Effects of Inventory Policies on Supply Chain Design

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Shuo Xu

Abstract

This thesis is motivated by an industrial problem faced by Bombardier Inc. in designing a two-echelon supply chain. The upper echelon is a plant that operates under batch ordering inventory policy. The lower echelon is a set of service centers which operate under base stock inventory policy. The problem is to decide which service centers to open and how to assign customers to open service centers based on their preference. The stock level at each open service center is decided to meet a specified mean target response time requirement. The objective is to minimize the total cost including the location-allocation cost, the ordering cost at the plant, the holding cost, and the backorder cost at both the plant and open service centers.

The inventory-location problem is formulated as a mixed-integer programing problem with stochastic variables and mean target response time constraint. A cutting-plane algorithm is proposed to solve the model. Numerical testing is performed on industry instances and instances from the literature to evaluate the effectiveness of the cutting-plane algorithm and to investigate the effects of different inventory replenishment policies.

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Dedication

This thesis is dedicated to my dearest parents, my lovely wife and my cute kitty and puppies.

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Chapter 1

Introduction

An inventory-location problem is studied in this thesis. The problem focuses on designing a supply chain for spare parts with customer preferences and response time constraints. Spare parts are used to replace or repair a malfunctioned system or part of it. This research is motivated by a similar work by Riaz [20]. We study a similar system which consists of a central manufacturing plant that has a limited production capacity and multiple service centers (SCs). The SCs need to meet customer demands by keeping stocks and replenishing from the plant. The SCs have a limited capacity and operate under a base stock (one-for-one) replenishment policy and the plant also has a limited capacity and operates under a batch ordering replenishment policy. Demands from the customers follow an independent Poisson process. Customers are assigned to open SCs based on their preference and customer orders must be satisfied within a target response time. The decisions to make are where to locate the open SCs, how to allocate customers to the open SCs, and what stock levels to keep at the plant and at the open SCs. The objective is to minimize the total cost including the location cost and the total inventory cost.

The solution to this supply chain design problem can be used to solve other problems in the industry, especially in relation to spare parts. Spare parts are used in the high technology field like computer, automotive and aerospace industry. It is important to study the spare parts industry because failure of a single part may cause a whole system to break down. In the spare parts industry, the base stock policy is widely used since spare parts are generally ordered in single unit and the holding cost is high while the order cost is relatively low at the service centers [27]. However at the plant, since demand is accumulated from all the SCs, it is relatively higher at the plant than that at the SCs [1]. Therefore, it is reasonable for the plant to operate a batch ordering policy. In this thesis, we assume a batch ordering policy at the plant. We also assume a fixed replenishment cost.

Response time is essential to guanrantee customer orders are satisfied within a specified target time. Since spare parts are critical for customer operation, reliable fulfillment of spare parts is a requirement for successful spare parts supply chain.

A possible way to satisfy customer demand within a certain time is to keep ample supply at SCs. However, for some expensive spare parts like those used in the aerospace industry, the demand is low and inventory holding costs are high, so it is not economical to keep ample supply at the SCs [20]. The major challenge for modern industries is how to balance the trade-offs between inventory holding costs and customer service. Manufacturers could make significant savings by designing an efficient supply chain network of spare parts.

In this thesis, a mixed-integer programming is formulated to solve a two-echelon inventory-location problem for spare part. In order to find the optimal solution, a cutting-plane algorithm is proposed. The algorithm separates the original problem into a master problem which is a location problem and a subproblem which is a inventory stocking problem. Numerical testing using an industrial dataset and a dataset from the literature is performed to test the effectiveness of the algorithm.

The rest of the thesis is organized as follows: Chapter 2 presents a literature review related to the facility location problem, the inventory stocking problem and the inventory-location problem. Chapter 3 presents the formulation of the inventory-location problem. Chapter 4 discusses the formulation of the inventory stocking problem and the steady state parameters, and gives the exact algorithm to solve the inventory stocking problem. Chapter 5 develops an exact cutting-plane algorithm to solve the inventory-location problem. Chapter 6 presents the results on the performance of the cutting plane algorithm and discusses the effects of base stock and batch ordering inventory policies. Finally the conclusion is provided in Chapter 7.

Chapter 2

Literature Review

There are three major research areas related to the inventory-location problem studied in this thesis, namely facility location, inventory and integrated inventory-location. The facility location problem is a critical step in the strategic planning for many industries. Many mathematical models are available in the literature. Similarly, there is a large body of literature that studies the inventory stocking decisions. On the other hand, the inventory-location problem attempts to make the location and inventory decisions simultaneously. In this chapter, we first review the literature on the facility location problem. Then we review the inventory stocking problem. Finally we review the integrated inventory-location problem.

2.1 The Facility Location Problem

The facility location problem has been widely studied in Operation Research literature. A general facility location problem is to locate facilities and allocate customers to open facilities to meet the customer demand so as to minimize the location-allocation cost. The simplest setting of this problem is to select p locations from candidate locations to minimize the total cost or distance while fulfilling customer demand. This is the p-median problem. This setting requires that all candidate places are equivalent. However, if the setup cost for locating a candidate place is different but fixed, this setting is called the uncapacitated facility location problem, known as UFLP. Daskin [6], Hamacher and Drezner [7] discusses the p-median problem while references of UFLP can be found in Mirchandani and Francis [14] and Revelle et al. [19]. There is also an important extension of UFLP which is called capacitated facility location problem (CFLP) where the demand that can be supplied from each candidate site is limited by its capacity [25]. More recent reviews are in Klose and Drexl [9] and Melo [12].

A typical location problem allocates customers to facilities based on minimum cost. However, based on the industry application, customers pay for shipment cost, so it is natural that customers are given the option to be served by their most preferred open service center. Similar to Riaz [20], we allocate customers based on their preference.

2.2 Inventory Stocking Problem

The inventory stocking problem focuses on determining optimal stocking policy to minimize the total cost, which may include the holding cost, the order cost and the backorder cost. Sherbrooke [23] constructed the METRIC (multi-echelon technique for recoverable item control) model. The METRIC model assumes that the stockouts at the service centers are fully backordered. The model finds an optimal stock level that minimizes the expected backorder at service centers subject to a budget constraint. This is the first multi-echelon model for the service part inventory management. Many works have been carried out to extend this model. Wang et al. [29] and Andersson et al. [2] used the METRIC model to deal with a two-echelon inventory system and specifically, Wang et al. [29] studied the impact of depot-replenishment lead times on system performance. Andersson et al. [2] optimized the base stock policy with one plant and an arbitrary number of service centers. In addition to the analytical models, simulation models are also used. Moinzadeh et al. [15] used the METRIC model and simulation to study a multi-echelon inventory system under a base stock policy where all the stocking locations have two options: replenishing the inventory through the normal channel or a more expensive emergency channel.

The inventory stocking problem we study is characterized by the mean target response time constraint which is considered to be a complicating constraint. Several works have been done to provide methods including heuristics, approximation algorithms and exact solution approach algorithms to deal with the complicating constraint and to find the solution. Basten et al. [3] proposed a step and check heuristic to find the near optimal solution with 0.2% error on average and a smart enumeration heuristic to find the exact optimal solution for a two-echelon distribution network with one plant and multiple service centers with each facing independent Poisson demand. Tsai et al. [28] presented a simulation optimization algorithm to solve the multi-echelon inventory problem subject to service level constraints. Lin Li et al. [10] presented solution approaches to find approximate inventory policies for a two echelon inventory system with stochastic demand, stochastic lead times, fixed order costs and customer service level requirements. Caggiano et al. [4] described a continuous review inventory model for a multi-item and multi-echelon service parts distribution system with time-based service level requirements. They also used an intelligent greedy algorithm to find near-optimal solutions to large-scale problems and a Lagrangian-based approach to find both near-optimal solutions and good lower bounds.

The most related work to our inventory stocking problem is by Topan and Bayindir [26]. Topan and Bayindir [26] developed greedy heuristic approaches in multi-product two-echelon spare parts inventory systems in order to minimize the system-wide inventory holding costs under aggregate mean response time constraint. Topan and Bayindir [27] also presented an exact branch-and-price algorithm to find the inventory control policy parameters that minimize the system-wide inventory holding and fixed order cost subject to an aggregate mean response time constraint at each facility. In both papers, they assumed a batch ordering policy in the plant and a base-stock policy at each service center.

2.3 Inventory-Location Problem

The integration of the location and inventory stocking decision is a challenging task. The integrated inventory-location problem balances the location and inventory stocking costs, but is a difficult problem to solve. Many models and several approximation and heuristic approaches are developed in the literature. Jia Shu et al. [24] proposed a scenario-based two-level stochastic model to address the design of the two-echelon network. They determine how many service centers to be opened, where to locate them and how to allocate these service centers to the customers. The near-optimal inventory replenishment policies for service centers and customers were used to minimize the total expected system-wide multi-echelon inventory, transportation, and facility location costs in order to save the CPU time. Sadjady et al. [22] considered a problem of designing a two-echelon supply chain network, which allowed multiple levels of capacities for the facilities at both echelons. They developed a Lagrangian-relaxation based heuristic solution to solve the given problem. Chen and Li [5] studied an inventory-location problem that optimized facility locations, customer allocations, and inventory management decisions where facilities were subject to disruption risks. They also developed a Lagrangian relaxation solution framework for this problem, including a polynomial-time exact algorithm for the relaxed nonlinear Lagrangian subproblem. Miranda et al. [13] proposed a novel inventory location model with stochastic capacity constraints based on a periodic inventory control policy, and they used an exhaustive algorithm to find the optimal solution for small instances. Jin et al. [18] studied the inventory-location problem with multiple-commodities, stochastic demands and capacity constraints. Zhang et al. [31] considered the integrated optimization problem of the location and inventory decisions of a distribution center, subject to a given customer service level constraint in a multi-product and multi-echelon supply chain. Yao et al. [30] studied an integrated facility location-allocation and inventory problem. The major difference is that the service center can be replenished by several plants together because of capabilities and capacities of plants.

