \exp(-\alpha_3 t) + G_1 \exp(-\alpha_1 t) + G_2 \exp(-\alpha_2 t) \\ + G_3 \sin \nu t + G_4 \cos \nu t + G_5 \end{aligned} \tag{5.24}$$

where

$$\begin{aligned} G_1 &= -\left[\frac{A_1 \beta_1}{\alpha_3 - \alpha_1}\right] \\ G_2 &= -\left[\frac{A_2 \beta_2}{\alpha_3 - \alpha_2}\right] \\ G_3 &= \frac{\overline{P}_m (\alpha_3 \cos \nu\theta + \nu \sin \nu\theta)}{\alpha_3^2 + \nu^2} \\ G_4 &= \frac{\overline{P}_m (\alpha_3 \sin \nu\theta + \nu \cos \nu\theta)}{\alpha_3^2 + \nu^2} \\ G_5 &= -\left[\frac{\beta_1 \overline{S}_C}{\alpha_1 \alpha_3} + \frac{\beta_2 \overline{S}_N}{\alpha_2 \alpha_3} + \frac{\gamma}{\alpha_3}\right] \\ A_3 &= x_3(0) - [G_1 - G_2 - G_4 - G_5] \end{aligned} \tag{5.25}$$

5.2.3 Generalized CBOD and NOD Decay Models

Recall that in Eq 5.4 the oxidation of CBOD and NOD is modeled as a 'first-order' reaction process in which the oxidation rate is proportional to the amount of CBOD and NOD present. The term 'first-order' arises from an implicit exponent of '1' on the x_1 and x_2 variables in these equations. However, since the environmental processes being considered will not necessarily decay in a first order manner, the coupled CBOD-NOD-DO equations

of Eq 5.4 should be modified to allow for more flexibility in the oxidation of CBOD (x_1) and NOD (x_2) as follows:

$$\frac{dx_1}{dt} = -(k_1 + \mathcal{L})x_1^{\lambda} + S_C$$

$$\frac{dx_2}{dt} = -k_2 x_2^{\mu} + S_N$$

$$\frac{dx_3}{dt} = k_3(O_S - x_3) - k_1 x_1^{\lambda} - k_2 x_2^{\mu} + P_m \sin[\nu(t+\theta)] - R$$

where the parameters λ and μ are now not necessarily 1. The above system of equations can be conveniently written in matrix form:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -(k_1+\mathcal{L}) & 0 & 0 \\ 0 & -k_2 & 0 \\ -k_1 & -k_2 & -k_3 \end{bmatrix} \begin{bmatrix} x_1^{\lambda} \\ x_2^{\mu} \\ x_3 \end{bmatrix} + \begin{bmatrix} S_C \\ S_N \\ P_m \sin[\nu(t+\theta)] - R + k_3 O_S \end{bmatrix} \tag{5.26}$$

The corresponding Îto SDEs in differential form are:

$$dx_1 = \left[-\left(\overline{k}_1 + \overline{\mathcal{L}}\right) x_1^{\lambda} + \overline{S}_C + \frac{\sigma_1^2 + \sigma_L^2}{2} x_1^{2\lambda-1}\right] dt - \sigma_1 x_1^{\lambda} dW_1(t) - \sigma_L x_1^{\lambda} dW_L(t) + \sigma_6 dW_6(t)$$

$$dx_2 = \left[-\overline{k}_2 x_2^{\mu} + \overline{S}_N + \frac{\sigma_2^2}{2} \mu x_2^{2\mu-1}\right] dt - \sigma_2 x_2^{\mu} dW_2(t) + \sigma_7 dW_7(t)$$

$$dx_3 = \left[\overline{k}_3 (O_S - x_3) - \overline{k}_1 x_1^{\lambda} - \overline{k}_2 x_2^{\mu} + \frac{\sigma_1^2}{2}\lambda x_1^{2\lambda-1} + \frac{\sigma_2^2}{2}\nu x_2^{2\nu-1} + \frac{\sigma_3^2 x_3}{2} + \overline{P}_m \sin\left[\nu(t+\theta)\right] - \overline{R}\right] dt - \sigma_1 x_1^{\lambda} dW_1(t) - \sigma_2 x_2^{\mu} dW_2(t) + \sigma_3 (O_S - x_3) dW_3(t) + \sigma_4 \overline{P}_m \sin\left[\nu(t+\theta)\right] dW_4(t) - \sigma_5 dW_5(t) \quad (5.27)$$

5.2.4 SEAL System and Water Quality Modeling

Sample paths of DO vs time are calculated in MATLAB using the SEAL (Stochastic Environmental Analysis for Lokahi) model. Details of how the SEAL system integrates the Îto SDEs and other code related to the SEAL model is provided in Appendix D. Typical output generated by the SEAL system for water quality management is shown in Fig 5.2 which illustrates four sample paths of DO vs time.

Given any specific location along a river, the SEAL system produces the probability density function (pdf) of DO that corresponds to this location. Fig 5.3 illustrates the DO density functions corresponding to locations x_1 and x_2. There are two main steps in employing the SEAL model for the analysis of stochastic water resources problems. First, hundreds, or even thousands of sample DO paths are simulated. Next, the SEAL model displays a probabilistic description of the DO sag curve at any point in space (or time).

This probabilistic analysis can be used to make inferences about both central tendencies and extreme events. For example, at location x_1 the mean of the DO density function is well above the critical value of DO (DO_{cr}) necessary to ensure the survival of aquatic organisms (see Fig 5.3). Of course the value of DO_{cr} is subjective and context dependent. Finally, Fig 5.3 illustrates that a small fraction of the total DO samples will likely have a DO level less than DO_{cr} (by considering the tails of the DO distributions in Fig 5.3).

Moreover, the minimum DO concentration (DO_{min}) is of great interest in calculating maximum assimilable organic loads. As the concentration of DO falls below DO_{cr}, fish and other organisms begin to die, and floating sludges predominate (due to increased activity of bacterial communities). If DO_{min} is anticipated to be significantly below DO_{cr} at a critical location, an environmental agency may be forced to take drastic actions to reduce the level of organic wastes that are released into the water body.

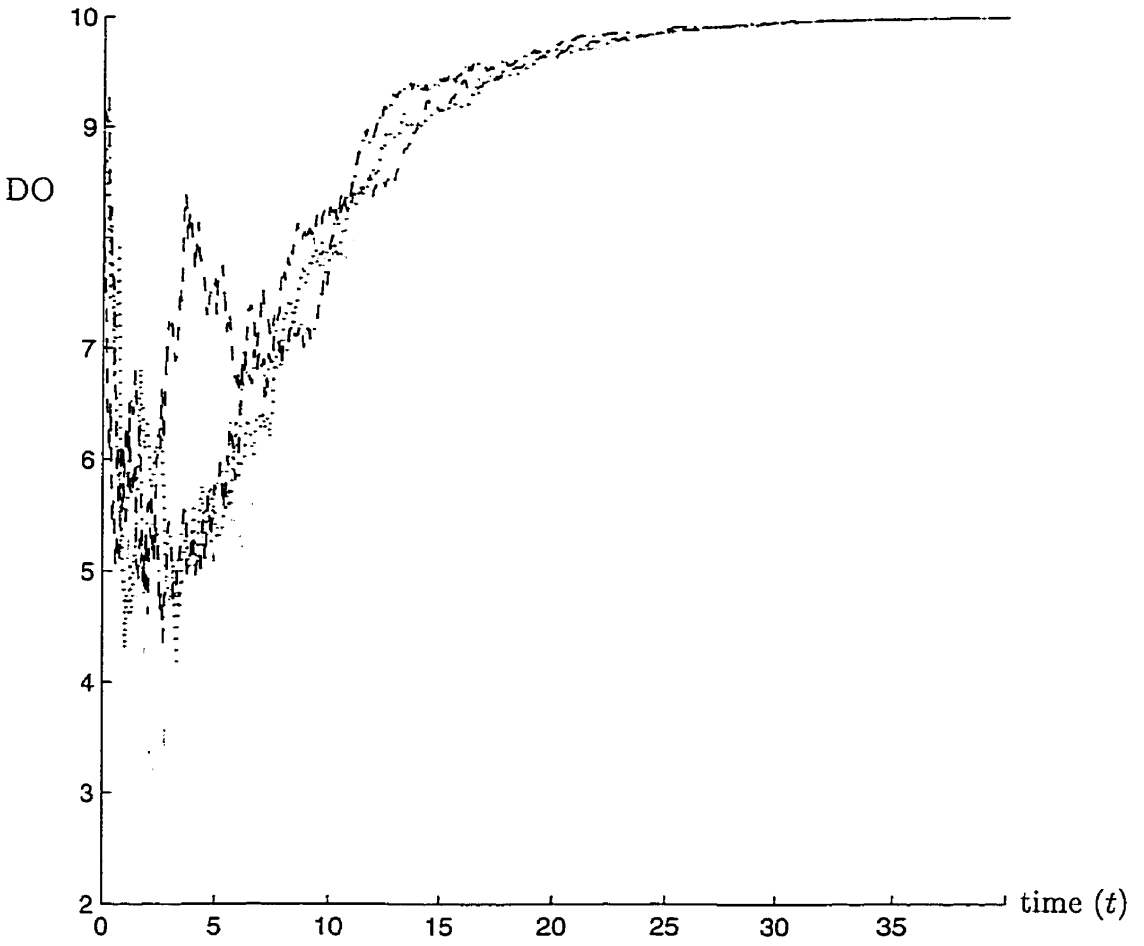


Figure 5.2: Four sample paths of Dissolved Oxygen (DO) vs time (t)

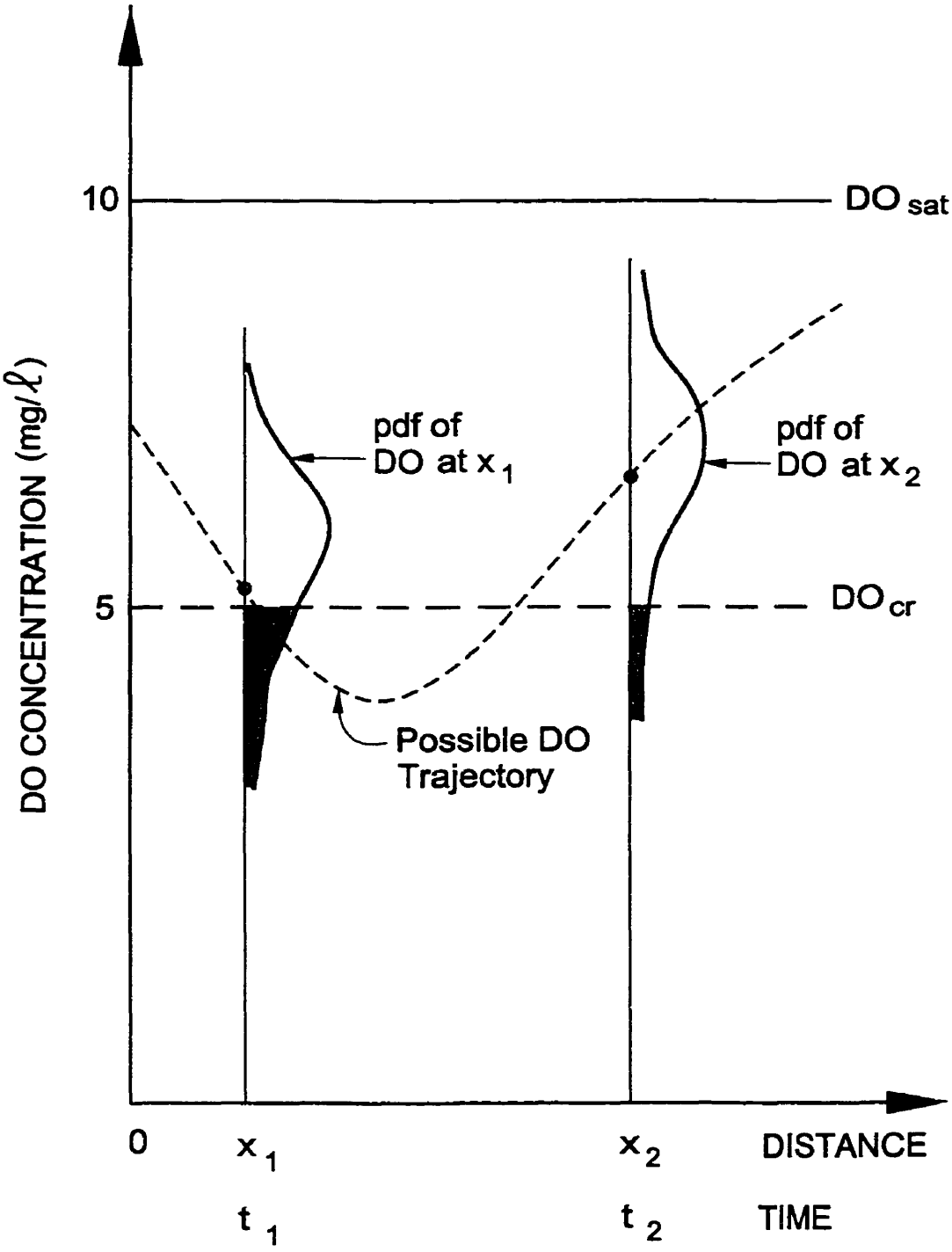


Figure 5.3: Probabilistic nature of DO sag curves

5.3 Colored Noise in Environmental Models

5.3.1 Limitations of White Noise Assumptions

The term "noise" was first used in communications engineering to describe the undesired acoustic affects accompanying spontaneous electric fluctuations in speakers. A "white noise" function $\xi(t)$ is theoretically conceived as a Gaussian stochastic process with mean zero in which the 'time scale of correlations is zero'. That is, white noise has a Dirac delta autocorrelation function $R_\xi(t_1, t_2)$ and a power spectral density $S_\xi(\omega)$ with constant value over the entire frequency (ω) spectrum. According to the Central Limit Theorem, the normality assumption is justifiable if the noise is composed of many small independent (or weakly dependent) random effects. The term *white* is borrowed from optics, where *white light* has been used to signify uniform energy distribution among the colors[4]. A white noise stochastic process $\xi(t)$ is formally assumed to satisfy the following properties, at least approximately:

- $\xi(t)$ is stationary
- $< \xi(t) > = 0$ for all t
- $S_\xi(\omega)$ is constant $\qquad -\infty < \omega < \infty$
- $\langle \xi(t_1)\,\xi(t_2) \rangle = \delta(t_1 - t_2)$

where $\delta(\cdot)$ denotes the Dirac delta function and $< \cdot >$ represents the average over the ensemble of the stochastic process.

Nonetheless, the white noise process $\xi(t)$ is a useful mathematical idealization for describing random influences that fluctuate rapidly and hence are virtually uncorrelated for different instants of time. For example, the force exerted on a particle immersed in a fluid may be usefully idealized as "white noise" since such a particle may undergo more than 10^{21} molecular

[4]Actually, the analogy is not correct since in optics the uniform energy distribution of white light is based on wavelength rather than frequency.

collisions per second from all directions. Other typical applications for white noise include modeling the thermal noise in electrical systems and the arrival of atomic particles at a Geiger counter. While the white noise assumption may be appropriate in these circumstances, replacing a real, physical, wide-band stationary process $\eta(t)$ (*i.e.* colored noise) by a delta-correlated process (white noise) means that the cutoff frequency of the actual process (ω_c) is not explicitly taken into account.

It is only permissible to approximate colored noise $\eta(t)$ with white noise $\xi(t)$ if the cutoff frequency (ω_c) is considerably larger than all other frequencies which are important for the system under consideration. Specifically, if the correlation time

$$\tau_{cor} \approx \frac{1}{\omega_c} = \frac{\int_{\infty}^{0} \tau R(\tau)\, d\tau}{\int_{\infty}^{0} R(\tau)\, d\tau} \tag{5.28}$$

is small relative to the other relative time constants of the system, we can realistically replace $\eta(t)$ with $\xi(t)$.

5.3.2 BOD under colored noise

Let $C(t)$ represent the concentration of BOD remaining at time t in mg/L. Elevated BOD levels frequently arise when an industrial plant is releasing organic wastes into a water body. Accurate modeling of the rate at which the BOD is exerted in the receiving water is important for determining downstream oxygen deficit. Conventionally, BOD exertion is modeled as a first-order decay process in which oxygen consumption is proportional to the concentration of BOD remaining at time t, $C(t)$:

$$\frac{dC}{dt} = -k(t)\, C(t) \qquad \text{with} \qquad C(0) = C_0 \tag{5.29}$$

Assuming that fluctuations in the environment (such as temperature and precipitation) manifest themselves as fluctuations in the BOD decay rate

constant $k(t)$, the stochastic BOD model will be:

$$\frac{dC}{dt} = -\left[\,\overline{k} + \sigma\,\eta(t)\,\right] C(t) \tag{5.30}$$

where $\overline{k}$ is the expected value of k, σ is the intensity of the fluctuations, and $\eta(t)$ is a colored noise (Ornstein–Uhlenbeck) process which is a more assumption than white noise.

Eq 5.29 can be solved by re-arranging the equation and integrating from 0 to t:

$$\int_0^t \frac{dC(s)}{C(s)} = -\int_0^t \left[\,\overline{k} + \sigma\,\eta(s)\,\right] ds \tag{5.31}$$

It follows that

$$\ln\left[\frac{C(t)}{C_0}\right] = -\overline{k}\,t - \sigma\int_0^t \eta(s)\,ds \tag{5.32}$$

It is now straightforward to solve for $C(t)$:

$$C(t) = C_0 \exp\left[-\overline{k}\,t - \sigma\int_0^t \eta(s)\,ds\right] \tag{5.33}$$

The colored noise process $\eta(t)$ has zero mean:

$$< \eta(t) > = 0 \tag{5.34}$$

with an autocorrelation function given by:

$$R_\eta(t_1, t_2) = \langle\,\eta(t_1)\,\eta(t_2)\,\rangle = \frac{\sigma^2}{2\alpha}\exp\left(-\alpha\,|\,s_1 - s_2\,|\,\right) \tag{5.35}$$

where $\langle\,\cdot\cdot\rangle$ represents the average over the ensemble of the stochastic process. Note that the autocorrelation function R_η is dependent only upon the time displacement, τ:

$$R_\eta(t_1, t_1 + \tau) = R_\eta(\tau) = \frac{\sigma^2}{2\alpha}\exp\left(-\alpha\,|\tau|\right) \tag{5.36}$$

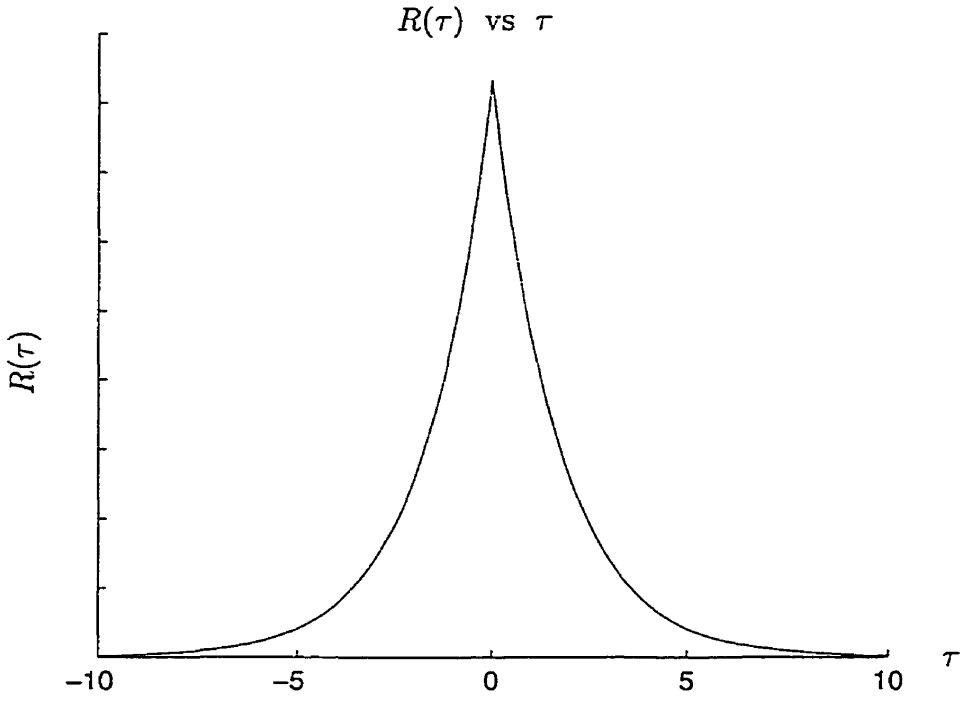


Figure 5.4: Autocorrelation function $R_\eta(\tau) = \frac{\sigma^2}{2\alpha} \exp(-\alpha|\tau|)$ for colored noise process $\eta(t)$ where $\tau = t_1 - t_2$

A plot of $R(\tau)$ for the colored noise process $\eta(t)$ is given in Fig 5.4. Note how this autocorrelation function differs from $R(\tau)$ of white noise shown in Fig ??. The process $\eta(t)$ is the solution of the stochastic differential equation (Uhlenbeck and Ornstein, 1954)

$$\frac{d\eta}{dt} = -\alpha\,\eta(t) + \sigma_0\,\xi(t) \qquad \alpha > 0 \tag{5.37}$$

where $\xi(t)$ denotes zero mean Gaussian white noise, formally the time derivative of the Wiener Process, $W(t)$:

$$\xi(t) = \frac{dW(t)}{dt} \qquad \text{and} \qquad W(t) = \int_0^t \xi(s)\,ds = \int_0^t dW(s) \tag{5.38}$$

Hence, Eq 5.37 can be written as

$$d\eta = -\alpha\,\eta(t)\,dt + \sigma_0\,dt \qquad \alpha > 0 \tag{5.39}$$

5.3.3 Approximate Colored Noise Solution

To facilitate practical use of the colored noise assumption, the following approximate solution is proposed to calculate moments of $C(t)$ when σ is small. First, note that Eq 5.33 can be written as:

$$C(t) = C_0 \exp\left(-\overline{k}\,t\right)\left[1 - \sigma\int_0^t \eta(s)\,ds + \frac{\sigma^2}{2!}\int_0^t\int_0^t \eta(s_1)\,\eta(s_2)\,ds_1 ds_2 - \cdots\right]$$

Keeping only terms of $O(\sigma^3)$ and taking expected value of both sides yields:

$$< C(t) > = < C_0 > \exp\left(-\overline{k}\,t\right)\left[1 - \sigma\int_0^t < \eta(s) > ds + \frac{\sigma^2}{2!}\int_0^t\int_0^t < \eta(s_1)\,\eta(s_2) > ds_1 ds_2\right] \tag{5.40}$$

Since $< \eta(t) > = 0$ and $R_\eta(s_1, s_2) = < \eta(s_1)\,\eta(s_2) >$ it follows that

$$< C(t) > = < C_0 > \exp\left(-\overline{k}\,t\right) \left[1 + \frac{\sigma^2}{2!} \int_0^t \int_0^t R_\eta(s_1, s_2)\, ds_1\, ds_2 \right] \quad (5.41)$$

Now, as $t \to \infty$

$$R_\eta(s_1, s_2) \to \frac{\sigma_0^2}{2\alpha} \exp\left(-\alpha \mid s_1 - s_2 \mid\right) \quad (5.42)$$

and Eq 5.41 becomes

$$< C(t) > = C_0 \exp\left(-\overline{k}\,t\right) \left[1 + \frac{(\sigma_0\,\sigma)^2}{4\,\alpha} \int_0^t \int_0^t \exp\left(-\alpha \mid s_1 - s_2 \mid\right) ds_1\, ds_2 \right] \quad (5.43)$$

Note that

$$\begin{aligned} & \int_0^t \int_0^t \exp\left(-\alpha \mid s_1 - s_2 \mid\right) ds_1\, ds_2 \\ = & \int_0^t \left[\int_0^{s_2} \exp\left(-\alpha\,(s_2 - s_1)\right) ds_1 + \int_{s_2}^t \exp\left(-\alpha\,(s_1 - s_2)\right) ds_1 \right] ds_2 \\ = & \frac{2}{\alpha^2} \left[\alpha\,t + \exp\left(-\alpha\,t\right) - 1\right] \end{aligned}$$

Accordingly, the first moment for $C(t)$ is

$$< C(t) > = C_0 \exp\left(-\overline{k}\,t\right) \left[1 + \frac{(\sigma_0\,\sigma)^2}{2\alpha^3} \left[\alpha\,t + \exp\left(-\alpha\,t\right) - 1\right] \right] \quad (5.44)$$

The variance of $C(t)$ can also be computed using the formula:

$$\mathrm{Var}[C(t)] = < C^2(t) > - [< C(t) >]^2 \quad (5.45)$$

To compute the first term on the right hand side of Eq 5.45 note that

$$\begin{aligned} C^2(t) = C_0^2 \exp\left(-2\overline{k}t\right) \Bigg[& 1 - 2\,\sigma \int_0^t \eta(s)\, ds + \frac{2\,\sigma^2}{2!} \int_0^t \int_0^t \eta(s_1)\,\eta(s_2)\, ds_1 ds_2 \\ & + \sigma^2 \int_0^t \int_0^t \eta(s_1)\,\eta(s_2)\, ds_1 ds_2 + O(\sigma^3) \Bigg] \end{aligned} \quad (5.46)$$

where the right hand side can be further simplified:

$$C^2(t) \;=\; C_0^2 \,\exp\left(-2\bar{k}t\right)\left[1 \;-\; 2\,\sigma\int_0^t \eta(s)\,ds \;+\; 2\,\sigma^2\int_0^t\int_0^t \eta(s_1)\,\eta(s_2)\,ds_1 ds_2\right]$$

where terms of $O(\sigma^3)$ and higher are neglected since σ is assumed to be small. Taking expectations of both sides of the above equation yields:

$$< C^2(t) > \;\;=\;\; < C_0^2 > \exp\left(-2\bar{k}t\right)\left[1 \;-\; 2\,\sigma\int_0^t < \eta(s) > \,ds \;+\; 2\,\sigma^2\int_0^t\int_0^t < \eta(s_1)\,\eta(s_2) > \,ds_1 ds_2\right]$$

which simplifies to

$$< C^2(t) > \;=\; C_0^2 \,\exp\left(-2\bar{k}t\right)\left[1 \;+\; 2\,\sigma^2\int_0^t\int_0^t R_\eta(s_1, s_2)\,ds_1 ds_2\right] \qquad (5.47)$$

since

$$< \eta(t) > \;=\; 0 \qquad \text{and} \qquad R_\eta(s_1, s_2) \;=< \eta(s_1)\,\eta(s_2) > \qquad (5.48)$$

It should be noted that

$$\int_0^t\int_0^t R_\eta(s_1, s_2)\,ds_1 ds_2 \;=\; \frac{\sigma_0^2}{\alpha^3}\left[\alpha\,t \;+\; \exp\left(-\alpha\,t\right) \;-\; 1\right] \qquad (5.49)$$

Hence, it follows that

$$< C^2(t) > \;=\; C_0^2 \,\exp\left(-2\bar{k}t\right)\left[1 \;+\; \frac{2\,(\sigma_0\,\sigma)^2}{\alpha^3}\left[\alpha\,t \;+\; \exp\left(-\alpha\,t\right) \;-\; 1\right]\right] \qquad (5.50)$$

and the variance of $C(t)$ can now be easily computed using Eq 5.45.

