Empirical Adequacy of Ranking Theory: A Behavioural and Theoretical Investigation of Human Uncertainty Representation

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Measuring and quantifying degrees of belief poses a fundamental challenge, prompting an exploration of how humans navigate uncertainty. This study investigated the application of ranking theory to human belief systems, focusing on its effectiveness in measuring degrees of disbelief across diverse contexts. Experiments 1 and 2 confirmed the reliability of negative ranks as a robust measure of disbelief in uncertain situations where belief might be shaped by personal experiences, societal, and cultural norms. Experiment 3 extended the framework by introducing positive ranks, providing a more comprehensive representation of belief and disbelief, and enabling finer distinctions between disbelief, neutrality, and belief. Experiment 4 examined ranking functions within a dynamic learning environment, demonstrating that ranks aligned more closely with objective probabilities than subjective probabilities when outcomes were clearly defined and consistently observed. Experiments 5 and 6 explored the Ellsberg Paradox, showing that ranking functions reduced ambiguity aversion and provided a more true reflection of participants' beliefs compared to traditional choice-based methods. Overall, the findings supported ranking theory as an effective framework for understanding belief systems, highlighting its potential to simplify the cognitive demands of uncertainty assessment and reduce biases commonly associated with subjective probability models. This work laid a foundation for future research to explore ranking theory's application in psychology as a metric for belief and its broader relevance in decision-making processes.

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Lastly, I share words of wisdom from my father that have deeply resonated with me:

everything has its time

summer cradles
the ripening watermelons

frost waits
for the persimmons to sweeten

so too,
do we each have our season to grow

to bloom
in our own time

with patience and sincerity
let us embrace

each day fully
one step at a time

this is the essence of life

Table of Contents

List of Tables **xii**

List of Figures **xiv**

1 Introduction **1**

1.1 The Current Model: Subjective Probability and Its Discontents 3

1.2 Current Issues with Belief Representation 7

1.3 Debate on Whether Probability Theory Can Capture Human Belief 10

1.4 What is Ranking Theory? 12

1.5 Thesis Outline Overview 14

2 Rules of Ranking Theory **17**

2.1 The Basics of Ranking Theory 17

2.2 Justification for the Preference of Negative Ranks in Ranking Theory 21

2.3 Probability-Ranking Translation 22

2.4 Summary of the Chapter 25

3 Experiment 1: Expressing Disbelief through Negative Ranking Functions **26**

3.1 Introduction 26

3.2 Methods 27

3.2.1 Participants 27

3.2.2 Task Environment 27

3.2.3 Procedure 28

3.2.4 Data Analysis 31

3.2.5 Ethical Considerations 32

3.3 Results . . . 32
3.3.1 Analysis of Negative Ranks . . . 32
3.3.2 Relationship Between Subjective Probabilities (via Open Sampling Format) and Negative Ranks . . . 41
3.3.3 Mathematical Model Fit . . . 43
3.3.4 Subjective Probabilities (via Open Sampling Format) and Subjective Probabilities (via Percentage — End of Task Survey) . . . 47
3.3.5 Examining the Law of Disjunction . . . 48
3.3.6 End-of-Task Survey on Task Difficulty . . . 56
3.4 Discussion . . . 59
3.4.1 The Utility of Negative Ranks . . . 60
3.4.2 Subjective Probabilities and Negative Ranks . . . 62
3.4.3 Open-Sampling Format for Capturing Subjective Probability . . . 65
3.4.4 Disjunction Judgements . . . 65
3.4.5 Perceived Difficulty in Grading Degrees of Disbelief . . . 67
3.4.6 Conclusion . . . 67

4 Experiment 2: In-Lab Replication of Negative Ranking Functions 69
4.1 Introduction . . . 69
4.2 Methods . . . 70
4.2.1 Participants . . . 70
4.2.2 Procedure . . . 70
4.2.3 Data Analysis . . . 70
4.2.4 Ethical Considerations . . . 70
4.3 Results . . . 71
4.3.1 Analysis of Negative Ranks . . . 71
4.3.2 Relationship Between Subjective Probabilities and Negative Ranks . . . 78
4.3.3 Mathematical Model Fit . . . 80
4.3.4 Subjective Probabilities (via Open Sampling Format) and Subjective Probabilities (via Percentage — End of Task Survey) . . . 83
4.3.5 Examining the Law of Disjunction . . . 84
4.3.6 End-of-Task Survey on Task Difficulty . . . 91
4.4 Discussion . . . 93

4.4.1 Validating the Utility of Negative Ranks Through Replication 94
4.4.2 Subjective Probabilities and Negative Ranks 95
4.4.3 Efficacy of Open Sampling Format in Subjective Probability Assessment 96
4.4.4 Disjunction Judgements 96
4.4.5 Task Difficulty and Preference for Belief vs. Disbelief 97
4.4.6 Conclusion and Future Direction 97

5 Experiment 3: Examining the Full Spectrum of Belief with Positive and Negative Ranking Functions 99

5.1 Introduction 99
5.2 Methods 100
5.2.1 Participants 100
5.2.2 Task Environment 100
5.2.3 Procedure 100
5.2.4 Ethical Considerations 103
5.3 Results 104
5.3.1 Analysis of Negative and Positive Ranks 104
5.3.2 Relationship Between Subjective Probabilities and Negative Ranks . 115
5.3.3 Mathematical Model Fit 117
5.3.4 Relationship Between Subjective Probabilities and Two-Sided Ranks . 120
5.3.5 Mathematical Model Fit 121
5.3.6 Examining the Law of Disjunction 125
5.4 Discussion 132
5.4.1 Adherence to the Law of Negation 133
5.4.2 Subjective Probabilities and Ranks 136
5.4.3 Law of Disjunction 137
5.4.4 Conclusion 139

6 Experiment 4: Evaluating Ranking Functions in a Probability Learning Task 140

6.1 Introduction 140
6.2 Methods 142
6.2.1 Participants 142

6.2.2 Stimuli . . . 143
6.2.3 Procedure . . . 145
6.2.4 Data Analysis . . . 149
6.2.5 Exclusion Criteria . . . 151
6.2.6 Ethical Considerations . . . 152
6.3 Results . . . 153
6.3.1 Data Exclusion . . . 153
6.3.2 Hide-and-Seek Gameplay — Participant Strategies . . . 153
6.3.3 Assessing Ranks: Correlations with Learned Probability Distributions 156
6.3.4 Relationship Between Subjective Probabilities and Negative Ranks . 159
6.3.5 Mathematical Model Fit . . . 160
6.3.6 Relationship Between Subjective Probabilities and Two-Sided Ranks . 164
6.3.7 Mathematical Model Fit . . . 165
6.3.8 Objective Probabilities and Subjective Probabilities . . . 169
6.3.9 Comparison of Subjective Probabilities and Ranks with Objective Probabilities . . . 171
6.4 Discussion . . . 175
6.4.1 Strategic Approaches in Hide-and-Seek Gameplay . . . 175
6.4.2 Ranking Functions in a Controlled Probability Framework . . . 176
6.4.3 Mathematical Similarity Between Subjective Probabilities And Ranks 178
6.4.4 Comparison of Subjective Probabilities and Ranks with Objective Probabilities . . . 182
6.4.5 Conclusion . . . 184

7 Experiment 5: Rethinking the Ellsberg Paradox through Negative Ranking Functions 185

7.1 Introduction . . . 185
7.2 Methods . . . 188
7.2.1 Participants . . . 188
7.2.2 Procedure . . . 188
7.2.3 Task Environment . . . 189
7.2.4 Data Analysis . . . 189
7.2.5 Ethical Considerations . . . 190

7.3 Results . . . 191
7.3.1 Ambiguity Aversion . . . 191
7.3.2 Ambiguity Attraction . . . 191
7.3.3 Consistent and Selective Neutrality . . . 191
7.3.4 Consistent Preferences . . . 192
7.3.5 Degrees of Disbelief Across Propositions . . . 196
7.4 Discussion . . . 198

8 Experiment 6: Comparing Traditional Betting Choices and Negative Ranks in the Ellsberg Paradox 202
8.1 Introduction . . . 202
8.2 Methods . . . 203
8.2.1 Participants . . . 203
8.2.2 Procedure . . . 203
8.2.3 Data Analysis . . . 204
8.2.4 Ethical Considerations . . . 205
8.3 Results . . . 206
8.3.1 Ambiguity Aversion . . . 206
8.3.2 Ambiguity Attraction . . . 208
8.3.3 Consistent Neutrality . . . 209
8.3.4 Selective Neutrality . . . 209
8.3.5 Consistent Preferences . . . 210
8.3.6 Degrees of Disbelief Across Propositions . . . 213
8.3.7 Relationship Between Disbelief and Betting Choices . . . 214
8.3.8 Comparison with Randomized Rank Responses . . . 218
8.4 Discussion . . . 219

9 General Discussion 224
9.1 The Utility of Ranks and Their Distinction from Subjective Probabilities . . . 224
9.2 Ranking Theory – A Natural Way of Representing Beliefs . . . 227
9.3 Application of Ranking Theory in Real-World Practical Settings and Future Directions . . . 231
9.3.1 Potential to Address Cognitive Biases . . . 231

9.3.2 Exploring the Dynamics of Belief Revision 234
9.3.3 Advancing Psychological Research through Ranking Theory 235
9.4 Conclusion 235

References **247**

Appendix **247**
Experiments 1 and 2 Materials 247
Experiment 3 Materials 249
Experiment 4 Materials 253
Experiments 5 and 6 Materials 256

List of Tables

3.1 Descriptive statistics for Language Demographics, showing the median and interquartile range (IQR) of negative rank scores for each language. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values. 35

3.2 Descriptive statistics for Temperature Ranges, showing the median and interquartile range (IQR) of negative rank scores for each temperature range. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values. 38

3.3 Descriptive statistics for Music Preferences, showing the median and interquartile range (IQR) of negative rank scores for each genre. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values. 41

3.4 Results of the linear mixed-effects model predicting disjunction ranks. This table displays the estimated coefficients, standard errors, t-values, and p-values for the arsinh-transformed minimum and maximum ranks, alongside questionnaire types and their interaction effects. 54

4.1 Descriptive statistics for Language Demographics, showing the median and interquartile range (IQR) of negative rank scores for each language. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values. 73

4.2 Descriptive statistics for Temperature Ranges, showing the median and interquartile range (IQR) of negative rank scores for each temperature range. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values. 75

4.3 Descriptive statistics for Music Preferences, showing the median and interquartile range (IQR) of negative rank scores for each genre. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values. 78

4.4 Results of the linear mixed-effects model predicting disjunction ranks. This table displays the estimated coefficients, standard errors, t-values, and p-values for the arsinh-transformed minimum and maximum ranks, alongside questionnaire types and their interaction effects. 89

5.1 Descriptive statistics for Temperature Ranges, showing the median and interquartile range (IQR) of negative rank scores for each temperature range and its negation. 106

5.2 Descriptive statistics for Music Preferences, showing the median and interquartile range (IQR) of negative rank scores for each genre and its negation. . . . 109

5.3 Descriptive statistics for Investment Outcomes, showing the median and interquartile range (IQR) of negative rank scores for each outcome and its negation. 112

5.4 Descriptive statistics for lottery outcomes, showing the median and interquartile range (IQR) of negative rank scores for each proposition and its negation. 115

5.5 Results of the linear mixed-effects model predicting disjunction ranks. This table displays the estimated coefficients, standard errors, t-values, and p-values for the arsinh-transformed minimum and maximum ranks, alongside questionnaire types and their interaction effects. 130

6.1 Opponents faced by participants in each condition. Each distribution is presented here in decreasing room probabilities, though probabilities were randomly assigned to rooms during the experiment. Opponent identities were also randomly assigned to each game without replacement, and the order of the distributions was randomized except for the practice, which always came first. 145

List of Figures

3.1 Boxplots showing the degrees of disbelief (negative rank) for encountering a native Quebec resident who speaks only one language, across different languages. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each language, with lower values indicating lower disbelief. French and English exhibited the lowest disbelief, indicating that participants expected these languages to be more commonly spoken. Conversely, languages such as Urdu and Japanese showed higher disbelief, reflecting their perceived rarity. 34

3.2 Boxplots showing the degrees of disbelief (negative rank) for various daytime high temperature ranges in Boston during September. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each temperature range, with lower values indicating lower disbelief. The temperature range of 11 to 20°C exhibits the lowest disbelief, indicating that participants expect this range to be typical for September (median score of 0). Conversely, extreme temperature ranges such as -29 to -20°C and 41 to 50°C show higher disbelief, reflecting their perceived rarity. 37

3.3 Boxplots showing the degrees of disbelief (negative rank) for various music genres that a coworker might sing along to at a karaoke party. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each genre, with lower values indicating lower disbelief. The genre of country music exhibits the lowest disbelief, indicating that participants expect this genre to be typical for the coworker. Conversely, genres such as opera show higher disbelief, reflecting their perceived rarity. 40

3.4 Scatter plots showing the relationship between subjective probabilities (via Open Sampling Format) and negative ranks (κ) across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis is scaled to display negative ranks effectively, while the labels show the original rank values for easier interpretation. The dotted blue line represents the threshold where disbelief is greater than 0. Instances above this line indicate varying degrees of disbelief. The plots suggest a nonlinear relationship between negative rank and subjective probability, with lower probabilities generally associated with higher degrees of disbelief. However, probabilities ranging from 0 to 1 may not always elicit surprise or disbelief, demonstrating the value of negative rank in explicitly reporting varying degrees of disbelief. 43

3.5 Scatter plot illustrating the relationship between subjective probabilities and negative ranks (κ) across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis was scaled to display negative ranks effectively, while the labels showed the original rank values for easier interpretation. A logarithmic model was fitted to the data, approximately explaining 3.90% of the variance in negative ranks. 45

3.6 Scatter plot showing the relationship between subjective probabilities and negative ranks (κ) across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis was scaled to effectively display negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with an exponential decay model, which accounts for approximately 8.35% of the variance observed in the negative ranks. 47

3.7 Scatterplot showing the relationship between subjective probabilities recorded via Open Sampling Format (OSF) and subjective probabilities recorded as percentages at the end of the task. The correlation coefficient is 0.641, indicating a strong relationship between the two measures. 48

3.8 Boxplots examining the law of disjunction for three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis is on a logarithmic scale to display a wider range of observations. The minimum rank represents the least disbelieved of the two propositions involved in the disjunction, while the maximum rank represents the more disbelieved of the two propositions. The disjunction rank corresponds to the logical "or" relationship between the two propositions, reflecting the combined disbelief that at least one of the propositions is true. The dots are jittered points representing individual participant responses. The disjunction rank is more disbelieved than the minimum rank but less disbelieved than the maximum rank for Language Demographics and Music Preferences. For Temperature Ranges, the disjunction rank has the same median as the minimum rank. . . 51

3.9 Predicted disjunction rank values (κ) for minimum rank across different questionnaire types. Temperature Ranges are shown in yellow, Music Preferences in green, and Language Demographics in red. Shaded areas denote confidence intervals. The plot demonstrates a significant positive relationship between minimum rank and disjunction rank for Temperature Ranges, with varying effects observed in other types. 55

3.10 Predicted disjunction rank values for the maximum rank of two propositions, segmented by questionnaire type. Language Demographics are shown in red, Temperature Ranges in yellow, and Music Preferences in green. Shaded areas indicate confidence intervals. This graph highlights the variation in the effect of maximum rank on disjunction rank across different questionnaire types, especially evident in Music Preferences and Language Demographics, where disjunction rank is predominantly influenced by the maximum of the two ranks. 56

3.11 Bar graph showing the distribution of participants' task difficulty ratings for grading degrees of disbelief. The most common response was moderately difficult (Rating 4). 58

3.12 Bar graph depicting the distribution of responses to the perceived ease of assigning degrees of belief over degrees of disbelief. The most frequent rating was neutral (Rating 4). 59

4.1 Boxplots showing the degrees of disbelief (negative rank) for encountering a native Quebec resident who speaks only one language, across different languages. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each language, with lower values indicating lower disbelief. French and English exhibit the lowest disbelief, indicating that participants expect these languages to be more commonly spoken. Conversely, languages such as Urdu and Japanese show higher disbelief, reflecting their perceived rarity. 72

4.2 Boxplots showing the degrees of disbelief (negative rank) for various daytime high temperature ranges in Boston during September. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each temperature range, with lower values indicating lower disbelief. The temperature range of 11 to 20°C exhibits the lowest disbelief, indicating that participants expect this range to be typical for September. Conversely, extreme temperature ranges such as -29 to -20°C and 41 to 50°C show higher disbelief, reflecting their perceived rarity. 74

4.3 Boxplots showing the degrees of disbelief (negative rank) for various music genres that a coworker might sing along to at a karaoke party. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each genre, with lower values indicating lower disbelief. The genre of country music exhibits the lowest disbelief, indicating that participants expect this genre to be typical for the coworker. Conversely, genres such as opera show higher disbelief, reflecting their perceived rarity. 77

4.4 Scatter plots showing the relationship between subjective probabilities (via Open Sampling Format) and negative ranks (κ) across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis is scaled to display negative ranks effectively, while the labels show the original rank values for easier interpretation. The dotted blue line represents the threshold where disbelief is greater than 0. Instances above this line indicate varying degrees of disbelief. The plots suggest a nonlinear relationship between negative rank and subjective probability, with lower probabilities generally associated with higher degrees of disbelief. However, low probability values may not always elicit surprise or disbelief, demonstrating the value of negative rank in explicitly reporting varying degrees of disbelief. 80

4.5 Scatter plot illustrating the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to display negative ranks effectively, with labels indicating the original rank values for easier interpretation. A logarithmic model is fitted to the data, but it was a poor fit as indicated by the pseudo R^2 value. 81

4.6 Scatter plot showing the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to effectively display negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with an exponential decay model, which accounts for approximately 40.50% of the variance observed in the negative ranks. 83

4.7 Scatterplot showing the relationship between subjective probabilities recorded via Open Sampling Format (OSF) and subjective probabilities recorded as percentages at the end of the task. The correlation coefficient is 0.728, indicating a strong relationship between the two measures. 84

4.8 Boxplots examining the law of disjunction for three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis is on a logarithmic scale to display a wider range of observations. The minimum rank represents the least disbelieved of the two propositions involved in the disjunction, while the maximum rank represents the more disbelieved of the two propositions. The disjunction rank corresponds to the logical "or" relationship between the two propositions, reflecting the combined disbelief that at least one of the propositions is true. The dots are jittered points representing individual participant responses. 86

4.9 Predicted disjunction rank values (κ) for minimum rank across different questionnaire types. Temperature Ranges are shown in yellow, Music Preferences in green, and Language Demographics in red. Shaded areas denote confidence intervals. The plot demonstrates a significant positive relationship between minimum rank and disjunction rank for Temperature Ranges, with varying effects observed in other types. 90

4.10 Predicted disjunction rank values for the maximum rank of two propositions, segmented by questionnaire type. Language Demographics are shown in red, Temperature Ranges in yellow, and Music Preferences in green. Shaded areas indicate confidence intervals. This graph highlights the variation in the effect of maximum rank on disjunction rank across different questionnaire types, especially evident in Music Preferences and Language Demographics, where disjunction rank is predominantly influenced by the maximum of the two ranks. 91

4.11 Bar graph showing the distribution of participants' task difficulty ratings for grading degrees of disbelief. The most common response was moderately difficult (Rating 4). 92

4.12 Bar graph depicting the distribution of responses to the perceived ease of assigning degrees of belief over degrees of disbelief. The most frequent rating was neutral (Rating 4). 93

5.1 Box plots of negative ranks (degrees of disbelief) for different temperature ranges in NYC's daytime high temperatures during September. The y-axis is on a logarithmic scale to accommodate the wide range of rank values. Yellow represents disbelief in the temperature ranges, while blue represents disbelief in the negation of the temperature ranges (interpreted as belief in the temperature ranges). 105

5.2 Box plots of negative ranks (degrees of disbelief) for various music genres and their negation among coworkers. The y-axis is on a logarithmic scale to display a wider range of observations. Green represents disbelief in the music genres, while orange represents disbelief in the negation of the music genres (interpreted as belief in the music genres), highlighting that genres like opera are more disbelieved while country music tends to be believed. 108

5.3 Boxplots showing the degrees of disbelief (negative rank) for various investment outcomes and their negations. The y-axis is on a logarithmic scale to display a wider range of observations. Turquoise represents disbelief in the investment outcomes, while purple represents disbelief in the negation of the investment outcomes (interpreted as belief in the investment outcomes). . . . 111

5.4 Boxplots showing the degrees of disbelief (negative rank) for various lottery outcomes. The y-axis is on a logarithmic scale to accommodate the wide range of rank values. Pink represents disbelief in the outcomes 'winning' and 'winning or losing,' while navy represents disbelief in the outcome 'losing (not winning),' interpreted as belief in 'winning.' 114

5.5 Scatter plots showing the relationship between subjective probabilities (via Open Sampling Format) and negative ranks (κ) across four questionnaires: Temperature Ranges, Music Preferences, Real-estate Investment, and Lottery. The y-axis is scaled to display negative ranks effectively, while the labels show the original rank values for easier interpretation. The dotted blue line signifies the threshold of significant disbelief. These plots highlight the non-linear and context-dependent nature of the relationship between subjective probabilities and negative ranks, with lower probabilities generally associated with higher disbelief. However, lower probabilities do not uniformly elicit disbelief, highlighting the importance of negative ranks in capturing the complexity of subjective disbelief. 116

5.6 Scatter plot illustrating the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to display negative ranks effectively, with labels indicating the original rank values for easier interpretation. A logarithmic model is fitted to the data, but it was a poor fit as indicated by the R^2 value. 118

5.7 Scatter plot showing the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to effectively display negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with an exponential decay model, which accounts for 4.98% of the variance observed in the negative ranks. 119

5.8 Scatterplot showing the relationship between subjective probabilities and two-sided ranks τ for each distribution configuration. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The dotted line represents a two-sided rank of zero, indicating neutral judgment. 121

5.9 Scatter plot showing the relationship between subjective probabilities and two-sided ranks τ across all questionnaires. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with a log-odds model, which captures 6.40% of the variance in the ranks. 123

5.10 Scatter plot showing the relationship between subjective probabilities and two-sided ranks τ across all questionnaires. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with a polynomial regression model, which captures 10.99% of the variance in the ranks. The fitted curve demonstrates how subjective probabilities predict two-sided ranks, capturing potential non-linearities in the data. 124

5.11 Boxplots examining the law of disjunction for four questionnaires: Temperature Ranges, Music Preferences, Investment Scenarios, and Lottery Outcomes. The y-axis is on a logarithmic scale to display a wider range of observations. The minimum rank represents the least disbelieved of the two propositions involved in the disjunction, while the maximum rank represents the more disbelieved of the two propositions. The disjunction rank corresponds to the logical "or" relationship involving the two propositions. The dots are jittered points representing individual participant responses. 127

5.12 Predicted disjunction rank values as a function of the minimum rank, which represents the least disbelieved of the two propositions in the disjunction ranks, across different questionnaire types. Shaded areas denote confidence intervals, demonstrating the significant positive relationship between minimum rank and disjunction rank across varying types of questionnaires. 131

5.13 Predicted disjunction rank values as a function of the maximum rank, which represents the more disbelieved of the two propositions predicting disjunction, across different questionnaire types. Shaded areas denote confidence intervals, reflecting how maximum ranks affect disjunction ranks across different contexts. The maximum rank did not significantly predict disjunction overall, but a significant interaction was observed for music preferences. 132

6.1 Participant view of the screen after selecting the middle room in a 'seek' trial within the three-room dollhouse condition. The selected room turns grey, revealing the child's location and presenting a notification indicating the outcome. In this example, the participant did not find Jill. The figure illustrates a trial in the hide-and-seek gameplay, where participants select one of three rooms in the dollhouse to seek the hidden child. 144

6.2 For seeking strategies defined by the equation $\alpha + \beta = 1$, error (indicated by red regions) is minimized when $\vec{\epsilon} = \vec{0}$. Numerous points on or near this line suggests a strong model fit, whereas clusters of points in areas with high error signal a poor fit. 151

6.3 Participants' strategy space of each distribution. Each plot shows the relationship between two strategies (maximizing and matching for seeking; minimizing and mismatching for hiding). The diagonal lines represent the optimal strategy combinations ($\alpha+\beta = 1$ for seeking; $\gamma+\delta = 1$ for hiding). Points represent individual participants' strategies, with histograms along the margins showing the frequency distribution of each strategy component. Participant strategies existing on or near line $\alpha + \beta = 1$ for seeking, $\gamma + \delta = 1$ for hiding implies near perfect model fit. 155

6.4 Violin plot illustrating participants' degrees of disbelief and belief for each probability distribution, adjusted using the arsinh transformation to scale values from -infinity (representing impossibility) to infinity (representing certainty). This plot compares the relationship between objective probabilities and the two-sided ranks, where negative values indicate disbelief and positive values indicate belief. The triangle within each violin plot represents the median, highlighting how participants' belief systems adapt to different probability distributions. 158

6.5 Scatterplot of subjective probabilities against negative ranks for each distribution configuration. 160

6.6 Scatter plot illustrating the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to display negative ranks effectively, while the labels show the original rank values for easier interpretation. A logarithmic model is fitted to the data, explaining 55.37% of the variance in negative ranks. 162

6.7 Scatter plot showing the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to effectively display negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with an exponential decay model, which accounts for 54.77% of the variance observed in the negative ranks. 163

6.8 Scatterplot showing the relationship between subjective probabilities and two-sided ranks τ for each distribution configuration. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The dotted line represents a two-sided rank of zero, indicating neutral judgment. 165

6.9 Scatter plot showing the relationship between subjective probabilities and two-sided ranks τ across all questionnaires. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with a log-odds model, which captures 54.83% of the variance in the ranks. 167

6.10 Scatter plot showing the relationship between subjective probabilities and two-sided ranks τ across all questionnaires. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with a polynomial regression model, which captures 56.18% of the variance in the ranks. 168

6.11 Violin plot illustrating participants' subjective probabilties against objective probabilities for each probability distribution. The triangle within each violinplot represents the median. The horizontal red dotted lines indicate the corresponding objective probabilities for each distribution. 171

6.12 A probability simplex (Δ_{n-1}) displays participants' subjective probabilities and disbelief rankings against a fixed objective probability [$33.\overline{3}$, $33.\overline{3}$, $33.\overline{3}$] marked in red. Blue dots, representing subjective probabilities, are widely dispersed, illustrating varied response densities from multiple participants. Yellow dots, indicating negative ranks, cluster at the centre, reflecting a more consistent evaluation of objective probability, with each room ranked similarly. Larger dots indicate more common responses, highlighting that rankings provide a clearer picture of the uncertainty structure compared to subjective probabilities. 174

7.1 This pie chart illustrates the behavioral patterns observed in the Ellsberg one-urn scneario. It shows variations in participants' responses, including ambiguity aversion, a preference for ambiguity, consistent and selective neutrality, and consistent preferences for red and black. These findings highlight the diversity of participant behaviour under ambiguous conditions. 194

7.2 The figure displays the distribution of preferences based on degrees of disbelief (negative ranks) across participants. The stacked bars in the figure show the distribution of preferences for (a) 'Red' versus 'Black', and (b) 'Red or Yellow' versus 'Black or Yellow', and situations of equal disbelief between the two. Preferences were determined by comparing the degrees of disbelief. For example, a preference for 'red' over 'black' was noted when disbelief in drawing a red ball was less than that for a 'black' ball, and vice versa. Situations where disbelief in 'red' and 'black' were equivalent are represented as "equal" preferences. The results show that a higher proportion of participants had equal disbelief between the two propositions. 195

7.3 This diagram shows how negative ranks (degrees of disbelief) are distributed across propositions. Negative ranks were adjusted on a logarithmic scale to include zero values, with zero represented by a small positive constant, epsilon. The plot uses boxplots, jittered points, and lines to show variations and trends in disbelief responses for each participant across propositions. The y-axis is transformed using a base 10 logarithmic scale. The data highlights both the central tendency and variability within each proposition group, providing insights into participants' degrees of disbelief, which did not differ significantly across the propositions. 197

8.1 Change in choice preference across rounds for Red vs. Black and Red or Yellow vs. Black or Yellow. The plots show the percentage of participants who switched their choices for each round (2, 3, 4). A large proportion of participants did not change their choices across four rounds, while some switched their choices to show neutrality or some degree of confidence in their choices. The y-axis represents whether participants switched their choices (True) or kept the same choices across rounds (False). 208

8.2 Comparative visualization of participant responses: The top row shows pie charts of betting choices and negative ranks in the one-urn scenario, while the bottom row presents the rarity one-urn scenario. These charts illustrate patterns of ambiguity aversion, attraction, neutrality, and preference, highlighting how these behaviours differed across modalities. 212

8.3 Visual Representation of Disbelief Scores Across Propositions and Urn Scenarios: This plot demonstrates the distribution of disbelief ranks on a logarithmic scale, utilizing boxplots to depict central tendencies and variability, jittered points to indicate individual responses, and connecting lines to trace changes in disbelief for each participant across different propositions and urn scenarios. The boxplots show a clear distinction in disbelief levels between one-urn and rarity-urn scenarios, with increased disbelief in the rarity scenario compare to the one-urn scenario. Importantly, within each scenario, no differences in disbelief scores were observed across the propositions. 214

8.4 Degrees of Disbelief and Choice Preferences with Randomized Data Points. This figure illustrates the relationship between participants' degrees of disbelief and their choices in the black versus red scenario. The y-axis represents the difference in negative ranks (κ) between black and red, with negative values indicating greater disbelief in red; while positive values indicate less disbelief in red. The x-axis displays the frequency of choosing red across four rounds, ranging from 0 to 4 times. Black dots represent actual participant data, while red dots indicate randomized data points generated with a fixed mean of 0 and SD that matches the observed data, but only varying in sample size based on each frequency count of choosing red. The box plots visualize the central tendency and dispersion of disbelief scores for each choice frequency, revealing no apparent correlation between degrees of disbelief and the frequency of choosing red. 216

8.5 Degrees of Disbelief and Choice Preferences with Randomized Data Points. This figure illustrates the relationship between participants' degrees of disbelief and their choices in the black or yellow versus red or yellow scenario. The y-axis represents the difference in negative ranks (κ) between red or yellow and black or yellow, with negative values indicating greater disbelief in black or yellow; while positive values indicate less disbelief in black or yellow. The x-axis displays the frequency of choosing black or yellow across four rounds, ranging from 0 to 4 times. Black dots represent actual participant data, while red dots indicate randomized data points generated with a fixed mean of 0 and SD that matches the observed data, but only varying in sample size based on each frequency count of choosing black or yellow. The box plots visualize the central tendency and dispersion of disbelief scores for each choice frequency, revealing no apparent correlation between degrees of disbelief and the frequency of choosing black or yellow. 217

8.6 The distribution of ambiguity aversion percentages for randomized negative ranks is presented by scenario condition (one-urn and rarity one-urn). The dotted vertical lines represent the observed empirical data for ambiguity aversion: 8.62% for the one-urn condition (blue line) and 5.17% for the rarity one-urn condition (red line). The histograms show the distribution of ambiguity aversion percentages across 1000 randomizations, where negative ranks were shuffled within each proposition ("Red," "Black," "Red or Yellow," "Black or Yellow") across participants. The observed empirical percentages varied from the central tendencies of the randomized distribution. 219

1 Ranking questionnaire for language demographics of Quebec. 247

2 Ranking questionnaire for September weather in Boston, Massachusetts, United States. 248

3 Ranking questionnaire for coworker's music genre preferences. 248

4 Open sampling format used to measure subjective probability, represented by a 10 by 10 grid of possible outcomes. In this example, the language demographics questionnaire shows French as the most expected, followed by English, with equal expectancy for the remaining languages. 249

5 Ranking questionnaire for September weather in New York City, New York, United State, used in Experiment 3. 250

6 Ranking questionnaire for coworker's music genre preferences, used in Experiment 3. 251

7 Ranking questionnaire for real estate investment outcomes. 252

8 Ranking questionnaire for lottery outcomes. 252

9 Ranking questionnaire for hiding condition. 253

10 Ranking questionnaire for seeking condition. 254

11 Probability questionnaire for hiding condition. 255

12 Probability questionnaire for seeking condition. 256

13 Ranking questionnaire for the one-urn scenario. 257

14 Participants chose between different gambles across four rounds, in Experiment 6. 258

15 Ranking questionnaire for the rarity one-urn scenario in Experiment 6. . . . 259

Chapter 1

Introduction

How do humans represent their beliefs about an uncertain world? The mechanisms underlying human uncertainty representation remain unclear. Understanding how humans interact with uncertainty and exploring the nature of belief provide valuable insights into how individuals perceive their environments and how the brain encodes uncertainty (Huettel et al., 2005; Mushtaq et al., 2011). For example, consider a hiker descending a mountain who must choose a path based on their assessment of the slope's steepness. In this case, uncertainty about the slope's gradient influences the decision. The hiker may gather more information by approaching the slope before selecting an alternative route. As new information becomes available, the hiker's belief about the optimal path will adjust. Here, uncertainty refers to the observer's limited knowledge of the environment, as humans cannot encompass all available information in their surroundings (Walker et al., 2022).

Humans often face uncertainty and struggle to determine the best course of action or what to believe (Bland and Schaefer, 2012). One of the most influential and widely used models for representing beliefs is probability theory, which assigns a probability to each possible

state of the world and uses probability laws to guide decision-making (Cosmides and Tooby, 1996; Sanborn and Chater, 2016). In this framework, degrees of belief are represented as real numbers in the range [0,1] (Wyer and Albarracin, 2005), with higher values indicating greater confidence in a proposition's truth (Huber, 2009).

Although probability distributions are one way humans may form beliefs, they impose strict constraints on how beliefs are combined and updated. For instance, the same subjective probability must lead to the same degree of belief regardless of context. Additionally, the probabilities of mutually exclusive events are additive, meaning the total probability is the sum of their individual probabilities (Kolmogorov, 2018). For independent events, the joint probability is the product of their individual probabilities (Feller, 1991). While these rules are mathematically rigorous, they may not fully capture the complexity of human belief systems. Spohn argues that belief cannot be equated with probability alone (Spohn, 2013), suggesting that probability functions are better viewed as tools for calculating expected utility in decision-making (Smets, 2002).

This thesis explored ranking theory, an alternative belief representation model grounded in formal epistemology and rationality principles. Ranking theory provides a *normative* account of belief dynamics that differs from traditional probabilistic approaches (Skovgaard-Olsen, 2016; Spohn, 2013). Instead of assigning probabilities to events, ranking theory assigns degrees of belief or disbelief to propositions based on their logical relations and available evidence (Skovgaard-Olsen, 2016). However, it has not been evaluated for its potential as a descriptive model — specifically, how well it explains real-world human behaviour when individuals form, update, or combine beliefs under uncertainty (Kern-Isberner et al., 2021).

To test the empirical adequacy of ranking theory, a series of behavioural experiments were conducted to examine how people represent their beliefs in various scenarios.

1.1 The Current Model: Subjective Probability and Its Discontents

Probability theory is a widely used formal model for representing beliefs in cognitive science and artificial intelligence literature (Chater et al., 2006; Colombo et al., 2016; Ghahramani, 2015). It is based on the idea that degrees of belief are represented as subjective probabilities (Smith et al., 1991). Probabilities are numerical values between 0 and 1 that express how likely a proposition is to be true, with a probability of 0 being impossible and a probability of 1 indicating certainty. The higher the probability of a proposition, the more strongly it is believed, and vice versa (Icard, 2016; Wyer and Albarracin, 2005).

Probability theory relies on specific rules to express logical relations between propositions. For instance, the probability of a conjunction $P(p \cap q)$ is the product of the individual probabilities if the events are independent:

$$P(p \cap q) = P(p) \cdot P(q)$$

Similarly, the probability of a disjunction $P(p \cup q)$ is given by:

$$P(p \cup q) = P(p) + P(q) - P(p \cap q)$$

This rule accounts for the overlap of p and q when they are not mutually exclusive (Feller, 1991).

Beliefs are updated in response to new evidence using the rule of conditionalization. If P represents the probability and E is new evidence, the updated probability of a proposition H given E is:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

This formula, known as Bayes' theorem, allows beliefs to be adjusted in light of new evidence (Bayes, 1763; Jaynes, 2003; Laplace, 1812).

Probability theory provides a structured framework for quantifying uncertainty and making inferences across various contexts. By assigning numerical probabilities to different outcomes, it enables comparisons and predictions. This framework not only allows for reasoning based on available evidence but also supports learning from data. Consequently, it facilitates the decision-making process by providing methods for updating probabilities with new information (Fox, 2003; Li et al., 2012).

Despite its strengths, probability theory has notable limitations. One major limitation is that it does not differentiate between degrees of belief and disbelief in a proposition (Spohn, 2012). Probability theory assigns a single value to each proposition, which represents its overall plausibility or likelihood. For example, a proposition with a probability of 0.6 is interpreted as being more likely to be true than false. However, this single value does not capture the different levels of belief people might have, as it is merely a likelihood estimate. Consequently, probability theory cannot fully account for certain characteristics of human belief representation, such as the distinction between acceptance and rejection of a proposition, or sensitivity to the strength or quality of evidence (Spohn, 2012).

Classical probability theory is limited in handling logical relationships between propositions. Probability assignments must coexist without contradictions. For instance, a coherent set of probabilities cannot assign a value greater than zero to a contradiction, such as both p (it is raining) and $\neg p$ (it is not raining) (Bueno-Soler and Carnielli, 2016). This constraint limits probability theory's utility in belief systems with inherent contradictions, as it cannot account for propositions that should logically have zero probability but are still considered in decision-making. Probability theory also assumes uncertainty is always quantifiable (Kolmogorov, 2018). However, as Keynes (2013) argued in his Principle of Indifference, when an agent lacks sufficient information — such as being indoors all day without knowledge of the weather — it is erroneous to assume that the absence of evidence implies equal probabilities (i.e., assigning a probability of 0.5). Keynes critiqued this approach, showing that ignorance or lack of external relevant evidence should not automatically lead to such equal probability assignments, as this could result in paradoxes and contradictions (Keynes, 2013). Instead, it reflects indecision or a lack of information, which classical probability theory struggles to capture effectively.

Although probability theory provides a formal structure for quantifying uncertainty, the relationship between numerical probabilities and human belief is neither clear nor intuitive (Spohn, 2009). Research shows that individuals often struggle to align probabilistic likelihood with their subjective sense of belief. For instance, Kahneman and Tversky demonstrated how people rely on heuristics, such as availability and representativeness, which can lead to systematic errors in judgment and inconsistencies between likelihood estimations and beliefs about outcomes (Kahneman, 2011; Tversky and Kahneman, 1974). This gap between

formal probability models and intuitive belief systems illustrates the complexity of applying probabilistic reasoning to real-world decision-making, where beliefs are influenced by factors beyond simple probability assessments (Fox, 2003; Spohn, 2012)

The gambling and lottery paradoxes further illustrate the difficulty of aligning mathematical probability with human cognition. Common fallacies, such as the gambler's fallacy, reveal how individuals mistakenly believe that past independent events influence future outcomes. For instance, after five consecutive 'tails' in a coin toss, one might expect 'heads' to be more likely, despite the independence of each flip. Similarly, the lottery paradox reflects how people misinterpret low probabilities. In a 1000-ticket lottery, each ticket has a low chance of winning (0.001). If we consider each ticket individually and sequentially, one might mistakenly believe that no single ticket will win due to its low probability. However, collectively, it is certain that one ticket must win. These paradoxes highlight the disconnect between normative probabilistic reasoning and subjective interpretations, where human reasoning often conflicts with probabilistic logic, which requires considering all outcomes collectively to ensure their probabilities sum to one (Clotfelter and Cook, 1993; Nelkin, 2000; Wheeler and Wheeler, 2007).

Juslin et al. (2009) demonstrated that cognitive biases, such as the conjunction fallacy and base-rate neglect, present substantial challenges in human reasoning. These biases show that individuals frequently overestimate the likelihood of combined events or favour specific details over general information. Such deviations from probabilistic norms may result from the complexity of multiplicative probability integration (Juslin et al., 2009; Spohn, 2012). Alternatively, these inconsistencies suggest that associating human belief exclusively with

probability theory may be problematic, indicating the need for a different framework.

Though people can learn probability distributions (Go et al., 2022), studies show that they do not always adopt optimal strategies. For example, in a hide-and-seek game against a computer assessing probability-matching and anti-matching behaviour, some individuals did not follow the maximizing strategy, which involves consistently choosing the option with the highest success probability (DiBerardino, 2023). These findings imply that probabilistic reasoning may rely more on implicit or procedural processes than on explicit or declarative ones. This is intriguing, because it suggests that probability is something we 'do' through our actions rather than something we 'think' through our words.

These examples emphasize that while individuals naturally deviate from probabilistic norms, the real challenge lies in interpreting these behaviours within a framework based on subjective probability. People easily make decisions that contradict probabilistic reasoning, suggesting that their cognitive processes function differently from what probability theory predicts. This raises the question of whether probabilities alone can sufficiently model human reasoning under uncertainty. People may simultaneously hold beliefs and probabilities, yet neither fully captures the other (Spohn, 2013). Therefore, we need to explore alternative accounts of graded belief.

1.2 Current Issues with Belief Representation

Understanding belief systems is crucial in cognitive research, as it influences how people think, learn, and make decisions. Belief representation encompasses various aspects, such as

how beliefs are formed, stored, remembered, updated, and used. Factors like logic, evidence, memory, attention, emotion, motivation, and social context all play a role in belief representation (Uso-Domenech and Nescolarde-Selva, 2016; Grayling, 2011; Halpern and Fagin, 1992; Wyer and Albarracin, 2005). The representation of beliefs profoundly influences cognitive outcomes and behaviours, particularly through judgment and inference, emphasizing that beliefs are not merely passive reflections but active engagements with truth, guided by normative standards of correctness. (Wedgwood, 2002).

One of the main challenges in belief representation is modelling it formally and rigorously. Cognitive science and artificial intelligence employ various models for this purpose, including probability theory, Dempster-Shafer theory, fuzzy reasoning, Bayesian reasoning, possibility theory, default reasoning, nonmonotonic reasoning, and ranking theory (Pearl, 1988; Kern-Isberner et al., 2021). Each model has distinct logical assumptions, methods, applications, strengths, and weaknesses. For instance, probability theory is more rigorous in its mathematical foundation, while Dempster-Shafer theory allows for a more flexible representation of uncertainty (particularly when evidence is incomplete) (Pearl, 1988; Shafer, 1976). Bayesian reasoning, commonly applied in probabilistic predictions by integrating prior knowledge with new evidence, and ranking theory can be more suitable for representing degrees of disbelief in nonmonotonic contexts (Griffiths and Tenenbaum, 2006; Spohn, 2012). These models also have unique relationships and comparisons; some are more flexible or have strict sets of rules, while others are more practical or theoretical.

Another challenge with belief representation is determining how to evaluate it empirically. In cognitive science and artificial intelligence, various methods have been developed to assess

belief representation, each with its own procedures, design, and measurement techniques, along with unique advantages and disadvantages. These methods differ in terms of validity and reliability. For instance, some methods are more controlled and standardized, such as laboratory experiments using formal tasks, which enhance reliability. Others are more naturalistic or ecologically valid, such as field studies observing real-world decision-making, which are crucial for generalizing findings to real-world situations (Childers et al., 2023; Kern-Isberner et al., 2021)

Cognitive neuroscientists have explored the neural representation of beliefs by investigating the brain regions and networks involved in processing them. Studies have identified key areas, including the frontal, parietal, and medial temporal lobes, as well as the hippocampus, which play significant roles in the formation, evaluation, and storage of beliefs (Harris et al., 2009; Seitz, 2022). However, regions associated with beliefs also interact with many lower-level cognitive processes, such as attention, perception, and memory, raising questions about their specificity to belief representation (Connors and Halligan, 2022). Additionally, distinct types of beliefs, such as religious and non-religious beliefs, recruit different neural substrates depending on their content, cultural context, and subjective certainty. For example, religious beliefs have been shown to activate regions such as the precuneus, anterior insula, ventral striatum, anterior cingulate cortex, and posterior medial cortex, reflecting their integration of emotional, self-referential, and cognitive conflict processing (Gaw, 2019; Harris et al., 2009; Seitz, 2022). In contrast, non-religious beliefs engage memory-related networks, particularly in the left hemisphere (Harris et al., 2009). It is also important to note that in many fMRI studies, participants were presented with propositions framed as

true or false and asked whether they believed them, rather than examining the degrees of belief (Seitz, 2022). Consequently, more fine-grained and systematic research is required to delineate the specific neural underpinnings of belief representation (Connors and Halligan, 2022; Seitz, 2022).

One way to understand belief is by developing a descriptive model that explains how beliefs are formed, represented, and used by people, based on theoretical and mathematical principles. A good descriptive model should be generalizable to various real-world contexts and flexible enough to modify beliefs easily when presented with new evidence, without requiring complex computations. By exploring the cognitive processes and mechanisms involved in belief formation and representation, we can move closer to understanding human belief representation.

1.3 Debate on Whether Probability Theory Can Capture Human Belief

Probability theory has traditionally been regarded as a framework for measuring uncertainty, linking evidence to hypotheses to promote rational belief formation based on empirical and logical foundations (Galavotti, 2017). However, this approach often fails to account for the subjective aspects of human belief, as it assumes that a complete and objective assessment of evidence will lead to a singular, rational probability assignment (Galavotti, 2017).

Critics argue that this interpretation is too rigid, overlooking the personal and context-

dependent nature of belief. Subjective interpretations, such as those proposed by Bruno de Finetti, emphasize that beliefs are shaped by individual perspectives and the specific information available to a person (de Finetti, 1975; Galavotti, 2017). De Finetti highlighted that evaluating probability involves both objective components, such as data and facts, and subjective components, such as opinions based on this evidence (Galavotti, 2017). He argued that subjective elements are essential for interpreting objective evidence, as the relevance of certain data may vary depending on the evaluator's experience and perspective. These views challenge the notion that objective probability can serve as a universally applicable measure of belief (Galavotti, 2017).

Evaluating probability is thus a complex process, incorporating both objective and subjective elements. While probability and environmental evidence play significant roles in belief formation, the absence of reliable statistics can limit the applicability of probability theory in capturing the complexities of human belief systems (Galavotti, 2017). Various interpretations of probability have been proposed, and the debate continues. The frequentist interpretation is widely used in natural sciences such as genetics and statistical mechanics, though it struggles with single events and everyday scenarios (Galavotti, 2017; von Mises, 1957). A key issue is determining the appropriate reference class, as the probability of an event depends on how it is classified (Galavotti, 2017). Epistemic theories, like logicism, are preferred by some philosophers but are less popular among scientists due to their formal complexity (Galavotti, 2017; Carnap, 1962). Subjectivism is more prevalent in economics and social sciences, where beliefs and expectations shape forecasts and models. However, in the natural sciences, which rely on objective probability, subjectivism is often viewed

with skepticism, sometimes misconceived as a mere expression of personal opinions without empirical grounding (Galavotti, 2017). Bayesian methods have gained prominence by offering a pluralistic approach that combines objective and subjective elements (Galavotti, 2017). Despite this, a significant limitation of both Bayesian and pluralistic methods is whether humans can realistically perform such complex calculations when adjusting their beliefs. Research suggests that cognitive load and other factors may hinder belief adjustment, raising questions about the real-world applicability of these models (Henckel et al., 2022).

Given these challenges, while logical interpretations of probability are insightful, they may not fully capture the complexities of human belief systems. Alternative models could offer a more comprehensive understanding of how individuals handle uncertainty. Rather than replacing subjective probability, models such as ranking theory may provide valuable insights in specific contexts, serving as complementary approaches that enhance our understanding of uncertainty (Spohn, 2012).

1.4 What is Ranking Theory?

Ranking theory, introduced by Wolfgang Spohn in 1983, provides an alternative to traditional probabilistic models for representing beliefs and uncertainties. Rather than assigning probabilities to events, it employs a system based on natural numbers and infinity to quantify degrees of disbelief or belief in propositions. This framework is rooted in formal epistemology and principles of rationality (Skovgaard-Olsen, 2016; Spohn, 2012, 2013).

Ranking theory expresses logical relations and supporting evidence by assigning ranks of disbelief or belief to propositions. This method effectively addresses paradoxes like the lottery paradox previously discussed. By assigning a high rank of disbelief to each ticket, ranking theory acknowledges the low chance of any single ticket winning while maintaining the conditional belief that one ticket must win. This approach allows varying degrees of belief to coexist without conflict, avoiding the need for constant probabilistic recalculations (Spohn, 2012). Ranking theory provides a more adaptable framework for managing beliefs, streamlining belief adjustments based on new evidence without the complexity involved in probability-based models. These practical advantages make ranking theory a valuable tool for representing human beliefs and uncertainties in a more practical manner (Spohn, 2012).

Despite its potential, ranking theory faces challenges. It can be complex to understand and apply due to its substantial conceptual shift from traditional probabilistic models. This shift requires a deeper understanding of formal logic and epistemology. Additionally, ranking theory involves managing a broad scale that includes natural number plus infinity, which presents unique challenges in both learning and practical applications. Furthermore, empirical validation is needed to confirm the applicability of this model in human belief modelling as it has not been tested in psychological or behavioural experiments. Lastly, integrating ranking theory with existing probabilistic models poses challenges that must be addressed for broader acceptance and application (Kern-Isberner et al., 2021; Skovgaard-Olsen, 2016).

This thesis aimed to address these challenges by conducting a series of behavioural experiments to examine the empirical adequacy of ranking theory in diverse scenarios and contexts. By exploring the utility of ranking theory as both a normative and descriptive model, this

study sought to advance our understanding of human belief representation, contributing to the development of a model that clarifies the cognitive processes and mechanisms involved.

1.5 Thesis Outline Overview

This dissertation consisted of six experiments that examined the empirical adequacy of ranking theory in modelling human belief systems. The chapters built upon each other, starting from foundational concepts and progressing to more complex applications, aiming to explore how ranking functions captured degrees of belief and disbelief across various contexts.

Experiments 1 and 2 examined whether humans could express degrees of disbelief using negative ranking functions in situations with uncertain probabilities. Participants assessed their degrees of disbelief regarding various propositions without relying on explicit probabilities. These experiments aimed to determine whether participants could consistently use negative ranks to express varying levels of disbelief across different propositions, even when objective probabilities were ambiguous. Additionally, they explored the relationship between negative ranks and subjective probabilities, as well as how participants judged disjunctive statements involving multiple propositions. These studies laid the groundwork for understanding how ranking functions capture degrees of disbelief in diverse contexts.

Having established the use of negative ranks, Experiment 3 expanded the scope by introducing positive ranks, allowing for the simultaneous assessment of belief and disbelief. This approach provided a more comprehensive view of the belief spectrum. By including positive ranks, the study sought to clarify the relationship between subjective probabilities

and ranking functions for both belief and disbelief, and to refine the understanding of how people process disjunctive statements.

Experiment 4 explored the applicability of ranking theory in a dynamic learning environment where participants engaged in a hide-and-seek game against a computer opponent following undisclosed probability distributions. Participants provided ranks and subjective probabilities for each possible outcome, reflecting their perceptions of the opponent's behaviours. The study aimed to examine the relationship between ranking functions, subjective probabilities, and objective probabilities to determine the extent to which ranking functions could capture belief systems under conditions where outcomes were well-defined and consistently observed.

Experiments 5 and 6 focused on the Ellsberg Paradox, a classic problem in decision theory that challenges traditional models by demonstrating ambiguity aversion in human decision-making under uncertainty. In Experiment 5, negative ranking functions were used to measure individuals' degrees of disbelief about the outcomes of drawing from an urn in the classic Ellsberg one-urn scenario. The aim was to examine whether ranking theory could provide insights into how people evaluate uncertainty and whether it could offer a more coherent framework for decision-making under ambiguity. Experiment 6 extended this investigation by reintroducing betting scenarios, allowing participants to express their degrees of disbelief prior to making choices involving hypothetical monetary stakes. The study sought to determine whether betting choices reflected participants' underlying beliefs, as suggested by Ellsberg, and to explore how ranking theory might influence or reduce paradoxical behaviour observed in traditional approaches.

Collectively, these experiments investigated the applicability of ranking theory in modelling human belief systems, exploring its potential as a tool for measuring belief and its application in understanding how individuals process uncertainty and make decisions in psychology and behavioural experiments.

Chapter 2

Rules of Ranking Theory

2.1 The Basics of Ranking Theory

In this chapter, the formal definitions and mathematical properties of ranking theory are introduced, drawing from Spohn's *The Laws of Belief* (2012). These properties will be examined in the empirical work presented in Chapters 3 through 8, focusing on whether individuals can apply the fundamental principles of ranking theory and on examining the theoretical relationship between probabilities and ranks. For most key features of ranking theory, the mathematical definitions are provided first, followed by an informal restatement to aid in building intuition.

Let us start with some brief formal explanations. Ranking theory is based on belief metrics, which quantify a grading of disbelief expressed by negative ranking functions, denoted by κ. The negative ranking function, κ, can be thought of as varying degrees of disbelief or surprise an observer has about a proposition. Ranking theory follows possible world semantics to represent propositions as a set of possible worlds or possibilities. Thus, consider

a non-empty set, W, which is a set of all possible worlds that are mutually exclusive and jointly exhaustive (Spohn, 2012).

Informally, one can think of an algebra as the rules specifying subsets of our space of all possible worlds, and rules for how those subsets can be combined and quantified. For example, when flipping a coin, the space of all possible worlds might include heads, tails, and the coin landing on its side. The algebra provides the operational details for how these subsets can be combined and measured. With this logic, we consider an algebra, $\mathcal{A}$, as subsets of W, which we refer to as propositions (Spohn, 2012).

Eq. 1: negative ranking function

Let $\mathcal{A}$ be an algebra over W. Then κ is a negative ranking function for $\mathcal{A}$ if and only if κ is a function from $\mathcal{A}$ into $\mathbb{N} \cup \{\infty\}$ (the set of natural numbers plus infinity) such that for all $A, B \in \mathcal{A}$:

$$\kappa(W) = 0 \quad \text{and} \quad \kappa(\emptyset) = \infty \tag{2.1}$$

A negative ranking function κ, in the standard interpretation, expresses a grading of disbelief. If $\kappa(A) = 0$, A is not disbelieved (not surprising); if $\kappa(A) > 0$, A is disbelieved (surprising); if $\kappa(A) = \infty$, A is considered impossible.

Eq. 2: law of negation

$$\text{Either } \kappa(A) = 0 \quad \text{or } \kappa(\bar{A}) = 0 \quad \text{or both} \tag{2.2}$$

If $\kappa(A) = 0$, A is not disbelieved at all. This allows that $\kappa(\bar{A}) = 0$, where $\bar{A}$ is the negation of A; in such cases, indifference or suspension of judgement regarding A is observed. The law of negation states that one cannot simultaneously disbelieve in both A and $\bar{A}$.

Imagine considering the event of a coin landing heads (A). If you do not disbelieve that the coin will land heads ($\kappa(A) = 0$), you are open to it happening. You cannot disbelieve both the coin landing heads and it not landing heads ($\bar{A}$) simultaneously, as this would be illogical. If neither $\kappa(A)$ nor $\kappa(\bar{A})$ holds disbelief (both are zero), it indicates neutrality or suspension of judgment.

Eq. 3: law of disjunction

$$\kappa(A \cup B) = \min\{\kappa(A), \kappa(B)\} \tag{2.3}$$

The law of disjunction states that the negative rank of the disjunction (logical OR) of two propositions A and B is equal to the minimum of the ranks of the individual propositions. This means that the disbelief in $A \cup B$ is determined by the lesser of the disbeliefs in A and B.

If you have minimal skepticism about either event A or B occurring, then your skepticism about the combined possibility $A \cup B$ (either A or B happening) aligns with the least skepticism towards the individual events. In contrast to classical probability, where the likelihood of $A \cup B$ is calculated by adding the probabilities of A and B and subtracting the overlap (if any), ranking theory simplifies to adopting the minimal disbelief between A and B. This approach focuses on the most plausible scenario rather than accumulating

probabilities.

Eq. 4: positive ranking function

$$\beta(A) = \kappa(\bar{A}) \tag{2.4}$$

A positive ranking function β expresses degrees of belief for A, defined by the disbelief in the negation of A. If $\beta(A) > 0$, A is believed (to some positive degree); if $\beta(A) = 0$, A is not believed; if $\beta(A) = \infty$, A is believed with absolute certainty.

Eq. 5: two-sided ranking function

τ is a two-sided ranking function for $\mathcal{A}$ if and only if τ is a function from $\mathcal{A}$ into $\mathbb{Z} \cup \{-\infty, \infty\}$, such that there is a negative ranking function κ and its positive counterpart β for all $A \in \mathcal{A}$:

$$\tau(A) = \kappa(\bar{A}) - \kappa(A) = \beta(A) - \kappa(A) \tag{2.5}$$

A negative ranking function measures the degree of disbelief in A, while a positive ranking function measures the degree of belief in A. Thus, if $\tau(A) > 0$, A is believed; if $\tau(A) < 0$, A is disbelieved (surprising); and if $\tau(A) = 0$, A is neither believed nor disbelieved.

A two-sided ranking function, τ, quantifies belief and disbelief in a proposition A by consolidating the degree of belief ($\beta(A)$) and disbelief ($\kappa(A)$) into a single measure. Positive values of $\tau(A)$ indicate confidence in A's truth, negative values suggest skepticism, and a zero value denotes neutrality, reflecting a balanced or undecided view on A. This method

simplifies the evaluation of one's stance towards A, aligning it efficiently with changes in evidence or perspective.

2.2 Justification for the Preference of Negative Ranks in Ranking Theory

Ranking theory, as developed by Wolfgang Spohn, highlights the preferred use of negative ranking functions over positive or two-sided ranking functions due to several operational advantages (Spohn, 2012). Historically, degrees of belief have been central to epistemological studies, exemplified by Cohen's inductive support gradings (Cohen, 1977) and Dubois and Prade's possibility measures (Dubois and Prade, 2023). While traditional models focus on positive beliefs, Spohn argues that positive ranking functions are inherently complex because they are set functions not reducible to point functions, assigning ranks to sets of outcomes. This complexity makes them less practical for dynamically updating beliefs (Spohn, 2012).