Two similar models to our problem are from Jeet et al. [8] and Mak and Shen [11]. Jeet et al. [8] presented a single-part integrated network design and inventory stocking problem for low-demand systems such as service parts logistic networks. They developed an exact scheme based on an outer-approximation shceme to find the upper and lower bound of the problem. Mak and Shen [11] presented a two-echelon supply chain model for spare parts with target response time constraint and proposed a Lagrangian relaxation method.

Our work is different from the papers mentioned so far because we consider a two-echelon problem with fixed single unit replenishment time and response time constraint. We also proposed an cutting-plane algorithm to find the exact solution of the problem. The most related work is from Riaz [20]. He considered a two-echelon inventory-location system for spare parts and assumed base stock replenishment policy at both plant and service centers. He developed a cutting-plane algorithm to solve the inventory-location problem. The only difference from our work is the replenishment policy at the plant. In our work, we assume a batch ordering replenishment policy at the plant.

Chapter 3

The Inventory-Location Problem

3.1 Problem Description

The problem we consider is to design a two-echelon supply chain system. The upper echelon is a plant and the lower echelon is a set of potential service centers. The plant's goal is to set stock levels to satisfy the demand from the service centers, while the service centers' goal is to set stock levels to satisfy stochastic demand from the customers. The stochastic demand of a customer follows a Poisson distribution.

Service centers use base stock replenishment policy with backordering: a customer places an order at its assigned service center. If there is inventory available, the customer order is fulfilled. Otherwise, the item is backordered. Whenever the service center receives an order from a customer, it makes an immediate order from the plant. The backorder is filled when a replenishment is received based on a first-in-first-out (FIFO) policy. The plant uses a batch ordering policy. It satisfies the order from each service center if it has enough inventory. Otherwise, the order is backordered. The plant place an order of size Q whenever the stocking level falls below the reorder point R. After receiving the replenishment order, the outstanding backorders at the plant are immediately satisfied according to a FIFO policy. The shipment time between the plant and the service centers, or lead time between the echelons, is fixed.

The demand process at each service center is a Poisson process and the demand process at the plant is a superposition of the service centers' ordering processes which is also a Poisson process. Customers have a preference of the SCs and are assigned to an open SC based on the preference. This also indicates that each customer will be assigned to the most preferred open SC and we can assume the customer to be served by a single source.

Finally, the time interval between when the customer places an order and when the customer order is fulfilled is called customer response time. Customer orders need to be satisfied within a mean target response time. In our model, we are interested in the long-run operation of the system and we are not interested in the risk of the decision. Therefore we consider the steady state behavior performance instead of using chance constraint to consider the risk. Figure 3.1 depicts the replenishment process.

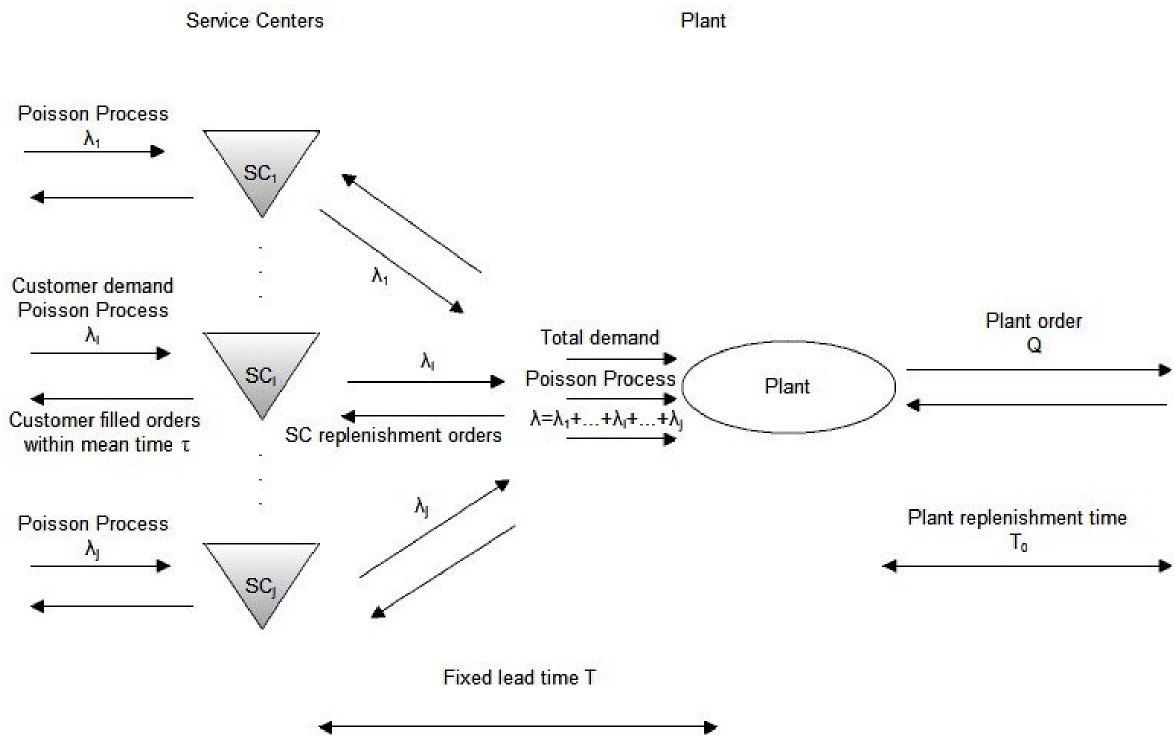


Figure 3.1: Replenishment process

The decisions to make are: locate the service centers, assign customers to the service centers, and determine the stock levels at each facility so that the response time constraint is satisfied. The objective of the problem is to minimize the total cost including the inventory holding cost and the backorder cost both at the service centers and at the plant, the order cost at the plant, and the location cost of all service centers.

3.2 Problem Formulation

We use the following notation to formulate our model.

Parameters:

I	: Set of customers.
$N \cup 0$	: N represents the set of potential SCs, 0 represents plant.
f_n	: Fixed cost of opening SC n, $n \in N$.
K	: Fixed order cost at the plant.
h_n	: Holding cost at SC n, $n \in N$.
h_0	: Holding cost at plant.
p_n	: Backorder cost at SC n, $n \in N$.
p_0	: Backorder cost at plant.
τ	: The mean target response time.
λ_i	: Demand rate of customer $i \in I$.
d_{max}	: Distance limit between customer and the assigned SC.
N_i	: The preference list of SC for customer i.
C_0	: The storage capacity at the plant.
C_n	: The storage capacity at SC n, $n \in N$.
Q_0	: The order size limit.
$\lambda = \sum_{i \in I} \lambda_i$	: Total customer demand.

Decision Variables:

$X_n =$ 1 if SC n is opened, 0 otherwise, $n \in N$.

$Y_{in} =$ 1 if customer i is assigned to SC n, 0 otherwise, $i \in I, n \in N$.

S_n : The base-stock level at SC, $n \in N$.

Q : Order size at plant.

R : Reorder Point at plant.

Auxiliary Variables:

$\overline{I}_n$: Expected on-hand inventory level at SC $n \in N$.

$\overline{I}_0$: Expected on-hand inventory level at plant.

$\overline{IP}_0$: Expected inventory position at the plant.

$\overline{X}_n$: Expected number of outstanding orders at SC $n \in N$.

$\overline{Y}_n$: Expected demand during lead time at SC $n \in N$.

$\overline{B}_n$: Expected backorder level at SC $n \in N$.

$\overline{B}_0$: Expected backorder level at the plant.

$\overline{B}_0^{(n)}$: Expected backorder level of SC n at the plant.

$\overline{W}_n$: Expected waiting time at SC $n \in N$.

$\overline{W}_0$: Expected waiting time at the plant.

We use $Z = \sum_n (h_n \overline{I}_n + p_n \overline{B}_n) + \lambda K/Q$ for $n \in N \cup 0$ to be the inventory ordering, holding and back order cost incurring in the whole system. The inventory-location model is formulated next.

$$
\begin{aligned}
\text{[ILP]}: \quad & \min \quad \sum_{n\in N} f_n X_n + Z && (3.1)\\
& \text{s.t.} \quad \sum_{n\in N} Y_{in} = 1 \qquad \forall i\in I, && (3.2)\\
& Y_{in} \le X_n, \qquad \forall i\in I, \forall n \in N_i, && (3.3)\\
& Y_{in} \ge X_n - \sum_{l=1}^{n-1} X_l, \qquad \forall i\in I,\ \forall n\in N_i, && (3.4)\\
& S_n \le C_n X_n \qquad \forall n\in N, && (3.5)\\
& R \le C_0 + Q_0, && (3.6)\\
& \overline{W}_n \le \tau \qquad \forall n\in N, && (3.7)\\
& Z = \sum_n (h_n \overline{I}_n + p_n \overline{B}_n) + \lambda K/Q, \qquad \forall n\in N\cup 0 && (3.8)\\
& S_n \ge 0 \qquad \forall n\in N, && (3.9)\\
& Q \ge 0, && (3.10)\\
& Q \le Q_0, && (3.11)\\
& R \ge -1, && (3.12)\\
& Z \ge 0, && (3.13)\\
& X_n \in \{0,1\} \qquad \forall n\in N, && (3.14)\\
& Y_{in} \in \{0,1\} \qquad \forall i\in I,\ \forall n\in N && (3.15)
\end{aligned}
$$

The objective function (3.1) minimizes the total cost of setting up the facilities and the inventory cost at both the service centers and the plant. Constraint (3.2) ensure that each customer must be assigned to exactly one service center. Constraint (3.3) enforces that the customer can only be assigned to an open facility. Constraint (3.4) is the preference constraint. It assigns each customer to the most preferred available service center based on the preference. Constraint (3.5) sets the maximum inventory level at the service center not exceed the capacity. Constraint (3.6) enforces the reorder point at the plant not exceed the maximum inventory position at the plant. Constraint (3.7) is the complicated constraint which says that the average waiting time at any service center should not exceed the mean target response time. The expected waiting time $\overline{W}_n$ is calculated based on Q, R and S. Constraints (3.9) - (3.15) are the non-negativity and the integrality constraints. In constraint (3.12), $R = -1$ means the reorder is made when a backorder occurs.

