Modelling Multiple Criteria-Multiple Participant Problems: The Integrative Approach

by

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Abstract

Many real-world problems involve multiple criteria and multiple participants. Conflicting criteria have been dealt with in multiple criteria decision making (MCDM) and opposing participants have been modelled in multiple participant decision making (MPDM) which includes areas such as game theory and conflict analysis. In order to deal with multiple criteria-multiple participant problems, MCDM and MPDM have attempted independently to incorporate their missing component by extending their existing methodologies. This extension approach has placed Multiple Criteria-Multiple Participant Decision Making (MCMP) as a subset or application area of the existing domains. As a result, MCMP has so far received only limited attention and has no foundation as an independent research domain. In this thesis, direct focus is placed on MCMP. By doing so, insight is gained from simultaneous consideration of MCDM and MPDM which contributes to the improvement of MCMP significantly.

The research objectives for this thesis are twofold. First, a classification scheme for MCMP solution approaches is proposed with the purpose of building a foundation in MCMP research. Then, new methodologies are developed to improve the quality of decision making associated with MCMP problems.

Three classes of MCMP solution approaches are identified and referred to as reduction, decomposition, and integrative. In this thesis, the *integrative approach* is shown to possess a high potential for providing improvement opportunities in MCMP. The integrative approach essentially deals with MCDM and MPDM components simultaneously. It can allow inclusion of more preference information in the decision making process than the other two MCMP approaches. Consequently, the possibility of missing a solution that is preferred by all parties, namely a Pareto optimal solution, can be greatly reduced by taking the integrative approach.

Based on the integrative approach, two new methodologies are developed. The first method is referred to as *candidate enumeration* for solving multicriteria games. Candidate

enumeration deals with multiple criteria and multiple parties simultaneously by enumerating all possible solution candidates based on a binary tree model to structure ordinal preferences. When compared to other multicriteria methods, the improvement made in candidate enumeration is the capability to explicitly include criteria in a game model and to utilize criterion importance information.

The second method is called *logrolling* which is for dealing with multi-issue negotiation. Logrolling is a trade-off procedure in which the exchange of loss in some issues for gain in others results in mutual overall gain. Logrolling integrates MCDM and MPDM by considering trade-offs among criteria and among parties simultaneously. The improvement in logrolling results from incorporating criterion weight information and MCDM-MPDM interaction into the trade-off process.

The investigation of logrolling and related concepts has proven to be particularly fruitful. Variations of the logrolling-based negotiation procedure are proposed and the properties of the efficient frontier are studied extensively. Also, interesting implications can be drawn in terms of negotiation behaviour and strategies. In addition, logrolling is shown to be effective in accommodating a specific type of non-linear preference. The introduction of non-linearity leads to more realistic human preferences, and therefore, can enhance the applicability and decision making quality of the logrolling-based negotiation methodology. An example is given to demonstrate the application of logrolling and related concepts in a three-issue two-party negotiation context with linear and non-linear preferences.

Many opportunities for future research exist based on the research completed here, including various extensions of the two methodologies. In closing, it is hoped that this thesis will encourage further growth of research in multiple criteria-multiple participant decision making.

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Dedication

To my parents, Hitomi and Ryoichi Tajima, who gave me support every step of the way

Okasan, Otosan, nagai aida iroiro domo arigato.

Also to Ralph Waldo Emerson's poem:

To laugh often and much,
to win the respect of intelligent people
and the affection of children;
to earn the appreciation of honest critics
and endure the betrayal of false friends;
to appreciate beauty,
to find the best in others;
to leave the world a bit better,
whether by a healthy child, a garden patch,
or a redeemed social condition;
to know even one life has breathed easier
because you have lived.
That is to have succeeded.

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Chapter 1

Introduction

1.1 Multiple Criteria–Multiple Participant Decision Making (MCMP)

Many real-world problems involve multiple criteria and multiple participants. Examples of such problems can be found in various decision making settings both in the private and public sectors. In the private sector, many business decision problems such as facility location, investment problems, labour-management negotiation, and corporate policy development involve multiple criteria and multiple participants. In business, the participants are sometimes firms in competition, departments within a company, management and union, or various levels of executives. The possible criteria include costs, profits, benefits, availability of resources, environmental impact, location, distance, efficiency, and reputation. In the public sector, capital project decisions such as pollution problems, water resource problems, and a decision about building a nuclear power plant or a new highway involve a number of objectives and a number of participants. Public sector participants include various interest groups and political parties. In addition to the previously mentioned criteria, public decision making may also include effectiveness, equity, and public interest. Complicated and interrelated global problems involving food, energy, natural resources, population growth, and economic growth are also examples of multiple criteria-multiple participant problems.

Here, countries around the globe and international organizations are the possible participants who are certainly concerned with many different criteria.

From these examples, one can realize that a good portion of real-life problems indeed deals with both multiple criteria and multiple participants. Moreover, the growing complexity of socio-economic problems may significantly increase the need to deal with such problems in the future. Therefore, the number of potential applications that can benefit from effective decision methodologies in the presence of multiple criteria and multiple parties is high.

Decision making by a single party whose preference is based on incommensurable and conflicting criteria, objectives, or issues has been dealt with in *Multiple Criteria Decision Making* (MCDM). Decision making in which two or more participants have the power to make a decision and their decisions affect one another may be called *Multiple Participant Decision Making* (MPDM). It has been modelled in areas such as game theory, bargaining theory, negotiation analysis, and conflict analysis. In MPDM, the participants exhibit a fully or partially conflicting interest against each other. When a decision making problem is both MCDM and MPDM, this type of problem may be called *Multiple Criteria-Multiple Participant Decision Making* (MCMP).

The basic elements of an MCDM problem, MPDM problem, and MCMP problem are summarized based on Hipel, Radford, and Fang (1993).

The elements of an MCDM problem:

- Single decision maker
- A set of criteria, $\{C_1, C_2, \ldots, C_m\}$
- A set of possible alternatives, or courses of action, $\{A_1, A_2, \ldots, A_p\}$
- A set of the decision maker's preferences, $\{P_{ij}\}$, with respect to criterion i for alternative j, $i = 1, \ldots, m, j = 1, \ldots, p$

The elements of an MPDM problem:

- A set of decision makers, $\{DM_1, DM_2, \ldots, DM_n\}$
- A set of states, $\{S_1, S_2, \ldots, S_q\}$, resulting from possible actions by the decision makers
- A set of preferences, $\{P_{i1}, P_{i2}, \ldots, P_{iq}\}$, for decision maker i, $i = 1, \ldots, n$, with respect to the set of states

The elements of an MCMP problem consist of the elements of its two main components, MCDM and MPDM.

The elements of an MCMP problem:

- A set of decision makers, $\{DM_i, i = 1, \ldots, n\}$
- A set of criteria, $\{C_{ij}, j = 1, \ldots, m_i\}$, for decision maker i, $i = 1, \ldots, n$
- A set of states, $\{S_k, k = 1, \ldots, q\}$, resulting from possible actions by the decision makers
- A set of preferences, $\{P_{ijk}\}$, for decision maker i, $i = 1, \ldots, n$, and criterion j, $j = 1, \ldots, m_i$, with respect to state k, $k = 1, \ldots, q$

MCDM and MPDM are both established areas of research, but their intersection, MCMP, remains as a relatively new, uncharted domain. Two major research paths have been identified in current MCMP studies, one originating from MCDM and the other from MPDM. A general overview of the origin of MCMP may be summarized as follows. Specific references are provided in the following chapter.

In MCDM, the studies of MCMP begin with the recognition of group decision making being a common practice in most organizations. MCMP studies based on MCDM are therefore referred to as multiple criteria (MC)-group decision making. The inclusion of multiple participants in multiple criteria decision models has been a growing interest among MCDM researchers because of the importance of group decision making both in understanding of the problem and in increasing the confidence in the quality of the final solution. The multiple participants in an MC-group decision making model are treated as another source of complexity besides multiple criteria. The focus of MC-group decision making is placed on how to aggregate the preferences over multiple criteria and over multiple participants.

On the other hand, in MPDM, studies of MCMP originate from the recognition of the existence of conflicting objectives within an individual. The inclusion of multiple criteria in multiple participant decision models has been pursued by the researchers in MPDM because it may be useful in understanding and resolving mixed motive situations. Most of the MCMP models based on MPDM are game models which are referred to as multicriteria games. The multiple criteria are introduced to a game model as a vector of payoffs for each outcome.

One can see that it was natural for the MCMP studies to be conceived and developed from either MCDM or MPDM. This course of evolution, however, has placed MCMP in the position of being a mere extension or an application area of one of these well established research domains. The relationships among MCDM, MPDM, and MCMP have been shown by Hipel, Radford, and Fang (1993). However, a review of the literature reveals no clear foundation on which MCMP research may evolve in the future. Without the foundation, the mission to enhance theory and practice of decision making in MCMP would be difficult.

1.2 Research Objectives

The main research objectives for this thesis are twofold. First, a classification scheme for MCMP solution approaches is proposed with the purpose of providing a foundation in MCMP research. Then, new methodologies are developed to improve the quality of decision making associated with MCMP problems. The specific goals and the achievements as reported in this research are listed below.

1. To propose a classification scheme for MCMP solution approaches:
   - three classes of solution approaches are identified as reduction, decomposition, and integrative,
   - advantages and disadvantages of the different types of solution approaches are discussed, and
   - advantages and disadvantages of the classification scheme are discussed.
2. To improve MCMP methodologies:
   - opportunities for improvement are identified by investigating the sources of difficulty in finding a solution that is preferred by all parties, namely a Pareto optimal solution,

- one of the solution approaches, the *integrative approach*, is shown to provide improvement opportunities by being able to simultaneously utilize preference information from MCDM and MPDM components of an MCMP problem, and
- based on the integrative approach, two techniques, *candidate enumeration* and *logrolling*, are developed and incorporated into the solution methodologies for multicriteria games and multi-issue negotiation, respectively.
- In addition to the development of the solution methodology, the general properties of the efficient frontier, behavioral and strategic implications, and linear and non-linear preference assumptions are investigated for the logrolling-based negotiation procedure.

1.3 Dissertation Outline

The motivation and objectives of this thesis were presented in the current chapter. The remainder of the dissertation is structured as follows. Chapter 2 presents a survey of the MCMP literature which is organized by different decision models. Based on the survey, a general classification scheme for MCMP solution approaches is proposed in Chapter 3. Here, the comparison among the different types of solution approaches, as well as the advantages and disadvantages of the classification scheme are discussed. Chapter 3 also contains an examination of possible improvement opportunities in MCMP. The discussion of the difficulty in finding a Pareto optimal solution in MCMP highlights the potential of one of the solution approaches, the integrative approach. Chapter 3 concludes by briefly introducing the two MCMP methodologies developed in the research, the candidate enumeration method and logrolling, which are described and studied in detail in the following four chapters.

Chapter 4 presents the detailed discussion of the development of the candidate enumeration method for multicriteria games. The key concepts, preference trees and candidates, are introduced in the context of bicriteria 2×2 game. Three solution analysis steps are discussed with an example.

Chapters 5, 6, and 7 discuss the research in logrolling. The core concepts of logrolling are developed in Chapter 5 based on linear preference assumption. Furthermore, the negotiation procedure based on logrolling is proposed, and the properties of the efficient frontier are studied for the two-issue two-party case. The results are then extended to the *m*-issue case. Various implications regarding negotiation behaviour and strategies are also discussed in this chapter. Chapter 6 demonstrates the application of logrolling and related concepts using an example. The sample solution analysis shows both intuitive and counter-intuitive results. Chapter 7 deals with a specific type of non-linear preference based on diminishing marginal utility in the context of logrolling. The negotiation procedure and the efficient frontier are investigated for two-issue two-party negotiation extending the results from linear case. For more than two issues, the expected difficulties in analysis are discussed.

Finally, Chapter 8 concludes the dissertation by presenting the summary of contents, main contributions, and suggestions for future research.

Chapter 2

Literature Review

The objective of this chapter is to provide an overview of the field. Due to the prescriptive nature of research contained in this thesis, the survey focuses on analytical and quantitative studies excluding descriptive research areas such as psychology and social sciences. The chapter is organized by four decision models: multiple criteria-group decision making (Section 2.1), game (Section 2.2), bargaining and negotiation (Section 2.3), and other models (Section 2.4). Based on the survey, three general solution approaches, which are collectively exhaustive and do not depend on the specific type of decision models, are identified.

2.1 Multiple Criteria-Group Decision Making Models

Various solution methods in MCDM evolved over time to accommodate the increasing recognition of group decision making as a common practice in most organizations. MCMP problems that are dealt with in this section are modelled as multiple criteria (MC)-group decision making problems. The decision making situations are cooperative in this case. The multiplicity of participants is essentially treated as another source of complexity in the multiple criteria decision making process. Typically, there is one stage for dealing with the MCDM component using the existing MCDM methods and another for the MPDM component which is dealt with by a variety of compromise facilitation techniques.

Compromise facilitation is described as either generation of the group objective or generation of the group consensus. Due to the two-phase structure, most of the solution methods for MC-group decision making problems can be described by identifying the MCDM methods and compromise facilitation techniques.

Here are some of the early studies in MC-group decision making. Thiriez and Houri (1975) used the Delphi technique (Dalkey, 1967) to determine the group weights. For evaluation of alternatives, the weighted average was computed. Tell (1977) used the revised Delphi technique to determine both group criteria and group criteria weights. In this study, factor analysis was applied to reduce the number of criteria. Irving and Conrath (1988) also used the Delphi technique to determine group criteria and criteria weights. They defined the median of the individual decision makers' scores as the aggregated score for each criterion. Bodily's (1978) model was based on multiattribute utility which incorporated the preferences of three interest groups. In his model, only the main interest of each group was incorporated. Ulvila and Snider (1980) studied the negotiation of international oil tanker standards using a multiattribute value model. They analyzed the patterns of weights of individual countries in order to identify possible agreements.

The majority of MC-group decision making models adopts an interactive approach. The interactive methods iterate between the MCDM and compromise facilitation stages. Iz and Gardiner (1993) identified two general types of MCDM problems to be multiple objective programming (MOP) problems and discrete alternative problems. For MOP-based MCMP problems, two interactive stages are generation of efficient or nondominated solutions and investigation of individual preferences and/or compromise facilitation among multiple participants. For discrete alternative-MCMP problems, the two stages are investigation of individual preferences and compromise facilitation among multiple participants. The interactive methods take information from the participants' local and unstructured preferences to provide insight into the problem structure, possible alternatives, and possible efficient level of objective achievement. They attempt to take all participants' preferences into account and iterate the decision making process until the participants express satisfaction with a solution. Hence, they enhance the acceptance of a compromise solution by the participants.

The interactive approach to MC-group decision making problems was adopted in the following studies. Wendell (1980) developed a theoretic approach for the bicriteria case, which obtained a group consensus by either direct or interactive approaches. Lewis and Butler (1993) proposed a procedure in which efficient solutions were generated by the Simplified Interactive Multiple Objective Linear Programming (SIMOLP) (Reeves and Franz, 1985) or the interactive weighted Tchebycheff procedure (Steuer and Choo, 1983). After generation of efficient solutions, the minimum regret heuristic (Beck and Lin, 1983) computed the consensus ranking of the alternatives. Lawrence and Marose (1993) studied planning for a mutual life insurance company. They applied the Analytical Hierarchy Process (AHP) (Saaty, 1980) to determine weights for the firm's multiple objectives and solved a goal programming model. Kwak et al. (1996) also used AHP to derive weights for decision makers' preferences in capital budgeting problems. In their study, the two-phased solution approach that iterates between the AHP and integer programming stages was developed.

Most of the interactive methods have been implemented in decision support systems. Schaeffers (1985) described the process of designing a decision support for regional water resource allocation problems, which consist of multiple objectives and multiple decision makers. Various issues in terms of decision support were discussed including objective formulation, model operationalization, application to a specific problem, and presentation of results. Iz and Gardiner (1993) provided a list of thirty-one multicriteria-group decision support systems (MCGDSS) according to their author and the interactive method used in the system. In this survey, a set of selective studies from Iz and Gardiner's (1993) list are presented. The studies are described by the MCDM methods, compromise facilitation techniques, and basic procedure involved.

For discrete alternative problems, some of the studies that have been implemented in MCGDSS are the following. Bui and Jarke (1984) used ELECTRE (Roy, 1968) as the individual preference investigating technique, and the Nominal Group Technique (NGT) (Van de Ven and Delbecq, 1971) and the Delphi technique (Dalkey, 1967) were combined for compromise facilitation. Liang and Wang's (1991) MCGDSS was based on fuzzy set theory. The group average was used to combine the participants' opinions which were

expressed in terms of fuzzy numbers. The overall ranking of the alternatives was achieved by Chen's method (1985) which ranks fuzzy numbers.

Saaty (1988) used AHP (Saaty, 1980) as the basis for his MCGDSS. Two methods were provided for obtaining the pairwise comparison of alternatives, which is the fundamental input for AHP. In the first method, the participants reached agreement on the pairwise comparison judgements by consensus voting. In the second, the individual judgements were aggregated by using the geometric mean. Vetschera's (1991) model was based on multiattribute utility, and it described the changes in individual opinions required to achieve the group consensus. A linear programming model was formulated to determine the required changes within the acceptable range for each participant, resulting in a better group agreement.

For MOP problems, the following are some of the studies which have been implemented in MCGDSS. Franz et al. (1986) used SIMOLP (Reeves and Franz, 1985) to generate a set of efficient solutions. The efficient solutions were ranked by each participant individually. The consensus ranking of the alternatives was then achieved by the Cook and Kress ranking method (1985) or the Cook and Seiford assignment method (1978). Iz (1992) proposed two MCGDSSs. Both systems used the interactive weighted Tchebycheff procedure (Steuer and Choo, 1983) to generate a set of efficient solutions. After generating the efficient solutions, the first system used the Cook and Kress ranking method (1985) to obtain the group consensus. In the second system, the participants used AHP to obtain the individual rankings of the alternatives, and the final group consensus was reached by the modified Delphi technique. Iz and Krajewski (1992) applied the Step Method (STEM) (Benayoun et al., 1971) or the Interactive Sequential Goal Programming (ISGP) Method (Masud and Hwang, 1981) as the technique for generating an efficient solution. The modified Delphi technique and the Nominal Group Technique were used to determine if the current solution was satisfactory to all participants, and if not, determine the changes in the objective values that were acceptable to each participant.

Korhonen et al. (1980) used the Zionts-Wallenius interactive multiple objective linear programming procedure (Zionts and Wallenius, 1976) as the solution generation technique. A conflict matrix was defined to measure the conflict among the participants. The

participant who exhibited the highest measure of conflict with others was asked to make a concession. This process could reach the consensus if the participants continued making concessions. In Korhonen et al. (1986), the Zionts-Wallenius procedure (Zionts and Wallenius, 1976) was modified to generate a sequence of solutions, each of which improved the group utility over its predecessor. Shimizu (1983) used the Borda count (1781) to approximate the group objective function from the estimates made by the participants. By the Interactive Constrained Simplex Method (ICSM) of Ichikawa (1980), the estimated group objective function was optimized in the nonlinear programming problem which was defined to represent the MC-group decision making problem. Finally, Reeves and Bordetski (1995) developed a framework for interactive MCGDSS, which was described as simpler and more flexible than previous approaches.

2.2 Game Models

MCMP problems in the context of game theory are referred to as multicriteria n-person games. Most of the MCMP models originated from MPDM are game models. A game model consists of three components: players, players' strategies, and payoffs for possible outcomes. Multicriteria n-person games differ in structure from single criterion n-person games by having a vector of payoffs, instead of a payoff, for each outcome. This results from the recognition of conflict within an individual while conflict among players is being modelled. Some MCMP problems are modelled based on a noncooperative perspective and some are modelled based on a cooperative perspective, as the solution concepts developed for classical game theory include both situations. A survey of solution methods in which the existing solution concepts are modified to accommodate the presence of multiple criteria is presented first.

Blackwell (1956) was the pioneer in multicriteria games. He analyzed multicriteria repeated games and established an asymptotic analog to the minimax theorem (von Neumann and Morgenstern, 1944) for two-person zero-sum games. Nieuwenhuis (1983) generalized the notions of minimax, maximin, and saddle point for vector-valued outcomes. He

discovered that the minimax and maximin solutions rarely have equality between them in the multicriteria case. Sakawa and Nishizaki (1994) examined the maximin solutions for fuzzy multicriteria games.

Alternative definitions of Nash equilibrium (Nash, 1951) in the presence of multiple criteria have been proposed by many studies. In most cases, the efficient strategy for each player was defined to be the strategy which yields the nondominated expected payoff vector for the player against the other players' mixed strategies. An equilibrium was then the result of all players' efficient strategies.

The studies based on two-person games are as follows. Shapley (1959) defined the strong and weak equilibrium points for multicriteria two-person zero-sum games using the concept of Pareto optimality, and showed that equilibria of both types always exist. Based on the same definition of an equilibrium point as Shapley (1959), Corley (1985) established the necessary and sufficient conditions for equilibria in multicriteria two-person games. Ghose and Prasad (1989) used the notions of the Pareto optimality and the security levels to define new solution concepts for multicriteria two-person games. Borm, Tijs, and Aarssen (1990) modified the Pareto equilibria so that their solution approach examined the nondominated solutions in the polytope of possible vector payoffs to one player when the other player's strategy was fixed. They showed that, for each multicriteria two-person game, the set of modified Pareto equilibria was non-empty.

Several studies investigated the general case, multicriteria n-person games. Charnes et al. (1987, 1990) developed sufficient and necessary conditions for nondominated equilibrium points in terms of normalized Kuhn-Tucker conditions. Wierzbicki (1990) discussed sufficient and necessary conditions as well as parametric characterizations of Pareto equilibria. Zhao (1991) defined and proved the existence of the hybrid solution and the quasi-hybrid solution for multicriteria n-person games. The essence of the hybrid solution was to assume competition across coalitions and cooperation within each coalition. The Nash equilibrium and the core solution for multicriteria n-person games were special cases of the hybrid solutions. Wang (1993) obtained several sufficient conditions to guarantee the existence of Pareto equilibria in multicriteria n-person games. Krus and Bronisz (1994) presented theoretical results on definitions of the noncooperative equilibria, a proof on the

existence of equilibria, and the relationships of the multicriteria game equilibria to the Nash equilibrium in single-criterion game.

The following solution methods essentially reduce a multicriteria n-person game to an MCDM problem, especially a mathematical programming problem, are presented. Contini, Olivetti, and Milano (1966) introduced two solution methods for multicriteria two-person games against Nature. One method attempted to maximize the expected payoff for each criterion, and this was formulated as a vector maximum problem. The second method sought a strategy that assured a minimum target payoff with the highest subjective probability, and the problem was formulated as a linear programming problem. Charnes et al. (1968a, 1968b), Cassidy et al. (1972), and Blau (1974) extended the study of multicriteria two-person zero-sum games. They used a satisficing model in which a player maximized a minimum target payoff subject to a prescribed confidence level of achieving that target. Their models led to formulation of a mathematical programming problem in which the payoff vectors were treated as random variables. Zeleny (1976) formulated a multicriteria two-person zero-sum game as a multiparametric programming problem. In this study, each vector payoff was replaced by a convex combination of the vector components.

Some methods essentially treat a multicriteria n-person game as an MPDM problem. Bergstresser and Yu (1977) proposed various solution concepts for multicriteria n-person games, which were based on the solution concepts for single criterion n-person games both in normal form and characteristic function form. Bronisz and Krus (1986) proposed the weighted nucleolus solution to multicriteria cooperative games, and it was incorporated in the interactive solution approach.

Conflict analysis, a set of practical methods for applying game theory to real-world conflicts, with the presence of multiple criteria was attempted in two studies. In Alexander and Saaty (1977) and Saaty (1983), the multiple criteria were basically aggregated into a single criterion by the AHP before the conflict among the multiple participants was also solved by the AHP. Alexander and Saaty (1977) included multiple participants at the highest level in the hierarchy while Saaty (1983) included the players' options at the highest level.

2.3 Bargaining and Negotiation Models

Some MCMP problems are modelled as bargaining or negotiation models. As an early attempt, Yager (1980) introduced powers of the preference functions to Nash's bargaining model (Nash, 1950) in order to include importance of decision makers. Each participant in Yager's model represented one objective. Darling et al. (1997) developed a method to support resource allocation negotiations. Their study focused on understanding disagreements by Pareto-efficiency analysis, as well as numerical and graphical analyses of feasible settlement spaces and efficient frontiers.

The following studies are based on an interactive approach as seen in MC-group decision making. Bronisz and Krus (1988) proposed an interactive procedure for multicriteria bargaining problems. In their model, each player acted separately first. Then the players modified their nondominated solutions until an agreement was reached. Korhonen and Wallenius (1990) presented a general approach to support individuals in multi-criteria multi-party negotiation. Their approach was based on multiple objective linear programming and the interactive procedure. Heiskanen (1998) proposed a decomposition method for computing Pareto optimal solutions in a multi-issue negotiation context. The decomposition resulted into an interactive procedure between the decision makers and a neutral mediator.

The Edgeworth box (Edgeworth, 1881) is a concept in microeconomics which describes the possible allocations of two goods between two parties in terms of their welfare. Edgeworth showed that Pareto optimal solutions exist at the tangent points of the parties' indifference curves, but no particular negotiation procedure was proposed for obtaining a Pareto solution from a non-Pareto point. In this model, it was assumed that all value function information was known. The following studies developed multi-issue negotiation methodologies based on the Edgeworth model.

Contini and Zionts (1976) were the first to discuss the use of the Edgeworth model. In their method, each party first proposed a set of feasible solutions that satisfied certain minimum utility requirements. At the second stage, each party relaxed his/her own constraints until a group feasible solution was reached. Teich et al. (1995) developed RAMONA, the Resource Allocation Multiple Objective Negotiation Approach, for multi-

issue negotiation. RAMONA first produces a piecewise linear approximation of the contract curve, or the efficient frontier, based on the Edgeworth model. Then, the parties work out an agreeable solution on the approximated curve. Ehtamo et al. (1997) proposed the constraint proposal methods in which joint tangents of the decision makers' indifference curves were searched by adjusting an artificial plane constraint. In their methods, the negotiation problem was decomposed into MCDM stage and group coordination stage.

In the following axiomatic studies of multi-issue bargaining or negotiation problems, the new axioms were introduced to accommodate the existence of multiple criteria. Peters (1986) studied simultaneous bargaining over two or more issues by two parties. A family of solutions was discussed for the sum of bargaining games, which was considered as simultaneous bargaining based on the super-additivity axioms. Ponsati and Watson (1994) introduced two new axioms, agenda independence and issue-by-issue individual monotonicity, and proved that a solution that satisfies axioms of efficiency, symmetry, invariance, and the two new axioms is equivalent to the Nash bargaining solution.

Raiffa (1982, pp.148-165) suggested a two-party negotiation procedure for generating a Pareto optimal solution based on the Edgeworth model. One party proposes several solution points that pass through one of his/her indifference curve. The second party chooses one that improves his/her overall value. Along the indifference curve that passes through the most recently selected solution, the second party selects some solutions and proposes them to the first party. This time, the first party chooses one that improves his/her overall value. The procedure continues until a Pareto optimal solution is reached. In this procedure, the trade-offs among multiple issues are implicitly taken into account.

Gupta (1989) presented a mathematical model for multi-issue bargaining problems. The model focused on trade-offs between criteria and issue authority, that is, the potential to influence the other party. The equivalence of the model to an axiomatic model of cooperative bargaining was also shown. Matwin et al. (1991) argued that negotiation rules could be learned and invented by genetic algorithms. The results of experiments with a prototype implementation were reported. Ehtamo et al. (1995) proposed a joint gain approach to finding Pareto optimal agreements in multi-issue two-party negotiation. Their search for a

solution is based on the set of locally defined, mutually acceptable trade-offs and the bisecting compromise principle.

Finally, some studies developed multiple criteria-negotiation support systems (MCNSS). Jarke et al. (1987) developed an MCNSS called MEDIATOR. The main purpose of MEDIATOR was to support the human mediator in a negotiation by allowing him/her to examine the admissible set of solutions for all players and by finding a solution through expanding and/or contracting the group goal target set. Kersten et al. (1991) proposed satisficing-based approaches in a negotiation context. Iz (1994) developed a group decision and negotiation support system for discrete multicriteria problems. It was designed to facilitate compromise among decision makers through a preference elicitation method based on fuzzy logic.

2.4 Other Models

Besides the MC-group decision making, game, and bargaining and negotiation models, there are some other models which have dealt with MCMP problems using different solution methodologies. The studies listed below typically transform the original MCMP problem into a single criterion-single participant or single criterion-multiple participant problem.

Buckley (1984) expressed an MCMP problem using fuzzy sets. Each participant first defined fuzzy sets over the alternatives for each criterion. The problem was then dealt with by aggregating all fuzzy sets into one fuzzy set. Peters and van der Stel (1990) defined a family of functions, which measures the degree of potential conflict among the participants. A particular choice for a potential conflict function depended on the context of application. The objective of the problem was to find a solution that minimizes the function. This approach was a close analogy of the potential function approach in physics. Wang and Archer (1994) used a neural network technique to determine an overall value function which was represented in the form of fuzzy functions. Intuitively, this technique could be described as a curve-fitting technique. Ryu (1997) studied commodity auction market problems with m sellers and n buyers with heterogeneous commodities. He modified an optimization model

based on hierarchical constraint reasoning to deal with different importance of the commodities. The trading partners were then determined based on the stable matching principle.

2.5 Summary

In this chapter, an overview of the current MCMP literature was presented, organized by distinct decision models used in MCMP. From a more general and structural perspective, the survey shows the existence of some basic solution approaches used in solving MCMP. For example, some MCMP problems are essentially converted to MCDM, MPDM, or single criterion-single party decision making. In multiple criteria-group decision making, most of the problems are solved in two stages, one that deals with MCDM and the other dealing with MPDM. Finally, in multicriteria games and bargaining problems, some solution concepts are developed by incorporating multiple criteria into the existing concepts. In the next chapter, three solution approaches are identified by specific terms and examined in detail.

Chapter 3

Improving MCMP

This chapter presents the underlying material for the rest of the research: classification of MCMP solution approaches, identification of a possible improvement opportunity, and introduction of means to actualize the identified opportunity. Sections 3.1 and 3.2 present the proposed classification and comparison of different types of solution approaches. The advantages and disadvantages of the classification scheme are also discussed. Section 3.3 studies the difficulty involved in MCMP and identifies an opportunity for improving the quality of decision making. In order to operationalize this opportunity, Section 3.4 introduces two solution methodologies that are developed in this research: the candidate enumeration method for multicriteria games and the logrolling procedure for multi-issue negotiation. Brief description of the two methods is given. Section 3.5 summarizes the contents in this chapter.

3.1 Classification of MCMP Solution Approaches

MCMP problems are diverse and numerous. Therefore, classification by problem type or decision model may lack the unification and comprehensiveness that are essential for building a solid research foundation. In order to provide a foundation that focuses on MCMP research, this thesis proposes categorization by the type of solution approach as the

classification scheme. By reviewing the literature, three classes of solution approach can be proposed. They are referred to as the following:

- Reduction,
- Decomposition, and
- Integrative.

Each class is described with some examples below.

3.1.1 Reduction Approach

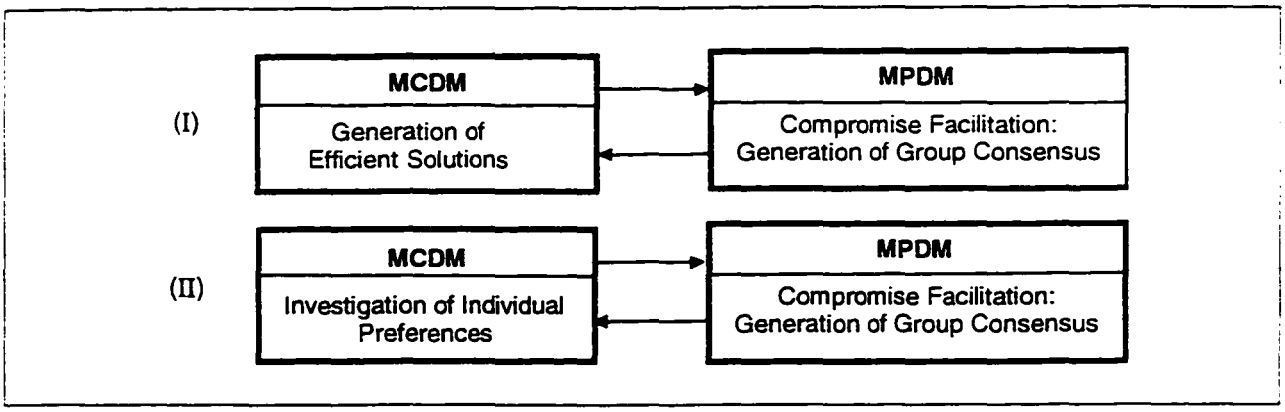
Some solution methods basically convert an MCMP problem into an MPDM or MCDM problem. This is called the *reduction* approach, and it is the simplest of all. Reduction is usually carried out in the form of aggregation. One way is through aggregation of multiple decision makers into one "group" decision maker as done by Tell (1977) and Wang and Archer (1994). Tell uses the Delphi technique (Dalkey, 1967) to determine the group criteria and the group criteria weights. Wang and Archer use a neural network technique to determine an overall value function which is represented in the form of fuzzy functions. Alternatively, the number of criteria can be reduced through aggregation. In Alexander and Saaty (1977), multiple criteria are aggregated into a single criterion by the Analytical Hierarchy Process (AHP) (Saaty, 1980). The conflict among the multiple parties is then solved again by AHP. In any case, the success of the reduction approach depends highly on which elements are aggregated and in what manner.

3.1.2 Decomposition Approach

A solution approach that is based on decomposing an MCMP problem into two parts, an MCDM stage and an MPDM stage, is called the *decomposition* approach. It is mainly used for solving multicriteria-group decision making problems. Two types of the decomposition approach have been identified by Iz and Gardiner (1993) as shown in Figure 3.1. In either case, the decomposed stages are solved separately in an iterative fashion, and hence, have an

input-output relationship with each other. This input-output relationship is indicated by the pair of arcs shown in Figure 3.1.

Figure 3.1 Two Types of Decomposition Approach for MC-group Decision Making Problems (Iz and Gardiner, 1993)



In diagram (I) of Figure 3.1, the MCDM stage is set up as a Multiple Objective Programming (MOP) problem. For MOP, the set of alternatives is defined by a set of constraints. Therefore, at the MCDM stage, a set of efficient solution alternatives is generated. At the MPDM stage, the efficient solutions are evaluated by the group of decision makers, and the group consensus is sought using compromise facilitation techniques. If the group consensus is not reached, then feedback is given from the MPDM stage to the MCDM stage and a revised set of efficient solutions is generated.

For instance, Franz et al. (1986) use SIMOLP (Reeves and Franz, 1985) to generate a set of efficient solutions. The efficient solutions are ranked by individuals, and the consensus ranking is achieved by the Cook and Kress ranking method (1985) or the Cook and Seiford assignment method (1978). The result of the consensus ranking becomes the input for the next round of SIMOLP computation. The process repeats until an overall consensus is reached or no more compromise solutions are generated by SIMOLP.

In diagram (II), on the other hand, the MCDM stage is set up as a discrete alternative problem. In a discrete alternative problem, the set of alternatives is explicitly known, so the generation of alternatives is not necessary. Instead, the MCDM stage deals with clarification of the individual preferences over the known alternatives. The MPDM stage seeks the group consensus using compromise facilitation techniques. If the consensus is not reached, then as

in (I), feedback is given from the MPDM stage to the MCDM stage. Here, the individual preferences are reconsidered based on feedback.

For example, Bui and Jarke (1984) use ELECTRE (Roy, 1968) as the preference investigating technique in the individual decision support systems (DSS). The Nominal Group Technique (Van de Ven and Delbecq, 1971), the Delphi technique (Dalkey, 1967), and a group version of ELECTRE are used for compromise facilitation in the group DSS. The main objective of the group ELECTRE is to identify possible concessions. The parties can discuss how to reach an agreement using NGT and the Delphi technique. If some concessions can be made, the individual ELECTRE modifies the evaluation scores accordingly. Iteration between the individual DSS and group DSS is therefore considered as sequential concession making.

3.1.3 Integrative Approach

Some solution methods do not reduce MCMP problems or do not solve them as two separate stages. This type of approach that considers the MCDM and MPDM components simultaneously is referred to as the *integrative* approach. In the MCMP literature, the integrative approach is most commonly applied to solving multicriteria games. In the context of games, the MCDM and MPDM components are integrated by modifying the definition of a solution concept, generally an equilibrium, to accommodate the presence of multiple criteria that are expressed in the form of vector payoffs.

For example, Shapley (1959) defines the strong and weak equilibrium for multicriteria two-person zero-sum games using the concept of Pareto optimality. Based on Shapley's definition, Corley (1985) establishes necessary and sufficient conditions for equilibria in multicriteria two-person games. Ghose and Prasad (1989) use the notions of the Pareto optimality and security levels to define new solution concepts for multicriteria two-person games. In general, the efficient strategies for each player are determined from vector maximization. An efficient strategy yields the nondominated expected payoff against other players' mixed strategies. A solution strategy is then simultaneously efficient for all players, yielding an equilibrium.

Some bargaining and negotiation methodologies also exhibit the integrative approach. In the studies that introduce new axioms in consideration of multiple criteria (Peters, 1986; Ponsati and Watson, 1994), the MCDM and MPDM components are dealt with simultaneously. In Gupta (1989) and Ehtamo et al. (1991), the trade-offs that are involved with both criteria and parties are considered. These solution methods are also said to exhibit an integrative approach.

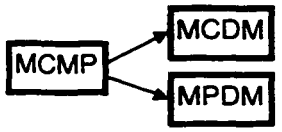
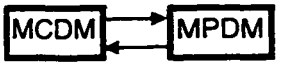
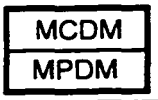
As the success of the reduction approach depends on aggregation techniques, the success of the integrative approach relies on how the MCDM and MPDM components are integrated in the decision making process. However, since the integrative approach has not been used extensively in MCMP, few integration techniques are known. At the present, the framework of integration seems to be wide open for further research. Ingenuity is certainly required for the invention of integration techniques.

In the behavioral studies of multi-issue negotiation, a solution is said to be integrative to the extent that it reconciles or integrates the negotiating parties' conflicting interests and thus provides high benefit to all parties (Pruitt, 1981). Here, the term integrative solution has originated from Follett's (1940) notion of "integration" in constructive conflict. While there has been relatively little research on the integrative solution in the behavioral studies (Weingart et al., 1993), many benefits are recognized by Pruitt (1981, 1983) and Pruitt and Lewis (1977). Note that, in this thesis, the term integrative is used in a broader and more general sense to imply the simultaneous treatment of MCDM and MPDM components.

3.2 Comparison of MCMP Solution Approaches

The three MCMP solution approaches are summarized in Table 3.1. Their advantages and disadvantages are discussed in terms of the following features: tractability of the solution approach, preservation of preference information in the decision making process, and misrepresentation of the problem. The comparison is made based on these features to reflect the quality of decision making supported by each solution approach. The three features are selected due to their obviousness and no requirement for any definite unit of measure.