In contrast, negative ranking functions can be reduced to point functions that directly rank individual outcomes, facilitating simpler and more efficient belief revisions (Spohn, 2009). As a result, negative ranking functions are favoured in environments requiring frequent belief updates. This simplification aligns them closely with probabilistic reasoning, as many operations in probability theory correspond to operations on negative ranks (Spohn, 2012). For instance, the sum and product of probabilities correspond to the minimum and addition of negative ranks, respectively. Thus, negative ranking functions are preferred in

ranking theory, especially given the importance of relating ranking theory to probability theory (Kern-Isberner et al., 2021).

Spohn also critiques two-sided ranking functions, which attempt to integrate both belief and disbelief, noting that they lack a simple axiomatic foundation. The differing behaviours of belief and disbelief under logical operations such as conjunction and disjunction make two-sided ranking functions impractical for applications that demand clear and consistent reasoning (Spohn, 2012).

By focusing on negative ranking functions, which directly represent degrees of disbelief, we adopt a more efficient and clearer method for updating beliefs in light of new information. The simplicity and computational efficiency of this approach make negative ranks ideal for models requiring continuous updates to belief structures, reflecting Spohn's broader perspective on the adaptability and robustness of ranking theory (Spohn, 2012).

2.3 Probability-Ranking Translation

While ranking theory and probability theory are distinct, a meaningful parallel emerges between negative ranking functions and probability distributions when comparing how ranks and probabilities combine (Spohn, 2013). For example, while the probability of two independent events, A and B, is the product of their individual probabilities, the corresponding negative ranks for these events are summed. This parallels how taking the logarithm of a product converts it to a sum of the logarithms of the individual probabilities. In this analogy, negative ranks can be understood as logarithms of probabilities with a very small base,

$b \in (0, 1)$ (Spohn, 2013). Similarly, the sum of probabilities corresponds to the minimum of ranks, and the quotient of probabilities to the difference of ranks (Raidl and Skovgaard-Olsen, 2017).

These translations preserve certain logical properties, such as the law of additivity for probabilities, the law of disjunction for ranks, the law of multiplication for probabilities, and the law of conjunction for ranks. This correspondence makes ranks more efficient to compute than probabilities, highlighting the potential of ranking theory as a descriptive model of belief (Raidl and Skovgaard-Olsen, 2017). However, despite this parallel, ranks should not be reduced to purely probabilistic terms (Spohn, 2013). Belief cannot always be consistently or deductively represented probabilistically, which supports the independent development of ranking theory as a distinct approach to belief representation (Kern-Isberner et al., 2021).

Raidl and Skovgaard-Olsen (2017) further highlighted the parallels between ranking and probability functions, introducing the equations below, which will be examined in detail in Chapters 3 through 6.

Eq. 6: probability to (real-valued) negative ranking translation

$$\kappa(A) = \log_b P(A) - \log_b \max_{S \in W} P(S), \quad \text{where} \quad b \in (0, 1) \tag{2.6}$$

$\kappa(A)$ denotes the negative ranking function for a proposition A. This function translates the probability $P(A)$ into a real-valued scale, quantifying the degree of disbelief. The term $\log_b P(A)$ is the logarithm (base b) of the probability $P(A)$, where $b \in (0, 1)$.

The term $\log_b \max_{S \in W} P(S)$ represents the logarithm (base b) of the maximum probability among all sets of propositions S within a specified set W. This serves as a normalization factor, ensuring that $\kappa(A)$ is scaled appropriately.

Eq. 7: probability to (real-valued) two-sided ranking translation

$$\tau(A) = \log_b \left(\frac{1 - P(A)}{P(A)} \right), \quad \text{where } b \in (0, 1) \tag{2.7}$$

This equation translates the probability $P(A)$ into a two-sided ranking function $\tau(A)$, quantifying both belief and disbelief on a unified scale. It represents the log-odds of event A, which is the logarithm of the ratio between the probabilities of A not occurring and occurring, where $b \in (0, 1)$.

Eq. 8: negative ranking to probability translation

$$P(A) = \frac{b^{\kappa(A)}}{\sum_{s \in W} b^{k(s)}}, \quad \text{where} \quad b \in (0, 1) \tag{2.8}$$

This equation translates the negative ranking function $\kappa(A)$ back into a probability measure $P(A)$. The normalization factor $\sum_{s \in W} b^{k(s)}$ ensures that the probabilities sum to one.

Since the following holds:

$$P(A) + P(\bar{A}) = 1 \tag{2.9}$$

Where the sum of the probability of a proposition A and its complement $\bar{A}$ equals one, ensuring completeness in the probability space.

$$\tau(A) = \kappa(\bar{A}) - \kappa(A) = \log_b(1 - P(A)) - \log_b P(A) \tag{2.10}$$

$\tau(A)$ represents the difference between the negative ranking of the complement of A and A itself, providing a two-sided ranking function.

Eq. 9: two-sided ranking to probability translation

$$P(A) = \frac{1}{b^{\tau(A)} + 1}, \quad \text{where } b \in (0, 1) \tag{2.11}$$

This equation translates the two-sided ranking function $\tau(A)$ back into a probability measure $P(A)$, ensuring consistency between the ranking and probabilistic interpretations.

2.4 Summary of the Chapter

This chapter introduced the mathematical properties of ranking theory, which will be tested empirically in Chapters 3 through 8. The focus is on whether individuals can apply the core principles of ranking theory. Additionally, the equations presented provide a theoretical framework for understanding the relationship between subjective probabilities and ranking functions. This framework will be explored in detail in Chapters 3 through 6 to assess how these theoretical relationships align with the empirical data.

Chapter 3

Experiment 1: Expressing Disbelief through Negative Ranking Functions

3.1 Introduction

Experiment 1 investigated whether humans could express degrees of disbelief using negative ranking functions and explored the relationship between negative ranks and subjective probabilities. Participants assessed their degrees of disbelief regarding various propositions without relying on explicit probabilities. Scenarios included topics such as language demographics in Quebec, September weather in a designated city, and coworkers' music preferences at a hypothetical karaoke party. These everyday situations with uncertain probabilities prevented participants from relying solely on pre-existing knowledge.

The design ensured that the objective probability of the scenarios was ambiguous, avoiding priming participants to think in probabilistic terms. By examining how participants used negative ranking functions and the relationship between these ranks and subjective probabilities, Experiment 1 aimed to provide empirical evidence supporting the application

of ranking theory in understanding human belief and uncertainty.

3.2 Methods

3.2.1 Participants

A total of 168 undergraduate students from the University of Waterloo participated in the online study, receiving course credits in exchange for their involvement. The study did not have any exclusion criteria. The median age was 20 years, with 101 females, 66 males, and 1 undisclosed.

3.2.2 Task Environment

The study utilized a custom-built online platform created with JavaScript to collect participant responses (de Leeuw et al., 2023). The platform featured an interface that enabled participants to enter their responses for negative ranks, as well as a 10 by 10 interactive grid where they could click on boxes to indicate possible event cards. Participants finished the study in under 30 minutes.

3.2.3 Procedure

Grading of Disbelief — Negative Ranking Functions

Participants began by completing a series of questionnaires designed to assess their degrees of disbelief toward various propositions. Each proposition was assigned a non-negative integer ranging from zero (not surprising) to infinity (impossible), representing varying levels of disbelief or surprise. Participants were to grade their degrees of disbelief: 0 for not surprising, a value greater than 0 for surprising, and infinity for impossible. The topics of these questionnaires included:

1. **Language Demographics in Quebec:** Participants graded their disbelief about meeting a Quebec native (born and raised) who speaks only one language. The languages included were French, Spanish, English, Greek, Polish, Korean, Urdu, Japanese, or the disjunction Spanish or Urdu.

2. **September Weather in a Designated Visiting City:** Participants graded their disbelief regarding daytime high temperatures for a hypothetical day trip to Boston, Massachusetts, United States in September. The temperature ranges included -29 to -20°C, -19 to -10°C, -9 to 0°C, 1 to 10°C, 11 to 20°C, 21 to 30°C, 31 to 40°C, 41 to 50°C, and the disjunction -9 to 0 or 1 to 10°C.

3. **Coworkers' Music Preferences During a Hypothetical Karaoke Party:** Participants imagined being employees at an automotive service centre invited to a work karaoke party. They were informed that their coworker Tom, a Caucasian male in his

20s who grew up in a rural area and loves wearing his cowboy hat, would be attending. Participants then graded their disbelief about Tom enjoying singing along to various music genres such as Pop, Rock, Hip Hop, Heavy Metal, Country, Jazz, Opera, Folk, or the disjunction Jazz or Opera.

The selected scenarios reflected everyday situations with unclear objective probabilities. Comprehensive details of the ranking questionnaires are documented in Appendix 9.4.

Open Sampling Format (OSF)

To measure participants' representation of probabilities, an open sampling format (OSF) was used following each questionnaire. Participants engaged with a 10x10 grid where each cell represented a possible outcome.

Initially, the grid was generated using a ranking-to-probability equation with a logarithmic base set between 0 and 1, initially at 0.5, see Equation 2.6. This arbitrary choice was based on theoretical predictions of a logarithmic relationship. Participants then adjusted the grid to reflect their reported degrees of disbelief from the questionnaires.

Each grid cell represented a possible outcome. Participants filled the grid according to their disbelief levels, with higher frequencies indicating greater perceived commonality. They were informed that some outcomes might be missing or appear more frequently than expected. Participants adjusted the number of event cards for each outcome according to their reported disbelief levels. They could select an outcome from a dropdown menu to replace a cell or choose not to make any changes if the event cards already reflected their

level of disbelief (see Figure 4).

This graphical representation aimed to capture subjective probabilities without directly asking about them. Adjusting the grid was intended to reduce the priming effect of thinking in terms of probability and provide a more accurate representation of expectations (Tiede et al., 2022). Literature suggests people make better decisions when probabilities are shown graphically rather than numerically. Unlike past studies where OSF probabilities were premade, this study allowed participants to create and modify the OSF themselves, introducing a novel, interactive method.

End of Task Survey

Subjective Probability Estimates: Following the experiment, participants completed the same three questionnaires as before, but this time using percentages. The questionnaires covered language demographics, weather in a designated city, and coworkers' music preferences. This step aimed to evaluate whether the OSF correlates with subjective probabilities. To avoid priming, subjective probability estimates were gathered at the end of the experiment.

Difficulty and Preference Assessment: Participants were then asked to evaluate the difficulty of quantifying their degrees of disbelief and their preference for belief versus disbelief assignments using a Likert scale. The questions assessed were:

1. The difficulty of quantifying degrees of disbelief/surprise.

2. Whether it would have been more natural or easier to assign degrees of belief rather than degrees of disbelief/surprise.

These questions aimed to capture participants' subjective experiences regarding the ease of the task and their preference for belief versus disbelief assignments.

3.2.4 Data Analysis

Data from the questionnaires and OSF tasks were analyzed using R (version 4.3.1) for statistical computing and visualization. To examine the relationship between observed negative ranks and subjective probabilities, a non-linear least squares regression was used. This regression model estimated the optimal logarithmic base b, optimizing the fit between the ranks and probabilities. Descriptive statistics, correlation analyses, and regression models were utilized to further explore the relationships between these and other measured variables. The ggplot2 package in R was used to generate figures providing visual representations of the data (Wickham, 2016).

Data Transformation

For data analysis involving statistical tests, such as curve fitting (non-linear least squares regression) and linear mixed-effects models (predicting disjunction scores), the inverse hyperbolic sine (arsinh) function was applied to the ranks. This transformation was used to minimize the impact of outliers, which are prevalent in the ranks due to their large values. While logarithmic transformation is typically employed for such cases, arsinh was preferred in our analysis because it can accommodate zero values, which are inherent in the ranking functions, as well as negative values for usages for two-sided ranking functions in subsequent

experiments. Unlike the logarithmic function, which cannot process zeros or negative values, arsinh provides a solution for handling extreme values in the data.

3.2.5 Ethical Considerations

The study received ethics clearance from the University of Waterloo Research Ethics Board (REB 44180) before it was conducted. All participants were informed about the purpose of the study and their right to withdraw at any time without facing any penalty. To protect participants' privacy, de-identified data was securely stored. All participants provided their informed consent before participating.

3.3 Results

3.3.1 Analysis of Negative Ranks

The negative ranks from the three questionnaires: language demographics, temperature ranges, and music preferences, were analyzed using descriptive statistics, focusing on the median and Interquartile Range (IQR). The IQR, representing the middle 50% of the data (25% on either side of the median), illustrated response variability. A lower IQR indicated more consensus, while a higher IQR suggested a broader range of disbelief. For each questionnaire, negative ranks for each proposition were visualized with the y-axis set to a logarithmic scale for better visibility.

Language Demographics

Figure 3.1 and Table 3.1 showed that French was widely accepted as the predominant language in Quebec, with a median disbelief of 0 and an IQR of 0, indicating no disbelief. English had a median disbelief of 1 and an IQR of 2, showing it was slightly surprising but had the lowest possible median disbelief.

In contrast, languages such as Polish and Japanese showed higher disbelief. Polish had a median disbelief of 9 and an IQR of 21.75, while Japanese had a median of 9 and an IQR of 36, indicating less consensus among participants. Urdu showed the highest disbelief with a median of 10 and an IQR of 45.25, yielding as the most surprising language for a Quebec native to speak exclusively. The varying IQRs highlighted different levels of variability in participants' disbelief responses, generally showed greater variability for propositions that elicited higher disbelief. For detailed statistics, refer to Table 3.1.

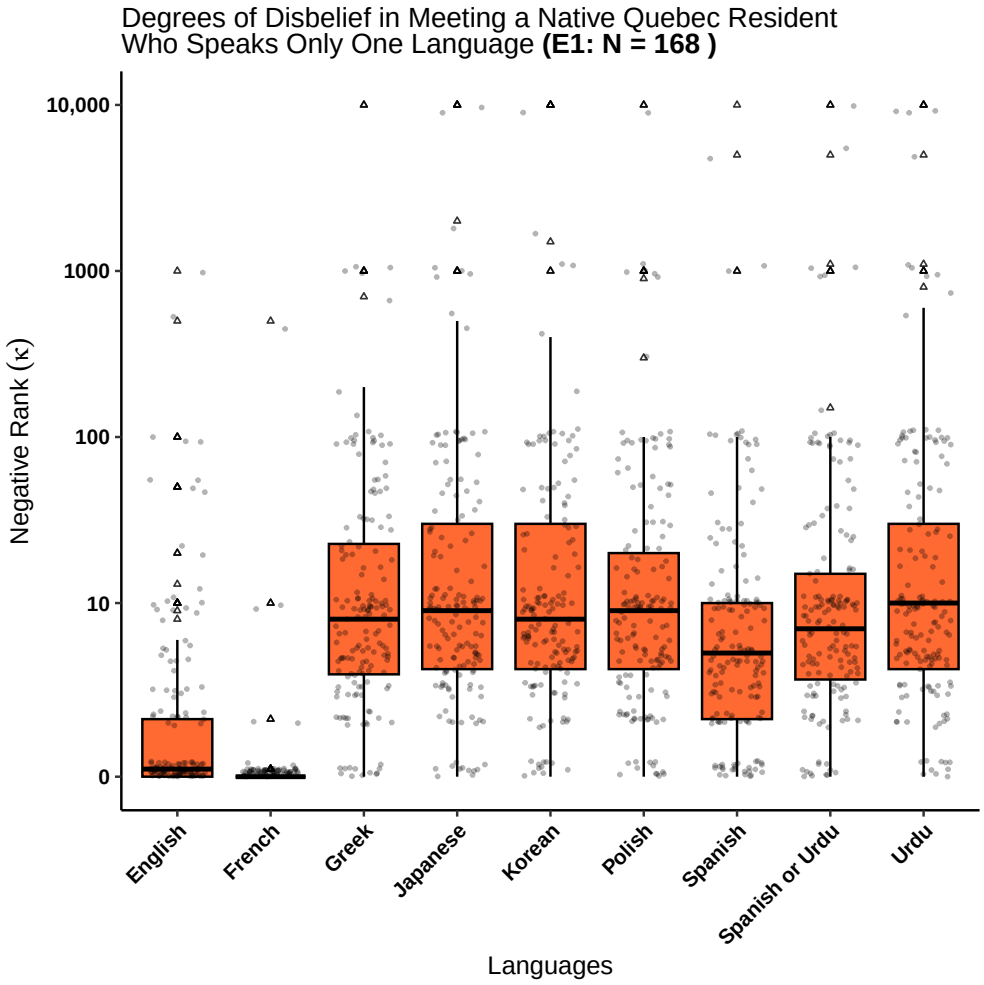


Figure 3.1: Boxplots showing the degrees of disbelief (negative rank) for encountering a native Quebec resident who speaks only one language, across different languages. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each language, with lower values indicating lower disbelief. French and English exhibited the lowest disbelief, indicating that participants expected these languages to be more commonly spoken. Conversely, languages such as Urdu and Japanese showed higher disbelief, reflecting their perceived rarity.

	Propositions	Median Negative Rank	IQR Negative Rank
1	English	1.00	2.00
2	French	0.00	0.00
3	Greek	8.50	26.00
4	Japanese	9.00	36.00
5	Korean	8.50	27.25
6	Polish	9.00	21.75
7	Spanish	5.00	8.00
8	Spanish or Urdu	8.00	21.25
9	Urdu	10.00	45.25

Table 3.1: Descriptive statistics for Language Demographics, showing the median and interquartile range (IQR) of negative rank scores for each language. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values.

Temperature Ranges

Figure 3.2 and Table 3.2 illustrated participants' levels of disbelief regarding daytime high temperatures in September. A U-shaped distribution was noted: moderate temperatures (11 to 20°C) with a median disbelief of 0 and an IQR of 2, showed broad consensus as typical and expected. Extreme temperatures, particularly -29 to -20°C and 41 to 50°C, exhibited the highest disbelief with median scores of 15 and 13.5, respectively. Both had IQRs of 94, indicating significant variability in surprise ratings among respondents.

For temperatures within the 21 to 30°C range and the 1 to 10°C range, both with a median of 2, there was some surprise, though less than for more extreme temperatures. These findings suggest that while these temperatures are somewhat surprising, they are still considered possible.

The broader IQRs, typically found in rarer outcomes, highlight the diversity in expectations and the magnitude of surprise, influenced by participants' personal experiences and regional climate knowledge. For detailed statistics on each temperature range, refer to Table 3.2.

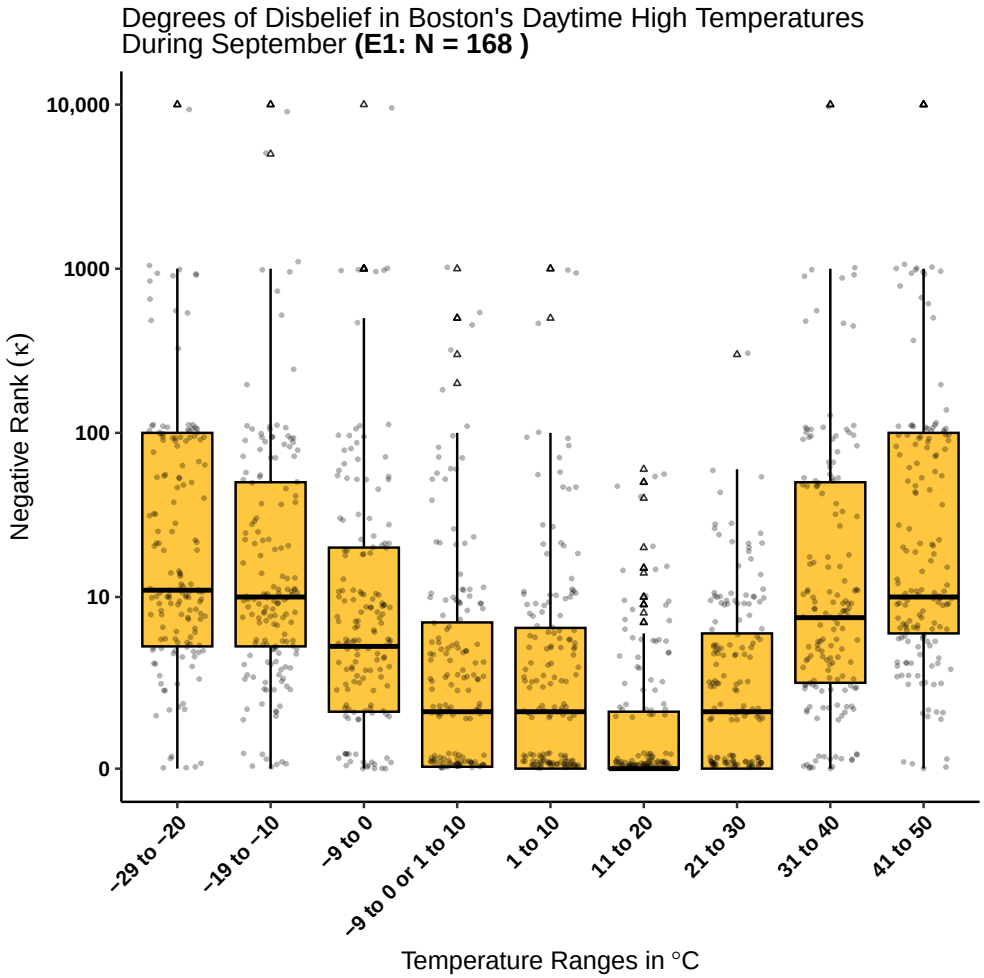


Figure 3.2: Boxplots showing the degrees of disbelief (negative rank) for various daytime high temperature ranges in Boston during September. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each temperature range, with lower values indicating lower disbelief. The temperature range of 11 to 20°C exhibits the lowest disbelief, indicating that participants expect this range to be typical for September (median score of 0). Conversely, extreme temperature ranges such as -29 to -20°C and 41 to 50°C show higher disbelief, reflecting their perceived rarity.

	Propositions	Median Negative Rank	IQR Negative Rank
1	-29 to -20	15.00	94.00
2	-19 to -10	10.00	57.50
3	-9 to 0	6.00	18.00
4	-9 to 0 or 1 to 10	2.00	7.25
5	1 to 10	2.00	7.00
6	11 to 20	0.00	2.00
7	21 to 30	2.00	6.00
8	31 to 40	8.00	47.00
9	41 to 50	13.50	94.00

Table 3.2: Descriptive statistics for Temperature Ranges, showing the median and interquartile range (IQR) of negative rank scores for each temperature range. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values.

Music Preferences

Figure 3.3 and Table 3.3 revealed participants' levels of disbelief regarding different music genres that Tom, a co-worker described in the survey, might sing along to at a karaoke party. The analysis showed a spectrum of disbelief shaped by cultural expectations.

Country music elicited the least disbelief among participants, with both median and IQR at 0, indicating an agreement in the expectation of this genre — participants would not be surprised. Pop, Rock, and Folk, with medians of 1, and IQRs of 5, elicited surprise but to a

minimal degree, thus could be considered plausible genres.

Conversely, Heavy Metal and Jazz or Opera presented higher disbelief, with medians of 5 and IQRs of 8 and 10.25, respectively, suggesting that these genres are less associated with Tom's profile. Opera, in particular, recorded the highest disbelief with a median of 6 and an IQR of 17, reflecting greater variability in participant responses.

These results highlighted how cultural expectations influenced perceptions of musical preferences. The hypothetical scenario demonstrated judgment formation based on cultural influences rather than solely on probabilities. Detailed statistics for each music genre are provided in Table 3.3.

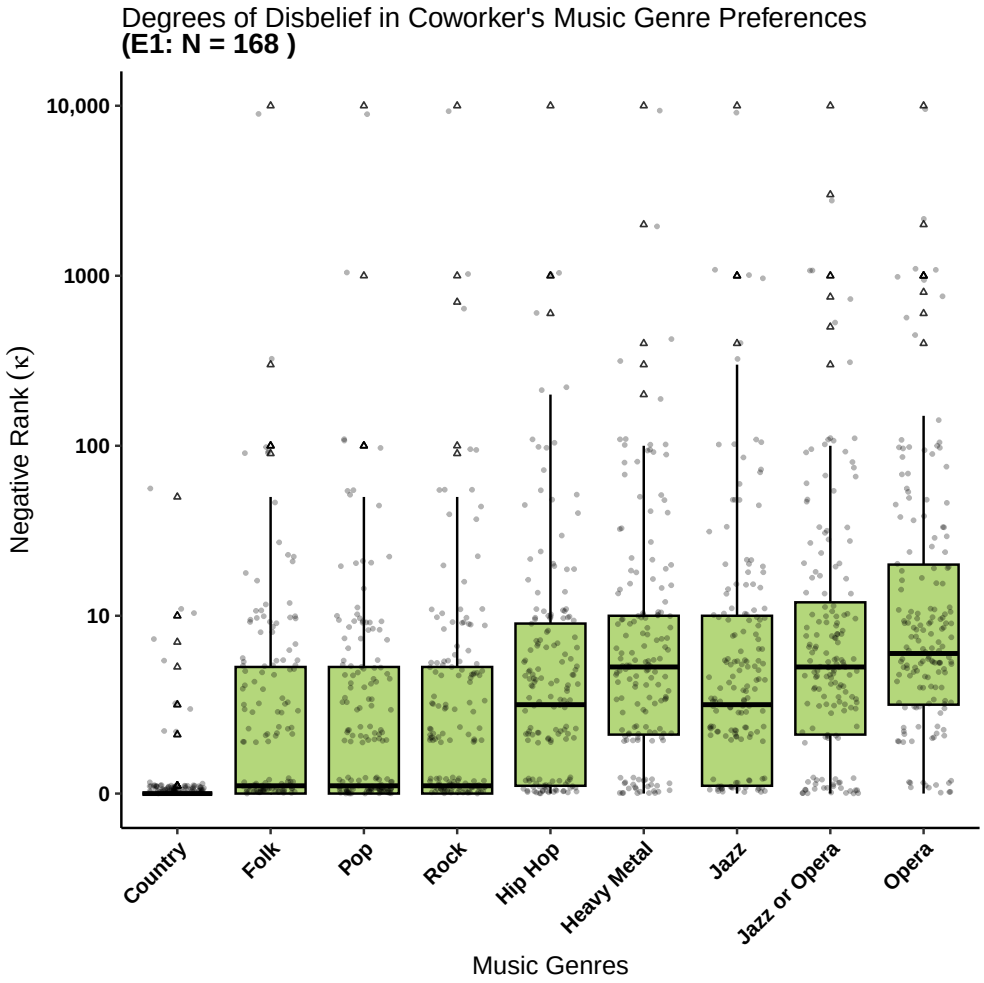


Figure 3.3: Boxplots showing the degrees of disbelief (negative rank) for various music genres that a coworker might sing along to at a karaoke party. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each genre, with lower values indicating lower disbelief. The genre of country music exhibits the lowest disbelief, indicating that participants expect this genre to be typical for the coworker. Conversely, genres such as opera show higher disbelief, reflecting their perceived rarity.

	Propositions	Median Negative Rank	IQR Negative Rank
1	Country	0.00	0.00
2	Folk	1.00	5.00
3	Heavy Metal	5.00	8.00
4	Hip Hop	3.00	9.00
5	Jazz	3.00	9.00
6	Jazz or Opera	5.00	10.25
7	Opera	6.00	17.00
8	Pop	1.00	5.00
9	Rock	1.00	5.00

Table 3.3: Descriptive statistics for Music Preferences, showing the median and interquartile range (IQR) of negative rank scores for each genre. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values.

3.3.2 Relationship Between Subjective Probabilities (via Open Sampling Format) and Negative Ranks

Visualization: The relationship between negative ranks and subjective probabilities was visualized across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. Based on Equation 2.6, this relationship was expected to resemble a logarithmic function, where the negative ranking function is the logarithm of the probability

with base b, where $b \in (0, 1)$.

For propositions that did not elicit disbelief (indicating no surprise and potential degrees of belief), participants provided a wide range of subjective probabilities from 0 to 1. The dotted blue line represents the threshold where disbelief is greater than 0. Instances above this line indicate varying degrees of disbelief, as shown in Figure 3.4.

The graph suggests a nonlinear relationship between negative rank and subjective probability. The threshold at which a proposition becomes surprising varied, highlighting the context-dependent nature of subjective probability thresholds for disbelief. Generally, high probability values were associated with a lack of surprise or disbelief. However, low probability values corresponded to a wide range of disbelief levels, emphasizing that negative rank offered granularity in assessing degrees of disbelief. This provided more detailed information compared to a simple probability value, such as 0.05.

Overall, lower probabilities were associated with greater degrees of disbelief. However, probabilities ranging from 0 to 1 may not always elicit surprise or disbelief, demonstrating the value of negative rank in explicitly reporting varying degrees of disbelief.

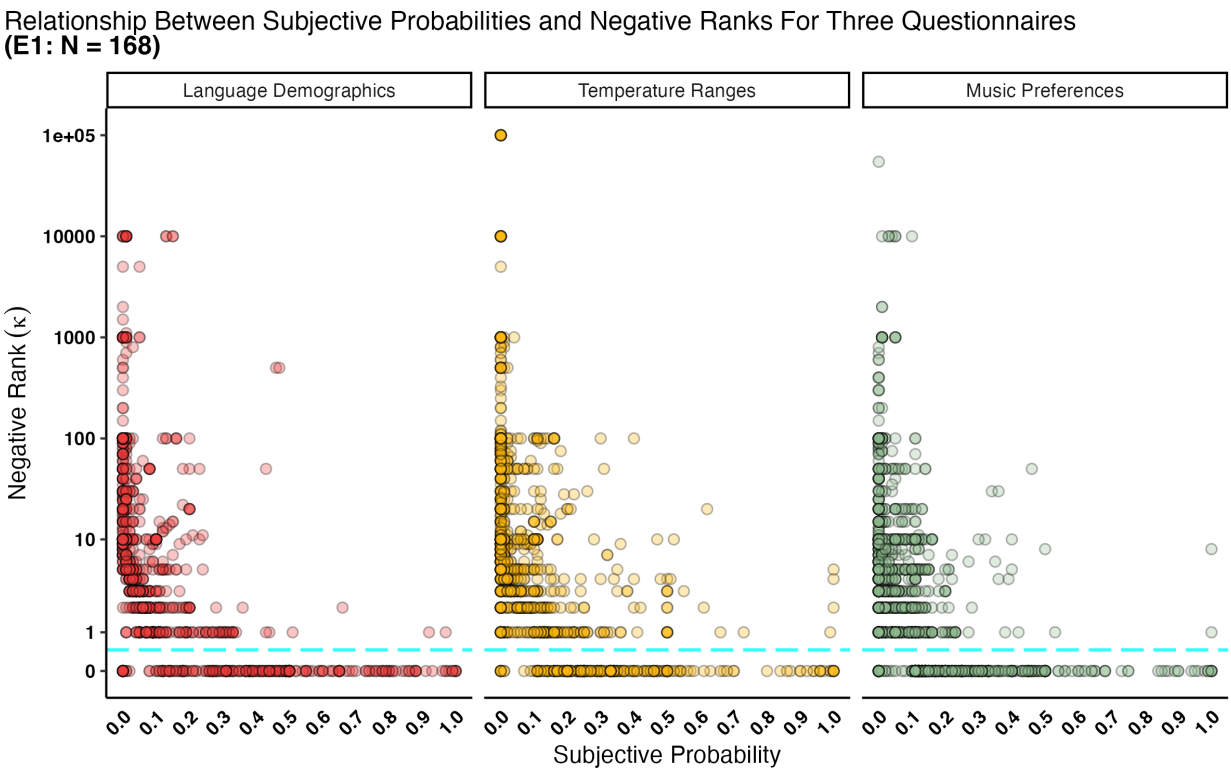


Figure 3.4: Scatter plots showing the relationship between subjective probabilities (via Open Sampling Format) and negative ranks (κ) across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis is scaled to display negative ranks effectively, while the labels show the original rank values for easier interpretation. The dotted blue line represents the threshold where disbelief is greater than 0. Instances above this line indicate varying degrees of disbelief. The plots suggest a nonlinear relationship between negative rank and subjective probability, with lower probabilities generally associated with higher degrees of disbelief. However, probabilities ranging from 0 to 1 may not always elicit surprise or disbelief, demonstrating the value of negative rank in explicitly reporting varying degrees of disbelief.

3.3.3 Mathematical Model Fit

Model 1: Logarithmic Model

The analysis investigated the relationship between subjective probabilities and negative

ranks as defined by Equation 2.6. This relationship was described by the equation:

$$\kappa(A) = \log_b(P(A)) - \log_b(\max_{S \in W} P(S))$$

where $P(A)$ represented the subjective probability of event A and b was the logarithmic base, ranging between 0 and 1. To manage extreme values and enhance the visual representation in plots, the inverse hyperbolic sine function, denoted as $\text{asinh}(\kappa)$, was applied to the negative ranks (κ). This function effectively stabilized extreme values without distorting the underlying data distribution, see rationale on page 31.

A non-linear least squares regression was employed to estimate the logarithmic base b:

$$b = 0.001, \quad \text{SE} = 0.0002, \quad t(1) = 4.91, p < .001$$

indicating a statistically significant logarithmic base fit. Pseudo R^2 was computed to provide an approximation of the variance explained by the model in the non-linear context, but is supplemented by the AIC and BIC in the later section. The model yielded a pseudo R^2 of 0.04, suggesting that approximately 3.90% of the variance in negative ranks was explained by this logarithmic relationship (see Figure 3.5).

The initial mathematical model theorized a relationship between negative ranks and subjective probabilities. However, this model did not explain much of the variance. To further explore this relationship, an exponential decay model was investigated as part of an exploratory analysis.

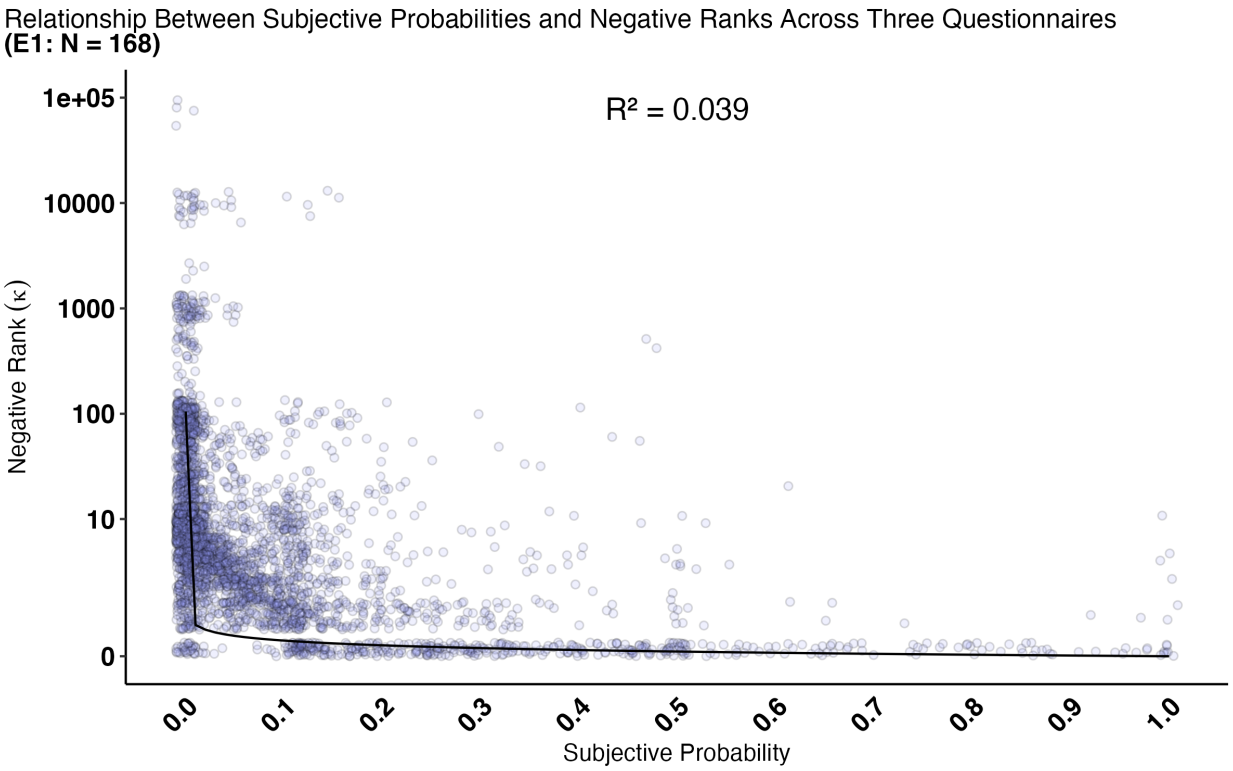


Figure 3.5: Scatter plot illustrating the relationship between subjective probabilities and negative ranks (κ) across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis was scaled to display negative ranks effectively, while the labels showed the original rank values for easier interpretation. A logarithmic model was fitted to the data, approximately explaining 3.90% of the variance in negative ranks.

Model 2: Exponential Decay An exponential decay model was used to further explore this nonlinear relationship, examining how disbelief might diminish exponentially as subjective probability increases, reflecting the natural decay of disbelief with increased certainty. The equation for this model is described as:

$$\kappa(A) = a \cdot e^{b \cdot P(A)}$$

where a and b were parameters to be estimated.

The model fitting process involved non-linear least squares regression, yielding statisti-

cally significant parameters as follows:

$$a = 4.73, \quad b = -12.97, \quad \text{SE}_a = 0.142, \quad \text{SE}_b = 1.318, \quad p_a < .001, \quad p_b < .001$$

The fit of the exponential model was also assessed, yielding a pseudo R^2 value of 0.084, indicating that approximately 8.35% of the variance in negative ranks was explained by this exponential relationship (see Figure 3.6).

Model Comparision

To determine which model more accurately described the relationship between subjective probability and negative ranks (arsinh transformed), we computed the AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) values. These criteria assessed model fit while penalizing the number of parameters to avoid overfitting. The logarithmic model had one parameter, as opposed to two parameters estimated in the exponential model. The results indicated that the exponential model provided a better fit, with an AIC of 25393.18 compared to 25582.68 for the logarithmic model, yielding a difference of 189.5. Similarly, the BIC for the exponential model was 25412.09, whereas it was 25595.28 for the logarithmic model, resulting in a difference of 183.2. This analysis showed that the exponential model fit the data better than the logarithmic model.

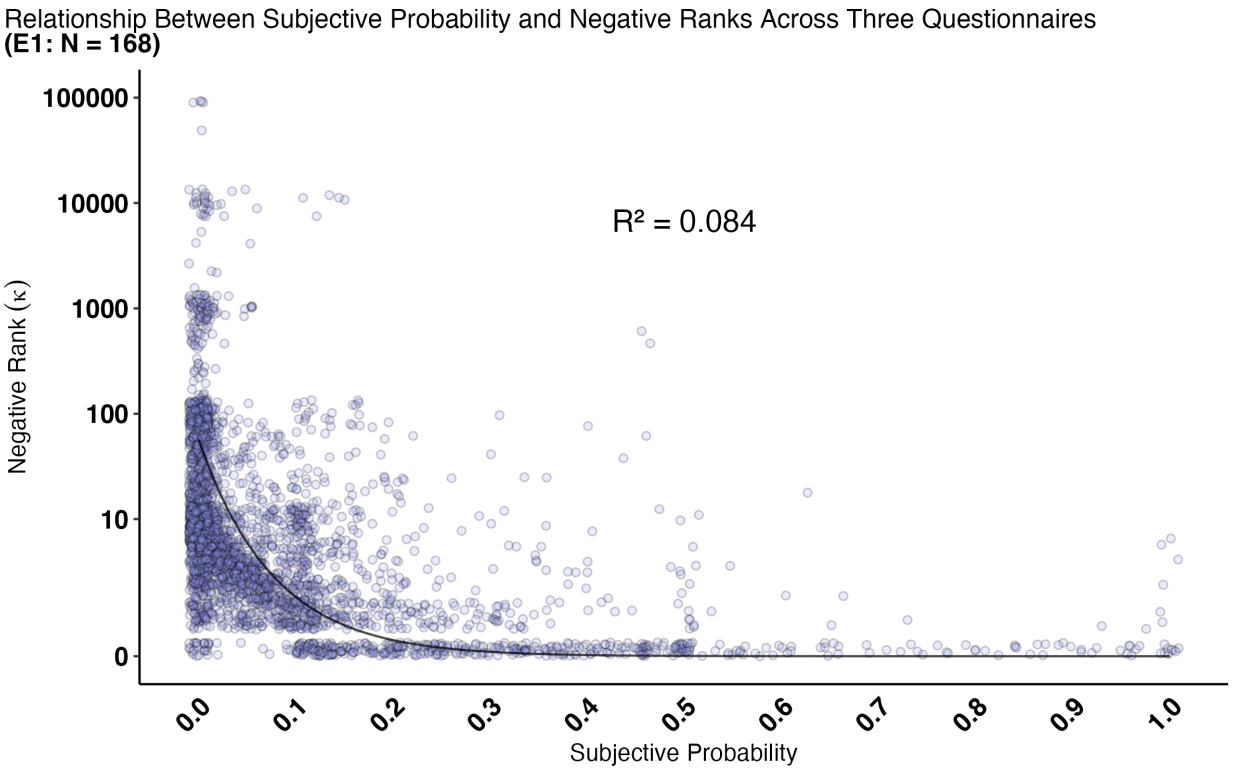


Figure 3.6: Scatter plot showing the relationship between subjective probabilities and negative ranks (κ) across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis was scaled to effectively display negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with an exponential decay model, which accounts for approximately 8.35% of the variance observed in the negative ranks.

3.3.4 Subjective Probabilities (via Open Sampling Format) and Subjective Probabilities (via Percentage — End of Task Survey)

This section examined the relationship between subjective probabilities recorded via the Open Sampling Format (OSF) immediately after each scenario and the subjective probabilities recorded as percentages at the end of the task. A correlation of 0.641 was observed

between these two measures ($p < .001$), indicating a strong relationship. This suggests that the OSF effectively captured participants' subjective probability responses in a manner moderately consistent with their end-of-task survey responses, Figure 3.7.

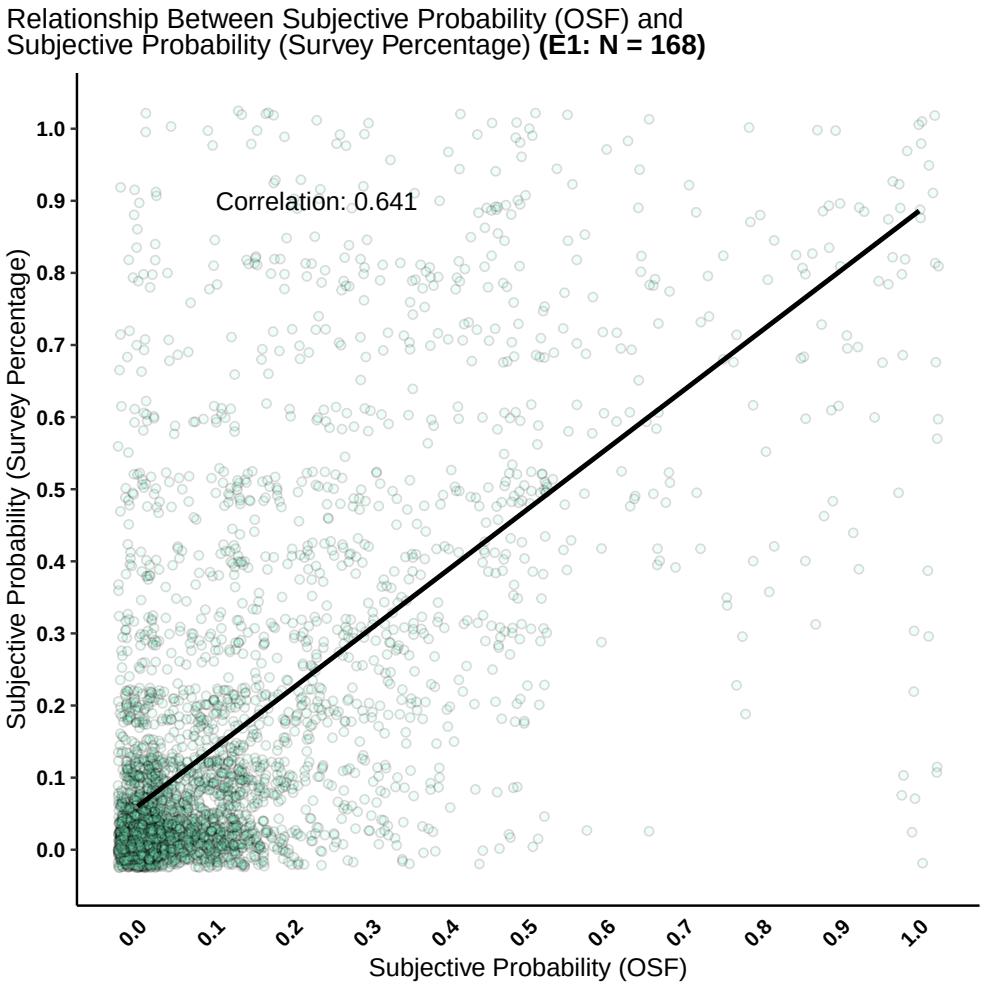


Figure 3.7: Scatterplot showing the relationship between subjective probabilities recorded via Open Sampling Format (OSF) and subjective probabilities recorded as percentages at the end of the task. The correlation coefficient is 0.641, indicating a strong relationship between the two measures.

3.3.5 Examining the Law of Disjunction

Visualization and Descriptives:

In the analysis of disjunctions, each proposition within a disjunction (a logical "or" relationship involving two propositions) was assigned a negative rank based on the level of disbelief. The minimum rank was the lower of the two negative ranks, representing the least disbelief or surprise. The maximum rank was the higher of the two, indicating greater disbelief or surprise. If the minimum and maximum ranks were equal, the propositions were equally disbelieved. The disjunction rank reflected the combined disbelief that at least one of the propositions is true. For example, if the propositions 'Spanish' and 'Urdu' received negative ranks of 6 and 20, respectively, then 'Spanish' was the minimum rank (6) and 'Urdu' the maximum rank (20).

The disjunction rank, which involves a combined assessment of both propositions (e.g., 'Spanish or Urdu'), was then compared to these individual ranks. In the provided example, if 'Spanish or Urdu' received a rank of 7, this disjunction rank was higher than the minimum rank (6) but lower than the maximum rank (20), suggesting that the disjunction is more surprising than 'Spanish' alone but less surprising than 'Urdu' alone.

The disjunction box plot, shown in Figure 3.8, illustrates these relationships. For the categories like language demographics and music preferences, it is typically observed that the disjunction rank falls between the minimum and maximum ranks, meaning the disjunction rank was more disbelieved than the minimum rank of the two propositions but less disbelieved than the maximum rank of the two propositions.

For the language demographics, the median negative rank for the minimum rank was 5 (IQR = 8), for the disjunction rank was 8 (IQR = 21.25), and for the maximum rank was

10 (IQR = 45.25).

For the temperature ranges, the median negative rank for the minimum rank was 2 (IQR = 6.25), for the disjunction rank was 2 (IQR = 7.25), and for the maximum rank was 6 (IQR = 18). The disjunction rank for the temperature ranges had the same median as the minimum rank, which aligns with Equation 2.3, the law of disjunction — stating that the disjunction should be the minimum κ of the two propositions.

For the music preferences, the median negative rank for the minimum rank was 3 (IQR = 9), for the disjunction rank was 5 (IQR = 10.25), and for the maximum rank was 6 (IQR = 17).

Lastly, percentage of participants who had their disjunction equal to the minimum was 38.69%, while 35.12% of participants were equal to the maximum, and 26.79% of the participant were between minimum and maximum.

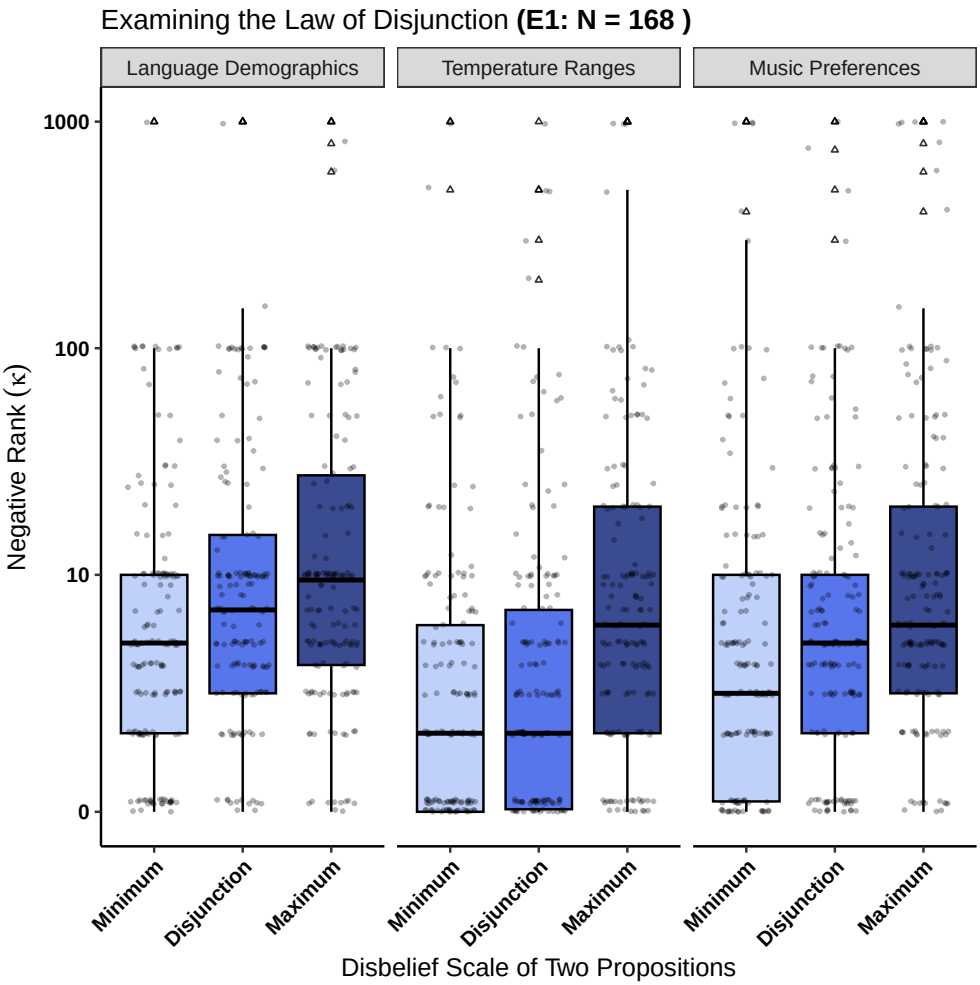


Figure 3.8: Boxplots examining the law of disjunction for three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis is on a logarithmic scale to display a wider range of observations. The minimum rank represents the least disbelieved of the two propositions involved in the disjunction, while the maximum rank represents the more disbelieved of the two propositions. The disjunction rank corresponds to the logical "or" relationship between the two propositions, reflecting the combined disbelief that at least one of the propositions is true. The dots are jittered points representing individual participant responses. The disjunction rank is more disbelieved than the minimum rank but less disbelieved than the maximum rank for Language Demographics and Music Preferences. For Temperature Ranges, the disjunction rank has the same median as the minimum rank.

Analysis of Linear Mixed-Effects Model:

A linear mixed-effects model was used to predict disjunction ranks in instances where

the minimum rank differed from the maximum rank among the propositions. Cases with ties between the minimum and maximum ranks were excluded from the analysis to identify the variable that best predicts the disjunction of ranks, which, theoretically, should be the minimum rank of the two. The number of observations removed due to ties between the minimum and maximum ranks of the two propositions was 26.79%.

Negative ranks were transformed using the inverse hyperbolic sine (arsinh) method to accommodate extreme values, which ranged from 0 to very high numbers (see rationale on page 31). Preliminary checks for multicollinearity were conducted between the two independent variables, minimum and maximum ranks. The Pearson correlation coefficient showed a high multicollinearity of 0.96, which was a cause for concern in the interpretation. Multicollinearity occurred because the two independent variables, minimum rank and maximum rank, were highly correlated, indicating that the two propositions in the disjunction had similar disbelief ratings. Despite the observed multicollinearity, the linear mixed-effects model was still performed.

A null model was conducted to predict disjunction (arsinh-transformed) with a random intercept for subjects. This model yielded an Intraclass Correlation Coefficient (ICC) of 0.74. This ICC value indicated that 74.03% of the total variance in disjunction scores was attributable to differences between subjects. This suggested that individual differences significantly predicted disjunction, necessitating the inclusion of random intercepts in subsequent models to account for subject variability.

A linear mixed-effects model was utilized to analyze disjunction ranks (arsinh-transformed),

incorporating fixed effects for the minimum and maximum ranks (both arsinh-transformed) of two propositions. Additionally, interactions between questionnaire type and both minimum and maximum ranks were examined. To account for individual variability, a random intercept for participants was included. Using temperature ranges as the reference category for questionnaire type, the estimated effect of the minimum rank (β = 1.05, SE = 0.03, t = 33.1, $p < .001$) reflected the relationship between the minimum rank and disjunction rank within the reference category of temperature ranges and when the maximum rank was 0. Similarly, the non-significant estimate for the maximum rank (β = 0.01, SE = 0.03, t = 0.48, $p = .632$) applied to this same condition. However, there were significant interactions where questionnaire type influenced both the minimum and maximum ranks. Specifically, the interactions with music preferences and language demographics significantly decreased the estimate for minimum rank (β = -0.92, SE = 0.08, t = -12.3, $p < .001$); (β = -1.06, SE = 0.09, t = -11.8, $p < .001$) and increased the estimate for maximum rank (β = 0.84, SE = 0.06, t = 13.81, $p < .001$); (β = 0.59, SE = 0.07, t = 8.71, $p < .001$). These interaction effects indicated that the relationship between minimum or maximum rank and disjunction varied significantly across questionnaire types. Refer to Table 3.4 for full details of the predicted model.

The figures below (Figure 3.9 and Figure 3.10) demonstrated that disjunction was effectively predicted by the minimum rank for temperature ranges, aligning closely with the minimum of the two proposition ranks. In contrast, for other questionnaires, the maximum rank seemed to better predict disjunction, although the estimated disjunction values were influenced by the context and typically were lower than the maximum rank. Consequently,

the simple effects for the reference category were not generalizable across all contexts. Significant interaction effects between the minimum rank and questionnaire type, particularly for temperature ranges and music preferences, indicated that the context significantly influenced the predictive power of the minimum and maximum ranks on disjunction.

	Est.	S.E.	t val.	d.f.	p
(Intercept)	0.09	0.09	0.95	356.52	0.34
Minimum Rank	1.05	0.03	33.10	359.99	0.00
Maximum Rank	0.01	0.03	0.48	358.64	0.63
Music Preferences	-0.37	0.19	-1.96	310.81	0.05
Language Demographics	1.00	0.13	7.63	280.67	0.00
Minimum Rank:Music Preferences	-0.92	0.08	-12.30	325.89	0.00
Minimum Rank:Language Demographics	-1.06	0.09	-11.80	346.13	0.00
Maximum Rank:Music Preferences	0.84	0.06	13.81	281.88	0.00
Maximum Rank:Language Demographics	0.59	0.07	8.71	347.12	0.00

Table 3.4: Results of the linear mixed-effects model predicting disjunction ranks. This table displays the estimated coefficients, standard errors, t-values, and p-values for the arsinh-transformed minimum and maximum ranks, alongside questionnaire types and their interaction effects.

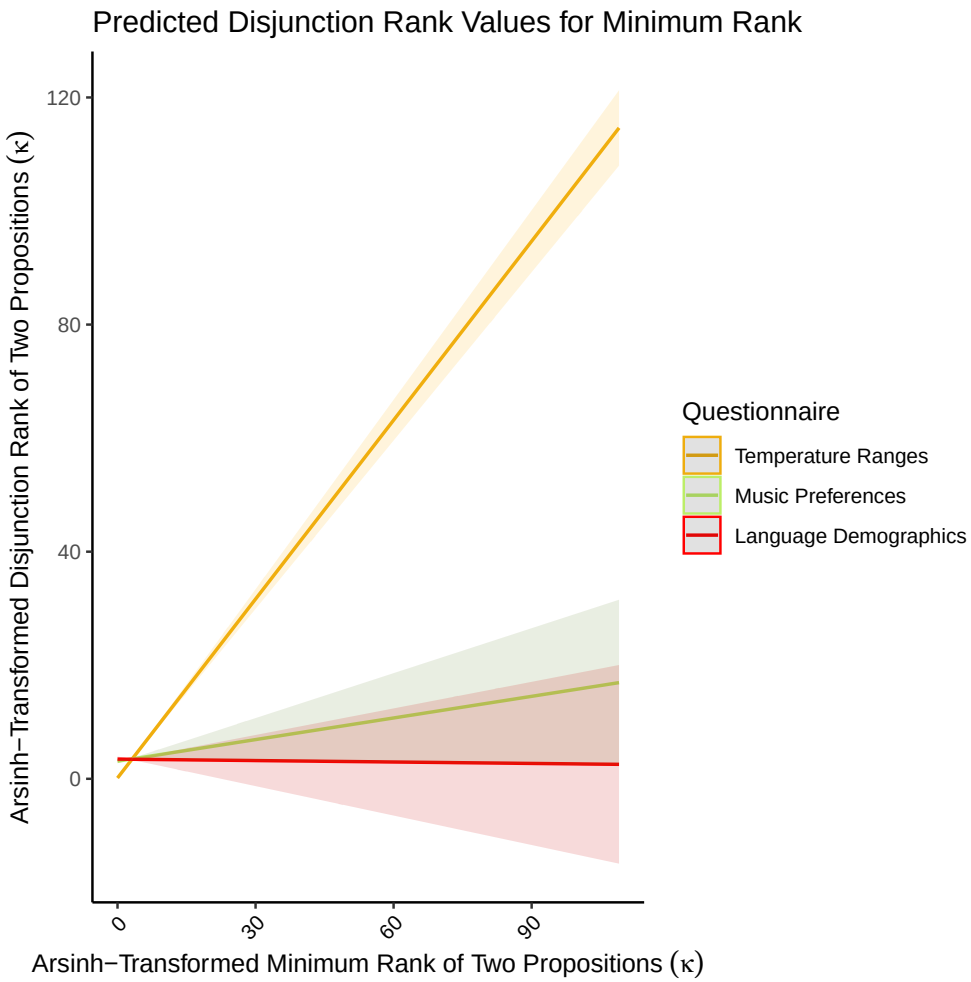


Figure 3.9: Predicted disjunction rank values (κ) for minimum rank across different questionnaire types. Temperature Ranges are shown in yellow, Music Preferences in green, and Language Demographics in red. Shaded areas denote confidence intervals. The plot demonstrates a significant positive relationship between minimum rank and disjunction rank for Temperature Ranges, with varying effects observed in other types.