Problem [ILP] is hard because of constraints (3.7). The mean response time $\overline{W}_n$ needs to be calculated using the mean backorder level and to find the mean backorder level, we need to know the inventory decision based on a certain location allocation decision, which makes [ILP] not solvable directly using commercial software. Mak and Shen [11] used Lagrangian relaxation to separate the constraint (3.7) from the other constraints. We use a similar idea as in Riaz [20]. A cutting-plane method is used to separate constraint (3.7).

When dropping constraints (3.5), (3.6), (3.7), (3.9), (3.10), (3.11) and (3.12), the problem becomes:

$$
\begin{aligned}
\text{[MP]}: \quad & \min \quad \sum_{n\in N} f_n X_n + Z && && (3.16)\\
& \text{s.t.} \quad \sum_{n\in N} Y_{in} = 1 && \forall i \in I && (3.17)\\
& Y_{in} \le X_n, && \forall i \in I,\ \forall n \in N_i && (3.18)\\
& Y_{in} \ge X_n - \sum_{l=1}^{n-1} X_l, && \forall i \in I,\ \forall n \in N_i && (3.19)\\
& Z \ge 0 && && (3.20)\\
& X_n \in \{0,1\} && \forall n \in N && (3.21)\\
& Y_{in} \in \{0,1\} && \forall i \in I,\ \forall n \in N && (3.22)
\end{aligned}
$$

The [MP] is a location allocation problem featured with the customer preference constraint and it can be solved by commercial software directly.

In the next chapter, we describe how to deal with the complicating constraint (3.7) and the exact method to calculate the mean response time $\overline{W}_n$. In Chapter 5, we detail the exact cutting-plane algorithm.

Chapter 4

The Inventory Stocking Problem

The $(Q,\ R)$ policy is a continuous review policy where we place an order of size Q whenever the inventory level drops to level R. It is also known as batch ordering policy. According to Muckstadt [16], the key assumption of the model is there is no more than one single order outstanding at any point in time. This implies the expectation of demand over a lead time never exceeds Q. We choose an appropriate R to satisfy the demand during the lead time. A large Q will decrease the order cost since it will decrease the number of orders in a period. However at the same time it will increase the average holding cost as the average in-stock inventory level is higher. Thus, selecting Q involves a trade-off between the order cost and the inventory holding cost.

The $(S,\ S-1)$ policy is also a continuous review policy known as base stock policy. By following the $(S,\ S-1)$ policy, an order is placed immediately whenever a demand occurs for one or more units of an item and the order quantity matches the size of the demand exactly. Therefore, the inventory position is constant over the infinite planning horizon.

In our model, the plant operates under $(Q,\ R)$ policy while the service centers operate under $(S,\ S-1)$ policy. The most related work to our problem is by Topan and Bayindir [27]. They minimize the total holding and order costs subject to an aggregate mean response time constraint. However, in our model, we also have to consider the backorder cost. We need to find the optimal $(S,\ S-1)$ policy for each service center and the optimal $(Q,\ R)$ policy for the plant.

4.1 Problem Formulation

Recall that K is the fixed order cost and λ is the mean demand faced by the plant. Let $\hat{N}$ denote the set of locations where a service center is open. The inventory stocking problem is formulated as:

$$
\begin{aligned}
\text{[SP]}: \quad & \min \quad Z = \sum_{n \in \hat{N} \cup 0} (h_n \overline{I}_n + p_n \overline{B}_n) + \lambda K / Q && (4.1) \\
& \text{s.t.} \quad S_n \le C_n, \qquad \forall n \in \hat{N} && (4.2) \\
& \qquad \overline{W}_n \le \tau, \qquad \forall n \in \hat{N} && (4.3) \\
& \qquad Q \le Q_0, && (4.4) \\
& \qquad R \le C_0 + Q_0, && (4.5) \\
& \qquad S_n \ge 0, \qquad S_n \text{ integer}, \forall n \in \hat{N} && (4.6) \\
& \qquad Q \ge 1, R \ge -1, \qquad Q, R \text{ integer} && (4.7)
\end{aligned}
$$

In the formulation, the objective function (4.1) is to minimize the total cost including the inventory holding and backorder cost both at the service centers and at the plant, as well as the order cost at the plant. Constraint (4.2) makes sure the inventory level at each service center lower than the capacity. Constraint (4.3) is the service constraint that says the mean waiting time at each facility can not exceed the target. Constraints (4.4) and (4.5) set upper bounds on Q and R. Constraints (4.8) and (4.9) are the integrality constraints for decision variables $Q, R,$ and S_n. The complicating constraint is (4.3). To deal with it, we analyze each SC as a queue. By Little's law, we have:

$$
\overline{W}_n = \frac{\overline{B}_n}{\lambda_n} \tag{4.8}
$$

Substituting (4.8) in expression (4.3) and get

$$\overline{B}_n \leq \tau\lambda_n \tag{4.9}$$

To solve the inventory problem, we need to evaluate the backorder term $\overline{B}_n$.

4.2 Inventory Level at the Plant and at the Service Centers

Table 4.1: Notations

n	Service center index, 0 represents the plant
Y_n	Demand during lead time at service center $n \in \hat{N}$
Y_0	Demand during lead time at the plant
T_0	Lead time at the plant
T	Lead time between service center n and the plant
$I_n(Q, R, S)$	On-hand inventory level at service center $n \in \hat{N}$
$I_0(Q, R, S)$	On-hand inventory level at the plant
$IP_0(Q, R, S)$	Inventory position at the plant
$B_n(Q, R, S)$	Backorder level at service center $n \in \hat{N}$
$B_0(Q, R, S)$	Backorder level at the plant
$O_n(Q, R, S)$	Outstanding orders at service center $n \in \hat{N}$
$B_0^n(Q, R, S)$	Backorder level of service center n at the plant

For sake of brevity, we rewrite the variables omitting the parameters that the variables depend on, e.g., $B_0(Q,\ R\ S)$ is denoted as B_0. An open service center operates under base stock policy so that a demand occurrence at a service center will automatically trigger an order at the plant. Since the demand at each service center follows a Poisson distribution, the plant faces demand with Poisson distribution as well. Muckstadt [17] shows that the inventory position at the plant which is IP_0 is uniformly distributed between R+1 and R+Q. Based on result from Topan et. al [27] for multiple item model, we derive the steady state distributions of parameters for the single item model as follow

$$I_0 - B_0 = IP_0 - Y_0 \tag{4.10}$$

Equation (4.10) holds when the replenishment time at the plant is constant. In our model, we assume the time to replenish one unit is a constant μ. For a given order size Q, the replenishment time is a constant $T_0 = Q\mu$. Therefore, the demand at the plant during replenishment time Y_0 has a Poisson distribution with mean $\lambda Q\mu$. According to (4.10), I_0 and B_0 are:

$$P(I_0 = x) = \begin{cases} \dfrac{1}{Q} \displaystyle\sum_{k=max(R+1,x)}^{R+Q} \mathrm{P}\{Y_0 = k - x\} & \text{for } 1 \leq x \leq R+Q \\ \dfrac{1}{Q} \displaystyle\sum_{k=r+1}^{R+Q} \mathrm{P}\{Y_0 \geq k\} & \text{for } x = 0 \end{cases} \tag{4.11}$$

$$P(B_0 = x) = \begin{cases} \dfrac{1}{Q} \displaystyle\sum_{k=R+1}^{R+Q} \mathrm{P}\{Y_0 = k + x\} & \text{for } x \geq 1 \\ \dfrac{1}{Q} \displaystyle\sum_{k=R+1}^{R+Q} \mathrm{P}\{Y_0 \leq k\} & \text{for } x = 0 \end{cases} \tag{4.12}$$

Then we need to have the steady state distribution of the inventory level as well as the backorder level at each service center. Since every service center uses a base stock policy, the inventory position is always a constant at the base stock level S. Therefore, the inventory position at time t is:

$$IP_n(t) = S_n = I_n(t) - B_n(t) + O_n(t) \tag{4.13}$$

In equation (4.13), we need to get the expression of I_n and O_n to calculate B_n. To find O_n, we see that the number of outstanding orders at time t at service center n is the sum of the number of backorders delivered to service center n at time $t-T$ and the demand during the lead time T. Thus, O_n can be expressed as:

$$O_n(t) = B_0^{(n)}(t-T) + Y_n(t-T, t) \tag{4.14}$$

In equation (4.14), we can calculate the first term $B_0^{(n)}$ using conditional probability on B_0 as:

$$\mathrm{P}\{B_0^{(n)} = x\} = \sum_{y=x}^{\infty} \mathrm{P}\{B_0^{(n)} = x | B_0 = y\} \times \mathrm{P}\{B_0 = y\} \quad \text{for } x \geq 0 \tag{4.15}$$

In this expression, $B_0^{(n)}|B_0$ follows a binomial distribution with parameters B_0 and $\frac{\lambda_n}{\lambda_0}$ [16]. If we use the same idea to deal with (4.14), we will see that:

$$\mathrm{P}\{O_n = x\} = \sum_{y=0}^{x} \mathrm{P}\{Y_n = y\} \times \mathrm{P}\{B_0^{(n)} = x - y\}, \quad \text{for } x \geq 0 \tag{4.16}$$

where Y_n follows a Poisson distribution with mean $\lambda_n T$.

We come back to equation (4.13). We could see that if there is any on-hand inventory, the backorder level must be 0 which says if I_n is positive, B_n must be 0. Now we can calculate the steady state distribution of I_n as:

$$P(I_n = x) = \begin{cases} \mathrm{P}\{O_n = S_n - x\} & \text{for } 1 \leq x \leq S_n \\ 1 - \sum_{x=1}^{S_n} \mathrm{P}\{I_n = x\} & \text{for } x = 0 \end{cases} \tag{4.17}$$

Now we have all the expressions we need to calculate the backorder level. Then we can find the expression of the backorder level as:

$$\begin{aligned} \overline{B}_0 &= \overline{Y}_0 - R - \frac{Q+1}{2} + \overline{I}_0 \\ \overline{B}_n &= \overline{O}_n - S_n + \overline{I}_n \end{aligned} \tag{4.18}$$

Then we can use the expression of the backorder in constraint (4.9).