Table 3.1 Characteristics of MCMP Solution Approaches

Type	Decision Making Process
Reduction	• by reducing two components to one 
Decomposition	• by dealing with two components alternately 
Integrative	• by dealing with two components simultaneously 

Tractability. By studying the decision making process, the main advantage of the reduction and decomposition approaches can be identified as their tractability. They reduce the complexity of MCMP problems, and hence make the problems more manageable and comprehensible for the decision makers. They also take full advantage of the existing MCDM and MPDM solution methods. Consequently, ease of implementation may be expected. In case of the decomposition approach, since its process resembles that of the interactive methods, it inherits the benefits of the interactive methods. The interactive methods consist of alternating computation steps and dialogue with the decision maker (Vincke, 1992). They are known to generate satisfactory compromises, enhance acceptance of the final solution, and improve insight into the problem structure. Compared to the reduction and decomposition approaches, the integrative approach is less tractable. The decision makers must deal directly with the complexity of MCMP problems. The decision making process is less structured and less transparent. For solving problems, either the existing MCDM or MPDM solution methods must be modified or new methods must be developed.

Preservation of preference information. In either the reduction or decomposition approach, aggregation of the individual interests into a group objective or aggregation of criteria may be applied as a part of the decision making process. With any type of

aggregation, some information is expected to be lost in modelling of the MCMP problem (e.g., Bergstresser and Yu, 1977). Moreover, the structure of the decision making process for the reduction or decomposition approach restricts simultaneous consideration of all preference information. That is, at any point in time, all possible information may not be available to the decision makers. Absent preference information implies a possibility of inadequate modelling of MCMP problems. Inadequate modelling can hinder attainment of a good solution. On the other hand, the integrative approach reduces a risk of inadequate modelling by avoiding aggregation from the process of decision making. By dealing with the MCDM and MPDM components simultaneously, it also retains more preference information concurrently in the decision making process. Furthermore, since the MCDM and MPDM components are considered simultaneously, the integrative approach allows interaction between the two components. The additional information from the interaction can only be captured by the integrative approach.

Misrepresentation. When MCDM and MPDM are not considered simultaneously in the solution approach, in addition to lost or absent information, misrepresentation may occur. The MCMP problem may be misrepresented because the original decision making goals are falsified or only partially expressed. In the reduction approach, only one type of the MCDM and MPDM goals is treated as the main decision making goal. The reduction process may not be able to sustain some part of the original goals. For example, when the attitude among the multiple parties is extremely competitive, the group objective can significantly falsify some or all parties' interests. In the decomposition approach, MCDM and MPDM goals are pursued alternately. This may cause difficulty in expressing and hence satisfying all goals simultaneously because the scope of the decision making perspective is always somewhat limited. In either case, there is a possibility of misrepresenting the problem to a sufficient degree that the attained solution may not be useful. The integrative approach avoids this issue by dealing with MCDM and MPDM simultaneously.

The discussion of the advantages and disadvantages of the solution approaches suggests that the integrative approach has a strong edge over the other two approaches. It cannot be concluded, however, that the integrative approach is always better than the other approaches. For example, if manageability of the problem is extremely important to the decision makers,

then the tractability of the reduction or decomposition approach is certainly attractive. What can be concluded is that, in general, the integrative approach seems more desirable than the other approaches by promising the most preservation of information and satisfactory representation of the problem. Poor tractability may be compensated by increasing the tractability of an individual solution method that uses the integrative approach. For example, one can try to simplify the decision making procedure of a solution method as much as possible.

At this point, the advantages and disadvantages of the proposed classification scheme are reviewed. The classification scheme that consists of the reduction, decomposition, and integrative approaches has several desirable features. It is based on the structural properties of solution approaches that are elementary for all types of MCMP problems. Consequently, the classification scheme is universal and comprehensive. In fact, the three types of solution approaches are also collectively exhaustive since there are no other structures available to describe an approach based on the MCDM and MPDM components.

In terms of decision support, the proposed classification scheme can improve MCMP decision making by assisting with identification of a starting point for solving various MCMP problems. This is a definite benefit for the decision makers who have little or no experience with MCMP problems. The presentation of three approaches offers a choice regarding how to proceed in solving the problem. For the participants who have previously dealt with either MCDM or MPDM problems exclusively, the classification helps expand their perspective to acknowledge other approaches.

From the decision support perspective, the main limitation of the scheme is that the comparison of the three approaches is simplistic at this point. Refinement of the classification may be an asset for more elaborate and practical decision support. So far, the integrative approach is simply defined as a solution approach that considers MCDM and MPDM simultaneously. Within the integrative approach, different structures may exist. If so, identification of distinct types would provide further refinement to the current classification and also provide different ways to compare solution approaches. Hence, further research is required to provide sufficient information in order to devise a useful decision support system for MCMP problems.

3.3 Difficulty in MCMP

Consider the following difficulty discussed in the negotiation literature. Having multiple issues increases complexity enormously in the negotiation situation. The negotiators may not be able to reach an agreement due to the difficulty of identifying and understanding the possible solutions (UNISYS, 1987). Even if an agreement is reached, it may not be the best solution possible for any of the parties. In fact, empirical research suggests that negotiators often settle for suboptimal compromises (Neale and Northcraft, 1991; Pruitt, 1981; Roth, 1985; Thompson, 1991). Due to the conflicting criteria and participants, optimization in the overall context of MCMP is rather meaningless. However, one can at least attempt to avoid leaving a better solution undiscovered. In other words, to improve the quality of decision making, one must try to reduce the possibility of missing a solution that is preferred by all parties, namely, an efficient or Pareto optimal solution.

One of the reasons for reaching an inefficient decision is an insufficient understanding of the feasible solution set. This can be attributed to the complexity of MCMP problems. The individual decision makers may have difficulty assessing their own preferences in terms of multiple criteria. Having multiple participants who exhibit a competitive and distrusting relationship may also hinder sharing of preference information. In general, it is likely that the decision makers do not have a good comprehension of the feasible set of their MCMP problem. It is also likely that they make their decision even without being aware of this lack of knowledge regarding the feasible set. Moreover, indiscriminate application of an MCMP solution approach may result in the distorted feasible set caused by loss or absent of information or misrepresentation of the problem.

In a classical example of a quarrel between two sisters over an orange (Fisher and Ury, 1981), the sisters agree on a compromise to split the fruit in half. Then one sister squeezes her portion for juice while the other uses the peel from her portion in a cake. This example illustrates how the existence of a better solution that the first sister gets all the juice and the second one gets all the peel can be left undiscovered by the insufficient understanding of the feasible set. If the sisters had better comprehension of each other's preference and the actual feasible set, the solution would have been obvious. Compromise is a common alternative

that stands part way between the parties' preferred positions (Pruitt and Rubin, 1986). However, in this example, the compromise principle fails to account for the existence of two issues and the difference in the sisters' preferences, and therefore, results in an inefficient solution.

In order to assist the decision makers with a more accurate and complete assessment of the feasible set, the increased incorporation of preference information becomes crucial. Among the three types of MCMP solution approaches, the integrative approach is best suited for this purpose. The reduction and decomposition approaches are usually involved with some type of aggregation, and they may experience absent preference information as a result. They also restrict the simultaneous utilization of information due to a two-phase or alternating decision making process. The integrative approach, on the other hand, allows the most concurrent intake of preference information in the decision making process by considering MCDM and MPDM simultaneously. In order to increase the incorporation of preference information, the integrative approach has the most suitable structure, and therefore, can provide improvement opportunities where other approaches have difficulty doing so. In this aspect of improving the quality of MCMP, the integrative approach shows definite advantage over the other two approaches.

3.4 Improved Solution Methods

Based on the integrative approach, two improved MCMP solution methods are proposed. Each method is briefly described below.

The solution methods that have been used for solving multicriteria games basically represent multiple criteria by vector payoffs. These methods are not designed to deal with conflicting criteria explicitly or utilize criterion importance as part of the preference information perhaps because multiple criteria and their importance have been traditionally foreign concepts in game theory. To incorporate the additional preference information, that is, the existence of criteria and their importance, in the analysis, an alternative solution method based on the integrative approach for multicriteria games is proposed. In the

proposed method, ordinal player preference and ordinal criterion importance are considered. Ordinal information offers some useful advantages (Fraser, 1994). For instance, it is relatively easy to obtain from a decision maker, and it is sufficiently meaningful without disclosing too much especially in the presence of other opposing parties.

Instead of dealing with one game with vector payoffs, the method proposed here enumerates single-payoff games, called candidates, corresponding to *all* possible combinations of players' different criteria and criterion importance. Candidate enumeration is based on the properties of preference trees (Fraser, 1994). A preference tree is a binary tree model to structure ordinal preferences. More detail is given in the following chapter. Since the candidate enumeration method takes the ordinal preference information into account, the enumeration algorithm produces a finite number of such single-payoff games or candidates. These games are not created as separate subproblems. As a set, they show players' criteria with different levels of criterion importance and the interdependence among the players. Once the set of candidates is generated, the dominant strategies and equilibria are examined. This is an integrative approach since it considers MCDM and MPDM simultaneously. It incorporates the existence of criteria and criterion importance explicitly.

Criterion importance can have a richer content if it is represented by a criterion weight. Another proposed solution method based on the integrative approach includes a set of criteria weights as part of preference information in multi-issue negotiation. Many of the current multi-issue negotiation methodologies, that take the integrative approach, do not utilize criterion weight information explicitly or do not utilize it at all. Criteria weights provide trade-off information among criteria and also among parties. The proposed method seeks possible trade-offs that result in a better solution for all parties. The integrity of MCMP is conserved by considering both trade-offs among criteria and among parties simultaneously.

The proposed trade-off procedure is called *logrolling* which is the exchange of loss in some issues for gain in others resulting in mutual overall gain. The key to identifying mutual gain is recognition of the exchange rates that permit logrolling. The range of such exchange rates can be determined from the parties' criteria weights and is referred to as the *logrolling range*. The justification and the existence of the logrolling range are based on the underlying concept of Keeney and Raiffa's (1976) weight assessment technique using trade-offs. Detail

is shown in Chapter 5. The logrolling procedure guarantees Pareto optimal outcomes. For more than two issues, the logrolling procedure that deals with two issues at a time is proposed in order to simplify the trade-off process for the decision makers. This is an example of an attempt to increase tractability of the integrative approach.

In reality, obtaining more information can be a difficult practice among competing and distrusting participants. The assessment of an individual decision maker's criteria weights can be a troublesome practice itself (e.g., Boulet and Fraser, 1995; vonWinterfeld and Edwards, 1986). However, the benefits of having more information as discussed in this chapter hopefully encourage the decision makers to overcome the possible drawbacks.

3.5 Summary

This chapter essentially lays the backbone of the research. The classification scheme that consists of the reduction, decomposition, and integrative approaches is proposed and the features of each approach are discussed. An opportunity for improvement is identified from studying the apparent difficulty in the multi-issue negotiation literature. It is shown that this improvement opportunity is realizable through the integrative approach. Two MCMP solution methodologies, candidate enumeration and logrolling, are then proposed based on the integrative approach. Candidate enumeration deals with ordinal preference information and discrete alternatives. Logrolling deals with cardinal information and continuous alternatives. The full description of the two methods can be found in the following chapters.

The classification scheme proposed in this chapter has played a crucial role in shaping the perspective of the MCMP research. Without the framework provided by the three solution approaches, the design of the two methodologies specifically intended to be the integrative approach would have been difficult.

Chapter 4

An Integrative Solution Method for Multicriteria Games

Conflict among multiple parties has been modelled using the concept of games. In game theory, the recognition of conflict within an individual has been introduced as vector payoffs. This chapter proposes a game theory-based integrative method for MCMP problems, which does not express multiple criteria in the vector form of payoffs. An alternative way to include criteria in an MPDM model is proposed for the following purposes: to deal with conflicting criteria explicitly, to achieve a satisfactory overall picture of the decision making situation, and to allow utilization of criterion importance information which is not attempted by most of the existing multicriteria game methodologies. In particular, the solution analysis of bicriteria ordinal 2×2 games is investigated. Bicriteria 2×2 games are the smallest possible multicriteria n-person games. They may seem restrictive, but they still possess the fundamental characteristics of multicriteria n-person games. The simplicity of bicriteria 2×2 games can be considered as an advantage rather than a disadvantage for exploratory purposes.

The elements of the decision model are described first in Section 4.1. Then the key concepts, preference trees and candidates, are discussed in Sections 4.2 and 4.3. Based on these concepts, the solution analysis is developed. There are three steps involved in the analysis. Each of the three steps is discussed in turn in Sections 4.4, 4.5, and 4.6. The summary of this chapter is given in Section 4.7.

4.1 Bicriteria Ordinal 2×2 Games

The elements associated with a game model are as follows. There are two players, arbitrarily called Player 1 and Player 2, who are assumed to be noncooperative with each other. That is, no preplay communication is allowed between the players. The solution analysis is asymmetrically prescriptive, that is, the analysis is prescriptive from one player's point of view and descriptive from the other's point of view (Raiffa, 1982). Player 1's perspective is chosen to be the perspective of the analysis. The two players have one option each, and they decide either to take or not to take the option. In this case, the number of strategies is two for each player, and the number of possible outcomes is four.

Options can be considered as the building blocks of strategies (Fraser, 1993*b*). In general, a strategy for a player consists of a set of possible options, each of which is decided to be taken or not taken by the player. Options are not necessarily mutually exclusive, as strategies are. It is assumed that the players' preferences for outcomes are given and known to all players. The players' preferences are expressed using a strict ordinal scale of 1, 2, 3, and 4. 4 is assigned to the most preferred outcome, 3 is to the second most preferred, 2 is to the third most preferred, and 1 is to the least preferred outcome. Ordinal preferences are known to be easier to obtain and more meaningful than cardinal preferences, and descriptive without being specific (Fraser, 1994). It is also known that ordinal preferences require fewer assumptions than cardinal preferences do. For example, transitivity may or may not be required for ordinal preferences. *Preference trees* are used to model ordinal preferences. They are discussed further in the following section.

The elements from an MCDM model incorporated into the decision model are criteria and their importance. Two criteria are assumed to exist for each player. They are not necessarily identical for the two players. In the existing MCMP models based on game theory, criteria are incorporated in the vector form of payoffs within single game. In the proposed model, instead of vectors of payoffs, two criteria are separately expressed in two games with a single payoff as an outcome. This is not supposed to imply the creation of two subproblems to the original problem. The concept of candidates provides the connection between the two games

corresponding to two criteria and the original bicriteria game. The description of candidates is given in Section 4.3. Criterion importance is also included in the solution analysis. The existing methods for multicriteria games do not consider criterion importance in their models except Wang (1993) in which the weight Nash equilibrium is defined based on the inner product of a weight vector and a payoff vector.

In the next two sections, the two key concepts of the proposed solution method, preference trees and candidates, are described. They are the basis of the development of the solution analysis.

4.2 Preference Trees

A preference tree is a binary tree model to structure ordinal preferences. More precisely, a preference tree is a binary tree in which the leaves of the tree represent an ordered set of alternatives, and each node bifurcates a subset of the alternatives according to a logical preference statement (Fraser, 1993*a*). In an MPDM problem, a preference statement is obtained by specifying the following three features:

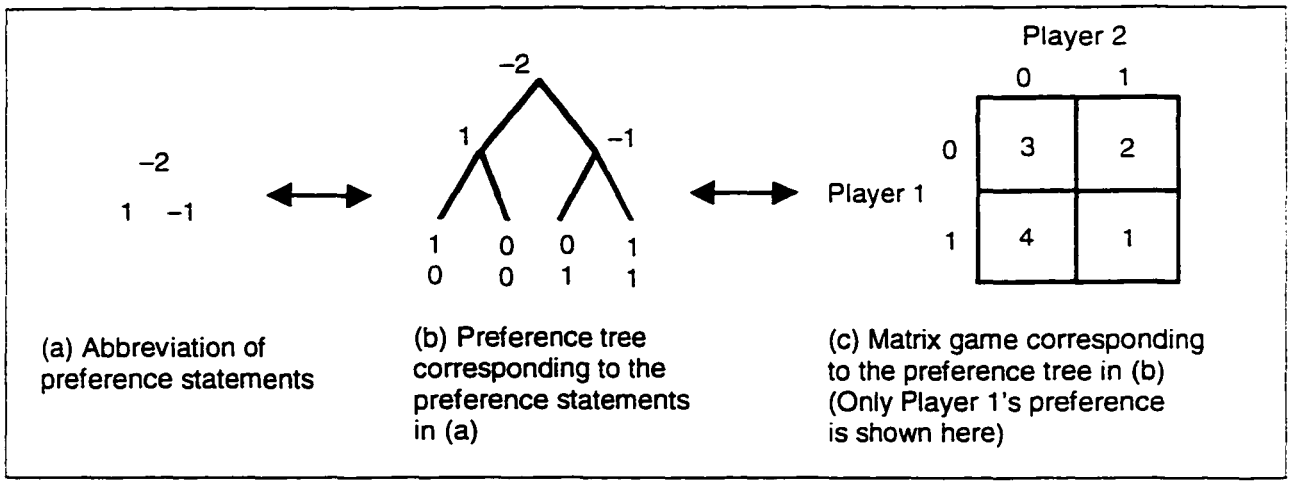
- the ordering of all options according to their *importance*,
- the *desirability* of each option, either to take or not to take the option, and
- *preferential conditions* if any exists, such as option 1 is taken if and only if option 2 is not taken.

A set of preference statements is converted to a binary tree by assigning the options to nodes of the tree according to their importance ordering with an indication of whether an option is taken or not taken using a plus or minus number, respectively. The root node is for the most important option, the next subsequent node is for the second most important option, and so on. The assignment of the options to the nodes also depends on preferential conditions if any exists. The leaves of the tree are the columns of 0's and 1's specifying the desirability of each option: 0 if the option is not taken and 1 if the option is taken. The columns represent

the outcomes, and they are ordered so that the column on the most left is the most preferred outcome by the player, and the second from the left is the second most preferred outcome, and so on.

An example of preference trees is given in Figure 4.1 corresponding to the following set of preference statements. "Player 1 has option 1 and Player 2 has option 2. To Player 1, option 2 is more important than option 1. Player 1 prefers Player 2 not taking option 2 than taking it. Player 1 prefers not to take option 1 if and only if option 2 is taken by Player 2." Figure 4.1*a* shows the abbreviation of the preference statements. Option 2 is on top of option 1 to indicate its greater importance to Player 1, and plus and minus numbers show the desirability of the options. Option 1 on the bottom has two different signs to indicate a preferential condition. Figure 4.1*b* is the preference tree corresponding to the set of preference statements. Figure 4.1*c* is the matrix form of the game corresponding to the preference tree.

Figure 4.1 An Example of Preference Trees



For this study, preference trees with symmetric shape are studied. That is, only the trees with a simple preference statement are studied. A simple preference statement includes information about each option's importance and desirability, and preferential conditions, if any exists, with two options involved in the "if and only if" statement. That is, "1 if and only if 2" is an example of a simple preference statement whereas "1 if and only if 2 or 3" is not. For

the total of m options for all n players, the number of preference trees with symmetric shape, denoted T, is computed by the following expression:

$$T = \prod_{i=0}^{m-1} (m-i)\left[\binom{2^i}{0} + \binom{2^i}{1} + \cdots + \binom{2^i}{2^i}\right].$$

The first term, $(m-i)$, essentially keeps track of the number of options available for a position in importance ranking, and the second term keeps track of the number of different combinations of desirability, including possible preferential conditions once an option is assigned to a position in importance ranking. According to this expression, 16 distinct preference trees with symmetric shape exist for a decision model with a total of 2 options.

In essence, a preference tree captures the underlying preferential information in a considerably compact manner by making use of its lexicographic structure. Also, by dealing with the importance and desirability of options, and not outcomes, it can represent human preferences at a higher cognitive level than the simple ordering of outcomes (Fraser, 1994). Compact preference representation at a high cognitive level is the reason that the operations among preference trees described in the next section are possible and meaningful for producing candidates.

4.3 Candidates

This section concentrates on the description of candidates. For two players having two strategies, there are four possible outcomes. It should be straightforward to rank the four outcomes with respect to one criterion. Consequently, the preference rankings of outcomes represented by two preference trees corresponding to two criteria should be obtained with little effort. In the MCDM methdologies, the next commonly taken step is the aggregation procedure. Instead of aggregating the two preference trees, the possible *candidates* for the aggregated preference tree are considered. The main purpose of introducing candidates is to conserve the integrity of MCMP problem structure without any form of aggregation.

In MCDM, by assigning different sets of criteria weights, one can have different aggregated scores for the alternatives. Similarly, the rankings of the alternatives according to multiple criteria can be aggregated into one ranking in many different ways by assigning the different sets of criteria weights. In other words, without specifying a set of criteria weights, one cannot have *the* aggregated ranking of the alternatives. One is left with many possible rankings which may be the aggregated ranking if a set of criteria weights is given. These possible rankings, before the set of criteria weights is specified, are represented by the preference trees called the candidates. According to the expression of T in Section 4.2, for the decision model with two options in total, there exist at most 16 candidates since the number of distinct preference trees with symmetric shape is 16.

By studying all possible candidates, the loss of information through aggregation is avoided, and the actual assignment of criteria weights is not required. More importantly, studying all candidates for both players conserves the integrity of MCMP problem structure by dealing with MCDM and MPDM components simultaneously. Therefore, the proposed solution method is referred to as the *candidate enumeration* method. The formal definition of a candidate is given below. It is based on the features that describe a preference tree.

Definition 4.1 The *structural features* of a preference tree are the importance of an option, the desirability of an option, and preferential conditions if any exists.

Definition 4.2 A *candidate* for the aggregated tree of two preference trees is a preference tree which satisfies the following two conditions.

> **Condition 1.** If the two preference trees possess the same structural feature, then a candidate has the same feature as well.
>
> **Condition 2.** If the two preference trees have different features, then a candidate possesses a feature from one tree or the other, or a feature derived from combining the two trees' features.

Examples for Conditions 1 and 2 are as follows. Suppose that option 1 is most important in preference tree 1 and preference tree 2. Then Condition 1 states that a candidate also exhibits option 1 as the most important option. In fact, all possible candidates must have option 1 as the most important option according to Condition 1. Now suppose that option 1

is most important for preference tree 1 and option 2 is most important for preference tree 2. Then Condition 2 states that a candidate can have option 1 or option 2 as its most important option. If option 1 is preferred to be taken for tree 1 and not for tree 2, then, by Condition 2, a candidate can have the same desirability as in tree 1 or tree 2, or can have preferential conditions which are considered to be derived from combining the two trees' different desirability. The two original trees are candidates for the aggregated tree as well.

There is an underlying principle in the definition of a candidate. It is the principle of heredity. In the biological sense, heredity is meant passing on of physical or mental characteristics genetically. A simple illustration is given for the origin of the two conditions in the definition of a candidate. Suppose that two parents both have brown eyes, then their child will most likely have brown eyes. Condition 1 is originated from this idea. If one parent has brown eyes and the other has blue eyes, then their child may likely to have either brown or blue eyes, or there is some possibility that the child may have some different colour, say hazel. Condition 2 is based on this idea.

Heredity is chosen to be the principle for producing candidates due to an assumption that a candidate should take after the two preference trees in some systematic manner. Since it is easy to consider two preference trees as two parents and a candidate as a child of the parents, heredity can explain the similarity of a candidate to the two preference trees as the similarity of a child to the parents based on the genetic principles.

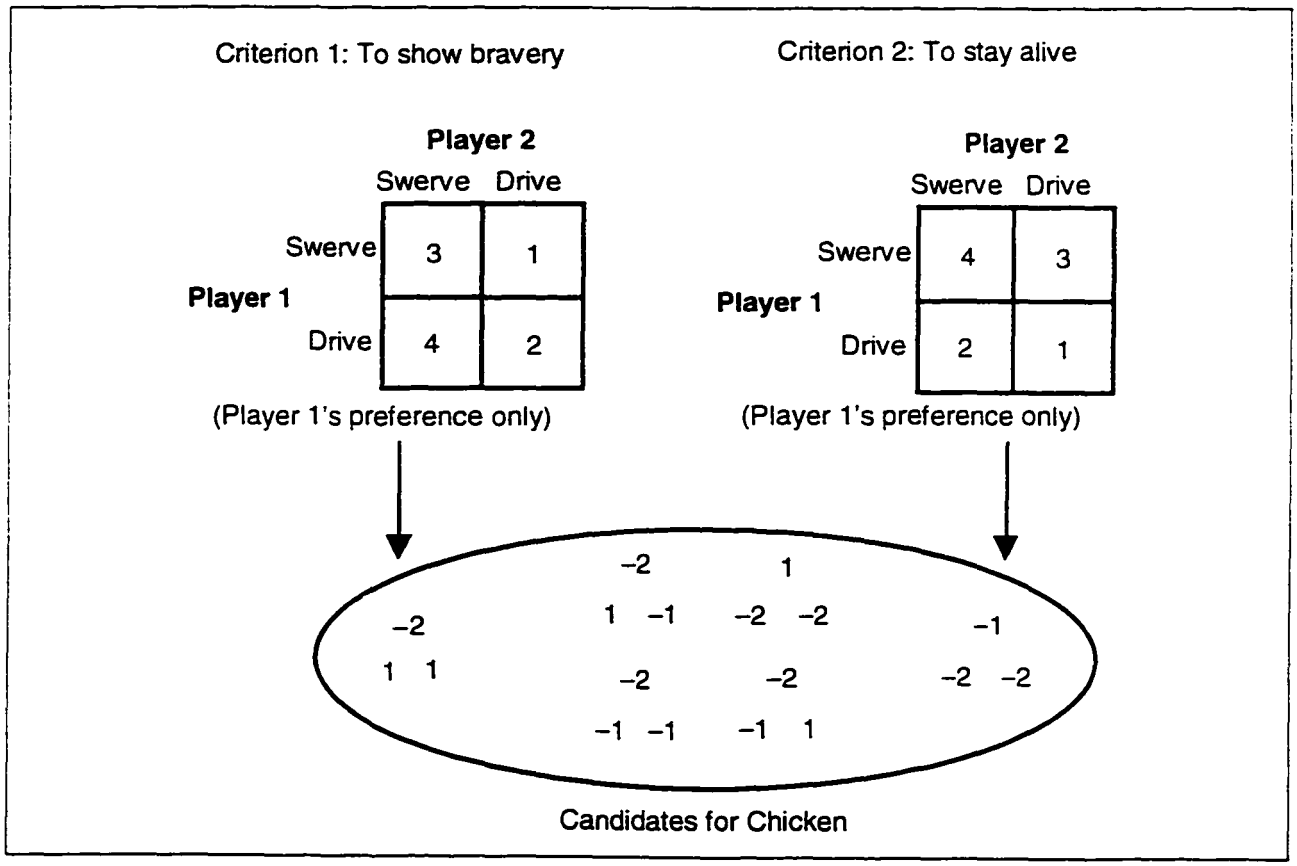
An algorithm has been developed using the concept of candidates as a basis. It is referred to as the *candidate enumeration algorithm* and is presented in Appendix A.1. The algorithm is designed to produce all possible candidates for the aggregated preference tree.

Example: Chicken

Throughout the remaining sections in Chapter 4, the game of Chicken is used as an example. Chicken is known as the game to compare two people's courage often by driving a car towards each other at high speeds. Whoever swerves first is the "chicken" and loses. If both swerve at the same time, then they lose some face to their peers, but if both continue to drive,

they will likely seriously injure themselves or even die. Chicken has been modelled as a 2×2 game. Two players both have an option to continue driving. That is, each player has two strategies either to continue driving or not to continue driving. Let Player 1's option be called option 1 and Player 2's option be called option 2. In addition to the classical model, two criteria are explicitly revealed. To show bravery is one criterion, called criterion 1, and to stay alive is the other criterion, called criterion 2. The two criteria are assumed to be the same for both players. Sample ordinal preferences for the four outcomes are obtained with respect to each criterion separately. These preferences are modelled by two preference trees. The candidates are then produced using the candidate enumeration algorithm from these two trees. Figure 4.2 shows all candidates of the game of Chicken for Player 1 in the abbreviated form of preference trees.

Figure 4.2 The Game of Chicken and Its Candidates for Player 1



Based on the concept of candidates, the solution analysis is developed. There are three steps. The following three sections correspond to the three steps of the solution analysis.

4.4 Solution Analysis Step 1: Dominant Strategies

By using the candidate enumeration algorithm, all possible candidates for the aggregated preference tree of two preference trees corresponding to two criteria can be produced. Recall that a candidate is a preference tree which represents a ranking of outcomes. Since the problem is modelled as a game problem, a solution concept in game theory is applied to the games produced by matching the candidates of one player with the candidates of the other player. The games produced by matching the candidates of two players are called the *candidate games*. The existence of equilibria or Nash equilibria (Nash, 1951) as defined below is investigated for the candidate games.

> **Definition 4.3** An outcome is *equilibrium* if it is rational for all players. An outcome is *rational* (or Nash rational) if it is the player's best outcome given the other players' strategy choices.

Investigation of equilibria is actually the second step in the solution analysis. Before equilibria are examined, dominant strategies are investigated for Player 1's candidate games. Recall that Player 1's perspective is arbitrarily chosen to be the perspective of the analysis. The rational outcomes for one player can be determined without the other player's preference information, and studying the rational outcomes determines the existence of a dominant strategy.

> **Definition 4.4** A strategy is a *dominant strategy* of one player if and only if, by taking the strategy, the outcome is always more preferred for the player than taking other strategies no matter what the other player's strategy may be.

The investigation of rational outcomes is the first stage of solution analysis which is at the individual level without the players' interaction. The existence of a dominant strategy

examined for all candidate games can provide significant information for determining the solution strategy.

Suppose that Player 1 has option 1 and Player 2 has option 2. By focusing on the existence of a dominant strategy for one player, the following lemma is deduced from the structural features of preference trees.

Lemma 4.1 Consider a preference tree and the game whose outcome ranking is represented by that preference tree. There exist three types of relationship between the *desirability* of a player's option and the existence of a dominant strategy for the associated game.

The three possible cases are illustrated using Player 1 and his/her option. option 1. The notation of the outcomes used here is the representation of the outcomes in the preference tree as shown in Figure 4.1*b*. Note that the importance of option 1 is not relevant in any of the cases.

(i) If Player 1 prefers to take option 1, then the strategy of taking option 1 is dominant.

Proof. If Player 1 prefers to take option 1, then $\begin{matrix}1\\0\end{matrix}$ is preferred to $\begin{matrix}0\\0\end{matrix}$, and therefore, $\begin{matrix}1\\0\end{matrix}$ is rational by Definition 4.3. Also, $\begin{matrix}1\\1\end{matrix}$ is preferred to $\begin{matrix}0\\1\end{matrix}$, and $\begin{matrix}1\\1\end{matrix}$ is rational by Definition 4.3. The strategy of taking option 1 yields a rational outcome no matter what the other player's strategy may be. Therefore, taking option 1 is a dominant strategy by Definition 4.4. ■

(ii) If Player 1 prefers not to take option 1, then the strategy of not taking option 1 is dominant.

Proof. Same argument as above. ■

(iii) If there is a preferential condition on Player 1's option, then a dominant strategy does not exist.

Proof. A preferential condition on option 1 implies that taking option 1 is preferred to not taking option 1 depending on Player 2's strategy. That is, none of Player 1's strategies *always* yield a rational outcome. Therefore, a dominant strategy does not exist for Player 1. ■

The preference trees can be classified into three different groups in terms of the desirability of a player's option. The definition for these groups are given below in terms of Player 1 and option 1. The groups are collectively exhaustive.

Definition 4.5 A preference tree in which taking option 1 is preferred to not taking option 1 belongs to *group 1*. A preference tree in which not taking option 1 is preferred to taking option 1 belongs to *group −1*. A preference tree in which a preferential condition exists on option 1 belongs to *group 0*.

By Lemma 4.1, the following conclusion can be made on each group of preference trees. The games whose outcome rankings are represented by the preference trees in group 1 have a dominant strategy of taking option 1. The games whose outcome rankings are represented by the trees in group −1 have a dominant strategy of not taking option 1. The games whose outcome rankings are represented by the trees in group 0 have no dominant strategies.

The next series of lemmas is the basis for analyzing the dominant strategies of the candidate games. Lemmas 4.2.1, 4.2.2, and 4.2.3 identify the group type(s) of candidates. Then a conclusion is made regarding the existence of a dominant strategy based on Lemma 4.1 and Definition 4.5.

Let *C1* be the preference tree corresponding to the ranking of outcomes with respect to criterion 1, and *C2* be the preference tree corresponding to the ranking of outcomes with respect to criterion 2. Specification of the group types of *C1* and *C2* determines the group(s) in which the candidates belong.

Lemma 4.2.1 If *C1* and *C2* both belong to group i, then all candidates also belong to group i, $i = 1, -1, 0$.

Proof. For $i = 1, -1$, there are three cases to consider: (i) option 1 is most important option for *C1* and *C2*, (ii) option 1 is second most important option for *C1* and *C2*, and (iii) option 1 is most important option for one of *C1* and *C2*, and second most important for the other. For any of these cases, the candidates always have the same desirability of option 1 as in *C1* and *C2* based on the candidate enumeration algorithm. This can be shown by following the steps involved in the algorithm. The detail is contained in Appendix A.2. Therefore, all candidates also belong to the same group as *C1* and *C2*.

For $i = 0$, there are two cases to consider: (iv) $C1$ and $C2$ have the same preferential condition, and (v) $C1$ and $C2$ have different preferential conditions. For case (iv), the candidates always have the same preferential condition on option 1 as in $C1$ and $C2$, and for case (v), the candidates either have the preference condition on option 1 as in $C1$ or as in $C2$ according to the candidate enumeration algorithm. The detail is available in Appendix A.2. Therefore, all candidates also belong to group 0 for either case. ■

Lemma 4.2.2 If one of $C1$ and $C2$ belongs to group i and the other belongs to group 0, then some candidates belong to group i and some belong to group 0, $i = 1, -1$.

Proof. As seen in the proof of Lemma 4.2.1, by considering the possible cases for option 1's importance in $C1$ and $C2$ and by following the steps involved in the candidate enumeration algorithm, one can obtain the desired result. The detail is omitted. ■

Lemma 4.2.3 If one of $C1$ and $C2$ belongs to group 1 and the other belongs to group -1, then the candidates belong to any of the three groups.

Proof. Same argument as above. ■

The implication of Lemmas 4.2.1–4.2.3 is as follows. By specifying the group types of $C1$ and $C2$, the type(s) of the candidates can be identified. If all candidates belong to either group 1 or group -1, this implies that a unique dominant strategy exists for all candidate games based on Lemma 4.1 and Definition 4.5. When there is a unique strategy for all candidate games, it is then the solution strategy for the original bicriteria game. For any other case, there is no unique dominant strategy for all candidate games, and hence, the solution strategy is indeterminate at this point. The solution analysis then proceeds to the next stage: to include the other player's preferences and strategies, that is, to examine equilibria. Even with the identification of a unique solution strategy at this point, without the other player's preference information, the existence of equilibria remains undetermined. Therefore, some useful information may still be gained by proceeding to the next stage of analysis in any case.

Example: Chicken

All candidates for the game of Chicken are shown in Figure 4.2 in the previous section. According to the definitions of the groups, $C1$ for Chicken belongs to group 1 and $C2$ belongs to group -1. By Lemma 4.2.3, no specific conclusion can be made on the type of the candidates. Consequently, no unique dominant strategy exists for the candidates games, and therefore, no unique dominant strategy is identified for Player 1 at this stage. Further analysis including the other player's preferences and strategies is in order.

4.5 Solution Analysis Step 2: Equilibria

If Player 1 has a unique dominant strategy for all candidate games, then this dominant strategy is the solution strategy for Player 1. When a unique dominant strategy does not exist, the next step is to produce the *matrix of candidates*. By matching the candidates of one player with the candidates of the other player, the games with a single payoff as an outcome are produced. These games can be organized in the matrix form, and hence the term matrix of candidates is given. A matrix of candidates is constructed for the example of Chicken in Figure 4.3 at the end of this section.

By considering all candidates for both players, the two players' two criteria are present in the model and included in the analysis of games. The matrix of candidates succeeds in showing an overall picture of the decision making situation including the existence of equilibria. It also maintains the integrity of MCMP problem structure. This is mainly possible due to the ordinal preference structure. Players' preferences among possible outcomes and criterion importance are all ordinal in the candidate enumeration method. Ordinality allows generation of a finite number of candidates, and consequently, a finite number of candidate games, by the candidate enumeration algorithm.

The second step of the solution analysis investigates the existence of a unique solution strategy by examining the matrix of candidates. Since an equilibrium is the outcome which is

rational for all players, if one exists, it can provide significant information regarding the solution strategy in the absence of a dominant strategy. Information from all candidate games may help determining the solution strategy for the original bicriteria 2×2 game. The following observations made on the existence of equilibria for the games in the matrix of candidates can be used to trace out a sequence of analysis steps.

Observation 4.1 Suppose that Player 1 does not have a unique dominant strategy across the candidate games. If Player 2 has a dominant strategy for all candidate games in the matrix of candidates, then the existence of one equilibrium is guaranteed for every game.

If the equilibria in the matrix of candidates identify a unique solution strategy for Player 1, then this unique strategy is the solution strategy for the original game.

If the equilibria in the matrix of candidates do not identify a unique solution strategy for Player 1, then the solution strategy for the original game is indeterminate at this point. The solution analysis proceeds to the next step which is described in Section 4.6.

Observation 4.2 Suppose that Player 1 does not have a unique dominant strategy across the candidate games. If Player 2 does not have a dominant strategy for all candidate games in the matrix of candidates, then the existence of one equilibrium for each game is not guaranteed. An equilibrium may or may not exist for all candidate games.

If an equilibrium exists for every candidate game in the matrix of candidates, then one can follow the same analysis steps described in Observation 1.

If an equilibrium does not exist for all candidate games in the matrix of candidates, there must be no equilibria or two equilibria for one or more games in the matrix of candidates. The solution strategy for the original game is indeterminate, and the solution analysis proceeds to the next step in Section 4.6.