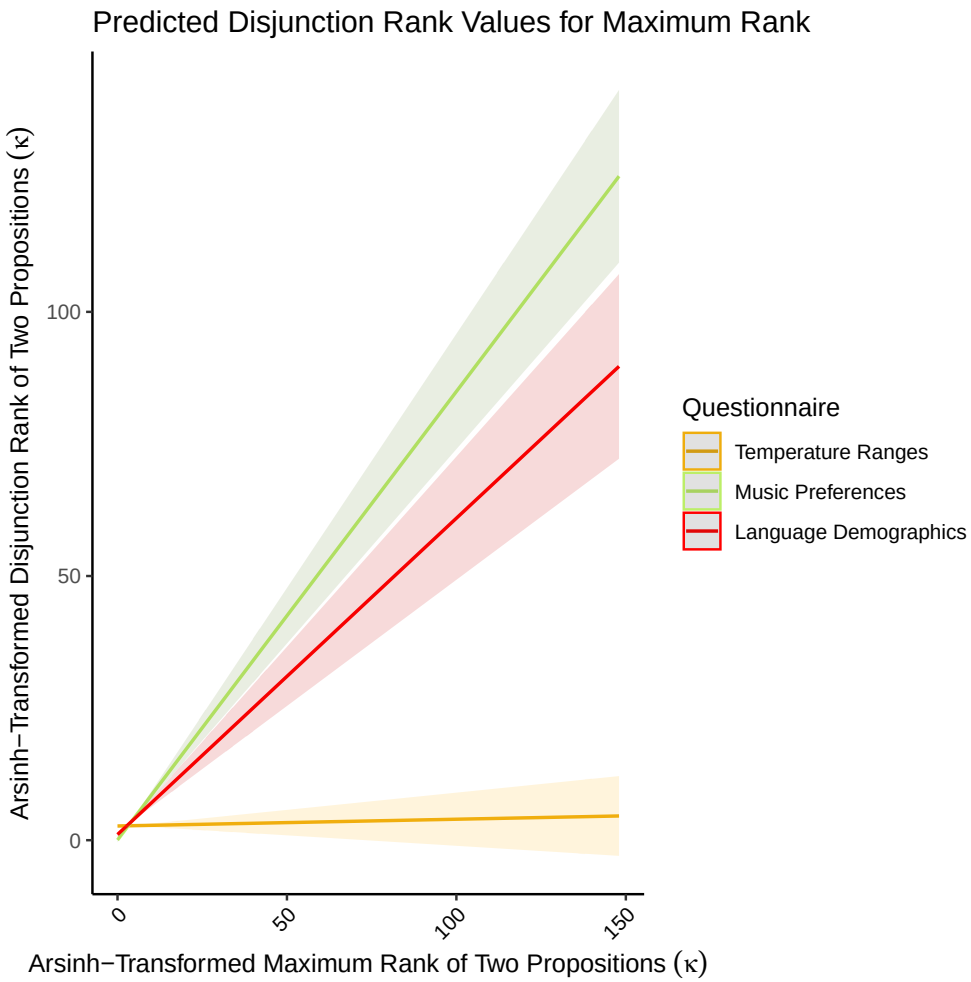


Figure 3.10: Predicted disjunction rank values for the maximum rank of two propositions, segmented by questionnaire type. Language Demographics are shown in red, Temperature Ranges in yellow, and Music Preferences in green. Shaded areas indicate confidence intervals. This graph highlights the variation in the effect of maximum rank on disjunction rank across different questionnaire types, especially evident in Music Preferences and Language Demographics, where disjunction rank is predominantly influenced by the maximum of the two ranks.

3.3.6 End-of-Task Survey on Task Difficulty

At the conclusion of the task, participants were asked to rate the difficulty of grading degrees of disbelief and to hypothetically evaluate whether it would be easier to assign degrees of

belief instead.

Task Difficulty Ratings: Most participants found the task moderately challenging, with the most common rating being a 4 on a scale from 1 (Very Easy) to 7 (Very Difficult). This rating, chosen by approximately 30% of participants, suggests a balanced complexity of the task. The fewer responses at the extremes of the scale, with ratings of 1 and 7, imply that participants neither found the task as very easy or very difficult. This distribution is depicted in Figure 3.11.

Perceived Ease in Assigning Degrees of Belief: When asked hypothetically about the ease of assigning degrees of belief over disbelief, the majority of responses centred around a neutral rating of 4, 'Neither Agree nor Disagree'. This suggests a neutral stance by most participants, as illustrated in Figure 3.12.

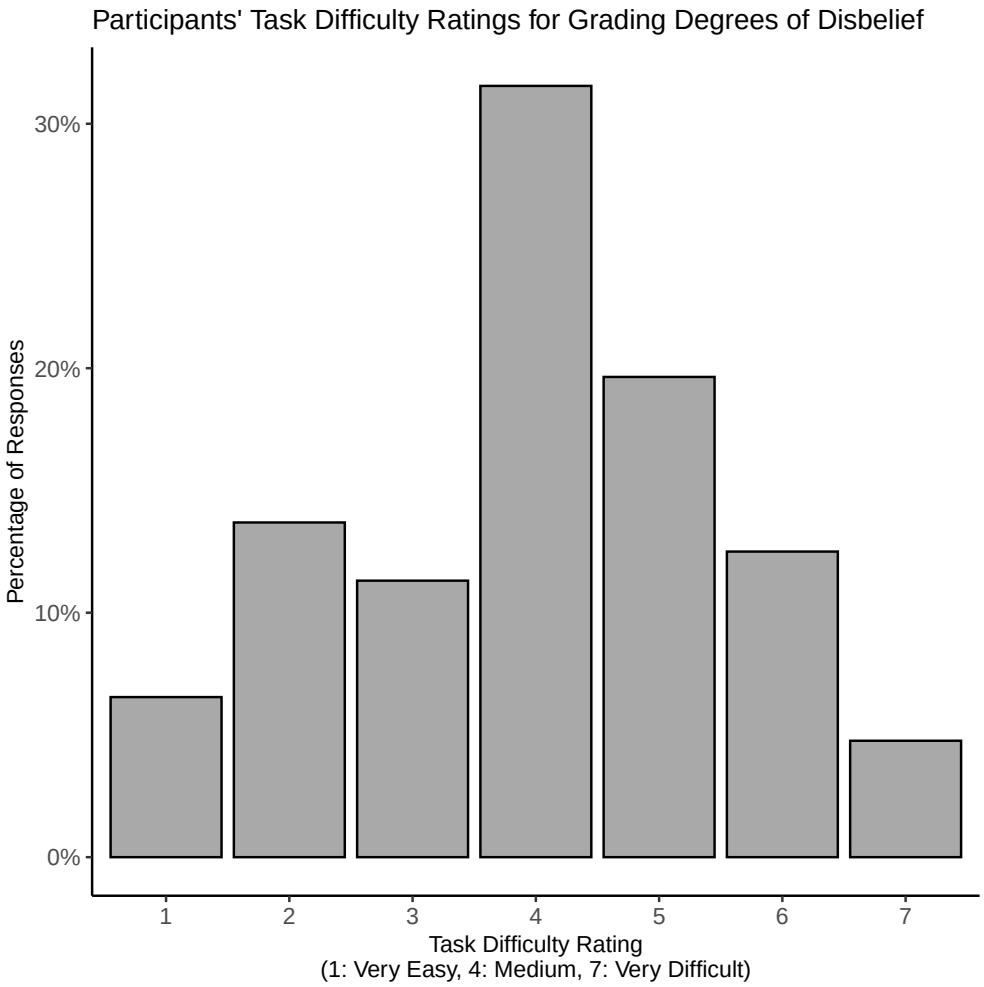


Figure 3.11: Bar graph showing the distribution of participants' task difficulty ratings for grading degrees of disbelief. The most common response was moderately difficult (Rating 4).

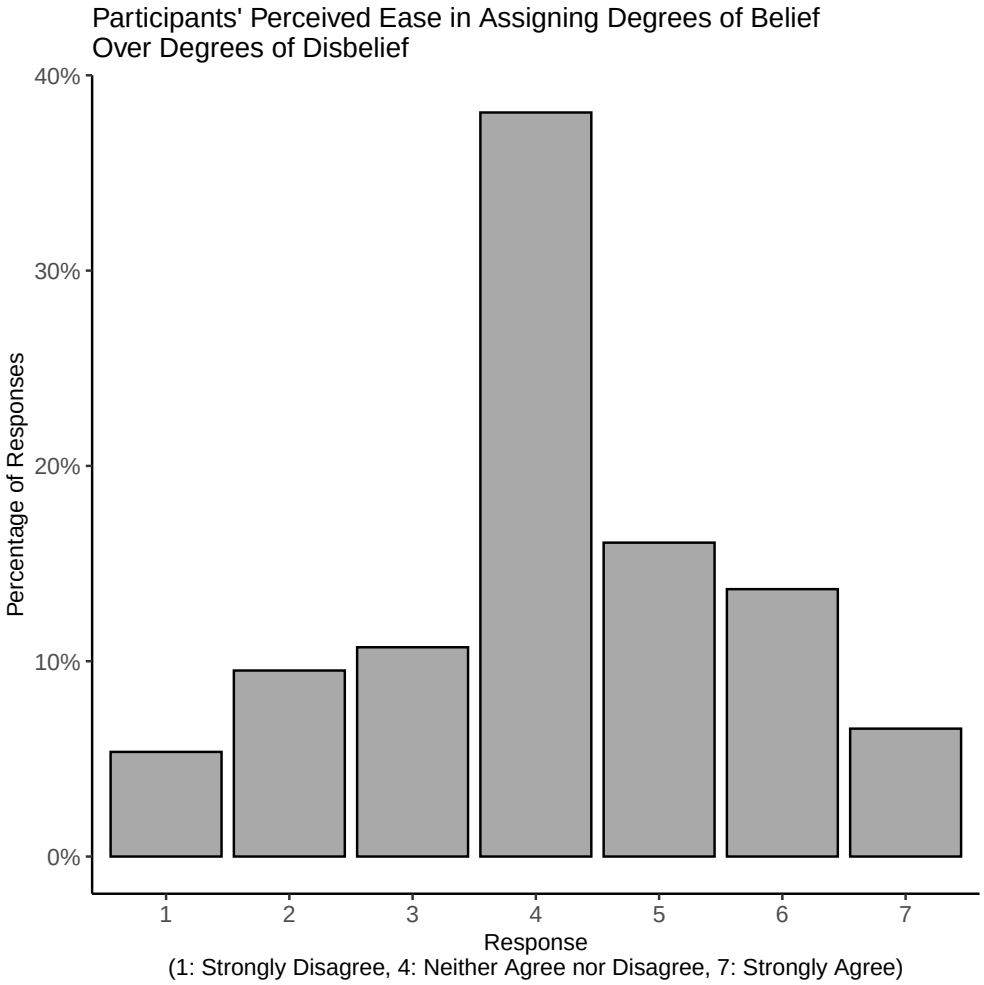


Figure 3.12: Bar graph depicting the distribution of responses to the perceived ease of assigning degrees of belief over degrees of disbelief. The most frequent rating was neutral (Rating 4).

3.4 Discussion

The study aimed to integrate ranking theory into behavioural experiments, demonstrating that participants could consistently express disbelief through negative ranks in situations with unclear probabilities. These findings emphasized the effectiveness of negative ranking functions across three distinct domains: language demographics, temperature ranges, and

music preferences, where beliefs may have been shaped by personal experiences and cultural norms.

3.4.1 The Utility of Negative Ranks

The use of negative ranks enabled exploration beyond the conventional 0 to 1 probability range, capturing a spectrum of disbelief from 0 to infinity. This scale is definitively bounded at 0, marking non-surprising events, with values greater than 0 indicating increasing surprise, and reaching towards infinity for perceived impossibilities. Despite the absence of qualitative regions like those in probability scales, the observed median and range patterns across various scenarios suggest that negative ranks can measure degrees of disbelief. This method, which does not require the sums of all events to equal 1, potentially aligns with how people naturally assess the relative surprise of events in a 'rank-based' system, without assigning specific likelihood estimates to each scenario.

The range of disbelief varied among participants; some used a scale from 0 to 10, while others opted for a broader range, such as 0 to 100. This scale variability could explain the increased IQR values when the median surprise rating for a proposition was high. For instance, introducing rare propositions, particularly in scenarios such as extreme temperature ranges, resulted in larger Interquartile Range (IQR) values. Such increased variability could be attributed to the inherently personal nature of surprise; for example, a surprise rating of 100 does not uniformly resonate with all individuals. Negative ranks address this by allowing for relative comparisons across different propositions, each tailored to an individual

participant's perspective. The large IQR values for rare events could also stem from individual differences in the degree of surprise attributed to such events. Some participants might perceive these as highly surprising due to their rarity, while others might not, depending on their personal experiences or expectations.

Conversely, lower IQRs where a rating of 0 indicated non-surprising events suggest low variability for expected outcomes. This highlights a general consensus among participants and the ease of identifying non-surprising events. Another possible explanation for the low variability is a floor effect, since 0 is the minimum score. A negative rank of 0 also suggests that participants might hold some degree of belief towards that proposition, which could be further assessed through positive ranks. For instance, although there was a general consensus that the French language would not be surprising, the degree of belief regarding participants' expectations towards French may vary across individuals.

The negative ranks across propositions and different questionnaires adhered to expected patterns. For instance, there was a U-shaped distribution in disbelief about temperature: 11 to 20°C was the least disbelieved, while temperatures diverging from this range faced increasing disbelief. This alignment reflected the average temperature range for Boston in September. However, estimating probabilities precisely would have been more challenging, especially for someone who had never visited Boston in September or did not know the geographical location. Furthermore, the hypothetical co-worker's music preferences were inferred from the described character's attributes. Thus, beliefs about this character, Tom, were shaped by cultural and societal expectations, and the disbelief gradings appeared to align with these cultural norms. Regarding language demographics, since French is the most

common language in Quebec, being monolingual in French was not surprising, while other languages were surprising.

Overall, the results from the three questionnaire scenarios demonstrated that participants consistently used negative ranking functions to express varying levels of disbelief across different propositions. In scenarios where the probability of an outcome was unclear, negative ranking functions proved effective. These results indicated that ranking theory can capture and distinguish people's disbeliefs in diverse settings influenced by societal, environmental, and personal factors. Given that this was an initial experiment and the first evidence of how people might use negative ranks, we explored a replication of Experiment 1 in the next chapter (Experiment 2).

3.4.2 Subjective Probabilities and Negative Ranks

The relationship between subjective probabilities and negative ranks demonstrated that lower probabilities were linked to higher degrees of disbelief, while higher probabilities were associated with reduced or no disbelief. This analysis revealed a nonlinear relationship, suggesting that the likelihood of an event does not directly correspond to the levels of disbelief associated with it, indicating that probability metrics and negative ranks measure different aspects: probability metrics quantify event likelihood, while negative ranks capture levels of disbelief. The varying thresholds at which an event becomes surprising were context-dependent, highlighting the limitations of using subjective probabilities alone to capture beliefs.

Although subjective probabilities are useful for providing the likelihood of events, they show minimal correspondence with negative ranks under conditions of uncertainty, highlighting their inadequacy in capturing the full spectrum of disbelief. The varying probability thresholds for when something becomes surprising across different contexts suggest that subjective probabilities alone might not fully capture beliefs. Negative ranks reveal a broad dispersion of disbelief levels for lower subjective probabilities and illustrate that the same probability can correspond to different degrees of disbelief depending on the context. Moreover, probabilities ranging from 0 to 1 may not always elicit surprise or disbelief, demonstrating the value of negative ranks in explicitly reporting varying degrees of disbelief. This detail is crucial for understanding how individuals process information and form expectations, particularly concerning the likelihood and impact of different events. This discrepancy demonstrates that directly inferring belief from subjective probabilities may be inaccurate, suggesting a reevaluation of how beliefs are inferred from these metrics.

Spohn theorized that negative ranks logarithmically relate to probabilities, with an infinitesimal base, because this mapping neatly aligns the operations used in each theory: sums of probabilities translate to minimums of ranks, products to sums, and quotients to differences (Spohn, 2009; Raidl and Skovgaard-Olsen, 2017). This approach provides a systematic way to compare probabilistic and ranking methods. This theory formed the foundation for investigating the relationship between these two approaches in modelling belief systems.

Thus, two models were examined: a logarithmic model and an exponential decay model as exploratory approaches. Despite the strong theoretical foundation derived from Spohn's ranking theory, the logarithmic model showed limited ability to effectively explain the re-

lationship between subjective probabilities and negative ranks. This suggests that negative ranks fundamentally differ from probabilities and are not simply a result of a logarithmic transformation in scenarios with unclear probabilities. It is possible that belief formation in such cases may rely on factors other than subjective probabilities, or there may be no logarithmic relationship at all. This will be explored further in Chapter 6 as we reevaluate this theorized relationship in a setting where belief formation is solely based on probabilities.

In contrast, the exponential decay model allowed us to examine whether disbelief decayed as probability increased. It demonstrated a slightly better fit in our analysis with the additional parameter, as evidenced by lower AIC and BIC scores. Although AIC does not penalize the number of parameters as strongly as BIC, the interpretation for model comparison remained the same. According to Raftery (1995), a BIC difference of 10 corresponds to a Bayes factor of 150, indicating strong evidence that the model with the lowest BIC fits best. These metrics showed robust evidence that the exponential model more accurately described the relationship between subjective probability and negative ranks. However, the overall variance explained by this model was still limited. It is important to note that the pseudo-R^2 was only an approximate estimate. This is because, in nonlinear settings, R^2 may not provide an accurate measure of model fit and is rather used as a supplement to the AIC and BIC (Spiess and Neumeyer, 2010).

These results suggest that in environments where probability is not directly available, negative ranks provide value as people can 'rank' their level of surprise for a given outcome, which is different from assigning a probability value to an event. Thus, negative ranks are especially useful in situations where probabilistic information alone falls short. This study

contributes to the ongoing effort to compare and contrast these theoretical frameworks by empirically testing their relationship.

3.4.3 Open-Sampling Format for Capturing Subjective Probability

A novel method was implemented to graphically capture subjective probability, allowing participants to modify event cards to reflect their expectations for each proposition. The manipulation check, which compared the Open Sampling Format (OSF) to the recorded percentages at the task's conclusion, showed a strong correlation, indicating that the OSF effectively captured participants' subjective probability responses. However, this correlation's robustness might be limited due to the delay in collecting percentage responses at the task's end. This delay was introduced to avoid priming participants to think in probabilistic terms, without explicitly asking for probabilities after their disbelief responses.

3.4.4 Disjunction Judgements

The law of disjunction posits that the rank of a disjunction should be the minimum disbelief of the involved propositions, to maintain logical consistency. In practical terms, if a proposition 'the sky is blue' is universally accepted (rank of 0) and another 'the sky is green' is largely disbelieved (rank of 15), the disjunction 'the sky is blue or the sky is green' should not be surprising.

Our analysis indicated that the influence of the less disbelieved proposition on disjunction judgments varied by context. Specifically, in the temperature scenario, the least dis-

believed proposition played a significant role, aligning with the theory that less surprising outcomes predicted the disjunctive judgments. However, for language demographics and music preferences, the more disbelieved proposition had predictive power, suggesting that participants sometimes considered both propositions (minimum and maximum) when forming their judgments. High intra-correlation between propositions may have reduced the distinction between the two variables, complicating interpretation.

This prompted a refinement in a subsequent study (Chapter 5) to address the high multicollinearity. By including disjunctions with a greater discrepancy in disbelief values between the two propositions involved, we aimed to lower the correlation and clarify their individual contributions to disjunction judgments, thereby addressing the confounding effects observed in this study.

Instances where participants' judgments reflected an average rank between propositions suggest that they considered both disbelief values rather than simply choosing the minimum. This indicates a more complex processing of disjunctive statements than what the law of disjunction alone would suggest.

These findings highlight the need to revisit theoretical frameworks for understanding disjunction judgments, particularly considering the role of context and proposition distinctiveness in how people process and integrate competing information. The observed tendency of some individuals to prioritize the more disbelieved proposition suggests potential challenges in processing disjunctive statements, which may disrupt traditional views of logical consistency in disjunctive reasoning. Further studies could lead to refined theoretical frame-

works that better account for the complexities observed in judgment processes.

3.4.5 Perceived Difficulty in Grading Degrees of Disbelief

Participants generally found grading disbelief to be of medium difficulty, indicating it required cognitive effort but was manageable. The distribution of perceived difficulty responses was fairly normal, suggesting the task was neither overly challenging nor too easy for most. When asked about the perceived difficulty of grading belief instead, the majority remained neutral, showing no perceived difference in cognitive effort. This neutrality supports the use of negative ranks as an effective and user-friendly measure in psychological experiments, aligning with Spohn's (2012) recommendation of negative ranks for ranking theory.

3.4.6 Conclusion

This study marks the first application of ranking theory in a behavioural context, providing valuable insights into how individuals process and represent degrees of disbelief. Negative ranks fundamentally differ from traditional probabilistic models, particularly in environments with unclear objective probabilities. They are not redundant to probability — neither merely a logarithmic transformation nor a replacement for probabilities. Instead, negative ranks serve as a consistent and effective tool for measuring disbelief (or surprise).

These insights highlight the potential for applying negative ranks in behavioural experiments, offering a deeper understanding of the degrees of disbelief that individuals may express. Further research is necessary to replicate these findings and ensure consistent results

regarding negative ranks, as this is the first application of ranking theory in a psychological experiment. Such replication efforts will be crucial in validating the robustness and generalizability of the observed relationships and enhancing the theoretical framework of ranking theory.

Chapter 4

Experiment 2: In-Lab Replication of Negative Ranking Functions

4.1 Introduction

Experiment 2 was an in-lab replication of Experiment 1, conducted to validate the findings from the online study. Using the same stimuli and procedures but with a different sample, the study aimed to examine the consistency of participants' use of negative ranks and the robustness of the observed relationships between negative ranks and subjective probabilities. This was important, as the previous chapter demonstrated the first usage of negative ranks in a behavioural experiment.

4.2 Methods

4.2.1 Participants

63 undergraduate students from the University of Waterloo participated in the in-lab study, receiving course credits for their involvement. The median age was 19 years, with 48 females, 13 males, and 2 who preferred not to answer. Data collection was conducted in a subsequent semester to Experiment 1.

4.2.2 Procedure

The procedure was the same as that of Experiment 1. Participants completed the study on lab computers using identical JavaScript code. The procedure is documented in Section 3.2.3.

4.2.3 Data Analysis

Data analysis followed the same approach as Experiment 1, investigating negative ranks across three questionnaire scenarios, the relationship between subjective probability and negative ranks, and examining the law of disjunction (see Section 3.2.4).

4.2.4 Ethical Considerations

This research study obtained ethics clearance from the University of Waterloo Research Ethics Board (REB 44180) prior to conducting the experiment. All participants were in-

formed about the study's purpose and their right to withdraw at any time without penalty. To ensure privacy, we securely stored de-identified data, and all participants provided informed consent before participating.

4.3 Results

4.3.1 Analysis of Negative Ranks

The negative ranks from the three questionnaires were analyzed to examine participants' disbelief. The median and interquartile range (IQR) were calculated for each proposition. Visualization techniques were consistent with those in Experiment 1 (page 32).

Language Demographics

Figure 4.1 and Table 4.1 summarize the negative ranks for encountering a Quebec native who speaks only one language. The findings replicated those of Experiment 1 (as shown on page 34), demonstrating that French and English were associated with lower disbelief, while languages such as Urdu and Japanese elicited higher disbelief. Like Experiment 1, French was not disbelieved by the majority of participants (median = 0, IQR = 0), reflecting its acceptance as the predominant language in Quebec.

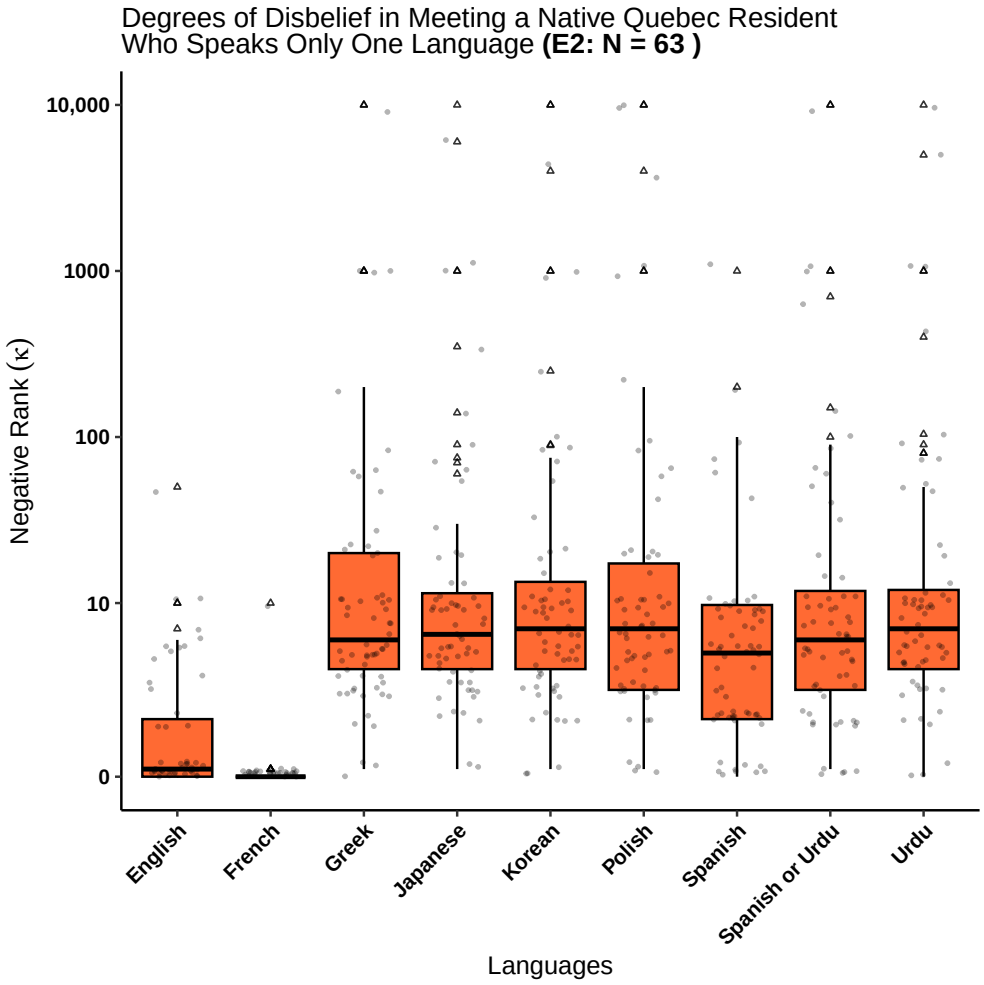


Figure 4.1: Boxplots showing the degrees of disbelief (negative rank) for encountering a native Quebec resident who speaks only one language, across different languages. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each language, with lower values indicating lower disbelief. French and English exhibit the lowest disbelief, indicating that participants expect these languages to be more commonly spoken. Conversely, languages such as Urdu and Japanese show higher disbelief, reflecting their perceived rarity.

	Propositions	Median Negative Rank	IQR Negative Rank
1	English	1.00	2.00
2	French	0.00	0.00
3	Greek	6.00	16.00
4	Japanese	7.00	9.00
5	Korean	7.00	9.50
6	Polish	7.00	14.50
7	Spanish	5.00	8.00
8	Spanish or Urdu	6.00	9.00
9	Urdu	7.00	8.50

Table 4.1: Descriptive statistics for Language Demographics, showing the median and interquartile range (IQR) of negative rank scores for each language. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values.

Temperature Ranges

Figure 4.2 and Table 4.2 present participants' levels of disbelief regarding various daytime high temperatures for a day trip to Boston in September. Consistent with Experiment 1 (see page 37), extreme temperatures elicited higher disbelief, while moderate temperatures resulted in lower disbelief. A similar U-shaped distribution of negative rank scores was observed. Specifically, temperatures between 11 and 20°C elicited no disbelief, with participants agreeing that these temperatures are typical for September in Boston. Additionally,

extreme temperatures were associated with high IQR values, consistent with Experiment 1, indicating considerable variation in disbelief among participants.

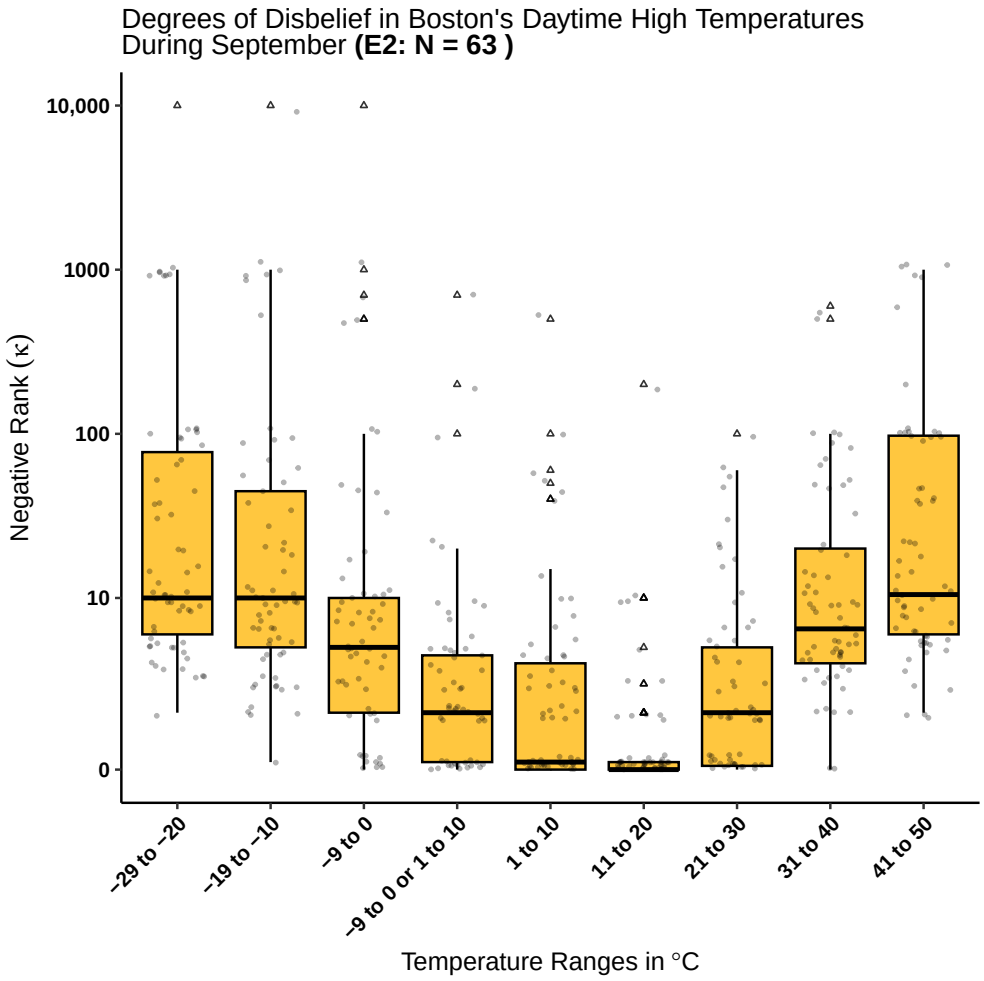


Figure 4.2: Boxplots showing the degrees of disbelief (negative rank) for various daytime high temperature ranges in Boston during September. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each temperature range, with lower values indicating lower disbelief. The temperature range of 11 to 20°C exhibits the lowest disbelief, indicating that participants expect this range to be typical for September. Conversely, extreme temperature ranges such as -29 to -20°C and 41 to 50°C show higher disbelief, reflecting their perceived rarity.

	Propositions	Median Negative Rank	IQR Negative Rank
1	-29 to -20	10.00	84.00
2	-19 to -10	10.00	40.00
3	-9 to 0	5.00	8.00
4	-9 to 0 or 1 to 10	2.00	3.50
5	1 to 10	1.00	4.00
6	11 to 20	0.00	1.00
7	21 to 30	2.00	4.50
8	31 to 40	7.00	16.00
9	41 to 50	11.00	94.00

Table 4.2: Descriptive statistics for Temperature Ranges, showing the median and interquartile range (IQR) of negative rank scores for each temperature range. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values.

Music Preferences

Figure 4.3 and Table 4.3 illustrate participants' levels of disbelief regarding various music genres a co-worker might sing along to at a karaoke party. The results mirrored those of Experiment 1 (see page 40). Genres such as country music elicited the least disbelief (median = 0, IQR = 0), indicating a strong consensus among participants. In contrast, opera yielded the highest disbelief scores (median = 7, IQR = 9.5), suggesting it was the most surprising genre. As in Experiment 1, all other genres elicited greater disbelief than country music but

less than opera.

When comparing the box plot visualizations across the three questionnaires, it was observed that temperature ranges produced larger IQR values compared to language demographics and music preferences, indicating more variability with some participants viewing extreme temperatures as rarer. The results from Experiment 1 and Experiment 2 were very consistent in terms of median and IQR values. This consistency across different samples reinforced the robustness of our findings.

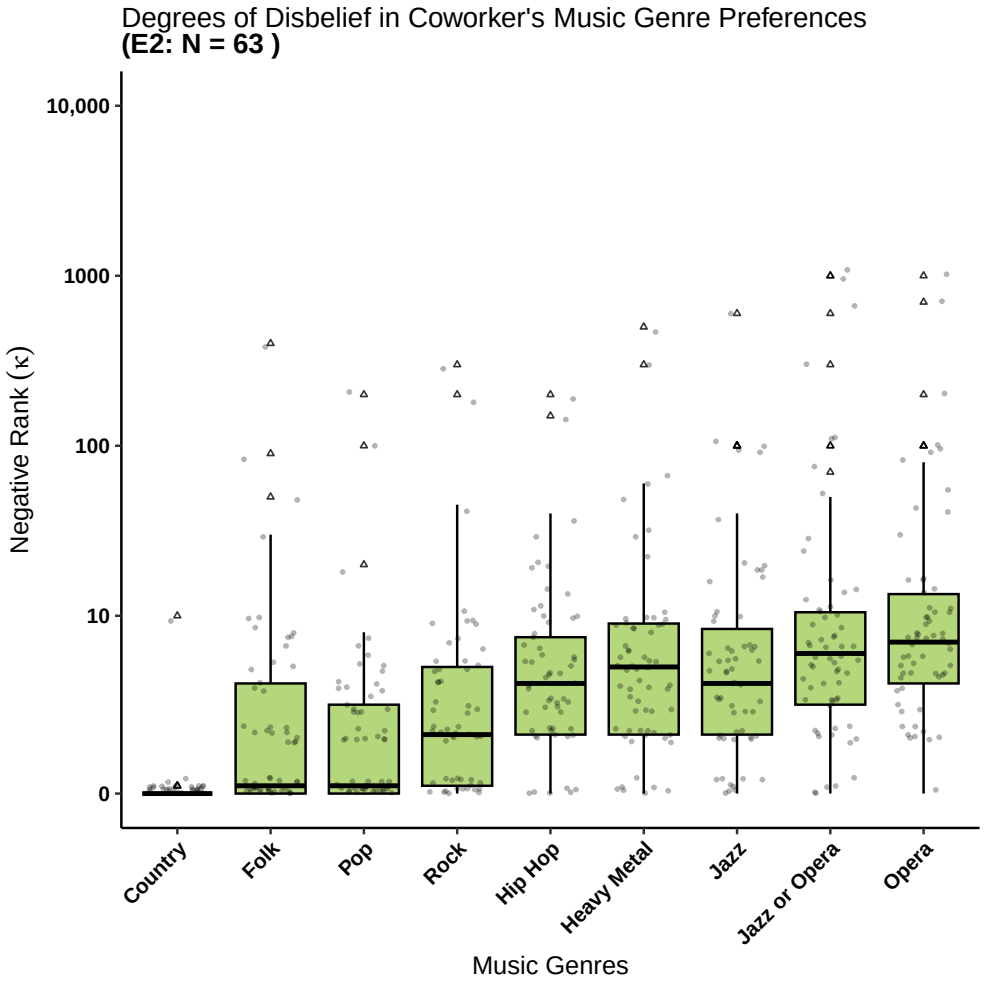


Figure 4.3: Boxplots showing the degrees of disbelief (negative rank) for various music genres that a coworker might sing along to at a karaoke party. The y-axis is on a logarithmic scale to display a wider range of observations. The median and interquartile range (IQR) are indicated for each genre, with lower values indicating lower disbelief. The genre of country music exhibits the lowest disbelief, indicating that participants expect this genre to be typical for the coworker. Conversely, genres such as opera show higher disbelief, reflecting their perceived rarity.

	Propositions	Median Negative Rank	IQR Negative Rank
1	Country	0.00	0.00
2	Folk	1.00	4.00
3	Heavy Metal	5.00	7.00
4	Hip Hop	4.00	5.50
5	Jazz	4.00	6.50
6	Jazz or Opera	6.00	7.50
7	Opera	7.00	9.50
8	Pop	1.00	3.00
9	Rock	2.00	4.00

Table 4.3: Descriptive statistics for Music Preferences, showing the median and interquartile range (IQR) of negative rank scores for each genre. The IQR represents the middle 50% of the data, indicating more consensus with smaller values and greater variability in disbelief scores with larger values.

4.3.2 Relationship Between Subjective Probabilities and Negative Ranks

Figure 4.4 revealed a nonlinear relationship between subjective probabilities and negative ranks across the three questionnaires. This pattern was similar to what was observed in Experiment 1 (see page 43), where lower probabilities were generally linked with higher degrees of disbelief. However, unlike in Experiment 1 — where a negative rank of 0 extended

across the entire probability scale — Experiment 2 displayed a distinct pattern: subjective probabilities approaching or at 0 consistently corresponded with negative ranks of 1 or higher, indicating disbelief. This pattern clearly demonstrated that extremely low probability values were reliably associated with higher degrees of disbelief.

However, a negative rank of 0 — which indicated a lack of disbelief, and potentially belief — corresponded to subjective probabilities ranging from 0.1 to 1. Additionally, probabilities greater than 0.5 were frequently associated with a negative rank of 0, signifying the absence of disbelief. This relationship suggested that higher subjective probabilities were typically not linked with surprise.

Across the three questionnaires, the point at which a subjective probability was associated with disbelief varied. Specifically, for music preferences, a subjective probability of less than 0.2 was linked to disbelief, while for temperature ranges, a subjective probability of less than 0.35 elicited disbelief for most participants. This variation highlighted that the subjective probabilities considered surprising or disbelieved depended on the specific context.

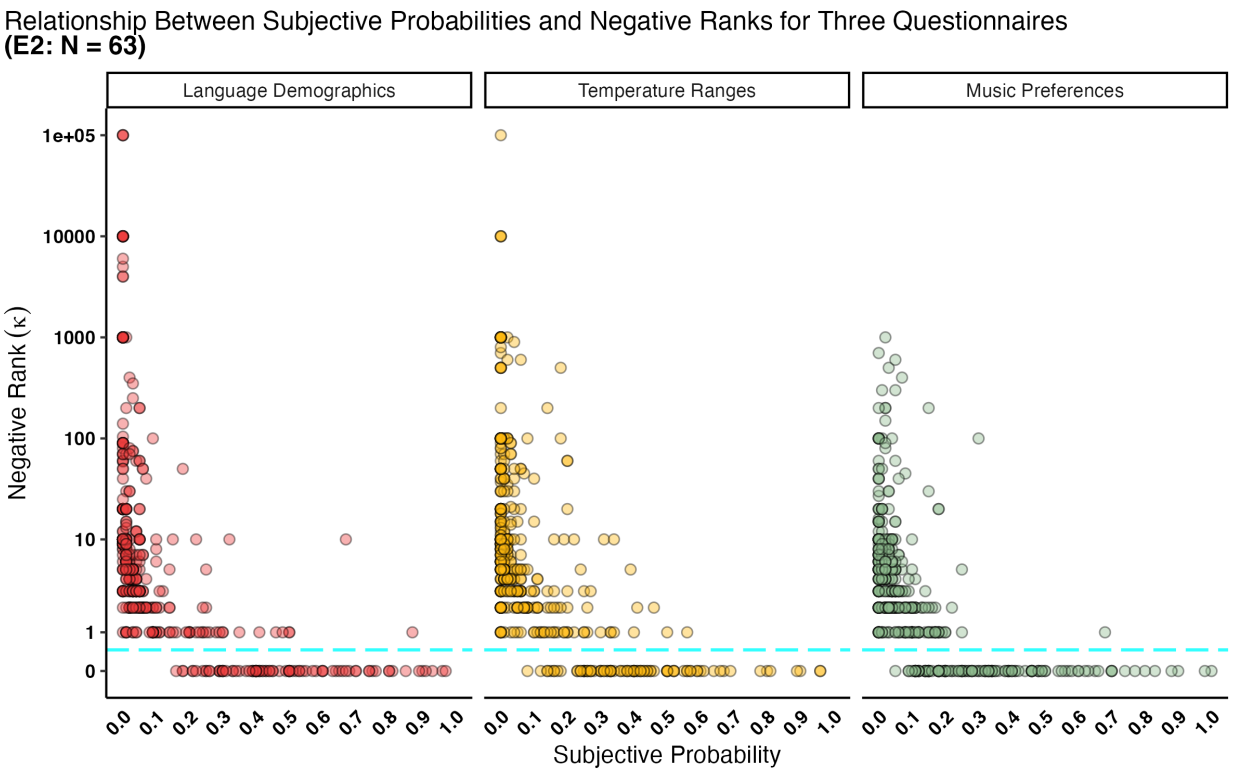


Figure 4.4: Scatter plots showing the relationship between subjective probabilities (via Open Sampling Format) and negative ranks (κ) across three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis is scaled to display negative ranks effectively, while the labels show the original rank values for easier interpretation. The dotted blue line represents the threshold where disbelief is greater than 0. Instances above this line indicate varying degrees of disbelief. The plots suggest a nonlinear relationship between negative rank and subjective probability, with lower probabilities generally associated with higher degrees of disbelief. However, low probability values may not always elicit surprise or disbelief, demonstrating the value of negative rank in explicitly reporting varying degrees of disbelief.

4.3.3 Mathematical Model Fit

As in Experiment 1, two models were tested to fit the relationship between subjective probabilities and negative ranks: a logarithmic model and an exponential decay model (see rationale on page 43). It is important to note that the negative ranks were transformed using the arsinh function (see rationale on page 31).

Model 1: Logarithmic Model

A non-linear least squares regression estimated a logarithmic base b:

$$b = 0.0002, \quad \text{SE} = 0.00004, \quad t(1) = 4.81, p < .001$$

This indicated a statistically significant logarithmic base fit. However, the overall model fit was poor, as reflected by a pseudo R^2 of -0.06. A negative pseudo R^2 value suggests that the model does not explain the variance in negative ranks and performs worse than a horizontal line (through the mean value of negative ranks), implying that the logarithmic model is not suitable for this data, (see Figure 4.5).

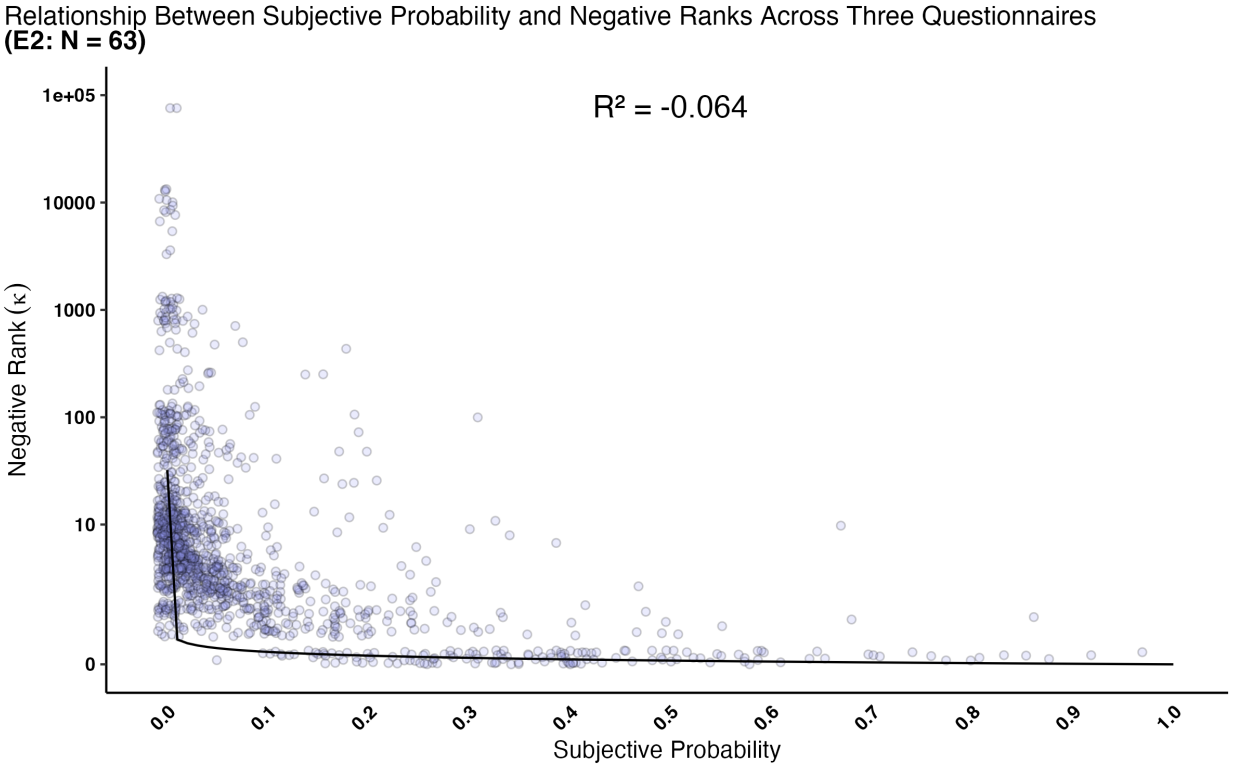


Figure 4.5: Scatter plot illustrating the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to display negative ranks effectively, with labels indicating the original rank values for easier interpretation. A logarithmic model is fitted to the data, but it was a poor fit as indicated by the pseudo R^2 value.

Model 2: Exponential Decay

The exponential decay model yielded the following parameters using non-linear least squares regression:

$$a = 3.54, \quad b = -9.01, \quad \text{SE}_a = 0.062, \quad \text{SE}_b = 0.599, \quad p_a < .001, \quad p_b < .001$$

The fit of the exponential model yielded in a pseudo R^2 value of 0.405. This indicates that approximately 40.50% of the variance in negative ranks was explained by this exponential relationship, demonstrated a strong fit for the data (see Figure 4.6).

Model Comparison

AIC and BIC values were calculated to compare the models explaining the relationship between subjective probability and negative ranks (arsinh transformed). The results showed that the exponential model provided a substantially better fit, with an AIC of 5592.55 compared to 6469.18 for the logarithmic model, yielding a difference of 876.63. Similarly, the BIC for the exponential model was 5608.51, whereas it was 6479.82 for the logarithmic model, resulting in a difference of 871.31.

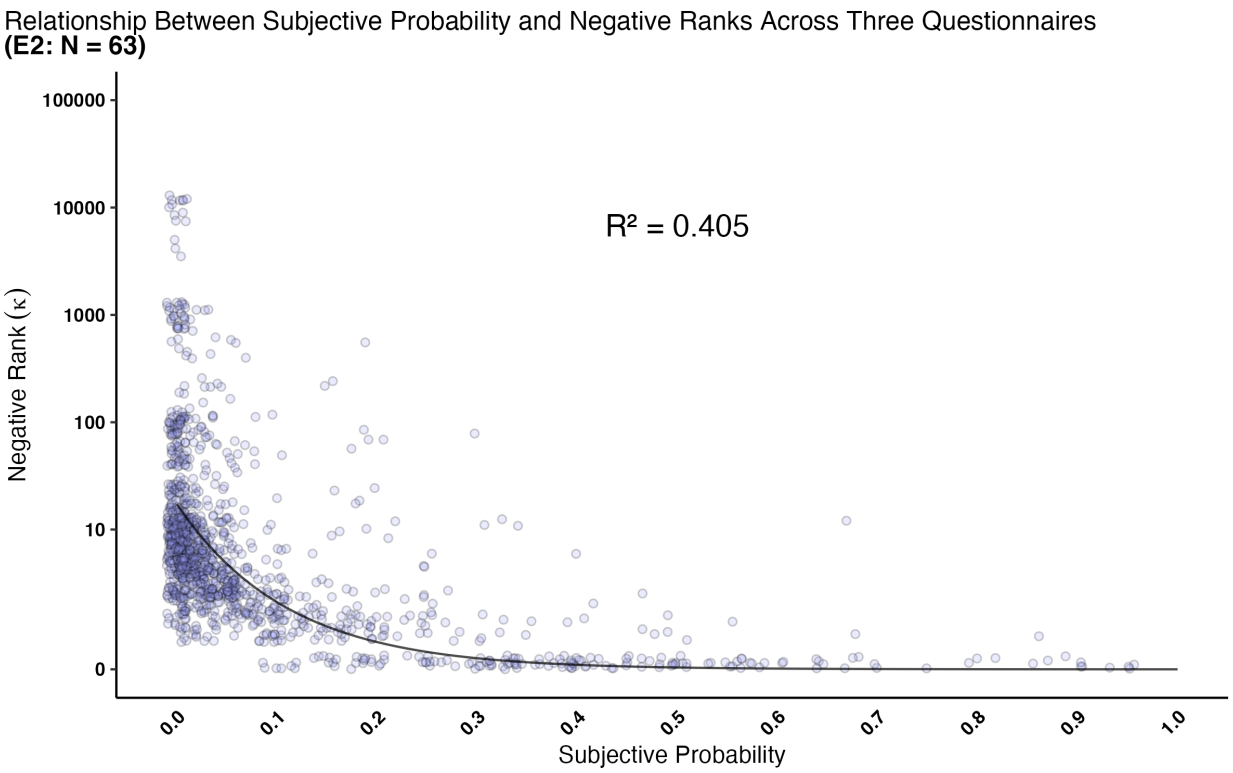


Figure 4.6: Scatter plot showing the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to effectively display negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with an exponential decay model, which accounts for approximately 40.50% of the variance observed in the negative ranks.

4.3.4 Subjective Probabilities (via Open Sampling Format) and Subjective Probabilities (via Percentage — End of Task Survey)

There was a strong correlation between subjective probabilities recorded via the Open Sampling Format (OSF) immediately after each scenario and those recorded as percentages at the end of the task (r = 0.728, $p < .001$). This finding, consistent with Experiment 1, indicates that the OSF effectively captures participants' subjective probability responses (see

Figure 4.7).

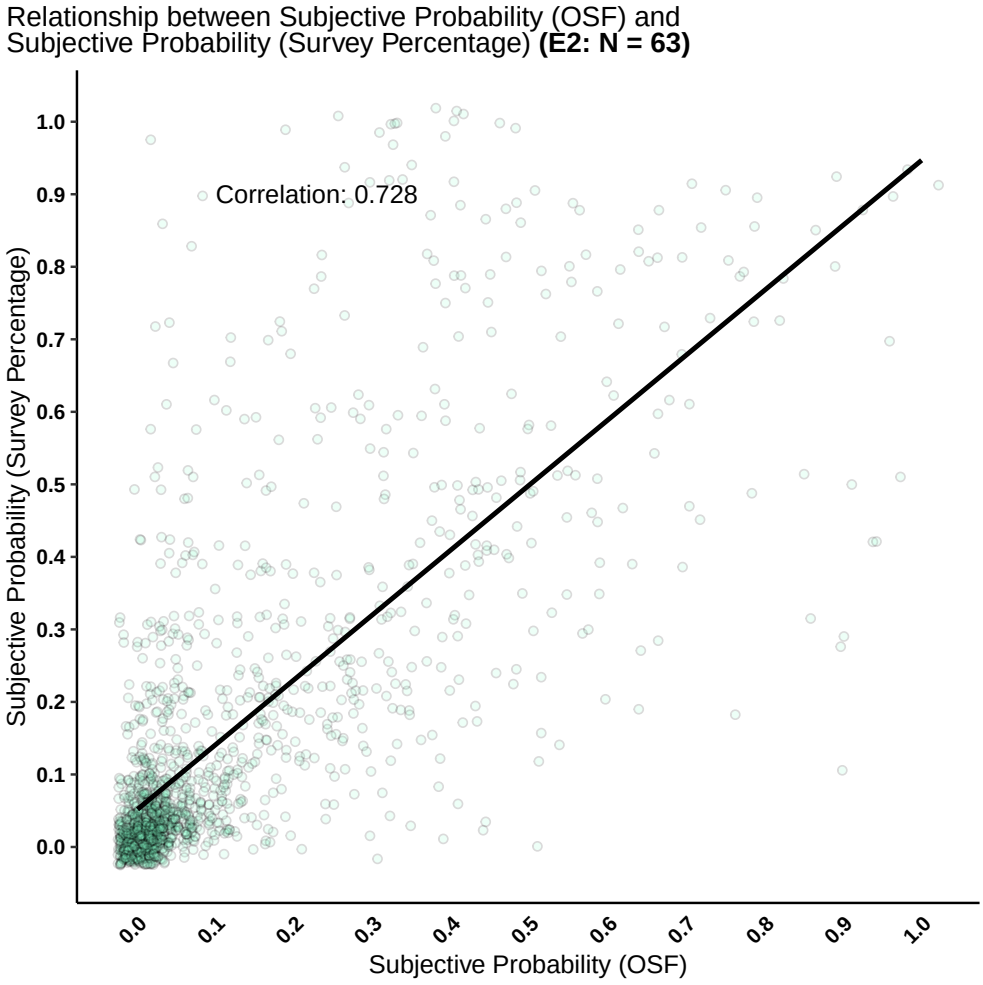


Figure 4.7: Scatterplot showing the relationship between subjective probabilities recorded via Open Sampling Format (OSF) and subjective probabilities recorded as percentages at the end of the task. The correlation coefficient is 0.728, indicating a strong relationship between the two measures.

4.3.5 Examining the Law of Disjunction

Visualization and Descriptives:

The analysis focused on the hypothesized relationship that disjunction values would align with the minimal ranks (see rationale on page 48). Preliminary results showed that the me-

dian disjunction values generally fell between the minimum and maximum ranks. This distribution suggests a variation from the hypothesized alignment with minimal ranks. Figure 4.8 visually represents these findings.

For language demographics, medians and interquartile ranges (IQR) revealed variability in responses: the median for the minimum rank was 5 (IQR = 8). The median for the disjunction rank was slightly higher at 6 (IQR = 9), , while the maximum rank had a median of 7 (IQR = 8.5).

For temperature ranges, data clustered more narrowly: the minimum rank median was 1 (IQR = 4), the disjunction rank median was 2 (IQR = 3.5), and the maximum rank median was 5 (IQR = 8).

For music preferences, the data exhibited the broadest spread, with a minimum rank median of 4 (IQR = 6.5), a disjunction rank median of 6 (IQR = 7.5), and a maximum rank median of 7 (IQR = 11).

Participant disjunction rankings showed variability: 38.62% aligned with the minimum, while 39.15% with the maximum, and 25.93% were in-between. These results illustrate the diverse judgments of participants when grading their disbelief for disjunctions.

