

Semi-Analytical Framework for Thermo-Mechanical Analysis of Energy Piles in Elastic and Elastoplastic Soils

by

Abhisek Paul

A thesis

presented to the University of Waterloo

in fulfillment of the

thesis requirement for the degree of

Doctor of Philosophy

in

Civil Engineering

Waterloo, Ontario, Canada, 2024

© Abhisek Paul 2024

Examining Committee Membership

The following served on the Examining Committee for this thesis.

External Examiner	Dr. Omid Ghasemi Fare Associate Professor Department of Civil & Environmental Engineering, J. B. Speed School of Engineering, University of Louisville
Supervisor	Dr. Dipanjan Basu Professor Department of Civil and Environmental Engineering, University of Waterloo
Internal Member	Dr. Shunde Yin Professor Department of Civil and Environmental Engineering, University of Waterloo
Internal Member	Dr. Jitendrapal Sharma Professor, Inaugural Department Chair Department of Civil Engineering, Lassonde School of Engineering, York University
Internal-External Member	Dr. Cliff Butcher Associate Professor Department of Mechanical and Mechatronics Engineering, University of Waterloo

Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Energy piles or geothermal piles are used to reduce the energy demand of a building by helping in the heating/cooling of the building as required. The advantage of energy piles over other ground heat exchangers is that piles are an integrated part of the foundation of a building to carry the superstructure load. The same piles can be used to extract/inject heat from/to the ground and that heat can be used in the cooling/heating of the building. Thus, the energy demand of the building for heating/cooling is reduced. Energy piles are subjected to mechanical load that comes from the superstructure as well as thermal load (temperature change) caused by the heat exchange operation. The combined mechanical and thermal load changes the behavior of the pile foundation. Because the length of the pile is much higher compared to its diameter, the temperature change affects the axial behavior of the pile rather than its lateral response. The settlement of an energy pile is different than a conventional pile foundation as heating or cooling of the pile causes extension in some parts of the pile and compression in other parts of the pile. The axial response of an energy pile under mechanical and thermal loads in terms of vertical settlement, strain, and stress in the pile is calculated in the available literature where the pile-soil interaction has been considered by modeling the soil with equivalent linear and nonlinear soil springs. The representation of soil as springs does not take into account the effect of three-dimensional pile-structure interaction. Analysis of the energy pile considering the three-dimensional pile-structure interaction has been done in the literature using numerical methods which are computationally expensive. Apart from that, the thermo-mechanical behavior of soil needs to be considered in the energy pile analysis because of the temperature change of the pile and soil, and heat exchange between the pile and the soil. In this context, the thermo-mechanical soil constitutive model should satisfy the laws of thermodynamics. The analysis of the energy pile with a thermodynamically acceptable thermo-mechanical soil constitutive model is lacking in the available literature.

In this thesis, a continuum-based semi-analytical framework for the analysis of the energy pile that takes into account the three-dimensional pile-structure interaction is proposed. First, the semi-analytical framework for an energy pile that is embedded in multi-layered soil and subjected to mechanical axial load and thermal load is developed using the variational principle of mechanics by minimizing the potential energy of the pile-soil continuum where both the pile material and soil are considered to behave as a linear elastic material.

In the next part of the thesis, a semi-analytical framework for the same energy pile is developed where the soil is modeled as an elastoplastic material. The analysis framework for the energy pile in elastoplastic soil is developed from the laws of thermodynamics with the energy potential and dissipation function where the plastic behavior of soil is taken into account through the dissipation function. The derived analytical framework is used for elastoplastic soil response with the Drucker-Prager constitutive model. The results from the present analysis are verified with available results in the literature from experiments and numerical analysis. The Drucker-Prager constitutive model does not take into account the effect of temperature on the stress-strain response of the soil. So, this constitutive model is most suitable to model the thermo-mechanical behavior of sand as the effect of temperature on the thermo-mechanical response of sand is not significant. Therefore, energy piles in sand can be analyzed using the present framework with the Drucker-Prager constitutive model. Energy pile in clay needs to be represented with a soil constitutive model that can capture the effect of temperature on the thermo-mechanical response of clay. In the third part of the thesis, the present analytical framework for an energy pile in elastoplastic soil is used to analyze the pile in clay with a thermo-mechanical constitutive model that takes into account the change in the mechanical response of clay due to temperature change. The thermo-mechanical model used in the analysis was developed using the hyperplasticity formalism, which satisfies the laws of thermodynamics. The present framework in all cases predicts results with acceptable accuracy. The present analytical framework predicts the response of energy pile under different loads (mechanical and thermal) in terms of vertical displacement, stress (with <10% difference between the results from the present analysis and finite element analysis/field tests) with acceptable accuracy in less computational time. The run time for the present analysis is approximately 10 and 5 times faster than axisymmetric finite element analysis for elastic and elastoplastic cases, respectively.

In the final part of the thesis, the stresses developed in an energy pile under mechanical and thermal loads are observed for different soil properties. A parametric study is conducted on the developed stresses in an energy pile in single and two-layer soils under multiple loading conditions. A correlation between the applied mechanical and thermal load is established, and under these loading conditions, the effects of soil layering and material properties on the developed stresses are observed. The development of the tensile zone and its range in an energy pile for certain conditions are concluded from the parametric study.

Acknowledgements

First and foremost, I would like to express my deepest gratitude to my supervisor Dr. Dipanjan Basu for providing me the opportunity to pursue doctoral study at the University of Waterloo, and for his unwavering support, insightful feedback, and guidance. His guidance was instrumental in shaping my research and personal growth throughout this journey.

I would also like to extend my heartfelt thanks to my committee members, Dr. Omid Ghasemi Fare, Dr. Shunde Yin, Dr. Jitendrapal Sharma, Dr. Cliff Butcher, and Dr. Giovanni Cascante for their invaluable suggestions, and the time to review my work. Their insightful advice was vital in refining and strengthening this thesis. I would also like to acknowledge the faculty members at the University of Waterloo whose courses benefited my research work.

I would like to recognize the financial support (Graduate Research Studentship, International Doctoral Student Award, Hira and Renu Ahuja Scholarship) that has been provided to me during my years at the University of Waterloo. I would also like to convey my thanks to the undergraduate classes where I served as a Teaching Assistant and received two TA rewards (TA Excellence Award, and Laurence Hamlin Memorial Award) which feels like a rewarding milestone. I am also thankful to the university for providing me with the Doctoral Thesis Completion Award.

I would like to thank my friends Dr. Indranil Hazra, Dr. Pranav Agrawal, Rituraj Bhadra, and Dr. Upasana Pandey whose companionship and moral support have been a source of comfort and laughter through the highs and lows of this academic journey.

Lastly, I want to express my profound gratitude to my parents for their unconditional love, sacrifices, support, and belief in my decisions. I would also like to thank my wife Arkamitra for her unwavering support, patience, and understanding that held me up during the most challenging times.

Thank you all for making this journey possible.

Dedication

To my family and to those who may find the work useful.

Table of Contents

Examining Committee Membership.....	ii
Author’s Declaration	iii
Abstract	iv
Acknowledgements	vi
Dedication	vii
List of Figures	xii
List of Tables.....	xvi
Chapter 1 Introduction.....	1
1.1 Motivation	1
1.2 Literature review	7
1.2.1 Thermo-mechanical analysis of geothermal pile.....	7
1.2.2 Thermo-mechanical constitutive model for soil.....	10
1.3 Research limitations	13
1.4 Objective of this research	14
1.5 Outline of the thesis.....	14
Chapter 2 Analysis of Energy Pile in Elastic Soil Continuum.....	16
2.1 Introduction	16
2.2 Problem statement	16
2.3 Thermal stress and thermal strain in a continuum.....	17
2.4 Analysis.....	18
2.4.1 Displacement in pile and soil	18
2.4.2 Strain-displacement relationships in soil.....	18
2.4.3 Strain energy of the Pile-Soil Continuum.....	19

2.4.4 Minimization of the strain energy of the Pile-Soil Continuum	20
2.4.5 Temperature change in soil.....	23
2.4.6 Governing equations and boundary conditions for vertical displacement of pile	24
2.4.7 Governing equations and boundary conditions for soil displacement shape functions.....	25
2.5 Solution	26
2.5.1 Finite difference (FD) discretization for vertical displacement of pile $u_p(z)$	27
2.5.2 Finite difference (FD) discretization for soil displacement shape function ϕ	30
2.6 Solution algorithm.....	32
2.7 Modification in soil modulus.....	33
2.8 Results	35
2.8.1 Validation with experimental study.....	35
2.8.2 Verification with numerical analysis.....	39
2.9 Summary	44
Chapter 3 Analysis of Energy Pile in Elastoplastic Soil Continuum.....	45
3.1 Introduction	45
3.2 Motivation	45
3.3 Internal energy calculation for a dissipative medium.....	47
3.4 Analysis of energy pile.....	52
3.4.1 Governing equations and boundary conditions for vertical displacement of pile	53
3.4.2 Governing equations and boundary conditions for soil displacement shape functions.....	56
3.5 Solution	57
3.5.1 Finite difference (FD) discretization of governing equations and boundary conditions	58
3.6 Drucker-Prager yield criteria.....	61
3.6.1 Formulation of tangential stiffness tensor	61

3.7 Solution algorithm.....	62
3.7.1 Subroutine-1	63
3.8 Results	64
3.8.1 Validation with experimental study.....	64
3.8.2 Verification with numerical analysis.....	65
3.9 Summary	71
Chapter 4 Analysis of Energy Pile in Clay with a Thermo- mechanical Constitutive Model.....	73
4.1 Introduction	73
4.2 Motivation	73
4.3 Details of thermo-mechanical constitutive model by Golchin et al. (2022b).....	75
4.4 Stress integration scheme for the thermo-mechanical constitutive model	78
4.4.1 Residual equation for strain.....	79
4.4.2 Residual equation for pre-consolidation pressure.....	80
4.4.3 Residual equation for the consistency condition	81
4.4.4 Linearization of the residual equations.....	81
4.4.5 Consistent tangent stiffness matrix.....	83
4.4.6 Subroutine for stress integration.....	83
4.5 Solution procedure	85
4.6 Validation of the constitutive model	85
4.6.1 Abuel-Naga et al. (2006)	86
4.6.2 Uchaipichat and Khalili (2009)	87
4.6.3 Graham et al. (2001).....	90
4.6.4 Cekerevac and Laloui (2004)	91
4.6.5 Yu (1998)	93

4.7 Validation of the present framework for energy piles	95
4.7.1 Bourne-Webb et al. (2009)	95
4.7.2 Goode and McCartney (2015)	103
4.7.3 Laloui <i>et. al.</i> (2006).....	104
4.8 Summary	107
Chapter 5 Parametric study on the thermo-mechanical response of energy pile.....	108
5.1 Introduction	108
5.2 Calculation of equivalent loads	108
5.3 Results	109
5.3.1 Energy pile in single-layer soil.....	109
5.3.2 Energy pile in two-layer soil	110
5.4 Summary	117
Chapter 6 Summary and Recommendations	119
6.1 Summary	119
6.2 Conclusions	120
6.3 Recommendations for future work.....	120
References	122
Appendix A	131

List of Figures

Figure 1-1 Usage of different energy geo-structures over time (Laloui & Rotta Loria, 2020)	2
Figure 1-2 Schematic diagram of geothermal piles.....	3
Figure 1-3 Pile reinforcement with absorber pipes (Brandl, 2006)	3
Figure 1-4 Axial strain and stress distribution profiles for a floating energy pile under (a) mechanical loading only, (b) heating only, (c) mechanical loading and heating, (d) cooling only, and (e) mechanical loading and cooling	5
Figure 2-1 Circular pile embedded in multilayered soil.....	17
Figure 2-2 Finite line source system	23
Figure 2-3 Discretization of soil domain for finite difference analysis.....	27
Figure 2-4 Pile with soil layering and properties	33
Figure 2-5 Vertical displacement profiles of the pile obtained from the present analysis and FEA under thermal load ($\Delta T_p = +30^\circ \text{C}$).....	34
Figure 2-6 Vertical displacement profiles of the pile obtained from the present analysis (with modified soil modulus) and FEA under thermal load ($\Delta T_p = +30^\circ \text{C}$)	35
Figure 2-7 Vertical displacement profiles of the pile obtained from the present analysis, numerical analysis, and field test under thermal load ($\Delta T_p = +18^\circ \text{C}$)	36
Figure 2-8 Axial strain profiles along the pile for a) thermal loading, and b) combined (mechanical and thermal) loading obtained from the present analysis and field data	38
Figure 2-9 Axial stress profile along the pile for mechanical ($Q = 1300 \text{ kN}$), thermal ($\Delta T = +15^\circ \text{C}$), and combined loading obtained from present analysis and field data.....	39
Figure 2-10 Finite element model of the energy pile	40
Figure 2-11 Comparison of results for pile between present analysis and FEA under mechanical, thermal ($\Delta T_p = +30^\circ \text{C}$), and combined loadings in terms of a) Displacement, and b) stress along the length of the pile.....	41
Figure 2-12 Comparison of results for pile between present analysis and FEA under mechanical, thermal ($\Delta T_p = -10^\circ \text{C}$), and combined loadings in terms of a) Displacement, and b) stress along the length of the pile.....	43
Figure 3-1 Axial stress profile along the pile for mechanical ($Q = 1300 \text{ kN}$), thermal ($\Delta T = 15^\circ \text{C}$), and combined loading obtained from present analysis and field data.....	47
Figure 3-2 A continuum with applied traction and heat flow.....	49

Figure 3-3- Free body diagram for pile and soil showing the forces acting on pile and soil and heat flow from the pile to the soil.....	52
Figure 3-4 Flow chart of the solution scheme	64
Figure 3-5 Vertical displacement profiles of the pile obtained from the present analysis, and field test under thermal load ($\Delta T_p = +18^\circ \text{C}$)	65
Figure 3-6 Pile with soil layering and properties	66
Figure 3-7 Comparison of results for energy pile between present analysis and FEA under mechanical, thermal ($\Delta T_p = +25^\circ \text{C}$), and combined loadings in terms of a) Displacement, and b) stress along the length of the pile.....	67
Figure 3-8 Comparison of results for pile between present analysis and FEA under mechanical, thermal ($\Delta T_p = -15^\circ \text{C}$), and combined loadings in terms of a) Displacement, and b) stress along the length of the pile	69
Figure 3-9 Vertical stress profiles of the pile obtained from the present analysis and numerical analysis for different loading conditions	71
Figure 4-1 Axial stress profile along the pile under mechanical ($Q = 1300 \text{ kN}$), thermal ($\Delta T = +15^\circ \text{C}$), and combined loading obtained from present analysis and field data	75
Figure 4-2 Comparison of model predictions of volumetric strain vs. mean stress, during isotropic compression at different temperatures, with experimental results of Abuel-Naga et al. (2006).	87
Figure 4-3 Comparison of model predictions of deviatoric stress vs. shear strain at different temperatures with experimental results of Uchaipichat & Khalili (2009) for a) OCR = 1.33, b) OCR = 2, c) OCR = 4	90
Figure 4-4 Comparison of model predictions of deviatoric stress vs. axial strain with results from Graham et al. (2001).....	91
Figure 4-5 Comparison of model predictions of deviatoric stress vs. axial strain for different OCR with experimental results of Cekerevac & Laloui (2004) at temperature a) 22° , and b) 90°	93
Figure 4-6 Comparison of model predictions of deviatoric stress vs. axial strain with a) experimental results, and b) results from numerical analysis given by Yu (1998)	95
Figure 4-7 Vertical strain profiles along the pile length a) under mechanical load only, b) at the end of cooling ($\Delta T_p = -20^\circ \text{C}$), and c) at the end of heating ($\Delta T_p = 30^\circ \text{C}$) as obtained from the present analysis and field test	99

Figure 4-8 Axial load profiles along the pile length a) under mechanical load only, b) at the end of cooling ($\Delta T_p = -20^\circ \text{ C}$), and c) at the end of heating ($\Delta T_p = 30^\circ \text{ C}$) as obtained from the present analysis and field test	101
Figure 4-9 a) Axial strain, and b) axial load profiles of the pile under thermal load ($\Delta T_p = 29.4^\circ \text{ C}$), as obtained from the present analysis and field test.....	102
Figure 4-10 Vertical displacement profiles of the pile obtained from the present analysis, and field test subjected to thermal load ($\Delta T_p = +18^\circ \text{ C}$)	103
Figure 4-11 Vertical strain profiles of the pile obtained from the present analysis and field test under thermal load (temperature change).....	105
Figure 4-12 Comparison of vertical stress profiles of an energy pile obtained from the present analysis (using elastic, elastoplastic (DP), and thermo-mechanical (TM) soil constitutive model) and field test under a) mechanical load only, b) thermal load only, and (c) combined load.....	106
Figure 5-1 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads. a) $\Delta T_p > 0$, b) $\Delta T_p < 0$	110
Figure 5-2 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p > 0$) for a) $\alpha_{s2}/\alpha_{s1} = 0.1$, and b) $\alpha_{s2}/\alpha_{s1} = 10$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\kappa_{s2}/\kappa_{s1} = 1$).....	111
Figure 5-3 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p < 0$) for a) $\alpha_{s2}/\alpha_{s1} = 0.1$, and b) $\alpha_{s2}/\alpha_{s1} = 0.05$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\kappa_{s2}/\kappa_{s1} = 1$).....	112
Figure 5-4 Normalized vertical stress profile plotted against the normalized pile length for an energy pile for different ratios of thermal expansion coefficient of soil for $\eta = 4$ for a) $\Delta T_p < 0$ b) $\Delta T_p > 0$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\kappa_{s2}/\kappa_{s1} = 1$)	112
Figure 5-5 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p > 0$) for a) $\kappa_{s2}/\kappa_{s1} = 0.5$, and b) $\kappa_{s2}/\kappa_{s1} = 2$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\alpha_{s2}/\alpha_{s1} = 1$).....	113
Figure 5-6 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p < 0$) for a) $\kappa_{s2}/\kappa_{s1} = 1.5$, and b) $\kappa_{s2}/\kappa_{s1} = 2$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\alpha_{s2}/\alpha_{s1} = 1$).....	114

Figure 5-7 Normalized vertical stress profile plotted against the normalized pile length for an energy pile for different ratios of thermal conductivity of soil for $\eta = 4$ for a) $\Delta T_p < 0$ b) $\Delta T_p > 0$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\alpha_{s2}/\alpha_{s1} = 1$).....	115
Figure 5-8 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p > 0$) for a) $G_{s2}/G_{s1} = 0.5$, and b) $G_{s2}/G_{s1} = 5$ when other parameters are kept constants, ($\alpha_{s2}/\alpha_{s1} = 1$, $\kappa_{s2}/\kappa_{s1} = 1$).....	116
Figure 5-9 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p < 0$) for a) $G_{s2}/G_{s1} = 0.5$, and b) $G_{s2}/G_{s1} = 5$ when other parameters are kept constants, ($\alpha_{s2}/\alpha_{s1} = 1$, $\kappa_{s2}/\kappa_{s1} = 1$).....	116
Figure 5-10 Normalized vertical stress profile plotted against the normalized pile length for an energy pile for different ratios of the shear modulus of soil for $\eta = 4$ for a) $\Delta T_p < 0$ b) $\Delta T_p > 0$ when other parameters are kept constants, ($\alpha_{s2}/\alpha_{s1} = 1$, $\kappa_{s2}/\kappa_{s1} = 1$)	117

List of Tables

Table 2-1- Pile dimension and material properties of soil and pile material (Goode & McCartney, 2015)	
.....	36
Table 2-2- Material properties of soil and pile material derived from Laloui et al., (2006)	37
Table 3-1- Soil properties given by Laloui et al., (2006)	46
Table 3-2- Pile dimension and material properties of soil and pile material (Saggu & Chakraborty, 2017).....	70
Table 4-1- Constitutive model parameters	86
Table 4-2- Soil properties and constitutive model parameters	96

Chapter 1

Introduction

1.1 Motivation

The energy demand in the world is increasing because of the growing population, infrastructure development, and enhanced living standards. Alternative sources of energy in the form of renewable energies are required to fulfill these energy demands (International Energy Agency, 2016) if the impacts of climate change have to be mitigated within 2050. Geothermal energy is one form of renewable energy that has growing popularity and has the potential to cover approximately 8.3% of total world electricity production (Dauncey, 2001). The concept of geothermal energy is based on the extraction of heat from and ejection of heat to the soil and rock (where the temperature remains more or less constant throughout the year), respectively, by sending fluids through conduits buried into the ground. These conduits may be boreholes (e.g., those used for Ground Source Heat Pumps), or structural components of buildings embedded into the subsurface (e.g., energy piles or basements), which help in heat transmissions. These geothermal systems reduce the total energy demands of residential and commercial buildings for heating and cooling purposes. The energy pile or geothermal pile is one type of geothermal energy system, as shown in Figure 1-2, which is an energy geo-structure. The advantage of using energy piles is that piles are often integral components of buildings that are used for transferring superstructure loads to the ground and, therefore, there can be significant cost savings if these geo-structures are used as heat exchangers instead of installing a separate geothermal system. As a result, the growth in the usage of energy piles among all the energy geo-structures has been the maximum in the last two decades (Figure 1-1).

A schematic diagram (Brandl, 2006; Laloui & Rotta Loria, 2020) of energy piles with components used for heating/cooling of a building is shown in Figure 1-2. The heat pump sends the cold/hot water to inlet pipes embedded inside the pile where the heat transmission occurs between the pipe and pile material (concrete in general) and between pile material and soil and the hot/cold water comes out through outlet pipes to the heat pump and helps in heating/cooling of the building which results in less electricity consumption than it would have if this supplement process was not there.

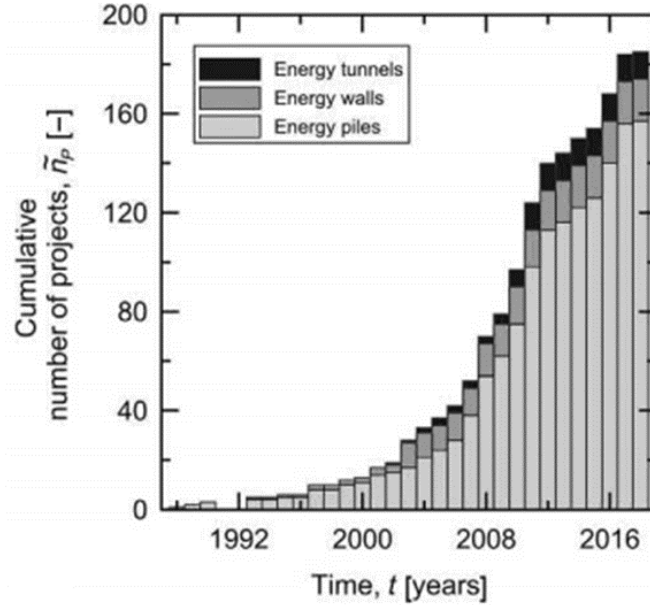


Figure 1-1 Usage of different energy geo-structures over time (Laloui & Rotta Loria, 2020)

Geothermal piles are generally bored piles where the absorber pipes (that carry heat exchanger fluid) are fixed with the reinforcement cage (Figure 1-3). The connecting end of the pipe is protected by a tube at the pile head. A manometer is fixed at the pipe head to maintain pressure in the pipe at 8 bar for integrity check (Brandl, 2006). Before the year 2000, prefabricated reinforced concrete piles with absorber pipes were mostly used as geothermal piles (Brandl, 2006). In recent years, usage of bored cast in situ large diameter piles has increased. The pipes inside the pile are protected by providing a stiff reinforcement cage. The whole process of heating/cooling by geothermal pile consists of two circuits connected by the heat pump (Brandl, 2006). The primary circuit contains closed pipework in the pile through which a heat carrier fluid (e.g., water, water with antifreeze like glycol, saline solution etc.) is pumped that exchanges heat from the building with the ground. The piping within the pile is individually joined to a header block. The secondary circuit is a closed fluid-based building heating or cooling network (secondary pipework) embedded in the floors and walls of the structure.

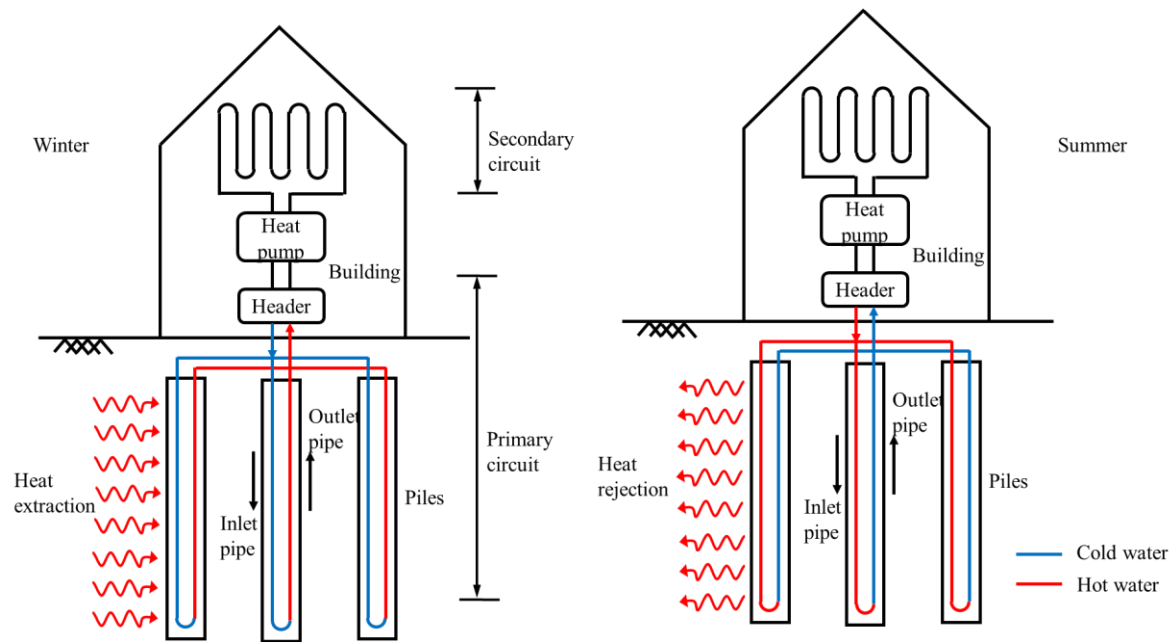


Figure 1-2 Schematic diagram of geothermal piles



Figure 1-3 Pile reinforcement with absorber pipes (Brandl, 2006)

Analysis of geothermal piles should be done differently from that of conventional piles because temperature change in an energy pile caused by heat transmission generates additional stresses and alters the interaction of the pile with the surrounding soil (Laloui & Rotta Loria, 2020). In any pile foundation, compressive stress is developed within the pile shaft because of the vertical superstructure load. In energy piles, the heating/cooling processes generate a tendency of elongation/shortening of the pile shaft. As the surrounding soil provides resistance against the elongation/shortening (restraining the thermal strain) in the form of side resistance (skin friction) and base resistance, thermal compressive/tensile stresses are developed in the pile in addition to mechanical compressive stress caused by the superstructure loading. The additional stresses generated by thermal loading depend on side and base resistances that arise from the complex thermo-mechanical interaction between the pile and the surrounding soil.

For an ideal condition of only thermal loading (Figure 1-4) with both cooling and heating cycles, where there is no restraint at the pile head (i.e. when there is no superstructure load), the restrained thermal strain is maximum around the mid-depth of an ideal floating pile (where the base resistance is zero) and decreases with an increase in vertical distance from the mid-depth and becomes zero at the two ends (head and base). For an ideal end-bearing pile (where side resistance is zero) with no restraint at the pile head (i.e., with no superstructure load), the strain is maximum at the base of the pile, decreases with an increase in vertical distance from the base, and becomes zero at the pile head. In contrast, in a conventional pile with a vertical superstructure load, maximum strain and stress both occur at the pile head and decrease with depth (Salgado, 2008). Therefore, a combination of mechanical and thermal loads produces a different strain and stress distribution along the length of the pile. Under thermal loading, the pile deforms with respect to a point that is situated along the length of the pile. This point is called the null point around which the stress within the pile changes from compressive to tensile. Determination of the null point is important for determining the mobilized stress (Bourne-Webb et al., 2009). Clearly, the interaction of the pile with the surrounding soil changes when the thermal load acts, and this makes the analysis and design of energy piles different from that of conventional piles.

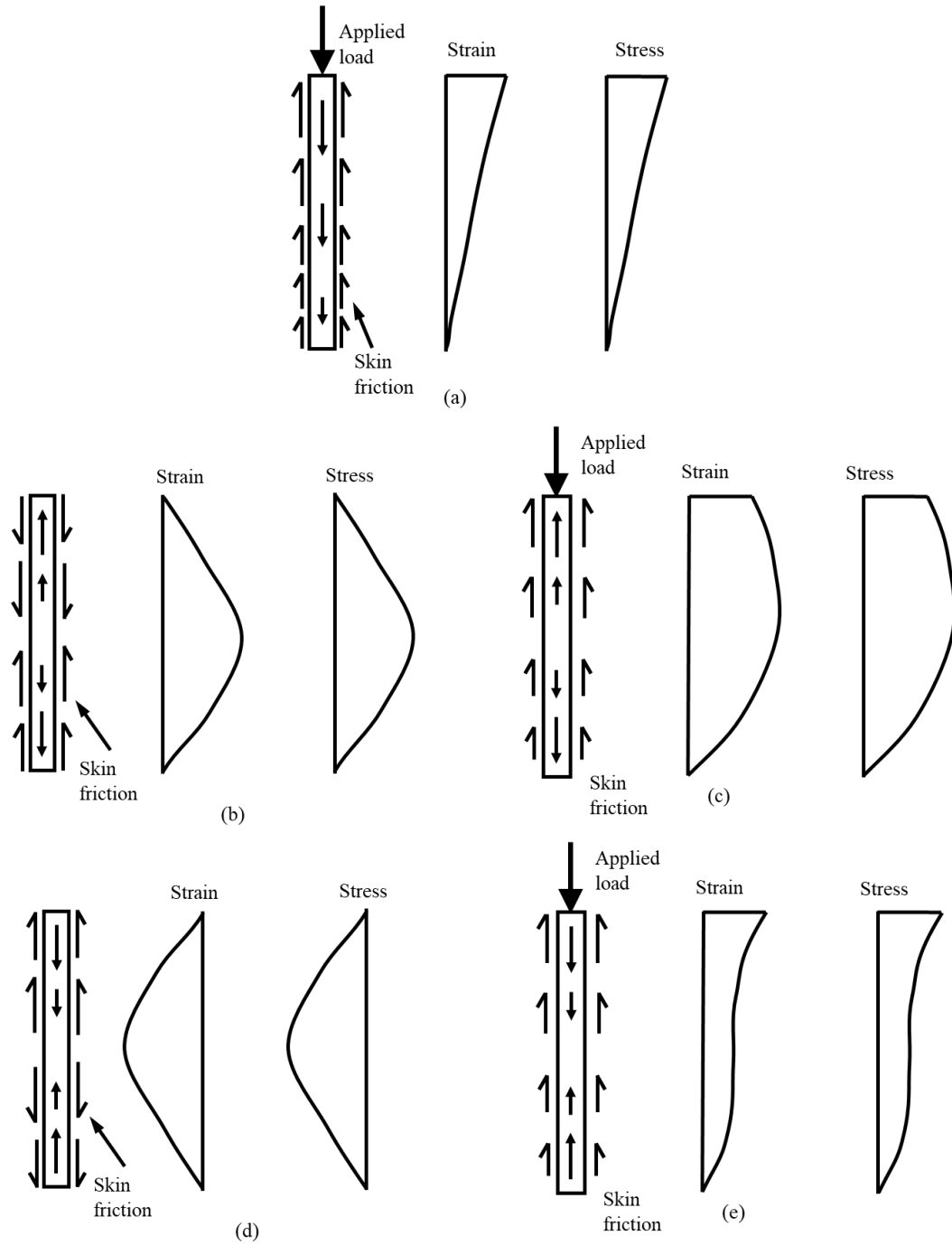


Figure 1-4 Axial strain and stress distribution profiles for a floating energy pile under (a) mechanical loading only, (b) heating only, (c) mechanical loading and heating, (d) cooling only, and (e) mechanical loading and cooling

The dimension of a single pile in the axial direction is much higher compared to its lateral dimensions (diameter). As a result, temperature change within a pile has a negligible effect on the lateral response of the pile. The temperature change affects the displacement, strain, and stress in the axial direction of the pile (Amatya et al., 2012; Bourne-Webb et al., 2009, 2013; Goode & McCartney, 2015; Laloui et al., 2006; Laloui & Rotta Loria, 2020; Stewart, 2010). Therefore, in this study, the axial response of the energy pile is determined with a proper thermo-mechanical analysis.

Axially loaded piles are designed against the ultimate limit state of ultimate axial capacity (usually, corresponding to 10% relative settlement) and the serviceability limit state of a maximum tolerable settlement (Salgado, 2008). Ultimate axial pile capacity, which is a summation of ultimate base capacity and limit shaft capacity, may change for an energy pile (from the corresponding capacities of a conventional pile of the same dimension) because of a change in soil shear strength caused by temperature change. At the same time, the settlement of an energy pile may be different from that of a conventional pile (of the same dimension) because of the altered pile-soil interaction caused by the heat flow and temperature change. This research study concerns the investigation of energy pile settlement under different thermal loading conditions.

Pile settlement is conventionally estimated using soil-spring-based t - z analysis in which nonlinear springs representing soil resistance t are attached to the pile shaft and the pile displacement z is calculated iteratively by solving the pile settlement differential equation in which the nonlinear relationship (curve) between t and z are given as inputs (Berardi & Bovolenta, 2005; Guo, 2000; Halder & Sivakumar Babu, 2008; Misra & Roberts, 2009). Although continuum-based analyses including finite element analysis may be used to calculate pile settlement (Basu et al., 2009; Guo & Lee, 2001) and are conceptually superior to the spring-based approaches, the t - z method is the most popular because it is easier to use than the continuum-based methods. The accuracy of t - z method depends on the accuracy of the nonlinear t - z relationship used in the analysis. For energy piles, t - z relationship depends also on the thermo-mechanical pile-soil interaction. However, temperature-dependent t - z relationships available in the literature (Chen & McCartney, 2017; M. E. Suryatriyastuti et al., 2014; Xiao & Suleiman, 2015) have been derived either by pile-soil spring analysis or by calculating curve-fitted parameters with the help of experimental or numerical results. As a result, a reliable t - z curve derived from rigorous continuum-based analysis with proper thermo-mechanical soil constitutive model and with consideration of thermo-mechanical pile-soil interaction for energy piles is not available.

In this study, a continuum-based analysis framework for settlement estimation of energy piles is developed following the principles of continuum mechanics and thermodynamics. The goal of the study is to develop a rigorous thermo-mechanical elastoplastic analytical framework for energy piles. The response of the energy pile is obtained with the soil modeled by a thermodynamically viable thermo-mechanical constitutive relationship. The reason behind analyzing the energy pile with this specific type of constitutive model is to incorporate the effect of temperature on soil behavior by satisfying the fundamental laws of thermodynamics. Another objective of this study is to investigate the changes in axial stress in the pile for different soil and loading conditions that need to be taken into account in the design of the pile.