In summary, once it is realized that Player 1 does not have a unique dominant strategy for the candidate games, Player 2's preferences are included in the solution analysis by producing the matrix of games. The existence of equilibria for the candidate games in the matrix of candidates is examined first, and the existence of a unique solution strategy for the original game is considered. If the solution strategy is indeterminate at the end of this analysis stage, then the analysis proceeds to the next stage: analysis of criterion importance (Section 4.6). The analysis of criterion importance is the last stage of the solution analysis. The main

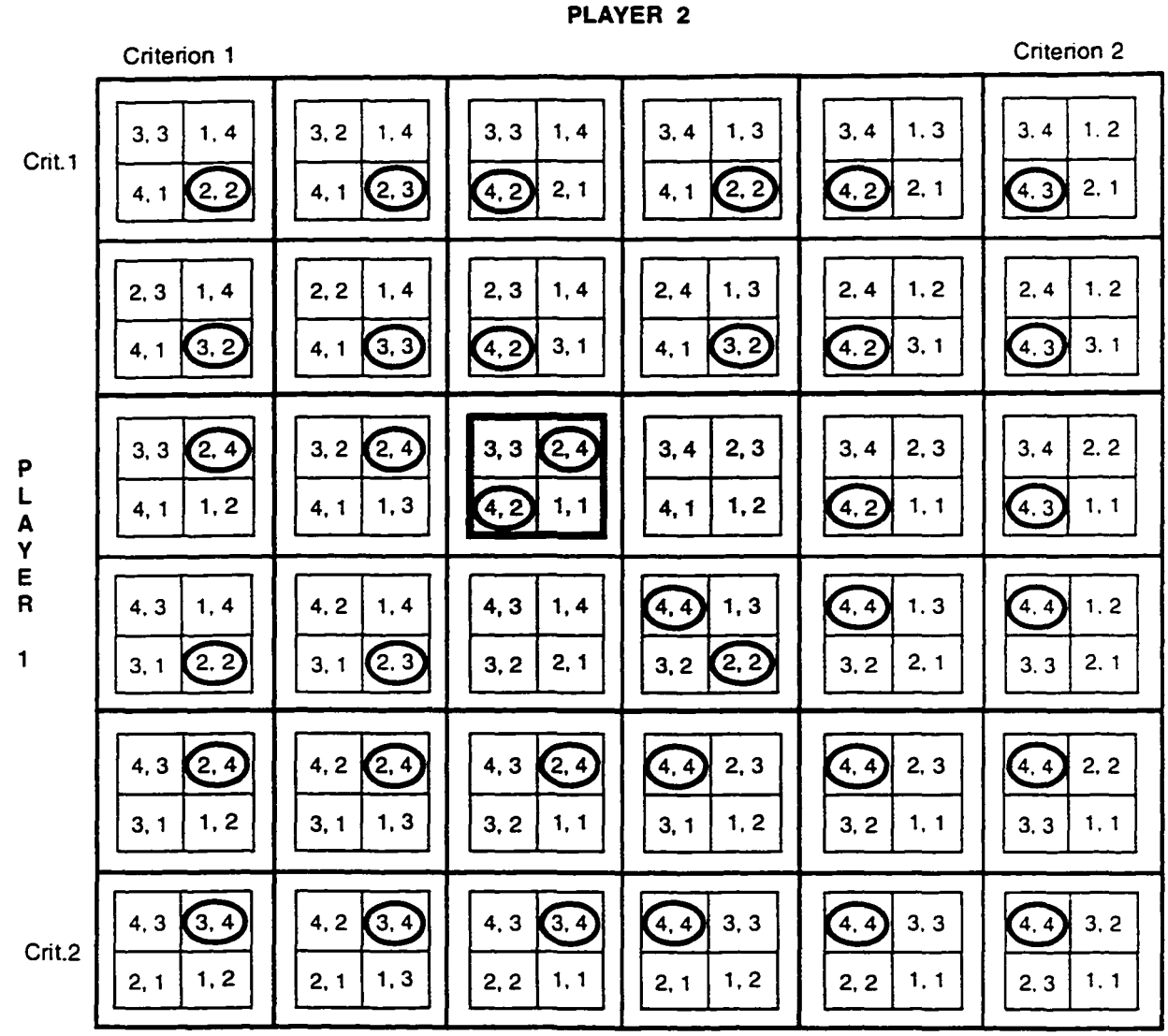
purpose is to reduce the size of the matrix of candidates if possible in order to narrow down the focus of the analysis and to gain further insight.

Example: Chicken

It has been shown that there is no unique dominant strategy for Player 1's candidate games for the game of Chicken. As the next step in the solution analysis, the matrix of candidates is constructed. Figure 4.3 shows an example of the matrix of candidates for Chicken. This matrix consists of six rows and six columns. The number of rows corresponds to the total number of candidates for Player 1 as shown in Figure 4.2, and the number of columns corresponds to the number of candidates for Player 2 which can be determined in a similar manner. The ordering of candidates in the matrix is arranged by different groups as defined in Definition 4.5. There is a reason for this arrangement, and it is discussed further in the following section. The circled outcomes are the equilibria of the candidate games. The shaded game with the thick border represents the classical 2×2 game of Chicken.

It is observed that Player 2 does not have a dominant strategy for all candidate games, so the steps described in Observation 2 are followed. Then, the existence of zero or two equilibria are identified for the games that are shaded in Figure 4.3. Therefore, the solution strategy for the original game is indeterminate, and the analysis proceeds to the next stage.

Figure 4.3 The Matrix of Candidates for Chicken



4.6 Solution Analysis Step 3: Criterion Importance

When the examination of the matrix of candidates does not identify the solution strategy, the concept of criterion importance is introduced to the analysis. The main purpose of this step is to reduce the size of the matrix of candidates as much as possible. As the size of the matrix of candidates is reduced, the number of possible solution strategies either does not change or

decreases. Reduction in size, however, does not guarantee a unique solution strategy. Even so, the final decision, which must be made with or without the solution strategy derived from the analysis, may be facilitated by considering a smaller number of possible solutions. The concept of candidates is based on not assigning criteria weights in order to produce all possible candidates and conserve the integrity of MCMP problem structure. Therefore, the utilization of ordinal criterion importance is proposed. The ordinal information of criterion importance is the knowledge of one criterion being more important than another criterion.

Let $C1$ be the preference tree corresponding to the ranking of outcomes with respect to criterion 1, and $C2$ be the preference tree corresponding to the ranking of outcomes with respect to criterion 2. $C1$ and $C2$ can be identified in terms of groups defined in Section 4.4. Recall that, by Definition 4.5, a preference tree associated with the information that taking option 1 is preferred to not taking option 1 belongs to group 1. If not taking option 1 is preferred, then the corresponding preference tree belongs to group -1. A preference tree with a preferential condition on option 1 belongs to group 0. It is assumed that the ordinal criterion importance information for Player 1 can be obtained from Player 1's general preference on the desirability of option 1 since the desirability is a cruicial feature in determining a dominant strategy. More specifically, the following assumption is stated.

Assumption 4.1 Suppose that $C1$ belongs to group i and $C2$ belongs to group j. $i, j = 1, 0, -1$. If, in general, Player 1 agrees with the information contained in the definition of group $i(j)$, then it is concluded that $C1$ ($C2$) is more important than $C2$ ($C1$). If Player 1 is indifferent or uncertain about, or disagrees with, the information contained in groups i and j, then the importance of $C1$ and $C2$ with respect to each other is indeterminate.

For example, suppose that $C1$ belongs to group 1 and $C2$ belongs to group -1. If Player 1 prefers to take option 1 in general, then it is concluded by Assumption 4.1 that $C1$ is more important than $C2$ to player 1. If Player 1 is indifferent between taking option 1 and not taking option 1, then, by the assumption, criterion importance is not specified.

As the importance of criteria is specified based on Assumption 4.1, the games with the candidates which belong to the group corresponding to the more important criterion are

examined exclusively for the existence of a unique solution strategy. The size of the matrix of candidates can be reduced through this process.

If Player 1 is indifferent or uncertain about taking or not taking option 1, the following step is proposed. If there exist candidates which belong to a group or groups other than group i or j, then the games with those candidates are examined exclusively for the existence of a unique solution strategy. The size of the matrix of candidates can be still reduced this way. If all candidates belong to either group i or j, then the games with the candidates which belong to group i may be studied separately from the games with the candidates which belong to the group j, and vice versa, to see if more insight is gained. In addition, a similar grouping of candidate games for Player 2 may help further reduce the size of the matrix of candidates.

The analysis of criterion importance is the last step in the solution analysis. If the reduction in the size of the matrix of candidates does not identify a unique solution strategy, some ad hoc procedures may be used to continue with the search of solution strategy. One suggestion is the analysis of the individual games. Analyzing the individual games in the reduced matrix of candidates may provide further insight into the original multicriteria game. Collection of more information is an alternative suggestion. For example, more information on the other player's preferences is valuable in determining Player 1's strategy.

Example: Chicken

Since the examination of the matrix of candidates does not reveal a unique solution strategy for Player 1, criterion importance is introduced to the analysis of the solution strategy. Recall first that *C1* belongs to group 1 and *C2* belongs to group −1 for Player 1 for the game of Chicken from the previous section. The first two rows of the matrix of candidates belong to group 1, the two rows in the middle belong to group 0, and the last two rows belong to group −1. Figure 4.4 shows the classification of the individual games into the three groups.

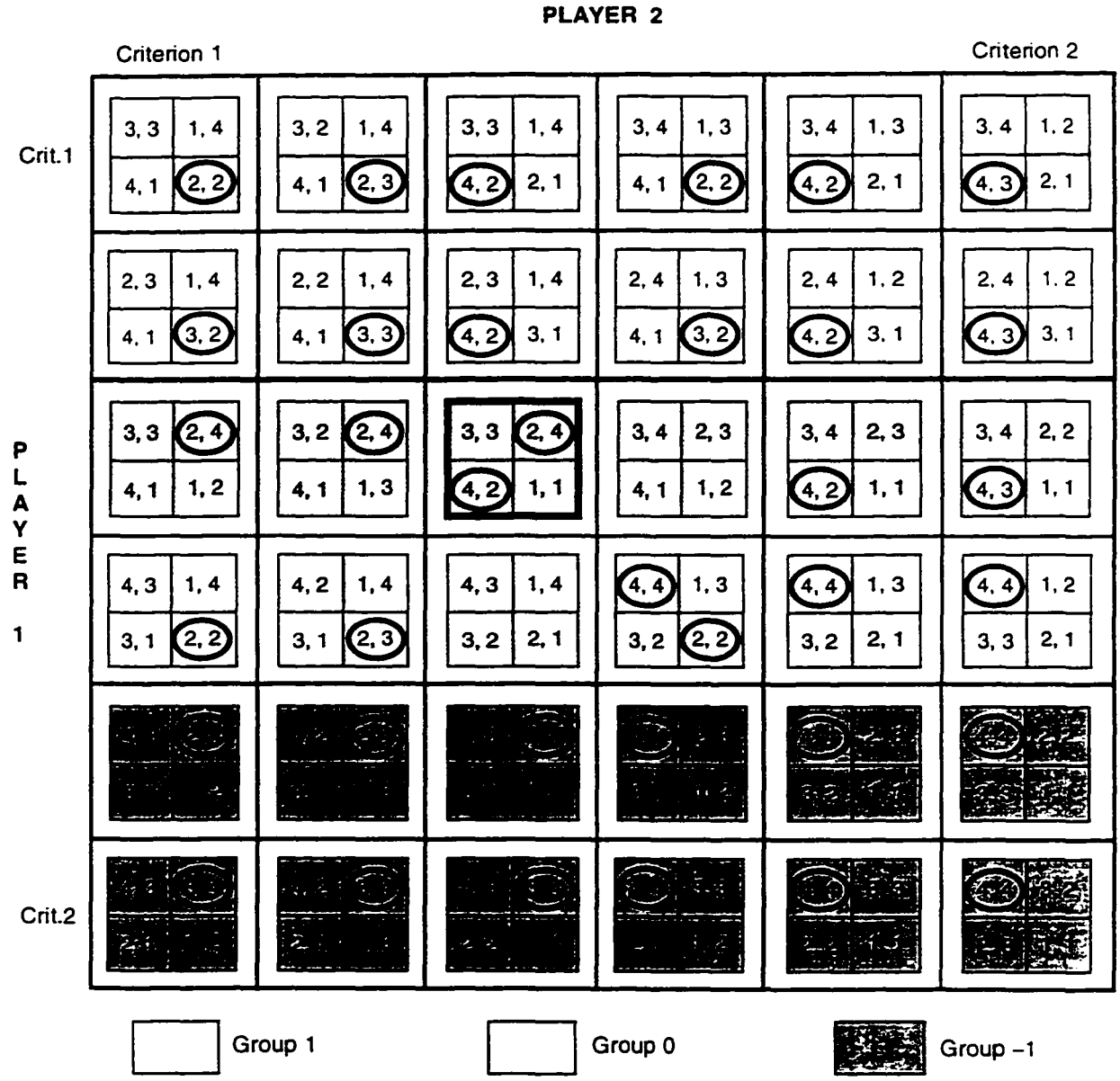
Suppose that Player 1 prefers taking option 1 than not taking option 1 in general. That is, Player 1 prefers to show off to his/her peers than not to show off. This implies that *C1* is preferred to *C2*, and therefore, the games in group 1 are examined exclusively for the

existence of solution strategy. The size of the matrix of candidates is reduced to one-third of the original size in this case. If Player 1 prefers not taking option 1 to taking option 1, then it is concluded that *C2* is more important than *C1*, and the games in group −1 are examined. If Player 1 is indifferent about taking or not taking option 1 or not sure which is more important to him/her, then in this case, there is an option of examining the games in group 0 which neither belongs to group 1 nor group −1.

The examination of group 1 or group −1 leads to a unique dominant strategy for Player 1. In group 1, taking option 1, that is, to continue driving, is the unique dominant strategy, and hence, the solution strategy for Player 1. In group −1, not taking option 1 or to swerve is the unique dominant strategy, and hence, the solution strategy. However, the examination of group 0 does not lead to a unique dominant strategy. In this case, the analysis of individual games in group 0 and collection of more information are suggested as the possible further analysis.

Lastly, although the candidate games are arranged in an orderly fashion in the matrix of candidates, the arrangement does not correspond to any scale such as one based on the criteria weights. The candidates are determined from the players' ordinal preferences which do not supply cardinal preference information.

Figure 4.4 Classification of the Games in the Matrix of Candidates for Chicken



4.7 Summary

Candidate enumeration is an integrative method used for solving multicriteria games. The key concepts, preference trees and candidates, and the solution analysis steps have been discussed extensively in this chapter. The fundamental principle of candidate enumeration can be

described as a visible approach. All key components are shown simultaneously in the matrix of candidates including the players, criteria for each player, and interaction between the MCDM and MPDM components in terms of the existence of equilibria. This is possible due to the ordinal preference structure. The solution analysis procedure and contributions of the candidate enumeration method are summarized separately as follows.

4.7.1 Summary of Solution Analysis

A summary of the solution analysis involved in the candidate enumeration method is given below.

The elements of the decision model:

- 2 criteria for each player
- 2×2 games, 4 possible outcomes
- 1 option, 2 strategies (take the option or not)
- Noncooperative games
- Strict ordinal preferences

Step 1: Examine dominant strategies for the candidate games.

First, all candidates are produced using the candidate enumeration algorithm. By specifying the types of two criteria according to Definition 4.5, the group type(s) of the candidates is identified (Lemma 4.2.1–4.2.3). The group type(s) of candidates indicates the existence of a unique dominant strategy for all candidate games (Lemma 4.1). If a unique dominant strategy exists, then that is the solution strategy for Player 1. If a unique dominant strategy does not exist for all candidate games, then the analysis proceeds to Step 2.

Step 2: Examine equilibria for the candidate games in the matrix of candidates.

The matrix of candidates is produced by matching the candidates of one player with the candidates of the other player. The existence of equilibria is examined for the candidate games in the matrix of candidates using a sequence of steps identified in Observations 4.1 and 4.2. If a unique solution strategy is identified after the examination of equilibria, then no further analysis is required. If a unique solution strategy is not identified, then the analysis proceeds to Step 3.

Step 3: Determine ordinal criterion importance, and use the information to reduce the size of the matrix of candidates.

The ordinal information of criterion importance is determined based on Assumption 4.1. As the importance of criteria is specified, the size of the matrix of candidates is reduced by concentrating the analysis on a certain set of candidate games in the matrix of candidates. If a unique solution strategy is identified in the reduced matrix of candidates, then that strategy is suggested as a solution. If a unique solution strategy is not identified, some ad hoc procedures are suggested to be used.

4.7.2 Advantages

The candidate enumeration method has several advantages over the existing game theory-based MCMP methods. The list of advantages is presented below.

- The concepts of candidates, candidate games, and the matrix of candidates have succeeded in maintaining the integrity of MCMP problem structure.
- These concepts provide an alternative way to display the overall decision making situation. Conflicting criteria are dealt with explicitly. Also, the aggregation over multiple criteria is not required, so the loss of information from the aggregation process is avoided.

- The proposed solution method is unique in its capability of including criterion importance information which is rarely taken into account in the other existing methods for multicriteria games.
- Ordinal preferences have the advantages such that they are easily obtained, meaningful, and descriptive without being specific.
- The candidates are generated in a systematic and objective manner based on the underlying structure of preference trees. Therefore, they are reasonable representation of the preferences with multiple criteria. The candidates also provide consistency and completeness in analysis.
- The analytical procedure is based on the existing solution concepts. It is simple, and its steps follow a logical order.
- The candidate enumeration method does not guarantee a unique strategy solution. From a decision mediation point of view, however, the matrix of candidates can assist in promoting cooperative behaviour by offering decision makers an opportunity to understand each other and an opportunity to recognize jointly rational solutions if one exists.
- The matrix of candidates can also provide the decision makers more opportunities to resolve or avoid deadlocks by allowing the players to better understand the situation.

In this chapter, the inclusion of ordinal criterion importance was attempted in order to increase the amount of preference information compared to the existing multicriteria integrative methodologies. As mentioned in Chapter 3, criterion importance in the form of weights can have a richer content. Criteria weights include both preference ranking and preference intensity information, from which they can provide trade-off information among criteria and also among participants. Trade-off is a natural and fundamental concept for bargaining or negotiation problems but not for game models. In the next chapter, in order to utilize criterion weight information, a multi-issue negotiation methodology is proposed.

Chapter 5

An Integrative Solution Method for Multi-Issue Negotiation

5.1 Multi-Issue Negotiation Methodology

In politics, business, and other international and interpersonal relations, people negotiate and make agreements that involve many issues. Multi-issue negotiation represents one type of MCMP problem. It is an important and challenging task whose outcome, success or failure, often has significant consequences. The fundamental objective of a multi-issue negotiation methodology is to aid negotiators in making better decisions by dealing with the complexity caused by multiple issues and multiple decision makers. Many of the current multi-issue negotiation methodologies that take the integrative approach do not consider criterion weight information. In this chapter, a methodology based on the integrative approach is proposed to improve the quality of decision making by increasing the amount of preference information incorporated in the decision making process. The proposed method utilizes criterion weight information in extracting useful trade-off information.

In the behavioral studies of multi-issue negotiation, the integrative approach has been discussed with a more specific meaning than dealing with the MCDM and MPDM components simultaneously. Pruitt (1981) considers the integrative approach as a way to find

a high benefit solution to all parties by reconciling, or integrating, the parties' interests. In the same context, the integrative approach is known to offer many benefits such as speedy settlement, stable bargaining outcomes, and improved relationship among the parties (Pruitt, 1983; Pruitt, 1981; Pruitt and Lewis, 1977). One of the methods for achieving integrative solutions introduced by Pruitt (1981) is called *logrolling*.

In general, logrolling means the exchanging of assistance or favours. The term originated from an early American custom of neighbours assisting one another in rolling logs into a pile for burning. It also means the trading of votes by legislators (Webster On-Line Dictionary, 1996). One known area of research in logrolling is voting (Bernholz, 1973; Miller, 1977; Tullock, 1959; and others). The other acknowledged area is in the psychology of bargaining behaviour, which discusses logrolling as one of the bargaining strategies (Froman and Cohen, 1970; Roloff and Jordan, 1991; Thompson, 1990*a*; and others). In this chapter, logrolling is considered in the bargaining context and its definition and analysis are strictly quantitative. The quantitative solution method using logrolling, which is designed to achieve mutually beneficial solutions, is a novel application of the concept of logrolling.

Gupta (1989) and Ehtamo et al. (1995) also apply the integrative approach to bargaining problems in a quantitative fashion. Gupta (1989) considers the trade-offs between interests (same as criteria weights) and issue authority, or potential to influence, so that the parties try to give up authority over the issues of less importance but gain authority over more important issues. The solution method based on logrolling considers the explicit trade-offs among issues that are natural and fundamental to the concept of negotiation or bargaining. Moreover, logrolling focuses on the added value increase for each party, which is likely to be an incentive for all parties to comply.

Ehtamo et al. (1995) consider a joint gain approach to finding Pareto optimal agreements in multi-issue two-party negotiation. Their search is based on the set of locally defined, mutually acceptable trade-offs. This is the same principle applied in the logrolling-based solution method. Their approach, however, differs from logrolling in the following aspects. The mutually acceptable trade-offs in Ehtamo et al. (1995) are subjectively determined, that is, by asking the decision makers which agreements are preferred. The logrolling-based solution

method identifies all possible mutually acceptable trade-offs based on the parties' criterion weight information. In Ehtamo et al., a "mediator" proposes a fair compromise based on the direction of improvement indicated by the parties. In logrolling, the parties themselves negotiate on the rate of trade-off to be used in the procedure. Overall, the solution procedure based on logrolling may better represent the real-life negotiation practice, and therefore, provide opportunities to gain insight into negotiation behaviour and strategies.

The remainder of this chapter is organized as follows. In Section 5.2 to 5.5, logrolling and the related concepts are defined and discussed for *two-issue two-party* negotiation with *linear* preference. Section 5.2 defines logrolling and the logrolling range. Section 5.3 discusses *trade-off simultaneity*, which is an interesting feature of MCMP problems and is the basis of the concept of logrolling. Section 5.4 proposed a negotiation procedure based on logrolling. Section 5.5 investigates the general properties of the efficient frontier produced by logrolling. The discussion is then extended to *m*-issue negotiation in Sections 5.6, 5.7, and 5.8. The two-issue sequential procedure is proposed in Section 5.6. The properties of the efficient frontier are modified in Section 5.7 based on the two-issue results. Section 5.8 extends the discussion further from 2-issue to *k*-issue sequential logrolling. Section 5.9 concludes this chapter.

5.2 Logrolling

Logrolling in 2-issue 2-party negotiation is the exchange of loss in one issue for gain in the other resulting in an increase of the overall value for *both* parties. Let i denote the issue which increases and j denote the issue which decreases in an exchange. Assume that the type of bargaining is distributive over each issue and that maximization is the objective for both parties. That is, regarding each issue, "more is better" for both parties. Then the *exchange rate* for Party 1 between issues i and j is the ratio of the increase in i to the decrease in j and is denoted by S_{1i}/S_{1j}, S_{1i}, $S_{1j}>0$. The exchange rate for Party 2 is S_{2j}/S_{2i}, S_{2i}, $S_{2j}>0$, and $S_{1i}=S_{2i}$ and $S_{1j}=S_{2j}$. In other words, for example, $S_{1i}=S_{2i}$ implies that the amount Party 1 gains in issue i is equal to the amount Party 2 loses in issue i. S_{1i}, S_{1j}, S_{2i}, and S_{2j} are assumed to be in the

original units of measurement. Using this idea of exchange rate, the definition of logrolling is given as follows.

Definition 5.1 *Logrolling* is the exchange based on the exchange rates, S_{1i}/S_{1j} for Party 1 and S_{2j}/S_{2i} for Party 2, which satisfies

$$V_1(x_{1i} + S_{1i}, x_{1j} - S_{1j}) > V_1(x_{1i}, x_{1j}) \text{ and } V_2(x_{2i} - S_{2i}, x_{2j} + S_{2j}) > V_2(x_{2i}, x_{2j}).$$

$V_k(X)$, $X \in \Re^2$, is the value function for party k, and (x_{ki}, x_{kj}) is the starting point for party k, $k = 1, 2$. The value function is assumed to exist for each party, but it does not need to be identified completely.

By definition, logrolling provides mutual gain. The key to identifying mutual gain is recognition of the exchange rates which permit logrolling. The range of such exchange rates can be determined using the parties' criteria weights and is called the *logrolling range*.

Definition 5.2 Suppose S_{1i}/S_{1j} and S_{2j}/S_{2i} are Party 1 and Party 2's exchange rates; w_i and w_j are Party 1's weights for issues i and j; and z_i and z_j are Party 2's weights for i and j.

(i) Two parties have different priorities if and only if $w_i > w_j$ and $z_i < z_j$. In this case, the logrolling ranges for S_{1i}/S_{1j} and S_{2j}/S_{2i} are as follows.

$$\frac{w_j}{w_i} < \frac{S_{1i}}{S_{1j}} < \frac{z_j}{z_i} \quad \text{and} \quad \frac{z_i}{z_j} < \frac{S_{2j}}{S_{2i}} < \frac{w_i}{w_j}$$

These ranges exist if both parties gain in their more important issue and lose in the less important issue.

(ii) Two parties have the same priority but at different preference intensity if and only if $(w_i/w_j) \neq (z_i/z_j)$ where $w_i > w_j$ and $z_i > z_j$. In this case, the logrolling ranges for S_{1i}/S_{1j} and S_{2j}/S_{2i} are as follows.

$$\text{If } (w_i / w_j) > (z_i / z_j), \quad \frac{w_j}{w_i} < \frac{S_{1i}}{S_{1j}} < \frac{z_j}{z_i} \quad \text{and} \quad \frac{z_i}{z_j} < \frac{S_{2j}}{S_{2i}} < \frac{w_i}{w_j}.$$

$$\text{If } (w_i / w_j) < (z_i / z_j), \quad \frac{w_i}{w_j} < \frac{S_{1j}}{S_{1i}} < \frac{z_i}{z_j} \quad \text{and} \quad \frac{z_j}{z_i} < \frac{S_{2i}}{S_{2j}} < \frac{w_j}{w_i}.$$

The ranges exist if the party having the higher (lower) preference intensity gains in issue i (j) and loses in issue j (i).

The only case in which logrolling range cannot exist is when two parties have the same priority at the same preference intensity, that is, $(w_i/w_j) = (z_i/z_j)$. In other words, the difference in the negotiating parties' preferences allows logrolling and facilitates the discovery of mutual gain. Even if the parties prefer the same issue, as long as there is a difference in how much they prefer, logrolling can still apply. Since the difference is usually seen as the basis of disagreement among opposing parties, the existence of the logrolling range is counter-intuitive and interesting.

The justification and the existence of the logrolling range are based on the underlying concept of the weight assessment technique using trade-offs (Keeney and Raiffa, 1976). Let y_i and y_j be the amount of change in issues i and j. Keeney and Raiffa's trade-off method assures theoretically valid weights by determining the amount y_j that a decision maker is willing to give up for obtaining y_i in return. The trade-off ratio used by Keeney and Raiffa (1976) can be expressed as follows:

$$\frac{w_j}{w_i} = \frac{y_i}{y_j} \qquad \text{where } V(x_i + y_i, x_j - y_j) - V(x_i, x_j) = 0 \qquad (*1)$$

w_i and w_j are the weights for criteria i and j, and (x_i, x_j) is the starting point. In weight assessment, enough questions are asked to the decision maker to obtain a sufficient number of acceptable trade-off ratios. The ratios are then used to solve for a unique set of weights.

Definition 5.1 indicates that the concept of logrolling does not depend on the form of the value functions. However, the concept of the logrolling range assumes that $V_1(X)$ and $V_2(X)$ are the linear weighted average of issues since Keeney and Raiffa's trade-off method (1976) holds this assumption. This further implies that $V_1(X)$ and $V_2(X)$ are linear. Conceptually, linear preference implies some proportional increase in overall value as gain in an issue is observed. More precisely, a value function $V:\Re^2 \rightarrow \Re^2$ is linear if the following holds:

$V(a\boldsymbol{x} + b\boldsymbol{y}) = aV(\boldsymbol{x}) + bV(\boldsymbol{y})$ $\forall a, b \in \Re$ and $\forall \boldsymbol{x}, \boldsymbol{y} \in \Re^2$.

The weights implied by Keeney and Raiffa's trade-off method are constants of scale, that is, if one changes the unit in which an issue is expressed, its weight changes (Vincke, 1992). In this case, multiplication of weights by a positive scalar is allowed, and the weights constitute a ratio scale (Vincke, 1992). Consequently, comparison in an ordinal manner is possible among the weights.

The trade-off method is chosen to be the basis of the logrolling range for the following reasons:

- its theoretical validity,
- the ratio y_i/y_j is the same as the definition of exchange rate, and
- the trade-off ratio provides a relationship between criteria weights and exchange rates.

The formal justification and existence of the logrolling range are given below.

(i) Two parties have different priorities: $w_i > w_j$ and $z_i < z_j$.

Justification. It is shown that any exchange based on the exchange rate within the logrolling range results in overall gain for both parties. The following is developed based on the expression used in (*1). Recall from Definition 5.2 that S_{1i}/S_{1j} and S_{2j}/S_{2i} are Party 1 and Party 2's exchange rates, w_i and w_j are Party 1's weights, and z_i and z_j are Party 2's weights.

$$V_1(x_{1i} + S_{1i}, x_{1j} - S_{1j}) > V_1(x_{1i}, x_{1j}) \qquad \text{(condition for improvement)}$$

$$\Leftrightarrow V_1(x_{1i}, x_{1j}) + w_i S_{1i} - w_j S_{1j} > V_1(x_{1i}, x_{1j}) \qquad (V_1 \text{ linear weighted average})$$

$$\Leftrightarrow w_i S_{1i} - w_j S_{1j} > 0$$

$$\Leftrightarrow (S_{1i}/S_{1j}) > (w_j/w_i) \qquad (*2)$$

(*2) is Party 1's necessary and sufficient condition for improvement where V_1 is the linear weighted average of issues. Similarly, $(S_{2j}/S_{2i}) > (z_i/z_j)$ is Party 2's necessary and sufficient condition for improvement. Using the distributive bargaining property, $S_{1i} = S_{2i}$ and $S_{1j} = S_{2j}$, it follows that

$$(S_{2j}/S_{2i}) > (z_i/z_j) \quad \Leftrightarrow \quad (S_{1j}/S_{1i}) > (z_i/z_j) \quad \Leftrightarrow \quad (S_{1i}/S_{1j}) < (z_j/z_i). \qquad (*3)$$

From (*2) and (*3), the exchange rate, S_{1i}/S_{1j}, that satisfies $(w_j/w_i)<(S_{1i}/S_{1j})<(z_j/z_i)$ provides overall gain to both parties. Therefore, $(w_j/w_i)<(S_{1i}/S_{1j})<(z_j/z_i)$ is the logrolling range for Party 1. Using the distributive bargaining property, the logrolling range for Party 2 is $(z_i/z_j)<(S_{2j}/S_{2i})<(w_i/w_j)$.

Existence. Suppose that issue i has higher priority than issue j for party 1. That is, $w_i>w_j$, which implies $z_j>z_i$ in this case. Then, the following is true:

$$w_i > w_j \Leftrightarrow w_j/w_i < 1 \qquad \text{and} \qquad z_j > z_i \Leftrightarrow z_j/z_i > 1.$$

This guarantees the existence of an interval between the two values, (w_j/w_i) and (z_j/z_i), and thus, the existence of an exchange rate, S_{1i}/S_{1j}, such as $(w_j/w_i)<(S_{1i}/S_{1j})<(z_j/z_i)$. The existence of an exchange rate, S_{1i}/S_{1j}, guarantees the existence of an exchange rate, S_{2j}/S_{2i}, such as $(z_i/z_j)<(S_{2j}/S_{2i})<(w_i/w_j)$ by the distributive bargaining property.

(ii) Two parties have the same priority but at different intensity: $w_i>w_j$ and $z_i>z_j$.

Justification. Same argument as above for two cases: $(w_i/w_j) > (z_i/z_j)$ and $(w_i/w_j) < (z_i/z_j)$.

Existence. If $(w_i/w_j) > (z_i/z_j)$, it guarantees the existence of an interval between the two values, w_i/w_j and z_i/z_j, and it further guarantees the existence of the exchange rate, S_{2j}/S_{2i}, such as $(z_i/z_j)<(S_{2j}/S_{2i})<(w_i/w_j)$. The existence of an exchange rate, S_{2j}/S_{2i}, guarantees the existence of an exchange rate, S_{1i}/S_{1j}, such as $(w_j/w_i)<(S_{1i}/S_{1j})<(z_j/z_i)$ by the distributive bargaining property. A similar argument applies to the case of $(w_i/w_j)<(z_i/z_j)$.

5.3 Trade-Off Simultaneity

The concept of logrolling is based on an interesting feature of MCMP problems that is worth further discussion. This feature is referred to as *trade-off simultaneity*, and is described as follows. In general, trade-offs among criteria are perceived on an individual basis, and trade-offs among decision makers are perceived as concession making over a single issue or criterion. In a multi-issue negotiation context, when decision makers consider a trade-off between two criteria, for example, it can be also equivalent to making a trade-off with the other decision makers over the two criteria. In other words, the trade-off among criteria and the trade-off among decision makers imply each other. Trade-off simultaneity is, therefore, the key to conserve the integrity of MCMP by considering both trade-offs among criteria and among parties simultaneously. It also accounts for the interaction between MCDM and MPDM, which is considered to be additional preference information incorporated into the decision making process.

Trade-off simultaneity has two important implications in improving MCMP. The first is regarding utilization of differences in preferences. In multi-issue negotiation, some studies indicate that the parties typically assume that most crucial issues to one party are also crucial to the other parties (Walton and McKersie, 1965; Raiffa, 1982). Consequently, they fail to trade off issues which they value differently (Thompson, 1990*a*) and settle for a compromise solution while a better solution is left undiscovered. This is related to the insufficient understanding of the feasible set as discussed in Chapter 3. Differences are generally assumed to create a conflict, but they can also create a solution (Fisher and Ury, 1981). Understanding of trade-off simultaneity is crucial in the recognition and utilization of the differences in the decision makers' preferences, and hence, promises significant contribution toward improvement of MCMP. Trade-off simultaneity has played a vital role in conceiving the concepts of logrolling and the logrolling range.

The second important implication of trade-off simultaneity is regarding the mutual consequence experienced by the decision makers. Mutuality contributes to many psychological benefits associated with an improved relationship. Mutuality provides a strong

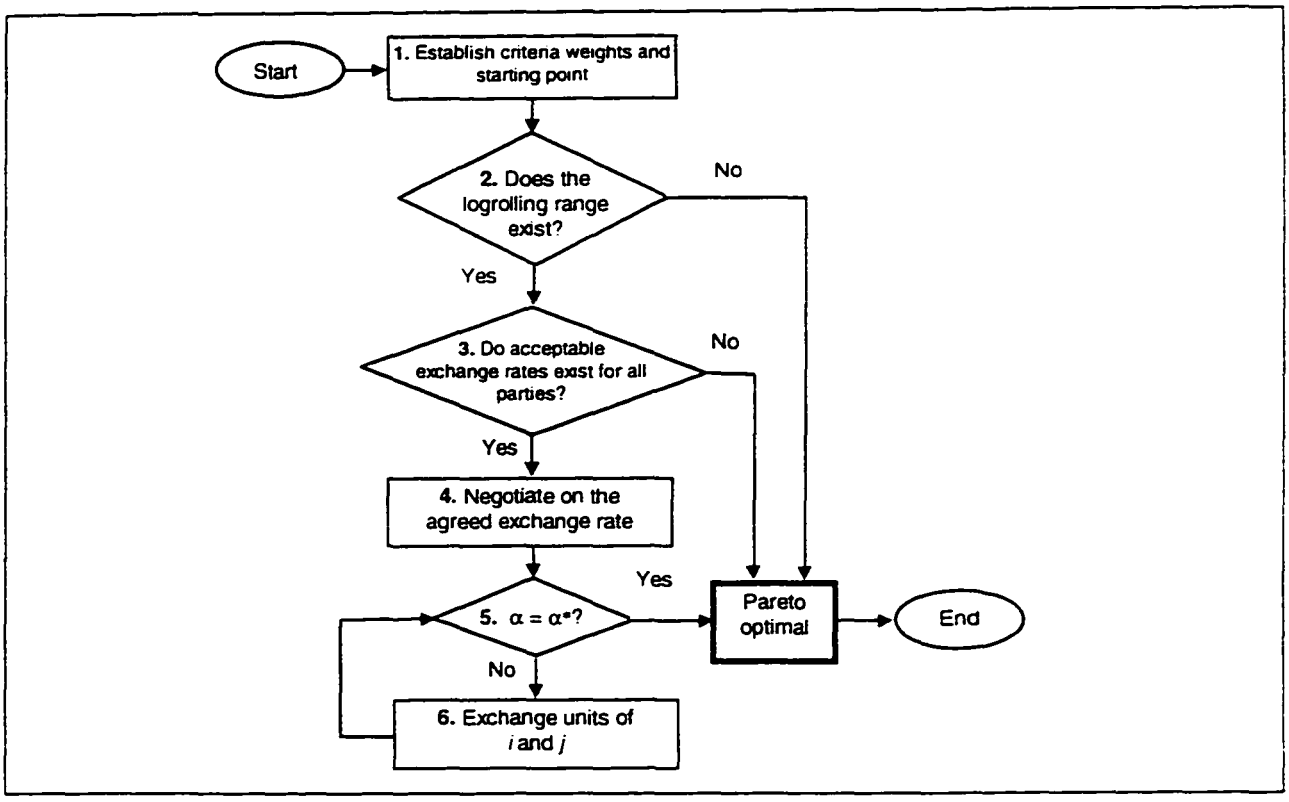
and explicit incentive for the decision makers to comply. It encourages joint effort to find a solution that is acceptable by all (Pruitt and Rubin, 1986). It also encourages a shared perspective to view conflict as an opportunity for improvement, reconciliation, and group unity (Pruitt and Rubin, 1986). The discussion of the implications highlights the importance of acknowledging the existence of trade-off simultaneity.

In essence, trade-off simultaneity utilizes differences among the parties' preferences and it facilitates mutual gain. In the reduction approach, one of the MCDM and MPDM components is absorbed into the other, so the trade-offs among criteria and among decision makers cannot be considered together. In the decomposition approach, the two types of trade-offs are made separately in an iterative fashion without any explicit relationship defined between them. Therefore, trade-off simultaneity can only be recognized by taking the integrative approach. Due to the valuable implications of trade-off simultaneity in terms of improving MCMP, the ability to provide a framework that captures trade-off simultaneity is considered to be another main advantage of the integrative approach.

5.4 Two-Issue Logrolling Procedure

The potential for mutual gain is indicated by the logrolling range and is realized by the actual exchange of issues. Here, a logrolling-based negotiation procedure that produces a Pareto optimal solution as the end result is presented (Figure 5.1).

Figure 5.1 Negotiation Procedure Using Logrolling



Notes on the terminology used in Figure 5.1:

Step 1. Each party's criteria weights are assumed to be obtainable. The starting point is assessed from the status quo.

Step 3. The exchange rate is *acceptable* for party k if it yields $\Delta V_k > A_k$ when it is used in logrolling. ΔV_k is the difference in the value before and after logrolling. A_k is the activation cost of negotiation. $k = 1, 2$.

Step 4. Agreed exchange rate is the exchange rate that is acceptable and agreed by all parties in the application of logrolling. The agreed exchange rate is assumed to remain constant over one session of logrolling.

Steps 5 and 6. α is a multiplying factor, $\alpha \geq 0$, which is used to express the total amount of change in issues. Let s_{1i}/s_{1j} be Party 1's agreed exchange rate with particular values. Then αs_{1i} and αs_{1j} are the total amount of change in issues i and j for Party 1. This way, the exchange rate remains constant ($\alpha s_{1i}/\alpha s_{1j} = s_{1i}/s_{1j}$), and the value after logrolling is expressed

as $V_1(x_{1i} + \alpha s_{1i}, x_{1j} - \alpha s_{1j})$. It can be shown below that $V_1(x_{1i} + \alpha s_{1i}, x_{1j} - \alpha s_{1j})$ obtains the maximum when α takes its maximum, α^*.

$$\begin{aligned} V_1(x_{1i} + \alpha s_{1i}, x_{1j} - \alpha s_{1j}) &= V_1[(x_{1i}, x_{1j}) + \alpha \cdot (s_{1i}, -s_{1j})] \\ &= V_1(x_{1i}, x_{1j}) + \alpha \cdot V_1(s_{1i}, -s_{1j}) \quad \text{(by linearity of } V_1\text{)} \end{aligned}$$

Here, $V_1(x_{1i}, x_{1j})$ and $V_1(s_{1i}, -s_{1j})$ are constant. Therefore, $V_1(x_{1i} + \alpha s_{1i}, x_{1j} - \alpha s_{1j})$ obtains the maximum value when α takes its maximum possible value.

The value of α^* can be determined from how the exchange ends. The exchange stops when one of the following conditions is met.