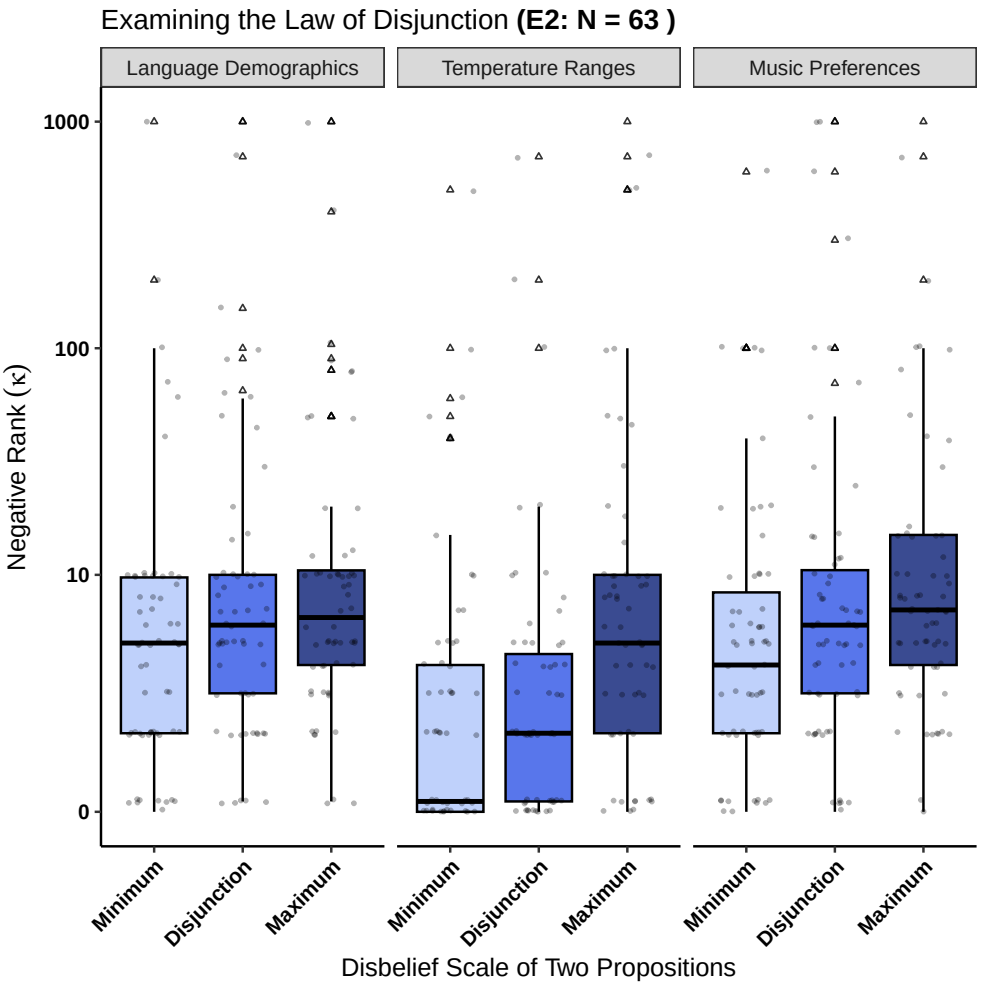


Figure 4.8: Boxplots examining the law of disjunction for three questionnaires: Language Demographics, Temperature Ranges, and Music Preferences. The y-axis is on a logarithmic scale to display a wider range of observations. The minimum rank represents the least disbelieved of the two propositions involved in the disjunction, while the maximum rank represents the more disbelieved of the two propositions. The disjunction rank corresponds to the logical "or" relationship between the two propositions, reflecting the combined disbelief that at least one of the propositions is true. The dots are jittered points representing individual participant responses.

Analysis of Linear Mixed-Effects Model:

A linear mixed-effects model was used to predict disjunction ranks (arsinh-transformed) based on differing minimum and maximum proposition ranks (both arsinh-transformed; see

rationale on page 31). To enhance predictive accuracy, observations involving ties, which accounted for 25.4%, of the data, were excluded. Preliminary checks revealed a high Pearson correlation coefficient of 0.73 between the two independent variables, indicating considerable multicollinearity and similarity in disbelief ratings between the minimum and the maximum. Despite these challenges, the analysis proceeded.

A null model, incorporating a random intercept for subjects, revealed an Intraclass Correlation Coefficient (ICC) of 0.36, indicating that 36.42% of the variance in disjunction scores was due to differences between subjects. This significant variation justified the inclusion of random intercepts in further models to account for subject variability.

Subsequent analysis with a linear mixed-effects model included fixed effects for minimum and maximum ranks, and their interactions with questionnaire type. Using temperature ranges as the reference category for questionnaire type, the estimated effect of the minimum rank (β = 0.49, SE = 0.1, t = 5.01, $p < .001$) represented the relationship between the minimum rank and disjunction rank within the reference category and when the maximum rank was 0. With the same reference category, the maximum rank under the same condition did not significantly predict the disjunction ranks (β = 0.1, SE = 0.08, t = 1.21, p = .227). However, significant interactions indicated that questionnaire type influenced both the minimum and maximum ranks. Specifically, interactions with music preferences and language demographics significantly decreased the estimate for minimum rank (β = -0.64, SE = 0.2, t = -3.17, p = .002); (β = -0.43, SE = 0.15, t = -2.96, p = .004) and increased the estimate for the maximum rank (β = 1.17, SE = 0.2, t = 5.96, $p < .001$); (β = 0.7, SE = 0.11, t = 6.39, $p < .001$). These interaction effects indicated that the relationships between

minimum or maximum rank and disjunction varied significantly across questionnaire types, as observed in Experiment 1. Refer to Table 4.4 for full model details.

Figures 4.9 and 4.10 demonstrated that disjunction was significantly predicted by the minimum rank for temperature ranges, with a moderate predictive effect. Specifically, a predicted score estimate of 0.49 suggested that as the minimum rank increased by one unit, the disjunction rank increased by approximately 0.49 units, based on the arsinh-transformed data. For other questionnaires, the maximum rank appeared to be a stronger predictor of disjunction; however, the predicted disjunction values were context-dependent and generally fell below the maximum rank. These findings emphasized that the main conclusions were based on interaction effects rather than the simple effects of the reference category, which were not generalizable across all contexts.

	Est.	S.E.	t val.	d.f.	p
(Intercept)	0.67	0.21	3.13	131.96	0.00
Minimum Rank	0.49	0.10	5.01	131.74	0.00
Maximum Rank	0.10	0.08	1.21	131.94	0.23
Music Preferences	-1.43	0.37	-3.83	111.71	0.00
Language Demographics	-0.47	0.31	-1.48	111.16	0.14
Minimum Rank:Music Preferences	-0.64	0.20	-3.17	131.97	0.00
Minimum Rank:Language Demographics	-0.43	0.15	-2.96	118.14	0.00
Maximum Rank:Music Preferences	1.17	0.20	5.96	131.83	0.00
Maximum Rank:Language Demographics	0.70	0.11	6.39	100.53	0.00

Table 4.4: Results of the linear mixed-effects model predicting disjunction ranks. This table displays the estimated coefficients, standard errors, t-values, and p-values for the arsinh-transformed minimum and maximum ranks, alongside questionnaire types and their interaction effects.

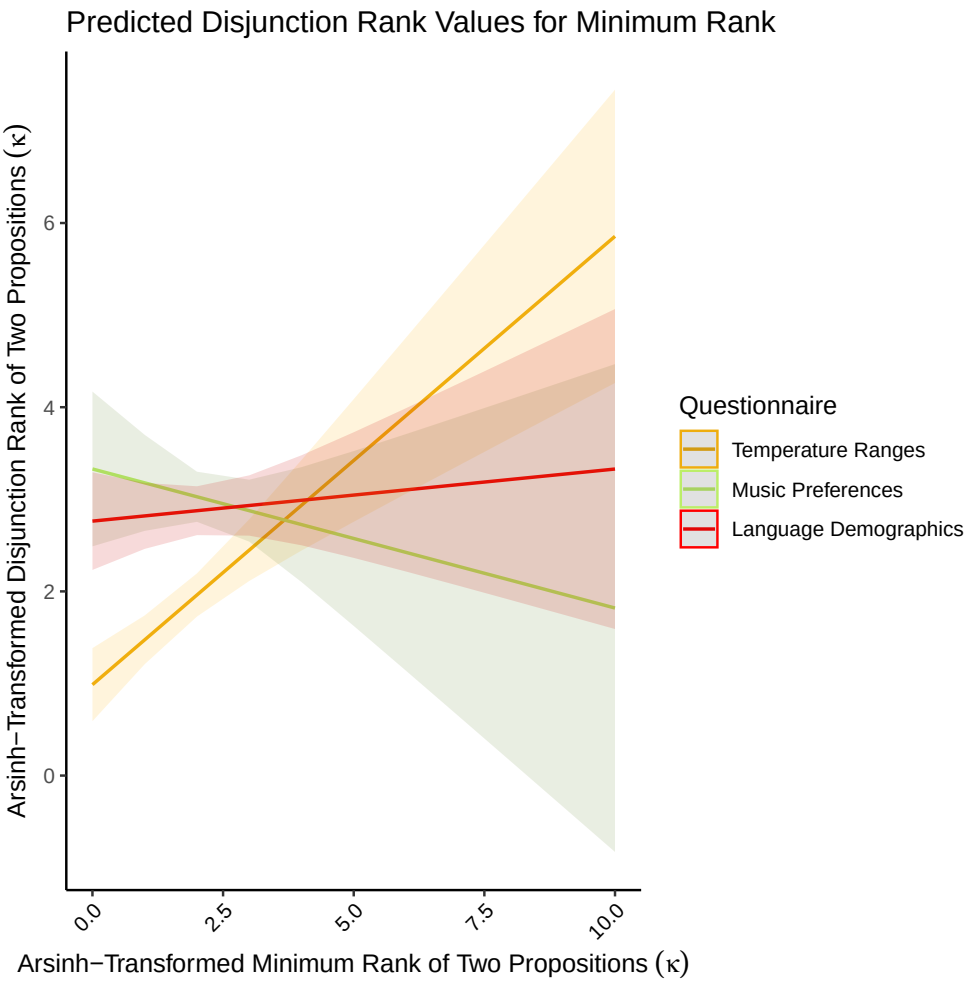


Figure 4.9: Predicted disjunction rank values (κ) for minimum rank across different questionnaire types. Temperature Ranges are shown in yellow, Music Preferences in green, and Language Demographics in red. Shaded areas denote confidence intervals. The plot demonstrates a significant positive relationship between minimum rank and disjunction rank for Temperature Ranges, with varying effects observed in other types.

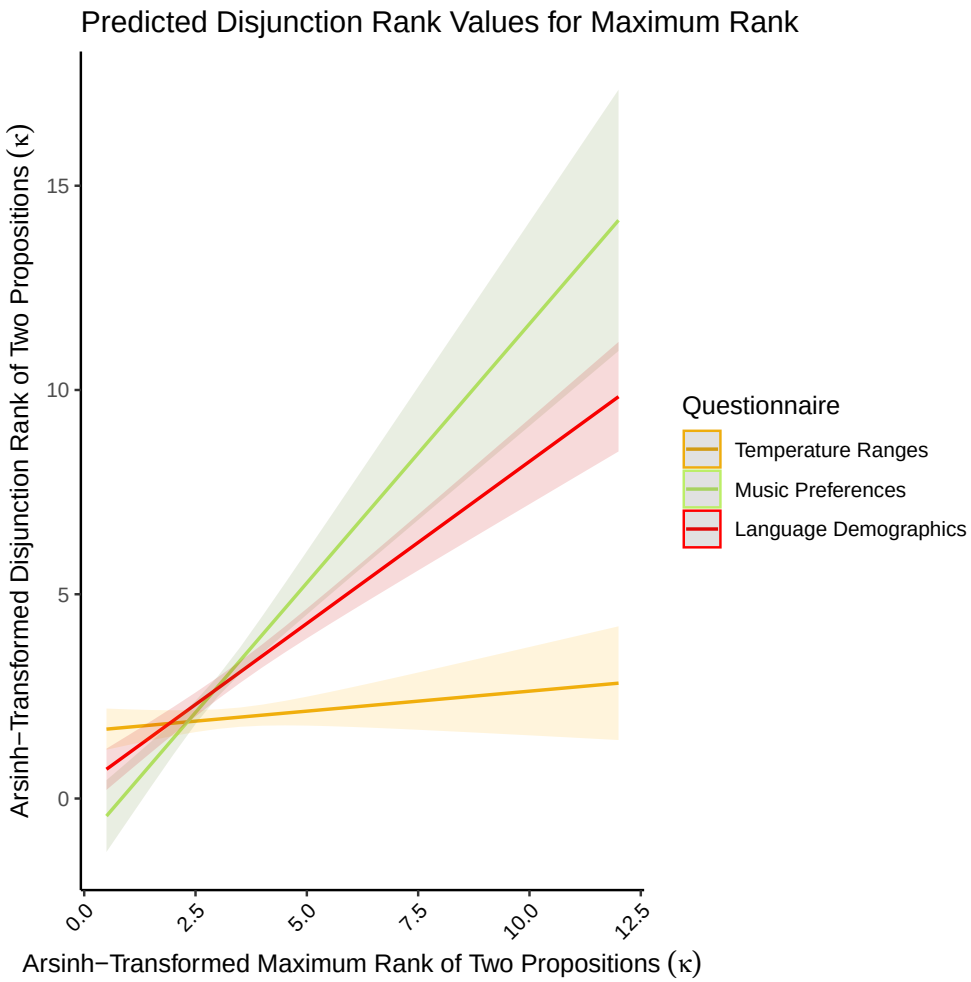


Figure 4.10: Predicted disjunction rank values for the maximum rank of two propositions, segmented by questionnaire type. Language Demographics are shown in red, Temperature Ranges in yellow, and Music Preferences in green. Shaded areas indicate confidence intervals. This graph highlights the variation in the effect of maximum rank on disjunction rank across different questionnaire types, especially evident in Music Preferences and Language Demographics, where disjunction rank is predominantly influenced by the maximum of the two ranks.

4.3.6 End-of-Task Survey on Task Difficulty

Task Difficulty Ratings: Figure 4.11 displayed participants' ratings on the difficulty of assessing disbelief levels. Ratings ranged from 1 "very easy," to 7 "very difficult," with the

majority clustering between 2 and 5 and a modal rating of 4 observed in Experiment 1 (see page 58). This pattern indicated a general moderate challenge faced by participants.

Perceived Ease in Assigning Degrees of Belief: Figure 4.12 illustrated participants' perceptions regarding the ease of assigning belief levels compared to disbelief levels. The neutral rating (4), predominated, aligning with findings from Experiment 1 (see page 59), suggesting no marked preference for either scale.

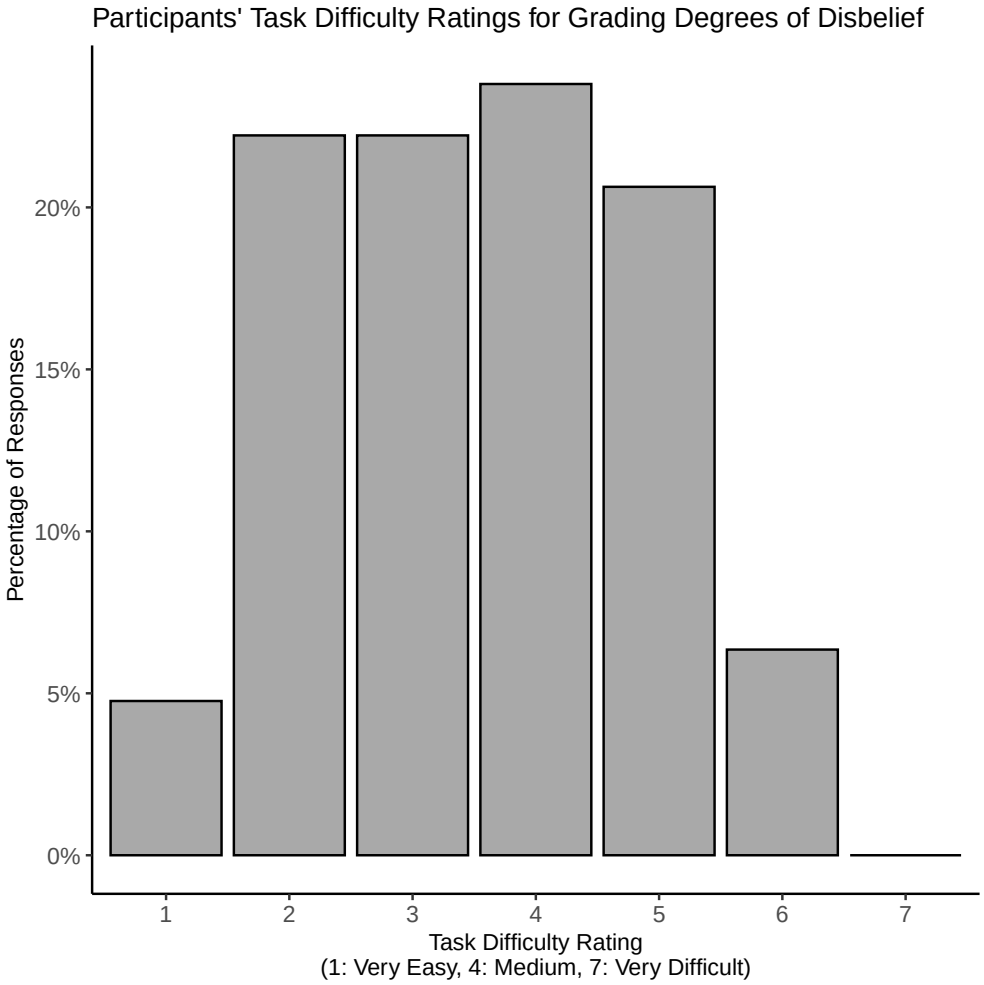


Figure 4.11: Bar graph showing the distribution of participants' task difficulty ratings for grading degrees of disbelief. The most common response was moderately difficult (Rating 4).

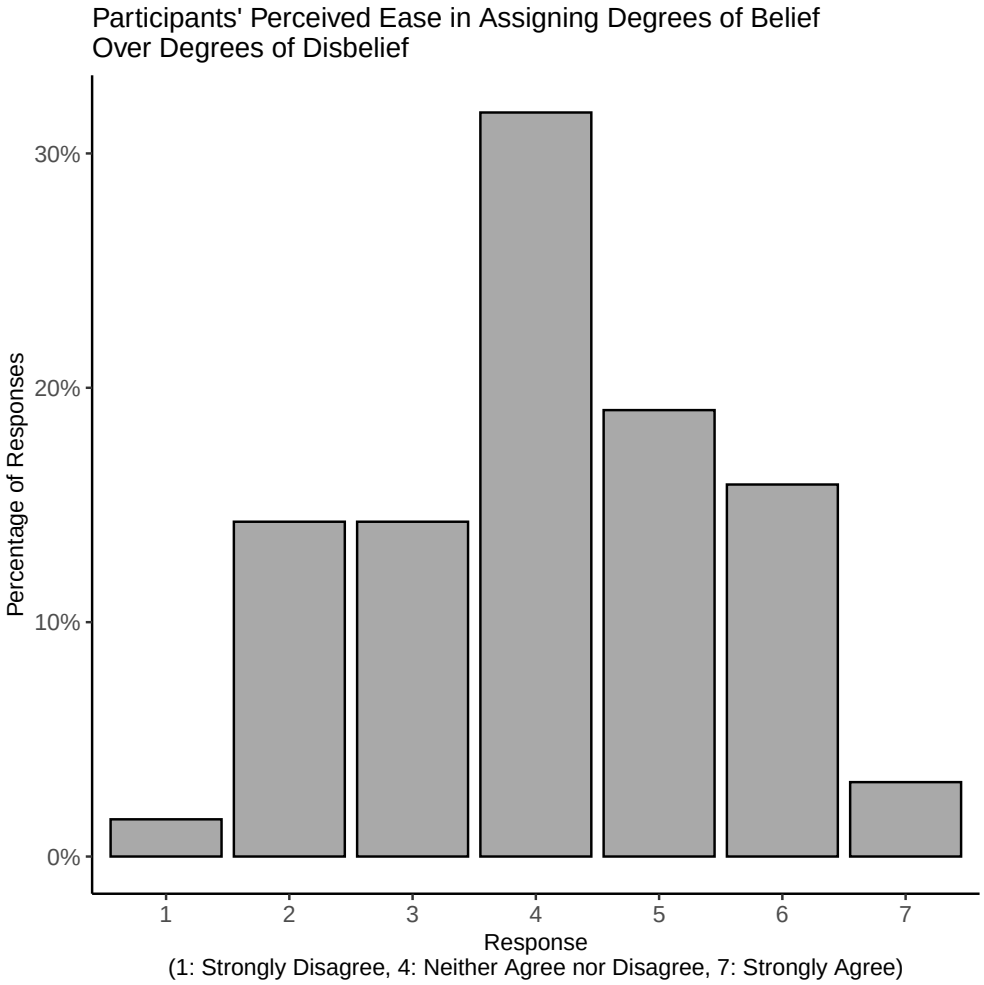


Figure 4.12: Bar graph depicting the distribution of responses to the perceived ease of assigning degrees of belief over degrees of disbelief. The most frequent rating was neutral (Rating 4).

4.4 Discussion

Experiment 2 served as a controlled in-lab replication of Experiment 1 to validate the robustness and reliability of negative ranking functions in expressing disbelief. The aim was also to examine the relationship between negative ranks and subjective probabilities. The replication study confirmed several key patterns observed in Experiment 1, thereby reinforcing

the applicability of negative ranking functions in capturing degrees of disbelief.

4.4.1 Validating the Utility of Negative Ranks Through Replication

One of the main challenges in employing ranking theory is the significant conceptual shift from traditional probabilistic models to a framework that includes a broad scale — from natural numbers to infinity — posing unique challenges in practical contexts (Skovgaard-Olsen, 2016). This shift demands a deeper understanding of formal logic and epistemology, which can complicate its application and acceptance. This replication was essential for evaluating the consistency of applying negative ranks across various settings without relying on traditional probabilistic models. Specifically, we sought to determine whether negative ranks, extending from zero to infinity, could be intuitively and consistently utilized by participants.

The results from various scenarios — language demographics, temperature variations, and music preferences — mirroring the median and interquartile range values from Experiment 1, reinforce the effectiveness of the negative ranking functions. Furthermore, the larger IQR values observed for extreme temperature ranges were comparable to Experiment 1, highlighting the varying magnitude of scales used and the varied perspectives on rare outcomes. This replication confirmed the method's capability to reliably measure varying levels of disbelief across diverse contexts and maintain consistent outcomes across different participant groups.

Future research should explore the application of negative ranks in belief updating, potentially incorporating negative rank measures over time to assess their consistency in responses

and how they evolve with new evidence. This study, while significant, was limited in this regard, as it measured participants' prior beliefs based solely on the information provided for specific scenarios.

This replication highlights the potential of negative ranks to provide valuable insights in studies where traditional probabilistic methods may fall short. The findings encourage further investigation into how ranking theory can be integrated into broader psychological and behavioural research, particularly in studies requiring subjective measures of disbelief or surprise.

4.4.2 Subjective Probabilities and Negative Ranks

This study further supported the lack of a systematic logarithmic relationship between subjective probabilities and negative ranks, reinforcing that negative ranks are fundamentally different from probabilities, particularly in contexts where beliefs may be formed independently of probabilistic reasoning. Both experiments revealed a nonlinear relationship; lower probabilities were consistently linked to higher disbelief levels, and higher probabilities to diminished surprise levels. The exponential decay model better captured this relationship, explaining a greater variance in negative ranks compared to the logarithmic model, highlighting that disbelief diminishes exponentially as subjective probability increases. Unlike in Experiment 1, this model accounted for a significant proportion of the variance, possibly due to the fact that this sample consistently associated very low probabilities with some levels of disbelief, whereas in Experiment 1, very low probabilities were not always surprising to

some. Furthermore, the exponential model, with an additional parameter, offers a more flexible explanation of the relationship compared to the logarithmic model. This detailed assessment through negative ranks was valuable in understanding disbelief systems under uncertainty.

4.4.3 Efficacy of Open Sampling Format in Subjective Probability Assessment

Experiment 2 replicated the strong correlation observed in the initial study between the Open Sampling Format (OSF) and the subjective probability measures recorded at the task's conclusion. This consistency supports the OSF's utility as an interactive method for assessing subjective probabilities. The findings confirmed the reliability of OSF in capturing participants' probabilistic assessments across different settings, indicating its potential for use in various experimental contexts.

4.4.4 Disjunction Judgements

Interpreting the results requires caution due to the potential impact of multicollinearity, which made it difficult to determine whether the less disbelieved or more disbelieved proposition predicted the outcome of a disjunction. The similarity in disbelief levels between the two propositions within the disjunction contributed to this challenge. In scenarios with a more clear separation in disbelief values, such as those involving temperature ranges, the least disbelieved proposition significantly predicted disjunction disbelief ratings. However,

in scenarios with minimal differences in disbelief levels, such as language demographics and music preferences, the more disbelieved proposition appeared to contribute to the disjunction disbelief ratings.

These findings were consistent with the initial experiment. The recurrence of these patterns suggests a need for further investigation into context-dependent effects and the challenges posed by multicollinearity. Addressing these issues is necessary for accurately testing the law of disjunction and understanding the underlying cognitive mechanisms.

The subsequent experiment focused on scenarios where the propositions in the disjunction showed greater differences in disbelief values. By reducing overlapping influences, the study aimed to re-examine how people judge disjunctions. This is important because we often encounter 'or' statements in everyday situations. This insight would clarify how individuals evaluate or interpret disjunctive statements.

4.4.5 Task Difficulty and Preference for Belief vs. Disbelief

Participants again found the task of grading disbelief moderately challenging. They expressed no clear preference for assigning degrees of belief over disbelief.

4.4.6 Conclusion and Future Direction

This in-lab replication of Experiment 1 provided strong evidence supporting the use of negative ranking functions in psychological research. The consistency in findings across both studies demonstrated that negative ranks effectively capture degrees of disbelief, offering

deeper insights into human belief and uncertainty representation than traditional probability measures. Participants consistently used negative ranking functions to express disbelief across various propositions and scenarios, with the observed non-linear relationship emphasizing the detailed distinctions that ranking functions provide compared to conventional subjective probability assessments.

The study indicated that the rank of a disjunction between propositions generally reflected the least disbelieved proposition, though in some contexts, the more disbelieved proposition significantly affected the outcome. Persistent issues with multicollinearity and data ties highlighted the need for ongoing methodological improvements.

While the focus was on negative ranks, the absence of a mechanism to capture positive ranks limited our understanding of the relationship between subjective probability and ranks. Therefore, we examined positive and two-sided ranks in subsequent experiments. Introducing positive ranks would complement our current understanding of disbelief, offering a more comprehensive view of the relationship with subjective probabilities. Subsequent studies also explored the application of ranking theory in different contexts to further validate its theoretical and empirical utility. Expanding this methodology could enhance our tools for detecting subtle shifts in both disbelief and belief, thereby improving the precision and depth of psychological research in judgment and decision-making. By including both positive and negative ranking functions, future research could provide a richer human belief framework, offering deeper insights into how beliefs shape our perceptions, judgments, and decisions in various contexts.

Chapter 5

Experiment 3: Examining the Full Spectrum of Belief with Positive and Negative Ranking Functions

5.1 Introduction

Experiment 3 expanded upon earlier findings by introducing positive ranks alongside negative ranks, enabling the computation of a two-sided ranking function. This approach allowed for a more detailed analysis of belief and disbelief. By including both types of ranks, Experiment 3 aimed to offer a more comprehensive understanding of the belief spectrum, enhancing insights into how individuals represent and reason about uncertainty in various scenarios.

5.2 Methods

5.2.1 Participants

A total of 200 undergraduate students from the University of Waterloo participated in this online experiment in exchange for course credit. No exclusion criteria were applied (median age: 19 years; 156 females; 44 males).

5.2.2 Task Environment

The study employed a custom-built online platform developed with JavaScript, similar to those used in Experiments 1 and 2 (de Leeuw et al., 2023). This platform featured an interface that allowed participants to input responses for both negative and positive ranks. Additionally, a 10x10 interactive grid enabled participants to indicate possible event cards by clicking on boxes. The study was completed in under 30 minutes.

5.2.3 Procedure

Experiment 3 extended the methodologies from Experiments 1 and 2 by integrating two new scenarios: 'real estate investment' and 'lottery ticket.' These new scenarios were added to the pre-existing 'temperature ranges' and 'coworker's music preferences' scenarios. Participants were required to provide positive ranks in addition to the existing negative ranks. To integrate these new scenarios without extending the study duration, the 'language demographics' questionnaire from Experiments 1 and 2 was omitted.

Grading of Disbelief (Negative Ranking Functions) and Grading of Belief (Positive Ranking Functions)

In this study, a positive ranking function was defined as $\beta(A)$, where positive ranks indicated the degree of belief and corresponded to the degree of disbelief in the negation of a proposition, $\kappa(\bar{A})$ (Equation 2.4). Adhering to Spohn's preferred method (Spohn, 2013), this study measured belief by focusing on the disbelief in a proposition's negation. This approach facilitated a comparative analysis with subjective probabilities, which are traditionally used to measure belief and uncertainty.

The experiment refined propositions to encompass all possible outcomes or 'worlds', thereby capturing the full spectrum of participants' beliefs. Participants assessed their disbelief toward various propositions and their negations, assigning non-negative integers ranging from zero (not surprising) to infinity (impossible) to indicate varying levels of disbelief or surprise. The questionnaires covered the following topics:

1. **September Weather in a Designated Visiting City:** Participants graded their disbelief towards daytime high temperatures for a hypothetical day trip to New York City, United States, in September (negative ranks: below -19°C, -19 to -10°C, -9 to 0°C, 1 to 10°C, 11 to 20°C, 21 to 30°C, above 30°C; positive ranks: not below -19°C, not -19 to -10°C, not -9 to 0°C, not 1 to 10°C, not 11 to 20°C, not 21 to 30°C, not above 30°C, and their disjunction, -19 to 10°C or 11 to 20°C).

2. **Coworkers' Music Preferences During a Hypothetical Karaoke Party:** Participants were presented with a scenario involving a coworker, Tom, and graded their

disbelief if they learned that Tom enjoyed singing along to various music genres (negative ranks: Pop, Rock, Hip Hop, Country, Opera, Folk; positive ranks: a type other than the mentioned genres, (e.g., a type other than Pop), and the disjunction, Rock or Opera).

3. **Real Estate Investment:** Participants graded their disbelief regarding the value of a Toronto property 5 years from today (negative ranks: increased, decreased, remained the same; positive ranks: did not increase, did not decrease, did not remain the same, and the disjunction, increased or remained the same).

4. **Lottery Ticket Scenario:** Participants graded their degrees of disbelief if they won the lottery after purchasing 900 out of 1000 tickets (negative ranks: won the lottery; positive ranks: lost the lottery, and the disjunction: won or lost the lottery).

These scenarios represented a range of everyday situations with varying levels of objective probability clarity. While the lottery ticket scenario was explicitly based on frequency probabilities (900 out of 1000 tickets), the other scenarios relied on broader forms of prior knowledge, including personal experiences and cultural expectations. This design aimed to broaden the contextual variety, building upon previous experiments. Detailed descriptions of the ranking questionnaires are available in Appendix 9.4.

Manipulation Check

To verify the consistency in the use of positive and negative ranks, participants responded to a simple question for two questionnaires:

1. **Real Estate Investment:** Do you believe the value of your Toronto property will increase 5 years from today? Yes or no.

2. **Lottery Ticket Scenario:** Do you believe you will win the lottery? Yes or no.

These yes/no responses were compared with the computed two-sided ranks from positive and negative ranks. Two-sided ranks consist of a negative ranking function and its positive counterpart (belief), allowing for the expression of disbelief and belief on a unified scale, (Equation 2.5).

The Law of Disjunction

Modifications were made to reduce collinearity between the less disbelieved and more disbelieved propositions in the disjunction and to decrease the number of ties in disbelief values. Specifically, two propositions in the disjunction were selected to have greater discrepancy in their disbelief values across the four questionnaires. For example, in Experiments 1 and 2, the disjunction for temperature ranges considered -9 to 0°C or 1 to 10°C, which had similar disbelief levels. In this experiment, the disjunction was -19 to -10°C or 11 to 20°C, with -19 to -10°C previously shown as surprising and 11 to 20°C as least surprising.

5.2.4 Ethical Considerations

The study received ethics clearance from the University of Waterloo Research Ethics Board (REB 44848) before data collection. Participants were informed of the study's purpose and

their right to withdraw at any time without penalty. De-identified data were securely stored to protect privacy, and all participants provided informed consent before participating.

5.3 Results

5.3.1 Analysis of Negative and Positive Ranks

Negative and positive ranks from four questionnaires were examined using median and interquartile range (IQR) to assess disbelief and belief. Visualization techniques, such as box plots on a logarithmic y-axis, were utilized to effectively display and interpret the wide range of rank values.

Temperature Ranges

Figure 5.1 and Table 5.1 illustrated participants' levels of disbelief and belief regarding various daytime high temperatures for a September day trip to New York City. Extreme temperatures elicited noticeably higher disbelief, while moderate temperatures were generally accepted, exhibiting a U-shaped distribution of disbelief that was consistent with previous experiments (see pages 37 and 74.)

The disbelief in the negation of the temperature ranges (interpreted as belief in temperature ranges) revealed an inverted U-shaped distribution, with the greatest disbelief median value at the 'not' 11 to 20°C range (Figure 5.1). This pattern aligned with the law of negation (Equation 2.2), demonstrating that participants logically did not believe and disbelieve in the occurrence of the same temperature range simultaneously. Approximately 60.64% of

responses adhered to this principle, indicating general adherence to rational expectations.

For detailed statistics, including the median and IQR for each temperature range and its negation, refer to Table 5.1.

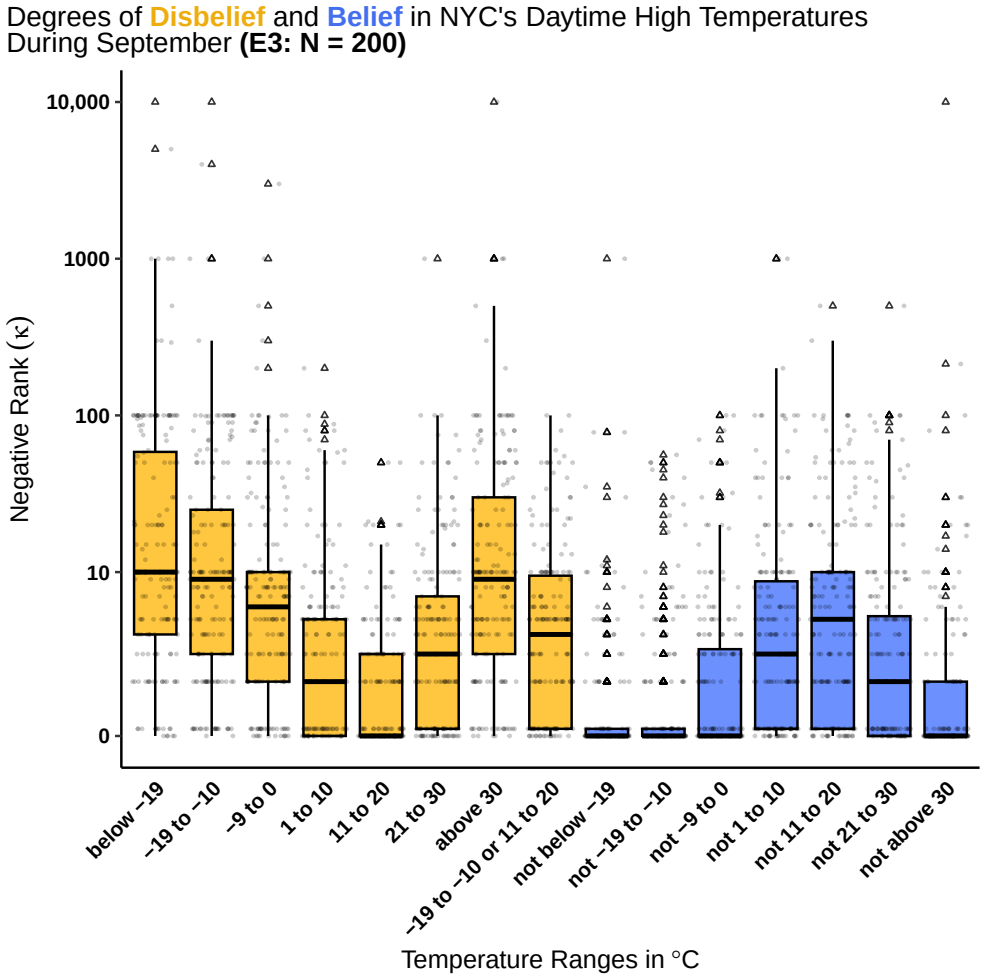


Figure 5.1: Box plots of negative ranks (degrees of disbelief) for different temperature ranges in NYC's daytime high temperatures during September. The y-axis is on a logarithmic scale to accommodate the wide range of rank values. Yellow represents disbelief in the temperature ranges, while blue represents disbelief in the negation of the temperature ranges (interpreted as belief in the temperature ranges).

	Propositions	Median Negative Rank	IQR Negative Rank
1	below -19	10.00	79.75
2	-19 to -10	9.00	29.50
3	-9 to 0	6.00	8.50
4	1 to 10	2.00	5.00
5	11 to 20	0.00	3.00
6	21 to 30	3.00	6.25
7	above 30	9.50	27.00
8	-19 to -10 or 11 to 20	4.00	9.00
9	not below -19	0.00	1.00
10	not -19 to -10	0.00	1.00
11	not -9 to 0	0.00	3.25
12	not 1 to 10	3.00	8.25
13	not 11 to 20	5.00	9.00
14	not 21 to 30	2.00	5.25
15	not above 30	0.00	2.00

Table 5.1: Descriptive statistics for Temperature Ranges, showing the median and interquartile range (IQR) of negative rank scores for each temperature range and its negation.

Music Preferences

Figure 5.2 and Table 5.2 displayed participants' levels of disbelief and belief regarding

a coworker's music genre preferences. Country music, marked by an absence of disbelief, suggests that general acceptance and potential belief in this genre is common for a coworker based on their description. This finding was consistent with the results from earlier experiments (see pages 40 and 77). Conversely, the significant degrees of disbelief associated with the negated proposition 'not country' emphasized a strong belief in the prevalence of country music for a coworker.

In contrast, opera yielded the highest levels of disbelief, with considerable variability in participants' responses. The disbelief in the negation of this genre, 'not opera,' was characterized by an absence of disbelief. This indicated that participants generally found the idea of their coworker preferring opera surprising.

This pattern was expected if participants followed the law of negation, which states that you cannot disbelieve both the proposition and its negation (Equation 2.2). Based on the percentage of responses that were in agreement with this finding for the music questionnaire, 69.42% of responses obeyed the law of negation. This result further demonstrated participants' general consistency in grading degrees of disbelief, ensuring that neither a proposition nor its negation was simultaneously disbelieved.

For complete details on the median and IQR values for each music genre and its negation, refer to Table 5.2.

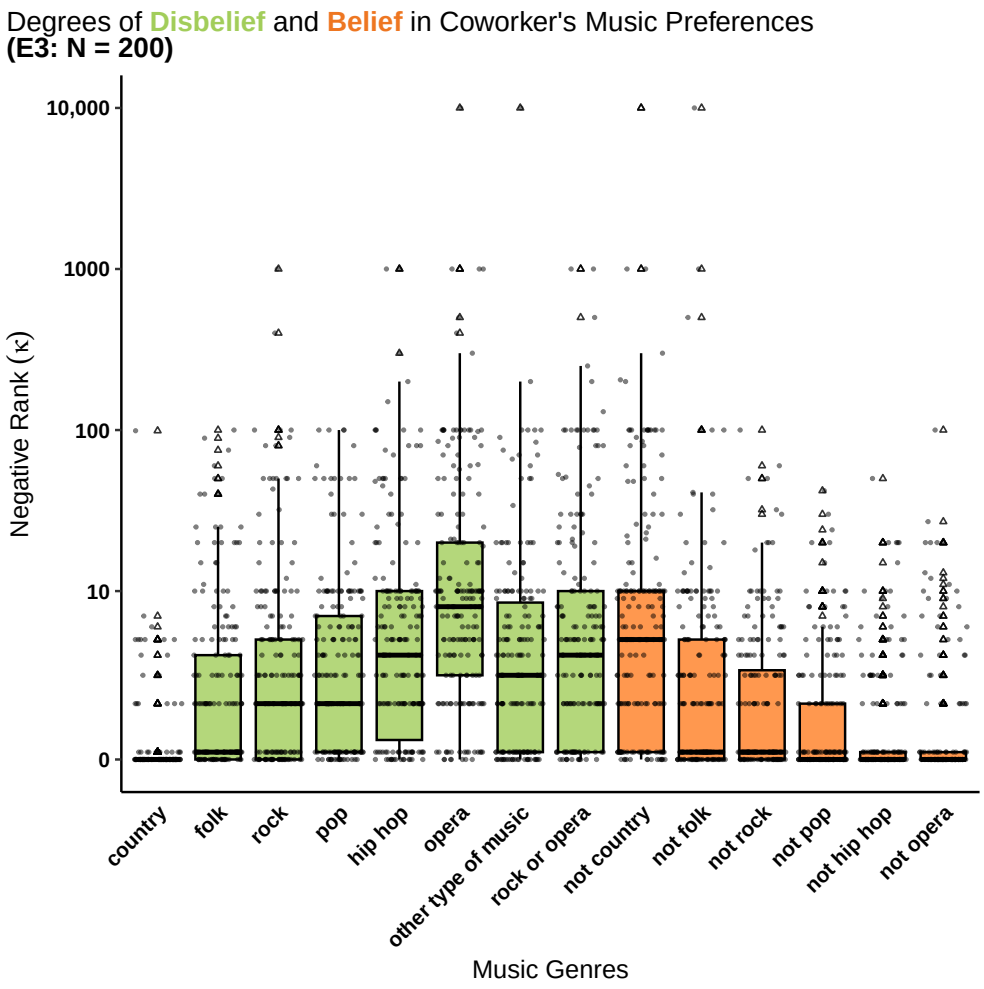


Figure 5.2: Box plots of negative ranks (degrees of disbelief) for various music genres and their negation among coworkers. The y-axis is on a logarithmic scale to display a wider range of observations. Green represents disbelief in the music genres, while orange represents disbelief in the negation of the music genres (interpreted as belief in the music genres), highlighting that genres like opera are more disbelieved while country music tends to be believed.

	Propositions	Median Negative Rank	IQR Negative Rank
1	country	0.00	0.00
2	folk	1.00	4.00
3	rock	2.00	5.00
4	pop	2.00	6.00
5	hip hop	4.00	8.25
6	opera	8.00	18.00
7	other type of music	3.00	8.00
8	rock or opera	4.00	9.25
9	not country	5.00	9.25
10	not folk	1.00	5.00
11	not rock	1.00	3.25
12	not pop	0.00	2.00
13	not hip hop	0.00	1.00
14	not opera	0.00	1.00

Table 5.2: Descriptive statistics for Music Preferences, showing the median and interquartile range (IQR) of negative rank scores for each genre and its negation.

Investment Scenarios

Figure 5.3 and Table 5.3 presented participants' levels of disbelief regarding various investment outcomes. An increase in real estate property value was met with no disbelief,

indicating that participants commonly expected property values to rise. Conversely, outcomes such as a decrease in property value and the status of 'remain the same' were met with considerable disbelief, suggesting these scenarios were less anticipated.

The disbelief in the negation of the investment outcomes, interpreted as belief in these outcomes, showed a consistent pattern. 'Not decrease' and 'not remain the same' were characterized by no disbelief, suggesting that participants would not be surprised if the property's price did not decrease nor remained unchanged. In contrast, 'not increase' was associated with higher disbelief, indicating a strong belief that the property's price would increase.

This pattern indicated that the majority of participants adhered to the law of negation, with approximately 69.5% of responses having obeyed the law of negation, indicating a relatively consistent application of logical reasoning.

Additionally, in the investment scenario, participants were asked whether they believed their real estate property value would increase, by answering "yes" or "no". Their responses were then compared to their given two-sided ranks, which were calculated based on their disbelief towards the proposition (property value increasing) and its negation (property value not increasing). This comparison revealed that 92.97% of participants' responses were consistent with their expressed belief or disbelief, thereby affirming the robustness of the measurement approach.

For complete details on the median and IQR values for each investment outcome and its negation, refer to Table 5.3.

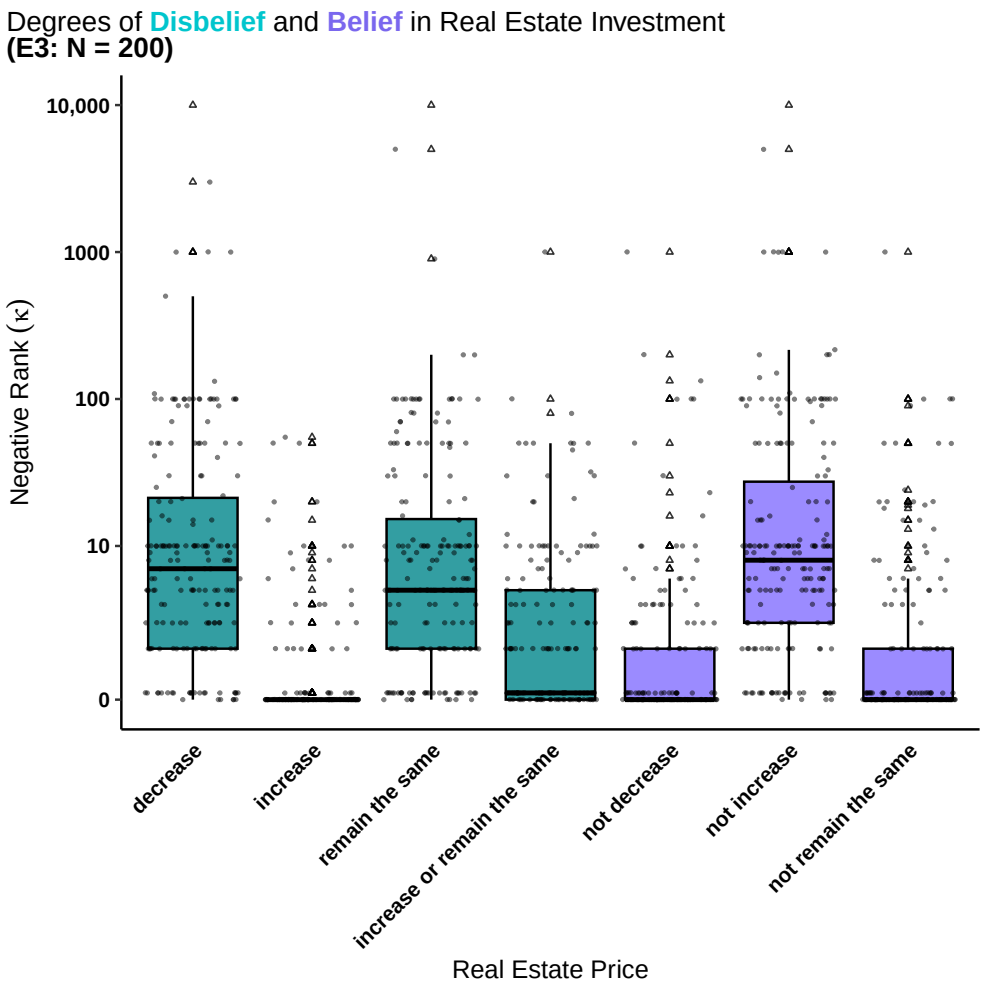


Figure 5.3: Boxplots showing the degrees of disbelief (negative rank) for various investment outcomes and their negations. The y-axis is on a logarithmic scale to display a wider range of observations. Turquoise represents disbelief in the investment outcomes, while purple represents disbelief in the negation of the investment outcomes (interpreted as belief in the investment outcomes).

	Propositions	Median Negative Rank	IQR Negative Rank
1	decrease	8.00	28.00
2	increase	0.00	0.00
3	remain the same	5.00	18.00
4	increase or remain the same	1.00	5.00
5	not decrease	0.00	2.00
6	not increase	8.00	27.00
7	not remain the same	0.00	2.00

Table 5.3: Descriptive statistics for Investment Outcomes, showing the median and interquartile range (IQR) of negative rank scores for each outcome and its negation.

Lottery Outcomes

Figure 5.4 and Table 5.4 illustrated participants' disbelief regarding lottery outcomes. In a scenario where participants had imagined purchasing 900 lottery tickets from a pool of 1000, the median values for disbelief across all possible outcomes — winning, winning or losing, and losing — were greater than 0. This indicated a persistent skepticism even when considering all possible lottery outcomes.

Only 41.5% of observations adhered to the law of negation, with many participants disbelieving both the possibility of winning and losing the lottery. This suggested that a significant portion of participants maintained skepticism towards both potential outcomes, thus violating the law of negation.

Furthermore, in the lottery scenario, participants were asked to state whether they believed they would win the lottery by answering "yes" or "no." Their responses were then compared to their two-sided ranks, which had been calculated based on their disbelief in winning and not winning the lottery. This comparison revealed that 86.24% of the participants' responses aligned with their expressed belief or disbelief, again affirming the reliability of the belief and disbelief measurements.

For complete details on the median and IQR values for each lottery outcome, refer to Table 5.4.

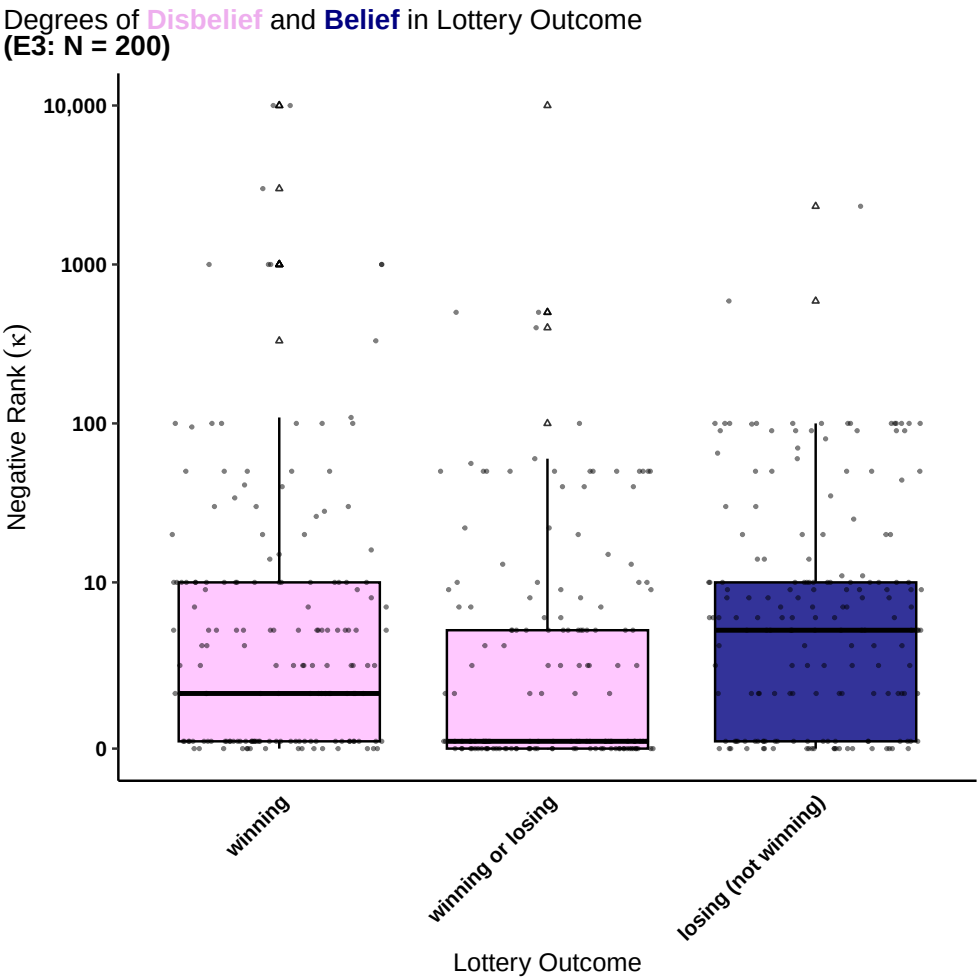


Figure 5.4: Boxplots showing the degrees of disbelief (negative rank) for various lottery outcomes. The y-axis is on a logarithmic scale to accommodate the wide range of rank values. Pink represents disbelief in the outcomes 'winning' and 'winning or losing,' while navy represents disbelief in the outcome 'losing (not winning),' interpreted as belief in 'winning.'

	Propositions	Median Negative Rank	IQR Negative Rank
1	winning	2.00	9.00
2	winning or losing	1.00	5.00
3	losing (not winning)	5.00	10.00

Table 5.4: Descriptive statistics for lottery outcomes, showing the median and interquartile range (IQR) of negative rank scores for each proposition and its negation.

5.3.2 Relationship Between Subjective Probabilities and Negative Ranks

Figure 5.5 illustrated the relationship between subjective probabilities and negative ranks across four distinct questionnaires. The scatter plots revealed a generally nonlinear relationship where lower subjective probabilities were frequently associated with higher negative ranks, indicating greater disbelief. Notably, in the lottery context, even at higher subjective probabilities, disbelief persisted. For instance, a 90% chance of winning still elicited skepticism. This underlines the influence of context on the interpretation of subjective probabilities, which was further demonstrated by the variation in the threshold of disbelief. Points above the dotted blue line in Figure 5.5 marked this threshold (greater than 0), showing that subjective probabilities alone might not accurately reflect disbelief, and emphasizing the importance of considering contextual factors in disbelief assessments.

Furthermore, a wide range of subjective probabilities for a negative rank of 0 implied that

unsurprising events were not limited to high probabilities but rather covered a wide spectrum from 0 to 1. This distribution showed that although high subjective probabilities were generally associated with non-disbelief, lower extreme probabilities could also be associated with non-disbelief, highlighting the complex relationship between disbelief and probability.

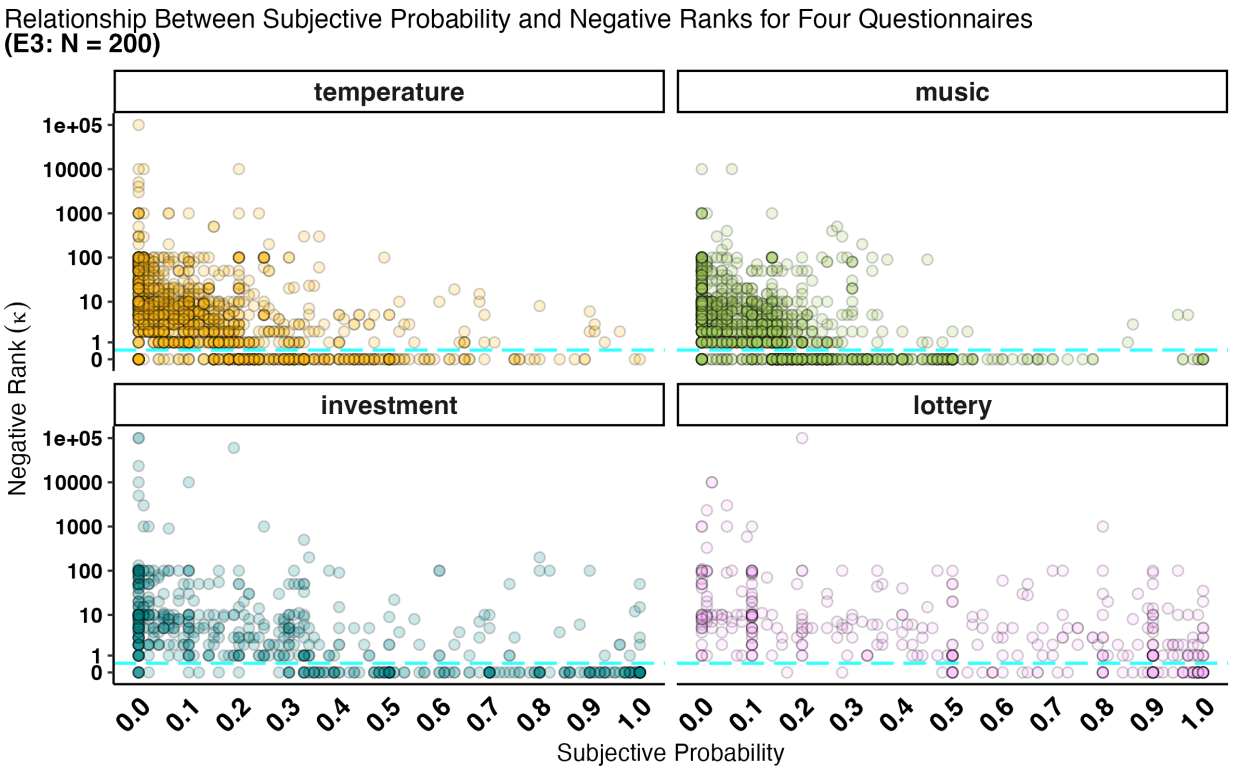


Figure 5.5: Scatter plots showing the relationship between subjective probabilities (via Open Sampling Format) and negative ranks (κ) across four questionnaires: Temperature Ranges, Music Preferences, Real-estate Investment, and Lottery. The y-axis is scaled to display negative ranks effectively, while the labels show the original rank values for easier interpretation. The dotted blue line signifies the threshold of significant disbelief. These plots highlight the non-linear and context-dependent nature of the relationship between subjective probabilities and negative ranks, with lower probabilities generally associated with higher disbelief. However, lower probabilities do not uniformly elicit disbelief, highlighting the importance of negative ranks in capturing the complexity of subjective disbelief.

5.3.3 Mathematical Model Fit

As in Experiments 1 and 2, two models were employed to investigate the relationship between subjective probabilities and negative ranks: a logarithmic model and an exponential decay model. To reduce skewness and handle extreme values, the inverse hyperbolic sine function $\text{asinh}(\kappa)$ was applied to the negative ranks. For detailed justification of these models, see the explanation on page 43.

Model 1: Logarithmic Model

For the logarithmic model, the base b was estimated using non-linear least squares regression:

$$b = 0.0009, \quad \text{SE} = 0.0003, \quad t(1) = 3.38, p < .001$$

However, the logarithmic model provided a poor fit, with a pseudo R^2 of -0.02, indicating it did not adequately explain the variance in negative ranks (Figure 5.6). This result was consistent with the findings from Experiments 1 and 2.