1.2 Literature review

1.2.1 Thermo-mechanical analysis of geothermal pile

Analytical, numerical, and physical modeling of geothermal piles has been performed by several researchers. Analytical solutions for geothermal piles have been done by Knellwolf et al., (2011); Najma & Sharma, (2021); Olia & Perić, (2021); Perić et al., (2020). Numerical modeling usually involves finite element and finite difference (Knellwolf et al., 2011) analysis. Physical modeling includes field tests, centrifuge tests, and reduced-scale laboratory tests (Goode & McCartney, 2015; McCartney & Rosenberg, 2011; Suryatriyastuti et al., 2014; Wang et al., 2015). The effect of temperature change in energy pile affects the settlement/axial displacement, axial strain, axial stress, pile-soil interface stress distribution, and axial load capacity of the pile which have been observed in the studies mentioned in this section.

The axial displacement, strain, stress, and mobilized shear forces distribution along the length of the pile under thermal load are different compared to that under mechanical load. In the case of energy piles, the piles are subjected to both mechanical and thermal loading. Changes that happen to the quantities mentioned previously have been studied by different researchers and are described in this section. A qualitative distribution of strain, stress, and mobilized skin friction under both temperature change and mechanical load for both floating and end-bearing pile have been given by Amatya et al., (2012); Bourne-Webb et al., (2009); Bourne-Webb et al., (2013) based on a full-scale field test on an energy pile. The soil was idealized as a rigid plastic material with a uniform shaft resistance. The global equilibrium equations for axial displacement, shear stress, and axial stress based on linear thermo-elasticity theory have been provided by Rotta Loria & Laloui, (2019), which upon solving provides the

distribution diagram proposed by Amatya et al., (2012); Bourne-Webb et al., (2009); Bourne-Webb et al., (2013).

Analysis for the calculation of axial strain, stress, load, and displacement profiles of energy pile under mechanical and thermal loading with the surrounding soil modeled as springs have been carried out by Iodice et al., (2020a); Knellwolf et al., (2011); Najma & Sharma, (2021a); Olia & Perić, (2021a); Perić et al., (2020). An analytical solution for heat exchanger end-bearing piles has been developed by Perić et al., (2020) with the soil modeled as linear springs where closed-form solutions for head displacement, axial stress, and strain distribution along the length of the pile under both axial and thermal load was provided. As an extension of the previous work that includes the determination of the null point and location of the maximum stress point along the length of the pile has been given by Olia & Perić, (2021). Another soil spring model for energy pile with isotropic, elastic one-layer, and two-layer soil with variable modulus has been given by Iodice et al., (2020a) where mobilized soil resistance along the shaft is provided in terms of side friction coefficient calibrated with the help of finite element analysis. The distribution of axial load, strain, and stress along the pile has also been shown using the closed-form solution. Knellwolf et al., (2011) developed a soil-spring-based pile settlement analysis model using the t - z curve given by Frank & Zhao, (1982) for cyclic loading which has been developed without taking into account the thermal effect on soil. Incremental load transfer analysis has been performed by Najma & Sharma, (2021) for cyclic loading with soil springs that follow nonlinear hyperbolic behavior. The effects of imposed mechanical load, temperature change, and position of null point have been discussed by the authors. The occurrence of critical design situations in terms of capacity and settlement of the energy pile under both thermal and mechanical loads has been described by the authors through their developed solution method. Thermo-hydro-mechanical numerical modeling with the Mohr-Coulomb soil constitutive model has been done by Rotta Loria & Donna, et al., (2015) to model a centrifuge test (Rotta Loria et al., 2015) where the importance of null point in an energy pile and the influencing factors of the null point were discussed. Axisymmetric numerical analysis of geothermal piles with Mohr-Coulomb soil constitutive model (Saggu & Chakraborty, 2015a) and CASM soil constitutive model (Saggu & Chakraborty, 2015b, 2017) have been developed and parametric studies have been performed (Olia & Perić, 2021; Saggu & Chakraborty, 2015a, 2017) to describe pile displacement, strain, stress developed at the pile-soil interface.

The procedures described in this section to calculate the response of energy piles in terms of axial displacement, strain, stress, mobilized soil friction etc. have been done analytically by modeling the soil as discrete springs and numerically by implementing axisymmetric finite element method. While the soil spring models provide results quickly, the springs cannot properly represent the continuum-based multi-axial stress-strain response of soil in which normal and shear stresses play important roles in pile-soil interaction. Finite element or other numerical analysis simulates the soil continuum much more accurately; however, these methods are computationally expensive and require specialized knowledge regarding model development. None of these approaches considers the thermodynamics and mechanics in an integrated framework, which should be considered for energy pile analysis.

Energy piles can undergo multiple heating and cooling cycles. Depending on the seasonal variation of ambient temperature during different seasons, the nature of the thermal load on the energy pile changes (the pile is heated during summer due to the supply of hot liquid and vice versa during winter (Figure 1-2). Studies on energy piles considering cyclic thermal loading have been conducted by Di Donna & Laloui, (2015); Iodice et al., (2020a); McCartney & Rosenberg, (2011); Ng et al., (2019); Suryatriyastuti et al., (2014); Wang et al., (2015); You et al., (2016). The effect of cyclic thermal load on the capacity, which is one of the major aspects of energy pile, has been studied in laboratory tests by McCartney & Rosenberg, (2011); Ng et al., (2019) and in field tests by Wang et al., (2015); You et al., (2016). The capacity of energy piles increases with heating and returns to its initial capacity when allowed to cool naturally (Wang et al., 2015; You et al., 2016). This increase in the capacity of the energy pile caused by heating has been reported as 40% of its initial capacity for the temperature changes from 15° to 60° (McCartney & Rosenberg, 2011), 13% and 30% of its initial capacity for temperatures 37° and 52° respectively (Ng et al., 2019). The effect of cyclic thermal loading on both single pile and pile groups considering soil thermo-plasticity has been studied by Di Donna & Laloui, (2015b) with ACMEG-T soil constitutive model (which belongs to the Cam clay family) developed by Laloui & François, (2009). They found that the thermal load generates additional pile displacement and the rate of change of additional displacement diminishes as the number of heating-cooling cycles increases. Similar observations were made by Suryatriyastuti et al., (2014), who used a load transfer function and a t - z curve by calibrating the parameters with the help of numerical simulations. Axial load distribution along the pile and settlement at the pile head under cyclic thermal loading was studied by Iodice et al., (2020b) using finite element method with different soil constitutive models like Mohr-

Coulomb, modified Cam Clay (MCC), and hypoplastic model. It has been found by the authors that the accumulation of irreversible pile settlements occurs under cyclic thermal loading. The effect of cyclic thermal loadings on the energy pile with a thermo-elasto-plastic soil constitutive model can produce more accurate results for different loading and soil conditions.

The importance of t - z curves for the design of the energy pile has been mentioned in the previous section. Studies on the development of t - z curves for energy piles have also been done by McCartney & Rosenberg, (2011); Pasten & Santamarina, (2014); Suryatriyastuti et al., (2014). The calibrated plots for base resistance-base displacement and side friction-side displacement have been developed with the help of the results of full-scale and geotechnical centrifuge tests including the temperature effects by McCartney & Rosenberg, (2011). Suryatriyastuti et al. (2014) proposed a cyclic t - z function that comprises a cyclic hardening/softening mechanism based on the response of nonlinear soil behavior. The thermal hardening/softening affects the mobilized soil friction along the shaft of the energy pile. Their load transfer function can account for the degradation of the soil-pile capacity during two-way cyclic thermal loading in the energy pile. The t - z design charts developed in the literature as mentioned in this section have been developed by calibrating the parameters using the data from field tests and numerical simulations. The stress developed in an energy pile due to the temperature change can be different than that of the conventional piles as suggested by Amatya et al., (2012); Bourne-Webb et al., (2009, 2013); Laloui & Rotta Loria, (2020). As a result, nature of axial stress can get altered and tension may develop in the pile. In this context, a study on the determination of the null point and tensile zone within the pile is important in the aspects of the design of the pile.

1.2.2 Thermo-mechanical constitutive model for soil

Proper analysis and design of energy piles require modeling of pile-soil interaction considering the thermo-mechanical behavior of soil. To do that, an understanding of the thermo-mechanical soil behavior and the use of a proper thermo-elasto-plastic soil constitutive model are necessary.

The response of soil under thermal and mechanical loads has been studied using three types of loading paths: thermal, isothermal-mechanical, and thermo-mechanical (Laloui, 2001). Thermal loading involves the application of temperature change at constant stress, isothermal-mechanical loading involves the application of mechanical loading at constant temperature and thermo-mechanical loading involves the application of both thermal and mechanical loads simultaneously (Laloui, 2001). The application of thermal load in the form of temperature change has effects on the different physical properties (e.g., volume and strength) of soil in many different ways. Temperature change induces a

change in the void ratio (or volume) which impacts the stress and, in turn, affects the constitutive behavior of soil. Normally consolidated (NC) clays contract when it is heated and a significant part of this deformation is irreversible upon cooling and the opposite behavior has been observed for overconsolidated clays (Abuel-Naga et al., 2009; Cui et al., 2000; Laloui & Cekerevac, 2003). The volume change in loose to medium dense sand and dense sand is analogous to normally consolidated and overconsolidated clay, respectively, but the volumetric strain in loose sand caused by the temperature change is much less compared to the strain developed for the same temperature change in normally consolidated clay (Ng, Wang, et al., 2016). For normally consolidated clay, thermally induced volume contraction decreases with an increase in temperature (Campanella & Mitchell, 1968). The strength of the soil may reduce or increase when the soil is heated (Cui et al., 2018; Modaressi & Laloui, 1997). Some experimental results showed that the critical state friction angle either increases or decreases or remains unchanged with temperature which reflects an increase or decrease in the strength of soil (Laloui, 2001). The hydraulic conductivity of sandy soil is very high. Therefore, the groundwater dissipates the temperature change by heat transfer through the advection phenomenon which results in a low thermal gradient in sands and hence, results in low volumetric deformation (Suryatriyastuti, 2013). As a result, the effect of temperature change is higher on the thermo-mechanical response of clay. The inclusion of the change in shear strength of soil in the modeling of the thermo-mechanical pile-soil interaction of energy piles is important, which can be achieved by using a proper thermo-mechanical soil constitutive model in a numerical or analytical framework.

Various thermo-mechanical soil constitutive models developed by different researchers that describe the evolution of yield surface for temperature loading with strain hardening and thermal hardening/softening are briefly stated in this section. A thermoplastic constitutive model based on the concept of critical state soil mechanics in which the yield surface is a function of temperature, thermal softening, and strain hardening has been developed by Hueckel & Borsetto, (1990). Based on the available experimental results following thermal, isothermal-mechanical, and thermo-mechanical loading paths, a cyclic thermo-viscoplastic soil constitutive model, which includes thermal hardening and evolution of yield surface with temperature developed by Modaressi & Laloui, (1997). Cui et al., (2000) extended the work by Hueckel & Borsetto, (1990) to investigate the effect of overconsolidation ratio (OCR) by incorporating a coupled mechanical and thermal hardening phenomenon in the model for saturated clay. Based on the model by Cui et al., (2000), a thermomechanical soil constitutive model has been developed by Hong et al., (2016) based on two-surface plasticity theory for saturated clays

with calibrated parameters from available data. The Cam clay model has been modified by Graham et al., (2001) to include the effect of temperature by calibrating the parameters for elastic deformation, hardening function, yield function, and peak and critical strength. The time-dependent secondary compression of clay which is governed by the plastic strain depends on the creep rate coefficient. The modification in creep rate due to temperature has been made by Kurz et al., (2016) in a semi empirical elastic-thermo-visco-plastic model. The model has been further modified by Fathalikhani et al., (2022) where a modification has been made by the assumption of elastic unloading at a short timeframe and elastic and visco-plastic reloading. A thermo-mechanical model has been developed by Laloui & Cekerevac, (2003) and Laloui et al., (2006) based on experimental results on the thermal behavior of soil which includes the thermal effect on void ratio and pre-consolidation pressure. An experimentally calibrated model has been proposed by Mašin & Khalili, (2012) based on the concept of incrementally nonlinear hypoplastic model for saturated clay. A critical state-based cyclic thermo-elastoplastic model named LTVP has been developed by Laloui, (2001) where the strain was decomposed into nonlinear thermoelastic and thermoplastic components. A critical state-based thermo-elastoplastic model was developed by Abuel-Naga et al., (2009) which described the thermally induced volume change of saturated clay in NC and OC conditions in triaxial stress space by applying some modifications to the modified Cam Clay model. Coccia & McCartney, (2016) presented a constitutive model for volume change of soil due to temperature change using the concept of thermally-accelerated secondary compression. Thermo-mechanical soil constitutive models based on bounding surface plasticity theory have also been developed by Yao & Zhou, (2013); Zhou & Ng, (2015); Zhou et al., (2017); Zhou & Ng, (2018). Further, thermodynamically consistent thermo-mechanical models for fine-grained soils have been developed by (Golchin et al., 2022b, 2022a; Houlsby & Puzrin, 2007).

Constitutive models established using the thermodynamics framework provides the confidence in giving results as they are not thermodynamically unreasonable (Houlsby & Puzrin, 2007). The elastoplastic constitutive model of soil with or without temperature change can be derived using the thermodynamics principle within hyperplasticity framework. In that way, these models satisfy the conventional plasticity approach as well as the laws of thermodynamics. The thermodynamic approach to the constitutive behavior of material was first established by Ziegler, (1983). He described how the state of a material depends on a state variable and an internal variable, and the change of internal energy can be described in the form of state function and dissipation. The formulation of the constitutive model using thermodynamics, also known as the hyperplasticity approach, was described in detail by Houlsby

& Puzrin, (2007). The principles of thermodynamics are capable of developing a framework that accurately captures the behavior of geomaterials (Houlsby & Puzrin, 2007). The elastoplastic and thermo-elastoplastic constitutive models of soil discussed in the previous section comply with the plasticity theory and are capable of producing reasonably accurate results, but most of them do not follow the hyperplasticity formalism and therefore do not satisfy thermodynamics (Golchin et al., 2022b). Recently, Golchin et al., (2022b, 2022a); Houlsby & Puzrin, (2007) developed thermo-mechanical models for fine-grained soils based on the hyperplasticity formalism. Thermo-mechanical constitutive model for clay with two yield surfaces including thermal, isotropic, and kinematic hardening (Golchin et al., 2022a); and single yield surfaces with thermal, and isotropic hardening (Golchin et al., 2022b) have been developed for isothermal and non-isothermal conditions using energy potential and dissipation potential which comply with the laws of thermodynamics.

1.3 Research limitations

The following gaps were identified from the literature review:

1. Soil continuum-based settlement calculation methods are superior to the spring-based methods but usually require numerical methods (e.g., finite element method) for solution and, therefore, are computationally intensive.
2. Approaches that integrate both the mechanics and thermodynamics aspects in a single framework, which is important for energy piles, are lacking.
3. No analysis of energy piles is available in which a thermodynamically consistent soil constitutive model has been used.

1.4 Objective of this research

Based on the research gaps, the following objectives have been identified for the present research:

- 1) Develop an efficient soil continuum-based semi-analytical framework for settlement calculation of energy piles that is based on the principles of mechanics with soil modeled as an elastic material.
- 2) Develop a continuum-based semi-analytical framework for settlement calculation of energy piles that is based on the principles of thermodynamics and mechanics and incorporates the elastoplastic behavior of soil.
- 3) Analyze the response of energy pile in clay using the elastoplastic analytical framework by modeling the clay with a hyperplasticity based thermo-mechanical constitutive model.
- 4) Perform a parametric study on the stress developed within energy piles to find out the effect of different loads and soil conditions.

1.5 Outline of the thesis

This research proposal is divided into three chapters.

Chapter 1 presents the motivation for the research, and literature review on the thermo-mechanical response of energy piles followed by the identification of research gaps and objectives.

Chapter 2 presents the development of a semi-analytical framework for an energy pile in a multi-layered soil continuum subjected to both mechanical and thermal load (temperature change) where both the pile and the soil are considered to behave as a linear elastic material. The framework is developed based on the principle of minimization of the potential energy of the pile-soil continuum using the variational calculus. The results from the present analysis are verified with the available experimental results in the literature and with the results from equivalent finite element analysis.

Chapter 3 provides the development of the semi-analytical framework for an energy pile in a multi-layered soil continuum subjected to both mechanical and thermal load (temperature change) where the soil is considered to behave as an elastoplastic material. The elastoplastic behavior of soil modifies the pile-soil interaction and hence, affects the thermo-mechanical response of the pile. The framework is

derived using the laws of thermodynamics and minimizing the internal energy of the pile-soil continuum. The Drucker-Prager soil constitutive model is used to represent the elastoplastic soil behavior. Comparisons between the results from the present analysis and results from field experiments and finite element analysis are made to show the accuracy of the present framework

Chapter 4 describes the present framework with a thermo-mechanical soil constitutive model that represents the elastoplastic behavior of clayey soil including the effect of temperature on the mechanical response of the clayey soil. The effect of temperature is more pronounced on clay than sand and therefore the inclusion of a proper thermo-mechanical soil constitutive model was required. The constitutive model was developed by following the hyperplasticity formalism (satisfying the laws of thermodynamics). The predicted response of soil elements under different loading conditions using the constitutive model is verified with the results that are available in the literature. The response of energy pile that is obtained from the present framework including the thermo-mechanical constitutive model is verified with the available experimental results from the literature.

Chapter 5 presents the parametric study of an energy pile in different soil and loading conditions. The stress distribution along the length of an energy pile differs from that of a conventional pile because of the temperature change. Therefore, a parametric study on an energy pile that is embedded in single-layer, and two-layer soils and subjected to mechanical and multiple combined loading conditions is conducted. The effect of strength parameters and thermal properties of soil on the thermo-mechanical response of the pile is observed and conclusions are made regarding tensile zones that develop in the pile in certain conditions that need to be considered in the design of the pile.

Chapter 6 provides the conclusions resulting from the present work along with recommendations for future research.

Chapter 2

Analysis of Energy Pile in Elastic Soil Continuum

2.1 Introduction

In this chapter, a semi-analytical approach to analyze an energy pile embedded in a multilayered elastic soil deposit subjected to both axial and thermal load is discussed. The governing differential equations and boundary conditions for the pile under these loadings are derived by minimizing the potential energy of the pile-soil continuum using the variational principle. The pile-soil system is assumed as a continuum where no slip occurs at the pile-soil interface. The results obtained in the analysis are compared with the experimental results available in the literature (Goode & McCartney, 2015; Laloui et al., 2006) and the results obtained from the finite element analysis (FEA) conducted by the author. The comparisons show a good match between the results from the literature and FEA with the results from the present analytical approach.

2.2 Problem statement

A circular pile of length L_p , diameter D_p and Young's modulus E_p is embedded in a multilayered soil deposit as shown in Figure 2-1. The pile is subjected to an axial load Q_p and it undergoes a temperature change ΔT_p . The temperature change along the length and cross-section of the pile is assumed to be constant (Perić et al., 2020) because the temperature difference along the depth of the pile has been observed to vary within $\pm 2^\circ\text{C}$ (Knellwolf et al., 2011) which does not have a substantial effect on the response of the pile. The soil is assumed to be extended to a very large distance in both horizontal and vertical directions. The depth of i -th soil layer from the ground level is denoted as H_i as shown in Figure 2-1. It is assumed that both the pile material and the soil in each layer behave as a linear elastic, isotropic, and homogeneous material. The soil is characterized by Lamé's constants λ_s and G_s respectively. A right-handed cylindrical (r, θ, z) coordinate is used for the analysis as shown in Figure 2-1. The origin of the coordinate system is at the center of the pile head and the positive z axis points downward along the axis of the pile.

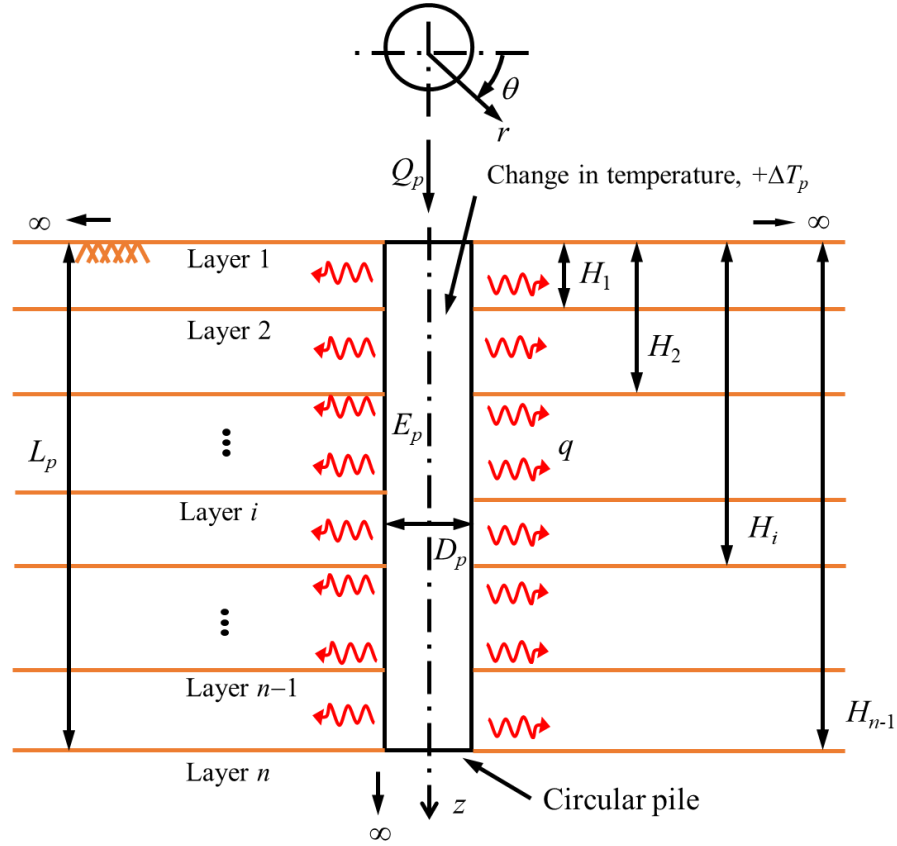


Figure 2-1 Circular pile embedded in multilayered soil

2.3 Thermal stress and thermal strain in a continuum

The strain developed in any continuum (with movement restrained at boundaries) that undergoes temperature change, (described as thermal strain) ε_{ij}^T (superscript T denotes strain due to change in temperature) is calculated as (Laloui & Rotta Loria, 2020; Perić et al., 2020),

$$\varepsilon_{ij}^T = \varepsilon_{ij} - \alpha_{ii} \Delta T \quad (2.1)$$

$$\varepsilon_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \quad (2.1a)$$

where α_{ii} is the coefficient of thermal expansion, ΔT is the temperature difference that occurs in a continuum ($\Delta T = \text{final temperature} - \text{initial temperature}$) and u is the displacement vector. In equation (2.1) the second term in the right-hand side is called thermal free strain which yields in a material if it

undergoes a temperature change (Bourne-Webb et al., 2013). If displacement u occurs due to temperature change in an elastic continuum which results in different strain than that of thermal free strain, then the stress generated within the continuum (known as thermal stress) is calculated from strain ε_{ij}^T using the relation, $\sigma_{ij} = C_{ijkl} \varepsilon_{kl}^T$ where C_{ijkl} is the stiffness tensor, and i, j, k , and l are the dummy indices. Depending on the final and initial temperatures developed, stress can be either tensile or compressive and it contributes only to the hydrostatic part of the stress tensor. Coefficient of thermal expansion α_{ii} can be different in three orthogonal directions.

2.4 Analysis

2.4.1 Displacement in pile and soil

The vertical displacement of the pile and the soil column below the pile is denoted as $u_p(z)$ which varies along the depth of the pile. The displacement of soil surrounding the pile is expressed as (Seo et al., 2009),

$$u_r = 0 \quad (2.2a)$$

$$u_\theta = 0 \quad (2.2b)$$

$$u_z = u_p(z)\phi(r) \quad (2.2c)$$

where u_r , u_θ , and u_z are the soil displacements in the radial, tangential, and vertical directions, respectively; and ϕ is the dimensionless displacement shape function of soil associated with u_p . The function ϕ varies along the radial distance r with its values equal to 1 at the pile-soil interface (i.e., $\phi = 1$ at $r = r_p$) and 0 at large horizontal distances away from the pile (i.e., $\phi = 0$ at $r = \infty$). u_z becomes zero at very large depth below the pile. The assumed soil displacement field ensures continuity and compatibility of the displacements and strains in soil by taking into account the displacement of the pile.

2.4.2 Strain-displacement relationships in soil

The strain tensors for soil can be calculated from pile displacement as described below;

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ \gamma_{r\theta} \\ \gamma_{rz} \\ \gamma_{\theta z} \end{bmatrix} = - \begin{bmatrix} \frac{\partial u_r}{\partial r} \\ \frac{u_r}{r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} \\ \frac{\partial u_z}{\partial z} \\ 2 \times \frac{1}{2} \left(\frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right) \\ 2 \times \frac{1}{2} \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z} \right) \\ 2 \times \frac{1}{2} \left(\frac{1}{r} \frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z} \right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\frac{du_p(z)}{dz} \phi(r) \\ 0 \\ -u_p \frac{d\phi(r)}{dr} \\ 0 \end{bmatrix} \quad (2.3a)$$

Using equation (2.1), the thermal strain in soil is given by

$$\boldsymbol{\varepsilon}^T = \begin{bmatrix} -\alpha_s \Delta T_s(z, r) \\ -\alpha_s \Delta T_s(z, r) \\ -\frac{du_p(z)}{dz} \phi(r) - \alpha_s \Delta T_s(z, r) \\ 0 \\ -u_p \frac{d\phi(r)}{dr} \\ 0 \end{bmatrix} \quad (2.3b)$$

where α_s is the coefficient for thermal expansion for all the normal directions and $\Delta T_s(z, r)$ represents the temperature change in the soil continuum which varies both along the depth and radial distance. The coefficient of thermal expansion α_s is considered the same in all three orthogonal directions.

2.4.3 Strain energy of the Pile-Soil Continuum

The governing equations for $u_p(z)$ and $\phi(r)$ are obtained by minimizing the potential energy of the pile-soil system using the variational principle (Reddy, 2002). The total potential energy of the pile-soil system is given by

$$\pi = \int_{\Omega_p} \frac{1}{2} \sigma_{ij} \varepsilon_{ij} d\Omega_p + \int_{\Omega_s} \frac{1}{2} \sigma_{ij} \varepsilon_{ij} d\Omega_s - Q_p u_p \Big|_{z=0} \quad (2.4)$$

where σ_{ij} and ε_{ij} are the strain and stress tensors, respectively. Ω_p and Ω_s are the domains for the pile and the soil, respectively. For an isotropic elastic material, σ_{ij} is calculated from ε_{ij} using the relation, $\sigma_{ij} = C_{ijkl}^{(e)} \varepsilon_{kl}$. The elastic stiffness tensor for soil $\mathbf{C}_e$ or $C_{ijkl}^{(e)}$ is given by

$$\mathbf{C}_e = \begin{bmatrix} \lambda_s + 2G_s & \lambda_s & \lambda_s & 0 & 0 & 0 \\ \lambda_s & \lambda_s + 2G_s & \lambda_s & 0 & 0 & 0 \\ \lambda_s & \lambda_s & \lambda_s + 2G_s & 0 & 0 & 0 \\ 0 & 0 & 0 & G_s & 0 & 0 \\ 0 & 0 & 0 & 0 & G_s & 0 \\ 0 & 0 & 0 & 0 & 0 & G_s \end{bmatrix} \quad (2.5)$$

The strain energy of the pile is calculated only for the strain in the axial direction (z-direction). Expressing the stress in terms of the strains and the Lamé's constants, strains in terms of the displacement and ignoring the higher order terms with α_s ($\because \alpha_s \ll 1$) equation (2.4) can be rewritten as

$$\begin{aligned} \pi = & \frac{1}{2} E_p A_p \int_0^{L_p} \left(\frac{du_p}{dz} - \alpha_p \Delta T_p \right)^2 dz + \frac{1}{2} \int_0^\infty \int_0^{2\pi} \int_{r_p}^\infty \left[(\lambda_s + 2G_s) \left\{ \left(\frac{du_p}{dz} \phi \right)^2 - 2 \left(\frac{du_p}{dz} \right) \phi \alpha_s \Delta T_s \right\} \right. \\ & \left. - 4\lambda_s \left(\frac{du_p}{dz} \right) \phi \alpha_s \Delta T_s + G_s \left(u_p \frac{d\phi}{dr} \right)^2 \right] r dr d\theta dz + \frac{1}{2} \int_{L_p}^\infty \int_0^{2\pi} \int_0^{r_p} \left[(\lambda_s + 2G_s) \left\{ \left(\frac{du_p}{dz} \right)^2 \right. \right. \\ & \left. \left. - 2 \left(\frac{du_p}{dz} \right) \alpha_s \Delta T_s \right\} - 4\lambda_s \left(\frac{du_p}{dz} \right) \alpha_s \Delta T_s \right] r dr d\theta dz - Q_p u_p \Big|_{z=0} \end{aligned} \quad (2.6)$$

where, A_p and α_p are the cross-sectional area and coefficient of thermal expansion of the pile.

2.4.4 Minimization of the strain energy of the Pile-Soil Continuum

The variational operator is used on equation (2.6) to minimize the strain energy of the pile-soil continuum to obtain the following form,

$$\begin{aligned}
\delta\pi = & E_p A_p \int_0^{L_p} \left(\frac{du_p}{dz} - \alpha_p \Delta T_p \right) \delta \left(\frac{du_p}{dz} \right) dz + 2\pi \int_0^\infty \int_{r_p}^\infty \left[(\lambda_s + 2G_s) \left\{ \phi^2 \frac{du_p}{dz} \delta \left(\frac{du_p}{dz} \right) \right. \right. \\
& + \left. \left(\frac{du_p}{dz} \right)^2 \phi \delta \phi \right\} + (\lambda_s - 2G_s) \alpha_s \Delta T_s \left\{ \delta \left(\frac{du_p}{dz} \right) \phi + \frac{du_p}{dz} \delta \phi \right\} + G_s \left\{ \left(\frac{d\phi}{dr} \right)^2 u_p \delta u_p \right. \\
& + \left. u_p^2 \frac{d\phi}{dr} \delta \left(\frac{d\phi}{dr} \right) \right\} \Big] r dr dz + 2\pi \int_{L_p}^\infty \int_0^{r_p} \left[(\lambda_s + 2G_s) \frac{du_p}{dz} \delta \left(\frac{du_p}{dz} \right) \right. \\
& + \left. (\lambda_s - 2G_s) \alpha_s \Delta T_s \delta \left(\frac{du_p}{dz} \right) \right] r dr dz - Q_p \delta u_p \Big|_{z=0}
\end{aligned} \tag{2.7}$$

Variation of temperature change in pile is not considered because of the assumption made for pile temperature being same all through its length and cross-section. The temperature change in soil is also known to priori and therefore, the variation of temperature change in soil is also zero. The changes in temperature in soil at different radial and vertical distances from the pile are calculated following the method suggested by Zeng et al., (2002) for finite line heat source where the pile is considered as a finite length line heat source (given in section 2.4.5).

Performing integration by parts and incorporating the soil layering as shown in Figure 2-1 with the suffix i denoting the i^{th} layer ($i = 1, 2, \dots, n$), equation (2.7) can be expressed as,

$$\begin{aligned}
\delta\pi = & E_p A_p \sum_{i=1}^{n-1} \left[\left(\frac{du_p^{(i)}}{dz} - \alpha_p \Delta T_p \right) \delta u_p^{(i)} \right]_{H_{i-1}}^{H_i} - \int_{H_{i-1}}^{H_i} \frac{d^2 u_p^{(i)}}{dz^2} \delta u_p^{(i)} dz \\
& + \sum_{i=1}^n \left[2\pi (\lambda_{si} + 2G_{si}) \left(\int_{r_p}^{\infty} \phi^2 r dr \right) \left\{ \frac{du_p^{(i)}}{dz} \delta u_p^{(i)} \right\}_{H_{i-1}}^{H_i} - \int_{H_{i-1}}^{H_i} \frac{d^2 u_p^{(i)}}{dz^2} \delta u_p^{(i)} dz \right] \\
& + \sum_{i=1}^n \left\{ 2\pi (\lambda_{si} + 2G_{si}) \int_{H_{i-1}}^{H_i} \left(\frac{du_p^{(i)}}{dz} \right)^2 dz \right\} \int_{r_p}^{\infty} \phi r \delta \phi dr \\
& + \sum_{i=1}^n \left[2\pi (\lambda_{si} - 2G_{si}) \alpha_{si} \left\{ \left(\int_{r_p}^{\infty} \phi r \Delta T_s dr \right) \delta u_p^{(i)} \right\}_{H_{i-1}}^{H_i} - \int_{H_{i-1}}^{H_i} \frac{d}{dz} \left(\int_{r_p}^{\infty} \phi r \Delta T_s dr \right) \delta u_p^{(i)} dz \right] \\
& + \int_{r_p}^{\infty} \sum_{i=1}^n \left\{ 2\pi (\lambda_{si} - 2G_{si}) \alpha_{si} \left(\int_{H_{i-1}}^{H_i} \frac{du_p^{(i)}}{dz} \Delta T_s dz \right) \right\} r \delta \phi dr \\
& + \sum_{i=1}^n \left\{ 2\pi G_{si} \int_{r_p}^{\infty} \left(\frac{d\phi}{dr} \right)^2 r dr \right\} \left(\int_{H_{i-1}}^{H_i} u_p^{(i)} \delta u_p^{(i)} dz \right) \\
& + \sum_{i=1}^n \left[2\pi G_{si} \int_{H_{i-1}}^{H_i} \{u_p^{(i)}\}^2 dz \right] \left\{ r \frac{d\phi}{dr} \delta \phi \right\}_{r_p}^{\infty} - \int_{r_p}^{\infty} \left(\frac{d\phi}{dr} + r \frac{d^2 \phi}{dr^2} \right) \delta \phi dr \\
& + 2\pi (\lambda_{sn} + 2G_{sn}) \left\{ \frac{du_p^{(n)}}{dz} \delta u_p^{(n)} \right\}_{L_p}^{\infty} - \int_{L_p}^{\infty} \frac{d^2 u_p^{(n)}}{dz^2} \delta u_p^{(n)} dz \\
& + 2\pi (\lambda_{sn} - 2G_{sn}) \alpha_{sn} \left[\left(\int_{r_p}^{\infty} r \Delta T_s dr \right) \delta u_p^{(n)} \right]_{L_p}^{\infty} - \int_{L_p}^{\infty} \frac{d}{dz} \left(\int_{r_p}^{\infty} r \Delta T_s dr \right) \delta u_p^{(n)} dz - Q_p \delta u_p^{(1)} \Big|_{z=0}
\end{aligned} \tag{2.8}$$

where $u_p^{(i)}$ is the function $u_p(z)$ within the i^{th} layer; λ_{si} and G_{si} are the Lamé's constants for the i^{th} layer. Equation (2.8) takes the form $\int F_1(u_p) \delta u_p dz + \int F_2(\phi) \delta \phi dr = 0$ where δu_p and $\delta \phi$ are the first variation; $F_1(\bullet)$ and $F_2(\bullet)$ are the differentials of u_p and ϕ respectively. Considering the variation of u_p and ϕ separately governing equations and boundary conditions for pile and soil responses are obtained.