(i) Issue i reaches maximum for Party 1: $\bar{x}_i = x_{1i} + \alpha s_{1i} \Leftrightarrow \alpha = (\bar{x}_i - x_{1i})/s_{1i}$

(ii) Issue j reaches minimum for Party 1: $\underline{x}_j = x_{1j} - \alpha s_{1j} \Leftrightarrow \alpha = (x_{1j} - \underline{x}_j)/s_{1j}$

where $(\bar{x}_i, \bar{x}_j)$ represents a set of maximum values and $(\underline{x}_i, \underline{x}_j)$ represents a set of minimum values for the two issues. Then, $\alpha^* = min\{(\bar{x}_i - x_{1i})/s_{1i}, (x_{1j} - \underline{x}_j)/s_{1j}\}$. The total amount of change in i is $min\{\alpha^* s_{1i}, (\bar{x}_i - x_{1i})\}$ and in j is $max\{\alpha^* s_{1j}, (x_{1j} - \underline{x}_j)\}$ depending on the value of α^*.

5.5 Efficient Frontier for Two-Issue Logrolling

In actual negotiations, the parties must arrive at an agreed exchange rate to proceed with logrolling. Once chosen, the agreed exchange rate determines the trade-off between the issues and between the parties at the same time. Without a specific agreed exchange rate, logrolling does not suggest a unique solution. However, one can alternatively study the efficient frontier over the logrolling range in order to further analyze the possible solutions. The efficient frontier is defined as follows.

Definition 5.3 Let $v = (V_1(X), V_2(X))$ be the value vector for Party 1 and Party 2. $v^0 = (V_1(X^0), V_2(X^0))$ is *Pareto optimal*, or *efficient*, if there is no $v = (V_1(X), V_2(X))$, $X \neq X^0$, such that $V_1(X) \geq V_1(X^0)$ and $V_2(X) \geq V_2(X^0)$ with at least one inequality. $v, v^0 \in V$. V is the value space. $X^0, X \in N$. N is the negotiation outcome space. $X^0, X \in \Re^2$.

Definition 5.4 The *efficient frontier* for V is the set of all Pareto optimal solutions in V.

5.5.1 Linear Value Functions

In order to study the efficient frontier produced by logrolling, the parties' values as functions of the exchange rate, not of the quantities of issues, are first identified. Considering Party 1 first, $V_1(x_{1i} + \alpha S_{1i}, x_{1j} - \alpha S_{1j})$ is the end result of logrolling using the exchange rate S_{1i}/S_{1j}. $V_1(x_{1i} + \alpha S_{1i}, x_{1j} - \alpha S_{1j})$ is further specified to be $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ if logrolling stops when issue j reaches minimum for Party 1 or $V_1(\bar{x}_i, x_{1j} - \alpha S_{1j})$ if logrolling stops when issue i reaches maximum for Party 1.

In $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$, αS_{1i} indicates the amount of increase in issue i as the exchange rate S_{1i}/S_{1j} increases. Note that αS_{1i} is the only changing quantity in $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$. Assume that issues i and j are independent of each other. Then $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ can be treated as a function of αS_{1i} over the logrolling range. Now let us find the slope of $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ by letting αS_{1i} increase by a small amount ε, $\varepsilon > 0$.

$$\frac{V_1(x_{1i} + \alpha S_{1i} + \varepsilon, \underline{x}_j) - V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)}{(\alpha s_i + \varepsilon) - \alpha s_i}$$

$$= \frac{V_1[(x_{1i} + \alpha S_{1i} + \varepsilon, \underline{x}_j) - (x_{1i} + \alpha S_{1i}, \underline{x}_j)]}{\varepsilon} \quad \text{(by } V_1(X) \text{ linear)}$$

$$= \frac{V_1(\varepsilon, 0)}{\varepsilon}$$

Since the slope is constant, it is concluded that $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$, as a function of the exchange rate, is linear over the logrolling range.

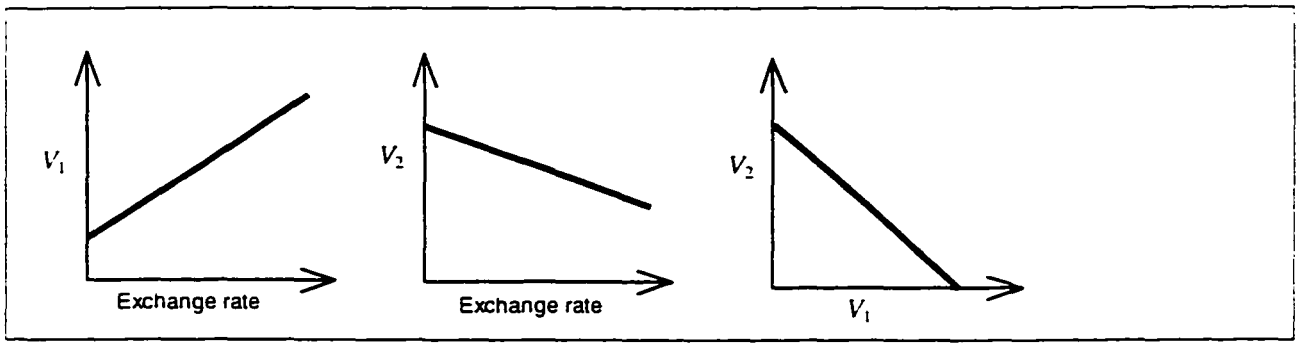
Following the same procedure above, the other possible end result, $V_1(\bar{x}_i, x_{1j} - \alpha S_{1j})$, can be shown to be linear. The slope of $V_1(\bar{x}_i, x_{1j} - \alpha S_{1j})$ by letting $-\alpha S_{1j}$ increase by a small amount δ, $\delta>0$, is $V_1(0,\delta)/\delta$. The same procedure also applies to Party 2's value functions.

5.5.2 Linear Efficient Frontier

Sometimes logrolling ends only by reaching issue j's minimum (case 1) or by reaching issue i's maximum (case 2) over the logrolling range. The combination case is also possible, but let us discuss cases 1 and 2 first.

In case 1, Party 1's value function is $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ and it is linearly increasing over the logrolling range. Party 2's value function is $V_2(x_{2i} - \alpha S_{2i}, \bar{x}_j)$, and it can be shown linearly decreasing over the logrolling range. Then as V_1 increases, corresponding V_2 decreases at a fixed rate. Therefore, the efficient frontier is linear (Figure 5.2).

Figure 5.2 Value Functions and the Efficient Frontier for Case 1



The same argument above applies to case 2 by replacing Party 1's value function with $V_1(\bar{x}_i, x_{1j} - \alpha S_{1j})$ and Party 2's value function with $V_2(\underline{x}_i, x_{2j} + \alpha S_{2j})$. The efficient frontier is shown to be linear in the same manner.

5.5.3 Piecewise Linear Value Functions

In the combination case, as the exchange rate increases, Party 1's value function switches from $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ to $V_1(\overline{x}_i, x_{1j} - \alpha S_{1j})$. This implies that the slope changes from $V_1(\varepsilon,0)/\varepsilon$ to $V_1(0,\delta)/\delta$. Here, the units of ε and δ may not be the same because the units of issues i and j can be different. However, scaling does not affect the basic expression of the nominators having one issue equal to 0 and the other equal to a positive score. So the slope of $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ can be written as $V_1(\varepsilon',0)$ and the slope of $V_1(\overline{x}_i, x_{1j} - \alpha S_{1j})$ as $V_1(0,\delta')$ over a common denominator. Now these expressions can be compared. By assuming issues i and j are independent, one of these cases holds.

$V_1(\varepsilon', 0) > V_1(0, \delta')$ if issue i has higher priority than issue j. (1)

$V_1(\varepsilon', 0) < V_1(0, \delta')$ if issue j has higher priority than issue i. (2)

In either case, the difference between the two slopes creates a corner point, or *break point*, at where the two functions intersect. Note that the break point occurs only once over the logrolling range and occurs simultaneously for two parties. The existence of the break point indicates that $V_1(x_{1i} + \alpha S_{1i}, x_{1j} - \alpha S_{1j})$ is piecewise linear over the logrolling range since it is composed of two linear functions, $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ and $V_1(\overline{x}_i, x_{1j} - \alpha S_{1j})$, which have different slopes.

5.5.4 Piecewise Linear Efficient Frontier

Two cases are considered here regarding the efficient frontier which corresponds to the piecewise linear value functions: the case in which two parties have the different priorities (case 3) and the case in which the parties have the same priority but at different preference intensity (case 4).

Case 3. Suppose that issue i has higher priority than issue j for Party 1 and suppose the opposite for Party 2. Then the slopes of $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ and $V_1(\overline{x}_i, x_{1j} - \alpha S_{1j})$ have

relationship (3) and the slopes of $V_2(x_{2i} - \alpha S_{1i}, \bar{x}_j)$ and $V_2(\underline{x}_i, x_{2j} + \alpha S_{1j})$ have relationship (4) over the logrolling range based on relationships (1) and (2).

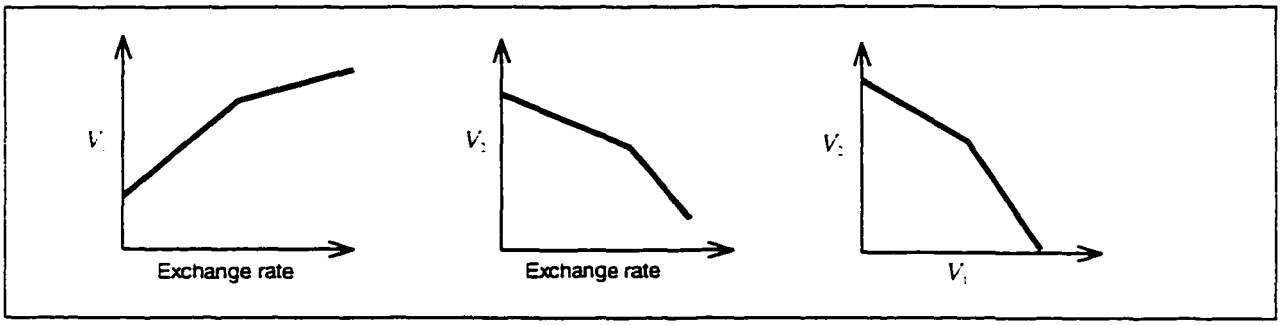
$$V_1(\varepsilon', 0) > V_1(0, \delta') \text{ where } V_1(\varepsilon', 0) \text{ and } V_1(0, \delta') > 0. \quad (3)$$

$$V_2(\varepsilon'', 0) > V_2(0, \delta'') \text{ where } V_2(\varepsilon'', 0) \text{ and } V_2(0, \delta'') < 0. \quad (4)$$

The slope of the efficient frontier before the break point is $[V_2(\varepsilon'',0)/V_1(\varepsilon',0)]<0$, while the slope of the efficient frontier after the break point is $[V_2(0,\delta'')/V_1(0,\delta')]<0$. From (3) and (4), the following relationship is true. Figure 5.3 illustrates this result.

$$\frac{V_2(0,\delta'')}{V_1(0,\delta')} < \frac{V_2(\varepsilon'',0)}{V_1(\varepsilon',0)} < 0 .$$

Figure 5.3 Value Functions and the Efficient Frontier for Case 3



Case 4. Suppose that issue i has higher priority than issue j for both parties. Also suppose that Party 1 has higher preference intensity than Party 2. Then the slopes of $V_1(x_{1i} + \alpha S_{1i}, \underline{x}_j)$ and $V_1(\bar{x}_i, x_{1j} - \alpha S_{1j})$ have relationship (5) and the slopes of $V_2(x_{2i} - \alpha S_{1i}, \bar{x}_j)$ and $V_2(\underline{x}_i, x_{2j} + \alpha S_{1j})$ have relationship (6).

$$V_1(\varepsilon',0) > V_1(0,\delta') \text{ where } V_1(\varepsilon',0),\ V_1(0,\delta')>0, \text{ and } V_1(0,\delta') = 1/a \cdot V_1(\varepsilon',0) \quad (5)$$

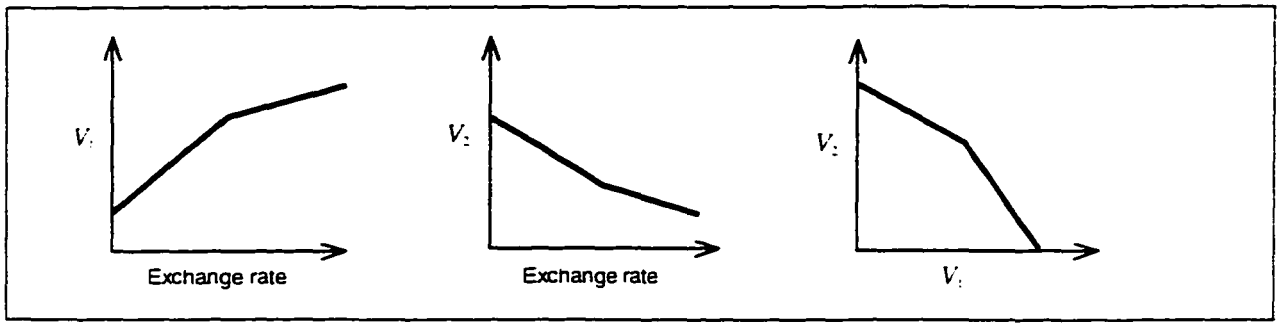
$$V_2(\varepsilon'',0) < V_2(0,\delta'') \text{ where } V_2(\varepsilon'',0),\ V_2(0,\delta'')<0, \text{ and } V_2(0,\delta'') = 1/b \cdot V_2(\varepsilon'',0) \quad (6)$$

In (5) and (6), $a,b>0$ and $a>b$ which implies $a/b>1$. The slope of the efficient frontier before the break point, $[V_2(\varepsilon'',0)/V_1(\varepsilon',0)]<0$, and the slope of the efficient frontier after the

break point. $[V_2(0.\delta'')/V_1(0.\delta')]<0$. have the following relationship based on (5) and (6). Figure 5.4 summarizes this result.

$$\left(\frac{V_2(0,\delta'')}{V_1(0.\delta')} = \frac{1/b}{1/a}\cdot\frac{V_2(\varepsilon'',0)}{V_1(\varepsilon',0)} = a/b\cdot\frac{V_2(\varepsilon'',0)}{V_1(\varepsilon',0)}\right) < \frac{V_2(\varepsilon'',0)}{V_1(\varepsilon',0)} < 0$$

Figure 5.4 Value Functions and the Efficient Frontier for Case 4



Cases 1 to 4 show that the efficient frontier for two-issue logrolling is linear or piecewise linear. The shape of the piecewise linear efficient frontier is shown to be concave.

5.6 Two-Issue Sequential Logrolling

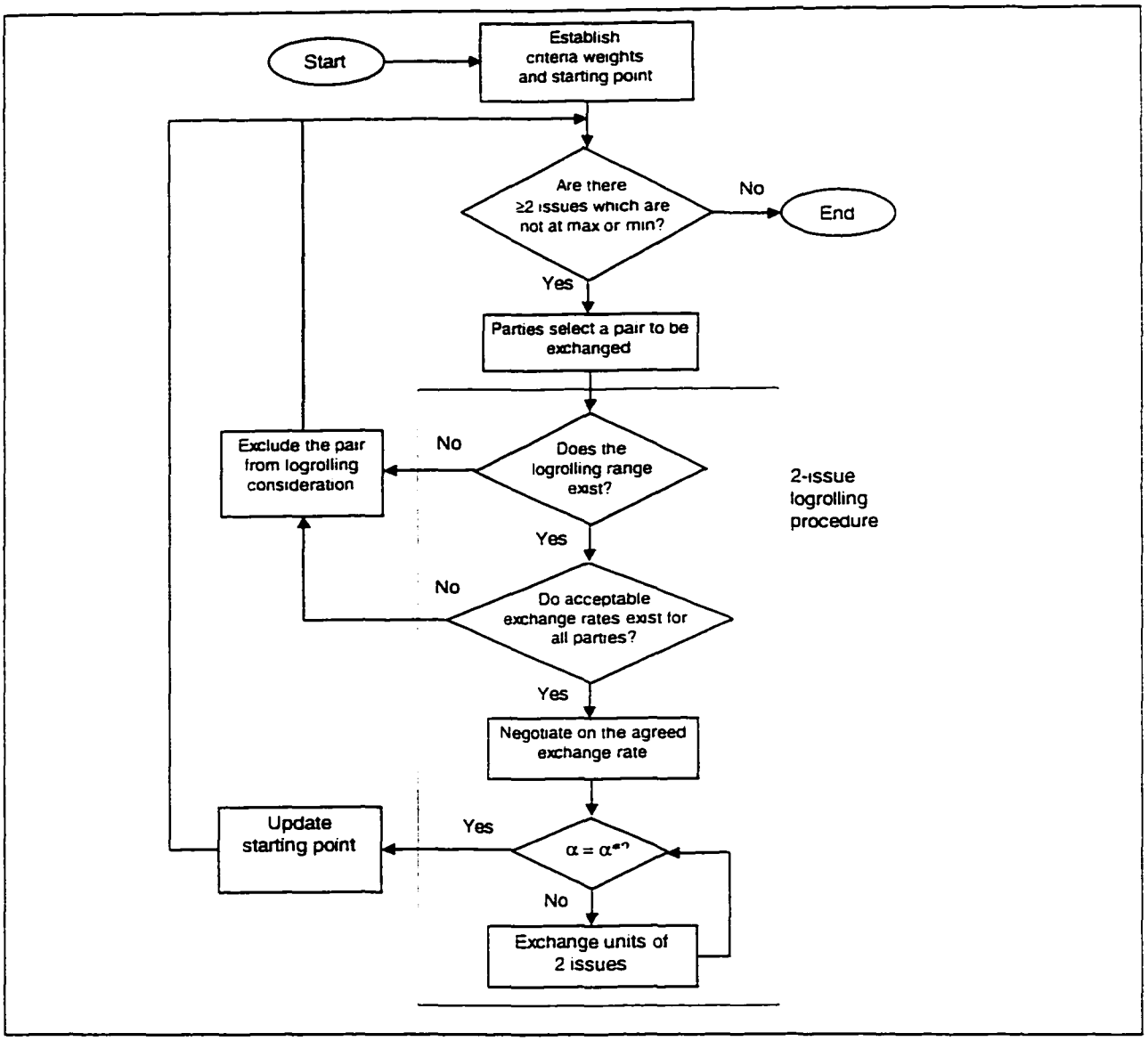
Up to this point, logrolling and related concepts have been discussed for 2-issue 2-party negotiation. The study is now extended from 2-issue to m-issue negotiation. In order to deal with m issues, the following modified procedure is discussed. A set of k issues ($2 \le k \le m$) is involved at a time in logrolling, and there are several iterations of exchanging until all possible logrolling opportunities are exhausted. This procedure is referred to as *sequential* logrolling. In practice, the value of k may vary from one iteration to the next. When the value of k is fixed at 2 throughout the entire negotiation process, it is called 2-issue sequential logrolling, and it is a direct extension of 2-issue logrolling.

Definition 5.5 When two issues are involved at each iteration and there are $(m-1)$ iterations of logrolling, it is called *2-issue sequential logrolling*.

5.6.1 Sequential Procedure

From the definition of the logrolling range, logrolling is possible for any two issues as long as $w_i/w_j \neq z_i/z_j$, $i,j = 1, 2, \ldots, m$. Therefore, the sequential procedure consists of three main parts: a selection process which determines the two issues to be exchanged, 2-issue logrolling, and updating of the starting point. Each 2-issue logrolling ends when one of the issues reaches its maximum or minimum. The sequential procedure eliminates the maximized or minimized issue from the further logrolling consideration and ends when there are no more pairs of issues left which are not at their maximum or minimum. The overall procedure is shown in Figure 5.5.

Figure 5.5 Negotiation Procedure Using 2-issue Sequential Logrolling



5.6.2 Updating the Starting Point

The updating procedure is based on the general formula, (*new value*) = (*starting point*) + (*amount of change*). Consider 3-issue case. Denote the three issues by i, j, and k. Suppose that i increases against j and k and j increases against k, and suppose that issues i and j are

exchanged first. After the first logrolling, the outcome is $(\bar{x}_i, x_{1j}', x_{1k})$ [1] or $(x_{1i}', \underline{x}_j, x_{1k})$ [2] where

x_{1j}' = (starting point) + (amount of change) = $x_{1j} + (x_{1i} - \bar{x}_i)/(s_{1i}/s_{1j})$ and
x_{1i}' = (starting point) + (amount of change) = $x_{1i} + (x_{1j} - \underline{x}_j)\cdot(s_{1i}/s_{1j})$.

The amount of change is either positive or negative. This is based on 2-issue logrolling.

By continuing from the first logrolling result, [1] yields $(\bar{x}_i, \bar{x}_j, x_{1k}')$ or $(\bar{x}_i, x_{1j}'', \underline{x}_k)$ and [2] yields $(\bar{x}_i, \underline{x}_j, x_{1k}')$ or $(x_{1i}'', \underline{x}_j, \underline{x}_k)$ where the general formula, (new value) = (starting point) + (amount of change), computes the following:

$$x_{1i}'' = x_{1i}' + (x_{1i} - \underline{x}_k)\cdot(s_{1i}/s_{1k}) = [x_{1i} + (x_{1j} - \underline{x}_j)\cdot(s_{1i}/s_{1j})] + (x_{1i} - \underline{x}_k)\cdot(s_{1i}/s_{1k}).$$

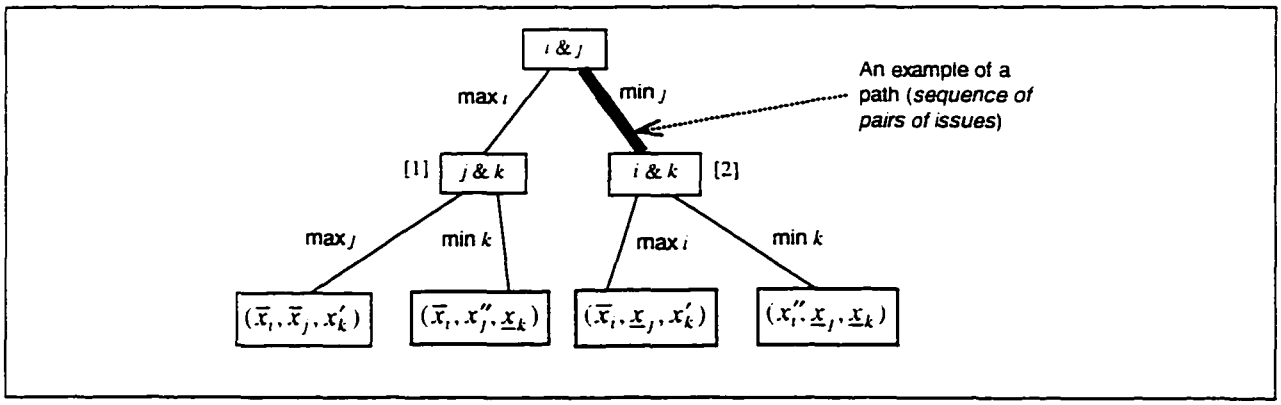
$$x_{1j}'' = x_{1j}' + (x_{1k} - \underline{x}_k)\cdot(s_{1j}/s_{1k}) = [x_{1j} + (x_{1i} - \bar{x}_i)/(s_{1i}/s_{1j})] + (x_{1k} - \underline{x}_k)\cdot(s_{1j}/s_{1k}).$$

$$x_{1k}'[1] = x_{1k} + (x_{1j}' - \bar{x}_j)/(s_{1j}/s_{1k}) = x_{1k} + ([x_{1j} + (x_{1i} - \bar{x}_i)/(s_{1i}/s_{1j})] - \bar{x}_j)/(s_{1j}/s_{1k}). \text{ and}$$

$$x_{1k}'[2] = x_{1k} + (x_{1i}' - \bar{x}_i)/(s_{1i}/s_{1k}) = x_{1k} + ([x_{1i} + (x_{1j} - \underline{x}_j)\cdot(s_{1i}/s_{1j})] - \bar{x}_j)/(s_{1i}/s_{1k}).$$

The outcomes of the 3-issue example are summarized in the tree diagram below (Figure 5.6).

Figure 5.6 Tree Diagram for the Outcomes of a 3-Issue Example



A set of pairs on a particular path in the tree diagram is referred to as a *sequence of pairs of issues*. In writing, the sequence is denoted by [identification of a pair](how the logrolling

ends)-[the next pair](how the logrolling ends)-·····-[the last pair]. For example, the sample sequence in Figure 5.6 is denoted by [*i*&*j*](min *j*)-[*i*&*k*].

Another key term is the exchange pattern. In order to achieve logrolling, the criteria weights determine which issue increases or decreases between a pair of issues based on the definition of the logrolling range. This increase/decrease pattern is referred to as the *exchange pattern*.

5.7 Efficient Frontier for Two-Issue Sequential Logrolling

The efficient frontier for 2-issue logrolling was shown to be linear or piecewise linear with 2 segments in Section 5.5. Let us refer to it as the 2-issue efficient frontier. In 2-issue sequential logrolling, two issues are exchanged at each iteration and the outcome of the exchange becomes the new starting point for the next logrolling. The efficient frontier *at each iteration*, therefore, is a 2-issue efficient frontier which is linear or piecewise linear.

Now the efficient frontier for all possible negotiation outcomes resulting from 2-issue sequential logrolling is examined. Assume that the overall value functions are linear for both parties and $w_i/w_j \neq z_i/z_j$ for any pair of issues i and j, $i, j=1, 2, \ldots, m$, $i \neq j$. That is, logrolling is possible for any pair.

5.7.1 Formal Representation

Let us define the following.

$$\mathbf{E} = \left\{ (V_1(X), V_2(X)) \;\middle|\; \begin{array}{l} V_1(X) = V_1(X_1, X_2, \ldots, X_m) \\ V_2(X) = V_2(\bar{x}_1 - X_1, \bar{x}_2 - X_2, \ldots, \bar{x}_m - X_m) \\ X \in E \end{array} \right\}$$

$X = \{X^k \mid X \text{ on the } k\text{-th segment}, k = 1, 2, \ldots, m\}$

$$E = \left\{ X^k = (X_1, X_2, \ldots, X_m) \text{ for all } k \;\middle|\; \begin{array}{ll} x_i < X_i \le \bar{x}_i & \text{for } i = 1 \\ X_i = \bar{x}_i & \text{for } 1 < i < k \\ \underline{x}_i \le X_i \le \bar{x}_i & \text{for } i = k \\ X_i = \underline{x}_i & \text{for } k < i < m-1 \\ \underline{x}_i \le X_i < x_i & \text{for } i = m \end{array} \right\}$$

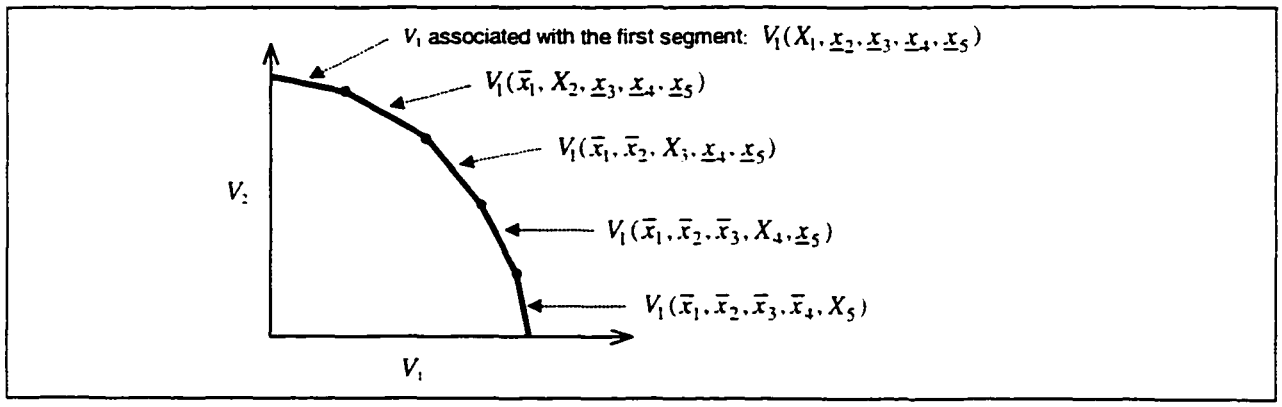
With the assumption that $V_1(X)$ and $V_2(X)$ are linear, the above notation defines **E** to be a concave piecewise linear function with m segments and X on the k-th segment to have the general expression:

$$X^k = (\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1}, \ldots, \underline{x}_m) \text{ where } \underline{x}_k \le X_k \le \bar{x}_k .$$

The general idea is shown in Figure 5.7.

The ordering or numbering of m issues, and therefore segments, is determined from the pairwise exchange patterns of issues. Set the ordering so that issue i increases against issue j if i precedes j in the ordering, that is, $i < j$. For instance, once the ordering is set, issue 1 increases against any other issue j, $j = 2, \ldots, m$. Issue 2 increases against j, $j = 3, \ldots, m$, but decreases against issue 1 in a pairwise exchange.

Figure 5.7 Efficient Frontier for 2-Issue Sequential Logrolling ($m = 5$)



Proposition 5.1 **E** is the unique efficient frontier for 2-issue sequential logrolling.

Proof. Recall the definitions of a Pareto optimal, or efficient, solution and the efficient frontier from Section 5.5. N is the negotiation outcome space. The proof consists of four parts. (1) In **E**, $X \in E$. First show $E \subseteq N$. (2) Show $(V_1(X), V_2(X))$ such that $X \in N/E$ is inefficient. (3) Show **E** is a set of Pareto optimal solutions. Together with (2), this result indicates that **E** is *the* set of Pareto optimal solutions, and therefore, **E** is the efficient frontier. Finally, (4) show **E** is the unique efficient frontier.

(1) Show $E \subseteq N$. $E \subseteq N$ if $X^k \in E$ is the outcome generated by the sequential logrolling procedure. Therefore, identification of the sequences of pairs of issues, which generate $X^k \in E$ in the logrolling procedure, proves $E \subseteq N$.

(1a) For k=1, a sequence of pairs of issues which generates $X^1 = (X_1, \underline{x}_2, \ldots, \underline{x}_m)$ with $x_1 < X_1 \leq \bar{x}_1$ is easily identified as follows: [2&3](min 3)-[2&4](min 4)-·····-[2&m](min m)-[1&2]. Similarly for k=m, a sequence which generates $X^m = (\bar{x}_1, \ldots, \bar{x}_{m-1}, X_m)$ with $\underline{x}_m \leq X_m < \bar{x}_m$ is [1&m−1](max 1)-[2&m−1](max 2)-·····-[m−2&m−1](max m−2)-[m−1&m].

(1b) Consider the general expression, $X^k = (\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1}, \ldots, \underline{x}_m)$, $2 \leq k \leq m-1$. A sequence can be identified with the following conditions:

(i) issues 1 to k−2 are maximized in the previous logrolling iterations,
(ii) issues k+2 to m are minimized in the previous logrolling iterations,
(iii) the second last pair of issues exchanged are k and k+1,
(iv) the last pair of issues exchanged are k−1 and k, and
(v) issue k has not been involved in any logrolling before the second last one.

This sequence is referred to as Sequence (*). For m=7, an example of such a sequence is [1&7](max 1)-[2&6](max 2)-[3&6](min 6)-[3&7](min 7)-[4&5](min 5)-[3&4] where k=4.

After the second last exchange of issues, Sequence (*) generates $(\bar{x}_1, \ldots, X_{k-1}, X_k, \underline{x}_{k+1}, \underline{x}_{k+2}, \ldots, \underline{x}_m)$ with $\underline{x}_{k-1} \leq X_{k-1} \leq \bar{x}_{k-1}$ and $x_k' \leq X_k \leq \bar{x}_k$. Then it generates $(\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1},$

.... $\underline{x}_m$) with $\underline{x}_k \le X_k \le x_k''$ after the last exchange of issues. Let X_k^s be the starting point of issue k before the last logrolling. Then,

$x_k' = X_k + (X_{k+1} - \underline{x}_{k+1})\cdot(w_{k+1}/w_k)$. Let $Y_1 = (X_{k+1} - \underline{x}_{k+1})\cdot(w_{k+1}/w_k)$.

$x_k'' = X_k^s - (\bar{x}_{k-1} - X_{k-1})/(z_k/z_{k-1})$. Let $Y_2 = (\bar{x}_{k-1} - X_{k-1})/(z_k/z_{k-1})$.

w_i and z_i are Party 1 and Party 2's weights for issues i, $i = 1, \ldots, m$. From the definition of the logrolling range, w_{k+1}/w_k is the worst exchange rate for Party 1 between issues k and $k+1$, and therefore, Y_1 represents the minimum amount of increase in issue k. Similarly, z_k/z_{k-1} is the best exchange rate between $k-1$ and k, and Y_2 represents the minimum amount of decrease in k.

If $Y_1 \le Y_2$, then a starting point such as $X_k^s = x_k + Y_2$ implies that $x_k'' = (x_k + Y_2) - Y_2 = x_k$. If $Y_1 > Y_2$, then a starting point such as $X_k^s = x_k + Y_1$ implies that $x_k'' = (x_k + Y_1) - Y_2 > x_k$. Therefore, Sequence (*) generates ($\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1}, \ldots, \underline{x}_m$) with $\underline{x}_k \le X_k \le x_k$.

By modifying conditions (iii) and (iv) of Sequence (*) to: (iii) the second last pair of issues exchanged are $k-1$ and k, and (iv) the last pair of issues exchanged are k and $k+1$, the previous argument shows that the modified sequence generates ($\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1}, \ldots, \underline{x}_m$) with $x_k \le X_k \le \bar{x}_k$. Hence, Sequence (*) and the modified one together generate $X^k = (\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1}, \ldots, \underline{x}_m)$ with $\underline{x}_k \le X_k \le \bar{x}_k$.

From (1*a*) and (1*b*), sequences of pairs of issues exist which generate $X^k = (\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1}, \ldots, \underline{x}_m)$ with $\underline{x}_k \le X_k \le \bar{x}_k$ for all k. This concludes that $E \subseteq N$.

Note that the $(k-1)$-th and k-th segments of **E** can be considered as the 2-issue efficient frontier generated from the last exchange of Sequence (*). Since the efficient frontier of 2-issue logrolling is piecewise linear and concave, the $(k-1)$-th and k-th segments must also be concave. The same argument applies to $k = 2, \ldots, m$. Therefore, the concavity of any two adjacent line segments contributes to the overall concavity of **E**.

(2) Show $(V_1(X), V_2(X))$ *such that* $X \in N/E$ *is inefficient.* Let us consider $X \in E$, ($\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1}, \ldots, \underline{x}_m$). Suppose $X' \in N/E$ exists such that $V_1(X') \ge V_1(X)$ and $V_2(X') \ge V_2(X)$

with at least one inequality. Then there must be an additional logrolling opportunity available from $X \in E$. A further exchange from X may be possible between i and j where $i \in \{1, \ldots, k\}$ and $j \in \{k, \ldots, m\}$, $i \neq j$, if issue i decreases and j increases. However, the ordering of m segments is determined from the pairwise exchange patterns of issues so that issue i increases against issue j if $i < j$. Therefore, any further exchange made by decreasing i and increasing j does not achieve logrolling because either (i) both parties give up a more important issue for a less important one, or (ii) the party with the higher preference intensity gives up the more important issue. Hence, the sequential logrolling procedure cannot produce X' such that $V_1(X') \geq V_1(X)$ and $V_2(X') \geq V_2(X)$, so then any $X \in N/E$ must be inefficient.

(3) Show **E** *is the set of Pareto optimal solutions.* Suppose X, $X^0 \in E$. It can be seen from the definition of **E** and E that if $V_1(X) > V_1(X^0)$, then $V_2(X) < V_2(X^0)$, and if $V_1(X) < V_1(X^0)$, then $V_2(X) > V_2(X^0)$. Therefore, **E** where $X \in E$ is a set of Pareto optimal solutions. It has been shown in (2) that if $X \in N/E$, the corresponding $(V_1(X), V_2(X))$ is inefficient. Therefore, **E** is the set of Pareto optimal solutions.

(4) Show **E** *is unique.* The uniqueness of the efficient frontier is given by the uniqueness of the ordering of the m line segments. The ordering is determined by the pairwise exchange patterns of issues with a condition that issue i increases against issue j if $i < j$. Suppose that there exist two different orderings which satisfy this condition. Then it is possible to identify a pair of issues whose ordering is reversed. This reversal implies a reversal of the pairwise exchange pattern. However, between any two issues, there is only one pattern of exchange which is determined from the criteria weights. Therefore, the ordering of the m line segments is unique. This proves the uniqueness of the efficient frontier for 2-issue sequential logrolling. ■

5.7.2 Implications

The efficient frontier for the logrolling procedure is shown to be piecewise linear. Although 2-issue or 2-issue sequential logrolling does not suggest a unique prescriptive solution, with

the general characteristics of the efficient frontier, one can examine the existing bargaining solutions such as the Nash solution (Nash, 1950) and the Kalai solution (Kalai, 1977).

In the sequential procedure, the negotiating parties have some freedom to choose when and which two issues are exchanged. For example, the assumption that logrolling is possible between any two issues implies that there are $\binom{m}{2}=\frac{m\cdot(m-1)}{2}$ pairs to choose from at the first iteration. The existence of a unique efficient frontier does not suggest any clear universal strategic advantage to either one of the parties of choosing a particular sequence of pairs of issues over the other. Perhaps sensitivity analysis on exchange rates, criteria weights, and sequences of pairs of issues can provide insight into the case-by-case individual strategies.

The illustration for 3-issue case (Figure 5.8) clearly shows that the sequential procedure may lead to an inefficient outcome. This is a main deviation from 2-issue logrolling case in which the logrolling outcome is guaranteed to be Pareto optimal. However, by realizing any inefficient outcome can be considered as a new starting point, an additional logrolling procedure exploits the mutual gain left undiscovered in the first round. The current procedure can easily accommodate the additional logrolling by introducing an investigation stage where all pairs are checked for the further logrolling possibility before the procedure ends. The modified procedure then guarantees a Pareto optimal solution. Figure 5.9 illustrates the multi-phase modified procedure.