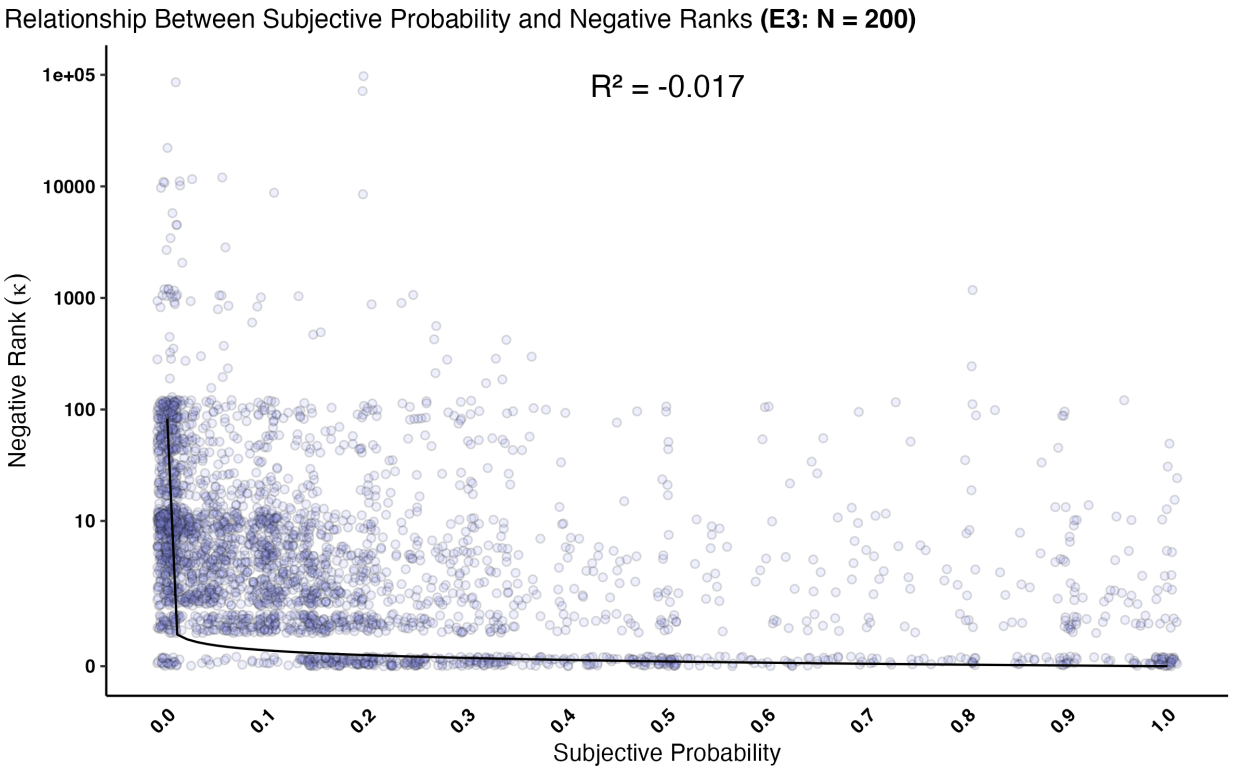


Figure 5.6: Scatter plot illustrating the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to display negative ranks effectively, with labels indicating the original rank values for easier interpretation. A logarithmic model is fitted to the data, but it was a poor fit as indicated by the R^2 value.

Model 2: Exponential Decay

For the exponential decay model, parameters a and b were also estimated using non-linear least squares regression:

$$a = 4.24, \quad b = -5.56, \quad \text{SE}_a = 0.169, \quad \text{SE}_b = 0.593, \quad p_a < .001, \quad p_b < .001$$

The pseudo R^2 value was 0.050, indicating that approximately 4.98% of the variance in negative ranks was explained by this model.

Model Comparison

To determine which model better explained the relationship between subjective probability and negative ranks (arsinh transformed), the AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) values were computed. The comparison indicated that the exponential model provided a better fit, with an AIC of 23970.52, compared to 24226.72 for the logarithmic model, yielding a difference of 256.2. Similarly, the BIC for the exponential model was 23989.25, whereas it was 24239.2 for the logarithmic model, resulting in a difference of 249.96.

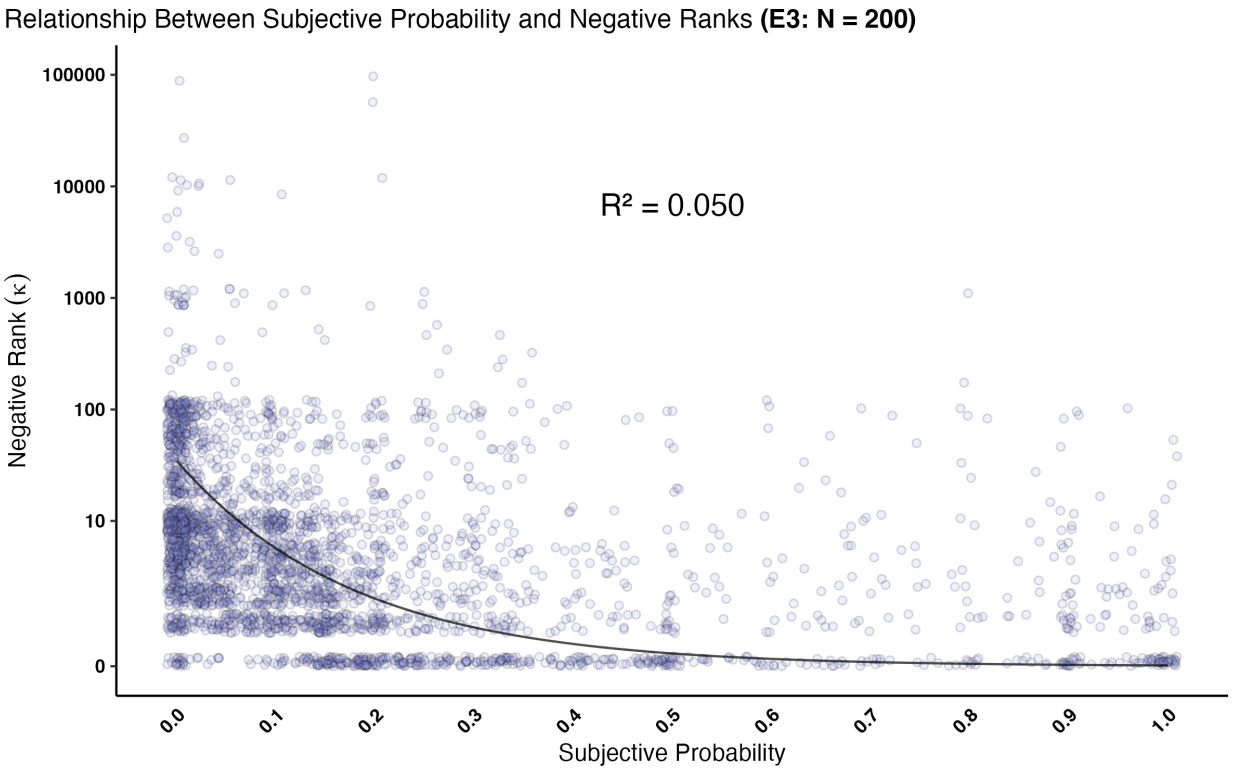


Figure 5.7: Scatter plot showing the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to effectively display negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with an exponential decay model, which accounts for 4.98% of the variance observed in the negative ranks.

5.3.4 Relationship Between Subjective Probabilities and Two-Sided Ranks

This section examined the relationship between subjective probabilities and two-sided ranks. Two-sided ranks were computed using both negative and positive ranking functions, allowing for the expression of disbelief and belief on a unified scale. Negative values indicated disbelief, positive values indicated belief, and a value of 0 indicated neutrality. Negative infinity represented impossibility, while infinity represented absolute certainty. This scale allowed for capturing the full spectrum of belief, distinguishing between neutral judgments and varying degrees of belief and disbelief.

Figure 5.8 illustrated the relationship between subjective probabilities (x-axis) and two-sided ranks (arsinh transformed) on the y-axis across four questionnaires. Generally, low probabilities aligned with disbelief, and high probabilities aligned with belief. However, temperature ranges and music preferences showed some noise, with very low probabilities (e.g., $p = 0$) occasionally linked to belief. In contrast, investment and lottery questionnaires showed a more consistent relationship between lower probabilities and disbelief, and higher probabilities and belief.

In comparison to a negative rank, a two-sided rank allowed for the differentiation between neutral judgments (neither believed nor disbelieved). Whereas a previous zero value in a negative ranking function implied "not disbelieved" (and potentially "believed"), the two-sided rank clearly distinguished between states of neither disbelieved nor believed and believed.

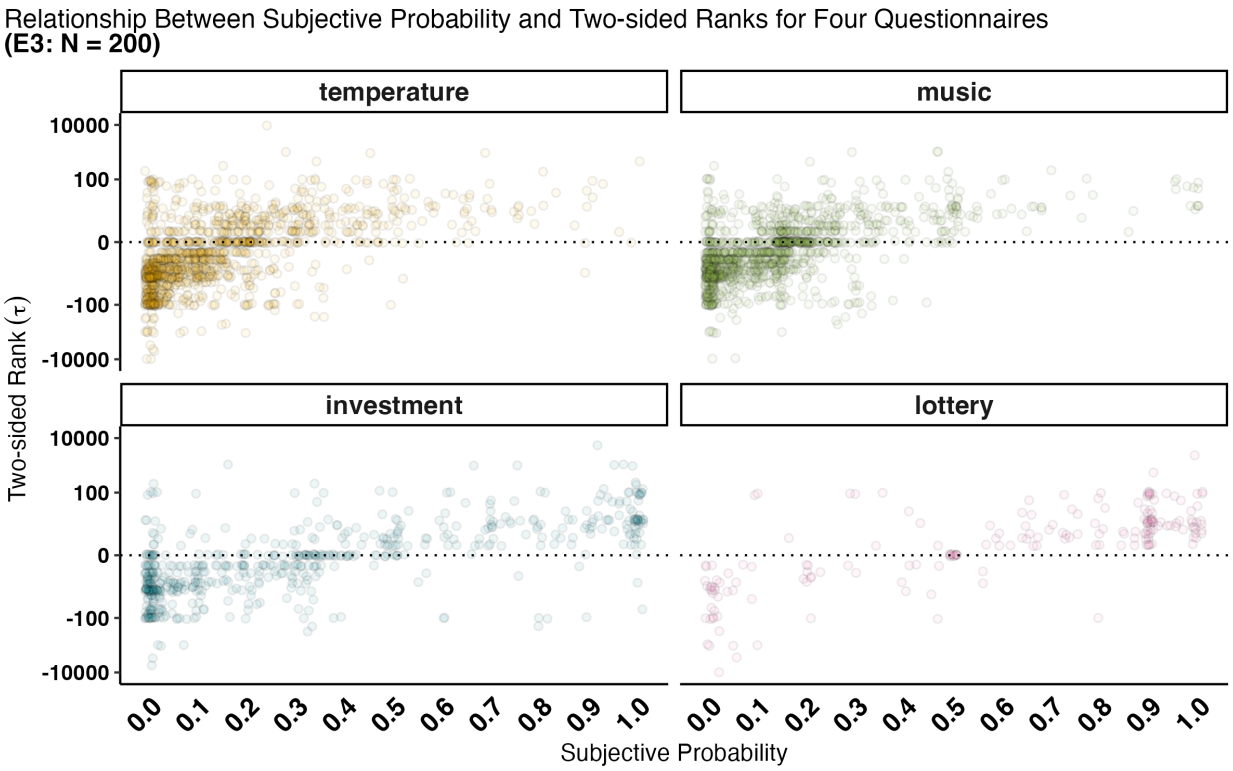


Figure 5.8: Scatterplot showing the relationship between subjective probabilities and two-sided ranks τ for each distribution configuration. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The dotted line represents a two-sided rank of zero, indicating neutral judgment.

5.3.5 Mathematical Model Fit

A log-odds model and a polynomial regression model were used to explore the relationship between subjective probabilities and two-sided ranks. The inverse hyperbolic sine function $\text{asinh}(\tau)$ was applied to reduce skewness and handle extreme values in the two-sided ranks. This transformation handled both positive and negative values, compressing large values and spreading out smaller ones, resulting in a more symmetrical distribution for analysis.

Model 1: Log-odds Model

The exploration of the relationship between subjective probabilities and two-sided ranks was based on a mathematical model that explains the theorized relationship, as defined in Equation 2.7. This model hypothesized that probabilities could be represented on a two-sided rank scale using the equation:

$$\tau(A) = \log_b \left(\frac{1 - P(A)}{P(A)} \right), \quad \text{where} \quad b \in (0, 1)$$

To analyze this quantitatively, non-linear least squares regression was used to fit the model. This approach transformed the log-odds of subjective probabilities into two-sided ranks using an optimal logarithmic base. It tested whether the mathematical representation aligned with empirical data, examining if the relationship between subjective probabilities and two-sided ranks could be effectively explained by this expression.

The non-linear least squares regression analysis determined the logarithmic base b as:

$$b = 0.0002, \quad \text{SE} = 0.0001, \quad t(1) = 2.24, p = .025$$

This parameter was statistically significant, with a pseudo R^2 of 0.06, suggesting that approximately 6.40% of the variance in the participants' two-sided rank responses could be explained by this model. While the pseudo R^2 value was modest, it indicated that the log-odds model captured some of the variance in subjective probabilities, providing a basis for further investigation into this relationship.

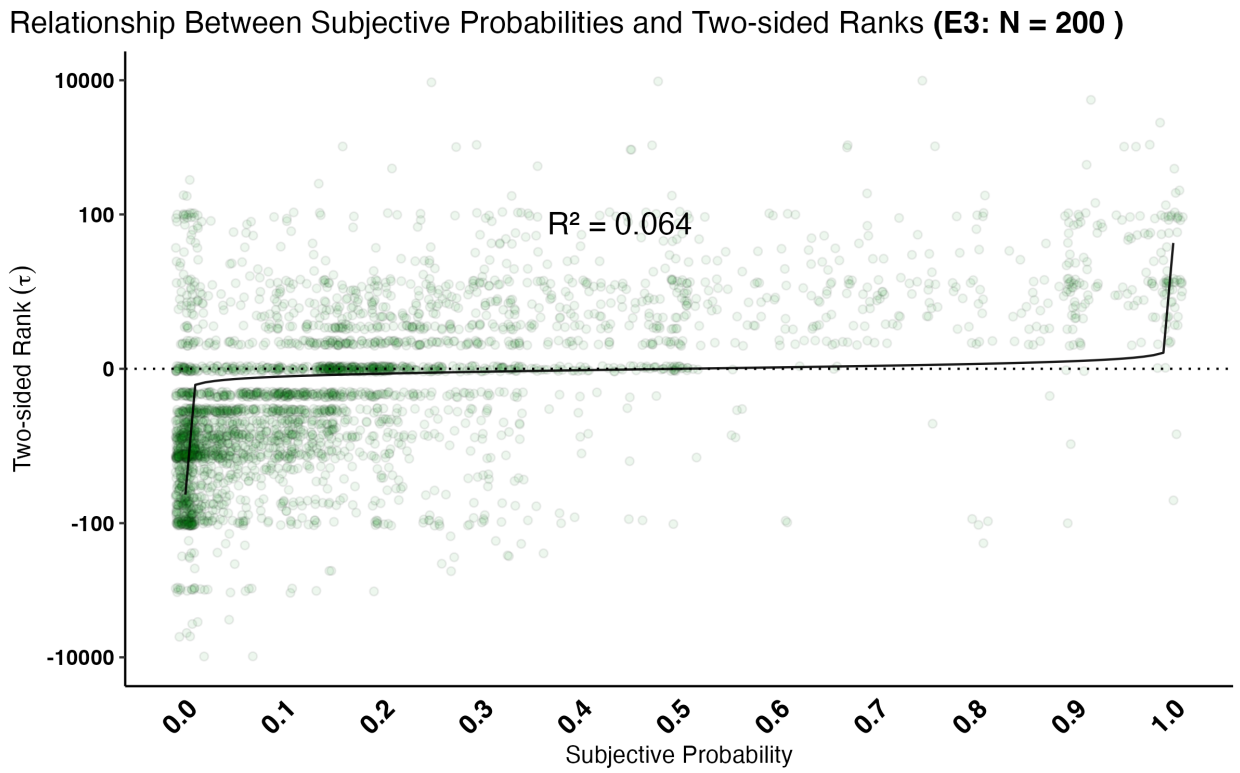


Figure 5.9: Scatter plot showing the relationship between subjective probabilities and two-sided ranks τ across all questionnaires. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with a log-odds model, which captures 6.40% of the variance in the ranks.

Model 2: Polynomial Regression Model

The second model explored the relationship using polynomial regression to flexibly capture potential non-linearities. The model was specified as:

$$\tau(A) = \beta_0 + \beta_1 P(A) + \beta_2 P(A)^2 + \beta_3 P(A)^3$$

where $\beta_0, \beta_1, \beta_2$, and β_3 are parameters that were to be estimated.

The model fitting process also used non-linear least squares regression, and the parameters were found to be statistically significant:

$$\beta_0 = -3.64, \quad \beta_1 = 23.09, \quad \beta_2 = -30.2, \quad \beta_3 = 14.43,$$

$$SE_{\beta_0} = 0.18, \quad SE_{\beta_1} = 2.58, \quad SE_{\beta_2} = 7.93, \quad SE_{\beta_3} = 5.96,$$

$$p_{\beta_0} < .001, \quad p_{\beta_1} < .001, \quad p_{\beta_2} < .001, \quad p_{\beta_3} = .015$$

The fit of the polynomial model was also assessed, yielding a pseudo R^2 value of 0.11, indicating that approximately 10.99% of the variance in two-sided ranks was explained by this polynomial relationship (see Figure 5.10).

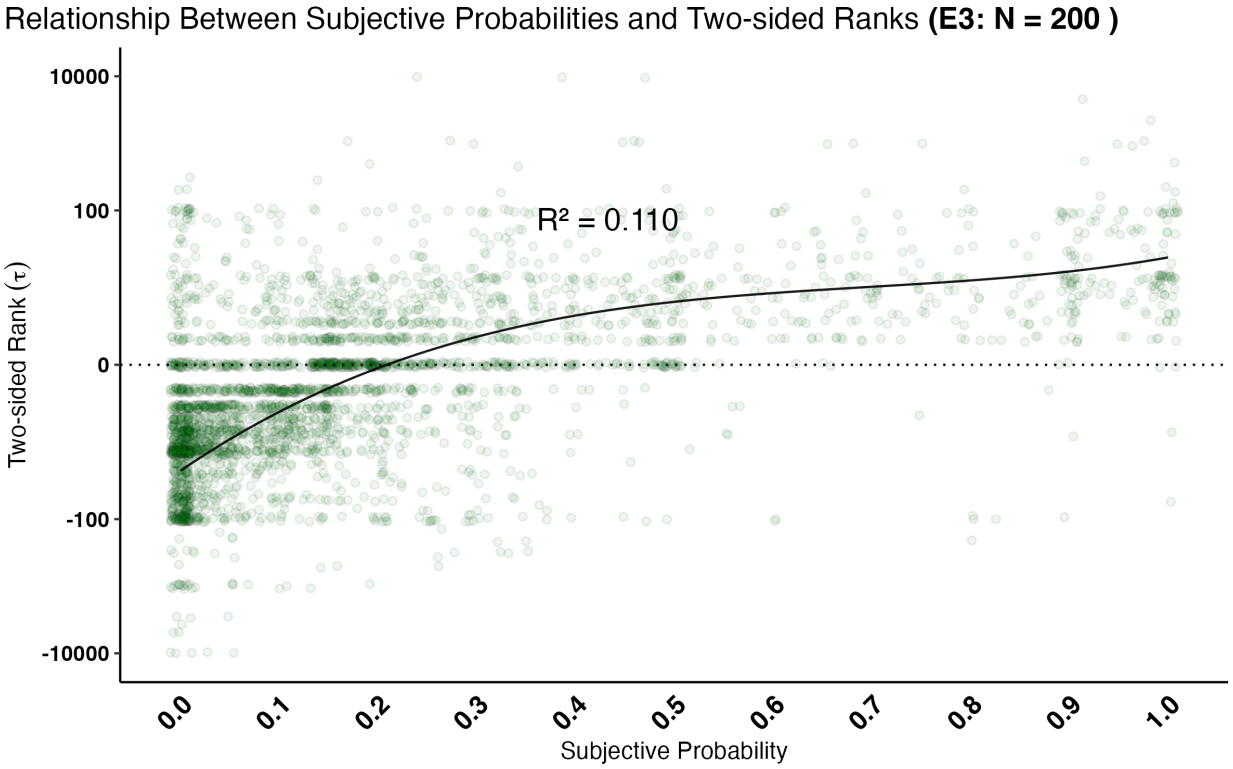


Figure 5.10: Scatter plot showing the relationship between subjective probabilities and two-sided ranks τ across all questionnaires. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with a polynomial regression model, which captures 10.99% of the variance in the ranks. The fitted curve demonstrates how subjective probabilities predict two-sided ranks, capturing potential non-linearities in the data.

Model Comparison

To compare the two models and determine which one best explained the relationship

between subjective probability and two-sided ranks (arsinh transformed), both the AIC and BIC were calculated. The comparison of the two models using AIC and BIC suggested that the polynomial model had a better fit. The AIC for the polynomial model was 23508.09, compared to 23683.05 for the log-odds model, resulting in a difference of 174.96. Similarly, the BIC for the polynomial model was 23539.04 compared to 23695.43 for the log-odds model, with a difference of 156.39. While these differences indicated that the polynomial model was preferable, the log-odds model was still reasonably close in describing the relationship. Thus, the log-odds model could still be considered given the theoretical alignment and its simplicity, as it estimated only one parameter.

5.3.6 Examining the Law of Disjunction

Visualization and Descriptives:

The analysis of disjunctions examined the hypothesized minimum relation (see rationale, page 48). Figure 5.11 illustrated these relationships. Typically, the median disjunction values lay between the median values of the minimum and maximum ranks of the individual propositions, except in certain cases, such as with lottery outcomes.

For temperature ranges, the median negative ranks were as follows: minimum rank: 0 (IQR = 2), disjunction rank: 4 (IQR = 9), maximum rank: 9 (IQR = 28.5).

For music preferences, the median negative ranks were: minimum rank: 2 (IQR = 5), disjunction rank: 4 (IQR = 9.25), maximum rank: 8.5 (IQR = 19).

For investment scenarios, the median negative ranks were: minimum rank: 0 (IQR = 0),

disjunction rank: 1 (IQR = 5), maximum rank: 6 (IQR = 18.25).

For lottery outcomes, the median negative ranks were: minimum rank: 1 (IQR = 3), disjunction rank: 1 (IQR = 5), maximum rank: 9 (IQR = 57.25).

The percentages of participants whose disjunctions equalled the minimum or maximum, or fell between these extremes, represented a notable change compared to previous experiments. Specifically, 31.25% of participants ranked the disjunction equal to the minimum, 15.75% equal to the maximum, and 37% ranked the disjunction between the minimum and maximum.

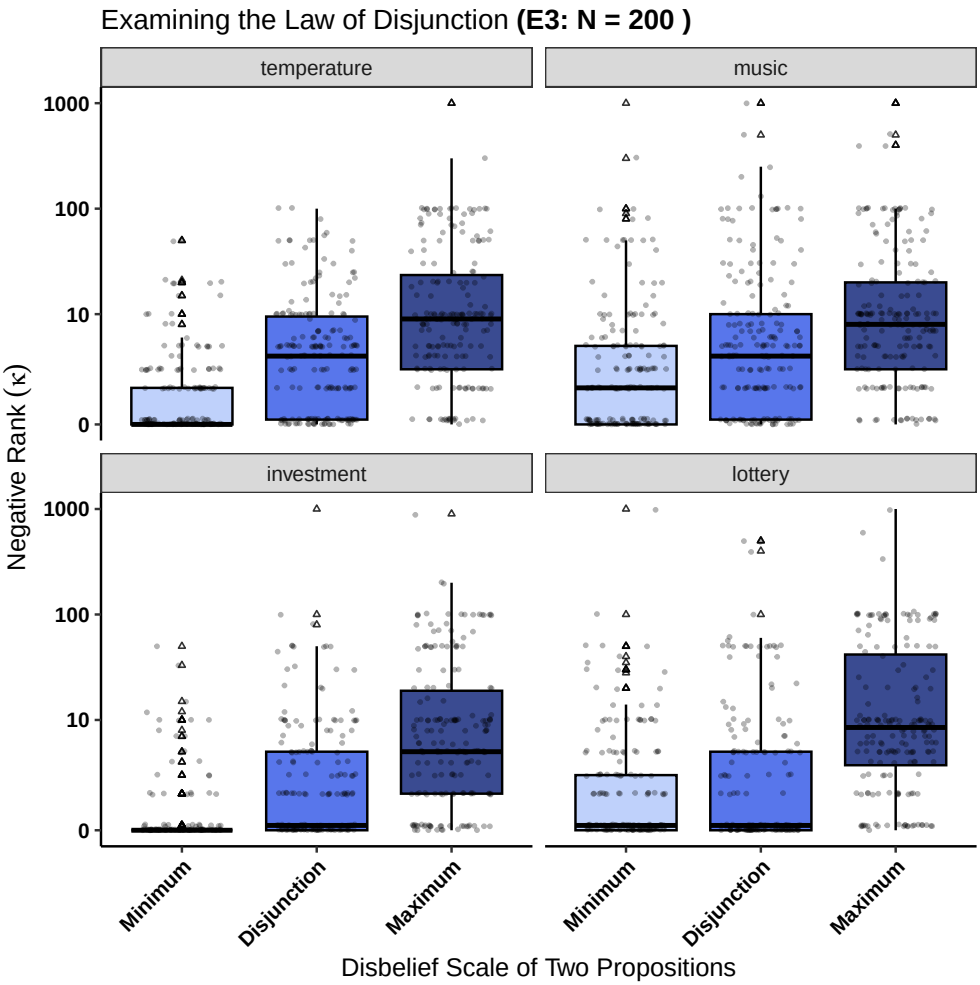


Figure 5.11: Boxplots examining the law of disjunction for four questionnaires: Temperature Ranges, Music Preferences, Investment Scenarios, and Lottery Outcomes. The y-axis is on a logarithmic scale to display a wider range of observations. The minimum rank represents the least disbelieved of the two propositions involved in the disjunction, while the maximum rank represents the more disbelieved of the two propositions. The disjunction rank corresponds to the logical "or" relationship involving the two propositions. The dots are jittered points representing individual participant responses.

Analysis of Linear Mixed-Effects Model:

A linear mixed-effects model was used to predict disjunction ranks in instances where the minimum rank differed from the maximum rank among the propositions. Cases with ties between the minimum and the maximum rank were excluded from the analysis, removing

9.25% of observations, a number markedly lower than that observed in the previous two experiments. Negative ranks were transformed using the inverse hyperbolic sine (arsinh) method to accommodate a wide range of values (see rationale on page 31).

Preliminary assessments for multicollinearity between the transformed minimum and maximum ranks revealed a Pearson correlation coefficient of only 0.07. This low degree of multicollinearity confirmed that the two propositions in the disjunction had distinct disbelief ratings.

A null model with a random intercept for subjects was conducted to predict disjunction (arsinh-transformed). This model yielded an Intraclass Correlation Coefficient (ICC) of 0.35, indicating that 34.88% of the total variance in disjunction scores was attributable to differences between subjects.

The linear mixed-effects model incorporated fixed effects for the minimum and maximum ranks of two propositions and examined interactions between questionnaire type and both ranks. A random intercept for participants was included to account for individual variability. Using real estate investment as the reference category, the minimum rank significantly predicted disjunction rank ($\beta = 0.44$, SE = 0.12, t = 3.54, $p < .001$), but the maximum rank did not ($\beta = 0.05$, SE = 0.05, t = 1.06, $p = .292$). These estimates reflected the simple effects of the minimum and maximum ranks within the reference category.

A significant interaction effect was observed for music preferences only, which enhanced the predictive power of the maximum rank ($\beta = 0.31$, SE = 0.07, t = 4.68, $p < .001$). However, no significant interaction emerged for minimum ranks across questionnaire types

(e.g., lottery, music preferences, and temperature ranges). This finding suggested that the simple effect of the minimum rank on disjunction values, based on the reference category (real estate investment), did not significantly vary across questionnaire types. Refer to Table 5.5 for full details of the predicted model. Figures 5.12 and 5.13 illustrated the predictive model for both the minimum and the maximum rank, providing visual insights into the differential impact of these factors across questionnaire types.

	Est.	S.E.	t val.	d.f.	p
(Intercept)	1.04	0.20	5.15	713.02	0.00
Minimum Rank	0.44	0.12	3.54	656.60	0.00
Maximum Rank	0.05	0.05	1.06	686.22	0.29
Lottery	-0.45	0.24	-1.84	572.85	0.07
Music	-0.48	0.25	-1.90	568.57	0.06
Temperature	0.74	0.23	3.21	612.55	0.00
Minimum Rank:Lottery	-0.09	0.14	-0.64	640.83	0.52
Minimum Rank:Music	-0.05	0.15	-0.36	635.79	0.72
Minimum Rank:Temperature	-0.12	0.15	-0.81	636.70	0.42
Maximum Rank:Lottery	0.04	0.06	0.71	629.41	0.48
Maximum Rank:Music	0.31	0.07	4.68	579.37	0.00
Maximum Rank:Temperature	-0.05	0.05	-0.91	675.06	0.37

Table 5.5: Results of the linear mixed-effects model predicting disjunction ranks. This table displays the estimated coefficients, standard errors, t-values, and p-values for the arsinh-transformed minimum and maximum ranks, alongside questionnaire types and their interaction effects.

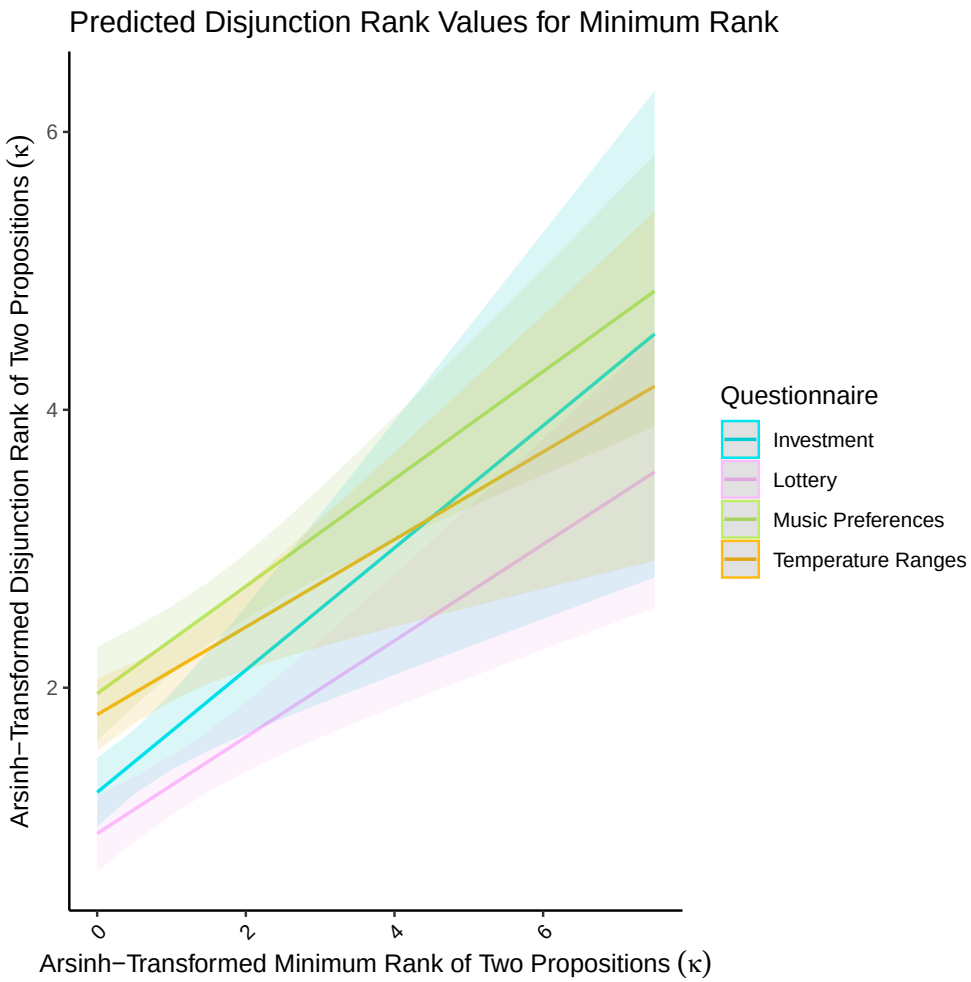


Figure 5.12: Predicted disjunction rank values as a function of the minimum rank, which represents the least disbelieved of the two propositions in the disjunction ranks, across different questionnaire types. Shaded areas denote confidence intervals, demonstrating the significant positive relationship between minimum rank and disjunction rank across varying types of questionnaires.

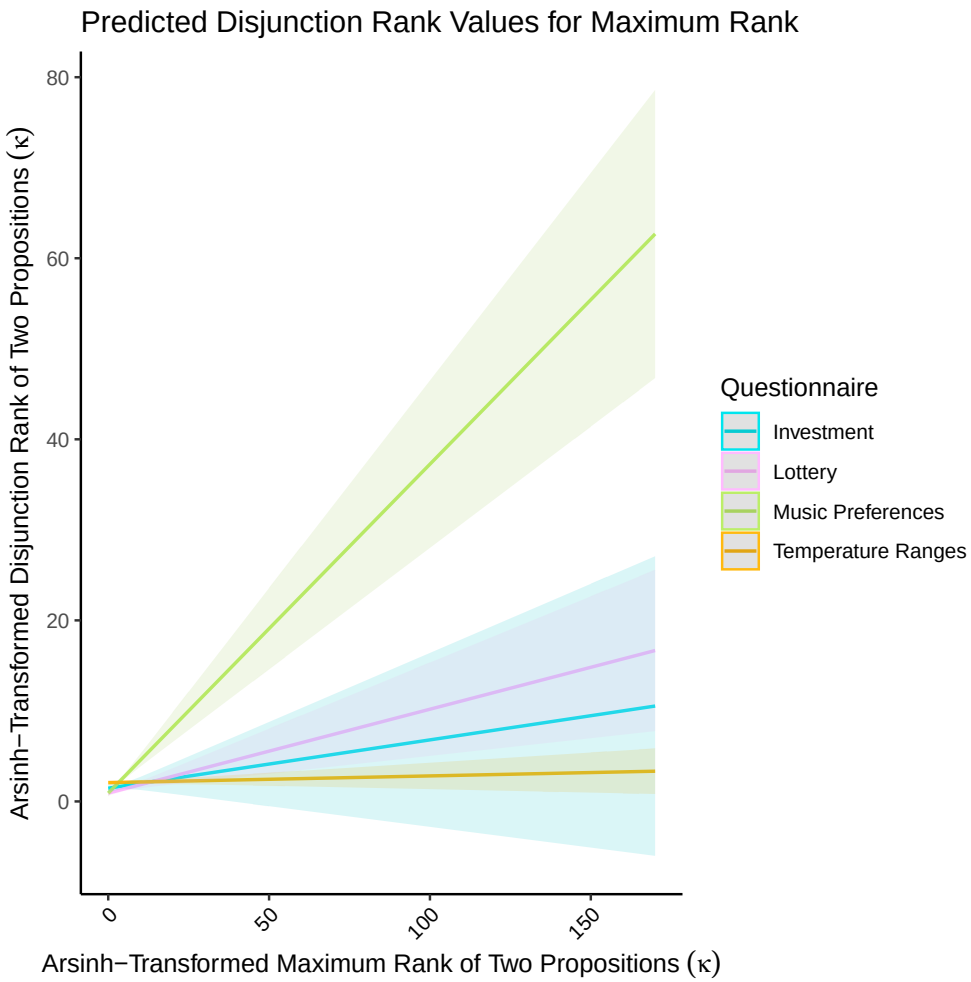


Figure 5.13: Predicted disjunction rank values as a function of the maximum rank, which represents the more disbelieved of the two propositions predicting disjunction, across different questionnaire types. Shaded areas denote confidence intervals, reflecting how maximum ranks affect disjunction ranks across different contexts. The maximum rank did not significantly predict disjunction overall, but a significant interaction was observed for music preferences.

5.4 Discussion

Experiment 3 expanded upon our previous studies by incorporating both positive and negative ranks, providing a comprehensive approach to examining belief and disbelief concur-

rently, offering a deeper understanding of how people navigate uncertainty across various scenarios.

5.4.1 Adherence to the Law of Negation

This study demonstrated a general adherence to the law of negation across most scenarios, reinforcing that participants typically did not hold contradictory beliefs about a proposition and its negation, even though they were unaware of the formal principles of ranking theory. Participants naturally graded their disbelief and belief in ways consistent with these principles. However, in the lottery scenario, a significant portion of responses expressed disbelief in both winning and losing. This pattern likely reflects skepticism specific to gambling contexts rather than an inability to logically assign ranks. This interpretation is supported by participants' binary 'yes' or 'no' responses regarding their belief in whether they would win the lottery, as these responses aligned with their assigned disbelief and belief. Despite this exception, the consistent application of the law of negation in other scenarios validated the use of ranking functions to capture coherent and logical belief systems.

The introduction of positive ranks in this study further enriched the analysis of belief systems, providing a fuller spectrum of participant responses — from disbelief through neutrality to belief. Scenarios such as temperature ranges and music preferences, previously examined only with negative ranks, demonstrated a consistent pattern that closely replicated findings from Experiments 1 and 2. The integration of positive ranks revealed an inverted pattern in the negation of propositions, relative to the original propositions, with

the notable exception of the lottery scenario. For instance, in propositions that previously elicited no surprise (i.e., negative rank of 0), positive ranks clarified the distinction between a lack of disbelief and varying degrees of belief. This differentiation is crucial as it captures the full range of participant responses, from disbelief to belief.

Although this study generally observed adherence to the law of negation, participants sometimes violated this law, particularly in scenarios with low surprise levels, such as temperatures ranging from 1 to 10°C. These scenarios led to ambiguous reactions, described as 'somewhat surprising' or 'believable.' In contrast, scenarios with predictable outcomes, such as opera music being very surprising or increases in property value being predictable, consistently followed the law of negation. The variation suggests that the degree of surprise influences how closely participants adhere to expected belief norms, with ambiguous scenarios more prone to mixed responses.

Despite the general adherence to the law of negation observed, participants sometimes violated this law, particularly in scenarios with low surprise levels. Such situations, like temperature ranges between 1 to 10°C, occasionally led to violations where outcomes were deemed 'somewhat surprising' or 'somewhat believable.' Conversely, scenarios with clear-cut expectations — like opera music (very surprising) and increases in property value (not surprising) — tended to elicit more definitive beliefs or disbeliefs, aligning more consistently with the law of negation. This pattern suggests that the degree of surprise inherent in a scenario may impact participants' adherence to expected belief norms, with ambiguous or low-surprise scenarios more likely to result in mixed beliefs and disbeliefs.

Notably, assigning a rank of 0, indicating no disbelief, participants inherently adhered to the law of negation. At this end of the scale, as long as the proposition or its negation is 0, by definition, one cannot violate this law. However, when participants found themselves in the middle of the scale, reflecting a state of uncertainty, they were more susceptible to violating the law of negation. This susceptibility is often influenced by secondary assumptions that participants may implicitly include in their evaluations. These variables may represent unspoken assumptions about the context or scope of a question, which can influence responses.

For instance, when asked about their surprise in encountering a specific scenario, participants might interpret the question as pertaining to individual encounters rather than reflecting a general population trend. This distinction between individual instances and overall population characteristics can lead to apparent violations of the law of negation if not explicitly clarified. For example, participants might consider all possible temperature ranges for September in a designated city, but their judgments may only reflect the weather during their actual visit, not the entire month. This subsidiary variable can lead to discrepancies that deviate from the expected mathematical relationships outlined by the law of negation. Exploring these variations can enhance our understanding of how individuals process and respond to different levels of surprise.

To address this in future studies, it is crucial to differentiate between evaluating general trends and individual occurrences. By clarifying the scope of inquiry — whether it focuses on general population characteristics or specific, isolated instances — researchers can minimize the influence of subsidiary variables, thereby clarifying subjective belief assessments and

increasing the reliability of experimental conclusions.

5.4.2 Subjective Probabilities and Ranks

This study demonstrated a nonlinear relationship between subjective probabilities and negative ranks across various contexts. Lower subjective probabilities consistently corresponded with higher negative ranks, indicating increased disbelief. These findings were consistent with findings from previous experiments. The inclusion of positive ranks clarified the relationship between subjective probabilities and ranks, showing that high subjective probabilities, previously linked to a negative rank of zero (indicating no disbelief), actually represented varying degrees of belief when assessed using two-sided ranks. This highlighted the utility of positive ranking functions in psychological assessments.

The influence of context on the threshold for disbelief was also evident, reinforcing the idea that disbelief was subjective and varied across scenarios. The varying thresholds for disbelief, marked by negative ranks greater than zero, emphasized the role of context in shaping reasoning and the expression of disbelief. This variation complicated the use of subjective probabilities to infer disbelief, emphasizing the importance of considering contextual factors in belief assessments.

Despite attempts to model the relationship between subjective probabilities and negative ranks, the results suggested that negative ranks were not simply a transformation of subjective probabilities. The limited variance explained by the models tested (i.e., logarithmic and exponential decay) further supported this conclusion.

For the analysis of two-sided ranks, the log-odds model, which predicted lower probabilities to correspond with degrees of disbelief and higher probabilities with degrees of belief, captured only a limited amount of variance. The polynomial regression model showed a slight improvement, capturing more variance than the log-odds model. The inclusion of additional parameters — three more in the polynomial regression — allowed for more flexible data point capture. Overall, like the negative ranks, the relationship was not easily represented by straightforward mathematical transformations.

The complexity observed in the relationship between subjective probabilities and ranks, particularly in contexts where probabilities were not explicitly clear, motivated the design of the next experiment (Experiment 4). This subsequent study aimed to investigate this relationship within a probability learning framework, providing participants with clearer guidelines on probability estimation. This approach sought to explore how probabilities directly correlated with ranks. Overall, the current study revealed an evident non-linearity between subjective probabilities and ranks, highlighting the complex nature of this relationship.

5.4.3 Law of Disjunction

This study refined the experimental approach to disjunction questions, addressing issues observed in previous experiments. Earlier studies found that the disjunction was predicted by the minimum rank in specific contexts and by the maximum rank in others. However, these experiments were complicated by high multicollinearity between the less and more disbelieved ranks within the disjunction and a higher percentage of ties, which could obscure

true effects.

Thus, methodological adjustments were made to reduce multicollinearity and the frequency of ties between the computed minimum and maximum propositions in each disjunction. As a result, the findings were more definitive across all four questionnaire scenarios. The analysis revealed that the less disbelieved proposition (minimum rank) generally predicted the disjunction rank across most scenarios. Specifically, the minimum rank significantly predicted the disjunction rank for real estate investment, with no significant interactions for the minimum rank across questionnaire types. This suggests that when individuals consider a disjunction, they are more influenced by the proposition they find least disbelieved rather than the one that invokes greater disbelief.

The maximum rank generally did not predict the disjunction across most scenarios. However, there was a significant interaction effect in one specific context – the music preference questionnaire. In this case, the maximum rank contributed to the prediction of the disjunction. This indicates that in specific contexts, the more disbelieved proposition can still play a role in how disjunctions are perceived.

These findings emphasized how contextual factors play a significant role in disjunctive reasoning. The minimum ranks play an important role in disjunctive reasoning; however, maximum ranks can have varying impacts depending on the scenario. This insight is crucial for understanding how people process combined propositions ("or" statements) in everyday reasoning.

5.4.4 Conclusion

Experiment 3 advanced our understanding of how individuals navigate uncertainty by using both positive and negative ranks to assess belief and disbelief. This methodology confirmed the effectiveness of ranking functions for a comprehensive evaluation of participant responses across different scenarios. The study highlighted the role of context in influencing beliefs and disbeliefs, demonstrating the adaptability of ranking theory to diverse experimental settings. These insights are particularly valuable for behavioural experiments, as they allow for a measure of the full spectrum of belief. This approach would enhance the accuracy of subjective assessments in psychological studies dealing with uncertainty.

Chapter 6

Experiment 4: Evaluating Ranking Functions in a Probability Learning Task

6.1 Introduction

Experiment 4 explored the dynamics of belief and disbelief using ranking theory within a learning context, focusing specifically on scenarios where probability distributions guided participant interactions but remained undisclosed. This study aimed to understand how individuals represented uncertainty and belief systems regarding the likelihood of different outcomes.

Research commonly integrates methods that prompt participants to adapt to and learn from probability distributions, assessing their internal models of belief regarding environmental outcomes (Go et al., 2022; Yu and Lagnado, 2012). Such studies typically require participants to make subjective probability judgments about outcome distributions, intending to track the alignment of these judgments with objective probabilities. However, equating belief with subjective probabilities is problematic, as their direct correspondence is debatable

(Spohn, 2013). Therefore, a closer examination of their relationship is necessary.

In Experiment 4, participants engaged in a children's hide-and-seek strategy game against a computer opponent, following a methodology adapted from DiBerardino (2023). Each opponent was allocated a specific probability distribution across three rooms, undisclosed to the participants. For instance, an opponent might choose Room A to hide 80% of the time, Room B 15%, and Room C 5%. Participants then provided their ranks and subjective probabilities for each room, reflecting their perceptions of the opponents' hiding and seeking behaviours.

This experiment sought to analyze the relationship between ranking functions and objective probabilities, offering insights into how different probability distributions influence belief and disbelief. The study evaluated which measures were more closely aligned with the ground truth by comparing participants' negative ranking functions and subjective probability assessments with objective probabilities. This analysis aimed to determine the consistency of ranking functions and subjective probability judgments with objective probabilities, thereby enhancing our understanding of how effectively ranking theory represents belief dynamics compared to traditional probabilistic judgments.

We postulated that if human belief is fundamentally represented through negative ranking functions, these should correspond more closely with objective probabilities than subjective probability assessments. This hypothesis is grounded in the premise that translating rank-based representations into probability measures could introduce errors due to the intrinsic differences between these formats. Such errors may reflect the potential inaccuracies inherent

in converting a rank-based system into a probability-based one. This supports the notion from ranking theory literature that negative ranks might represent a more straightforward cognitive expression of disbelief due to their simpler computational demands when managing environmental uncertainties (Spohn, 2013).

These considerations provoke further reflection on the nature of "belief" itself. Is it possible that what we commonly interpret as "belief" is actually an emergent property of simpler, more fundamental cognitive processes as described by ranking theory? If belief is indeed represented by ranks, this would suggest a different cognitive architecture from what is typically assumed in models based on subjective probabilities.

This study sought to explore these theoretical implications while contributing to the ongoing discourse on how best to model belief and disbelief within psychological and cognitive science research. By investigating these concepts through empirical methods, we aim to provide new insights into the cognitive foundations of uncertainty systems, particularly emphasizing how structured learning environments can enhance our understanding of belief formation based on well-defined and trackable events.

6.2 Methods

6.2.1 Participants

We recruited 268 undergraduate students from the University of Waterloo to participate in the experiment for course credit, with options for online or in-person. After excluding 67

participants for not adhering to instructions, a total of 201 participants were included in the analysis. The median age of the participants was 20 years, with 140 females, 56 males, and 5 undisclosed. Furthermore, we had 188 participants from the online sample and 13 participants from the in-person sample for the analysis.

6.2.2 Stimuli

We adapted a computerized version of the children's hide-and-seek game, in which participants played against a computer opponent (DiBerardino, 2023). Each computer opponent used a consistent probability distribution to select their hiding and seeking locations, which was unknown to the participant. The computer opponent selected hiding and seeking locations using the same probability distribution. The game interface featured a cartoon-style dollhouse with three visible rooms, each assigned a probability that summed up to 1, shown in Figure 6.1. The assignment of probability distributions to rooms varied randomly per game, as detailed in Table 6.1. A progress bar at the top of the screen displayed the participant's completion progress. We used the JsPsych JavaScript library to implement and collect online data (de Leeuw et al., 2023).

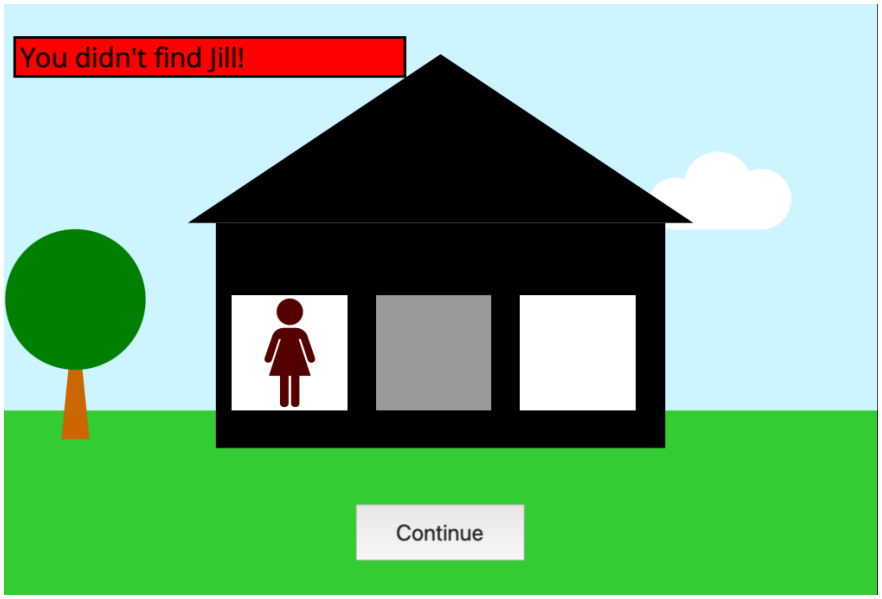


Figure 6.1: Participant view of the screen after selecting the middle room in a 'seek' trial within the three-room dollhouse condition. The selected room turns grey, revealing the child's location and presenting a notification indicating the outcome. In this example, the participant did not find Jill. The figure illustrates a trial in the hide-and-seek gameplay, where participants select one of three rooms in the dollhouse to seek the hidden child.

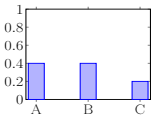
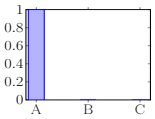
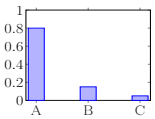
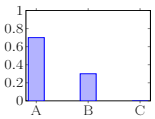
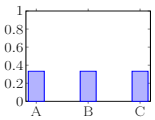
Opponent	Opponent Image	Distribution Identity	Distribution Graph
(Practice) Sally		40-40-20	
Bo		100-0-0	
Jill		80-15-5	
Kala		70-30-0	
Toby		33.33-33.33-33.33	

Table 6.1: Opponents faced by participants in each condition. Each distribution is presented here in decreasing room probabilities, though probabilities were randomly assigned to rooms during the experiment. Opponent identities were also randomly assigned to each game without replacement, and the order of the distributions was randomized except for the practice, which always came first.

6.2.3 Procedure

The experiment was conducted either online or in-person at the University of Waterloo, with identical procedures for both settings. Participants first completed demographic surveys detailing their biological sex, age, faculty, program name, and current term of study.

The experiment began with practice gameplay, during which participants engaged in

three seeking turns followed by three hiding turns against a computer opponent. Participants faced one of five simulated opponents – Sally, Kala, Bo, Jill, or Toby (see Table 6.1). As in standard hide-and-seek games, each participant was instructed to find each opponent while seeking and avoid each opponent while hiding. During the practice round, the probabilities of being assigned to Rooms A, B, and C were distributed randomly following a [40,40,20] distribution. This meant that the exact assignment of these probabilities varied, so while one room had a 20% probability of selection, the other two rooms each had a 40% probability, but the specific rooms these probabilities applied to were not fixed and could change (e.g., Room A: 40%, Room B: 20%, Room C: 40% or any other permutation).

Participants then progressed to complete the experiment trials. Each individual action during hide or seek was considered a 'trial'. The experiment alternated between 'seeking' and 'hiding' rounds, starting with a seeking round. Each set of 10 trials formed a 'round', and participants engaged in 7 seeking rounds and 7 hiding rounds per game, totalling 70 seeking and 70 hiding trials per game.

In each game, a different computer opponent was randomly assigned one of the predefined probability distributions: [100,0,0], [70,30,0], [80,15,5], [33.$\overline{3}$, 33.$\overline{3}$, 33.$\overline{3}$]. These distributions were also randomly assigned to the rooms, ensuring variability in the specific distribution per game. For example, if Sally was assigned the [100,0,0] distribution for one game, the remaining characters (Kala, Bo, Jill, Toby) would receive either [70,30,0], [80,15,5], or [33.$\overline{3}$, 33.$\overline{3}$, 33.$\overline{3}$] for their respective games. This randomized distribution of probabilities not only dictated the predictable behaviour pattern of each character for the duration of the game but also introduced an element of unpredictability in how participants could strategize against

different opponents.

The uniform distribution [$33.\overline{3}$, $33.\overline{3}$, $33.\overline{3}$] was specifically designed to induce equal beliefs among participants regarding the likelihood of their opponent hiding or seeking in any of the rooms. A distribution including a zero probability room aimed to maximize participants' degrees of disbelief or surprise about the opponent hiding or seeking in that room. After each game, participants provided their degrees of disbelief and belief and their subjective probability estimates for each room, counterbalanced to reflect their internal model of the computer opponent's seeking and hiding behaviour.

Seeking Phase: When seeking, participants were informed via a dialogue box ("Look for [child]!") that an opponent, referred to as "[child]", was hidden in one of the rooms. Participants chose a room to search by clicking on it, turning the room light grey. Success ("You found [child]!", in green) or failure ("You didn't find [child]!", in red) notifications were displayed accordingly, with the opponent's location revealed post-attempt, as shown in Figure 6.1.

Hiding Phase: When hiding, participants received a prompt ("Hide from [child]!") and selected a room to hide in using the mouse. The outcome was revealed through either a success ("[child] didn't find you!", in green) or failure ("[child] found you!", in red) notification, with the opponent's icon shown in the selected room.

In both the seek and hiding phases, we utilized the colours green and red to inform participants if they had successfully hidden or sought their opponent (in green) or had failed to hide or seek their opponent (in red). This was done to give additional visual feedback in

addition to the box dialogue.

Experiment Questionnaires: After completing the hide-and-seek game with each computer opponent, the participants were asked to fill out two questionnaires — the "Ranking Questionnaire" and the "Subjective Probability Questionnaire." The order in which they received these questionnaires was randomized. Each type addressed both "hiding" and "seeking" behaviours of a computer opponent in a simulated hide-and-seek game. For each item on the questionnaires, participants had to assign a numerical value that represented their level of belief/disbelief or their subjective probability estimates.

Ranking Questionnaire: Participants were asked to grade their degrees of disbelief of the opponent hiding or seeking in each of the three rooms, as well as the disbelief regarding the opponent not being in each of the room — negation of these propositions. Negation was measured to compute two-sided ranking functions in the data analysis. Each item required a non-negative integer value, with "0" indicating no disbelief or surprise, positive integers indicating increasing degrees of disbelief, and "infinity" signifying an impossible event. Additionally, participants were asked about disjunctions, or the disbelief that the opponent was in any of the rooms. Comprehensive details of both the hiding and seeking ranking questionnaires are documented in Appendix 9.4.

Subjective Probability Questionnaire: Participants were also asked to estimate the probability of the opponent hiding or seeking in each of the rooms, along with the probability of the opponent not being in each room (negation). Probabilities for disjunction scenarios, in which the opponent could be hiding or seeking in any combination of two rooms, were

additionally collected. Participants were instructed to give a value of "0" for an impossible event, while the probability of a certain event was expressed as "1." Additionally, guidelines required that the probability estimates for Rooms A, B, and C must sum to 1. Comprehensive details of both the hiding and seeking probability questionnaires are documented in Appendix 9.4.

Finally, participants completed a final survey assessing the difficulty of grading their subjective probability estimates and disbelief levels.

6.2.4 Data Analysis

Strategic Approaches

In the hide-and-seek context, participants employed two primary strategic approaches while seeking: maximizing and matching (DiBerardino, 2023). The maximizing strategy (α) involves seeking in the location with the highest probability of finding the opponent, thus directly maximizing the chances of success. On the other hand, the matching strategy (β) requires participants to align their search efforts proportionally with the known distribution of the opponent's hiding probabilities. Traditionally, a maximizing strategy is considered optimal for seeking because it directly targets the most likely hiding spots (DiBerardino, 2023).

Conversely, when hiding, participants utilized minimizing and mismatching strategies. The minimizing strategy (γ) entails always choosing the least probable hiding spot according to the opponent's searching pattern, thereby minimizing the likelihood of being discovered.

The mismatching strategy (δ) involves selecting a hiding spot in a manner that deliberately diverges from any predictable pattern, thereby attempting to be unpredictable. The optimal hiding strategy is typically minimizing, as it systematically reduces the chance of detection by focusing on the least likely locations (DiBerardino, 2023).

Fitting the Line on the Diagonal and Error Minimization: The strategy plots for both seeking and hiding were analyzed using a linear combination of optimal and non-optimal strategies, offset by an error term. Algebraically, for seeking strategies, this is represented as $\vec{s} = \alpha\vec{x} + \beta\vec{m} - \vec{\epsilon}$, where $\vec{x}$ represents the maximizing strategy, $\vec{m}$ the matching strategy, and $\vec{\epsilon}$ the error vector. Similarly, for hiding strategies, this relationship is denoted as $\vec{h} = \gamma\vec{y} + \delta\vec{n} - \vec{\epsilon}$, with $\vec{y}$ indicating the minimizing strategy, $\vec{n}$ the mismatching strategy, and $\vec{\epsilon}$ the error vector. The strategic fit is optimal (i.e., error is minimized) when $\alpha + \beta = 1$ for seeking and $\gamma + \delta = 1$ for hiding, indicating that participant strategies align perfectly along the diagonal line from (1,0) to (0,1) in the strategy space (DiBerardino, 2023), see Figure 6.2.

As $\alpha + \beta$ and $\gamma + \delta$ approach unity, the error represented by the residual sums of squares reduces, implying a better fit to the model where strategies are explained exclusively by the combinations of $\vec{x}$ and $\vec{m}$ for seeking, and $\vec{y}$ and $\vec{n}$ for hiding. This relationship is visible in the strategy space plots, where deviations from the diagonal line signify higher error levels, suggesting that other factors may be influencing participant decisions (DiBerardino, 2023).