2.4.5 Temperature change in soil

The temperature change at any point $X(r, z)$ (Figure 2-2) in soil ΔT_s is calculated as per the recommendation by Zeng et al., (2002) for finite line source of heat and is given by

$$\Delta T_s = \frac{q_l}{4\kappa_s \pi} \left\{ \frac{\operatorname{erfc} \left(\frac{\sqrt{r^2 + (z-h)^2}}{2\sqrt{a_s \tau}} \right)}{\sqrt{r^2 + (z-h)^2}} - \frac{\operatorname{erfc} \left(\frac{\sqrt{r^2 + (z+h)^2}}{2\sqrt{a_s \tau}} \right)}{\sqrt{r^2 + (z+h)^2}} \right\} dh \quad (2.9)$$

where q_l is the heating rate per length of the line source, κ_s is the thermal conductivity of soil, a_s is the thermal diffusivity of soil, h is the integrating parameter, and τ is the time for which the heat flow occurs. In the present analysis, both single-layered and multi-layered soils are considered for different cases in contrast with equation (2.9) which has been developed for homogeneous soil only. However, this assumption does not severely affect the mechanical response of the pile which is evident from section 2.8.

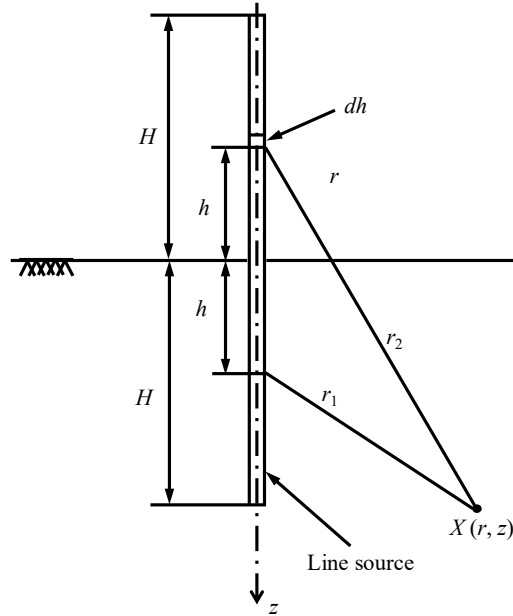


Figure 2-2 Finite line source system

2.4.6 Governing equations and boundary conditions for vertical displacement of pile

Considering the variation of u_p in equation (2.8) for domain $H_{i-1} \leq z \leq H_i$ where $H_i \leq L_p$, the differential equation for u_p for any layer i can be obtained as

$$\left(E_p A_p + 2t_i\right) \frac{d^2 u_p^{(i)}}{dz^2} - k_i u_p^{(i)} = F_i \quad (2.10)$$

where

$$t_i = \pi \left(\lambda_{si} + 2G_{si} \right) \left(\int_{r_p}^{\infty} \phi^2 r dr \right) \quad (2.10a)$$

$$k_i = 2\pi G_{si} \left\{ \int_{r_p}^{\infty} \left(\frac{d\phi}{dr} \right)^2 r dr \right\} \quad (2.10b)$$

$$F_i = -2\pi \left(\lambda_{si} - 2G_{si} \right) \alpha_{si} \left\{ \frac{d}{dz} \left(\int_{r_p}^{\infty} \phi r \Delta T_s dr \right) \right\} \quad (2.10c)$$

In equation (2.10), the terms k_i and t_i represent the compressive and shear resistance of soil. The term in the right-hand side of equation (2.10) F_i represents a distributed force along the shaft developed because of the change in the temperature of the soil initiated by the change in the temperature of the pile.

The associated boundary conditions at the top of the pile ($z = H_1 = 0$) (obtained from the variational form at $z = 0$) are

$$u_p^{(1)} \Big|_{z=0} = \text{constant or,} \quad (2.11a)$$

$$\left(E_p A_p + 2t_1\right) \frac{du_p^{(1)}}{dz} = -Q_p + E_p A_p \alpha_p \Delta T_p + F_{z1} \quad (2.11b)$$

At the intermediate layer ($z = H_i = H_{i+1}$), the boundary conditions (obtained from the variational form) are

$$u_p^{(i)} \Big|_{z=H_i} = u_p^{(i+1)} \Big|_{z=H_i} \quad (2.11c)$$

$$\left\{ \left(E_p A_p + 2t_i \right) \frac{du_p^{(i)}}{dz} - F_{zi} \right\} \bigg|_{z=H_i} = \left\{ \left(E_p A_p + 2t_{i+1} \right) \frac{du_p^{(i+1)}}{dz} - F_{z(i+1)} \right\} \bigg|_{z=H_i} \quad (2.11d)$$

Boundary conditions at the tip of the pile ($z = H_{n-1} = L_p$) are

$$u_p^{(n-1)} \big|_{z=0} = u_p^{(n)} \big|_{z=0} = \text{constant} \quad (2.11e)$$

$$\left(E_p A_p + 2t_{n-1} \right) \frac{du_p^{(n-1)}}{dz} - F_{z(n-1)} = \left\{ (\lambda_{sn} + 2G_{sn}) A_p + 2t_n \right\} \frac{du_p^{(n)}}{dz} - F_{zn}' + E_p A_p \alpha_p \Delta T_p \quad (2.11f)$$

where

$$F_{zi} = -2\pi (\lambda_{si} - 2G_{si}) \alpha_{si} \left(\int_{r_p}^{\infty} \phi r \Delta T_s dr \right) \quad (2.11g)$$

$$F_{zn}' = -2\pi (\lambda_{sn} - 2G_{sn}) \alpha_{sn} \left(\int_{r_p}^{\infty} \phi r \Delta T_s dr + \int_0^{r_p} r \Delta T_s dr \right) \quad (2.11h)$$

The differential equation for the axial displacement of the soil column below the pile is obtained by considering the first variation of $u_p^{(n)}$ from equation (2.8) as

$$\left\{ (\lambda_{sn} + 2G_{sn}) A_p + 2t_n \right\} \frac{d^2 u_p^{(n)}}{dz^2} - k_n u_p^{(n)} = F_n' \quad (2.12)$$

with boundary condition $u_p^{(n)}|_{z=\infty} = 0$. F_n' in equation (2.12) is given by

$$F_n' = -2\pi (\lambda_{sn} - 2G_{sn}) \alpha_{sn} \left\{ \frac{d}{dz} \left(\int_{r_p}^{\infty} \phi r \Delta T_s dr + \int_0^{r_p} r \Delta T_s dr \right) \right\} \quad (2.12a)$$

2.4.7 Governing equations and boundary conditions for soil displacement shape functions

Considering the variation of ϕ in equation (2.8) for $r_p \leq r \leq \infty$, the differential equation for the soil displacement function is obtained as

$$\frac{d^2\phi}{dr^2} + \frac{1}{r} \frac{d\phi}{dr} - \gamma\phi = -\frac{F_r}{m_{s1}} \quad (2.13)$$

where

$$\gamma = \frac{m_{s2}}{m_{s1}} \quad (2.14a)$$

$$m_{s1} = \sum_{i=1}^n \left[G_{si} \int_{H_{i-1}}^{H_i} \left\{ u_p^{(i)} \right\}^2 dz \right] \quad (2.14b)$$

$$m_{s2} = \sum_{i=1}^n \left[(\lambda_{si} + 2G_{si}) \int_{H_{i-1}}^{H_i} \left\{ \frac{du_p^{(i)}}{dz} \right\}^2 dz \right] \quad (2.14c)$$

$$F_r = -\sum_{i=1}^n \left\{ (\lambda_{si} - 2G_{si}) \alpha_{si} \left(\int_{H_{i-1}}^{H_i} \frac{du_p^{(i)}}{dz} \Delta T_s dz \right) \right\} \quad (2.14d)$$

2.5 Solution

The equations, (2.10) and (2.12) along with the boundary conditions given by equations (2.11a)-(2.11f) for pile displacement and equation (2.13) for soil displacement shape function are solved using the finite difference (FD) method. The horizontal and vertical extent of soil is considered as $60D_p$ from the face of the pile and $2.5L_p$ from the bottom of the pile, respectively to minimize the effect of boundary on the analysis. These distances have been calculated by performing convergence study for different piles in different loading conditions. The soil domain in radial direction is divided into n_r number of nodes where the distances between the nodes are varied as distance from the face of the pile is increased to reduce the analysis time. Same procedure is done below the pile in z direction where the soil domain is divided into n_z number of nodes. This operation is done due to the fact that the response against the loading reduces as distance from the pile is increased. The total length of pile is divided into segments having equal length Δz . The discretized domain of pile-soil system is shown in Figure 2-3. The values of displacement along the pile length (u_p), and soil displacement shape function (ϕ) at each node are calculated by simultaneously solving equations (2.10)-(2.12), (2.11a)–(2.11f) and (2.13).

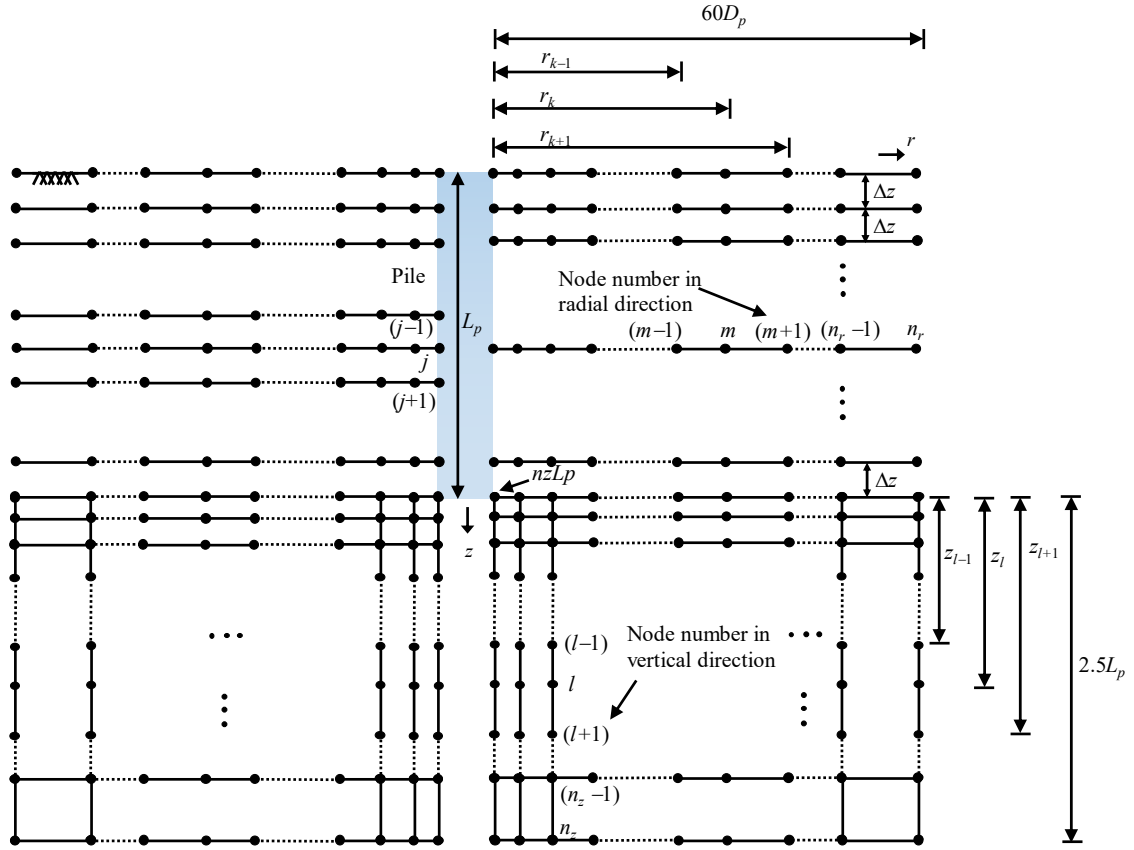


Figure 2-3 Discretization of soil domain for finite difference analysis

2.5.1 Finite difference (FD) discretization for vertical displacement of pile $u_p(z)$

Central FD method is used to discretize the governing differential equations (equations (2.10) and (2.12)) and boundary conditions (equations (2.11a)-(2.11f)) of the vertical displacement function. The discretized form of equation (2.10) for vertical displacement at any node along the pile length denoted as j can be written as

$$\left(E_p A_p + 2t_j\right) \left[\frac{u_p^{(j+1)} - 2u_p^{(j)} + u_p^{(j-1)}}{(\Delta z)^2} \right] - k_j u_p^{(j)} = F_j \quad (2.15)$$

where, $u_p^{(j+1)}$, $u_p^{(j)}$, and $u_p^{(j-1)}$ are the values of u_p at nodes j , $(j+1)$, and $(j-1)$, respectively; t_j , k_j , and F_j are the values of t , k , and F , respectively at the j -th node calculated using equations (2.10a)-(2.10c).

Applying the boundary conditions given by (2.11a) and (2.11f), equation (2.15) for node 1 and node $nzLp$ can be rewritten respectively as,

$$2(E_p A_p + 2t_1) \left[\frac{u_p^{(2)} - u_p^{(1)}}{(\Delta z)^2} \right] - k_1 u_p^{(1)} = F_1^{(1)} + \frac{2}{\Delta z} (-Q_p + E_p A_p \alpha_p \Delta T_p + F_{z1}^{(1)}) \quad (2.16a)$$

$$2(E_p A_p + 2t_n) \left[\frac{u_p^{(nzLp-1)} - u_p^{(nzLp)}}{(\Delta z)^2} \right] - k_n u_p^{(nzLp)} = F_n^{(nzLp)} - \frac{2(E_p A_p + 2t_n) E_p A_p \alpha_p \Delta T_p}{\Delta z \{ (E_p A_p + 2t_n) - A_p (\lambda_{sn} + 2G_{sn}) \}} \quad (2.16b)$$

The differential equation for the domain $L_p < z < \infty$ that describes the displacement of the soil column below the pile (equation (2.12)) can also be rewritten using central difference method for any node l as

$$\{(\lambda_{sl} + 2G_{sl}) A_p + 2t_l\} \left\{ \frac{2u_p^{(l+1)}}{(z_{l+1} - z_l)(z_{l+1} - z_{l-1}))} - \frac{2u_p^{(l)}}{(z_{l+1} - z_l)(z_l - z_{l-1}))} + \frac{2u_p^{(l-1)}}{(z_{l+1} - z_{l-1})(z_l - z_{l-1}))} \right\} - k_l u_p^{(l)} = F_n^{(l)} \quad (2.17)$$

where λ_{sl} , G_{sl} , t_l , k_l , and $F_n^{(l)}$ are the values of λ_s , G_s , t , k , and F_n at l -th node in the soil layer(s) below the pile.

Equations (2.15)-(2.17) can be expressed in the form $[K^{(u)}]_{(nz-1) \times (nz-1)} \{u\}_{(nz-1)} = \{f^{(u)}\}_{(nz-1)}$ which is given by

$$\begin{bmatrix} K_{1,1}^{(u)} & K_{1,2}^{(u)} & 0 & \dots & 0 & \dots & 0 \\ K_{2,1}^{(u)} & K_{2,2}^{(u)} & K_{2,3}^{(u)} & 0 & & & 0 \\ 0 & \ddots & \ddots & \ddots & 0 & \dots & 0 \\ \vdots & & 0 & K_{j,j-1}^{(u)} & K_{j,j}^{(u)} & K_{j,j+1}^{(u)} & 0 \\ 0 & \dots & & \ddots & \ddots & \ddots & \ddots \\ \vdots & & & 0 & K_{nzLp,(nzLp-1)}^{(u)} & K_{(nzLp-1),(nzLp-1)}^{(u)} & 0 \\ 0 & \dots & \dots & \ddots & \ddots & \ddots & \ddots \\ 0 & \dots & & \dots & 0 & K_{(nz-2),(nz-1)}^{(u)} & K_{(nz-1),(nz-1)}^{(u)} \end{bmatrix} \begin{bmatrix} u^{(1)} \\ u^{(2)} \\ \vdots \\ u^{(j)} \\ \vdots \\ u^{(nzLp)} \\ \vdots \\ u^{(nz)} \end{bmatrix} = \begin{bmatrix} f_1^{(u)} \\ f_2^{(u)} \\ \vdots \\ f_j^{(u)} \\ \vdots \\ f_{nzLp}^{(u)} \\ \vdots \\ f_{nz}^{(u)} \end{bmatrix}$$

$$(2.18)$$

where $[K^{(u)}]_{(nz-1) \times (nz-1)}$ is a tri-diagonal matrix comprises of the terms E_p , A_p , λ_{sl} , G_{sl} , k_j , and t_j ; $\{u\}$ is the unknown vector of vertical displacement; and $\{f^{(u)}\}$ is the vector comprises of Q_p , E_p , A_p , α_p , ΔT_p , and the integrals associated with ΔT_s . The non-zero elements of matrix $[K^{(u)}]_{(nz-1) \times (nz-1)}$ are given by, (j varies from 2 to $(nzLp-1)$, l varies from $(nzLp+1)$ to (n_z-2))

$$K_{j,j}^{(u)} = -\frac{2(E_p A_p + 2t_j)}{(\Delta z)^2} - k_j \quad (2.19a)$$

$$K_{j,j-1}^{(u)} = K_{j,j+1}^{(u)} = \frac{(E_p A_p + 2t_j)}{(\Delta z)^2} \quad (2.19b)$$

$$K_{1,1}^{(u)} = -\frac{2(E_p A_p + 2t_1)}{(\Delta z)^2} - k_1 \quad (2.19c)$$

$$K_{1,2}^{(u)} = \frac{2(E_p A_p + 2t_1)}{(\Delta z)^2} \quad (2.19d)$$

$$K_{nzLp,nzLp}^{(u)} = -\frac{2(E_p A_p + 2t_n)}{(\Delta z)^2} - k_n \quad (2.19e)$$

$$K_{nzLp,nzLp-1}^{(u)} = \frac{2(E_p A_p + 2t_n)}{(\Delta z)^2} \quad (2.19f)$$

$$K_{l,l}^{(u)} = -\frac{2\{(\lambda_{sl} + 2G_{sl})A_p + 2t_l\}}{(z_{l+1} - z_l)(z_l - z_{l-1})} - k_l \quad (2.19g)$$

$$K_{l,l-1}^{(u)} = \frac{2\{(\lambda_{sl} + 2G_{sl})A_p + 2t_l\}}{(z_{l+1} - z_{l-1})(z_l - z_{l-1})} \quad (2.19h)$$

$$K_{l,l+1}^{(u)} = \frac{2\{(\lambda_{sl} + 2G_{sl})A_p + 2t_l\}}{(z_{l+1} - z_l)(z_{l+1} - z_{l-1})} \quad (2.19i)$$

$$K_{n_z-1, n_z-1}^{(u)} = -\frac{2\left\{\left(\lambda_{s(n_z-1)} + 2G_{s(n_z-1)}\right)A_p + 2t_{(n_z-1)}\right\}}{\left(z_{n_z} - z_{n_z-1}\right)\left(z_{n_z-1} - z_{n_z-2}\right)} - k_{(n_z-1)} \quad (2.19j)$$

$$K_{n_z-1, n_z-2}^{(u)} = \frac{2\left\{\left(\lambda_{s(n_z-1)} + 2G_{s(n_z-1)}\right)A_p + 2t_{(n_z-1)}\right\}}{\left(z_{n_z} - z_{n_z-2}\right)\left(z_{n_z-1} - z_{n_z-1}\right)} \quad (2.19k)$$

The elements of vector $\{F^{(u)}\}_{(nz-1)}$ are given by,

$$f_{j,1}^{(u)} = F_j \quad (2.20a)$$

$$f_{1,1}^{(u)} = F_1^{(1)} + \frac{2}{\Delta z} \left(-Q_p + E_p A_p \alpha_p \Delta T_p + F_{z1}^{(1)} \right) \quad (2.20b)$$

$$f_{nzLp,1}^{(u)} = F_n^{(nzLp)} - \frac{2\left(E_p A_p + 2t_n\right)E_p A_p \alpha_p \Delta T_p}{\Delta z \left\{ \left(E_p A_p + 2t_n\right) - A_p \left(\lambda_{sn} + 2G_{sn}\right) \right\}} \quad (2.20c)$$

$$f_{l,1}^{(u)} = F_n^{(l)} \quad (2.20d)$$

$$f_{n_z,1}^{(u)} = F_n^{(n_z)} \quad (2.20e)$$

2.5.2 Finite difference (FD) discretization for soil displacement shape function ϕ

Equation (2.13) which describes the shape function for soil is also solved using finite difference method. Applying the central difference technique for any intermediate node m , the discretized form of equations (2.13) can be written as

$$\left\{ \frac{r_{m+1} + r_m}{r_m (r_{m+1} - r_m) (r_{m+1} - r_{m-1})} \right\} \phi^{(m+1)} + \left\{ -\frac{2}{(r_{m+1} - r_m) (r_m - r_{m-1})} - \gamma \right\} \phi^{(m)} + \left\{ \frac{r_m + r_{m-1}}{r_m (r_{m+1} - r_m) (r_{m+1} - r_{m-1})} \right\} \phi^{(m-1)} = \frac{F_r^{(m)}}{m_{s1}} \quad (2.21)$$

Equation (2.21) can be expressed in the form $[K^{(\phi)}]\{\phi\} = \{F^{(\phi)}\}$ where $[K^{(\phi)}]_{(nr-2) \times (nr-2)}$ is a tri-diagonal matrix; $\{\phi\}$ is the unknown vector of soil displacement shape function; and $\{F^{(\phi)}\}$ is the vector

comprises of terms associated with ΔT_s . The non-zero elements of $[K^{(\phi)}]_{(nr-2) \times (nr-2)}$ are given by (m varies from 3 to (n_r-3))

$$K_{m,m}^{(\phi)} = -\frac{2}{(r_{m+1}-r_m)(r_m-r_{m-1})} - \gamma \quad (2.22a)$$

$$K_{m,m-1}^{(\phi)} = \frac{r_m + r_{m-1}}{(r_{m+1}-r_m)(r_{m+1}-r_{m-1})} \quad (2.22b)$$

$$K_{m,m+1}^{(\phi)} = \frac{r_{m+1} + r_m}{r_m(r_{m+1}-r_m)(r_{m+1}-r_{m-1})} \quad (2.22c)$$

$$K_{1,1}^{(\phi)} = -\frac{2}{(r_3-r_2)(r_2-r_1)} - \gamma \quad (2.22d)$$

$$K_{1,2}^{(\phi)} = \frac{r_3 + r_2}{r_2(r_3-r_2)(r_3-r_1)} \quad (2.22e)$$

$$K_{n_r-2,n_r-2}^{(\phi)} = -\frac{2}{(r_{n_r}-r_{n_r-1})(r_{n_r-1}-r_{n_r-2})} - \gamma \quad (2.22f)$$

$$K_{n_r-2,n_r-3}^{(\phi)} = \frac{r_{n_r-1} + r_{n_r-2}}{(r_{n_r}-r_{n_r-1})(r_{n_r}-r_{n_r-2})} \quad (2.22g)$$

The non-zero elements of $[f^{(\phi)}]_{(nr-2) \times 1}$ are given by

$$f_{m,1}^{(\phi)} = \frac{F_r^{(m)}}{m_{s1}} \quad (2.23a)$$

$$f_{1,1}^{(\phi)} = \frac{F_r^{(1)}}{m_{s1}} - \frac{r_2 + r_1}{r_2(r_3-r_2)(r_3-r_1)} \quad (2.23b)$$

$$f_{n_r-2,1}^{(\phi)} = \frac{F_r^{(n_r-1)}}{m_{s1}} \quad (2.23c)$$

2.6 Solution algorithm

The input data required to perform the analysis are pile radius r_p , pile length L_p , elastic modulus E_p , coefficient of thermal expansion α_p , thermal conductivity κ_p of the pile material; details of soil stratification n , H_1 , H_2 , ... H_{n-1} , soil elastic constants λ_{si} and G_{si} , coefficient of thermal expansion of soil α_{si} , thermal conductivity of soil κ_s ; applied mechanical load Q_p and temperature change ΔT_p . It can be observed from equations (2.10), (2.10a)-(2.10c), (2.11a)-(2.11f), (2.12), (2.12a), (2.13), and (2.14a)-(2.14d) that the equations for pile displacement (u_p) and the soil displacement shape function (ϕ) are interdependent on each other. Thus, the equations (2.10)-(2.12) and (2.13) are solved simultaneously following an iterative algorithm.

The pile and the soil are discretized as shown in Figure 2-3. The discretization along the vertical and the radial direction are made by performing convergence study considering different node-to-node distances. The distance between nodes near the pile-soil interface is considered to be small. As the distance from the pile-soil interface increases in both the directions, the node-to-node distance increases and these distances are higher at the farthest horizontal and vertical boundaries. The change in soil temperature ΔT_s is calculated at each node in the soil domain using equation (2.9). The heat flow per unit length of pile q_l is calculated by multiplying the thermal conductivity of pile material κ_p with the thermal gradient caused by the temperature change in pile ΔT_p ($q_l = \kappa_p \times (\Delta T_p / r_p) \times (2\pi r_p)$). At the beginning of the algorithm, the vector $\{\phi\}_{nr \times 1}$ is assumed where ϕ decreases linearly from 1 at $r = r_p$ to 0 at $r = \infty$. Next, With the values of assumed ϕ , its numerical derivative, and the calculated ΔT_s at each node in the soil, the parameters k_j , t_j and F_j are calculated using equations (2.10a), (2.10b), and (2.10c), respectively with which the displacement vector $\{u\}_{(nz-1)}$ is calculated from equation (2.18). The calculated value of $\{u\}_{(nz-1)}$ is used to calculate γ using equations (2.14a). Subsequently, the newly calculated γ and $\{u\}_{(nz-1)}$ are used to iteratively recalculate ϕ . With the newly calculated values of ϕ , the parameters k_i , t_i and F_i ; $\{u\}_{(nz-1)}$; and γ are sequentially recalculated. A convergence criterion on γ is set as $\left| \gamma^{(\text{previous})} - \gamma^{(\text{new})} \right| / \gamma^{(\text{previous})} \leq 0.001$ where $\gamma^{(\text{previous})}$ and $\gamma^{(\text{new})}$ are the values of γ at the previous and current iterations, respectively. Iterations on γ are continued until convergence is reached at which point the calculated $\{u\}_{(nz-1)}$ and $\{\phi\}_{nr \times 1}$ are reported as the final values of pile displacement and soil displacement function, respectively.

2.7 Modification in soil modulus

The response of the pile embedded in multilayered soil as shown in Figure 2-4 is analyzed under thermal load and the vertical displacement profiles obtained from the present analysis and the FEA are shown in Figure 2-5. It is observed from Figure 2-5 that, the present method underestimates the displacement. The difference between the head displacements obtained from the present method and the FEA is 20%.

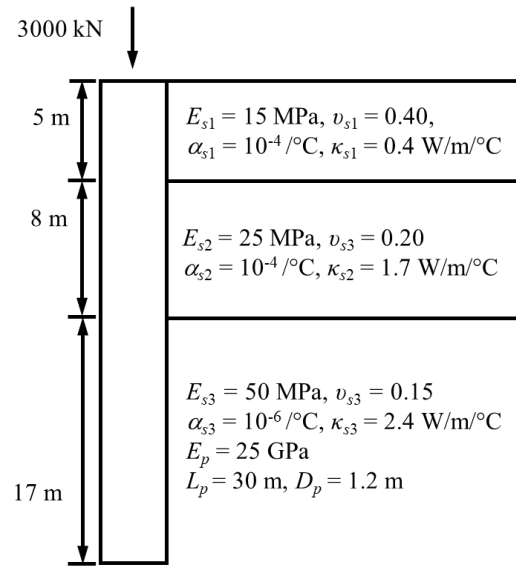


Figure 2-4 Pile with soil layering and properties

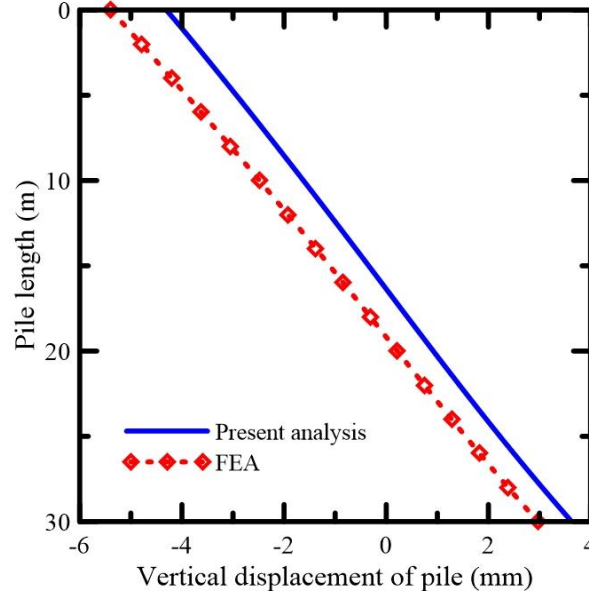


Figure 2-5 Vertical displacement profiles of the pile obtained from the present analysis and FEA under thermal load ($\Delta T_p = +30^\circ \text{C}$)

The soil displacement given by equation (2.13) provides a stiffer response for the pile due to the fact that the displacement in the r direction is restrained in the formulation which is not entirely true when compared to the practical behavior. Similar observations have been made by other researchers and recommendations have been provided to improve the accuracy of this method (Guo & Lee, 2001). In this analysis, a modification in soil modulus is made by changing $\lambda_{si} = 0$ and $G_{si} = 0.75G_{si}(1+1.25\nu_{si}^2)$ suggested by Seo et al., (2009). ν_{si} is the Poisson's ratio of soil in i -th layer. The same pile described in Figure 2-4 is analyzed with the modified soil modulus and the result from the present analysis improves as evident from Figure 2-6 with the difference in head displacement obtained from the present analysis and the FEA is 4%. In Figure 2-6 the vertical displacement profile of the pile without the modified soil modulus is also shown. Therefore, the modified shear modulus of soil is used further in the present analysis.

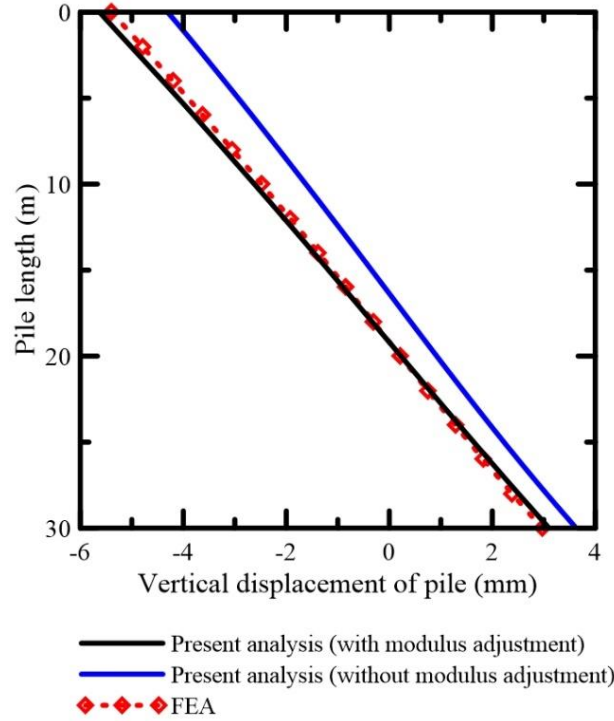


Figure 2-6 Vertical displacement profiles of the pile obtained from the present analysis (with modified soil modulus) and FEA under thermal load ($\Delta T_p = +30^\circ \text{C}$)

2.8 Results

2.8.1 Validation with experimental study

The accuracy of the present analysis is verified with the experimental results available in the literature by Goode & McCartney, (2015) and Laloui et al., (2006). The details of pile dimensions, loading conditions, and material properties used in the analysis are calculated with the data provided by the authors.

2.8.1.1 Goode and McCartney (2015)

The first validation for the present method is done by comparing the displacement profiles of an energy pile obtained from the present analysis with the experimental results provided by Goode & McCartney, (2015). The details of pile dimensions, properties of the pile material, and soil are given in Table 2-1.

Table 2-1- Pile dimension and material properties of soil and pile material (Goode & McCartney, 2015)

	Young's modulus, E_p or E_s (MPa)	Poisson's ratio, ν_p or ν_s	Coefficient of thermal expansion, α_p or α_s ($1/^\circ\text{C}$)	Thermal conductivity, κ_p or κ_s (W/m/ $^\circ\text{C}$)	Thermal diffusivity, a_s (m^2/s)	
Pile	33000	0.2	1.6×10^{-5}	2.100	-	$D_p = 1.50 \text{ m}$ $L_p = 8.20 \text{ m}$
Layer -1	78	0.3	1.7×10^{-5}	0.265	7.5×10^{-7}	$H_1 \rightarrow \infty$

The vertical displacement profile obtained from the present analysis for a temperature increase of 18°C is plotted in Figure 2-7 against those obtained by Goode & McCartney, (2015) in their experiment and the response from the present analysis shows a good match with the test. The vertical displacement profile of the same pile obtained from the numerical analysis by Saggu & Chakraborty, (2015) is also plotted in Figure 2-7. The difference between the head displacements obtained from the present analysis and the test is 4.6%.

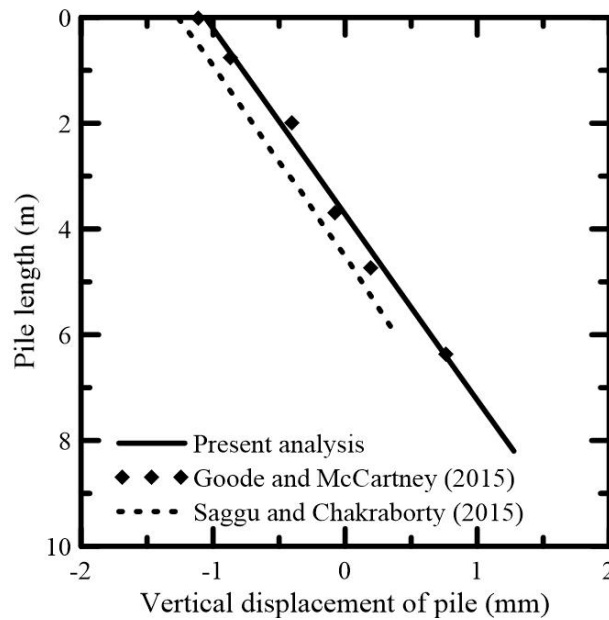


Figure 2-7 Vertical displacement profiles of the pile obtained from the present analysis, numerical analysis, and field test under thermal load ($\Delta T_p = +18^\circ\text{C}$)

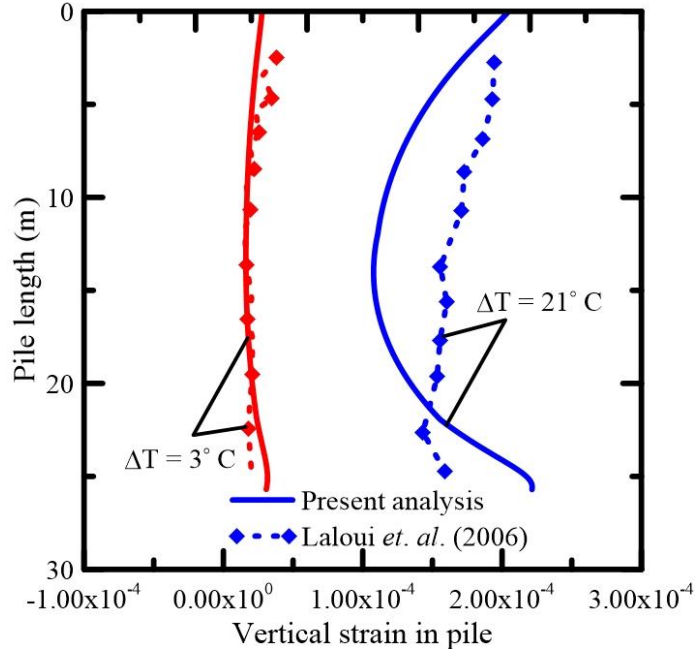
2.8.1.2 Laloui *et. al.* (2006)

The second validation for the present method is done by comparing the strain and stress profiles of an energy pile obtained from the present analysis with the experimental results provided by Laloui *et al.*, (2006). The details of pile dimensions, properties of the pile material, and soil are given Table 2-2.

