

Design, Implementation, and Control of a Magnetic Levitated Planar Motor

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Magnetic levitated planar motors employ electromagnetic force to levitate and manipulate the movers. They have the characteristics of multi-degrees-of-freedom motion, contactless operation, high precision and high acceleration. They have shown great potential in the area of flexible manufacturing, material transportation and photolithography. Maglev Microrobotics Laboratory at the University of Waterloo has proposed a novel electromagnetic design with stationary square coil arrays and a 2D Halbach array mover. With this electromagnetic design, two platforms are fabricated and tested, the maglev test bench and the maglev floor. The maglev test bench is a 64-coil platform with a levitation area of 0.6 m by 0.6 m, which is a pilot project of the maglev floor. The maglev floor is a 768-coil platform, whose levitation area is 2.4 m by 1.8 m. The maglev floor allows the tests to be closer to real-life manufacturing. The contributions of this thesis include the mover design, force and torque modeling between the mover and the coils, system integration, levitation implementation and advanced control for mover path tracking.

This thesis proposes a novel lookup-table-free data-driven method to model the force and torque on a 2D Halbach array when the magnet array is above a stationary coil. One critical contribution is the rapid and accurate force and torque modeling when the coil is located near the edge of the 2D magnet array. First, the area under the 2D Halbach array is divided into a central region, edge regions, and corner regions. Then, nonlinear force and torque components are identified and modeled with a modified third-order Fourier series for force and torque modeling. The proposed model is implemented on an industrial server, and the average computation time is measured to be $2.3 \mu s$. This model is universal for symmetric coils, verified through experimental measurements with circular and square coils of different sizes. The measurement result agrees well with the proposed modeling method, showing that the proposed method is suitable for real-time control of the maglev planar motor.

With accurate force and torque modeling, a new 2D Halbach mover is designed and fabricated. The coil dimensions are identified by the Fabry Factor, optimizing its magnetic flux density over the working area. For the mover, instead of directly aligning a magnetic pole with the coil center, a study area is identified to represent the entire levitation area. Then, the mover dimensions are identified through the analysis of energy efficiency, load-carrying ability and maximum horizontal force. A lightweight mover case with a cross feature on the top is designed to control the mover deformation down to 0.02 mm. Vicon visual markers are attached to the mover for the camera to measure the mover's location.

The modular test bench, including current amplifiers, coils, programmable logic controller and Vicon motion cameras, is then integrated and tested for levitation. A direct

wrench and active coil selection method is used to decouple the force and torque and determine the coil currents. With the force and torque model and the active coil selection method, the mover can move across different sets of active coils for long-stroke movement. Lastly, experimental validation results are presented. On the maglev test bench, the motion range is ± 120 mm in the horizontal directions. The platform is validated to have a full 360° range of yaw motion. The motion precision reaches 0.01 mm for the translational directions. The motion precision for the roll and pitch is 5 mdeg, and for yaw is 2 mdeg. Experiments are also conducted on the maglev floor. The mover's motion range is validated to be 2000 mm by 700 mm in the horizontal plane with a full yaw rotation range. The motion precision is 0.03 mm for the translational directions and 10 mdeg for the rotational directions. The maximum speed on both platforms are validated to be 500 mm/s for the translational motion and 75 deg/s for the yaw motion.

A novel predictor-based model predictive control is proposed to improve the tracking performance of the maglev planar motor. A Kalman filter and finite spectrum assignment based on an augmented system model are employed to forecast mover's location by compensating for the system delay due to data transmission and processing. The predicted location is fed back to model predictive control for control optimization under the limit of coil currents. Hildreth's quadratic algorithm is then applied to solve for the optimal control effort within the constraints. Comparative experiments demonstrate the effectiveness of the proposed control strategy, the total tracking error of which is much smaller than the result of a proportional-integral-derivative controller and standard model predictive control in all tests. Additionally, experimental results show that the proposed predictor-based model predictive control can tolerate higher speeds of the mover while retaining mover's steadiness. The experimental results suggest the proposed predictor-based model predictive control is an effective path-tracking strategy for magnetic levitated planar motors subject to delays.

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Dedication

This is dedicated to my parents, whose love and support have been a constant source of inspiration and strength throughout my journey.

Table of Contents

Examining Committee	ii
Author's Declaration	iii
Abstract	iv
Acknowledgements	vi
Dedication	vii
List of Figures	xii
List of Tables	xv
1 Introduction	1
1.1 Introduction to Flexible Manufacturing	1
1.2 Introduction to Magnetic Levitation	4
1.3 Objectives and Contributions	5
1.3.1 Objectives	5
1.3.2 Contributions	6
1.4 Thesis Outline	7

2	Maglev Planar Motors in Literature	9
2.1	System Design	9
2.1.1	Disk Magnet Mover	9
2.1.2	1D Halbach Array Mover	10
2.1.3	2D Halbach Array Mover	12
2.1.4	2D Permanent Magnet Array Mover	13
2.1.5	Rotary Mover	13
2.2	Force and Torque Modeling	15
3	Principles Of Maglev	18
3.1	System Coordinates	18
3.2	System Parameters	20
3.3	Magnetic Flux Density Modeling	20
3.3.1	Magnetic Nodes	20
3.3.2	Harmonic Expressions	22
3.3.3	Magnetic Flux Density Verification	24
3.4	Lorentz Force and Torque	24
4	Data-Drive Force and Torque Modeling	27
4.1	Zoning and Data Collection	27
4.2	Force	28
4.2.1	Central Region	28
4.2.2	Edge Region	30
4.2.3	Corner Region	34
4.3	Torque	36
4.3.1	Effective Torque Arm	36
4.3.2	Central Region	37
4.3.3	Edge Region	39

4.3.4	Corner Region	43
4.4	Validation, Analysis and Generalization	46
4.4.1	Experimental Validation	46
4.4.2	Computation Time and Limitations	47
4.4.3	Model Generalization	48
5	Electromagnetic Design	50
5.1	Coil Design	50
5.2	Mover Design	51
5.2.1	Mover Magnet Design	51
5.2.2	Mover Case Design	54
6	System Integration	56
6.1	Maglev Test Bench	56
6.1.1	Camera System	56
6.1.2	Modular Electromagnet Stator	58
6.2	Maglev Floor in Robohub	59
6.3	PID Control	60
6.3.1	System Model	60
6.3.2	Controller Design	61
6.4	Active Coil Selection and Current Decoupling	62
6.5	System Specifications	65
6.5.1	Test Bench Results	65
6.5.2	Maglev Floor Results	68
7	Advanced Control: Predictor-Based Model Predictive Control	78
7.1	Motivation	78
7.2	System Description	80

7.3	PMPC Control Design	80
7.3.1	Finite Spectrum Assignment Design	81
7.3.2	State and Output Prediction	82
7.3.3	Cost Function	84
7.3.4	Constraint Analysis	84
7.3.5	Hildreth's Quadratic Procedure	87
7.4	Experimental Validation	89
7.4.1	Step Response	90
7.4.2	Ramp Response	91
7.4.3	Circular Trajectory	92
7.4.4	Discussion	93
8	Conclusion and Recommendations	97
8.1	Conclusion	97
8.1.1	Data-Driven Modeling	97
8.1.2	System Design & Integration	98
8.1.3	Predictor-Based Model Predictive Control	98
8.2	Recommendations	98
8.2.1	Coil Temperature Estimation and Optimization	98
8.2.2	Multi-Mover Levitation and Collaboration	99
8.2.3	Operation Bandwidth Improvement	99
	References	100
	APPENDICES	110
A	List of Publications	111
A.1	Published Journal Articles	111
A.2	In-Process Journal Articles	111

List of Figures

1.1	Designs for flexible conveyors.	2
1.2	Designs for micro maglev conveyors.	3
1.3	Commercial MLPM.	4
2.1	Maglev systems with disk magnet movers.	10
2.2	Maglev systems with 1D Halbach array movers.	11
2.3	Maglev systems with 2D Halbach array movers.	14
2.4	2D permanent magnet array systems [7, 98].	15
2.5	Maglev rotatory motors.	16
3.1	(a) Schematic of a 1D Halbach array layout and its magnetic field [1] (b) Schematic of a 2D Halbach array layout and its magnetic field.	19
3.2	System coordinates, top view.	20
3.3	System layout with a single coil (a)bottom view (b)side view.	21
3.4	Magnetic nodes model on a cuboidal magnet with Y direction magnetization.	22
3.5	Comparison between magnetic field between ANSYS, magnetic node and harmonic model along the same line.	25
3.6	Schematic of the current density in a coil.	26
4.1	Magnetic flux density of the 2D Halbach array along $Y_m = \tau/2$	28
4.2	Regions under the 2D Halbach array, top view.	29
4.3	System layout showing the variables.	30

4.4	F_x in edge region 1 at 1.5 mm air gap.	32
4.5	f_1 , f_2 and f_3 value plane and lines at 1.5 mm air gap.	33
4.6	f_1 , f_2 and f_3 values at three different air gap.	34
4.7	Force and modeling error in edge region 1 at 5 mm air gap.	35
4.8	Fx in corner region 2 at 1.5 mm air gap.	36
4.9	Force and modeling error in corner region 2 at 5 mm air gap.	37
4.10	ETA in the central region at 1.5 mm air gap.	38
4.11	Torque and modeling error in the central region at 5 mm air gap.	40
4.12	ETA in edge region 1 at 1.5 mm air gap.	41
4.13	t_1 , t_2 , t_3 and t_4 value at 1.5 mm air gap.	42
4.14	t_1 , t_2 , t_3 and t_4 values at different air gap.	43
4.15	Torque and modeling error in edge region 1 at 5 mm air gap.	44
4.16	Torque and modeling error corner region 2 at 5 mm air gap.	45
4.17	Measurement setup (a) top view (b) side view.	46
4.18	Comparison of force and torque measurement and the model of the square coil.	47
4.19	Square and circular coils tested in this study.	48
4.20	Comparison of force and torque measurement and the model of the circular coil.	49
5.1	Fabry Factor distribution.	51
5.2	Study area for the mover geometry optimization.	54
5.3	Mover case design.	55
5.4	Simulation of the mover deformation by the internal force of the 2D Halbach mover.	55
6.1	Equipment setup with camera placement.	57
6.2	2D Halbach array mover with 6 reflective markers.	58
6.3	System stator overview (a) Modular electromagnet stator (b) Coil module.	59

6.4	Hardware connection diagram.	60
6.5	Maglev floor with coils lifted.	61
6.6	Maglev floor in operation.	62
6.7	Control diagram.	63
6.8	Coil activation with different mover locations. Active coils are shown in green. (a)25 coils activated (b)36 coils activated (c)29 coils activated. . . .	64
6.9	Step response in X_g direction.	66
6.10	Step response in Z_g direction.	67
6.11	Step response in γ direction.	68
6.12	Circular path tracking.	69
6.13	Maximum mover speed for long stroke movement.	70
6.14	Full 360° range of yaw motion.	71
6.15	Measurement precision of each axis.	72
6.16	Step response in the X_g direction on maglev floor.	73
6.17	Step response in the Z_g direction on maglev floor.	74
6.18	Step response in the γ direction on maglev floor.	75
6.19	Precision steps on each axis on the maglev floor.	76
6.20	A figure-eight path is tracked by the mover at 300 mm/s.	77
6.21	The maximum speed is tested to be 500 mm/s on the maglev floor.	77
7.1	Scheme of the proposed control structure.	81
7.2	Illustration of the m_x^g position prediction with 10 ms delay.	82
7.3	Examples of F_x and F_y constraints when the mover is at different locations along a short moving trajectory.	86
7.4	Step response comparisons between predictor-based MPC, MPC and PID.	90
7.5	Control effort comparison of three controllers during the step response.	91
7.6	Ramp response in X direction with the speed of 300 mm/s (a) X position during the ramp (b) position of other five axes during the motion.	95
7.7	Circular motion in the horizontal plane with the speed of 500 mm/s (a) m_x^g and m_y^g position (b) position of other four axes during the motion.	96

List of Tables

4.1	Dimensions of the primary system variables.	31
5.1	Coil dimensions.	52
5.2	Optimization results for a mover with 1" Z-magnet width.	53
5.3	Optimization results for a mover with 1.5" Z-magnet width.	53
5.4	Final mover dimensions.	53
6.1	Vicon V5 camera specifications.	57
6.2	PID controller parameters.	63
6.3	Tracking errors of each axis during the circular path.	67
7.1	Controller specifications of the step responses.	91
7.2	Controller performance specifications of the ramp responses.	92
7.3	Controller performance specifications of the circular trajectory tracking. . .	93

Chapter 1

Introduction

In this chapter, an introduction to existing flexible manufacturing technologies and magnetic levitation are given. The goal of the thesis is to investigate using the magnetic levitated motors for flexible conveying. The details goals and contribution are then presented in [section 1.3](#). Lastly, the layout of this thesis is introduced in [section 1.4](#).

1.1 Introduction to Flexible Manufacturing

A flexible manufacturing system is designed to provide manufacturers with the ability to quickly adjust their production schedules and products to meet market demands. Flexible manufacturing systems offer flexibility in several key aspects [92]: 1. Part variety: the capability of the system to produce different kinds of parts in different volumes. 2. Schedule change: if the system can perform different manufacturing tasks in changing schedules. 3. Error recovery: if the system can rapidly detect errors and if the system can elegantly recover from a breakdown. 4. New part test: if new part designs can be easily added to the existing production line. Compared to traditional manufacturing systems, flexible manufacturing systems offer significant advantages in adaptability and responsiveness to changes in product specifications, production volumes, and customer demands. It suits cases involving lower volume production, highly customizable products, and a dynamic market environment.

A flexible manufacturing system often contains the following elements: a central computer, machining stations, assembly stations, loading/ unloading systems, storage systems, conveying systems, human-machine interface and human-in-the-loop. The conveying system is crucial for system flexibility. It is expensive and time-consuming to modify the

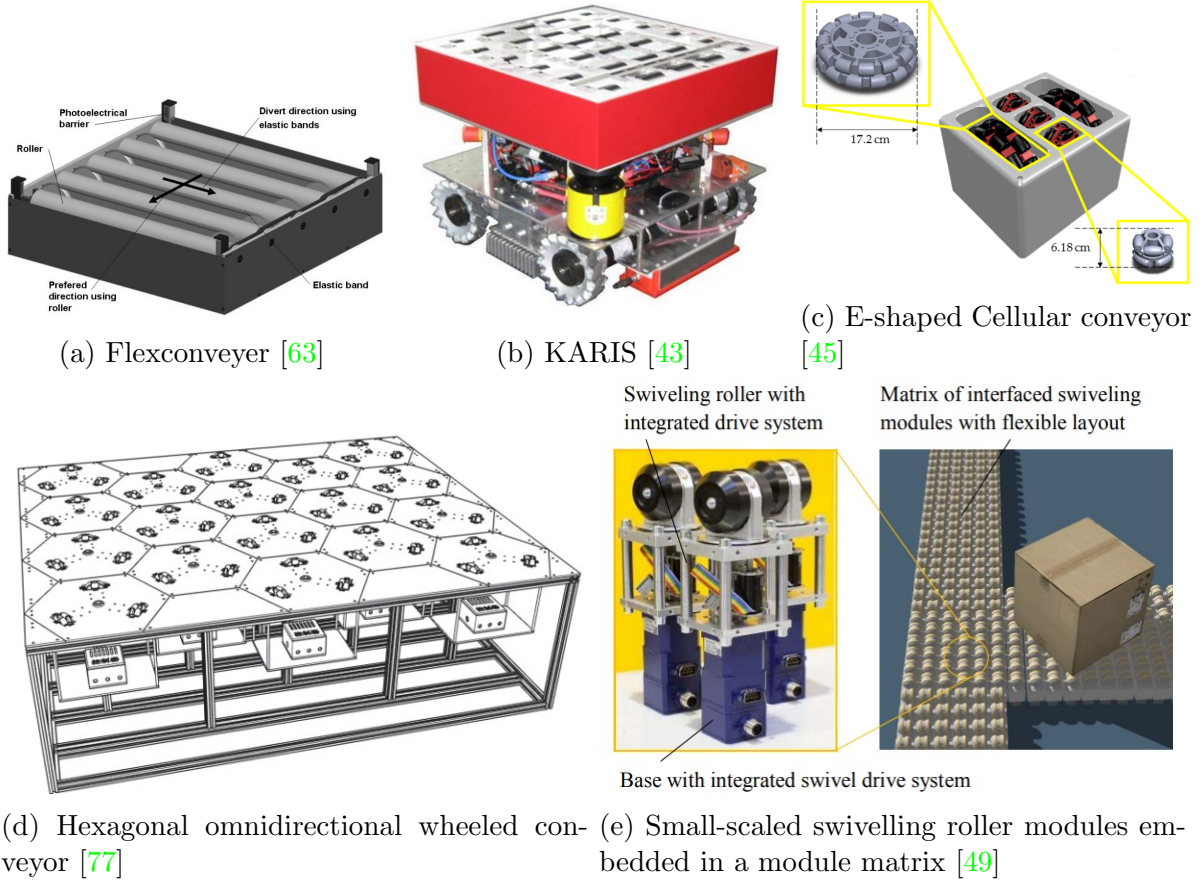


Figure 1.1: Designs for flexible conveyors.

layout of traditional conveyors since this process requires complicated mechanical and electrical modifications. Therefore, various flexible conveying systems have been designed. As shown in Figure 1.1a, Stephan [63] proposed a modular conveyor system named Flexconveyor. Each module can convey the four cardinal directions via belts. The modules can be randomly connected and can continuously exchange information with the adjacent modules to rapidly show routing information and achieve decentralized control. As shown in Figure 1.1b, KARIS is one of the extended designs of Flexconveyor [43]. With a similar belt conveying design on the top, each module is equipped with omnidirectional wheels, allowing rapid change of the overall layout of the conveying system.

Other than belt conveying systems, there are Cellular systems to deliver objects with Omni wheels. In Figure 1.1c, an e-pattern cell is presented [45]. This particular design

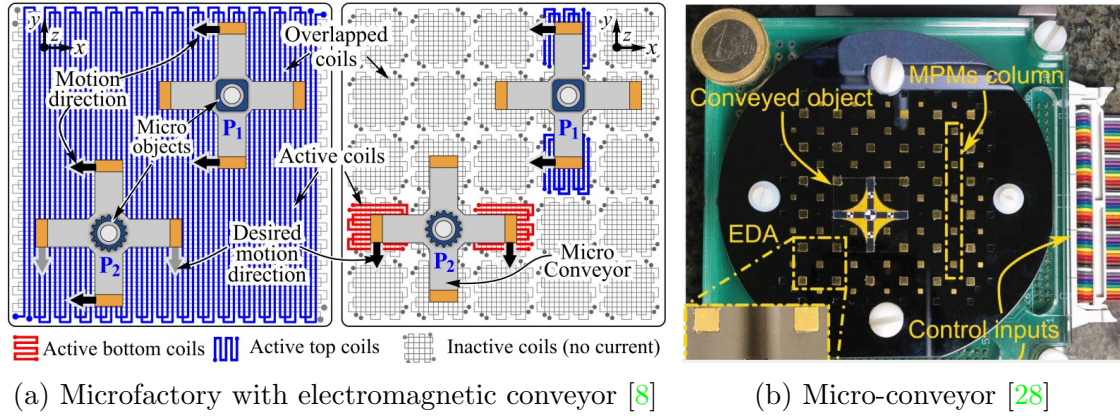
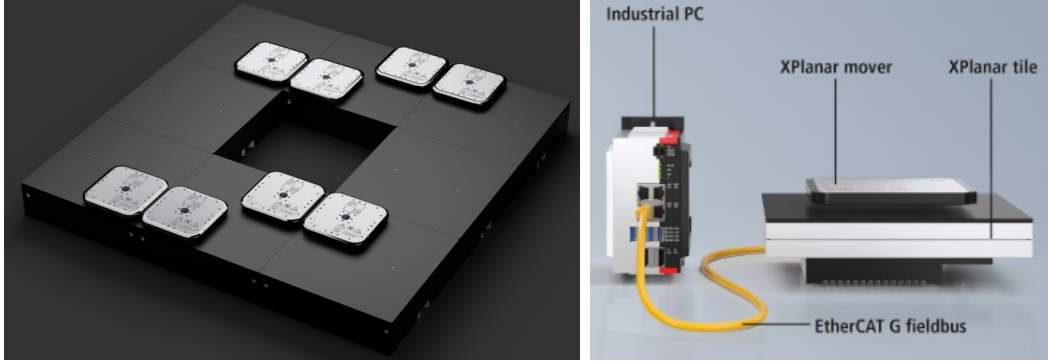


Figure 1.2: Designs for micro maglev conveyors.

allows the system to deliver the objects in x and y direction with a centralized control type. Similarly, a hexagonal omnidirectional wheeled conveyor is shown in Figure 1.1d [77, 30]. Lastly, there is a conveyor with individual small swivelling rollers design [49]. The size of each module is much smaller than the goods, and each module has multi-directional functionality for maximum flexibility. This system is shown in Figure 1.1e.

Magnetic levitation (maglev) has the unique character of no friction, high precision and low maintenance. Therefore, there are active investigations into using electromagnetic platforms for flexible manufacturing. Roberval Laboratory presented a smart surface with 1D Halbach arrays and linear coil stators [20, 8]. The surface can support multiple movers with three degrees of freedom (DOF). The total displacement area is 70 mm by 70 mm, the maximum rotation of each mover is $\pm 13^\circ$, and the maximum travel speed is 12 mm/s at a current of 0.8 A. This design is shown in Figure 1.2a. Another design for a micro-factory conveyor is shown in Figure 1.2b [28, 78]. This design applies linear PCB coils and a small cuboidal magnet array. This design has demonstrated limited motion range, and the position accuracy is around 0.5 mm.

There is also research regarding magnetic conveyor systems on a bigger scale. Zhang [100] proposed a maglev precision stage named Magtable, which can achieve six-DOF motion with a long stroke in the horizontal direction. The mover consists of disk magnets, and the stator is square coils. The position resolution of this system can reach 0.1 mm. The motion range is demonstrated to be 400 mm by 200 mm in the horizontal plane. The levitation height is validated to be 20 mm. Lu [58] proposed a six-DOF planar (MLPM) with an extendable motion range in the horizontal plane. This design can be used to deliver objects in lab or manufacturing settings. Other than academic research, there are



(a) Planar motor by Planar Motor Inc. [2]

(b) Xplanar by Beckhoff [3]

Figure 1.3: Commercial MLPM.

also commercial products promoting maglev technology for flexible manufacturing [2, 3] shown in Figure 1.3. These products offer flexible assembly and routing. They can achieve a maximum speed of 2m/s with a levitation height of 4 mm. The maximum load depends on the size of the mover. For a 23 cm by 23 cm mover, the maximum load is about 4 kg.

1.2 Introduction to Magnetic Levitation

Maglev uses electromagnetic force to manipulate the end effector. The principle of maglev is converting the magnetic energy to the kinematic energy of the manipulated object remotely via the magnetic field. The electromagnetic force can counter the gravitational force of the manipulated object to achieve levitation. They have special characteristics of non-contact, frictionless and highly precise. Therefore, they have wide application in magnetic bearing [75, 33], 3D printing [50, 51, 32], manipulation of microbot [26, 44, 52], transportation [55, 65], haptic devices [11, 68, 67] and precision stages [24, 37].

The focus of this thesis is on the MLPM. The MLPM can be divided into two kinds based on the mover type: permanent magnets mover [100] or coil mover[99, 16]. The coil mover system could provide accurate positioning and easy force decoupling. However, a power cable must be attached to the mover so that this design has only a limited range in the horizontal direction. Besides, an additional design for heat dissipation would add weight to the mover, which requires larger power to levitate the mover. The maximum power of the coils is usually limited. A planar motor with permanent magnet movers is more suitable for the purpose of flexible manufacturing. With no power cables attached to

the mover, the movers can travel expandable motion ranges. The collaboration between the movers will be easier since the problem of tangled power cables is avoided. Besides, the coils will be stationary. This allows the usage of bigger-sized, higher-powered coils with cooling design for higher-system performance.

1.3 Objectives and Contributions

In this thesis, a 2D Halbach array is designed for levitation with stator of square coils. Then, the electromagnetic system is integrated for the levitation experiments. Lastly, the system path tracking performance is improved with the PMPC controller.

The design of the electromagnetic stator is completed as prior work by Dr. Xiaodong Zhang and Dr. Chanuphon Trakarnchaiyo. The fabrication and the assembly of the two platforms were completed by Dr. Xiaodong Zhang, Dr. Chanuphon Trakarnchaiyo, Curtis Stewart, Dr. Hassan Askari, Dr. Moslem Sadeghigoughari and myself. The work of system integration, including control algorithm implementation, current decoupling algorithm integration and motion capture via Vicon camera, is a collaborative work by Curtis Stewart and myself.

1.3.1 Objectives

This thesis focuses on using the MLPM as a conveyor for flexible manufacturing. To achieve this goal, the following research objectives should be fulfilled:

1. A mover should be designed and made to fulfill the optimization goal.
 - (a) Develop a method to compute the force and torque between the 2D Halbach array and a coil.
 - (b) Identify the mover geometry satisfying the optimization goal given the coil geometry.
 - (c) Design a lightweight mover case to reduce mover deformation.
2. Complete the system integration of maglev test bench and maglev floor and conduct levitation experiments.
 - (a) Integrate the system hardware, including coils, power amplifiers, Vicon motion cameras and the Beckhoff industrial computer.

- (b) Implement the current decoupling algorithm.
 - (c) Conduct preliminary levitation experiments on maglev test bench.
 - (d) Implement the levitation strategy on maglev floor.
3. A proper control strategy for the mover to track a prescribed path should be implemented.
- (a) The controller should compensate for the system delays.
 - (b) The controller should find the control effort for the path tracking within the critical limitations of the coil current.

1.3.2 Contributions

The following contributions are achieved in this research:

Contribution 1: A novel real-time data-driven force and torque modeling on a 2D Halbach array by a symmetric coil is proposed and validated.

Finding the force and torque contribution of each coil on the mover is critical for the control of a MLPM. In the existing literature on 2D Halbach movers, the force and torque can be found by either a lookup table or in real-time with the harmonic model of magnetic flux density. However, a lookup table for multi-dimensional motion control occupies large system memory, and the harmonic model cannot model the end effect of the 2D Halbach array. Therefore, an accurate and fast force and torque model, including the end effect, is highly desired. A novel data-driven method is proposed, and it meets all the requirements. It is found that each computation only takes $2.3 \mu\text{s}$, and the measurements agree well with the computation result, making this method effective for the control of a MLPM.

Contribution 2: A novel mover geometry optimization method is proposed. A mover is designed and levitated using this method.

A common optimization method for the mover geometry is to place a coil right under a south or north pole of the mover, then identify the parameters to maximize the force. However, the edge effect of the 2D Halbach array, the current decoupling method and the relative location of the coil in the mover coordinates are not considered in the existing analysis. The newly proposed optimization method first identifies a study area, and solves for all the current maps. Then, the average power consumption and the average maximum horizontal can be identified accordingly. A mover is designed and fabricated with the

proposed method. Levitation experiments are conducted, and the results agree well with the optimization expectations.

Contribution 3: A novel active coil selection method is incorporated to the current decoupling, achieving extendable planar motion range and infinite yaw rotation of a 2D Halbach mover.

For MLPM with 2D Halbach mover, it is found in the literature that they can only travel for a limited motion range. A method for extendable planar motion range is desired. Therefore, a novel active coil selection method is proposed, incorporating the inverse wrench decoupling method. The method considers the contribution of each coil individually, enabling compensation for coil position error and gaps in coil modules, loosening fabrication tolerances. Experimental results show the proposed method can switch the mover between different active coil sets in the planar direction. Also, it is validated that this method allows infinite yaw rotation of the mover.

Contribution 4: A predictor-based model predictive controller (PMPC) is implemented and validated to improve the system tracking performance, as well as the steadiness and maximum speed.

For a MLPM, accurately tracking a prescribed path is desired. One thing to notice is that the control strategy should incorporate the critical coil current limits. Therefore, the model predictive control (MPC) is implemented on the MLPM since it can solve the optimal control effort with the optimization window under the coil current constraints. A predictor is added to the MPC design to compensate for the constant system delay. Comparative experiments show that the proposed PMPC effectively reduces the tracking error. Additionally, experimental results show that the proposed PMPC control can tolerate higher speeds of the mover while retaining the mover's steadiness.

1.4 Thesis Outline

The remaining chapters in this thesis are organized as follows:

Chapter 2 reviews the literature regarding various MLPMS by the mover, including disk magnet mover, 1D Halbach array mover, 2D Halbach array and rotary mover. Then, the force and torque modeling method between the coils and the permanent magnets is introduced.

Chapter 3 explains the levitation principles of the maglev system. First, three coordinates are introduced to describe the system. Then, magnetic node and harmonic expressions are introduced to calculate the magnetic flux density of a 2D Halbach array. Lorentz force rule is used to solve for the force and torque on the mover.

Chapter 4 presents the real-time data-driven force and torque modeling method. Based on the characteristics of the 2D Halbach array, the area under the array is divided into a center region, four edge regions and four corner regions. The force and torque equations are given for each region. Lastly, the proposed method is validated by comparing the model prediction, numerical results and actual measurements.

Chapter 5 shows the electromagnetic design of the coil and the 2D Halbach array. The coil dimension is optimized with the Fabry Factor. Then, the 2D Halbach array geometry is selected based on the energy efficiency, maximum load and average horizontal thrust within the area of study. Lastly, a lightweight mover case is designed to prevent the mover from bending.

Chapter 6 demonstrates the system integration details. The details regarding the camera system, modular stator design, proportional–integral–derivative (PID) controller design, and current decoupling methods are explained. Then, the levitation specifications on the test bench and the maglev floor are presented.

Chapter 7 shows the PMPC control implementation on the maglev system. Kalman filters are used to estimate the mover location and speed. Then, the system delay is compensated by finite spectrum assignment. With the delay-free state prediction, a MPC is employed to solve for the optimal control effort within the optimization window under the critical coil constraints. Comparative experiments are conducted to show the effectiveness of the proposed system in path tracking.

Chapter 8 concludes the current research and suggests future works.

Chapter 2

Maglev Planar Motors in Literature

In this chapter, a comprehensive literature regarding the magnetic levitated stages are presented. Then, methods used to compute the force and torque between permanent magnets and coils are reviewed.

2.1 System Design

2.1.1 Disk Magnet Mover

A disk magnet mover system refers to a maglev system whose mover consists of one or more disk magnets. Maglev systems with disk magnets are investigated because disk magnet movers have much bigger levitation height than Halbach arrays. The maximum levitation height found in the literature is 30 mm [100]. Therefore, maglev systems with disk magnets mover offer more flexibility in the motion range and rotation capabilities, and it can be more beneficial for applications like haptic devices. Due to the axial symmetric feature of a disk magnet, a single magnet mover has no control over the yaw rotation. A mover with more than one disk magnet can achieve full six-DOF control.

As shown in Figure 2.1a, Berkelman [12] has proposed a maglev system with two disk magnets as the mover with circular coils as the stator. This system has a motion range of 80 mm by 80 mm by 25mm. The maximum current of each circular coil is 3 A in DC. A motion capture system with LED infrared lights is used. In [13], the sensing system of this setup is redesigned with photodiodes to reduce the cost. In [64], their system is redesigned

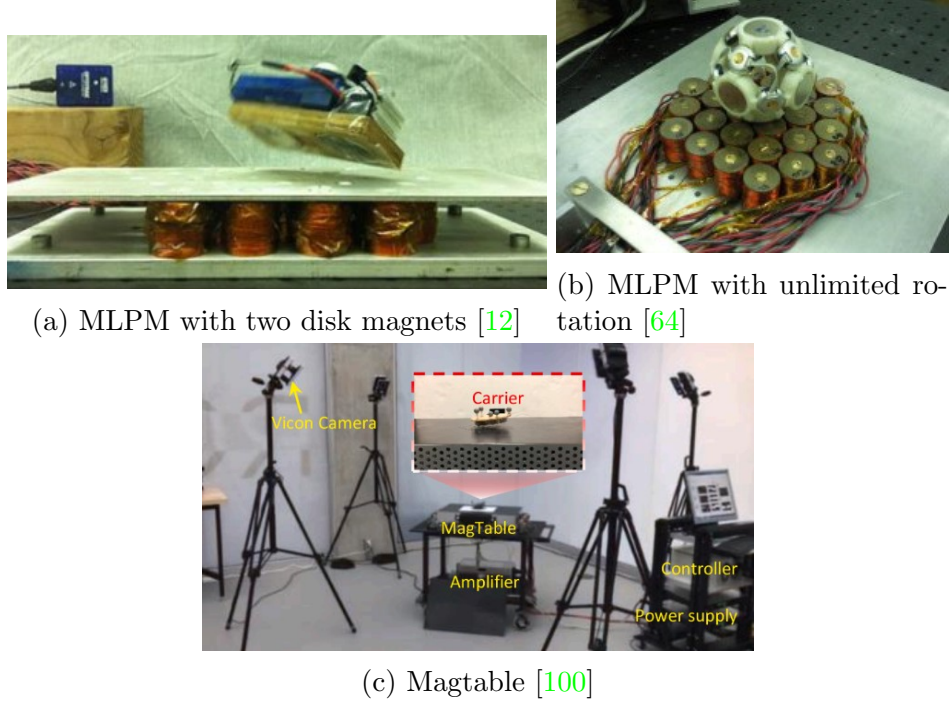


Figure 2.1: Maglev systems with disk magnet movers.

to a ball-shaped mover with six disk magnets on each side, shown in Figure 2.1b. This design is proven to have an unlimited rotation range.

Magtable [100] is proposed by the University of Waterloo, shown in Figure 2.1c. The stator is redesigned to square coils to enhance the magnetic flux density over the working area. This system achieved a working area of 400 mm by 200 mm 30 mm. The rotation range is $\pm 6^\circ$ in roll and pitch and $\pm 12^\circ$ in yaw rotation. Lai [54] from National Taiwan University also validated the design with a mover of 29 disk magnets and circular coils.