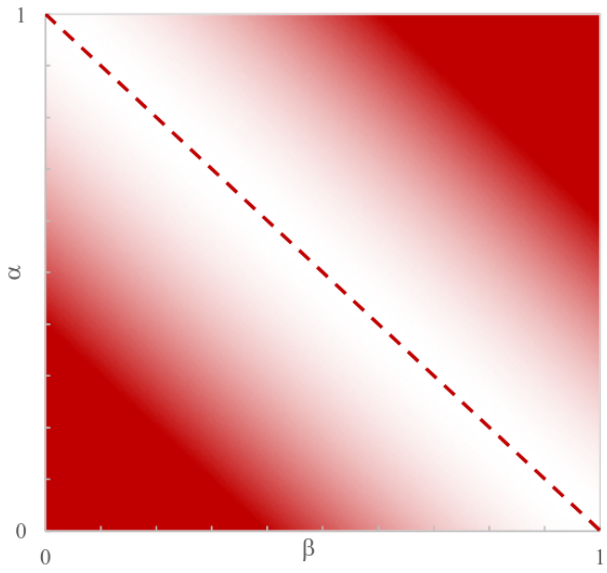


Figure 6.2: For seeking strategies defined by the equation $\alpha + \beta = 1$, error (indicated by red regions) is minimized when $\vec{\epsilon} = \vec{0}$. Numerous points on or near this line suggests a strong model fit, whereas clusters of points in areas with high error signal a poor fit.

6.2.5 Exclusion Criteria

Exclusion criteria were determined based on extreme violations of the instructions. For the probability condition, participants were instructed that the probabilities for Rooms A, B, and C should sum to 1. However, the JavaScript program did not enforce this rule to avoid biasing participants' responses and accommodate those with difficulties in basic arithmetic. Allowing responses to range between 0 and 1 captured natural variability in probabilistic judgments, which was essential for comparing subjective probability estimates with ranking theory performance.

Participants were excluded if their responses for the probabilities of Rooms A, B, and C summed to exactly 3 in any of the eight probability entries or if their responses summed to

a value close to 0 (e.g., less than or equal to 1×10^{-10}) more than 50% of the time (i.e., five or more out of the eight total probability questionnaires: four distributions with two entries each for the 'hiding' and 'seeking' conditions). This exclusion criterion addressed patterns suggestive of disengagement or attempts to bypass task requirements.

Although this approach was not a perfect system, it was necessary to mitigate issues arising from participants entering all 1s or 0s, or using arrow keys to produce extremely small values close to 0, as these methods were faster and easier than manually typing decimal probabilities. Repeated instances of such patterns indicated a lack of genuine engagement.

Unlike shorter tasks in previous experiments, this study required participants to complete 140 trials of hide-and-seek and provide subjective probability and ranking responses for four distributions. The sustained cognitive effort and repetitive nature of the task made such violations indicative of noncompliance. During the questionnaire, participants were required to provide a response to proceed to the next page; thus, an exclusion criterion was necessary to ensure data quality. Exclusion criteria were applied solely to participants' extreme errors in subjective probability responses, as the study aimed to assess the validity of ranking theory measurement. Therefore, the exclusion process avoids biasing results in favour of the ranking responses, and may instead favour the results of the subjective probabilities.

6.2.6 Ethical Considerations

The study received ethics clearance from the University of Waterloo Research Ethics Board (REB 45091) before data collection. Participants were informed about the study's purpose

and their right to withdraw at any time without penalty. To ensure privacy, all data were securely stored, and participants provided informed consent prior to participation.

6.3 Results

6.3.1 Data Exclusion

While most participants adhered to the instruction that the probabilities should sum to 1, a minority provided sums close to 0 or 3 across Rooms A, B, and C. Consequently, these non-compliant responses led to the exclusion of 25% of the data from the analysis.

6.3.2 Hide-and-Seek Gameplay — Participant Strategies

Participants exhibited various strategies in the hide-and-seek game as seekers or hiders. The strategy plots across distributions (see Figure 6.3) served as a manipulation check to assess whether participants accurately learned and adapted to their opponents' patterns.

For the [70, 30, 0] distribution, the seek strategy plot showed participants clustering around the diagonal line ($\alpha+\beta = 1$), indicating a mix of maximizing and matching strategies focused on the 70% hiding spot. The hide strategy plot also clustered around the diagonal line ($\gamma + \delta = 1$), suggesting participants employed minimizing and mismatching strategies to avoid the most probable locations.

In the [80, 15, 5] distribution, the seek strategy plot displayed a wider spread of points around the diagonal line, reflecting a mix of maximizing and matching strategies in line with

the probabilities. The hide strategy plot also showed clustering around the diagonal line ($\gamma + \delta = 1$), but with more deviation, indicating greater strategic diversity and challenges in predicting opponents' behaviour.

In the uniform distribution [$33.\overline{3}$, $33.\overline{3}$, $33.\overline{3}$], both seek and hide strategy plots showed a single point at (0.00, 1.00), indicating that any combination of maximizing/matching or minimizing/mismatching strategies was optimal, as each hiding spot was equally likely.

For the extreme [100, 0, 0] distribution, the strategies of maximizing and matching were identical, as participants generally selected the room with a 100% probability. This is evident from the clustering along the diagonal line ($\alpha + \beta = 1$) on the strategy plots, indicating a clear understanding of the distribution. Deviations from this line may reflect exploratory behaviour or the learning process during earlier trials. Similarly, the hiding strategies of minimizing and mismatching converged, as both involved selecting one of the two rooms with zero probability.

Overall, the strategy plots demonstrated that participants generally learned the distributions well, with points clustering around the diagonal lines ($\alpha + \beta = 1$ for seeking, $\gamma + \delta = 1$ for hiding). Deviations from these lines suggested either a lack of learning or strategic uncertainty, potentially leading to random choices. The histograms along the margins provided additional insights into the frequency of specific strategy components, revealing common strategies among participants (see Figure 6.3).

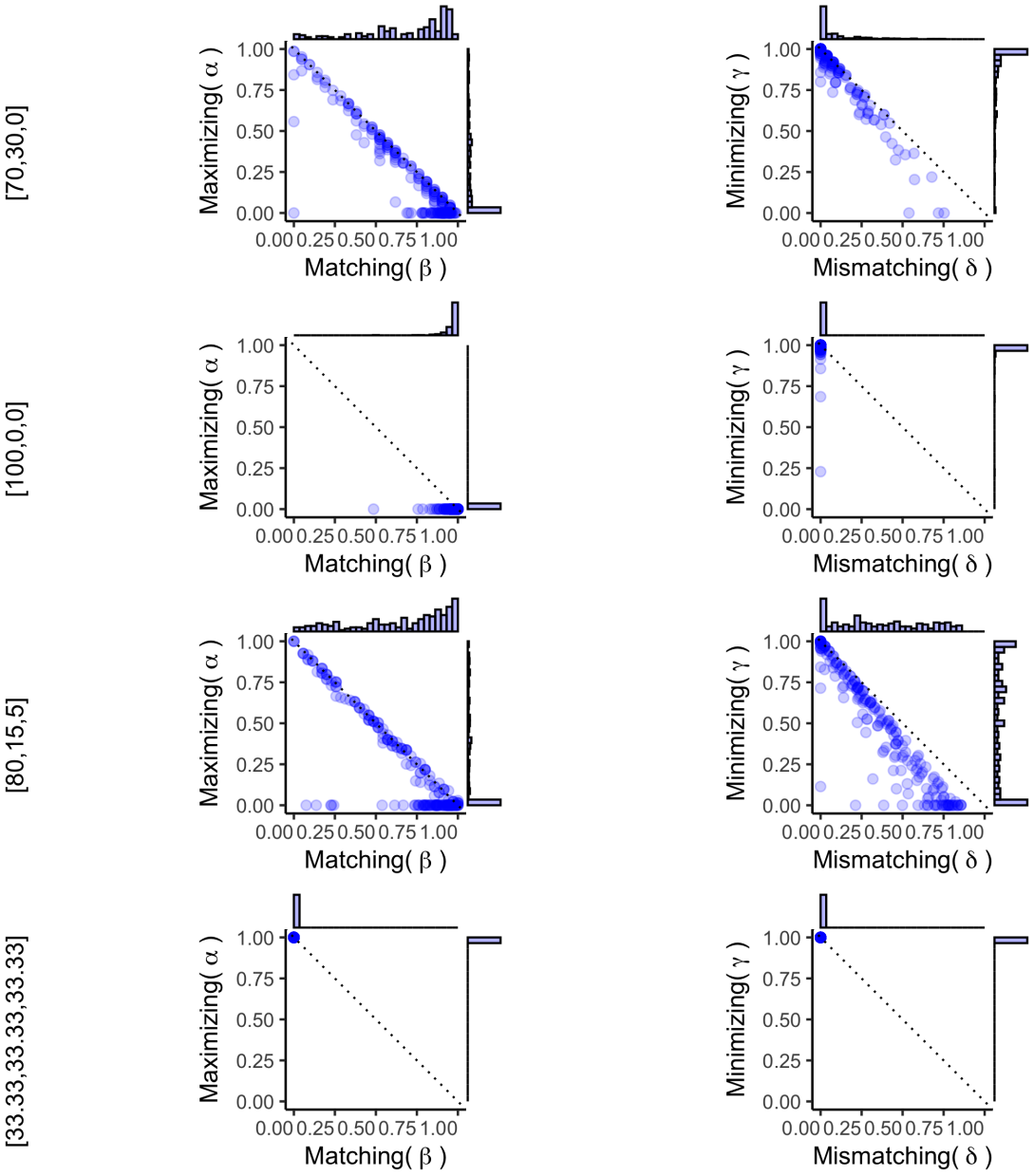


Figure 6.3: Participants' strategy space of each distribution. Each plot shows the relationship between two strategies (maximizing and matching for seeking; minimizing and mismatching for hiding). The diagonal lines represent the optimal strategy combinations ($\alpha + \beta = 1$ for seeking; $\gamma + \delta = 1$ for hiding). Points represent individual participants' strategies, with histograms along the margins showing the frequency distribution of each strategy component. Participant strategies existing on or near line $\alpha + \beta = 1$ for seeking, $\gamma + \delta = 1$ for hiding implies near perfect model fit.

6.3.3 Assessing Ranks: Correlations with Learned Probability Distributions

We measured participants' belief and disbelief after completing games with specific probability distributions of an opponent's hiding and seeking behaviours. Participants graded their disbelief for each room and its negation, which were then computed into the two-sided ranking function. The violin plot in Figure 6.4 illustrates the two-sided ranks across the probability distributions.

For the [100-0-0] distribution, participants generally associated the room with a 100% probability of containing the opponent as a certainty, denoted by a two-sided rank of infinity. Conversely, rooms with a 0% probability were considered impossible, marked by -infinity. This demonstrated that participants predominantly used the maximum and minimum values of the ranking scale, directly reflecting the absolute probabilities of 1 and 0.

The occurrence of a two-sided rank of 0 was particularly prevalent in the uniform [$33.\overline{3}$, $33.\overline{3}$, $33.\overline{3}$] distribution, suggesting neutrality across all rooms. This suggests that participants had a balanced perception regarding the presence or absence of the opponent in any room. It indicates that no single room was definitively perceived as more or less likely than the others, reflecting an equilibrium in participants' belief systems when faced with equal probabilities.

In the [80-15-5] and [70-30-0] distributions, the rooms with the highest probabilities (80% and 70%) elicited strong belief, while intermediate probabilities (30% and 15%) yielded in reduced belief, with 15% room being associated with some disbelief. The least probable

outcomes (0% and 5%) were met with strong disbelief, indicated by a median of impossibility for 0% and substantial disbelief for 5%. These findings suggest that participants graded their disbelief and belief in alignment with the probability distributions they had learned.

Relationship Between Objective Probabilities and Two-Sided Ranks:

The correlation between two-sided ranking functions and objective probabilities was examined. The Spearman's correlation coefficient between objective probabilities and two-sided ranks was 0.77, indicating a strong positive monotonic relationship. This finding suggests that as objective probabilities increase, there is also an increase in the degree of belief towards the proposition, as reflected by the computed two-sided ranks. The correlation was statistically significant ($p < .001$), strongly suggesting that the observed results were unlikely to be due to random variation.

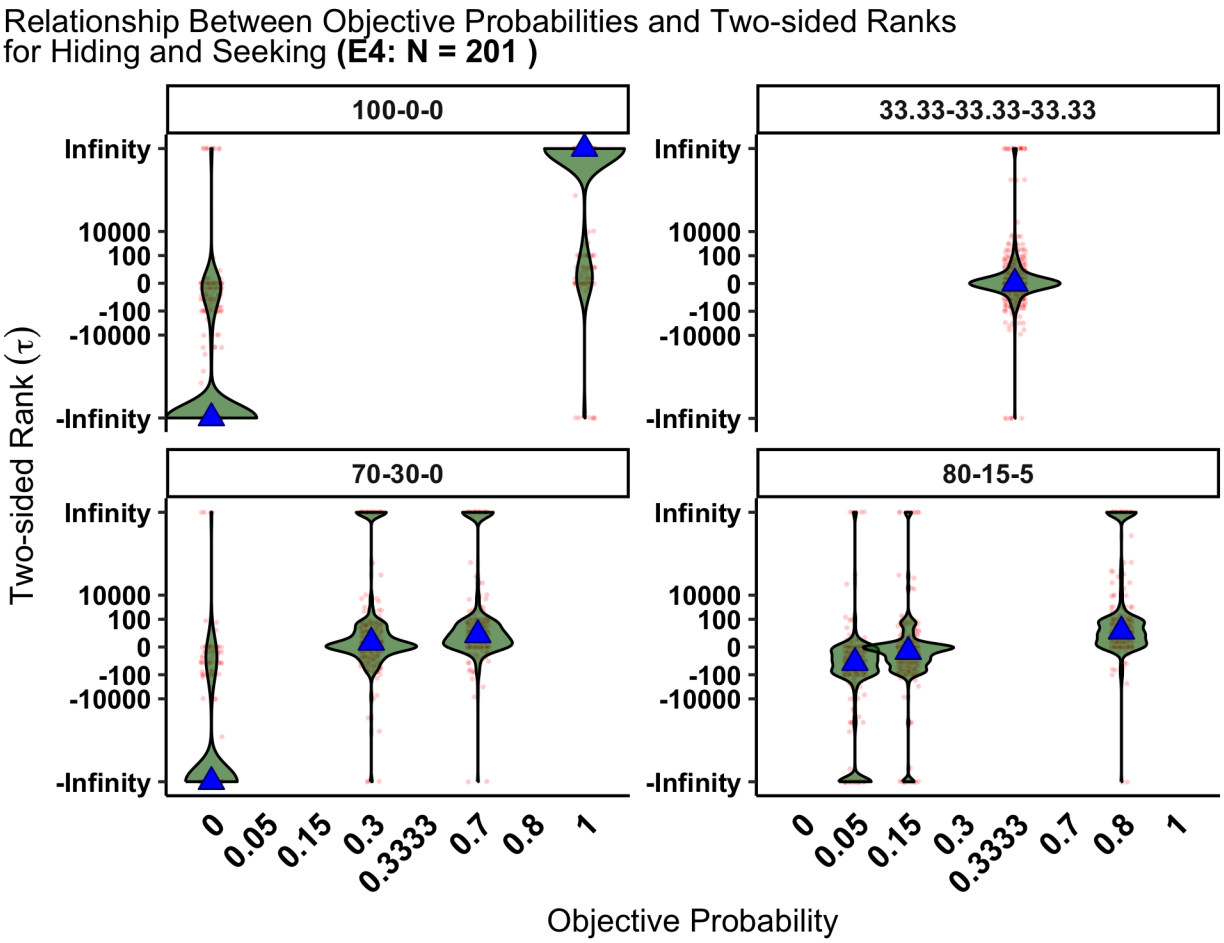


Figure 6.4: Violin plot illustrating participants' degrees of disbelief and belief for each probability distribution, adjusted using the arsinh transformation to scale values from -infinity (representing impossibility) to infinity (representing certainty). This plot compares the relationship between objective probabilities and the two-sided ranks, where negative values indicate disbelief and positive values indicate belief. The triangle within each violin plot represents the median, highlighting how participants' belief systems adapt to different probability distributions.

6.3.4 Relationship Between Subjective Probabilities and Negative Ranks

This analysis explored how participants' subjective probabilities corresponded with their negative ranks following hide-and-seek games. Initial analyses involved plotting subjective probabilities against negative ranks for each probability distribution configuration.

In the [100-0-0] distribution, participants typically assigned a subjective probability of 1 to rooms with a guaranteed presence of the opponent, corresponding with a negative rank of 0, signalling complete certainty. In contrast, rooms with a 0% likelihood saw subjective probabilities of 0, matched with maximum disbelief (-infinity), indicating perceived impossibility. However, a notable exception involved a few participants who assigned non-infinite negative ranks to these rooms, suggesting a perception of unlikelihood rather than impossibility, reflecting a degree of skepticism toward the opponent's strategy.

Similar patterns emerged in the [70-30-0] distribution, where zero-probability rooms generally received maximum disbelief ratings. Notably, the [70-30-0] and [80-15-5] distributions exhibited a broader range of subjective probability responses than the [100-0-0] distribution, suggesting increased difficulty in subjective probability estimates of the opponent's probability distribution. Furthermore, disbelief appeared to decrease non-linearly as subjective probability increased.

In the uniform distribution [$33.\overline{3}$, $33.\overline{3}$, $33.\overline{3}$], while many probability estimates were near $\frac{1}{3}$, a wide range of subjective estimates was observed, suggesting that some participants struggled to represent the environmental statistics accurately through probabilities. Addi-

tionally, many participants indicated it would not be surprising to find the opponent in any room, while others expressed varying levels of surprise.

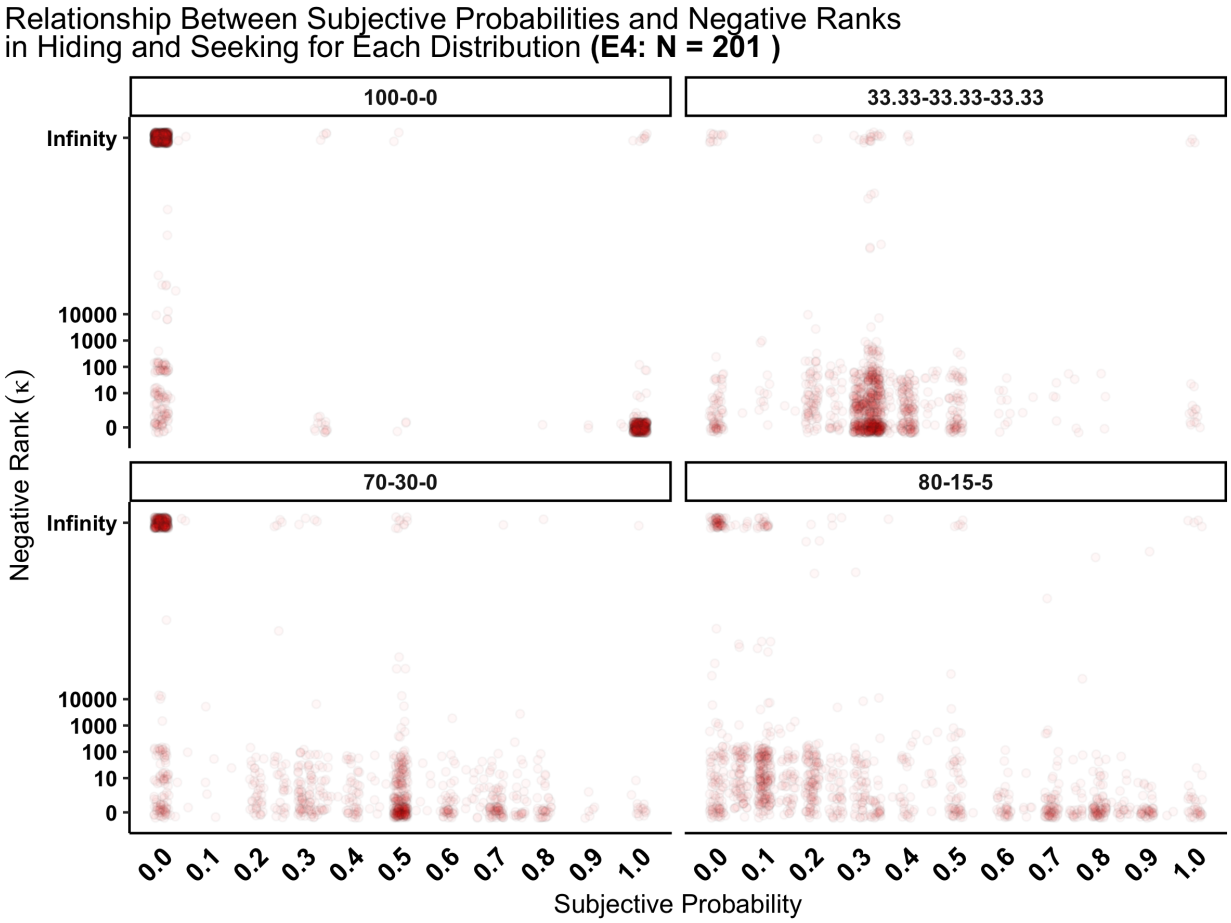


Figure 6.5: Scatterplot of subjective probabilities against negative ranks for each distribution configuration.

6.3.5 Mathematical Model Fit

As in Experiments 1 through 3, we employed two models — the logarithmic and exponential decay models — to examine the relationship between subjective probabilities and negative ranks. The inverse hyperbolic sine function $\text{asinh}(\kappa)$ was applied to the negative ranks to address skewness and extreme values. For model justification, refer to page 43. In this

experiment, the focus was on exploring this relationship within a probability learning context.

Model 1: Logarithmic Model

A non-linear least squares regression was used to estimate the logarithmic base b:

$$b = 0.16, \quad \text{SE} = 0.003, \quad t(1) = 59.28, p < .001$$

The model fit was robust, indicated by a pseudo R^2 of 0.55, suggesting that 55.37% of the variance in negative ranks was explained by this logarithmic relationship (see Figure 6.6).

The analysis demonstrated that lower subjective probabilities were generally associated with higher negative ranks, consistent with the theoretical expectation of an inverse relationship between event likelihood and perceived disbelief in the hide-and-seek game context.

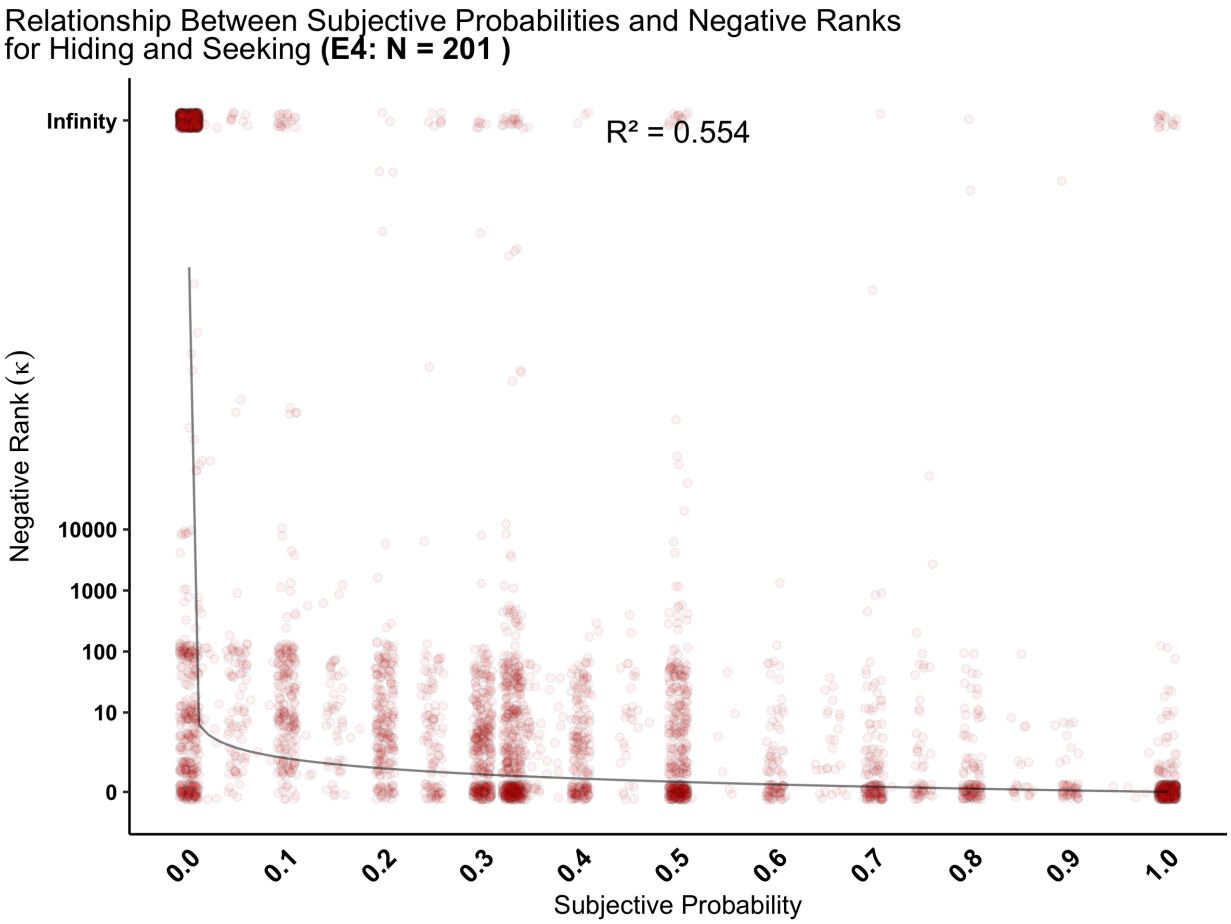


Figure 6.6: Scatter plot illustrating the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to display negative ranks effectively, while the labels show the original rank values for easier interpretation. A logarithmic model is fitted to the data, explaining 55.37% of the variance in negative ranks.

Model 2: Exponential Decay

For the exponential decay model, parameters a and b were estimated using non-linear least squares regression:

$$a = 18.98, \quad b = -10.44, \quad \text{SE}_a = 0.181, \quad \text{SE}_b = 0.428, \quad p_a < .001, \quad p_b < .001$$

The pseudo R^2 value was 0.548, indicating that 54.77% of the variance in negative ranks was explained by this exponential model, see Figure 6.7.

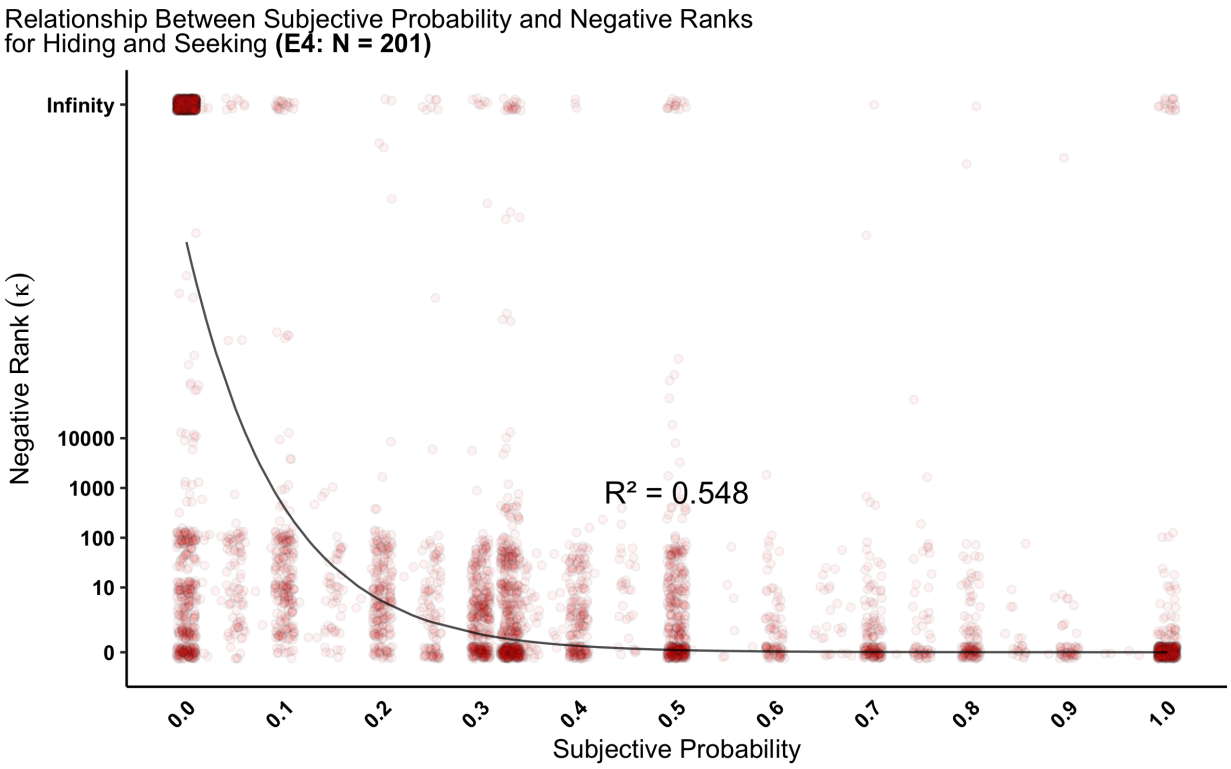


Figure 6.7: Scatter plot showing the relationship between subjective probabilities and negative ranks (κ) across all questionnaires. The y-axis is scaled to effectively display negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with an exponential decay model, which accounts for 54.77% of the variance observed in the negative ranks.

Model Comparison

To evaluate which model better explained the relationship between subjective probability and negative ranks, we compared the logarithmic and exponential models using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The logarithmic model showed a superior fit, with a lower AIC of 32292.26 compared to 32359.05 for the exponential model — a difference of 66.78. The BIC results similarly favoured the logarithmic model, with a lower BIC of 32305.23 compared to 32378.49 — a difference of 73.27. These findings deviate from those in Experiments 1 through 3, suggesting that the logarithmic

model better explained the relationship between subjective probabilities and negative ranks in a probability context.

6.3.6 Relationship Between Subjective Probabilities and Two-Sided Ranks

Computing the two-sided ranks clarified the relationship between subjective probabilities and participants' beliefs across distributions, as shown in Figure 6.8. Consistent patterns were observed, where a subjective probability of 0 typically corresponded to perceptions of impossibility (-infinity), probabilities around 0.33 to 0.5 corresponded with neutrality ($= 0$), and probabilities above 0.5 generally indicated belief (> 0). The certainty associated with a probability of 1 was reflected in the strongest belief (infinity), highlighting the model's fit to the non-linear responses observed in participant data.

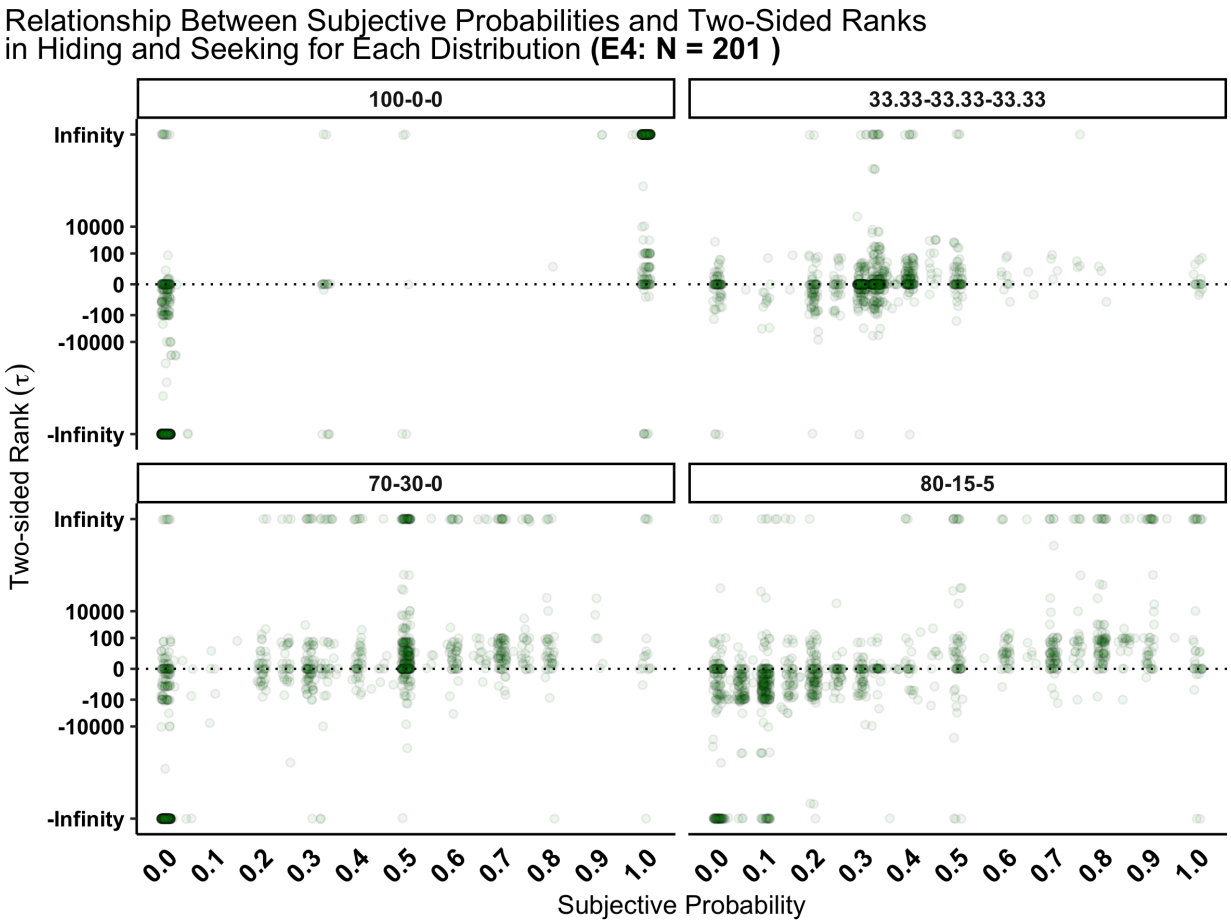


Figure 6.8: Scatterplot showing the relationship between subjective probabilities and two-sided ranks τ for each distribution configuration. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The dotted line represents a two-sided rank of zero, indicating neutral judgment.

6.3.7 Mathematical Model Fit

A log-odds model and a polynomial regression model were used to explore the mathematical relationship between subjective probabilities and two-sided ranks. The inverse hyperbolic sine function asinh(τ) was applied to reduce skewness and manage extreme values in the two-sided ranks. For details on the models, see the rationale on page 121.

Model 1: Log-odds Model

The non-linear least squares regression analysis, based on the equation

$$\tau(A) = \log_b \left(\frac{1 - P(A)}{P(A)} \right), \quad \text{where} \quad b \in (0, 1),$$

determined the logarithmic base b as:

$$b = 0.14, \quad \text{SE} = 0.003, \quad t(1) = 39.29, p < .001$$

This parameter was statistically significant, with a pseudo R^2 of 0.55, which suggested that approximately 54.83% of the variance in the participants' two-sided rank responses could be explained by this model. This indicated that the log-odds model effectively captured a meaningful proportion of the variance based on their subjective probabilities. This marked improvement, compared to Experiment 3 (see page 121), suggests that the log-odds model better captured this relationship in the probability context.

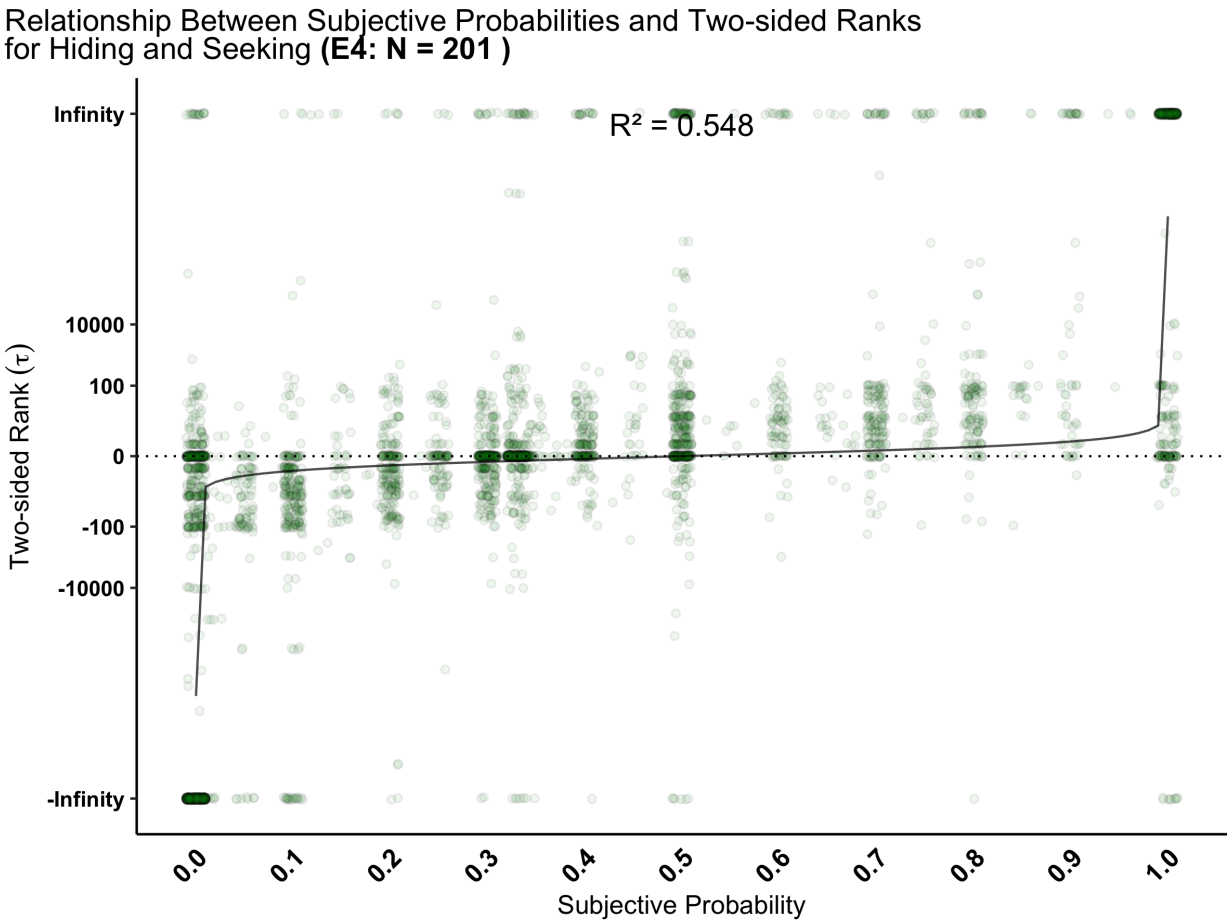


Figure 6.9: Scatter plot showing the relationship between subjective probabilities and two-sided ranks τ across all questionnaires. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with a log-odds model, which captures 54.83% of the variance in the ranks.

Model 2: Polynomial Regression

In line with Experiment 3, the second model used polynomial regression to capture potential non-linearities. The model was specified as:

$$\tau(A) = \beta_0 + \beta_1 P(A) + \beta_2 P(A)^2 + \beta_3 P(A)^3$$

where $\beta_0, \beta_1, \beta_2$, and β_3 were parameters to be estimated. The model was fitted using non-

linear least squares regression, and the parameters were statistically significant:

$$\beta_0 = -16.74, \quad \beta_1 = 113.08, \quad \beta_2 = -213.04, \quad \beta_3 = 134.96,$$

$$SE_{\beta_0} = 0.25, \quad SE_{\beta_1} = 3.11, \quad SE_{\beta_2} = 9.07, \quad SE_{\beta_3} = 6.39,$$

$$p_{\beta_0} < .001, \quad p_{\beta_1} < .001, \quad p_{\beta_2} < .001, \quad p_{\beta_3} < .001$$

The fit of the polynomial model was also assessed, yielding a pseudo R^2 value of 0.56, indicating that approximately 56.18% of the variance in two-sided ranks was explained by this polynomial relationship (see Figure 6.10). This model captured the variance in the probability context more effectively than in Experiment 3 (see page 123).

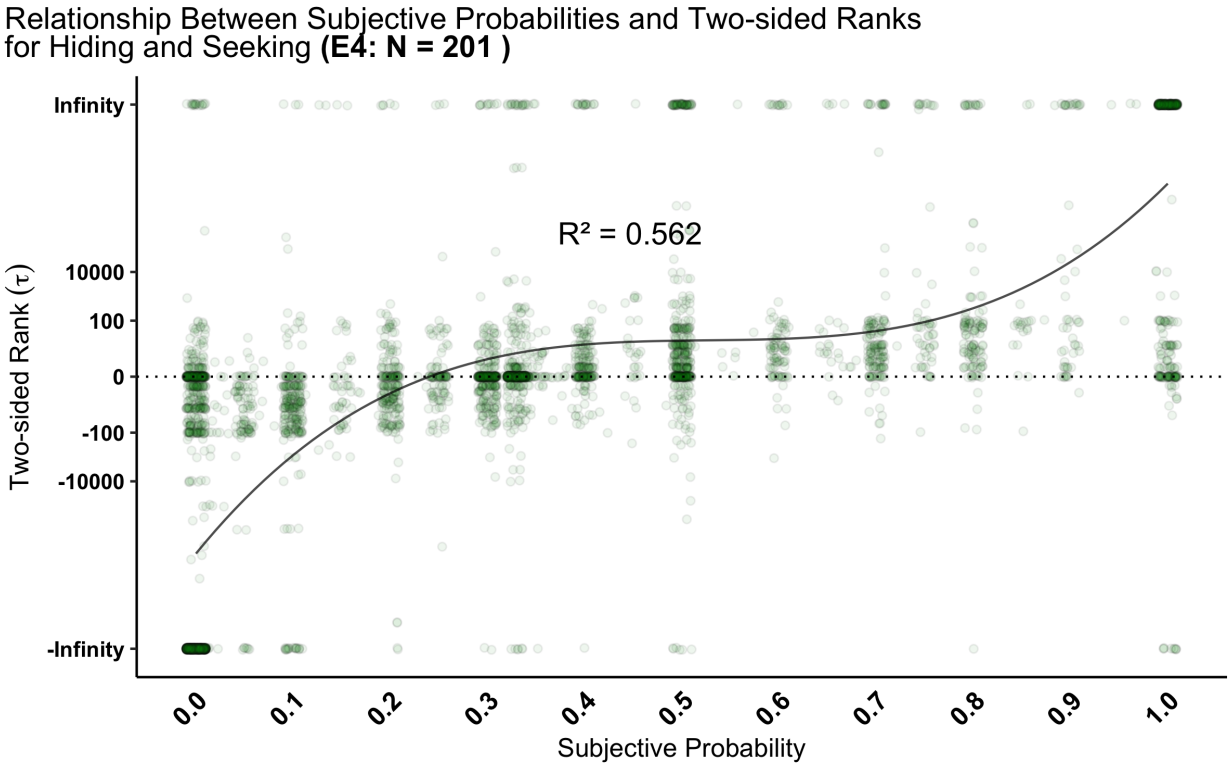


Figure 6.10: Scatter plot showing the relationship between subjective probabilities and two-sided ranks τ across all questionnaires. The y-axis is scaled to effectively display both positive and negative ranks, with labels representing the original rank values to aid interpretation. The plot is fitted with a polynomial regression model, which captures 56.18% of the variance in the ranks.

Model Comparison

To compare the two models and determine which one best explained the relationship between subjective probability and two-sided ranks (arsinh transformed), we calculated both the AIC and BIC. The comparison of the two models using AIC and BIC indicated that the polynomial regression was the better model based on both metrics. The AIC for the polynomial model was 35672.87, compared to 35813.79 for the log-odds model, resulting in a difference of 140.92. Similarly, the BIC for the polynomial model was 35705.27 compared to 35826.75 for the log-odds model, with a difference of 121.48.

The polynomial model performed better than the log-odds model in terms of overall fit, as shown by the lower AIC and BIC values. However, the log-odds model provided a strong theoretical framework, particularly for understanding behaviours at the extreme ends of probability. It suggested a balanced relationship where extreme probabilities — both high and low — were associated with larger magnitudes of two-sided ranks, representing a wide range of belief or disbelief, respectively.

6.3.8 Objective Probabilities and Subjective Probabilities

Given the expected positive linear relationship between objective and subjective probabilities, assuming participants could accurately learn the distributions, a Pearson correlation analysis was performed. This yielded a correlation coefficient of 0.87, $p < .001$, indicating a strong positive association. As depicted in Figure 6.11, participants accurately estimated the extreme probabilities (probabilities of 1 and 0). Although there was greater variance in esti-

mates for less extreme probabilities, the median subjective probabilities did not consistently align with the objective probabilities across various distributions.

It is important to note that this figure is not meant to be a direct comparison to Figure 6.4. The correlation in the previous graph was based on a different scale of two-sided rank, rather than subjective probability. Here, we are measuring the correlation between estimated probabilities and objective probabilities, not participants' beliefs. The two-sided rank reflects the degree of belief towards a proposition (see page 157), while the subjective probability measures participants' estimated likelihoods.

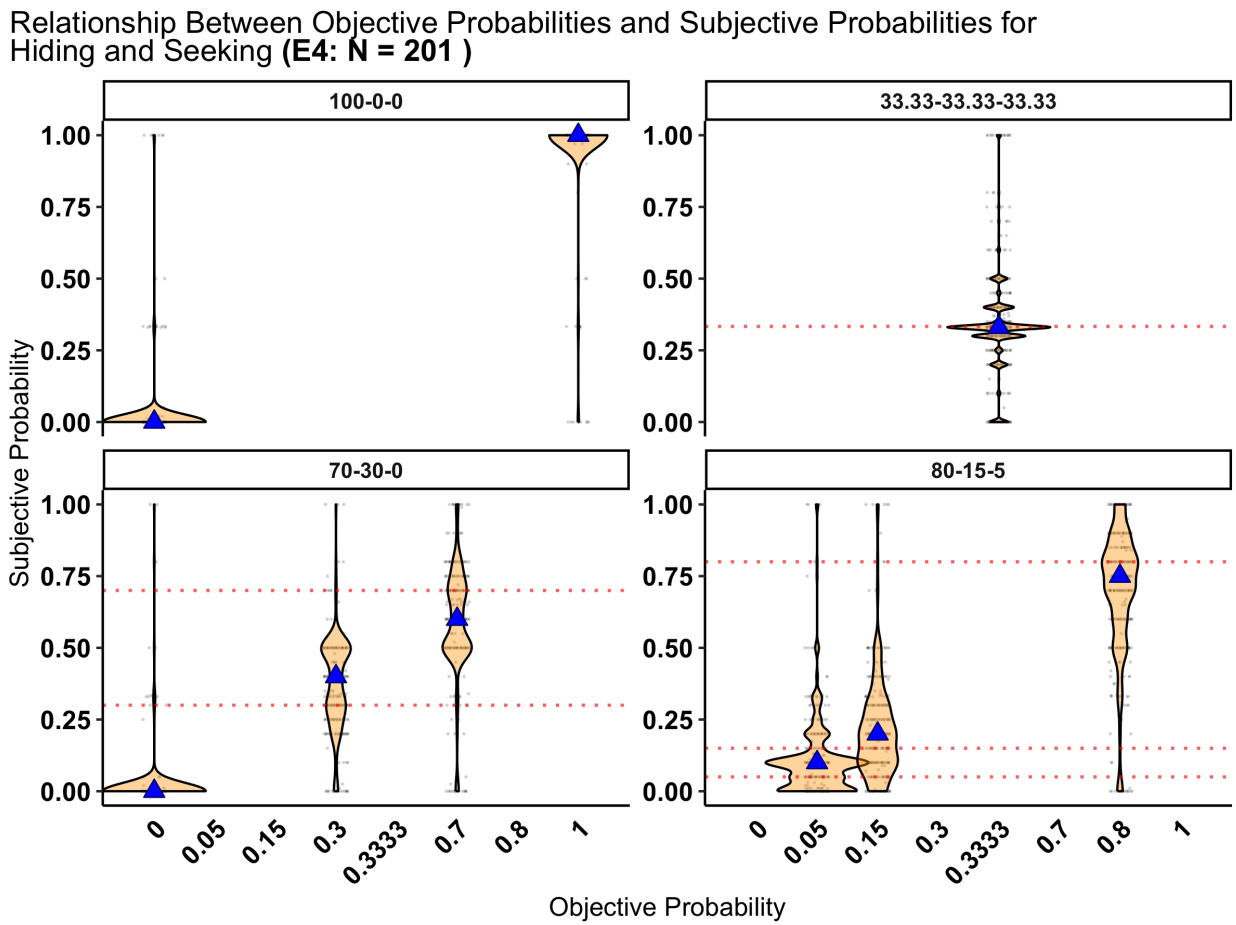


Figure 6.11: Violin plot illustrating participants' subjective probabilties against objective probabilities for each probability distribution. The triangle within each violinplot represents the median. The horizontal red dotted lines indicate the corresponding objective probabilities for each distribution.

6.3.9 Comparison of Subjective Probabilities and Ranks with Objective Probabilities

The comparison of subjective probabilities and negative ranks to objective probabilities was visualized using a probability simplex (Δ^{n-1}). This geometric representation, particularly useful in a three-outcome scenario ($n = 3$), plotted the distribution of probabilities as points

within an equilateral triangle, where each vertex corresponded to a 100% probability of selecting one of the three rooms. This 2D simplex illustrated how each participant's estimates mapped against the expected probabilities.

The study focused on the uniform distribution condition, where objective probabilities were fixed at [33.$\overline{3}$, 33.$\overline{3}$, 33.$\overline{3}$]. Initially, the plan was to extend this comparative analysis across all four distributions. However, the unique characteristics of the uniform distribution allowed for direct translation into the simplex space without assumptions about the logarithmic base necessary for other distributions. Therefore, this direct comparison was only feasible for the uniform condition. This allowed for a clear comparison of subjective probabilities and negative ranks directly against these fixed probabilities.

The visualization on the probability simplex focused on participants' subjective probability estimates for rooms A, B, and C, along with the corresponding negative ranks reflecting disbelief. Negations and two-sided ranks were excluded to ensure a clear, straightforward visual comparison of subjective probabilities and disbelief responses with the fixed objective probabilities of the uniform distribution.

Visualization and Data Interpretation Figure 6.12 presents a probability simplex (Δ^{n-1}) where each point represents participants' responses plotted against the objective probability fixed at [33.$\overline{3}$, 33.$\overline{3}$, 33.$\overline{3}$], marked in red. Here, subjective probabilities (blue dots) showed significant dispersion across the simplex, reflecting a varied understanding and estimation of the probabilities by different participants. In contrast, ranks (yellow dots), which denote negative ranks (degrees of disbelief), clustered more centrally and consistently

around the objective probability, indicating a more uniform assessment across responses. The size of each dot was proportional to the frequency of that response, with larger dots denoting more common responses. This highlighted that while subjective probabilities varied widely, ranks offered a more coherent representation of participants' assessments relative to the objective probabilities.

Quantitative Findings The quantitative analysis revealed a notable discrepancy in the accuracy of ranks compared to subjective probabilities. Specifically, in the hiding condition, 56.22% of participants' negative ranks aligned with the objective probabilities, in contrast to only 31.34% for subjective probabilities, considering a tolerance of 1.0%. This tolerance implied that a participant's probability response of 0.33 for each room would be deemed accurate, eliminating the need for more precise decimal representation. Similarly, in the seeking condition, 58.21% of participants' negative ranks were in agreement with the objective probabilities, compared to 34.33% for subjective probability, with the same tolerance applied. The findings showed that participants' negative ranks, which reflect their levels of disbelief, consistently corresponded to the actual probability distributions more closely than their subjective probability estimates. This indicated a fundamental difference in how participants perceive and measure probabilities, emphasizing the more reliable nature of ranked responses in capturing true probabilistic assessments.

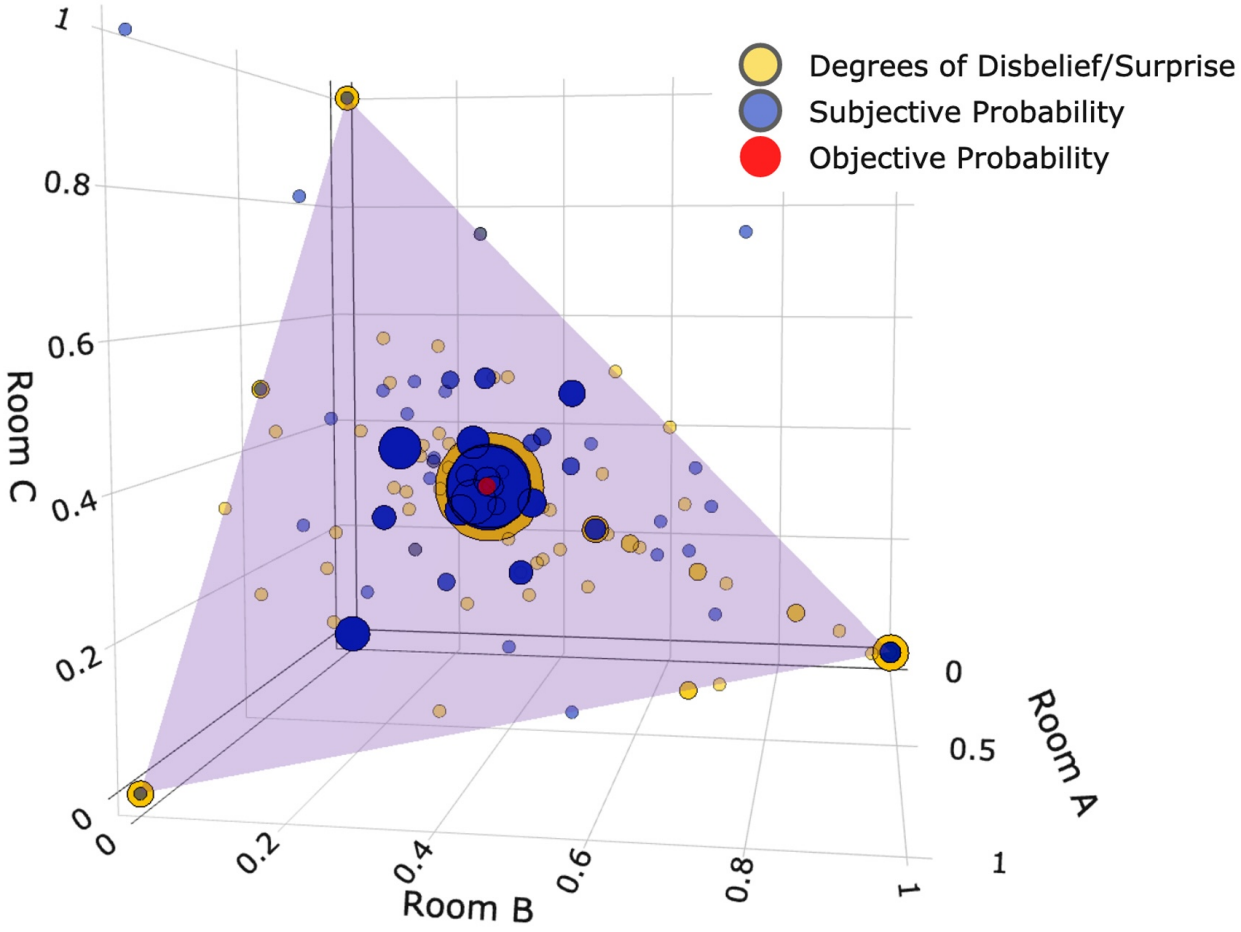


Figure 6.12: A probability simplex (Δ_{n-1}) displays participants' subjective probabilities and disbelief rankings against a fixed objective probability [$33.\overline{3}$, $33.\overline{3}$, $33.\overline{3}$] marked in red. Blue dots, representing subjective probabilities, are widely dispersed, illustrating varied response densities from multiple participants. Yellow dots, indicating negative ranks, cluster at the centre, reflecting a more consistent evaluation of objective probability, with each room ranked similarly. Larger dots indicate more common responses, highlighting that rankings provide a clearer picture of the uncertainty structure compared to subjective probabilities.

6.4 Discussion

This study aimed to investigate how people processed belief and disbelief in a learning scenario using ranking theory, specifically focusing on an opponent's hiding and seeking behaviours that followed a distinct probability distribution. Building on previous work from Experiments 1–3, where objective probabilities were unknown, this research explored the relationship between ranking functions, subjective probabilities, and objective probabilities.

6.4.1 Strategic Approaches in Hide-and-Seek Gameplay

The findings demonstrated that participants adapted their strategies based on opponents' behaviours across 70 hiding and 70 seeking trials. The strategic space of each distribution served as a benchmark for verifying participants' understanding. Notably, the [70,30,0] and [80,15,5] distributions were particularly relevant for observing a mix of optimal and non-optimal strategies. In both distributions, participants employed a combination of matching and maximizing strategies when seeking, with a noticeable degree of non-optimal probability matching.

During the hiding trials, a similar mix of strategies emerged, although the optimal minimizing strategy was more prevalent. In the [70,30,0] distribution, participants predominantly used the optimal minimizing strategy, consistently selecting the least likely room with a 0% probability. However, in the [80,15,5] distribution, the minimizing strategy was less evident due to the unclear differentiation between the 15% and 5% rooms.

Deviations from the diagonal strategy line may be due to the initial learning phase,

where participants' behaviour did not fully align with the presented distribution, as well as exploratory or random selections. Research suggests that participants typically learn and adapt to new probability distributions within a few trials (Go et al., 2022). This study's findings align with DiBerardino (2023), which indicated that participants were more likely to employ the optimal minimizing strategy when hiding than the maximizing strategy when seeking. DiBerardino (2023) provides an evolutionary explanation for this pattern, noting that failing to hide effectively often leads to more severe consequences (e.g., being caught by a predator) than failing to seek (e.g., remaining hungry). This may shift the payoff matrix to favour successful hiding over seeking.

6.4.2 Ranking Functions in a Controlled Probability Framework

The study aggregated subjective responses across hiding and seeking conditions without distinguishing between them. This approach was chosen because the computer opponent used the same probability distribution for both conditions, and no notable differences in participants' ranking responses between the two conditions (hiding and seeking) were evident in the data visualizations.