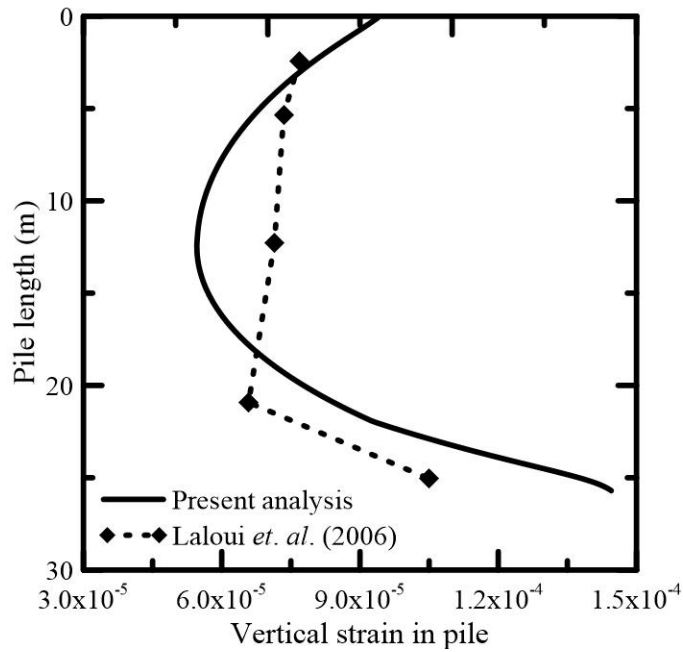
The vertical strain and stress profiles along the length of the pile obtained from the present analysis for different loading conditions are plotted against those obtained by Laloui *et al.*, (2006) in Figure 2-8 and Figure 2-9 and it is observed that results from the current method exhibit good agreement with the field results in terms of axial stress and strain distribution over the pile length. In Figure 2-8(a) the vertical strain profile along the pile length under thermal loads $\Delta T_p = +21^\circ\text{C}$ and $+3^\circ\text{C}$ are plotted against those obtained from the field test by Laloui *et al.*, (2006). Vertical strain profile of the pile obtained from the present analysis and the field test under combined loading ($Q_p = 1000 \text{ kN}$ and $\Delta T_p = +13.4^\circ\text{C}$) are shown in Figure 2-8(b). Both the figures show good agreement between the results obtained from the present analysis and field data. For $\Delta T_p = +21^\circ\text{C}$, there is a difference in the strain values at certain parts of the pile.

Table 2-2- Material properties of soil and pile material derived from Laloui *et al.*, (2006)

	Density (kg/m^3)	Young's modulus, E_s (MPa)	Poisson's ratio, ν_s	Coefficient of thermal expansion, α_s ($1/^\circ\text{C}$)	Thermal conductivity, κ_s (W/m/ $^\circ\text{C}$)	Specific heat, C_s (J/ $^\circ\text{C}$)	
Pile	2500	29200	0.200	10^{-5}	2.1	800.00	$D_p = 1.00 \text{ m}$ $L_p = 25.80 \text{ m}$
Layer -1	2000	259	0.146	10^{-5}	1.8	1200.00	$H_1 = 5.50 \text{ m}$
Layer -2	1950	259	0.146	10^{-5}	1.8	1230.77	$H_2 = 12.00 \text{ m}$
Layer -3	2000	451	0.200	10^{-4}	1.8	1200.00	$H_3 = 21.80 \text{ m}$
Layer -4	2200	634	0.200	10^{-4}	1.8	1090.90	$H_4 \rightarrow \infty$



(a)



(b)

Figure 2-8 Axial strain profiles along the pile for a) thermal loading, and b) combined (mechanical and thermal) loading obtained from the present analysis and field data

The vertical stress profiles under mechanical load ($Q = 1300$ kN), thermal load ($\Delta T_p = +15^\circ\text{C}$), and combined load (mechanical + thermal load) are shown in Figure 2-9. The stress profiles match well with those obtained from the field test except for the stress values at the pile tip under thermal and combined loading. The difference in maximum stress values obtained from the present analysis and field test under combined loading is less than 8%. The present analysis can produce fairly accurate results as evident from the displacement, strain, and stress profiles shown in Figure 2-7 to Figure 2-9.

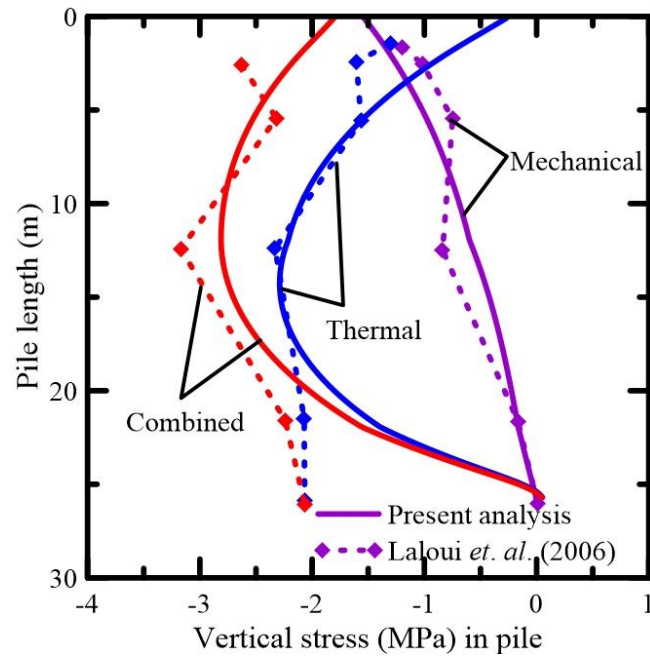


Figure 2-9 Axial stress profile along the pile for mechanical ($Q = 1300$ kN), thermal ($\Delta T = +15^\circ\text{C}$), and combined loading obtained from present analysis and field data

2.8.2 Verification with numerical analysis

The accuracy of the present analysis is verified with the results from FEA conducted in Abaqus. A comparison of the results obtained for a single pile in elastic soil medium under mechanical, thermal, and combined (mechanical + thermal) load in terms of displacement, and stress distribution along the length of the pile from present analysis and axisymmetric FEA in Abaqus is shown in Figure 2-11 and Figure 2-12. The input for the analysis is given in Figure 2-4.

It is observed from Figure 2-11 and Figure 2-12 that the displacement, and stress profiles along the length of the pile obtained using the present method produce very good match with the results from

FEA. FEA is carried out in Abaqus using axisymmetric linear coupled temperature-displacement (CAX4T) elements. The soil domain in the horizontal and vertical direction is taken up to the extent of $60D_p$ from the pile-soil interface and $2.5L_p$ from the pile base respectively to diminish the boundary effect. The horizontal and vertical boundary are assigned the fixed boundary condition. The finite element model is shown in Figure 2-10. The pile and the soil are constructed as different parts. The interface of the pile and the soil are tied (using the constraint tie in the interaction module) to ensure no slippage or separation to keep parity with the assumption of the present framework. Edge seeding along the horizontal and vertical directions is done from the pile-soil interface for meshing. The thermal interaction at the pile-soil interface is defined by the thermal conductivity of the soil. A temperature boundary condition is applied to the pile with a ΔT_p of $+30^\circ\text{C}$, and -10°C , respectively for two different cases.

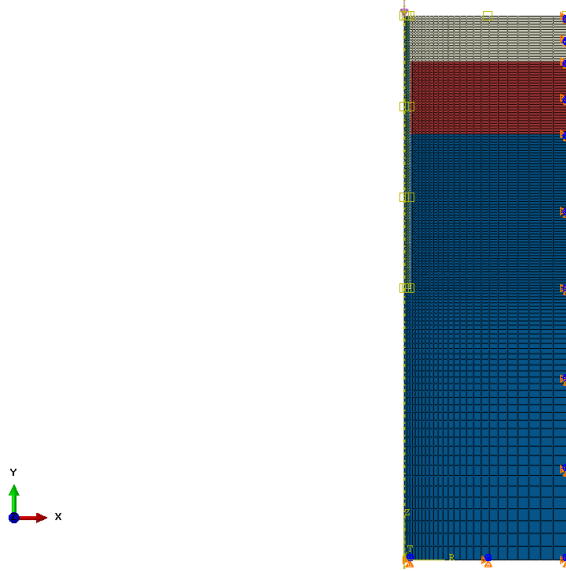
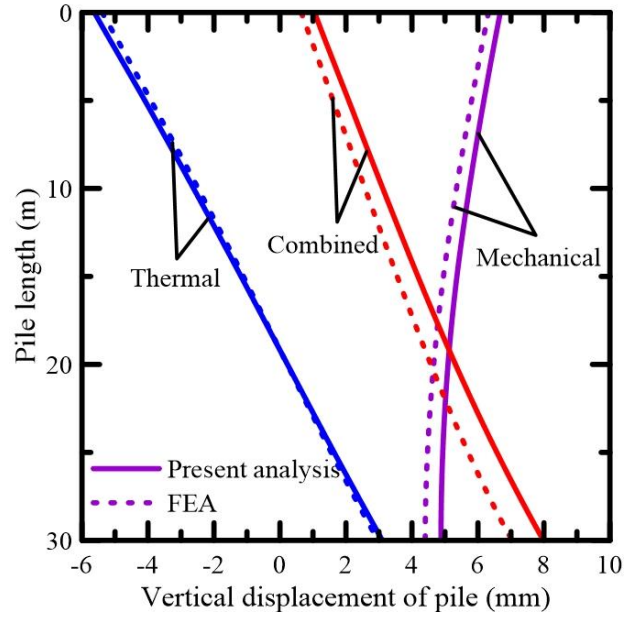
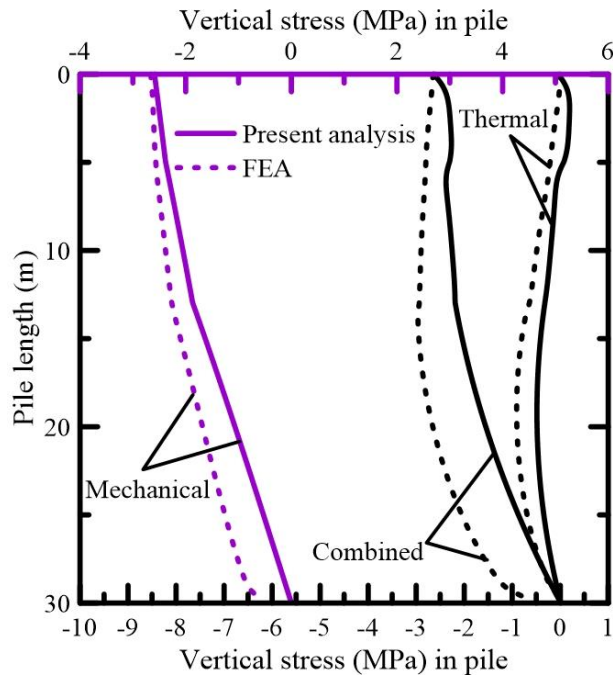


Figure 2-10 Finite element model of the energy pile

Figure 2-11(a)-(b) show the response of the pile under mechanical load and a temperature increase of 30°C whereas Figure 2-12(a)-(b) show the response of the same pile under the same mechanical load but a temperature reduction of 10°C . Displacement in the horizontal direction and vertical directions are restrained at the far end vertical, and bottom boundaries, respectively. Heat flow is considered zero at all the boundaries. The vertical and horizontal boundaries of soil far from the pile are taken at $60D_p$ from the face of the pile and $2.5L_p$ from the toe of the pile in the present analysis where the value of shape function (ϕ), and vertical displacement function (u_p) are zero.



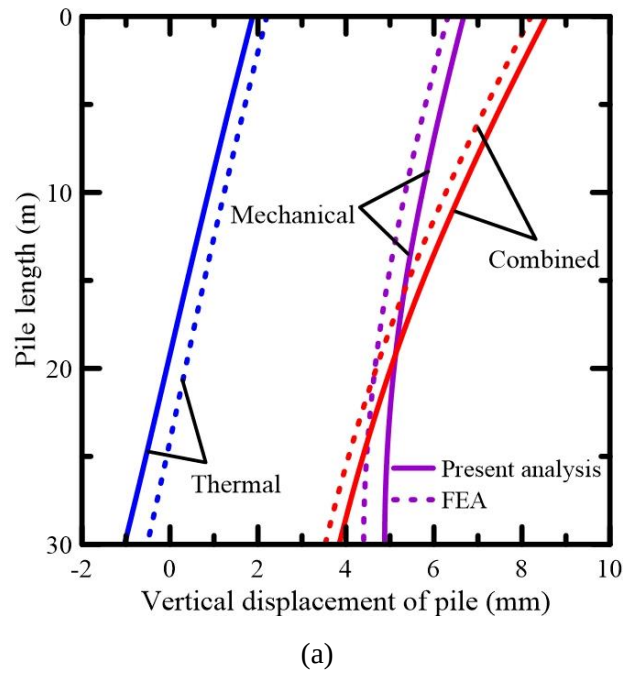
(a)

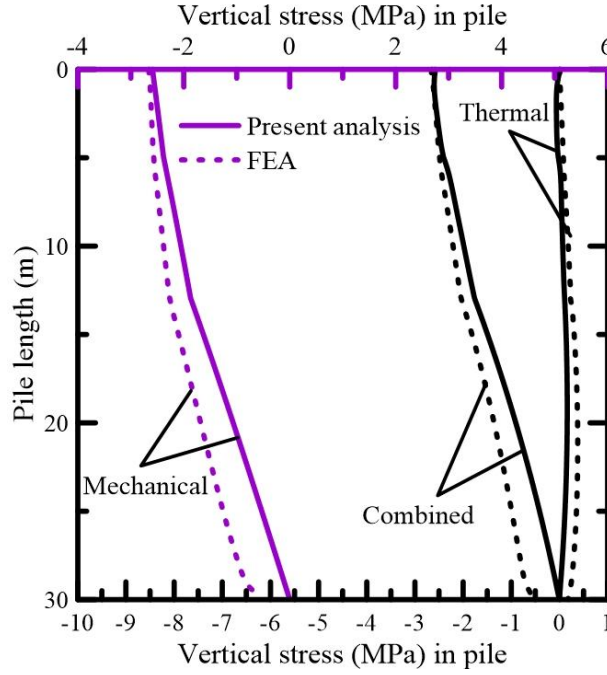


(b)

Figure 2-11 Comparison of results for pile between present analysis and FEA under mechanical, thermal ($\Delta T_p = +30^\circ\text{C}$), and combined loadings in terms of a) Displacement, and b) stress along the length of the pile

From Figure 2-11(a), the head displacements obtained from the present analysis and the FEA for mechanical, thermal, and combined loadings are 6.67 mm, −5.62 mm, and 1.05 mm; and 6.32 mm, −5.39 mm, and 0.92 mm, respectively with corresponding differences between the head displacement are 5%, 4%, and 12%, respectively. The maximum vertical (axial) stress at the head of the pile under mechanical and combined loading (Figure 2-11(b)) are obtained from the present analysis are 2.60 MPa, 2.621 MPa; and 2.649 MPa, 2.651 MPa, respectively. The differences in the stress values between both analyses are 2%, and 1%, respectively. The present analysis underestimates the axial stress at the bottom and at the mid-span of the pile under axial, and combined loadings, respectively. Also, the temperature change considered in the verification is higher compared to the normal temperature increase in an energy pile.





(b)

Figure 2-12 Comparison of results for pile between present analysis and FEA under mechanical, thermal ($\Delta T_p = -10^\circ \text{C}$), and combined loadings in terms of a) Displacement, and b) stress along the length of the pile

From Figure 2-12(a), the head displacements obtained from the present analysis and the FEA for thermal, and combined loadings are 1.98 mm and 8.55 mm; and 2.10 mm and 8.22 mm, respectively with corresponding differences between the head displacements are 6% and 4%, respectively. The maximum vertical (axial) stress at the head of the pile under combined loading (Figure 2-12(b)) are obtained from the present analysis are 2.58 MPa and 2.649 MPa, respectively. The difference in the stress values between both analyses is 3%. The time required to run the present analysis is nearly eight times less than that required for the FEA of the same energy pile in a computer with Intel(R) Core(TM) i7-9700 CPU and 32 GB RAM. Apart from that, results can be easily obtained from the present analysis by just inputting the pile dimensions, soil layering, soil properties, and applied loads in the algorithm in comparison to FEA where the proper model preparation and mesh generation are time-consuming.

2.9 Summary

A semi-analytical framework for an energy pile embedded in layered elastic soil and subjected to an axial mechanical and thermal load is presented. This model considers the pile and the soil as a continuum and allows no slippage or separation between the pile and the soil. The vertical displacement of the pile is considered in the analysis and the soil displacement field is defined as products of separable functions. The soil displacement field is defined by ensuring the continuity of displacements and compatibility of strains between the cylindrical pile and the surrounding soil. The change in soil temperature caused by the temperature change in the pile is calculated by assuming the pile as a finite length line heat source.

The principle of minimum potential energy and calculus of variations are used to derive the differential equations that describe the pile and soil displacements. The strains developed in the pile-soil continuum are due to the application of axial mechanical load and temperature change. Stresses in the pile-soil continuum caused by the temperature change are calculated from the restrained thermal strain. Differential equations describing the vertical pile and soil column displacement u_p and soil displacement shape function ϕ are obtained and solved using the one-dimensional finite difference method. The differential equations are coupled and therefore an iterative solution algorithm is used.

The accuracy of the present analysis is verified against pile responses (vertical displacement, strain, and stress) obtained using FEA and field tests. The present analysis produces quite accurate results for the pile that is subjected to axial mechanical load and undergoes temperature change. A modulus adjustment is done in the analysis as suggested in the available literature to take into account the restrained degree of freedom of soil displacements. The present method can produce an accurate response of energy piles in multilayered elastic soil under different loads in a fast and efficient manner (approximately 10 times faster than FEA in the same computer). The constitutive response of soil under high load can become elastoplastic and the change in the pile-soil interaction caused by the elastoplastic behavior of soil can affect the response of the pile. Therefore, the effect of the elastoplastic response of soil on the axial response of an energy pile is observed in the next chapter.

Chapter 3

Analysis of Energy Pile in Elastoplastic Soil Continuum

3.1 Introduction

In this chapter, a semi-analytical framework for analyzing an energy pile embedded in a multilayered soil deposit and subjected to both axial and thermal loads is developed by considering the elastoplastic behavior of soil. The framework is developed by satisfying the laws of thermodynamics. The governing differential equations and boundary conditions associated with the response of the pile under said loadings are derived by minimizing the internal energy (including the dissipation) of the pile-soil continuum using variational principles. The plastic deformation of the soil is taken into account through the dissipation function. The behavior of pile material is considered to be linear elastic. The results obtained from the analysis are compared with the results available in the literature (Goode & McCartney, 2015; Laloui et al., 2006; Saggu & Chakraborty, 2017) and the results obtained from equivalent FEA conducted by the author. In this chapter, the soil is modeled as an elastic perfectly plastic material that follows the Drucker-Prager constitutive relationship.

3.2 Motivation

Traditional elastic analysis which is done in Chapter 2 provides the response of an energy pile under both mechanical and thermal loads quite well. In that analysis, an adjustment in the soil modulus has been made to obtain the exact response of the energy pile. An axially loaded pile under a large load can push the soil beyond its elastic limit. Therefore, modeling the soil as an elastic material can significantly underestimate the settlement of an axially loaded pile, leading to an unoptimized design of the pile. An elastoplastic analysis, which considers both elastic and plastic behavior, offers a more accurate and comprehensive understanding of pile performance under realistic loading conditions. The interaction between a pile and the surrounding soil is non-linear, especially under high loads.

Further, it is evident from vertical stress profiles (shown in Figure 3-1) along the length of the pile obtained from the present analysis for different loading conditions (mechanical, thermal, and combined) which are plotted against those obtained by Laloui et al., (2006). In Figure 3-1, stress profiles obtained from the present analysis with the soil modeled as an elastic as well as an elastoplastic (modeled by the Drucker-Prager soil constitutive model) material are presented. The results with the elastic soil model do not include the modulus adjustment. It is observed from Figure 3-1 that, the

maximum vertical stress developed in the pile under different loading conditions is underestimated in the elastic analysis when compared with the experimental results. The results from the elastoplastic soil model provide more accurate values of maximum stress under different loading conditions (vertical stress is maximum at the head of the pile under mechanical load and around the mid-span of the pile for thermal and combined loads). Therefore, the elastoplastic modeling of soil in the present framework is required. The present framework development with the elastoplastic behavior of soil is described in the following sections. The additional soil parameters that are required to model the soil using the Drucker-Prager soil constitutive model for the pile response (shown in Figure 3-1) are given in Table 3-1.

Table 3-1- Soil properties given by Laloui et al., (2006)

	Cohesion, c_s (kPa)	Friction angle, ϕ_s ($^\circ$)
Layer -1	5	30
Layer -2	3	27
Layer -3	6	23
Layer -4	20	27

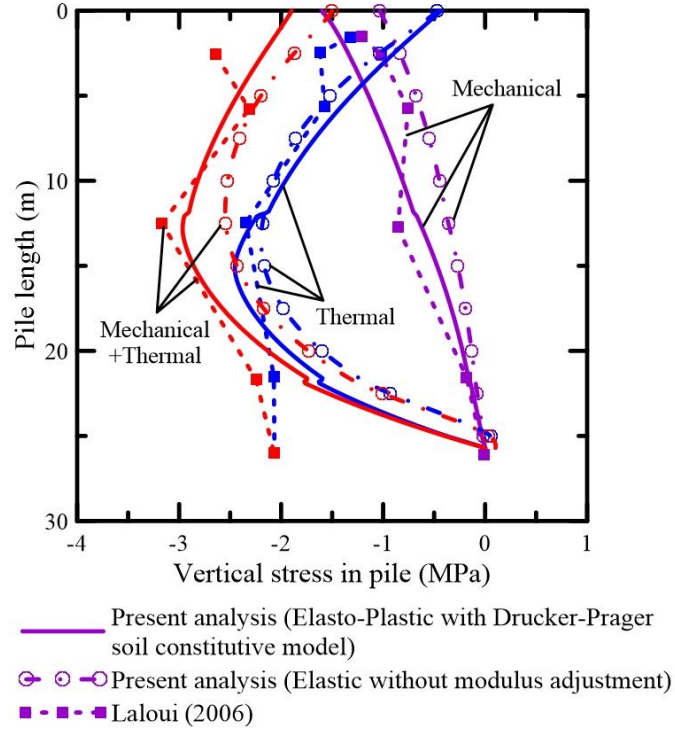


Figure 3-1 Axial stress profile along the pile for mechanical ($Q = 1300$ kN), thermal ($\Delta T = 15^\circ\text{C}$), and combined loading obtained from present analysis and field data

3.3 Internal energy calculation for a dissipative medium

The first law of thermodynamics that describes the conservation of energy is given by

$$\dot{U} = \dot{W} + Q \quad (3.1)$$

where $\dot{U}$ is the rate of change in internal energy, $\dot{W}$ is the rate of work done on the system and Q is the heat flux supplied to or extracted from the system. The second law of thermodynamics states that,

$$\dot{S} \geq \frac{Q}{\theta} \quad (3.2)$$

where $\dot{S}$ is the rate of change in entropy of the system and θ is the temperature of the system. For an irreversible process, the entropy can be divided into two parts, $\dot{S}^{(r)}$ and $\dot{S}^{(i)}$, where the superscripts r and i represent reversible and irreversible parts of the process. The reversible part is the entropy supply

from outside in the form of heat transfer and irreversible part is the entropy production inside the system (Ziegler, 1983). Therefore, the total rate of change of entropy $\dot{S}$ can be written as,

$$\dot{S} = \dot{S}^{(r)} + \dot{S}^{(i)} \quad (3.2a)$$

For any process to be reversible the equality in equation (3.2) should be satisfied and $\dot{S}^{(i)} = 0$. Therefore, for a reversible process, $\theta \dot{S}^{(r)} = Q$. If Q is replaced by $\theta \dot{S}^{(r)}$ in equation (3.2), then $\dot{S}^{(i)} \geq 0$ for any irreversible process. Substituting Q into equation (3.1) and using the relation given by equation (3.2a), equation (3.1) can be rewritten as,

$$\dot{U} = \dot{W} + \theta \dot{S} - \theta \dot{S}^{(i)} \quad (3.3)$$

An independent state variable a_k is assumed which describes the state of a system. U and S are the dependents state variables or state functions that depend on the independent state variable. The rate of changes of U and S are also state functions. Therefore, equations (3.3) can be rewritten as,

$$\begin{aligned} \frac{dU}{da_k} \dot{a}_k &= \frac{dW}{da_k} \dot{a}_k + \theta \frac{dS}{da_k} \dot{a}_k - \theta \dot{S}^{(i)} \\ \Rightarrow \frac{dW}{da_k} \dot{a}_k &= \left(\frac{dU}{da_k} - \theta \frac{dS}{da_k} \right) \dot{a}_k + \dot{\Phi} \end{aligned} \quad (3.4)$$

where $\dot{\Phi}$ is the energy dissipation from the system. $\frac{dW}{da_k}$ can be written as A_k which is equivalent to a

force like quantity. Dissipation $\dot{\Phi}$ can be written as $A_k^{(d)} \dot{a}_k$ where $A_k^{(d)}$ represents equivalent dissipative force. Dissipation can be defined as the part of work that is converted to heat and it is irreversible in nature. In the elastoplastic deformation of a body, the plastic deformation is always unrecoverable, so plastic deformation participates in the dissipation process. As the heat converted from work through dissipation goes out from the body, therefore, applying the notation of thermodynamics, the sign of $\dot{\Phi}$ is negative as shown in the following equation. Equation (3.4) then becomes

$$\begin{aligned} A_k \dot{a}_k &= \left(\frac{dU}{da_k} - \theta \frac{dS}{da_k} \right) \dot{a}_k - A_k^{(d)} \dot{a}_k = A_k^{(q)} \dot{a}_k - A_k^{(d)} \dot{a}_k \\ \Rightarrow A_k &= A_k^{(q)} - A_k^{(d)} \end{aligned} \quad (3.5)$$

where $A_k^{(q)} = \left(\frac{dU}{da_k} - \theta \frac{dS}{da_k} \right)$ that includes externally applied forces as well as internal forces including the effect of temperature. Expressing $\dot{W}$ in terms of A_k and $\dot{a}_k$ equation (3.1) can be rewritten as,

$$\begin{aligned}
 \dot{U} &= A_k \dot{a}_k + Q \\
 &= \{ A_k^{(q)} - A_k^{(d)} \} \dot{a}_k + Q \\
 &= A_k^{(q)} \dot{a}_k - A_k^{(d)} \dot{a}_k + Q \\
 &= A_k^{(q)} \dot{a}_k + Q - \dot{\Phi}
 \end{aligned} \tag{3.6}$$

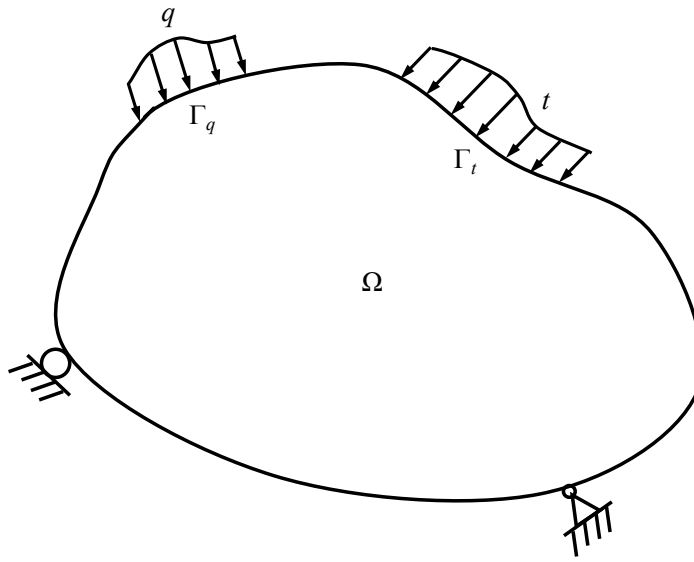


Figure 3-2 A continuum with applied traction and heat flow

In Figure 3-2, a continuum with domain Ω is considered. Γ_t and Γ_q are the boundary on which the traction t and heat flux q are applied, respectively. The sign conventions for work done and heat flow are the same as that is used in thermodynamics. Work done on the body by the external force is considered negative and work done by the internal force (stress) is considered positive. When heat is flowing into the body, it is considered positive and vice versa.

$$A_k^{(q)} \dot{a}_k = \int_{\Omega} \sigma_{ij} \dot{\epsilon}_{ij} d\Omega - \int_{\Gamma_t} t \dot{w} d\Gamma_t \quad (3.7a)$$

$$Q = \int_{\Gamma_q} q d\Gamma_q \quad (3.7b)$$

$$\dot{\Phi} = \int_{\Omega} \sigma_{ij}^{(d)} \dot{\epsilon}_{ij}^{(p)} d\Omega \quad (3.7c)$$

where σ_{ij} is the stress generated in the body because of the applied traction t over the boundary Γ_t and heat supply q over the boundary Γ_q , $\dot{\epsilon}_{ij}$ is the strain increment, $\sigma_{ij}^{(d)}$ is the generalized stress that takes part in the dissipation which is a part of total stress σ_{ij} , $\dot{\epsilon}_{ij}^{(p)}$ is the rate of change of plastic strain, $\dot{w}$ is the rate of change in displacement over the boundary Γ_t caused by the external traction t , and i, j are dummy indices. Substituting equations (3.7a)-(3.7c) into equation (3.6), equation (3.6) can be expressed as,

$$\dot{U} = \left[\int_{\Omega} \sigma_{ij} \dot{\epsilon}_{ij} d\Omega - \int_{\Gamma_t} t \dot{w} d\Gamma_t \right] + \left[\int_{\Gamma_q} q d\Gamma_q \right] - \left[\int_{\Omega} \sigma_{ij}^{(d)} \dot{\epsilon}_{ij}^{(p)} d\Omega \right] \quad (3.8)$$

$\dot{\epsilon}_{ij}$ can be decomposed into two parts, an elastic part ($\dot{\epsilon}_{ij}^{(e)}$) and a plastic part ($\dot{\epsilon}_{ij}^{(p)}$) so that $\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{(e)} + \dot{\epsilon}_{ij}^{(p)}$. Therefore, $\dot{\epsilon}_{ij}^{(p)}$ can be written as $\dot{\epsilon}_{ij}^{(p)} = (\dot{\epsilon}_{ij} - \dot{\epsilon}_{ij}^{(e)})$. Substituting $\dot{\epsilon}_{ij}^{(p)}$ into equation (3.8), the following is obtained:

$$\dot{U} = \left[\int_{\Omega} \sigma_{ij} \dot{\epsilon}_{ij} d\Omega - \int_{\Gamma_t} t \dot{w} d\Gamma_t \right] + \left[\int_{\Gamma_q} q d\Gamma_q \right] - \left[\int_{\Omega} \sigma_{ij}^{(d)} (\dot{\epsilon}_{ij} - \dot{\epsilon}_{ij}^{(e)}) d\Omega \right] \quad (3.8a)$$

As the dissipation part of equation (3.6) is described as the irreversible or unrecoverable part, only the plastic strain contributes there. Thus, the $\sigma_{ij}^{(d)} \dot{\epsilon}_{ij}^{(e)}$ on the right-hand side of equation (3.8a) does not comply with the definition of dissipation and therefore that term should be zero. Then equation (3.8a) can be written as

$$\dot{U} = \left[\int_{\Omega} \left(\sigma_{ij} - \sigma_{ij}^{(d)} \right) \dot{\varepsilon}_{ij} d\Omega - \int_{\Gamma_t} t \dot{w} d\Gamma_t \right] + \left[\int_{\Gamma_q} q d\Gamma_q \right] \quad (3.8b)$$

The stress tensor σ_{ij} at can be expressed as $\sigma_{ij} = C_{ijkl} \dot{\varepsilon}_{kl}$. Following the same notation if $\sigma_{ij}^{(d)}$ is written as $\sigma_{ij}^{(d)} = C_{ijkl}^{(p)} \dot{\varepsilon}_{kl}$ where ε_{kl} is the strain at time t_i and $C_{ijkl}^{(p)}$ is an equivalent stiffness tensor that relates strain with the generalized stress, then equation (3.8b) becomes

$$\dot{U} = \left[\int_{\Omega} \left(C_{ijkl} - C_{ijkl}^{(p)} \right) \varepsilon_{kl} \dot{\varepsilon}_{mn} d\Omega - \int_{\Gamma_t} t \dot{w} d\Gamma_t \right] + \left[\int_{\Gamma_q} q d\Gamma_q \right] \quad (3.8c)$$

If it is considered that the load is applied for a total time t and it is divided into n numbers of increment such that the time steps become $t_0 (= 0)$, t_1 , t_2 , ..., t_i , t_{i+1} , ..., $t_n (= t)$, then integrating equation (3.8c) over a time interval t_i - t_{i+1} gives,

$$\begin{aligned} \int_{t_i}^{t_{i+1}} \dot{U} dt &= \int_{t_i}^{t_{i+1}} \left[\left\{ \int_{\Omega} \left(C_{ijkl} - C_{ijkl}^{(p)} \right) \varepsilon_{kl} \dot{\varepsilon}_{mn} d\Omega - \int_{\Gamma_t} t \dot{w} d\Gamma_t \right\} + \left\{ \int_{\Gamma_q} q d\Gamma_q \right\} \right] dt \\ \Rightarrow U &= \left\{ \int_{\Omega} \left(C_{ijkl} - C_{ijkl}^{(p)} \right) \varepsilon_{kl} \varepsilon_{mn} d\Omega - \int_{\Gamma_t} t w d\Gamma_t \right\} + \int_{t_i}^{t_{i+1}} \left\{ \int_{\Gamma_q} q d\Gamma_q \right\} dt \end{aligned} \quad (3.9)$$

To obtain the governing equations and boundary conditions, the variational operator δ on equation (3.9) is applied to minimize the internal energy and making it equal to zero which provides the following form:

$$\delta U = \left[\int_{\Omega} \left(C_{ijkl} - C_{ijkl}^{(p)} \right) \varepsilon_{kl} \delta \varepsilon_{mn} d\Omega - \int_{\Gamma_t} t \delta w d\Gamma_t \right] + \int_{t_i}^{t_{i+1}} \left[\int_{\Gamma_q} \delta q d\Gamma_q \right] dt = 0 \quad (3.10)$$

In equation (3.10) the multiplier of strain, $(C_{ijkl} - C_{ijkl}^{(p)})$ is equivalent to the elastoplastic tangent stiffness tensor (C_{ijkl}^{ep}). When there is no plastic deformation the tangential stiffness tensor is the elastic stiffness tensor and in that case, $C_{ijkl}^{(p)} = 0$. Also, in the analysis of pile, the heat flux q is given as input and as a result, the last term in equation (3.10) disappears.