5.4 Conclusions

Stochastic models are developed for environmental systems, allowing for the order of occurrence of probabilistic events to be taken into account. This chapter uses stochastic water quality models to capture the interactions among biochemical oxygen demand (BOD), dissolved oxygen (DO), and other environmental variables. The temporal moment equations are derived for carbonaceous BOD (CBOD), nitrogenous BOD (NBOD), and DO. In addition, the Streeter—Phelps equations are generalized to more realistically model hydrologic processes. Finally, a practical **colored noise approximation** is put forth and used to replace the abstract mathematical concept of 'white' (theoretical) noise. The temporal expectation and variance of BOD is then computed under colored noise.

The SEAL model is developed to solve stochastic differential equations. The author programmed an explicit order 1.5 strong scheme to integrate Îto SDEs. The algorithm is found in Section 11.2 of Kloeden and Platen (1992). The SEAL model is used to produce a probability density function corresponding to the time (or location) at which DO_{min} occurs. This knowledge is valuable since it can help to predict when and where maximum aquatic stress will occur. In the context of aquaculture pond management, predicting the time of low DO events is critical since ponds are mechanically aerated during periods of low DO: successful aeration is dependent upon supplying enough oxygen to fulfill the respiratory needs of the animals and plants within the pond. The SEAL model is sufficiently flexible to provide a variety of inferences about DO, CBOD, and NBOD levels.

Moreover, the minimum DO concentration (DO_{min}) is of great interest in calculating maximum assimilable organic loads. As the concentration of DO falls below DO_{cr}, fish and other organisms begin to die, and floating sludges predominate (due to increased activity of bacterial communities). If DO_{min} is anticipated to be significantly below DO_{cr} at a critical location, an environmental agency may be forced to take drastic actions to reduce the level of organic wastes that are released into the water body.

Chapter 6

Uncertainty and Species Extinction

This chapter considers a class of problems that be categorized best under the heading 'First Passage Time'. By modeling first passage time, one investigates the time it takes (deterministically, or on average) for a certain event to occur (for example the time it takes for an oil spill to reach shore, a species to become extinct, or a nuclear accident to occur). Knowledge of this first passage time can help in the scientific management of natural resources, particularly from a *Nature Ephemeral* perspective, where grave events such as species extinction are not a question of if — but when. The first passage time problem can address many other important issues in environmental management, such as the impact of human activities (overharvesting, deforestation, *etc.*) on the behavior of ecological systems.

6.1 Analytical Equations for First Passage Time

The one-dimensional first passage time problem is examined first, where the region under consideration is an interval $x_1 \leq x_0 \leq x_2$. We are interested in the time T it takes the process $x(t)$ starting at x_0 to first reach the boundary $x = x_1$ or $x = x_2$, as shown in Fig 6.1. This so-called *first passage* time

varies from realization to realization, so the mean (expected) first passage time $M(x_0)$ is of interest. Other notations for $M(x_0)$ include $\mathrm{E}[T_x]$.

Consider a practical first passage time example: a bomb has exploded a few miles outside of a city. It is of interest to estimate the expected time it will take the dispersing molecules of poisonous gas to first reach the boundary of the city under the molecular bombardment of ordinary air molecules. As another example, consider the recent oil spill off the coast of Ecuador. It is of interest to estimate the first time that the oil will reach the Galapagos Islands or other ecologically sensitive areas.

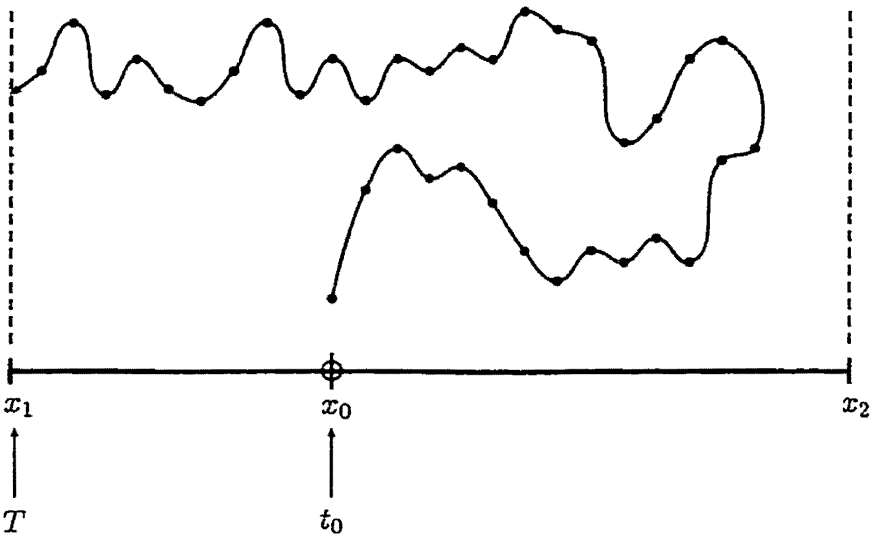


Figure 6.1: Determination of first passage time T to reach boundary $x = x_1$ or $x = x_2$ starting at x_0

6.1.1 Safe Domains

Three commonly used 'safe-domains' characterize first-passage problems: type–B barrier; type–D barrier; and type–E barrier.

Type–B barrier

In the first case, the safe domain is characterized by a single barrier or passage level $x = b$, *i.e.* any value of $x(t)$ such that $x < b$ is safe. The first-passage problem in this case consists of obtaining the probability distribution of the time T at which $x(t)$ first takes on a value $x \geq b$. A possible sample $x(t)$ with its corresponding first-passage time T is sketched in Fig 6.2a corresponding to an upper barrier. In Fig 6.2b the first passage time $T^{\dagger}$ corresponding to a lower-level barrier is illustrated. These problems involve a 'single-passage level', known as a type–B barrier (Crandall et al., 1966). It is quantitatively described by the magnitude of b.

Type–D barrier

This case is similar to the type–B barrier except that the safe domain is characterized by the double barrier $x = \pm b$ *i.e.* any value of x such that $|x| < b$ is safe. The first passage problem in this case consists of obtaining the probability distribution of the time T at which x first takes on a value $|x| \geq b$. The symmetric double-passage barrier of Fig 6.3 is known as a type–D barrier (Crandall et al., 1966).

Type–E Barriers

For the third case, consider a passage level not for the process $x(t)$ itself, but for its envelope $a(t)$ where

$$a^2 = x^2 + \frac{\dot{x}^2}{\omega_n^2} \tag{6.1}$$

The safe domain $a < b$ is therefore a circle of radius b in the phase plane where $\dot{x}/\omega_n$ is plotted against x. An envelope passage level is known as a type–E barrier (Crandall et al., 1966). The first-passage problem here consists of obtaining the probability distribution of the time T at which the envelope a first takes on a value $a \geq b$. The use of type–E barriers and envelope processes $a(t)$ are described in more detail in Section 6.1.4.

Let $x(t)$ be a Markov process with initial value $x_0 = x(t_0)$. If a Markov process is *continuous* and $p(x, t \mid x_0, t_0)$ denotes the transitional probability density that the system is in the neighborhood of state x at time t given that it was at x_0 at time t_0 ($t > t_0$), then p satisfies the well-known Kolmogorov backward equation

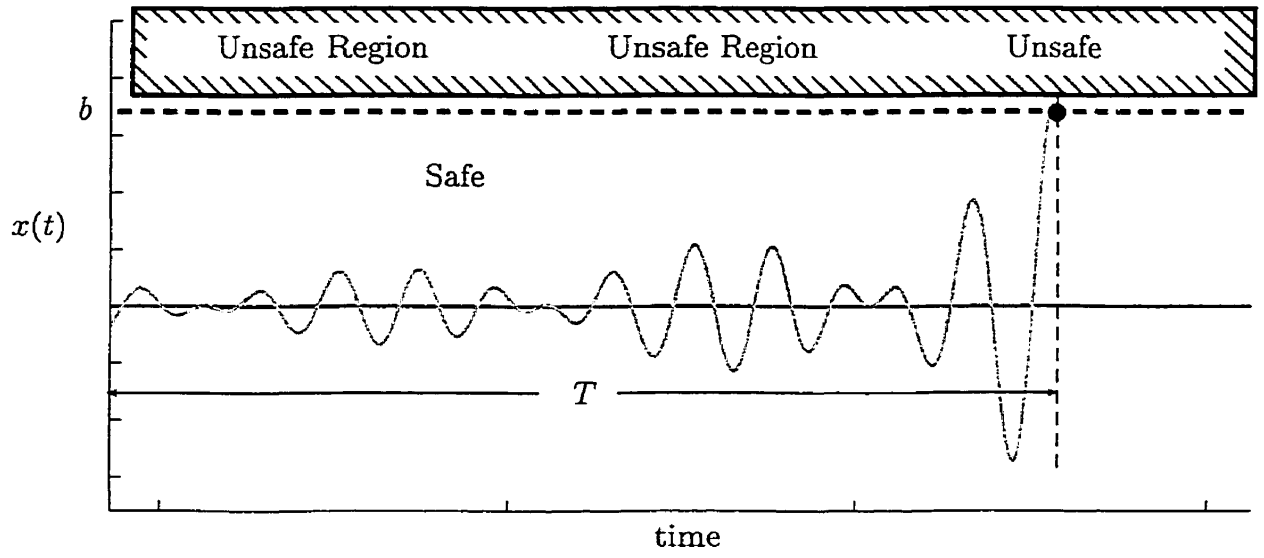
$$\frac{\partial p}{\partial t} = \frac{K_2(x_0, t_0)}{2} \frac{\partial^2 p}{\partial x_0^2} + K_1(x_0, t_0) \frac{\partial p}{\partial x_0} \tag{6.2}$$

To find an equation satisfied by $M(x_0)$, Stratonovich (1967b) derived the Pontriagin equation:

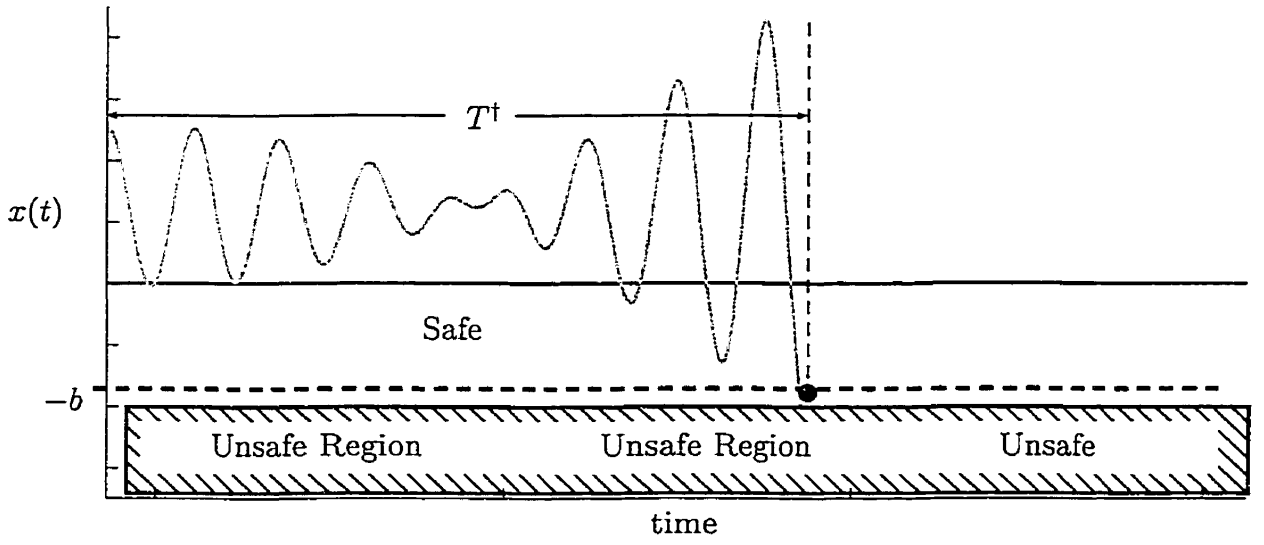
$$\frac{K_2(x_0)}{2} \frac{d^2 M(x_0)}{dx_0^2} + K_1(x_0) \frac{dM(x_0)}{dx_0} = -1 \tag{6.3}$$

Consider the one-dimensional case in which the boundary is composed of two endpoints, x_1 and x_2. If the initial point x_0 lies at the boundary itself, then the boundary is "reached immediately" and the mean first-passage time is zero. In this case Eq 6.3 satisfies the boundary condition

$$M(x_1) = M(x_2) = 0 \quad \text{where} \quad x_0 \in [x_1, x_2] \tag{6.4}$$



(a) First-passage time T for type-B barrier ($x < b$ is safe)



(b) First-passage time $T^{\dagger}$ for type-B barrier ($x > -b$ is safe)

Figure 6.2: First Passage Times for type-B Barriers

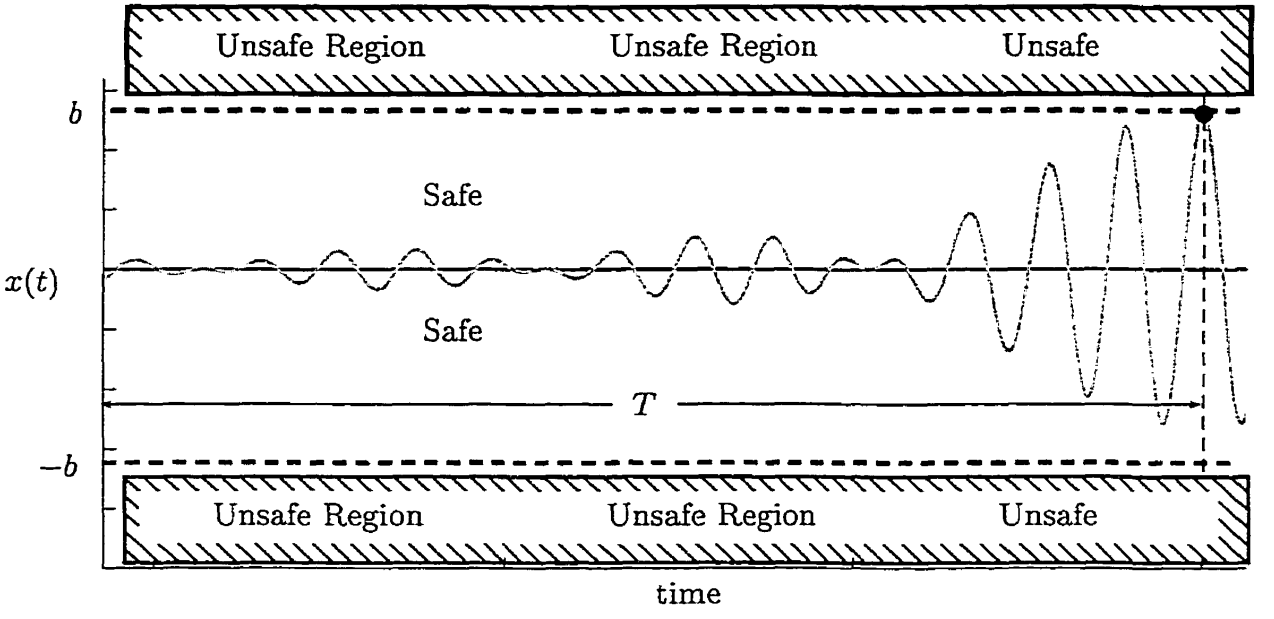


Figure 6.3: First Passage Times for type-D Barriers

6.1.2 Wiener Process (Brownian Motion)

In this section the Pontriagin equation (Eq 6.3) is applied to the Wiener Process (which was described in detail in Section 5.1.1). Consider the equation

$$\frac{dx}{dt} = \sigma\xi(t)$$

Since $\xi(t) = \frac{dW}{dt}$ we have

$$dx(t) = \sigma\, dW(t)$$

Using the Pontriagin equation (Eq 6.3), Stratonovich (1967b) shows that for the Wiener Process the first passage time $M(x_0)$ satisfies:

$$\frac{\sigma^2}{2}\frac{d^2M(x_0)}{dx_0^2} = -1 \tag{6.5}$$

and so

$$\frac{d^2M(x_0)}{dx_0^2} = -\frac{2}{\sigma^2} \tag{6.6}$$

Integrating both sides yields:

$$\begin{aligned} \frac{dM(x_0)}{dx_0} &= -\frac{2}{\sigma^2}x + C \\ M(x_0) &= -\frac{1}{\sigma^2}x_0^2 + C\,x_0 + D \end{aligned} \tag{6.7}$$

Using the boundary conditions $M(x_1) = 0$ and $M(x_2) = 0$ we have the following two equations:

$$\begin{aligned} 0 &= -\frac{1}{\sigma^2}x_1^2 + C\,x_1 + D \\ 0 &= -\frac{1}{\sigma^2}x_2^2 + C\,x_2 + D \end{aligned} \tag{6.8}$$

Solving for C and D yields

$$C = \frac{x_1 + x_2}{\sigma^2} \quad \text{and} \quad D = -\frac{x_1\,x_2}{\sigma^2} \tag{6.9}$$

Hence, from Eq 6.7 we have

$$\begin{aligned} M(x_0) &= -\frac{1}{\sigma^2}x_0^2 + \left(\frac{x_1 + x_2}{\sigma^2}\right)x_0 - \frac{x_1\,x_2}{\sigma^2} \\ &= -\frac{1}{\sigma^2}\left[x_0^2 - (x_1 + x_2)\,x_0 + x_1\,x_2\right] \end{aligned} \tag{6.10}$$

First Passage Time for Wiener Process with Drift

We say that $W(t)$ is a Wiener process with drift coefficient μ and variance parameter σ^2 if

- $W(0) = 0$
- $W(t)$ has stationary and independent increments

- $W(t)$ is normally distributed with mean μt and variance $\sigma^2 t$

Let T_b represent the time it takes for a Wiener process with drift coefficient $\mu > 0$, variance parameter σ^2, and $W(0) = 0$ to hit b. For such a Wiener process the Laplace transform of the probability density of T_b is given by:

$$\mathrm{E}\left[\exp\left(-\theta T_b\right)\right] = \exp\left(-\frac{b}{\sigma^2}\left(\sqrt{\mu^2 + 2\theta\sigma^2} - \mu\right)\right) \tag{6.11}$$

for $\mu \geq 0$, $\theta > 0$, and $b > 0$. This is also the moment generating function of T_b. Differentiating both sides with respect to θ yields

$$\mathrm{E}\left[-T_b \exp\left(-\theta T_b\right)\right] = \frac{-b}{\sqrt{\mu^2 + 2\sigma^2\theta}} \cdot \exp\left(-\frac{b}{\sigma^2}\left(\sqrt{\mu^2 + 2\theta\sigma^2} - \mu\right)\right) \tag{6.12}$$

Setting $\theta = 0$ yields

$$\mathrm{E}\left[T_b\right] = -\frac{b}{\mu} \tag{6.13}$$

Now differentiating both sides of Eq 6.12 yields

$$\begin{aligned}\mathrm{E}\left[-T_b^2 \exp\left(-\theta T_b\right)\right] = {} & \frac{b^2}{\mu^2 + 2\sigma^2\theta} \cdot \exp\left(-\frac{b}{\sigma^2}\left(\sqrt{\mu^2 + 2\theta\sigma^2} - \mu\right)\right) \\ & + \frac{b\sigma^2}{\left(\mu^2 + 2\sigma^2\theta\right)^{3/2}} \cdot \exp\left(-\frac{b}{\sigma^2}\left(\sqrt{\mu^2 + 2\theta\sigma^2} - \mu\right)\right)\end{aligned} \tag{6.14}$$

Again setting $\theta = 0$ yields

$$\mathrm{E}\left[T_b^2\right] = \frac{\mu b^2 + b\sigma^2}{\mu^3} \tag{6.15}$$

Now the variance of T_b, $\mathrm{Var}[T_b]$ can be determined by using the relationship

$$\begin{aligned}\mathrm{Var}[T_b] &= \mathrm{E}\left[T_b^2\right] - \left(\mathrm{E}\left[T_b\right]\right)^2 \\ &= \frac{\mu b^2 + b\sigma^2}{\mu^3} - \left(\frac{b}{\mu}\right)^2 \\ &= \frac{b\sigma^2}{\mu^3}\end{aligned} \tag{6.16}$$

6.1.3 Ornstein-Uhlenbeck Process

The Ornstein-Uhlenbeck Process was introduced in Section ??:

$$\frac{dx}{dt} + \alpha x(t) = \sigma \xi(t), \qquad \alpha > 0 \tag{6.17}$$

where $\xi(t)$ is Gaussian white noise with the property that $\xi(t) = dW(t)/dt$. Using the Pontriagin equation (Eq 6.3) Stratonovich (1967b) shows that for the Ornstein-Uhlenbeck Process the first passage time $M(x_0)$ satisfies:

$$\frac{\sigma^2}{2}\frac{d^2M(x_0)}{dx_0^2} - \alpha\frac{dM(x_0)}{dx_0} = -1 \tag{6.18}$$

Standard integration techniques can be used to solve Eq 6.18:

$$M(x_0) = A + B\exp\left(\frac{2\alpha x_0}{\sigma^2}\right) + \frac{x_0}{\alpha} \tag{6.19}$$

Using the boundary conditions $M(x_1) = 0$ and $M(x_2) = 0$ we have the following two equations:

$$\begin{aligned} A + B\cdot\exp\left(\frac{2\alpha x_1}{\sigma^2}\right) + \frac{x_1}{\alpha} &= 0 \\ A + B\cdot\exp\left(\frac{2\alpha x_2}{\sigma^2}\right) + \frac{x_2}{\alpha} &= 0 \end{aligned} \tag{6.20}$$

Solving for A and B yields:

$$\begin{aligned} A &= \frac{x_1\exp\left(\frac{2\alpha x_2}{\sigma^2}\right) - x_2\exp\left(\frac{2\alpha x_1}{\sigma^2}\right)}{\alpha\left\{\exp\left(\frac{2\alpha x_1}{\sigma^2}\right) - \exp\left(\frac{2\alpha x_2}{\sigma^2}\right)\right\}} \\ B &= \frac{x_1 - x_2}{\alpha\left\{\exp\left(\frac{2\alpha x_2}{\sigma^2}\right) - \exp\left(\frac{2\alpha x_1}{\sigma^2}\right)\right\}} \end{aligned} \tag{6.21}$$

6.1.4 Linear Oscillators

First passage time problems have been studied extensively with respect to electrical and mechanical systems. Consider the linear oscillator whose response $y(t)$ is related to the wide-band random excitation $F(t)$ by the differential equation:

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = F(t)$$

where the constants ω_n and ζ represent, respectively, the undamped natural frequency and the damping ratio of the vibratory system. The excitation $F(t)$ is taken to be a wide-band random process with zero mean. A commonly studied first passage time problem for linear oscillators is to determine the probability distribution of the time T that it takes for $y(t)$ starting from an initial amplitude level r to reach the barrier R (see Fig 6.4). The exact analytical solution to this problem is not available; hence, an approach for obtaining an approximate solution was derived by Ariaratnam and Pi (1973). The envelope $a(t)$ of $y(t)$ is shown in Fig 6.4. The approach of Ariaratnam and Pi (1973) is thus to determine the "first-passage time for envelope crossing for a linear oscillator". In Fig 6.4 this is denoted by t_a^*. This is used to estimate the first-passage time for $y(t)$, t_y^*. Note from Fig 6.4 that t_y^* appears to be a good approximation to t_a^* for large ω_n.

Electrical and Mechanical Examples

Consider an electrical system in which an inductor L of 0.5 Henry is connected in series with a 6 ohms resistor R, a capacitor C of 0.02 farads, an alternating voltage $E(t)$ given by $24\,sin10t$, and a switch as shown in Fig 6.5a. The voltage drop across the resistor, inductor, and capacitor is $6I$, $0.5\,dI/dt$, and $50Q$ respectively.