By examining negative ranks in relation to objective probabilities, the study gained insight into how participants ranked propositions and their negations within a structured framework. A strong monotonic relationship between objective probabilities and ranks suggests that participants' responses were influenced by the probability-learning context. This controlled environment allowed for the exploration of ranking theory, where participants

formed beliefs based solely on the manipulated probability distribution during gameplay against computer opponents. In contrast, previous experiments often required participants to rely on everyday reasoning and their pre-existing knowledge about the world.

Interestingly, the ends of the probability scale (0 and 1) were designed to elicit maximum disbelief and belief in participants. These values generally corresponded to the extremes of the two-sided ranking functions, ranging from negative infinity to positive infinity. However, there were instances where participants expressed finite responses. For example, some indicated moderate to strong disbelief for a probability of 0 and moderate to strong belief for a probability of 1. These deviations suggest that participants' belief systems may not strictly adhere to extreme probabilistic reasoning. Such responses do not necessarily indicate a lack of understanding of environmental statistics but rather reflect a more reserved approach with less degree of certainty. Although participants did not observe their opponent hiding or seeking in certain rooms, their belief systems may still account for the possibility. While surprising, they did not entirely rule out the chance that their opponent might seek or hide in such locations.

Furthermore, participants' neutrality in belief systems — neither believed nor disbelieved was clearly observed in the uniform condition [$33.\overline{3}$, $33.\overline{3}$, $33.\overline{3}$], where each of the three rooms was equally probable as a hiding and seeking spot. This probability of $\frac{1}{3}$ was unique to the three-room condition. In configurations with two or four rooms, participants are expected to exhibit such neutrality (two-sided rank of 0) for probabilities of 0.5 and 0.25, respectively, suggesting valuable directions for future research. It is hypothesized that increasing the dimensionality of room configurations (N-dimensions) could place greater cognitive demands

on participants. This increased complexity may impede their ability to learn the ground truth within a limited number of trials, thereby affecting their judgment and decision-making accuracy.

6.4.3 Mathematical Similarity Between Subjective Probabilities And Ranks

Observation of Mathematical Similarity in Controlled Settings:

This study provided valuable insights into how structured probabilistic environments shape human belief systems. It demonstrated a clear mathematical relationship between subjective probabilities and both negative and two-sided ranks in controlled settings, a relationship that had not emerged in the less structured environments of Experiments 1-3. The controlled nature of the hide and seek game, where outcomes were predictable and probabilities fixed, likely facilitated this similarity. Over 140 trials, participants directly observed and internalized the frequency of room choices, which were governed by distinct probability distributions (i.e., [70, 30, 0]). This structured setup enabled participants to base their probability estimates and ranks on observed frequencies. Consequently, this finding may have broader applicability to real-world learning scenarios where individuals adjust beliefs through repeated exposure to outcomes.

Role of Logarithmic Model In Explaining Negative Ranks and Probabilities:

The logarithmic model explained the relationship between subjective probabilities and negative ranks. Compared to the exponential model, it captured more variance at lower

probabilities nearing zero. This model aligns with Spohn's (2012) theoretical framework, suggesting that when external conditions provide consistent and observable cues, there is a greater mathematical similarity between ranks and probabilities. This relationship emphasizes the psychological benefits of processing probabilistic information in logarithmic terms (Skovgaard-Olsen, 2016). If humans naturally represent probabilistic information logarithmically to express degrees of disbelief, this method simplifies complex calculations by replacing multiplications and divisions with easier additions and subtractions, leading to probability judgments that are more consistent with normative responses (Skovgaard-Olsen, 2016). This simplification helps reduce cognitive biases, such as base-rate neglect, making logarithmic representations particularly advantageous in cognitive tasks (Juslin et al., 2011). The simplicity and flexibility of negative ranks may reflect how humans could naturally encode information in a rank-based format.

Extension to Two-Sided Ranks:

Both the log-odds model and the polynomial regression model accounted for a substantial variance in participants' two-sided ranking responses in probabilistic contexts. The polynomial model, consistent with findings from Experiments 1-3, explained significant variance in two-sided ranks, demonstrating the strength of this relationship. Similar to the log-odds model, lower probabilities predicted disbelief, while higher probabilities indicated degrees of belief. Although the polynomial model provided a better empirical fit in some cases, the log-odds model offered valuable theoretical insights, particularly at the extremes of the probability spectrum, where both high and low probabilities corresponded to larger magnitudes of two-sided ranks, encompassing a broad range of belief and disbelief.

However, the log-odds model's representation of neutrality at a probability of 0.5, a standard in theoretical frameworks implying equal likelihoods of competing outcomes, lacks the flexibility observed under different experimental conditions. Specifically, in the three-room scenario previously discussed, participants displayed neutrality at a probability of $\frac{1}{3}$, due to each room's equal likelihood of being chosen. According to the model, $\tau(A)$ is always 0 when P(A) = 0.5 because the logarithm of 1 is zero in any base between 0 and 1, simplifying to $\log_b(1)$ under these conditions (see Equation 2.7). Our findings demonstrated that neutrality can shift depending on context, and the direct translation between probability and ranks is not always straightforward. Overall, the polynomial model's additional parameters enabled it to better represent the complexity of the relationship, demonstrating flexibility across different contexts.

Contrast with Less Structured Scenarios and Real-World Applications:

In less structured real-world scenarios (as observed in Experiments 1-3), such as making judgement on their hypothetical coworker's music preferences, the relationship between ranks and subjective probabilities revealed a fundamental distinction in how belief systems were measured. While individuals were able to provide subjective probability estimates based on indirect cues and personal experiences, these estimates primarily reflected perceived likelihoods given the scenarios. In contrast, ranks in such scenarios were a metric of disbelief and belief regarding outcomes, capturing a broader array of elements that shaped belief — including probabilities, personal experiences, stereotypes, and cultural or societal norms. This means that in environments where a clear statistical probability structure was unavailable, ranks and subjective probabilities diverged due to their fundamentally different natures.

In structured environments, such as the hide and seek game, the probability distribution was the sole variable manipulated, directly influencing participants' subjective probabilities and their beliefs about their opponents' likelihood of seeking and hiding in certain rooms. The strong positive association between subjective probabilities and objective probabilities, along with participants' corresponding behaviours, supported that they understood their environment well. However, deviations from subjective probabilities still occurred, which can be attributed to memory-related errors (Sundh and Juslin, 2018). Biases in these scenarios may be explained by cognitive heuristics such as the availability heuristic (Tversky and Kahneman, 1974) or random noise in retrieval processes (Costello and Watts, 2016). Costello and Watts (2014) suggest that while people's knowledge of probabilities adheres to probability theory, their actual probability judgments are perturbed by random noise during memory retrieval.

Additionally, deviations in subjective probability estimates were more pronounced for intermediate probabilities (e.g., 80%, 70%, 30%, and 15%) compared to extreme ends. This pattern is well-documented in the probabilistic reasoning literature, where individuals often struggle with estimating probabilities that fall in the middle of the spectrum (Khaw et al., 2021; Prelec, 1998; Tversky and Kahneman, 1992). Cognitive models, such as Prospect Theory, explain this phenomenon by showing that people tend to overestimate low-probability events and underestimate high-probability ones (Smith et al., 2008). This non-linear perception of probabilities highlights the complexities in human decision-making and probability assessments.

The contrast between structured and unstructured scenarios demonstrated the adaptabil-

ity of negative ranks in understanding belief systems across various contexts. In controlled environments, the mathematical relationship between ranks and subjective probabilities provided insight into how ranks relate to probability. In everyday, unstructured situations, negative ranks accounted for the influence of external factors and personal cognition, highlighting how cultural and personal biases shape belief formation. This distinction deepens our understanding of cognitive processes and suggests new research avenues for examining how individuals adapt their beliefs in response to environmental cues.

Future research could investigate transitions between different probabilistic environments to better understand how individuals recalibrate their belief systems via ranks in response to changing external cues. Additional studies may explore the application of various mathematical models to provide deeper empirical insights, capturing the complexities of belief and disbelief across different scenarios. This continued investigation will improve the understanding of how ranking systems can be used to measure belief and disbelief in psychological research.

6.4.4 Comparison of Subjective Probabilities and Ranks with Objective Probabilities

The comparison between subjective probabilities, negative ranks, and objective probabilities highlighted key differences in how individuals assess probabilistic information. While subjective probabilities varied among participants, ranks consistently aligned more closely with objective probabilities, suggesting they more accurately reflect the ground truth. This

alignment indicates that participants often struggled with accurate probability estimation. Ranks offered a more straightforward and less error-prone tool for evaluating uncertainties. This supports the theoretical benefits of ranking functions as simpler and less cognitively demanding than traditional probability assessments.

Negative ranks offered a practical approach for understanding probabilistic scenarios, particularly in experimental settings. This method required fewer computational steps than traditional probability theory. Negative ranks can function on a relative scale of disbelief, avoiding the precision typical of probability estimates. For instance, in the evenly distributed scenario [$33.\overline{3}$, $33.\overline{3}$, $33.\overline{3}$], negative ranks reflected a uniform level of disbelief across all rooms, though individual ranks varied. Some participants perceived the three rooms as equally surprising (e.g., negative rank of 1,1,1), while others viewed them as equally unsurprising (e.g., negative rank of 0,0,0). In contrast, in the [70-30-0] distribution, the 30% room, despite having a probability near $\frac{1}{3}$, had different negative ranks due to its relation to the more frequently chosen 70% room and the never chosen 0% room, indicating a moderate level of surprise. This was greater than for the 70% room but much less than for the 0% room. These findings suggest that negative ranks may be less cognitively demanding than calculating precise probabilities, which often require more mental resources and are prone to errors due to difficulties in estimating randomness and handling probabilistic information.

Implications for Ranking Theory and Cognitive Processes

The results of this experiment have significant implications for understanding the cognitive processes underlying belief and disbelief representations. The alignment of negative ranks

with objective probabilities suggests that ranking functions may offer a more fundamental or direct cognitive representation of disbelief compared to subjective probabilities. This finding supports the hypothesis that belief, as represented by ranking theory, may be inherently rank-based, simplifying the computational demands of assessing environmental uncertainties.

The differences between subjective probabilities and ranks that have been observed require further research into the cognitive mechanisms that drive these variations. Understanding how individuals process and react to probabilities in different situations can provide valuable insights into decision-making and uncertainty management. Ultimately, this contributes to the broader discussion on modelling belief and disbelief within psychological and cognitive science research.

6.4.5 Conclusion

In summary, Experiment 4 showed that ranking theory offers a strong framework for understanding how people perceive and represent uncertainty. The fact that negative ranks align more closely with objective probabilities demonstrates the potential of ranking functions as a dependable measure of belief dynamics. These results enhance our understanding of belief systems and their relationship with objective data, opening up opportunities for further research to investigate the cognitive structure of belief and disbelief representations.

Chapter 7

Experiment 5: Rethinking the Ellsberg Paradox through Negative Ranking Functions

7.1 Introduction

Previous experiments demonstrated that individuals can use ranks to express both uncertainty and beliefs, which is not a simple transformation of subjective probability. Ranks provide a sensitivity to context that subjective probabilities lack and may more accurately reflect environmental frequencies. To further investigate its relevance in decision-making under uncertainty, we explored its value in settings where subjective probability fails to maintain coherence.

The Ellsberg paradox exemplifies a scenario where subjective probability lacks coherence and challenges classical decision theory, providing insights into human behaviour under uncertainty. Although Ellsberg proposed several thought-experiment decision problems, the most frequently cited is the Ellsberg (three-colour) one-urn problem (Ghirardato and Mari-

nacci, 2002; Slovic and Tversky, 1974; Oechssler and Roomets, 2015). This involves an urn filled with 90 balls: 30 are red, while the remaining 60 are either black or yellow in unknown proportions, ranging anywhere from all black to all yellow. Decision-makers are presented with two pairs of bets based on the colour of the drawn ball. Typically, individuals prefer bets with known probabilities over ambiguous ones (Ellsberg, 1961; Oechssler and Roomets, 2015).

For example, a bet on a 'red' ball offers a definite $\frac{1}{3}$ chance of winning a \$100 prize, which is preferred over a bet on a 'black' ball where the probability of winning varies ambiguously between 0 and $\frac{2}{3}$. Despite the average probability of a bet on a 'black' ball potentially being favourable, individuals opt for the certainty of the known $\frac{1}{3}$ probability. Similarly, a bet on a 'black or yellow' ball guarantees a $\frac{2}{3}$ chance of winning, in contrast to a bet on a 'red or yellow' ball, where the win probability could be anywhere from $\frac{1}{3}$ to 1 (Ellsberg, 1961).

This preference for known probabilities, termed ambiguity aversion, challenges expected utility theory (Ellsberg, 1961; Machina and Siniscalchi, 2014; Savage, 1972). Expected utility theory asserts that decision-makers maximize expected utility based on subjective probabilities derived from their preferences (Halevy, 2007; Savage, 1972). However, Ellsberg's findings revealed a strong preference for known risks, contradicting this framework and the Sure-Thing Principle, a core component of expected utility theory (Aerts and Sozzo, 2011; Savage, 1972). This principle posits that if an individual prefers one option in a certain situation, they should continue to prefer it even when an event affecting both options equally is introduced, regardless of the event's outcome (Aerts and Sozzo, 2011; Savage, 1972).

Ellsberg (1961) demonstrated that in real-world decision-making, individuals often exhibit non-neutral attitudes toward ambiguity, differentiating between risk, where probabilities are known, and uncertainty, where probabilities are unknown. This distinction has practical implications, particularly in economics, where decision-making models typically rely on subjective probabilities to define and solve problems (Halevy, 2007). Ellsberg's findings show that decision-makers do not always exhibit probabilistic sophistication, revealing descriptive limitations within expected utility theory and suggesting the need for models that account for non-standard attitudes toward ambiguity (Halevy, 2007).

Ellsberg's experiments explored people's subjective beliefs by observing their choices (Ellsberg, 1961). However, inferring these beliefs from choices may be challenging, especially when choices do not allow for expressions of indifference or uncertainty. For example, choosing a bet on a red ball over a black one does not necessarily indicate a stronger belief in drawing a red ball if the option to express neutrality is absent. Additionally, the presence of monetary stakes (hypothetical or real) may further complicate these interpretations, as decision-makers could prioritize potential gains over their true beliefs about outcomes. It is important to consider that choice preferences, even when indifference options are available, may not accurately reflect beliefs but could instead represent strategic responses to the incentives of the experimental setup (Kovářík et al., 2016).

To address these challenges, we incorporated ranking theory into the one-urn Ellsberg scenario. Our approach employed a negative ranking function to directly measure individuals' degrees of disbelief about the outcomes of drawing from an urn. By using this function, participants could articulate their levels of disbelief regarding a proposition, allowing them

to assess varying degrees of uncertainty across different outcomes. Through these measures of disbelief, we aimed to examine the implications of the Ellsberg scenario for expected utility theory and its assumptions about human behaviour, potentially revealing new insights into how people evaluate risk and uncertainty.

In our experiment, we simplified the measurement process by using negative ranking functions that focused solely on disbelief in a proposition, rather than both the proposition and its negation. This approach aligned more closely with the Ellsberg study's betting scenarios, reflecting the original intent of the one-urn scenario.

7.2 Methods

7.2.1 Participants

We conducted an online experiment and recruited 209 undergraduate students from the University of Waterloo to participate in exchange for course credit. The participants' median age was 21 years. There were 152 females, 51 males, and 6 preferred not to answer.

7.2.2 Procedure

Participants were presented with a hypothetical scenario involving an urn filled with coloured balls. The urn contained a total of 90 balls, which were of three different colours: red, black, and yellow. The number of red balls in the urn was fixed at 30 and was known to the participants. However, the distribution of the remaining 60 black and yellow balls was not

disclosed, which created an element of uncertainty.

The participants were informed that one ball would be randomly drawn from this urn. They were then asked to provide their degree of disbelief if the ball drawn was 'red', 'black', the disjunction, 'red or yellow', and 'black or yellow'. The input options provided in the task environment were used to record their disbelief responses (see Figure 13). Finally, the participants were asked to rate the difficulty of quantifying their disbelief, providing insights into the perceived challenges posed by the task.

7.2.3 Task Environment

The study used a custom-built online platform developed using JavaScript to capture participant responses (de Leeuw et al., 2023). The platform provided an interface that allowed participants to input their answers as whole numbers, starting from 0 and going up to any positive value, with an option to select 'infinity' via a checkbox. Participants completed the study in less than 15 minutes.

7.2.4 Data Analysis

The principal objective of this analysis was to evaluate the use of rank-based assessments in measuring subjective beliefs towards the one-urn paradox, specifically to determine whether the inconsistency in subjective probabilities, as observed in the original Ellsberg experiment, still existed when using ranks. By doing so, we aimed to gain insight into the effectiveness of rank-based assessments in contexts where subjective probability fails to maintain coherence.

Given that ranks provide a sensitivity to context that subjective probability lacks, we sought to determine whether they would more accurately capture participants' beliefs towards the contents of the urn.

Analysis of Disbelief Responses

Descriptive statistics were computed for the degrees of disbelief associated with each proposition (red, black, red or yellow, and black or yellow). We analyzed the proportion of participants displaying ambiguity aversion, attraction, indifference, and consistent preferences, as these tendencies are crucial for understanding decision-making under uncertainty. To identify significant differences in degrees of disbelief across propositions, a one-way analysis of variance (ANOVA) was conducted. Visualization techniques, using the ggplot2 package in R (Wickham, 2016), helped clarify the distribution of responses indicating ambiguity aversion and attraction.

7.2.5 Ethical Considerations

The study obtained ethics clearance from the University of Waterloo Research Ethics Board (REB 45165). Participants were informed about the study's purpose and their right to withdraw from the study at any time without penalty. To protect the privacy of the participants, all data were stored securely with no identifying information.

7.3 Results

7.3.1 Ambiguity Aversion

In assessing the expression of disbelief through negative ranking functions in the one-urn scenario, a small percentage of participants, 6.22%, exhibited paradoxical behaviour. Such paradoxical behaviour can be explained by ambiguity aversion when the participants were less surprised if a 'red' ball was drawn versus a 'black' ball. Similarly, they were also less surprised when a 'black or yellow' ball was drawn than a 'red or yellow' ball. These proportions are illustrated in Figure 7.1.

7.3.2 Ambiguity Attraction

Another finding from the study was the relatively rare attraction to ambiguous conditions among participants. The results revealed that a small percentage, 3.35%, showed a preference for ambiguity. This preference was evident when participants were less surprised by drawing a 'black' ball compared to a 'red' ball, and similarly less surprised when a 'red or yellow' is drawn compared to 'black or yellow' ball. This observation, which is depicted in Figure 7.1, suggests a unique and uncommon response to ambiguous conditions among participants.

7.3.3 Consistent and Selective Neutrality

A significant portion of participants expressed neutrality, which we classified into two distinct types: Consistent Neutrality and Selective Neutrality.

Consistent Neutrality was observed when participants displayed neutrality across both proposition comparisons: 'red versus black' and 'red or yellow versus black or yellow.' In this category, 43.54% of participants showed no difference in their degrees of disbelief between the propositions, indicating a general tendency to remain neutral, regardless of whether the probabilities were known or uncertain.

Selective Neutrality occurred when participants exhibited neutrality in one comparison but expressed a difference in disbelief in the other. 12.44% of participants were neutral in the simple probability propositions ('red versus black'), but differed in their degrees of disbelief for the compound probabilities ('red or yellow versus black or yellow'). Conversely, 14.83% participants exhibited neutrality in the compound probability propositions but not in the simple comparisons. This suggests that participants experienced varying levels of uncertainty when evaluating disjunctions (e.g., 'red or yellow') compared to simple propositions (e.g., 'red'). Figure 7.2 illustrates the proportions of participants exhibiting neutrality in each proposition comparison.

7.3.4 Consistent Preferences

Our study identified consistent patterns of disbelief when participants evaluated propositions in both comparisons: 'red versus black' and 'red or yellow versus black or yellow.'

Consistent Disbelief in Red: Participants in this category consistently expressed less surprise when a 'red' ball was drawn compared to a 'black' ball, and similarly, less surprise when a 'red or yellow' ball was drawn compared to a 'black or yellow' ball. Specifically,

12.92% consistently demonstrated such preference.

Consistent Disbelief in Black: Similarly, some participants consistently expressed less surprise when a 'black' ball was drawn compared to a 'red' ball, and less surprise when a 'black or yellow' ball was drawn compared to a 'red or yellow' ball. 6.7% consistently demonstrated such preference.

These findings suggest that while participants were less surprised by one colour in the 'red versus black' comparison, some maintained that pattern of disbelief in the 'red or yellow versus black or yellow' comparison. The consistency of these disbelief patterns is illustrated in Figure 7.1.

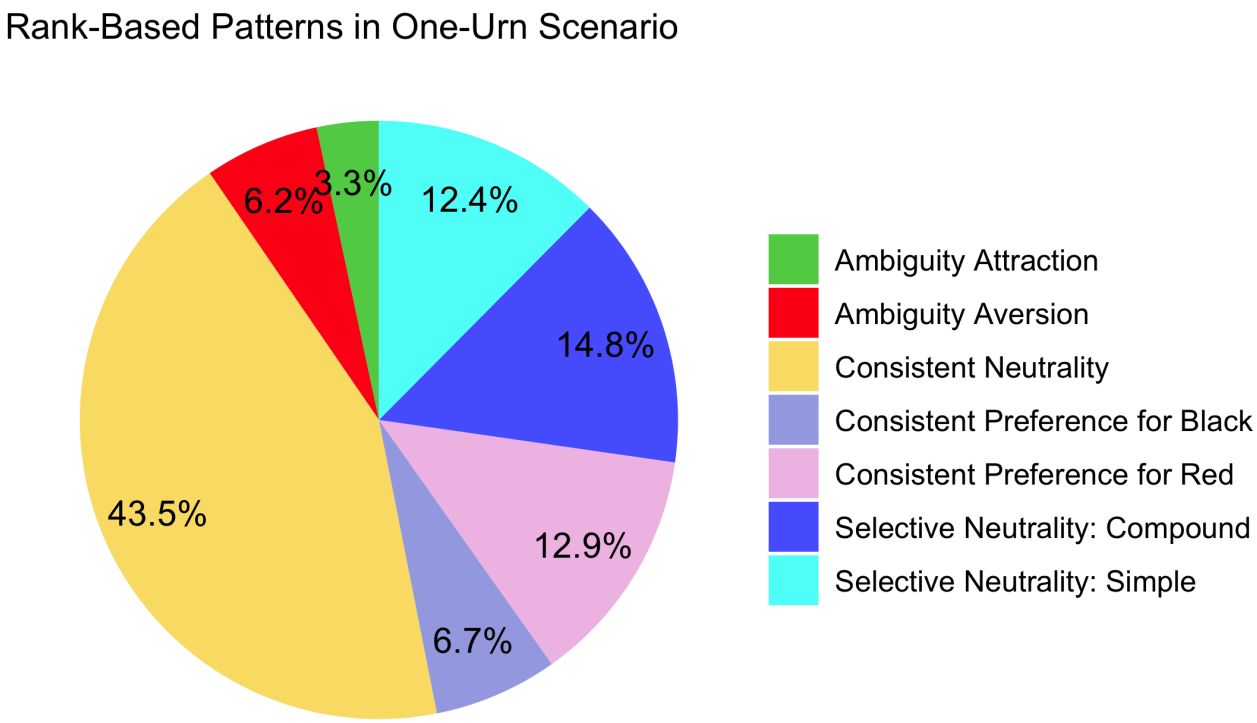


Figure 7.1: This pie chart illustrates the behavioral patterns observed in the Ellsberg one-urn scneario. It shows variations in participants' responses, including ambiguity aversion, a preference for ambiguity, consistent and selective neutrality, and consistent preferences for red and black. These findings highlight the diversity of participant behaviour under ambiguous conditions.

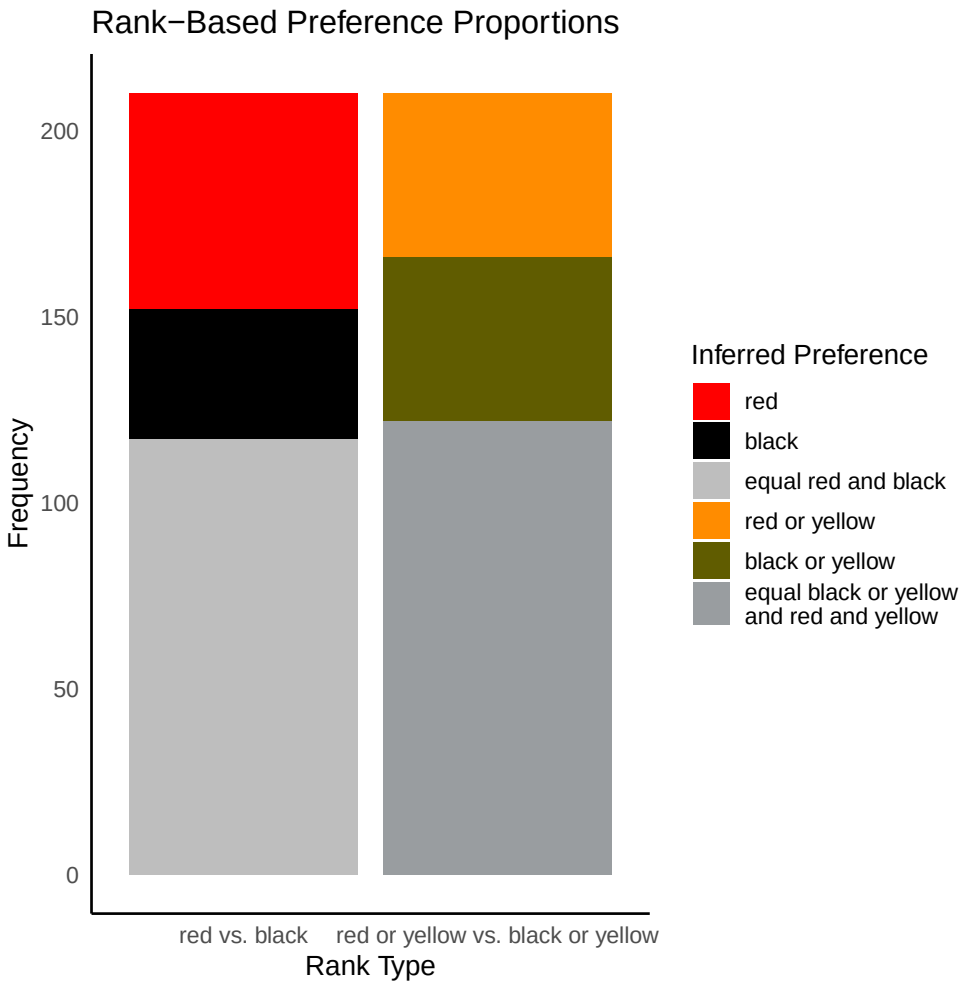


Figure 7.2: The figure displays the distribution of preferences based on degrees of disbelief (negative ranks) across participants. The stacked bars in the figure show the distribution of preferences for (a) 'Red' versus 'Black', and (b) 'Red or Yellow' versus 'Black or Yellow', and situations of equal disbelief between the two. Preferences were determined by comparing the degrees of disbelief. For example, a preference for 'red' over 'black' was noted when disbelief in drawing a red ball was less than that for a 'black' ball, and vice versa. Situations where disbelief in 'red' and 'black' were equivalent are represented as "equal" preferences. The results show that a higher proportion of participants had equal disbelief between the two propositions.

7.3.5 Degrees of Disbelief Across Propositions

The statistical analyses began with an assessment of the underlying assumptions for one-way ANOVA. Infinity rank values, indicating impossibility, were excluded as outliers. These values were illogical, given the urn contained 90 balls in total, with 30 red balls and 60 either black or yellow. This exclusion accounted for 5.12% of data. Homogeneity of variances was confirmed via Levene's test, yielding no significant differences across the groups, $F(3, 793) = 0.1344$, $p = .940$. However, the Shapiro-Wilk test indicated a failure to meet the normality assumption, with disbelief measures showing significant deviation from normal distribution $W = 0.0405$, $p < .001$. A subsequent log transformation attempted to correct this skewness was also unsuccessful in achieving normality $W = 0.831$, $p < .001$.

A one-way ANOVA was carried out despite the challenges. The reason for proceeding with ANOVA was because of its robustness to the normality assumption, particularly in cases of large sample sizes. This robustness is important because even if the data does not follow normal distribution, the impact on type I error rate is usually minimal (Schmider et al., 2010). The study's large sample size supports this reasoning, as it provides some protection against the increase of type I error rates. The results of the ANOVA indicated that there was no significant difference in disbelief scores among the various propositions, $F(3, 793) = 0.13$, $MSE = 139410403623.81$, $p = .940$, $\eta_p^2 < .001$.

To ensure the robustness of these findings, a non-parametric Kruskal-Wallis test was also conducted, suitable for data that does not meet the assumption of normality. This test similarly indicated no significant differences across groups, $H(3) = 3.03$, $p = .387$.

The consistency of non-significant findings across both parametric and nonparametric tests suggested that there was no evidence to support a difference in disbelief measures among the propositions, as visualized in Figure 7.3.

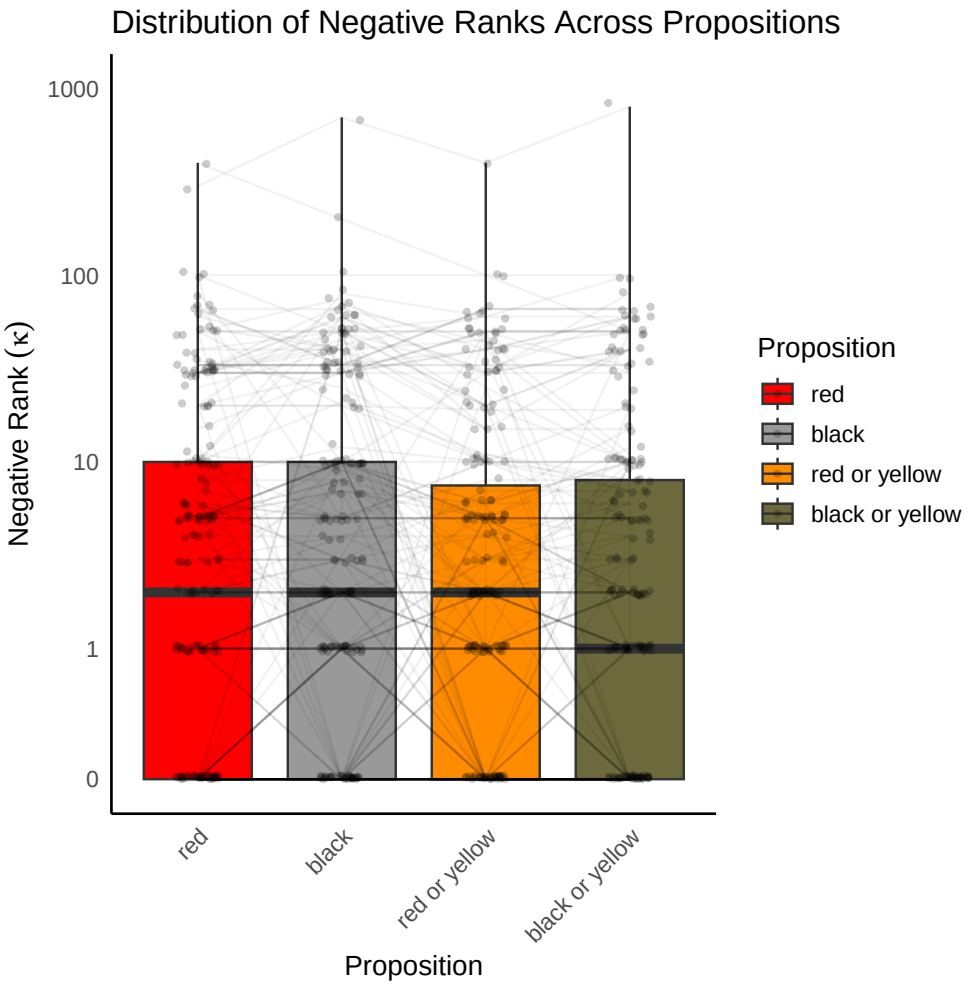


Figure 7.3: This diagram shows how negative ranks (degrees of disbelief) are distributed across propositions. Negative ranks were adjusted on a logarithmic scale to include zero values, with zero represented by a small positive constant, epsilon. The plot uses boxplots, jittered points, and lines to show variations and trends in disbelief responses for each participant across propositions. The y-axis is transformed using a base 10 logarithmic scale. The data highlights both the central tendency and variability within each proposition group, providing insights into participants' degrees of disbelief, which did not differ significantly across the propositions.

7.4 Discussion

This study contributes to a deeper understanding of the Ellsberg Paradox, which traditionally highlights a discrepancy between empirical choices and the predictions of expected utility theory. Ellsberg demonstrated that individuals tend to favour bets with known probabilities over those with unknown probabilities, reflecting a widespread aversion to ambiguity (Ellsberg, 1961). Such behaviour is termed "paradoxical" because it contradicts the subjective probability hypothesis (Machina and Siniscalchi, 2014). However, our findings challenge this view, showing that ambiguity aversion was less prevalent than previously reported.

Participants in previous studies showed varying degrees of ambiguity aversion, with some preferring known risks over ambiguous ones, illustrating different responses to known and unknown risks (Binmore et al., 2012; Slovic and Tversky, 1974). A comprehensive review indicated that, on average, slightly more than 50% of subjects in 40 experimental studies on the Ellsberg urn problem exhibited ambiguity aversion (Oechssler and Roomets, 2015). In contrast, our study revealed that only 6.22% of participants displayed ambiguity aversion when utilizing negative ranking functions to assess degrees of disbelief.

Attitudes toward ambiguity can be categorized into three behavioural responses: aversion, attraction, and neutrality (Dominiak et al., 2012). We found that neutrality was the most prevalent response, with a smaller proportion of participants displaying ambiguity aversion or attraction. Participants demonstrated consistent neutrality across both simple (e.g., 'red vs. black') and compound probabilities (e.g., 'red or yellow vs. black or yellow'), indicating logical consistency regardless of ambiguity.

A notable finding was the identification of selective neutrality, where participants exhibited neutrality in either simple or compound probability comparisons but not consistently in both. This suggests a form of logical inconsistency and a violation of the Sure-Thing Principle. In this scenario, the Sure-Thing Principle holds that if individuals are indifferent between two simple lotteries L_1 and L_2, they should also remain indifferent when these lotteries are mixed with a third lottery L_3 with the same probability (Aerts and Sozzo, 2011). The selective application of neutrality suggests that participants' preferences were influenced by how probabilities were framed.

The shift in preferences between simple and compound probability comparisons may be attributed to the added complexity introduced by disjunctions. Shafir (1994) suggests that individuals often struggle to account for underlying disjunctions in uncertain situations, failing to fully consider each possible outcome. Our findings align with this, as the introduction of an additional element in compound scenarios, such as the 'yellow' ball, likely increased complexity and uncertainty, leading to changes in beliefs. This is because disjunctions represent multiple possible states — either one state will occur, or another. To make decisions under uncertainty, individuals must account for the outcomes of each potential state (Shafir, 1994). The increased uncertainty surrounding combined outcome probabilities likely contributed to the observed shifts in beliefs. The difficulty of reasoning through disjunctions is well-documented, with researchers consistently noting the challenges individuals face when evaluating disjunctive probabilities, particularly within probabilistic frameworks (Bar-Hillel, 1973; Carlson and Yates, 1989; Tversky and Kahneman, 1974).

Additionally, some participants consistently disbelieved outcomes such as black or red

balls, regardless of the probability structures involved. These individuals demonstrated logical consistency, indicating that their beliefs remained stable even in the presence of ambiguity. Their consistent preferences across different scenarios can be attributed to the Sure-Thing Principle.

Contrary to traditional findings, where ambiguity aversion was typically dominant, our results indicated that most participants showed logical consistency rather than ambiguity aversion. One possible explanation for this shift is our experimental design, which allowed participants to express neutrality through negative ranks, thereby reducing the ambiguity aversion typically observed in such studies. Ranks provided greater coherence in participants' belief systems, highlighting their utility in decision-making under uncertainty. While most participants adhered to Savage's Sure-Thing Principle, as evidenced by consistent neutrality and preferences, those who did not were primarily characterized by selective neutrality, rather than ambiguity aversion or attraction. These results challenged the assumption that the Ellsberg Paradox was inherently reflective of human cognitive processes, suggesting that ambiguity aversion may have been an artifact of specific experimental setups.

Our analytical approach, including one-way ANOVA and Kruskal-Wallis tests, confirmed the robustness of these findings. Despite normality assumptions not being met, these tests showed no significant differences in disbelief scores across the propositions.

While the experiment was designed to reduce complexity and focus purely on measuring disbelief, this simplification may have influenced the outcomes. The next chapter will investigate this by replicating the initial Ellsberg scenario, incorporating both choice preferences

and subjective belief measures through ranking functions. This approach aims to determine whether the observed indifference is representative of broader patterns or unique to the experimental conditions. These findings highlight the importance of carefully considering how experimental designs and methodologies influence participant responses. The question of whether beliefs can be inferred from choice preferences will be explored in greater detail in the following chapter.

Chapter 8

Experiment 6: Comparing Traditional Betting Choices and Negative Ranks in the Ellsberg Paradox

8.1 Introduction

Experiment 5 demonstrated that allowing participants to express neutrality reduced ambiguity aversion, thus challenging the traditional interpretation of the Ellsberg Paradox. However, Experiment 5 did not incorporate participants' betting choices, a critical element in understanding decision-making, as these choices form a core methodological component of Ellsberg's original experiment.

In Experiment 6, we addressed this limitation by reintroducing betting scenarios central to the Ellsberg Paradox. Participants were asked to express their degrees of disbelief prior to making their choices. This approach aimed to replicate the findings of Experiment 5, while also examining how beliefs influence choice preferences under uncertainty. The study further aimed to clarify the extent to which betting choices reflect participants' 'beliefs', as

suggested by Ellsberg in his original work (Ellsberg, 1961).

8.2 Methods

8.2.1 Participants

We recruited 301 undergraduate students from the University of Waterloo for an online experiment in exchange for a course credit. We excluded 11 participants who failed attention checks. Thus, 290 participants were included in the analysis (median age: 19 years; 228 females; 52 males; 1 other; 9 preferred not to answer).

8.2.2 Procedure

The three-coloured one-urn scenario replicated that of Experiment 5 (see page 188). In this replication, participants imagined an urn with 90 balls: 30 red, with the remaining 60 being a mix of black and yellow, whose exact distribution was unknown. A random ball would be drawn from the urn, and participants provided disbelief rankings for drawing a red, black, red or yellow, and black or yellow ball.

Novel to Experiment 6, participants were then prompted to envision the same urn with the same number of balls as before. They faced two new betting scenarios, where they had to choose between receiving $100 for a certain colour or nothing. The scenarios were: (a) red vs. black, and (b) red or yellow vs. black or yellow. Participants repeated these two scenarios across four rounds to capture higher-order confidence in their choices. All four

rounds were displayed together on the screen, allowing participants to make selections for all rounds simultaneously. This setup enabled participants to express neutrality by choosing the same gamble twice in each scenario across the four rounds, a design choice informed by the absence of an option for indifference in Ellsberg's (1961) one-urn paradox (see Figure 14).

The study further introduced a rarity one-urn paradox scenario, where participants imagined an urn with 1000 balls of four colours: red, black, yellow, and green. There was one red ball, two balls of either black or yellow, and 997 green balls. Participants assigned degrees of disbelief to this scenario, maintaining the red-to-black/yellow ratio while increasing rarity to elicit stronger degrees of disbelief or surprise (see Figure 15).

To ensure attentiveness, an exclusion criterion was employed. Participants were required to answer questions about the colours and total number of balls in the urn, serving as a check for engagement. Those who failed this question were excluded from the analysis.

8.2.3 Data Analysis

Relationship Between Disbelief and Betting Choices:

This analysis explored the relationship between participants' degrees of disbelief and their choices in two distinct betting scenarios: (a) choosing between red and black, and (b) choosing between red or yellow and black or yellow.

The differences in negative ranks for the propositions were calculated and compared against participants' preferences, which were recorded over four rounds. Preferences were quantified by the number of times participants chose red in scenario (a) and black or yellow

in scenario (b), with counts ranging from 0 to 4. A count of 0 indicated no choice of the colour across all rounds, while a count of 4 indicated a consistent preference.

For scenario (a), the difference in negative ranks was computed as $\kappa(black) - \kappa(red)$. A negative value suggested a higher disbelief in red relative to black, while a positive value suggested lower disbelief in red. A similar computation was used for scenario (b) with κ(red or yellow) $- \kappa$(black or yellow).

Pearson's correlation analyses were conducted to assess the relationship between betting choices and degrees of disbelief. If a relationship existed, positive correlations were expected between the participants' disbelief and their betting choices. A neutral count of 2 would indicate no preference, aligning with a disbelief difference value of zero, indicating equal disbelief in both options.

Box plots were used to visualize the relationship between disbelief differences and betting choices. For the black vs. red scenario, the y-axis represented disbelief differences, and the x-axis indicated the frequency of choosing red. The same approach was applied to scenario (b). A positive linear trend would indicate alignment between participants' beliefs and their betting choices.

8.2.4 Ethical Considerations

The study obtained ethics clearance from the University of Waterloo Research Ethics Board (REB 45629). Participants were informed about the study's purpose and their right to withdraw from the study at any time without any penalty. To protect the privacy of the

participants, all data were stored securely with no identifying information.

8.3 Results

8.3.1 Ambiguity Aversion

Degrees of Disbelief via Negative Ranks:

When participants expressed their degrees of disbelief using negative ranking functions, 8.62% exhibited ambiguity aversion, traditionally viewed as paradoxical. For the rarity urn scenario, 5.17% of participants demonstrated this behaviour.

There was no significant difference in the level of ambiguity aversion between the one-urn and rarity-urn scenarios $\chi^2(1) = 1.85$, $p = .174$. The lack of difference in participants' degrees of disbelief between the one-urn and rarity-urn scenarios — both of which maintained the same proportion of red to black or yellow balls — suggests that the minimal occurrence of ambiguity aversion cannot be explained by a floor effect. While a floor effect was anticipated in the one-urn scenario, where outcomes were generally unsurprising to some, the rarity-urn scenario elicited more surprising outcomes (as depicted in Figure 8.3). Despite this, very few participants exhibited ambiguity aversion in either scenario, which was comparable to the findings from Experiment 5 (see page 191).

Betting Choices:

The analysis of participant betting choices revealed varying levels of ambiguity aversion across the rounds. In Round 1, 48.97% of participants displayed ambiguity aversion (i.e.,

chose 'red' and 'black or yellow'). Across four rounds, 31.38% of participants exhibited ambiguity aversion at least three times across the four rounds.

A chi-squared test for independence revealed a significant decrease in participants exhibiting ambiguity aversion over four betting rounds, $\chi^2(1) = 17.93$, $p < .001$. The four betting rounds allowed participants to express neutrality in their choice preferences. Figure 8.1 illustrates how participants' betting choices changed or remained the same across the four betting rounds for both scenarios. The y-axis indicates whether participants switched their choices (True) or retained their previous choices (False). This methodology enabled participants to modify their preferences across rounds, while others maintained consistent choices throughout.

Comparative Analysis:

The chi-squared test for independence revealed a significant reduction in the number of participants exhibiting ambiguity aversion when comparing negative ranking responses to betting choices across four rounds, $\chi^2(1) = 45.53$, $p < .001$. This result suggests that when participants were given the opportunity to explicitly express their degrees of disbelief using negative ranks, they were less likely to exhibit ambiguity aversion compared to when they made binary betting choices across rounds.

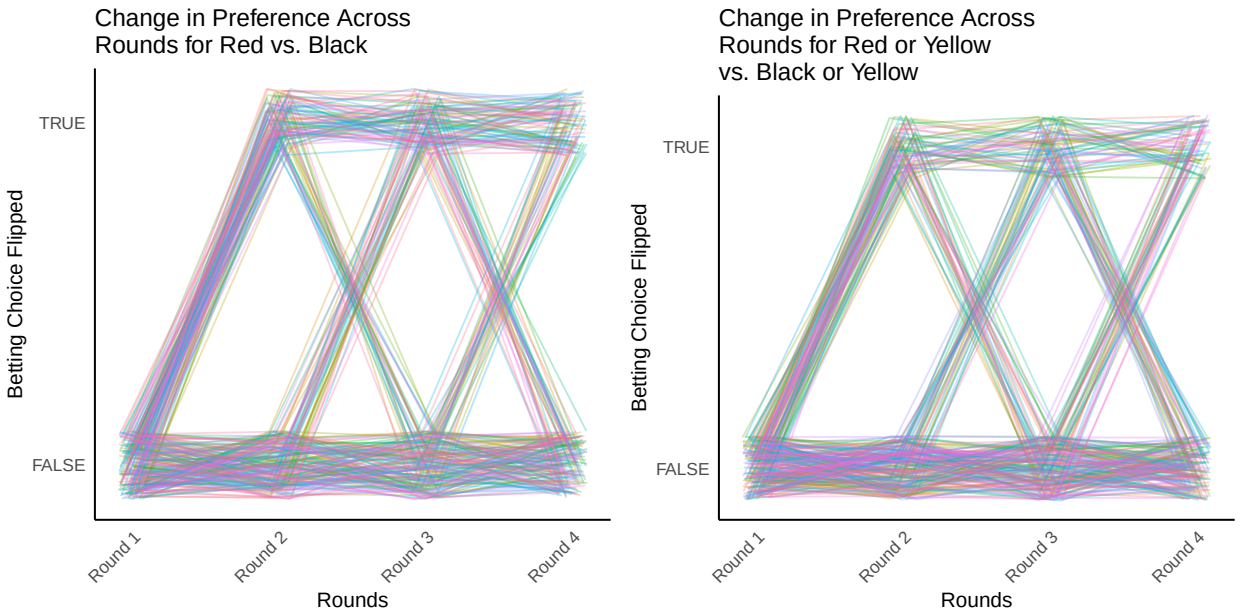


Figure 8.1: Change in choice preference across rounds for Red vs. Black and Red or Yellow vs. Black or Yellow. The plots show the percentage of participants who switched their choices for each round (2, 3, 4). A large proportion of participants did not change their choices across four rounds, while some switched their choices to show neutrality or some degree of confidence in their choices. The y-axis represents whether participants switched their choices (True) or kept the same choices across rounds (False).

8.3.2 Ambiguity Attraction

Degrees of Disbelief via Negative Ranks:

The analysis of negative ranks showed that attraction to ambiguous conditions among participants was limited. Only 3.79% exhibited ambiguity attraction, which was comparable to the findings from Experiment 5 (see page 191). Similarly, in the rarity urn scenario, only 5.86% of participants exhibited ambiguity attraction.

Betting Choices:

When examining choices under conditions of ambiguity, a small percentage of participants

— specifically, 6.55% — showed a preference for ambiguity by choosing 'black' and 'red or yellow' in the first round. However, this attraction to ambiguity decreased to 0.69% for participants who showed ambiguity attraction in at least three of the four rounds.

8.3.3 Consistent Neutrality

Degrees of Disbelief via Negative Ranks:

Consistent neutrality was observed when participants displayed neutrality across both proposition comparisons. In this category, 33.1% of participants showed no difference in their degrees of disbelief across propositions. For the rarity urn scenario, a large proportion of individuals, 44.14%, exhibited consistent neutrality. This proportion was comparable to that of Experiment 5 (see page 191).

Betting Choices:

In the analysis of betting choices over four rounds, participants were given the opportunity to express consistent neutrality (or indifference) towards the outcomes by selecting each option twice out of the four rounds. This approach revealed that 23.1% of participants expressed consistent neutrality, demonstrating the effectiveness of this methodological approach compared to Ellsberg's original framework, which did not allow for the expression of neutrality or indifference towards choices.

8.3.4 Selective Neutrality

Degrees of Disbelief via Negative Ranks:

Selective neutrality was observed when participants exhibited neutrality in one comparison but showed a difference in disbelief in the other. Specifically, 7.59% of participants were neutral in the simple probability propositions ('red versus black'), but differed in their degrees of disbelief for the compound probabilities ('red or yellow versus black or yellow'). Conversely, 14.14% were neutral in the compound probabilities but not in the simple comparisons. This indicates that participants experienced varying levels of uncertainty when evaluating disjunctions compared to simple propositions.

In the rarity urn scenario, 10.34% of participants neutral in the simple propositions but showing differences in the compound comparisons, and 19.66% neutral in the compound comparisons but not the simple ones. Selective neutrality in negative ranks accounted for much of the logical inconsistency observed in this study, as it did in Experiment 5 (see page 191).

Betting Choices:

In the betting choices analysis over four rounds, 13.45% of participants were neutral in the simple propositions, but not in the compound probabilities. Conversely, 7.24% were neutral in the compound probabilities but not in the simple ones. This behavioural pattern highlighted the logical inconsistency in participants' choices.

8.3.5 Consistent Preferences

Degrees of Disbelief via Negative Ranks:

11.38% of participants consistently expected red (and red or yellow) to be more likely

than black (and black or yellow), while 21.38% consistently expected black (and black or yellow) to be more likely, expressing disbelief towards red.

In the rarity urn scenario, 4.83% expected red to be more likely than black, while 10% expected black to be more likely.

Betting Choices:

In the betting choices analysis over four rounds, 13.79% of participants consistently chose red (and red or yellow), while 10.34% consistently chose black (and black or yellow) in at least three of the four rounds.

See Figure 8.2 for a comprehensive overview of participant behaviours across the different conditions.

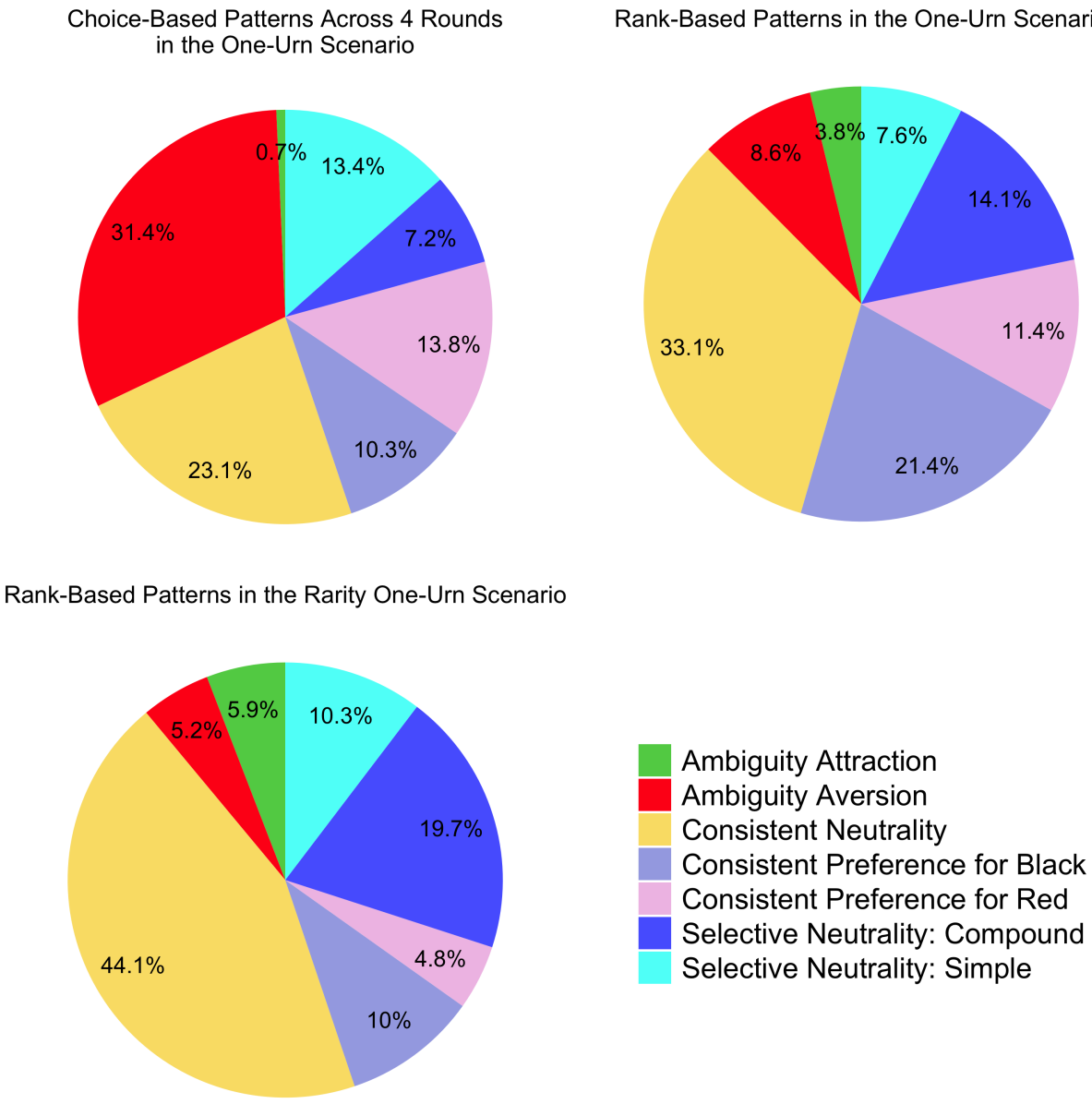


Figure 8.2: Comparative visualization of participant responses: The top row shows pie charts of betting choices and negative ranks in the one-urn scenario, while the bottom row presents the rarity one-urn scenario. These charts illustrate patterns of ambiguity aversion, attraction, neutrality, and preference, highlighting how these behaviours differed across modalities.

8.3.6 Degrees of Disbelief Across Propositions

We conducted a two-way ANOVA to explore differences in disbelief scores across different propositions and urn-scenario types (one-urn and rarity urn). Values indicating impossibility, classified as infinity ranks, were excluded as outliers, as they were considered illogical given that the urn contained balls of each color specified in the questionnaire. The two-way ANOVA revealed no difference in disbelief scores among different propositions, $F(3, 1996) = 0$, $p > .999$, $\eta_p^2 < .001$. However, there was a significant effect of urn-scenario, $F(1, 1996) = 5.21$, $p = .023$, $\eta_p^2 = .003$; overall degrees of disbelief were greater for the rarity-urn scenario ($Mdn = 98$) than for the one-urn scenario ($Mdn = 3$). Additionally, the interaction between proposition and urn-scenario was examined but was not significant, $F(3, 1996) = 0.01$, $p = .999$, $\eta_p^2 < .001$. Thus, the data do not provide sufficient evidence of an urn-scenario effect on the propositions' impact on disbelief scores.

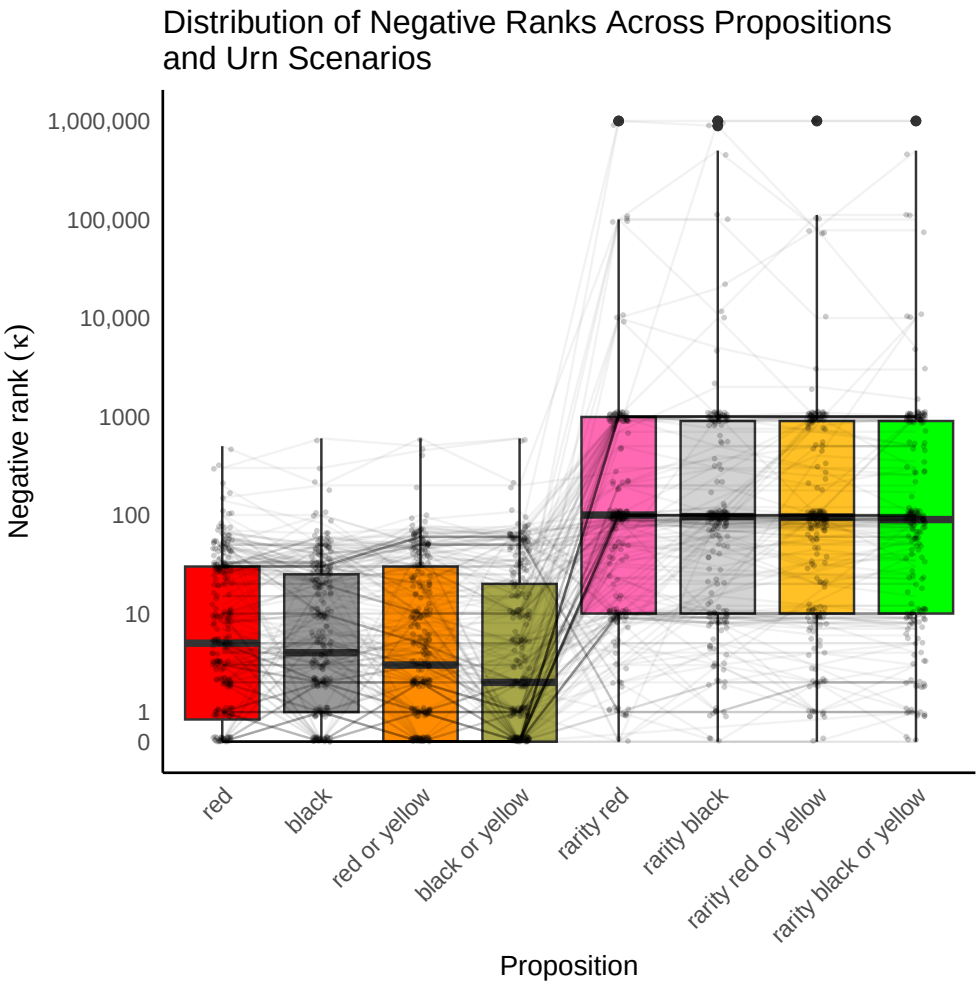


Figure 8.3: Visual Representation of Disbelief Scores Across Propositions and Urn Scenarios: This plot demonstrates the distribution of disbelief ranks on a logarithmic scale, utilizing boxplots to depict central tendencies and variability, jittered points to indicate individual responses, and connecting lines to trace changes in disbelief for each participant across different propositions and urn scenarios. The boxplots show a clear distinction in disbelief levels between one-urn and rarity-urn scenarios, with increased disbelief in the rarity scenario compare to the one-urn scenario. Importantly, within each scenario, no differences in disbelief scores were observed across the propositions.