Figure 5.8 Efficient Frontier for 3-Issue Case

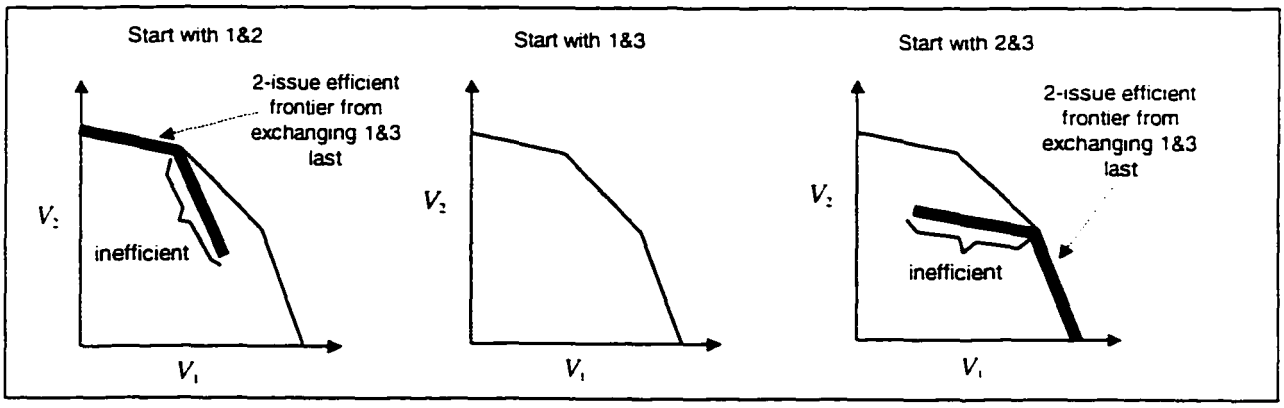
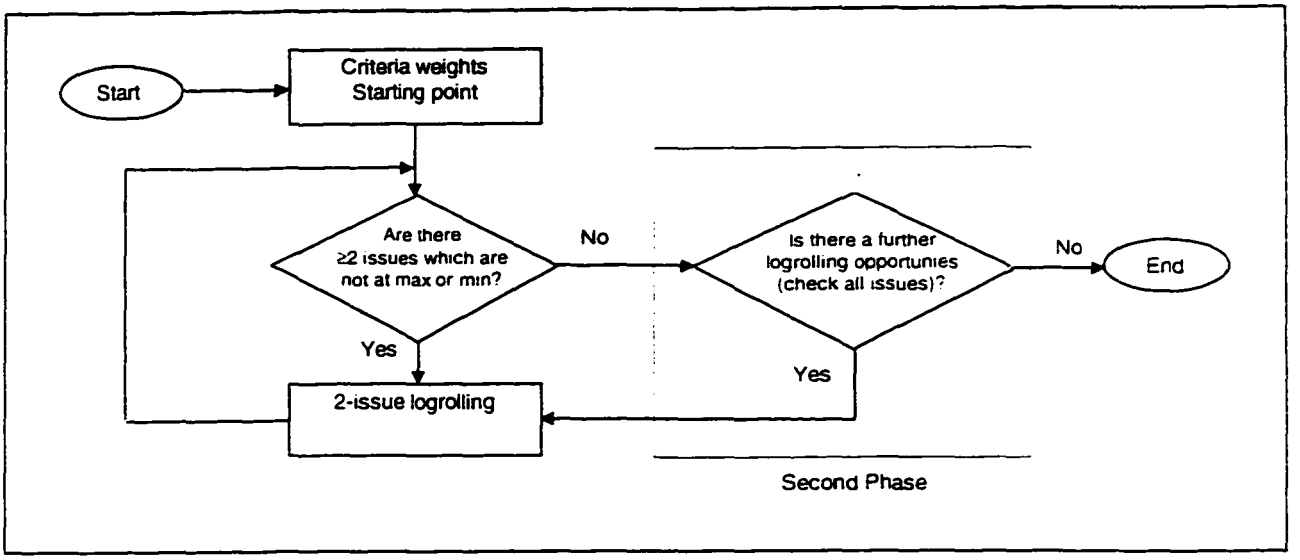


Figure 5.9 Multi-Phase 2-issue Sequential Logrolling



5.8 Extension to *k*-issue Sequential Logrolling

If the number of issues to be exchanged is not restricted to 2, then the sequential procedure is referred to as *k*-issue sequential logrolling ($2 \le k \le m$). In practice, the extension from 2-issue to *k*-issue adds flexibility to the negotiation procedure. The basis of the sequential procedure remains the same as in 2-issue sequential case (Figure 5.5). The difference is the value of *k* and how it may vary from one logrolling iteration to the next. The efficient frontier is examined based on that for 2-issue sequential logrolling.

5.8.1 Efficient Frontier

Proposition 5.2 The efficient frontier for *k*-issue sequential logrolling, EF_k, is equivalent to that for 2-issue sequential logrolling, EF_2, $2 \le k \le m$.

Proof. *(1) Show* $EF_2 \subseteq EF_k$. 2-issue sequential logrolling is a special case of *k*-issue sequential logrolling, so the outcomes of 2-issue case can be generated from setting $k = 2$.

This concludes that $EF_2 \subseteq EF_k$. Next, it is shown that EF_k does not contain outcomes which are more efficient than the outcomes in EF_2.

(2) Show $EF_2 \not\subset EF_k$. 2-issue sequential logrolling cannot generate outcomes which are more efficient than EF_2 because exchanging 2 issues further means (i) both parties give up a more important issue for a less important, and therefore, both parties experience a decrease in the value, or (ii) the party with the lower preference intensity increases his or her value at the expense of the other party who has the higher preference intensity. It is shown that k-issue sequential logrolling ($2<k\leq m$) cannot generate outcomes which are more efficient than EF_2 based on (i) and (ii).

Let a point in EF_2 be the starting point. For any k-issue exchange, two issues exist such that one increases (denoted A) and the other decreases (B). The exchange of A and B would result in (i) or (ii). Suppose (i) first. In order to increase the value, both parties need additional value contribution from other issues. Suppose each party gains in one issue: C for Party 1 and D for Party 2. Then there is a decrease in D for Party 1 and in C for Party 2 because of distributive bargaining, and C and D can be considered as another exchange of issues. Since any pair of issues cannot generate additional value from a point in EF_2, C and D do not provide a logrolling opportunity. This argument shows that k-issue exchange with such A and B cannot increase value for both parties.

Now suppose the exchange of A and B would result in (ii): A increases against B for Party 1. Then Party 1's value increases from the starting point in EF_2, but Party 2's value decreases. Suppose that Party 2 gains in an issue (C). Based on Party 1's general expression of an outcome in EF_2, $(\bar{x}_1, \ldots, \bar{x}_{k-1}, X_k, \underline{x}_{k+1}, \ldots, \underline{x}_m)$, it must be that $B,C \in \{1, \ldots, k\}$ and $A \in \{k, \ldots, m\}$, $B \neq A$ and $C \neq A$, since A increases and B and C decrease from the starting point. Considering A, B, and C as a 3-issue exchange, if A maximizes first, then A and B, or A and C, can always provide further logrolling because $A \in \{k, \ldots, m\}$ and $B,C \in \{1, \ldots, k\}$. Likewise, if B minimizes first, A and B provide further logrolling, and if C minimizes first, A and C provide logrolling. Therefore, the 3-issue exchange is inefficient, and again k-issue exchange with such A and B cannot increase value for both parties.

The results of (1) and (2) conclude that k-issue sequential logrolling cannot generate outcomes which are more efficient than EF_2. Therefore, the efficient frontier for k-issue sequential logrolling is equivalent to that for 2-issue sequential logrolling. ■

5.8.2 Implications

Since the efficient frontier for k-issue sequential logrolling is equivalent to that for 2-issue sequential logrolling, these two procedures do not differ in generating a set of Pareto optimal solutions. In this case, 2-issue procedure may be preferred to k-issue one based on its ease and familiarity to exchange between two issues. In general, 2-issue or k-issue sequential logrolling has an advantage over the two conventional negotiation procedures: sequential and simultaneous.

In the literature, the term sequential is reserved for describing the negotiation procedure which deals with one issue at a time. When all issues are considered together, it is referred to as the simultaneous negotiation (Balakrishnan et al., 1993; Weingart et al., 1993). The main advantage of the sequential procedure is that it sets boundaries, timing and order of discussion on otherwise complex multi-issue negotiation (Thompson et al., 1988). The disadvantage, on the other hand, is that the outcome may be inefficient and not mutually beneficial. The main advantage of the simultaneous procedure is its capacity to utilize different priorities among the negotiating parties. With the simultaneous procedure, integrative solutions are realized through trade-offs (Thompson, 1990*a*). The disadvantage is its procedural complexity which may prohibits the parties from negotiating effectively and efficiently.

In sequential logrolling, its iterative procedure provides an order to the negotiation discussion process. At the same time, its logrolling process facilitates the trade-offs among the issues, and by doing so, possible mutual benefit is exploited. Sequential logrolling combines the best features of the two different negotiation procedures providing an advantage over the two separately.

5.9 Summary

Multi-issue negotiation is modelled using logrolling in this chapter. Logrolling is essentially a type of exchange in which loss in some issues for gain in others results in mutual overall gain. It is shown that logrolling is possible due to the differences in the negotiating parties' preferences. In fact, the definition of the logrolling range implies that, even if the parties prefer the same issue over another, a logrolling opportunity may still exists as long as there is a difference in how much they prefer. The logrolling-based procedure does not suggest a unique solution to multi-issue negotiation, but it can be used to construct the efficient frontier for further analysis. The efficient frontier was shown to be linear or piecewise linear when the parties' value functions are linear. Two-issue two-party negotiation was considered first, and the extension on the number of issues from 2 to m was investigated based on the two-issue results. The k-issue sequential logrolling procedure ($2 \le k \le m$), an extension of 2-issue logrolling, demonstrates a procedural benefit which combines the best features from the sequential and simultaneous negotiation procedures.

Logrolling and related concepts presented here are applied to an example next. Benefits and possible application areas of logrolling are discussed at the end of the following chapter.

Chapter 6

Logrolling Illustration

The previous chapter introduced several concepts related to logrolling in abstract terms. The purpose of the current chapter is demonstration of these concepts using a small example with more concrete terms. The logrolling procedure, efficient frontier, and some solution analysis are presented for a specific problem in order to strengthen the understanding of logrolling. This chapter is not intended to be an in-depth case study. The level of analysis, therefore, is elementary.

This chapter is organized as follows. The example problem is defined in Section 6.1. Section 6.2 shows the efficient frontier for this particular problem based on Proposition 5.1. Then the simplified logrolling procedure is described in Section 6.3 for a better overall understanding of how the exchange of issues occurs. Section 6.4 contains a sample solution analysis. Finally, Section 6.5 concludes this chapter with the discussion of contributions and possible application areas of logrolling.

6.1 Problem Definition

The example used throughout this chapter is a 3-issue 2-party negotiation problem. 3-issue negotiation is sufficient to show details and implications of various aspects of logrolling. It also provides computational ease. The scenario is commonly observed in the working

environment: negotiation taking place between a newly recruited employee (Party 1) and the personnel department of a firm (Party 2) regarding the employment agreement. Three issues, starting salary, number of vacation days in the first year, and amount of medical benefit coverage, are being discussed and negotiated. Table 6.1 presents information on the parties' preferences. Each issue has maximum and minimum values that are known and fixed prior to negotiation. The possible agreement in each issue can take any value between the maximum and minimum. The parties' preferences are assumed to be continuous and are expressed by "points" which take into account the issues' relative importance. This data set is taken and modified from Thompson (1990*b*).

Table 6.1 Preference Data

	Salary	Vacation	Medical
1. Employee	Max \$24,000 (= 660 points)	Max 21 days (= 240 points)	Max 100% (= 100 points)
	Min \$20,000 (= 0 points)	Min 7 days (= 0 points)	Min 20% (= 0 points)
2. Personnel	Max \$24,000 (= 0 points)	Max 21 days (= 0 points)	Max 100% (= 0 points)
	Min \$20,000 (= 100 points)	Min 7 days (= 240 points)	Min 20% (= 660 points)

Over each issue, the parties have opposite preferences. For example, Party 1, the new employee, prefers higher starting salary, but Party 2, the employer, prefers lower starting salary. Furthermore, if Party 1 succeeds in attaining a higher salary than the starting offer, then his or her gain implies loss to the employer. On the other hand, if negotiation results in a lower starting salary for Party 1, then his or her loss implies gain to the employer. Therefore, for all issues, the type of bargaining is distributive.

Let $x_1^{(0)}$, $x_2^{(0)}$, and $x_3^{(0)}$ be the starting point of salary, vacation, and medical issues, respectively. The starting offer is assumed to be the midpoint, that is, $x_1^{(0)}$ = \$22,000, $x_2^{(0)}$ = 14 days, and $x_3^{(0)}$ = 60%. Let w_1, w_2, and w_3 be Party 1's criteria weights for salary, vacation, and medical issues, respectively. Similarly, let z_1, z_2, and z_3 be Party 2's weights. The weight for each issue is arbitrary chosen to be the ratio of maximum attainable points to total points available from all three issues. That is, for example, w_1 = 660/(660+240+100) =

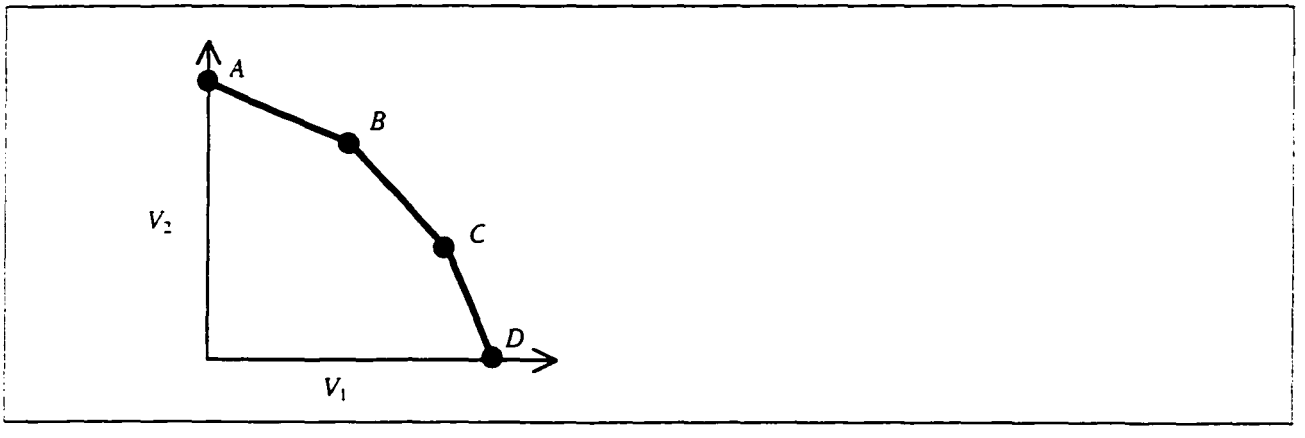
660/1000 = 0.66. The remaining weights are computed in the same manner: $w_2 = 0.24$, $w_3 = 0.1$, $z_1 = 0.1$, $z_2 = 0.24$, and $z_3 = 0.66$. Assume that the value function for each issue is linear with the minimum and maximum values based on the points indicated in Table 6.1. The overall value is assumed to be additive.

Since $w_1 > w_2 > w_3$ and $z_1 < z_2 < z_3$, the two parties' preferences are opposite over the three issues. In this case, a logrolling opportunity exists for any pair of issues. Recall that even if $w_1 > w_2 > w_3$ and $z_1 > z_2 > z_3$, logrolling opportunities may still exist as long as there is a difference in how much the parties prefer one issue over another.

6.2 Efficient Frontier

According to Proposition 5.1, the efficient frontier for this negotiation problem is piecewise linear with three linear segments as shown in Figure 6.1. This is a general result that does not depend on specific starting points or agreed exchange rates.

Figure 6.1 Efficient Frontier for the Employment Agreement Negotiation Problem



Assuming that the overall value function is additive, the coordinates of the end points and the corner points, *break points*, can be determined specifically. Party 1's value at points *A*, *B*, *C*, and *D* are computed as follows:

$V_1(A) = V_1(\underline{x}_1, \underline{x}_2, \underline{x}_3) = 0 + 0 + 0 = 0.$
$V_1(B) = V_1(\overline{x}_1, \underline{x}_2, \underline{x}_3) = 660 + 0 + 0 = 660.$
$V_1(C) = V_1(\overline{x}_1, \overline{x}_2, \underline{x}_3) = 660 + 240 + 0 = 900.$ and
$V_1(D) = V_1(\overline{x}_1, \overline{x}_2, \overline{x}_3) = 660 + 240 + 100 = 1000.$

Similarly, Party 2's values at various points are:

$V_2(A) = V_2(\overline{x}_1, \overline{x}_2, \overline{x}_3) = 100 + 240 + 660 = 1000.$
$V_2(B) = V_2(\underline{x}_1, \overline{x}_2, \overline{x}_3) = 0 + 240 + 660 = 900.$
$V_2(C) = V_2(\underline{x}_1, \underline{x}_2, \overline{x}_3) = 0 + 0 + 660 = 660,$ and
$V_2(D) = V_2(\underline{x}_1, \underline{x}_2, \underline{x}_3) = 0 + 0 + 0 = 0.$

The coordinates of the points are hence A = (0, 1000), B = (660, 900), C = (900, 660), and D = (1000, 0).

6.3 Logrolling Procedure

The basic description of the 2-issue sequential logrolling procedure for three issues, i, j, and k, can be summarized as follows. Activation cost of negotiation is assumed to be negligible.

Given: w_i, w_j, w_k, z_i, z_j, z_k, $x_i^{(0)}$, $x_j^{(0)}$, and $x_k^{(0)}$.

1. Choose two issues to be exchanged: i and j.

2. Compute the logrolling range (for Party 1).

   The minimum and maximum values of the logrolling range are computed as follows:
   If $(w_j/w_i) < (z_j/z_i)$, then set Min = w_j/w_i and Max = z_j/z_i.
   If $(w_j/w_i) > (z_j/z_i)$, then set Min = w_i/w_j, Max = z_i/z_j.

3. Negotiate on the agreed exchange rate. This part is analogous to traditional negotiation over single issue whose outcome highly depends on many factors including the parties' negotiation skills. For this chapter, the agreed exchange rate for each pair of issues is arbitrarily fixed at the midpoint of the corresponding logrolling range in order to avoid

subjective assessment. If $(w_j/w_i) < (z_j/z_i)$, then let the agreed exchange rate be s_j/s_i. If $(w_j/w_i) > (z_j/z_i)$, then let the agreed exchange rate be s_i/s_j.

4. Exchange the two issues until the increasing issue reaches its maximum or the decreasing issue reaches minimum for one party.

5. Update the starting point:

If $(w_j/w_i) < (z_j/z_i)$, then

$$x_i^{(1)} = Min\{x_i^{(0)} + (x_j^{(0)} - \underline{x}_j) \cdot (s_i/s_j), \bar{x}_i\}$$
$$x_j^{(1)} = Max\{x_j^{(0)} - (\bar{x}_i - x_i^{(0)})/(s_i/s_j), \underline{x}_j\}$$
$$x_k^{(1)} = x_k^{(0)}$$

If $(w_j/w_i) > (z_j/z_i)$, then

$$x_i^{(1)} = Max\{x_i^{(0)} - (\bar{x}_j - x_j^{(0)})/(s_j/s_i), \underline{x}_i\}$$
$$x_j^{(1)} = Min\{x_j^{(0)} + (x_i^{(0)} - \underline{x}_i) \cdot (s_j/s_i), \bar{x}_j\}$$
$$x_k^{(1)} = x_k^{(0)}$$

6. The next two issues to be exchanged are determined based on the following conditions. Repeat 2-5 with these issues.

If $(w_j/w_i) < (z_j/z_i)$ and $x_j^{(1)} = \underline{x}_j$, then exchange i and k next.
If $(w_j/w_i) < (z_j/z_i)$ and $x_i^{(1)} = \bar{x}_i$, then exchange j and k next.
If $(w_j/w_i) > (z_j/z_i)$ and $x_i^{(1)} = \underline{x}_i$, then exchange j and k next.
If $(w_j/w_i) > (z_j/z_i)$ and $x_j^{(1)} = \bar{x}_j$, then exchange i and k next.

7. Repeat 2-5 with the third pair of issues that has not been exchanged before. This step may be redundant for some situations, but it guarantees the final outcome to be Pareto optimal. The final outcome is the most recently updated starting point: $(x_i^{(2)}, x_j^{(2)}, x_k^{(2)})$.

End of the procedure.

Based on the procedure above, an EXCEL spreadsheet can be set up for calculation of the final outcome. Appendix B.1 contains sample EXCEL worksheets used in actual calculation. Note that the scaling problem may be generated due to the different magnitudes of numerical quantities. For example, the vacation days are in order of 10 whereas the starting salary is in

order of 10,000. In order to avoid the scaling problem, the data used for actual calculation are normalized values of the original numerical figures.

6.4 Solution Analysis

Logrolling does not prescribe a unique solution to a multi-issue negotiation problem. However, for 3-issue negotiation, a unique outcome to a particular scenario can be determined if specific values of the starting point, criteria weights, and agreed exchange rate are known as well as which pair of issues is exchanged first. Then, these items can be considered as the parameters of the problem, and one can perform parameter analysis in order to gain insight and confidence in the solution choice process. Here, three scenarios are examined for illustration: varying the starting point, varying the criteria weights, and varying the agreed exchange rate. Each case is discussed in turn below.

6.4.1 Parameter Analysis: Starting Point

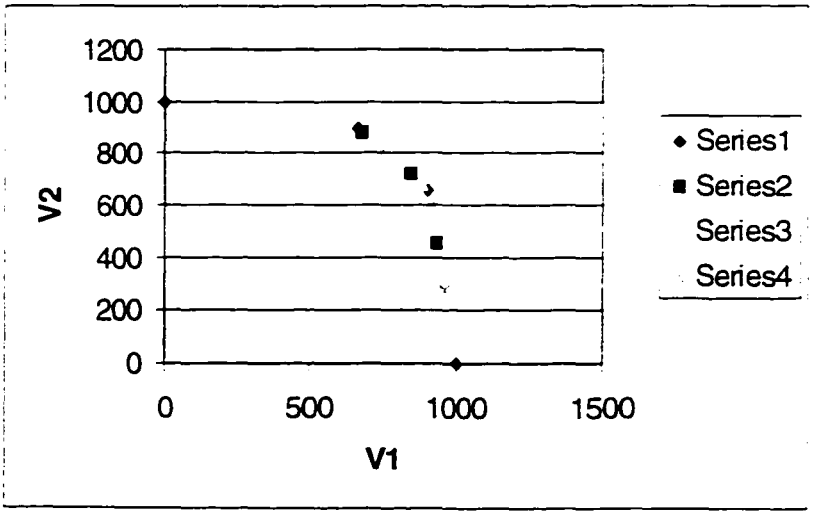
In order to examine the effect of different starting points, three levels, low, medium, and high, of the starting point in each issue are combined in various ways. Table 6.2 shows the combinations used in this analysis. The final outcome is computed for each scenario with three different pairs of issues to start logrolling: pairs 1 and 2, 1 and 3, and 2 and 3. The parties' criteria weights are fixed at $w_1 = 0.66$, $w_2 = 0.24$, $w_3 = 0.1$, $z_1 = 0.1$, $z_2 = 0.24$, and $z_3 = 0.66$ as specified in Section 6.1. The agreed exchange rate for each pair of issues is arbitrarily fixed at the midpoint of the corresponding logrolling range.

Table 6.2 Different Low-Medium-High Combinations of the Issues' Starting Level

Salary	Vacation (days)	Medical (%)
Low ($20,800)	Low (9.8) Medium (14) High (18.2)	Low (36) Medium (60) High (84)
Medium ($22,000)	Low (9.8) Medium (14) High (18.2)	Low (36) Medium (60) High (84)
High ($23,200)	Low (9.8) Medium (14) High (18.2)	Low (36) Medium (60) High (84)

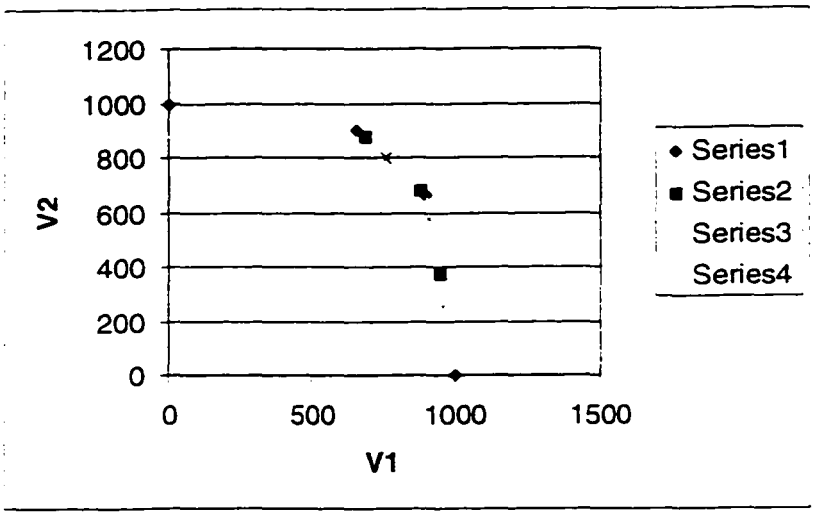
Figures 6.2, 6.3, and 6.4 summarize the results. All numerical results can be found in Appendix B.2.

Figure 6.2 Varying the Starting Point (Starting with Pair 1 and 2)



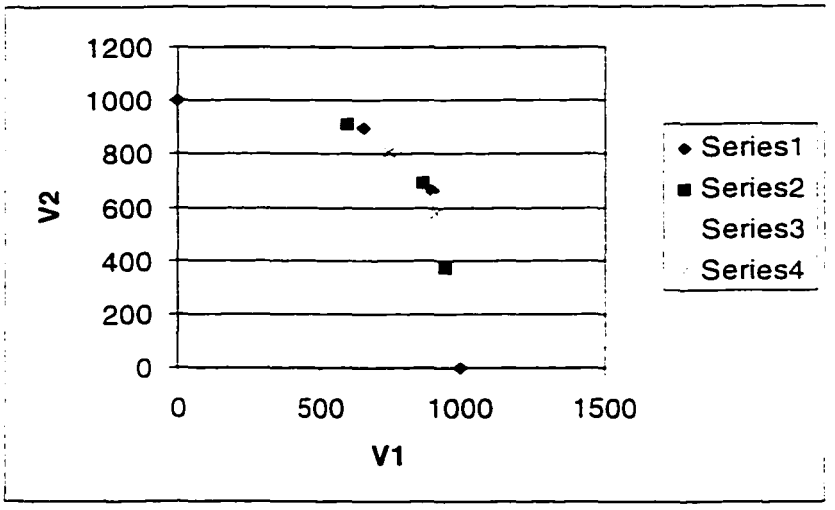
Series 1: Corner points of the theoretical efficient frontier (Figure 6.1)
Series 2: Low salary combinations
Series 3: Medium salary combinations
Series 4: High salary combinations

Figure 6.3 Varying the Starting Point (Starting with Pair 1 and 3).



Series 1: Corner points of the theoretical efficient frontier (Figure 6.1)
Series 2: Low salary combinations
Series 3: Medium salary combinations
Series 4: High salary combinations

Figure 6.4 Varying the Starting Point (Starting with Pair 2 and 3).



Series 1: Corner points of the theoretical efficient frontier (Figure 6.1)
Series 2: Low salary combinations
Series 3: Medium salary combinations
Series 4: High salary combinations

A general conclusion is that the better starting point yields the better final outcome. Since logrolling is designed to always increase value, the advantage of having a superior starting point over the other party seems to be maintained over the course of negotiation.

From the observation across Figures 6.2, 6.3, and 6.4, the final outcomes with different starting pairs of issues do not appear to differentiate from each other in any special way. In other words, the first pair of issues in the exchange does not have any significant effect in determining the outcome for this negotiation problem. This result is in accordance with the implication of Proposition 5.1 that there is no clear advantage to a party choosing a particular pair of issues over the other.

6.4.2 Parameter Analysis: Criteria Weights

In this section, the effect of changing Party 1's criteria weights is examined while keeping Party 2's weights constant at $z_1 = 0.1$, $z_2 = 0.24$, and $z_3 = 0.66$. In terms of ordinal preference, six distinct combinations exist over three issues. These six combinations as well as having one extremely important issue, having two important issues, and having three equally important issues are the scenarios to be examined. The combinations of weights representing the various scenarios are shown in Table 6.3. The starting point is fixed at $x_1^{(0)} = \$22{,}000$, $x_2^{(0)} = 14$ days, and $x_3^{(0)} = 60\%$, and the agreed exchange rate for each pair of issues is set at the midpoint of the corresponding logrolling range. All cases are assumed to begin with issues 1 and 2.

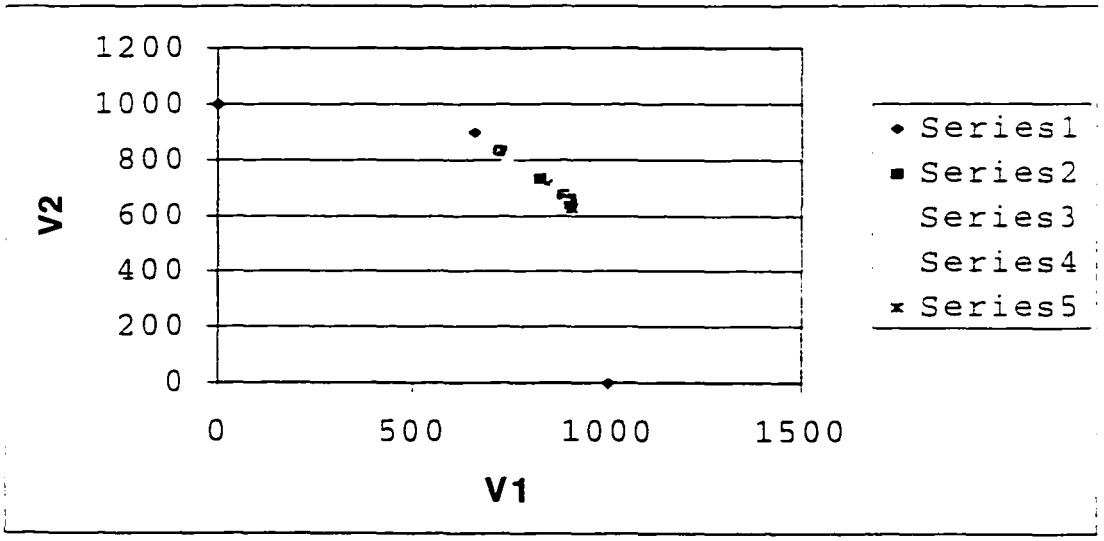
Note that the two parties' preferences are essentially identical in one case where $\{w_1 = 0.1, w_2 = 0.3, w_3 = 0.6\}$ and $\{z_1 = 0.1, z_2 = 0.24, \text{and } z_3 = 0.66\}$. Logrolling is technically still possible since there are differences in preference intensity.

Table 6.3 Different Combinations of Party 1's Criteria Weights

Personnel (fixed)	0.1	0.24	0.66
	Salary	Vacation	Medical
Six different ordinal preferences	0.66	0.24	0.1
	0.6	0.1	0.3
	0.3	0.6	0.1
	0.3	0.1	0.6
	0.1	0.6	0.3
	0.1	0.3	0.6
One extremely important	0.9	0.06	0.04
	0.06	0.9	0.04
	0.04	0.06	0.9
Two important	0.5	0.4	0.1
	0.1	0.5	0.4
	0.4	0.1	0.5
Three equally important	0.34	0.33	0.33

All logrolling results are summarized in Figure 6.5. The numerical results are available in Appendix B.2.

Figure 6.5 Varying the Criteria Weights (Starting with Pair 1 and 2)



Series 1: Corner points of the theoretical efficient frontier (Figure 6.1)
Series 2: Six different combinations of ordinal preferences
Series 3: One extremely important issue
Series 4: Two important issues
Series 5: Three equally important issues

The results are rather surprising because there is no indication of any significant patterns forming. If Party 1's least preferred issue is Party 2's most preferred and Party 1's most preferred issue is Party 2's least preferred, then one may conjecture that this type of situation produces most value for both parties from logrolling. In principle, logrolling exploits the parties' differences in preference and searches mutual gain. On the other hand, similar preferences between the parties must represent a tougher competition, and hence, less value may be yielded from logrolling for both. As shown in Figure 6.5, evidence to support these assertions is not found.

The existence of one extremely important issue is expected to dominate the party's negotiation behaviour in such a way that he or she would use all available resources to achieve as much of the most important issue as possible. Based on the same argument, having two important issues then may distribute the party's effort over the two issues. With three equally important issues, the outcome may depend more on the other party's preference. Again Figure 6.5 supports none of these apparently sound assertions. Perhaps the interpretation of the different weight patterns is inaccurate. In any case, additional analysis with greater detail may reveal further insight.

6.4.3 Parameter Analysis: Agreed Exchange Rate

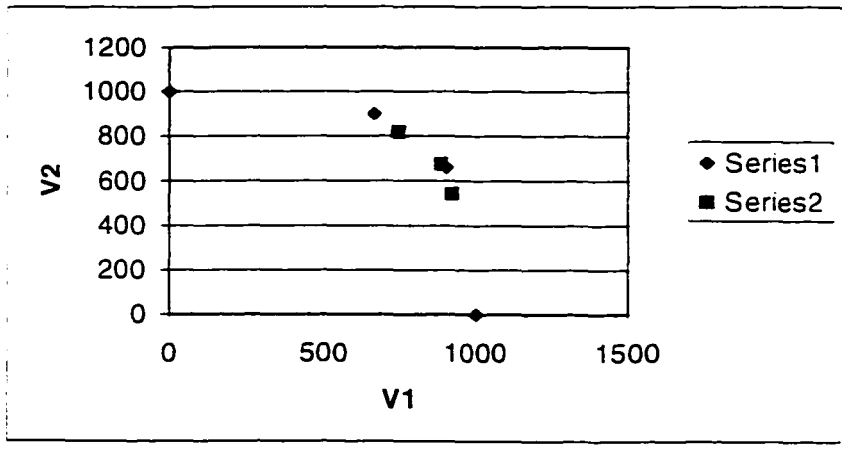
In the actual negotiation procedure, determining the agreed exchange rate is analogous to traditional negotiation over single issue. It is a subjective process that depends on many factors. One of the obvious factors is the parties' negotiation capability. By assuming that a party with superior skills can attain a better result, varying the agreed exchange rate may investigate how the parties' negotiation skills affect the outcome. Table 6.4 shows arbitrarily chosen exchange rates to represent scenarios that Party 1's skills are inferior, equal, and superior to Party 2's skills.

Table 6.4 Agreed Exchange Rate at Different Levels

Logrolling Range	Max = 2.4 Min = 0.363636	Max = 6.6 Min = 0.151515	Max = 2.75 Min = 0.416667
Starting Pair	1&2	1&3	2&3
Inferior	0.7	2	0.8
Equal	1.381818	3.375758	1.583333
Superior	2.1	5	2.3

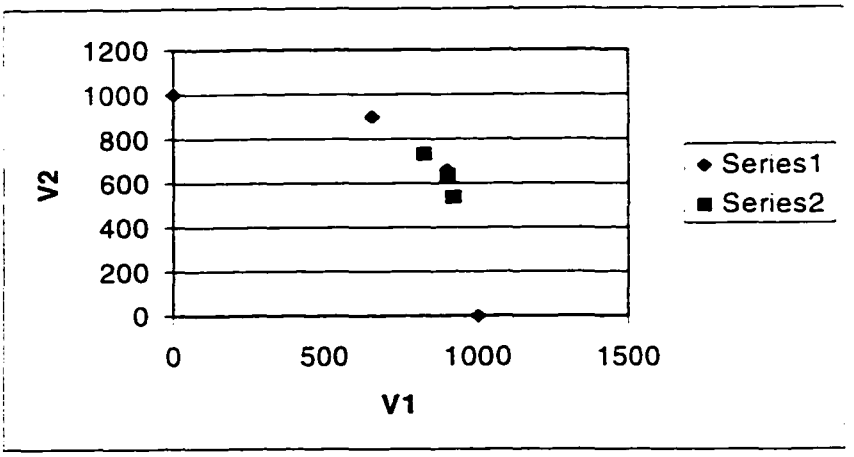
The starting point is fixed at $x_1^{(0)}$ = $22,000, $x_2^{(0)}$ = 14 days, and $x_3^{(0)}$ = 60%, and the parties' criteria weights are fixed at $w_1 = 0.66$, $w_2 = 0.24$, $w_3 = 0.1$, $z_1 = 0.1$, $z_2 = 0.24$, and $z_3 = 0.66$ as specified in Section 1. The outcomes corresponding to the three scenarios with different starting pairs of issues are computed. Figures 6.6, 6.7, and 6.8 exhibit the results. The numerical results are available in Appendix B.2.

Figure 6.6 Varying the Agreed Exchange Rate (Starting with Pair 1 and 2)



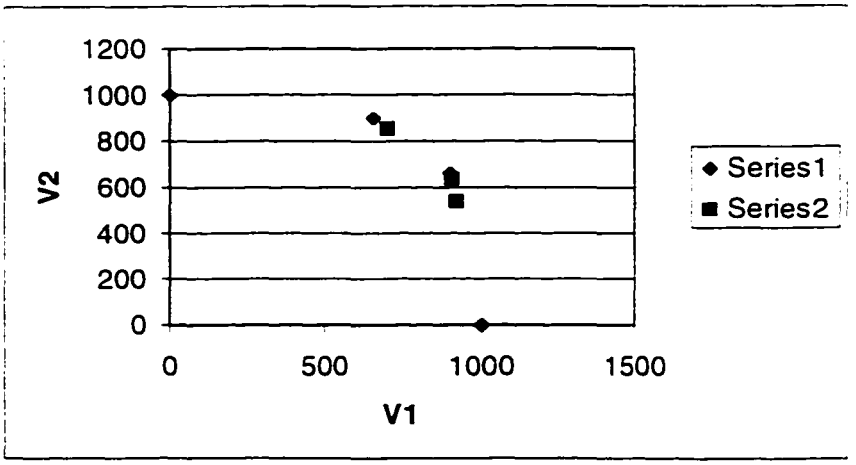
Series 1: Corner points of the theoretical efficient frontier (Figure 6.1)
Series 2: Inferior, equal, and superior agreed exchange rates (from left to right)

Figure 6.7 Varying the Agreed Exchange Rate (Starting with Pair 1 and 3)



Series 1: Corner points of the theoretical efficient frontier (Figure 6.1)
Series 2: Inferior, equal, and superior agreed exchange rates (from left to right)

Figure 6.8 Varying the Agreed Exchange Rate (Starting with Pair 2 and 3)



Series 1: Corner points of the theoretical efficient frontier (Figure 6.1)
Series 2: Inferior, equal, and superior agreed exchange rates (from left to right)

In general, the higher skill level yields the better final outcome. The observation across Figures 6.6, 6.7, and 6.8 indicates that if Party 1 is equal or superior to Party 2, then the order of negotiation, that is, which pair of issues is exchanged first, does not have any significant effect on the outcome. However, if Party 1's negotiation skills are inferior to Party 2's, then starting with issues 1 and 3 seems to have a slight advantage since the worst outcome for Party 1 is better than the worst outcomes of other cases.

6.5 Implementation Issues

The examination of the example problem provides a valuable opportunity to identify some possible difficulties at the implementation stage. Although not all of these issues can be addressed in a satisfactory manner at present, it is worthwhile to acknowledge their existence. Five such issues are described below. The discussion includes some suggestions for future improvement wherever possible.

Identical Set of Criteria Weights. When two parties claim to exhibit exactly the same criteria weights, that is, $w_i = z_i$ for all i, logrolling is theoretically not possible. The identical set of criteria weights does not allow mutual increase in value for all parties. However, since any small amount of difference in preference is sufficient to initiate the procedure, one can modify the weights slightly to allow logrolling and analysis. Alternatively, additional issues that differentiate the parties' preferences may be identified.

Information Manipulation. Logrolling requires disclosure of criteria weight information from all parties. Is there any advantage to falsifying one's weights? Change in criteria weights directly affects the logrolling range. Consequently, change in the logrolling range may or may not affect the choice of the agreed exchange rate. An extensive what-if analysis may be able to explain some aspects. Further investigation is definitely required to answer this question.

Negotiation Over the Agreed Exchange Rate. As mentioned in Section 6.4.3, determining the agreed exchange rate is a highly subjective process. What-if analysis may be useful as an asymmetric prescriptive analysis on one party's negotiation strategy. In practice, some procedure or guideline may be necessary to avoid deadlocks. No suggestion is available at this point.

Choice of Issues. For 3-issue negotiation, the parties have choice in which pair of issues to be exchanged first. With more than 3 issues, freedom increases in choosing which pair to be exchanged and when. Proposition 5.1 suggests no clear advantage to choosing one particular sequence of pairs of issues over another. However, in practice, one pair must be chosen in order to proceed with logrolling. Further investigation is required to determine a recommended procedure for choosing a suitable pair of issues. One possible suggestion is

that issues with different priorities may be exchanged before the issues with the same priority. If it is obvious to the parties that they have difference in preference, then it may be psychologically easier for them to comply. The difference in priority is more obvious than the difference in preference intensity.