2.1.2 1D Halbach Array Mover

1D Halbach array has been investigated as a mover for the maglev system since the magnetic flux density of this design can be found analytically. Accordingly, the analytical force and the torque of each coil can be found in real-time. This mover usually consists of four 1D Halbach arrays. Each 1D Halbach array is considered as a forcer, which can provide a vertical force and a horizontal force. With four forcers, the mover can achieve six-DOF

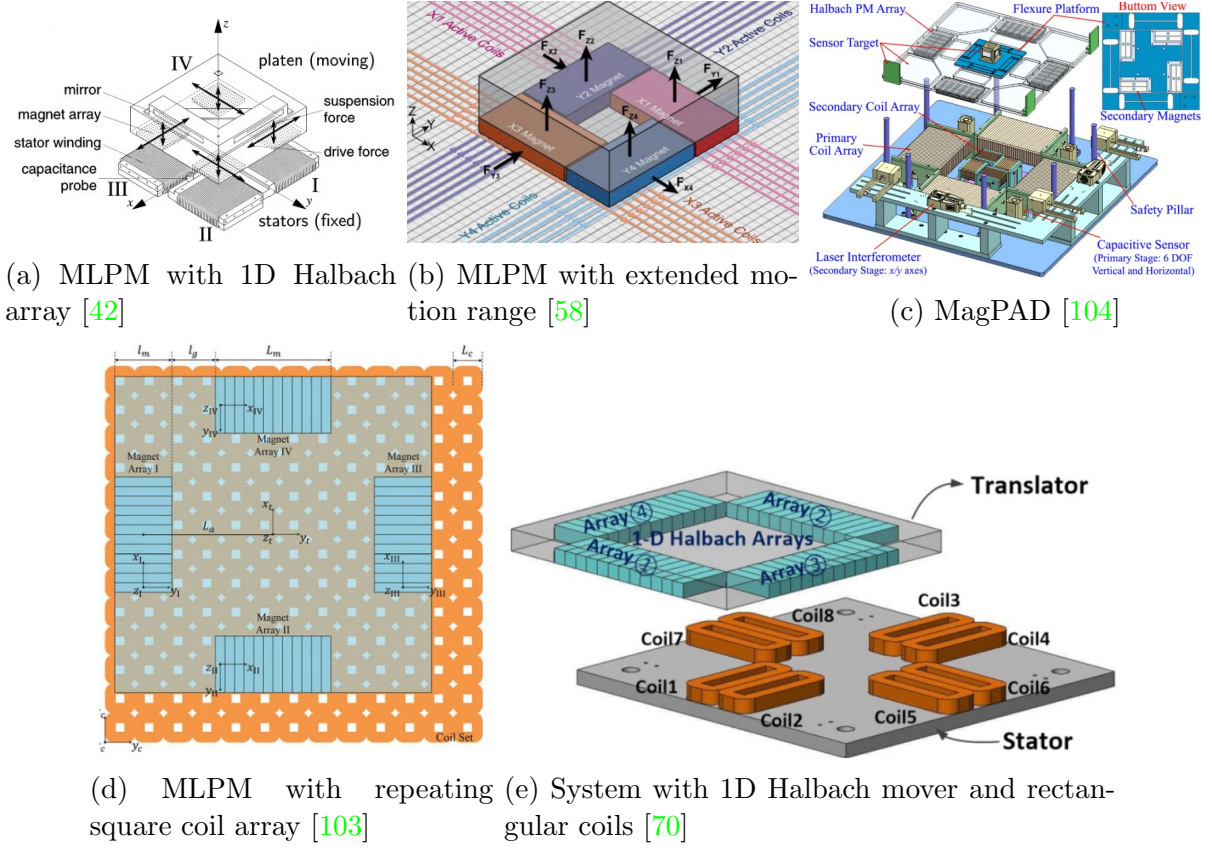


Figure 2.2: Maglev systems with 1D Halbach array movers.

movement with a simple current decoupling algorithm. However, the overall magnetic flux density is limited since there is an open area in the mover without any magnets.

As shown in Figure 2.2a, the world's first maglev stage that provides accurate positioning in six DOF was developed by Kim in 1999, [42]. The mover is made up of four orthogonal linear Halbach arrays, which can provide force in two directions with rectangular coils to achieve six-DOF motion. This system achieves a motion range of 50 mm by 50 mm. The stage operates with a positioning noise of 5 nm in the horizontal direction and can realize an acceleration of 1G. This system shows that the MLPM has great potential for next-generation semiconductor manufacturing.

This design was further improved by the Precision Mechatronics Lab at the University of British Columbia in 2012 [58] and A*STAR in the Singapore Institute of Manufacturing Technology in 2019 [104]. Precision Mechatronics Lab replaces the rectangular coil with the

linear printed circuit board (PCB) to extend the motion range in the horizontal direction to several meters, as shown in [Figure 2.2b](#). What’s more, the PCB contains several layers of coils so that more force can be provided. However, the levitation height is limited to around 2.5 mm above the stator. This long-stroke, six-DOF system allows more expansive application fields, including flexible manufacturing or clean room conveying. A*STAR’s design is about further improving the accuracy, as shown in [Figure 2.2c](#). Other than the four primary rectangular coils and linear Halbach array, four secondary rectangular coils and linear Halbach array are added, making the system a dual-stage system. Primary coils are responsible for coarse movement, and secondary coils are responsible for finer control. The primary and secondary Halbach arrays on the mover are connected with a spring damper system. In the test of a 2 mm linear motion profile, the primary position error is 4 μm , and the secondary position error is reduced to less than 0.5 μm .

Zhu [103] from A*STAR lab proposed a system with a 1D Halbach array and repeating square coils, shown in [Figure 2.2d](#). This design uses 8 phase coil currents to control the force over the mover. This mover can also travel across different active coil sets for unlimited stroke in the horizontal direction. The maximum speed tested for the mover in the horizontal direction is tested to be 100 mm/s. Besides using repeating square coils, Wuhan University proposed a design with rectangular coils [70] shown in [Figure 2.2e](#). However, this design can offer a limited motion range.

2.1.3 2D Halbach Array Mover

2D Halbach arrays have been widely investigated for maglev systems since this arrangement can concentrate the magnetic flux density over one side. Combining the 2D Halbach array with different coil designs, the planar movers can all achieve six-DOF movement with great energy efficiency. In 2004, Compter [18] proposed an electro-dynamic planar motor (EPM). As shown in [Figure 2.3a](#), this design combines the 2D Halbach array along with rectangular coils. Each coil group contains 3 coils and three-phase current is used to control the motion. A similar design is also seen in [99]. In 2008, a contactless planar actuator with a manipulator (COPAM) is developed by Boeij [22]. As shown in [Figure 2.3b](#), this design uses repeating circular coils.

The design was further improved in 2008 [40]. The rectangular coils align in a Herringbone layout for the best power dissipation and force and torque ripples, shown in [Figure 2.3c](#). The system is tested with a levitation height of 1.5 mm. The tracking error of the planar actuator is 30 μm and 100 μrad at full speed (1.4 m/s) and full acceleration (1.4 G). At standstill operation, the root mean square error (RMSE) of tracking is less than

0.1 μm and 1 μrad . The design of EPM, COPAM, and HPPA are compared and analyzed in [71]. The HPPA design has the smallest energy consumption among all the tasks and the deformation of its translator shows similar direction dependency.

Upon the HPPA layout, the University of Eindhoven designed a double-layer planar motor (DLPM) [74, 19]. The stator contains two layers of stationary coils, and the coils are optimized based on dissipated power and magnet plate deformation, shown in Figure 2.3d. From the simulation, the DLPM's dissipated power is 4% lower than HPPA during the static simulation and 8% higher than HPPA during the dynamic simulation. High-order harmonic content is very limited. The maximum temperature of DLPM is 2% lower than HPPA.

In 2008, Ueda [81] from the University of Tokyo presented a novel design for three-DOF motion (large yaw and translational displacement) with a planar Halbach array and directorial armature conductors. The current-carrying wires are arranged on a double-layered PCB as the stator. The horizontal gap between each conductor equals two times the magnet pitch. For this design, the force and torque can be analytically decoupled in real-time. In the experiment, the levitation height is 0.635 mm, with XY displacement of 20 mm and 10° yaw displacement.

2.1.4 2D Permanent Magnet Array Mover

Other than 2D Halbach array, 2D permanent magnet array can also be used as movers. The 2D permanent magnet array usually contains square magnets in alternating directions. The advantage of this design is it allows multi-phase current control and it has a lighter weight than 2D Halbach array. Two systems with different coil and magnets sizes [7, 98] are shown in Figure 2.4.

2.1.5 Rotary Mover

In 2017, Dyck [29] from the University of British Columbia proposed a rotary table with six-DOF motions for precision micromachining applications, shown in Figure 2.5b. The mover is a linear Halbach array in a circular configuration, and the stator is the linear current carrying wires on a printed circuit board (PCB) with five radial directions. Four three-phase amplifiers are used, and each amplifier can produce two forces in horizontal and vertical directions. The experiments suggest this design can offer limited motion range in three translational directions, limited rotation in the roll and pitch directions and infinite and fast rotation in the yaw direction. A larger bandwidth of 550 Hz is found in the

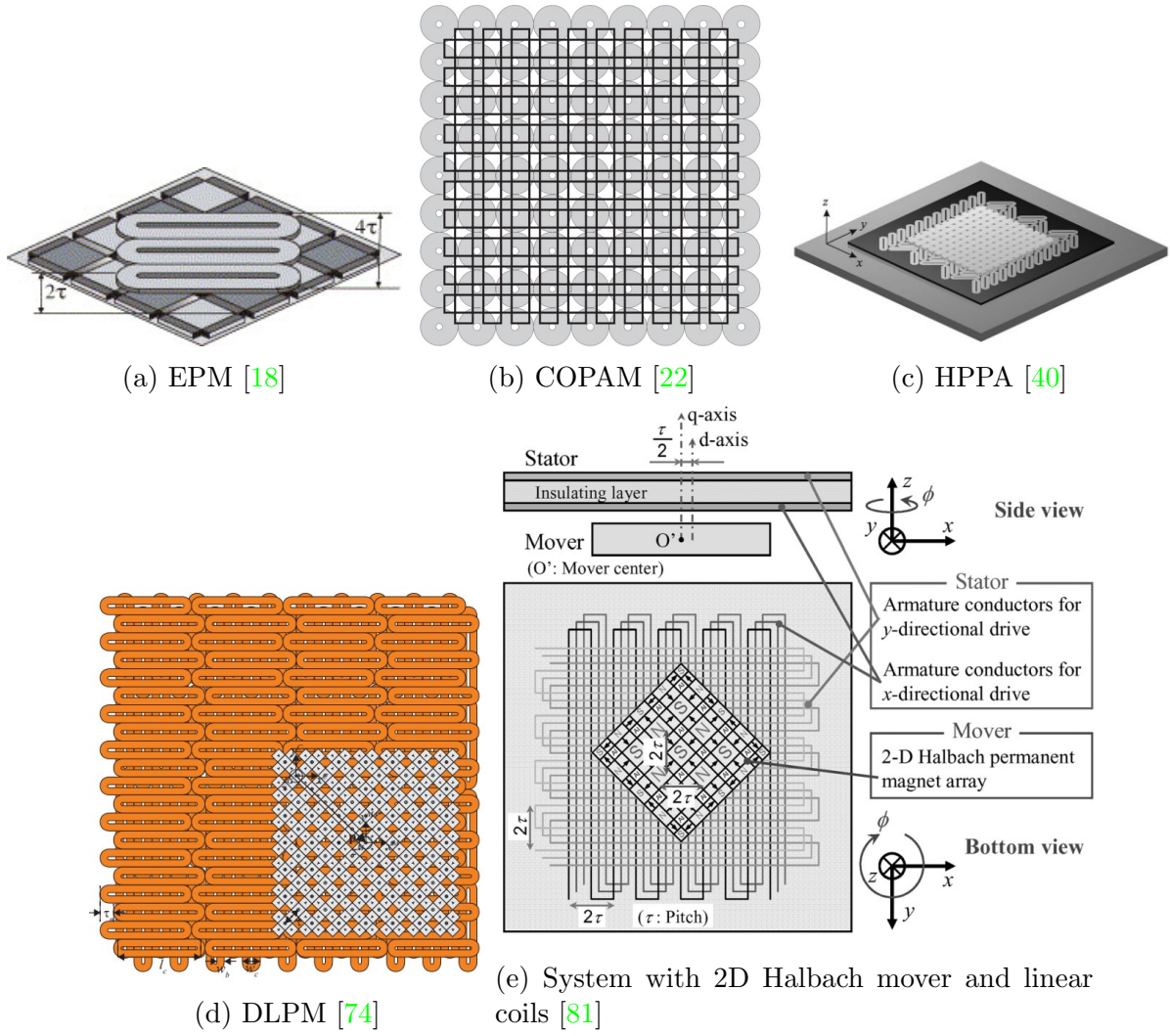


Figure 2.3: Maglev systems with 2D Halbach array movers.

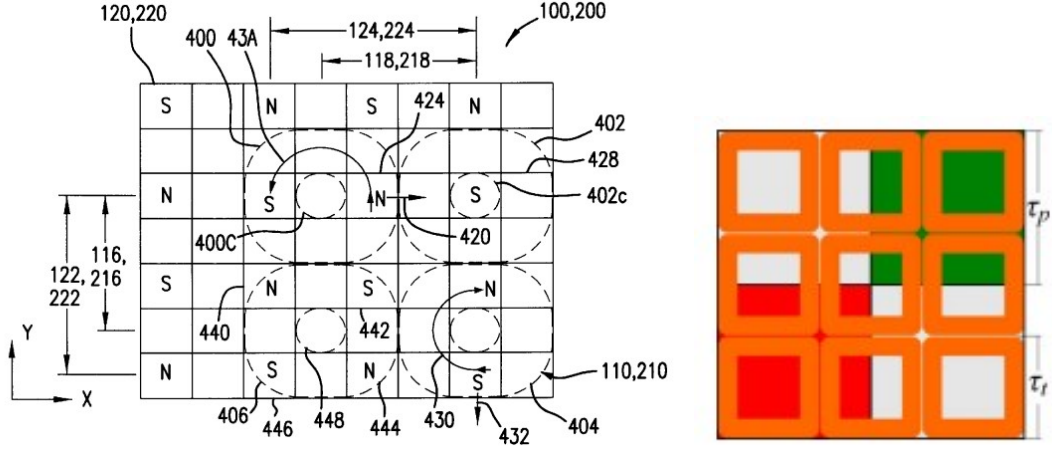


Figure 2.4: 2D permanent magnet array systems [7, 98].

yaw direction. As shown in Figure 2.5b, Wuhan University proposed a novel design with circular permanent magnet array and vertical rectangular coils [59]. This design allows simple current and force decoupling. There are 24 coils in total and 8-phase currents are given.

2.2 Force and Torque Modeling

For motion control of a MLPM, the current commutation algorithm decouples the force and torque and current at every sampling period. Therefore, accurate and real-time modeling of force and torque between driving coils and the mover is critical. For ironless type of MLPM, Lorentz force is used to model the force and torque on the mover. Lorentz force computes the force on the current carrying wires in the magnetic field. Therefore, the first step to compute the force and torque is to calculate the magnetic flux density of the permanent magnet. Bancel proposed a magnetic node method to calculate the magnetic flux density [9]. This method assumes the relative permeability of the permanent magnets to be unity, and it can compute the magnetic flux density to a good precision [93, 48]. The magnetic surface charge method offers higher precision and faster speed than the magnetic node method [6]. The magnetic node and the surface charge can be used to find the magnetic flux density of geometry. Due to the special geometry of the Halbach arrays, harmonic expressions can be used to estimate the magnetic flux density. For the 1D Halbach arrays, [89] and [47] provided the analytical solution. For the 2D Halbach

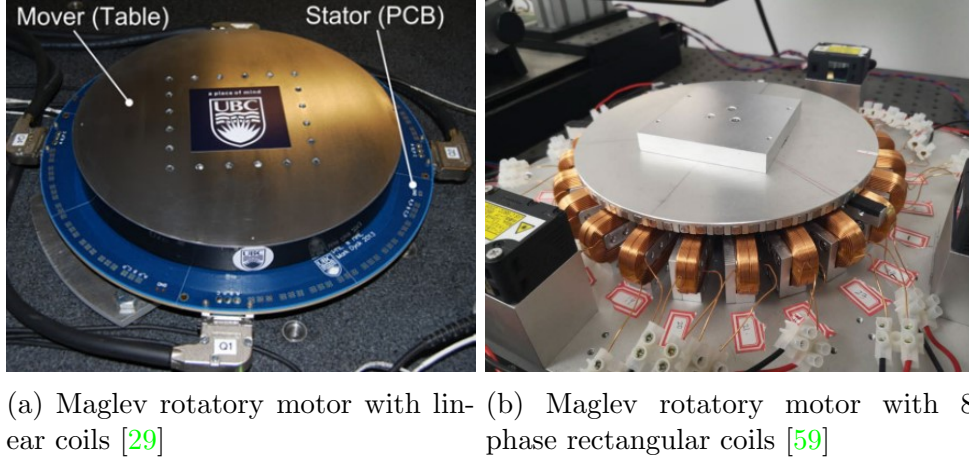


Figure 2.5: Maglev rotatory motors.

array, Boeij [21] proposed a harmonic expression to compute the magnetic flux density.

It is possible to calculate the Lorentz force and torque between the mover and the stator. However, this volume integral could be time-consuming to solve. Therefore, it is common to find the values and store them in a lookup table. In [100, 87], the magnetic flux density is found by the magnetic node and the Lorentz force is solved with Gaussian quadrature approximation. Besides, Rovers used the magnetic node to find the magnetic flux density and solve the Lorentz force by approximating the coil to be four straight segments [73]. Similarly, the magnetic flux density is found via the surface charge model and the Lorentz force is solved by the fine mesh of the coil in [39]. Other than computation, it is also possible to find the force and torque values through direct measurement [53] or finite element analysis [12]. The usage of lookup tables solves the problem of time-consuming force and torque computation. Also, the values will be more accurate since there is not any time limit when finding each entity. However, the size of the lookup table is influenced by the table mesh. Although the size of the table can be reduced based on the repeated magnet geometry [23], the table is still enormous to support the multi-DOF motion.

To avoid using lookup tables, many efforts are made to compute the force and torque in real-time. For maglev systems with 1D Halbach mover and 2D Halbach mover, there are some achievements. The force and torque between linear coils and 1D halbach array can be found in real time [58, 41]. This design is capable of extended motion range in the horizontal plane. Xu [85] solves the analytical force and torque between a carrier with 1D Halbach array and rectangular coils. An analytical solution including the yaw angles is included.

For 2D Halbach arrays, their magnetic flux density can be modeled with harmonic expressions. Therefore, the analytical force and torque can be found in real-time. One method is to simplify a rectangular coil to four straight segments [39, 94]. With the harmonic expressions of the magnetic flux density of a 2D Halbach array, [69] proposed a method to compute the circular segment of a coil, and [56] proposed a method to compute the force and torque of a circular coil in 15 μ s. A major limitation of the harmonic expressions is it cannot predict the edge effect of the 2D Halbach array. The usage of the coils in the edge area is not possible. There are also methods to compute the force and torque, including the edge effect. Numerical solutions of force and torque modeling were successfully implemented in real-time on field-programmable gate array (FPGA) [86]. However, it is difficult for FPGA-based systems to work with large-scale coil set. Besides, Yuan proposed an analytical expression for force and torque modeling under the end effect by modeling the magnetic flux density with Fourier series [95]. However, to achieve accurate modeling, high-order Fourier series is required, and the resultant analytical solution of the volume integral could be lengthy and time-consuming.

Chapter 3

Principles Of Maglev

The proposed system uses a square coil array as the stator and a 2D Halbach as the mover. The repulsive force from the stator coils levitates the 2D Halbach array for long-stroke motion. 2D Halbach array is applied as the mover because it can concentrate the magnetic flux density over one side, leading to higher energy efficiency. Schematics of the magnetic flux density of 1D and 2D Halbach arrays are shown in [Figure 3.1](#). In this chapter, the principles for the maglev planar motor are introduced. First, the coordinate systems and parameters needed are introduced in this section. Then, the magnetic flux density modeling and the Lorentz force rule are introduced.

3.1 System Coordinates

Three coordinate systems are needed for the proposed setup, shown in [Figure 3.2](#). The origin of global coordinate system $[X_g, Y_g, Z_g]$ is located at the center of the coil array, at the same height as the top surface of the coils. The location of the mover and a coil in the global coordinates are named as $\vec{m}^g = [m_x^g, m_y^g, m_z^g]$ and $\vec{c}^g = [c_x^g, c_y^g, c_z^g]$. The origin of the mover coordinate system $[X_m, Y_m, Z_m]$ is located at the center of the bottom surface of the 2D Halbach array. The location of a coil in the mover coordinates is named as $\vec{c}^m = [c_x^m, c_y^m, c_z^m]$. c_z^m is the vertical distance between the coil and the mover. On each coil, there is a coil coordinate system $[X_{ci}, Y_{ci}, Z_{ci}]$, $i \in \{1, 2 \dots n_c\}$. n_c is the total number of coils. The coil coordinates are located at the center of the top surface of each coil.

All coordinate systems are parallel. The mover has six-DOF motion. To find the global force and torque from each coil, the coil locations should be converted from global

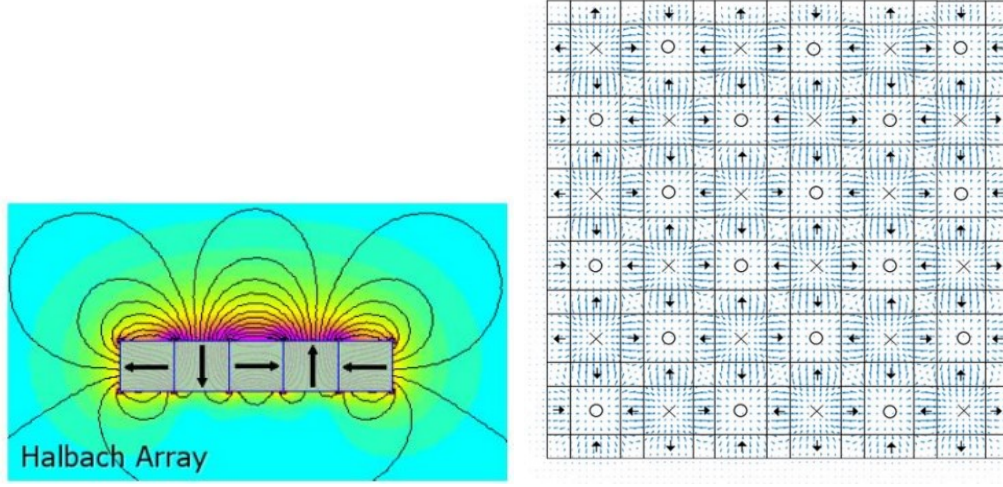


Figure 3.1: (a) Schematic of a 1D Halbach array layout and its magnetic field [1] (b) Schematic of a 2D Halbach array layout and its magnetic field.

coordinates to the mover coordinates. Euler angles are applied to describe this conversion. The roll (x-axis rotation), pitch (y-axis rotation) and yaw (z-axis rotation) angles are denoted with α , β and γ respectively. A composition of three intrinsic rotations is defined in the order of XYZ. The coil location in mover coordinates can be found as

$$\vec{c}^m = R_g^m(\vec{c}^g - \vec{m}^g), \quad (3.1)$$

where

$$R_g^m = R_m^g{}^T = (R_x R_y R_z)^T = R_z^T R_y^T R_x^T, \quad (3.2)$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix},$$

$$R_y = \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix},$$

$$R_z = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

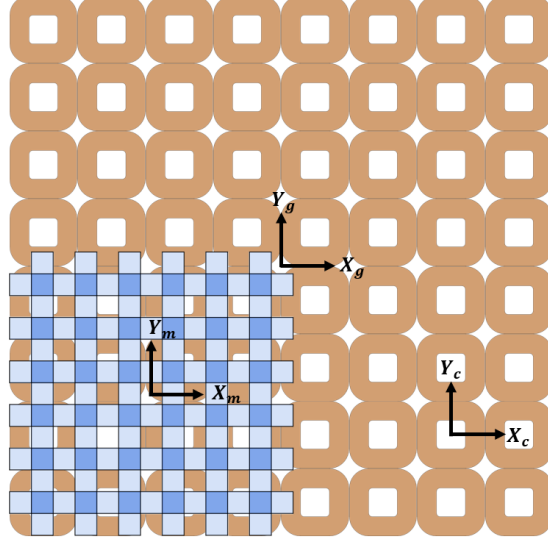


Figure 3.2: System coordinates, top view.

3.2 System Parameters

The geometry parameters are shown in [Figure 3.3](#). The mover has a six-by-six layout, and the height of the 2D Halbach array is h_m . The pole pitch distance is denoted as τ , and the width of the magnets with Z_m magnetization direction is denoted as τ_m . The remanence of the magnets is B_r . The height of the coil is denoted as h_c . The outer and inner width of the coil is denoted as l_c and s_c . The bundle width is w_c , and the number of turns is N .

3.3 Magnetic Flux Density Modeling

The magnetic nodes and the harmonic expressions are presented to model the magnetic flux density of a 2D Halbach array.

3.3.1 Magnetic Nodes

To obtain the magnetic flux density of the magnet array, the Halbach array is modeled with magnetic nodes, which was originally proposed in [\[9\]](#). The magnetic nodes model is deduced from the magnetic surface charge model [\[93\]](#) by assuming the relative permeability

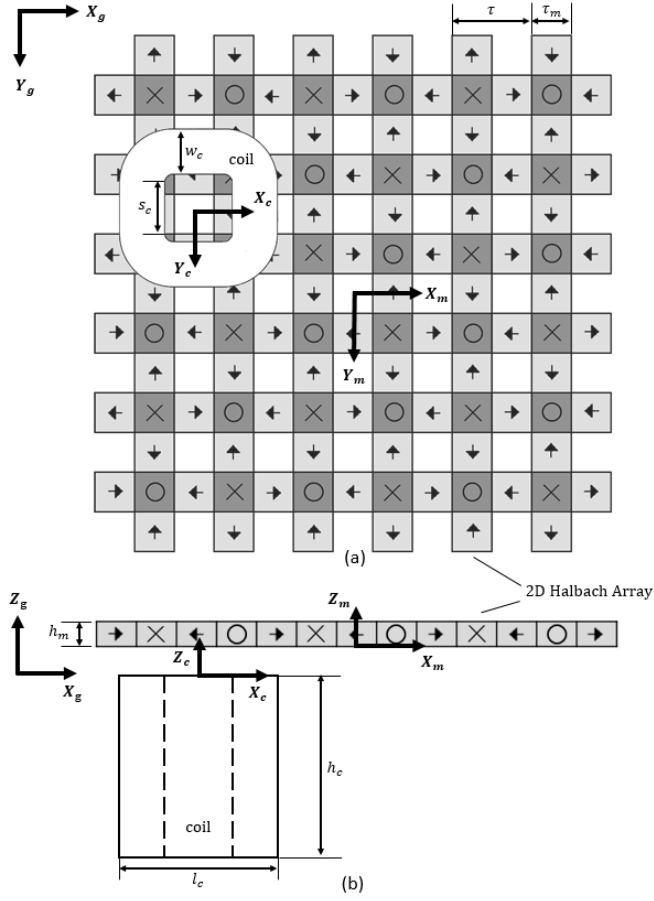


Figure 3.3: System layout with a single coil (a)bottom view (b)side view.

of the permanent magnets to be unity. Using the magnetic nodes method, one could analytically model the magnetic flux density of a Halbach array with good precision, including edge area. The magnetic nodes model for a single cuboidal magnet with Y direction magnetization (shown with arrow) is shown in Fig. 3.4. The equations for the magnetic flux density are

$$B_{xk} = \frac{\epsilon_k}{4\pi} \ln(-y_r + \sqrt{x_r^2 + y_r^2 + z_r^2}), \quad (3.3)$$

$$B_{yk} = \frac{\epsilon_k}{4\pi} \ln(-x_r + \sqrt{x_r^2 + y_r^2 + z_r^2}), \quad (3.4)$$

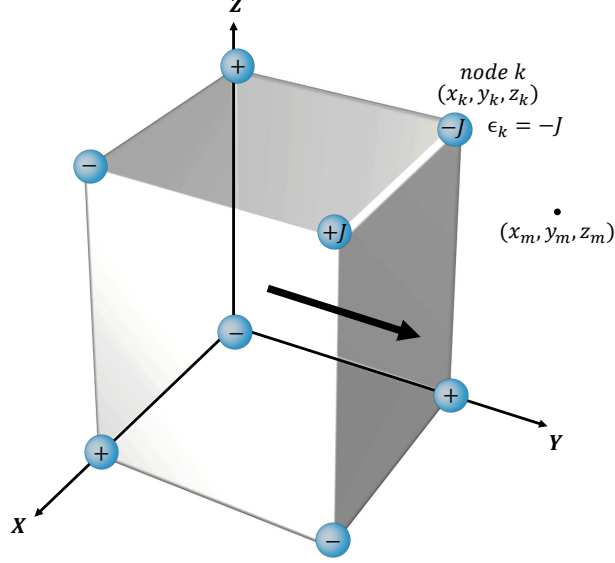


Figure 3.4: Magnetic nodes model on a cuboidal magnet with Y direction magnetization.

$$B_{zk} = \frac{\epsilon_k}{4\pi} \arctan \left(\frac{x_r y_r}{\sqrt{x_r^2 + y_r^2 + z_r^2}} \right), \quad (3.5)$$

where index k denotes each node, x_r , y_r and z_r are equal to $x_m - x_k$, $y_m - y_k$ and $z_m - z_k$ respectively. ϵ_k is the magnetic charge. Its absolute value equals B_r , the remanence of the permanent magnet, and its sign changes based on the location of the node. B_{xk} , B_{yk} and B_{zk} are the magnetic flux density of each node in the X_m , Y_m and Z_m direction.

3.3.2 Harmonic Expressions

Other than the magnetic nodes approach, the harmonic expressions can accurately predict the magnetic flux density only under the central region of a 2D Halbach array. The relative permeability μ_r of permanent magnets is assumed to be one. The actual μ_r of NdFeB permanent magnet is 1.03-1.05. μ_0 is the permeability of free space. The equations for harmonic expressions [39] at point p^m in the mover coordinate is

$$\vec{B}^m(\vec{p}^m) = -\mu_0 \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} K e^{\lambda p_z^m} \begin{bmatrix} \frac{k\pi}{\tau} \cos(\frac{k\pi p_x^m}{\tau}) \sin(\frac{l\pi p_y^m}{\tau}) \\ \frac{l\pi}{\tau} \sin(\frac{k\pi p_x^m}{\tau}) \cos(\frac{l\pi p_y^m}{\tau}) \\ \lambda \sin(\frac{k\pi p_x^m}{\tau}) \cos(\frac{l\pi p_y^m}{\tau}) \end{bmatrix}, \quad (3.6)$$

where

$$K(\mu_r = 1) = B_r(e^{-m_t \lambda} - e^{-m_b \lambda}) \frac{b_k b_l \pi (k^2 + l^2) + a_k b_l k \lambda \tau + a_l b_k l \lambda \tau}{2(k^2 + l^2) \pi \lambda \mu_0},$$

$$a_k = \frac{4}{k\pi} \cos(\frac{k\tau_m \pi}{2\tau}) \sin(\frac{k\pi}{2}),$$

$$a_l = \frac{4}{l\pi} \cos(\frac{l\tau_m \pi}{2\tau}) \sin(\frac{l\pi}{2}),$$

$$b_k = \frac{4}{k\pi} \sin(\frac{k\tau_m \pi}{2\tau}) \sin(\frac{k\pi}{2}),$$

$$b_l = \frac{4}{l\pi} \sin(\frac{l\tau_m \pi}{2\tau}) \sin(\frac{l\pi}{2}),$$

$$\lambda = \sqrt{(\frac{k\pi}{\tau})^2 + (\frac{l\pi}{\tau})^2}.$$

k and l are harmonic numbers. m_t and m_b are the height of the top and bottom of the Halbach array. This equation can be further simplified by assuming the harmonic number for both X and Y directions to be 1. Then, the model becomes:

$$\vec{B}^m(\vec{p}^m) = -\mu_0 K' e^{\frac{\sqrt{2}\pi p_z^m}{\tau}} \begin{bmatrix} \cos(\frac{\pi p_x^m}{\tau}) \sin(\frac{\pi p_y^m}{\tau}) \\ \sin(\frac{\pi p_x^m}{\tau}) \cos(\frac{\pi p_y^m}{\tau}) \\ \sqrt{2} \sin(\frac{\pi p_x^m}{\tau}) \cos(\frac{\pi p_y^m}{\tau}) \end{bmatrix}, \quad (3.7)$$

where

$$K' = \frac{4B_r}{\pi^4 \mu_0} (e^{-\frac{\sqrt{2}\pi m_t}{\tau}} - e^{-\frac{\sqrt{2}\pi m_b}{\tau}}) (\pi \sin \frac{\tau_m \pi}{\tau} - \sqrt{2} \cos \frac{\tau_m \pi}{\tau} + \sqrt{2} \tau).$$

3.3.3 Magnetic Flux Density Verification

The harmonic model and the magnetic nodes method are implemented on a six-by-six Halbach array, whose pole pitch is 1.5 inches, pole is 2 inches, and thickness is 0.5 inches. With the implementation of the methods, a 2D Halbach array of any size can be easily found. The magnetic flux density is calculated along an arbitrary line, whose starting point is $p_x^m = 3.5$ mm, $p_y^m = 10.5$ mm and ending point is $p_x^m = 192.5$ mm, $p_y^m = 37.5$ mm in the mover coordinates. The testing line is parallel to the mover with a height of -25mm. Three methods, magnetic node, first order harmonic and third order harmonic, are implemented. Their results are compared to ANSYS simulation results.

As shown in [Figure 3.5](#), the magnetic node can nicely predict the magnetic flux density along the entire line, while the harmonic model can only predict the magnetic flux density at the central end. The RMSE between the ANSYS simulation and the magnetic node model is 0.003T across the whole line. The harmonic model can only model the magnetic field under the central region (0-120 mm) of the Halbach array. In the central zone, the RMSE of the first and the third harmonics model are 0.0081T and 0.0033T, respectively. The third harmonics model's performance is similar to the magnetic node. The first-order harmonic model can predict magnetic flux density with some errors.

3.4 Lorentz Force and Torque

With the external magnetic field information, the Lorentz force $\mathbf{F}$ of a coil can be found by taking the volume integral of the cross product of current density $\mathbf{J}$ and the magnetic flux density $\mathbf{B}$. The force on the magnet array has the same amplitude and the opposite direction. The equation for force on the carrier is

$$\mathbf{F} = - \iiint \mathbf{J} \times \mathbf{B} dV, \quad (3.8)$$

where dV is the volume integral of the conductor and $\times$ is the cross product. The torque on the magnet array $\mathbf{T}$ is the integral of cross product of torque arm and the force. The equation for torque on the magnet array is

$$\mathbf{T} = - \iiint \mathbf{r} \times (\mathbf{J} \times \mathbf{B}) dV, \quad (3.9)$$

where $\mathbf{r}$ is the distance from the center of the computing element to the center of mass of the 2D Halbach array.

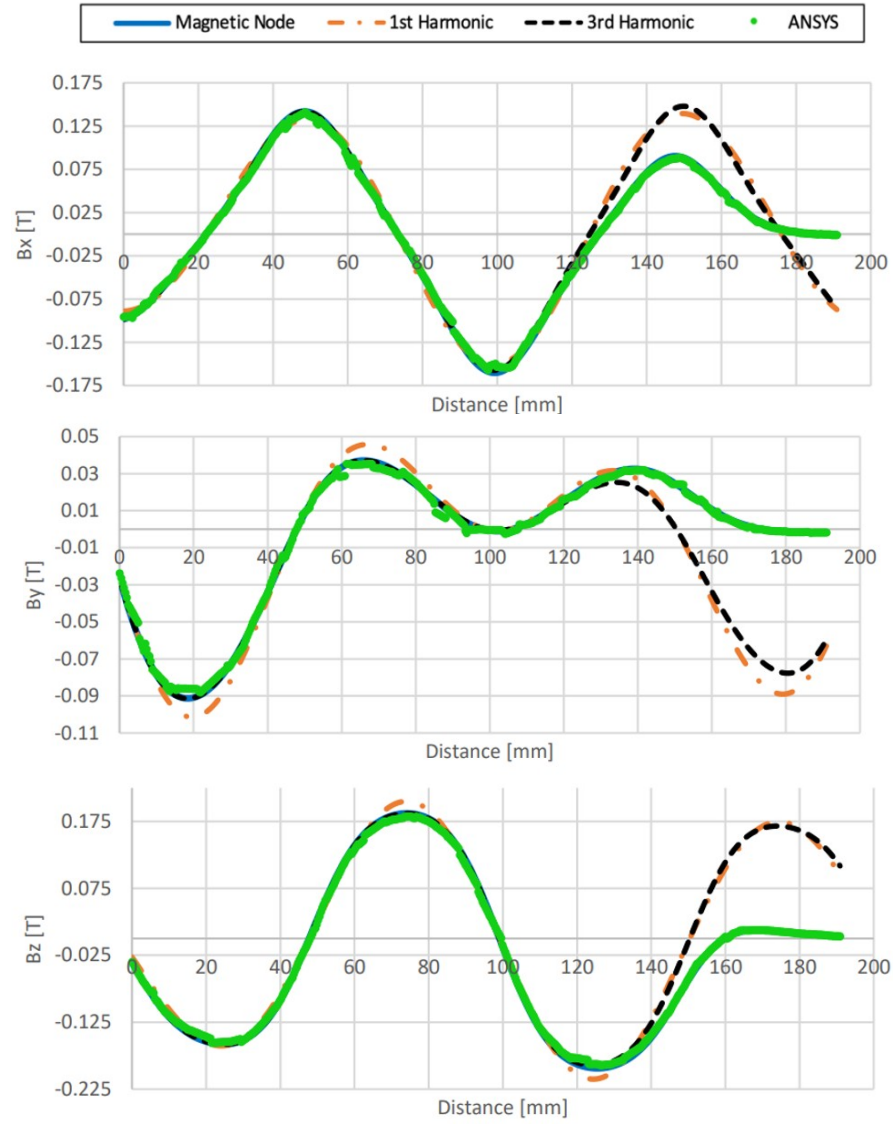


Figure 3.5: Comparison between magnetic field between ANSYS, magnetic node and harmonic model along the same line.