8.3.7 Relationship Between Disbelief and Betting Choices

For both betting scenarios (red vs. black) and (red or yellow vs. black or yellow), a Pearson's correlation analysis indicated that betting choices and disbelief were not significantly

correlated, $r(276) = .07$, $p = .264$, 95% CI [-0.05, 0.18] and $r(277) = .07$, $p = .278$, 95% CI [-0.05, 0.18], respectively. Thus, the null hypothesis of zero correlation was not rejected.

Visualization: Box plots showed no apparent correlation between disbelief and the frequency of choosing red in the black versus red scenario (see Figure 8.4) or in the black or yellow versus red or yellow scenario (see Figure 8.5). The medians of the box plots were relatively consistent across different frequencies of choices, though some variance was observed in the dispersion of participants' responses.

To assess whether the variance in participants' response was due to varying sample sizes across different choice behaviours, we plotted random data points (in red) with a fixed mean of 0 and a standard deviation matching the overall observed data. Sample sizes, such as $n = 111$ for choosing red four times, and $n = 16$ for choosing red once, influenced the observed variance, suggesting it may have been an artifact of sample size differences.

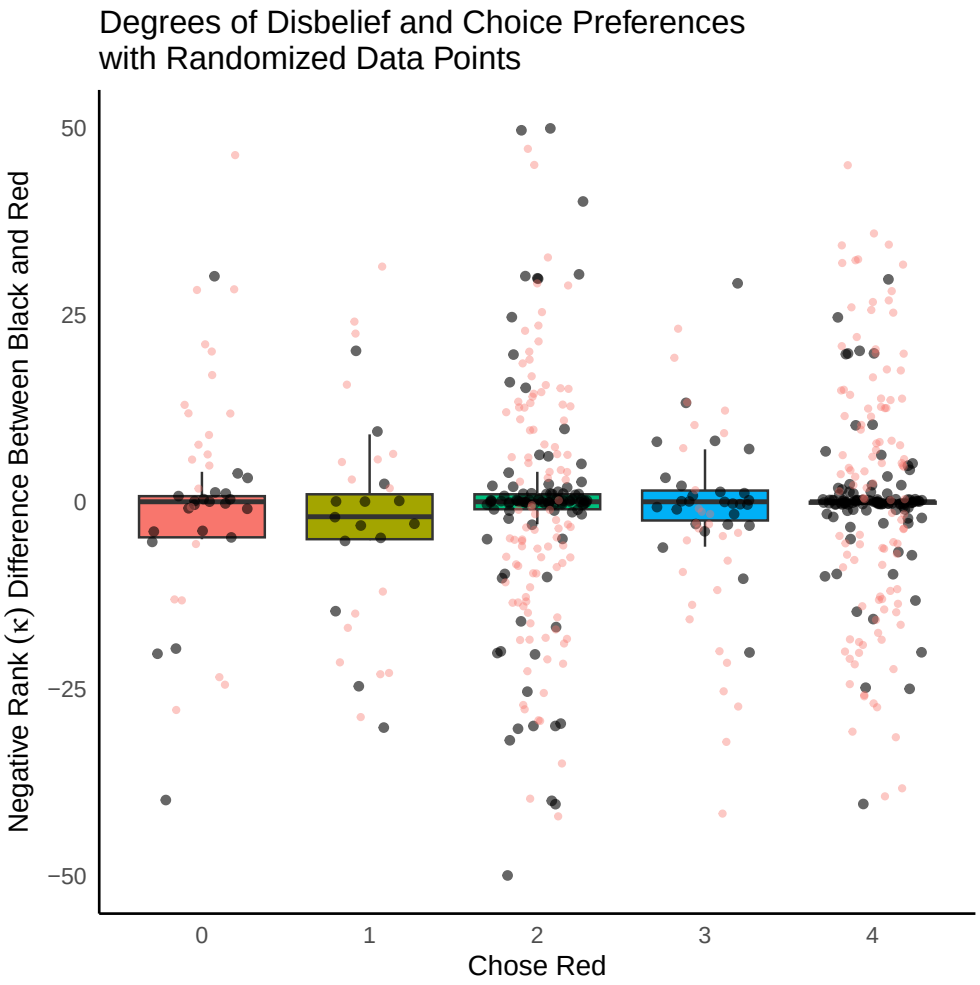


Figure 8.4: Degrees of Disbelief and Choice Preferences with Randomized Data Points. This figure illustrates the relationship between participants' degrees of disbelief and their choices in the black versus red scenario. The y-axis represents the difference in negative ranks (κ) between black and red, with negative values indicating greater disbelief in red; while positive values indicate less disbelief in red. The x-axis displays the frequency of choosing red across four rounds, ranging from 0 to 4 times. Black dots represent actual participant data, while red dots indicate randomized data points generated with a fixed mean of 0 and SD that matches the observed data, but only varying in sample size based on each frequency count of choosing red. The box plots visualize the central tendency and dispersion of disbelief scores for each choice frequency, revealing no apparent correlation between degrees of disbelief and the frequency of choosing red.

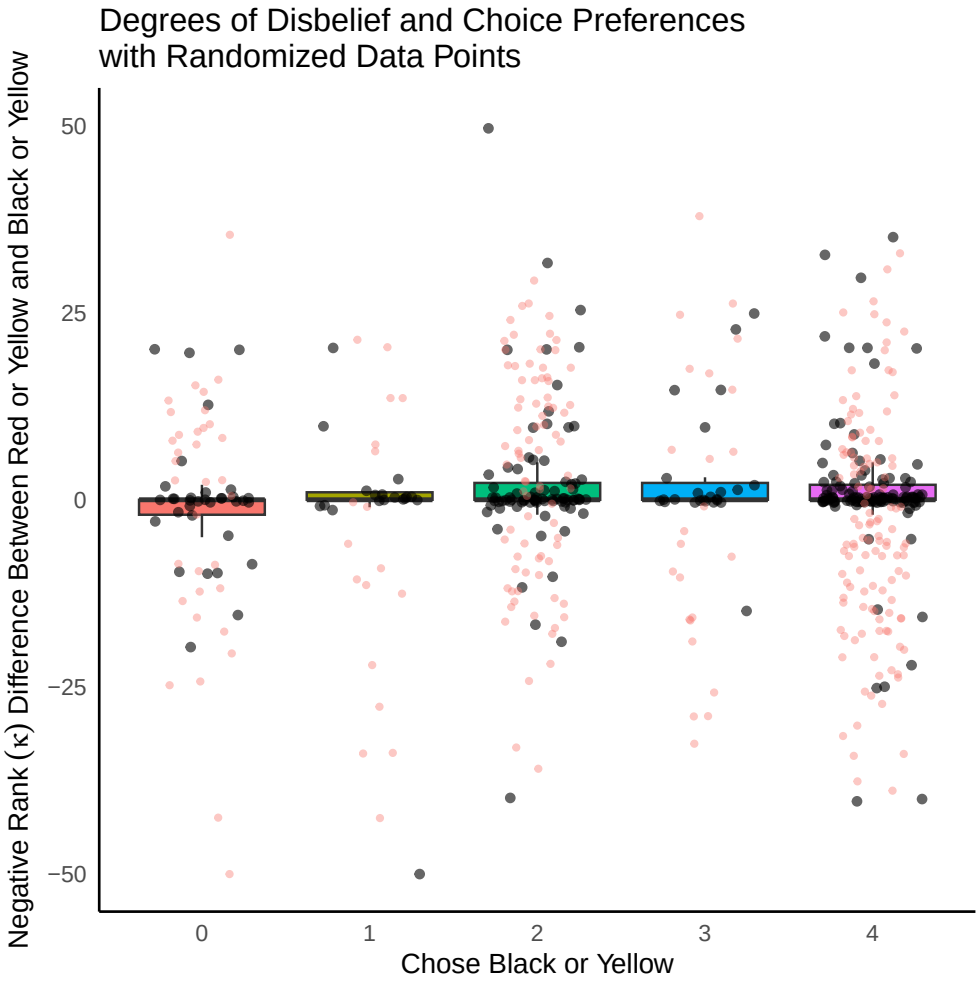


Figure 8.5: Degrees of Disbelief and Choice Preferences with Randomized Data Points. This figure illustrates the relationship between participants' degrees of disbelief and their choices in the black or yellow versus red or yellow scenario. The y-axis represents the difference in negative ranks (κ) between red or yellow and black or yellow, with negative values indicating greater disbelief in black or yellow; while positive values indicate less disbelief in black or yellow. The x-axis displays the frequency of choosing black or yellow across four rounds, ranging from 0 to 4 times. Black dots represent actual participant data, while red dots indicate randomized data points generated with a fixed mean of 0 and SD that matches the observed data, but only varying in sample size based on each frequency count of choosing black or yellow. The box plots visualize the central tendency and dispersion of disbelief scores for each choice frequency, revealing no apparent correlation between degrees of disbelief and the frequency of choosing black or yellow.

8.3.8 Comparison with Randomized Rank Responses

We conducted a randomization analysis to assess whether the observed levels of ambiguity aversion in negative ranking responses could be attributed to random variation. Specifically, we performed 1000 iterations of randomization by shuffling participants' negative ranks within each of the four propositions ("Red," "Black," "Red or Yellow," "Black or Yellow") while preserving the grouping by proposition. This approach tested whether the observed ambiguity aversion could result from random assignment of negative ranks. For each randomization, the percentage of ambiguity-averse responses was computed under two experimental conditions: the one-urn scenario and the rarity one-urn scenario.

The empirical data showed ambiguity aversion percentages for negative ranks of 8.62% for the one-urn condition and 5.17% for the rarity one-urn condition. These percentages were compared with the randomized distributions, which yielded mean percentages of 21.19% for the one-urn condition and 19.86% for the rarity one-urn condition. The observed empirical percentages were distinct from the central tendencies of the randomized distributions (see Figure 8.6). The higher percentages of ambiguity aversion in the randomized distributions further suggest that the observed reduction in ambiguity aversion with negative ranks was not merely due to random assignment of ranks across propositions.

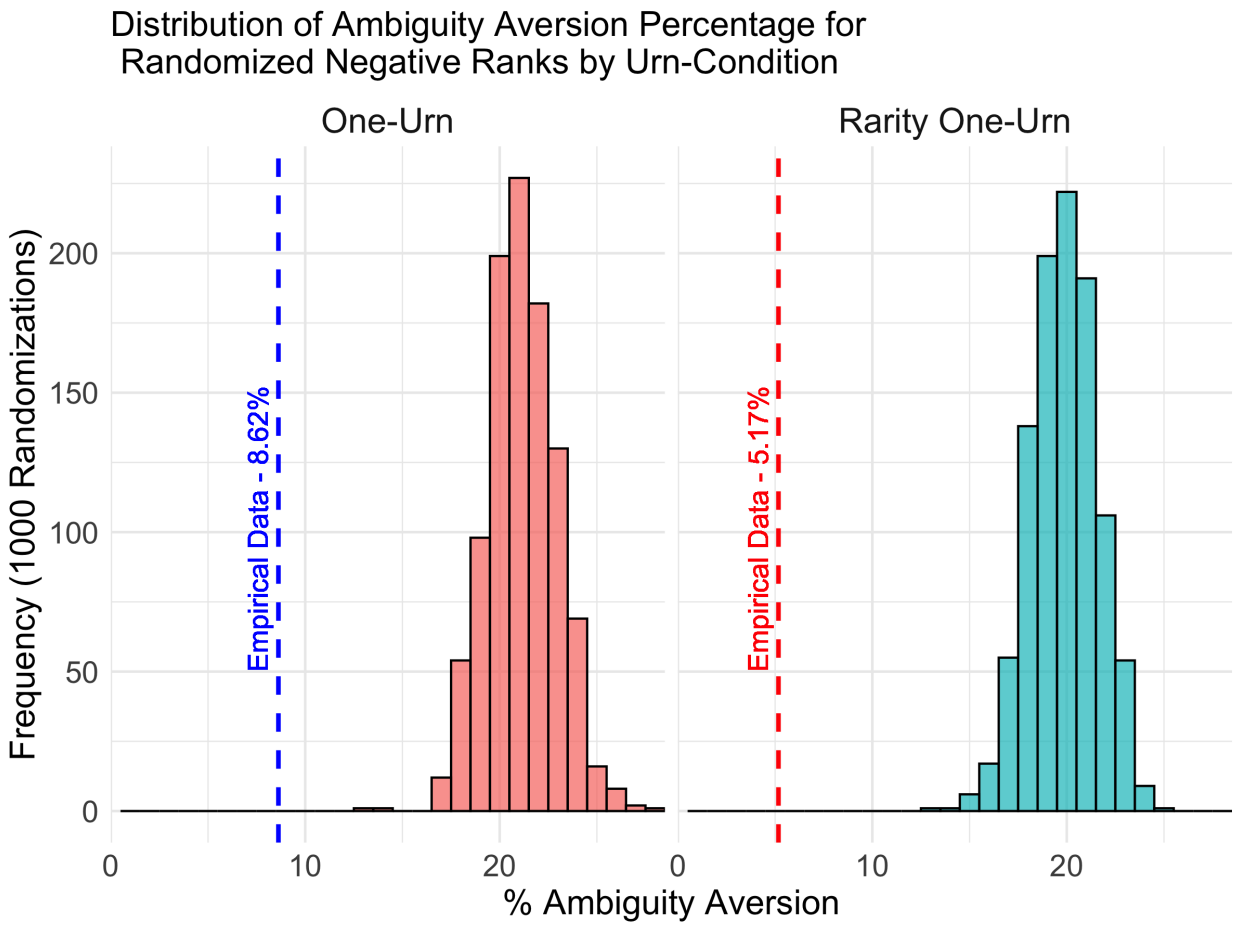


Figure 8.6: The distribution of ambiguity aversion percentages for randomized negative ranks is presented by scenario condition (one-urn and rarity one-urn). The dotted vertical lines represent the observed empirical data for ambiguity aversion: 8.62% for the one-urn condition (blue line) and 5.17% for the rarity one-urn condition (red line). The histograms show the distribution of ambiguity aversion percentages across 1000 randomizations, where negative ranks were shuffled within each proposition ("Red," "Black," "Red or Yellow," "Black or Yellow") across participants. The observed empirical percentages varied from the central tendencies of the randomized distribution.

8.4 Discussion

This study replicated the key findings from Experiment 5, with the addition of betting scenarios to explore the relationship between belief and decision-making under uncertainty.

Consistent with Experiment 5, when participants expressed their degrees of disbelief using negative ranking functions, ambiguity aversion was minimal. However, when participants made betting decisions, ambiguity aversion became more pronounced, with many favouring bets with known probabilities over those involving ambiguity, aligning with the range commonly reported from betting behaviour in the literature (Oechssler and Roomets, 2015).

Contrary to Ellsberg's proposition that gambling choices are influenced by, or 'reflect,' varying degrees of belief (Ellsberg, 1961), no significant relationship emerged between participants' disbelief ranks and their betting preferences. This suggests that betting choices were more susceptible to the disjunction effect, which emphasizes decision avoidance or alteration when faced with uncertainty (Aerts et al., 2014; Tversky and Shafir, 1992). Ambiguity aversion, alongside the disjunction effect, highlights a broader discomfort with uncertainty in decision-making. While ambiguity aversion demonstrates a preference for known risks over ambiguous ones, the disjunction effect reflects how the absence of complete information can lead to changes in decision-making, even when the missing information should not logically influence the choice (Tversky and Shafir, 1992).

Our findings further indicate that when using ranking theory, inconsistent choices — those often associated with ambiguity aversion — were significantly reduced. This suggests that ambiguity aversion may be driven more by the betting context and considerations of costs and benefits, rather than by participants' underlying epistemic estimates. The reduced ambiguity aversion observed in ranking tasks was unlikely to be explained by random assignment of ranks across propositions, as shown by the randomization analysis, which indicated that the observed effect of reduced ambiguity aversion was not a result of random variation.

The observed ambiguity aversion can be further explained by the comparative ignorance and competence hypotheses (Fox and Tversky, 1995; Heath and Tversky, 1991). According to Fox and Tversky (1995), comparative ignorance arises when individuals evaluate bets with known and unknown probabilities side by side, leading to a preference for known probabilities due to discomfort with ambiguity. However, when these bets are assessed in isolation, ambiguity aversion diminishes. In our experiment, participants provided their degrees of disbelief for different outcomes simultaneously, yet we did not observe the expected ambiguity aversion in their disbelief rankings as we did in their betting choices. This suggests that ranking theory allows for a more objective evaluation of uncertainty, one that is less influenced by the comparative evaluation process that typically heightens ambiguity aversion (Chow and Sarin, 2001; Fox and Tversky, 1995).

The competence hypothesis, as proposed by Heath and Tversky (1991), suggests that people's willingness to take risks in uncertain situations is influenced by their perceived competence in the domain. This often leads to reduced ambiguity aversion in areas where individuals feel knowledgeable. In the context of our study, the consistency of participants' disbelief rankings across different outcomes, regardless of whether the probabilities were known or unknown, may reflect perceived competence facilitated by the ranking theory framework. Unlike traditional decision-making contexts, where ambiguity aversion stems from a lack of competence in estimating probabilities, participants in this study demonstrated a more consistent approach to expressing disbelief, reinforcing the reliability of ranking theory as a dependable measure of belief under uncertainty.

This study demonstrated that many participants exhibited consistent neutrality in their

disbelief rankings, aligning with the findings from Experiment 5. Neutrality was observed across both simple and compound probability comparisons, suggesting that for some participants, their belief systems remained stable and unaffected by ambiguity in both the one-urn and rarity scenarios. However, in the betting scenarios, logical inconsistencies were more frequent, indicating that the method used to elicit preferences — whether through disbelief rankings or betting choices — markedly influenced participants' attitudes towards ambiguity and logical consistency.

The inclusion of four betting rounds gave participants the opportunity to refine their betting preferences and display greater confidence in their choices. This design also facilitated the observation of consistent and selective neutrality, which was absent from Ellsberg's original study (Ellsberg, 1961). As in Experiment 5, selective neutrality was accompanied by logical inconsistencies that violated the Sure-Thing Principle of expected utility theory (Savage, 1972). These inconsistencies, observed in both rank-based and choice-based patterns, suggest that some participants experienced varying levels of uncertainty when faced with combined probability outcomes, likely contributing to the shifts in belief and choice patterns (Shafir, 1994).

Overall, most participants demonstrated logical consistency in their negative rank-based assessments, whether through consistent neutrality or a preference for either red or black in both the one-urn and rarity-urn scenarios. However, in the betting rounds, their choice patterns showed a lack of this consistency. According to Expected Utility Theory, individuals assign probabilities to outcomes and choose the option that maximizes utility (Schoemaker, 1982). In our betting scenarios, the expected utility of all choices was identical ($100 payout),

and subjective probabilities — inferred from their bets based on the proportion of black and yellow balls — varied. While these probabilities were not directly recorded, they were derived from participants' preferences (Ellsberg, 1961). These findings suggest that ranking theory may better capture beliefs under uncertainty, reducing inconsistencies in attitudes towards ambiguity compared to traditional models.

One limitation of our study is that hypothetical monetary stakes were not included when participants provided negative ranking responses, unlike in the betting scenarios. We intentionally excluded monetary incentives in the ranking tasks to evaluate the coherence of belief systems without the influence of financial factors, presenting an interesting avenue for future research. Further exploration is needed to determine how incorporating hypothetical monetary incentives might affect belief assessments in the ranking theory framework. However, since participants' beliefs about the same imagined scenarios were measured in a trial before their betting choices, our findings suggest that even without directly reflecting the monetary conditions of the betting scenarios, the insights from ranking responses contribute valuable understanding to decision-making processes.

These findings have significant implications for decision-making theories under uncertainty and emphasize the need for careful experimental design in research on ambiguity and risk assessment. Ranking theory may provide a more reliable framework for measuring beliefs under uncertainty, as opposed to inferring them from betting behaviours. This approach is particularly relevant to behavioural economics (Halevy, 2007; Schröder and Gilboa Freedman, 2020).

Chapter 9

General Discussion

This research tested the normative model proposed by Wolfgang Spohn (2012) by evaluating the empirical adequacy of ranking theory for representing human belief systems. The study contributes to the ongoing discussion about how beliefs and uncertainties should be represented in cognitive science.

9.1 The Utility of Ranks and Their Distinction from Subjective Probabilities

The findings demonstrated that negative ranks reliably measured degrees of disbelief and can be applied to behavioural experiments. Negative ranks effectively captured disbelief across various contexts under uncertainty. Participants consistently used negative ranks in both online and lab environments, aligning with societal and cultural expectations. Currently,

paradigms using probabilities to measure belief are more established than those using ranking functions (Kern-Isberner et al., 2021). Negative ranks, which range from 0 to infinity, do not naturally divide into qualitative regions like probability scales do. Probability scales can be divided into regions such as low, middle, and high degrees of belief. In contrast, negative ranks can only be divided into regions of zero disbelief, above zero disbelief, and maximal disbelief (Kern-Isberner et al., 2021). Despite this, the consistent median and range patterns across experiments suggest that negative ranks effectively measure degrees of disbelief. This method compares disbelief scores relative to other propositions without needing all events to sum to 1. This could mirror natural judgment, where people judge one event as more surprising than another, like a 'rank-based' system, without assigning specific likelihood estimates to each scenario.

Negative ranks fundamentally differ from traditional probabilistic models, especially when objective probabilities are unclear. A non-linear relationship between negative ranks and subjective probabilities was observed. However, subjective probabilities had minimal predictive power on negative ranks under conditions of uncertainty. The exception was Experiment 2's exploratory exponential decay model, which was more flexible in explaining the variance, but not theoretically driven. This general pattern suggests that negative ranks and subjective probabilities capture fundamentally different aspects of uncertainty. Negative ranks showed a broad dispersion of disbelief levels for lower subjective probabilities. Context also influenced disbelief levels, as the same probability could correspond to different degrees of disbelief depending on the context. Therefore, directly inferring belief from subjective probabilities may be inaccurate since probabilities lack the notion of belief. Negative ranks provide a distinct

measure of disbelief, but they are not a direct replacement for probabilities (Spohn, 2009). While probabilities are useful for providing the likelihood of events, ranks offer insight into subjective beliefs and disbelief, indicating a need to reevaluate how beliefs are inferred from subjective probabilities.

In dynamic learning environments, the logarithmic relationship effectively explained the relationship between negative ranks and subjective probabilities. When frequencies are observed and easy to track based on recency, event numbers, and well-defined cases, negative ranks and probabilities are mathematically related. In these situations, the frequency of observed events can be used to directly estimate probabilities. Thus, belief formation in this scenario is influenced by these empirical probabilities rather than purely personal experiences or opinions. However, in broader reasoning under uncertainty, this relationship weakens. Therefore, the clear relationship between negative ranks and probabilities mainly applies to well-defined and trackable events. This finding highlights how structured learning environments can enhance our understanding of how individuals use probabilistic information to form beliefs. Spohn's (2012) work indicates that although disbelief can resemble the logarithm of probabilities mathematically, its interpretation fundamentally differs.

Incorporating both positive and negative ranks allowed for a neutral stance and captured the full belief spectrum, which traditional probabilistic assessments struggles to achieve. The cubic polynomial model best explained the relationship between two-sided ranks and subjective probabilities, showing that strong disbelief corresponds with very low probabilities, and strong belief with high probabilities. Interestingly, a neutral stance — neither disbelieved nor believed — spanned a wide range of probabilities [0,1]. Traditional models use a

0.5 probability to signify neutrality, indicating no bias or preference towards any outcome (Keynes, 2013). In Bayesian inference, this represents no strong prior knowledge favouring one outcome over another, as exemplified by a fair coin toss (Etz and Vandekerckhove, 2018). However, a neutral stance corresponded to any probability level, further highlighting that subjective probabilities and belief systems are fundamentally different.

Ranking theory reduced the paradoxical behaviour that emerged from the naive application of standard probability theory. The experiment showed that betting choices based on inferred subjective probabilities did not correlate with the expressed degrees of disbelief from negative ranks. This finding challenges the notion that traditional choice-based methods can reliably infer belief and emphasizes the complexity of belief formation in decision-making processes. The findings suggest that choices made when there are costs and benefits in play do not accurately reflect underlying beliefs, whereas ranks of disbelief offer a better indication of true beliefs. Thus, ranking theory helps us understand how beliefs are formed and acted upon in ambiguous situations.

9.2 Ranking Theory – A Natural Way of Representing Beliefs

Ranks may align with how human cognitive processes work. Individuals typically follow ranking theory principles, such as the laws of disjunction and negation, generally aligning with normative guidelines. Despite challenges in representing full beliefs through disbelief in

the negation of propositions (Kern-Isberner et al., 2021), adherence to the law of negation was observed. This is often challenging in traditional probability theory (Juslin et al., 2009). This suggests that ranking principles reflect natural cognitive mechanisms rather than merely being a heuristic simplification of the more subtle notion of subjective probability. Adopting a ranking-based framework could potentially enhance the precision of psychological models by better reflecting how humans process and express belief.

In dynamic learning environments, negative ranking functions offer a direct, fundamental cognitive representation of the underlying distribution, aligning more closely with objective probabilities than subjective assessments. This supports the hypothesis that belief systems, as modelled by ranking theory, could inherently operate on a rank-based system, simplifying the computational demands placed on individuals when assessing environmental uncertainties. Ranking theory captures the true states of belief and disbelief, reflecting a more natural and efficient process for managing probabilistic information. The close alignment of negative ranks with objective probabilities highlights ranking theory's potential. Additionally, ranking functions may reflect a hierarchical cognitive process, where propositions are initially categorized as surprising or unsurprising (or disbelieved or not disbelieved). Propositions can then be intuitively ordered based on their relative surprise or plausibility. When numerical values are required – such as in experimental tasks to assess belief – individuals may compute estimates, though this explicit computation might not occur in everyday reasoning. This flexibility in ordering and quantification, inherent to ranking theory, might more closely reflect how humans naturally represent environmental uncertainties, even in probabilistic settings, and could be a fundamental aspect of human cognition.

Empirical evidence from Juslin et al. (2011) indicates that logarithmic formats, such as those used in negative ranks, significantly reduce bias and enhance the intuitive understanding of probabilistic information. These formats simplify cognitive processes by making tasks additive rather than multiplicative, thereby reducing cognitive biases and aligning with natural cognitive functions. This transformation, exemplified by negative ranks in probabilistic settings, highlights the computational efficiency of ranking theory and its applicability to the laws of conjunction and disjunction (Spohn, 2012).

In Ellsberg's one-urn experiment, betting choices differed from subjective beliefs. Interpreting probability values as logarithmic transformations may enhance probabilistic reasoning, reduce ambiguity aversion, and align subjective beliefs more closely with actual probabilities. Streamlined formats, like negative ranks, seem to facilitate more accurate judgments. Even without a deep understanding of logarithmic metrics, the human cognitive system appears to benefit from this format in complex probabilistic reasoning (Juslin et al., 2011). This suggests that logarithmic transformations, such as negative ranks, might enhance the intuitive understanding and manipulation of probabilistic information across various decision-making contexts.

Beliefs are shaped by many factors, including cognition, emotions, social interactions, and motivations. Cognitively, beliefs are formed through mental processes such as memory, attitudes, opinions, judgments (Wyer and Albarracin, 2005), and probability reasoning (Halpern and Fagin, 1992), which help individuals interpret and respond to their environment. Emotionally, beliefs are influenced by personal experiences and affective states (Grayling, 2011). Socially, cultural norms and interpersonal interactions also shape beliefs (Grayling, 2011).

Additionally, motivational factors, such as the desire to maintain consistency in one's opinions and beliefs and the need to uphold a favourable conception of oneself and the world, play a critical role (Uso-Domenech and Nescolarde-Selva, 2016). Together, these diverse influences collectively form and reinforce personal and societal belief systems (Halpern and Fagin, 1992; Grayling, 2011; Uso-Domenech and Nescolarde-Selva, 2016; Wyer and Albarracin, 2005).

Ranking theory extends from a theoretical framework to a practical model that aligns with the complexities of human cognition and cultural constructs. While subjective probabilities remain valuable for forming beliefs and enabling rational decision-making, ranking theory offers a more flexible measurement that considers multiple factors influencing beliefs. This was evident in the lottery scenario in Experiment 3, where participants showed skepticism about both winning and losing the lottery via negative ranks, despite an objective probability of 90% for winning. This suggests that participants may rely more on personal experiences and general attitudes toward lotteries, which could be shaped by societal perceptions of lotteries as being less fair or predictable, leading to skepticism even with high probabilities of winning.

This study enhances our understanding of how humans handle uncertainty and express beliefs in various tasks and situations. The consistent findings support ranking theory as a valid alternative to probability theory in representing human beliefs. These results have important implications for cognitive science, suggesting that adopting a ranking-based framework could offer a distinct perspective on human belief systems, providing new insights beyond traditional probabilistic models.

9.3 Application of Ranking Theory in Real-World Practical Settings and Future Directions

9.3.1 Potential to Address Cognitive Biases

Subjective probabilities are often influenced by cognitive biases such as the gambler's fallacy, ambiguity aversion, loss aversion, and base-rate neglect (Spohn, 2012; Ellsberg, 1961; Tversky and Kahneman, 1983; Juslin et al., 2011). Reducing base-rate neglect using ranks is a notable application. Given the effectiveness of logarithmic transformations in simplifying probability judgments (Juslin et al., 2011), ranking theory could potentially enhance probability reasoning.

Base-rate neglect is the tendency to disregard the prior probability of an event, instead focusing on immediate evidence (Stengård et al., 2022). In Bayesian inference tasks like medical diagnostics, where base-rate neglect can lead to significant consequences, applying ranking theory could lead to more accurate and intuitively understandable diagnostic processes. By translating complex probability tasks into simpler additive tasks (logarithmic transformations), ranking theory could potentially reduce the cognitive load on decision-makers. If the logarithmic nature of negative ranking functions holds true (in probabilistic settings), negative ranks may facilitate incorporating base-rate information often overlooked in traditional probability assessments.

Building on the methodology used in Stengaard et al. (2022), ranks can be incorporated in addition to probabilities for comparison. A potential study could investigate this by

presenting participants with a scenario involving a newly discovered virus. The scenario would include a prevalence rate of 0.1%, a test sensitivity of 100%, and a false positive rate of 5%. Participants could be asked two questions: "What is the probability that a person who tested positive actually has the virus?" and "Grade your disbelief: how surprising would it be if this person actually has the virus?" This study could provide insights into whether ranking theory can reduce base-rate neglect and improve probabilistic reasoning in such scenarios.

Another interesting avenue of research where ranks could be applied is loss aversion. Prospect theory, developed by Kahneman and Tversky (1979), explains that people process information in a way that values losses and gains asymmetrically. They found that people perceive probabilities non-linearly in risky decision-making, avoiding losses more strongly than seeking equivalent gains Kahneman and Tversky (1979). To capture this non-linear perception, the probability weighting function transforms actual probabilities into subjective weights, reflecting individual perception. It follows an inverse S-shaped curve, concave for small probabilities and convex for medium and large probabilities (Gonzalez and Wu, 2022). This means that small probabilities are overweighted, and large probabilities are underweighted (Corr and McNaughton, 2012). Both the inverse S-shaped curve in two-sided ranks and prospect theory's weighting function highlight the non-linear complexity of probability perceptions in human cognition. Applying ranking functions can assess beliefs beyond traditional gambling choices, providing a pure assessment free from gains and losses. This approach is supported by findings that cost-benefit choices may not accurately reflect true beliefs, whereas disbelief ranks provide clearer insights, as demonstrated by our Ellsberg

experiment.

Lastly, ranking theory could be explored in the context of the conjunction fallacy, famously illustrated by the 'Linda the Bank Teller' study by Tversky and Kahneman (1983). Participants often incorrectly assessed that 'Linda is a bank teller and is active in the feminist movement' as more probable than 'Linda is a bank teller' alone, despite Bayesian principles indicating otherwise (Tversky and Kahneman, 1983). This contradicts the law of conjunction in probability calculus, where $P(A \cap B) = P(A) \times P(B \mid A)$ (Skovgaard-Olsen, 2016), making the conjunction less probable than the single event (Tversky and Kahneman, 1983).

Using ranks to evaluate each scenario independently, represented as $\kappa(A \cap B) = \kappa(A) + \kappa(B \mid A)$, may clarify the misconception of viewing a conjunctive scenario as more probable than its individual components. This additive measure of disbelief (κ) offers a more straightforward mathematical approach compared to the multiplicative nature of probability calculus (Skovgaard-Olsen, 2016). It may reduce the conjunction fallacy by simplifying probabilistic reasoning and aiding how we process information. For instance, if κ is 4 for 'Linda is a bank teller' (disbelieved) and 3 for 'Linda is active in the feminist movement given that she is a bank teller' (less disbelieved), then the conjunction would have a κ of 7. Thus, 'Linda is a bank teller and active in the feminist movement' should be more disbelieved (more surprising) compared to 'Linda is a bank teller' and, in probabilistic terms, less probable.

9.3.2 Exploring the Dynamics of Belief Revision

Future research should also focus on exploring belief revision through behavioural experiments. Belief revision theory examines how an ideal doxastic[1] agent should revise their beliefs upon receiving new information (Huber, 2013). Specifically, it addresses how beliefs should update in response to new evidence, following the normative guidelines set forth in Spohn's theories of belief revision. Ranking theory offers a robust framework for analyzing the dynamics of belief by providing insights into how people should adjust their beliefs in response to contradicting or supporting information (Spohn, 2009). This approach may align with the way humans naturally process information, where adjustments to beliefs do not necessarily require a 'recalibration' of the entire belief system but rather an intuitive updating of ranks relative to new evidence (Kern-Isberner et al., 2021; Spohn, 2009).

For example, consider the scenario described by Eichorn et al. (2018). They present two conditional beliefs: "If the car had been out of gas, then (usually) it would have stalled" and "If the car had not been out of gas, then (usually) it would not have stalled". Suppose new evidence reveals that "The car was out of gas." In this case, we only need to adjust the ranks of the possible worlds that are directly affected by this new evidence (Huber, 2013). Specifically, the ranks of the worlds where the car ran out of gas and either stalled or did not stall will be updated to reflect this new information. Conversely, the ranks of the worlds where the car did not run out of gas will be adjusted to reflect their decreased plausibility. This tailored adjustment illustrates an effective belief revision process. It updates specific beliefs without requiring a complete overhaul of the entire belief system.

[1]The term "doxastic" relates to belief; "doxastic agent" is an entity that holds beliefs.

9.3.3 Advancing Psychological Research through Ranking Theory

Psychological research predominantly uses probabilistic models to measure beliefs (Kern-Isberner et al., 2021). This work proposes a methodological shift towards incorporating ranking theory into psychological studies, which simplifies the calculations involved in updating and expressing beliefs. This shift may enhance the reliability of data, particularly in studies that infer or measure beliefs. Ranking theory, which has the potential to reduce cognitive biases, could be used alongside traditional measurement scales like the 7-point Likert scale or probability scales. This would enable a more direct assessment of degrees of belief and revisions, deepening our understanding of how beliefs are formed and modified. This thesis lays a solid groundwork for applying ranking theory to measure both disbelief and belief in behavioural experiments.

9.4 Conclusion

In conclusion, this research suggests that ranking theory is an effective framework for representing human belief systems, supporting its incorporation into behavioural studies. The experiments conducted throughout this thesis show that ranking theory addresses some of the limitations of subjective probabilities, offering an intuitive and computationally simpler framework for understanding human beliefs. By enhancing our understanding of how beliefs are formed, represented, and acted upon in an uncertain world, ranking-based approaches provide a distinct method for modelling subjective beliefs and have the potential to offer new insights into human cognition and behaviour across various fields.

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Appendix

Experiments 1 and 2 Materials

Participants completed questionnaires to express disbelief toward various propositions in Experiments 1 and 2.

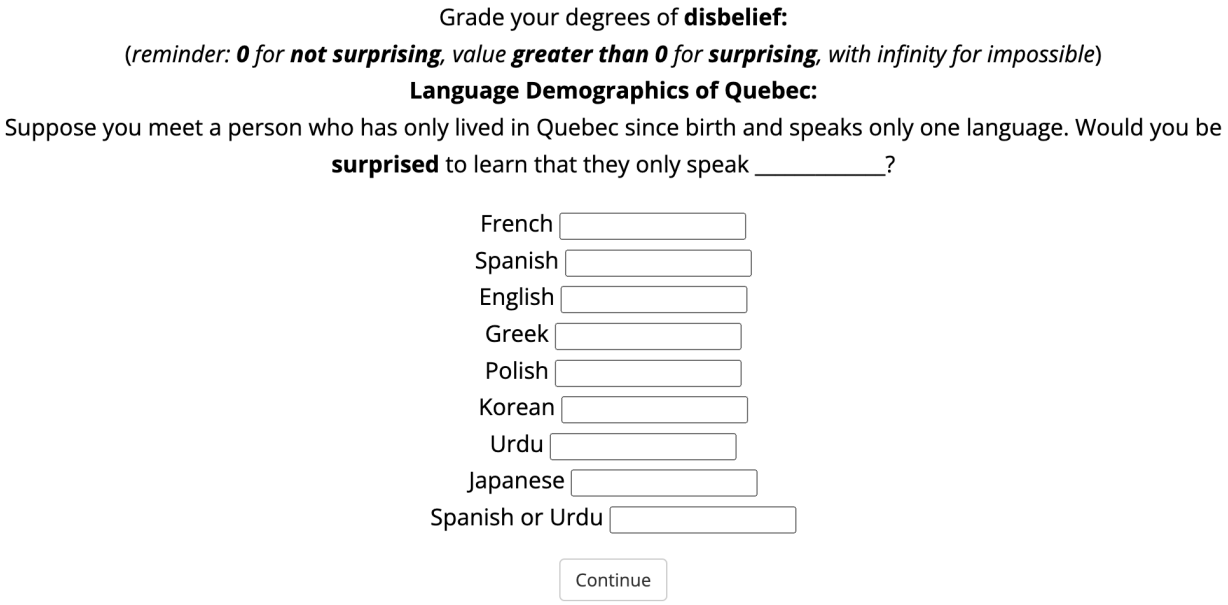


Figure 1: Ranking questionnaire for language demographics of Quebec.

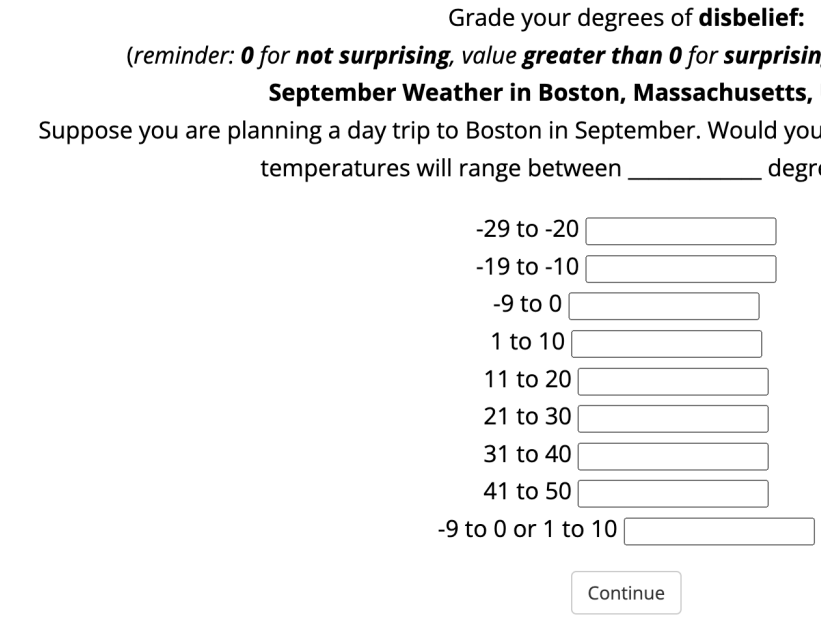


Figure 2: Ranking questionnaire for September weather in Boston, Massachusetts, United States.

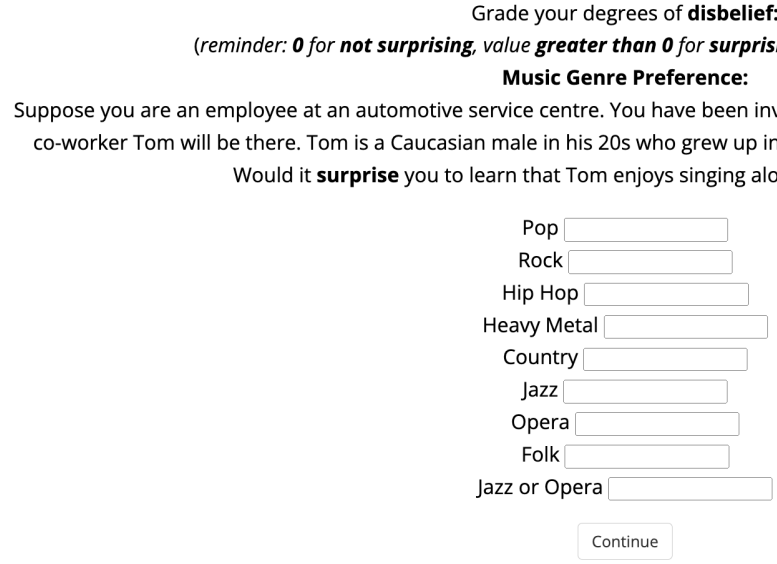


Figure 3: Ranking questionnaire for coworker's music genre preferences.

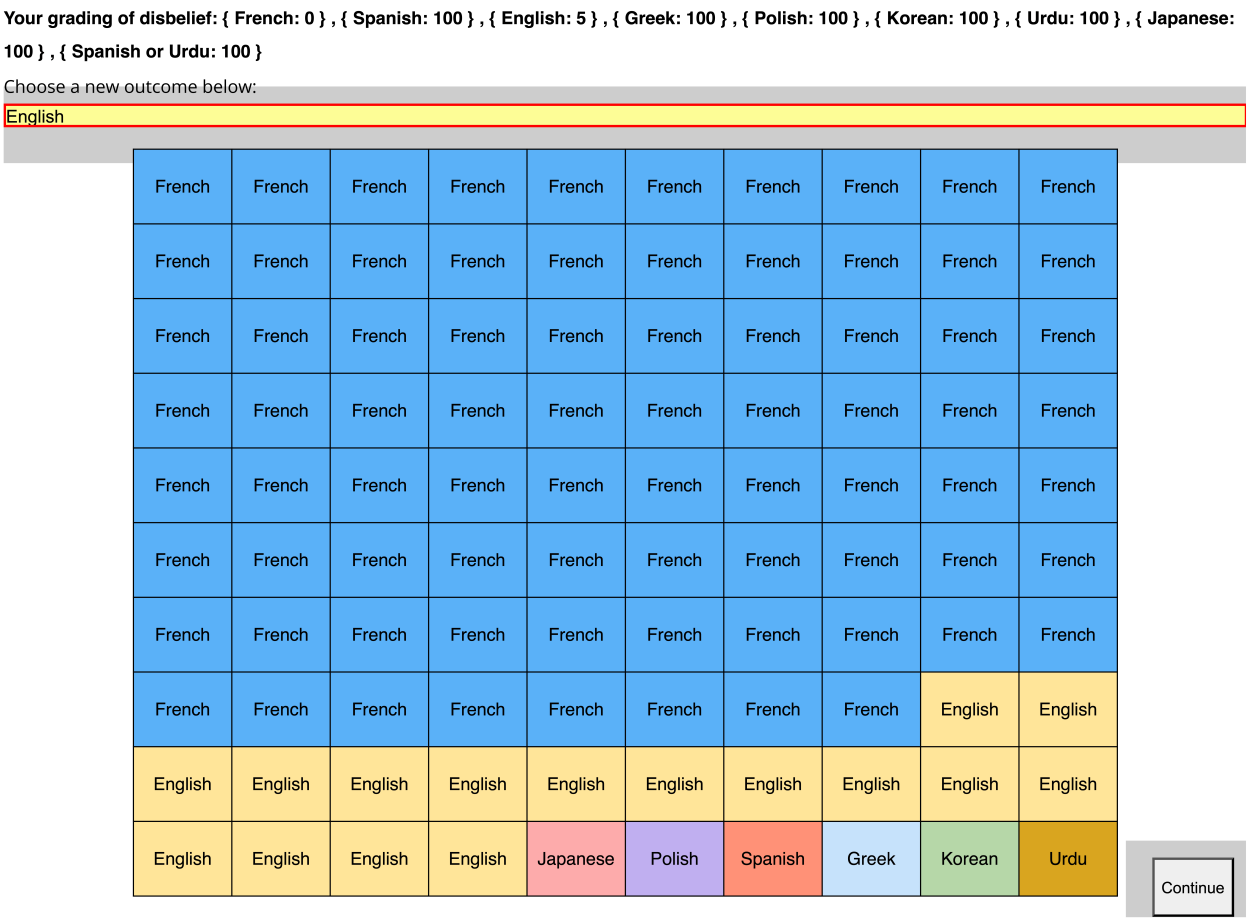


Figure 4: Open sampling format used to measure subjective probability, represented by a 10 by 10 grid of possible outcomes. In this example, the language demographics questionnaire shows French as the most expected, followed by English, with equal expectancy for the remaining languages.

Experiment 3 Materials

Participants completed questionnaires to express disbelief toward various propositions and its negation in Experiment 3.

Grade your degrees of **disbelief:**

(*reminder: **0** for **not surprising**, value **greater than 0** for **surprising**, with infinity for impossible*)

September Weather in New York City, New York, United States:

Suppose you are planning a day trip to New York City in September. How **surprised** would you be if the daytime high temperature on the day of your trip in New York City is _____ degrees Celsius (°C)?

below -19	
between -19 to -10	
between -9 to 0	
between 1 to 10	
between 11 to 20	
between 21 to 30	
above 30	
not below -19	
not between -19 to -10	
not between -9 to 0	
not between 1 to 10	
not between 11 to 20	
not between 21 to 30	
not above 30	
between -19 to -10 **or** 11 to 20	

Figure 5: Ranking questionnaire for September weather in New York City, New York, United State, used in Experiment 3.

Grade your degrees of **disbelief:**

(*reminder: **0** for **not surprising**, value **greater than 0** for **surprising**, with infinity for impossible*)

Music Genre Preference:

Tom is a Caucasian male in his 20s who grew up in a rural area and loves to wear his cowboy hat. Tom works at an automotive service centre and listens to music on his way to work every day. How **surprised** would you be if Tom listened to _____ on his way to work tomorrow?

Pop	
Rock	
Hip Hop	
Country	
Opera	
Folk	
any other type of music	
a type **other than** Pop	
a type **other than** Rock	
a type **other than** Hip Hop	
a type **other than** Country	
a type **other than** Opera	
a type **other than** Folk	
Rock **or** Opera	

Figure 6: Ranking questionnaire for coworker's music genre preferences, used in Experiment 3.

Grade your degrees of **disbelief:**

(*reminder: **0** for **not surprising**, value **greater than 0** for **surprising**, with infinity for impossible*)

Real Estate Investing:

Suppose you buy a **property** in Toronto, Ontario today. How **surprised** would you be if the property's value _____ 5 years from today?

increased	
decreased	
remained the same	
did **not** increase	
did **not** decrease	
did **not** remain the same	
increased **or** remained the same	

Do you believe that the value of your Toronto property will increase 5 years from today?

○ Yes

○ No

Figure 7: Ranking questionnaire for real estate investment outcomes.

Grade your degrees of **disbelief:**

(*reminder: **0** for **not surprising**, value **greater than 0** for **surprising**, with infinity for impossible*)

Lottery Ticket:

Suppose you purchase 900 lottery tickets from a pool of 1000 lottery tickets. How **surprised** would you be if you _____?

won the lottery	
lost the lottery	
won **or** lost the lottery	

Do you believe that you will win the lottery?

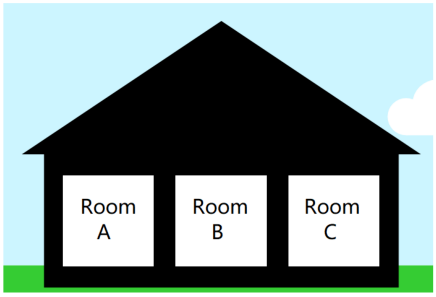
○ Yes

○ No

Figure 8: Ranking questionnaire for lottery outcomes.

Experiment 4 Materials

This section provides the materials for Experiment 4. Participants completed disbelief and subjective probability questionnaires regarding the opponent's seeking and hiding behaviour in a hide-and-seek game.



Grade your degrees of *disbelief* for each questionnaire, such that each question is assigned a value that is a **non-negative integer.**

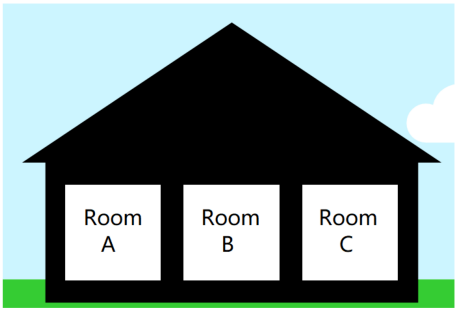
These values can be understood as **degrees of surprise ranging between 0 to infinity:**
0 for not surprising, value **greater than 0 for surprising (the greater the value, the greater your degree of surprise)**, with **infinity for impossible**.

How **surprised** would you be if Jill was **hiding** in _____?

Room A		Infinity ☐
Room B		Infinity ☐
Room C		Infinity ☐
Room A **or** B		Infinity ☐
Room A **or** C		Infinity ☐
Room B **or** C		Infinity ☐
not Room A		Infinity ☐
not Room B		Infinity ☐
not Room C		Infinity ☐

Continue

Figure 9: Ranking questionnaire for hiding condition.



Grade your degrees of *disbelief* for each questionnaire, such that each question is assigned a value that is a **non-negative integer.**

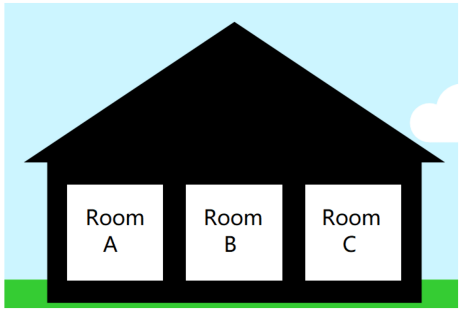
These values can be understood as **degrees of surprise ranging between 0 to infinity:**
0 for not surprising, value **greater than 0 for surprising (the greater the value, the greater your degree of disbelief)**, with **infinity for impossible**

How **surprised** would you be if Jill was **seeking** in _____?

Room A		Infinity ☐
Room B		Infinity ☐
Room C		Infinity ☐
Room A **or** B		Infinity ☐
Room A **or** C		Infinity ☐
Room B **or** C		Infinity ☐
not Room A		Infinity ☐
not Room B		Infinity ☐
not Room C		Infinity ☐

Continue

Figure 10: Ranking questionnaire for seeking condition.

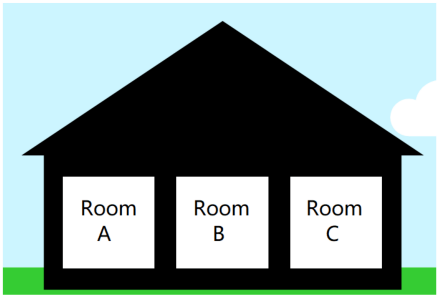


Give your probability estimate for each room, and it must be **between 0 and 1**.
The **sum** of the probabilities for Rooms A,B,and C **must be 1**. The probability of an **impossible** event is **0** and the probability of a **certain** event is **1**:
What is the **probability** that Jill is **hiding** in _____?

Room A
Room B
Room C
Room A **or** B
Room A **or** C
Room B **or** C
not Room A
not Room B
not Room C

Continue

Figure 11: Probability questionnaire for hiding condition.



Give your probability estimate for each room, and it must be **between 0 and 1**.
The **sum** of the probabilities for Rooms A,B,and C **must be 1**. The probability of an **impossible** event is **0** and the probability of a **certain** event is **1**:
What is the **probability** that Toby is **seeking** in _____?

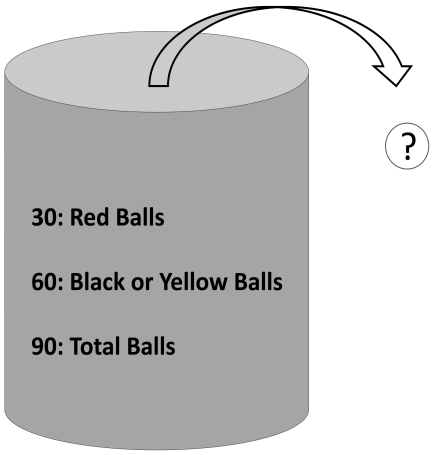
Room A
Room B
Room C
Room A **or** B
Room A **or** C
Room B **or** C
not Room A
not Room B
not Room C

Continue

Figure 12: Probability questionnaire for seeking condition.

Experiments 5 and 6 Materials

This section provides the materials for Experiments 5 and 6. In both experiments, participants graded their disbelief toward the one-urn Ellsberg scenario. Specifically for Experiment 6, participants also completed betting scenarios and the rarity-urn Ellsberg scenario.



Grade your degrees of *disbelief* for each questionnaire, such that each question is assigned a value that is a **non-negative integer.**

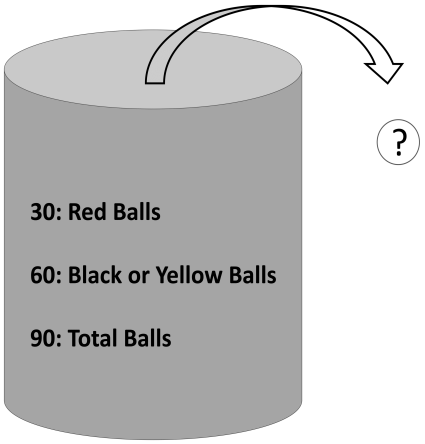
These values can be understood as **degrees of surprise ranging between 0 to infinity:**
0 for not surprising, value **greater than 0 for surprising (the greater the value, the greater your degree of surprise)**, with **infinity for impossible**.

Imagine an urn containing **90 balls** of three different colours: **red balls**, **black balls**, and **yellow balls**.
We know that the number of **red balls** is **30**, while the remaining of **60 balls** are either **black or yellow**, with **unknown** quantities of each.
One ball will be drawn at **random** from this urn.
How **surprised** would you be if the ball you drew was _____?

red ☐ *infinity* ☐
black ☐ *infinity* ☐

red **or** yellow ☐ *infinity* ☐
black **or** yellow ☐ *infinity* ☐

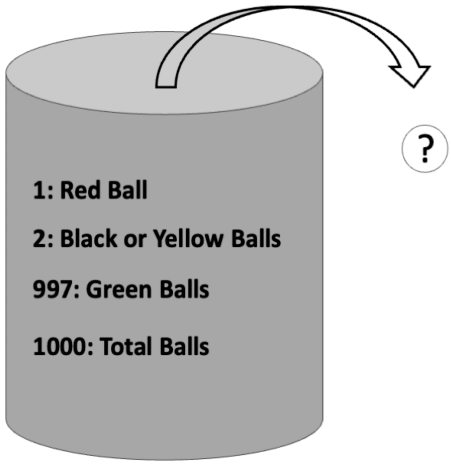
Figure 13: Ranking questionnaire for the one-urn scenario.



Imagine the **same** urn containing **90 balls** of three different colours: **red balls**, **black balls**, and **yellow balls**.
We know that the number of **red balls** is **30**, while the remaining of **60 balls** are either **black or yellow**, with **unknown** quantities of each.
One ball will be drawn at **random** from this urn.
Now, consider the following lotteries. For each of the **four rounds**, please **choose** one of the gambles for **each scenario**.

Round 1:	**Round 2:**
Scenario 1:	*Scenario 1:*
○ A: Receive $100 if Red, otherwise nothing	○ A: Receive $100 if Red, otherwise nothing
○ B: Receive $100 if Black, otherwise nothing	○ B: Receive $100 if Black, otherwise nothing
Scenario 2:	*Scenario 2:*
○ C: Receive $100 if Red or Yellow, otherwise nothing	○ C: Receive $100 if Red or Yellow, otherwise nothing
○ D: Receive $100 if Black or Yellow, otherwise nothing	○ D: Receive $100 if Black or Yellow, otherwise nothing
Round 3:	**Round 4:**
Scenario 1:	*Scenario 1:*
○ A: Receive $100 if Red, otherwise nothing	○ A: Receive $100 if Red, otherwise nothing
○ B: Receive $100 if Black, otherwise nothing	○ B: Receive $100 if Black, otherwise nothing
Scenario 2:	*Scenario 2:*
○ C: Receive $100 if Red or Yellow, otherwise nothing	○ C: Receive $100 if Red or Yellow, otherwise nothing
○ D: Receive $100 if Black or Yellow, otherwise nothing	○ D: Receive $100 if Black or Yellow, otherwise nothing

Figure 14: Participants chose between different gambles across four rounds, in Experiment 6.



Grade your degrees of *disbelief* for each questionnaire, such that each question is assigned a value that is a **non-negative integer.**
These values can be understood as **degrees of surprise ranging between 0 to infinity:**
0 for not surprising, value **greater than 0 for surprising (the greater the value, the greater your degree of surprise)**, with **infinity for impossible**.

Imagine an urn containing **1000 balls** of four different colours: **red balls**, **black balls**, **yellow balls**, and **green balls**.
Specifically, there is only **1 red ball**, and there are **2 balls that are either black or yellow**, with **unknown quantities** of each. The remaining **997 balls** in the urn are **green**.
One ball will be drawn at **random** from this urn.
How **surprised** would you be if the ball you drew was _____?

red _____ *infinity* ☐
black _____ *infinity* ☐

red **or** yellow _____ *infinity* ☐
black **or** yellow _____ *infinity* ☐

Figure 15: Ranking questionnaire for the rarity one-urn scenario in Experiment 6.