3.4 Analysis of energy pile

The free-body diagrams of an energy pile and the soil are shown in Figure 3-3. The length and diameter of the pile are L_p and D_p (radius r_p), respectively and the pile is subjected to an axial load Q_p and thermal load (temperature change) $+\Delta T_p$ as shown in Figure 3-3. The distributed reaction force along the length and at the tip of the pile are denoted as $t_p(z)$ and Q_b , respectively. The heat flux from pile to soil is q . In this discussion, the heat ejection from the energy pile is considered. Therefore, the direction of heat flow is from the pile to the soil. In the case of heat extraction, the direction of heat flow will be from the soil to the pile and the representation of the same will be opposite to that shown in Figure 3-3.

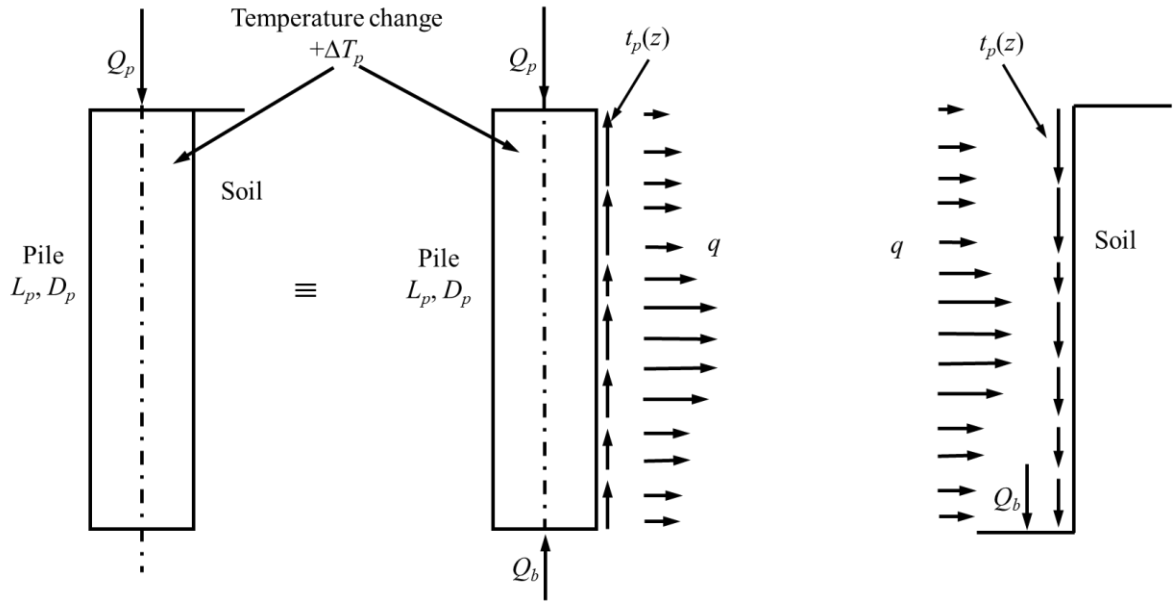


Figure 3-3- Free body diagram for pile and soil showing the forces acting on pile and soil and heat flow from the pile to the soil

The pile behavior is assumed to be elastic. For the axial response of pile only the stress and strain in the vertical direction (z -axis) is considered for the pile. The coordinate system used to calculate the internal energy of the pile and the soil is the same as shown in Figure 2-1. The equation for the variational form of internal energy increment in an energy pile can be written as,

$$\left[E_p A_p \int_0^{L_p} \left\{ \left(\frac{du_p}{dz} - \alpha_p \Delta T_p \right) - \left(\frac{du_p^{(0)}}{dz} - \alpha_p \Delta T_p^{(0)} \right) \right\} \delta \left(\frac{du_p}{dz} \right) dz + \int_0^{L_p} (t_p - t_{p0}) \delta u_p dz \right. \\ \left. - (Q_p - Q_{p0}) \delta u_p \Big|_{z=0} + (Q_b - Q_{b0}) \delta u_p \Big|_{z=L_p} \right] = 0 \quad (3.11a)$$

Also, the equation for the variational form of internal energy increment of an elastoplastic soil continuum in which the energy pile is embedded can be written as,

$$\int_{\Omega_s} \left[\left\{ - (C_{31}^{ep} + C_{32}^{ep}) \alpha_s (\Delta T_s - \Delta T_s^{(0)}) + C_{33}^{ep} \left(\frac{du_p}{dz} \phi - \frac{du_p^{(0)}}{dz} \phi_0 \right) + C_{35}^{ep} \left(u_p \frac{d\phi}{dr} - u_p^{(0)} \frac{d\phi_0}{dr} \right) \right\} \right. \\ \left. \delta \left(\frac{du_p}{dz} \phi \right) + \left\{ - (C_{51}^{ep} + C_{52}^{ep}) \alpha_s (\Delta T_s - \Delta T_s^{(0)}) + C_{53}^{ep} \left(\frac{du_p}{dz} \phi - \frac{du_p^{(0)}}{dz} \phi_0 \right) + C_{55}^{ep} \left(u_p \frac{d\phi}{dr} - u_p^{(0)} \frac{d\phi_0}{dr} \right) \right\} \right. \\ \left. \delta \left(u_p \frac{d\phi}{dr} \right) \right] d\Omega_s - \int_0^{L_p} (t_p - t_{p0}) \delta u_p dz + (Q_b - Q_{b0}) \delta u_p \Big|_{z=L_p} = 0 \quad (3.11b)$$

where u_p and $u_p^{(0)}$, ϕ and ϕ_0 , ΔT_s and $\Delta T_s^{(0)}$, Q_p and Q_{p0} , t_p and t_{p0} , Q_b and Q_{b0} are the vertical displacement, soil displacement shape function, temperature change in soil, applied load, shaft resistance, and base resistance at the beginning ($t = t_i$) and at the end ($t = t_{i+1}$) of a time increment, respectively. For the first increment $u_p^{(0)}$, ϕ_0 , $\Delta T_s^{(0)}$ and Q_0 is zero. C_{kl}^{ep} denotes the element in the k -th row and l -th column of the elastoplastic tangent stiffness tensor of soil. k and l are dummy indices that vary from 1 to 6. Variational calculus is applied to equation (3.11b) and then performing integration by parts equation the shaft resistance (t_p and t_{p0}), and base resistance (Q_b and Q_{b0}) can be represented in terms of C_{kl}^{ep} , vertical displacement (u_p and $u_p^{(0)}$), and soil displacement shape function (ϕ and ϕ_0). Substituting the shaft resistance (t_p and t_{p0}), and base resistance (Q_b and Q_{b0}) into equation (3.11a) the form $\int F_1(u_p) \delta u_p dz + \int F_2(\phi) \delta \phi dr = 0$ of equation (3.11a) is obtained from which the governing equations and boundary conditions for u_p and ϕ are obtained.

3.4.1 Governing equations and boundary conditions for vertical displacement of pile

Considering the first variation of u_p for domain $H_{i-1} \leq z \leq H_i$ where $H_i \leq L_p$, the differential equation for u_p for any layer i can be obtained as

$$\begin{aligned}
& \left(E_p A_p + t_{1i} \right) \frac{d^2 u_{pi}}{dz^2} + \left(\frac{dt_{1i}}{dz} + t_{2i} - t_{3i} \right) \frac{du_{pi}}{dz} - \left(t_{4i} - \frac{dt_{2i}}{dz} \right) u_{pi} = \\
& \left(E_p A_p + t_{1i}^{(0)} \right) \frac{d^2 u_{pi}^{(0)}}{dz^2} + \left(\frac{dt_{1i}^{(0)}}{dz} + t_{2i}^{(0)} - t_{3i}^{(0)} \right) \frac{du_{pi}^{(0)}}{dz} - \left(t_{4i}^{(0)} - \frac{dt_{2i}^{(0)}}{dz} \right) u_{pi}^{(0)} + (F_{1i} - F_{0i})
\end{aligned} \tag{3.12}$$

where

$$t_{1i} = 2\pi \left(\int_{r_p}^{\infty} C_{33}^{ep(i)} \phi^2 r dr \right) \tag{3.13a}$$

$$t_{1i}^{(0)} = 2\pi \left(\int_{r_p}^{\infty} C_{33}^{ep(i)} \phi_0 \phi r dr \right) \tag{3.13e}$$

$$t_{2i} = 2\pi \left(\int_{r_p}^{\infty} C_{35}^{ep(i)} \phi \frac{d\phi}{dr} r dr \right) \tag{3.13b}$$

$$t_{2i}^{(0)} = 2\pi \left(\int_{r_p}^{\infty} C_{35}^{ep(i)} \phi \frac{d\phi_0}{dr} r dr \right) \tag{3.13f}$$

$$t_{3i} = 2\pi \left(\int_{r_p}^{\infty} C_{53}^{ep(i)} \phi \frac{d\phi}{dr} r dr \right) \tag{3.13c}$$

$$t_{3i}^{(0)} = 2\pi \left(\int_{r_p}^{\infty} C_{53}^{ep(i)} \phi_0 \frac{d\phi}{dr} r dr \right) \tag{3.13g}$$

$$t_{4i} = 2\pi \left\{ \int_{r_p}^{\infty} C_{55}^{ep(i)} \left(\frac{d\phi}{dr} \right)^2 r dr \right\} \tag{3.13d}$$

$$t_{4i}^{(0)} = 2\pi \left\{ \int_{r_p}^{\infty} C_{55}^{ep(i)} \frac{d\phi_0}{dr} \frac{d\phi}{dr} r dr \right\} \tag{3.13h}$$

$$F_{1i} = 2\pi \alpha_{si} \left[\frac{d}{dz} \left\{ \int_{r_p}^{\infty} \left(C_{31}^{ep(i)} + C_{32}^{ep(i)} + C_{33}^{ep(i)} \right) \phi r \Delta T_s dr \right\} - \left\{ \int_{r_p}^{\infty} \left(C_{51}^{ep(i)} + C_{52}^{ep(i)} + C_{53}^{ep(i)} \right) \frac{d\phi}{dr} r \Delta T_s dr \right\} \right] \tag{3.13i}$$

$$\begin{aligned}
F_{0i} = 2\pi \alpha_{si} \left[\frac{d}{dz} \left\{ \int_{r_p}^{\infty} \left(C_{31}^{ep(i)} + C_{32}^{ep(i)} + C_{33}^{ep(i)} \right) \phi_0 r \Delta T_s^{(0)} dr \right\} - \left\{ \int_{r_p}^{\infty} \left(C_{51}^{ep(i)} + C_{52}^{ep(i)} + C_{53}^{ep(i)} \right) \frac{d\phi_0}{dr} r \Delta T_s^{(0)} dr \right\} \right] \\
\end{aligned} \tag{3.13j}$$

The boundary conditions at the top of the pile ($z = H_1 = 0$) are obtained by considering the first variation of u_p at $z = 0$ as,

$$u_p^{(1)} \Big|_{z=0} = \text{constant or,} \tag{3.14a}$$

$$\begin{aligned} \left(E_p A_p + t_{11}\right) \frac{du_{p1}}{dz} + t_{21} u_{p1} &= \left(E_p A_p + t_{11}^{(0)}\right) \frac{du_{p1}^{(0)}}{dz} + t_{21}^{(0)} u_{p1}^{(0)} + \left(F_{z0} - F_{z0}^{(0)}\right) \\ &+ E_p A_p \alpha_p \left(\Delta T_p - \Delta T_p^{(0)}\right) - \left(Q_p - Q_{p0}\right) \end{aligned} \quad (3.14b)$$

Similarly, at the intermediate layer ($z = H_i = H_{i-1}$), the boundary conditions are

$$u_p^{(i-1)} \Big|_{z=H_i} = u_p^{(i)} \Big|_{z=H_i} \quad (3.14c)$$

$$\begin{aligned} \left\{ \left(E_p A_p + t_{1i}\right) \frac{du_{pi}}{dz} + t_{2i} u_{pi} - F_{zi} \right\} - \left\{ \left(E_p A_p + t_{1(i-1)}\right) \frac{du_{p(i-1)}}{dz} + t_{2(i-1)} u_{p(i-1)} - F_{z(i-1)} \right\} = \\ \left\{ \left(E_p A_p + t_{1i}^{(0)}\right) \frac{du_{pi}^{(0)}}{dz} + t_{2i}^{(0)} u_{pi}^{(0)} - F_{zi}^{(0)} \right\} - \left\{ \left(E_p A_p + t_{1(i-1)}^{(0)}\right) \frac{du_{p(i-1)}^{(0)}}{dz} + t_{2(i-1)}^{(0)} u_{p(i-1)}^{(0)} - F_{z(i-1)}^{(0)} \right\} \end{aligned} \quad (3.14d)$$

Considering the first variation of u_p at $z = L_p$, boundary conditions at the tip of the pile ($z = H_{n-1} = L_p$) are obtained as,

$$u_p^{(n-1)} \Big|_{z=0} = u_p^{(n)} \Big|_{z=0} = \text{constant} \quad (3.14e)$$

$$\begin{aligned} \left\{ E_p A_p + \left(t_{1(n-1)} - t_{1n}\right) - A_p C_{33}^{ep(n)} \right\} \frac{du_{p(n-1)}}{dz} + \left(t_{2(n-1)} - t_{2n}\right) u_{p(n-1)} \\ = \left\{ E_p A_p + \left(t_{1(n-1)}^{(0)} - t_{1n}^{(0)}\right) - A_p C_{33}^{ep(n)} \right\} \frac{du_{p(n-1)}^{(0)}}{dz} + \left(t_{2(n-1)}^{(0)} - t_{2n}^{(0)}\right) u_{p(n-1)}^{(0)} \\ + \left(F_{z(n-1)} - F_{z(n-1)}^{(0)}\right) - \left(F_{zn} - F_{zn}^{(0)}\right) + E_p A_p \alpha_p \left(\Delta T_p - \Delta T_p^{(0)}\right) \end{aligned} \quad (3.14f)$$

where

$$F_{zi} = 2\pi\alpha_{si} \left\{ \int_{r_p}^{\infty} \left(C_{31}^{ep(i)} + C_{32}^{ep(i)} + C_{33}^{ep(i)}\right) \phi r \Delta T_s dr \right\} \quad (3.15a)$$

$$F_{zi}^{(0)} = 2\pi\alpha_{si} \left\{ \int_{r_p}^{\infty} \left(C_{31}^{ep(i)} + C_{32}^{ep(i)} + C_{33}^{ep(i)}\right) \phi_0 r \Delta T_s^{(0)} dr \right\} \quad (3.15b)$$

$$F'_{zn} = 2\pi\alpha_{sn} \left[\int_{r_p}^{\infty} (C_{31}^{ep(n)} + C_{32}^{ep(n)} + C_{33}^{ep(n)}) \phi r \Delta T_s dr + \int_0^{r_p} (C_{31}^{ep(n)} + C_{32}^{ep(n)} + C_{33}^{ep(n)}) r \Delta T_s dr \right] \quad (3.15c)$$

$$F'_{zn(0)} = 2\pi\alpha_{sn} \left[\int_{r_p}^{\infty} (C_{31}^{ep(n)} + C_{32}^{ep(n)} + C_{33}^{ep(n)}) \phi_0 r \Delta T_s^{(0)} dr + \int_0^{r_p} (C_{31}^{ep(n)} + C_{32}^{ep(n)} + C_{33}^{ep(n)}) r \Delta T_s^{(0)} dr \right] \quad (3.15d)$$

The differential equation for the axial displacement of the soil column below the pile is obtained by considering the first variation of $u_p^{(n)}$ as,

$$\begin{aligned} & \left(t_{1n} + A_p C_{33}^{ep(n)} \right) \frac{d^2 u_{pn}}{dz^2} + \left(\frac{dt_{1n}}{dz} + t_{2n} - t_{3n} + A_p \frac{dC_{33}^{ep(n)}}{dz} \right) \frac{du_{pn}}{dz} - \left(t_{4n} - \frac{dt_{2n}}{dz} \right) u_{pn} = \\ & \left(t_{1n}^{(0)} + A_p C_{33}^{ep(n)} \right) \frac{d^2 u_{pn}^{(0)}}{dz^2} + \left(\frac{dt_{1n}^{(0)}}{dz} + t_{2n}^{(0)} - t_{3n}^{(0)} + A_p \frac{dC_{33}^{ep(n)}}{dz} \right) \frac{du_{pn}^{(0)}}{dz} - \left(t_{4n}^{(0)} - \frac{dt_{2n}^{(0)}}{dz} \right) u_{pn}^{(0)} + (F_{1n} - F_{0n}) \end{aligned} \quad (3.16)$$

with boundary conditions $u_p^{(n)}|_{z=\infty} = 0$

3.4.2 Governing equations and boundary conditions for soil displacement shape functions

Considering the first variation of ϕ for $r_p \leq r \leq \infty$, the differential equation for soil displacement function is obtained as

$$\begin{aligned} & t_{2r} r \frac{d^2 \phi}{dr^2} + \left\{ t_{2r} + r \left(t_{1r} - t_{4r} + \frac{dt_{2r}}{dr} \right) \right\} \frac{d\phi}{dr} - \left\{ \left(t_{3r} - \frac{dt_{1r}}{dr} \right) r - t_{1r} \right\} \phi = \\ & t_{2r}^{(0)} r \frac{d^2 \phi_0}{dr^2} + \left\{ t_{2r}^{(0)} + r \left(t_{1r}^{(0)} - t_{4r}^{(0)} + \frac{dt_{2r}^{(0)}}{dr} \right) \right\} \frac{d\phi_0}{dr} - \left\{ \left(t_{3r}^{(0)} - \frac{dt_{1r}^{(0)}}{dr} \right) r - t_{1r}^{(0)} \right\} \phi_0 + (F_{r1} - F_{r2} r) \end{aligned} \quad (3.17)$$

where

$$t_{1r} = 2\pi \sum_{i=1}^n \left(\int_{H_{i-1}}^{H_i} C_{53}^{ep(i)} u_{pi} \frac{du_{pi}}{dz} dz \right) \quad (3.18a)$$

$$t_{1r}^{(0)} = 2\pi \sum_{i=1}^n \left(\int_{H_{i-1}}^{H_i} C_{53}^{ep(i)} u_{pi} \frac{du_{pi}^{(0)}}{dz} dz \right) \quad (3.18b)$$

$$t_{2r} = 2\pi \sum_{i=1}^n \left(\int_{H_{i-1}}^{H_i} C_{55}^{ep(i)} u_{pi}^2 dz \right) \quad (3.18c)$$

$$t_{2r}^{(0)} = 2\pi \sum_{i=1}^n \left(\int_{H_{i-1}}^{H_i} C_{55}^{ep(i)} u_{pi}^{(0)} u_{pi} dz \right) \quad (3.18d)$$

$$t_{3r} = 2\pi \sum_{i=1}^n \left\{ \int_{H_{i-1}}^{H_i} C_{33}^{ep(i)} \left(\frac{du_{pi}}{dz} \right)^2 dz \right\} \quad (3.18e)$$

$$t_{3r}^{(0)} = 2\pi \sum_{i=1}^n \left\{ \int_{H_{i-1}}^{H_i} C_{33}^{ep(i)} \frac{du_{pi}^{(0)}}{dz} \frac{du_{pi}}{dz} dz \right\} \quad (3.18f)$$

$$t_{4r} = 2\pi \sum_{i=1}^n \left(\int_{H_{i-1}}^{H_i} C_{35}^{ep(i)} u_{pi} \frac{du_{pi}}{dz} dz \right) \quad (3.18g)$$

$$t_{4r}^{(0)} = 2\pi \sum_{i=1}^n \left(\int_{H_{i-1}}^{H_i} C_{35}^{ep(i)} u_{pi}^{(0)} \frac{du_{pi}}{dz} dz \right) \quad (3.18h)$$

$$F_{r1} = 2\pi \sum_{i=1}^n \left\{ \alpha_{si} \left(C_{51}^{ep(i)} + C_{52}^{ep(i)} + C_{53}^{ep(i)} \right) \left(\int_{H_{i-1}}^{H_i} \Delta T_s u_p dz - \int_{H_{i-1}}^{H_i} \Delta T_s^{(0)} u_p dz \right) \right\} \quad (3.18i)$$

$$F_{r2} = 2\pi \sum_{i=1}^n \left\{ \alpha_{si} \left(C_{31}^{ep(i)} + C_{32}^{ep(i)} + C_{33}^{ep(i)} \right) \left(\int_{H_{i-1}}^{H_i} \Delta T_s \frac{du_p}{dz} dz - \int_{H_{i-1}}^{H_i} \Delta T_s^{(0)} \frac{du_p}{dz} dz \right) \right\} \quad (3.18j)$$

$$+ \frac{d}{dz} \left[\sum_{i=1}^n \left\{ 2\pi \alpha_{si} \left(C_{51}^{ep(i)} + C_{52}^{ep(i)} + C_{53}^{ep(i)} \right) \left(\int_{H_{i-1}}^{H_i} \Delta T_s u_p dz - \int_{H_{i-1}}^{H_i} \Delta T_s^{(0)} u_p dz \right) \right\} \right]$$

where $C_{mn}^{ep(i)}$ is the component of stiffness tensor of i -th soil layer and u_{pi} is the vertical displacement of the pile and soil column below the pile in i -th soil layer. The boundary conditions for the soil displacement shape function are $\phi = 0$ at $r = r_p$ and $\phi = 0$ at $r = \infty$.

3.5 Solution

Equation (3.12), alongwith boundary conditions (3.14a)-(3.14f) for pile displacement, equation (3.16) for soil column displacement, and equation (3.17) for soil displacement shape function are solved using the finite difference method. The horizontal and vertical extent of soil is considered as $60D_p$ from the face of the pile and $2.5L_p$ from the bottom of the pile to minimize the effect of boundary effect on the analysis as stated in Chapter 2. The discretization of the pile-soil domain is done as described in section 2.5 (refer Figure 2-3).

3.5.1 Finite difference (FD) discretization of governing equations and boundary conditions

Central FD method is used to discretize the governing equations for u_p (equation (3.12)). The boundary conditions at $z = 0$ and $z = L_p$ for vertical displacement u_p (equation (3.14b) and equation (3.14f), respectively) are discretized using the forward FD method. The discretized form of equation (3.12) for vertical displacement at any node along the pile length denoted as j is given by

$$\left\{ \frac{k_{1z(j)}}{(\Delta z)^2} + \frac{k_{2z(j)}}{2\Delta z} \right\} u_{j+1} - \left\{ \frac{2k_{1z(j)}}{(\Delta z)^2} + k_{3z(j)} \right\} u_j + \left\{ \frac{k_{1z(j)}}{(\Delta z)^2} - \frac{k_{2z(j)}}{2\Delta z} \right\} u_{j-1} = \left\{ \frac{k_{1z(j)}^{(0)}}{(\Delta z)^2} + \frac{k_{2z(j)}^{(0)}}{2\Delta z} \right\} u_{j+1}^{(0)} - \left\{ \frac{2k_{1z(j)}^{(0)}}{(\Delta z)^2} + k_{3z(j)}^{(0)} \right\} u_j^{(0)} + \left\{ \frac{k_{1z(j)}^{(0)}}{(\Delta z)^2} - \frac{k_{2z(j)}^{(0)}}{2\Delta z} \right\} u_{j-1}^{(0)} + \{F_{1(j)} - F_{0(j)}\} \quad (3.19a)$$

The discretized form of equation (3.14b) for node 1 at the ground level at the pile face, and equation (3.14f) for node $nzLp$ at the tip of the pile are respectively given by

$$\frac{k_{1z(1)}}{\Delta z} u_2 - \left\{ \frac{k_{1z(1)}}{\Delta z} - t_{2(1)} \right\} u_1 = \frac{k_{1z(1)}^{(0)}}{\Delta z} u_2^{(0)} - \left\{ \frac{k_{1z(1)}^{(0)}}{\Delta z} - t_{2(1)}^{(0)} \right\} u_1^{(0)} + \{F_{z0} - F_{z0}^{(0)}\} + E_p A_p \alpha_p (\Delta T_p - \Delta T_p^{(0)}) - (Q_p - Q_p^{(0)}) \quad (3.19b)$$

$$-\frac{k_{1z(nzLp)}}{\Delta z} u_{nzLp-1} + \left\{ \frac{k_{1z(nzLp)}}{\Delta z} - k_{2z(nzLp)} \right\} u_{nzLp} = -\frac{k_{1z(nzLp)}^{(0)}}{\Delta z} u_{nzLp-1}^{(0)} + \left\{ \frac{k_{1z(nzLp)}^{(0)}}{\Delta z} - k_{2z(nzLp)}^{(0)} \right\} u_{nzLp}^{(0)} + \{F_{z(nzLp-1)} - F_{z(nzLp-1)}^{(0)}\} - \{F'_{znzLp} - F'_{znzLp}{}^{(0)}\} + E_p A_p \alpha_p (\Delta T_p - \Delta T_p^{(0)}) \quad (3.19c)$$

Equation (3.14) that is discretized using the central FD method can be written as (where l denotes any node in the soil column below the pile)

$$\begin{aligned}
& \left\{ \frac{2k_{1zb(l)}}{(z_{l+1} - z_l)(z_{l+1} - z_{l-1})} + \frac{k_{2zb(l)}}{(z_{l+1} - z_{l-1})} \right\} u_{l+1} - \left\{ \frac{2k_{1zb(l)}}{(z_{l+1} - z_l)(z_l - z_{l-1})} + k_{3zb(l)} \right\} u_l \\
& + \left\{ \frac{2k_{1zb(l)}}{(z_l - z_{l-1})(z_{l+1} - z_{l-1})} - \frac{k_{2zb(l)}}{(z_{l+1} - z_{l-1})} \right\} u_{l-1} = \left\{ \frac{2k_{1zb(l)}^{(0)}}{(z_{l+1} - z_l)(z_{l+1} - z_{l-1})} + \frac{k_{2zb(l)}^{(0)}}{(z_{l+1} - z_{l-1})} \right\} u_{l+1}^{(0)} \\
& - \left\{ \frac{2k_{1zb(l)}^{(0)}}{(z_{l+1} - z_l)(z_l - z_{l-1})} + k_{3zb(l)}^{(0)} \right\} u_l^{(0)} + \left\{ \frac{2k_{1zb(l)}^{(0)}}{(z_l - z_{l-1})(z_{l+1} - z_{l-1})} - \frac{k_{2zb(l)}^{(0)}}{(z_{l+1} - z_{l-1})} \right\} u_{l-1}^{(0)} + \{F_{1(l)} - F_{0(l)}\}
\end{aligned} \tag{3.19d}$$

where

$$k_{1z(j)}^{(\bullet)} = \{E_p A_p + t_{1z(j)}^{(\bullet)}\} \tag{3.20a} \quad k_{1zb(l)}^{(\bullet)} = \{t_{1z(l)}^{(\bullet)} + C_{33(l)}^{ep} A_p\} \tag{3.20d}$$

$$k_{2z(j)}^{(\bullet)} = \left\{ \frac{dt_{1z(j)}^{(\bullet)}}{dz} + t_{2z(j)}^{(\bullet)} - t_{3z(j)}^{(\bullet)} \right\} \tag{3.20b} \quad k_{2zb(l)}^{(\bullet)} = \left\{ \frac{dt_{1z(l)}^{(\bullet)}}{dz} + t_{2z(l)}^{(\bullet)} - t_{3z(l)}^{(\bullet)} + A_p \frac{dC_{33(l)}^{ep}}{dz} \right\} \tag{3.20e}$$

$$k_{3z(j)}^{(\bullet)} = \left\{ t_{4z(j)}^{(\bullet)} - \frac{dt_{2z(j)}^{(\bullet)}}{dz} \right\} \tag{3.20c} \quad k_{3zb(l)}^{(\bullet)} = \left\{ t_{4z(l)}^{(\bullet)} - \frac{dt_{2z(l)}^{(\bullet)}}{dz} \right\} \tag{3.20f}$$

$$k_{1z(nzLp)}^{(\bullet)} = \{E_p A_p + (t_{1(n-1)}^{(\bullet)} - t_{1n}^{(\bullet)}) - A_p C_{33}^{(n)}\} \tag{3.20g}$$

$$k_{2z(nzLp)}^{(\bullet)} = (t_{2(n-1)}^{(\bullet)} - t_{2n}^{(\bullet)}) \tag{3.20h}$$

Equation (3.17) which describes the soil displacement shape function is also solved using central difference method. At any intermediate node m , the discretized form of equations (3.17) can be written as

$$\begin{aligned}
& \left\{ \frac{2t_{2r(k)}r}{(r_{k+1}-r_k)(r_{k+1}-r_{k-1})} + \frac{k_{1r(k)}}{(r_{k+1}-r_{k-1})} \right\} \phi_{k+1} - \left\{ \frac{2t_{2r(k)}r}{(r_{k+1}-r_k)(r_k-r_{k-1})} + k_{1r(k)} \right\} \phi_k \\
& + \left\{ \frac{2t_{2r(k)}r}{(r_k-r_{k-1})(r_{k+1}-r_{k-1})} - \frac{k_{1r(k)}}{(r_{k+1}-r_{k-1})} \right\} \phi_{k-1} = \left\{ \frac{2t_{2r(k)}^{(0)}}{(r_{k+1}-r_k)(r_{k+1}-r_{k-1})} + \frac{k_{1r(k)}^{(0)}}{(r_{k+1}-r_{k-1})} \right\} \phi_{k+1}^{(0)} \\
& - \left\{ \frac{2t_{2r(k)}^{(0)}}{(r_{k+1}-r_k)(r_k-r_{k-1})} + k_{1r(k)}^{(0)} \right\} \phi_k^{(0)} + \left\{ \frac{2t_{2r(k)}^{(0)}}{(r_k-r_{k-1})(r_{k+1}-r_{k-1})} - \frac{k_{1r(k)}^{(0)}}{(r_{k+1}-r_{k-1})} \right\} \phi_{k-1}^{(0)} \\
& + \left\{ F_{r1(k)} + F_{r2(k)} r_k \right\}
\end{aligned} \tag{3.21}$$

where,

$$k_{1z(j)}^{(\bullet)} = \left\{ E_p A_p + t_{1z(j)}^{(\bullet)} \right\} \tag{3.22a} \quad k_{1zb(l)}^{(\bullet)} = \left\{ t_{1z(l)}^{(\bullet)} + C_{33(l)}^{ep} A_p \right\} \tag{3.22d}$$

$$k_{2z(j)}^{(\bullet)} = \left\{ \frac{dt_{1z(j)}^{(\bullet)}}{dz} + t_{2z(j)}^{(\bullet)} - t_{3z(j)}^{(\bullet)} \right\} \tag{3.22b} \quad k_{2zb(l)}^{(\bullet)} = \left\{ \frac{dt_{1z(l)}^{(\bullet)}}{dz} + t_{2z(l)}^{(\bullet)} - t_{3z(l)}^{(\bullet)} + A_p \frac{dC_{33(l)}^{ep}}{dz} \right\} \tag{3.22e}$$

$$k_{3z(j)}^{(\bullet)} = \left\{ t_{4z(j)}^{(\bullet)} - \frac{dt_{2z(j)}^{(\bullet)}}{dz} \right\} \tag{3.22c} \quad k_{3zb(l)}^{(\bullet)} = \left\{ t_{4z(l)}^{(\bullet)} - \frac{dt_{2z(l)}^{(\bullet)}}{dz} \right\} \tag{3.22f}$$

$$k_{1z(nzLp)}^{(\bullet)} = \left\{ E_p A_p + \left(t_{1(n-1)}^{(\bullet)} - t_{1n}^{(\bullet)} \right) - A_p C_{33}^{(n)} \right\} \tag{3.22g}$$

$$k_{2z(nzLp)}^{(\bullet)} = \left(t_{2(n-1)}^{(\bullet)} - t_{2n}^{(\bullet)} \right) \tag{3.22h}$$

$$k_{1r(k)}^{(\bullet)} = \left\{ t_{2r(k)}^{(\bullet)} + r \left(t_{1r(k)}^{(\bullet)} - t_{4r(k)}^{(\bullet)} + \frac{dt_{2r(k)}^{(\bullet)}}{dr} \right) \right\} \tag{3.22i}$$

$$k_{2r(k)}^{(\bullet)} = \left\{ r \left(t_{3r(k)}^{(\bullet)} + \frac{dt_{1r(k)}^{(\bullet)}}{dr} \right) t_{2r(k)}^{(\bullet)} - t_{1r(k)}^{(\bullet)} \right\} \tag{3.22j}$$

3.6 Drucker-Prager yield criteria

The yield criteria that is used here is the smooth Drucker-Prager yield criterion where the apex is rounded to avoid the singularity following the geometric manipulation done by Abbo & Sloan, (1995). An associated flow rule is used here along with the yield criteria to define the direction of the plastic strain change. The yield criteria is given by (Yu, 2006)

$$F = \sqrt{J_2 + b^2 \alpha^2} - (\alpha I_1 + k_{DP}) \quad (3.23)$$

where J_2 is the second invariant of the deviatoric stress tensor (equation A.5), I_1 is the first invariant of the stress tensor ($\text{tr}(\boldsymbol{\sigma})$ where $\boldsymbol{\sigma}$ is the stress tensor). b is the distance between the original apex and the rounded apex as described by Abbo & Sloan, (1995) and calculated as per their recommendation which has produced results having very close parity with the original yield criteria.

$$b = 0.05c_s \cot \phi_s \quad (3.23a)$$

α and k_{DP} are material constants and are given by

$$\alpha = \frac{2 \sin \phi_s}{\sqrt{3}(3 - \sin \phi_s)} \quad (3.23b)$$

$$k_{DP} = \frac{6c_s \cos \phi_s}{\sqrt{3}(3 - \sin \phi_s)} \quad (3.23c)$$

where c_s and ϕ_s are the cohesion and angle of internal friction of soil respectively. The associated flow rule will be

$$\dot{\varepsilon}_{ij}^p = \dot{\lambda} \frac{\partial F}{\partial \sigma_{ij}} \quad (3.24)$$

where $\dot{\varepsilon}_{ij}^p$ is the plastic strain rate tensor and $\dot{\lambda}$ is the plastic multiplier.