Linear Preference. Based on linear preference, logrolling produces a distinctly characterized outcome. As exchanging continues until the minimum or maximum of an issue is reached, the outcome mostly contains "all or nothing" of each issue. Whether this is an acceptable and suitable solution to a real-world problem remains a question to be answered.

6.6 Summary

In this chapter, the employment agreement example is used to demonstrate the logrolling procedure, illustrate the efficient frontier, and study possible solution strategies based on parameter analysis. The results of analysis indicate that, even at the elementary level, some interesting insight can be gained in terms of desirable negotiation strategies. The discussion in concrete terms has also provided a valuable opportunity to identify some implementation issues.

In the rest of this section, contributions of the logrolling study are summarized. Some interesting application areas of logrolling are also presented.

6.6.1 Benefits

The following benefits are recognized for the logrolling-based solution method.

- The solution method using logrolling is an integrative approach that incorporates criteria weights and the interaction information, namely trade-off simultaneity.
- The quantitative method for achieving mutually beneficial solutions via trade-offs is a novel application of the concept of logrolling.

- Commonly, difference in opinion or preference is seen as an obstacle in reaching an agreement between the parties. An interesting implication of the existence of the logrolling range is that the difference in parties' priorities or preference intensities actually facilitates the search for mutual gain.
- 2-issue and 2-issue sequential procedures are proposed based on the logrolling principles. These procedures are based on the explicit trade-offs among issues which are natural and fundamental to real-life negotiation or bargaining.
- The logrolling procedure guarantees a Pareto optimal solution, which is not easily achieved in practice. However, a unique solution is determined only if all the agreed exchange rates are known.
- The general characteristics of the efficient frontier can be used for prescriptive analysis using the existing bargaining solutions such as the Nash and Kalai solutions.
- 2-issue and *k*-issue sequential logrolling procedures both have an advantage over conventional negotiation procedures. The equivalence of the efficient frontiers for 2-issue and *k*-issue sequential logrolling suggests that 2-issue sequential logrolling may be preferred to *k*-issue due to the ease and familiarity of the two-issue exchange.
- The implications of trade-off simultaneity indicate that logrolling offers mutual gain by utilizing differences in the parties' preferences. The mutual consequence experienced by all parties can contribute to psychological benefits associated with an improved relationship among the parties.
- Even at the elementary level, solution analysis based on the logrolling procedure can provide some interesting insight in terms of negotiation behaviour and desirable solution strategies.

6.6.2 Application Areas

Exchange of issues can apply in other than a traditional negotiation context such as buyer-seller bargaining or union-management negotiation. For example, it is reasonable to assume that, in project management, time, cost, and target performance are constantly traded off among the project team members who must meet the immediate and overall objectives within

the available resources. In distributive computing technology, task priority conflict may be resolved based on several criteria. Finally, health care management with doctors, patients, and administrative staff trading off different issues may be another excellent example of possible logrolling application.

In Chapters 5 and 6, the discussion of logrolling is based on the assumption of linear preference. The generalization from linear to non-linear preference is considered next. It is a natural extension of the research when the improvement in the applicability of the logrolling method is considered. The following chapter discusses motivation for studying non-linear preference in the context of multi-issue negotiation and presents the result of investigation.

Chapter 7

Logrolling for Non-Linear Preference

Let us consider the following negotiation situation. Two parties are negotiating over a large quantity of apples and bananas. Originally, Party 1 has more apples and Party 2 has more bananas. Party 1 loves bananas and Party 2 prefers apples over bananas, so an opportunity exists for both parties to gain by trading. Even if both parties prefer bananas, as long as there is a difference in how much they prefer, an opportunity for mutual gain still exists. This is based on the concepts of logrolling and the logrolling range as discussed in Chapters 5 and 6. By recognizing the logrolling opportunity, the two parties negotiate on the exchange rate, the ratio of bananas to apples, and start trading the fruits. After a while, Party 1 has gained a large quantity of bananas. How would Party 1 feel about the bananas at this point? Is it, "the more bananas and less apples the better," forever? According to the logrolling principle based on linear preference, the opportunity for mutual gain continues to exist until one of the parities runs out of fruit to trade.

The linearity is one of the most commonly adopted assumptions regarding a decision maker's preference. The linearity may be representative and acceptable for some value systems such as monetary value, but in reality, it is restrictive and may even falsely represent the underlying value or preference. In the example of bananas and apples, it is intuitive to

hypothesize that, as Party 1 obtains more bananas, its priority becomes less urgent and therefore lowered. The hypothesis implies a non-linear, rather than linear, preference.

This chapter investigates non-linear preference in the context of multi-issue negotiation. The investigation focuses on the *modelling* of non-linear preference within the framework of logrolling. The specific goals for this chapter are the following.

- Maintain the integrative approach by applying logrolling in order to continue utilizing the benefits of the approach.
- Increase the real-world applicability of the negotiation methodology by being able to model preferences in a versatile manner.
- Improve the quality of decision making by avoiding the false representation of some preferences that are not appropriately represented by the linear assumption.

The investigation in this chapter is structured as follows. First, the type of non-linear preference expected in a negotiation situation is discussed in Section 7.1. Section 7.2 then discusses how to model and incorporate non-linear preference in the framework of logrolling. In Section 7.3, a negotiation procedure for non-linear preference is proposed based on logrolling. The properties of the efficient frontier produced by the logrolling procedure are studied in Section 7.4. Section 7.5 discusses the extension on the number of issues, and Section 7.6 concludes this chapter with contributions and future research.

7.1 Non-Linear Preference

Research in multi-issue negotiation is relatively new. Some of the analytical studies in this field are Chaim (1990), Gupta (1989), Jarke, Jelassi, and Shakun (1987), Keeney and Raiffa (1991), Peters (1986), Ponsati and Watson (1994), and Teich et al. (1995). There are also some psychological studies done in the field (e.g., Froman and Cohen, 1970; Roloff and

Jordan, 1991; Thompson, 1990). None of the studies, however, discuss what type of non-linear preference has been observed or may be expected in a negotiation context.

The hypothesis that the priority of an issue decreases as more of it is obtained is analogous to the *law of diminishing marginal utility* in economics. It states that, as a person increases his or her consumption of a good, the additional satisfaction or value derived from each extra unit of that good, that is, marginal utility, will gradually diminish (Archer, 1978). The law was established to understand the nature of the demand for a good. A person would not be willing to pay as much for each successive unit of a good as for the previous ones if the marginal utility declines as consumption increases.

The validity of the law was based on introspection. Economists of an early generation examined their own psychological reactions to extra consumption of a good and verified the effect of diminishing marginal utility. The results of introspection seemed strengthened by the numerous psychological laboratory experiments in 1850s such as Weber-Fechner law of decreasing marginal effect. The law states that the greater the total weight the subject is already carrying, the less will be the effect of an extra unit of weight (Samuelson and Scott, 1966). The universal validity of the law, however, remains controversial (Ormazabal, 1995). Although it may seem intuitive and applicable, there are no empirical studies to confirm the negotiation behaviour based on the law of diminishing marginal utility. Nonetheless, this research assumes non-linear preference based on diminishing marginal utility in order to gain insight into negotiation behaviour.

7.2 How to Model Non-Linearity

In logrolling, the parties' criteria weights are used to determine the logrolling range, that is, the range of exchange rates which yield mutual gain. If the weights change as the preference changes, then the logrolling range also changes and affects the parties' trade-off behaviour. Therefore, the concept of the variable criteria weights is proposed as the basis for modelling

non-linearity. The idea of variable criteria weights fits well with the concepts of both diminishing marginal utility and logrolling.

It is assumed that change in weights occurs discretely, and it is modelled using a step function. A *step function* is a real-valued function defined on an interval $[a,b]$ with a partition $a = x_0 < x_1 < \cdots < x_n = b$ such that for each i the function assumes only one value in the interval: (x_i, x_{i+1}) (Royden, 1963). It seems reasonable to assume that the parties exchange for a while using a fixed exchange rate even if their preferences change slightly on a continuous basis. The parties re-negotiate on an exchange rate when the current exchange rate becomes unsatisfactory for at least one of the parties. With this type of the trade-off procedure, a step function seems to be a suitable model for the change of criteria weights and a sufficient approximation for a continuous case.

In multiple criteria decision making (MCDM) where the concept of criteria weights is rooted, the weights are parameters that are not subject to change once defined or assessed as a part of the problem description. The research topics involved with criteria weights are mostly concerned with elicitation of weights. A question arises whether or not changing weights is a sensible approach to modelling non-linear preference.

Zeleny (1981) states that a weight of issue i is a measure of its relative importance in a given decision situation with two components: (i) a relatively stable, a priori attribute importance which reflects an individual's cultural, genetic, psychological, societal, and environmental background, and (ii) a relatively unstable, context-dependent, and informational importance based on a particular set of feasible alternatives in a given decision situation. The non-linear preference that is studied here reflects the context-dependent and informational importance of the issues since diminishing marginal utility depends on the amount of the issues in possession. Therefore, it is argued that the weight change due to the context change is a reasonable assumption.

7.3 Logrolling Procedure for Non-Linear Preference

A procedure that incorporates the concepts of non-linear preference based on diminishing marginal utility, logrolling, and variable criteria weights is now proposed. Figure 7.1 illustrates the logrolling-based procedure for 2-issue 2-party negotiation. The proposed procedure either identifies the starting point as Pareto optimal or produces a Pareto optimal solution as the end result as in linear preference case. It is assumed that the change in criteria weights occurs discretely, and it is modelled using a step function as discussed in Section 7.2.

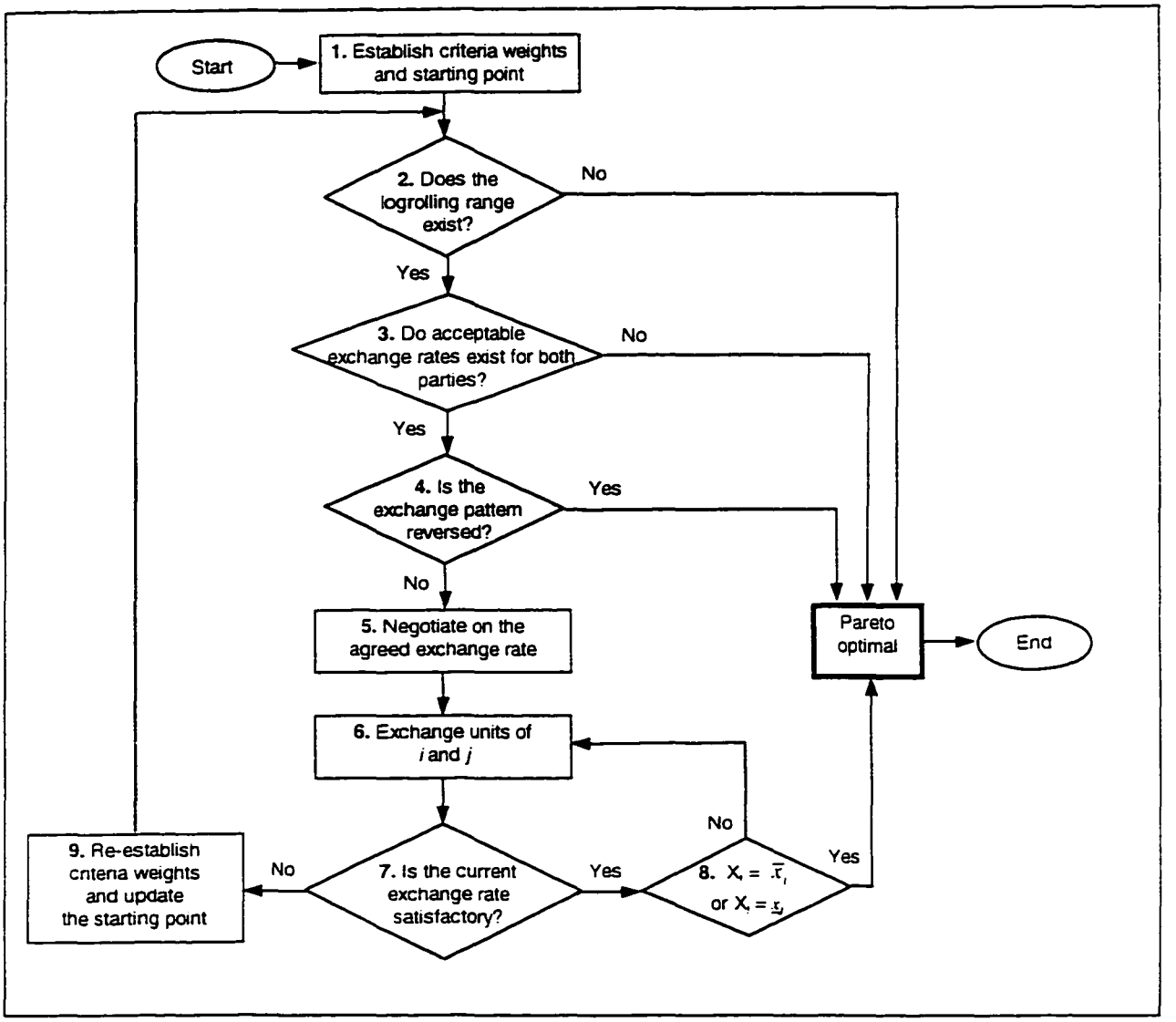
Notes on the new terminology used in Figure 7.1:

Step 4: the reversal of the exchange pattern. Suppose that issue i increases and j decreases in logrolling. The exchange continues until one of the following conditions is met: (i) issue i reaches maximum for Party 1 (step 8), (ii) issue j reaches minimum for Party 1 (step 8), or (iii) the current exchange rate becomes unsatisfactory. From (iii), the exchange may continue based on a new exchange rate as long as the logrolling range exists for the revised set of criteria weights. However, if the new logrolling range indicates that Party 1 should gain in issue j instead of i, then it is said that *the exchange pattern is reversed.* At that point, logrolling does not occur by further increasing issue i. Therefore, the reversal of the exchange pattern is another stopping condition.

Steps 7 and 9. Since the criteria weights change during the course of negotiation, the logrolling range must be re-identified each time the current exchange rate becomes unsatisfactory. The iterative operation shown by steps 7 and 9 deals with the change in criteria weights.

Step 8. $\bar{x}_i$ and $\underline{x}_j$ indicate the maximum and minimum values of issues i and j, respectively.

Figure 7.1 Negotiation Procedure Using Logrolling for 2-issue 2-party Negotiation with Non-Linear Preference



7.4 Efficient Frontier for Two-Issue Logrolling

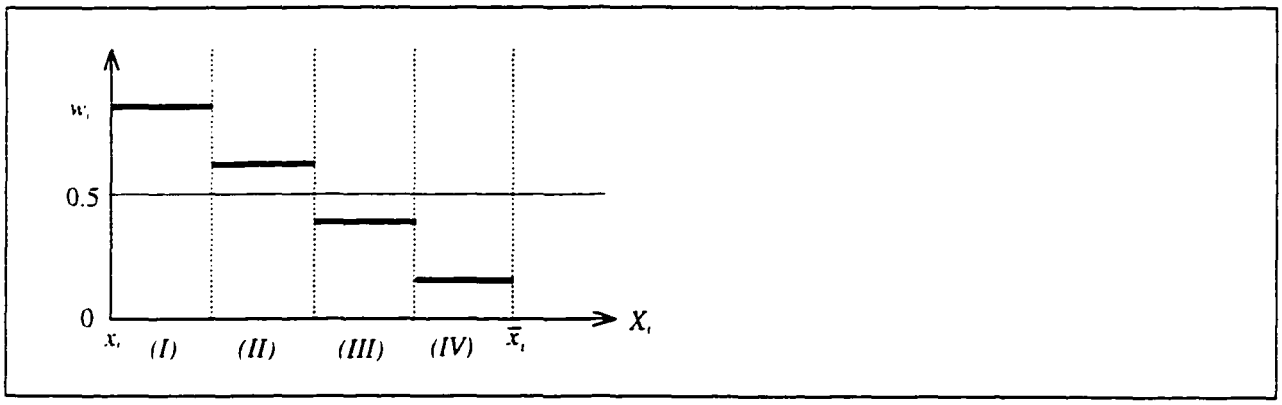
As in linear preference case, the logrolling-based procedure in Figure 7.1 produces a Pareto optimal solution. In this section, the general characteristics of the efficient frontier are examined based on the results from Chapter 5.

7.4.1 Basic Structure

Let i denote the issue which increases and j denote the issue which decreases in an exchange. w_i and w_j are Party 1's weights for issues i and j, and z_i and z_j are Party 2's weights for i and j. For investigation purposes, it is assumed that only Party 1 exhibits the non-linear preference. Party 1's weight for issue i is modelled using a step function. For 2-issue negotiation, the weights have a relationship, $w_i + w_j = 1$. Therefore, w_j can be determined directly from the value of w_i. This implies that the intervals are the same for the two issues. As for Party 2, it is assumed that the weights remain constant for the entire negotiation.

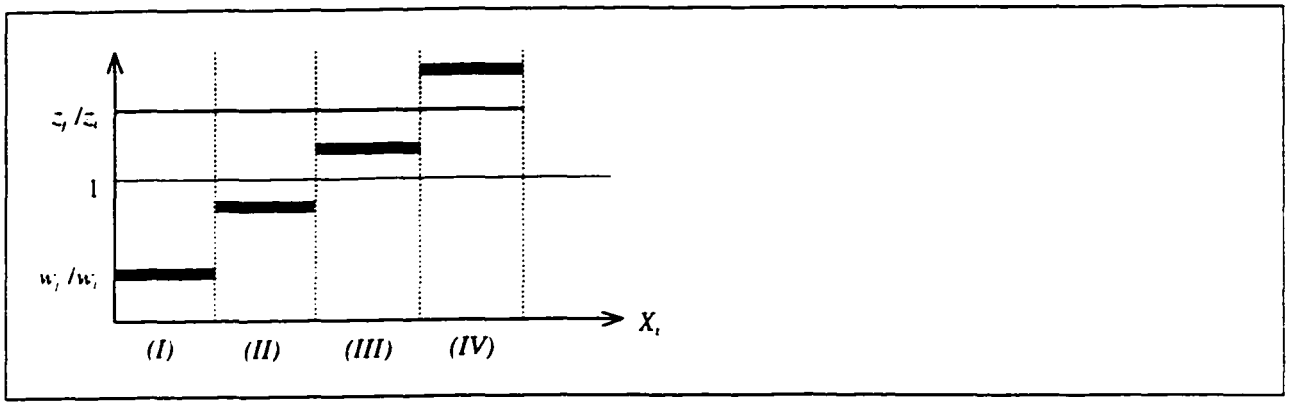
To provide a concrete illustration, the step function shown in Figure 7.2 is used to represent the change in Party 1's weight for issue i as it increases. The rest of this section is developed based on this step function. Due to diminishing marginal utility, the step function is monotone decreasing. In general, the equal interval length is not necessary, and the number of intervals needs not be four.

Figure 7.2 A Sample Step Function Used Throughout in Section 7.4



To see the change in the logrolling range, Figure 7.3 shows the ratios of weights for both parties.

Figure 7.3 Change in the Logrolling Range as Party 1's Weights Change



For intervals *(I)* and *(II)*, $w_i > w_j$ and $z_i < z_j$, therefore, the logrolling range for Party 1 is

$$\frac{w_j}{w_i} < \frac{S_{1i}}{S_{1j}} < \frac{z_j}{z_i}.$$

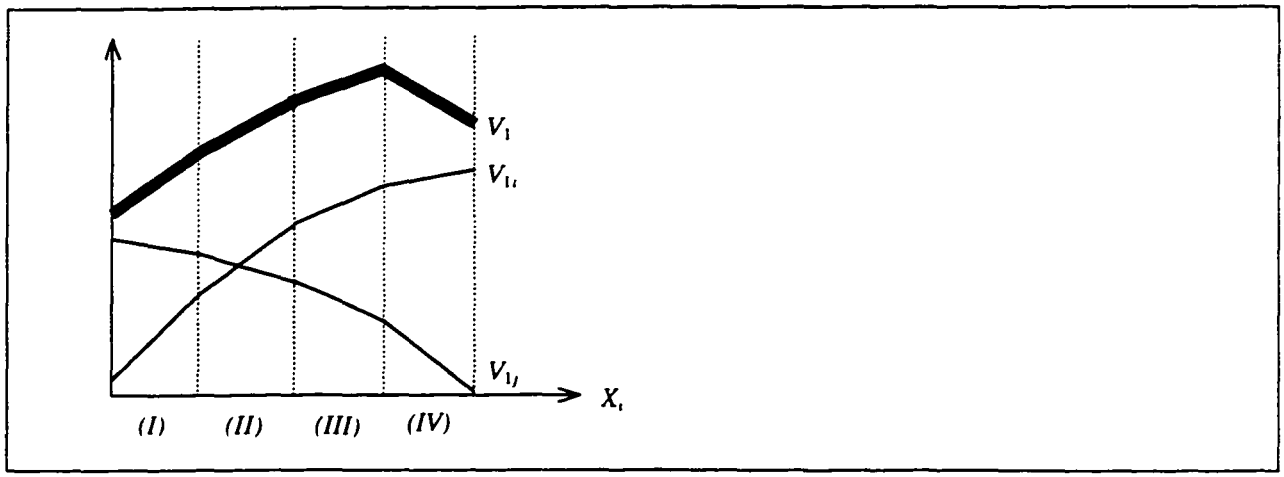
It indicates that Party 1 should gain in issue i and Party 2 should gain in j for mutual gain. In interval *(III)*, although issue j becomes more important to Party 1, that is, $w_i < w_j$, the relationship $(w_j/w_i) < (z_j/z_i)$ indicates that Party 1 should still gain in issue i in order to realize logrolling opportunities. In interval *(IV)*, however, Party 1 should gain in j and Party 2 should gain in i in order to achieve logrolling since $(w_j/w_i) > (z_j/z_i)$. Here, the reversal of the exchange pattern is observed.

7.4.2 Value Functions

Let $V_{1i}(X_i)$ and $V_{1j}(X_j)$ be the value functions for Party 1 with respect to issues i and j. Based on the underlying concept of the logrolling range, it is assumed that, during an interval, $V_{1i}(X_i)$ and $V_{1j}(X_j)$ are linear. It is further assumed that the overall value function for Party 1, $V_1(X_i, X_j)$, is an additive function of $V_{1i}(X_i)$ and $V_{1j}(X_j)$. The linearity and additivity of the value functions for issues imply that the overall value function is also linear during an interval. Figure 7.4 shows Party 1's value functions based on the worst exchange rate in the logrolling range. The horizontal axis represents the level of issue i, X_i. X_j is determined from X_i and the

exchange rate. Here, it is assumed that issue j does not reach its minimum before or during interval *(IV)*.

Figure 7.4 Party 1's Value Functions Corresponding to the Worst Exchange Rate in the Logrolling Range



In *(III)*, the relationship among the weights, $1 < (w_j/w_i) < (z_j/z_i)$, implies that both parties prefer issue j to i but Party 2's preference intensity is higher than Party 1's. Hence, the slope in *(III)* is still positive so that Party 1's value increases as issue i increases in the exchange.

In *(IV)*, the logrolling range indicates that Party 1 should increase issue j instead of i in order to achieve logrolling. Therefore, the slope in *(IV)* is negative to show that Party 1's value decreases if issue i continues to increase in the exchange.

The violation of the assumption that issue j does not reach its minimum before or during interval *(IV)* does not affect the general characteristics of the value functions. Figure 7.5 shows the sample cases where the assumption does not hold. The general shape of Figure 7.4 is present in these sample cases. Note that in Figure 7.5, the exchange cannot occur in interval *(IV)* because issue j is not available for an exchange.

Figure 7.5 Examples of Party 1's Value Functions When Issue *j* Reaches Its Minimum Before Interval *(IV)*

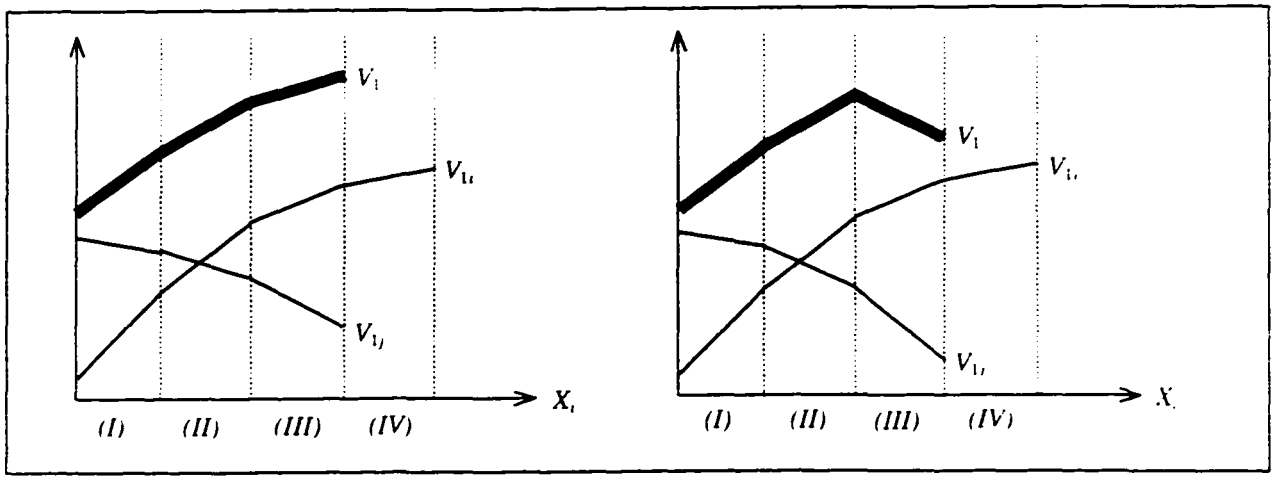
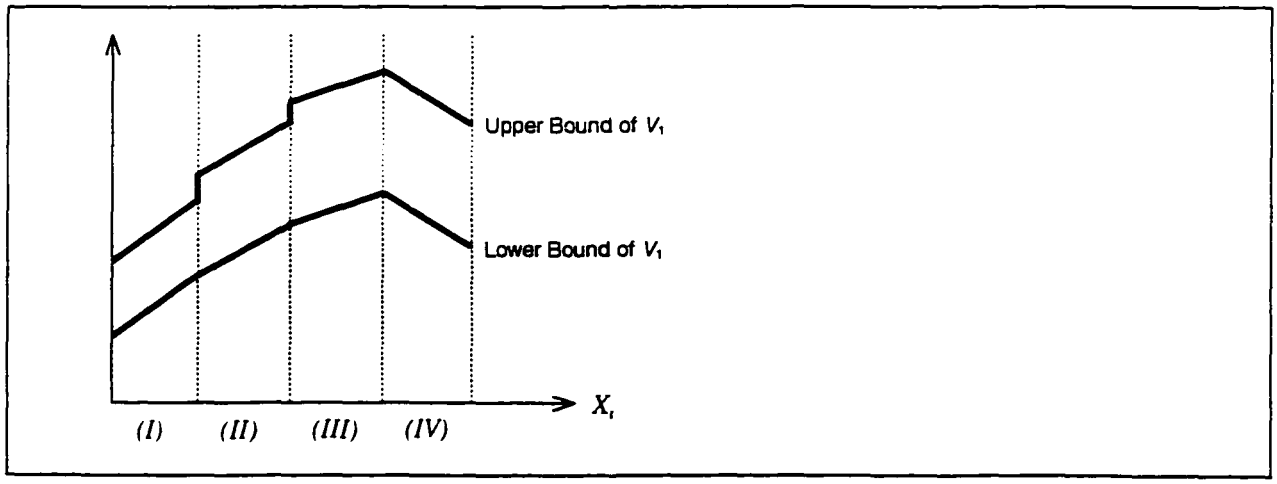


Figure 7.4 shows the overall value function based on the worst exchange rate. In order to include the possible value generated from other exchange rates in the logrolling range, the lower and upper bounds are introduced to the overall value function. Figure 7.4 provides the lower bound. For a fixed value of X_i, the overall value varies depending on the amount of issue j, X_j, after the exchange. The upper bound together with the lower bound provides a way to indicate this range of possible overall value in a two-dimensional graph. The unique overall value after a specific exchange is determined from a starting point and an exchange rate. Figure 7.6 shows both bounds for Party 1's overall value function.

Figure 7.6 Lower and Upper Bounds of Party 1's Overall Value Function



The upper bound is described as follows:

(i) In each interval, the upper bound is parallel to the lower bound. The following argument is for intervals *(I)*, *(II)*, and *(III)*, but a similar one can be applied to interval *(IV)*. Recall that the upper bound indicates the maximum possible overall value associated with a particular value of X_i. For any X_i within an interval, gaining one more unit of i through an exchange implies giving up some units of j. Depending on the exchange rate, there is a difference in the amount of j. The value difference contributed from this difference in j is equivalent to the difference in the possible overall value associated with a particular value of i. Symbolically, for the increase of *one more* unit of i, the amount of decrease in j with respect to the worst exchange rate is determined from the definition of an exchange rate as follows:

$$\frac{(amount\ of\ increase\ in\ i)}{(amount\ of\ decrease\ in\ j)} = (exchange\ rate) \Leftrightarrow \frac{1}{(amount\ of\ decrease\ in\ j)} = \frac{w_j}{w_i}$$

$$\Leftrightarrow (amount\ of\ decrease\ in\ j) = \frac{w_i}{w_j}$$

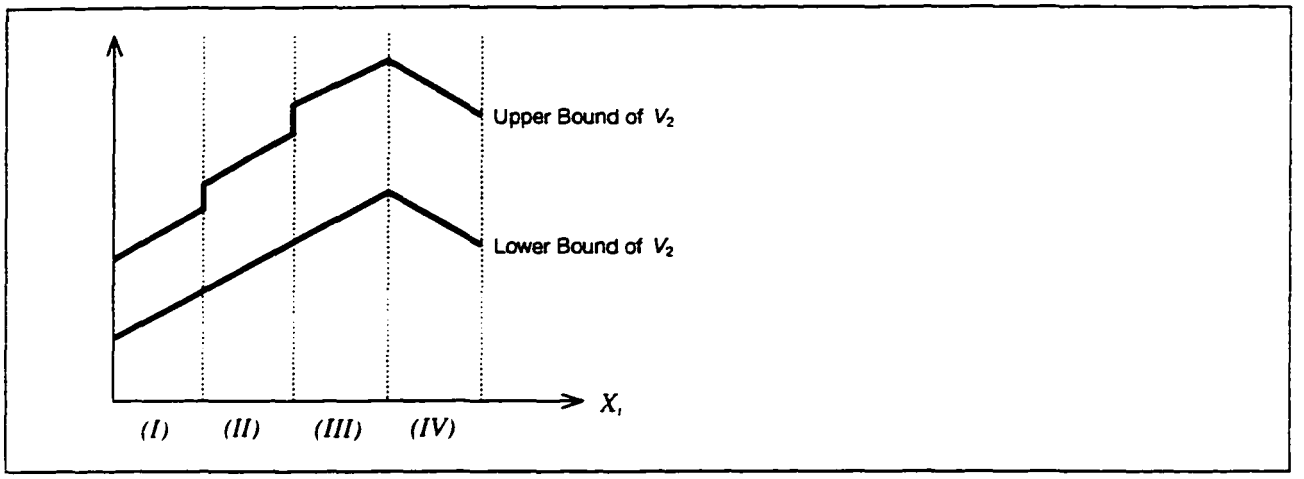
Similarly with respect to the best exchange rate, the amount of decrease in j is z_i/z_j. Then, $V_{1j}(w_i/w_j) - V_{1j}(z_i/z_j)$ is the difference in the possible overall value associated with a particular value of i. Since $V_{1j}(w_i/w_j) - V_{1j}(z_i/z_j)$ is a constant, the upper bound is parallel to the lower bound.

(ii) There is an instantaneous increase in the upper bound between *(I)* and *(II)*. The instantaneous increase is associated with the concept of logrolling. Consider the best possible starting point, (X_i, X_j), before interval *(II)* which corresponds to X_i on the border of *(I)* and *(II)* with X_j that provides the maximum overall value. From this point, logrolling in *(II)* allows an additional increase in overall value which is seen as an instantaneous jump between the upper bounds of intervals *(I)* and *(II)*. A similar argument applied to the instantaneous increase between *(II)* and *(III)*. There is no jump between *(III)* and *(IV)* because the exchange pattern is reversed and logrolling does not occur for Party 1 by increasing issue i.

Based on Figure 7.6, the upper and lower bounds of Party 2's overall value function are shown as follows (Figure 7.7). Note that since Party 2's weights are fixed, the intervals are

actually determined from Party 1's preference change, and the lower bound is linear from interval *(I)* to *(III)*. The slope in *(IV)* is also negative for Party 2 due to the reversal of the exchange pattern.

Figure 7.7 Lower and Upper Bounds of Party 2's Overall Value Function



7.4.3 Efficient Frontier

Based on Figures 7.6 and 7.7, the characteristics of the efficient frontier are investigated. Since the step function in Figure 7.2 implies that issue *i* does not reach its maximum until the end of interval *(IV)*, within each interval, logrolling continues until either Party 1's criteria weights change or there is no issue *j* left for the exchange. The efficient frontier is then examined for the following three cases:

1) Issue *j* does not reach its minimum during interval *(III)*;
2) Issue *j* reaches its minimum during interval *(III)*; and
3) Issue *j* reaches its minimum before interval *(III)*.

The result from each case is presented below.

Case 1

If issue j does not reach its minimum during interval *(III)*, then logrolling continues until X_i reaches the end of interval *(III)*. At this point, the logrolling range for interval *(IV)* indicates the reversal of the exchange pattern. The exchange stops here because both parties would lose value if they continue to exchange in the same fashion, that is, Party 1 gains in issue i and Party 2 gains in issue j. The final outcomes are then $(X_i^{(III)}, X_j)$ where $X_i^{(III)}$ is the level of issue i at the end of interval *(III)* and X_j depends on the starting point at the beginning of interval *(III)*, $(X_i^{(II)}, x_j)$, and the exchange rate used in interval *(III)*. More specifically, the range of X_j value can be identified as follows. The amount of j after the exchange is computed by subtracting the amount of decrease from the starting amount before the exchange. The amount of decrease can be computed from the definition of an exchange rate, that is, the ratio of the increase in i to the decrease in j. Then, the amount of j yielded using the worst exchange rate is:

$$X_j = x_j - (\textit{amount of decrease in } j) = x_j - \frac{(\textit{amount of increase in } i)}{(\textit{exchange rate})} = x_j - \frac{X_i^{(III)} - X_i^{(II)}}{w_j^{(III)} / w_i^{(III)}}.$$

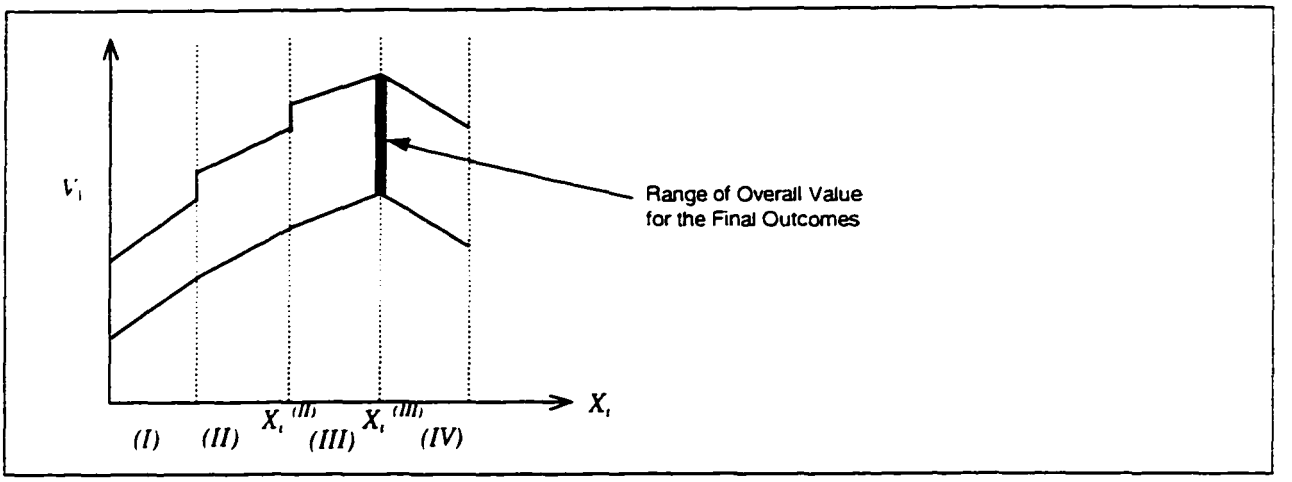
$w_i^{(III)}$ and $w_j^{(III)}$ are Party 1's weights for interval *(III)*. The amount of j yielded using the best exchange rate can be computed in a similar manner. These two quantities then identifies the range of X_j value as follows:

$$\left(x_j - \frac{X_i^{(III)} - X_i^{(II)}}{w_j^{(III)} / w_i^{(III)}} \right) < X_j < \left(x_j - \frac{X_i^{(III)} - X_i^{(II)}}{z_j^{(III)} / z_i^{(III)}} \right).$$

Bear in mind that the value of x_j varies depending on the logrolling result from interval *(II)*.

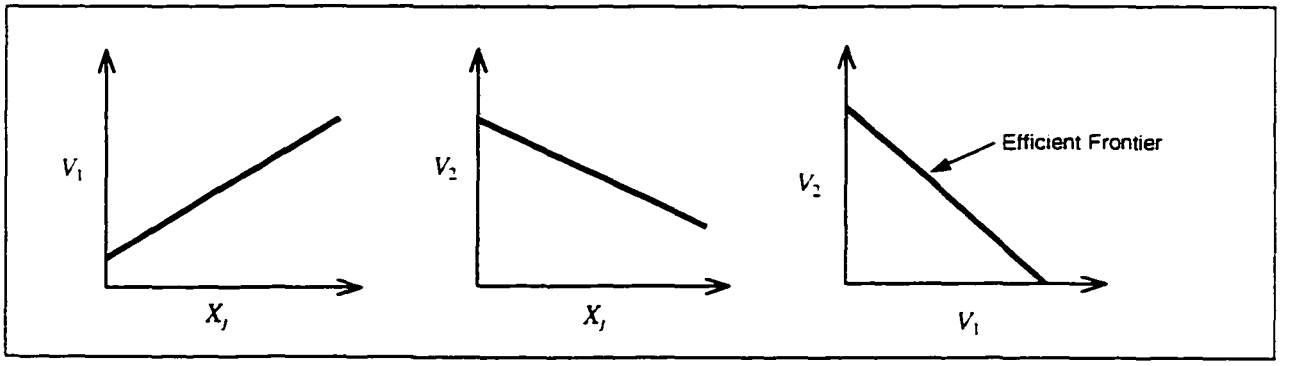
The overall value of the final outcomes is then expressed as $V_1(X_i, X_j) = V_{1i}(X_i^{(III)}) + V_{1j}(X_j)$ where X_j is shown as above. The different values of the final outcomes coincide with the border of intervals *(III)* and *(IV)* (see Figure 7.8).

Figure 7.8 Final Outcomes for Case 1



In this case, the two parties' value functions of the possible final outcomes are simply linear representing the competition between the parties over issue *j*. This implies that the efficient frontier is linear. Figure 7.9 shows the value functions of the final outcomes and the resulting efficient frontier.