In this study, numerical methods are applied to compute the volume integral. Cuboidal mesh is used for the straight sections of the coil, and the trapezoidal mesh is used at the rounded corners. The mesh size is within 1 mm, and the coil current input is 1 A for all the force and torque computations. The rectangle rule is used as the interpolating function for the numerical solution. This method has been proven effective in [100, 39]. A schematic of the current density distribution in a coil is shown in Figure 3.6. The computation time for the numerical method is found to be 5 s per coil on MATLAB.

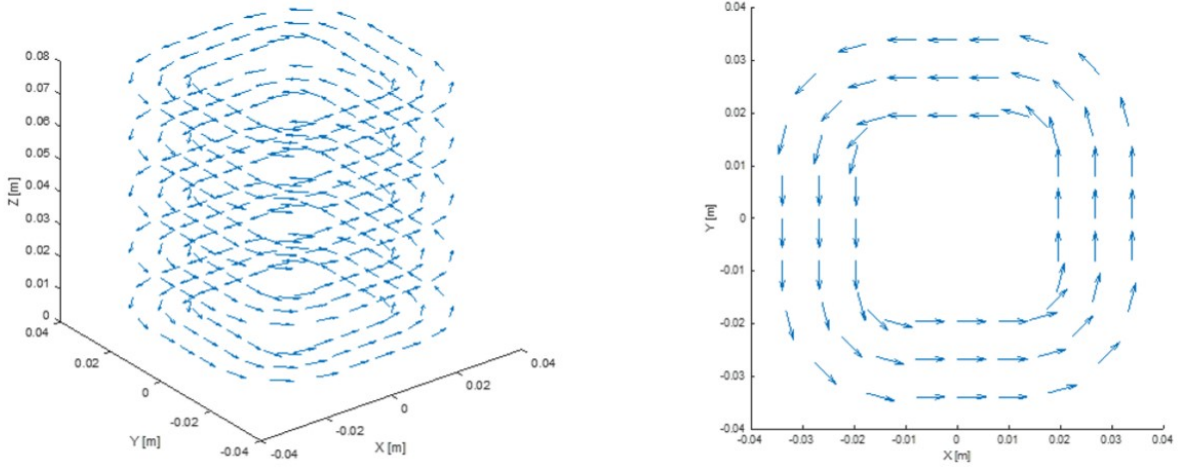


Figure 3.6: Schematic of the current density in a coil.

Chapter 4

Data-Drive Force and Torque Modeling

A continuous, table-free and accurate data-driven force and torque modeling method are presented in this chapter. The work in this chapter is published in [91].

4.1 Zoning and Data Collection

The area under the 2D Halbach array is divided into many regions for the data-driven modeling. To identify the region under the influence of the end effect, the magnetic flux density strength calculated by the magnetic nodes model is compared with the first harmonic expression along the line of $Y_m = \tau/2$ in Fig. 4.1, where τ is the pole pitch of a 2D Halbach array. It shows that in the region of -2.5τ to 2.5τ , magnetic flux density values from the two methods are very similar (peak-to-peak error within 2%). If the entire coil is located inside the -2.5τ to 2.5τ area, this coil is in the central region. The center location of the coil is applied to define the boundary of each region. Considering the size of the coil is about 1.5τ in this study, the central region limit for the coil is -1.75τ to 1.75τ . When the coil center moves to 4τ or -4τ , the magnetic flux linkage of the coil is almost zero. The outer boundary of the edge region is -4τ to 4τ . This method is applied in both X_m and Y_m directions, and the resultant zoning is shown in Fig.4.2. There is one central region, four edge regions and four corner regions.

As the coil moves across different areas under the 2D Halbach array, numerical solutions of force and torque are collected for further data fitting in 3D space. In the X_m and Y_m

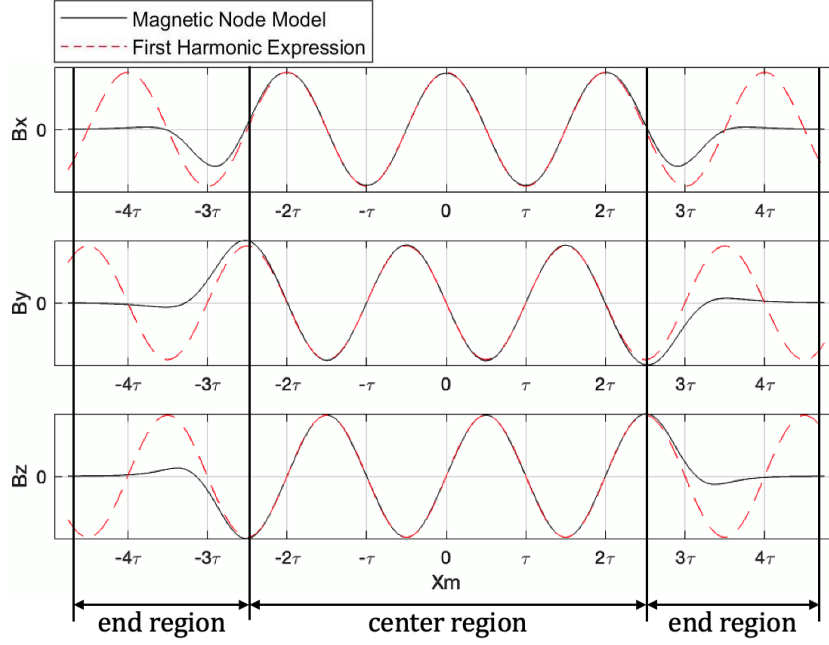


Figure 4.1: Magnetic flux density of the 2D Halbach array along $Y_m = \tau/2$.

directions, the location resolution is 1.016 mm from -4τ to 4τ . In the Z_m direction, the range of air gap covered is 0.5 mm to 7.5 mm, with a 1 mm resolution. The air gap range is limited by the electromagnetic design of the coil and the 2D Halbach array. The number of locations computed is 247808, 159104 and 102152 for the central region, one edge region and one corner region, respectively.

4.2 Force

In this section, methods of modeling the three-DOF force on the carrier by a coil, when the coil is located under different regions of the magnet array are presented.

4.2.1 Central Region

In the central region of the 2D Halbach array, the force on the magnet array by a coil in the central region is

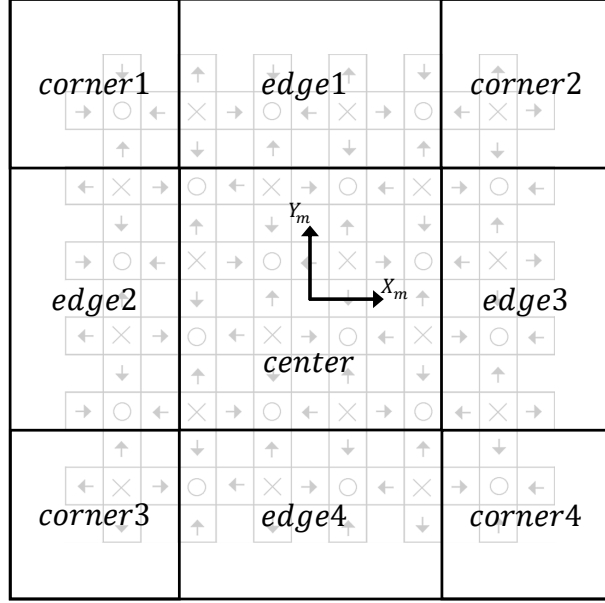


Figure 4.2: Regions under the 2D Halbach array, top view.

$$\vec{F}_{center}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} K_x \cos(\eta_x) \sin(\eta_y) \\ K_y \sin(\eta_x) \cos(\eta_y) \\ K_z \sin(\eta_x) \sin(\eta_y) \end{bmatrix}, \quad (4.1)$$

where $\eta_x = \pi c_x^m / \tau$ and $\eta_y = \pi c_y^m / \tau$ [14]. K_x , K_y and K_z are exponential decay functions related to the air gap, and their units are Newton per Ampere. Equations for K_x , K_y and K_z are:

$$K_x = \alpha_x e^{\beta_k c_z^m}, \quad (4.2)$$

$$K_y = \alpha_y e^{\beta_k c_z^m}, \quad (4.3)$$

$$K_z = \alpha_z e^{\beta_k c_z^m}, \quad (4.4)$$

where α_x , α_y , α_z and β_k are constants. Values of α_x , α_y , α_z and β_k are determined by the specific coil and carrier geometry. Their values can be found by fitting the data from the numerical force solution or analytically from the harmonic analysis. This chapter demon-

strates the proposed model with a specific carrier and coil setup. The system variables are shown in Figure 4.3, and values of each variable of this setup are shown in Table 4.1. α_x , α_y , α_z and β_k in (4.2)-(4.4) for the example setup are found by data fitting, and their values are 3.9021, 3.9021, 5.5209 and -87.14 respectively.

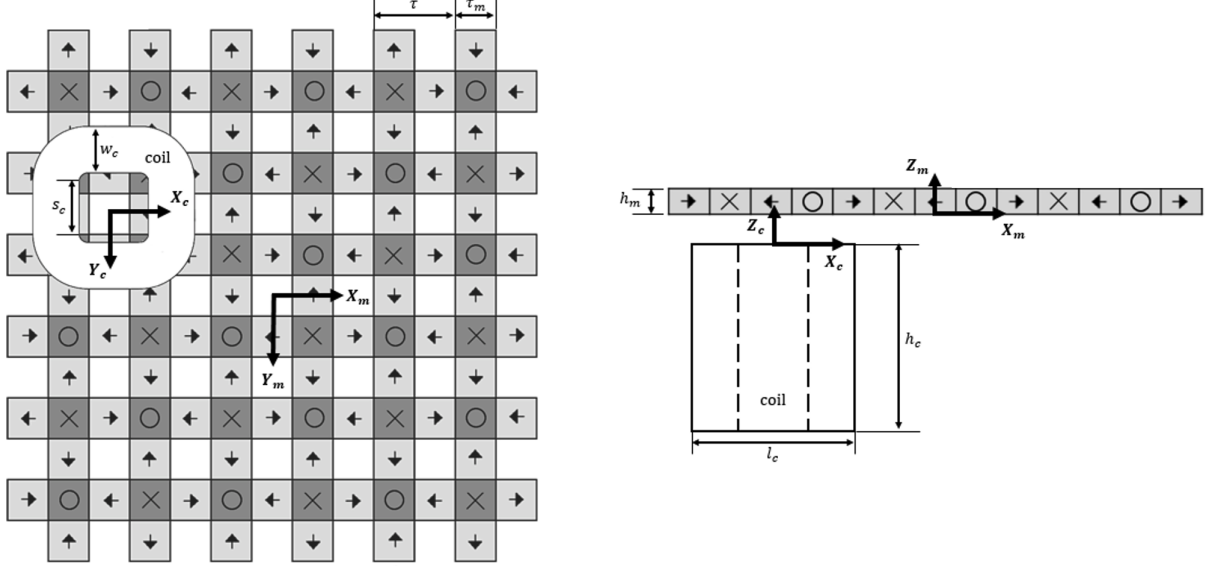


Figure 4.3: System layout showing the variables.

To examine the proposed model, numerical force and torque values at 5 mm air gap are computed with a location resolution of 1.016 mm in the X_m and Y_m direction. In the central region, numerical results at 30976 locations are computed at 5 mm. The range of F_x and F_y are both ± 2.5239 N, and the range of F_z is ± 3.571 N. The results from the proposed model are compared with the numerical solution at the 5 mm air gap. In the central region, the RMSE of F_x , F_y and F_z is 0.0143 N, 0.0143 N and 0.0203 N respectively.

4.2.2 Edge Region

F_x in edge region 1 at 1.5 mm air gap distance is shown in Fig. 4.4. In this figure, along the X_m direction, the force distribution has a sinusoidal shape, with the same phase as the central region and changing amplitude. Along the Y_m direction, the force distribution displays shapes of nonlinear waves. Each wave has a similar shape but different amplitudes. Similar trends of distribution are also found in F_y and F_z in edge region 1. Based on the

Table 4.1: Dimensions of the primary system variables.

Variables	Values
w_c	21.5 mm
s_c	24 mm
l_c	75 mm
h_c	86.5 mm
τ	50.8 mm
τ_m	25.4 mm
h_m	6.35 mm
N	972
I	1 A
B_r	1.32 T
c_z^m	0.5-7.5 mm

distribution characteristics, assumptions of force distributions in edge region 1 are made as:

1. Force distribution along the X_m direction is the same as the central region.
2. Force distribution along the Y_m direction is a 1D nonlinear wave at a certain height.

These nonlinear waves in the Y_m direction are named nonlinear force components, whose values are related to c_y^m and c_z^m . With this assumption, force equations in edge region 1 become

$$\vec{F}_{edge1}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} K_x \cos(\eta_x) f_1(c_y^m, c_z^m) \\ K_y \sin(\eta_x) f_2(c_y^m, c_z^m) \\ K_z \sin(\eta_x) f_3(c_y^m, c_z^m) \end{bmatrix}, \quad (4.5)$$

where K_x , K_y and K_z are the same as (4.2)-(4.4). f_1 , f_2 and f_3 are the nonlinear force components. f_1 , f_2 and f_3 are dimensionless.

At a certain height, the 2D distribution of nonlinear force components can be found by (4.5) using the corresponding numerical solutions. As an example, values of f_1 , f_2 and f_3 at air gap distance of 1.5 mm are shown in Fig. 4.5. The dimension of the nonlinear force components is reduced from 2D to 1D by averaging along the X_m direction.

Nonlinear force components are computed at each air gap distance from 0.5 mm to 7.5 mm. The variation of nonlinear force components across different air gap is shown in

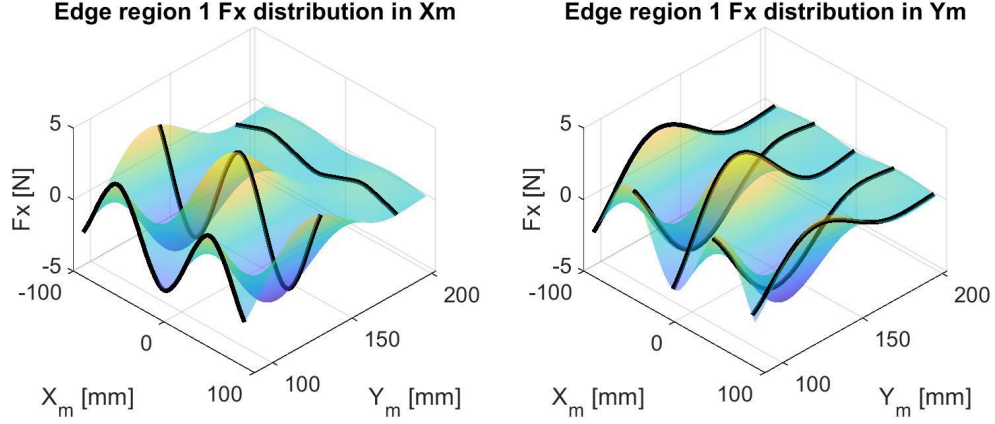


Figure 4.4: F_x in edge region 1 at 1.5 mm air gap.

Fig. 4.6. Knowing the values of nonlinear force components at different air gap, a fitting equation is needed to model their distributions. It is noticed that a regular third-order Fourier series can nicely model a nonlinear force component at a certain height. Changes are made to a regular third-order Fourier series by considering the fundamental frequency and harmonics amplitude linearly related to the air gap. The fitting equation is

$$f(x, c_z^m) = a_0 + a_1x + a_2c_z^m + (k_1c_z^m + b_1) \cos((k_0c_z^m + b_0)x + a_3) + (k_2c_z^m + b_2) \cos(2(k_0c_z^m + b_0)x + a_4) + (k_3c_z^m + b_3) \cos(3(k_0c_z^m + b_0)x + a_5). \quad (4.6)$$

In (4.6), c_z^m is the air gap, and x is the distance away from the central region. For example, c_y^m is plugged in for x in edge region 1. There are 14 parameters ($k_0, k_1 \dots k_3; a_0, a_1 \dots a_5; b_0, b_1 \dots b_3$) for each nonlinear force component. All parameters can be found via nonlinear data-fitting using *nlinfit* function in MATLAB. *nlinfit* estimates the specified model parameter in the least-square sense. For f_1, f_2 and f_3 in edge region 1, the peak-to-peak modeling error percentage for all nonlinear force components are found within 2.5% using (4.6).

Using the force symmetry feature of the 2D Halbach array, the force equations in other edge regions are shown in (4.7)-(4.9):

$$\vec{F}_{edge2}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} K_x f_2(-c_x^m, c_z^m) \sin(\eta_y) \\ -K_y f_1(-c_x^m, c_z^m) \cos(\eta_y) \\ -K_z f_3(-c_x^m, c_z^m) \sin(\eta_y) \end{bmatrix}, \quad (4.7)$$

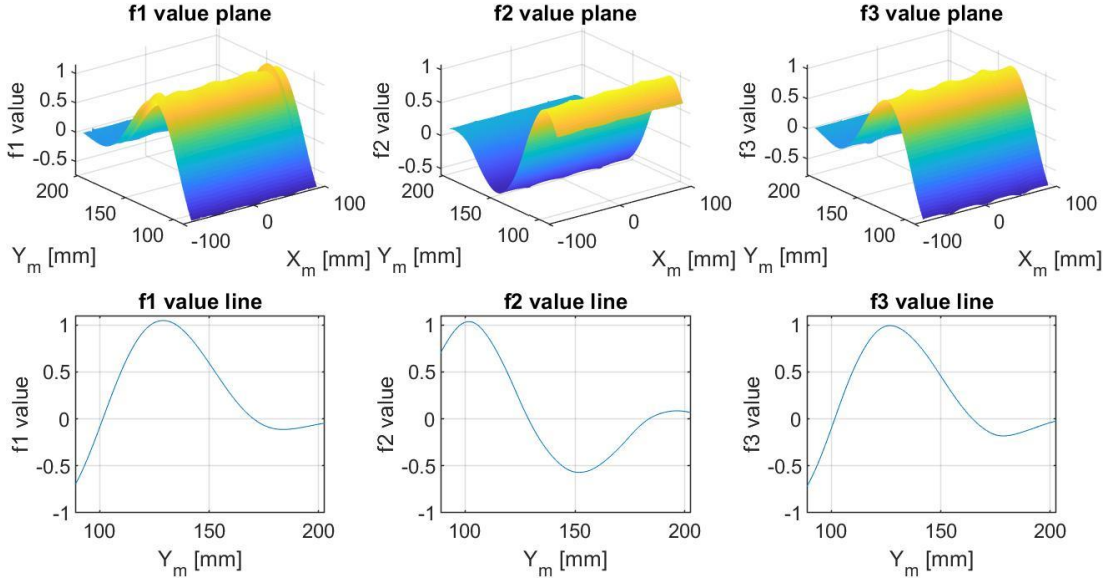


Figure 4.5: f_1 , f_2 and f_3 value plane and lines at 1.5 mm air gap.

$$\vec{F}_{edge3}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} K_x f_2(c_x^m, c_z^m) \sin(\eta_y) \\ K_y f_1(c_x^m, c_z^m) \cos(\eta_y) \\ K_z f_3(c_x^m, c_z^m) \sin(\eta_y) \end{bmatrix}, \quad (4.8)$$

$$\vec{F}_{edge4}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} -K_x \cos(\eta_x) f_1(-c_y^m, c_z^m) \\ K_y \sin(\eta_x) f_2(-c_y^m, c_z^m) \\ -K_z \sin(\eta_x) f_3(-c_y^m, c_z^m) \end{bmatrix}. \quad (4.9)$$

In (4.7)-(4.9), equations f_1 , f_2 and f_3 are the same as in edge region 1 with different values plugged in.

The proposed force model is compared with the numerical results at 5 mm air gap. In edge region 1 at 5 mm, numerical force values are collected at 19888 locations (1.016 mm location resolution) to examine the proposed model. The proposed model and the modeling error are shown in Fig. 4.7. The RMSE of the F_x , F_y and F_z in edge region 1 are 0.0127 N, 0.012 N and 0.0175 N respectively.

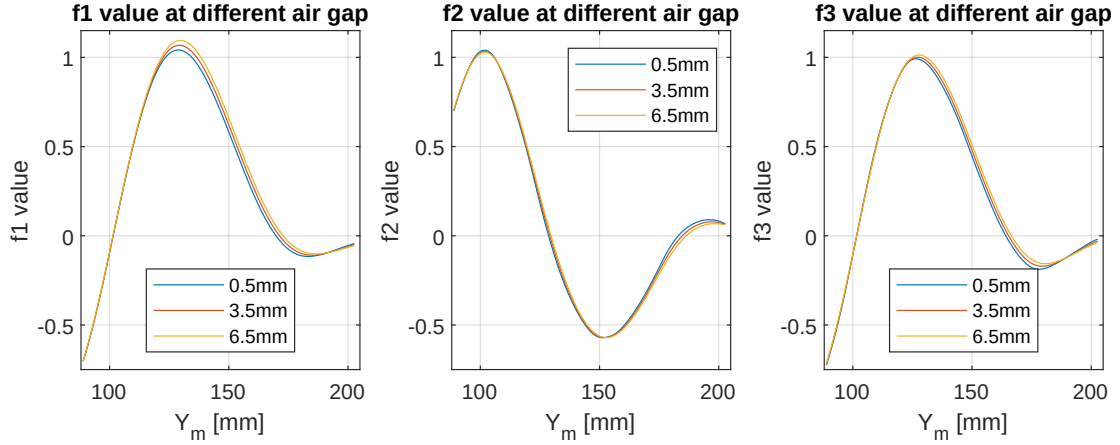


Figure 4.6: f_1 , f_2 and f_3 values at three different air gap.

4.2.3 Corner Region

Similarly to edge regions, force in corner regions is examined in X_m and Y_m directions. F_x in edge region 2 at 1.5 mm air gap is shown in Fig. 4.8. In X_m and Y_m directions, the force distribution both show shapes of nonlinear waves with changing amplitude. In X_m , the shape of the wave is highly similar to f_2 (F_x distribution in edge 3 along X_m direction). In Y_m , the shape of the wave is highly similar to f_1 (F_x distribution in edge 1 along Y_m direction). The same trends are also observed in F_y and F_z . Based on the distribution characteristics, assumptions of force distributions in corner region 2 are made as:

1. Force distribution in an edge region can be modeled by the nonlinear force components from the two neighboring edge regions using superposition.
2. Force distribution in corner region 2 along the X_m direction is the same as edge region 3.
3. Force distribution in corner region 2 along the Y_m direction is the same as edge region 1.

Therefore, the force equation for corner region 2 is

$$\vec{F}_{corner2}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} K_x f_2(c_x^m, c_z^m) f_1(c_y^m, c_z^m) \\ K_y f_1(c_x^m, c_z^m) f_2(c_y^m, c_z^m) \\ K_z f_3(c_x^m, c_z^m) f_3(c_y^m, c_z^m) \end{bmatrix}. \quad (4.10)$$

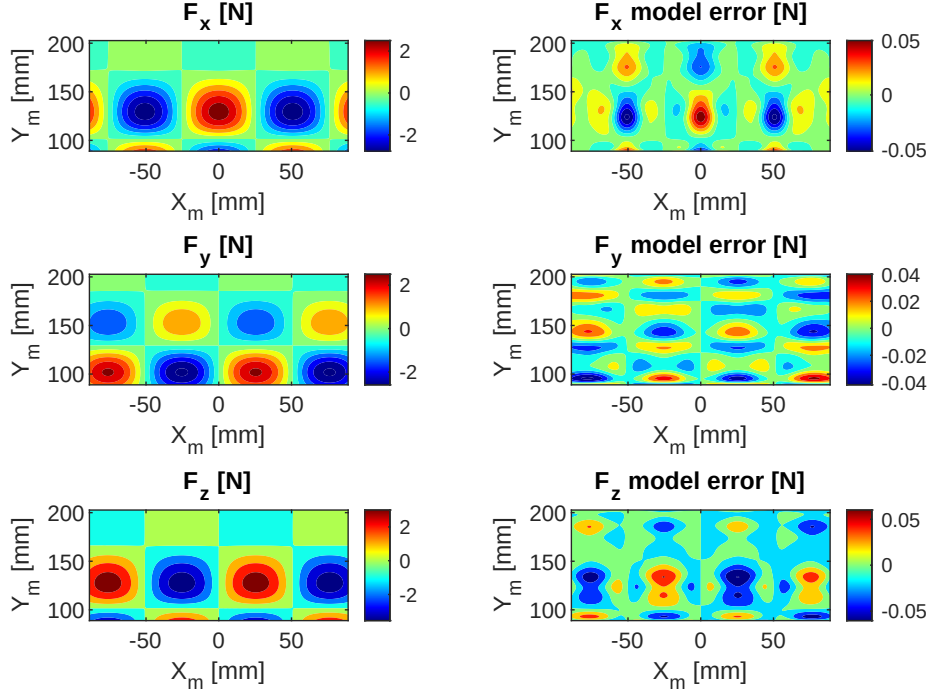


Figure 4.7: Force and modeling error in edge region 1 at 5 mm air gap.

In (4.10), K_x , K_y and K_z are defined in (4.2)-(4.4). Nonlinear force components are the same as in edge regions.

Similarly, the force equations for edge region 1, 3 and 4 are:

$$\vec{F}_{corner1}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} K_x f_2(-c_x^m, c_z^m) f_1(c_y^m, c_z^m) \\ -K_y f_1(-c_x^m, c_z^m) f_2(c_y^m, c_z^m) \\ -K_z f_3(-c_x^m, c_z^m) f_3(c_y^m, c_z^m) \end{bmatrix}, \quad (4.11)$$

$$\vec{F}_{corner3}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} -K_x f_2(-c_x^m, c_z^m) f_1(-c_y^m, c_z^m) \\ -K_y f_1(-c_x^m, c_z^m) f_2(-c_y^m, c_z^m) \\ K_z f_3(-c_x^m, c_z^m) f_3(-c_y^m, c_z^m) \end{bmatrix}, \quad (4.12)$$

$$\vec{F}_{corner4}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} -K_x f_2(c_x^m, c_z^m) f_1(-c_y^m, c_z^m) \\ K_y f_1(c_x^m, c_z^m) f_2(-c_y^m, c_z^m) \\ -K_z f_3(c_x^m, c_z^m) f_3(-c_y^m, c_z^m) \end{bmatrix}. \quad (4.13)$$

Nonlinear force components in (4.11)-(4.13) are the same as in edge regions. Therefore, for

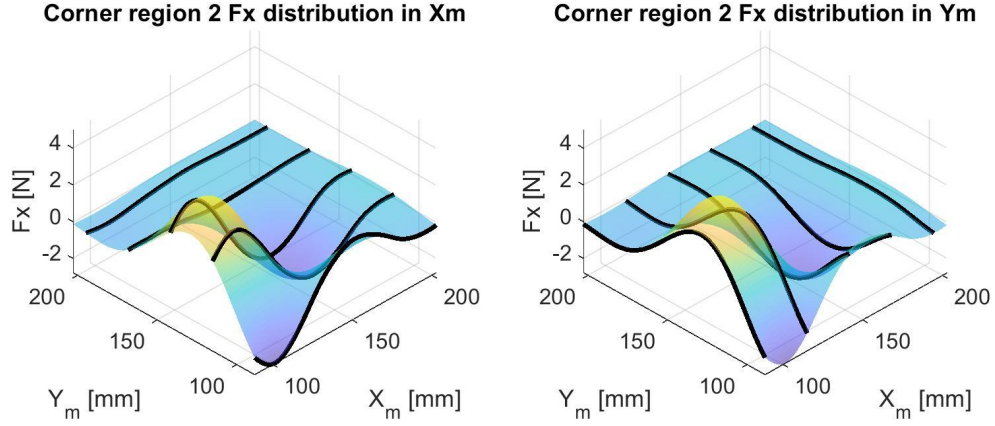


Figure 4.8: F_x in corner region 2 at 1.5 mm air gap.

all the edge and corner regions, only three equations are needed to model all the nonlinear force components.

The proposed model is compared with the numerical solution at 5 mm air gap. The location resolution of the numerical solution of 5 mm air gap is 1.016 mm, and there are 12769 locations computed in corner region 2. The modeled force, as well as the modeling error compared with the numerical solution, is shown in Fig. 4.9. The RMSE of the F_x , F_y and F_z in corner region 2 are 0.0291 N, 0.0291 N and 0.1005 N respectively

4.3 Torque

In this section, methods to compute the three-DOF torque on the magnet array by a coil located in different regions are presented.

4.3.1 Effective Torque Arm

From section 4.2, force on the 2D Halbach array can be modeled. If the length of the torque arm can be found, the torque is simply the cross product of the torque arm and force. However, there is no direct contact between a current-carrying coil and the 2D Halbach array. Effective torque arm (ETA) is defined to represent the length of the torque arm. The relationship between ETA and the torque is shown in (4.14):

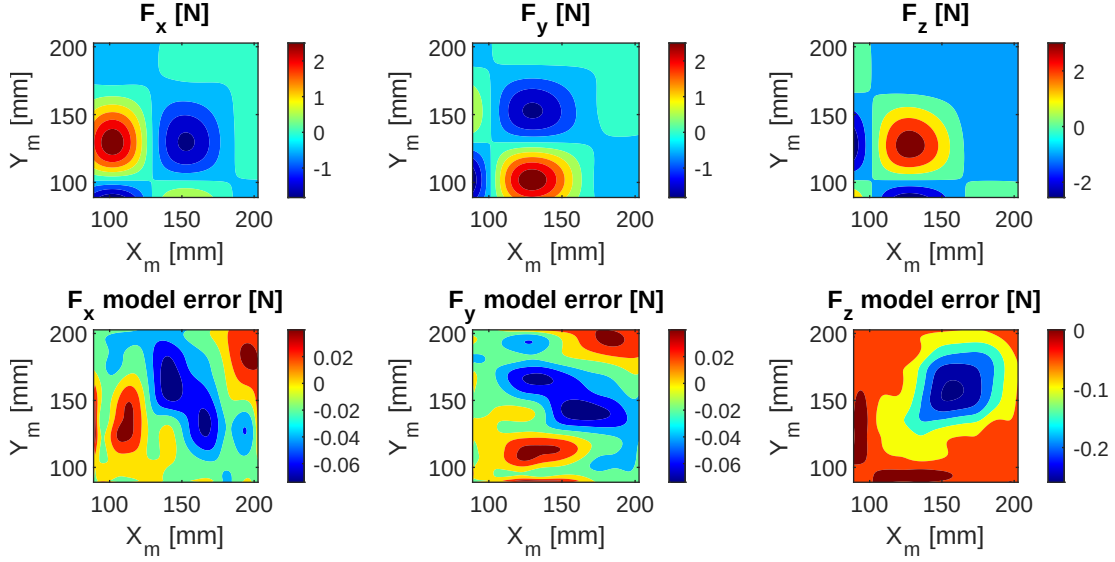


Figure 4.9: Force and modeling error in corner region 2 at 5 mm air gap.

$$\vec{T} = \begin{bmatrix} r_{y1}^{eff} F_z - r_{z1}^{eff} F_y \\ -r_{x1}^{eff} F_z + r_{z2}^{eff} F_x \\ r_{x2}^{eff} F_y - r_{y2}^{eff} F_x \end{bmatrix}, \quad (4.14)$$

where r_{x1}^{eff} , r_{x2}^{eff} , r_{y1}^{eff} , r_{y2}^{eff} , r_{z1}^{eff} and r_{z2}^{eff} are the ETA.

Torque in each direction has two components. Each torque component is named 1 and 2 based on the order in (4.14). For example, $r_{y1}^{eff} F_z$ is named as T_{x1} and $-r_{z1}^{eff} F_y$ is named as T_{x2} . At each coil location, the length of the ETA can be found through the numerical solution of force and torque. Instead of directly finding the total torque (T_x , T_y and T_z), each component of the torque (T_{x1} , T_{x2} , T_{y1} , T_{y2} , T_{z1} and T_{z2}) is first found. Then, the ETA can be found as a quotient of each torque component and the corresponding force. In the following sections, ETA in each region is found in 3D space with the same position resolution used for force analysis. Then, the characteristics of ETA are observed and used for torque modeling.

4.3.2 Central Region

The ETA r_{z1}^{eff} and r_{z2}^{eff} have a relatively constant distribution at each air gap distance in the central region. It is assumed that r_{z1}^{eff} and r_{z2}^{eff} are not relevant to c_x^m and c_y^m , only

relevant to air gap distance. Values of r_{z1}^{eff} and r_{z2}^{eff} are firstly found by averaging their values at each air gap distance. Then, their values are modeled to be linearly related to the air gap distance. Using the data from 0.5 mm to 7.5 mm, equations for r_{z1}^{eff} and r_{z2}^{eff} in the central region are found by data fitting using

$$r_{z1} = r_{z2} = -c_z^m + \alpha_{rz}, \quad (4.15)$$

where α_{rz} is a constant. In the example setup, value of α_{rz} is -0.0146 m.