3.6.1 Formulation of tangential stiffness tensor

The tangential stiffness tensor for soil is used in the analysis to reflect the elastoplastic behavior of soil. This stiffness tensor changes depending on the stress state in the soil once yielding occurs. The formulation for the updated stiffness tensor depending on the stress state is calculated using the concept

of classical plasticity theory (Potts, 1999). Total strain increment at any pseudo time increment ΔT can be decomposed into two parts, i) elastic strain increment ($\Delta \epsilon_e$), and ii) plastic strain increment ($\Delta \epsilon_p$). The elastic strain part can be related to the stress increment ($\Delta \sigma$), by the elastic stiffness tensor (C_e). Again, the yield criteria require the stress state to be on the yield surface during yield. Using the consistency condition, it can be written as

$$dF = \frac{\partial F}{\partial \sigma} \Delta \sigma \quad (3.25)$$

Expressing stress increment in terms of total strain increment and plastic multiplier and putting that expression into equation (3.25), the plastic multiplier can be calculated in terms of total strain increment, elastic stiffness matrix, and yield surface gradient. Putting the value of the plastic multiplier into the constitutive equation (stress-strain increment relation), the elastoplastic tangential stiffness tensor is calculated as

$$C_{ep} = C_e - \frac{C_e \left\{ \frac{\partial F}{\partial \sigma} \right\} \left\{ \frac{\partial F}{\partial \sigma} \right\}^T}{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T C_e \left\{ \frac{\partial F}{\partial \sigma} \right\}} \quad (3.26)$$

3.7 Solution algorithm

Equations (3.19a)-(3.19d) and (3.21) are solved using an incremental iterative algorithm. The input parameters required for the analysis are the pile dimensions, pile material property, soil property, applied load and temperature change of the pile. A pseudo time t is defined in the algorithm which is zero at the beginning of the analysis and 1 at the end of the analysis. The time t is increased in an iterative manner through the analysis by a time increment Δt which is determined by the convergence criteria. The load and temperature change those are applied to the pile at any time T is calculated as, $Q_{p(T)} = t \times Q_p$ and $\Delta T_{p(T)} = \Delta T_p \times t$ which at the beginning of the analysis will be zero. In the beginning of the analysis ($t = 0$), $\{\phi\}_{nr \times 1}$ at the nodes are assumed with its proper boundary condition ($\phi_1 = 1$ and $\phi_{nr} = 0$) and the geostatic stress in the soil is defined. Then for a time increment Δt , the vertical displacement $\{u\}_{nz \times 1}$ and shape function $\{\phi\}_{nr \times 1}$ are calculated using Subroutine-1 which is described in the next subsection. Using the values of $\{u\}_{nz \times 1}$ and $\{\phi\}_{nr \times 1}$ of current step and previous step the strain increment in the soil is calculated. This strain increment is used in a stress integration subroutine where

the stress integration in the soil nodes is done by Modified Euler Scheme with Substepping which also included negative plastic multiplier and yield surface correction (Abbo, 2005). The calculated stress is then used to update the elastoplastic stiffness tensor using equation (3.26). The vertical displacement $\{u\}_{nz \times 1}$, shape function $\{\phi\}_{nr \times 1}$ and the updated elastoplastic stiffness tensor are used in the calculation for the next time increment. t is updated as $t + \Delta t$ at the end of each time increment. The analysis ends when t becomes 1. The $\{u\}_{nzLp \times 1}$ and $\{\phi\}_{nr \times 1}$ obtained at the end of the analysis are the final pile displacement and shape function values respectively under the application of axial load Q_p and total temperature change of pile, ΔT_p . The solution algorithm is shown as a flow chart in Figure 3-4

3.7.1 Subroutine-1

The vertical displacement $\{u\}_{nz \times 1}$, shape function $\{\phi\}_{nr \times 1}$ and the stiffness tensor of soil, applied loads at the end of the previous step and applied loads at the end of the current step are required as the input of this subroutine algorithm. The terms described by the equations (3.13a)-(3.13j), (3.15a)-(3.15d), and (3.12a)-(3.13h) are calculated using $\{\phi\}_{nr \times 1}$, its derivative, temperature change in soil, and the stiffness tensor of the soil. These quantities are used to calculate $\{u\}_{nz \times 1}$ using equations (3.19a)-(3.19d). The calculated value of $\{u\}_{nz \times 1}$, its derivatives, and stiffness tensor are then used to calculate the coefficients described by equation (3.18a)-(3.18j) and (3.22a)-(3.22j). These quantities are used to calculate $\{\phi\}_{nr \times 1}$ from equation (3.21) satisfying the boundary conditions $\phi_1 = 1$ and $\phi_{nr} = 0$. From calculated $\{\phi\}_{nr \times 1}$ again the coefficients required to calculate $\{u\}_{nz \times 1}$ are recalculated as mentioned in the beginning and again $\{u\}_{nz \times 1}$ consecutively $\{\phi\}_{nr \times 1}$ are calculated. These iterations go on until the convergence criteria $\left\| \{u^{(i)}\} - \{u^{(i-1)}\} \right\| / \left\| \{u^{(i)}\} \right\| \leq 10^{-4}$ is achieved where $\{u^{(i)}\}$ is the displacement calculated in the i -th iteration of the subroutine. $\|\bullet\|$ denotes the L^2 norm of the vector. The $\{u\}_{nz \times 1}$ and $\{\phi\}_{nr \times 1}$ obtained in the final iteration of the subroutine are outputs that are further used in the main algorithm.

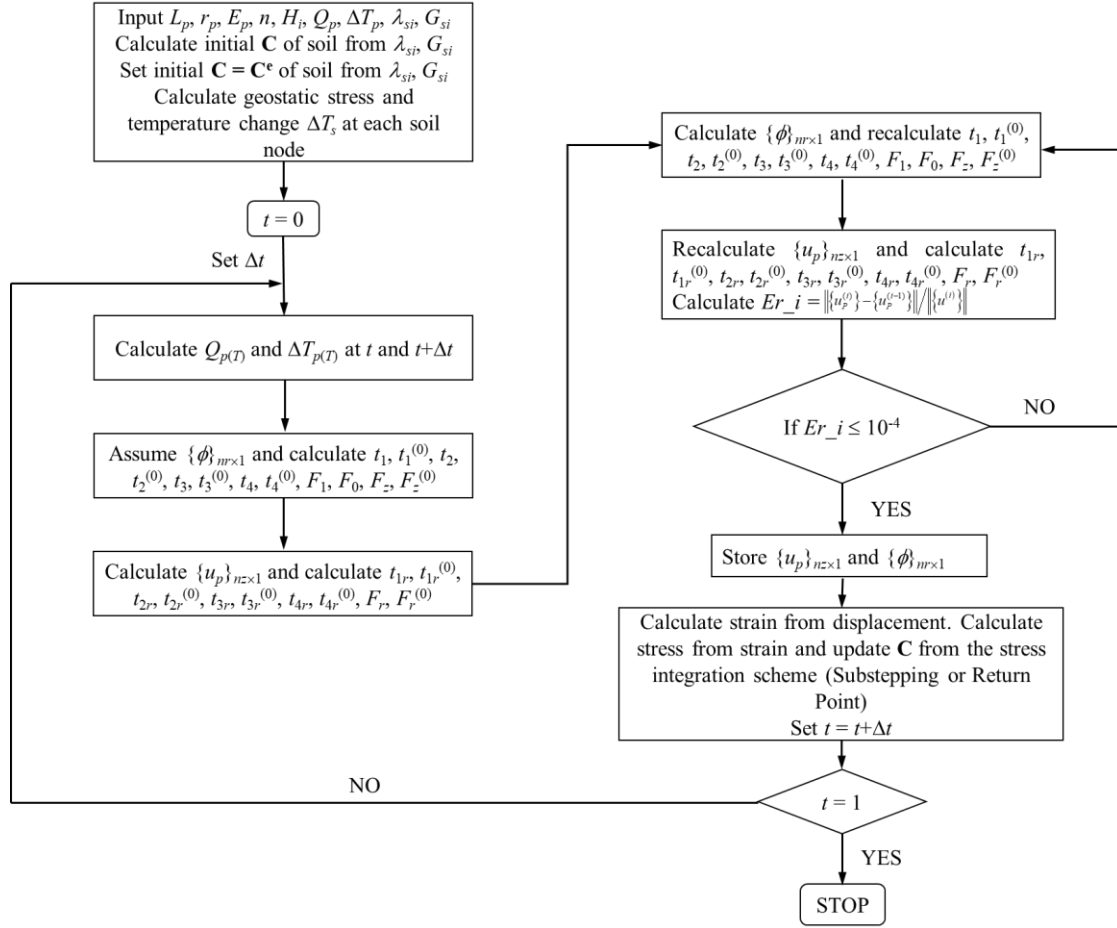


Figure 3-4 Flow chart of the solution scheme

3.8 Results

3.8.1 Validation with experimental study

The accuracy of the present analysis with the elastoplastic soil constitutive model is validated with the results from experimental data as stated in the section 2.8.1.

3.8.1.1 Goode and McCartney (2015)

The vertical displacement profile obtained from the present analysis is plotted in Figure 3-5 alongwith the results obtained by Goode & McCartney, (2015) in their experiment. The input for the analysis is given in Table 2-1. In the experiment, a temperature change of +18°C is applied to a concrete pile embedded in Nevada sand. The cohesion and the friction angle of the sand are considered as 0.5

kPa and 35° (provided by the authors), respectively. The displacement profile obtained from the present analysis matches well with the experimental result by Goode & McCartney, (2015) without any modification in the modulus which also proves the need to include an elastoplastic soil response in the present analysis. The difference between the head displacements obtained from the present analysis and the test is 8%.

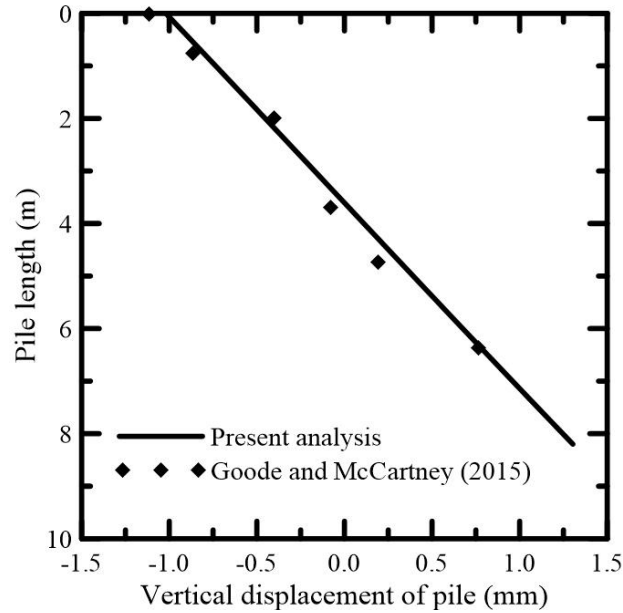


Figure 3-5 Vertical displacement profiles of the pile obtained from the present analysis, and field test under thermal load ($\Delta T_p = +18^\circ \text{C}$)

3.8.2 Verification with numerical analysis

3.8.2.1 Analysis performed by the author

The accuracy of the present analysis is verified with the results from FEA conducted in Abaqus. A comparison of the results obtained for a single pile in elastoplastic soil medium under mechanical, thermal, and combined (mechanical + thermal) load in terms of displacement, strain, and stress distribution along the length of the pile obtained from present analysis and axisymmetric FEA in Abaqus is shown in Figure 3-7 and Figure 3-8.

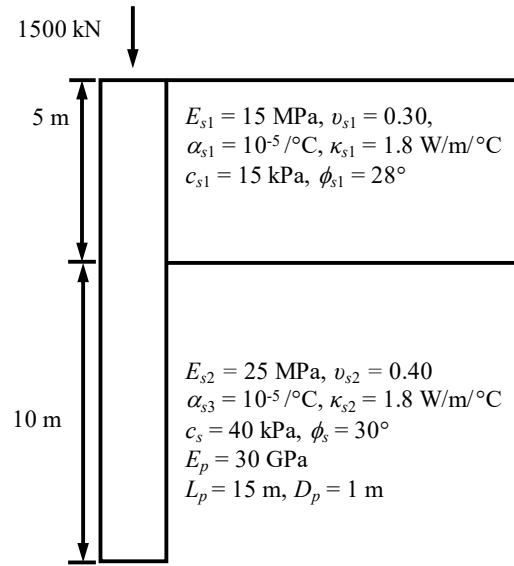
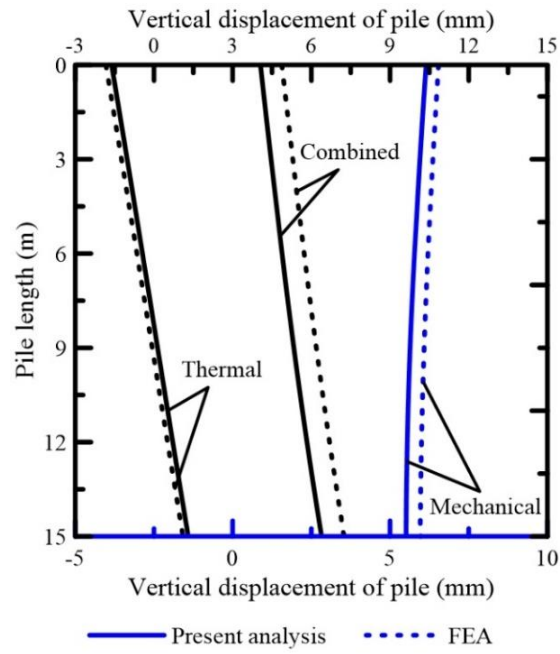
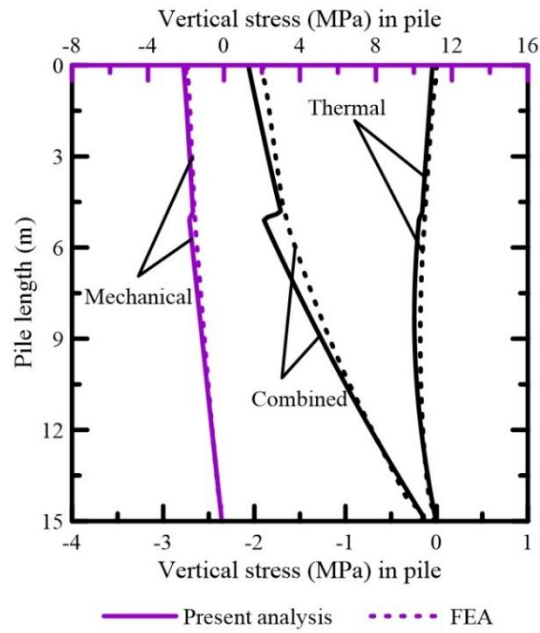


Figure 3-6 Pile with soil layering and properties

It is observed from Figure 3-7 and Figure 3-8 that the displacement, and stress profiles along the length of the pile obtained using the present method produce very good similarity with the results from the FEA. The details of the FE model are provided in the section 2.8.2. The only change that is made in the FE model by defining the soil material properties with the Drucker-Prager yield criteria instead of elastic properties only. A temperature boundary condition is applied to the pile with a ΔT_p of $+25^\circ\text{C}$, and -15°C , respectively for two different cases. Figure 3-7(a)-(b) show the response of the pile under mechanical load and a temperature increase of 25°C whereas Figure 3-8(a)-(b) show the response of the same pile under the same mechanical load but a decrease in temperature of 15°C .



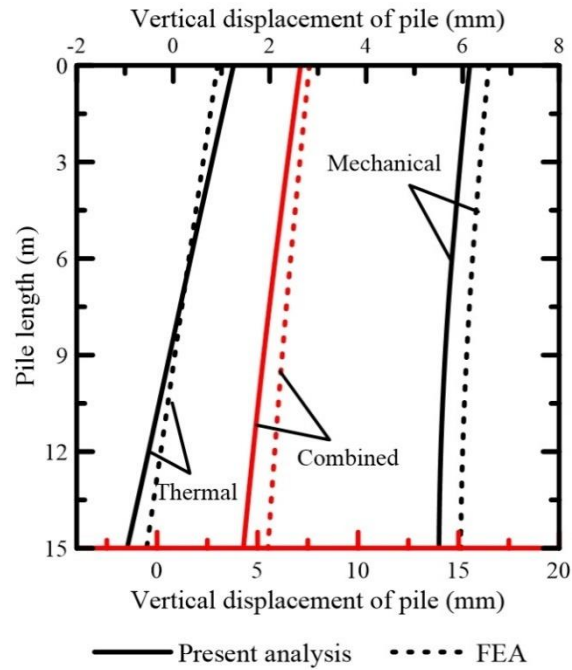
(a)



(b)

Figure 3-7 Comparison of results for energy pile between present analysis and FEA under mechanical, thermal ($\Delta T_p = +25^\circ \text{C}$), and combined loadings in terms of a) Displacement, and b) stress along the length of the pile

From Figure 3-7 (a), vertical head displacements obtained from the present analysis and the FEA for mechanical, thermal, and combined loadings are 6.14 mm, -1.61 mm, and 4.06 mm; and 6.54 mm, -1.73 mm and 4.57 mm, respectively with corresponding differences between the head displacements are 6%, 8%, and 10%, respectively. Maximum vertical (axial) stresses at the head of the pile under mechanical and combined loading (Figure 3-7 (c)) are obtained from the present analysis and the FEA are 2.12 MPa, 2.06 MPa; and 1.95 MPa, 1.90 MPa, respectively. The differences in the stress values between both the analyses are less than 10%.



(a)

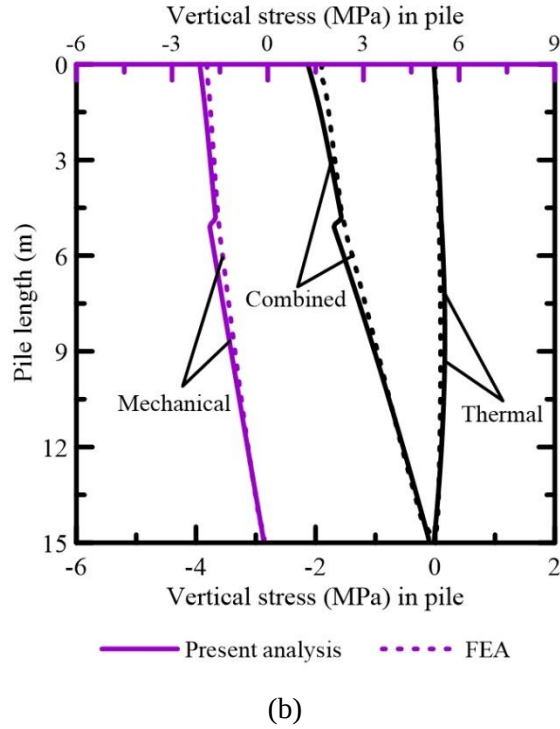


Figure 3-8 Comparison of results for pile between present analysis and FEA under mechanical, thermal ($\Delta T_p = -15^\circ \text{C}$), and combined loadings in terms of a) Displacement, and b) stress along the length of the pile

The vertical head displacements obtained from the present analysis and the FEA for thermal, and combined loadings are 1.14 mm and 7.13 mm; and 1.0 mm and 7.58 mm, respectively (Figure 3-8(a)) with corresponding differences between the head displacements are 14% and 6%, respectively. The maximum vertical (axial) stress at the head of the pile under combined loading (Figure 3-8(b)) are obtained from the present analysis are 1.91 MPa and 2.05 MPa, respectively. The difference in the stress values between both analyses is less than 10%. The time required to run the present analysis with the elastoplastic model is nearly five times less than that required for the FEA of the same energy pile in the computer specified in Chapter 2.

3.8.2.2 Saggu and Chakraborty (2017)

The accuracy of the present analysis is verified with the results from FEA conducted by Saggu & Chakraborty, (2017). A comparison of the vertical stress developed in an energy pile (calculated using the present method and obtained by Saggu & Chakraborty, (2017)) in Ottawa sand under

mechanical and combined (mechanical + thermal) is shown in Figure 3-9. The details of pile dimensions, properties of the pile material and soil are given in Table 3-2.

Table 3-2- Pile dimension and material properties of soil and pile material (Saggu & Chakraborty, 2017)

	Young's modulus, E_p or E_s (MPa)	Poisson's ratio, ν_p or ν_s	Coefficient of thermal expansion, α_p or α_s ($1/^\circ\text{C}$)	Thermal conductivity, κ_p or κ_s (W/m/ $^\circ\text{C}$)	Thermal diffusivity, a_s (m^2/s)	
Pile	33700	0.2	1×10^{-5}	2.100	-	$D_p = 0.50 \text{ m}$ $L_p = 20 \text{ m}$
Ottawa sand	85	0.3	1×10^{-4}	0.274	1.2×10^{-7}	$H_1 \rightarrow \infty$ $c_s = 0.5 \text{ kPa}$ $\phi_s = 40^\circ$

The vertical load and temperature change applied to the pile are 800 kN and $+21^\circ \text{C}$, respectively. The friction angle for Ottawa sand is suggested by Ghayoomi et al., (2018). The stress profiles obtained from the present analysis for both loading conditions match well with the results obtained from the numerical analysis.

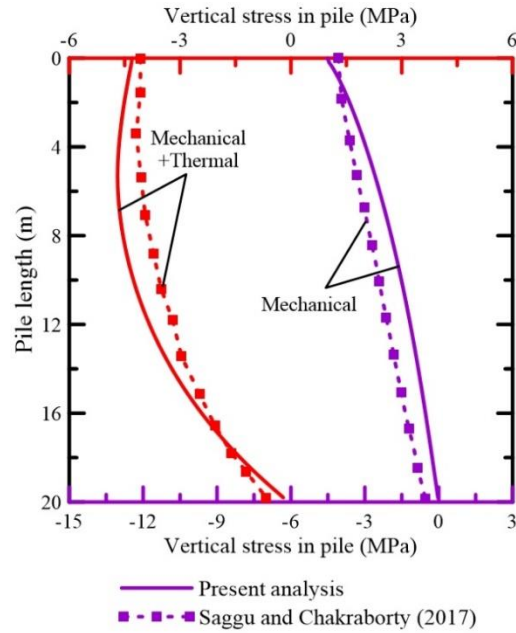


Figure 3-9 Vertical stress profiles of the pile obtained from the present analysis and numerical analysis for different loading conditions

3.9 Summary

The semi-analytical framework for the energy pile in elastoplastic soil is developed in this chapter based on the principles of thermodynamics and minimization of internal energy using variational calculus. The plastic behavior of soil is considered in the energy potential through the dissipation function. An incremental analysis is done where the current state variables are calculated on the basis of the variable in the previous increment. The stiffness tensor of soil is updated after each increment depending on the stress which is calculated using the modified Euler method with substepping stress integration scheme. The Drucker-Prager constitutive model is used to model the soil which is able to represent the elastoplastic behavior of sand as well as clay.

The results obtained from the present analysis are compared with the results obtained from the FEA in the form of axial displacement, and stress profile in the pile under mechanical, thermal, and combined loading. The time required to analyze the energy pile with an elastoplastic soil constitutive model using the present method is approximately 5 times less than equivalent numerical analysis. Further, a comparison with field data is also made where it provides a good match. The present method

produces pile response with good accuracy which is an improvement that is included in the present analysis by removing the requirement of modulus adjustment as described in Chapter 2 for elastic soil behavior. The present model which includes the elastoplastic behavior of soil can be applied for different types of soil (sand and clay). Although the Drucker-Prager constitutive model represents the behavior of both clay and sand well, the effect of temperature on the mechanical response of clay is greater than that on sand and hence, the effect of temperature on the constitutive relationship of clay needs to be considered more precisely in the analysis with a proper thermo-mechanical constitutive model which is further discussed in the next chapter. Therefore, the present framework with the Drucker-Prager constitutive model is more suitable for sandy soil.

Chapter 4

Analysis of Energy Pile in Clay with a Thermo- mechanical Constitutive Model

4.1 Introduction

In this chapter, the semi-analytical framework developed in Chapter 3 is utilized to calculate the response of the energy pile embedded in fine-grained soil (clay) which is represented by a thermodynamically compatible thermo-mechanical soil constitutive model suggested by Golchin et al., (2022b). The yield function and the flow rule of the thermo-mechanical constitutive model have been developed from the concept of hyperplasticity (derived from the fundamental laws of thermodynamics) using the free energy and dissipation potential. The constitutive model for fine-grained soil that is used in the analysis includes the effect of temperature on the pre-consolidation pressure which in turn affects the yield function and the flow rule. The elemental test (i.e. the stress-strain response of a single element under deformation) of the constitutive model is conducted and the results are validated with laboratory results available in the literature. The response of the energy pile under different loading conditions obtained from the present semi-analytical framework with that thermo-mechanical constitutive model is validated with the results from different field tests.

4.2 Motivation

The mechanical response of any material consists of two parts. Those are the volumetric behavior and the shear behavior. Granular materials are capable of carrying the compressive load only, and the shear behavior of the material plays a crucial role alongwith its densification caused by the compression in the determination of the strength of the granular material. The effect of temperature on the mechanical response of soil can also be considered as thermally affected volumetric part and shear part. Although the strain developed because of the change of temperature in the soil contributes only to the volumetric part, it affects both the volumetric and shear behavior of the soil. The hydraulic conductivity of sandy soil is very high. Therefore, the groundwater dissipates the imposed pile temperature change by heat transfer through the advection phenomenon which results in a low thermal gradient in sands and hence, results in low volumetric deformation (Suryatriyastuti, 2013). The effect of temperature on the mechanical behavior of clay is of more importance than sand. The pre-

consolidation pressure plays an important role in the constitutive behavior of clay that determines the threshold point between the elastic and plastic behavior of clay (Wood, 1991). The pre-consolidation pressure in clay changes with temperature. The pre-consolidation decreases as temperature increases which results in lowering the yield stress value (Abuel-Naga, Bergado, & Bouazza, 2007; Cekerevac & Laloui, 2004). The shear behavior of clay subjected to temperature increase has been observed to vary in several ways. The shear strength of clay may increase, remain unchanged, or decrease during heating (Abuel-Naga, Bergado, & Lim, 2007; Cekerevac & Laloui, 2004; Hueckel & Baldi, 1990; Towhata et al., 1993). The reduction in shear modulus has also been observed at high temperature (Zhou et al., 2015). The constitutive behavior of clay depends on the overconsolidation ratio (OCR) also. Normally and slightly overconsolidated clayey soil (low OCR values) undergoes permanent volume reduction when the temperature of the soil increases (Abuel-Naga, Bergado, & Bouazza, 2007; Cekerevac & Laloui, 2004; Di Donna & Laloui, 2015b; Sultan et al., 2002). With the increase in OCR, the thermally induced volume reduction decreases, and at high OCR the soil exhibits thermo-elasticity (Khalili et al., 2010). Normally and slightly overconsolidated soils are prone to higher volumetric contraction upon heating compared to remolded soil specimens because of the evenly distributed voids in the later (Ng, Cheng, et al., 2016). Soils that undergo heating-cooling cycles exhibit hardening behavior as their void ratios reduce and as a result, they reach a denser state (Di Donna & Laloui, 2015b; Ng, Cheng, et al., 2016). Therefore, the pre-consolidation pressure increases and the elastic domain becomes bigger. The response beyond yield for clay vanishes as thermal cycles induce thermo-elasticity in the clay. So, the analysis of energy pile embedded in clayey soil requires the soil to be modeled by a proper constitutive relation that can incorporate the effect of temperature change on the mechanical response of soil during heat exchange between the pile and the soil, which in turn will affect the pile-soil interaction and mechanical response of the pile (in terms of axial stress, axial strain). Also, many thermo-mechanical constitutive models do not follow the hyperplasticity formalism and therefore do not satisfy the laws of thermodynamics (section 1.2.2). Therefore, in the present analysis, the thermo-mechanical constitutive model for clay suggested by Golchin et al., (2022b) is used which complies with the laws of thermodynamics.

In this context, the energy pile, the response of which has been investigated by Laloui et al., (2006) (which has been used for the validation of the present method in section 2.8.1.2 and 3.2) is considered for analysis in this section incorporating the thermo-mechanical soil constitutive model to the present framework. The vertical stress profiles (Figure 4-1) along the length of a pile obtained from

the present analysis with the thermo-mechanical soil constitutive model (denoted as Present analysis (TM)) alongwith the stress profiles obtained from the present analysis with Drucker-Prager soil constitutive model (denoted as Present analysis (DP)) for different loading conditions are plotted against those obtained by Laloui et al., (2006). It is observed from Figure 4-1 that, the Present analysis (TM) predicts the maximum stress values in the pile for different loading conditions better than the Present analysis (DP). Therefore, representing the soil with a proper constitutive model that includes the temperature effect on the mechanical response of clay is required to be considered in the present analysis when clayey soil is present. The details of the constitutive model, the solution procedure, and the modification in the stress integration algorithm are described in following sections.

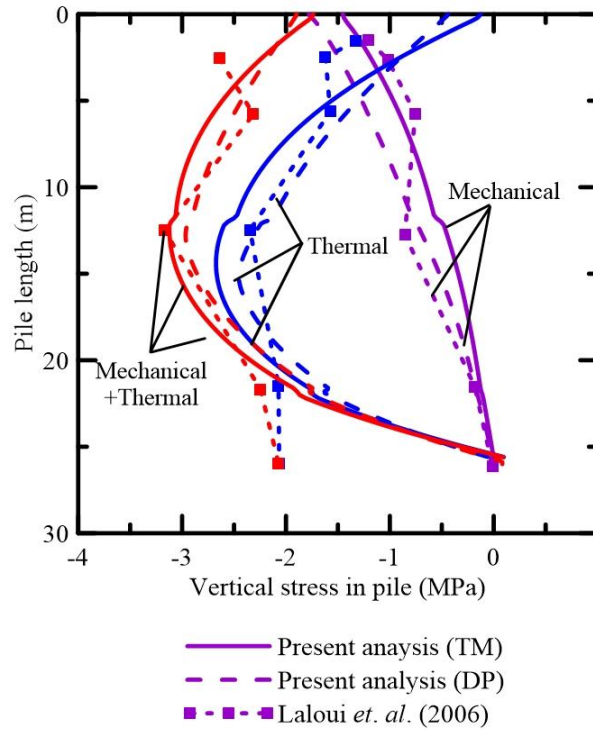


Figure 4-1 Axial stress profile along the pile under mechanical ($Q = 1300$ kN), thermal ($\Delta T = +15^\circ\text{C}$), and combined loading obtained from present analysis and field data

4.3 Details of thermo-mechanical constitutive model by Golchin et al. (2022b)

The thermo-mechanical constitutive model for clay developed by Golchin et al., (2022b) is used in the present framework to calculate the response of energy pile under mechanical and thermal loading conditions. The constitutive model has been developed on the hyperplasticity approach by

satisfying the laws of thermodynamics. The Gibbs energy potential that is considered in the development of the constitutive model includes the elastic part of the thermo-mechanical behavior through a Gibbs energy function which describes the energy caused by the elastic deformation. The plastic part of the thermo-mechanical behavior is considered in the Gibbs energy potential through the dissipation potential. The constitutive model includes the change of elastic domain for the developed plastic strain as well as the temperature change where the pre-consolidation pressure at any temperature (p_{cT}) varies depending on the temperature and the plastic strain and thus affects the size of the yield surface. The model also includes the effect of temperature on the shear strength at the critical state condition by changing the critical state stress ratio (M) with temperature.

The thermo-mechanical constitutive model considers the elastic behavior of clay to be nonlinear. Therefore, the pressure-dependent shear and bulk modulus are considered in the model as suggested by Houlsby et al., (2005) which is given by

$$\frac{K_s}{p_{ref}} = \bar{K}_s \left(\frac{p}{p_{ref}} \right)^{m_s} \quad (4.1a)$$

$$\frac{G_s}{p_{ref}} = \bar{G}_s \left(\frac{p}{p_{ref}} \right)^{n_s} \quad (4.1b)$$

where p is the mean effective stress (equation (A.6)); p_{ref} is the reference mean effective stress, which is generally considered 1 kPa, $\bar{K}_s$ and $\bar{G}_s$ are material constants in model defining bulk and shear modulus, respectively; and m_s and n_s are material constants defining the power dependency of bulk and shear modulus on pressure, respectively.

The yield function y of the thermo-mechanical constitutive model given by Golchin et al., (2022b) is described as,

$$y = B^2 (p - C)^2 + A^2 (q - \beta p)^2 - A^2 B^2 \quad (4.2)$$

where

$$A = (1 - \gamma) p + \frac{\gamma}{2} p_{cT} \quad (4.2a)$$

$$B = M \left\{ (1 - \alpha) p + \frac{\alpha \gamma}{2} p_{cT} \right\} \quad (4.2b)$$

$$C = \frac{\gamma}{2} p_{cT} \quad (4.2c)$$

where q is the deviatoric stress (equation (A.7)), β is the level of inclination of the yield surface, α and γ are parameters related to the shape of the yield surface, p_{cT} is the pre-consolidation pressure at any temperature T , and M is the critical State stress ratio.

The non-associated flow rule is given by

$$\begin{Bmatrix} \dot{\varepsilon}_v^p \\ \dot{\varepsilon}_s^p \end{Bmatrix} = \dot{\Lambda} \begin{Bmatrix} r_p \\ r_q \end{Bmatrix} = \dot{\Lambda} \begin{Bmatrix} 2B^2(p - C) - 2\beta A^2(q - \beta p) \\ 2A^2(q - \beta p) \end{Bmatrix} \quad (4.3)$$

where $\dot{\varepsilon}_v^p$ and $\dot{\varepsilon}_s^p$ are plastic volumetric and deviatoric strain increment, respectively; and r_p and r_q are the directions of plastic flow in p and q directions, respectively.

The variation of critical state stress ratio M may change depending on the variation of the temperature. The dependency of M on temperature has been considered as,

$$M = M_0 + \pi^* (T - T_0) \quad (4.4)$$

where π^* is the slope of the variation of M with respect to temperature T , and M_0 is the Critical State stress ratio at ambient temperature (T_0). The value of π^* depends on the mineralogy of the soil.

The pre-consolidation pressure p_{cT} without considering the variation with temperature (isothermal condition) was suggested by Houlsby & Puzrin, (2007). Considering the effect of temperature pre-consolidation pressure at any temperature T has been represented as,

$$p_{cT} = p_{c0} \exp\left(\frac{1 + e_0}{\lambda - \kappa} \varepsilon_v^p\right) \exp[-\mu(T - T_0)] \quad (4.5)$$

where p_{c0} is the initial pre-consolidation pressure, λ and κ are the slopes of the normal compression (NCL) and unloading lines in e (void ratio) - $\ln p$ space, respectively, e_0 is the initial void ratio, ε_v^p is the volumetric part of the plastic strain, and μ is the coefficient of thermal softening (Cui et al., 2000). The apparent pre-consolidation pressure hardens isotropically with plastic volumetric strain (through

the term $e^{\left(\frac{1+e_0}{\lambda-\kappa} \dot{\epsilon}_v^p\right)}$, and it softens thermally with an increase in temperature (through the term $e^{\left[-\mu(T-T_0)\right]}$). Thermal softening reduces the size of the elastic domain.

The isotropic hardening which is governed by the pre-consolidation pressure is given by the rate of equation (4.5) as,

$$\dot{p}_{cT} = p_{cT} \left[\left(\frac{1+e_0}{\lambda-\kappa} \dot{\epsilon}_v^p \right) - \mu \dot{T} \right] \quad (4.5a)$$

4.4 Stress integration scheme for the thermo-mechanical constitutive model

The stress integration scheme that is used for the present thermo-mechanical constitutive model is different than the one that has been used for the Drucker-Prager constitutive model in Chapter 3. The stress integration scheme that is given by Golchin, (2023) is based on implicit Euler backward integration method with a Newton-Raphson iterative procedure which is known as Return Stress algorithm. The stress integration is divided into two steps: an elastic predictor and a plastic corrector. First, in the elastic predictor step the trial stress is calculated (using the elastic trial strain) by considering the deformation to be completely elastic. Then, in the plastic corrector step, if the trial stress is found outside the yield surface, then it is returned back on the yield surface through an iterative calculation by calculating the developed plastic strain. For any time step between t_n and t_{n+1} (where $t_{n+1} = t_n + \Delta t$, where Δt is the time increment), it is assumed that the state variables (e.g. stress and strain), hardening variables (the pre-consolidation pressure p_{cT}) and the temperature at time t_n are known. The strain, stress and pre-consolidation pressure at any time t_n are denoted as $\boldsymbol{\sigma}_n$, $\boldsymbol{\epsilon}_n$, p_{cT}^n . For any boundary value problem, the strain and temperature increments (denoted as $\Delta \boldsymbol{\epsilon}_n$ and ΔT) at the end of time increment Δt are also known. The purpose of stress integration is to calculate the stress ($\boldsymbol{\sigma}_{n+1}$), hardening variable (the pre-consolidation pressure p_{cT}^{n+1}), and consistent tangent stiffness tensor ($\mathbf{C}_{ep}$) at the end of the time $t_n + \Delta t$ (i.e. t_{n+1}). The plastic behavior of the soil depends on the developed plastic strain which depends on the plastic multiplier. Therefore, the increment of the plastic multiplier (denoted as $\Delta \Lambda$) is also unknown at the end of the time increment Δt . The general method of implicit stress integration scheme involves the calculation of residuals and minimizing the same using an iterative approach such as the Newton-Raphson method. These residual equations are defined for the strain, pre-consolidation pressure, and plastic multiplier.