4.3 Search algorithm

In this section, an exact algorithm is proposed to solve the inventory stocking problem introduced in Section 4.1. This algorithm is based on the enumeration of all feasible solutions to calculate the optimal policy parameters $(Q,\ R)$ for the plant and the optimal policy parameter S_n for each service center that satisfy both response time and the capacity constraints.

The algorithm starts by setting Q and R at their upper limit. The upper limit for Q is the order size limit Q_0 and the upper limit for R is the plant capacity limit

C_0. Then it enumerates on all feasible values of S_n for each open service center with respect to the mean target response time constraint. We look for a value of S_n that gives the lowest total cost, including the total holding cost and the backorder cost at service centers. If setting S_n at its highest value C_n cannot satisfy the mean target response time constraint for any open service center, we conclude the inventory problem as infeasible. Once we find a local minimum solution for given $(Q,\ R)$, we keep Q the same, decrease R by one unit, and repeat the enumeration procedure. After R reaches the lower bound which is -1, we decrease Q by one unit and set R to its upper limit to go through this enumeration again until Q hits the lower bound value 0.

After generating all the local optimal solutions, we select the minimum to be the global optimal solution. If all local solutions are infeasible, we conclude [SP] is infeasible.

Before presenting the search algorithm, we define $\hat{Z} = \hat{Z}_0 + \sum_{n \in \hat{N}} \hat{Z}_n$ where $\hat{Z}_0 = \lambda K/Q + h_0 \overline{I}_0 + p_0 \overline{B}_0$ and $\hat{Z}_n = h_n \overline{I}_n + p_n \overline{B}_n$ for $n \in \hat{N}$.

The search algorithm is summarized as:

Search algorithm to determine $(Q,\ R)$ at the plant and $(S,\ S-1)$ at the SCs.

Initialize $Q = Q_0$

While $Q > 0$

Initialize $R = C_0 + Q_0$

While $R > -1$

Calculate $\hat{Z}_0$

For Each open service center n

Set $S_n = 0$

Calculate B_n

While $S_n < C_n$

If $B_n > \lambda_n * \tau$

$S_n = S_n + 1$, Calculate B_n

Else

Calculate $\hat{Z}_n$, Update $\hat{Z}_n$

End If

End While

Update $\hat{Z}$, Update $(Q,\ R,\ S_n)$

End For

$R = R - 1$

End While

$Q = Q - 1$

End While

4.4 Useful Lemmas

In order to narrow the search space and improve the algorithm, we could use the results from Topan et al. [27] to improve the lower bound on R based on stochastic dominance [21].

Definition 1. (First-Order Stochastic Dominance). A cumulative distribution A first-order stochastic dominates another distribution B iff

$$P(A \geq x) \geq P(B \geq x) \tag{4.19}$$

for all x with a strict inequality over some interval.

Definition 1 is equally saying A first-order stochastic dominates B iff $F_A(x) \leq F_B(x)$ where $F_A(x)$ is the cumulative distribution function of A and $F_B(x)$ is the cumulative distribution function of B.

Corollary 1 If A first-order stochastic dominates B

$$E_A(x) \geq E_B(x) \tag{4.20}$$

We use notation $\geq_{st}$ to denote the first-order stochastic dominance.

Lemma 1. For any $R^+ > R$

$$B_0(Q, R) \geq_{st} B_0(Q, R^+) \tag{4.21}$$

Proof. From (4.12), we have $P(B_0(Q,R) \le x) = \frac{1}{Q}\sum_{k=R+1}^{R+Q} P(Y_0 \le k+x)$. Increasing R will not change the summation interval. Denote $t = R+p+x$ where $1 \le p \le Q$ and $t^+ = R^+ + p + x$ where $1 \le p \le Q$ and therefore $t \le t^+$. Since $t \le t^+$ and $P(Y_0)$ is a cumulative distribution function of Y_0, $P(Y_0 \le t) \le P(Y_0 \le t^+)$ which result in the $\sum_{t=R+1+x}^{R+Q+x} P(Y_0 \le t) \le \sum_{t^+=R^++1+x}^{R^++Q+x} P(Y_0 \le t^+)$ for the same summation interval length. Therefore $P(B_0(Q,R) \le x) \le P(B_0(Q,R^+) \le x)$. By definition, it is the same as (4.21).

Lemma 2. For any $R^+ > R$

$$B_0^{(n)}(Q,R) \ge_{st} B_0^{(n)}(Q,R^+) \tag{4.22}$$

$$O_n(Q,R) \ge_{st} O_n(Q,R^+) \tag{4.23}$$

Inequality (4.22) can be directly derived from (4.21) and (4.15). Inequality (4.23) is a direct result from (4.14) and (4.22).

Lemma 3. For any $R^+ > R$

$$\mathrm{P}(I_n(Q,R,S_n) = 0) \ge \mathrm{P}(I_n(Q,R^+,S_n) = 0) \tag{4.24}$$

Proof. $\mathrm{P}(I_n(Q,R,S_n) = 0) = \mathrm{P}(O_n(Q,R,S_n) \ge S_n)$. According to (4.23) and Definition 1, $\mathrm{P}(O_n(Q,R,S_n) \ge S_n) \ge \mathrm{P}(O_n(Q,R^+,S_n) \ge S_n)$.

Lemma 4. For any $R^+ > R$, k is a positive integer parameter

$$\mathrm{P}(B_n(Q,R,S_n) \geq k) \geq \mathrm{P}(B_n(Q,R^+,S_n) \geq k) \tag{4.25}$$

$$B_n(Q,R,S_n) \geq_{st} B_n(Q,R^+,S_n) \tag{4.26}$$

Proof. $\mathrm{P}(I_n(Q,R,S_n) = 0) = \mathrm{P}(B_n(Q,R,S_n) \geq 0)$. By (4.24), $\mathrm{P}(B_n(Q,R,S_n) \geq 0) \geq \mathrm{P}(B_n(Q,R^+,S_n) \geq 0)$. For any other positive integer parameter k, $\mathrm{P}(B_n(Q,R,S_n) \geq k) = \mathrm{P}(O_n(Q,R,S_n) \geq S_n + k)$. Therefore, (4.25) is a direct result from (4.23) and (4.26) is a direct result of (4.24) and (4.25).

Corollary 2. For any $R^+ > R$

$$E(B_n(Q,R,S_n)) \geq E(B_n(Q,R^+,S_n)) \tag{4.27}$$

Corollary 2 is used in our search algorithm to reduce the search space since when the value of Q is fixed, if some larger value of R cannot provide a feasible solution, any smaller value R cannot provide a feasible solution either.

Chapter 5

Solution of the Inventory-Location Problem

The original formulation [ILP] is a stochastic mixed integer optimization program that is difficult to solve. In this chapter, a cutting-plane algorithm is provided to solve the problem.

The algorithm first drops constraints (3.5) -(3.12), the relaxed master problem [MP] is then a location allocation problem. The solution to the relaxed master problem gives a lower bound to the original problem. Given the assignment of the customers and the location decisions from the solution of the [MP], the subproblem is an inventory stocking problem [SP]. By solving [SP], we may obtain an upper bound on the original problem and valid cuts. These cuts are then added to the relaxed master problem. If the subproblem is feasible, we obtain an optimality cut from the subproblem and then the lower bound may be improved. If the subproblem is infeasible, then a feasibility cut is added. The algorithm iterates between the solution of [MP] and [SP] until the upper bound and the lower bound are equal.

5.1 Valid Cuts and Algorithm

The optimal solution of [MP], defined as $(\hat{X}, \hat{Y}, \hat{Z})$ gives the following information: $\hat{X}$ gives which service centers are open and $\hat{Y}$ gives the assignment of customers to open SCs. We define $\hat{N} = \{n : \hat{X}_n = 1\}$ to be the set of open service centers and $\hat{\lambda}_n = \sum\limits_{i \in I} \lambda_i \hat{Y}_{in}$ be the demand rate faced at service center n. We then solve the inventory stocking problem [SP] using $\hat{N}$ and $\hat{\lambda}$.

When [MP] is solved, in the first iteration, Z will be set to 0. After solving [SP] for a given $(\hat{X}, \hat{Y})$, Z is either updated to $\hat{Z}$ which means [SP] is feasible or it is concluded that [SP] is infeasible.

If the subproblem is feasible, an optimality cut is formulated as:

$$Z \geq \hat{Z} - \hat{Z}\Big(\sum_{n \in \hat{N}} (1 - X_n) + \sum_{n \notin \hat{N}} X_n\Big) \tag{5.1}$$

where $\hat{N}$ is the set of open service centers and $\hat{Z}$ is the minimum total inventory cost given $\hat{N}$.

If the same set of service centers is selected, the cut is reduced to:

$$Z \geq \hat{Z} \tag{5.2}$$

which forces Z to be $\hat{Z}$. If a different set of service center is selected, then $\sum_{n \in N}(1 - X_n) + \sum_{n \notin N} X_n$ is not 0 and the cut is redundant.

If the subproblem is infeasible, we add the feasibility cut defined as:

$$\sum_{n \in \hat{N}}(1 - X_n) + \sum_{n \notin \hat{N}} X_n \geq 1 \tag{5.3}$$

If the same set of service centers is open, the cut is:

$$\sum_{n \notin \hat{N}} X_n \geq 1 \tag{5.4}$$

which forces at least one more service center to open. In other words the current infeasible solution is removed from the search space.

Cuts (5.1) and (5.3) are valid since the two cuts do not remove any feasible solution and do remove the infeasible solutions. Therefore, the search space is narrowed by adding cuts and the solution will converge to optimality.