At 1.5 mm air gap distance, the ETA r_{x1}^{eff} , r_{x2}^{eff} , r_{y1}^{eff} and r_{y2}^{eff} in the central region are shown in Fig. 4.10. They can be modeled as inclined tangent plane and their values are

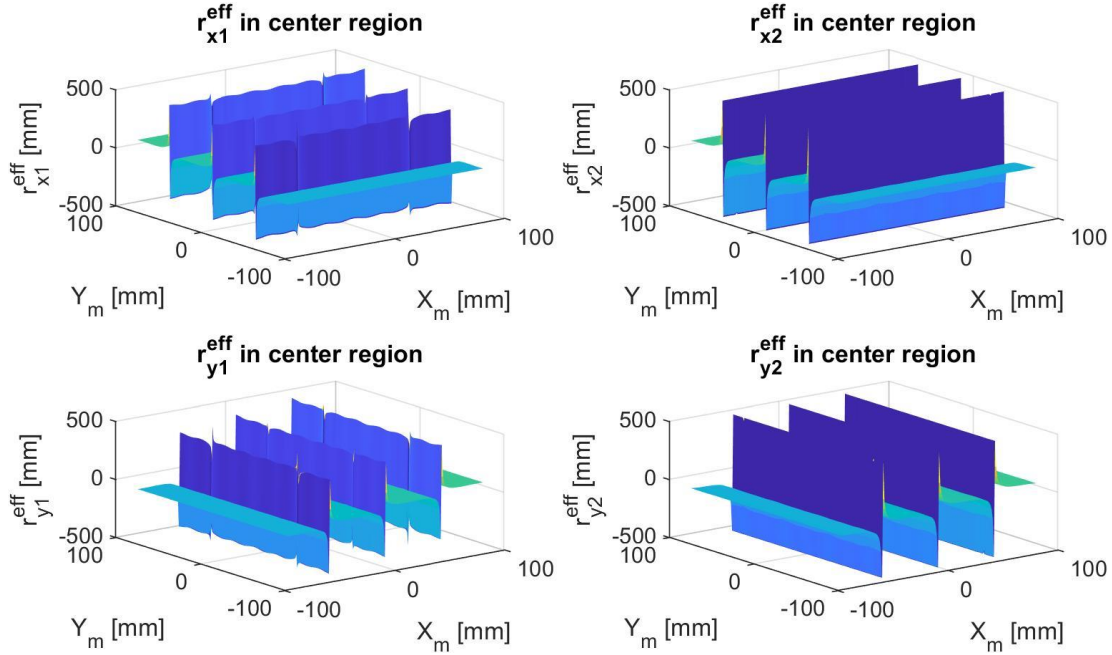


Figure 4.10: ETA in the central region at 1.5 mm air gap.

found by non-linear regression, shown in (4.16):

$$r_{ji}^{eff}(p_j) = p_j - m_i \tan(np_j + \pi/2), \quad (4.16)$$

where j stands for x or y, i is 1 or 2, meaning the index of ETA. Linear equations are applied to describe their changes with respect to air gap distance. n is the period parameter and equals 61.8424. This is the same as π/τ . m_i is the magnitude parameter, and values of m_i

are found from 0.5 mm to 7.5 mm. Values of m_i are firstly found at each air gap. Then, linear fitting equations are used to describe trends of m_i at different air gap distance as

$$m_1 = \alpha_{m1}c_z^m + \beta_{m1}, \quad (4.17)$$

$$m_2 = \alpha_{m2}c_z^m + \beta_{m2}, \quad (4.18)$$

where α_{m1} , β_{m1} , α_{m2} and β_{m2} are constants. Their values in the example setup are -0.0715, -0.123 m, -0.1046 and -0.0203 m.

Knowing the force and the ETA, the torque can be calculated by (4.14). Again, the model is built from information from 0.5 mm to 7.5 mm air gap distance. At 5 mm air gap distance, the resultant torque model is compared to the numerical solution. The resultant torque and the torque modeling error in the central region are shown in Fig. 4.11. The RMSE of T_x , T_y and T_z are 0.0014 Nm, 0.0014 Nm and 0.0011 Nm respectively.

4.3.3 Edge Region

The length of the ETA is found in the edge region. In edge region 1, r_{x1}^{eff} and r_{x2}^{eff} still follows similar pattern as the central region, whose trend can be modeled by (4.16). As shown in Fig. 4.12, r_{y1}^{eff} , r_{y2}^{eff} , r_{z1}^{eff} and r_{z2}^{eff} do not share the same feature as the central region, but their distribution is highly uniform along the X_m direction. They are assumed to be irrelevant to the X_m direction for simplicity.

There are six torque components in total. Methods are proposed to model these components. Combining the features of the force and ETA in edge regions, torque in edge regions can be modeled. The expression for T_{y1} is the same as the central region, since ETA r_{x1}^{eff} and F_z are known. The equation for T_{y1} is

$$T_{y1} = -r_{x1}^{eff} F_z. \quad (4.19)$$

T_{z1} in edge region 1 can be found in the same way. Different approach is proposed to model T_{x1} , T_{x2} , T_{y2} and T_{z2} as their ETA are different with central region. Take T_{x1} as an example, equation for T_{x1} is

$$T_{x1} = r_{y1}^{eff} F_z. \quad (4.20)$$

Plugging F_z from (4.5) into (4.20), T_{x1} becomes

$$T_{x1} = IK_z(c_z^m) \sin(\eta_x) r_{y1}^{eff}(c_y^m, c_z^m) f_3(c_y^m, c_z^m). \quad (4.21)$$

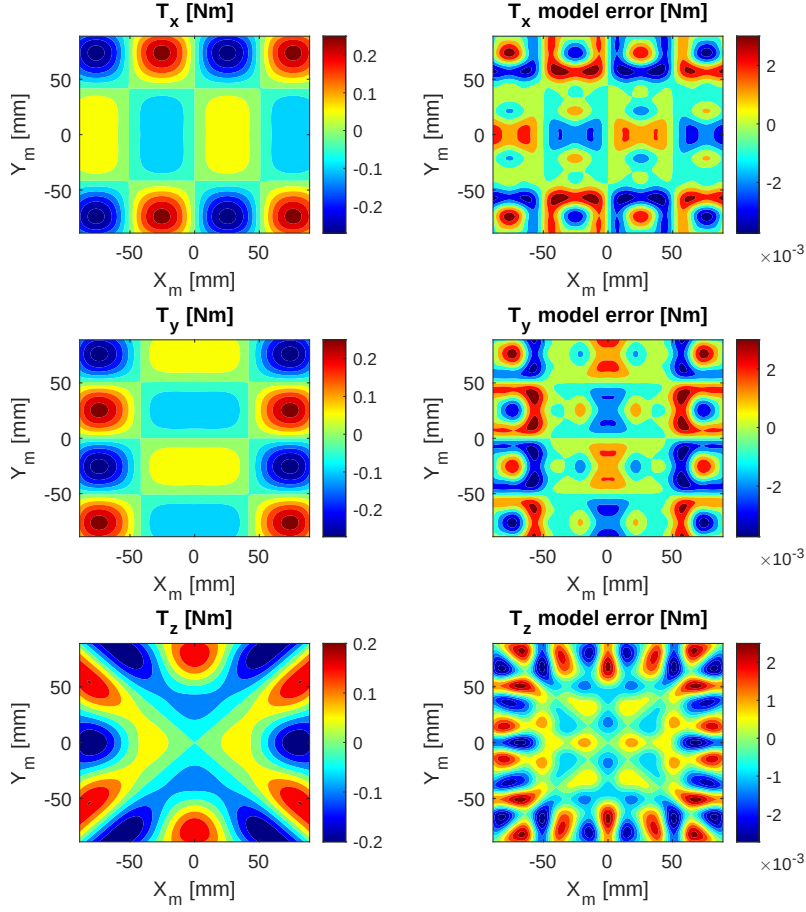


Figure 4.11: Torque and modeling error in the central region at 5 mm air gap.

In (4.21), r_{y1}^{eff} and $f_3(c_y^m, g)$ is related to c_z^m and c_y^m . K_z is only related to c_z^m . Nonlinear torque component, $t_1(c_y^m, g)$ is to defined to model the variation of T_{x1} in the Y_m and Z_m direction. Equation for T_{x1} becomes

$$T_{x1} = I \sin(\eta_x) t_1(c_y^m, c_z^m). \quad (4.22)$$

Same approach is applied for T_{x2} , T_{y2} and T_{z2} . The final equation for torque in edge region 1 is

$$\vec{T}_{edge1}(c_x^m, c_y^m, c_z^m) = \begin{bmatrix} I \sin(\eta_x) t_1(c_y^m, c_z^m) + I \sin(\eta_x) t_2(c_y^m, c_z^m) \\ -r_{x1}^{eff} F_z + I \cos(\eta_x) t_3(c_y^m, c_z^m) \\ r_{x2}^{eff} F_y + I \cos(\eta_x) t_4(c_y^m, c_z^m) \end{bmatrix}, \quad (4.23)$$

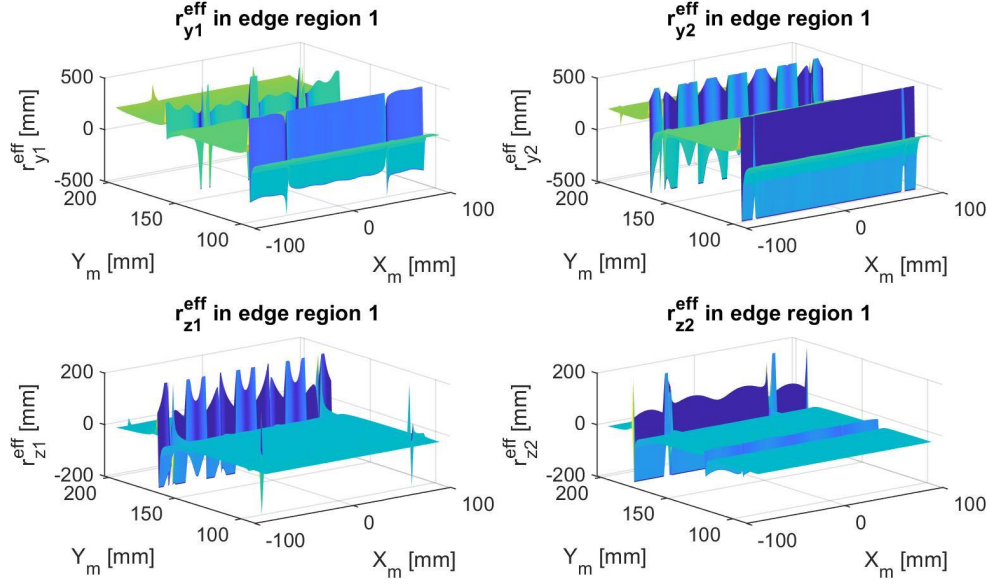


Figure 4.12: ETA in edge region 1 at 1.5 mm air gap.

where t_1 , t_2 , t_3 and t_4 are the nonlinear torque components. The unit of nonlinear torque components is Newton-meter per Ampere.

2D distribution of t_1 , t_2 , t_3 and t_4 in edge region 1 at 1.5 mm is shown in Fig. 4.13. Their values are highly uniform along the X_m direction. The dimension of all nonlinear torque components is reduced from 2D to 1D by averaging in the X_m direction at each air gap. Variation of t_1 , t_2 , t_3 and t_4 across different air gap distance is shown in Fig. 4.14. Distribution of t_1 , t_2 , t_3 and t_4 in the Y_m and Z_m direction can be modeled with (4.6) using the same method as the nonlinear force components. In the example setup, the peak-to-peak modeling errors of all nonlinear torque components are found within 3%.

With the equations of the nonlinear torque components and the 2D Halbach being symmetry, similar to edge region 1, the torque equations for edge region 2, 3, and 4 are presented in (4.24)-(4.26):

$$\vec{T}_{edge2}(c_x^m, c_y^m, c_z^m) = \begin{bmatrix} r_{y1}^{eff} F_z + It_3(-c_x^m, c_z^m) \cos(\eta_y) \\ -It_1(-c_x^m, c_z^m) \sin(\eta_y) - It_2(-c_x^m, c_z^m) \sin(\eta_y) \\ -It_4(-c_x^m, c_z^m) \cos(\eta_y) - r_{y2}^{eff} F_x \end{bmatrix}, \quad (4.24)$$

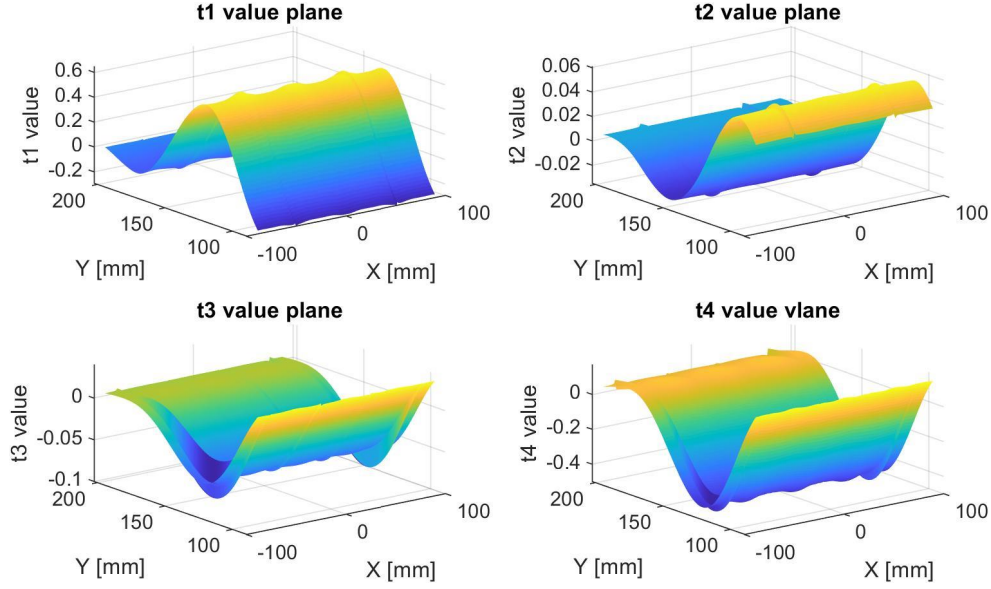


Figure 4.13: t_1 , t_2 , t_3 and t_4 value at 1.5 mm air gap.

$$\vec{T}_{edge3}(c_x^m, c_y^m, c_z^m) = \begin{bmatrix} r_{y1}^{eff} F_z - It_3(c_x^m, c_z^m) \cos(\eta_y) \\ -It_1(c_x^m, c_z^m) \sin(\eta_y) - It_2(c_x^m, c_z^m) \sin(\eta_y) \\ -It_4(c_x^m, c_z^m) \cos(\eta_y) - r_{y2}^{eff} F_x \end{bmatrix}, \quad (4.25)$$

$$\vec{T}_{edge4}(c_x^m, c_y^m, c_z^m) = \begin{bmatrix} I \sin(\eta_x) t_1(-c_y^m, c_z^m) + I \sin(\eta_x) t_2(-c_y^m, c_z^m) \\ -r_{x1}^{eff} F_z - I \cos(\eta_x) t_3(-c_y^m, c_z^m) \\ r_{x2}^{eff} F_y + I \cos(\eta_x) t_4(-c_y^m, c_z^m) \end{bmatrix}. \quad (4.26)$$

The nonlinear torque components in (4.24)-(4.26) are the same as in (4.23) with different values plugged in.

The torque in edge region 1 is modeled using the method mentioned above. The proposed model is compared with numerical results at 5 mm air gap. The The RMSE of T_x , T_y and T_z are 0.0043 Nm, 0.0047 Nm and 0.0019 Nm. T_x , T_y and T_z and the modeling error are shown in Fig. 4.15.

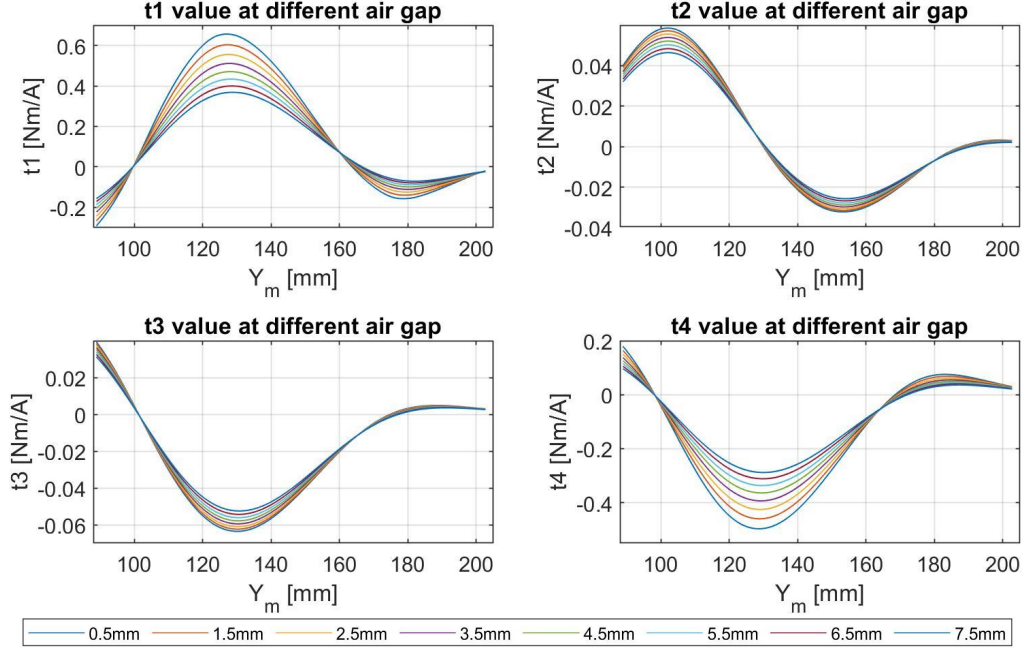


Figure 4.14: t_1 , t_2 , t_3 and t_4 values at different air gap.

4.3.4 Corner Region

The torque equation in each edge region can be represented by two neighbouring edge regions. The torque equation in corner region 1 is shown in (4.27). In (4.27), X_m component of T_{x1} , T_{y1} , T_{z1} and T_{z2} is the same as their distribution in edge region 2, and their distribution in Y_m is the same as distribution in edge region 1. For T_{x2} and T_{y2} , their distribution equation in edge region 1 and 2 are both described by nonlinear torque component, whose unit is Newton-meter per Ampere. To use their product, a normalizing function $n(g)$ is introduced. $n(g)$ is a linear fit of the inverse of the maximum value of t_2 across different air gap distances. The unit of $n(g)$ is m^{-1} . The equation of $n(g)$ is shown in (4.28).

$$\vec{T}_{corner1}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} -f_3(-c_x^m, c_z^m)t_1(c_y^m, c_z^m) + n(c_z^m)t_3(-c_x^m, c_z^m)t_2(c_y^m, c_z^m) \\ -t_1(-c_x^m, c_z^m)f_3(c_y^m, c_z^m) + n(c_z^m)t_2(-c_x^m, c_z^m)t_3(c_y^m, c_z^m) \\ -t_4(-c_x^m, c_z^m)f_2(c_y^m, c_z^m) + f_2(-c_x^m, c_z^m)t_4(c_y^m, c_z^m) \end{bmatrix}, \quad (4.27)$$

where

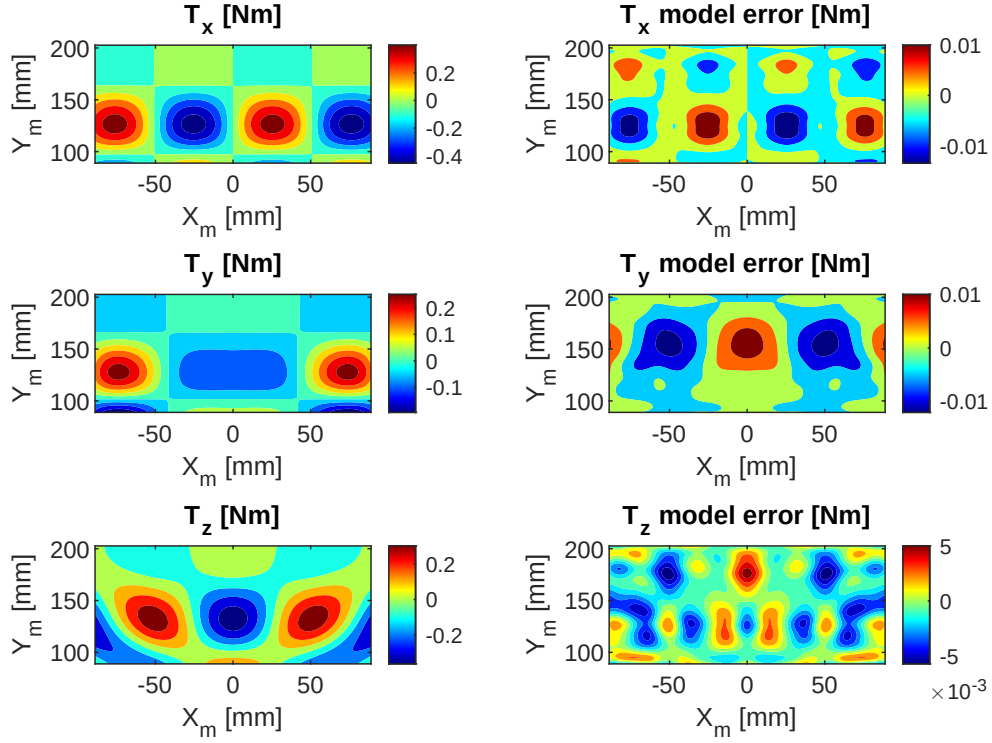


Figure 4.15: Torque and modeling error in edge region 1 at 5 mm air gap.

$$n(c_z^m) = \alpha_n c_z^m + \beta_n. \quad (4.28)$$

In (4.28), α_n and β_n are constants, and their values are found to be 634.41 m^{-2} and 16.484 m through data fitting in the example setup.

Similar to corner region 1, torque equations for corner region 2-4 are shown in (4.29)-(4.31):

$$\vec{T}_{corner2}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} f_3(c_x^m, c_z^m)t_1(c_y^m, c_z^m) - n(c_z^m)t_3(c_x^m, c_z^m)t_2(c_y^m, c_z^m) \\ -t_1(c_x^m, c_z^m)f_3(c_y^m, c_z^m) + n(c_z^m)t_2(c_x^m, c_z^m)t_3(c_y^m, c_z^m) \\ -t_4(c_x^m, c_z^m)f_2(c_y^m, c_z^m) + f_2(c_x^m, c_z^m)t_4(c_y^m, c_z^m) \end{bmatrix}, \quad (4.29)$$

$$\vec{T}_{corner3}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} -f_3(-c_x^m, c_z^m)t_1(-c_y^m, c_z^m) + n(c_z^m)t_3(-c_x^m, c_z^m)t_2(-c_y^m, c_z^m) \\ t_1(-c_x^m, c_z^m)f_3(-c_y^m, c_z^m) - n(c_z^m)t_2(-c_x^m, c_z^m)t_3(-c_y^m, c_z^m) \\ -t_4(-c_x^m, c_z^m)f_2(-c_y^m, c_z^m) + f_2(-c_x^m, c_z^m)t_4(-c_y^m, c_z^m) \end{bmatrix}, \quad (4.30)$$

$$\vec{T}_{corner4}(c_x^m, c_y^m, c_z^m) = I \begin{bmatrix} f_3(c_x^m, c_z^m)t_1(-c_y^m, c_z^m) - n(c_z^m)t_3(c_x^m, c_z^m)t_2(-c_y^m, c_z^m) \\ t_1(c_x^m, c_z^m)f_3(-c_y^m, c_z^m) - n(c_z^m)t_2(c_x^m, c_z^m)t_3(-c_y^m, c_z^m) \\ -t_4(c_x^m, c_z^m)f_2(-c_y^m, c_z^m) + f_2(c_x^m, c_z^m)t_4(-c_y^m, c_z^m) \end{bmatrix}. \quad (4.31)$$

Nonlinear torque components in (4.27)-(4.31) are the same equations used for (4.23). Only four equations for nonlinear torque components are required for all the torque modeling in edge and corner regions.

The torque in corner region 2 is calculated via the proposed method, and it is compared to the numerical solution at 5 mm air gap for validation. The modeled torque and error are shown in Fig. 4.16. The resultant RMSE of T_x , T_y and T_z are 0.0157 Nm, 0.0157 Nm and 0.0036 Nm.

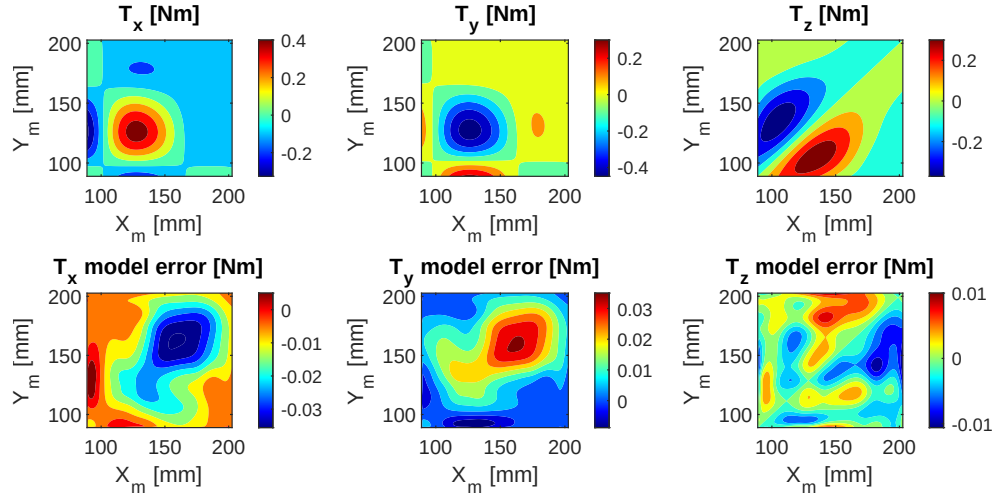


Figure 4.16: Torque and modeling error corner region 2 at 5 mm air gap.

4.4 Validation, Analysis and Generalization

4.4.1 Experimental Validation

The proposed force and torque model is verified through measurements. The measurement setup is shown in Fig. 4.17. The force and torque on the Halbach array are measured by ATI SI-65-5 6-axis load cell. The cable is attached onto the table to avoid the influence of its weight. The carrier and attachment weight are offset to 0 in the program. The coil followed a straight line on the frame and the line of measurement is $c_y^m = -0.5101c_x^m - 70.897$ mm. This line travels through edge region 2, central region, edge region 4 and corner region 4. The location of the coil is measured by Vicon motion cameras using the markers on the coil. The current is supplied by the DC power supply. The air gap distance between the magnet array and the coil is 5 mm.

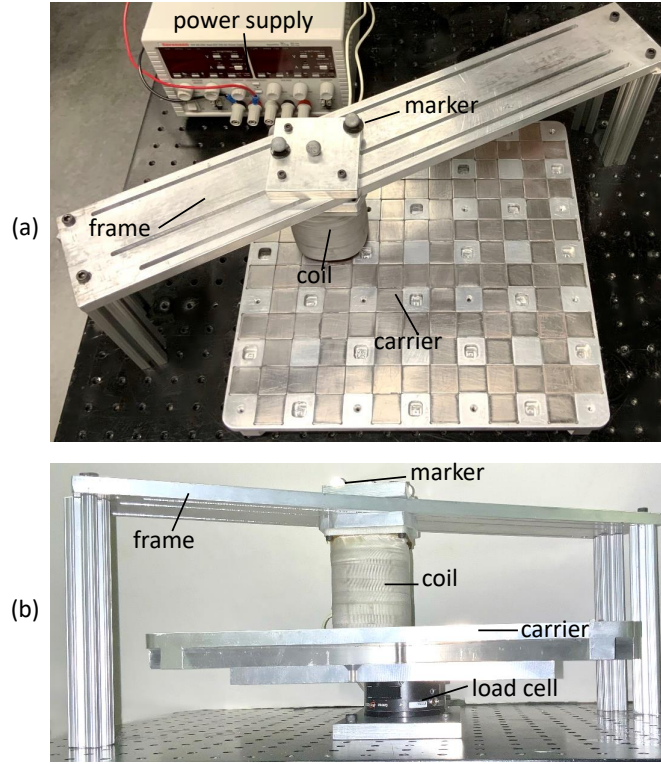


Figure 4.17: Measurement setup (a) top view (b) side view.

The measurement result is shown in Fig. 4.18, and it shows good agreement between

the proposed model and the actual measurement. The RMSE for F_x , F_y , F_z , T_x , T_y and T_z are 0.0948 N, 0.1094 N, 0.1687 N, 0.0181 Nm, 0.0209 Nm and 0.0142 Nm respectively.

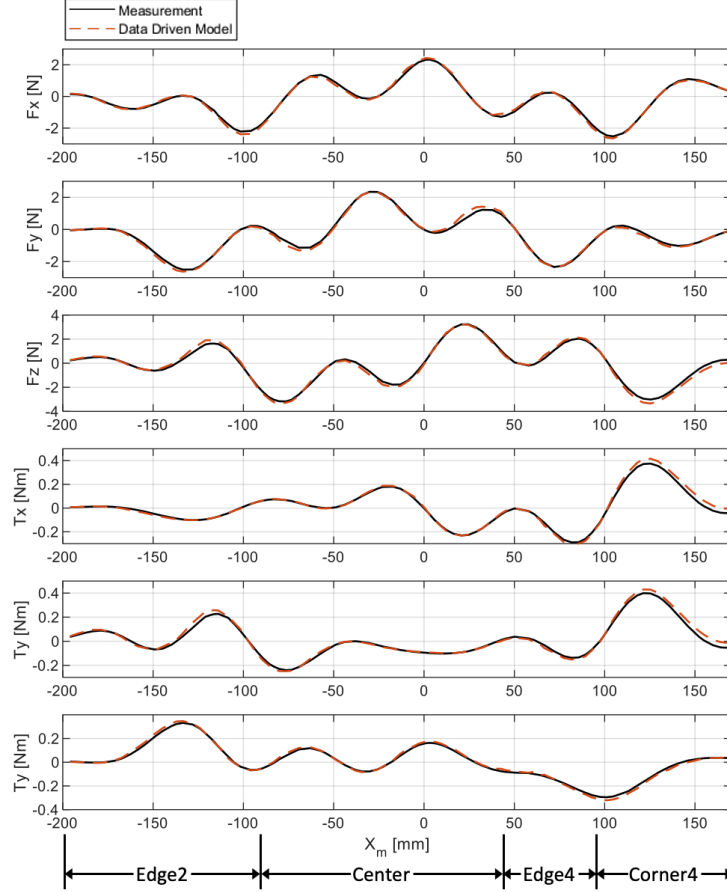


Figure 4.18: Comparison of force and torque measurement and the model of the square coil.

4.4.2 Computation Time and Limitations

The computation time of the proposed model is experimentally measured on Beckhoff C6670-0010 industrial server. In the central region, the equations are the simplest, and the average computation time is found to be $0.9 \mu s$. In edge regions, nonlinear force and torque components are calculated from one direction. The resultant average computation

time is $2.1 \mu s$. In corner regions, nonlinear force and torque components are calculated from two directions. The resultant average computation time is $3.2 \mu s$. Considering the area and computation speed of all regions, the overall computation time of the entire area under the 2D Halbach array is $2.3 \mu s$. This speed allows the proposed model to be used in real-time control of an MLPM system.

In the proposed model, the rotation angles between the carrier and the coil are all zero. When implementing this method in control of maglev systems, infinite translational motion in the X_g and Y_g direction can be achieved, while limited rotation can be supported.

4.4.3 Model Generalization

The proposed model is also tested on a circular coil of a different size. The inner diameter of the circular coil is 15 mm; the outer diameter is 40 mm; the height is 45 mm; the air gap is 3 mm, and there are 1000 turns in this coil. The two coils tested in this study are shown in Fig. 4.19.

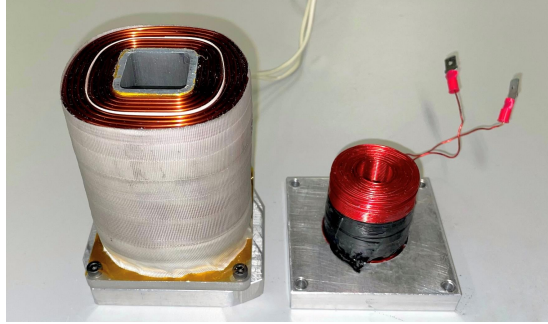


Figure 4.19: Square and circular coils tested in this study.

The force and torque on the carrier are measured along $c_y^m = -0.5101c_x^m - 70.897$ mm, which is the same line as the square coil. The measurement results are compared with the proposed model in Fig. 4.20. The RMSE of the circular coil for F_x , F_y , F_z , T_x , T_y and T_z are 0.1239 N, 0.1223 N, 0.1776 N, 0.0129 Nm, 0.0129 Nm and 0.0147 Nm respectively. The proposed modeling method works well with both circular and square coils of different sizes and air gap. This method is beneficial for predicting the force and torque of any circular and square coils of different sizes.

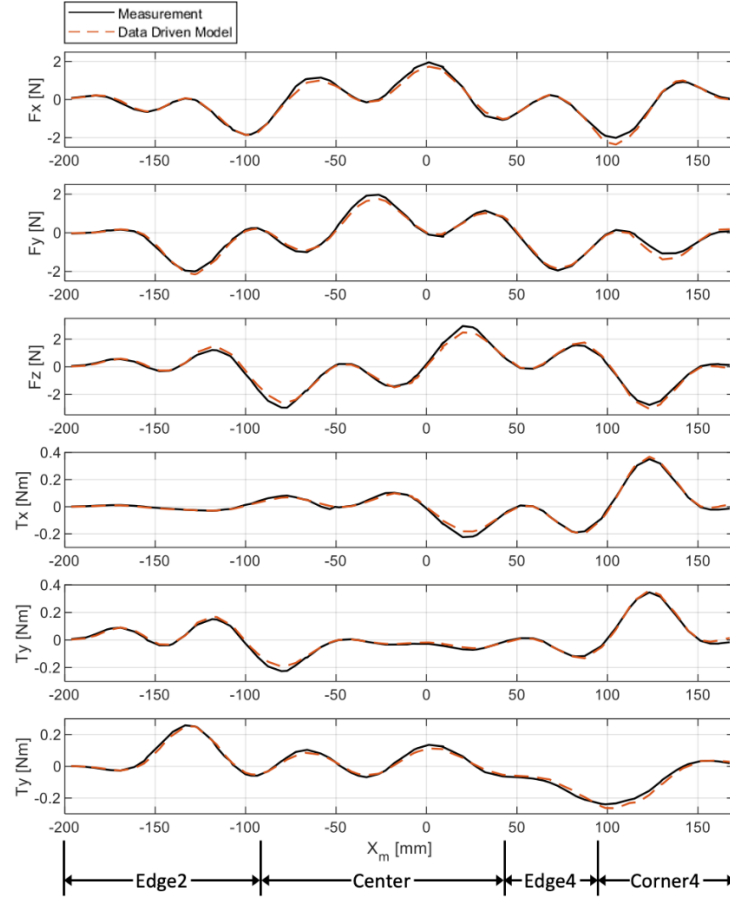


Figure 4.20: Comparison of force and torque measurement and the model of the circular coil.

Chapter 5

Electromagnetic Design

In this chapter, the design of coils and the magnetic mover will be explained. Currently, two systems with the same electromagnetic design are built. They are the maglev test bench and maglev floor.

5.1 Coil Design

Fabry Factor can measure the power efficiency of a square coil design based on if the coil geometry can produce a large magnetic flux density. The equation for Fabry Factor is [17]:

$$G(\alpha, \beta) = \frac{1}{5} \sqrt{\frac{2\pi\beta}{\alpha^2 - 1}} \ln\left(\frac{\alpha + \sqrt{\alpha^2 + \beta^2}}{1 + \sqrt{1 + \beta^2}}\right), \quad (5.1)$$

where $\alpha = l_c/(l_c - 2w_c)$ and $\beta = h_c/(l_c - 2w_c)$. The contour plot for the Fabry Factor is shown in Figure 5.1. To have the highest Fabry Factor value, $\alpha = 3.09$ and $\beta = 1.88$. This will result in a Fabry Factor of 0.179.

The final coil dimension is shown in Table 5.1. For the actual coil design, the coil inner width is chosen to be 32 mm, which is the width of the coil bobbin. The outer width of the coil is 75 mm, and the coil height is 86.5mm. The actual $\alpha = 2.34$ and $\beta = 2.7$. This coil dimension results in a Fabry Factor of 0.164. The maximum coil current is designed to be 10 A, and this results in a wire size of 16 AWG. From the coil dimension and wire gauge, this coil design has 976 turns. The final resistance of the coil is 2.5 Ω , and the inductance is 17 mH.