4.4.1 Residual equation for strain

Total strain $\boldsymbol{\varepsilon}$ in the soil under deformation can be decomposed as,

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^{\text{therm}} + \boldsymbol{\varepsilon}^p \quad (4.6a)$$

where $\boldsymbol{\varepsilon}^e$, $\boldsymbol{\varepsilon}^{\text{therm}}$, and $\boldsymbol{\varepsilon}^p$ are elastic strain, thermal strain, and plastic strain, respectively. Therefore, the rate equation for total strain can be written as,

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^{\text{therm}} + \dot{\boldsymbol{\varepsilon}}^p \quad (4.6b)$$

where $\dot{\boldsymbol{\varepsilon}}$, $\dot{\boldsymbol{\varepsilon}}^e$, $\dot{\boldsymbol{\varepsilon}}^{\text{therm}}$, and $\dot{\boldsymbol{\varepsilon}}^p$ are the rates of change of total strain, elastic strain, thermal strain, and plastic strain, respectively. Integrating equation (4.6b) over the time interval of $[t_n, t_{n+1}]$ results in,

$$\begin{aligned} \int_t^{t+\Delta t} \dot{\boldsymbol{\varepsilon}} dt &= \int_t^{t+\Delta t} \dot{\boldsymbol{\varepsilon}}^e dt + \int_t^{t+\Delta t} \dot{\boldsymbol{\varepsilon}}^{\text{therm}} dt + \int_t^{t+\Delta t} \dot{\boldsymbol{\varepsilon}}^p dt \\ \Rightarrow \Delta \boldsymbol{\varepsilon} &= \Delta \boldsymbol{\varepsilon}^e + \Delta \boldsymbol{\varepsilon}^{\text{therm}} + \Delta \boldsymbol{\varepsilon}^p \end{aligned} \quad (4.6c)$$

Again, from the known temperature increment ΔT

$$\Delta \boldsymbol{\varepsilon}^{\text{therm}} = \alpha_s \Delta T \mathbf{I} \quad (4.6d)$$

where $\mathbf{I} = \{1 \ 1 \ 1 \ 0 \ 0 \ 0\}^T$. The plastic strain increment ($\Delta \boldsymbol{\varepsilon}^p$) is given by

$$\Delta \boldsymbol{\varepsilon}^p = \Delta \Lambda \mathbf{r}(\boldsymbol{\sigma}_{n+1}, p_{cT}^{n+1}) \quad (4.6e)$$

where $\mathbf{r}(\boldsymbol{\sigma}_{n+1}, p_{cT}^{n+1})$ is the plastic flow tensor. The increment in elastic strain over the time interval $[t_n, t_{n+1}]$ can be given by

$$\Delta \boldsymbol{\varepsilon}^e = \boldsymbol{\varepsilon}_{n+1}^e - \boldsymbol{\varepsilon}_n^e \quad (4.6f)$$

where $\boldsymbol{\varepsilon}_n^e$, and $\boldsymbol{\varepsilon}_{n+1}^e$ are the elastic strain tensor at time t_n and t_{n+1} , respectively. The elastic strain at time t_{n+1} can be derived from the Gibbs free energy potential (g_1) (equation (A.2)) as

$$\boldsymbol{\varepsilon}_{n+1}^e = - \frac{\partial g_1(\boldsymbol{\sigma}_{n+1})}{\partial \boldsymbol{\sigma}_{n+1}} \quad (4.6g)$$

Substituting equations (4.6d)-(4.6g) in equation (4.6b) results in

$$-\frac{\partial g_1(\boldsymbol{\sigma}_{n+1})}{\partial \boldsymbol{\sigma}_{n+1}} + \Delta \Lambda \mathbf{r}(\boldsymbol{\sigma}_{n+1}, p_{cT}^{n+1}) - (\boldsymbol{\varepsilon}_n^e + \Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}^{\text{therm}}) = 0 \quad (4.6h)$$

The term at the right-most part in the left-hand side of equation (4.6h) that consists of $\boldsymbol{\varepsilon}_n^e$, $\Delta \boldsymbol{\varepsilon}$, and $\Delta \boldsymbol{\varepsilon}^{\text{therm}}$ are known terms. Other terms in equation (4.6h) are functions of $\boldsymbol{\sigma}_{n+1}$, p_{cT}^{n+1} , and $\Delta \Lambda$, which are the unknown terms at the end of the time increment Δt . Therefore, $\boldsymbol{\varepsilon}^{\text{e,trial}}$ is defined as trial strain that is given by

$$\boldsymbol{\varepsilon}^{\text{e,trial}} = (\boldsymbol{\varepsilon}_n^e + \Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}^{\text{therm}}) \quad (4.6i)$$

The trial strain remains unchanged for time increment Δt . But the unknown variables ($\boldsymbol{\sigma}_{n+1}$, p_{cT}^{n+1} , and $\Delta \Lambda$) are update iteratively to satisfy equation (4.6h). Thus, a tensor residual equation for strain is defined as

$$\mathbf{R}_1 = -\frac{\partial g_1(\boldsymbol{\sigma}_{n+1})}{\partial \boldsymbol{\sigma}_{n+1}} + \Delta \Lambda \mathbf{r}(\boldsymbol{\sigma}_{n+1}, p_{cT}^{n+1}) - \boldsymbol{\varepsilon}^{\text{e,trial}} \quad (4.7)$$

4.4.2 Residual equation for pre-consolidation pressure

The change in hardening variable (pre-consolidation pressure) over the time interval of $[t_n, t_{n+1}]$ is given by integrating equation (4.5) as

$$\int_t^{t+\Delta t} \frac{\dot{p}_{cT}}{p_{cT}} dt = \frac{1+e_0}{\lambda-\kappa} \int_t^{t+\Delta t} \dot{\varepsilon}_v^p dt - \mu \int_t^{t+\Delta t} \dot{T} dt \quad (4.8a)$$

where $\dot{p}_{cT}$ is the rate of change of pre-consolidation pressure, $\dot{\varepsilon}_v^p$ is the rate of change of plastic volumetric strain, $\dot{T}$ is the rate of change of temperature. Integrating equation (4.8a) the following equation is obtained as

$$p_{cT}^{n+1} = p_{cT}^n \exp\left(\frac{1+e_0}{\lambda-\kappa} \Delta \varepsilon_v^p - \mu \Delta T\right) \quad (4.8b)$$

where $\Delta \varepsilon_v^p$ is the change of plastic volumetric strain for the time increment Δt which is given by

$$\Delta \varepsilon_v^p = \Delta \Lambda r_p(\boldsymbol{\sigma}_{n+1}, p_{cT}^{n+1}) \quad (4.8c)$$

where r_p is the part of plastic flow defined from the mean stress p . Therefore, the residual equation for pre-consolidation pressure that needs to be minimized is given in dimensionless form as,

$$R_2 = \frac{p_{cT}^{n+1}}{p_{cT}^n} - \exp\left(\frac{1+e_0}{\lambda-\kappa} \Delta\Lambda r_p - \mu\Delta T\right) \quad (4.9)$$

4.4.3 Residual equation for the consistency condition

The definition of yield function requires the following consistency condition as,

$$y(\boldsymbol{\sigma}_{n+1}, p_{cT}^{n+1}) = 0 \quad (4.10)$$

If the stress state is outside the yield surface, then the value of $y > 0$. Therefore, for $\boldsymbol{\sigma}_{n+1}$ and p_{cT}^{n+1} the yield function y needs to be minimized which results in the following residual equation as,

$$R_3 = y(\boldsymbol{\sigma}_{n+1}, p_{cT}^{n+1}) \quad (4.11)$$

4.4.4 Linearization of the residual equations

The residual equations (equations (4.7), (4.9), and (4.11)) need to be minimized and solved simultaneously. Linearization of equations (4.7), (4.9), and (4.11) can be shown as

$$\begin{aligned} \ell R_1 &= \mathbf{R}_1 + \frac{\partial \mathbf{R}_1}{\partial \boldsymbol{\sigma}_{n+1}} \delta \boldsymbol{\sigma} + \frac{\partial \mathbf{R}_1}{\partial p_{cT}^{n+1}} \delta p_{cT} + \frac{\partial \mathbf{R}_1}{\partial \Delta\Lambda} \delta \Delta\Lambda = 0 \\ \ell R_2 &= R_2 + \frac{\partial R_2}{\partial \boldsymbol{\sigma}_{n+1}} \delta \boldsymbol{\sigma} + \frac{\partial R_2}{\partial p_{cT}^{n+1}} \delta p_{cT} + \frac{\partial R_2}{\partial \Delta\Lambda} \delta \Delta\Lambda = 0 \\ \ell R_3 &= R_3 + \frac{\partial R_3}{\partial \boldsymbol{\sigma}_{n+1}} \delta \boldsymbol{\sigma} + \frac{\partial R_3}{\partial p_{cT}^{n+1}} \delta p_{cT} + \frac{\partial R_3}{\partial \Delta\Lambda} \delta \Delta\Lambda = 0 \end{aligned} \quad (4.12)$$

where ℓ and δ represent, respectively, the linearization operator and the increment of the unknown variables.. Equation (4.12) can be rewritten as,

$$\begin{bmatrix} \frac{\partial \mathbf{R}_1}{\partial \boldsymbol{\sigma}_{n+1}} & \frac{\partial \mathbf{R}_1}{\partial p_{cT}^{n+1}} & \frac{\partial \mathbf{R}_1}{\partial \Delta\Lambda} \\ \frac{\partial R_2}{\partial \boldsymbol{\sigma}_{n+1}} & \frac{\partial R_2}{\partial p_{cT}^{n+1}} & \frac{\partial R_2}{\partial \Delta\Lambda} \\ \frac{\partial R_3}{\partial \boldsymbol{\sigma}_{n+1}} & \frac{\partial R_3}{\partial p_{cT}^{n+1}} & \frac{\partial R_3}{\partial \Delta\Lambda} \end{bmatrix} \begin{Bmatrix} \delta \boldsymbol{\sigma} \\ \delta p_{cT} \\ \delta \Delta\Lambda \end{Bmatrix} = - \begin{Bmatrix} \mathbf{R}_{11} \\ R_{21} \\ R_{31} \end{Bmatrix} \quad (4.13)$$

$$\Rightarrow \mathbf{J}\mathbf{v} = \mathbf{R}$$

where

$$\mathbf{J} = \begin{bmatrix} \frac{\partial \mathbf{R}_1}{\partial \boldsymbol{\sigma}_{n+1}} & \frac{\partial \mathbf{R}_1}{\partial p_{cT}^{n+1}} & \frac{\partial \mathbf{R}_1}{\partial \Delta\Lambda} \\ \frac{\partial R_2}{\partial \boldsymbol{\sigma}_{n+1}} & \frac{\partial R_2}{\partial p_{cT}^{n+1}} & \frac{\partial R_2}{\partial \Delta\Lambda} \\ \frac{\partial R_3}{\partial \boldsymbol{\sigma}_{n+1}} & \frac{\partial R_3}{\partial p_{cT}^{n+1}} & \frac{\partial R_3}{\partial \Delta\Lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} & \mathbf{J}_{13} \\ \mathbf{J}_{21} & J_{22} & J_{23} \\ \mathbf{J}_{31} & J_{32} & J_{33} \end{bmatrix} \quad (4.13a)$$

$$\mathbf{v} = \begin{Bmatrix} \delta \boldsymbol{\sigma} \\ \delta p_{cT} \\ \delta \Delta\Lambda \end{Bmatrix} \quad (4.13b)$$

$$\mathbf{R} = - \begin{Bmatrix} \mathbf{R}_1 \\ R_2 \\ R_3 \end{Bmatrix} \quad (4.13c)$$

where $\mathbf{J}_{11}$, $\mathbf{J}_{12}$, $\mathbf{J}_{13}$, $\mathbf{J}_{21}$, J_{22} , J_{23} , $\mathbf{J}_{31}$, J_{32} , and J_{33} are the derivatives of the residuals defined in Appendix A. The unknown increments at each iteration are solved as

$$\mathbf{v} = \mathbf{J}^{-1}\mathbf{R} \quad (4.14)$$

The unknown increments obtained from equation (4.14) are added to the values of $\boldsymbol{\sigma}_n^{(i)}$, $p_{cT}^{n(i)}$, and $\Delta\Lambda_{(i)}$, where i denotes the i -th iteration in the algorithm of minimizing the residuals for any time interval $[t_n, t_{n+1}]$ with time increment Δt . The iteration procedure continues until the yield function (y) comes within

an acceptable tolerance range (10^{-6} in this case). The stress integration algorithm is described in section 4.4.6.

4.4.5 Consistent tangent stiffness matrix

The consistent tangent elastoplastic stiffness matrix that is derived from the convergence of residual equations is given by Golchin, (2023) as,

$$\mathbf{C}_{ep} = \mathbf{w} - \mathbf{w} \frac{\boldsymbol{\chi} \mathbf{w} \mathbf{r}^*}{\bar{\chi} J_{32} J_{23} + \boldsymbol{\chi} \mathbf{w} \mathbf{r}^*} \quad (4.15)$$

where

$$\mathbf{w} = \left[\mathbf{J}_{11} - \frac{\mathbf{J}_{12} \mathbf{J}_{21}}{J_{22}} \right]^{-1} \quad (4.15a)$$

$$\boldsymbol{\chi} = \mathbf{J}_{31} - \left(\frac{J_{32}}{J_{22}} \right) \mathbf{J}_{21} \quad (4.15b)$$

$$\mathbf{r}^* = \mathbf{J}_{13} - \left(\frac{J_{23}}{J_{22}} \right) \mathbf{J}_{12} \quad (4.15c)$$

$$\bar{\chi} = \frac{1}{J_{22}} \quad (4.15d)$$

The elastoplastic consistent tangent matrix given by equation (4.15) at the end of any time increment t_n is used in the algorithm for the energy pile response under different loading conditions that is described in section 3.7.

4.4.6 Subroutine for stress integration

At the beginning of a typical time increment denoted as t_n , the stress $\boldsymbol{\sigma}_n$, pre-consolidation pressure p_{cT}^n , elastic strain $\boldsymbol{\epsilon}_n^e$, and temperature T_n are known. For time increment Δt ($t_{n+1} = t_n + \Delta t$), the strain increment $\Delta \boldsymbol{\epsilon}$ and temperature increment ΔT are known. The steps of the stress integration algorithm are given below,

START OF THE ALGORITHM

Step-1:

- i) Update the temperature for the step time Δt , $T_{n+1} = T_n + \Delta T$
- ii) Calculate the thermo-elastic strain ($\Delta \boldsymbol{\varepsilon}^{\text{therm}}$) using equation (4.6d)
- iii) Calculate the critical state stress ratio (M) using equation (4.3)

Step-2: Calculate the trial strain ($\boldsymbol{\varepsilon}^{\text{e,trial}}$) using equation (4.6i)

Elastic predictor:

Step-3: Calculate the trial stress ($\boldsymbol{\sigma}^{\text{trial}}$) as, $\boldsymbol{\sigma}^{\text{trial}} = \mathbf{C}_e \boldsymbol{\varepsilon}^{\text{e,trial}}$ where $\mathbf{C}_e$ is the elastic stiffness tensor

Step-4: Set the initial values of variables by assuming elastic conditions

- i) The increment of plastic multiplier is set to zero ($\Delta \Lambda = 0$)

Step-5: Check the yield function value with the trial stress ($\boldsymbol{\sigma}^{\text{trial}}$).

IF $y(\boldsymbol{\sigma}^{\text{trial}}) \leq \text{TOL}$, then

- i) $\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}^{\text{trial}}$
- ii) $p_{cT}^{n+1} = p_{cT}^n$
- iii) $\Delta \Lambda = 0$
- iv) $\mathbf{C}_{ep} = \mathbf{C}_e$

ELSE

Plastic corrector (iterative process):

1. DO WHILE $y(\boldsymbol{\sigma}^{\text{trial}}) \leq \text{TOL}$ (10^{-6})
 - i) Calculate trial stress invariants (p_i, q_i where i is the plastic correction iteration number)
 - ii) Calculate the flow rule
 - iii) Calculate the $\mathbf{J}$ matrix (Appendix A)
 - iv) Calculate the residual tensor ($\mathbf{R}$) (Appendix A)
 - v) Calculate the increments of unknowns ($\mathbf{v}$) using equation (4.14)
 - vi) Update the unknowns
 - a) $\boldsymbol{\sigma}_{i+1} = \boldsymbol{\sigma}_i + \delta \boldsymbol{\sigma}$
 - b) $p_{cT}^{i+1} = p_{cT}^i + \delta p_{cT}$
 - c) $\Delta \Lambda_{i+1} = \Delta \Lambda_i + \delta \Delta \Lambda$

END DO

2. $\sigma_{n+1} = \sigma_{i+1}$, and $p_{cT}^{n+1} = p_{cT}^{i+1}$ based on the values calculated in the last iteration that converges in the WHILE loop
3. Calculate C_{ep} using equation (4.15)

END IF

END OF THE ALGORITHM

Thus, the stress the stress σ_{n+1} , pre-consolidation pressure p_{cT}^{n+1} , consistent tangent operator C_{ep} , and elastic strain ϵ_{n+1}^e at time t_{n+1} are obtained.

4.5 Solution procedure

The solution procedure of the semi-analytical framework with the thermo-mechanical soil constitutive model is the same as described in section 3.7. The parameters $\bar{K}_s$ and $\bar{G}_s$ are required to perform the stress integration which can be calculated at the beginning of the global algorithm from calculating the geostatic stress. The mean stress p at each node can be calculated from the geostatic stress tensor at each node and then using equations (4.1a) and (4.1b), $\bar{K}_s$ and $\bar{G}_s$ can be calculated from the known initial bulk modulus (K_{s0}) and shear modulus (G_{s0}) values. For the nodes at the ground level, the geostatic stress can be calculated at a distance $0.5\Delta z$ from the ground. The initial value of p_{cT} is calculated from the geostatic stress and the overconsolidation ratio of the soil. The solution algorithm described for energy pile analysis with elastoplastic soil model in section 3.7 can be used with the present thermo-mechanical soil constitutive model also. The changes will be in the stress integration algorithm which is different than that mentioned in the section 3.7. The updated stress and consistent elastoplastic tangent tensor that is calculated using the algorithm described in section 4.4.6 is used in the solution algorithm (instead of the stress and elastoplastic tangent operator for Drucker-Prager model) given in section 3.7 to include the effect of the present thermo-mechanical response of soil in the energy pile analysis.

4.6 Validation of the constitutive model

The stress-strain response of different soil samples under thermo-mechanical loading that is obtained by different researchers in different laboratory experiment and numerical simulations are predicted using the present thermo-mechanical constitutive model developed by Golchin et al., (2022b) where the stress integration algorithm described in section 4.4 is used. The results are plotted in

following subsections with corresponding discussions. The input model parameters are provided by Golchin et al., (2022b) and are stated in Table 4-1. For the experiment and simulations conducted by Yu, (1998), the parameters are provided by the author.

Table 4-1- Constitutive model parameters

Parameters	Abuel-Naga <i>et. al.</i> (2006, 2007)	Graham <i>et. al.</i> (2001)	Cekerevac and Laloui (2004)	Uchaipichat and Khalili (2009)	Yu (1998)	
					Experimental	Numerical
α (-)	0.9	0.7	0.6	0.75	1	1
γ (-)	0.4	0.72	1.05	1.1	1	1
λ (-)	0.43	0.087	0.13	0.09	0.093	0.161
κ (-)	0.09	0.017	0.028	0.006	0.025	0.062
M_0 (-)	0.8	1.07	0.82	1.218	0.9	0.888
$\bar{K}_s$ (-)	17.5	59	500	75	65	27.5
$\bar{G}_s$ (-)	15	20	700	170	31	13
n_s (-)	0.7	1.0	0.35	0.95	1	1
m_s (-)	1.0	1.0	0.7	0.85	1	1
α_s (1/°C)	3.1×10^{-6}	1.5×10^{-4}	1.8×10^{-4}	3.4×10^{-5}	-	-
μ (1/°C)	7.7×10^{-3}	7×10^{-2}	7.4×10^{-4}	3.4×10^{-3}	-	-
π^* (1/°C)	0	0	1.05×10^{-3}	0	-	-

4.6.1 Abuel-Naga et al. (2006)

The results of triaxial compression test conducted on Bangkok clay by Abuel-Naga et al., (2006) in terms of mean stress and volumetric strain under isotropic compression are presented in Figure 4-2. The results shown in Figure 4-2 are presented for three different temperatures of 25, 70 and 90° C. In the test, the soil was isotropically compressed to a mean effective stress (p) of 300 kPa at room temperature (25°C) and then unloaded to a mean effective stress 25 kPa that results in an OCR value of 12. Then the sample was heated to 70°C and was recompressed. The same test procedure was followed for the elevated temperature of 90°C. The model parameters are given in Table 4-1. It is observed from Figure 4-2 that, the constitutive model predicts the soil behavior accurately at different temperatures. The elastoplastic behavior of soil at room temperature begins at $p = 300$ kPa whereas for higher temperatures, the elastoplastic response starts at lower p values indicating the shrinkage of yield

surface with the increase of temperature. The results obtained by Golchin et al., (2022b) are also shown in Figure 4-2. This proves the solution algorithm provides accurate results in the elemental tests.

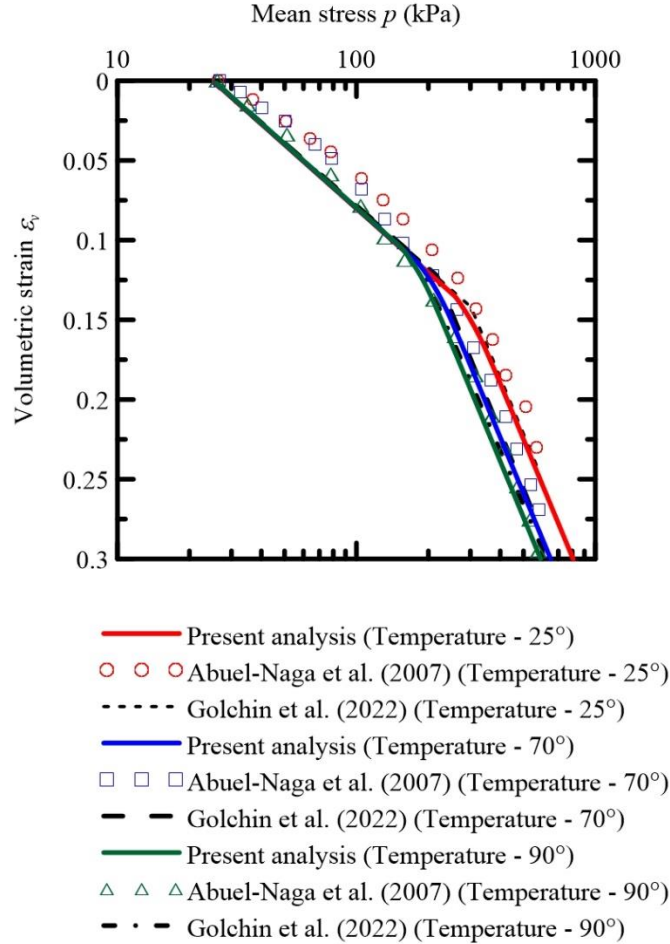
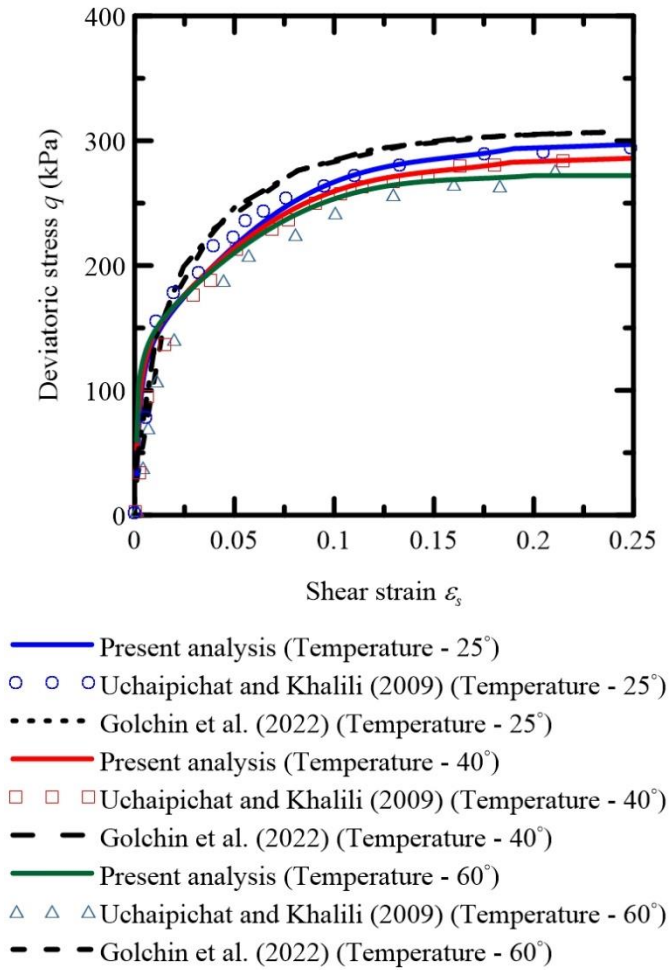


Figure 4-2 Comparison of model predictions of volumetric strain vs. mean stress, during isotropic compression at different temperatures, with experimental results of Abuel-Naga et al. (2006).

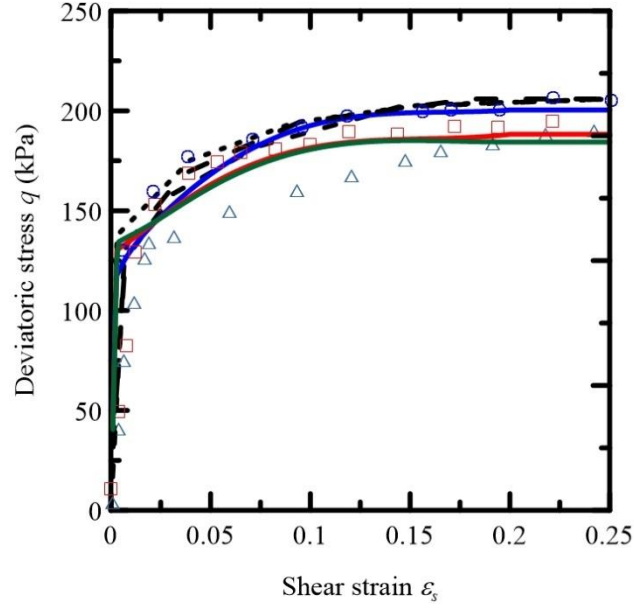
4.6.2 Uchaipichat and Khalili (2009)

The test results regarding the shear response (in terms of deviatoric stress and shear strain) of silty clay samples at different temperatures that has been obtained during the experiment by Uchaipichat & Khalili, (2009), are reproduced using the thermo-mechanical constitutive model and are shown in Figure 4-3. The pre-consolidation pressure in this case is 200 kPa. The samples were kept in loaded conditions with OCR values 1.33 and 2 at ambient temperature of 25°C. The temperature of the samples

was increased to 40 and 60°C. The shearing of the samples was done in drained condition. The present solution algorithm produces a good match with the experimental results as evident from Figure 4-3 at different temperatures for different OCRs. The results obtained by the developer of the constitutive model are also shown in Figure 4-3.



(a)



- Present analysis (Temperature - 25°)
- ○ Uchaipichat and Khalili (2009) (Temperature - 25°)
- Golchin et al. (2022) (Temperature - 25°)
- Present analysis (Temperature - 40°)
- □ Uchaipichat and Khalili (2009) (Temperature - 40°)
- Golchin et al. (2022) (Temperature - 40°)
- Present analysis (Temperature - 60°)
- △ △ Uchaipichat and Khalili (2009) (Temperature - 60°)
- Golchin et al. (2022) (Temperature - 60°)

(b)

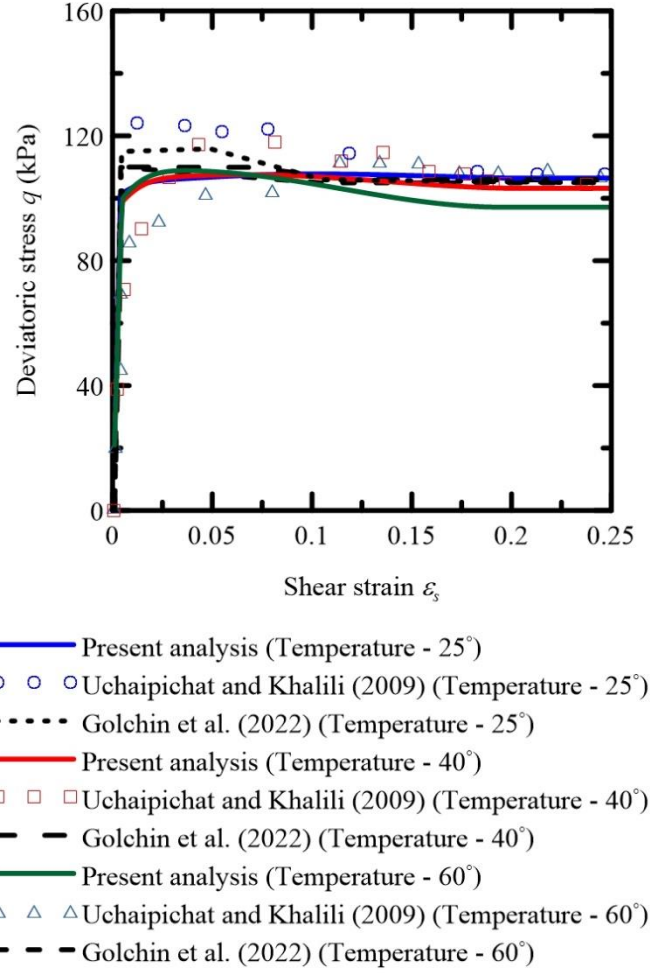


Figure 4-3 Comparison of model predictions of deviatoric stress vs. shear strain at different temperatures with experimental results of Uchaipichat & Khalili (2009) for a) OCR = 1.33, b) OCR = 2, c) OCR = 4

4.6.3 Graham et al. (2001)

The thermo-mechanical response of a clay that was described by Graham et al., (2001) is used to verify the accuracy of the present thermo-mechanical constitutive model. The clay sample in the analysis has a pre-consolidation pressure of 3000 kPa with an OCR value of 7.5. The sample was sheared at an elevated temperature of 65°C from its initial temperature of 28°C. The deviatoric stress - axial strain plot from the present constitutive model matches well with the results obtained by Graham

et al., (2001) (Figure 4-4). It is also to mention that, in the both the cases, the deviatoric stress are higher than the experimental values.

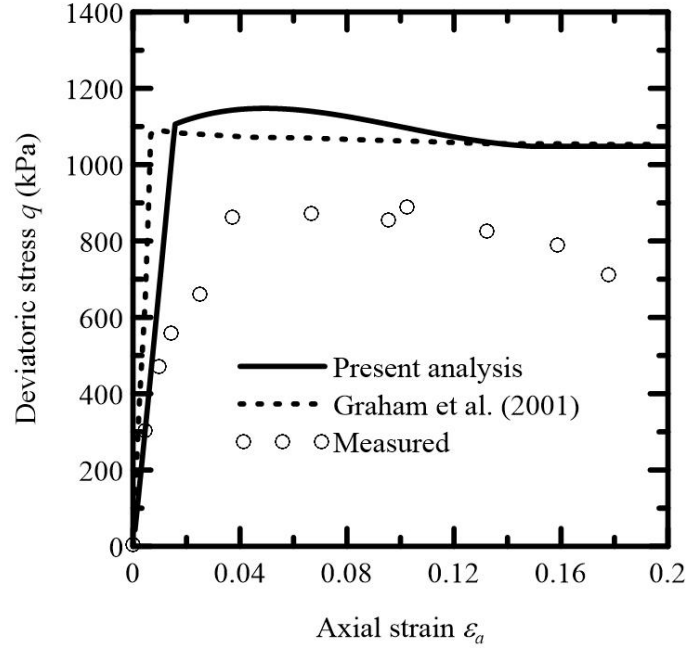
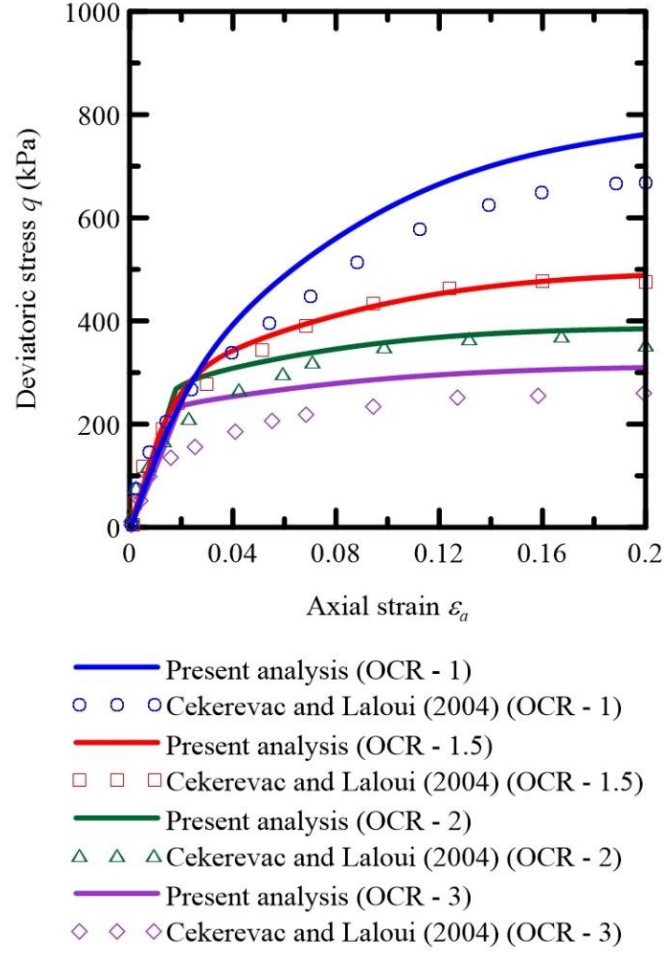


Figure 4-4 Comparison of model predictions of deviatoric stress vs. axial strain with results from Graham et al. (2001)

4.6.4 Cekerevac and Laloui (2004)

The thermo-mechanical response of kaolin clay that was observed by Cekerevac & Laloui, (2004) is reproduced with the present stress integration algorithm and shown in Figure 4-5(a) and Figure 4-5 (b). The pre-consolidation pressure of the clay sample at an ambient temperature of 22° C was 600 kPa. The soil has been unloaded to 400, 300, and 200 kPa to achieve OCR values of 1.5, 2, and 3, respectively. The temperature of the samples was increased from 22°C to 90°C under drained conditions and the samples were sheared. The model parameters provided by Golchin et al., (2022b) are given in Table 4-1. The model predictions for the two temperatures are compared with the experimental data in Figure 4-5(a) and Figure 4-5 (b). It is observed from Figure 4-5(a)-(b) that the failure shear stresses of samples increase at elevated temperature.