We decompose the original problem into the relaxed master problem [MP] which is a location-allocation problem and the subproblem which is an inventory stocking problem [SP]. The algorithm solves the relaxed master problem which gives a lower bound on the original problem. Given information from the solution of [MP], the algorithm solves the subproblem as well. If the subproblem is feasible, an optimality cut is added to [MP] and the upper bound is updated and if the subproblem is infeasible, a feasibility cut is added to [MP]. The algorithm performs the two steps iteratively until the lower bound is equal to the upper bound.

The cutting-plane algorithm is described below:

Cutting-Plane Algorithm [CP]

Initialize: $UB = \text{inf}$, $LB = 0$.

While $LB \neq UB$

Step 1. Solve [MP], obtain solution ($\hat{X}$, $\hat{Y}$, $\hat{Z}$), update LB,

Step 2. If $UB = LB$, Stop

Step 3. Solve [SP] using search algorithm

- If [SP] is feasible:

    - Construct feasible solution ($\hat{X}$, $\hat{Y}$, $\hat{Z}$), update UB.

    - Add optimality cut (5.1) to [MP].

- If [SP] is infeasible:

    - Add feasibility cut (5.3) to [MP].

- Go to step 1

Chapter 6

Numerical Testing and Sensitivity Analysis

In this chapter, we test the effectiveness of the cutting-plane algorithm [CP]. The algorithm is implemented in Matlab R2012 on a computer with Intel(R) Core(TM) i5-2500 CPU @3.30GHz, 16.00GB RAM and Windows 7. The master problem [MP] is solved by solver CPlex Version 12.6.

6.1 Performance of the Cutting Plane Algorithm

In this section we test the performance of [CP] using an industrial dataset from Bombardier Inc. and datasets from the literature.

6.1.1 The Industry Dataset

We use the same dataset as Riaz [20]. The dataset is obtained from Bombardier Inc. This dataset has 20 potential SC locations and 121 customers. The demand data has 20 different parts. We use one spare part which has the highest demand. Different parameters are set to form 4 instances to test the effectiveness of the algorithm under different scenarios.

The distance from customer i to facility j is in hours and the demand is monthly based. Lead time for every facility is 0.23 months and the order size limit is 5 units. We have two different versions of Bombardier instances. The differences are:

- The plant and service center capacity in version 1 is 10 units and in version 2 is 5 units;
- The replenishment time for one unit at the plant in version 1 is $\frac{0.9}{\lambda}$ and in version 2 is $\frac{0.5}{\lambda}$;
- The mean target response time requirement in version 1 is 0.025 months and in version 2 is 0.01 months;
- The distance requirement in version 1 is 40 hours and in version 2 is 25 hours.

Each instance is solved with 3 different values of the order cost: 0, 5, 10.

Version 2 is considered to be a more difficult version than version 1 because the capacity limit in version 2 is smaller which could cause a higher backorder level but the mean target response time is smaller. The distance requirement is also smaller in version 2 which may require more facilities to open to fulfill customer demand.

The notation used in Tables 6.1 and 6.2 is: Iter stands for the number of iterations of [CP]; # of SC stands for the number of open SCs; LB stand for lower bound; K is the fixed order cost at the plant; Q is the optimal reorder quantity at the plant, run time denotes the time to reach the optimal solution in clock seconds, # of F denotes the total number of feasibility cuts when reach optimal solution and CoF denotes the cost of first feasible solution.

Dataset	K	Iter	# of SC	LB	GAP	Q	Run Time	# of F	CoF
BBD V1	0	21	1	104680	0	1	215.3	0	104680
BBD V1	5	21	1	104694	0	4	212.3	0	104694
BBD V1	10	21	1	104710	0	5	211.8	0	104710
BBD V1	15	21	1	104799	0	5	211.8	0	104799
BBD V2	0	82	2	202580	0	1	620.4	0	202890
BBD V2	5	82	2	202602	0	4	608.1	0	202913
BBD V2	10	82	2	202610	0	5	602.1	0	202920
BBD V2	15	82	2	202809	0	5	602.1	0	202925

Table 6.1: Performance of [CP] for Bombardier Instances

Table 6.1 presents the results of running [CP] on Bombardier instances. Optimality gap is 0 for all instances. The time to get the optimal solution varies between 3 minutes and 10 minutes. When order cost is 0, the plant has an optimal reorder size 1 which means the plant is running under $(S,\ S-1)$ policy. Increasing the order cost will shorten the run time.

6.1.2 Daskin Instances

Daskin instances consist of a 49-node, a 88-node and a 150-node datasets. The data is based on the 1990 United Stated Census, where 49 nodes represent the 48 capital cities and Washington D.C.; 88 nodes represent the 50 most populated cities and the 48 capital cites; and 150 nodes represent the 150 largest cities. A node is either a SC location or a demand point. The following assumptions are made: the facility location costs f_j is the same for all j; the demand rates λ_i are obtained by dividing the census population figures by 10^6; the unit holding costs h_0 and h_n are 50; and backorder cost p is 150.

There are two versions where the shipment lead time α_j is obtained by dividing the distance from the potential SC location node to the demand location by a factor of 100 in version 1 and 1000 in version 2. The other differences are:

- The plant and SC capacities $C_0 = C_j = 10$ in version 1 and $C_0 = C_j = 5$ in version 2;
- The replenishment time for one unit at the plant is $\frac{0.9}{\lambda}$ in version 1 and $\frac{0.5}{\lambda}$ in version 2;
- The response time requirement $\tau = 5.5$ in version 1 and $\tau = 1.5$ in version 1;
- The distance requirement $d_{max} = 2000$ in version 1 and $d_{max} = 500$ in version 2.

Version 2 of the datasets are considered more difficult as the capacities, response time and distance requirement are set to be less than those of version 1. As a consequence, it is expected that more facilities will open in order to fulfill demand.

Dataset	K	Iter	# of SC	LB	GAP	Q	Run Time	# of F	CoF
Da49 V1	0	3	1	48826	0	1	3.43	0	48826
Da49 V1	5	3	1	49132	0	3	3.35	0	49132
Da49 V1	10	3	1	49320	0	5	3.26	0	49320
Da49 V1	15	3	1	49390	0	5	3.27	0	49392
Da49 V2	0	2	5	283670	0	1	2.43	0	283670
Da49 V2	5	2	5	283892	0	3	2.21	0	283892
Da49 V2	10	2	5	284170	0	5	2.18	0	284170
Da49 V2	15	2	5	284260	0	5	2.18	0	284160
Da88 V1	0	2	1	49226	0	1	2.27	0	49226
Da88 V1	5	2	1	49279	0	3	2.18	0	49279
Da88 V1	10	2	1	49316	0	5	2.10	0	40316
Da88 V1	15	2	1	49421	0	5	2.10	0	49421
Da88 V2	0	2	7	417840	0	1	2.42	0	417840
Da88 V2	5	2	7	417891	0	3	2.31	0	417891
Da88 V2	10	2	7	417930	0	5	2.12	0	417930
Da88 V2	15	2	7	417955	0	5	2.13	0	417955
Da150 V1	0	45	1	104430	0	1	2898.31	14	104571
Da150 V1	5	45	1	104967	0	3	2853.87	14	105192
Da150 V1	10	45	1	105400	0	5	2698.64	14	105925
Da150 V1	15	45	1	105465	0	5	2697.60	14	105990
Da150 V2	0	1075	6	607500	-	1	-	-	-
Da150 V2	5	-	-	-	-	-	-	-	-
Da150 V2	10	-	-	-	-	-	-	-	-

Table 6.2: Performance of [CP] for Daskin Instances

Table 6.2 presents the results of running [CP] on Daskin instances. The last two lines are not complete because it takes over 35 hours to get the optimal solutions. Optimality gap is 0 for all other instances and the run time varies between 2 seconds and 49 minutes.

Since Riaz [20] studied a two-echelon supply chain which operates a base stock policy at both the service centers and the plant while the system we study operates a base stock policy at the service centers and a batch ordering policy at the plant, the different policy settings make the algorithm perform differently.

Since we use the exact approach to find the solution, we need to compare our results with the exact solution result in Riaz [20]. However, the result in Riaz [20] is only for METRIC-like approximation and negative binomial approximation and the exact approach is expected to take longer than the approximation approaches. So, we compare our results with those in Riaz [20] that take the longer time among the Metric-like and negative binomial approximation.

The negative binomial approach for Bombardier V1 in Riaz [20] work takes 43 seconds and 21 iterations and in our model, it takes more than 200 seconds to find the optimal solution when the order cost is 0, which is theoretically the same system as Riaz [20]. This time difference is due to time of solving [SP]. In our model, we enumerate on both Q and R in every iteration which is very time consuming. This is also shown in Daskin 150 city instance Version 1, where the run time of our system performs almost 1000 seconds worse than Riaz [20] work. When Q_0 and C_0 increase, solution time increases exponentially.

From Tables 6.2 and 6.1, when the order cost is 10, the run time is relatively short compared to the 0 order cost case. This happens since in our enumeration process, if the order cost is positive, the search algorithm will not need to go through every possible Q and R. The difference is only within 0.5 seconds for Daskin 49 city version 1 but is almost 200 seconds for Daskin 150 city version 2.

To better understand how these parameters affect the system in terms of the total cost and run time, we perform experiments using different parameters in the next section.

6.2 Effect of Different Replenishment Policies

In the previous section, we showed that [CP] can solve the small industrial case in acceptable time. To better understand the effect of the policy change and the impact of the parameters, including plant and service center capacity and the ordering size, we run several experiments.

6.2.1 Effect of Capacity

To better understand the impact of changing the capacity of the SCs and the plant, we choose Bombardier case to test. The reason that the Bombardier case is chosen is because the number of iterations to get the optimal solution for both versions of Bombardier data set is reasonable. We also pick the cases with order cost 5 and

10.