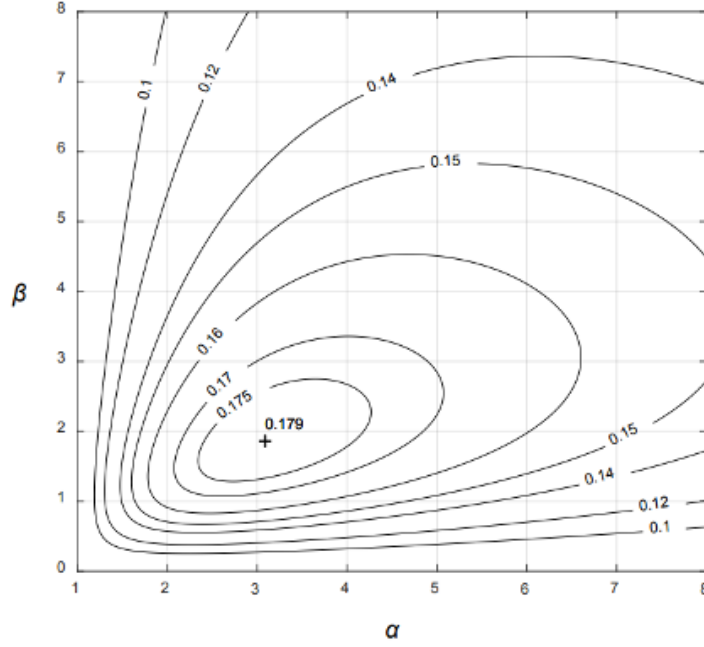


Figure 5.1: Fabry Factor distribution.

5.2 Mover Design

5.2.1 Mover Magnet Design

The Halbach array has been widely implemented in maglev systems because it concentrates the magnetic field on one side, and its magnetic flux density in the central region can be easily modeled. To design a 2D Halbach mover for the existing coil array, the overall energy efficiency and the average horizontal acceleration are considered. The following design constraints are used for the mover design: (1) The levitation height is 6.5 mm. (2) An aluminum case of 2.5 kg will be used to hold the magnets. (3) The maximum coil current is 10 A. (4) The pole pitch distance of the mover will be 2". The width of the Z-direction magnets and the height of the magnets are the two variables to optimize. A bigger size of magnets can enhance the strength of the magnetic field; however, it also adds more self-weight. A multi-variable analysis should be conducted to optimize energy efficiency and the average maximum horizontal acceleration.

The most common design method assumes the mover is levitated by one single coil [40, 71], while the coil is located right under a north or a south pole of the 2D Halbach

Table 5.1: Coil dimensions.

Parameter	Values
Outer width (l_c)	75 mm
Inner width (s_c)	32 mm
Height (h_c)	86.5 mm
Bundle width (w_c)	21.5mm
Wire gauge	16 AWG
Number of turns	976
Resistance	2.6Ω
Induction	17mH

mover. However, this method has the disadvantages of (1) Neglecting the impact of the commutation method and change in the number of active coils. (2) Neglecting the edge effect of the 2D Halbach array. (3) Neglecting the relative locations between the coil and the mover. In this thesis, a novel method is proposed to improve the problems of the previous method. First, the area of study is identified to include all the relative locations between the mover and the coil array. Due to the repetitive coils and coil symmetry, one-quarter of the coil (1.5" by 1.5" area) is identified as the study area, which is shown in [Figure 5.2](#). The average energy efficiency and the average acceleration in this area will be found. Then, the blue study area is divided into a five-by-five mesh area, and the midpoint of each mesh is used. At each midpoint, the current map of the coils is found using the direct wrench method and the active coil switching. The average energy efficiency is the average heat loss among all 25 locations when only vertical force is supplied to levitate the coil. The average maximum horizontal acceleration is also an average for all 25 locations. The maximum horizontal acceleration at each location is found by giving a vertical levitation force and the horizontal force that saturates the coil current.

For the width of the Z-magnets, there are two options, which are 1" and 1.5". For each width, the height of the carrier varies from 0.125" to 1". The results of 1" Z-magnet and 1.5" Z-magnet are shown in [Table 5.2](#) and [Table 5.3](#).

For both layouts, 0.25" carrier thickness provides the best energy efficiency, and the performance between the 1" pole and 1.5" pole distance are very similar. Compared with the 1.5" pole design, the magnetic flux density of the 1" pole design can be described with first-order harmonic equations, which makes the force and torque modeling much easier. Therefore, the pole distance is chosen to be 1," and the mover thickness is chosen to be 0.25". The final dimension of the mover is shown in [Table 5.4](#).

Table 5.2: Optimization results for a mover with 1" Z-magnet width.

Height [in]	Magnet weight [N]	Average power [W]	Horizontal acceleration [m/s ²]
0.125	18.07	436.39	0.71G
0.25	36.13	286.00	1.15G
0.5	72.28	292.52	1.10G
0.75	108.42	375.30	0.84G
1	144.53	499.80	0.66G

Table 5.3: Optimization results for a mover with 1.5" Z-magnet width.

Height [in]	Magnet weight [N]	Average power [W]	Horizontal acceleration [m/s ²]
0.125	21.66	409.26	0.75G
0.25	43.32	287.21	1.16G
0.5	86.65	306.64	1.08G
0.75	129.96	399.61	0.81G
1	173.28	536.14	0.57G

Table 5.4: Final mover dimensions.

Parameter	Values
Magnet height (h_c)	6.35 mm
Pole pitch (τ)	50.8 mm
Z magnet width (τ_m)	25.4mm
Number of Z magnet on one side	6

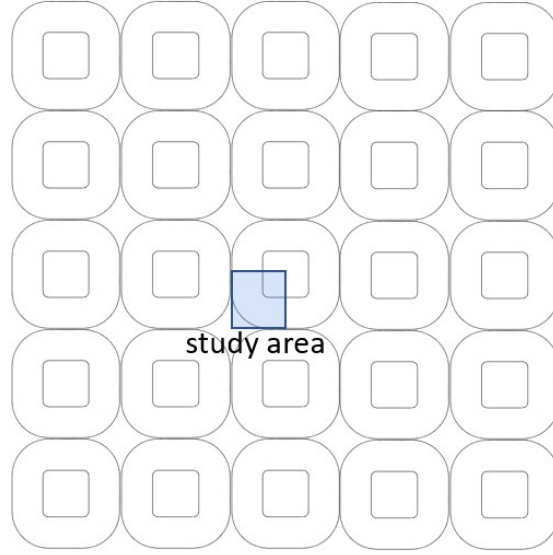


Figure 5.2: Study area for the mover geometry optimization.

5.2.2 Mover Case Design

As stated in [72], there exists a considerable amount of force between the magnets in a 2D Halbach array. The internal force will cause bending in the aluminum mover case. The case must be strong enough to control the bending in a relatively small range. Also, the mover case should be lightweight, and there should be mounting holes for Vicon camera markers as well as laser sensors. Using ANSYS, the forces on each magnet are computed. The magnitudes of the Z direction forces of each magnet are much higher than other forces in other directions. The maximum force is on Z-direction magnets in the central area, whose magnitudes reach 348 N. To overcome the bending caused by the magnets, a cross-shaped structure is designed over the mover, which is shown in Figure 5.3. Solidworks simulation is conducted to check the deformation of this design, and the simulation results are shown in Figure 5.4. The maximum formation is about 0.0089 mm. In the central area, there exists a 4" by 4" square, whose deformation is within 0.0015 mm.

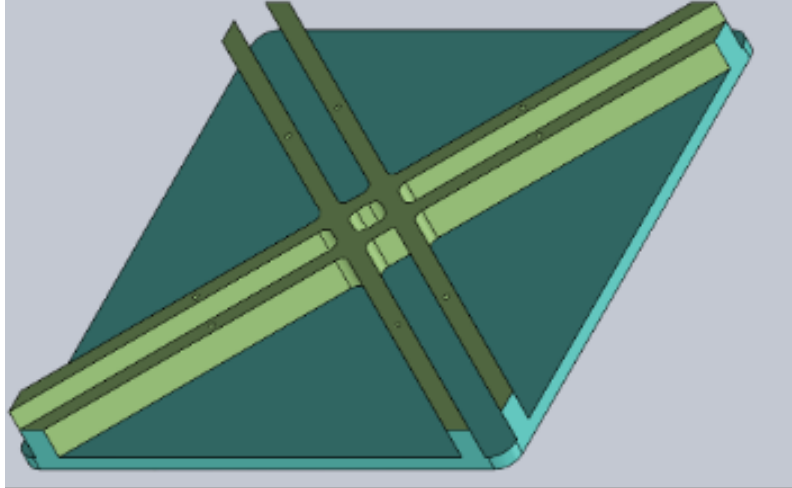


Figure 5.3: Mover case design.

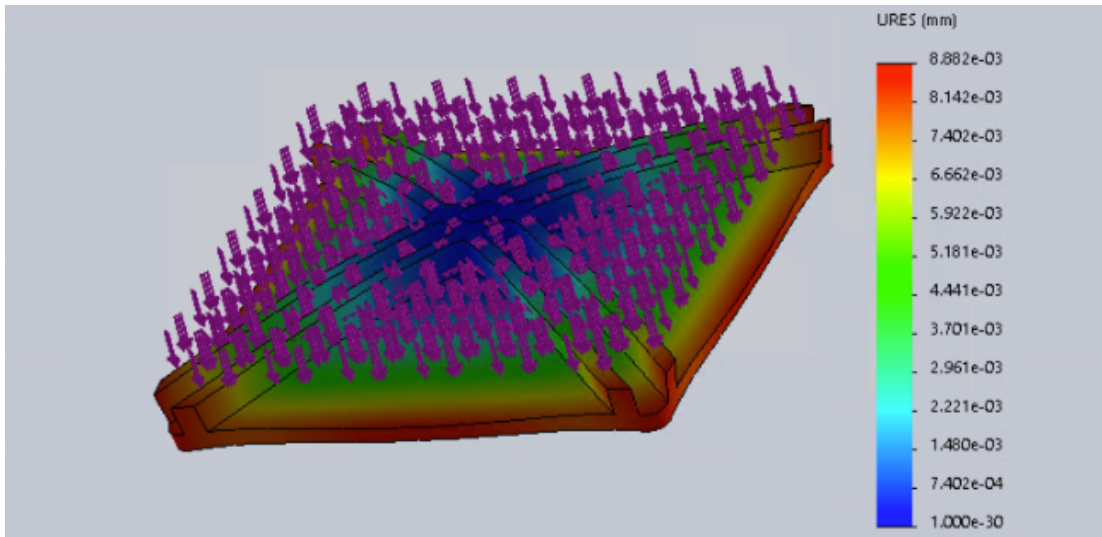


Figure 5.4: Simulation of the mover deformation by the internal force of the 2D Halbach mover.

Chapter 6

System Integration

With the same electromagnetic design, two systems, maglev test bench and maglev floor are developed. In this chapter, the details regarding the two levitation systems are introduced. With the proposed PID controller and the current decoupling method, levitation experiments are conducted over the two systems. System performance are validated.

6.1 Maglev Test Bench

6.1.1 Camera System

Similar to our previous work in [100], a Vicon camera motion tracking system is utilized to accommodate measurement feedback for long-stroke applications since cameras have much greater measurement ranges than the laser interferometers commonly used in previous literature. In a larger system, many cameras could be added with overlapping fields of view. With automatic camera switching, a theoretically infinite motion range can be accommodated while still providing sub-millimeter resolution. An alternative approach is to incorporate hall sensors throughout the assembly; however, maintaining accuracy with this strategy is difficult due to errors caused by higher harmonic components of the magnet array and manufacturing tolerances, so model parameters vary with position and between magnet arrays [5, 27].

In the experiment setup, four Vicon Vantage V5 cameras are placed approximately 1 m from the center of the stator, as shown in Figure 6.1. The cameras are set to High Speed Mode in the Vicon Tracker software, which uses subsampling to accommodate the 1000

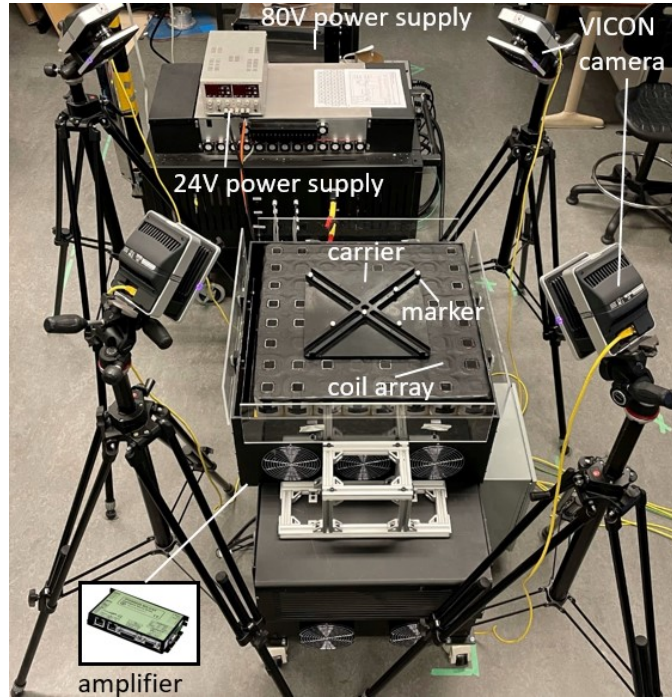


Figure 6.1: Equipment setup with camera placement.

fps frame rate without reducing the field of view. See [Table 6.1](#) for detailed specifications of the camera system.

Table 6.1: Vicon V5 camera specifications.

Component	Specification
Resolution (pixels)	2432 H x 2048 V
Set frame rate	1000 fps
Lens focal length	8.5 mm
Field of view (H x V) $^{\circ}$	63.5 x 55.1
Strobe	850 nm (IR)

The mover has six reflective markers mounted on its top surface for the IR camera system to detect. The three markers in the center form a 90° angle to align the mover's coordinate system, which is then moved to the center bottom of the magnet array in the Vicon software. Three additional markers are placed on standoffs of varying heights at the corners of the mover for a more precise measurement of the rotations of the mover. See

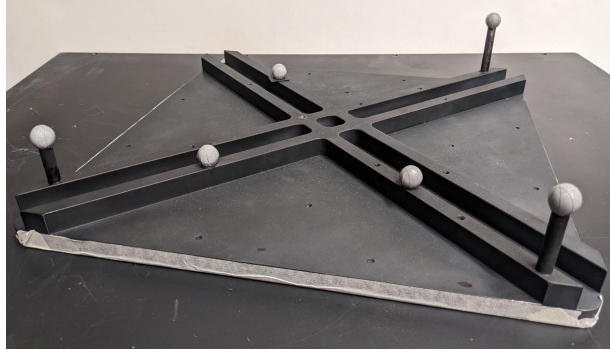


Figure 6.2: 2D Halbach array mover with 6 reflective markers.

Figure 6.2 for the marker setup on the mover.

6.1.2 Modular Electromagnet Stator

The overall design of the proposed system stator is represented in Figure 6.3a. The stator consists of an array of coil modules and a support structure.

The coil array comprises eight coil modules. Each coil module has eight square coils mounted on a module frame in rows with a 76.2 mm spacing between the centers of the coils as shown in Figure 6.3b. The total planar levitation area is 600 mm $\times$ 600 mm, approximately. The coil module frame is made from aluminum to reduce magnetic field absorption and hold the weight of eight coils. The square coils are wired to the power receptacle at the bottom of the module frame.

The steel support structure contains two levels. The upper-level frame is for coil module mounting with level adjustment mechanisms, while the lower level is a rigid frame with a built-in amplifier module. PWM current amplifiers are implemented to control the coil current. Each current amplifier is supplied with 80 VDC 10 A power supply and 24 VDC logic supply. There are two independent current coil loops within each amplifier, and the maximum current of each loop is set to 9 A. The amplifier module has 32 amplifier units mounted on a copper plate installed in the support structure with insulated connections. The copper plate is used to return power from amplifier units to the power source. EtherCAT is a protocol that provides high-speed communication and reduces wiring complexity between the controller and amplifiers in the system.

Figure 6.4 represents the system integration of the motion tracking system, modular maglev system, and control hardware. With the modular design of the electromagnet sta-

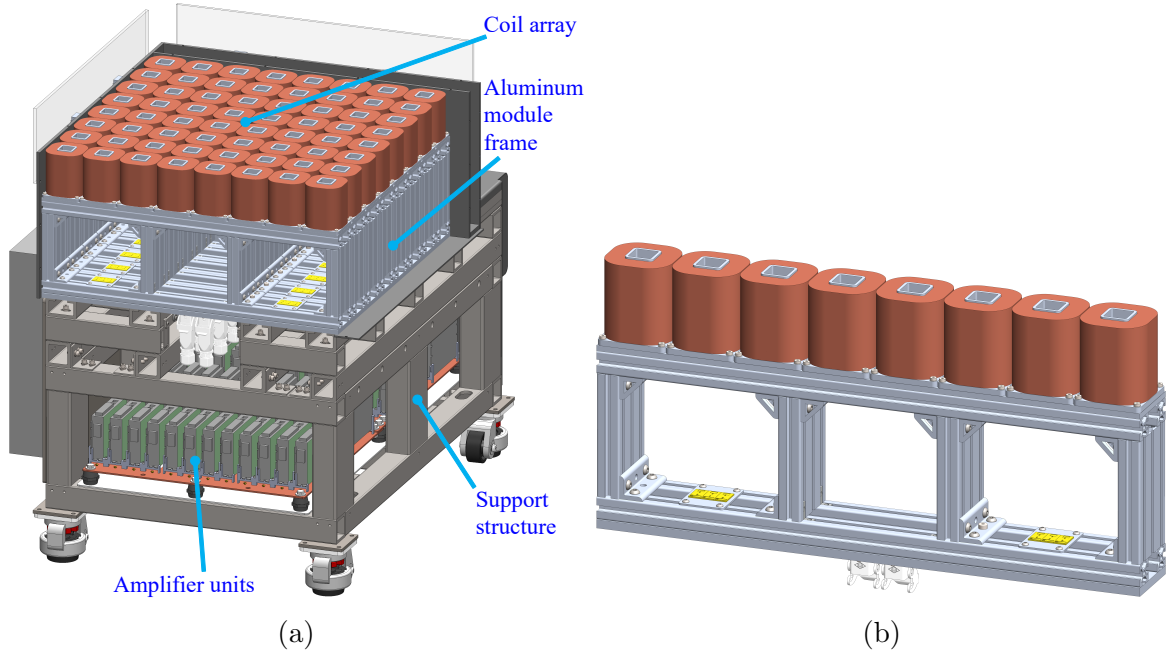


Figure 6.3: System stator overview (a) Modular electromagnet stator (b) Coil module.

tor, the stator can be flexibly extended to fit in the desired area. The motion tracking system using cameras provides a large tracking area. The EtherCAT communication protocol reduces the wiring complexity between controller and amplifiers. These features of the proposed system design enhance the flexibility of industrial applications.

6.2 Maglev Floor in Robohub

The aforementioned methods are also tested on the maglev floor. The maglev floor system is shown in Figure 6.6. The maglev floor contains eight tables. Each table supports the coil array of 12 by 8 coils. The same current amplifiers as the test bench are used on the maglev floor. There are 48 amplifiers on each table. A levitation area of 2.4 m by 1.8 m is created by a total of 768 coils. The same four Vicon motion cameras are used to measure the mover location. Since bigger levitation area is covered by the same camera, the measurement resolution is expected to decrease. The maglev floor stator was designed by Dr. Xiaodong Zhang and Dr. Chanuphon Trakarnchaiyo. The assembly of the maglev floor is a collaborative work by Dr. Xiaodong Zhang, Dr. Chanuphon Trakarnchaiyo,

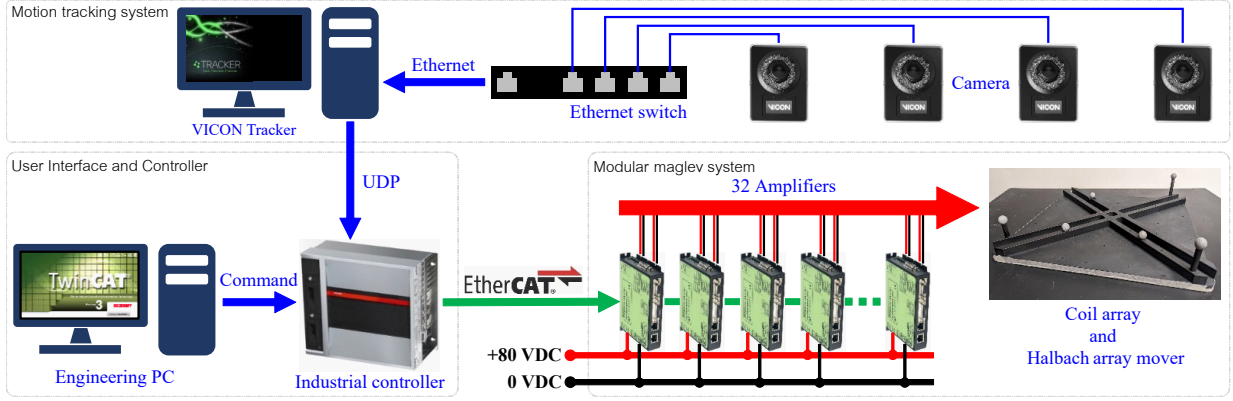


Figure 6.4: Hardware connection diagram.

Dr. Hassan Askari, Dr. Moslem Sadeghigoughari, Curtis Stewart and myself. The system testing, control implementation, and current decoupling implementation were completed by Curtis Stewart and myself.

The maglev floor is located on a hydraulic lift, and it has three operating height. When the floor is lifted all the way up, it is at its maintenance height, allowing easy access to all the components. The maintenance height is shown in Figure 6.5. With the hydraulic lift moving down, it reaches the working level, where the top of the coils flush with the floor level. This is shown in Figure 6.6. Lastly, when the hydraulic lift is at its lowest, the maglev is in the storage level. The entire maglev system can be stored underground, and the floor area can be used for any other purpose.

6.3 PID Control

6.3.1 System Model

The mover is levitated with no friction or damping in the plane and the air resistance is neglected in all directions. Since the proposed control strategy is implemented in the X_g and Y_g directions, the associated motion equations are

$$\begin{aligned} F_x &= ma_x^g, \\ F_y &= ma_y^g, \end{aligned} \tag{6.1}$$

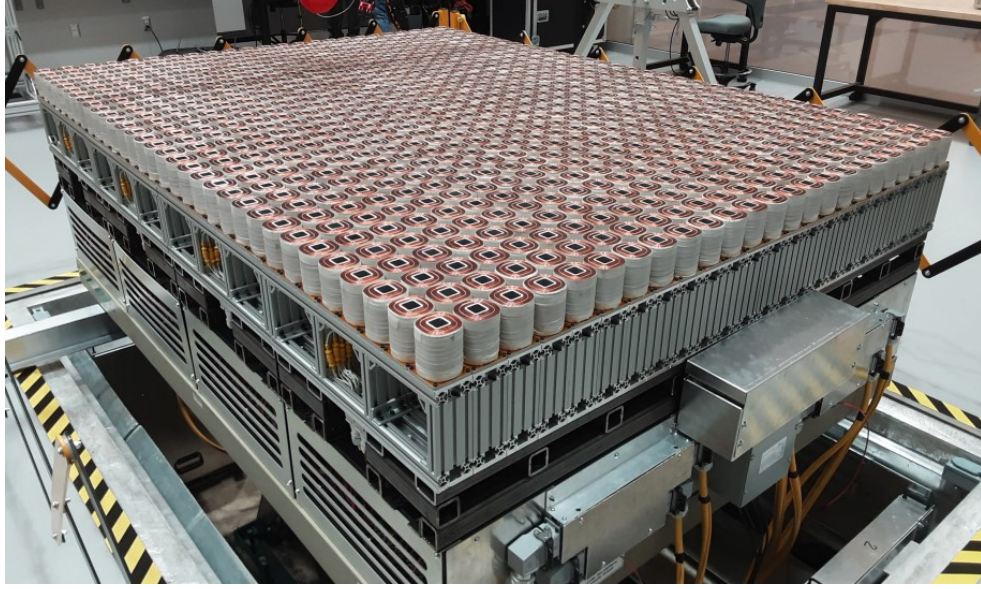


Figure 6.5: Maglev floor with coils lifted.

where m is the mass of the mover. a_x^g and a_y^g are the acceleration of the mover in the $X_g Y_g$ plane. From the motion equations above, the associated discrete state-space representation can be written as

$$\begin{aligned}\zeta(k+1) &= A\zeta(k) + Bu(k), \\ \rho(k) &= C\zeta(k),\end{aligned}\tag{6.2}$$

where the state vector is $\zeta = [m_x^g, v_x^g, m_y^g, v_y^g]^\top$, the force input is $u = [F_x, F_y]^\top$, output is $\rho = [m_x^g, m_y^g]^\top$, the sampling period is ΔT , and k is the time instant. v_x^g and v_y^g are the speed of the mover in the global coordinate. State matrix A , input matrix B and output matrix C are:

$$A = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{\Delta T^2}{2m} & 0 \\ \frac{\Delta T^2}{2m} & 0 \\ 0 & \frac{\Delta T^2}{2m} \\ 0 & \frac{\Delta T^2}{2m} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}^\top.$$

6.3.2 Controller Design

PID controllers are used to control the motion of the mover. Six individual control loops are used to control the six-DOF motion. The control diagram is shown in [Figure 6.7](#). From

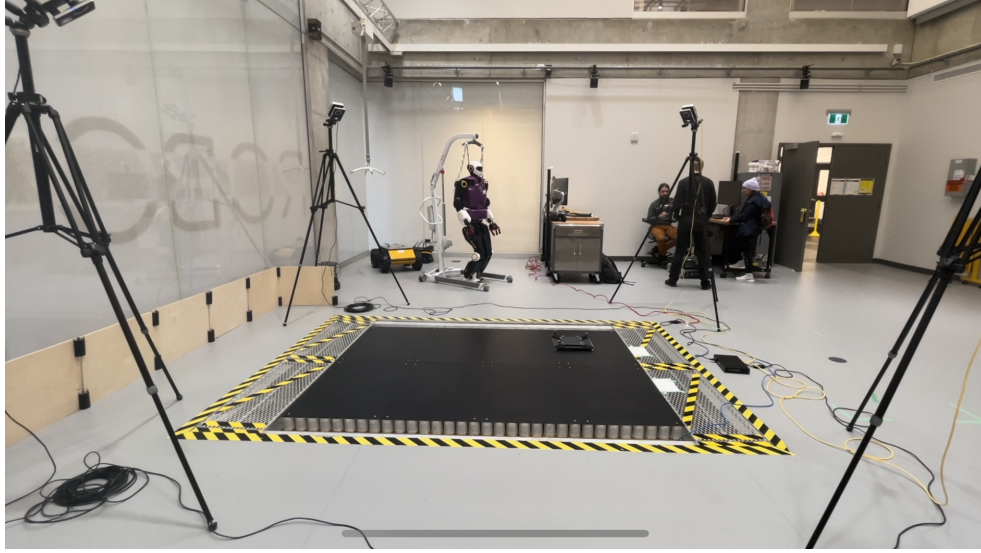


Figure 6.6: Maglev floor in operation.

the reference and the actual position measurement, the desired force and torque of each axis are solved. Then, the desired force and torque are converted to the control current using the direct wrench method. The weight of the carrier is 6.6 kg. The moments of inertia are 0.059 kgm^2 for X_m and Y_m , and 0.12 kgm^2 for Z_m . A combination of pole placement and online tuning is used to identify the PID parameters. First, the poles of each axis are selected based on a desired step response with a 2% settling time of 0.15 s and an overshoot of 15%. Then, the designed PID parameters are tested and tuned on the experimental setup. The PID control parameters used on maglev test bench and maglev floor are the same. The final PID control parameters are shown in [Table 6.2](#).

6.4 Active Coil Selection and Current Decoupling

The control force and torque are decoupled to coil current at each sampling period. As the mover moves across different areas, different coil sets are activated for the levitation. A coil will be activated if it is located inside a 16" by 16" square boundary centered and rotated with the mover since the coil outside this region can barely produce force and torque. This boundary corresponds to the edge of the coil touching the edge of the magnet array. At each sampling period, the coil locations in global coordinates will be converted into mover coordinates using [\(3.1\)](#). If the coil center is located in the boundary, this coil

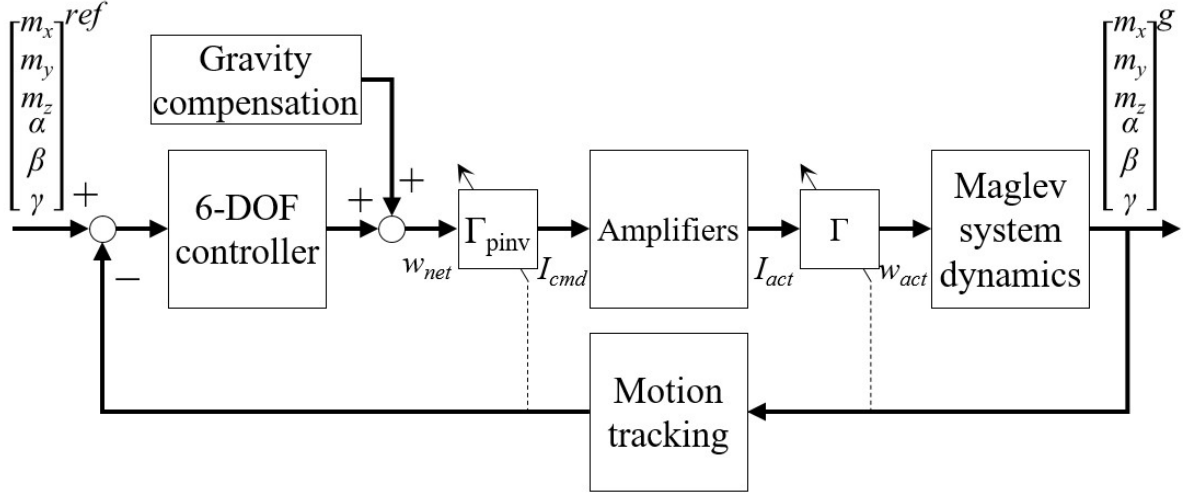


Figure 6.7: Control diagram.

Table 6.2: PID controller parameters.

DOF	PID controller					
	P gain		I gain		D gain	
X	10000	N/m	4800	N/m·s	350	N·s/m
Y	10000	N/m	4800	N/m·s	350	N·s/m
Z	11000	N/m	50000	N/m·s	325	N·s/m
Roll	180	$\frac{\text{N}\cdot\text{m}}{\text{rad}}$	400	$\frac{\text{N}\cdot\text{m}}{\text{rad}\cdot\text{s}}$	4	$\frac{\text{N}\cdot\text{m}}{\text{rad/s}}$
Pitch	180	$\frac{\text{N}\cdot\text{m}}{\text{rad}}$	400	$\frac{\text{N}\cdot\text{m}}{\text{rad}\cdot\text{s}}$	4	$\frac{\text{N}\cdot\text{m}}{\text{rad/s}}$
Yaw	200	$\frac{\text{N}\cdot\text{m}}{\text{rad}}$	180	$\frac{\text{N}\cdot\text{m}}{\text{rad}\cdot\text{s}}$	6.5	$\frac{\text{N}\cdot\text{m}}{\text{rad/s}}$

will be selected for activation. An example coil selection picture is shown in Figure 6.8, where the green coils are the activated coils. For the mover and coil in this chapter, 25 to 36 coils will be activated simultaneously.

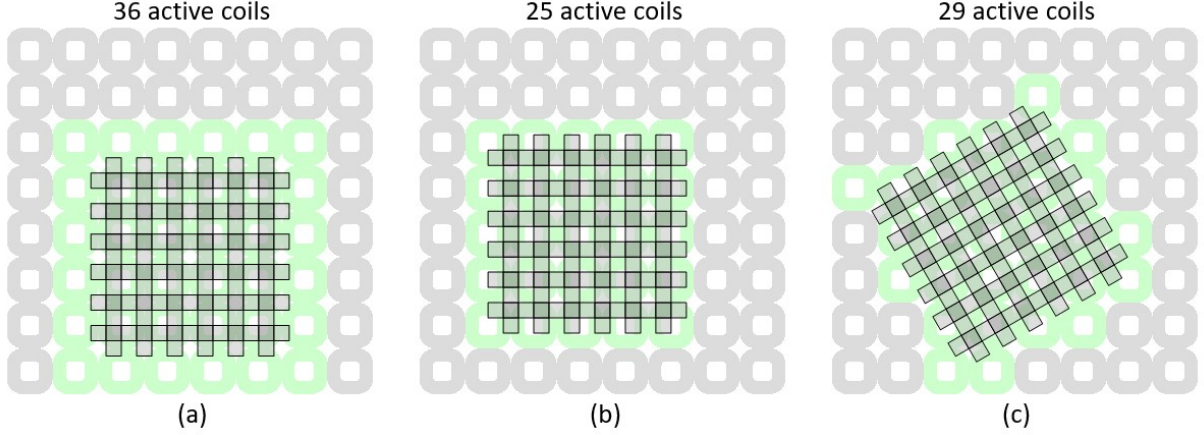


Figure 6.8: Coil activation with different mover locations. Active coils are shown in green. (a) 25 coils activated (b) 36 coils activated (c) 29 coils activated.

After identifying the active coil set, the direct wrench method is applied to decouple the desired force and torque to control the current. The individual force and torque contribution on the mover is denoted as $w_n^g = [F_{x,n}^g, F_{y,n}^g, F_{z,n}^g, T_{x,n}^g, T_{y,n}^g, T_{z,n}^g]^T$ when the coil current is 1 A. The coil index is n , and the range of n changes dynamically based on the number of coils activated. The desired net force and torque on mover is denoted as $w_{\text{net}}^g = [F_{x,\text{net}}^g, F_{y,\text{net}}^g, F_{z,\text{net}}^g, T_{x,\text{net}}^g, T_{y,\text{net}}^g, T_{z,\text{net}}^g]^T$. Then, the desired force and torque are converted to the mover coordinates by

$$w_{\text{net}}^m = R_g^m w_{\text{net}}^g \quad (6.3)$$

The relationship between the desired mover force and torque and coil control current vector I is

$$w_{\text{net}}^m = [w_1^m, w_2^m, \dots, w_n^m][I_1, I_2, \dots, I_n]^T = \Gamma(\vec{c}^m)I, \quad (6.4)$$

where $\Gamma(\vec{c}^m)$ contains all the force and torque elements from each coil at 1 A coil current input. The six-DOF motion is always over-actuated. The control current to the coil can be solved by the Moore-Penrose inverse to minimize the energy consumption. The equation for control current is

$$I = \Gamma^T (\Gamma \Gamma^T)^{-1} w_{\text{net}}^m. \quad (6.5)$$

The mover movement range can easily increase as the coil array expands by applying the proposed active coil selection strategy and current decoupling method. When building force and torque matrix Γ , the measured coil locations are used. The coil location error from setup construction can be included.

6.5 System Specifications

6.5.1 Test Bench Results

This section presents step responses, path tracking, motion range, and positioning resolution on the test bench.

Decoupled Six-DOF Motion

A step response of 8 mm in X_g direction is shown in [Figure 6.9](#). The 2% settling time is 0.088 s with an overshoot smaller than 1%. During the movement, the maximum acceleration is around 1.4 gravitational acceleration. When a large force is used in the X_g direction, clear oscillation is observed on other five axes. Therefore, there are still cross-coupling effect. During the movement, the position fluctuation in the Z_g direction is around ± 0.15 mm, and the fluctuation in β is around ± 0.1 mm. No significant fluctuations are observed in other axes. When the mover is at stationary levitation, only vertical force is supplied. The average power consumption is 270 W, and the maximum current is 5.9 A. During the step response, a large horizontal force is supplied to achieve fast dynamics. The system reaches maximum power during the step response. The maximum power is 887 W, and the maximum current used is 9 A.