Hence, by Kirchhoff's law,

$$6I + 0.5\frac{dI}{dt} + +50Q = 24\sin 10t \tag{6.22}$$

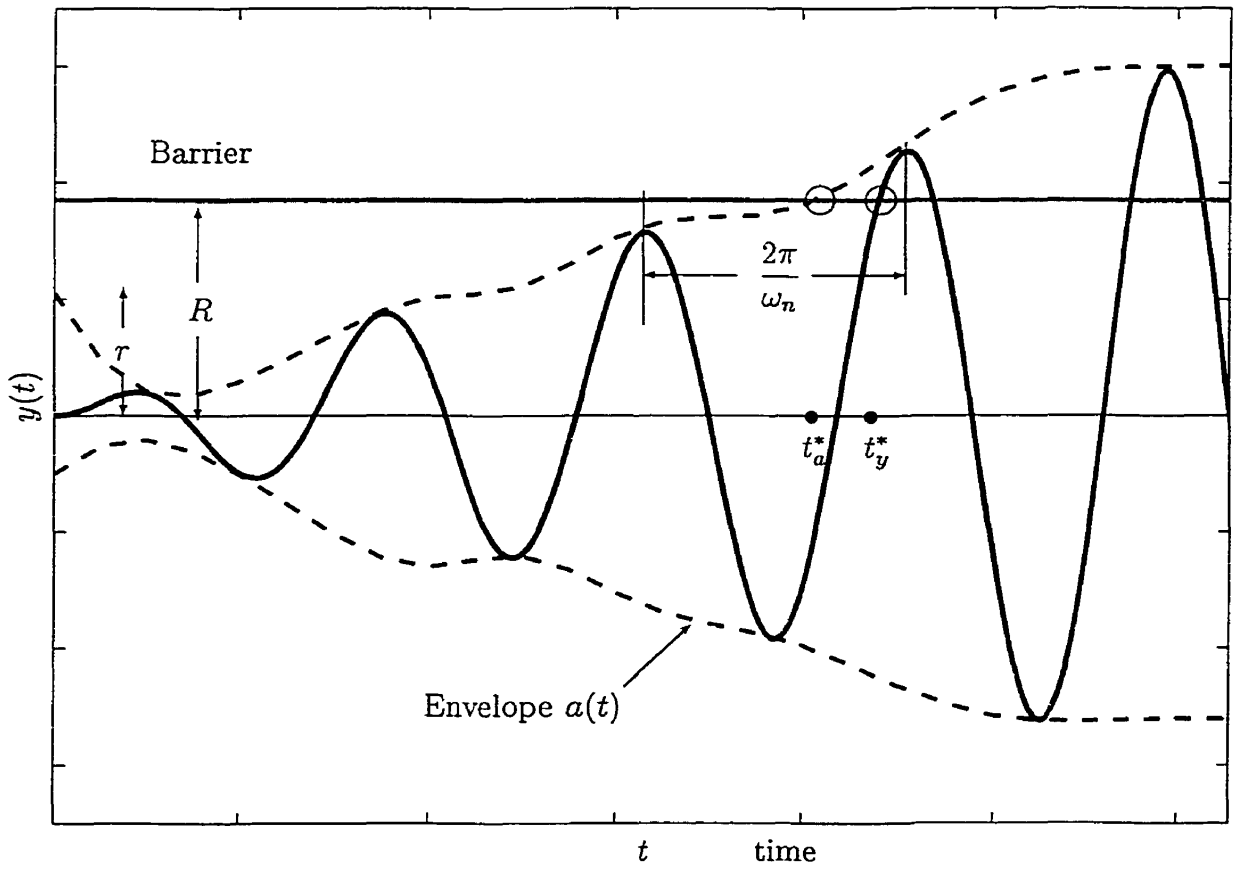


Figure 6.4: Sketch of $y(t)$ and $a(t)$ crossing the barrier R

where I is the instantaneous current and $I = \frac{dQ}{dt}$. Accordingly,

$$6\frac{dI}{dt} + 0.5\frac{d^2I}{dt^2} + 50I = 240\cos 10t \tag{6.23}$$

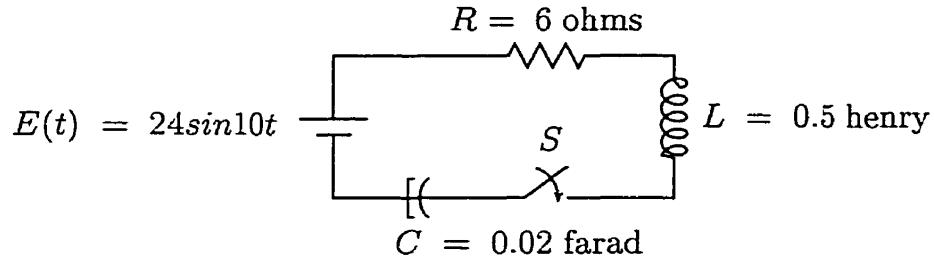
We are often interested in determining the first time that the current reaches a maximum level (above a threshold level of current, the circuit may blow a fuse). This problem may be solved using the first passage time envelope method of Ariaratnam and Pi (1973) where $240\cos 10t$ is the applied potential (excitation) in volts and $I(t)$ is the resulting current in amperes.

As another example, consider a simple mechanical structure that can be represented by a a mass, a spring, and a dashpot. Assume that the motion of the mass is restricted to translation in only one direction as shown in Fig 6.5(b). Here, k is a spring constant, c is a viscous damping coefficient, and m is a mass. Assume that the input of interest is a force applied to the mass, F, and $y(t)$ is the resulting output displacement of the mass. Since the sum of all forces acting on the mass must equal zero it follows that the equation of motion for this system is:

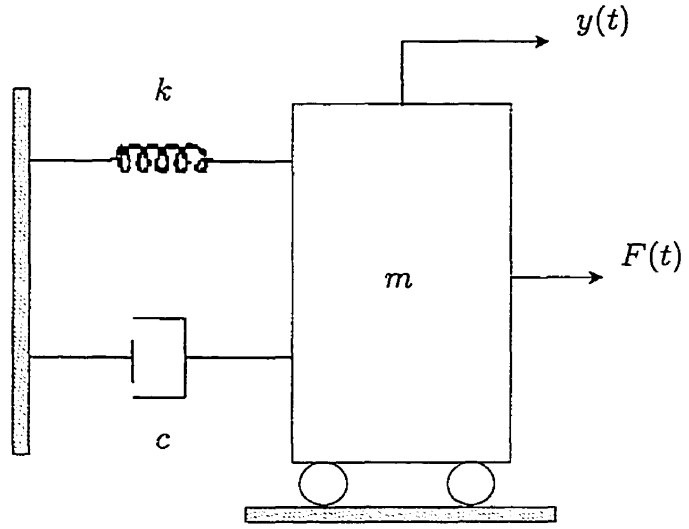
$$m\ddot{y}(t) + c\dot{y}(t) + ky = F(t) \tag{6.24}$$

since the spring force is $-k\,y(t)$, the damping force is $-c\,\dot{y}(t)$, and the inertial force is $-m\ddot{y}(t)$. Eq 6.24 connects the desired response quantity $y(t)$ with the excitation $F(t)$. In order to standardize the analysis it is conventional to introduce the notation

$$\frac{k}{m} = \omega_n^2 \qquad \text{and} \qquad \frac{c}{\sqrt{k\,m}} = 2\zeta$$



(a) Circuit Diagram



(b) Linear Oscillator

Figure 6.5: (a)Electrical system with voltage input; (b)Simple mechanical system with mass m, spring, dashpot and a force input

6.2 Uncertainty and Extinction in Fisheries

Uncertainty is perhaps the most ubiquitous theme in fisheries management (Hilborn, 1997). Commercial fishers must cope with considerable uncertainty about the price they will receive, the costs of fishing (including fuel, interest rates, and license fees), the abundance of fish, and political constraints, such as the length of fishing seasons. Recreational anglers also face changing regulations and fluctuating stock levels. Moreover, fisheries managers are obliged to make complex decisions under uncertainty, such as estimates of stock size and appropriate quotas for recreational and commercial fisheries: uncertainty is pervasive in the estimation of model parameters used to predict sustainable yield levels (including mortality rates, growth rates, and stock recruitment). At higher levels of government, fisheries officials are beset by a fickle electorate, fluctuating budgets, and the impact of mammals and foreign vessels on fisheries.

Ludwig et al. (1993) note that in every instance of a major fisheries collapse there has been no general agreement about the causes of these failures: classic examples include the overharvesting of the Pacific sardine (off the coast of California) and northern cod (off the coast of Newfoundland). Many scientists are still in denial about these catastrophes (arguing that it is virtually impossible to overfish a pelagic species); other scientists blame seals, foreign vessels, or climatic conditions. While El Niño events certainly played a role in the most spectacular fisheries collapse of all time — the Peruvian anchovy fishery — there remains no general agreement about the role of overfishing and oceanographic events as causes of this well studied disaster. Fisheries agencies have been generally unable to cope with the uncertainty in the stock assessment process: overfishing has led to the spectacular collapse of several major fisheries.

6.3 Extinction Risk and the SEAL Model

Learning about fisheries sustainability requires systematically and experimentally harvesting from it. Yet, as was previously emphasized, both underharvesting and overharvesting are often politically, economically, and environmentally infeasible. Accordingly, stochastic simulations with the SEAL model are of great value to calculate the "risk" of stock collapse at low abundance or high fishing pressure. The SEAL model requires a population threshold below which the species may be gravely threatened. Such information is subjective, but estimates are available. For example, fisheries managers in South Africa feel that their pelagic fishery faces "unacceptable risks" if the spawning biomass falls below 20% of the mean pre-exploitation level within a 20-year period (Butterworth et al., 1997).

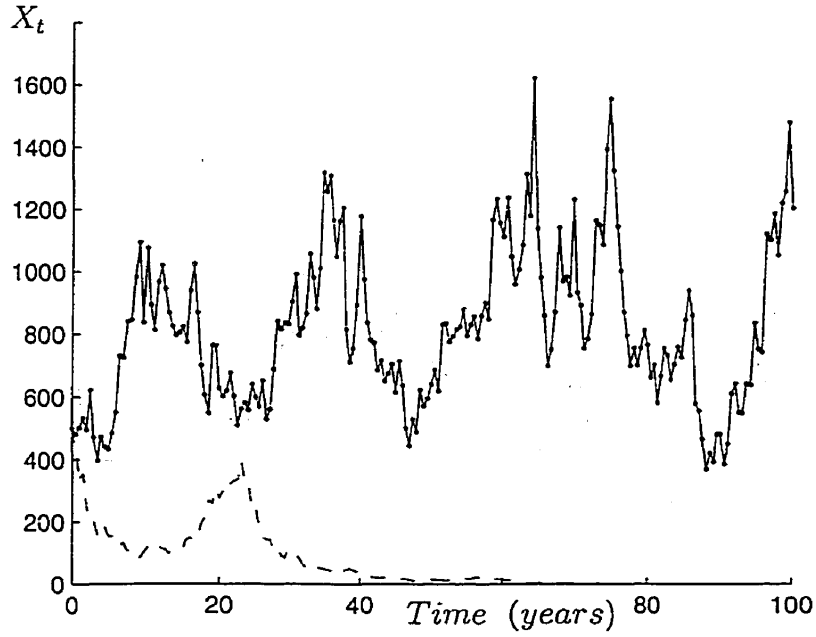


Figure 6.6: Sample Functions

The simplest deterministic model of population growth is the exponential equation $dN/dt = \mu(t)N$, where $\mu(t)$ is the Malthusian growth coefficient,

which may vary in sign and magnitude with t to account for seasonal variations and vagaries of the environment. Hence $\mu(t)$ may fluctuate randomly as $r + \sigma\xi(t)$. In Eq ?? it was shown that when σ and r are constants and $\xi(t)$ is a zero mean Gaussian white noise process a linear Îto stochastic differential equation was obtained:

$$dN(t) \;=\; r\,N(t)\,dt \;+\; \sigma\,N(t)\,dW(t) \tag{6.25}$$

In this model, the exponential population growth is unbounded. Under conditions of limited resources, however, there exists a finite carrying capacity K with the population decreasing whenever it exceeds this value. This feature can be incorporated into deterministic models by replacing the growth constant r by the linear factor $r\,[K - N(t)]$. Then we obtain the linear-quadratic Verhulst equation

$$\frac{dN(t)}{dt} \;=\; r\,N(t)\,[K - N(t)] \qquad N(0) = N_0 > 0 \tag{6.26}$$

The corresponding nonlinear stochastic differential equation is

$$dN(t) \;=\; r\,N(t)\,[K - N(t)]\,dt + \sigma\,N(t)\,dW(t) \qquad N(0) = N_0 > 0 \tag{6.27}$$

where the constant $K > 0$ is called the carrying capacity of the environment, the constant $r \in \Re$ is a measure of the quality of the environment, and the constant σ is a measure of the size of the noise in the system. Gard (1988) proves that

$$N(t) \;=\; \frac{N_0 \exp\left[\left(r\,K \;-\; \frac{1}{2}\sigma^2\right)t + \sigma\,W(t)\right]}{1 \;+\; N_0\,r \int\limits_0^t \exp\left[\left(r\,K \;-\; \frac{1}{2}\sigma^2\right)s + \sigma W(s)\right]\,ds} \tag{6.28}$$

is the solution of Eq 6.26 with $t \geq 0$.

The above Verhulst equation (Eq 6.26) is often written simply as

$$\frac{dN(t)}{dt} \;= \lambda\,N(t) \;-\; N^2(t) \tag{6.29}$$

On randomizing the parameter λ to $\lambda + \sigma\xi(t)$ one obtains the stochastic differential equation

$$dN(t) = \left[\lambda N(t) - N^2(t)\right] dt + \sigma N(t)\, dW(t) \tag{6.30}$$

using the Îto interpretation, which again can be explicitly solved (see, for example Kloeden and Platen (1992)):

$$N(t) = \frac{N_0 \exp\left[\left(\lambda N(t) - \frac{1}{2}\sigma^2\right) t + \sigma W(t)\right]}{1 + N_0 \int_0^t \exp\left[\left(\lambda - \frac{1}{2}\sigma^2\right) s + \sigma W(s)\right] ds} \tag{6.31}$$

One of the simplest deterministic models of critical depensation is given by the following equation:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right)\left(\frac{x}{m} - 1\right) - qEx \tag{6.32}$$

where qEx represents the harvested yield. The vagaries of the environment can be modelled by allowing r to vary randomly as $r + \sigma\xi(t)$. This yields the stochastic differential equation

$$dX_t = \left[rx\left(1 - \frac{x}{K}\right)\left(\frac{x}{m} - 1\right) - qEx\right] dt + \sigma X_t dW_t \tag{6.33}$$

using the Îto interpretation where σ represents the intensity of the fluctuations and $dW_t = \xi(t)\, dt$. Note that the stochastic differential equation given in Eq. 6.33 represents population growth with critical depensation and harvesting. Three realizations of this stochastic process given are shown in Fig 6.6. Here, it is assumed that the effort level E is 300 vessels, and that the intensity of fluctuations σ is 0.25. In addition, $x(0) = 500$. That is, the population at time 0 is 500. Finally, note that species extinction occurs when the population falls below 200 individuals, i.e. $x_m = 200$.

Note that for the three samples illustrated in Fig 6.6 only one 'is driven to extinction' before $t = 100$. In the SEAL model, the mean first passage time is calculated in two ways (giving rise to the symbols μ_1 and μ_2):

- All samples (realizations) are included (μ_1). This is achieved by assuming that every sample faces extinction either before $t = 100$ (as calculated by the SEAL model) or at $t = 100$.
- Only those samples (realizations) that became extinct before $t = 100$ are included (μ_2).

Both means are illustrated in Fig 6.7. Note that μ_2 will always be less than or equal to μ_1.

6.4 Precautionary Principle and the SEAL model

At the 1987 Conference on the Protection of the North Sea, the precautionary principle was put forward in an attempt to shift the burden of proof from the regulatory agency to the marine polluter. A pollution-related version of the precautionary principle is that "potentially damaging pollution emissions should be reduced even when there is no scientific evidence to prove a causal link between emissions and effects" (Peterman and M'Gonigle, 1992). The obvious fisheries variation on this statement is that catches should be reduced unless there is good evidence that the current catch is sustainable. This is the opposite of what Ludwig et al. (1993) consider the norm — not reducing catch levels until there is compelling evidence of a collapse.

In a developing fishery, the precautionary principle suggests a slow development of the fishing industry and caution in expanding catches. More specifically, caution in expanding fishing capacity is called for to minimize the chance of stock depletion and economic overdependence. While biological caution may have economic costs in foregone yield, in a developing fishery it should not lead to the large-scale dislocation of existing fishers. On the other hand, when a stock is fully developed or exploited (and often overexploited), precautionary reductions in catch may reduce the stock collapse risk at the

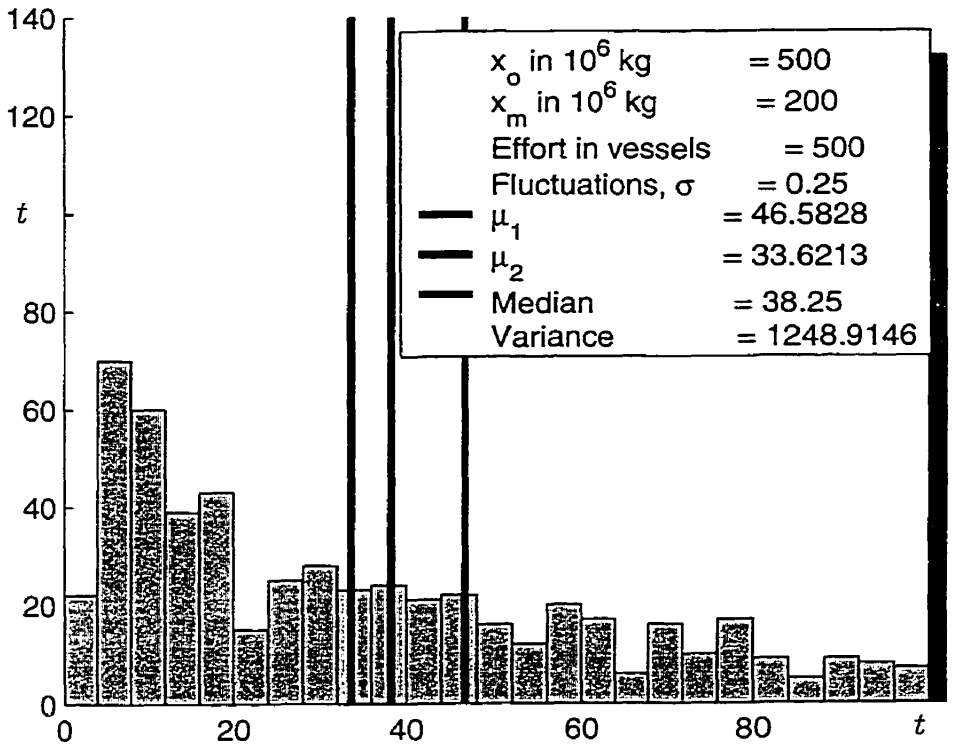


Figure 6.7: Histogram of First Passage Time to reach x_m

expense of disruptive social and economic change in the community of people who harvest the fish.

Recent years have witnessed the extension of the precautionary principle to the protection of fishery resources. At the international level, there have been attempts to apply the precautionary principle to the Code of Conduct for Responsible Fisheries, the Rio Declaration of the U.N. Conference on Environment and Development, and the U.N. Convention of the Law of the Sea Relating to the Conservation of Straddling Stocks and Highly Migratory Fish Stocks. Although each of these agreements promotes the precautionary principle's use, an operational definition of the term eludes each of them. While the precautionary principle "implies the commitment of resources now to safeguard against the potentially adverse future outcomes of some decision" (Perrings, 1991), it does not suggest how much resources to allocate or which future outcomes are most important.

To address these issues, the SEAL model is used to quantify the risk of species extinction caused by harvesting activities. Specifically the role of 'fishing effort' (number of boats, traps, *etc.*) is examined to see how an increase in fishing vessels affects the risk of species extinction. The relationship between 'fishing effort' and the first passage time to stock collapse is illustrated in Fig 6.8. As the number of boats increases from 300 to 400 to 800 to finally 1500, the mean first passage time to extinction (defined by μ_1) decreases from 24.4 to 23.0 to 22.0 to 17.7 respectively. The precautionary principle and the SEAL model should be applied to all fisheries resources: greater uncertainty regarding the productivity of a stock should correspond to greater caution in setting target harvest levels.

6.5 Conclusion

The theory and practice of first passage time problems is discussed in the context of mechanical and electrical systems. The knowledge is then applied to environmental problems, specifically fisheries management. Uncertainty

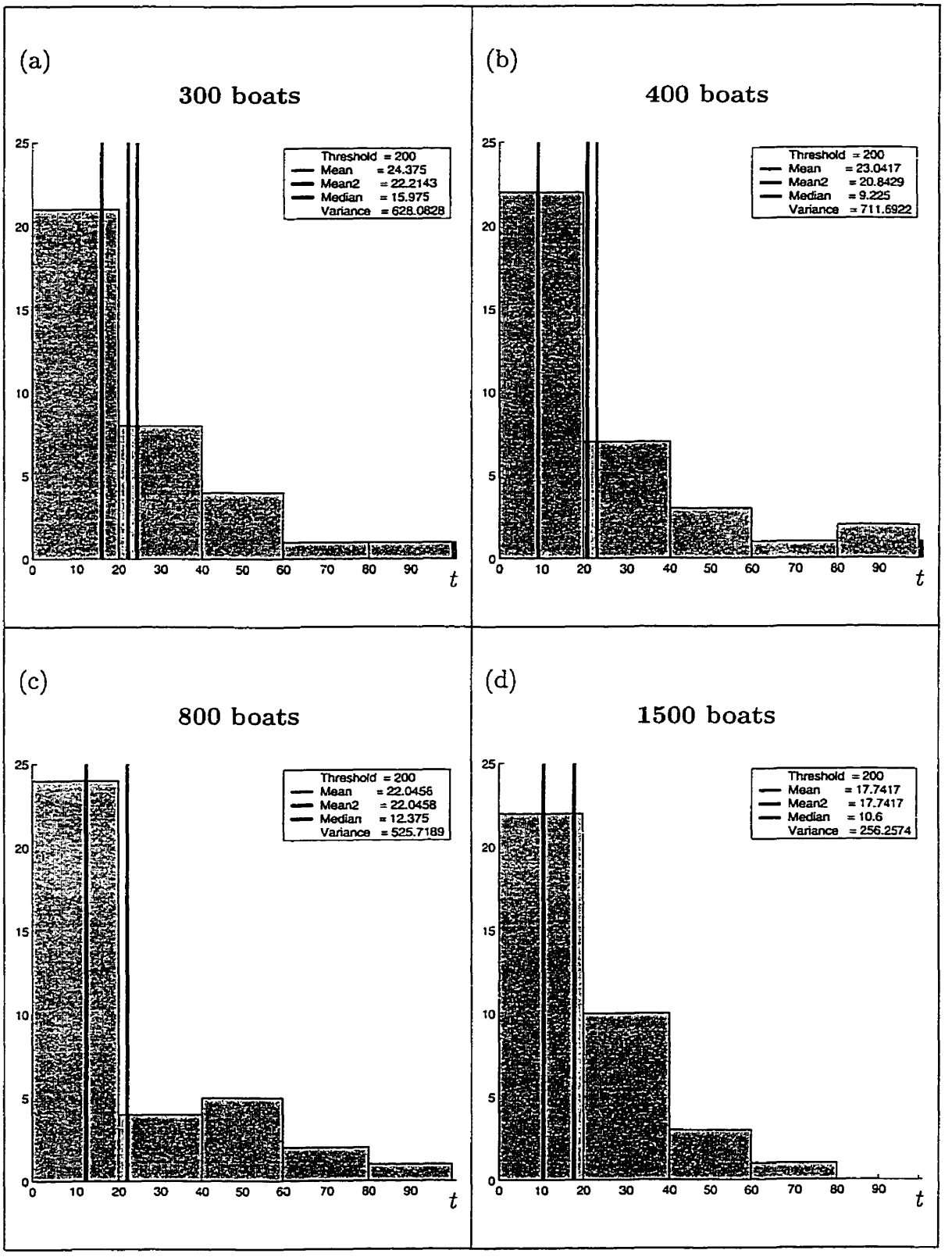


Figure 6.8: Effort vs First Passage Time

and species extinction are perhaps the most ubiquitous themes in fisheries (Hilborn, 1997). Since MSY levels are, in fact, often unsustainable (Larkin, 1977), the long term health of fisheries is a grave concern. Resource managers must cope with considerable uncertainty about appropriate quotas for fishers, estimates of stock size, the social implications of harvesting, and political considerations such as the length of fishing seasons.

The risk of extinction was formally modelled. The SEAL decision support system was used to advise the Risk Institute (University of Waterloo, Civil Engineering) and the Government of Canada (Department of Fisheries and Oceans (DFO) about the relationship between 'fishing effort' and stock extinction. It is shown that as the the number of boats decreases by a factor of five, the mean time time to extinction increases by approximately 38%. It is concluded that the precautionary principle and the SEAL model should be applied to all fluctuating fisheries resources.

Chapter 7

Convex Models and MCDA under uncertainty

Recognizing that environmental conditions, social systems, and management objectives will change over time, Section 7.1 formally models the robustness, flexibility, and adaptability of the systems we monitor, analyze, design, and operate. Particular emphasis is placed on notions of flexibility (Section 7.1.1), robustness (Section 7.1.2), and adaptability (Section 7.1.3). The general absence of operational measures for these concepts is "impeding academic progress" in decision analysis (Kumar, 1986). The use of robustness analyses is illustrated with examples from urban planning to whale management at the International Whaling Commission.