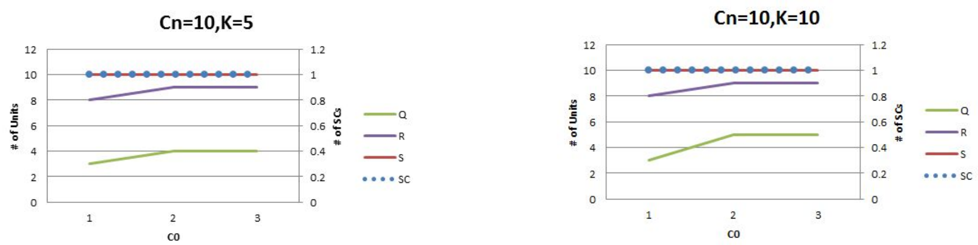


Figure 6.1: Optimal Decisions Under Different C_0 for BBD V1

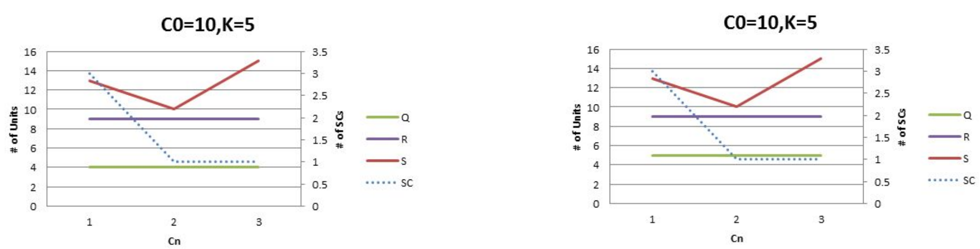


Figure 6.2: Optimal Decisions Under Different C_n for BBD V1

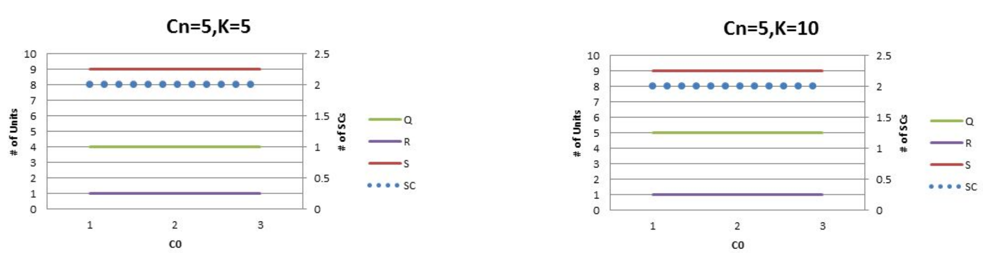
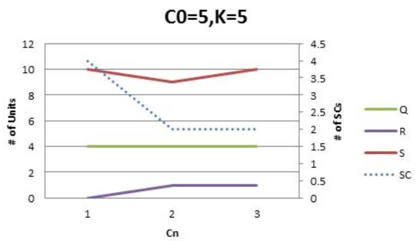


Figure 6.3: Optimal Decisions Under Different C_0 for BBD V2



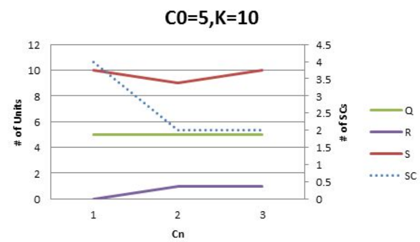


Figure 6.4: Optimal Decisions Under Different C_n for BBD V2

When more than one SC are needed, S_n is the total stock level over all open SCs, $\overline{I}_n$ and $\overline{W}_n$ is the inventory decision at the service center which has the longest mean target response time.

According to Table A.16 and Figures 6.1-6.4, it is shown that changing the replenishment policy at the plant doesn't change the location decision of the supply chain when the order cost is small compare to the fixed facility cost but it changes the inventory decision at the plant. It further shows that the location decision is affected by the service center's capacity. The plant capacity may affect the optimal order size and reorder point. For Bombardier dataset version 1, when the plant's capacity drops to 5, even when the order cost is large, the plant orders 3 units and keeps the reorder point at the maximum. When the plant's capacity is 10, the order size is affected by the order cost: a larger order cost gives a larger order size so as to decrease the number of orders. The reorder point is higher than base stock level in Riaz [20] and leads to a larger holding cost. When the plant's capacity increases to 15, since the location decision and the inventory decision at the service center are not changed, the plant works the same way as when the capacity is 10. On the other hand, when the capacity of the service center changes, the location decision and the inventory decision at both the plant and the service centers may change. When the capacity of the service center drops to 5, a single service center cannot satisfy customer demand subject to the mean target response time constraint even when the service center keeps inventory at its maximum level. Therefore, more service centers are needed under this scenario. Since the location decision changes, the inventory decision at each service center changes also. However, since the plant faces the same aggregate demand from the service centers, the inventory decision at the plant does not change. When the capacity of the service center increases to 15, the reorder point at the plant drops to 2 and the reorder point at the service center increases to 15. A possible explanation is that the system shifts inventory from the plant to the service centers in order to save on holding cost.

In conclusion, we found that when changing the replenishment policy at the plant, the location decision and the service center's inventory decision are not affected but the inventory decision at the plant changes. Fewer orders are needed but higher ordering and inventory holding costs are incurred. We also found that the plant's capacity may affect the inventory decision but makes no impact on the location decisions and the service center's inventory decisions. However, the service center's capacity may affect both the location decision and the inventory decisions at both the service centers and the plant.

6.2.2 Effect of the Order Size and Order Cost

In this section, we will discuss the impact of the order size and the order cost parameters. To see the impacts, we choose the Bombardier cases with order cost 5,10 and 15 to test.

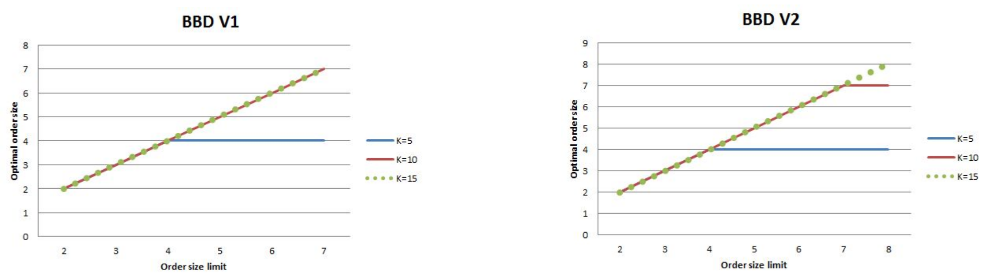


Figure 6.5: Relation Between Q and Q_0 Under Different K

Table A.15 and Figure 6.5 show that, changing of either the order cost or the order limit affects the inventory decision at the plant. For Bombardier dataset version 1, when the order cost drops to 5, the order size at the plant is either the allowable order size or 4. When the allowable order size decreases to 2, the reorder point at the plant decreases to 8 and it decreases the inventory holding cost at the plant. However, the trade-off is the order cost, since number of orders increases, and the backorder cost increases. When the allowable order size increases to 5 or more, the order size does not change. This is because if the plant orders more, although the order cost drops, the plant needs to keep more safety stock since the replenishment time for the order increases as well and the trade-off is a larger expense on holding inventory. Increasing the allowable order size either decreases the total cost or keeps the total cost unchanged. Theoretically, increasing maximum order size implies to looser constraint (3.11), if the constraint was binding, a better solution is obtained, otherwise the solution remains unchanged.

In conclusion, changing of the order cost and the maximum order size will affect the plant's inventory decisions. Increasing the maximum order size either improves the inventory decision or keeps the inventory decision the same.

6.2.3 Effect of Large Order Cost

Recall Table A.16, when the capacity of the plant is 5, and the capacity of the service center is 10, although the order cost is different, the optimal order size is the same at 3. However, This may not be the case when the order cost is large compared to facility location cost. In this section, further tests on larger order cost values are presented.

We use dataset from Bombardier version 1 to see how the large order cost will affect the supply chain design decision when the capacity of the plant is limited.

Figure 6.6: Location Decision Under Different Order Cost for BBD V1

Figure 6.6 shows that when the order cost jumps from 20 to 3000, it changes the location allocation decisions and the inventory decision at both the plant and the service centers. The reason is when the order cost is large enough, it will force the plant to decrease order times which will increase the order size. However, ordering more means the lead time is longer and the required safety stock is more. For the decision to open 1 service center, keeping most stock at the plant can not satisfy the demand from the service center and will not satisfy the mean target response time constraint. Therefore, the system decides to open more service centers and to keep more stock at the service centers. This is the decision balances the order cost and the location cost.

Next, we use dataset from Bombardier version 2 to see how the large order cost will affect the supply chain design decision under different plant lead times. The parameters are $C_0 = C_n = 5$, $Q_0 = 50$. Notation μ represents the lead time of replenishing a single unit. i.e. $\mu = 0.5$ if the lead time of replenishing one unit is $0.5/\lambda$.

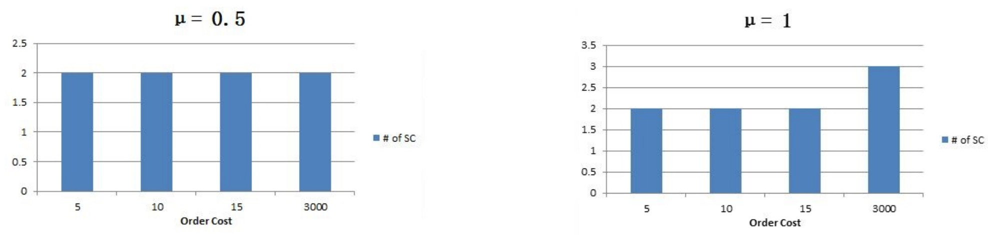


Figure 6.7: Location Decision Under Different Order Cost for BBD V2

Figure 6.7 shows that increasing the order cost to a large value will force the plant to order fewer times but to order more every time. When the lead time for replenishing a single unit is short, ordering more will not increase the total lead time significantly. Therefore the plant can still keep enough safety stock for the demand during lead time. However, when the lead time for replenishing a single unit is long, ordering more will increase the total lead time significantly. Therefore according to Figure 6.7, the plant may not increase the order size when the order cost is not large. However, when the order cost is arbitrarily large, the plant will certainly order more which results in a significant time increase. In that case, the plant may not be able to hold enough stock to satisfy the demand from the service centers and the service centers are not able to satisfy the mean target response time constraint. Therefore the system needs to open one more service centers and to keep more stock in order to satisfy the mean target response time constraint. This is a trade off between order cost and facility location cost.