A step response of 1 mm in the Z_g direction is shown in [Figure 6.10](#). A bigger steady-state error is found in the Z_g direction. The 2% settling time is 0.631 s with a 14.7% overshoot. Other axes are stable while the mover moves. The step response of γ is tested. The 2% settling time is 0.101 s with an overshoot of 4.6%. The other axes were stable while the mover travels. It can be observed from [Figure 6.9](#) to [Figure 6.11](#) that the proposed system has decoupled six-DOF motion. Compared with other axes, the Z_g direction shows more cross-coupling effects.

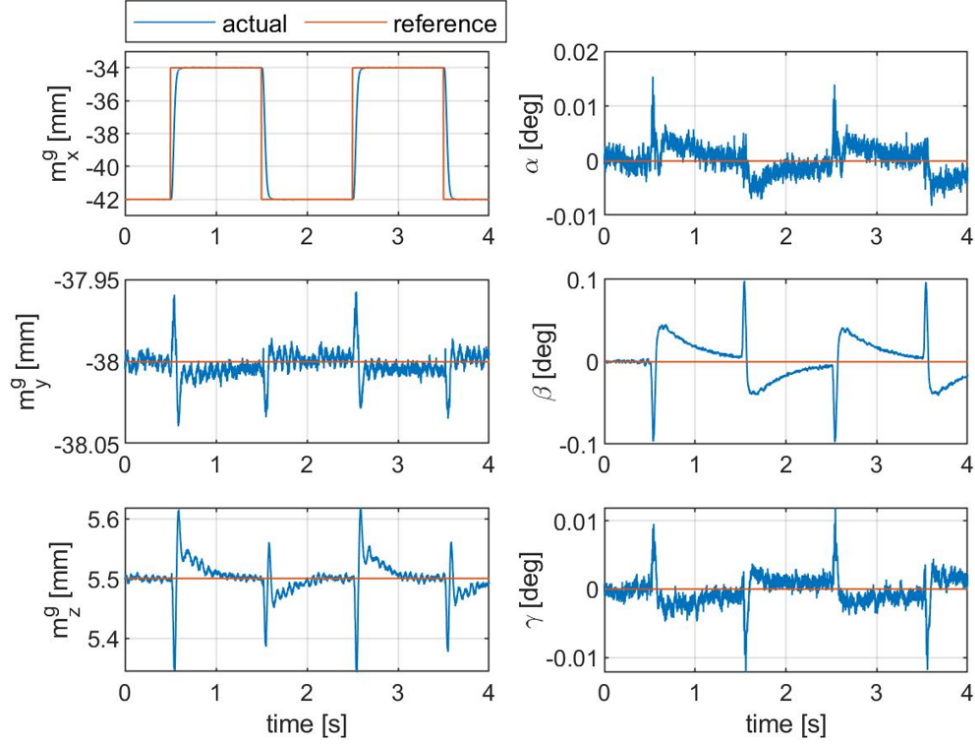


Figure 6.9: Step response in X_g direction.

Path Tracking and Maximum Speed

A circular path is tested in the $X_g Y_g$ plane. The diameter of the circular path is 240 mm. The mover follows the circular path with a speed of 100 mm/s, 200 mm/s and 300 mm/s, which is shown in [Figure 6.12](#). RMSE of each axis during the motion is shown in [Table 7.3](#). The RMSE grow as the speed increases. When moving in a circular path at a speed of 100 mm/s, the system consumes an average power of 316 W and reaches a maximum current of 6.2 A. As the speed doubles to 200 mm/s, the average power consumption rises to 338 W, accompanied by a maximum current of 6.3 A. Finally, at the fastest speed of 300 mm/s, the average power consumption reaches 377 W, and the maximum current peaks at 6.7 A.

The maximum linear speed is found to be 500 mm/s, which is shown in [Figure 6.13](#). The mean absolute position error between the reference and the actual position is 2.5877 mm during constant speed motion. The levitation height fluctuates between 5.6 mm to 7.2 mm. The range of β is around $\pm 0.2^\circ$. No significant variations are found in other axes.

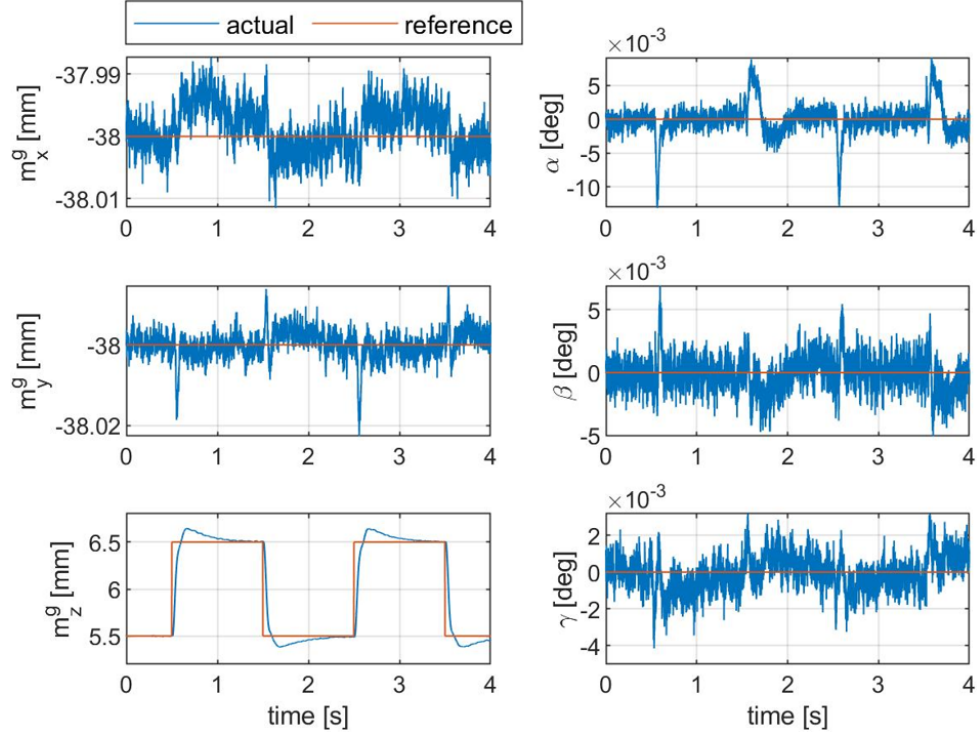


Figure 6.10: Step response in Z_g direction.

Table 6.3: Tracking errors of each axis during the circular path.

Speed [mm/s]	100	200	300
$RSME_{xy}$ [mm]	0.260	0.633	0.924
$RSME_z$ [mm]	0.103	0.116	0.146
$RSME_\alpha$ [deg]	0.038	0.057	0.090
$RSME_\beta$ [deg]	0.031	0.050	0.083
$RSME_\gamma$ [deg]	0.010	0.014	0.021

When the mover speed exceeds 500 mm/s, the motion becomes unstable.

Motion Range and Resolution

As shown in Figure 6.12, the X_gY_g motion range is ± 120 mm. The mover can switch between different active coil sets for an expandable motion range. The range of levitation

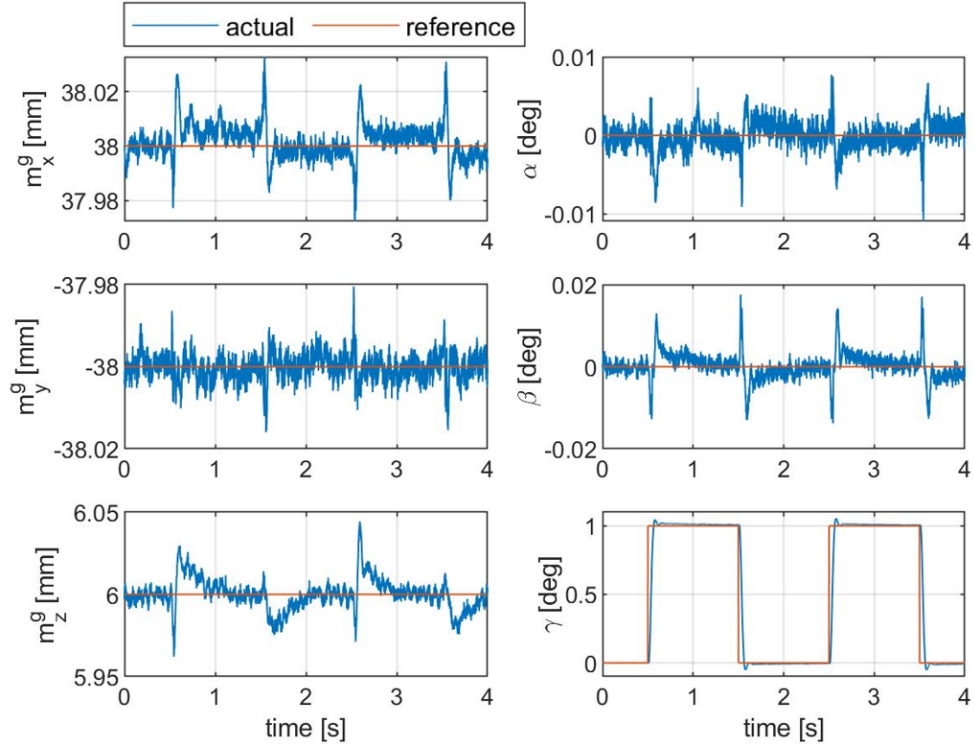


Figure 6.11: Step response in γ direction.

height is 4.5 mm to 7.5mm. The motion of the α and β is limited by the size of the mover and levitation height, which is $\pm 0.2^\circ$. The γ ramp response is shown in Figure 6.14, and the proposed system is found to have a full 360° range of yaw motion.

The motion resolution of the system is limited by the sensing method. The motion precision of each axis is measured. As shown in Figure 6.15, the motion precision in $X_g Y_g Z_g$ direction is 0.01 mm. The motion precision for α and β are 5 mdeg, and the precision for γ is 2 mdeg.

6.5.2 Maglev Floor Results

Decoupled Motion

A step response of 8 mm in X_g direction is shown in Figure 6.16. The 2% settling time is 0.558 s with a percentage overshoot of 13.8%. Due to the cross-coupling effect, m_z^g had

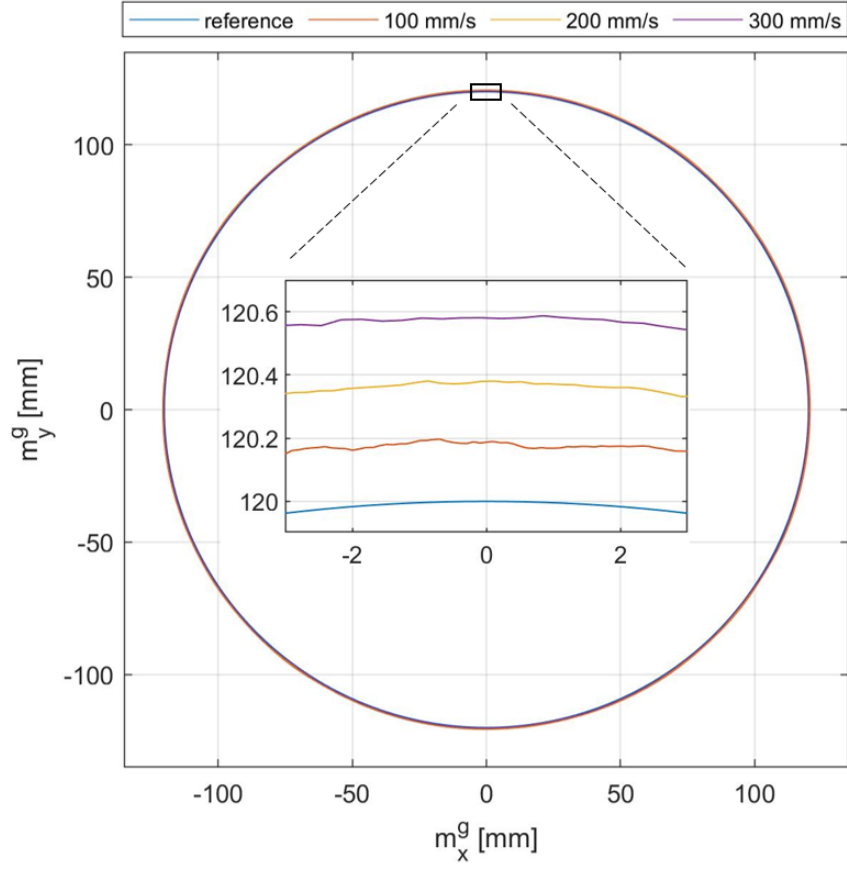


Figure 6.12: Circular path tracking.

0.25 mm fluctuation and β had 0.1° fluctuation. Similar power consumption and maximum coil current as the test bench are observed.

A step response in the Z_g direction is shown in [Figure 6.17](#). The 2% settling time is 0.463 s with a percentage overshoot of 24.8%. No fluctuations were observed in other axes.

The step response of the yaw angle is tested. The 2% settling time is 0.731 s with an overshoot of 14.6%. The other axes remained stable while the mover travels.

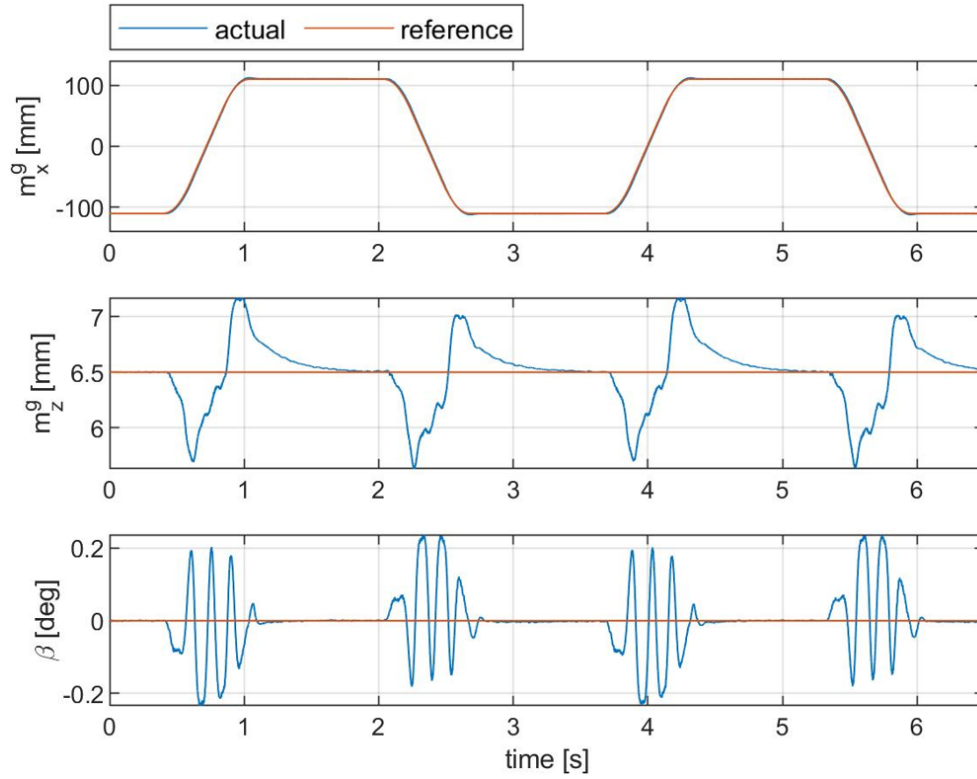


Figure 6.13: Maximum mover speed for long stroke movement.

Motion Precision

The motion resolution of the system is limited by the sensing method. The motion precision of each axis is measured. As shown in [Figure 6.19](#), the motion precision in $X_g Y_g Z_g$ direction is 0.01 mm. The motion precision for the rotational axes are 10 mdeg.

Motion Range and Maximum Speed

A figure-eight path is tested on the maglev floor with a speed of 300 mm/s. This figure-eight has a dimension of 2060 mm by 690 mm in the $X_g Y_g$ directions, which covers the entire levitation area. It is observed that the mean absolute error in the $X_g Y_g$ direction is 0.128mm and 0.184mm.

The maximum linear speed is measured to be 500 mm/s on the maglev floor, which is shown in [Figure 6.21](#). The mean absolute position error between the reference and the

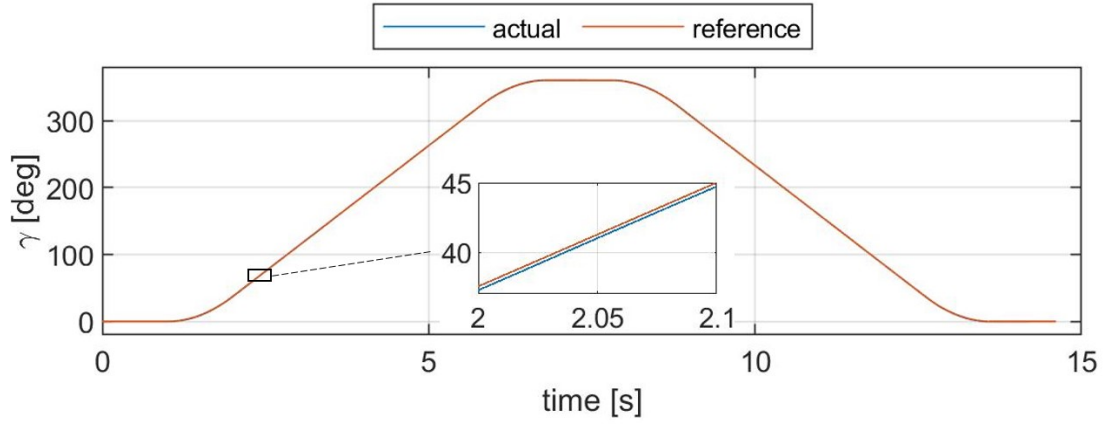


Figure 6.14: Full 360° range of yaw motion.

actual position is 3.217 mm during constant speed motion. The levitation height fluctuates between 4.9 mm to 6.9 mm. The range of β is around $\pm 0.3^\circ$. No significant variations are found in other axes. When the mover speed exceeds 500 mm/s, the motion becomes unstable.

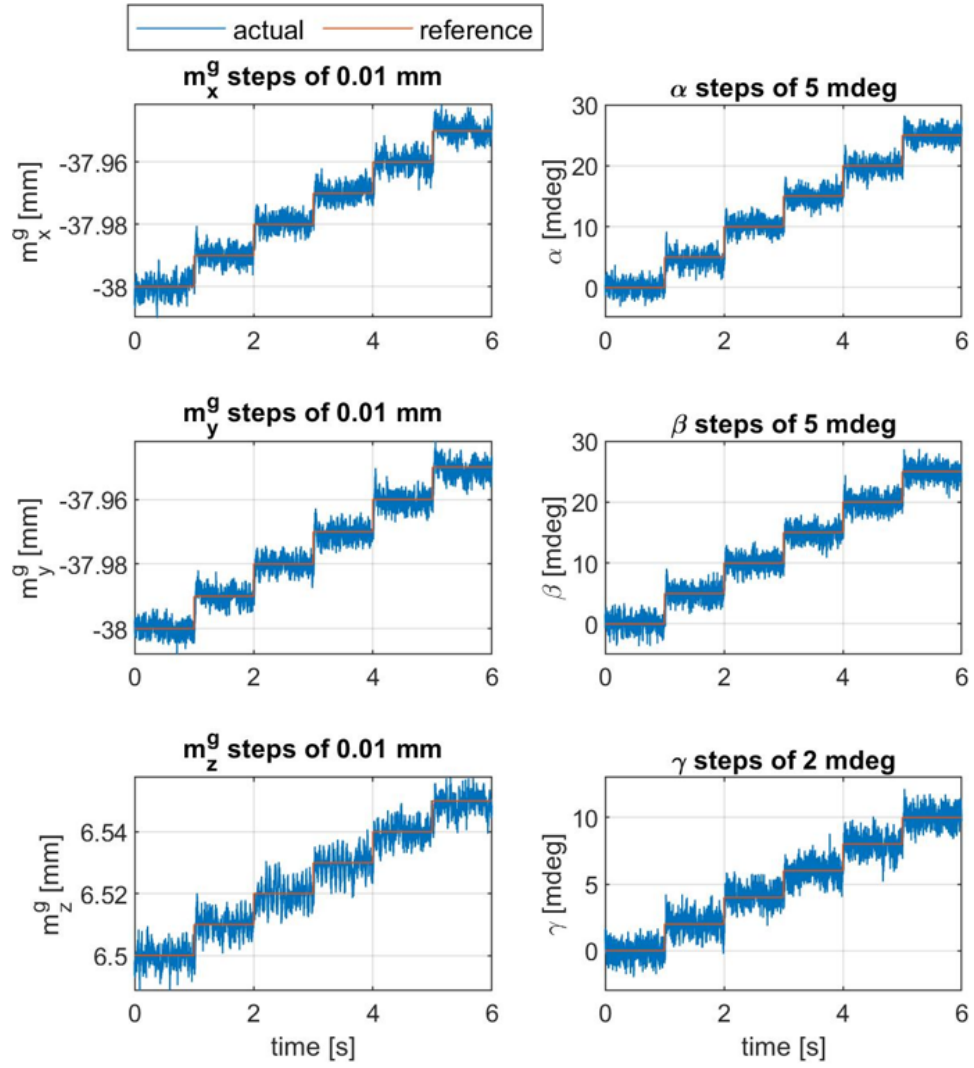


Figure 6.15: Measurement precision of each axis.

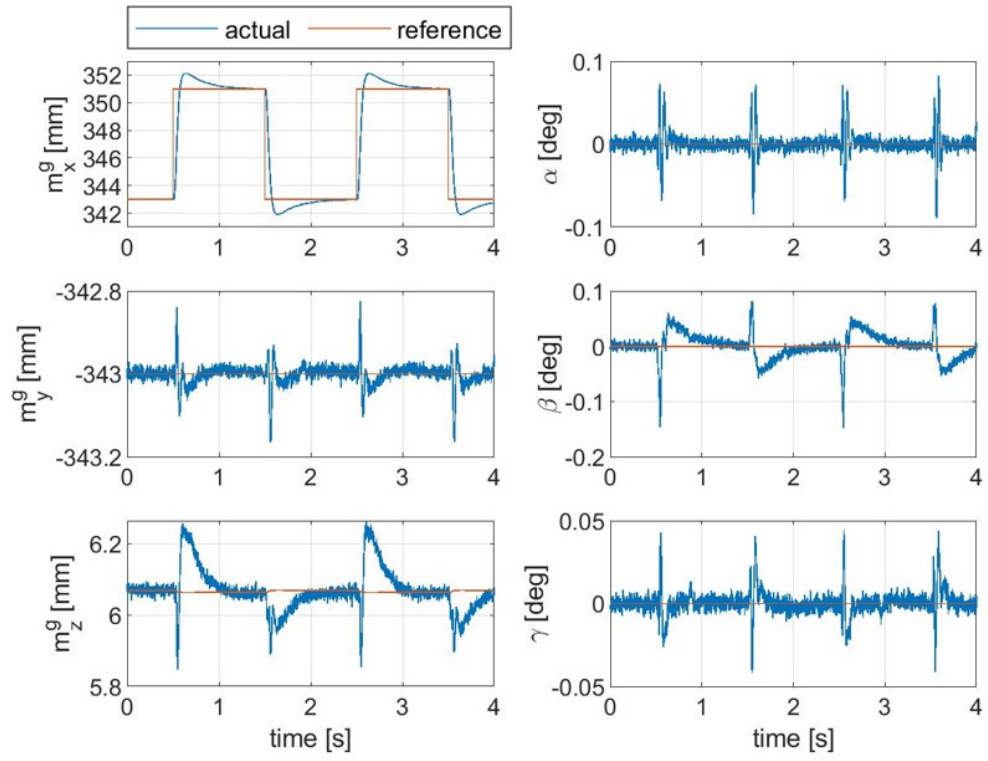


Figure 6.16: Step response in the X_g direction on maglev floor.

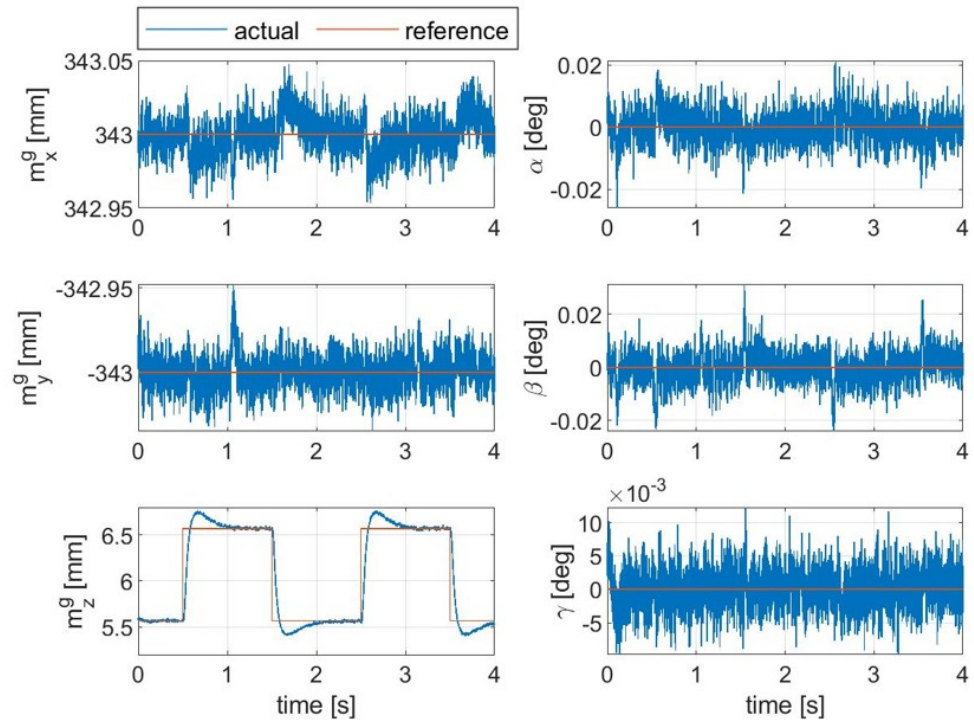


Figure 6.17: Step response in the Z_g direction on maglev floor.

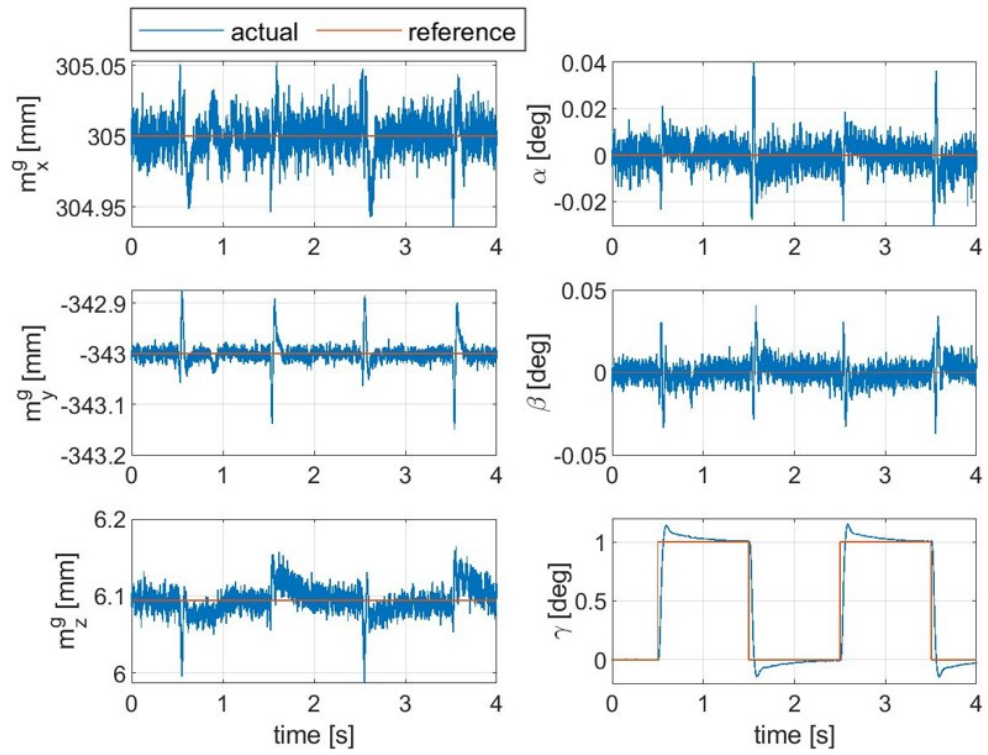


Figure 6.18: Step response in the γ direction on maglev floor.

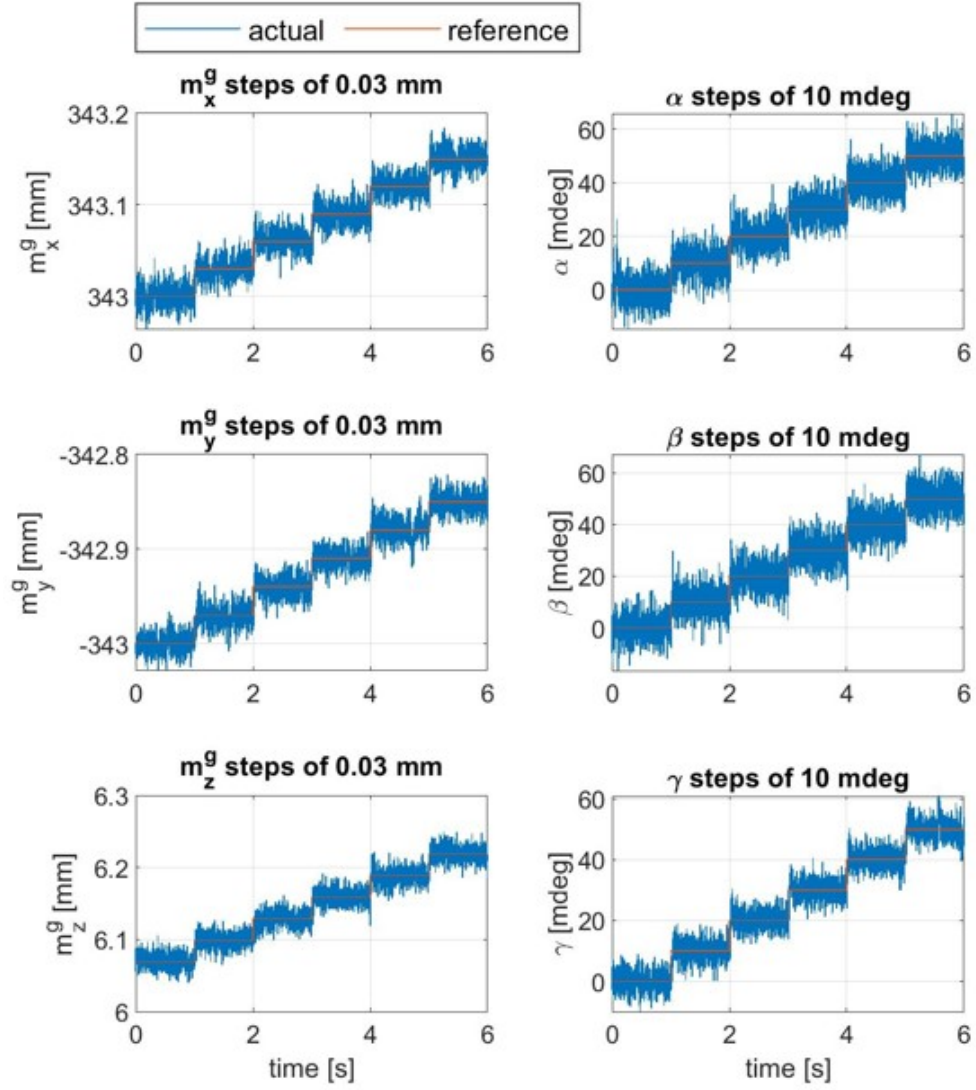


Figure 6.19: Precision steps on each axis on the maglev floor.

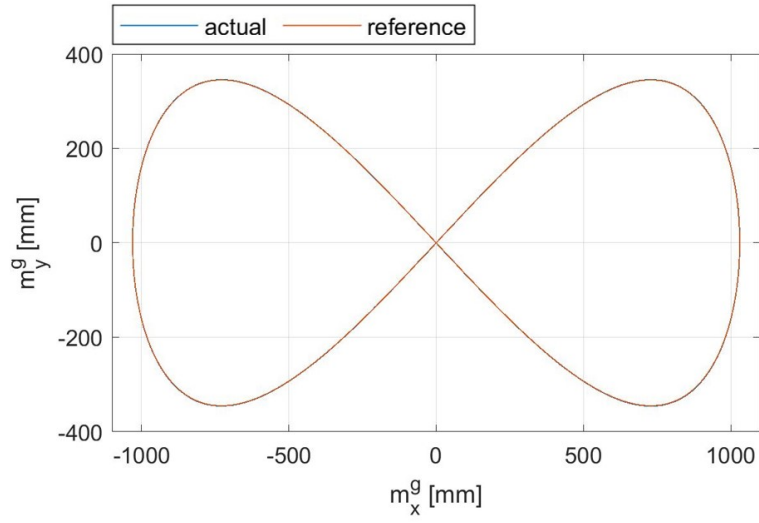


Figure 6.20: A figure-eight path is tracked by the mover at 300 mm/s.

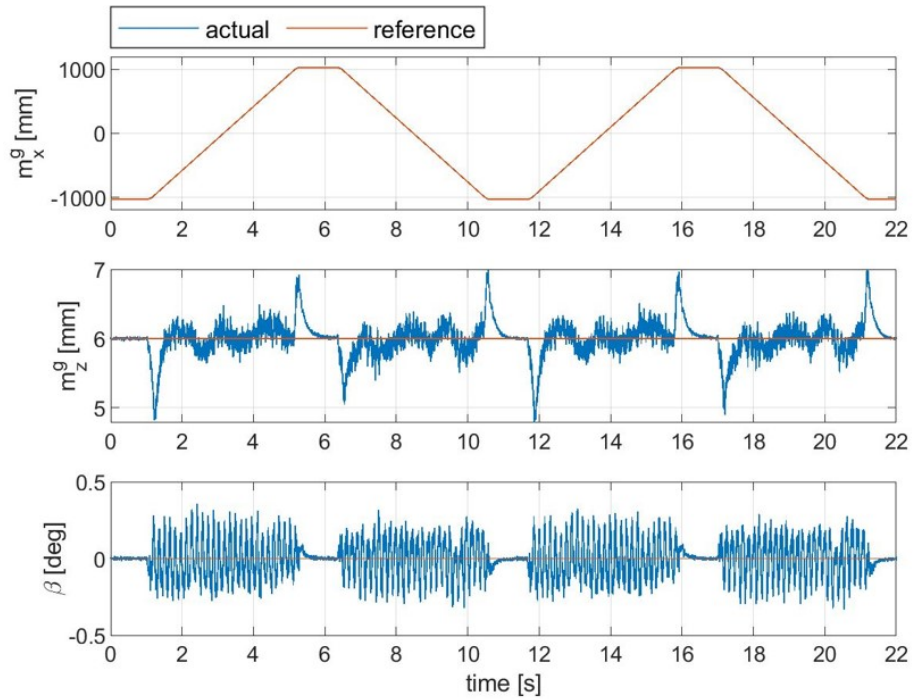


Figure 6.21: The maximum speed is tested to be 500 mm/s on the maglev floor.

Chapter 7

Advanced Control: Predictor-Based Model Predictive Control

In this chapter, a PMPC control is implemented on the MLPM. This work was published in [90].