Given the extreme uncertainty and complexity of many strategic decision problems, humans often use "bounded models of rationality". This is described in Section 7.2 with special emphasis on Nobel Laureate H.A. Simon's bounded rationality framework known as *satisficing*. It is explained why satisficing (a combination of the words satisfactory and sufficient) is the most appropriate decision strategy for addressing complex, large-scale problems.

In Section 7.3, the aforementioned ideas of ecological resilience, 'bounded rationality', uncertainty, and multiple criteria decision making are considered by means a new, non-probabilistic approach to decision analysis known as

"convex models" (Ben-Haim, 1998). This methodology is introduced in the context of improving water quality (Section 7.3.1). The theory of convex models is discussed (Section 7.3.2) and, for the first time, convex models are extended to include multiple criteria problems (Section 7.3.3). A key contribution of this chapter is the identification of environmental policy alternatives that are robust to uncertainty (Section 7.3.4). Finally, the proposed methodology is used to select alternatives in a multiple criteria forest management problem in New Brunswick, Canada (Section 7.3.5).

7.1 Flexibility and Robustness

In recent years Holling's concept of ecological resilience has been borrowed by those researchers searching for increased flexibility, robustness, and adaptability in their decision making: applications range from water resources ("safe-fail systems") to financial management (portfolio hedging and asset liquidity). Planners need rigorous methods for judging whether a particular policy or system is more "robust" than another, along with useful metrics for quantifying these features precisely. The most valuable strategies "are somehow robust in the face of rapidly changing circumstances and unanticipated outcomes" (Chávez and Shachter, 1998). Ideally, a robust policy should be able to respond to uncertainties that we do not explicitly model at the time of system design.

7.1.1 Decision Flexibility

The marketplace is changing so rapidly (changing product lines, changing technical characteristics of products, and changing market demands) that industries are increasingly turning to flexibility and robustness analyses (Kumar, 1987; Mandelbaum, 1978; Zelenovic, 1982) to cope with uncertainty: Gerwin (1985) identified seven different sets of uncertainties in one specific manufacturing process. The arena of environmental management which is also fraught with a wide range of uncertainties. Authors such as Falkenmark

(1997) and Kundzewicz (1997) point out that environmental strategies must be flexible, robust, and adaptive to deal with impacts of human activities such as the widespread pollution of surface and ground water supplies.

Flexibility is important in decision-making because strategies and policies should be responsive to a large range of uncertain outcomes.

A flexible solution may be preferred to one which is optimal, because the decision maker does not have total confidence in the model. "Optimization" may be misguided in situations characterized by extreme uncertainty, highly non-linear interrelationships, turbulent dynamics, and inevitable changes in key decision makers over the strategic time horizon. For example, strategic decisions concerning investments in future U.S. space transportation vehicles require developmental lead times of over 10 years during which unexpected changes in technology, funding, and leadership (and hence priorities, policies, and desired capabilities) are likely to occur (Richards, 1996).

Stigler (1939) presents a useful and intuitive notion of flexibility (often referred to as 'robust flexibility') which has been applied and extended by a number of researchers (Marschak and Nelson, 1962; Merkhofer, 1975; Epstein, 1980; Jones and Ostroy, 1984; Shachter and Mandelbaum, 1999). The notion of robust flexibility is explained in Fig 7.1(a) which shows average cost curves $v(d, \mathbf{X})$ for two factories, A and B. Note that both factories achieve the minimum average cost at the same level of output x^*. While factory A achieves a lower average cost than B at x^*, it is quite sensitive to the uncertain quantity $\mathbf{X}$. Intuitively, factory B is more robust to uncertainty in the value of $\mathbf{X}$ because of the protection it provides (from high values of average cost) over a broad range of possible output values. Stigler (1939) characterizes the "robust flexibility" of these factories using the second derivative of their average cost curves: a lower second derivative corresponds to increased flexibility.

Fig 7.1(b) formally defines Stigler's problem in decision analytic terms. Here, the value function $v(d, \mathbf{X})$ specifies the value that results when action d is taken and action $x \in \mathbf{X}$ obtains. Note that there are three alternatives:

d^+, which yields a moderate payoff $v(d^+, \mathbf{X})$ for all values of $\mathbf{X}$; d^*, which suffers a significant drop in $v(d^*, \mathbf{X})$ for high values of $\mathbf{X}$; and d^-, which yields low values of v for small $\mathbf{X}$. Inuitively, d^+ appears to be the most flexible alternative in that its value remains nearly constant over the entire range of $\mathbf{X}$, whereas both d^- and d^* have value curves that are characterized by low dips.

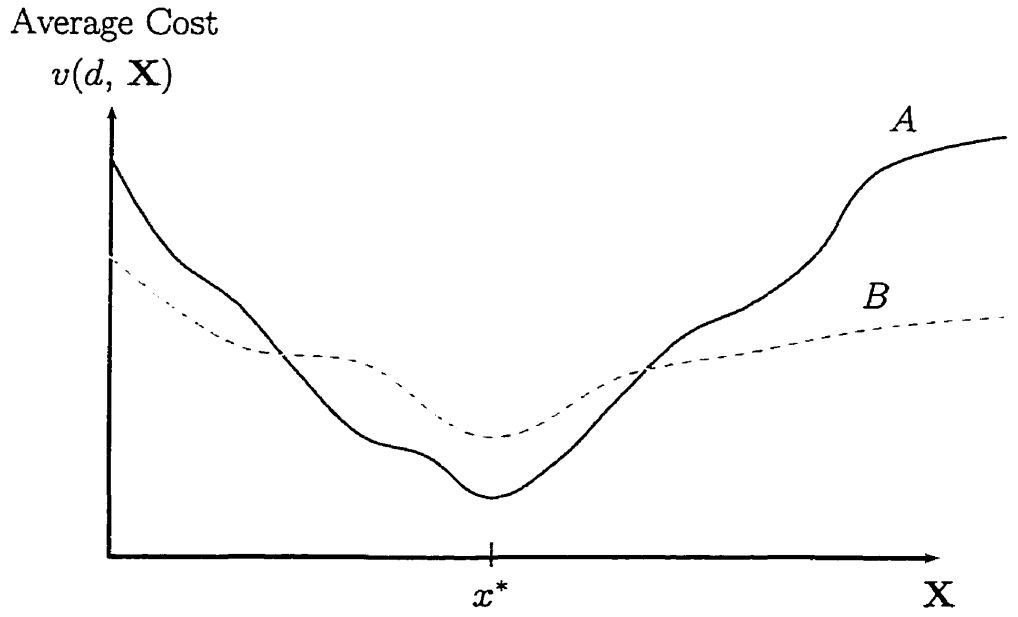
More precisely, d^+ appears to be most flexible (or least brittle) of the three alternatives in the sense that it minimizes the cumulative distance between itself and the upper boundary of the value curves for each of the available alternatives. Because $\mathbf{X}$ is given probabilistically, the notation $P(\mathbf{X} \mid \xi)$ is used to specify the probability distribution of $\mathbf{X}$ conditional on ξ (the prior state of knowledge). Using these definitions, Chávez and Shachter (1998) define the flexibility of action d_i with respect to the random variable $\mathbf{X}$ as:

$$E_{\mathbf{X}}\left[\max_{d} v(d, \mathbf{X}) - v(d_i, \mathbf{X}) \mid \xi\right] \tag{7.1}$$

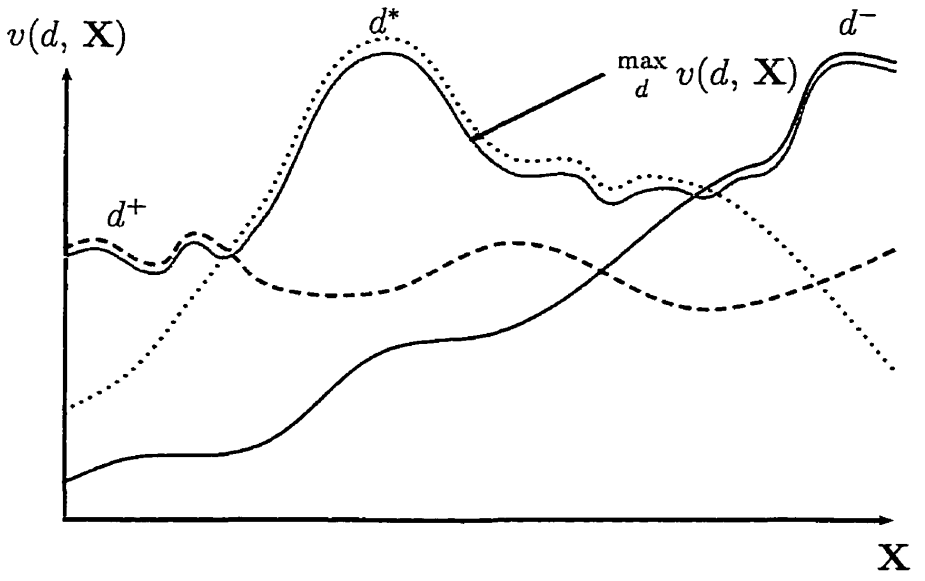
where subscripting E by $\mathbf{X}$ indicates that the expectation is taken with respect to $\mathbf{X}$. The most flexible (least brittle) alternative/action minimizes the quantity in Eq 8.6. Finally, it is useful to observe the duality between flexibility and brittleness (flexible strategies are less brittle because they do not suffer 'dips in value' across a wide range of outcomes).

7.1.2 Robustness

Robustness may be viewed as an insurance policy against uncertainty: "Unlike a traditional insurance policy, however, robustness is almost certain to pay off, but the amount of the payoff is not guaranteed" (Richards, 1996). Gupta and Rosenhead (1968) argue that a robustness analysis "abandons the search for optimality" in an unknowable future in favor of "the more modest and practical goal of future flexibility."



(a) Stigler's Robust Flexibility



(b) Stigler-type flexibility for a decision problem with three alternatives

Figure 7.1: Stigler's approach to flexibility: (a) Definition of Robust Flexibility; (b) Decision Problem with Three Alternatives.

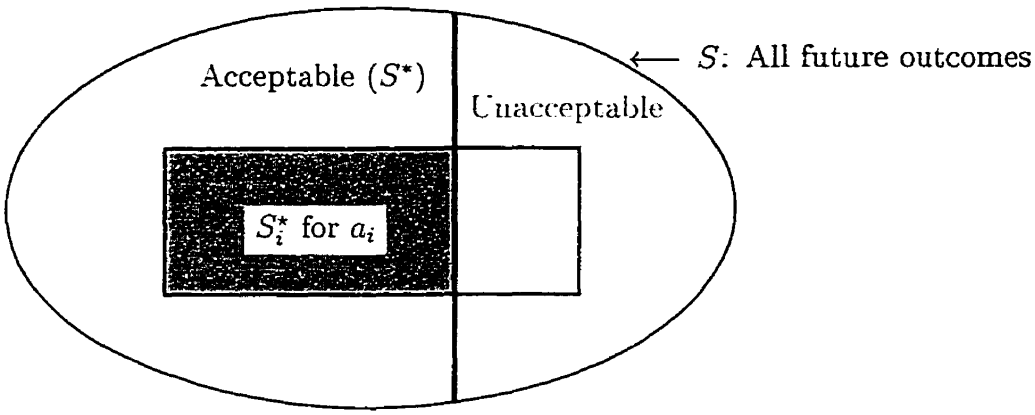
To some, robustness can be conceptualized as a counterpart to risk: "robustness represents desirable variability in a decision process as opposed to the undesirable variability implied by riskiness" (Richards, 1996). To others, robustness is reflected in the adage: "keep your options open" (Erlandson, 1981). Since robustness has to do with variability, not with point estimates, many of the methodologies currently used to evaluate technological, policy, and environmental systems are not not applicable.

The robustness concept implies a different policy-formulation paradigm from that offered by traditional optimization techniques, including return-on-investment approaches and cost/benefit analysis. A robustness approach will not provide the "optimal" answer; rather, it offers insights that can lead to more adaptive and flexible strategies. Furthermore, robustness is inherently an inclusive, process-oriented concept: decision robustness is evaluated at each stage of the strategic planning process, ideally in a participatory, community-building framework. There is a large literature pertaining to robust decision making in military planning and strategy; for instance, "deployment versatility" is necessary to cope with battlefield uncertainty and surprise.

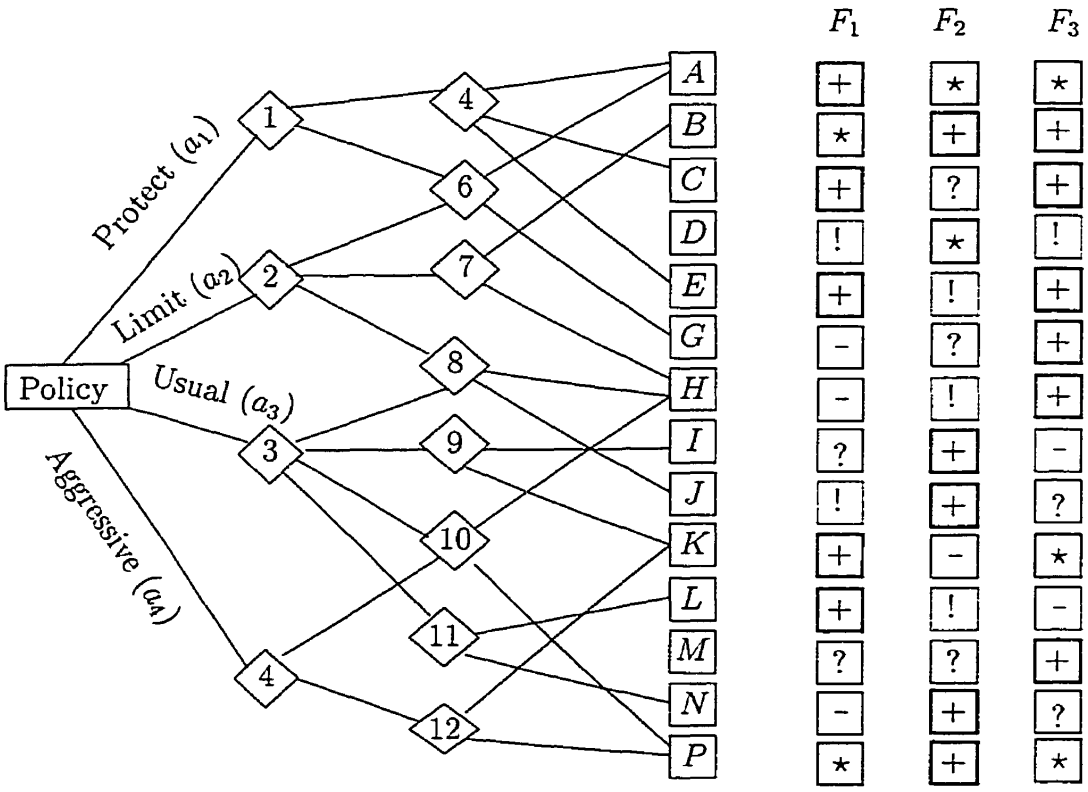
Robustness Measures

The simplest set of robustness measures is the ratio of the number of "good" options left open after selecting an action to the number of "good" options prior to taking the action (Gupta and Rosenhead, 1968)[1]. Consider a planning situation in which the set of all outcomes is denoted by S, and the set of all acceptable outcomes is represented by S^* (a subset of S). The set of outcomes attainable if the ith alternative (a_i) is selected is denoted by S_i, and S_i^* represents the set of all acceptable outcomes if a_i is chosen. In Fig 7.2(a), S_i^*, a subset of S_i, is shaded.

[1]The notion that a desirable current action is one that gives rise to good outcomes in the future has appeared frequently in the development of micro-economic theory (Marschak and Nelson, 1962).



(a) Categorization of Future Options (Rosenhead, 1989)



(b) Three-stage urban planning problem with endstate valuation

Figure 7.2: (a) Rosenhead's Definition of Robust Flexibility; (b) City of Waterloo Urban Planning Problem

Rosenhead (1989) defines the robustness of an action (alternative) a_i as:

$$\text{robustness}(a_i) = \frac{n(S_i^*)}{n(S^*)} \tag{7.2}$$

where n(·) is the number of elements in the relevant set. Since

$$n(S_i^*) \leq n(S^*), \tag{7.3}$$

the robustness of any initial decision must lie between 0 and 1. The higher the robustness of an initial decision, the more acceptable outcomes that remain open. Rosenhead's robustness analysis has been conveniently applied to discrete multiple criteria problems (Levy et al., 2000b).

To illustrate the use of a robustness analysis, consider the "Waterloo Urban Planning" problem: the City of Waterloo, in Ontario, Canada, must decide how much development to permit in the Laurel Creek Watershed. This problem was outlined in the Laurel Creek Watershed study (see for example Grand River Conservation Authority (1993) and WATgreen Advisory Committee (1996)). Here we consider only a caricature of the "Waterloo Urban Planning" problem. Assume that the City of Waterloo has four courses of action they can pursue, as shown in Fig 7.2(b):

- Proactively protect the environment (a_1),
- Limit future development (a_2),
- Business as Usual (a_3), or
- Aggressively promote development (a_4).

Assume that these four alternatives give rise to the fourteen outcomes A through P in Fig 7.2(b). In general, these fourteen outcomes are described by multiple attributes such as housing needs, flooding risk, and ecological issues (water and land quality). Assume that

- ! represents ideal endstates (housing needs are met with no additional degradation to water resources and no additional risk of flooding),

- $+$ represents desirable outcomes,
- $\star$ represents acceptable endstates,
- ? represents questionable outcomes, and
- $-$ represents undesirable outcomes.

The "futures" F_1, F_2, and F_3 in Figure 7.2(b) represent possible future social, economic, and environmental scenarios. For example, F_1 might represent the situation in which the Microsoft Corporation moves their headquarters to Waterloo Region and the population grows significantly. F_2 might represent a future with increased global warming and a more vigorous hydrological cycle, increasing the risk of floods, *etc.*

Figure 7.2(b) is summarized in Table 7.1(a). If only preferred endstates are taken into account (that is $\star$, $+$, and !), then no action dominates all others when futures F_1, F_2, and F_3 are considered. However, using the maximin criteria of Wald (1950), *Limiting Development* (action a_2) offers the most flexibility. On the other hand, under the minimax regret criterion of Savage (1954), the *Business as Usual* alternative would be selected.

By the same token, the concept of 'debility' (Caplin and Kornbluth, 1975) can be used to model the 'undesirability' of an alternative. Here, debility is defined as the number of undesirable outcomes that can arise after selecting a particular course of action, expressed as a ratio of all such undesirable endstates (analogous to Eq 7.2). The debility results are shown in Table 7.1(b). Note that the action *Protect* (a_1) dominates all others (since a lower debility score is preferred).

Alternative (a_i)	Futures F_1	F_2	F_3	Decision Criteria
Protect (a_1)	3/9	2/10	4/10	
Limit (a_2)	3/9	4/10	4/10	$\Longleftarrow$ maximin
Usual (a_3)	4/9	6/10	3/10	$\Longleftarrow$ minimax regret
Aggressive (a_4)	2/9	2/10	3/10	

(a) Robustness Matrix

Alternative (a_i)	Futures F_1	F_2	F_3	Decision Criteria
Protect (a_1)	1/3	0/1	0/2	$\Longleftarrow$ dominance
Limit (a_2)	2/3	0/1	0/2	
Usual (a_3)	2/3	1/1	2/2	
Aggressive (a_4)	1/3	1/1	0/2	

(b) Debility Matrix

Table 7.1: Using Decision Criteria to select among Alternatives: (a) Robustness Matrix; (b) Debility Matrix.

7.1.3 Adaptability

Adaptability is similar to the concepts of flexibility and robustness, in that it implies the ability to cope with and respond to perturbations, while often adding the ingredient of learning or evolution. Adaptability theory *sensu* Conrad (1983) uses entropy measures (Kumar, 1986) to describe the capacity of an ecosystem to persist in an uncertain environment. Conrad (1983) holds that in order to survive, biological systems must be capable of functioning in an uncertain environment; the *adaptability* of the biota is given by the entropy of the most uncertain environment which does not "inevitably cause a catastrophic change in biota". Many long-term ecological research studies, including the most influential, Hubbard Brook (Bormann and Likens, 1979), have clearly demonstrated that the aggregate behavior of environmental systems is highly uncertain and dynamic.

The information theoretic approach of Conrad (1983) is now briefly presented. Ω represents the transition scheme of an ecosystem: this is the set of probabilities which determine the state of the biota (β) and environment (ε) at time $t+\tau$ given their states at time t. The transition scheme is

$$\Omega = \{p\left[\beta^u\left(t+\tau\right), \varepsilon^v\left(t+\tau\right) \mid \beta^r(t), \varepsilon^s(t)\right] \mid u, r \in I, v, s \in J\} \tag{7.4}$$

where t is time, τ is a definite time interval, I is the index set of the biota, J is the index set of the environment.

The entropy of the ecosystem transition scheme is given by:

$$H(\Omega) = -\sum p\left[\beta^r(t), \varepsilon^s(t)\right] \times p\left[\beta^u\left(t+\tau\right), \varepsilon^v(t+\tau) \mid \beta^r(t), \varepsilon^s(t)\right] \times \log p\left[\beta^u\left(t+\tau\right), \varepsilon^v(t+\tau) \mid \beta^r(t), \varepsilon^s(t)\right] \tag{7.5}$$

where the sum is taken over all $u, r \in I$ and $v, s \in J$ and H is defined as 'entropy' (in the Shannon-Weaver sense):

$$H = -\sum_{i=1}^{n} p_i \ln(p_i) \tag{7.6}$$

where H decreases as one event becomes increasingly likely and it is written as $H(\Omega)$ rather than as a function of all its arguments.

Conrad (1983) also defines the average uncertainty in the behavior of the biota given the initial state of the biota and the initial state of the environment (uncertainty of biota transition scheme):

$$H(\omega) = -\sum p\left[\beta^r(t), \varepsilon^s(t)\right] \times p\left[\beta^u\left(t+\tau\right) \mid \beta^r(t), \varepsilon^s(t)\right] \times \log p\left[\beta^u\left(t+\tau\right) \mid \beta^r(t), \varepsilon^s(t)\right] \tag{7.7}$$

and the sum runs over u, r, and s. Likewise, Conrad defines a conditional entropy for the behavior of the environment:

$$H(\omega^*) = -\sum p\left[\beta^r(t), \varepsilon^s(t)\right] \times p\left[\varepsilon^v(t+\tau) \mid \beta^r(t), \varepsilon^s(t)\right] \times \log p\left[\varepsilon^v(t+\tau) \mid \beta^r(t), \varepsilon^s(t)\right] \tag{7.8}$$

The adaptability of the biota is given by:

$$H(\widehat{w}^*) \equiv \max_{w^*}\left[H(w^*) \text{ s.t. } \mathcal{A}\right], \tag{7.9}$$

where w^* is the transition scheme of the actual environment that the system encounters and the condition $\mathcal{A}$ is that "the half-life of the biota is not decreased at all". In other words, for the biota to persist, the adaptability of the living system must be greater than or equal to the actual uncertainty of the environment: $H(\widehat{w}^*) \geq H(w^*)$. Conrad argues that organisms can adapt to their environment by a variety of mechanisms, such as *selective indifference*, avoiding parts of the environment for "good reasons" (geographical regions with many predators, water with high pH, *etc.*)[2].

Adaptive Management

Holling (1978) argues that robust, innovative solutions require adaptive management and that it is necessary to design for uncertainty:

[2]M. Conrad and T. E. Creese make the distinction between selective and non-selective indifference, where the latter refers to avoiding parts of the environment *when you should not be avoiding them.*

> While efforts to reduce uncertainty are admirable ... if not accompanied by an equal effort to *design for uncertainty* and obtain benefits from the unexpected, the best of predictive methods will only lead to larger problems arising more quickly and more often. This view is the heart of adaptive environmental management — an interactive process using techniques that not only reduce uncertainty but also benefit from it. The goal is to develop more resilient policies.

Adaptive management is a process of adjusting actions, as appropriate, in light of new information and on our progress toward meeting objectives. Its basic premise is that "if human understanding of nature is imperfect, then human interactions with nature (*e.g.* policies) should be experimental" (Lee, 1995). Management discussions can be viewed as experiments, subject to modification — but with goals clearly in mind. Adaptive management stresses the need to review and revise management approaches because of the ever changing natural environment coupled with our incomplete knowledge base. While adaptive management is an appropriate response to biological uncertainty, it is time consuming and can give grossly inaccurate results when relevant variables are either ignored or not held constant (Smith, 1997).

7.2 Satisficing and Bounded Rationality

There have been many important phenomena observed in human decision making that are not explained by the theory of utility or profit maximization. Nobel Laureate H.A. Simon (Simon, 1957; 1958; 1979) proposed a rationality framework for decision analysis called *satisficing* (a combination of the words satisfactory and sufficient). A decision maker who chooses the best available alternative according to some criterion is said to optimize; one who chooses an alternative the meets (or exceeds) specified criteria, is said to satisfice. Of course the satisficing solution is not guaranteed to be either unique or in

any sense 'best'[3].

March (1978), Simon (1979), and March and Simon (1958) have both noted that satisficing involves a *bounded rationality*, in that:

- decisions occur in limited time frames,
- decision makers are unable to acquire all the information they need,
- decision makers are not aware of all of the things they need to know to make a decision.

The term 'bounded rationality' implies "somewhat less than perfect rationality" (Lewandowski et al., 1989). However, the evidence suggests that this approach represents not bounded, but simply culturally different rationality (Simon, 1997). The satisficing framework proposes that decision makers satisfice not only because of the difficulty of optimization but also due to the inherent complexity of many real-world decision situations. As summarized by Eilon (1971): "optimizing is the science of the ultimate and satisficing is the art of feasible."