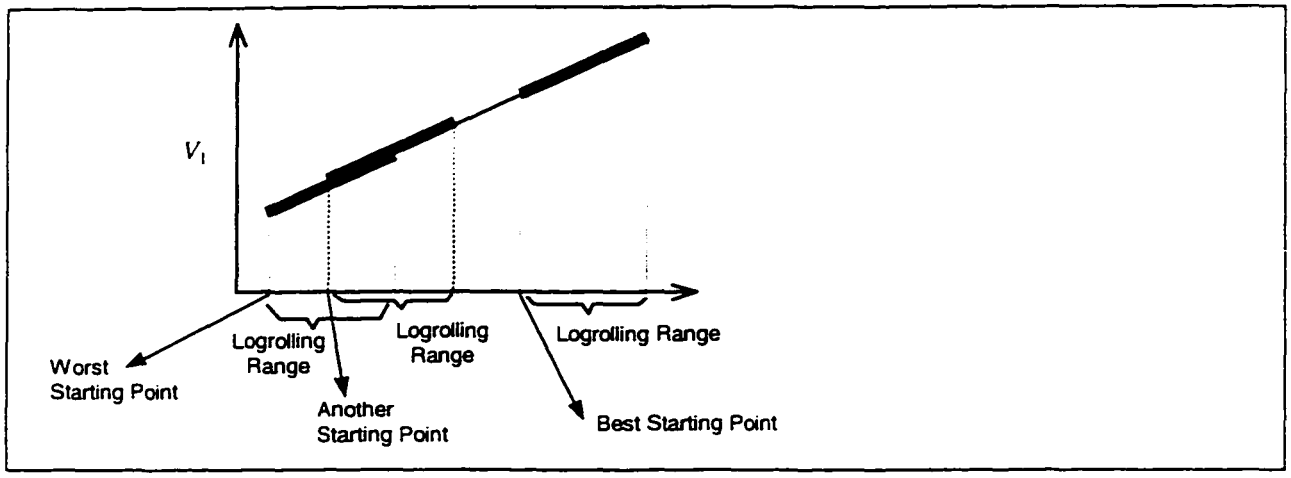
Figure 7.9 Value Functions With Respect To the Final Outcomes and the Efficient Frontier (Case 1)



The close-up of the value function in Figure 7.9 is given in Figure 7.10 to clarify some details. For a fixed starting point, a range of final outcomes is possible due to the different exchange rates in the logrolling range. Then the overall value of these final outcomes is represented as a segment of the value function shown in Figure 7.9. There are numerous starting points at the beginning of interval *(III)*. Hence, the overall values of all of the

negotiation outcomes are represented as a set of different segments overlapping to form a linear value function. The final outcomes are efficient solutions because the value increase for one party from any outcome would mean the value decrease for the other party.

Figure 7.10 Details of Party 1's Value Function Shown in Figure 7.9



Case 2

When issue j reaches its minimum during interval *(III)*, logrolling stops at that point because there is nothing left to trade for the further increase of issue i. Consider a starting point at the beginning of *(III)* with which issue j reaches its minimum during the interval for any exchange rate in the logrolling range. Suppose that the starting point is $(X_i^{(II)}, x_j)$. Bear in mind that the starting point varies depending on the logrolling result from interval *(II)*. The final outcomes from this particular starting point are $(X_i, \underline{x}_j)$ where X_i has a value in the following range which is computed from the definition of an exchange rate as shown in Case 1:

$$\left(X_i^{(II)} + x_j \cdot \frac{w_j}{w_i} \right) < X_i < \left(X_i^{(II)} + x_j \cdot \frac{z_j}{z_i} \right).$$

$w_i^{(III)}$ and $w_j^{(III)}$ are Party 1's weights for interval *(III)* and $z_i^{(III)}$ and $z_j^{(III)}$ are Party 2's weights for interval *(III)*. The overall value of the final outcomes is $V_1(X_i, X_j) = V_{1i}(X_i) + V_{1j}(\underline{x}_j)$ where

X_i is shown as above. Note that the increase in V_1 is according to $V_{1i}(X_i)$ since $V_{1j}(\underline{x}_j)$ is a constant.

With some starting points, issue j may reach its minimum for some of the exchange rates in the logrolling range. This case is a combination of Cases 1 and 2. Figure 7.11 shows a general view of Case 1, 2, and the combination case all together.

Figure 7.11 Final Outcomes for Case 2

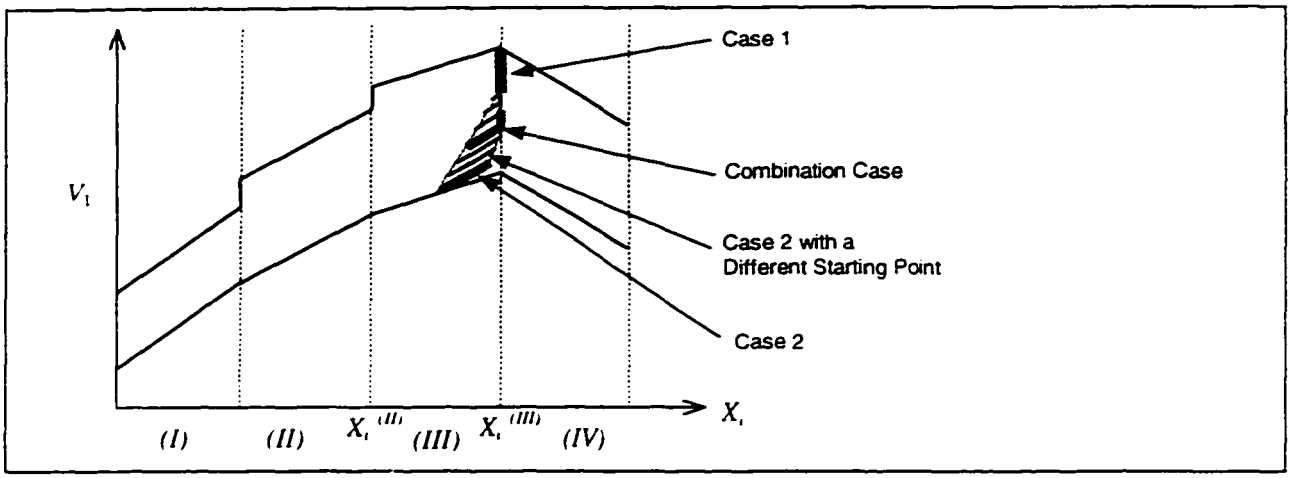
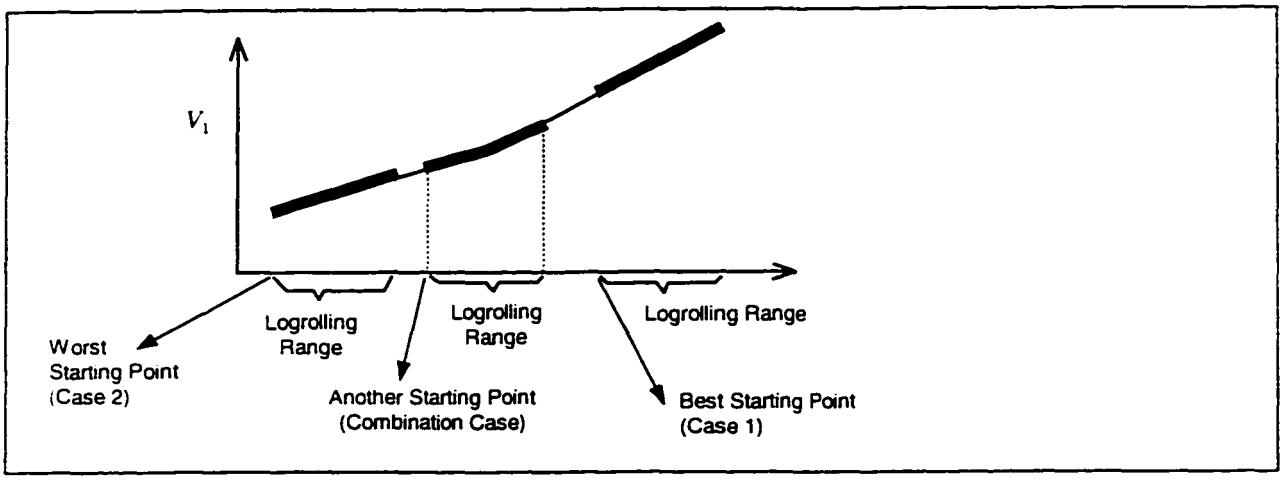


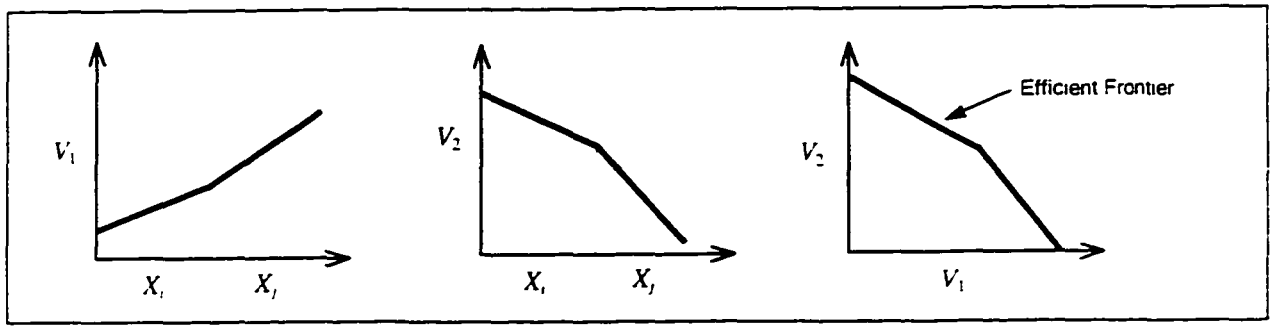
Figure 7.11 indicates that each party's value function of the possible final outcomes has two components: the part where the value increase is solely contributed from issue i (case 2) and the other part where the value increase is contributed from issue j (case 1). Figure 7.12 helps visualize the overall value of all of the negotiation outcomes consisting of a set of different segments overlapping to form a piecewise linear value function. Note that the value function in Figure 7.12 is convex rather than concave because issue j's weight is greater than issue i's for Party 1 in interval *(III)*.

Figure 7.12 Details of Party 1's Value Function With Respect To the Final Outcomes (Case 2)



Based on Figure 7.12, Figure 7.13 shows the parties' value functions which are both piecewise linear. The efficient frontier implied by these parties' value functions is also shown in Figure 7.13 to be piecewise linear, and moreover, concave. The rationale behind the form of the efficient frontier is first developed for linear preference in Chapter 5, but it is directly applicable here. The basic argument is reviewed as follows. The efficient frontier consists of two parts. In one part, a range of V_1 contributed from issue i corresponds to a range of V_2 also contributed from issue i. In the second part, a range of V_1 contributed from issue j corresponds to a range of V_2 contributed from issue j. Each part of the efficient frontier is linear representing the competition between the parties over one issue at a time. Therefore, the efficient frontier is piecewise linear. The concavity of the efficient frontier becomes apparent when the slopes of the two linear segments are compared. It can be shown that the slope of the segment corresponding to the value contribution from issue j is more negative than the slope of the segment corresponding to the value contribution from issue i, hence, implying that the efficient frontier is concave.

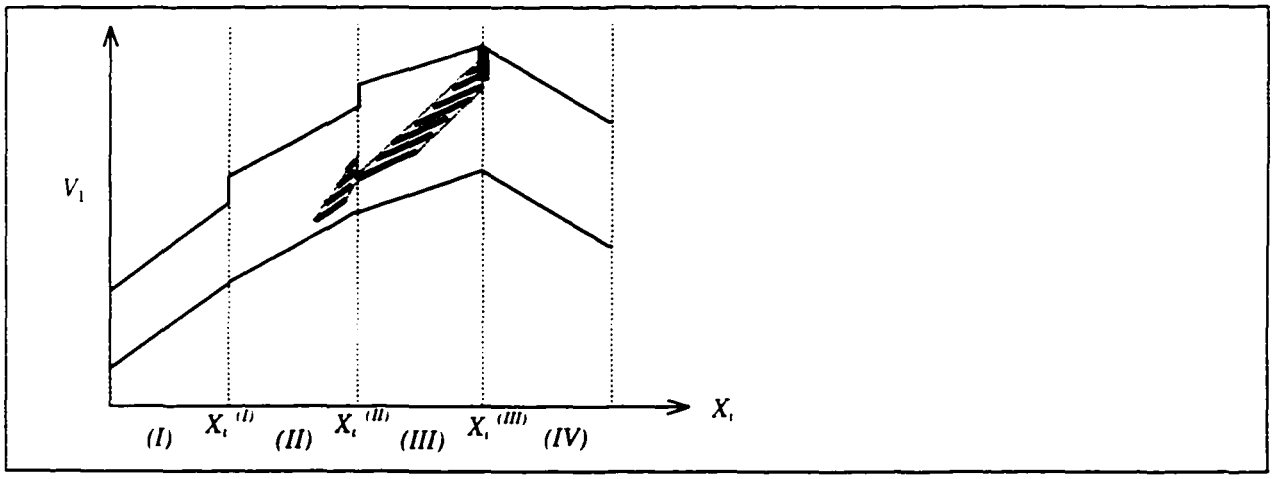
Figure 7.13 Value Functions With Respect To the Final Outcomes and the Efficient Frontier (Case 2)



Case 3

If issue j reaches its minimum before interval *(III)*, then at the end of interval *(II)*, the outcomes resemble Figure 7.11. Logrolling continues in the next interval for the outcomes where issue j is still available for an exchange. The basic concept of Figure 7.11 is extended in Figure 7.14.

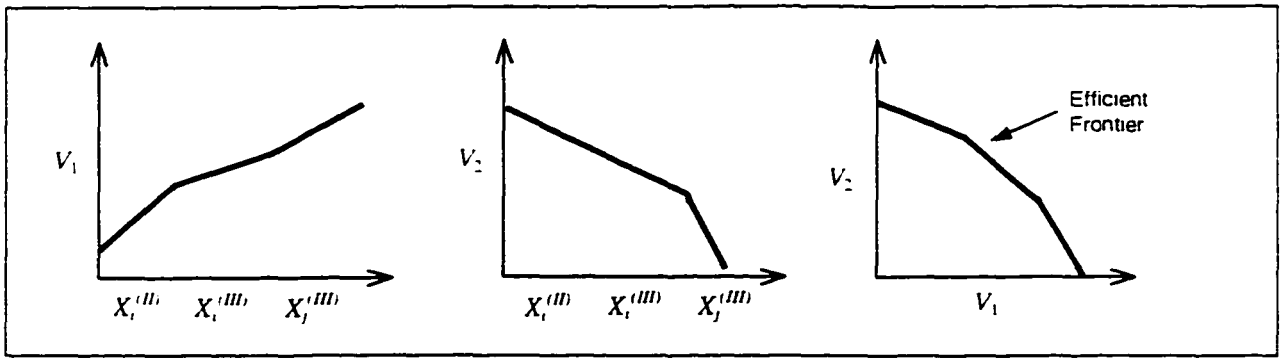
Figure 7.14 Final Outcomes for Case 3



Based on cases 1 and 2, it can be deduced that Party 1's value function of the possible final outcomes now has three components: two parts where the value increase is contributed from issue i with two different importance weights and the third part where the value increase is contributed from issue j. On the other hand, Party 2's value function has two components:

the part where the value increase is contributed from i and the other part where the value increase is contributed from j. Figure 7.15 shows the corresponding value functions and the efficient frontier. The form of the efficient frontier, piecewise linear and concave, can be justified based on the same argument used in Case 2.

Figure 7.15 Value Functions With Respect To the Final Outcomes and the Efficient Frontier (Case 3)



If the preference changes frequently, the final outcomes may go across several intervals. The number of segments can be as many as one more than the number of intervals with 2-issue logrolling.

In summary, three cases are examined regarding the characteristics of the efficient frontier for non-linear preference based on diminishing marginal utility. Assuming that the parties' value functions are linear within each interval, 2-issue logrolling produces an efficient frontier that is linear or piecewise linear. The assumption that Party 2's weights remain constant does not affect the general characteristics of the efficient frontier. Figures 7.16 and 7.17 demonstrate that the general shape of the upper and lower bounds of the overall value function as seen in Figure 7.6 is retained even if both parties have non-linear preference. Therefore, similar conclusions are expected regarding the efficient frontier. Finally, although these results are for 2-issue 2-party negotiation, the derivation process presented in this section provides a useful basis for developing the efficient frontier for more general cases that may include more non-linearity in the model.

Figure 7.16 Both Parties Have the Same Intervals (Example)

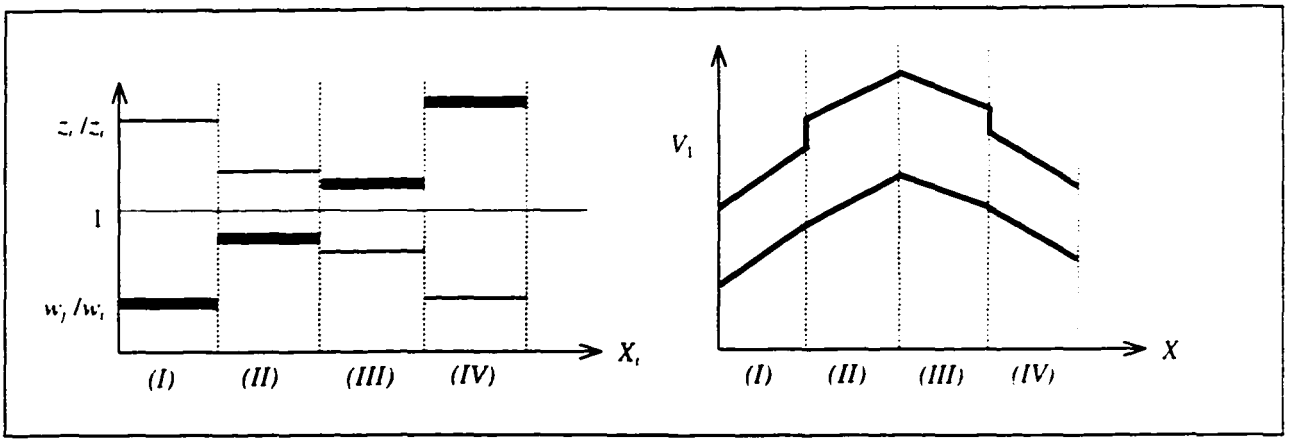
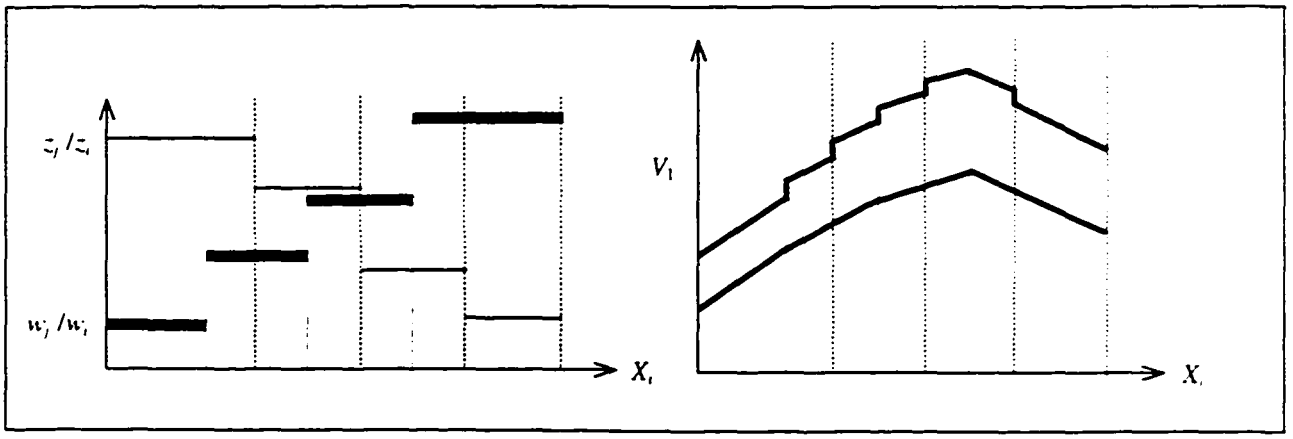


Figure 7.17 Two Parties Have Different Intervals (Example)



7.5 Extension to More Than Two Issues

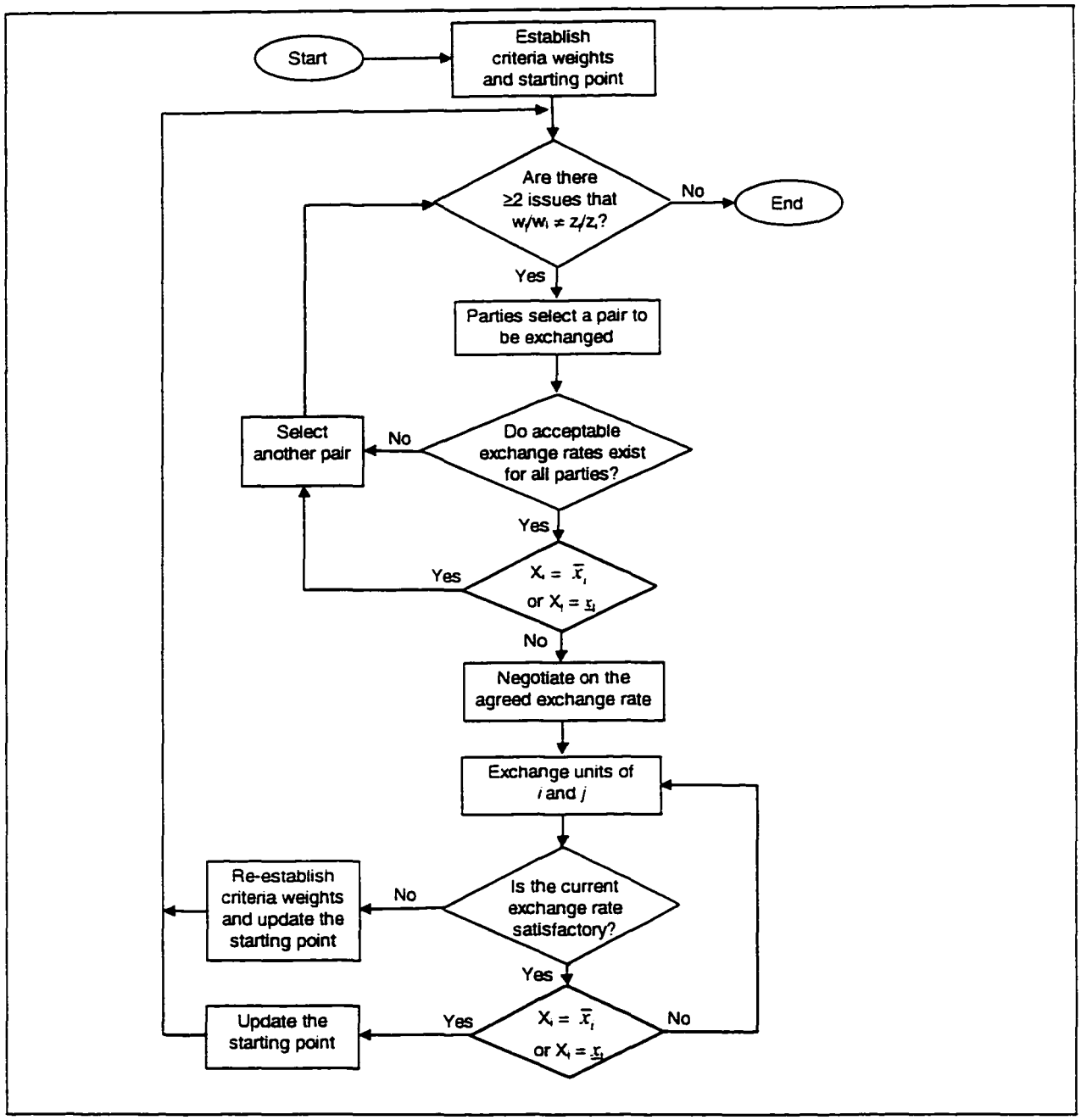
In this section, the discussion is extended to more than 2 issues. The 2-issue sequential logrolling procedure developed for linear case can be modified to accommodate non-linear preference. This proposed procedure is shown first. Then the discussion continues with the complications encountered in generalization of the efficient frontier.

7.5.1 Sequential Logrolling Procedure

For m-issue negotiation with linear preference ($m \geq 3$), the sequential logrolling procedure based on the 2-issue exchange was developed and applied. In this section, the 2-issue sequential procedure is modified for the non-linear case. Suppose that issue i increases and j decreases for Party 1 in an exchange session. For non-linear preference based on diminishing marginal utility, one exchange session with a fixed exchange rate continues until one of the following conditions is met: (i) issue i reaches maximum, (ii) issue j reaches minimum, or (iii) the current exchange rate becomes unsatisfactory for at least one of the parties. If the exchange ends with condition (iii), instead of considering a different pair, it is assumed for the sake of simplifying the modified procedure that the same pair of issues is exchanged with a re-negotiated exchange rate.

The discussion for 2-issue case in Sections 7.3 and 7.4 was based on the discrete change in weights for investigation purposes. In this case, the reversal of the exchange pattern indicated the end of the exchange session between two issues. However, technically, any small difference in the two parties' weights creates the logrolling range and allows mutual gain exchange of issues. For m-issue case, it is assumed that the change in weights is continuous. This assumption implies that the exchange between a particular pair of issues ends if the increasing issue reaches its maximum, the decreasing issue reaches its minimum, or the two parties' exchange rates become identical. The identical exchange rates imply that logrolling range does not exist for the pair of issues, that is, no more logrolling opportunities exist. The entire procedure should end when no logrolling opportunities exist for *all* possible pairs of issues. The following procedure (Figure 7.18) is proposed to include the necessary modification.

Figure 7.18 Modified Procedure Using 2-issue Sequential Logrolling with Non-Linear Preference



Since the change of weights based on diminishing marginal utility may allow the same pair of issues to be exchanged several times during the course of negotiation, the total number of pairs to be considered for logrolling is not fixed and not known. It is then not obvious that

the proposed procedure will end eventually. In order to guarantee that the procedure will stop, it is shown that cycling does not occur.

Consider dealing with three issues, i, j, and k. An example of cycling is the repetition of the following three 2-issue logrolling sessions: first, j decreases and i increases; next, i decreases and k increases; then, k decreases and j increases. It is cycling only if each issue increases and decreases the same amount. By definition, logrolling provides overall gain. Therefore, cycling is conceptually impossible from the logrolling procedure since it cannot provide continuous increase in value from the limited resources. In fact, examination of the stopping condition for a 2-issue exchange session confirms that the logrolling procedure would not produce cycling. Let $w_i^{(t)}$, $w_j^{(t)}$, and $w_k^{(t)}$ be Party 1's weights before t-th logrolling session. Assume that Party 2's weights remain constant at z_i, z_j, and z_k. For the first exchange between i and j, the logrolling range for Party 1 must be that:

$$\frac{w_j^{(1)}}{w_i^{(1)}} < \frac{S_{1i}}{S_{1j}} < \frac{z_j}{z_i} \quad . \tag{1}$$

S_{1i}/S_{1j} is the ratio of the increase in i to the decrease in j for Party 1. Similarly, for the second exchange between i and k, the logrolling range must be that:

$$\frac{w_i^{(2)}}{w_k^{(2)}} < \frac{S_{1k}}{S_{1i}} < \frac{z_i}{z_k} \quad . \tag{2}$$

From (1) and (2), it is deduced that:

$$\left(\frac{S_{1i}}{S_{1j}}\right)\cdot\left(\frac{S_{1k}}{S_{1i}}\right) < \left(\frac{z_j}{z_i}\right)\cdot\left(\frac{z_i}{z_k}\right) \quad \Leftrightarrow \quad \frac{S_{1k}}{S_{1j}} < \frac{z_j}{z_k} \quad \Leftrightarrow \quad \frac{S_{1j}}{S_{1k}} > \frac{z_k}{z_j} \quad . \tag{3}$$

For cycling to occur, the logrolling range for the third exchange between j and k must be that:

$$\frac{w_k^{(3)}}{w_j^{(3)}} < \frac{S_{1j}}{S_{1k}} < \frac{z_k}{z_j} \quad . \tag{4}$$

However, (3) and (4) are inconsistent. Therefore, cycling cannot occur. In terms of the logrolling procedure, it may continue with j and k with the opposite exchanging pattern, continue with a different pair of issues, or end because the logrolling range does not exist for any pair of issues. This concludes that, even if the same pair of issues is exchanged several times during negotiation due to the changes in criteria weights, the procedure proposed in Figure 7.18 will stop since cycling does not occur.

7.5.2 Changes in Criteria Weights

In order to study the general properties of the efficient frontier based on the sequential procedure, one must first understand how the criteria weights change during the negotiation process. Between two issues, one increases and the other decreases in an exchange. In this case, as the weight of the increasing issue changes according to diminishing marginal utility, the other issue's weight is changed correspondingly based on the assumption that the two weights sum to one. Consider three issues with a pair of issues in an exchange at a time. The increasing issue's weight changes according to diminishing marginal utility as in the 2-issue case, but a question is raised as to what sort of changes are expected for the other two issues.

Several scenarios can be considered. Let issues i and j be the ones in the exchange. Issue i increases against j. The third issue k is not involved in the exchange. Since i increases, its weight, w_i, decreases based on diminishing marginal utility. By considering all weights sum to one and j and k's weights either increase, decrease, or do not change, five possible weight change combinations are identified (Table 7.1). In all cases, the rate of change in weights is subjective that depends on a particular negotiator's preference in a particular negotiation context.

Table 7.1 Possible Weight Change Combinations for 3 issues

	w_i	w_j	w_k
(1)	↓	↑	no change
(2)	↓	↑	↑
(3)	↓	↑	↓
(4)	↓	no change	↑
(5)	↓	↓	↑

(1) represents the case in which the third issue is considered to be independent of the exchange of the other two issues. (2) and (3) represent a dynamic and global relationship among the three issues with the assumption that the decreasing issue's weight increases as the result of the exchange. (4) and (5) are the other possible combinations. (1), (2), and (3) may be interpreted as exhibiting the opposite of diminishing marginal utility for the decreasing issue, j. Between issues j and k, the relationship shown in (2) may correspond to the idea similar to complementary commodities. When j and k are complementary in nature, it may be reasonable to assume that both issues increase in importance against the decrease of importance in issue i. On the other hand, when j and k are substitutable, the relationship shown in (3) or (5) may be expected.

Further interpretation in terms of the negotiation behaviour is unknown at this time. Empirical studies may clarify the feasibility and suitability of each weight change pattern in a negotiation context.

7.6 Example Revisited

The employment agreement negotiation between a newly recruited employee (Party 1) and the personnel department of a firm (Party 2) from Chapter 6 is revisited in this section for the following purposes. First, it is used to illustrate the 2-issue sequential negotiation procedure with a focus on the change in criteria weights. Secondly, the example is used to highlight the

difficulties involved in generalization of the efficient frontier from 2-issue to *m*-issue negotiation. The key input data are presented in Table 7.2.

Table 7.2 Input Data for the Employment Agreement Example

Parameters	Starting Salary	Vacation in the First Year	Medical Coverage
Range of agreement	Max = $24,000 Min = $20,000	Max = 21 days Min = 7 days	Max = 100% Min = 20%
Starting Point	$22,000	14 days	60%
Party 1's Criteria Weights (at start)	0.66	0.24	0.1
Party 2's Criteria Weights (fixed)	0.1	0.24	0.66
Agreed Exchange Rate	Midpoint of the corresponding logrolling range for each pair of issues		

It is assumed that Party 1 exhibits non-linear preference. Party 2's preference is assumed to be linear through the negotiation process. In order to accommodate non-linear preference, variation in Party 1's criteria weights is modelled by weight change pattern (1) from Table 7.1. The change is introduced in the following manner. For all three issues, starting salary, vacation, and medical coverage, the weight change due to diminishing marginal utility is assumed to occur after achieving 85%. For example, the possible starting salary ranges from $20,000 to $24,000. The 85% level for this issue is then equivalent to achieving an agreement at $23,400. After reaching 85%, the change in the three weights is assumed to occur based on weight change patterns (1).

More precisely, let $w_i^{(t)}$, $w_j^{(t)}$, and $w_k^{(t)}$ be Party 1's most recent weights for issues, i, j, and k. $t \geq 0$ and integer. The changes due to weight change pattern 1 after exchanging i and j are expressed as follows. r is the reduction rate.

$$w_i^{(t+1)} = w_i^{(t)} - r \cdot w_i^{(t)}$$
$$w_j^{(t+1)} = w_j^{(t)} + r \cdot w_i^{(t)}$$
$$w_k^{(t+1)} = w_k^{(t)}$$

For this example, 50% reduction rate is arbitrarily chosen and used for computation with this example.

The negotiation outcomes are computed step by step using the 2-issue sequential logrolling procedure for two cases: Party 1 exhibits (i) linear preference and (ii) non-linear preference with weight change pattern 1. An EXCEL worksheet is designed for actual computation based on the worksheet used in Chapter 6. Appendix B.1 contains a sample EXCEL worksheet. A change in weight is detected by setting the maximum value of the increasing issue to be at the 85% level. If the current logrolling session ends with the increasing issue reaching 85%, then Party 1's weights are modified before proceeding with the next exchanging session. Since this is a small example, Party 1's weights are manually updated based on the weight change assumptions made above.

Table 7.3 presents the course of negotiation observed under linear preference and non-linear preference with weight change pattern (1) for the employment agreement example.

Table 7.3 Sample Negotiation Sequence with Linear Preference

	Pair in Exchange		Salary ($)	Vacation (days)	Medical (%)
Start			22,000	14	60
Round 1	1&2	*Weight*	*0.66*	*0.24*	*0.1*
		Outcome	24,000	8.934212	60
Round 2	2&3	*Weight*	*0.66*	*0.24*	*0.1*
		Outcome	24,000	20.01755	20

Table 7.4 Negotiation Sequence with Non-Linear Preference: Weight Change Pattern (1)

	Pair in Exchange		Salary ($)	Vacation (days)	Medical (%)
Start			22,000	14	60
Round 1	1&2	*Weight*	*0.66*	*0.24*	*0.1*
		Outcome	23,400	10.45395	60
Round 2	2&3	*Weight*	*0.33*	*0.57*	*0.1*
		Outcome	23,400	18.9	27.00456
Round 3	2&3	*Weight*	*0.33*	*0.285*	*0.385*
		Outcome	23,400	21	21.15216
Round 4	1&3	*Weight*	*0.33*	*0.285*	*0.385*
		Outcome	23,624	21	20
Round 5	1&2	*Weight*	*0.33*	*0.285*	*0.385*
		Outcome	24,000	20.19291	20

Table 7.4 indicates that the non-linear case can be more involved in the negotiation process than the linear case even for a small example. In Table 7.3, once the pair of issues in Round 1 is selected, the logrolling procedure determines the pair in Round 2. However, for non-linear case, the negotiating parties would have had to select a pair of issues to exchange in Rounds 1, 2, 3, and 4 due to the weight change. In this example, the pairs are randomly chosen. It is also observed from Tables 7.3 and 7.4 that issues 1 and 2, and issues 2 and 3, are exchanged only once with the linear case, but are exchanged more than once in the non-linear case.

The main concern associated with the generalization of the efficient frontier from 2-issue to *m*-issue case is that even a 3-issue case is difficult to observe thoroughly. The following factors contribute to the complication. First, as shown in Section 7.4, the identification of an outcome from exchanging a pair of issues is already complicated by a series of exchanging sessions based on multiple exchange rates. For linear case, the number of line segments in a piecewise efficient frontier is at most two for 2-issue negotiation. For non-linear case, it can be much greater than two depending on how frequently the criteria weights change.

Secondly, as discussed earlier in this section, the feasible and suitable weight change patterns are not known at this point. Thirdly, as illustrated in the employment agreement

example (Table 7.4), non-linear preference allows more flexibility in selecting a pair of issues to be exchanged. The selection process is highly subjective, and it would be challenging to generalize the properties of all possible outcomes. Finally, the change of weights may allow the same pair of issues to be exchanged a number of times as also seen in Table 7.4. This implies that the total number of possible pairs for logrolling is an unknown variable. All these complications make enumeration of all possible outcomes a difficult task.

Some exploratory work may help obtain insight into the general properties of the efficient frontier for logrolling with non-linear preference. Simulation with specific weight change pattern, random ordering of pairs of issues, and random exchange rates may be a good starting point.

7.7 Summary

Even for a small example of trading apples and bananas, the assumption of linear preference can be unrealistic. In this chapter, non-linear preference in multi-issue negotiation is considered. From a simple yet reasonable observation, non-linear preference based on diminishing marginal utility is assumed. It is modelled by the variable criteria weights and incorporated into the logrolling procedure. Logrolling, a specific type of trade-off process that provides mutual gain, is first introduced for linear preference in Chapter 5. The efficient frontier produced by the logrolling procedure is also studied. With the discrete change in criteria weights and the linear assumption within each interval, it is shown that 2-issue logrolling yields the linear or piecewise linear efficient frontier. In terms of the extension to m-issue negotiation, the possibility of different weight change patterns and the complications involved with the generalization of the efficient frontier are discussed.

Benefits

The following are recognized as the benefits of the study presented in this chapter.

- The logrolling-based negotiation procedure is modified to accommodate non-linear preference. It adopts diminishing marginal utility and the variable criteria weights well, and therefore, it is effective in accommodating this type of non-linear preference.
- As in linear preference case, the modified logrolling procedure guarantees a Pareto optimal solution.
- For modelling non-linear preference, a new approach, namely, the combination of variable criteria weights and diminishing marginal utility, is developed.
- The logrolling solution method succeeds in relating a behavioral aspect of negotiation to a conceptual model.
- The logrolling method can model various and more realistic situations by accounting for non-linear preference. This contributes to the increase in applicability of the logrolling methodology.
- Also, the improvement in the quality of decision making is expected by avoiding the misrepresentation of non-linear preference.
- In terms of an analytical technique, the concept of an upper bound and a lower bound of a value function (Section 7.4.2) provides a way to indicate the range of possible overall value corresponding to outcomes with different starting points and different exchange rates all in a two-dimensional graph.
- Although the result from Section 7.4.3 regarding the characteristics of the efficient frontier is restrictive to 2-issue 2-party case, the derivation process should be useful as a basis for developing more general cases.

This concludes the presentation of the results of the research. In the final chapter, the contents and key contributions are summarized. The chapter then ends with various suggestions for future research raised throughout the body of the thesis.

Chapter 8

Conclusion

8.1 Summary of Contents

This research deals with multiple criteria-multiple participant decision making (MCMP), a domain that is associated with both MCDM and MPDM. Its history provides an insight into the origin of MCMP, and it also indicates a need for a foundation on which MCMP research can evolve in the future. A classification scheme for the solution approaches used in MCMP is proposed for the purpose of building a foundation. The reduction, decomposition, and integrative approaches are identified and compared. The difficulty of finding a Pareto optimal solution is discussed in order to improve MCMP. Then the suitability of the integrative approach for dealing with this type of difficulty is shown. Based on the integrative approach, two MCMP solution methods are developed.

The first solution method is referred to as the candidate enumeration method. It provides an alternative way to model multicriteria games, which presents all essential components in the matrix of candidates. Preference trees and candidates are the basic concepts for developing the three solution analysis steps. The first step is the analysis of dominant strategies at the individual level. The second step investigates the existence of equilibria among the candidate games. The final step incorporates criterion importance information.

The second solution method, logrolling, is a specific type of trade-off procedure for multi-issue negotiation. In logrolling, the exchange of loss in some issues for gain in others

results in mutual gain. The key to identifying mutual gain is recognition of the exchange rates that permit logrolling. The range of such exchange rates is referred to as the logrolling range, and it is determined from the parties' criterion weight information. The efficient frontier produced by logrolling is studied for 2-issue and *m*-issue negotiation. An example of employment agreement negotiation is used to demonstrate logrolling and related concepts in concrete terms.