7.1 Motivation

To satisfy the high-speed and precision requirements in these applications, a fast-responding controller of the MLPM to achieve desired tracking performance needs to be designed. Numerous studies of control strategies applied to MLPM have been conducted using either conventional or advanced approaches. Conventional methods, such as PID control [100], lead-lag control [46] and linear quadratic regulator [76], require an effortful tuning process although they are straightforward to be implemented. More advanced controllers for MLPM are also investigated, such as H-infinity [82], iterative learning control [10, 101], neural-network-based control [66], fuzzy logic control [102], and learning adaptive robust control [36]. Observers are often employed to estimate the system internal states [80, 4]. However, these methods cannot incorporate the current limit in the control design, which is critical in MLPM.

MPC has been demonstrated to be an effective control algorithm for path-tracking problems [83, 34, 60]. It can solve the constrained optimization problem online, making it a desirable candidate to control the MLPM. In recent studies, MPC has been investigated on a MLPM with a mover of four 1D Halbach arrays and eight stationary coils [96, 88, 97].

In [96], a nonlinear MPC based on an observer is implemented in real-time for three translation directions. However, the computation time of the nonlinear MPC is relatively long. A linear MPC with a disturbance estimator [88, 97] is also investigated to have faster computation and better system performance compared with the nonlinear MPC. Implementation of the linear MPC to maglev systems with a moving 2D Halbach array is still challenging as the maglev systems are subject to a varying number of coils and extendable motion ranges.

When designing a controller for maglev systems, time delay needs to be considered. It reduces the stability margin of the closed-loop system [31] and results in a nontrivial position measurement error between the actual and measured positions of the mover, particularly, at a high-speed travel. Given that many MLPM are required to respond to reference changes within 0.1 seconds or less, developing a strategy to compensate for the system delay can significantly improve the response of the closed-loop systems as well as their achievable maximum speed.

Predictor-based control computes system states at future instants and solves the control effort based on the state prediction [25], which has been employed in networked control systems [61] and multi-agent systems [57]. For systems with constant delays, finite spectrum assignment (FSA) can efficiently leverage the predicted state to feedback control design [62], resulting in a delay-free system.

A PMPC is proposed and validated for MLPM. The six-DOF mover actuation and the control current are decoupled via the direct wrench method. FSA is implemented for system delay compensation and state prediction. An augmented linear motion model of the X_gY_g plane is then formulated using the delay-free state for the MPC design, and the associated constraints are discussed. Hildreth's quadratic procedure is adopted to solve the constrained MPC. The main contributions are:

1. A PMPC control is introduced and experimentally validated to have desirable transient performance and steadiness in MLPM.
2. The augmented model used is demonstrated to be effective in improving the tracking performance of the MLPM.
3. A method of decoupling the coil current limits to the force constraints in the X_gY_g plane is introduced and used for the MPC synthesis.

7.2 System Description

The state space model of the system is presented in [subsection 6.3.1](#). Similarly to [\[83\]](#), an augmented state-space model is constructed to remove the steady-state error by embedding integrators. The control input of the augmented system becomes the control increment Δu , defined as

$$\Delta u(k) = u(k) - u(k-1), \quad (7.1)$$

and the state vector ζ_a becomes $\zeta_a = [\Delta\zeta^\top, \rho^\top]^\top$. The state-space representation of the augmented system model is

$$\begin{aligned} \zeta_a(k+1) &= A_a \zeta_a(k) + B_a \Delta u(k), \\ \rho(k) &= C_a \zeta_a(k), \end{aligned} \quad (7.2)$$

where

$$A_a = \begin{bmatrix} A & 0_n^\top \\ CA & I_p \end{bmatrix}, B_a = \begin{bmatrix} B \\ CB \end{bmatrix}, C_a = [0_n \quad I_p]. \quad (7.3)$$

n , p and q are the number of control inputs, outputs and the number of states in [Equation 6.2](#). 0_n is a $p \times q$ zero matrix and I_p is a $p \times p$ identity matrix. The characteristic polynomial equation of the augmented model is

$$\begin{aligned} \det(\lambda I_{q+p} - A_a) &= \det \begin{bmatrix} \lambda I_q - A & 0_n^\top \\ -CA & (\lambda - 1)I_p \end{bmatrix} \\ &= (\lambda - 1)^p \det(\lambda I_q - A) = 0. \end{aligned} \quad (7.4)$$

[Equation 7.4](#) explicitly shows that p integrators are embedded into the augmented system model, which implies the steady state errors due to slow-varying and constant disturbances can be eliminated.

7.3 PMPC Control Design

The proposed control structure is shown in [Figure 7.1](#). The system is decoupled by the inverse wrench method. The inputs are the force and torque applied to the mover instead of the coil currents. In the X_g and Y_g directions, the delayed augmented states $\hat{\zeta}_a$ are estimated by a Kalman filter. The delay-free states ζ_f can be then calculated from the FSA, which are used by MPC to calculate F_x and F_y under the force constraints. PID controllers are used to control the motion of other axes.

Remark 1 The proposed PMPC is used in the X_g and Y_g directions for two reasons. One is that the planar motion control is more critical than the other axes for path-tracking purposes which is the main scope of this work. The other reason is that using PMPC only on X_g and Y_g directions can effectively reduce the computation burden compared with using PMPC on all axes.

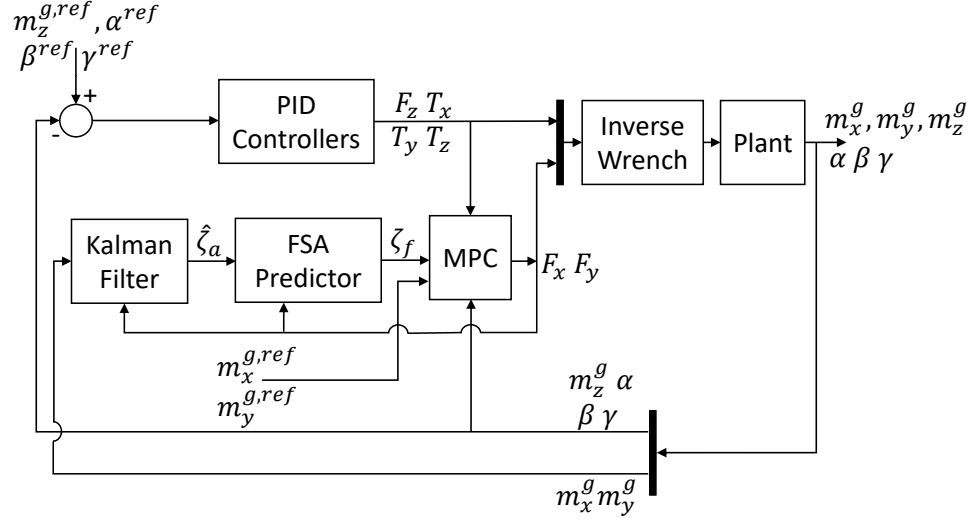


Figure 7.1: Scheme of the proposed control structure.

7.3.1 Finite Spectrum Assignment Design

For the system under investigation, constant input and output delays are identified to be the time needed for data processing and transmission through the sensor and actuator, respectively. Because the measured outputs to update the Kalman filter are actually delayed due to the input delay and the output delay also needs to be compensated, finite spectrum assignment is employed by introducing delay-free states ζ_f , where $\zeta_f = \hat{\zeta}_a(k + k_{tot}|k)$ and k_{tot} are the total instants of input and output delays.

The state prediction of FSA is given as [25]

$$X(t+h) = e^{A_c h} X(t) + \int_{t-h}^t e^{A_c(t-s)} B_c U(s) ds, \quad (7.5)$$

where A_c and B_c are the state and input matrix of a continuous system, respectively, and $h > 0$ is the time delay. By discretizing Equation 7.5 and incorporating the past control,

the delay-free states ζ_f can be obtained to predict the future values at the given time instant k ,

$$\begin{aligned}\zeta_f(k) = \hat{\zeta}_a(k + k_{tot}|k) &= A_a^{k_{tot}} \hat{\zeta}_a(k) + A_a^{k_{tot}-1} B_a \Delta u(k) \\ &+ \dots + B_a \Delta u(k + k_{tot} - 1).\end{aligned}\quad (7.6)$$

Figure 7.2 shows the illustration of the m_x^g position prediction by Equation 7.6. In Figure 7.2, the mover is traveling towards the positive X_g direction. The measured location is lagged behind by 10 ms due to the input and output delays. The predicted location is delay-free and closer to the actual mover location. The comparison between the measured location and the predicted location is shown.

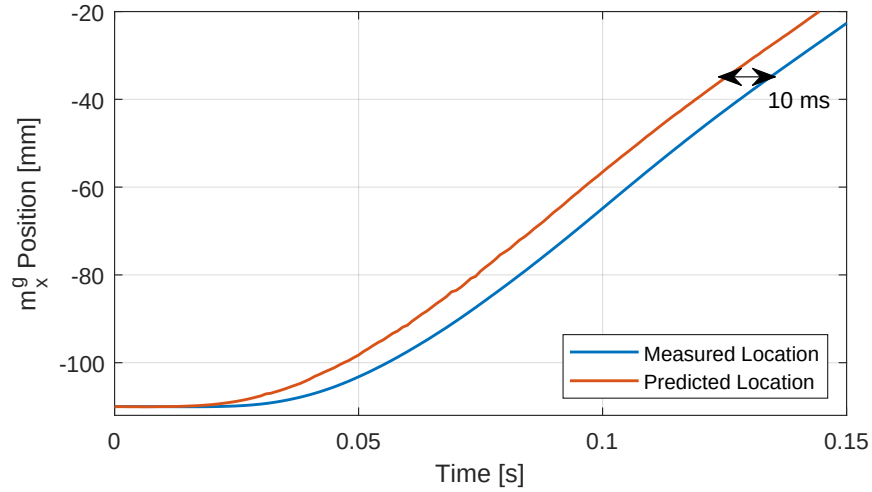


Figure 7.2: Illustration of the m_x^g position prediction with 10 ms delay.

7.3.2 State and Output Prediction

Given the system model, future state, future control candidate and the optimization window, the plant output can be predicted and synthesized into a cost function to solve out an optimal control trajectory satisfying the constraints. The prediction horizon is denoted as H_p and the control horizon is denoted as H_c . The future control increment trajectory is defined as

$$\Delta U_c = [\Delta u(k) \quad \Delta u(k+1) \quad \dots \quad \Delta u(k+H_c-1)]^\top. \quad (7.7)$$

The control increment is assumed to be zero after the control horizon.

The future state X_c is defined as

$$X_c = [\zeta_f(k+1|k) \quad \zeta_f(k+2|k) \quad \cdots \quad \zeta_f(k+H_p|k)]^\top, \quad (7.8)$$

where $\zeta_f(k+i|k)$, $i = 1, \dots, H_p$, is the delay-free state predicted at $k+i$ with the given information $\zeta_f(k)$. Based on the system model, the sequence of the future delay-free states can be obtained by using the future control increment trajectory ΔU_c ,

$$\begin{aligned} \zeta_f(k+1|k) &= A_a \zeta_f(k) + B_a \Delta u(k), \\ \zeta_f(k+2|k) &= A_a^2 \zeta_f(k) + A_a B_a \Delta u(k) + B_a \Delta u(k+1), \\ &\vdots \\ \zeta_f(k+H_c|k) &= A_a^{H_c} \zeta_f(k) + A_a^{H_c-1} B_a \Delta u(k) + \cdots \\ &\quad + B_a \Delta u(k+H_c-1), \\ &\vdots \\ \zeta_f(k+H_p|k) &= A_a^{H_p} \zeta_f(k) + A_a^{H_p-1} B_a \Delta u(k) + \cdots \\ &\quad + A_a^{H_p-H_c} B_a \Delta u(k+H_c-1). \end{aligned} \quad (7.9)$$

The predicted output Y_c is defined as

$$Y_c = [\rho(k+1|k) \quad \rho(k+2|k) \quad \cdots \quad \rho(k+H_p|k)]^\top. \quad (7.10)$$

Using [Equation 7.9](#) and [Equation 7.2](#), Y_c can be written in a compact form as

$$Y_c = \Lambda X_c + \Phi \Delta U_c, \quad (7.11)$$

where Λ and Φ are

$$\begin{aligned} \Lambda &= [C_a A_a \quad C_a A_a^2 \quad C_a A_a^3 \quad \cdots \quad C_a A_a^{H_p}]^\top, \\ \Phi &= \begin{bmatrix} C_a B_a & 0 & \cdots & 0 \\ C_a A_a B_a & C_a B_a & \cdots & 0 \\ C_a A_a^2 B_a & C_a A_a B_a & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_a A_a^{H_p-1} B_a & C_a A_a^{H_p-2} B_a & \cdots & C_a A_a^{H_p-H_c} B_a \end{bmatrix}. \end{aligned} \quad (7.12)$$

7.3.3 Cost Function

The objective of the MPC is to track the prescribed path in the $X_g Y_g$ plane within the optimization window. The reference R_r is defined as

$$R_r = [x_r(k+1) \quad y_r(k+1) \quad x_r(k+2) \quad y_r(k+2) \quad \cdots \\ x_r(k+H_p) \quad y_r(k+H_p)]^\top, \quad (7.13)$$

where $x_r(k+i)$ and $y_r(k+i)$, $i = 1, \dots, H_p$, are the future references in X_g and Y_g directions at time instant $k+i$. The cost function J is chosen as the sum of the squared tracking error and weighted norm of the control increment

$$J = (R_r - Y_c)^\top (R_r - Y_c) + \Delta U_c^\top \bar{R} \Delta U_c, \quad (7.14)$$

where $\bar{R} = r_w I_n$ is the penalty over the future control increment. By substituting [Equation 7.11](#) into [Equation 7.14](#), the cost function can be expressed with ΔU ,

$$J = \Delta U_c^\top (\Phi^\top \Phi + \bar{R}) \Delta U_c - 2 \Delta U_c^\top \Phi^\top (R_r - \Lambda X_c) \\ + (R_r - \Lambda X_c)^\top (R_r - \Lambda X_c). \quad (7.15)$$

Since the algorithm aims to minimize J with respect to ΔU , the cost function can be further written as

$$J = \Delta U_c^\top (\Phi^\top \Phi + \bar{R}) \Delta U_c - 2 \Delta U_c^\top \Phi^\top (R_r - \Lambda X_c). \quad (7.16)$$

7.3.4 Constraint Analysis

The constraint of the system is the current limit of each coil. The current limit can be equivalently converted to the limitation of force and torque applied. As m_z^g, α, β and γ are controlled with PID controllers, their force and torque inputs F_z, T_x, T_y and T_z can be predetermined. For the maglev planar motor in this study, the force constraints is determined by the mover location. To find the constraints of F_x and F_y within the control horizon, the following assumptions are made:

1. the positions of m_z^g, α, β , and γ remain the same in the control horizon, that is, F_z, T_x, T_y and T_z are constant.
2. the mover will travel at a constant speed in the X_g and Y_g directions within the control horizon.

The predicted mover location m_g at time instant $k + i$, $i = 1, \dots, H_c$, by the given k can be denoted as

$$m_g(k + i|k) = [m_x^g + i\Delta T v_x^g \quad m_y^g + i\Delta T v_y^g \quad m_z^g \quad \alpha \quad \beta \quad \gamma]^\top, \quad (7.17)$$

where it is worth noting that m_x^g and m_y^g are the delay-free states from the ζ_f . The force and torque on the mover is

$$w_{k+i} = [F_x(k + i) \quad F_y(k + i) \quad F_z \quad T_x \quad T_y \quad T_z]^\top. \quad (7.18)$$

By using the inverse wrench method, Equation 7.17 and Equation 7.18, current limits at time instant $k + i$, $i = 1, \dots, H_c$, can be equivalently written as

$$I_{c,min} \leq \Gamma_{k+i}^\top (\Gamma_{k+i} \Gamma_{k+i}^\top)^{-1} w_{k+i} \leq I_{c,max}, \quad (7.19)$$

where Γ_{k+i} is the concatenated matrix in the function of $m_g(k + i|k)$, and $I_{c,max}$ and $I_{c,min}$ are the column vectors of the current limits. The length of the constraint equation equals two times the active coil number. The linear equation set Equation 7.19 can be simplified to four to eight equations by removing the redundant constraints

$$G_{k+i} [F_x(k + i) \quad F_y(k + i)]^\top \leq L_{k+i}, \quad (7.20)$$

where G_{k+i} and L_{k+i} are the constraint matrices at time instant $k + i$. The force constraints in Equation 7.20 form a polygon in the plane of F_x and F_y . As implied in Equation 7.19, the limits of F_x and F_y vary with the mover's location. Examples of F_x and F_y constraints along a short moving trajectory are shown in Figure 7.3 from $m_x^g = 288$ mm and $m_y^g = 340$ mm to $m_x^g = 292.5$ mm and $m_y^g = 343$ mm.

At each time instant, the force constraint can be found as

$$\begin{aligned} G_k(u(k - 1) + \Delta u(k)) &\leq L_k, \\ G_{k+1}(u(k - 1) + \Delta u(k) + \Delta u(k + 1)) &\leq L_{k+1}, \\ &\vdots \\ G_{k+H_c}(u(k - 1) + \Delta u(k) + \Delta u(k + 1) + \dots \\ &\quad + \Delta u(k + H_c)) \leq L_{k+H_c}. \end{aligned} \quad (7.21)$$

Equation 7.21 can be written as

$$C_1 u(k - 1) + C_2 \Delta U_c \leq C_3, \quad (7.22)$$

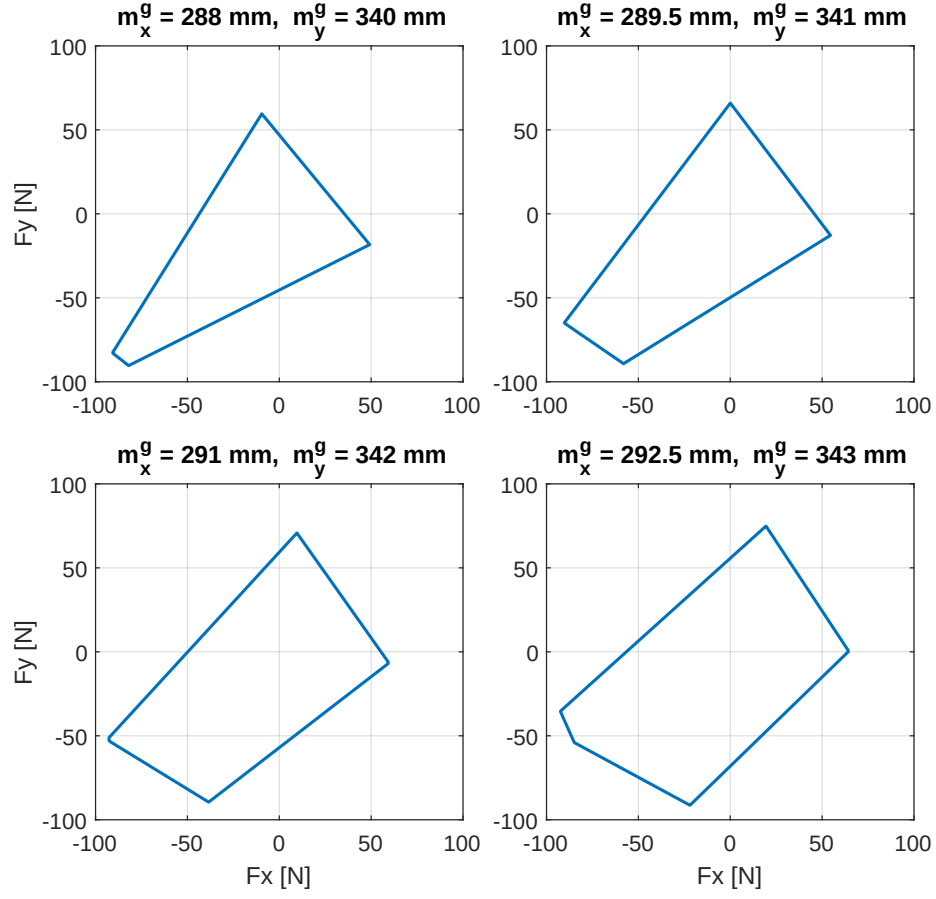


Figure 7.3: Examples of F_x and F_y constraints when the mover is at different locations along a short moving trajectory.

where

$$\begin{aligned}
 C_1 &= [G_k \ G_{k+1} \ G_{k+2} \ \cdots \ G_{k+H_c}]^\top, \\
 C_2 &= \begin{bmatrix} G_k & 0 & 0 & 0 & \cdots & 0 \\ G_{k+1} & G_{k+1} & 0 & 0 & \cdots & 0 \\ G_{k+1} & G_{k+2} & G_{k+2} & 0 & \cdots & 0 \\ \vdots & & & & & \\ G_{k+H_c} & G_{k+H_c} & G_{k+H_c} & G_{k+H_c} & \cdots & G_{k+H_c} \end{bmatrix}, \\
 C_3 &= [L_k \ L_{k+1} \ L_{k+2} \ \cdots \ L_{k+H_c}]^\top.
 \end{aligned}$$

Equation 7.22 can be further written in a compact form,

$$M\Delta U_c \leq N, \quad (7.23)$$

where M is C_2 and N is $C_3 - C_1 u(k-1)$.

7.3.5 Hildreth's Quadratic Procedure

Combining the cost function Equation 7.16 and the constraints Equation 7.23, the optimization problem can be formulated to minimize

$$J = \frac{1}{2} \Delta U_c^\top E \Delta U_c + \Delta U_c^\top F, \quad (7.24)$$

subject to

$$M\Delta U_c \leq N, \quad (7.25)$$

where

$$\begin{aligned} E &= 2(\Phi^\top \Phi + \bar{R}), \\ F &= -2\Phi^\top (R_r - \Lambda X_c). \end{aligned} \quad (7.26)$$

The Lagrange multiplier λ is introduced to minimize Equation 7.24 under Equation 7.25,

$$J_L = \frac{1}{2} \Delta U_c^\top E \Delta U_c + \Delta U_c^\top F + \lambda^\top (M\Delta U_c - N). \quad (7.27)$$

The optimal λ and ΔU can be found as

$$\begin{aligned} \lambda &= -(ME^{-1}M^\top)^{-1}(N + ME^{-1}F), \\ \Delta U &= -E^{-1}F - E^{-1}M^\top \lambda. \end{aligned} \quad (7.28)$$

A primal-dual method is used to find the active constraints. The primal problem is equivalent to

$$\max_{\lambda \geq 0} \min_x \left[\frac{1}{2} \Delta U^\top E \Delta U + \Delta U^\top F + \lambda^\top (M\Delta U - N) \right]. \quad (7.29)$$

Plugging Equation 7.28 into Equation 7.29, the dual problem with λ as the decision variable can be written as

$$\min_{\lambda \geq 0} \left[\frac{1}{2} \lambda^\top H \lambda + \lambda^\top K + \frac{1}{2} N^\top E^{-1} N \right], \quad (7.30)$$

where

$$\begin{aligned} H &= ME^{-1}M^\top, \\ K &= N + ME^{-1}F. \end{aligned} \tag{7.31}$$

Hildreth's quadratic programming procedure is applied to solve the dual problem [35]. The decision variable is firstly initialized to 0. Then, the value in λ^* is calculated in an iterative manner. If λ_i is smaller than 0, $\lambda_i = 0$. If λ_i is greater or equal to 0, the value of λ_i will be updated. The pseudo code is shown in Algorithm 1, where λ_i^{a+1} is the i th term of λ at iteration $a + 1$. The equation for updating a candidate of λ , denoting as v , is

Algorithm 1 Demonstration of the Hildreth's Quadratic Procedure.

Require: H, K

Ensure: λ

$n = \text{length}(K)$

$\lambda^0 = \text{zeros}(n, 1)$

for $a = 0 : \text{max_iteration}$ **do**

for $i = 1 : n$ **do**

$v_i^{a+1} = f(H, K, i, j, \lambda)$

$\lambda_i^{a+1} = \max(0, v_i^{a+1})$

▷ Equation 7.32

end for

if λ converges **then**

 return

end if

end for

$$v_i^{a+1} = -\frac{1}{h_{ii}}(k_i + \sum_{j=1}^{i-1} h_{ij}\lambda_j^{a+1} + \sum_{j=i+1}^n h_{ij}\lambda_j^a). \tag{7.32}$$

h_{ij} denotes the ij th element of H and k_i denotes i th element of vector K . The equation to update λ_i^{a+1} is

$$\lambda_i^{a+1} = \max(0, v_i^{a+1}). \tag{7.33}$$

Once the algorithm converges to λ^* , the control effort can be found as

$$\Delta U = -E^{-1}F - E^{-1}M^\top\lambda^*. \tag{7.34}$$

The λ^* will converge if the constraints are linearly independent and the number of active constraints is equal or less than the number of decision variables [84]. The computation will

stop after a certain number of iterations. It always returns a compromised, near-optimal solution within the constraint. A safe and continuous operation of the algorithm can be ensured.

7.4 Experimental Validation

The proposed PMPC is examined by comparing its performance to PID controller and baseline MPC method through different tests, while the same PID controllers are used on the m_z^g, α, β and γ directions. The same Kalman filters are used on all three cases to estimate the mover's location and speed. The detailed configuration of the three controllers are shown as follows:

PMPC: This is the proposed control method. The prediction horizon H_p and control horizon H_c are chosen to be 45 and 7, respectively. The energy penalty r_w is selected as 1.5E-8 for optimal tracking performance. The constraints are added on the first 5 control steps. The computation time of the proposed controller is measured to be 680 μ s.

MPC: This is the baseline MPC method. Same as the PMPC method, this MPC is only employed in the X_g and Y_g directions. The prediction horizon H_p and control horizon H_c are chosen to be 70 and 8, respectively. The energy penalty r_w is selected as 3E-6. The constraints are also added to the first 5 control steps. Since longer prediction and control horizon are used, the computation time of MPC is longer, which is measured to be 790 μ s.

PID: The PID controller used in X_g and Y_g directions is:

$$C_{\text{PID}}(s) = 9000 + 400s + \frac{5000}{s}. \quad (7.35)$$

Step response, ramp response and circular path are tested with the three controllers. The path tracking performance of all the controllers is measured with the root mean square error (RMSE):

$$e_{rmse,\theta} = \sqrt{\frac{\sum_{k=1}^N (\theta(k) - \theta_{ref}(k))^2}{N}}, \quad (7.36)$$

and the maximum absolute error (MAE):

$$e_{mae,\theta} = \max |\theta(k) - \theta_{ref}(k)|, \quad (7.37)$$

where θ is the mover position ($m_x^g, m_y^g, m_z^g, \alpha, \beta$ or γ), θ_{ref} is the reference and N is the

number of samples.

In practice, the mover's motion is coupled in all directions. Uncertain and nonlinear effects along any axis can lead to the mover oscillating across all axes when moving in the $X_g Y_g$ plane. Therefore, the overall oscillations impose limitations on the maximum speeds achievable by the mover in the X_g and Y_g directions. In this study, the motion of the mover is considered to be steady if the tracking errors of α and β are controlled within $\pm 0.35^\circ$ and z is controlled within ± 1 mm during the entire movement.

Remark 2 *The control parameters of the controllers are tuned to have minimal RMSE, short rise time and 2% settling time as listed in Table 7.4.1, respectively.*

7.4.1 Step Response

Step responses of 5 mm distance in the X_g and Y_g directions are tested. The responses are highly similar in the two directions, and the responses in the X_g direction are shown in Figure 7.4. The performance is evaluated by 2% settling time, percentage overshoot, rise

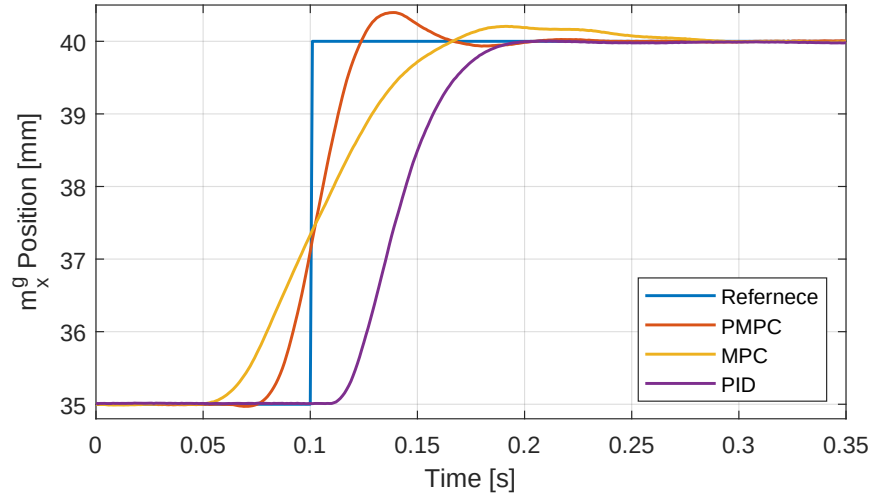


Figure 7.4: Step response comparisons between predictor-based MPC, MPC and PID.

time (10% to 90%) and the RMSE of the process. The results of the three controllers are shown in Table 7.4.1. MPC and PMPC start the motion earlier than PID since they can consider future reference. It can be concluded that the proposed PMPC has the fastest settling time and rise time with a higher overshoot of 7.94%. The PMPC has the smallest

Table 7.1: Controller specifications of the step responses.

	PMPC	PID	MPC
Rise Time [s]	0.030	0.047	0.071
Overshoot	7.94%	4.12%	0.42%
2% Settling time [s]	0.059	0.086	0.143
$e_{rms,e,x}$ [mm]	0.552	1.781	0.838
$e_{mae,x}$ [mm]	2.580	4.991	2.602

RMSE and MAE between the actual performance and the reference. The control efforts of three controller are shown in Figure 7.5. PMPC and PID have similar highest control effort, which is around 50 N. The maximum control effort of the MPC is around 20 N.

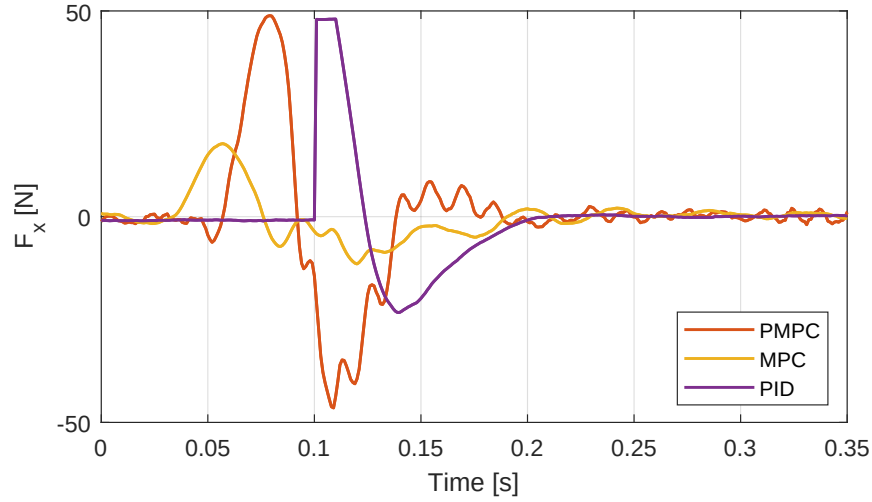


Figure 7.5: Control effort comparison of three controllers during the step response.

7.4.2 Ramp Response

A ramp response with the speed of 300 mm/s in the X_g direction is tested. The total displacement distance is 220 mm. The responses are shown in Figure 7.6, where Figure 7.6a shows the m_x^g of the carrier and Figure 7.6b shows the responses of the other five axes ($m_y^g, m_z^g, \alpha, \beta$ and γ). The RMSE of all axes is shown in Table 7.2.

At the start of the ramp response, PMPC and MPC approach the reference much faster than PID, while the PMPC is the closest to the reference. At the end of the ramp response,

Table 7.2: Controller performance specifications of the ramp responses.

Speed [mm/s]	300	300	300	800
Controller	PMPC	PID	MPC	PMPC
$e_{rmse,x}$ [mm]	0.194	1.493	0.631	1.482
$e_{rmse,y}$ [mm]	0.009	0.064	0.044	0.200
$e_{rmse,z}$ [mm]	0.076	0.252	0.305	0.165
$e_{rmse,\alpha}$ [deg]	0.013	0.031	0.052	0.032
$e_{rmse,\beta}$ [deg]	0.045	0.117	0.111	0.099
$e_{rmse,\gamma}$ [deg]	0.011	0.031	0.034	0.036
$e_{mae,x}$ [mm]	1.254	5.753	2.783	3.810
$e_{mae,y}$ [mm]	0.029	0.143	0.105	0.824
$e_{mae,z}$ [mm]	0.180	0.858	0.842	0.415
$e_{mae,\alpha}$ [deg]	0.031	0.084	0.093	0.080
$e_{mae,\beta}$ [deg]	0.109	0.320	0.240	0.306
$e_{mae,\gamma}$ [deg]	0.038	0.143	0.085	0.085

all controllers have overshoot. The PID controller has a relatively big overshoot of 5.753 mm. MPC has an overshoot of 0.819 mm, while the overshoot of PMPC is only 0.598 mm. The tracking RMSE in X_g direction is greatly reduced by the proposed method. It is reduced by 87% compared to PID and 69% compared to MPC. The PMPC also has the smallest MAE among all three comparing controllers.

The predictor also helps with the control of other axes. Although the exact same controllers are used on m_z^g, α, β and γ , the performance of the PMPC is significantly better than PID and MPC. The predictors reduce the positioning error caused by the delay, and the inverse wrench algorithm decouples the current more accurately. The maximum ramp speed is examined, and the maximum speed is limited by if the motion is stable. The maximum ramp speed is 300 mm/s for PID. The maximum speed is improved to 500 mm/s by MPC, and the maximum ramp speed is 800 mm/s for the PMPC. The performance of the PMPC 800 mm/s ramp is shown in Table 7.2 and the overshoot distance for this test is 3.633 mm.

7.4.3 Circular Trajectory

A circular trajectory tracking of 500 mm/s is tested. The diameter of the circle is 230 mm. The response is shown in Figure 7.7, where Figure 7.7a is the response in the horizontal

Table 7.3: Controller performance specifications of the circular trajectory tracking.

Speed [mm/s]	500	500	500	800
Controller	PMPC	PID	MPC	PMPC
$e_{rmse,xy}$ [mm]	0.031	1.615	0.445	0.107
$e_{rmse,z}$ [mm]	0.124	0.169	0.149	0.195
$e_{rmse,\alpha}$ [deg]	0.042	0.130	0.146	0.070
$e_{rmse,\beta}$ [deg]	0.027	0.118	0.135	0.047
$e_{rmse,\gamma}$ [deg]	0.012	0.032	0.037	0.016
$e_{mae,xy}$ [mm]	0.104	2.452	0.783	0.681
$e_{mae,z}$ [mm]	0.308	0.486	0.379	0.374
$e_{mae,\alpha}$ [deg]	0.097	0.244	0.299	0.141
$e_{mae,\beta}$ [deg]	0.066	0.243	0.266	0.128
$e_{mae,\gamma}$ [deg]	0.038	0.079	0.091	0.041

plane and Figure 7.7b shows the responses of other four axes. The RMSE of each controller is shown in Table 7.3.