One variation on the satisficing theme is the 'organizational slack' hypothesis of Cyert and March (1963): firms will settle for 'satisfactory' profits, and it is only when these thresholds are not met that an organization searches for an improved product or more efficient operation. Cyert and March (1963) define organizational slack as "the difference between total resources and necessary payments", *i.e.* uncommitted capital that can be used as a 'buffer' for hard times. They continue,

> Many interesting phenomena within the firm occur because slack is typically not zero ... (Slack) seems to be useful in dealing with the adjustment of firms to gross shifts in the external environment ... When the environment becomes less favorable, organizational slack represents a cushion ... (permitting) firms to

[3]The term 'satisfice', which appears in the Oxford English Dictionary as a Northumbrian synonym for 'satisfy', was borrowed for this new use by Simon (1956).

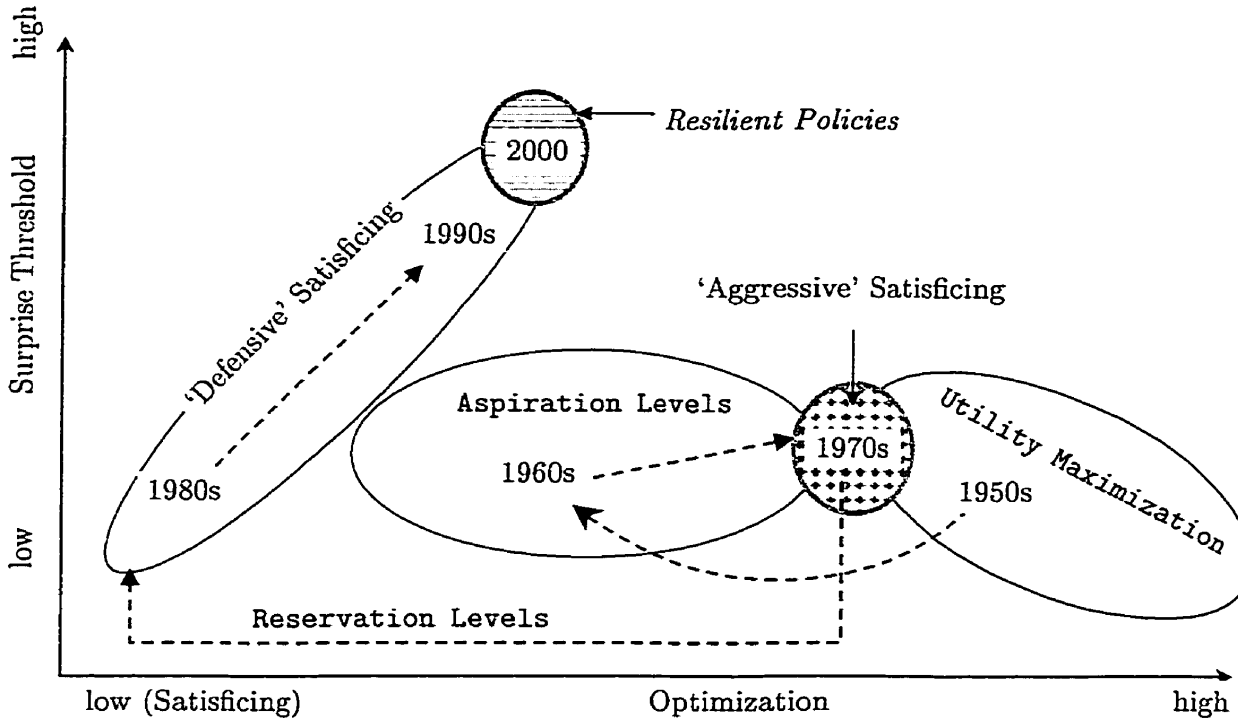
> survive in the face of adversity ... (It) absorbs a substantial share of the potential variability in the firm's environment ... (playing) both a stabilizing and adaptive role (pp 36-38)

And they conclude that organizational slack is not imposed by management but generated spontaneously in the healthy dynamics of a robust business environment.

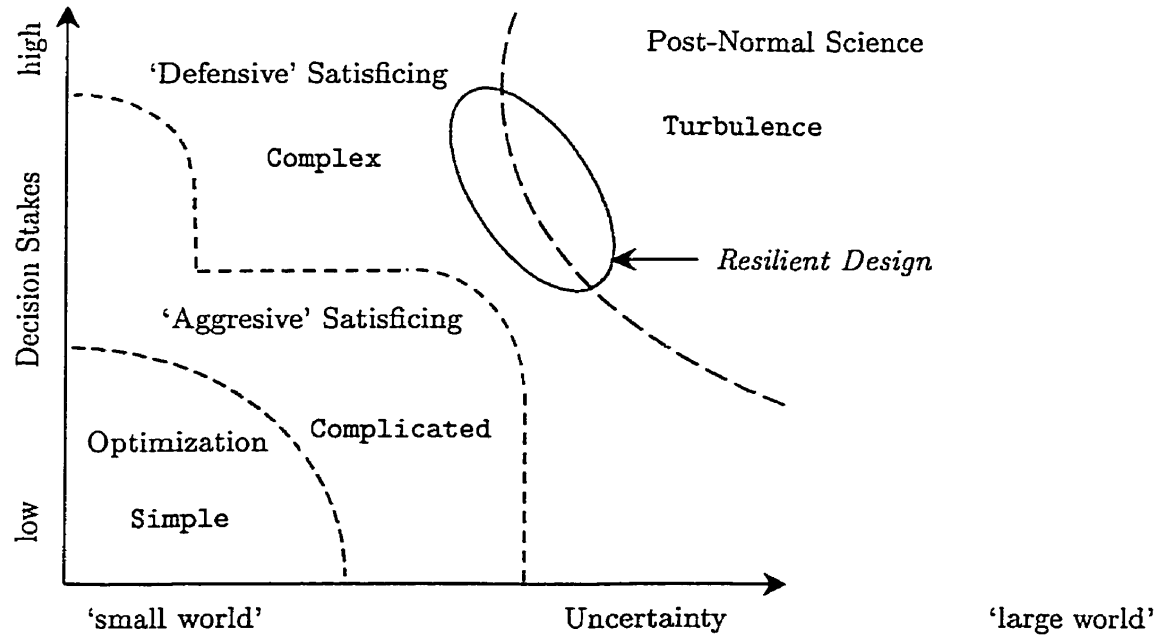
The present emphasis on decision making under uncertainty can at least partly be seen as a reaction to the mainstream determinism of the 1960s and early 1970s. Tracing research endeavors back into the nineteenth century Beck (1987) and Brush (1995) argue that the recent emphasis on uncertainty can be interpreted as a swing of the pendulum "away from determinism and toward indeterminism". Fig 7.3a illustrates the author's interpretation of how decision making paradigms have evolved from the use of utility maximization in the 1950s, to 'aggressive' satisficing in the 1970s and 'defensive' satisficing in the early 21st century. The move toward 'defensive satisficing' reflects the fact that real-world problems are pressure-packed, ill-defined, dynamic, and inherently complex: the number of components and the number of ways of combining them are so large that searching for optimality is unrealistic.

'Defensive' satisficing is the best approach for designing policies that are robust to uncertainty (possess a high 'surprise threshold') and safeguard against ecological degradation (Williams et al., 1997; Prato, 1999). This is particularly important as decision stakes in modern problems are often quite high (Fig 7.3b). Satisficing is a central theme in the behavioral approach to economics, which studies the actual decision making process. For many firms, organizations, and individual decision makers, a solution that is as close as possible to a goal is more acceptable than an optimal one: decision makers routinely reject apparent optimal solutions for those that provide a minimum standard of satisfaction, often referred to as "aspiration levels" (March, 1978).

While utility maximization may be a useful tool for representing "mass economic behavior" (Lewandowski et al., 1989) it has many limitations as a predictor of individual behavior, see for example Erlandson (1981) and Horsky and Rao (1984). For example, extensive studies of decisions to purchase (or not to purchase) flood insurance reveal behavior that cannot be reconciled with the maximization of utility. Rather, Kunreuther et al. (1978) found that people tend to ignore (hence, not insure against) low-probability, high-consequence events, unless they have had "rather direct past personal experience of them" (Simon, 1997). However, maximizing utility may be an appropriate for well-structured repetitive tasks, with well-trained decision makers, such as oil drilling decisions (Schoemaker, 1982).



(a) Evolution of Decision Making Frameworks



(b) Decision Making in a Turbulent Environment

Figure 7.3: Categories of Decision Making

There is a large body of research showing that satisficing is particularly relevant for MCDA (Hogarth, 1980; Kahneman et al., 1982). A standard experimental paradigm presents subjects with a number of multi-attributed alternatives, and allows them to obtain additional information about each until they make a choice (select an alternative) or exhaust the available information. Experiments show that decision makers usually satisfice, both in terms of failing to examine all the available information and in the sense of choosing an alternative as soon as one has been found to meet minimum requirements against the criteria of concern. Additional evidence suggests that business firms (Cyert and March, 1963; Bromiley, 1986) and individual decision makers (Clarkson, 1962; Bouwman and Ungston, 1982; Soelberg, 1966) rarely examine all alternatives in real-world situations, or pay attention to all potentially relevant variables. For example, when searching for their first job, Soelberg (1966) showed that business school students use a variety of rules of thumb to limit the list of firms they contacted and to choose among those who made offers to them. The very notion of *heuristics* to address a problem is inspired by the idea of satisficing, since "an acceptable solution in hand is better than an optimal solution in the bush" (Rajabi, 1997).

7.3 Convex Models of Uncertainty

The overwhelming volume of economic and environmental literature on the subject of decision making under uncertainty is probabilistic. However, a large body of evidence indicates that competent decision-makers often violate the axioms of expected utility and may lack the information, expertise, or time to perform a probabilistic analysis (Knight, 1921). In this section, uncertainty is viewed as an information gap: the disparity between what is known and what needs to be known in order to make a perfect decision. In environmental planning, industrial management, medical diagnosis and other areas, this gap is often quite substantial. But how can we measure the size of this gap? And is it possible to get a meaningful quantification of

uncertainty? Finally, are non-probabilistic approaches capable of modeling robustness to uncertainty?

Convex models of uncertainty are used to answer these questions. Convex models require fewer assumptions and less data than probabilistic models for their formulation and verification. Accordingly, somewhat weaker assertions, with "starker interpretations" (Ben-Haim, 1999), will be accessible with convex models than with probabilistic models. Nonetheless, meaningful results can be obtained. An additional motivation for the non-probabilistic quantification of uncertainty arises in situations where critical events have extremely low probabilities, which are difficult for decision makers to interpret. Though the mathematical formulation of convex models emerged in control theory (Schweppe, 1973), seismic design of structures (Drenick, 1968), nuclear measurements (Ben-Haim, 1985), and mechanical analysis (Ben-Haim and Elishakoff, 1990), it matches an intuition of uncertainty which is prevalent among economists and environmental managers.

Some scholars, such as K.W. Hipel of the University of Waterloo, Canada and Y. Ben-Haim, Technion, Israel prefer the term "information-gap" (or "info-gap") models (in place of convex models) to emphasize the fact that "uncertainty is the complement of knowledge. It is the gap between what is known and what needs to be known to make correct decisions" (Mack, 1971). John Kenneth Galbraith explains the importance of this information gap in the context of complex industrial organizations: "the difference between the amount of information required to perform the task and the amount of information already possessed by the organization" (Galbraith, 1973). Convex models can formally model uncertainty and provide insights into the problems that were once thought too complex and unpredictable to analyze.

7.3.1 Lake modeling and Convex Models

Consider the following, not atypical, modeling problem: assessing the impact of alternative policies to improve the water quality in Lake Erie by reducing

eutrophication and algae production (Nelson, 1976; Simon, 1997). A common approach to addressing this problem is the construction of a complex simulation model of the lake and the adjoining watershed (a total area of some thousands of square miles). Next, a grid can be imposed on this watershed area and equations constructed for each cell to estimate phosphate production (on land) and algal growth (in the lake). The grid will need to be quite fine to account for important point sources (*e.g.* city sewer outfalls) and complex currents that redistribute material over large distances. Such a model will not only be enormous (tens of thousands of parameters) but it will also contain equations that represent poorly understood mechanisms (such as biochemical aspects of eutrophication processes).

Moreover, data on phosphate sources, such as agricultural runoff are notoriously inaccurate. And developing a forecasting model which operates dynamically over time adds additional complexity. Information on socio-economic patterns and the biological effects of pollution is so inexact it makes no sense to attempt more than order-of-magnitude estimates. Given the degree of crudeness in biological and social models, it is unwise to model the entire system in great detail. Modeling in such complex situations may call for "little more than back-of-an-envelope estimates" (U.S. National Academy of Sciences and National Academy of Engineering, 1974). To make 'sense' of overwhelming model complexity in the Lake Erie system, Kay and Regier (1997) used a heuristic qualitative model in the form of an "impressionistic sketch" based on their two-attractor catastrophe model (Regier and Kay, 1996). Using concepts from 'complex systems thinking' (involving terminology such as multiple equilibria, positive feedback, and attractors) Kay and Regier (1997) qualitatively described two alternative 'systemic states' of Lake Erie.

7.3.2 Theory of Convex Modeling

The convex modeling approach to analyzing phosphorus levels in Lake Erie is now described. Assume that the nominal (typical/anticipated) phospho-

rus levels in Lake Erie are given by $\overline{P}(t)$, a known function. The actual phosphorus level, $P(t)$, deviates by an unknown amount from the expected phosphorus level $\overline{P}(t)$. This information may be quantified in an information-gap model of uncertainty. Consider the set of all phosphorous-functions $P(t)$ whose deviation from the nominal function $\overline{P}(t)$ is bounded by α:

$$\mathcal{R}(\alpha, \overline{P}) = \left\{P(t) : \left|P(t) - \overline{P}(t)\right| \leq \alpha\right\}, \quad \alpha \geq 0 \tag{7.10}$$

$\mathcal{R}(\alpha, \overline{P})$ is a set of functions that contains all phosphorus functions consistent with our prior information, where α is the *uncertainty parameter*, expressing the (unknown) phosphorus level.

As explained by Ben-Haim (1996) this information-gap uncertainty model, $\mathcal{R}(\alpha, \overline{P})$, is a family of nested sets for $\alpha \geq 0$. This means that $\mathcal{R}(\alpha, \overline{P}) \subseteq \mathcal{R}(\beta, \overline{P})$ if $\alpha \leq \beta$. For fixed α, the set $\mathcal{R}(\alpha, \overline{P})$ represents a degree of uncertain variability in the lake's phosphorous level $P(t)$. The greater the value of α, the greater the possible variation of phosphorus, so α, the uncertainty parameter, expresses the information gap between what is known ($\overline{P}(t)$) and what needs to be known for an ideal solution (the exact function $P(t)$). Robustness to uncertainty underlies the convex modeling appraoch: specified goals are attained, while at the same time the decision-maker's immunity to uncertainty is maximized.

Convex modeling is a stark theory of uncertainty, motivated by a severe lack of information. It does, however, have its own particular subtlety. It is facile enough to express the idea that uncertainty may be either pernicious or propitious. That is, uncertain variations may be either adverse or favorable: the *robustness function* is the greatest level of uncertainty consistent with no-failure; while the *opportunity function* is the least level of uncertainty which entails the possibility of sweeping success. If q is a vector of parameters such as time, design variables, and model parameters, the robustness and opportunity functions can be expressed as the maximum or minimum of a

set of α-values:

$$\widehat{\alpha}(q) = \max\{\alpha : \text{ minimal requirements are satisfied}\} \quad (7.11)$$

$$\widehat{\beta}(q) = \min\{\alpha : \text{ sweeping success is obtained } \} \quad (7.12)$$

The robustness function $\widehat{\alpha}(q)$ is the immunity against failure, so a large value of $\widehat{\alpha}(q)$ is desirable. In contrast, the opportunity function $\widehat{\beta}(q)$ is the immunity against sweeping success, so a small value of $\widehat{\beta}(q)$ is desirable.

Quite often the degree of success is assessed by a scalar reward function $R(q, u)$ which depends on the vector q of actions, decisions and model parameters as well as on an uncertain quantity u whose variations are described by an information-gap model $\mathcal{U}(\alpha, \overline{u})$. The minimal requirement in Eq 8.8 is that the reward be no less than a critical value r_c. Likewise, the sweeping success in Eq 8.9 is attainment of the "wildest dream" reward r_w. The robustness and opportunity functions can now be expressed more explicitly:

$$\widehat{\alpha}(q, r_c) = \max\left\{\alpha : \min_{u \in \mathcal{U}(\alpha, \overline{u})} R(q, u) \geq r_c\right\} \quad (7.13)$$

$$\widehat{\beta}(q, r_w) = \min\left\{\alpha : \max_{u \in \mathcal{U}(\alpha, \overline{u})} R(q, u) \geq r_w\right\} \quad (7.14)$$

As explained elsewhere (Ben-Haim, 1998), the robustness function $\widehat{\alpha}(q, r_c)$ decreases monotonically in the minimum required reward r_c. This expresses the trade-off between demanded reward and immunity to uncertainty: if large reward is required then only low immunity to uncertainty is possible. Conversely, the opportunity function $\widehat{\beta}(q, r_c)$ increases monotonically in wildest-dream reward r_w: sweeping success cannot be attained at low levels of ambient uncertainty. This is illustrated in Fig 7.4.

7.3.3 Convex Models and MCDA

Suppose that prior knowledge exists about nominal (anticipated) attribute levels for an alternative j, i.e.: the vector $\overline{x}_j = (\overline{x}_{1j}, \overline{x}_{2j}, \cdots, \overline{x}_{nj})$ is known, but very little is known about how the actual attribute values will deviate

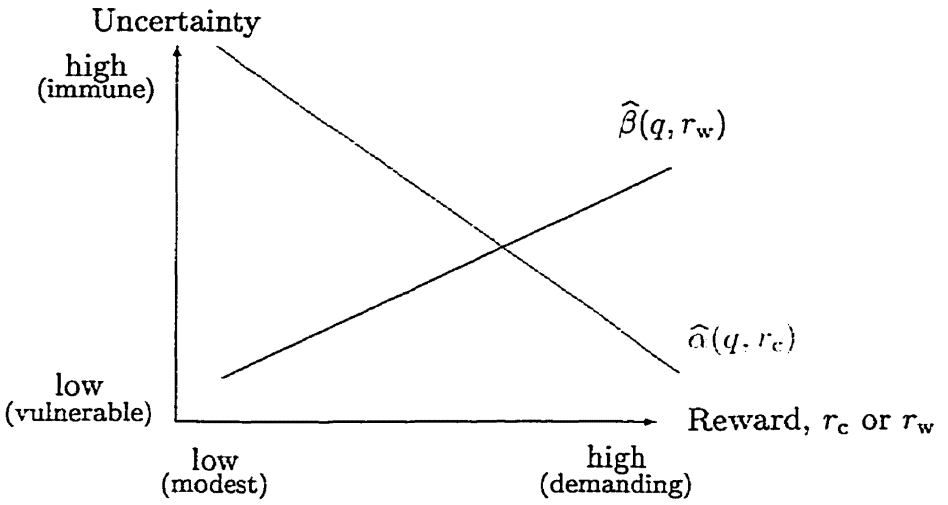


Figure 7.4: Robustness and opportunity curves (schematic).

from these nominal levels. A convex model determines the robustness to variability of the jth policy alternative by considering the following three components:

- A **decision model** to evaluate the overall value of each alternative, $V(\boldsymbol{x}_j)$. In this section an additive value function model is employed.
- The **failure criterion**, or conditions under which the alternative does not meet minimum requirements: the failure region may be written as $V(\boldsymbol{x}_j) < u_{cr}$.
- The **uncertainty model**, or quantification of the variability inherent in the attribute levels. For the jth alternative, uncertainty can be modelled as a solid sphere centered at the point $\overline{\boldsymbol{x}}_j = (\overline{x}_{1j}, \overline{x}_{2j}, \cdots, \overline{x}_{nj})$, with radius α. This set, of the form

$$S_j(\alpha, \overline{x}_{ij}) = \left\{ \boldsymbol{x}_j : \sum_{i=1}^{n} (x_{ij} - \overline{x}_{ij})^2 \leq \alpha^2 \right\}. \tag{7.15}$$

  is more and more likely to contain the actual attribute levels as α increases.

Define J to be the set of decision alternatives, of which the decision maker must select one. The evaluation of policy alternative $j \in J$ is described by a vector of indicators, $\boldsymbol{x}_j = (x_{1j}, x_{2j}, \cdots, x_{nj})$, where n is the number of indicators, and x_{ij} is the performance level of alternative j on indicator i. Let $v_i(\cdot)$ be the value function for indicator i. An amalgamation rule combines consistently scaled component (marginal) value functions, $v_i(x_{ij})$, into an overall index of value or worth, $V(\boldsymbol{x}_j)$. This is achieved most often using a linear additive model, in which overall value is the weighted sum of scaled indicators:

$$V(\boldsymbol{x}_j) = \sum_{i=1}^{n} k_i\, v_i(x_{ij}), \qquad j = 1, \cdots, m. \tag{7.16}$$

The constants k_i rescale the indicators to be comparable, while at the same time indicating their relative importance. In order for the linear additive model to be a valid representation of the overall objective, the indicators should be preferentially independent (Keeney and Raiffa, 1976), meaning that the level of any specific indicator does not depend on the levels of the other indicators.

7.3.4 Robustness of Policy Alternatives

Here the uncertainty model is described in more detail. Suppose that we have some prior knowledge about nominal (anticipated) indicator outcomes for an alternative j, i.e.: the vector $\overline{\boldsymbol{x}}_j = (\overline{x}_{1j}, \overline{x}_{2j}, \cdots, \overline{x}_{nj})$ is known, but that we know very little about how the actual indicator values will deviate from the nominal. A simple uncertainty model based on this information states that each outcome, x_{ij}, may deviate by an unknown fraction of its nominal value, $\overline{x}_{ij}$. Consider the following "uniform-bound" information-gap model for uncertain indicator levels, $\mathcal{H}_j(\alpha)$, a family of nested sets for each $\alpha \geq 0$ defined by

$$\mathcal{H}_j(\alpha) = \left\{ \boldsymbol{x}_j : \ \frac{|x_{ij} - \overline{x}_{ij}|}{\overline{x}_{ij}} \leq w_i \alpha, \quad i = 1, \ldots, n \right\}. \tag{7.17}$$

Each element x_{ij} of the vector $\boldsymbol{x}_j \in \mathcal{H}_j(\alpha)$, representing the level of the ith indicator for alternative j, must lie within a specific interval according to

$$\overline{x}_{ij} - w_i \overline{x}_{ij} \alpha \;\leq\; x_{ij} \;\leq\; \overline{x}_{ij} + w_i \overline{x}_{ij} \alpha. \tag{7.18}$$

In other words, $\mathcal{H}_j(\alpha)$ is the set of $\boldsymbol{x}_j$-vectors (attribute-vectors for alternative j), whose elements x_{ij} vary from their nominal values by no more than a fraction $w_i\alpha$. The uncertainty weights, $w_1, \ldots, w_n$ are positive numbers that express our prior information about the relative variability of the indicators. If no prior information about the relative variability of the attributes exist, then all the w_i's will equal unity. In the case where the ith attribute varies more than the others, then its uncertainty coefficient, w_i, should exceed 1. The uncertainty parameter, α, expresses the (unknown) degree of variation of the actual indicator outcomes x_{ij}. For $\alpha \geq 0$, the allowable uncertainty in x_{ij} increases with α. This is expressed by the "nesting" of sets, i.e. if $\alpha < \beta$, then $\mathcal{H}_j(\alpha) \subset \mathcal{H}_j(\beta)$. Since these sets are convex, this information-gap model is called a convex model of uncertainty.

The robustness of a policy alternative is the greatest value of the uncertainty parameter, α, that is consistent with the minimum required overall value (u_{cr}). We evaluate this as follows. First we define a set of "acceptable" α-values: those values that do not allow failure if the jth alternative is implemented. Consider the set

$$\mathcal{A}_j(u_{\text{cr}}) = \left\{ \alpha : \; \min \sum_{i=1}^{n} k_i \, v_i(x_{ij}) \geq u_{\text{cr}} \quad \forall \quad \boldsymbol{x}_j \in \mathcal{H}_j(\alpha) \right\}. \tag{7.19}$$

$\mathcal{A}_j(u_{\text{cr}})$ is the set of α-values for which all indicator-vectors $\boldsymbol{x}_j$ in $\mathcal{H}_j(\alpha)$ have overall value greater than or equal to u_{cr} for alternative j. The robustness of an alternative is the greatest acceptable value of the uncertainty parameter

$$\widehat{\alpha}_j = \max \{ \alpha : \quad \alpha \in \mathcal{A}_j(u_{\text{cr}}) \}; \tag{7.20}$$

thus, $\widehat{\alpha}_j$ is the maximum α-value consistent with achieving u_{cr} for all indicator outcomes in the uncertainty-set $\mathcal{H}_j(\alpha)$, if alternative j is selected. An alternative is 'robust' to uncertainty if, despite high levels of attribute uncertainty, it can achieve a minimum overall value (u_{cr}).