In conclusion, increasing the order cost under some conditions, i.e. limited capacity of the plant and longer replenishment time, will affect the location decision of the supply chain.

Chapter 7

Conclusion

The problem presented in this thesis is a two-echelon inventory location problem. The SCs face stochastic demands that are independent Poisson processes. The replenishment rate for a single item at plant is μ. The SCs operate under base stock policy and the plant operates under batch ordering policy. The SCs need to satisfy the mean target response time requirement for customers.

To the best of our knowledge, our paper is the first to find an optimal solution to a two-echelon inventory-location problem when the plant operates under batch ordering policy with fixed single unit replenishment time and the service centers operate under base stock policy with Poisson demand. We consider customer preference constraints when making the location-allocation decision and time-based service constraints when making the inventory decision. In the work of Topan et.al [27], the replenishment time at the plant is fixed but the replenishment time at the plant in our model varies based on the order size. The mean target response time constraint makes the problem difficult to solve. The cutting-plane algorithm is to break the problem into two parts which are location allocation problem and inventory stocking problem. We solve the location-allocation problem as master problem and solve the inventory stocking problem as subproblem to generate valid cuts. We provide an exact solution approach to the original inventory-location problem.

We used industrial data set obtained from Bombardier Inc. and data sets from the literature to test the cutting-plane algorithm. The results show that when the search space is relatively small, the cutting-plane algorithm finds the optimal solution in a reasonable time. The algorithm is time consuming when the maximum order size and plant capacity are relatively large.

Appendix A

Numerical Results

A.1 Bombardier Results

Iteration	#SC	LB	UB	GAP	Q
1	1	50000	104680	1.0936	1
2	1	50000	104680	1.0936	1
3	1	50000	104680	1.0936	1
4	1	50000	104680	1.0936	1
5	1	73346	128020	0.74548	1
6	1	85012	139690	0.64318	1
7	1	94168	148850	0.58064	1
8	1	94168	148850	0.58064	1
9	1	94168	148850	0.58064	1
10	1	94168	148850	0.58064	1
11	1	94168	148850	0.58064	1
12	1	94168	148850	0.58064	1
13	1	94168	148850	0.58064	1
14	2	100000	171540	0.71545	1
15	2	100000	171020	0.71024	1
16	2	100000	178690	0.78691	1
17	2	100000	178290	0.78289	1
18	2	100000	171680	0.7168	1
19	2	100000	171020	0.71024	1
20	1	100000	154680	0.54678	1
21	1	104680	104680	0	1

Table A.1: Bombardier V1 / order cost = 0

iteration	#of SC	Lb	Ub	GAP	Q
1	1	50000.0	104710.0	1.0943	5
2	1	50000.0	104710.0	1.0943	5
3	1	50000.0	104710.0	1.0943	5
4	1	50000.0	104710.0	1.0943	5
5	1	73346.0	128060.0	0.74595	5
6	1	85012.0	139730.0	0.64359	5
7	1	94168.0	148880.0	0.58101	5
8	1	94168.0	148880.0	0.58101	5
9	1	94168.0	148880.0	0.58101	5
10	1	94168.0	148880.0	0.58101	5
11	1	94168.0	148880.0	0.58101	5
12	1	94168.0	148880.0	0.58101	5
13	1	94168.0	148880.0	0.58101	5
14	2	100000.0	171580.0	0.71579	5
15	2	100000.0	171060.0	0.71058	5
16	2	100000.0	178730.0	0.78726	5
17	2	100000.0	178320.0	0.78324	5
18	2	100000.0	171060.0	0.71058	5
19	2	100000.0	171710.0	0.71715	5
20	1	100000.0	154710.0	0.54713	5
21	1	104710.0	104710.0	0	5]

Table A.2: Bombardier V1 / order cost = 10

Iteration	#SC	LB	UB	GAP	Q
1	2	100000	202890	1.0289	1
2	2	100000	202580	1.0258	1
3	2	100000	202580	1.0258	1
4	2	123350	226210	0.83394	1
5	2	135010	236930	0.75489	1
6	2	144170	254640	0.76626	1
7	2	144170	247070	0.71377	1
8	2	144170	247070	0.71377	1
9	2	144170	246750	0.71151	1
10	2	144170	246290	0.70839	1
11	2	144170	246750	0.71151	1
12	2	144170	246750	0.71151	1
13	2	144170	246750	0.71151	1
14	2	144170	246350	0.70874	1
15	2	144170	247060	0.71366	1
16	3	150000	247060	0.64703	1
17	3	150000	272780	0.81851	1
18	3	150000	272780	0.81851	1
19	2	158360	261250	0.64973	1
20	2	163210	265780	0.62851	1
21	2	163210	265780	0.62851	1
22	2	163210	266090	0.63041	1
23	2	167510	278470	0.66235	1
24	2	168660	271240	0.60818	1
25	2	168660	271240	0.60818	1
26	2	168660	271240	0.60818	1
27	3	173350	303280	0.74954	1
28	3	173350	303280	0.74954	1
29	3	173350	303340	0.74992	1
30	3	173350	303340	0.74992	1
31	3	173350	295770	0.70626	1
32	3	173350	295660	0.70559	1
33	2	175650	278560	0.58583	1
34	2	177610	279790	0.57529	1
35	2	179180	281310	0.56997	1
36	2	179180	282080	0.5743	1
37	2	179180	282080	0.5743	1
38	3	185010	306710	0.65779	1
39	3	185010	306880	0.65869	1
40	3	185010	306880	0.65869	1
41	2	188340	298810	0.58656	1
42	2	188340	290910	0.54465	1
43	2	188340	291240	0.54638	1
44	2	188340	290910	0.54465	1

Table A.3: Bombardier V2 / order cost = 0

45	2	188340	291240	0.54638	1
46	2	188340	290910	0.54465	1
47	3	194170	324100	0.66916	1
48	3	194170	324100	0.66916	1
49	3	194170	324100	0.66916	1
50	3	194170	324100	0.66916	1
51	3	194170	324100	0.66916	1
52	3	194170	324180	0.66958	1
53	3	194170	316400	0.62952	1
54	3	194170	316400	0.62952	1
55	3	194170	316370	0.62937	1
56	3	194170	316370	0.62937	1
57	3	194170	316370	0.62937	1
58	3	194170	316940	0.63232	1
59	3	194170	324210	0.66976	1
60	3	194170	316940	0.63232	1
61	3	194170	324180	0.66958	1
62	3	194170	324100	0.66916	1
63	3	194170	324140	0.66936	1
64	3	194170	324140	0.66936	1
65	3	194170	324140	0.66936	1
66	3	194170	316940	0.63232	1
67	3	194170	324160	0.6695	1
68	3	194170	316480	0.62993	1
69	3	194170	324230	0.66986	1
70	3	194170	316940	0.63232	1
71	3	194170	316940	0.63232	1
72	3	194170	324160	0.6695	1
73	3	194170	316480	0.62993	1
74	3	194170	316680	0.63098	1
75	3	194170	316680	0.63098	1
76	3	194170	316940	0.63232	1
77	2	198220	300350	0.51522	1
78	3	200000	330000	0.65	1
79	4	200000	330000	0.65	1
80	3	200000	330000	0.65	1
81	3	200000	330100	0.65052	1
82	2	202580	202580	0	1

Table A.4: Bombardier V2 / order cost = 0

Iteration	# of SC	LB	UB	GAP	Q
1	2	100000	202920	1.0292	5
2	2	100000	202610	1.0261	5
3	2	100000	202610	1.0261	5
4	2	123350	226240	0.83423	5
5	2	135010	236970	0.75515	5
6	2	144170	254670	0.7665	5
7	2	144170	247110	0.71402	5
8	2	144170	246780	0.71175	5
9	2	144170	246780	0.71175	5
10	2	144170	246780	0.71175	5
11	2	144170	247110	0.71402	5
12	2	144170	246780	0.71175	5
13	2	144170	247090	0.7139	5
14	2	144170	246380	0.70898	5
15	2	144170	246330	0.70863	5
16	3	150000	246330	0.6422	5
17	3	150000	272810	0.81874	5
18	3	150000	272810	0.81874	5
19	2	158360	261280	0.64995	5
20	2	163210	265820	0.62873	5
21	2	163210	265820	0.62873	5
22	2	163210	266130	0.63063	5
23	2	167510	278500	0.66256	5
24	2	168660	271270	0.60839	5
25	2	168660	271270	0.60839	5
26	2	168660	271270	0.60839	5
27	3	173350	303310	0.74974	5
28	3	173350	303310	0.74974	5
29	3	173350	303380	0.75012	5
30	3	173350	303380	0.75012	5
31	3	173350	295690	0.70579	5
32	3	173350	295810	0.70646	5
33	2	175650	278590	0.58603	5
34	2	177610	279820	0.57549	5
35	2	179180	282120	0.5745	5
36	2	179180	281340	0.57016	5
37	2	179180	282120	0.5745	5
38	3	185010	282120	0.52486	5
39	3	185010	306910	0.65888	5
40	3	185010	306750	0.65798	5
41	2	188340	298840	0.58674	5
42	2	188340	290950	0.54484	5
43	2	188340	291270	0.54657	5
44	2	188340	291270	0.54657	5