Finally, diminishing marginal utility and variable criteria weights are used to model non-linear preference in the framework of logrolling. The efficient frontier produced by the modified logrolling procedure is studied for 2-issue negotiation. In terms of *m*-issue negotiation, the difficulties associated with generalization of the efficient frontier are discussed.

8.2 Key Contributions

Detailed discussions of benefits of the research are presented at the end of each major section. Here, the key contributions to the field of MCMP are summarized.

- A classification scheme based on the type of solution approaches has set the framework on how to perceive MCMP problems. This classification scheme has helped identify an improvement opportunity and suggest a means to achieve it. In this sense, this research has been built on the foundation laid by the proposed classification scheme.
- The integrative approach has been shown to possess significant potential in providing improvement opportunities by allowing the simultaneous incorporation of preference information without reduction or decomposition of the MCDM and MPDM components.
- Candidate enumeration as an integrative method that is capable of including criterion importance information is developed. This highly visible approach can offer decision makers an opportunity to understand each other, avoid deadlocks, and recognize jointly

rational solutions by presenting all key components simultaneously in the matrix of candidates.

- Logrolling, a unique quantitative method for achieving mutually beneficial solutions via trade-offs, is developed. It incorporates criteria weights and the interaction information, namely trade-off simultaneity, in the decision making process. Logrolling focuses on the individuals and their improvement. It exploits the difference in opinion and avoids connotation such as compromise, sacrifice, or concession making. These features of logrolling contribute not only to effective decision making, but also to conflict resolution. The proposed 2-issue and sequential procedures contribute to practical implementation of logrolling.
- The logrolling study has been extended to consider non-linear preference that is modelled by diminishing marginal utility and the variable criteria weights. The logrolling-based negotiation procedure is shown to be effective in accommodating this type of non-linear preference. By accounting for non-linearity, the logrolling method can model various and more realistic situations and increase its applicability.

8.3 Suggestions for Future Research

The results presented in this thesis are fundamental results. Many directions for further research exist based on these results including various extensions, validations, and some interesting ideas. Some specific suggestions for extending candidate enumeration and logrolling and suggestions for general future research are discussed below.

8.3.1 Candidate Enumeration

By using this research as a steppingstone, the candidate enumeration method can be extended in several directions. The multicriteria game model in this study is the smallest possible case. While dealing with the smallest case is ideal for exploratory purposes, it is restrictive in practice. A more general case must be considered for providing practical decision support.

Three types of extensions are possible: extension to more than one option, to more than two players, and to more than two criteria. Each type of extension is discussed in turn.

Extension to More Than One Option. In the current multicriteria game model, players have one option each. When a player has more than one option, four issues need to be reconsidered. First, the candidate enumeration algorithm must be modified since it does not include some obvious candidates. For example, the most important option with respect to one criterion is the third most important option for the other criterion. In this case, it seems natural to include this option as the second most important option for some candidates based on the concept of heredity. At present, the algorithm does not produce such candidates. The candidate enumeration algorithm must be able to produce all suitable candidates.

Secondly, grouping of candidates must be reconsidered. The grouping of candidates discussed in Section 4.4 is based on the existence of a dominant strategy implied by the candidates. When a player's strategy consists of more than one option, a dominant strategy depends on the various combinations of taking and not taking several options. Therefore, the grouping of candidates must be defined accordingly. Consequently, Definition 4.5 and Lemma 4.1 must be restated.

Third, the conclusions reached in Section 4.4 regarding the group type of candidates (Lemma 4.2.1–4.2.3) must be re-examined, as they are related to Definition 4.5 and Lemma 4.1. Finally, obtaining the ordinal criterion information from the option preferences in Section 4.6 must be reviewed. Since the number of groups, which is at least the number of strategies for a player, is greater than three, one must consider what should be done when the player is indifferent or unsure about the option preferences.

Further, one should keep in mind that the number of all possible candidates increases rapidly as the number of options increase. The number of preference trees with symmetric shape for three options is 768, and 786,432 for four options. The number of candidates produced by the algorithm can be as large as the number of symmetric trees, which may quickly become computationally intractable.

Extension to More Than Two Players. The additional players increase the number of outcomes in total. This increase should not affect the principle and formation of candidates and the solution strategy analysis, but the presentation of candidates must be reconsidered

due to the added complexity to the matrix form. If the analysis is kept asymmetrically prescriptive, then the matrix presentation can be modified by listing all possible games formed against one candidate of Player 1 in one row. This produces a matrix of candidates in which Player 1's unique dominant strategies are easily detected. Furthermore, when there are more than two players involved, coalition formation should be investigated as to how it may affect the concept of candidates.

Extension to More Than Two Criteria. The addition of criteria increases the number of candidates, and consequently, it increases the number of outcomes. Based on the independence assumption of criteria, all candidates can be produced by using the candidate enumeration algorithm. The algorithm is designed for two criteria, so one way of producing all candidates is as follows. One can proceed by producing candidates for two criteria first, and continue producing candidates between each of the candidates already produced and one new criterion. In addition, the presentation of all candidates needs to be considered. Ideally, the candidates with the same dominant strategy should be presented close together in order to help analyze equilibria in the matrix of candidates. This helps in detecting the pattern of the solution strategies across the matrix, and therefore, increases analysis efficiency.

Multicriteria Conflict Analysis. By extending the solution analysis to include conflict analysis, the applicability of the candidate enumeration method may be enhanced. To begin the investigation with multicriteria conflict analysis, the solution analysis steps proposed in this research must be modified. The dominant strategy analysis step is replaced by the analysis of individual stability, the equilibrium analysis step is replaced by the group stability analysis, and the third remains to be the analysis of criterion importance.

8.3.2 Logrolling

The logrolling study also provides many possible extensions. The concepts such as the exchange rate and the logrolling range are currently based on the exchange between two issues. Since 2-issue exchange is a common practice, its application in the negotiation context seems natural and practical. However, if the issues or criteria are not independent, it is suspected that 2-issue exchange may fail to identify some mutual gain where simultaneous

multi-issue exchange can. Hence, an investigation of the concept of multi-issue exchange may yield an additional benefit to the logrolling methodology.

The existence of a unique efficient frontier for m-issue sequential logrolling implies no clear universal strategic advantages realized by manipulating the sequences of pairs of issues. The result of solution analysis using a concrete example seems to be in agreement with this implication. Even so, sensitivity analysis on the elements such as criteria weights, exchange rates, and sequences of pairs of issues may provide insight into planning the case-by-case strategies. It may also be useful as a training, preparation, or evaluation tool for multi-issue negotiation. This suggests the development of a logrolling-based negotiation support system as a possible future project.

A generalization of the study to consider n-party negotiation is another possible future research direction. When multiple parties are involved, a possibility of coalition formation becomes inevitable. Such varying degrees of competitiveness and cooperation among the parties are likely to affect the fundamental concepts. The extent of the possible effect must be investigated.

Applicability is an important quality of a multi-issue negotiation methodology. Therefore, the acceptability and quality of its solutions must be validated through cases and field studies. On the other hand, an axiomatic approach may be able to strengthen the rationalization of the use of logrolling.

The extension from linear to non-linear preference has been dealt with in Chapter 7. The questions raised as the result of the investigation are as follows. Although the rationale behind the two concepts, diminishing marginal utility and variable criteria weights, seems legitimate, further investigation is desirable for validation purposes. Diminishing marginal utility in a negotiation context may be validated through empirical studies. As for the variable criteria weights, to verify its use in negotiation, one must look into how the stable and unstable (context-dependent) components actually make up a weight for individual negotiators. This would be an interesting research topic even in general decision making settings. It may be also worthwhile to explore other representations of non-linearity.

Another area of future research is to continue with m-issue negotiation under non-linear preference. In this case, the change in weights must be investigated further due to the

possible existence of different changing patterns as discussed in Section 7.5. The weight change may affect all issues more collectively than independently. This would be another reason to investigate multi-issue simultaneous exchange.

8.3.3 General Issues

Logrolling has many attractive features, but it is for a rather specific type of MCMP problems with the following characteristics: cardinal preferences, continuous alternatives, and distributive bargaining. The discussion of ordinal preferences, discrete alternatives, and group decision making clarifies that logrolling is not appropriate for all MCMP problems. It also provides suggestions for further research in a more general context.

Ordinal Preferences. When preferences are expressed in an ordinal manner instead of cardinal, the fundamental principle of logrolling is still applicable. However, the search for mutual gain becomes limited since the difference in preference intensity cannot be considered in the context of ordinal preferences.

The advantages of ordinal preference are emphasized in the context of multicriteria games. In terms of logrolling, the benefits of cardinal preference are exploited. It would be interesting to see if one can utilize the best features of the two types of preference representation through the use of a solution methodology that combines the features from candidate enumeration and logrolling.

Discrete Alternatives. One obvious drawback of logrolling applied to discrete alternatives is that the result of exchange may not correspond to an existing alternative. For example, consider the purchase of a car. The buyer is currently considering a $15,000 car with an expected life of 6 years. It is likely that the higher price is more important than the expected life of a car to the car dealer. If the longer expected life is more important than the price to the buyer, then a logrolling opportunity exists between the two parties. For extra $1000, the buyer expects at least an increase of two years in the expected life. However, on the dealer's lot, there is no single car that satisfies this trade-off ratio. This illustrates that, with discrete alternatives, even if a logrolling opportunity exists based on the parties' criteria, there may not be an alternative that corresponds to the result of the exchange. It may still be

possible to use logrolling as a screening tool. In general, further investigation is required for evaluating the applicability of logrolling for discrete alternatives.

When the number of alternatives is finite as in discrete alternative case, the difficulty of not reaching a Pareto optimal solution is not a serious concern any longer since one can examine the alternatives individually for their efficiency. Opportunities for improving MCMP must be reconsidered in this case. As for dealing with discrete alternatives more effectively than logrolling, one may be able to make use of the existing MCDM concepts such as the outranking method (Roy, 1974) as well as the concepts in MPDM such as voting.

Group Decision Making. In distributive bargaining, the relationship among the parties is regarded as competitive since one's gain implies another's loss. In a more cooperative decision making setting as referred to as group decision making, there is a difference in the overall objective and perception of the situation. Rather than individual optimization in the presence of others, consensus formulation becomes the group goal. In this case, similarities of the group members are the important attributes instead of differences that logrolling attempts to exploit. Therefore, the solution methods that focus on how to combine or aggregate individuals' preferences are more suitable for group decision making. A possible future research then is to investigate the applicability of trade-off simultaneity in facilitating consensus formation among group members.

Finally, one interesting concept to examine in the context of MCMP is Arrow's Impossibility Theorem (Arrow, 1951) in social choice theory. In principle, the theorem states that it is not possible to determine a group preference ranking from individual orderings of at least three alternatives without violation of Arrow's five axioms. Since it is possible for logrolling to exploit the differences among participants and achieve mutually beneficial outcome. Perhaps consideration from the MCMP perspective may provide a means to avoid the limitation imposed by Arrow's Impossibility Theorem.

8.4 Conclusion

As seen in the final section, many opportunities exist in MCMP research. This research provides a basis for further development of MCMP methodologies, especially in the areas of multicriteria games and multi-issue negotiation, as well as various applications based on the candidate enumeration and logrolling methods. The research in MCMP has been challenging, stimulating, and certainly rewarding as it is realized that multiple criteria-multiple participant decision making is a part of everyday business or personal activities. It is hoped that the research presented in this thesis will increase the awareness of the need to deal with MCMP problems and encourage further growth of this area.

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Appendices

A. Bicriteria Ordinal 2×2 Games

A.1 Candidate Enumeration Algorithm

This is an algorithm to produce all possible candidates for two preference trees with respect to two criteria.

Notation

Let Θ_{ij} = The j-th important option with respect to criterion i, $i = 1,2$, $j = 1,2$.
Let Θ_{cj} = The j-th important option for a candidate, $j = 1,2$.
Let $A=B$ imply that option A and option B are identical.
Let $|A|=|B|$ imply that option A and option B are identical except their desirability.
Let $|A|=|B_{cond}|$ imply option A and option B are identical except that option B has a preferential condition that option A does not have.
Let $(\Theta_{c1}, \Theta_{c2})$ represent a candidate: the first and the second components are the most important and the second most important options of the candidate, respectively. The second component may include a preferential condition.

Procedure

The most important option for a candidate can be of two types. Step 1 and 2 produce a set of candidates whose most important option is either the most important option for criterion 1 or 2 (type 1). Let this set be S_{1i}. Step 3 and 4 produce another set of candidates whose most important option is other than the most important option for criterion 1 or 2 (type 2). Let this set be S_{2i}. Then the final output of the algorithm is the union of S_{1i} and S_{2i} for all i.

Given: Θ_{11}, Θ_{12}, Θ_{21}, and Θ_{22}.

FOR $i = 1,2$, $j = 1,2$, $i \neq j$, DO step 1 to 4.

Step 1. Determine the possible most important option of type 1 for a candidate (Θ_{c1}).

$\Theta_{c1} = \Theta_{i1}$.

Step 2. Determine the possible second most important option for a candidate (Θ_{c2}) whose most important option is type 1.

IF Θ_{i2} is unconditional,

IF $\Theta_{i2} = \Theta_{j1}$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2})\}$.

ELSEIF $|\Theta_{i2}| = |\Theta_{j1}|$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2}), (\Theta_{i1}, \Theta_{j1}), (\Theta_{i1}, \Theta_{i2} \text{ iff } \Theta_{c1}), (\Theta_{i1}, \Theta_{i2} \text{ iff } -\Theta_{c1})\}$.

ELSEIF $\Theta_{i2} = \Theta_{j2}$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2})\}$.

ELSEIF $|\Theta_{i2}| = |\Theta_{j2}|$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2}), (\Theta_{i1}, \Theta_{j2}), (\Theta_{i1}, \Theta_{i2} \text{ iff } \Theta_{c1}), (\Theta_{i1}, \Theta_{i2} \text{ iff } -\Theta_{c1})\}$.

ELSEIF $|\Theta_{i2}| = |\Theta_{j2\ cond}|$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2}), (\Theta_{i1}, \Theta_{j2})\}$.

ENDIF

ELSEIF Θ_{i2} is conditional,

IF $|\Theta_{i2\ cond}| = |\Theta_{j1}|$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2}), (\Theta_{i1}, \Theta_{j1})\}$.

ELSEIF $\Theta_{i2} = \Theta_{j2}$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2})\}$.

ELSEIF$|\Theta_{i2\ cond}| = |\Theta_{j2}|$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2}), (\Theta_{i1}, \Theta_{j2})\}$.

ELSEIF $|\Theta_{i2\ cond}| = |\Theta_{j2\ cond}|$, THEN
$S_{1i} = \{(\Theta_{i1}, \Theta_{i2}), (\Theta_{i1}, \Theta_{j2})\}$.

ENDIF

ENDIF

Step 3. Determine the possible most important option of type 2 for a candidate (Θ_{c1}).

IF $|\Theta_{i2}| = |\Theta_{j1}|$, THEN
$\Theta_{c1} = \Theta_{i2}$.
GO TO Step 4.

ELSE
$S_{2i} = \varnothing$.
GO TO Step 5.

ENDIF

Step 4. Determine the possible second most important option for a candidate (Θ_{c2}) whose most important option is type 2.

IF$\Theta_{i1} = \Theta_{j2}$, THEN
$S_{2i} = \{(\Theta_{i2}, \Theta_{i1})\}$.

ELSEIF $|\Theta_{i1}| = |\Theta_{j2}|$, THEN
$S_{2i} = \{(\Theta_{i2}, \Theta_{i1}), (\Theta_{i2}, \Theta_{j2}), (\Theta_{i2}, \Theta_{i1}$ iff $\Theta_{c1}), (\Theta_{i2}, \Theta_{i1}$ iff $-\Theta_{c1})\}$.

ELSEIF$|\Theta_{i1}| = |\Theta_{j2\ cond}|$, THEN
$S_{2i} = \{(\Theta_{i2}, \Theta_{i1}), (\Theta_{i2}, \Theta_{j2})\}$.

ENDIF

Step 5. The final output of the algorithm is $\cup\{S_{1i}, S_{2i}\}$ for all i, which is the set of all candidates for the two preference trees with respect to the two criteria given.

End of the algorithm.

A.2 Proof of Lemma 4.2.1

Lemma 4.2.1 If *C1* and *C2* both belong to group i, then all candidates also belong to group i, $i = 1, -1, 0$.

Proof. For *C1* and *C2* which both belong to group 1 or -1, there are three cases to consider: (i) option 1 is most important option for *C1* and *C2*, (ii) option 1 is second most important

option for *C1* and *C2*, and (iii) option 1 is most important option for one of *C1* and *C2*, and second most important for the other. The proof is done for each case. The detail is shown for *C1* and *C2* in group 1. The same procedure is applied to the proof for group −1.

Note that all proofs are based on the candidate enumeration algorithm in Appendix A.1.

Case (i): Option 1 is most important option for *C1* and *C2*.

Step 1 in the algorithm sets $\Theta_{c1} = \Theta_{i1} = 1$ for $i = 1, 2$. In step 2, Θ_{c2} deals only with option 2. Therefore, the desirability of option 1 for all candidates contained in S_{1i} depends on Θ_{i1}. Since $\Theta_{i1} = 1$ for $i = 1, 2$, the desirability of option 1 for all candidates contained in S_{1i} is to take option 1.

In step 3, $|\Theta_{i2}| = |\Theta_{j1}|$ does not hold for $i = 1, 2$, since Θ_{i2} only deals with option 2 and Θ_{j1} only deals with option 1. Therefore, $S_{2i} = \varnothing$ for all i.

Therefore, all candidates in case (i) belong to group 1.

Case (ii): Option 1 is second most important option for *C1* and *C2*.

In step 1, $\Theta_{c1} = \Theta_{i1} = 2$ or -2 for $i = 1.2$. Since Θ_{i2} is unconditional and $\Theta_{i2} = \Theta_{j2} = 1$, in step 2, $S_{1i} = \{(\Theta_{i1}, \Theta_{i2})\} = \{(\Theta_{i1}, 1)\}$ for $i = 1, 2$. Therefore, the desirability of option 1 for all candidates contained in S_{i1} is to take option 1.

In step 3, $|\Theta_{i2}| = |\Theta_{j1}|$ does not hold for $i = 1, 2$, since Θ_{i2} only deals with option 1 and Θ_{j1} only deals with option 2. Therefore, $S_{2i} = \varnothing$ for all i.

Therefore, all candidates in case (ii) belong to group 1.

Case (iii): Option 1 is most important option for one of *C1* and *C2*, and second most important for the other.

Assume that option 1 is most important for *C1* and second most important for *C2*. Then, $\Theta_{11} = \Theta_{22} = 1$.

For $i = 1$, step 1 sets $\Theta_{c1} = \Theta_{11} = 1$. In step 2, Θ_{c2} deals only with option 2. Therefore, the desirability of option 1 for all candidates contained in S_{1i} depends on Θ_{11}. Since $\Theta_{11} = 1$, the desirability of option 1 for all candidates contained in S_{11} is to take option 1.

In step 3, if $|\Theta_{12\ cond}| = |\Theta_{21}|$, then $S_{ij} = \varnothing$. If $|\Theta_{12}| = |\Theta_{21}|$, then $\Theta_{c1} = \Theta_{12} = 2$ or -2. Following step 4, since $\Theta_{11} = \Theta_{22} = 1$, $S_{21} = \{(\Theta_{12}, \Theta_{11})\} = \{(\Theta_{12}, 1)\}$. Therefore, the desirability of option 1 for all candidates contained in S_{21} is to take option 1.

For $i = 2$, step 1 sets $\Theta_{c1} = \Theta_{21} = 2$ or -2. Since Θ_{22} is unconditional and $\Theta_{22} = \Theta_{11} = 1$, in step 2, $S_{12} = \{(\Theta_{21}, \Theta_{22})\} = \{(\Theta_{21}, 1)\}$. Therefore, the desirability of option 1 for all candidates contained in S_{ij} is to take option 1.

In step 3, since $\Theta_{11} = \Theta_{22} = 1$, $S_{22} = \varnothing$.

Therefore, all candidates in case (iii) belong to group 1.

In all three cases, it is shown that all candidates belong to group 1.

By applying the same procedure above to *C1* and *C2* in group -1, it can be shown that all candidates belong to group -1 in this case. Therefore, if *C1* and *C2* both belong to group i, then all candidates also belong to group i, $i = 1, -1$.

For *C1* and *C2* which both belong to group 0, there are two cases to consider: (iv) *C1* and *C2* have the same preferential condition on option 1, and (v) *C1* and *C2* have different preferential conditions on option 1.

Case (iv): *C1* and *C2* have the same preferential condition on option 1.

In step 1, $\Theta_{c1} = \Theta_{i1} = 2$ or -2 for $i = 1,2$. Since $\Theta_{i2} = \Theta_{j2}$, $S_{1i} = \{(\Theta_{i1}, \Theta_{i2})\}$ for all i. Therefore, option 1 for all candidates in S_{1i} has the same preferential condition as Θ_{i2}.

In step 3, $|\Theta_{i2}| = |\Theta_{j1}|$ does not hold for $i = 1, 2$, since Θ_{i2} has a preferential condition on option 1. Therefore, $S_{2i} = \varnothing$ for all i.

Therefore, all candidates in case (iv) belong to group 0.

Case (v): *C1* and *C2* have different preferential conditions on option 1.

In step 1, $\Theta_{c1} = \Theta_{i1}$ for $i = 1,2$. Since $|\Theta_{i2\ cond}| = |\Theta_{j2\ cond}|$, $S_{1i} = \{(\Theta_{i1}, \Theta_{i2}), (\Theta_{i1}, \Theta_{j2})\}$ for $i = 1,2$. Θ_{i2} and Θ_{j2} both have a preferential condition on option 1, so option 1 for all candidates in S_{1i} has the same preferential condition as Θ_{i2} or Θ_{j2}.

In step 3, $|\Theta_{i2}| = |\Theta_{j1}|$ does not hold for $i = 1, 2$, since Θ_{i2} has a preferential condition on option 1. Therefore, $S_{2i} = \varnothing$ for all i.

Therefore, all candidates in case (v) belong to group 0.

From cases (iv) and (v), if *C1* and *C2* both belong to group 0, then all candidates also belong to group 0. ■

B. Logrolling Illustration

B.1 EXCEL Worksheets

Logrolling: Starting with Issues 1 and 2 . . . . . Page 154

Sample Formulas for Logrolling: Starting with 1 and 2 . . . . . Page 155

Logrolling for Non-Linear Preferences . . . . . Page 159

	A	B	C	D	E	F	G
1	**Logrolling: Starting with Issues 1 and 2**						
2							
3	1) Data		Salary	Vacation	Medical		
4		Max	1	1	1		
5		Min	0	0	0		
6							
7	2) Starting Point		0.5	0.5	0.5		
8							
9	3) Criteria Weights						
10							
11		Employee	0.66	0.24	0.1		
12		Personnel	0.1	0.24	0.66		
13							
14	4) Logrolling Range				Employee		
15			Min	Max	gains in		
16		1&2	0.363636	2.4	1		
17		1&3	0.151515	6.6	1		
18		2&3	0.416667	2.75	2		
19							
20	5) Agreed Exchange Rate						
21							
22		1&2	1.381818				
23		1&3	3.375758				
24		2&3	1.583333				
25							
26	6) Exchange Result						
27							
28	Round 1:	EX1	Salary	Vacation	Medical		
29		1.381818	1	0.138158	0.5		
30							
31							
32	Round 2:	EX2	Salary	Vacation	Medical		
33		1.583333	1	0.929825	0		
34							
35							
36	Round 3:	EX3	Salary	Vacation	Medical		
37		3.375758	1	0.929825	0		
38							
39			Outcome in Original Units			V1	V2
40		Final	24	20.01754	20	883.1579	676.8421
41		Start	22	14	60	500	500

	A	B
1	**Sample Formulas for**	**Logrolling: starting with 1 and 2**
2		
3	1) Data	
4		Max
5		Min
6		
7	2) Starting Point	
8		
9	3) Criteria Weights	
10		
11		Employee
12		Personnel
13		
14	4) Logrolling Range	
15		
16		1&2
17		1&3
18		2&3
19		
20	5) Agreed Exchange Rate	
21		
22		1&2
23		1&3
24		2&3
25		
26	6) Exchange Result	
27		
28	Round 1:	EX1
29		=$C$22
30		
31		
32	Round 2:	EX2
33		=IF($C$29>=$C$4, $C$24, $C$23)
34		
35		
36	Round 3:	EX3
37		=IF($C$29>=$C$4, $C$23, $C$24)
38		
39		
40		Final
41		Start

	C
1	
2	
3	Salary
4	1
5	0
6	
7	0.5
8	
9	
10	
11	0.66
12	0.1
13	
14	
15	Min
16	=IF($D$11/$C$11<$D$12/$C$12, $D$11/$C$11, $C$11/$D$11)
17	=IF($E$11/$C$11<$E$12/$C$12, $E$11/$C$11, $C$11/$E$11)
18	=IF($E$11/$D$11<$E$12/$D$12, $E$11/$D$11, $D$11/$E$11)
19	
20	
21	
22	=$C$16+($D$16-$C$16)*0.5
23	=$C$17+($D$17-$C$17)*0.5
24	=$C$18+($D$18-$C$18)*0.5
25	
26	
27	
28	Salary
29	=MIN($C$7+ABS($D$5-$D$7)*B29, $C$4)
30	
31	
32	Salary
33	=IF(C29>=$C$4, C29, MIN(C29+ABS($E$5-E29)*B33, $C$4))
34	
35	
36	Salary
37	=IF($B$37=$C$24, C33, MIN(C33+ABS($E$5-E33)*B37, $C$4))
38	
39	Outcome in Original Units
40	=4*C37+20
41	=4*C7+20

	D
1	
2	
3	Vacation
4	1
5	0
6	
7	0.5
8	
9	
10	
11	0.24
12	0.24
13	
14	
15	Max
16	=IF($D$11/$C$11<$D$12/$C$12, $D$12/$C$12, $C$12/$D$12)
17	=IF($E$11/$C$11<$E$12/$C$12, $E$12/$C$12, $C$12/$E$12)
18	=IF($E$11/$D$11<$E$12/$D$12, $E$12/$D$12, $D$12/$E$12)
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	Vacation
29	=MAX($D$7-ABS($C$4-$C$7)/B29, $D$5)
30	
31	
32	Vacation
33	=IF(C29>=$C$4, MIN(D29+ABS($E$5-E29)*B33, $D$4), D29)
34	
35	
36	Vacation
37	=IF($B$37=$C$24, MIN(D33+ABS($E$5-E33)*B37, $D$4), D33)
38	
39	
40	=14*D37+7
41	=14*D7+7

	E
1	
2	
3	Medical
4	1
5	0
6	
7	0.5
8	
9	
10	
11	0.1
12	0.66
13	
14	Employee
15	gains in
16	=IF($D$11/$C$11<$D$12/$C$12, 1, 2)
17	=IF($E$11/$C$11<$E$12/$C$12, 1, 3)
18	=IF($E$11/$D$11<$E$12/$D$12, 2, 3)
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	Medical
29	=$E$7
30	
31	
32	Medical
33	=IF(C29>=$C$4, MAX(E29-ABS($D$4-D29)/B33, $E$5), MAX(E29-ABS($C$4-C29)/B33, $E$5)
34	
35	
36	Medical
37	=IF($B$37=$C$24, MAX(E33-ABS($D$4-D33)/B37, $E$5), MAX(E33-ABS($C$4-C33)/B37, E
38	
39	
40	=80*E37+20
41	=80*E7+20

	A	B	C	D	E	F
1	**Logrolling for Non-Linear Preference**					
2						
3	1) Data		Salary	Vacation	Medical	
4		Max	1	0.85	0.85	
5		Min	0	0	0	
6						
7	2) Starting Point		0.85	0.246711	0.5	
8						
9	3) Criteria Weights					
10						Sum
11		Employee	0.33	0.57	0.1	1
12		Personnel	0.1	0.24	0.66	1
13						
14	4) Logrolling Range				Employee	
15			Min	Max	gains in	
16		1&2	1.727273	2.4	1	
17		1&3	0.30303	6.6	1	
18		2&3	0.175439	2.75	2	
19						
20	5) Agreed Exchange Rate					
21						
22		1&2	2.063636			
23		1&3	3.451515			
24		2&3	1.462719			
25						
26	6) Exchange Result					
27						
28	Start with					
29	1&2:	EX1	Salary	Vacation	Medical	
30		2.063636	1	0.174024	0.5	
31						
32	1&3:	EX2	Salary	Vacation	Medical	
33		3.451515	1	0.246711	0.35	
34						
35	2&3:	EX3	Salary	Vacation	Medical	
36		1.462719	0.85	0.85	0.087557	
37						
38						
39	Start with (opposite exchanging pattern)					
40	2&1:	EX1	Salary	Vacation	Medical	
41		2.063636	0.557657	0.85	0.5	
42						
43	3&1:	EX2	Salary	Vacation	Medical	
44		3.451515	0.748595	0.246711	0.85	
45						
46	3&2:	EX3	Salary	Vacation	Medical	
47		1.462719	0.85	0.007431	0.85	

B.2 Numerical Data

Table B.2.1 Varying the Starting Point (Starting with Pair 1 and 2)

Table B.2.2 Varying the Starting Point (Starting with Pair 1 and 3)

Table B.2.3 Varying the Starting Point (Starting with Pair 2 and 3)

Table B.2.4 Varying the Criteria Weights (Starting with Pair 1 and 2)

Table B.2.5 Varying the Agreed Exchange Rate (Starting with Pair 1 and 2)

Table B.2.6 Varying the Agreed Exchange Rate (Starting with Pair 1 and 3)

Table B.2.7 Varying the Agreed Exchange Rate (Starting with Pair 2 and 3)

Table B.2.1 Varying the Starting Point (Starting with Pair 1 and 2)

	Starting Point			Starting Value		Final Outcome			Final Value	
	Salary	Vac	Med	V1	V2	Salary	Vac	Med	V1	V2
1	20.8	9.8	36	200	800	24	7.99	20	677.1	882.9
2	20.8	14	60	302	530	24	17.37	20	837.7	772.2
3	20.8	18.2	84	404	260	24	21	44.64	930.8	456.7
4	22	9.8	36	398	770	24	9.96	20	710.8	849.2
5	22	14	60	500	500	24	20.02	20	883.2	676.8
6	22	18.2	84	602	230	24	21	55.61	944.5	366.2
7	23.2	9.8	36	596	740	24	12.21	20	749.3	810.7
8	23.2	14	60	698	470	24	21	27.42	909.3	598.8
9	23.2	18.2	84	800	200	24	21	66.58	958.2	275.7

Table B.2.2 Varying the Starting Point (Starting with Pair 1 and 3)

	Starting Point			Starting Value		Final Outcome			Final Value	
	Salary	Vac	Med	V1	V2	Salary	Vac	Med	V1	V2
1	20.8	9.8	36	200	800	24	8.54	20	686.3	873.7
2	20.8	14	60	302	530	24	19.83	20	879.9	680.1
3	20.8	18.2	84	404	260	24	21	54.94	943.7	371.8
4	22	9.8	36	398	770	24	10.95	20	727.7	832.3
5	22	14	60	500	500	24	21	22.89	903.6	636.2
6	22	18.2	84	602	230	24	21	62.05	952.6	313.1
7	23.2	9.8	36	596	740	24	12.92	20	761.5	798.5
8	23.2	14	60	698	470	24	21	29.99	912.5	577.5
9	23.2	18.2	84	800	200	24	21	69.16	961.4	254.5

Table B.2.3 Varying the Starting Point (Starting with Pair 2 and 3)

	Starting Point			Starting Value		Final Outcome			Final Value	
	Salary	Vac	Med	V1	V2	Salary	Vac	Med	V1	V2
1	20.8	9.8	36	200	800	23.66	7	20	603.2	908.6
2	20.8	14	60	302	530	24	19.19	20	869.1	690.9
3	20.8	18.2	84	404	260	24	21	54.94	943.7	371.8
4	22	9.8	36	398	770	24	9.17	20	697.2	862.8
5	22	14	60	500	500	24	21	22.89	903.6	636.2
6	22	18.2	84	602	230	24	21	62.05	952.6	313.1
7	23.2	9.8	36	596	740	24	12.21	20	749.3	810.7
8	23.2	14	60	698	470	24	21	29.99	912.5	577.5
9	23.2	18.2	84	800	200	24	21	69.16	961.4	254.5

Table B.2.4 Varying the Criteria Weights (Starting with Pair 1 and 2)

	Weights			Final Outcome			Final Value	
	Salary	Vac	Med	Salary	Vac	Med	V1	V2
Six ordinal comb.	0.66	0.24	0.1	24	20.017	20	883.15	676.84
	0.6	0.1	0.3	24	10.984	20	728.31	831.68
	0.3	0.6	0.1	24	21	20.103	900.12	659.14
	0.3	0.1	0.6	24	10.734	20	724.01	835.98
	0.1	0.6	0.3	24	16.696	20	826.22	733.77
	0.1	0.3	0.6	24	21	22.347	902.93	640.63
One extreme	0.9	0.06	0.04	24	20.282	20	887.70	672.29
	0.06	0.9	0.04	24	14.739	20	792.67	767.32
	0.04	0.06	0.9	24	11.916	20	744.27	815.72
Two similar	0.5	0.4	0.1	24	20.125	20	885	675
	0.1	0.5	0.4	24	17.803	20	845.20	714.79
	0.4	0.1	0.5	24	10.689	20	723.25	836.74
Three similar	0.34	0.33	0.33	24	21	26.008	907.51	610.43

Table B.2.5 Varying the Agreed Exchange Rate (Starting with Pair 1 and 2)

	Agreed Exchange Rate			Final Outcome			Final Value	
Skill Level	1&2	1&3	2&3	Salary	Vac	Med	V1	V2
Inferior	0.7	2	0.8	24	11.76	20	741.6	818.4
Equal	1.381	3.375	1.583	24	20.01	20	883.1	676.8
Superior	2.1	5	2.3	24	21	34.32	917.9	541.8

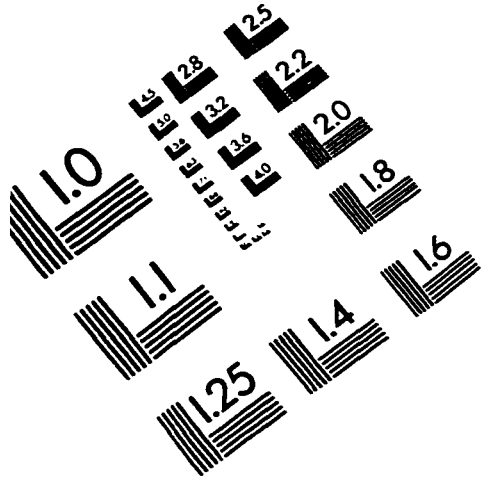
Table B.2.6 Varying the Agreed Exchange Rate (Starting with Pair 1 and 3)

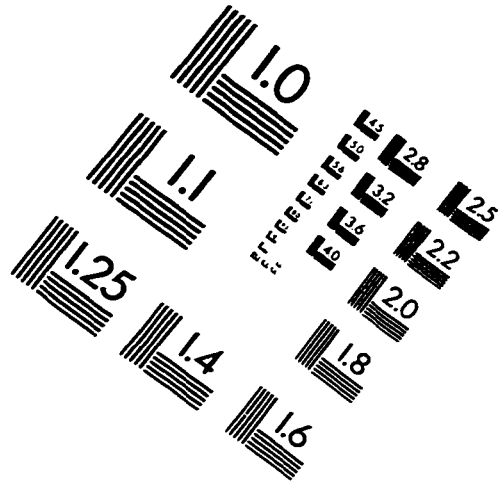
	Agreed Exchange Rate			Final Outcome			Final Value	
Skill Level	1&2	1&3	2&3	Salary	Vac	Med	V1	V2
Inferior	0.7	2	0.8	24	16.8	20	828	732
Equal	1.381	3.375	1.583	24	21	22.88	903.6	636.1
Superior	2.1	5	2.3	24	21	34.60	918.2	539.4

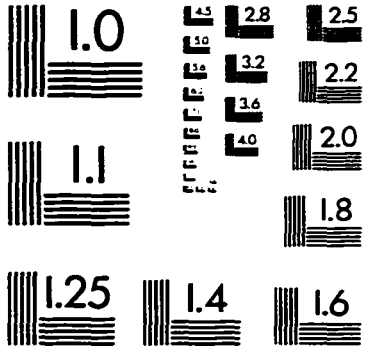
Table B.2.7 Varying the Agreed Exchange Rate (Starting with Pair 2 and 3)

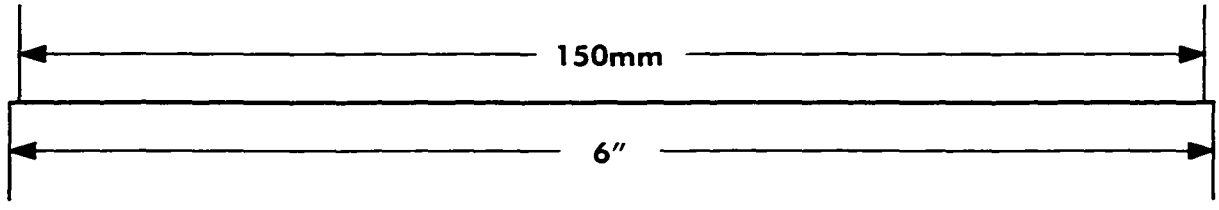
	Agreed Exchange Rate			Final Outcome			Final Value	
Skill Level	1&2	1&3	2&3	Salary	Vac	Med	V1	V2
Inferior	0.7	2	0.8	24	9.6	20	704.5	855.4
Equal	1.381	3.375	1.583	24	21	22.88	903.6	636.1
Superior	2.1	5	2.3	24	21	34.60	918.2	539.4

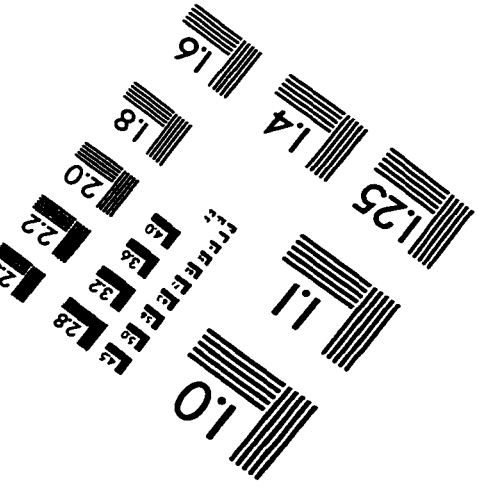
IMAGE EVALUATION
TEST TARGET (QA-3)

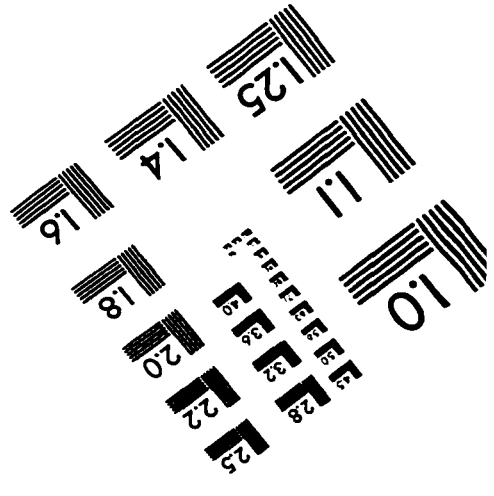












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