Simple Illustration of Convex Models

Consider two attributes, x_1 and x_2 with nominal attribute values $\overline{x}_{1j}$ and $\overline{x}_{2j}$, respectively on alternative j. Assume that there is twice as much variability in x_1 as in x_2, then $w_1 = 2$ and $w_2 = 1$. As a visualization aid, let $\alpha = 0.15$. Hence, from Eq 7.18, the allowable variation in attribute 1 is: $\overline{x}_{1j} - 0.3\overline{x}_{1j} \leq x_{1j} \leq \overline{x}_{1j} + 0.3\overline{x}_{1j}$. Similarly, the allowable variation in attribute 2 would be: $\overline{x}_{2j} - 0.15\overline{x}_{2j} \leq x_{2j} \leq \overline{x}_{2j} + 0.15\overline{x}_{2j}$. This gives rise to the rectangles in Fig 7.5. In other words, $\mathcal{H}_j(0.15)$ is the set of $\boldsymbol{x}_j$-vectors (attribute-vectors for alternative j), whose elements x_{1j} and x_{2j} vary from their nominal values by no more than the fractions 0.3 and 0.15, respectively. Similarly, $\mathcal{H}_j(0.2)$ is the set of $\boldsymbol{x}_j$-vectors whose elements x_{1j} and x_{2j} vary from their nominal values by no more than the fractions 0.4 and 0.2, respectively.

Since $\mathcal{A}_j(u_{\text{cr}})$ is the set of α-values for which all indicator-vectors $\boldsymbol{x}_j$ in $\mathcal{H}_j(\alpha)$ have overall value greater than or equal to u_{cr} for alternative j, the situation shown in Fig 7.5 has $\mathcal{A}_j(u_{\text{cr}}) = \{\alpha : \ 0 \leq \alpha \leq 0.3\}$. This implies that $\widehat{\alpha}_j = 0.3$. Recall that the alternative with the highest $\widehat{\alpha}$ will be the most robust to variability.

7.3.5 Forest Management and Convex Models

Researchers have shown that the coniferous forest ecosystem of New Brunswick consists of multiple stable states (Clark et al., 1979; Ludwig et al., 1978; Holling, 1988): the system moves in a discontinuous fashion between maximum foliage just before an outbreak and minimum foliage immediately after the outbreak. Essentially, a lower equilibrium density for budworm is established by insectivorous birds whose populations control budworm populations in younger stands. However, as the trees mature, the accumulation of foliage volume impedes the birds' ability to search for budworm. Eventually, a higher equilibrium density for budworm (an outbreak) is established, followed by a budworm dieoff (low equilibrium density).

Nominal indicator (performance) levels and value functions for each in-

dicator in this problem were taken from the research of Clark et al. (1979). Levy et al. (2000a) provides additional detail about the spruce budworm case study as it pertains to convex models. Specifically, the robustness to uncertainty of three policy alternatives were considered: Historical Management ($\widehat{\alpha}_1$), Winkler-Dantzig ($\widehat{\alpha}_2$), and Branch-Density Hybrid ($\widehat{\alpha}_3$). It is assumed that the uncertainty weights are selected as $w_1 = 2$, $w_2 = 1$, $w_3 = 1$, $w_4 = 6$, and $w_5 = 1$ (*i.e.* Forest Volume and Recreational Quality indicators tend to vary two and six times more than the other attributes).

Robustness curves are generated by the REAL (Robust Environmental Analysis for Lokahi) decision support system. Fig 7.6 shows that the *Branch-Density Hybrid* alternative clearly dominates the other two with respect to immunity-to-uncertainty when $u_{\text{cr}} > 0.45$ ($u_{\text{cr}} \cong 0.45$ at the upper crossover point), while for lower values of u_{cr}, the *Historical* alternative is most robust ($u_{\text{cr}} \cong 0.38$ at the lower crossover point). It is also important to recognize that $\widehat{\alpha}_j$ is a decreasing function of u_{cr}: this represents a tradeoff between minimum required overall value and immunity to uncertainty (Ben-Haim, 1998).

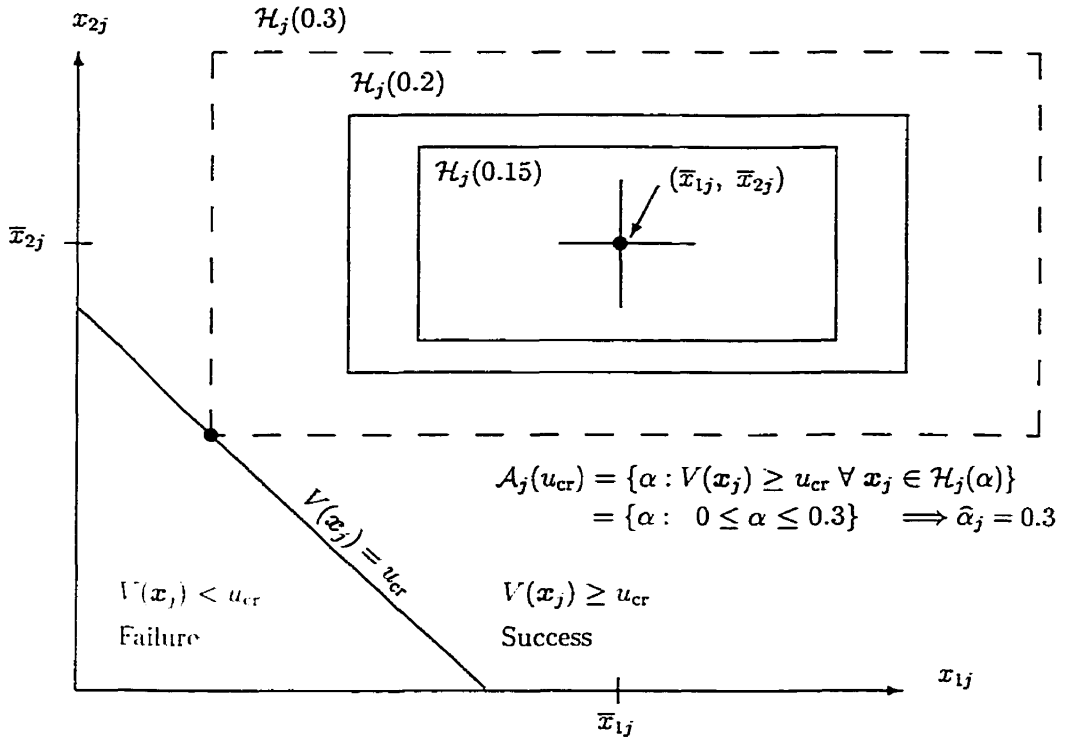


Figure 7.5: Graphical interpretation of the convex model for the jth alternative

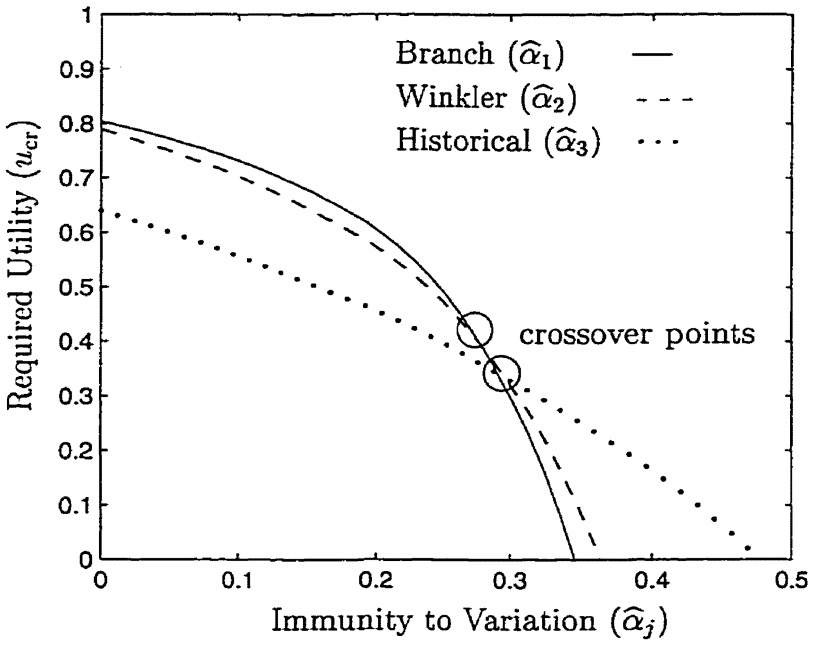


Figure 7.6: Minimum Required Utility (u_{cr}) vs. Maximum Robustness ($\widehat{\alpha}_j$)

7.3.6 Conclusions

Recognizing that environmental conditions, political structures, and management objectives may change significantly over time, this chapter formally models the robustness, flexibility, and adaptability of complex, large-scale systems. An multiple criteria information-gap procedure is put forth for identifying policy alternatives that, while capable of coping with attribute variability, still achieve minimum socio-economic and ecological requirements. The management of water quality in Lake Erie and spruce-budworm outbreaks in New Brunswick are used as illustrative examples. Numerical and theoretical results show how the minimum required return and the available prior information determine which policy alternative can best cope with uncertainty in environmental variables. Moreover, the constructed robustness curves assess the global sensitivity of alternatives to uncertainty.

The proposed multiple criteria info-gap model is entirely non-probabilistic and constitutes a viable medium for integrating environmental indicators, conflicting objectives, and ambient uncertainty in a complex decision context. In addition, the info-gap approach operationalizes the notion of 'satisficing' (finding a solution that is 'good enough' for the problem at hand). Particularly under high levels of uncertainty, the art of the feasible (satisficing) may be more helpful than the art of the ultimate (optimization).

Chapter 8

Conclusions and Future Research

Social change and technological uncertainty continue to accelerate along the information superhighway of modernity: "technological upheaval", "information revolution", "organizational instability", and "economic disequilibrium" are the norm, often contributing to social malaise, violent conflict, and economic paralysis (Schon, 1971; Rifkin, 1981). While periods of upheaval can be disconcerting, they provide fertile soil for the growth for new ideas: the Newtonian model arose at a time when feudalism in Western Europe was crumbling — the social system, was, so to speak, "far from equilibrium".

8.1 Main Contributions of the Thesis

Dyer et al. (1992) emphasize that one of the most difficult and important topics in operations research is the development of new techniques to realistically model uncertainty in a multiple criteria context. To this end, new MCDA uncertainty approaches and computer tools were developed in a variety of fields, from Bayesian statistics and stochastic calculus to info-gap models. Specific contributions are as follows:

- An overview of environmental optimal control theory
- An overview of discrete MCDA under certainty and uncertainty
- The inclusion of extinction risk in optimal control theory
- The inclusion of a 'preservation value' in optimal control theory
- A determination of the relationship between cost, price, discount rate and optimal population level; a determination of how the optimal population level changes over time
- The development and application of sustainability indicators
- The implementation of MCDA interval methods
- A review of adaptive management, robustness, resilience, and flexibility from a decision analytic perspective
- Generalizations of Streeter-Phelps equations under uncertainty
- Formulation of moment equations for Streeter-Phelps equations under uncertainty
- Development of the SEAL model to solve stochastic differential equations. The author programmed an explicit order 1.5 strong scheme to integrate Îto SDEs. The algorithm is found in Section 11.2 of Kloeden and Platen (1992).
- Use of SEAL model to identify the expected minimum DO level. This information is then used to determine the maximum organic waste material that can be discharged into a receiving water.
- The design of MCDA info-gap models
- An exploration of the axiomatic foundations of MCDA info-gap models

- The use of MCDA info-gap models to capture risk attitudes and aspiration levels
- A discussion of non-probabilistic approaches to modeling uncertainty
- The use of Bayesian parameter estimation in the context of BOD decay
- The use of the MEAL decision support system to elicit to elicit marginal distributions given a posterior distribution
- The development of an approximation scheme to replace white (theoretical) noise with colored (physical) noise.
- The application of Îto and Stratonovich calculus to model the water quality models of Streeter and Phelps (1925)
- The derivation of temporal expectation models for BOD, NOD, and DO
- The use of the SEAL model to solve a wide range of environmental problems, from water quality modeling to the extinction of fish species.
- A comprehensive analysis of the 'first passage time problem' for environmental systems.

8.2 Compare and Contrast Techniques

It is a valuable exercise to compare and contrast the various MCDA techniques used in this thesis in order to

- identify the relative strengths and weaknesses of the approaches
- determine which approaches are complementary

Recent years have witnessed a growing interest in developing multistrategy MCDA systems that can integrate two or more techniques into a single system. Such systems have the potential for greater competence and versatility

than monostrategy approaches because they can solve a wider range of problems using the complementarity of individual methodologies. Since human learning is clearly multistrategy, decision support systems which allow for a variety of MCDA approaches are most likely to accurately replicate the heuristics of human problem solving. The author agrees with the philosophy of Hämäläinen and Salo (1997) that "the separation of competing [multiple criteria] methodologies into isolated schools of thought is regrettable". In this section, a number of techniques are compared pairwise in order to determine their relative strengths and weaknesses, including:

- Frequentist and Bayesian Approach
- CBA and MAVT
- MAVT and the Elimination Method
- MAVT and Interval Methods
- MAUT and ELECTRE
- MAUT and MAVT
- Satisficing and Optimization
- Probabilistic Approaches and info-gap methods

8.2.1 Frequentist and Bayesian Inferencing

In the frequentist interpretation of statistical inferencing, probability is the result of an infinite series of trials conducted under identical conditions. Whereas to a Bayesian, probability represents the observer's degree of belief (or an organized appraisal in light of the data). Hence, Bayesian inferencing is particularly useful when data are hard to obtain or when consensus cannot be achieved (experts may legitimately assign different probabilities to the same event).

A central assumption underlying the frequentist approach is that there is a true, fixed value for each parameter of interest, and the expected value of this parameter is the average value obtained by repeated sampling. On the other hand, the Bayesian school assumes that parameters are random variables. Using the Bayesian approach the author determined the joint probability density function for the parameters in the BOD decay function.

Other distinctions between the Bayesian and frequentist approach include the notion of a "k% confidence interval". In the frequentist view, a k% confidence interval will include the true value of a given parameter in k% of all possible samples. Whereas, the Bayesian interpretation emphasizes that k% of the possible parameter values will fall within the confidence interval.

8.2.2 Comparison of CBA and MAVT

CBA requires *monetary values* for environmental, recreational, and social considerations. For this reason a CBA analysis is highly tractable and all criteria can be easily compared. However, it is difficult to assign a monetary value for many criteria: specifically, how to put a dollar value on the life of a human?

Multi-attribute value theory (MAVT), on the other hand, allows each criteria to be measured in different units (lives saved, acres of forest logged, *etc.*) by constructing value functions for each criteria. Value functions map the scores profile of an alternative into a value, usually normalized from 0 to 1. The 0 and 1 values are associated with two real or hypothetical score profiles, which represent the best and worst situations considered. In summary, MAVT is often used to convert different kinds of measurements to a standardized scale. Then the standardized variables can be manipulated as if they were a common measure. In the simplest case, individual value functions can be combined with an additive weighted combination.

There are several commonly cited advantages of MAVT over CBA (Naiman et al., 1997; Cameron, 1997). First, CBA compromises the "authenticity, richness, and quality" (Prato, 1999) of decision making since an inherently

multiple criteria problem (with socio-cultural dimensions) must be analyzed with a single monetary criterion (net present value). On the other hand, MAVT encourages one to reflect upon personal values which can aid in the systematic creation and evaluation of meaningful alternatives for a decision problem.

Second, CBA is often accomplished using Willingness to Pay (WTP) estimates of ecological services elicited with Contingent Valuation (CV) methods (Feather et al., 1995). However, respondents often have trouble stating their WTP and CV approaches are often controversial (Cameron, 1997). In addition, estimating dollar values for goods and services is particularly impractical in developing countries where most business transactions and social activities occur outside of any formal market setting (Bjornstad and Kahn, 1996).

Third, CBA requires discounting all benefits and costs of investments that occur over a given planning horizon. However, discounting cash flows from ecological investments can be problematic, particularly those that have long-term, uncertain benefits and high, short-term costs. For these investments, discounting reduces their net present value and may result in underinvestment in ecological protection or preservation. This problem arises, for example, when evaluating policies to reduce greenhouse gas emissions (Perrings, 1994).

Fourth, MAVT is capable of addressing fairness issues while CBA cannot. It is well-known that many citizens of the Third World (particularly women, children, and the elderly) do not have adequate access to health care, food supplies, and employment opportunities. In developed nations, some segments of the population (often minorities, the poor, or rural residents) are disproportionately exposed to hazardous substances (Mohai, 1995).

Fifth, CBA does not address 'procedural fairness' (Lind et al., 1990): what are the procedures by which decisions are reached? Joubert et al. (1997) argue that MAVT is a more appropriate tool than CBA with respect to procedural justice, particularly for evaluating environmental projects that

generate significant social and ecological externalities. In summary, MAVT is a superior technique to CBA for environmental assessment and management since the former can capture the complex social and ecological dimensions of sustainable development while the latter is a single criterion tool primarily used to measure economic efficiency.

8.2.3 Comparison of MAVT and the Elimination Method

In many decision situations alternatives cannot be evaluated entirely in quantitative form. In addition, numeric weighting factors are often unavailable to express the priorities of the objectives. In this situation, noncompensatory models such as the Elimination Method offer some capability of placing a number of alternatives in an order of preference.

MAVT is limited to compensatory situations (in which one criterion can be traded off against another to arrive at an overall score). In MAVT, individual value functions are combined into an overall value function and weights must be assessed. On the other hand, in the Elimination Method, one attribute has overriding importance: decisions are made on the basis of it alone.

A disadvantage of lexicographic preference is that significant emphasis is placed on the primary attribute, to the exclusion of other issues. Hence, the Elimination Method and other lexicographic methods should be used judiciously to ensure that all possible information is used in the analysis. Necessary conditions for use of the Elimination Method include ordinal or cardinal preferences for alternatives (for each objective) and an ordinal ranking of the criteria.

8.2.4 Comparison of MAVT and Interval Methods

Interval judgments can be used to capture the uncertainty in a decision maker's preferences. This allows the decision maker to make approximate ratio statements as intervals of values on a ratio scale. With interval judgments,

the decision maker can make 'strength of preference' statements consistent with 'natural' verbal expressions such as "the i-th subattribute is two to three times more important than the j-th subattribute"; this is abbreviated as $I_{ij} = [l_{ij}, u_{ij}] = [2, 3]$.

It appears that interval methods are more flexible than MAVT approaches because they can explicitly allow for uncertainty in the weights and scores of a decision problem. Of course, the robustness of an MAVT solution can be investigated by performing a a traditional sensitivity analysis (investigating how small changes to weights, scores, and other input variables affect the final ranking of alternatives). However, sensitivity analyses are often *ad hoc* and inadequate for two important reasons. First, traditional sensitivity methods occupy a separate phase of the MCDA cycle, usually at the "back end" (Chávez and Shachter, 1998). They inform the decision maker which uncertainties are important, but do not provide the necessary feedback to the decision maker ("at the front end"). Second, while a sensitivity analysis can provide clues as to how changes in model inputs will impact the recommended action, they do not represent a comprehensive basis for measuring the robustness of a proposed ranking.

Arbel (1989; 1991) interpret interval judgments as linear constraints on the local weights. Arbel emphasizes the definition of interval judgments: A given local weight vector $\boldsymbol{w} = (w_1, \cdots, w_n)$ is consistent with the judgment $I_{ij} = [l_{ij}, u_{ij}]$ only if it satisfies the constraints

$$l_{ij} w_j \leq w_i \leq u_{ij} w_j \tag{8.1}$$

In practice, some analysts have combined MAVT with the interval techniques. As an example one might perform a tradition MAVT analysis allowing for 'interval uncertainty' in the weights. Consider an example with three weights: w_1 (cost factors); w_2 (environmental quality); and w_3 (social issues). Fig 8.1 shows the feasible region S (shaded area) based on the following verbal statements:

- "The social factor is at least as important, but no more than two times

more important than the cost factor"; this is abbreviated as $I_{31} = [l_{31}, u_{31}] = [1, 2]$ and implies that $w_1 \leq w_3 \leq 2w_1$

- "The social factor is at least as important, but no more than three times more important than the environmental factor"; this is abbreviated as $I_{32} = [l_{32}, u_{32}] = [1, 3]$, implying $w_2 \leq w_3 \leq w_2$
- The above two constraints can be combined to obtain interval judgments comparing cost (w_1) and quality (w_2): cost is at least half but no more than three times as important an attribute as quality". This statement is derived from $w_1 \leq 3w_2$ and $w_2 \leq 2w_1$, *i.e.* $0.5w_2 \leq w_1 \leq 3w_2$ which implies $I_{12} = [0.5, 3]$.

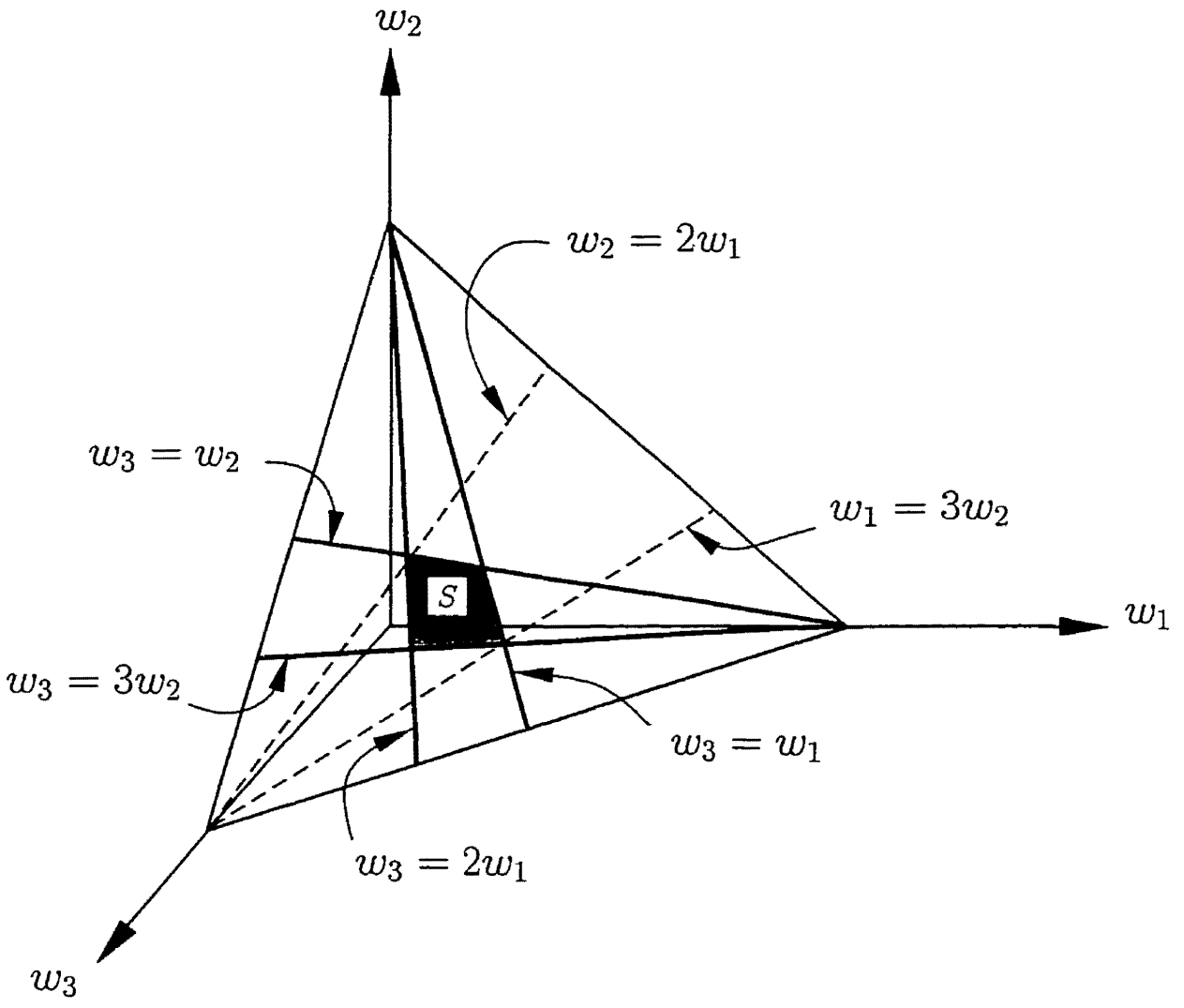


Figure 8.1: Illustration of feasible region

8.2.5 Contrast MAUT and MAVT

Two fundamentally different measurement approaches are used to model multi-attribute preferences. The first, multi-attribute value function theory

(MAVT), founded on difference measurement, asks for judgments about the strength of preference to derive a *value function*, $v(x)$. Value function theory, often referred to as "riskless" decision analysis has its roots in the theory of ordered value differences (Frisch, 1926; Pareto, 1927; Dyer and Sarin, 1979a). Both ordinal and cardinal value functions are widely used.

The second approach to modeling multiattribute preferences is based on EU theory to handle multiattribute decisions under "risk": preferences among gambles are used to construct a *utility function* over multiattribute outcomes. This approach is known as Multi-attribute Utility Theory (MAUT). MAUT theory has been widely accepted as the preeminent normative standard for rational decision making under risk and uncertainty. Under MAUT, a decision maker who subscribes to the von Neumann and Morgenstern (1947) axioms will select an alternative that maximizes her expected utility. von Neumann and Morgenstern proved that if a person's preferences conform to the axioms of complete ordering, transitivity, independence, and continuity then two important consequences follow. First, a decision maker's *utility function* can be inferred by observing her choices among risky prospects. Second, a decision maker's choices can be described as if she were following a decision rule of maximizing expected utility.

Hence, MAUT applies to outcomes of decision alternatives that are uncertain, while MAVT applies only to decision making under certainty (the outcomes are known) and hence does not require probability distributions of outcomes (Debreu, 1959; Frisch, 1964; Alt, 1971). As summarized by Krzystofowicz (1983), "[a value function] v results from an axiomatization of riskless decision making, while [a utility function] u holds under an axiomatization of risky decision making." However, uncertainty can be included in a value function analysis by performing an *ex-post facto* sensitivity analysis (van Herwignen et al., 1995; Insua and French, 1991; Insua, 1990). In short, MAVT uses value functions under certainty, denoted $v(x)$, while MAUT uses utility functions under risk, $u(x)$.