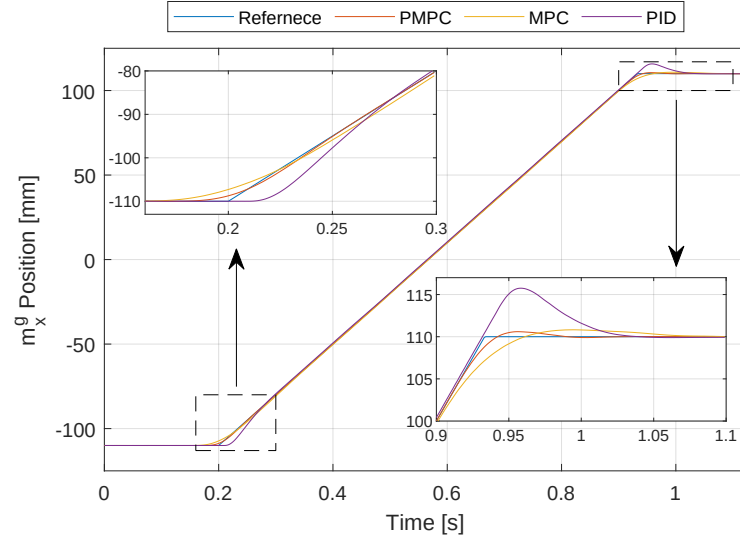
During the circular motion of 500 mm/s, the performance of the PMPC is significantly better than the other two controllers. The PMPC reduces the tracking RMSE of PID by 98% and MPC by 93%. The tracking errors of all other axes are also greatly reduced by PMPC. The maximum speed of the circular tracking is examined. The maximum speed of both PID and MPC are measured to be 600 mm/s. The maximum speed of PMPC is improved to 800 mm/s. It can be seen from Table 7.3 that PMPC still offers great tracking performance even with higher speed.

7.4.4 Discussion

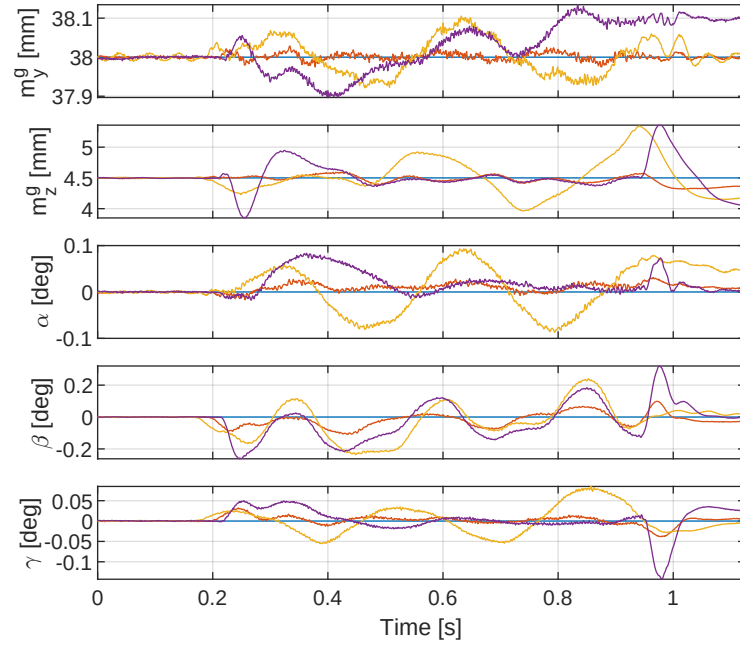
In all three tests, PMPC and MPC outperform PID in effectively reducing tracking errors. This superiority is rooted in two key reasons. First, PMPC and MPC have the ability to anticipate future reference changes. Consequently, they initiate responses ahead of reference changes, while PID will be behind the reference due to its dependence on feedback tracking errors. Second, both PMPC and MPC can solve the optimal solution within force constraints, ensuring the optimal use of available power and avoiding power overconsumption.

The experimental results demonstrate the superiority of the proposed PMPC in path tracking while maintaining the system steadiness. A main feature of the proposed PMPC

is that it adopts the delay-free states prepared by FSA ahead for control optimization. This incorporation provides more accurate predicted states and simplifies the computation. This could account for the shorter optimal prediction and control horizon lengths in PMPC, resulting in reduced computation time. Additionally, the delay-compensated estimates are more accurate in updating the predictions of the MPC and adjusting its control actions over a specified prediction horizon to achieve the desired trajectory. Another feature of the proposed PMPC is the use of an augmented model. The added integral attribute can eliminate the steady-state errors arising from low-varying and constant disturbances.



(a)



(b)

Figure 7.6: Ramp response in X direction with the speed of 300 mm/s (a) X position during the ramp (b) position of other five axes during the motion.

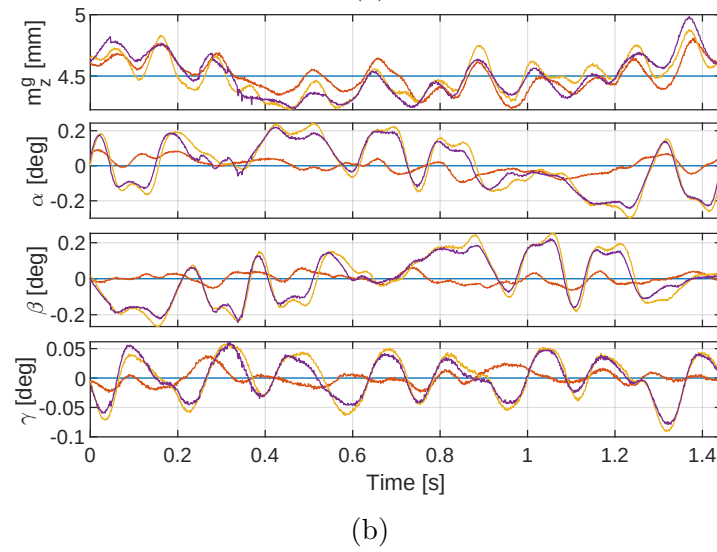
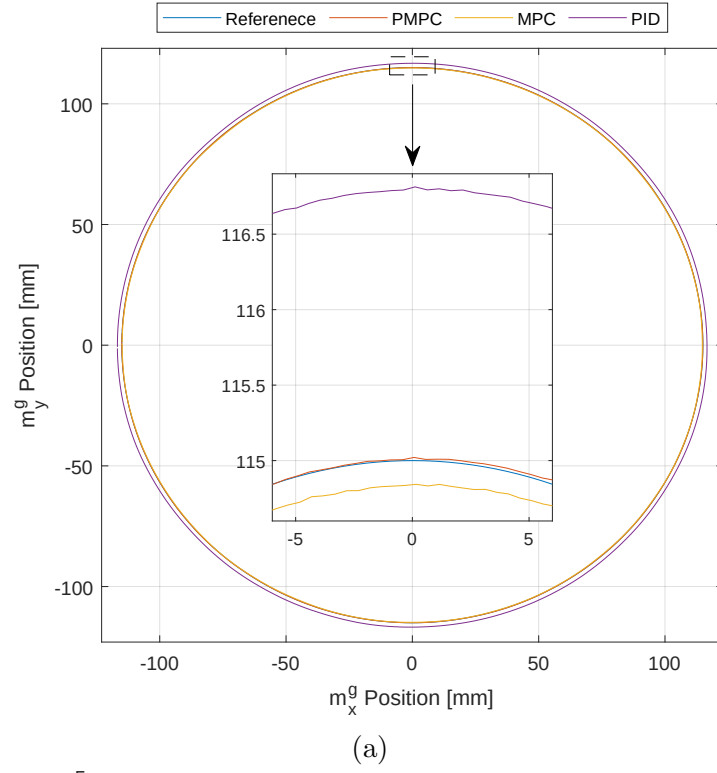


Figure 7.7: Circular motion in the horizontal plane with the speed of 500 mm/s (a) m_x^g and m_y^g position (b) position of other four axes during the motion.

Chapter 8

Conclusion and Recommendations

In this chapter, the conclusions regarding the modeling, design, integration and control are presented. Then, suggestions for future works are discussed.

8.1 Conclusion

8.1.1 Data-Driven Modeling

This thesis presents a real-time, lookup-table-free and continuous force and torque modeling method for MLPM with a 2D Halbach array and square/circular coils. The force and torque of the coil at each location are calculated by the Lorentz rule and numerical method. ETA is introduced to model the torque. Then, the modified Fourier series is used to model the seven nonlinear force and torque components in all edge and corner regions. The method's speed is experimentally tested on an industrial server, and the average computation time of force and torque at each location is $2.3 \mu s$. The proposed method is beneficial for both square and circular coils. It is experimentally validated for a square and a circular coil with different sizes, and the measurements match well with the model. In conclusion, the proposed method is real-time, accurate, continuous and lookup-table-free, making it suitable for real-time controls of MLPM.

8.1.2 System Design & Integration

In this paper, a modular maglev system is proposed and demonstrated. Square coils and a 2D Halbach array are selected for higher energy efficiency. The experimental setup is designed and built. Vicon cameras and a modular frame design are implemented, allowing the motion range to be expandable. Experimental validations are conducted to verify the proposed method and design. Combined with active coil selection and direct wrench decoupling, the mover has six-DOF decoupled motions. The mover can switch between different active coil sets to achieve long horizontal translational and infinite yaw motion.

8.1.3 Predictor-Based Model Predictive Control

A novel PMPC is proposed to control MLPM. A Kalman filter and finite spectrum assignment are applied to estimate the location of the mover under a known delay. Augmented-model-based MPC with delay-free states is implemented to control the planar motion of the mover. Experimental results demonstrate the effectiveness of the proposed PMPC over a PID controller and a baseline MPC in tracking step, ramp, and circular trajectories. Experiments further highlight that the proposed PMPC can accommodate higher mover speeds while maintaining system steadiness. The findings suggest that the proposed PMPC can be used as an efficient control strategy for MLPM in high-speed path-tracking applications.

8.2 Recommendations

In this section, possible future improvements are suggested.

8.2.1 Coil Temperature Estimation and Optimization

One critical limitation of the MLPM is the coil heat. The temperature of the coil could rapidly increase. As a result, the coil resistance will increase accordingly, and the system efficiency and levitation power will decay. Therefore, taking the coil temperature optimization into consideration can greatly improve the system's performance. On the current system of maglev test bench and maglev floor, around 16% of the coils are embedded with thermocouples for temperature measurement. However, there are still many coils whose

temperatures are not available for direct measurement. To incorporate the coil temperature control, two goals should be achieved. First, a method to accurately and rapidly compute the coil temperature should be developed. Second, a current decoupling method, including the coil temperature control, should be developed.

8.2.2 Multi-Mover Levitation and Collaboration

In this thesis, only one mover is investigated for levitation. For the purpose of flexible manufacturing, it is important that multiple movers can be levitated and cooperate with each other or other machines. To achieve this goal, two problems should be solved, which are force and torque modeling between magnetic movers and path planning for multiple movers on various tasks.

8.2.3 Operation Bandwidth Improvement

For a MLPM, higher bandwidth is important for the manufacturing process. In this thesis, tests are only conducted with lower frequency. The system frequency response should be identified, and different control strategies could be implemented to improve the system bandwidth.

For the kinematic model used in this thesis, the system dynamics are ignored. The system resonance could be excited in practice, which limits the tracking performance and system steadiness and potentially damages the system. Therefore, the structure dynamics should to be considered in control design, and it is possible to solve this issue via input shaping.

References

- [1] Halbach Arrays. <https://www.kjmagnetics.com/blog.asp?p=halbach-arrays>.
- [2] Planar Motor. <https://www.planarmotor.com/en>.
- [3] XPlanar—Planar motor system. https://www.beckhoff.com/en-en/products/motion/xplanar-planar-motor-system/?pk_campaign=AdWords-AdWordsSearch-xplanar_en&pk_kwd=planar%20motor.
- [4] Karim Ahmadi, Davood Asadi, Abdelrazzak Merheb, Seyed-Yaser Nabavi-Chashmi, and Onder Tutsoy. Active fault-tolerant control of quadrotor uavs with nonlinear observer-based sliding mode control validated through hardware in the loop experiments. *Control Engineering Practice*, 137:105557, 2023.
- [5] Hyeong Joon Ahn and Kyoung Rock Kim. 2d hall sensor array for measuring the position of a magnet matrix. *International Journal of Precision Engineering and Manufacturing-Green Technology*, 1:125–129, 2014.
- [6] G. Akoun and J.-P. Yonnet. 3d analytical calculation of the forces exerted between two cuboidal magnets. *IEEE Transactions on Magnetics*, 20(5):1962–1964, 1984.
- [7] Michael Binnard Andrew Hazelton and Jean-Marc Gery. Electric motors and positioning devices having moving magnet arrays and six degrees of freedom, US 2001/006208045B1, Mar 27 2001.
- [8] Neha Arora, Muneeb Ullah Khan, Laurent Petit, Frédéric Lamarque, and Christine Prelle. Design and development of a planar electromagnetic conveyor for the microfactory. *IEEE/ASME Transactions on Mechatronics*, 24(4):1723–1731, 2019.
- [9] F Bancel. Magnetic nodes. *Journal of Physics D: Applied Physics*, 32(17):2155–2161, aug 1999.

- [10] Sergei Basovich, Shai A. Arogeti, Yonattan Menaker, and Ziv Brand. Magnetically levitated six-dof precision positioning stage with uncertain payload. *IEEE/ASME Transactions on Mechatronics*, 21(2):660–673, 2016.
- [11] Peter Berkelman and Hamza Abdul-Ghani. Electromagnetic haptic feedback system for use with a graphical display using flat coils and sensor array. *IEEE Robotics and Automation Letters*, 5(2):1618–1625, 2020.
- [12] Peter Berkelman and Michael Dzadovsky. Magnetic levitation over large translation and rotation ranges in all directions. *IEEE/ASME Transactions on Mechatronics*, 18(1):44–52, 2013.
- [13] Peter Berkelman and Yu-Sheng Lu. Long range six degree-of-freedom magnetic levitation using low cost sensing and control. *Journal of Robotics and Mechatronics*, 32(3):683–691, 2020.
- [14] Michael Binnard. Six degree of freedom control of planar motors, US 2003/0085676A1, May 8 2003.
- [15] Peng Cheng, Hai Wang, Vladimir Stojanovic, Fei Liu, Shuping He, and Kaibo Shi. Dissipativity-based finite-time asynchronous output feedback control for wind turbine system via a hidden markov model. *International Journal of Systems Science*, 53(15):3177–3189, 2022.
- [16] Han-Sam Cho and Hyun-Kyo Jung. Analysis and design of synchronous permanent-magnet planar motors. *IEEE Transactions on Energy Conversion*, 17(4):492–499, 2002.
- [17] J. D. Cockcroft. The design of coils for the production of magnetic fields. *Philosophical Transactions of the Royal Society Series A, Mathematical, physical and engineering sciences*, 1928.
- [18] Ir.J.C. Compter. Electro-dynamic planar motor. *Precision Engineering*, 28(2):171–180, 2004.
- [19] C. H. H. M. Custers, J. W. Jansen, and E. A. Lomonova. Static decoupling of force and torque components in a moving-magnet planar motor. In *2017 IEEE International Electric Machines and Drives Conference (IEMDC)*, pages 1–8, 2017.
- [20] The Anh Tuan Dang, Magali Bosch-Mauchand, Neha Arora, Christine Prelle, and Joanna Daaboul. Electromagnetic modular smart surface architecture and control

in a microfactory context. *Computers in Industry*, 81:152–170, 2016. Emerging ICT concepts for smart, safe and sustainable industrial systems.

- [21] J. de Boeij, E. Lomonova, and A. Vandenput. Modeling ironless permanent-magnet planar actuator structures. *IEEE Transactions on Magnetics*, 42(8):2009–2016, 2006.
- [22] Jeroen de Boeij, Elena Lomonova, and Jorge Duarte. Contactless planar actuator with manipulator: a motion system without cables and physical contact between the mover and the fixed world. In *2008 IEEE Industry Applications Society Annual Meeting*, pages 1–8, 2008.
- [23] Jeroen de Boeij, Elena Lomonova, and Andre J.A. Vandenput. Look-up table based real-time commutation of 6-dof planar actuators. In *2007 IEEE International Conference on Control Applications*, pages 1118–1123, 2007.
- [24] Angelo C. P. de Klerk, Georgo Z. Angelis, and Jan van Eijk. Design of a next generation 6 dof stage for scanning application in vacuum with nanometer accuracy and mgauss magnetic stray field. 2004.
- [25] Yang Deng, Vincent Léchappé, Emmanuel Moulay, Zhang Chen, Bin Liang, Franck Plestan, and Qing-Long Han. Predictor-based control of time-delay systems: a survey. *International Journal of Systems Science*, 53(12):2496–2534, 2022.
- [26] Eric Diller, Joshua Giltinan, Guo Zhan Lum, Zhou Ye, and Metin Sitti. Six-degree-of-freedom magnetic actuation for wireless microrobotics. *The International Journal of Robotics Research*, 35(1-3):114–128, 2016.
- [27] Shengwu Du, Jinchun Hu, Yu Zhu, and Ming Zhang. A hall sensor-based position measurement with on-line model parameters computation for permanent magnet synchronous linear motor. *IEEE Sensors Journal*, 18:5245–5255, 7 2018.
- [28] Simon Duque Tisnes, Atif Tasneem, Laurent Petit, and Christine Prella. Trajectory and conveyance validation of a micro conveyor based on a digital electromagnetic actuators array for the micro-factory. *Applied Sciences*, 11(24), 2021.
- [29] Mark Dyck, Xiaodong Lu, and Yusuf Altintas. Magnetically levitated rotary table with six degrees of freedom. *IEEE/ASME Transactions on Mechatronics*, 22(1):530–540, 2017.
- [30] Mohamed E. El-sayed, Arsany W. Youssef, Omar M. Shehata, Lamia A. Shihata, and Eman Azab. Computer vision for package tracking on omnidirectional wheeled

conveyor: Case study. *Engineering Applications of Artificial Intelligence*, 116:105438, 2022.

- [31] Emilia Fridman. *Introduction to Time-Delay Systems*. Springer, 2014.
- [32] Farzad Ghayoor and A Swanson. Modelling and analysis of electrodynamic suspension of an aluminum disc as a complex engineering problem. *International Journal of Electrical Engineering & Education*, 55(2):91–108, 2018.
- [33] H. Habermann and G. Liard. An active magnetic bearing system. *Tribology International*, 13(2):85, 1980.
- [34] Peng He, Jiwei Wen, Vladimir Stojanovic, Fei Liu, and Xiaoli Luan. Finite-time control of discrete-time semi-markov jump linear systems: A self-triggered mpc approach. *Journal of the Franklin Institute*, 359(13):6939–6957, 2022.
- [35] Clifford Hildreth. A quadratic programming procedure. *Naval Research Logistics Quarterly*, 4(1):79–85, 1957.
- [36] Chuxiong Hu, Ze Wang, Yu Zhu, Ming Zhang, and Hao Liu. Performance-oriented precision larc tracking motion control of a magnetically levitated planar motor with comparative experiments. *IEEE Transactions on Industrial Electronics*, 63(9):5763–5773, 2016.
- [37] Tiejun Hu and Won-Jong Kim. Extended range six-dof high-precision positioner for wafer processing. *IEEE/ASME Transactions on Mechatronics*, 11(6):682–689, 2006.
- [38] J W Jansen. *Magnetically levitated planar actuator with moving magnets : electromechanical analysis and design*. PhD thesis, Electrical Engineering, 2007.
- [39] J. W. Jansen, C. M. M. van Lierop, E. A. Lomonova, and A. J. A. Vandenput. Modeling of magnetically levitated planar actuators with moving magnets. *IEEE Transactions on Magnetics*, 43(1):15–25, 2007.
- [40] J. W. Jansen, C. M. M. van Lierop, Elena A. Lomonova, and André J. A. Vandenput. Magnetically levitated planar actuator with moving magnets. *IEEE Transactions on Industry Applications*, 44(4):1108–1115, 2008.
- [41] Hao Jiang, Xueliang Huang, Gan Zhou, Yubin Wang, and Zheng Wang. Analytical force calculations for high-precision planar actuator with halbach magnet array. *IEEE Transactions on Magnetics*, 45(10):4543–4546, 2009.

- [42] Won jong Kim and David L. Trumper. High-precision magnetic levitation stage for photolithography. *Precision Engineering*, 22(2):66–77, 1998.
- [43] Frank Schonung Kai Furmans and Kevin R. Gur. Plug-and-work material handling systems. *11th IMHRC Proceedings*, 2010.
- [44] Pooriya Kazemzadeh Heris and Mir Behrad Khamesee. Design and fabrication of a magnetic actuator for torque and force control estimated by the ann/sa algorithm. *Micromachines*, 13(2), 2022.
- [45] Joe Siang Keek, Ser Lee Loh, and Shin Horng Chong. Design and control system setup of an e-pattern omniwheeled cellular conveyor. *Machines*, 9(2), 2021.
- [46] Won-Jong Kim, D.L. Trumper, and J.H. Lang. Modeling and vector control of planar magnetic levitator. *IEEE Transactions on Industry Applications*, 34(6):1254–1262, 1998.
- [47] Baoquan Kou, Feng Xing, Lu Zhang, Chaoning Zhang, and Yiheng Zhou. A real-time computation model of the electromagnetic force and torque for a maglev planar motor with the concentric winding. *Applied Sciences*, 7(1), 2017.
- [48] Maarten F. J. Kremers, Johannes J. H. Paulides, Esin Ilhan, Jeroen L. G. Janssen, and Elena A. Lomonova. Relative permeability in a 3d analytical surface charge model of permanent magnets. *IEEE Transactions on Magnetics*, 49(5):2299–2302, 2013.
- [49] Tobias Krühn, Sascha Falkenberg, and Ludger Overmeyer. Decentralized control for small-scaled conveyor modules with cellular automata. In *2010 IEEE International Conference on Automation and Logistics*, pages 237–242, 2010.
- [50] Parichit Kumar, Yuze Huang, Ehsan Toyserkani, and Mir Behrad Khamesee. Development of a magnetic levitation system for additive manufacturing: Simulation analyses. *IEEE Transactions on Magnetics*, 56(8):1–7, 2020.
- [51] Parichit Kumar, Saksham Malik, Ehsan Toyserkani, and Mir Behrad Khamesee. Development of an electromagnetic micromanipulator levitation system for metal additive manufacturing applications. *Micromachines*, 13(4), 2022.
- [52] Michael P. Kummer, Jake J. Abbott, Bradley E. Kratochvil, Ruedi Borer, Ali Sengul, and Bradley J. Nelson. Octomag: An electromagnetic system for 5-dof wireless micromanipulation. *IEEE Transactions on Robotics*, 26(6):1006–1017, 2010.

- [53] Yi-Chih Lai, Ye-Ling Lee, and Jia-Yush Yen. Design and servo control of a single-deck planar maglev stage. *IEEE Transactions on Magnetics*, 43(6):2600–2602, 2007.
- [54] Yi-Chih Lai and Jia-Yush Yen. Design of a novel 6-dof planar maglev system. *Journal of Magnetism and Magnetic Materials*, 304(1):e386–e390, 2006. International Symposium on Spintronics and Advanced Magnetic Technologies and International Symposium on Magnetic Materials and Applications 2005.
- [55] Hyung-Woo Lee, Ki-Chan Kim, and Ju Lee. Review of maglev train technologies. *IEEE Transactions on Magnetics*, 42(7):1917–1925, 2006.
- [56] Feng Liu, Ming Zhang, Yu Zhu, and Chuxiong Hu. A real-time model of iron-less planar motors with stationary circular coils. *IEEE Transactions on Magnetics*, 51(7):1–10, 2015.
- [57] Qingsong Liu. Pseudo-predictor feedback control for multiagent systems with both state and input delays. *IEEE/CAA Journal of Automatica Sinica*, 8(11):1827–1836, 2021.
- [58] Xiaodong Lu and Irfan ur-rab Usman. 6d direct-drive technology for planar motion stages. *CIRP Annals*, 61(1):359–362, 2012.
- [59] Xing Lu, Fengqiu Xu, Xianze Xu, Tong Zheng, and Ruotong Peng. Directed-driven 8-phase magnetically levitated rotary table based on an analytical-numerical model. *IEEE Access*, 8:31159–31170, 2020.
- [60] Yukun Lu, Amir Khajepour, Amir Soltani, Ruilong Li, Ran Zhen, Yegang Liu, and Minghui Wang. Gain-adaptive skyhook-lqr: a coordinated controller for improving truck cabin dynamics. *Control Engineering Practice*, 130:105365, 2023.
- [61] Vincent Léchappé, Emmanuel Moulay, Franck Plestan, and Qing-Long Han. Discrete predictor-based event-triggered control of networked control systems. *Automatica*, 107:281–288, 2019.
- [62] A. Manitius and A. Olbrot. Finite spectrum assignment problem for systems with delays. *IEEE Transactions on Automatic Control*, 24(4):541–552, 1979.
- [63] Stephan H. Mayer. *Development of a completely decentralized control system for modular continuous conveyors*. PhD thesis, 2009.
- [64] Muneaki Miyasaka and Peter Berkelman. Magnetic levitation with unlimited omnidirectional rotation range. *Mechatronics*, 24(3):252–264, 2014.

- [65] M. Ono, S. Koga, and H. Ohtsuki. Japan’s superconducting maglev train. *IEEE Instrumentation Measurement Magazine*, 5(1):9–15, 2002.
- [66] Tiansheng Ou, Chuxiong Hu, Yu Zhu, Ming Zhang, and Limin Zhu. Intelligent feed-forward compensation motion control of maglev planar motor with precise reference modification prediction. *IEEE Transactions on Industrial Electronics*, 68(9):7768–7777, 2021.
- [67] Fabrizio Pece, Juan Jose Zarate, Velko Vechev, Nadine Besse, Olexandr Gudozhnik, Herbert Shea, and Otmar Hilliges. Magtics: Flexible and thin form factor magnetic actuators for dynamic and wearable haptic feedback. In *Proceedings of the 30th Annual ACM Symposium on User Interface Software and Technology*, UIST ’17, page 143–154, New York, NY, USA, 2017. Association for Computing Machinery.
- [68] Sahba Aghajani Pedram, Roberta L. Klatzky, and Peter Berkelman. Torque contribution to haptic rendering of virtual textures. *IEEE Transactions on Haptics*, 10(4):567–579, 2017.
- [69] Junrong Peng, Yunfei Zhou, and Guangdou Liu. Calculation of a new real-time control model for the magnetically levitated ironless planar motor. *IEEE Transactions on Magnetics*, 49(4):1416–1422, 2013.
- [70] Ruotong Peng, Tong Zheng, Xing Lu, Xianze Xu, and Fengqiu Xu. Simulation of a synchronous planar magnetically levitated motion system based on a real-time analytical force model. *Energies*, 13(23), 2020.
- [71] J. M. M. Rovers, J. W. Jansen, and E. A. Lomonova. Multiphysical analysis of moving-magnet planar motor topologies. *IEEE Transactions on Magnetics*, 49(12):5730–5741, 2013.
- [72] J. M. M. Rovers, J. W. Jansen, E. A. Lomonova, and M. J. C. Ronde. Calculation of the static forces among the permanent magnets in a halbach array. *IEEE Transactions on Magnetics*, 45(10):4372–4375, 2009.
- [73] J. M. M. Rovers, J. W. Jansen, and Elena A. Lomonova. Analytical calculation of the force between a rectangular coil and a cuboidal permanent magnet. *IEEE Transactions on Magnetics*, 46(6):1656–1659, 2010.
- [74] J.M.M. Rovers, J.W. Jansen, and E.A. Lomonova. Design and measurements of the double layer planar motor. In *2013 International Electric Machines Drives Conference*, pages 204–211, 2013.

- [75] Pranab Samanta and Harish Hirani. Magnetic bearing configurations: Theoretical and experimental studies. *IEEE Transactions on Magnetics*, 44(2):292–300, 2008.
- [76] Jose Christian Silva-Rivas and Won-jong Kim. Multivariable control and optimization of a compact 6-dof precision positioner with hybrid H_2/H_∞ and digital filtering. *IEEE Transactions on Control Systems Technology*, 21(5):1641–1651, 2013.
- [77] Tao Sun, Youpan Zhang, Hui Zhang, Pei Wang, Yongguo Zhao, and Guangliang Liu. Three-wheel driven omnidirectional reconfigurable conveyor belt design. In *2019 Chinese Automation Congress (CAC)*, pages 101–105, 2019.
- [78] S. Duque Tisnes, Z. Shi, E. Herth, S. Edmond, L. Petit, E. Martincic, C. Prella, J. Terrien, J. Moulin, E. Lefeuvre, and F. Lamarque. Long-range planar conveyance device based on a digital electromagnetic actuator array. In *2020 Symposium on Design, Test, Integration Packaging of MEMS and MOEMS (DTIP)*, pages 1–5, 2020.
- [79] Chanuphon Trakarnchaiyo, Yang Wang, and Mir Behrad Khamesee. Design of a compact planar magnetic levitation system with wrench-current decoupling enhancement. *Applied Sciences*, 13(4), 2023.
- [80] Onder Tutsoy, Davood Asadi, Karim Ahmadi, and Seyed-Yaser Nabavi-Chasmi. Robust reduced order thau observer with the adaptive fault estimator for the unmanned air vehicles. *IEEE Transactions on Vehicular Technology*, 72(2):1601–1610, 2023.
- [81] Yasuhito Ueda and Hiroyuki Ohsaki. A planar actuator with a small mover traveling over large yaw and translational displacements. *IEEE Transactions on Magnetics*, 44(5):609–616, 2008.
- [82] Marc van de Wal, Gregor van Baars, Frank Sperling, and Okko Bosgra. Multivariable H_∞/μ feedback control design for high-precision wafer stage motion. *Control Engineering Practice*, 10(7):739–755, 2002.
- [83] Liuping Wang. *Model Predictive Control System Design and Implementation Using MATLAB*. Springer, 2009.
- [84] David Wismer and R. Chattergy. *Introduction to Nonlinear Optimization: A Problem Solving Approach*. North Holland, New York, 1979.
- [85] Fengqiu Xu, Xing Lu, Tong Zheng, and Xianze Xu. Motion control of a magnetic levitation actuator based on a wrench model considering yaw angle. *IEEE Transactions on Industrial Electronics*, 67(10):8545–8554, 2020.

- [86] Fengqiu Xu, Yun Lv, Xianze Xu, and Venkata Dinavahi. Fpga-based real-time wrench model of direct current driven magnetic levitation actuator. *IEEE Transactions on Industrial Electronics*, 65(12):9635–9645, 2018.
- [87] Fengqiu Xu, Ruotong Peng, Tong Zheng, and Xianze Xu. Development and validation of numerical magnetic force and torque model for magnetically levitated actuator. *IEEE Transactions on Magnetics*, 55(1):1–9, 2019.
- [88] Fengqiu Xu, Yang Shi, Kaiyang Zhang, and Xianze Xu. Real-time application of robust offset-free mpc in maglev planar machine. *IEEE Transactions on Industrial Electronics*, 70(6):6121–6130, 2023.
- [89] Lei Xu, Mingyao Lin, Xinghe Fu, Kai Liu, and Baocheng Guo. Analytical calculation of the magnetic field distribution in a linear and rotary machine with an orthogonally arrayed permanent magnet. *Energies*, 10(4), 2017.
- [90] Zhenchuan Xu, Yang Wang, Jie Wang, and Mir Behrad Khamesee. Predictor-based model predictive control for maglev planar motor with a 2d halbach array mover. *Control Engineering Practice*, 141:105731, 2023.
- [91] Zhenchuan Xu, Xiaodong Zhang, and Mir Behrad Khamesee. Real-time data-driven force and torque modeling on a 2-d halbach array by a symmetric coil considering end effect. *IEEE Transactions on Magnetics*, 58(11):1–10, 2022.
- [92] Anupma Yadav and S.C. Jayswal. Modelling of flexible manufacturing system: a review. *International Journal of Production Research*, 56(7):2464–2487, 2018.
- [93] Jean-Paul Yonnet and Hicham Allag. Three-dimensional analytical calculation of permanent magnet interactions by “magnetic node” representation. *IEEE Transactions on Magnetics*, 47(8):2050–2055, 2011.
- [94] Ding Yuan, Ming Zhang, Yu Zhu, Xin Li, Leijie Wang, and Chuxiong Hu. Performance evaluation of the wrench models of ironless permanent magnet planar motor with different simplifications. *IEEE Transactions on Magnetics*, 54(12):1–8, 2018.
- [95] Ding Yuan, Ming Zhang, Yu Zhu, Leijie Wang, Xin Li, and Chuxiong Hu. Extended unified wrench model suitable for the end effect of the ironless permanent magnet planar motor. *IET Electr. Power Appl.*, 13:402–409, 03 2019.
- [96] Kaiyang Zhang, Fengqiu Xu, and Xianze Xu. Observer-based fast nonlinear mpc for multi-dof maglev positioning system: Theory and experiment. *Control Engineering Practice*, 114:104860, 2021.

- [97] Kaiyang Zhang, Fengqiu Xu, and Xianze Xu. Robust iterative learning model predictive control for repetitive motion of maglev planar motor. *IET Electric Power Applications*, 16(10):1189–1200, 2022.
- [98] Lu Zhang, Baoquan Kou, he Zhang, and Shoulun Guo. Characteristic analysis of a long-stroke synchronous permanent magnet planar motor. *IEEE Transactions on Magnetics*, 48:4658–4661, 11 2012.
- [99] Shengguo Zhang, Yu Zhu, Haihua Mu, Kaiming Yang, and Wensheng Yin. Decoupling and levitation control of a six-degree-of-freedom magnetically levitated stage with moving coils based on commutation of coil array. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 226(7):875–886, 2012.
- [100] Xiaodong Zhang, Chanuphon Trakarnchaiyo, Heng Zhang, and Mir Khamesee. Magtable: A tabletop system for 6-dof large range and completely contactless operation using magnetic levitation. *Mechatronics*, 77:102600, 08 2021.
- [101] Chenhui Zhou, Hongfeng Tao, Yiyang Chen, Vladimir Stojanovic, and Wojciech Paszke. Robust point-to-point iterative learning control for constrained systems: A minimum energy approach. *International Journal of Robust and Nonlinear Control*, 32(18):10139–10161, 2022.
- [102] Haibo Zhou, Hua Deng, and Ji’an Duan. Hybrid fuzzy decoupling control for a precision maglev motion system. *IEEE/ASME Transactions on Mechatronics*, 23(1):389–401, 2018.
- [103] Haiyue Zhu, Tat Joo Teo, and Chee Khiang Pang. Design and modeling of a six-degree-of-freedom magnetically levitated positioner using square coils and 1-d halbach arrays. *IEEE Transactions on Industrial Electronics*, 64(1):440–450, 2017.
- [104] Haiyue Zhu, Tat Joo Teo, and Chee Khiang Pang. Magnetically levitated parallel actuated dual-stage (maglev-pad) system for six-axis precision positioning. *IEEE/ASME Transactions on Mechatronics*, 24(4):1829–1838, 2019.

APPENDICES

Appendix A

List of Publications

A.1 Published Journal Articles

1. Zhenchuan Xu, Xiaodong Zhang and Mir Behrad Khamesee, "Real-time data-driven force and torque modeling on a 2-D Halbach array by a symmetric coil considering end effect," in *IEEE Transactions on Magnetics*, vol. 58, no. 11, pp. 1-10, Nov. 2022
2. Zhenchuan Xu, Yang Wang, Jie Wang and Mir Behrad Khamesee, "Predictor-based model predictive control for maglev planar motor with a 2D Halbach array mover," in *Control Engineering Practice*, vol. 141, p. 105731, Dec. 2023

A.2 In-Process Journal Articles

1. Zhenchuan Xu, Chanuphon Trakarnchaiyo, Curtis Stewart and Mir Behrad Khamesee, "Modular maglev: Design and implementation of a modular magnetic levitation system to levitate a 2D Halbach array," submitted to *Mechatronics*
2. Yang Wang, Zhenchuan Xu and Mir Behrad Khamesee, "Magnet-coil role-switching real-time wrench model for magnetic levitated planar motors with extendable motion range," submitted to *IEEE Transactions on Industrial Electronics*