Table A.5: Bombardier V2 / order cost = 10

45	2	188340	290950	0.54484	5
46	2	188340	290950	0.54484	5
47	3	194170	324130	0.66934	5
48	3	194170	324130	0.66934	5
49	3	194170	324130	0.66934	5
50	3	194170	324210	0.66976	5
51	3	194170	324130	0.66934	5
52	3	194170	316980	0.6325	5
53	3	194170	324210	0.66976	5
54	3	194170	324130	0.66934	5
55	3	194170	324130	0.66934	5
56	3	194170	316440	0.6297	5
57	3	194170	316980	0.6325	5
58	3	194170	324130	0.66934	5
59	3	194170	324170	0.66954	5
60	3	194170	324170	0.66954	5
61	3	194170	316410	0.62955	5
62	3	194170	316410	0.62955	5
63	3	194170	324200	0.66968	5
64	3	194170	316720	0.63116	5
65	3	194170	324170	0.66954	5
66	3	194170	324250	0.66994	5
67	3	194170	324250	0.66994	5
68	3	194170	324270	0.67004	5
69	3	194170	316980	0.6325	5
70	3	194170	316980	0.6325	5
71	3	194170	316980	0.6325	5
72	3	194170	316980	0.6325	5
73	3	194170	316510	0.63011	5
74	3	194170	324200	0.66968	5
75	3	194170	316720	0.63116	5
76	3	194170	316510	0.63011	5
77	2	198220	300380	0.5154	5
78	3	200000	330030	0.65017	5
79	4	200000	330030	0.65017	5
80	3	200000	330030	0.65017	5
81	3	200000	330140	0.6507	5
82	2	202610	202610	0	5

Table A.6: Bombardier V2 / order cost = 10

A.2 Daskin Results

Iteration	# of SC	LB	UB	GAP	Q
1	1	48400	48826	0.008809	1
2	1	48800	49226	0.008736	1
3	1	48826	48826	0	1

Table A.7: Daskin 49 V1 / order cost = 0

iteration	# of SC	LB	UB	GAP	Q
1	1	48400	49320	0.019017	5
2	1	48800	49720	0.018861	5
3	1	49320	49320	0	5

Table A.8: Daskin 49 V1 / order cost = 10

Iteration	# of SC	LB	UB	GAP	Q
1	5	282200	283670	0.005224	1
2	5	283670	283670	0	1

Table A.9: Daskin 49 V2 / order cost = 0

iteration	# of SC	LB	UB	GAP	Q
1	5	282200	284170	0.006975	5
2	5	284170	284170	0	5

Table A.10: Daskin 49 V2 / order cost = 10

iteration	# of SC	LB	UB	GAP	Q
1	1	48800	49226	0.008736	1
2	1	49226	49226	0	1

Table A.11: Daskin 88 V1 / order cost = 0

Iteration	# of SC	LB	UB	GAP	Q
1	1	48800	49316	0.010574	5
2	1	49316	49316	0	5

Table A.12: Daskin 88 V1 / order cost = 10

Iteration	# of SC	LB	UB	GAP	Q
1	7	417000	417840	0.002022	1
2	7	417840	417840	0	1

Table A.13: Daskin 88 V2 / order cost = 0

Iteration	# of SC	LB	UB	GAP	Q
1	7	417000	417930	0.002237	5
2	7	417930	417930	0	5

Table A.14: Daskin 88 V2 / order cost = 10

A.3 Sensitivity Analysis Result

Table A.15: Effects on order size and Cost

Instance	K	Q_0	Q	R	S_n	$\overline{I}_0$	$\overline{B}_0$	Total Cost
BBD V1	5	2	2	8	10	2.76	3.84	104706
BBD V1	5	3	3	9	10	3.53	3.25	104701
BBD V1	5	4	4	9	10	3.54	3.21	104694
BBD V1	5	5	4	9	10	3.54	3.21	104694
BBD V1	5	6	4	9	10	3.54	3.21	104694
BBD V1	5	7	4	9	10	3.54	3.21	104694
BBD V2	5	2	2	0	9	0	0	202608
BBD V2	5	3	3	1	9	0.14	0	202604
BBD V2	5	4	4	1	9	0.15	0	202602
BBD V2	5	5	4	1	9	0.15	0	202602
BBD V2	5	6	4	1	9	0.15	0	202602
BBD V2	5	7	4	1	9	0.15	0	202602
BBD V1	10	2	2	8	10	2.76	3.84	104731
BBD V1	10	3	3	9	10	3.53	3.25	104728
BBD V1	10	4	4	9	10	3.54	3.21	104720
BBD V1	10	5	5	9	10	3.56	3.18	104710
BBD V1	10	6	6	9	10	3.57	3.16	104701
BBD V1	10	7	7	10	10	4.18	2.97	104693
BBD V2	10	2	2	0	9	0	0	202711
BBD V2	10	3	3	1	9	0.14	0	202695
BBD V2	10	4	4	1	9	0.15	0	202649
BBD V2	10	5	5	1	9	0.15	0	202610
BBD V2	10	6	6	1	9	0.16	0	202553
BBD V2	10	7	7	2	9	0.77	0	202512
BBD V2	10	8	7	2	9	0.77	0	202512
BBD V1	15	2	2	8	10	2.76	3.84	104818
BBD V1	15	3	3	9	10	3.53	3.25	104811
BBD V1	15	4	4	9	10	3.54	3.21	104805
BBD V1	15	5	5	9	10	3.56	3.18	104799
BBD V1	15	6	6	9	10	3.57	3.16	104787
BBD V1	15	7	7	10	10	4.18	2.97	104779
BBD V2	15	2	2	0	9	0	0	202832
BBD V2	15	3	3	1	9	0.14	0	202826
BBD V2	15	4	4	1	9	0.15	0	202818
BBD V2	15	5	5	1	9	0.15	0	202809
BBD V2	15	6	6	1	9	0.16	0	202798
BBD V2	15	7	7	2	9	0.77	0	202786
BBD V2	15	8	8	3	9	1.14	0	202772

Table A.16: Effects on Capacity Change

Instance	K	C_0	C_n	# of SC	Q	R	Sn	$\overline{I}_0$	$\overline{B}_0$	$\overline{I}_n$	$\overline{W}_n$
BBD V1	5	5	10	1	3	8	10	2.77	3.61	2.94	0.018
BBD V1	10	5	10	1	3	8	10	2.77	3.61	2.94	0.018
BBD V1	5	10	10	1	4	9	10	3.54	3.21	2.94	0.018
BBD V1	10	10	10	1	5	9	10	3.56	3.18	2.94	0.018
BBD V1	5	15	10	1	4	9	10	3.54	3.21	2.94	0.018
BBD V1	10	15	10	1	5	9	10	3.56	3.18	2.94	0.018
BBD V1	5	10	5	3	4	9	13	3.54	3.21	1.71	0.023
BBD V1	10	10	5	3	5	9	13	3.56	3.18	1.71	0.023
BBD V1	5	10	10	1	4	9	10	3.54	3.21	2.94	0.018
BBD V1	10	10	10	1	5	9	10	3.56	3.18	2.94	0.018
BBD V1	5	10	15	1	4	2	15	0	0	4.59	0.014
BBD V1	10	10	15	1	5	2	15	0	0	4.59	0.014
BBD V2	5	4	5	2	4	1	9	0.15	0	1.55	0.024
BBD V2	10	4	5	2	5	1	9	0.15	0	1.55	0.024
BBD V2	5	5	5	2	4	1	9	0.15	0	1.55	0.024
BBD V2	10	5	5	2	5	1	9	0.15	0	1.55	0.024
BBD V2	5	6	5	2	4	1	9	0.15	0	1.55	0.024
BBD V2	10	6	5	2	5	1	9	0.15	0	1.55	0.024
BBD V2	5	5	4	4	4	0	10	0	0	0.41	0.023
BBD V2	10	5	4	4	5	0	10	0	0	0.41	0.023
BBD V2	5	5	5	2	4	1	9	0.15	0	1.55	0.024
BBD V2	10	5	5	2	5	1	9	0.15	0	1.55	0.024
BBD V2	5	5	6	2	4	1	10	0.15	0	1.55	0.024
BBD V2	10	5	6	2	5	1	10	0.15	0	1.55	0.024

Appendix B

Worst Case Scenario of the Cutting-Plane Algorithm

In our thesis, we proposed an cutting-plane algorithm to find the optimal solution to an inventory-location problem. The algorithm defines on two nested procedures. The outer procedure searches through all possible location allocation decisions and the inner procedure searches for the optimal inventory decision for a given location allocation decision and then generates cut for the outer procedure. In this section, we will going to find the maximum number of iterations that will be needed to find the optimal solution to the inventory-location problem in the worst case scenario.

First, in our model, the inner procedure is an enumeration algorithm that searches through all feasible combinations of (Q, R, S). In the worst case, the enumeration algorithm will need to search through all combinations of (Q, R, S). For a given standard of order size limit and the capacity of both the plant and the service centers, the maximum value of Q, R and S is fixed. Therefore the maximum number of iterations to find the optimal inventory decision for one service center as well as the plant is fixed. We call this number "M". However, when more than one service center is open, for example j service centers are open, in the worst case, the maximum number of iterations to find the optimal inventory decision is jM.

Second, the outer procedure is to find the location allocation decision, in the worst case, it will need to search through all possible combinations of open service centers, i.e, if there are n potential service centers, in the worst case, the procedure will need to iterates $2^n - 1$ times.

Finally, combining the result from above, for a problem with n potential service centers, and with given standard of order size and capacity of the plant and the service centers, in the worst case, the maximum number of iterations to find the optimal solution is given as:

$$N = C_n^1 M + 2C_n^2 M + 3C_n^3 M + ... + nC_n^n M \tag{B.1}$$

According to (B.1), since M is not depending on n, we can factor it out as:

$$N = (C_n^1 + 2C_n^2 + 3C_n^3 + ... + nC_n^n)M \tag{B.2}$$

As known,

$$kC_n^k = \frac{kn!}{k!(n-k)!} = nC_{n-1}^{k-1} \tag{B.3}$$

Therefore,

$$C_n^1 + 2C_n^2 + 3C_n^3 + ... + nC_n^n = n(C_{n-1}^0 + C_{n-1}^1 + C_{n-1}^2 + ... + C_{n-1}^{n-1}) = n2^{n-1} \tag{B.4}$$

According to (B.4), (B.2) can be reduced to:

$$N = n2^{n-1}M \tag{B.5}$$

Therefore, in the worst case, as n increases, the maximum number of iterations will increase exponentially.

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