One weakness of MAVT is that it cannot capture preference information

and risk attitudes. On the other hand, there are several advantages of MAVT to MAUT.

First, value functions are easier to elicit from decision makers than utility functions. The use of MAUT is an arduous, time consuming process (Stewart, 1995; Insua and French, 1991) and involves "non-intuitive probabilistic concepts of gambling and indifference between lotteries" (Barzilai, 1996).

Second, when applying MAUT, analysts consistently assume that criteria are *utility-independent* (Keeney and Raiffa, 1976). In practice, the elicitation of utility functions is a difficult cognitive task even under the assumption of utility independence so weaker assumptions are unlikely to be invoked very often, if at all. Third, in group decision making, it may be desirable only to have members' inputs regarding the $v(x)$ functions, but not their risk attitudes (which are often centrally determined).

Fourth, the experimental evidence has shown that MAUT is a poor descriptive theory of decision making under uncertainty (Schoemaker, 1980; French, 1986): informed and educated decision makers persistently and systematically violate the axioms of EU theory. As MacCrimmon and Larsson (1979) note: "many careful, intelligent decision makers seem to violate some axioms of expected utility theory, even upon reflection of their choices." Morgenstern (1972) himself identifies an imposing research agenda that he feels must be addressed before EU theory can be generally applicable. These researchers have concluded that MAUT, a normative tool, should be complemented by descriptive theory, aimed at capturing actual cognitive, social, and behavioral decision processes (Kahneman and Tversky, 1979; Weber and Borcherding, 1993).

Comparing Utility and Value functions

Theoretical relationships between decomposed utility u and value functions v are herein described. To illustrate, define the vector of attributes, $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$, where n is the number of attributes, and x_i is the performance level of the ith attributes. Let $u_i(\cdot)$ be the marginal utility function

for attribute i. Several decompositions of $u(\boldsymbol{x})$ have been developed (Keeney and Raiffa, 1976) of which the additive and multiplicative are most frequently employed:

$$u(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} k_i\, u_i(x_i), \tag{8.2}$$

$$1 + ku(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} [1 + k\, k_i\, u_i(x_i)] \tag{8.3}$$

where

$0 \leq u_i(\cdot) \leq 1$	is a single attribute utility function
$0 \leq k_i \leq 1$	is a scaling constant
$-1 < k < 0;\ \ k > 0$	is a parameter
$u(\cdot)$	is the overall utility

These decompositions rest on the assumption that preferential independence of attribute pairs and utility independence are satisfied. The additive model further requires additive independence. A value function v can broken down into additive and multiplicative representations similar to the decompositions of utility functions (Dyer and Sarin, 1979b):

$$v(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} w_i\, v_i(x_i), \tag{8.4}$$

$$1 + wv(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} [1 + w\, w_i\, v_i(x_i)] \tag{8.5}$$

where

$0 \leq v_i(\cdot) \leq 1$	is a single attribute difference value function
$0 \leq w_i \leq 1$	is a scaling constant
$-1 < w < 0;\ \ w > 0$	is a parameter
$v(\cdot)$	is the overall difference value function

Note the similarities between the additive and multiplicative forms of $v(\boldsymbol{x})$ and $u(\boldsymbol{x})$. In spite of these obvious structural similarities, there are no a priori reasons to assume that they are related by a simple functional form. In principle, the shape and aggregation form of u and v can be quite different. For example, v may be additive, while u may be multiplicative, or not decomposable at all. Establishing closed form functional relationships is, however, possible when special decompositional forms such as equations 8.2 to 8.5 are assumed. von Winterfeldt (1979) proved that value and utility functions must be related by a linear, logarithmic, or exponential transformation, if both are either additive or multiplicative. These results are summarized in Table 8.1.

Table 8.1: Theoretical relationships between decomposed utility and value functions

		Value Function, $v(\boldsymbol{x})$, is	
		Additive	Multiplicative
utility function, $u(\boldsymbol{x})$, is	Additive	$u = v$	$u = \frac{\ln(1+wv)}{\ln(1+w)}$
	Multiplicative	$1 + ku = (1+k)^v$	$1 + ku = (1+wv)^{\frac{\ln(1+k)}{ln(1+w)}}$

8.2.6 Comparison of MAUT and ELECTRE

The ELECTRE technique was developed by Benayoun et al. (1966) and Roy (1973); the name ELECTRE is an acronym for ELimination Et (and) Choice TRanslating algorithm. Proponents of outranking methods argue that their lack of an axiomatic foundation is compensated by their descriptive reality (Bouyssou, 1993). In ELECTRE, the decision maker must provide both a set of weights reflecting the relative importance of the objectives and numerical scores evaluating the alternatives. Information pertaining to the 'discordance' and 'concordance' indices is also required.

Several common criticisms of ELECTRE are:

- there are often no alternatives selected by ELECTRE;
- the methods lack a strong axiomatic basis;
- many input parameters are required which may have little intuitive meaning (such as the discordance and concordance thresholds);
- a consultant often adjusts the thresholds and the weights in order to obtain the desired solution;
- the method is quite complicated;
- if one possesses the information necessary for building a linear utility function, MAUT can be used in place of ELECTRE

While both ELECTRE and MAUT are time consuming and elaborate techniques MAUT can capture risk attitudes while ELECTRE cannot. On the other hand, ELECTRE techniques provide significant insights into the problem solving *process* that is not possible with MAUT. In addition, fuzzy information is readily incorporated into ELECTRE, while probabilistic information is required for MAUT.

8.2.7 Satisficing vs Optimizing

The info-gap concept implies a different policy-formulation paradigm from that offered by traditional optimization techniques, including return-on-investment approaches and cost/benefit analysis. Info-gap models promote a "robustness approach" which cannot provide the "optimal" answer or the alternative with the largest utility; rather, it suggests strategies that are adaptive and flexible.

Nobel Laureate H.A. Simon (Simon, 1957; 1958; 1979) argues that many important phenomena observed in human decision making are not explained by the theory of utility or profit maximization. Accordingly, he proposed a rationality framework for decision analysis called *satisficing* (a combination of the words satisfactory and sufficient). A decision maker who chooses the best available alternative according to some criterion is said to optimize; one who chooses an alternative the meets (or exceeds) specified criteria, is said to satisfice. Of course the satisficing solution is not guaranteed to be either unique or in any sense 'best'[1]. Info-gap modeling formally captures the satisficing framework.

Since the capacity of the human mind for formulating and solving complex problems is limited, Simon (1955) proposed that humans use 'levels of aspiration' so that decision tasks can be simplified. March (1978), Simon (1979), and March and Simon (1958) have both noted that satisficing involves a *bounded rationality*, in that:

- decisions occur in limited time frames,
- decision makers are unable to acquire all the information they need,
- decision makers are not aware of all of the things they need to know to make a decision.

[1]The term 'satisfice', which appears in the Oxford English Dictionary as a Northumbrian synonym for 'satisfy', was borrowed for this new use by Simon (1956).

The phenomena of satisficing runs counter to the assumption that humans maximize expected utility. As summarized by Eilon (1971): "optimizing is the science of the ultimate and satisficing is the art of feasible."

8.2.8 Probabilistic models vs Info-gap models

Flexibility Analysis and Info-gap models

Both flexibility analysis (Jones and Ostroy, 1984; Shachter and Mandelbaum, 1999) and info-gap models (Ben-Haim, 1998) emphasize notions of robustness to uncertainty: a flexible, robust solution may be preferred to one which is optimal in situations of extreme uncertainty and turbulent dynamics. In a flexibility analysis, a value function $v(d, \mathbf{X})$ specifies the score that results when action d is taken and action $x \in \mathbf{X}$ obtains. Because $\mathbf{X}$ is given probabilistically, the notation $P(\mathbf{X} \mid \xi)$ is used to specify the probability distribution of $\mathbf{X}$ conditional on ξ (the prior state of knowledge). Using these definitions, Chávez and Shachter (1998) define the flexibility of action d_i with respect to the random variable $\mathbf{X}$ as:

$$E_{\mathbf{X}}\left[\max_{d} v(d, \mathbf{X}) - v(d_i, \mathbf{X}) \mid \xi\right] \tag{8.6}$$

where subscripting E by $\mathbf{X}$ indicates that the expectation is taken with respect to $\mathbf{X}$. The most flexible (least brittle) alternative/action minimizes the quantity in Eq 8.6. Note that a flexibility analysis is designed for only probabilistic uncertainty, while an info-gap model is primarily designed for non-probabilistic information.

Info-gap models and EU theory

The overwhelming volume of economic and environmental literature on the subject of decision making under uncertainty is probabilistic. For example EU theory considers the situation in which a person will receive, for $i = 1, 2, \ldots, n$, a *reward* r_i with *probability* p_i. This is denoted as the *lottery* L:

$$L = (p_1, r_1;\ p_2, r_2;\ p_3, r_3;\ \ldots\ p_n, r_n). \tag{8.7}$$

However, a large body of evidence indicates that competent decision-makers may lack the information, expertise, or time to perform a probabilistic analysis (Knight, 1921). Info-gap models, on the other hand, require fewer assumptions and less data than probabilistic models for their formulation and verification. Accordingly, somewhat weaker assertions, with "starker interpretations" (Ben-Haim, 1999) are accessible with info-gap models than with probabilistic models. An additional motivation for the non-probabilistic quantification of uncertainty arises in situations where critical events have extremely low probabilities, since decision makers may have difficulty interpreting the difference between, say, the risk of 10^{-8} and 10^{-9}.

Info-gap set models measure uncertainty in terms of the size of the uncertainty parameter α. However, this is much weaker information than in probability theory or possibility theory where the distribution functions indicate recurrence-frequency. In info-gap set models of uncertainty the emphasis is on "cluster-thinking" (Ben-Haim, 1998) rather than on recurrence or likelihood. Given a particular piece of information an info-gap modeler might ask: what is the "cloud of possibilities" (Ben-Haim, 1999) consistent with this information? How does this cloud shrink, expand and shift as our information changes? What is the gap between what is known and what could be known?

(Ben-Haim, 1998) shows that the robustness function $\widehat{\alpha}(q, r_c)$ decreases monotonically in the critical reward r_c. This proposition expresses the trade-off between the critical reward r_c required by the decision maker and the immunity-to-failure $\widehat{\alpha}$ which he also demands. A large value of the critical reward entails a low value of immunity. Conversely, high demanded immunity leads to low guaranteed reward. This monotonic decrease in robustness is illustrated schematically in Fig 8.2 for the robustness functions of $\mathcal{X}$ and $\mathcal{Y}$.

As shown in Fig 8.2, employing an information system to replace $\mathcal{X}$ by $\mathcal{Y}$ results in an increase in the critical reward r_c which can be demanded at any given level of ambient uncertainty α. This increase, $\Delta r_c(\alpha)$, is the *gain* in critical reward obtained by employing an information system. $\Delta r_c(\alpha)$ is the

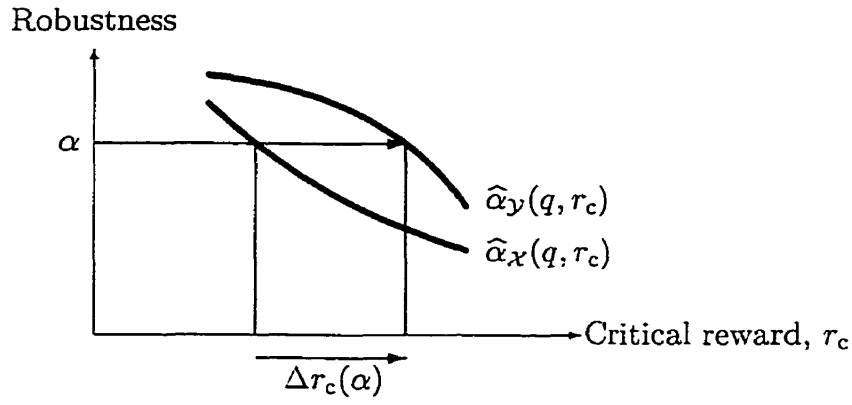


Figure 8.2: Robustness versus critical reward for two different information systems. Illustrating the demand value of an information system.

least gain in reward which the decision maker can be assured of obtaining if the ambient uncertainty does not exceed α. $\Delta r_c(\alpha)$ is also the greatest amount which a cautious decision maker would be willing to expend on an information system for moving from $\mathcal{X}$ to $\mathcal{Y}$, since by doing so he cannot guarantee an increase in revenue any greater than $\Delta r_c(\alpha)$.

Risk Attitudes

In EU theory, it can be easily shown that a decision maker with a utility function $u(x)$ is

1. *Risk-averse* if and only if $u(x)$ is strictly concave.
2. *Risk-neutral* if and only if $u(x)$ is a linear function.
3. *Risk-seeking* if and only if $u(x)$ is strictly convex (Arrow, 1971).

In many situations, it is of interest to measure the degree of risk aversion associated with a utility function $u(x)$ and to determine how risk aversion depends on the level of a decision maker's wealth. To this end, Pratt (1964)

proposed the absolute risk aversion (ARA) measure, the negative ratio of the second to first derivative:

$$\mathrm{ARA}(x) = -\frac{u''(x)}{u'(x)}$$

Info-gap modeling can also incorporate risk attitudes of the decision maker: uncertain variations of state variables may be perceived as dangerous (the decision maker is risk averse) or favourable (the decision maker is a risk seeker). In the context of info-gap modeling the *robustness function* captures risk aversion: the greatest level of uncertainty consistent with no-failure. On the other hand risk seeking behavior is modeled with the *opportunity function*, the least level of uncertainty which entails the possibility of sweeping success. If q is a vector of parameters such as time, design variables, and model parameters, the robustness and opportunity functions can be expressed as the maximum or minimum of a set of α-values:

$$\widehat{\alpha}(q, r_c) = \max\{\alpha : \text{ minimal requirements are satisfied}\} \quad (8.8)$$

$$\widehat{\beta}(q, r_w) = \min\{\alpha : \text{ sweeping success is obtained } \} \quad (8.9)$$

The robustness function $\widehat{\alpha}(q)$ is the immunity against failure, so a large value of $\widehat{\alpha}(q)$ is desirable. In contrast, the opportunity function $\widehat{\beta}(q)$ is the immunity against sweeping success, so a small value of $\widehat{\beta}(q)$ is desirable. The minimal requirement in Eq 8.8 is that the reward be no less than a critical value r_c. Likewise, the sweeping success in Eq 8.9 is attainment of the "wildest dream" reward r_w. The location of the robustness and opportunity curves on the uncertainty-vs.-reward plane reveal the type of gambling which is expressed by these trade-offs.

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8.2.9 Strengths and Weaknesses of Taylor Series Approximations in Uncertainty Analyses

For all except the simplest cases, such as linear combinations of normal variables, Springer (1979) argues that exact analytic methods for the propagation

of uncertainty are intractable (or require sophisticated numerical integration techniques). However, there are a variety of well-known approximate analytic techniques based on Taylor series expansions (Cheney, 1966). These techniques are sometimes called *Method of Moments* because they analyze uncertainty using the mean, variance, and sometimes higher order moments of a random (output) variable which is itself a function of one or more random (input) variables.

Both the first order approximation (Gaussian) and higher order approximations (method of moments) have been applied quite widely to the analysis of complex problems in engineering and the physical sciences. These analytic methods have a number of advantages: numerical calculations are relatively simple (once the algebraic analysis has been completed); the contribution of each input toward the variance in the output is clearly illustrated, and the entire probability distribution of the input parameters does not require specification (only the first few moments, typically the mean and variance). However, these analytic methods suffer from a number of disadvantages:

1. If the model is complex and higher order terms are necessary, the algebra can become intractable.

2. It is difficult to obtain estimates for the tails of the output distribution (usually only the mean and variance are conveniently calculated).

3. Large uncertainties in the input variables will cause significant inaccuracies since the Method of Moments is a "local approach".

4. First order approximations replace the actual function by a linear one (a hyperplane tangent to the response surface at the nominal scenario).

5. The method breaks down if the response surface has discontinuities or important covariance terms are omitted.

As in so many fields, the late-1970s were a time of great expectations for environmental modeling: it would not be long, or so many believed, before

the relatively ill-defined problems of large-scale environmental systems could nevertheless be precisely modeled with the help of powerful computers and the already vast array of methods available from applied mathematics and control theory (which had been so decisively successful in their application, for example, to the analysis of aerospace systems). Such a time has still to come to pass, at least for complex multivariate models of more than, say, three or four state variables.

8.3 Suggestions for Future Research

8.3.1 Stochastic Modeling of CBOD-NOD-DO system

Based on the theory of stochastic differential equations, temporal variance and higher order moments should be obtained through the use of the moment equations for the CBOD-NBOD-DO system (derived from the Îto stochastic differential rule). Preliminary work on the second order moments has begun, yet a thorough analysis is necessary due to the subtleties of Îto's lemma and stochastic calculus in general.

8.3.2 Quantify Fisheries Extinction Risk

Using the SEAL model, future researchers may wish to consider how varying parameters such as α, β, and Effort, E affect the mean time to extinction in the following critical depensation growth equation

$$F(x) = rx^{\alpha}\left(1 - \frac{x}{K}\right)\left(\frac{x}{m} - 1\right) \tag{8.10}$$

and harvesting equation

$$h = qEx^{\beta} \tag{8.11}$$

Finally, for increased realism, a multi-species fishery should be simulated (Pradhan and Chaudhuri, 1999). Although, it is difficult to construct a realistic and analytically tractable model in the multi-species case: acute problems lie in the estimation of the various interaction coefficients.

8.3.3 Test Descriptive Validity of Convex Models

A host of theoretical and applied studies have shown that EU theory often lacks both descriptive validity and normative acceptability. The last two decades has seen a profusion of new models of choice under uncertainty, all offered as alternatives to the classical "expected utility" model of von Neumann-Morgenstern, and all designed to explain — or at least to accommodate — a growing body of observed violations of the assumptions and predictions of the expected utility model[2]. In order to qualify as a legitimate substitute to EU theory, convex models should be empirically tested for descriptive validity. Here, both robustness ("maximum variability in state variables before failure") and its opposite, propitiousness ("minimum uncertainty required for sweeping success") should be considered.

8.3.4 Gibbs Sampler and Cryptosporidium

In recent years, the protozoan pathogen Cryptosporidium parvum has been the causative agent of several outbreaks of waterborne illness. Currently there is no effective treatment for cryptosporidiosis. Consequently, these organisms are of significant concern to public health and the drinking water industry. Random sampling errors of Cryptosporidium oocysts result from the fact that the entire water body is not evaluated (ideally the oocyst count would be determined from an infinite number of random samples). This is illustrated in Fig 8.3, where a true concentration of 1 oocyst/500 L of water is assumed. If three random 500-L samples were collected, the actual number of oocysts (N) in them might differ; in this case, $N_1 = 1$ oocyst, $N_2 = 2$ oocysts, and $N_3 = 0$ oocysts; as the number of samples approaches infinity, the average number of oocysts in a sample approaches 1. Given the imperfections of analytical methods, the oocyst counts observed after

[2]The theory of utility maximization has many drawbacks in real world situations outside the controlled laboratory environment. However, even "data from laboratory shows that under different circumstances of choice, subjects depart from the predictions of the subjective expected utility model in diametrically opposite directions" (Simon, 1997)

Cryptosporidium processing probably represent only a portion the number originally present in the sample; only the recovered fraction of oocysts (p) is observed by microscopy (Fig 8.4).

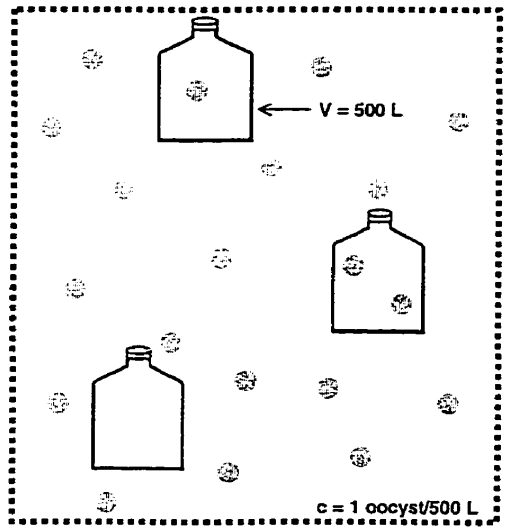

Figure 8.3: Sampling from a water body (personal communication with Dr. Monica B. Emelko and Dr. Park M. Reilly)

The joint probability density function for the parameters c (true oocyte concentration), N (true count of oocytes in a sample), and p (recovery probability, *i.e.* the probability that an oocyst in the sample will be observed on the slide (e.g. Fisher and van Belle, 1993)); where $0 \leq p \leq 1$ are shown in Eq 8.12.

$$Df(c, N_i, p_i|X_i) \propto$$

$$\frac{1}{c}\prod_{i=1}^{n} p_i^{a-1}(1-p_i)^{b-1}\,\frac{(cV_i)^{N_i}\,\exp(-cV_i)}{N_i!}\,\frac{N_i!}{X_i!(N_i-X_i)!}p_i^{X_i}(1-p_i)^{N_i-X_i} \tag{8.12}$$

Eq 8.12 describes c, N, and p given knowledge of the data X where $c > 0$, $N \leq X$, $0 \leq p \leq 1$ and a and b are constants. The Gibbs Sampler can be

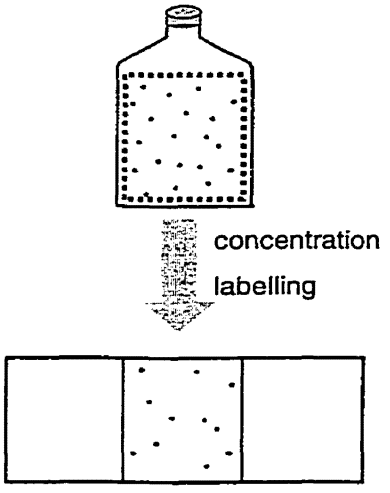

Figure 8.4: Effect of sample preparation on number of observed oocysts (personal communication with Dr. Monica B. Emelko and Dr. Park M. Reilly)

used to determine marginal distributions for the parameters (c, N, and p given knowledge of the data X). This is accomplished by sampling from the distributions described by the conditional probability density functions of each of the parameters.

8.3.5 Stochastic Optimal Control

Future work will involve implementing the ideas developed in this thesis (preservation value, allee effect) in the context of stochastic optimal control. The field of stochastic environmental management is relatively new and increasingly important. Future analysis will incorporate the following theorem of stochastic harvesting by Alvarez and Shepp (1998):

Theorem 1 (Alvarez-Shepp) *(i) If $\mu \geq r$, then the population is immediately driven to extinction (that is $Z(0) = x$ and $\tau^* = 0$ almost surely for all $x \in \mathbf{R}_+$) and the value of harvesting is $V(x) = x$ for all $x \in \mathbf{R}_+$.*

(ii) *If* $0 < \mu < r$, *then there exists an unique threshold density* $x^* \in (0, \infty)$ *satisfying the smooth-fit condition* $\psi''(x^*) = 0$ *and defined as* $x^* = \inf\{x \in \mathbf{R}_+ : \psi''(x) = 0\}$, *where* $\psi(x) = x^{\theta^+} M(\theta^+, 2\theta^+ + \frac{2r}{\sigma^2}, \frac{2r\gamma}{\sigma^2} x), \theta^+ = \frac{1}{2} - \frac{r}{\sigma^2}\mu\sigma^2$, *and* M *is the confluent hypergeometric function. For* a, b $\in \mathbf{R}_+$ M *is defined as*

$$M(a, b, cx) = \sum_{n=0}^{\infty} \frac{(a)_n (cx)^n}{(b)_n n!} = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} \int_0^1 e^{cxt} t^{a-1} (1-t)^{b-a-1} dt,$$

where $(a)_n = a(a+1)...(a+n-1)$ *and* $(a)_0 = 1$. *In this case, the value of harvesting corresponding to the optimal harvesting strategy is*

$$V(x) = \frac{\psi(x)}{\psi'(x^*)}, \quad if x < x^*$$
$$x - x^* + \frac{\psi(x^*)}{\psi'(x^*)}, \quad if x \geq x^*.$$

Moreover, the population does not get depleted in a finite time under the optimal harvesting strategy (i.e. $\tau^* = \infty$*).*

Appendix A

Multiple Objective Mathematical Programming (MOMP)

Mathematically, the MOMP problem is stated as

$$\begin{aligned} \max\ f(\boldsymbol{x}) &= [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ \ldots\ , f_q(\boldsymbol{x})] \\ s.t.\ \ \boldsymbol{x} \in \boldsymbol{X} &= \{\boldsymbol{x} \mid g_j(\boldsymbol{x}) \leq 0,\ j = 1, 2, \ldots, m;\ \ \boldsymbol{x} \geq 0\} \end{aligned} \tag{A.1}$$

where $\boldsymbol{x}$ is an n-dimensional vector of decision variables. The functions $f_1, f_2, \ldots, f_q$ are the q-real valued attribute functions defining the attribute of relevance and $\boldsymbol{x}$ is a non-empty, closed, and compact set defined by a set of m constraints dictated by the physical processes and resource endowments. The feasible set $\boldsymbol{X}$ is convex and the solution method often aggregates the attributes f_i by some rule. Unlike in scalar optimization problems, there is usually no single "optimum" solution to vector maximization problems because a solution that maximizes one objective will not, in general, maximize any of the other objectives. The vector maximization problem identifies a set of efficient solutions $\boldsymbol{x}^*$ in the decision space, or equivalently, non-inferior solution $f(\boldsymbol{x}^*)$ in the criterion space.

Definition 1 (Non-dominated solution concept) *A point* $\boldsymbol{x}^* \in \boldsymbol{X}$ *is defined to be an efficient (non-dominated) solution if and only if there does not exist another solution* $\boldsymbol{x}^0 \in \boldsymbol{X}$ *such that:*

$$\begin{aligned} f_i(\boldsymbol{x}^0) &\geq f_i(\boldsymbol{x}^*) \ \forall i = 1, \ldots, q \\ \textit{and} \quad f_i(\boldsymbol{x}^0) &> f_i(\boldsymbol{x}^*) \ \textit{for at least one } i \end{aligned}$$

That is, any solution for which none of the criterion functions can be improved without causing a degradation in any other is a non-inferior solution. In this thesis the terms efficient (Pareto-optimal) solutions, non-dominated solutions and non-inferior solutions will be used interchangeably and denoted by $f(\boldsymbol{x}^*)$.

Distance-based methods are favored by many researchers to solve the problem posed in A.1 because of their simplicity and their relationship to the theoretically appealing multi-attribute utility theory. The many solution algorithms available for the distance method all share the common approach of minimizing some measure of weighted distance from a reference point. Depending on the choice of distance metric and the definition of reference point, the methods differ.

Of course the operational definition of distance depends on the nature of the data to be analyzed, such as real numbers, vectors, functions, *etc.* . However, a distance function, d has to respect some general properties in order to be used as a distance measurement. Consider the vectors $\boldsymbol{a} = (a_1, a_2, \ldots, a_n)$ and $\boldsymbol{b} = (b_1, b_2, \ldots, b_n)$. Let S be a generic set of objects and $d : S \times S \rightarrow \Re$ a real-valued function. The function d is called a distance metric if it satisfies the following properties

1. $d(\boldsymbol{a}, \boldsymbol{b}) \geq 0$ for every $\boldsymbol{a}, \boldsymbol{b} \in S$
2. $d(\boldsymbol{a}, \boldsymbol{a}) = 0$ for every $\boldsymbol{a} \in S$
3. $d(\boldsymbol{a}, \boldsymbol{b}) = d(\boldsymbol{b}, \boldsymbol{a})$ for every $\boldsymbol{a}, \boldsymbol{b} \in S$
4. $d(\boldsymbol{a}, \boldsymbol{b}) \leq d(\boldsymbol{a}, \boldsymbol{c}) + d(\boldsymbol{c}, \boldsymbol{b})$ for every $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in S$

The simplest example of a distance function is the distance between real numbers: $S = \Re$ is the set of real numbers and $d(\boldsymbol{a}, \boldsymbol{b}) = |\boldsymbol{a} - \boldsymbol{b}|$ is the absolute difference.

The L_α distance metric (Minkovsky metric) is the most commonly employed "measure of closeness". It is represented in its general form as

$$L_\alpha = \left[\sum_{i=1}^{n} |a_i - b_i|^\alpha\right]^{1/\alpha}, \qquad 1 \leq p \leq \infty \tag{A.2}$$

where $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ are the coordinates of the two points, the distance between them being minimized. In a multi-objective problem context, the distance between the objective and its reference point is minimized. If the reference point is some notion of "ideal levels" (utopian points) f_i^* for each of the attributes, then it is solved as a *compromise programming* problem (Tecle et al., 1998; Tecle, 1992; Levy et al., 2000c). Alternatively, if the reference points are goal levels G_i for each of the attributes, then it is solved as a *goal programming* problem (Charness and Cooper, 1961). Both goal and compromise programming have been widely used in agriculture planning, energy policy, water resources management and other aspects of environmental planning (Zeleny, 1982; Lakshminarayan et al., 1991; 1995; Romero, 1991).

If the goal for the ith objective is denoted G_i then the general weighted goal programming formulation is:

$$\text{Minimize} \quad L_\alpha = \left[\sum_{i=1}^{n} w_i^\alpha \left|\frac{G_i - f_i(\boldsymbol{x})}{k_i}\right|^\alpha\right]^{\frac{1}{\alpha}}, \quad \alpha \in \{1, 2, 3, \ldots\} \cup \{\infty\} \tag{A.3}$$

where w_i is the importance of the deviation from the goal on the ith criterion and k_i is the normalizing constant for the ith criterion. The distance from an ideal solution L_α, is a function of the distance metric exponent, α. Often, Euclidean distance ($p = 2$) is used to penalize significant deviations from the ideal point. The Hamming distance ($\alpha = 1$) allows one to minimize the sum

of individual regrets; there is perfect competition among criteria. If the ideal (utopian) values f_i^* replace the goals G_i, the traditional weighted compromise programming problem of A.2 becomes

$$\text{Minimize} \quad d = \sum_{i=1}^{n} w_i \left| \frac{f_i^* - f_i(\boldsymbol{x})}{k_i} \right| \quad \text{subject to } \boldsymbol{x} \in \boldsymbol{X} \tag{A.4}$$

At the other extreme, the Chebychev distance ($\alpha = \infty$) results in no compensation among criteria as the largest deviation from the ideal dominates the assessment. The choice of a particular value of α depends on the degree of conflict between decision makers and the desired solution. Detailed analysis of the effect of α on the compromise solution to a multi-objective problem may be found in Goicechea et al. (1982).

By carefully selecting weights, distance metric and reference point one can the selected distance method to the multi-attribute utility theory (MAUT). For example, Hannan (1984) emphasizes that goal programming is related to MAUT in that it embodies an additively separable preference structure.

MOMP methods can be categorized according to articulation of preference structure: A priori preference assessment, Progressive preference elicitation, and Posterior preference information methods.

A priori preference information methods of MOMP begin with an investigation of the decision maker's value function. Once the preference structure of the decision maker has been assessed, all criteria are aggregated into one, thereby transforming the problem into a more tractable single-objective optimization. In most cases, assessing the Decision Maker's value is not only quite difficult but also highly subjective. Popular a priori preference information methods include compromise programming Zeleny (1982) and goal programming Charness and Cooper (1961), which is perhaps the first formal MOMP technique.

Due to the great difficulty of explicitly determining a decision maker's preferences, many procedures try to elicit them progressively. Methods that alternate between analysis and interaction with the decision maker are called interactive. The process begins with little preference information, and at each

iteration, a set of solutions is presented to the decision maker. After examining each solution, the decision maker updates her preference information. The process is complete when the decision maker is satisfied with the solution proposed by the model. STEM, the first interactive method, was proposed by Benayoun et al. (1971). Although originally proposed for solving linear programming problems it has been applied to integer and nonlinear problems. Other techniques for progressive preference assessment include the methods of Geoffrion et al. (1972) and Zionts and Wallenius (1976).

Posterior preference information methods begin by solving the decision problem without exploring the preference structure; this is usually achieved by vector optimization, by which a set of efficient solutions, or a subset, is generated. Three main approaches for generating efficient solutions are the *weighted approach*, the *kth objective* ϵ*-constraint* method, and the *Lagrangian approach.*

The concept of "non-dominance" is a common approach to determine the feasible solutions of MOMP problems; specifically, a non-dominated solution exists if there is no other feasible solution that will cause improvement in any one of the objectives without making at least one other objective worse.

Appendix B

Evaluation of Point Estimators

Let $\mathbf{X} = (X_1, \ldots X_n)$ be a sample from a population whose distribution is specified up to an unknown parameter θ; and let $d = d(\mathbf{X})$ be an estimator of θ. One way to determine the worth of an estimator θ is to consider the square of the difference between $d(\mathbf{X})$ and θ. However, since $(d(\mathbf{X}) - \theta)^2$ is a random variable, the standard approach is to consider the *mean square error* of the estimator d:

$$r(d, \theta) = E\left[(d(\mathbf{X}) - \theta)^2\right] \tag{B.1}$$

Although minimum mean square estimators rarely exist, it is sometimes possible to find an estimator having the smallest mean square error among all estimators that satisfy the property of unbiasedness:

$$b_\theta(d) = E\left[d(\mathbf{X})\right] - \theta \tag{B.2}$$

is called the bias of d as an estimator of θ. If $b_\theta(d) = 0$ for all θ, then d is said to be an unbiased estimator of θ. In other words, an estimator is unbiased if its expected value always equals the value of the parameter it is attempting to estimate. Moreover, it can be shown that the mean square error of an unbiased estimator is equal to its variance:

$$\begin{aligned} r(d,\theta) &= E\left[(d(\mathbf{X}) - \theta)^2\right] \\ &= E\left[(d(\mathbf{X}) - E[d(\mathbf{X})])^2\right] \\ &= \text{Var}\,(d(\mathbf{X})) \end{aligned} \tag{B.3}$$

It is well-known that the maximum likelihood estimator performs well in a variety of practical situations. In fact, it can be shown the maximum likelihood estimator will have an asymptotically smaller mean square error than any other estimator as the sample size increases to ∞. That is, it can be shown, subject to certain regularity conditions, that if d_n represents the maximum likelihood estimator of θ from a sample of size n and d_n^* is any other estimator of θ based on a sample of size n then

$$\lim_{n\to\infty} \frac{r(d_n,\theta)}{r(d_n^*,\theta)} \leq 1 \tag{B.4}$$

However, in smaller samples, even when the regularity conditions hold, it is sometimes possible to improve upon maximum likelihood estimators.

Appendix C

Comparing First and Second Order BOD Decay Models

C.1 Second Order BOD Decay Model

The MATLAB code in Fig C.1 and Fig C.2 finds values for k and L_0 that produce the minimum sum of squared differences (SSD) between the measured and predicted values of $y(t)$ (BOD exerted at time t) for the *Second Order BOD Decay Model.* Also, the *root mean square error* (RMSE) for the Second Order BOD Decay Model is calculated. The data is taken from the raw influent and primary effluent of the Waterloo Pollution Control Plant (Constable and McBean, 1977).

The MATLAB code in Fig C.3 and Fig C.4 finds values for k and L_0 that produce the minimum sum of squared differences (SSD) between the measured and predicted values of $y(t)$ (BOD exerted at time t) for the *First Order BOD Decay Model.* Also, the root mean square error (RMSE) for the First Order BOD Decay Model is calculated.

$$t := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \qquad y := \begin{pmatrix} 8.3 \\ 14.2 \\ 16.5 \\ 18.9 \\ 20.6 \\ 21.4 \\ 23.2 \end{pmatrix}$$

t is time in days

y is measured BOD exerted in mg/L

$n := rows(t)$

Second-order BOD equation

$$y_{2nd}(k_3, L_{2nd}, t) := \frac{\left(k_3 \cdot {L_{2nd}}^2 \cdot t\right)}{1 + k_3 \cdot L_{2nd} \cdot t}$$

Sum of squared differences using second-order BOD equation

$$SSD_{2nd}(k_3, L_{2nd}, n) := \sum_{i=0}^{n-1} \left(y_i - y_{2nd}(k_3, L_{2nd}, t_i)\right)^2$$

Root mean square error (RMSE) for the second-order BOD model

$$RMSE_{2nd}(k_3, L_{2nd}, n) := \sqrt{\frac{SSD_{2nd}(k_3, L_{2nd}, n)}{n}}$$

Figure C.1: Equations for sum of square differences (SSD) and root mean square error (RMSE) for second order model

Root mean square error (RMSE) for the second-order BOD model

$$RMSE_{2nd}(k_3, L_{2nd}, n) := \sqrt{\frac{SSD_{2nd}(k_3, L_{2nd}, n)}{n}}$$

Initial guesses for k_3 and L_{2nd}

$k_3 := 0.02 \qquad L_{2nd} := 40$

Given

$SSD_{2nd}(k_3, L_{2nd}, n) = 0$

$Solution := Minerr(k_3, L_{2nd})$

$$Solution = \begin{pmatrix} 0.012 \\ 31.405 \end{pmatrix}$$

$k_3 := Solution_0$

$k_3 = 0.012$

$L_{2nd} := Solution_1$

$L_{2nd} = 31.405$

$RMSE_{2nd}(k_3, L_{2nd}, n) = 0.361$

Figure C.2: Root Mean Square Error (RMSE) for second order model

First-order BOD equation

$$y_{1st}(k_1, L_{1st}, t) := L_{1st} \cdot \left(1 - e^{-k_1 \cdot t}\right)$$

Sum of squared differences using first-order BOD equation

$$SSD_{1st}(k_1, L_{1st}, n) := \sum_{i=0}^{n-1} \left(y_i - y_{1st}(k_1, L_{1st}, t_i)\right)^2$$

Root mean square error (RMSE) for the first-order BOD model

$$RMSE_{1st}(k_1, L_{1st}, n) := \sqrt{\frac{SSD_{1st}(k_1, L_{1st}, n)}{n}}$$

Initial guesses for k_1 and L_{1st}

$$k_1 := 0.4$$

$$L_{1st} := 20$$

Figure C.3: Equations for sum of square differences (SSD) and root mean square error (RMSE) for the first order model

This block finds values for k_1 and L_0 that produce the minimum of the squared error between the measured and predicted values of y (BOD exerted)

Given

$$SSD_{1st}(k_1, L_{1st}, n) = 0$$

$$Solution := Minerr(k_1, L_{1st})$$

$$Solution = \begin{pmatrix} 0.334 \\ 21.979 \end{pmatrix}$$

$$k_1 := Solution_0$$

$$k_1 = 0.334$$

$$L_{1st} := Solution_1$$

$$L_{1st} = 21.979$$

$$RMSE_{1st}(k_1, L_{1st}, n) = 1.159$$

Figure C.4: Root Mean Square Error (RMSE) for the first order model

Appendix D

SEAL Decision Support System: MATHCAD Source Code for Stochastic Algorithms

D.0.1 Explicit Order 1.5 Strong Scheme to Integrate Îto SDEs with a single noise term

This section contains the code for an explicit order 1.5 strong scheme to integrate Îto SDEs. The algorithm is found in Section 11.2 of Kloeden and Platen (1992).

```
function [x,w]=sde1(afun,bfun,ts,x0)
% Author: Jason K. Levy
% function [x,w]=sde1(afun,bfun,ts,x0)
% First-order and a half (1.5 order) accurate scheme to integrate the Ito SDE
%        dx = a(x,t)dt + b(x,t)dW
% for a realisation of one (scalar) Wiener noise W(t).
%
% x0   - column vector of initial condition
% afun - user supplied function afun(x,t) computes the
```

```
%        coefficient column vector of the dt term
% bfun - user supplied function bfun(x,t) computes the
%        coefficient column vector of the dW term
% ts   - row vector [t0 t1 ... tfin] of times at which
%        x(t) is computed using steps of size diff(ts)
% x    - columns are the solution at times given in ts
% w    - row of the Wiener process at times ts (w0=0)
%
% Ref: Kloeden & Platen, "Numerical solution of
% stochastic differential equations", Section 11.2
%
% Corresponding Stratonovich SDE is dx=(a-bb'/2)dt+b.dW
%
%

d=length(x0);            % dimensionality of the problem
dt=diff(ts);             % vector of delta t
rdt=\sqrt(dt);                % square root delta t
fdt=0.5*(\sqrt(dt))^3;
nt=length(dt);           % number of time steps
dw=randn$(1,nt).*rdt;     % increments of the noise process
dz=$(randn(1,nt)+randn(1,nt)/sqrt(3)).*fdt;
dd=(dw.^2-dt)./(2*rdt); % auxilary factors
f1=(dz./(2*rdt);
f2=(dt/4);
f3=(dw.^2-dt)./(4*rdt);
f4=(dw.*dt-dz)./(2*dt);
f5=(((1/3)*dw.^2-dt)./(4*dt))*dw;

%
x=zeros(d,nt+1);
```

```
x(:,1)=x0;
for n=1:nt
   a=feval(afun,x(:,n),ts(n));
   b=feval(bfun,x(:,n),ts(n));
   ap=feval(afun,x(:,n)+a*dt(n)+b*rdt(n),ts(n));
   an=feval(afun,x(:,n)+a*dt(n)-b*rdt(n),ts(n));
    bp=feval(bfun,x(:,n)+a*dt(n)+b*rdt(n),ts(n));
   bn=feval(bfun,x(:,n)+a*dt(n)-b*rdt(n),ts(n));

   zbp=x(:,n)+a*dt(n)+b*rdt(n)+ ...
      feval(bfun,x(:,n)+a*dt(n)+b*rdt(n),ts(n))*rdt(n);
   zbn=x(:,n)+a*dt(n)+b*rdt(n)- ...
      feval(bfun,x(:,n)+a*dt(n)+b*rdt(n),ts(n))*rdt(n);

    x(:,n+1)=x(:,n)+b*dw(n)+ ...
     (ap-an)*f1(n) + ...
     (ap+2*a+a*an)*f2(n) + ...
     (bp-bn)*f3(n) + ...
     (bp-2*b+bn)*f4(n) + ...
      (zbp-zbn-bp+bn)*f5(n)

end
%
w=cumsum([0 dw]);
end;
```

D.0.2 Code to determine 'Number of Crossings'

```
function times = ncrossings(t, x, threshold, uplo)

times = 0;
```

```
if x(1) < threshold
     below = 1;    above = 0;
else
     below = 0;    above = 1;
end

for i = 2:length(t)

     if strcmp(uplo, 'upper')
          if (x(i) >=threshold) & below
               times = times + 1;
               below = 0;
          end

          if (x(i) < threshold)
               below = 1;
          end
     end

     %-------------------------------------------------------

     if strcmp(uplo, 'lower')
          if (x(i) < threshold) & above
               times = times + 1;
               above = 0;
          end

          if (x(i) >= threshold)
               above = 1;
          end
     end
```

```
end
```

D.0.3 Code to determine the global minimum DO level

```
function [time, space] = global_minimum(t, x)

                [space, i]              = min(x);

          time =      t(i);
```

D.0.4 Code to determine the first passage time

```
function time = first_passage_time(t, x, threshold, uplo)

k = 0;

for i=1:length(t)

     if (x(i) >= threshold)  &  (k==0)  &  strcmp(uplo,'upper')

          k = i;

     end

     if (x(i) <= threshold)  &  (k==0)  &  strcmp(uplo,'lower')

          k = i;

     end

end
```

```
if k == 0
     k = length(t);
end

time = t(k);
```

D.0.5 Explicit Order 1.5 Strong Scheme to Integrate Îto SDEs with multiple noise terms

```
d=length(x0);              % dimension of state-space
[a,b]=feval(abfun,x0,ts(1));
m=size(b,2);               % number of independent noises
dt=diff(ts);               % vector of delta t
rdt=sqrt(dt);              % square root delta t
nt=length(dt);             % number of times
dw=randn(m,nt).*rdt(ones(m,1),:); % increments of the noises
%
x=zeros(d,nt+1);
x(:,1)=x0;
for n=1:nt
   %
   % compute the I matrix
   p=ceil(abs(pind)/dt(n));
   xi=dw(:,n)/rdt(n);
   mu=randn(m,1);
   if p>0,
      rr=1 ./(1:p);
      rrs=rr(ones(m,1),:);
      rhop=1/12-sum(rr.^2)/(2*pi^2);
      eta=randn(m,p);
```

```
      zet=randn(m,p);
      zetxi=(zet*rr')*xi';
      zetet=zet*(rrs.*eta)';
   else rhop=1/12; end
   muxi=(mu*xi')*sqrt(rhop);
   ip=xi*xi'/2;
   if p>=0, ip=ip +(muxi-muxi'); end
   if p>0,
      ip=ip+( (zetxi-zetxi')/sqrt(2) +(zetet-zetet')/2 )/pi;
   end
   ip=dt(n)*(ip -diag(0.5*ones(m,1)) );
   %
   % supporting values
   y=(x(:,n)+a*dt(n))*ones(1,m) +b*rdt(n);
   bb=zeros(d,1);
   for j1=1:m %sum
      [ay,by]=feval(abfun,y(:,j1),ts(n));
      bb=bb+(by-b)*ip(j1,:)';
   end
   %
   % time step
   x(:,n+1)=x(:,n)+a*dt(n)+b*dw(:,n) +bb/rdt(n);
   if n~=nt, [a,b]=feval(abfun,x(:,n+1),ts(n+1)); end
end
%
w=cumsum([zeros(m,1) dw]')';
end
```

D.0.6 Code to create histogram

```
function [frequency, x] = histogram_discrete(data, datalim, cutoff, color)
```

```
%%% nbin  = round(sqrt(length(data)));
    nbin  =             length( [datalim(1) : 1 : datalim(2)] );

width = (datalim(2) - datalim(1))/...
              (nbin - 1);

for bin=1:nbin
     x(bin) = datalim(1) + (bin-1)*width;
end

%-------------------------------------------------------------

frequency = zeros(nbin, 1);

for i=1:length(data)

     for bin=1:nbin-1
          if  (x(bin) <= data(i)) & (data(i) < x(bin+1))
               frequency(bin) = frequency(bin) + 1;
          end
     end

     for bin=nbin
          if   x(bin) <= data(i)
               frequency(bin) = frequency(bin) + 1;
          end
     end

end

%-------------------------------------------------------------
```

```
hold_was_off = ~ishold;

if hold_was_off,    cla,    end

hold on

%-------------------------------------------------------------

x = x - 0.5;

for bin=1:nbin-1
     h(bin) = patch([x(bin)   x(bin)+width   x(bin)+width    x(bin)          ],
                    [0        0              frequency(bin)  frequency(bin) ],
end

for bin=nbin
     h(bin) = patch([x(bin)   cutoff         cutoff           x(bin)         ],
                    [0        0              frequency(bin)   frequency(bin) ],
end

x = x + 0.5;

%-------------------------------------------------------------

if hold_was_off,    hold off,    end
```

D.0.7 Code to determine statistics of DO, BOD, and NOD curves

```
function sde
```

```
%k_1=0.2;
%k_2=0.6;
%k_3=0.001
%C_s=10;
%L_o=30;

%Second Order

%af = '0.6*(10-x) - ((0.001*30^2)/(1+0.001*30*t)^2)';
%bf = '0.15*(10-x)';

%First Order

%af = 'k_2*(C_s-x) - (k_1*L_o*exp(-k_1*t))';

%bf = '0.15*(C_s-x)';

%Second Order

%af = '0.6*(10-x) - (0.2*30*exp(-0.2*t))';
%bf = '0.15*(10-x)';

%af = 'k_2*(C_s-x) - (k_1*L_o*exp(-k_1*t))';

%bf = '0.15*(C_s-x)';


%af = inline( vectorize(af), 'x', 't' );
%bf = inline( vectorize(bf), 'x', 't' );
```

```
%-------------------------------------------------------------------

t0     = 0;        % GUI
%tstep  = 0.05;    % GUI

tstep  = 0.05;

tfinal = 50.0;      % GUI
textra = 1.0;      % GUI (maybe)

t      = [t0 : tstep : tfinal];

x0 = [9];

%[x, w] = sde1(af, bf, t, x0);

%-------------------------------------------------------------------

figure(1);


[x,w]=sde1m('dofun',t,x0,1);
      hold on
        plot(t,x, 'g-')
        %plot(t,w, 'g-')

      [x,w]=sde1m('dofun',t,x0,1);
        plot(t,x, 'b-.')
        %plot(t,w, 'b.')

        [x,w]=sde1m('dofun',t,x0,1);
```

```
        plot(t,x, 'r--')
        %plot(t,w, 'r.-')

      [x,w]=sde1m('dofun',t,x0,1);
        plot(t,x, 'k:')

get(gcf);
set(gcf, 'PaperPosition', [0.25, 2.5,6,6]);
print -depsc do1.eps;

hold off

%-----------------------------------------------------------------

%threshold = 5;    % GUI

%nr = 121 ;            % GUI
%nr = 676 ;
%nr = 2601 ;
%nr = 36;
nr = 10;

for r=1:nr
     [x,w]=sde1m('dofun',t,x0,1);
     [tt(r), xx(r)] = global_minimum(t,x);
end

r = [1 : nr];

tgraph=tfinal/10
%-----------------------------------------------------------------
```

```
for hist=1:2

figure(1+hist);

if hist == 2
     tt=xx;
     t0=0;
     tgraph=x0;
     textra=1;
end

[frequency, x] = histogram(tt, [t0 tgraph], tgraph + textra, 'gr')   %;

set(gca, 'xlim', [t0  tgraph+textra])
hold on

ylim= get(gca,'ylim');
ymax=ylim(2);


h0 = plot([mean(tt)          mean(tt)       ],          [0 ymax], 'r-');
set(h0, 'linewidth', 2.5)

h1 = plot([median(tt)        median(tt)     ],          [0 ymax], 'b-');
set(h1, 'linewidth', 2.5)

h2 = plot(t0, 0, '.');
set(h2, 'visible', 'off')
set(h2, 'color', [1 1 1])
```

```
h3 = plot(t0, 0, '.');
set(h3, 'visible', 'off')
set(h3, 'color', [1 1 1])

h4 = plot(t0, 0, '.');
set(h4, 'visible', 'off')
set(h4, 'color', [1 1 1])

if hist == 1
     title('PDF of t_{cr}')
else
     title('PDF of Minimum DO')
end

hold off

%-----------------------------------------------------

thehistogram = gca;
figure(4);   clf

     boxplot(tt, 0, '+', 0)
     h_boxplot = get(gca, 'children');

     adjust(h_boxplot, ymax)
     copyobj(h_boxplot, thehistogram)

delete(4)
figure(1+hist);  axes(thehistogram)

%-----------------------------------------------------
```

```
legend([h0 h1 h2 h3 h4],                                    ...
  ['Mean    = ' num2str( mean(tt)       )], ...
  ['Median    = ' num2str( median(tt)           )], ...
  ['COV       = ' num2str( mean(tt)/std(tt)   )], ...
  ['Skewness    = ' num2str( skewness(tt)         )], ...
  ['Kurtosis    = ' num2str( kurtosis(tt)           )], 1 )

set(gcf, 'PaperPosition', [0.25, 2.5,6,6]);

if hist == 1
     print -depsc do2.eps;
else
     print -depsc do3.eps;
end

end
```

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