(a)

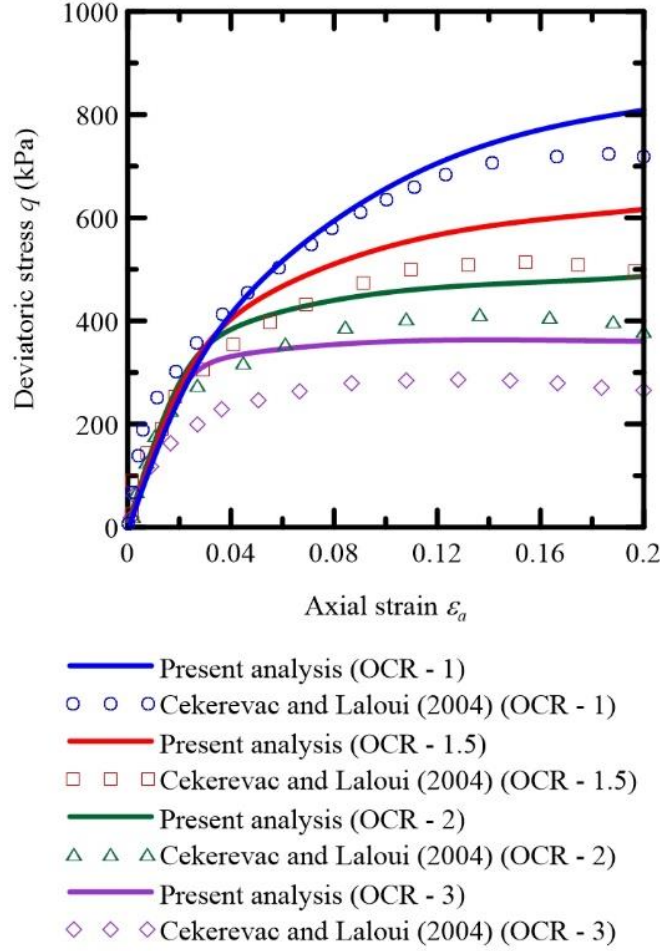


Figure 4-5 Comparison of model predictions of deviatoric stress vs. axial strain for different OCR with experimental results of Cekerevac & Laloui (2004) at temperature a) 22°, and b) 90°

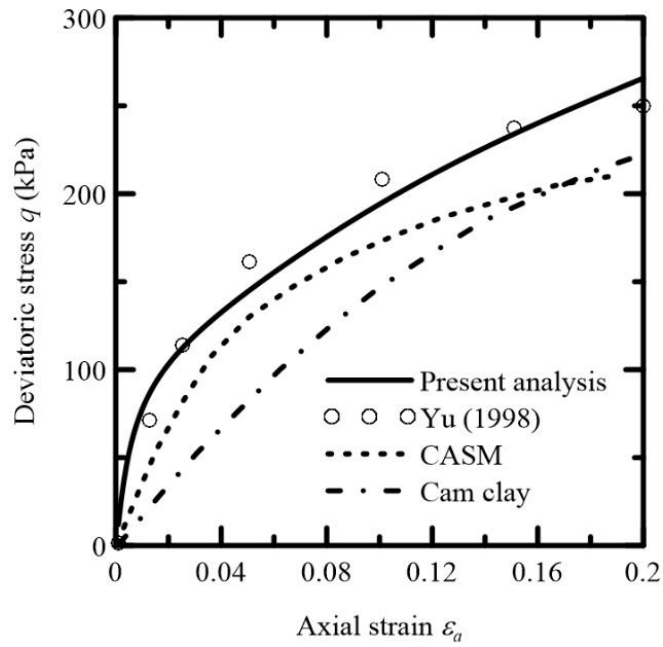
4.6.5 Yu (1998)

The experimental and simulated values of shear stress with axial strain that was obtained by Yu, (1998) are presented in Figure 4-6(a) and Figure 4-6(b), respectively with the results obtained from the present thermo-mechanical constitutive model. The results with CASM and Cam-clay constitutive model given by Yu, (1998) are also shown in Figure 4-6(a). The model parameters presented in

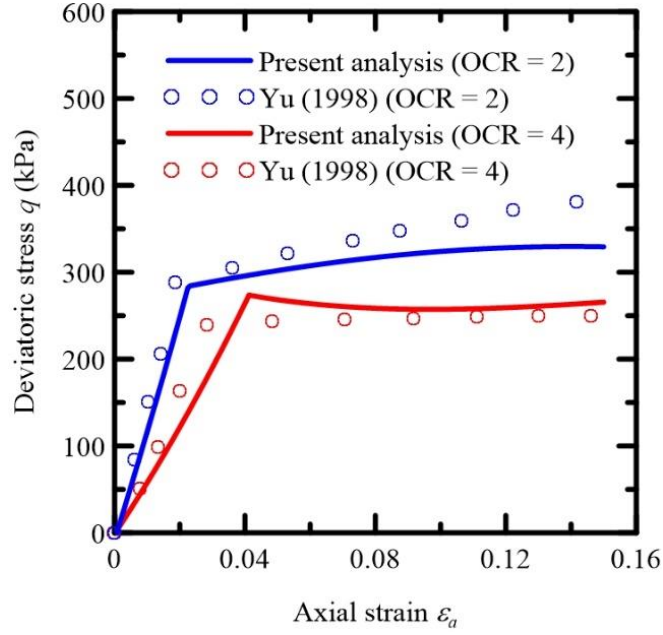
Table 4-1 are derived from the data provided by Yu, (1998). The model parameters (α^* , μ , π^*) associated with the thermal part of the constitutive model are considered zero as no temperature variation is present in the experiment. Both the stress dependency parameters (m , n) are considered 1.

The results shown in Figure 4-6(a) are for normally consolidated clay. The present thermo-mechanical model represents the shear response quite well when compared with the experimental results.

Figure 4-6(b) shows the shear stress - axial strain plot for clay with different OCR values (2, 4) as obtained from the simulations by Yu, (1998) and from the present analysis. The input model parameters are given in Table 4-1. The present analysis predicts the shear response quite accurately for different OCRs as evident from Figure 4-6(b).



(a)



(b)

Figure 4-6 Comparison of model predictions of deviatoric stress vs. axial strain with a) experimental results, and b) results from numerical analysis given by Yu (1998)

4.7 Validation of the present framework for energy piles

The response of the energy pile obtained from the present semi-analytical framework with the soil modeled by the thermo-mechanical constitutive relationship is compared with the experimental results available in the literature. Thus, the accuracy of the present method with a more complex soil constitutive model is verified.

4.7.1 Bourne-Webb et al. (2009)

The results from the field test on the energy pile conducted by Bourne-Webb et al., (2009) is compared with the results obtained from the present analysis to validate the accuracy of the present method with the soil behavior modelled by the thermo-mechanical constitutive relationship. An energy pile of length (L_p) 23 m and diameter (D_p) 0.55 m with Young's modulus (E_p) 40 GPa, thermal expansion coefficient (α_p) 8.5×10^{-6} /°C, and thermal conductivity (κ_p) 2.33 W/m/°C is embedded primarily in London clay. There is a made-up ground of height 1.5m below the ground level (layer-1), the behavior of which is considered to be elastic. Below that made-up ground, a terrace gravel layer of

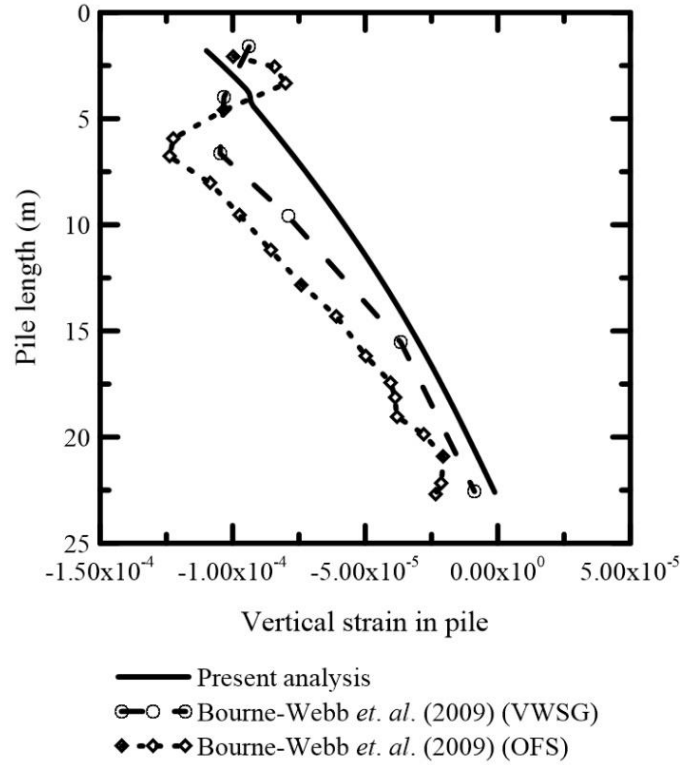
height 2.5 m is present (layer-2) which is modelled to behave as nonlinear elastic material where the initial shear modulus (G_{s0}) and bulk modulus (K_{s0}) are provided (Gawecka et al., 2016) (the modulus change with mean stress (p) as given by equations (4.1a) and (4.1b)). Layer-3 soil is London clay that extends upto a great depth. The thermal and mechanical properties of soil and the pile is suggested by Gawecka et al., (2016). The OCR is considered 2 in the analysis. The critical state properties of soil are obtained from the data provided by Gasparre, (2005) . The input details for the analysis are given in Table 4-2.

Table 4-2- Soil properties and constitutive model parameters

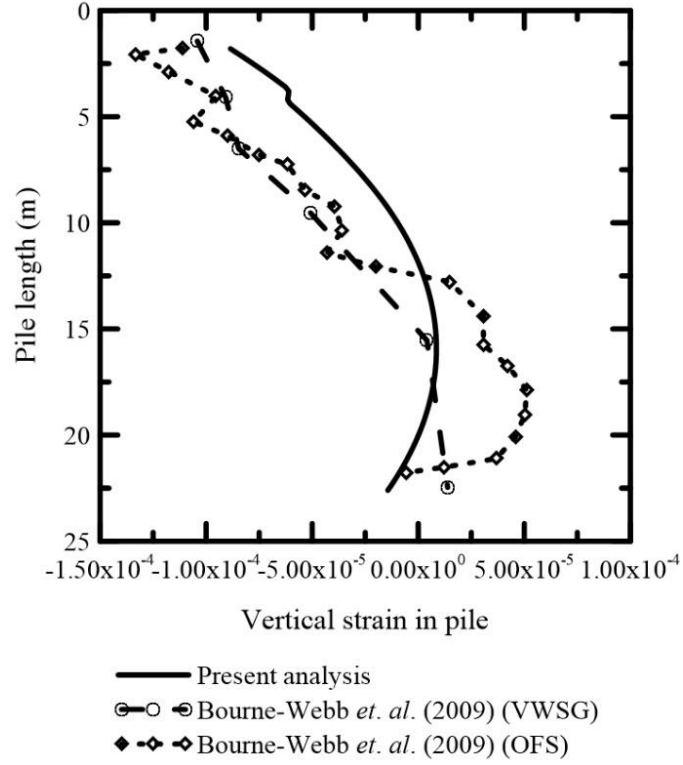
Parameters	Layer-1 (made-up ground)	Layer-2 (terrace gravel)	Layer-3 (London clay)
Unit weight γ_s (kN/m ³)	18	20	20
κ_s (W/m/°C)	1.40	1.40	1.79
a_s (m ² /s)	7.4×10^{-7}	7.4×10^{-7}	9.8×10^{-7}
α (-)	-	-	1
γ (-)	-	-	1
λ (-)	-	-	0.17
κ (-)	-	-	0.06
M_0 (-)	-	-	0.83
K_{s0} (MPa)	5.6	50	27
G_{s0} (MPa)	4.2	42	52
n_s (-)	-	1.0	1.0
m_s (-)	-	1.0	1.0
α_s (1/°C)	1.7×10^{-5}	1.7×10^{-5}	1.7×10^{-5}
μ (1/°C)	-	-	8×10^{-3}
π^* (1/°C)	-	-	0

The pile is loaded to a maximum axial load of 1800 kN and then it is unloaded to 1200 kN. The thermal load of 20°C is applied to the pile ($\Delta T_p = -20^\circ\text{C}$) while the mechanical load remains constant at 1200 kN. Then the pile is heated, and a temperature increase of +29°C is applied. The vertical strain

profiles along the length of the pile obtained from the present analysis alongwith the results from the field experiment (obtained by both vibratory wire strain gauge (VWSG) and optical fiber sensors (OFS)) are plotted in Figure 4-7(a)-(c) for mechanical load only, at the end of the cooling stage (mechanical load + ($\Delta T_p = -20^\circ\text{C}$), and at the end of the heating stage (mechanical load + ($\Delta T_p = 29^\circ\text{C}$)). The strain profiles from the present analysis match quite well with the experimental results. The top two layers of soils are not fine-grained soil. Therefore, those are modelled as elastic material. As a result, some differences in analytical and experimental results are obtained at the top of the pile. Also, from the experimental results, it is observed that the made-up ground provides some negative skin friction along the top most part of the pile which is out of the scope of the present model.



(a)



(b)

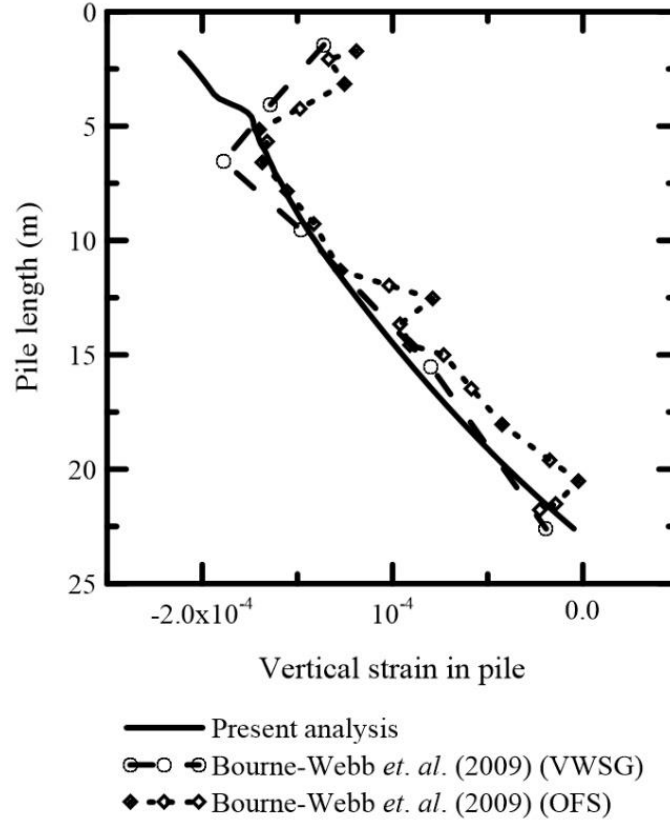
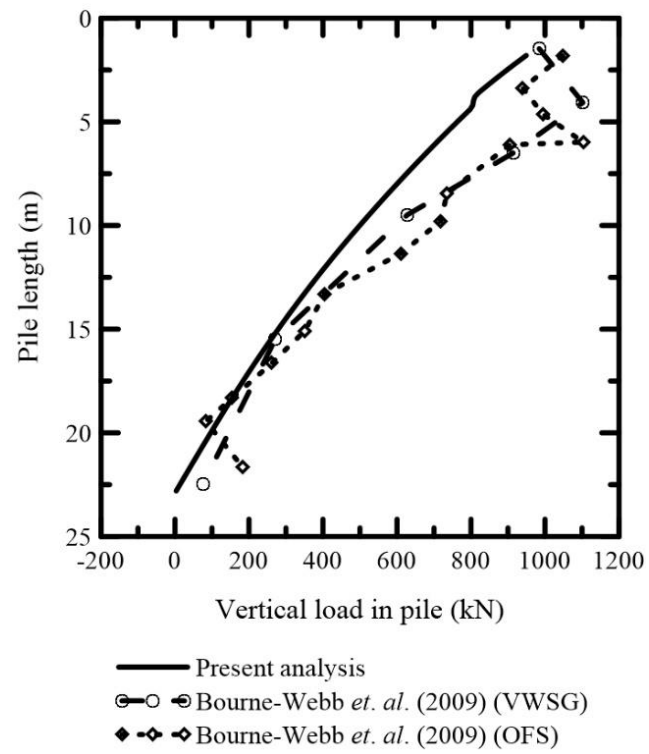
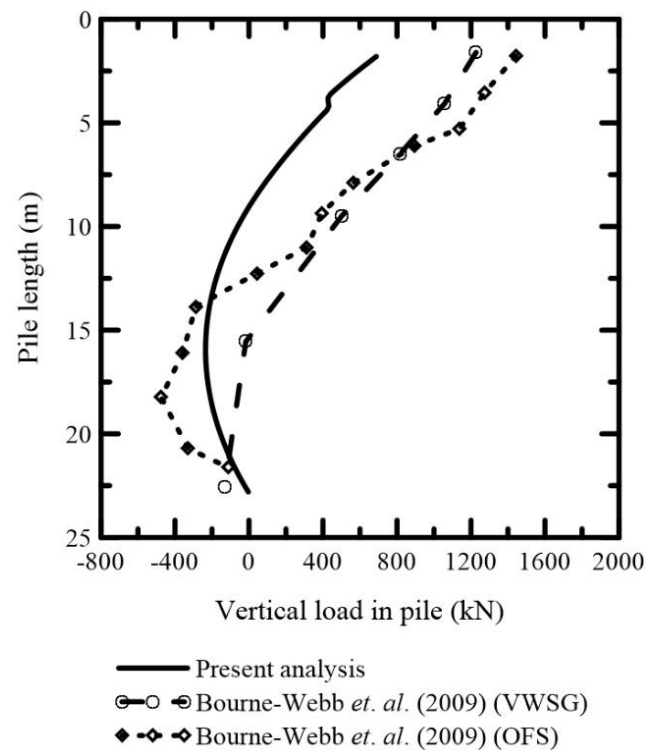


Figure 4-7 Vertical strain profiles along the pile length a) under mechanical load only, b) at the end of cooling ($\Delta T_p = -20^\circ\text{C}$), and c) at the end of heating ($\Delta T_p = 30^\circ\text{C}$) as obtained from the present analysis and field test

The axial load profiles along the length of the pile are plotted in Figure 4-8(a)-(c) for both the analytical solution and the field results. The differences in maximum axial loads obtained from the analytical solution and field test are $<10\%$ for mechanical and mechanical + heating loading conditions. For cooling, the present model underestimates the maximum axial load.



(a)



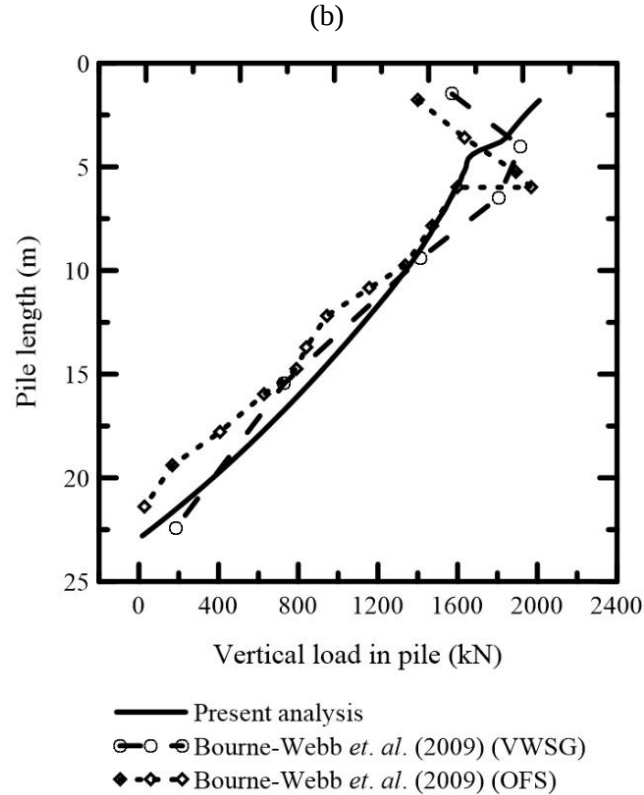
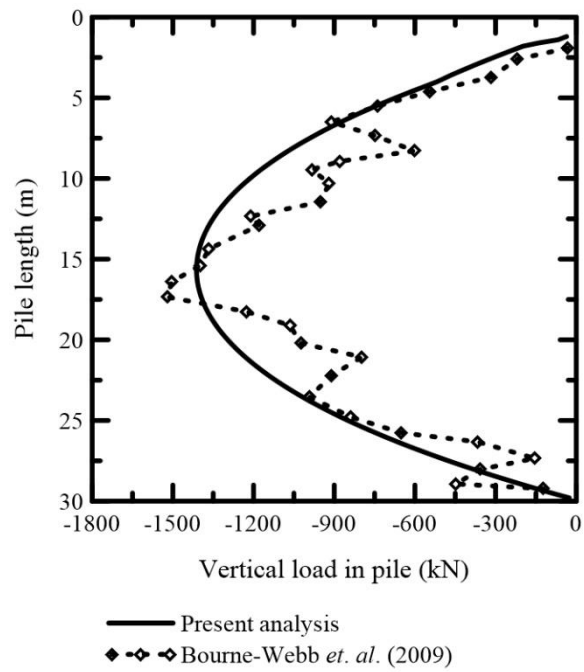
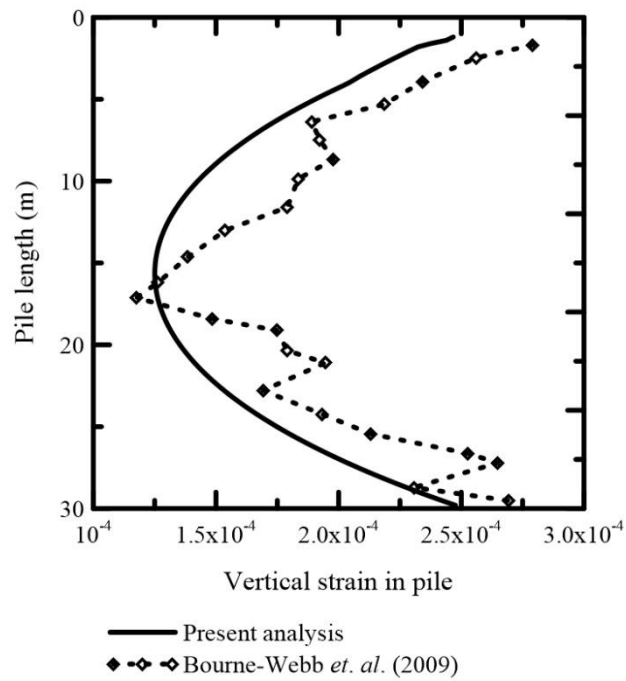


Figure 4-8 Axial load profiles along the pile length a) under mechanical load only, b) at the end of cooling ($\Delta T_p = -20^\circ \text{C}$), and c) at the end of heating ($\Delta T_p = 30^\circ \text{C}$) as obtained from the present analysis and field test

The vertical load and strain profile along the length of sink pile located near the test pile (results of which are shown in Figure 4-7(a)-(c) and Figure 4-8(a)-(c)) are shown in Figure 4-9(a), and Figure 4-9(b), respectively. The sink pile is under thermal load only. The results from the field test are also plotted in Figure 4-9(a), and Figure 4-9(b). Under the thermal loading only, the results from the present method show good match with the field result. The maximum vertical load and strain predicted by present analysis are similar to those obtained during the field test.



(a)



(b)

Figure 4-9 a) Axial strain, and b) axial load profiles of the pile under thermal load ($\Delta T_p = 29.4^\circ$ C), as obtained from the present analysis and field test

4.7.2 Goode and McCartney (2015)

The experimental result obtained by Goode & McCartney, (2015) for a pile foundation embedded in bonny silt and subjected to thermal load is predicted using the present method. In this experiment a 8 m long and 1.5 m diameter concrete pile with Young's modulus (E_p) 33 GPa, thermal expansion coefficient (α_p) 1.6×10^{-5} /°C, and thermal conductivity (κ_p) 2.1 W/m/°C is embedded in Bonny silt and undergoes a temperature change of +18°C. The constitutive model parameters for the analysis are $\alpha = 1$, $\gamma = 1$, $\lambda = 0.0065$, $\kappa = 0.00074$, $M_0 = 1.305$, $K_{s0} = 120$ MPa, $G_{s0} = 40$ MPa, $n_s = 1$, $m_s = 1$, $\alpha_s = 10^{-5}$, $\mu = 7.7 \times 10^{-3}$ /°C, and $\pi^* = 0$. The critical state properties λ and κ are calculated by dividing compression index (C_c) and recompression index (C_r) by 2.3, respectively (Wood, 1991). The M_0 value has been provided by the authors. The initial values of shear modulus and bulk modulus are given by Stewart, (2010). The thermal conductivity of the soil (κ_s) as given by Goode & McCartney, (2015) is 1.147 W/m/°C. The unit weight of soil is 16.9 kN/m³.

The axial displacement along the pile (caused by the temperature increase) predicted by the present analysis with the thermo-mechanical constitutive model match very well with the experimental results. The difference between the head displacement obtained from the present analysis and from the experiment is 8.5%.

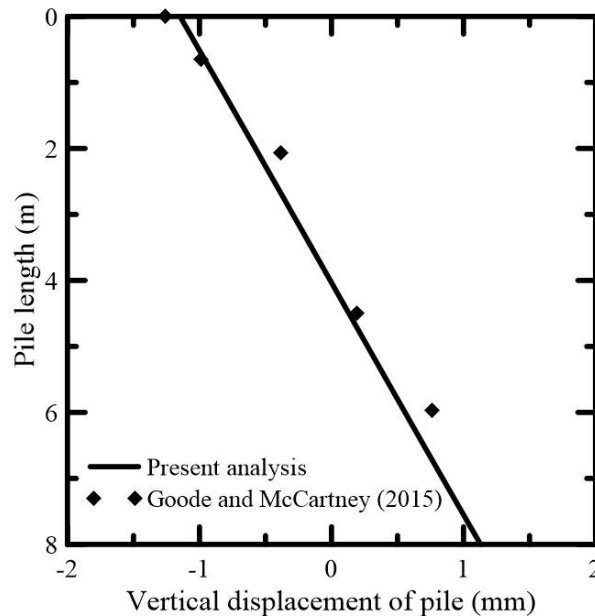


Figure 4-10 Vertical displacement profiles of the pile obtained from the present analysis, and field test subjected to thermal load ($\Delta T_p = +18^\circ \text{C}$)

4.7.3 Laloui et. al. (2006)

The results of the experiment on energy pile conducted by Laloui et al., (2006) has been used for the validation purpose in section 2.8.1.2 (for analysis with elastic soil behavior) and section 3.2 (for analysis with elastoplastic soil behavior). The same experimental results have been used in section 4.2 where the soil is represented by the thermo-mechanical constitutive model. The material properties of pile and soils, thermal properties of pile material and soils, and density of soils (for different layers) are given in Table 2-2. The soils in layer-3 and layer-4 are coarse grained in nature and therefore, in the present analysis it is modelled as nonlinear elastic as the thermo-mechanical constitutive model is suitable for fine-grained soil only. The constitutive mode parameters that are used in analysis are $\alpha = 1$, $\gamma = 1$, $\lambda_1 = \lambda_2 = 0.025$, $\kappa_1 = \kappa_2 = 0.005$, $M_0 = 1$, $K_{s0}^{(1)} = K_{s0}^{(2)} = 122$ MPa, $G_{s0}^{(1)} = G_{s0}^{(2)} = 113$ MPa, $n_s = 1$, $m_s = 1$, $\mu = 9 \times 10^{-3} / ^\circ\text{C}$, and $\pi^* = 0$. The critical state parameter λ and κ are calculated from C_c and C_r , respectively. C_c values are calculated using the equations given by Rendon-Herrero, (1980), that correlates the void ratio and the compression index which is given by $C_c = 0.141G_s \left[(1 + e_0) / G_s \right]^{2.382}$ where $G_s = 2.65$. The void ratio of soils in different layers are provided by Laloui et al., (2006). C_r is calculated as $C_r = 0.15C_c$ (Bowles, 1996).

The vertical strain profiles along the length of the pile under different thermal loadings ($\Delta T_p = +18^\circ\text{C}$ and $\Delta T_p = +3^\circ\text{C}$) are shown in Figure 4-11. The experimental results are also presented in Figure 4-11. The present method with the thermo-mechanical constitutive model produces accurate results for small temperature increment in this problem whereas for higher temperature increment differences exist between the analytical and experimental results although the analytical method can predict the maximum strain accurately.

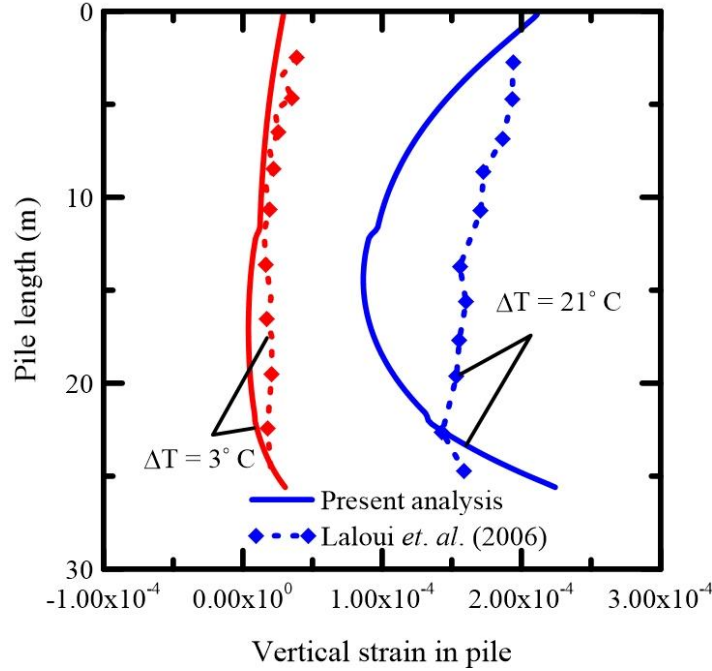


Figure 4-11 Vertical strain profiles of the pile obtained from the present analysis and field test under thermal load (temperature change)

The stress profiles from both the analytical and experimental results are shown in Figure 4-1. The analytical method predicts the stresses quite well except the tip of the pile under thermal and combined loading conditions. Under all the loading conditions the present method predicts the magnitude and location of the maximum stresses accurately, which is crucial for the structural design purpose.

A comparison between the vertical stress profiles obtained from the present analysis (both elastic and elastoplastic) with the field result is shown in Figure 4-12(a)-(c). The results from elastic analysis do not include the modification in shear modulus. The results from elastoplastic analysis that are shown in Figure 4-12 are conducted for both Drucker-Prager (this constitutive model does not include the effect of temperature on the stress-strain behavior of soil) and thermo-mechanical soil constitutive model. It is evident from Figure 4-12(a)-(c) that elastoplastic analysis predicts the pile response better than the elastic analysis by calculating the maximum stress more accurately. Also, the thermo-mechanical soil constitutive model which includes the temperature effect on the stress-strain relationship of the soil predicts more accurate results than the conventional soil constitutive model

(Drucker-Prager). Therefore, the inclusion of a proper thermo-mechanical soil constitutive model is necessary to analyze energy piles.

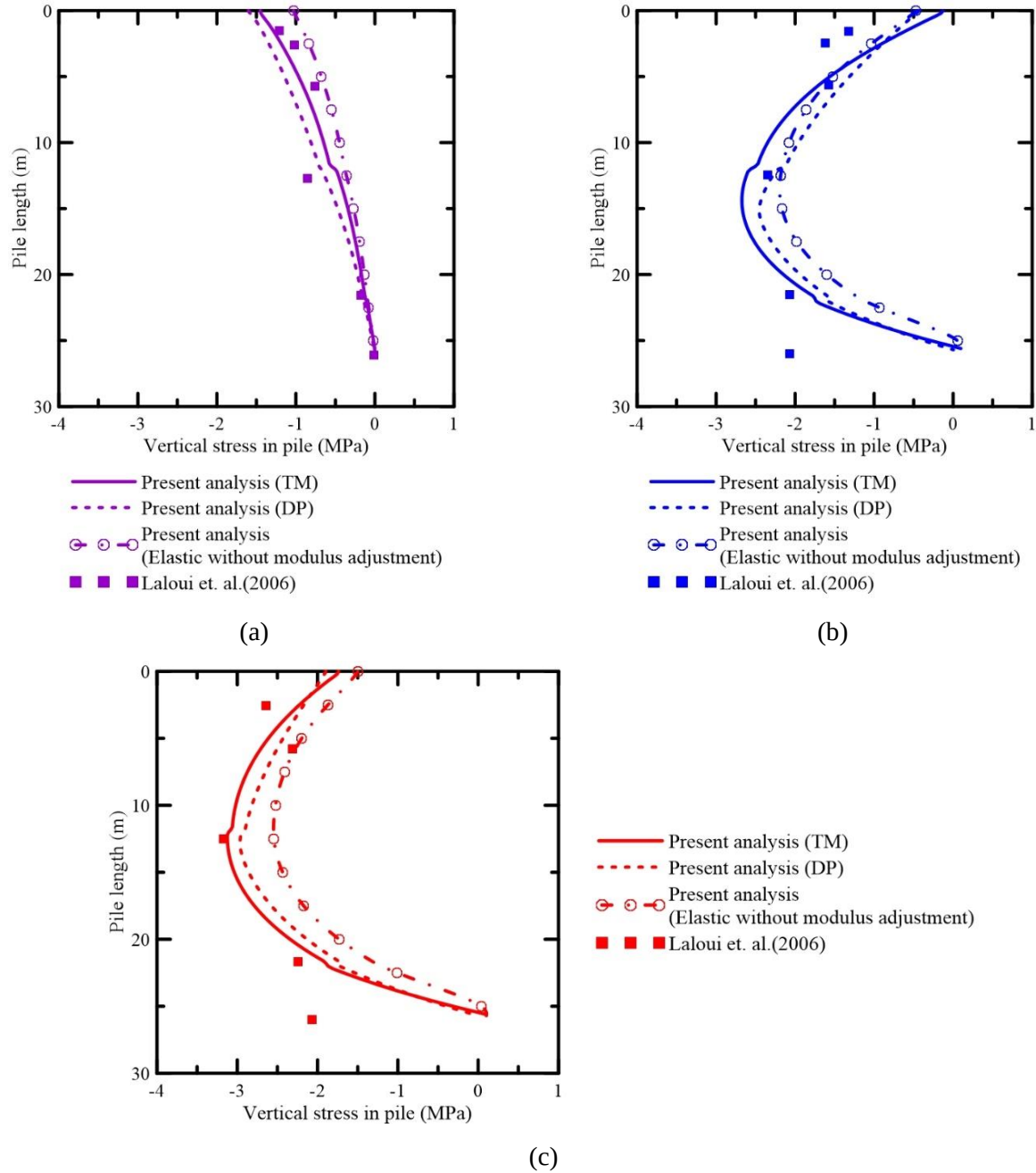


Figure 4-12 Comparison of vertical stress profiles of an energy pile obtained from the present analysis (using elastic, elastoplastic (DP), and thermo-mechanical (TM) soil constitutive model) and field test under a) mechanical load only, b) thermal load only, and (c) combined load.

4.8 Summary

The semi-analytical framework developed for energy pile in elastoplastic soil under different loading conditions in Chapter 3 is improved by representing the clayey soil with a thermo-mechanical constitutive model (Golchin et al., 2022b) that includes the effect of temperature specifically on the mechanical behavior of clay. The temperature change has substantial effects on the mechanical behavior of clay as suggested in available literature. Therefore, it is important to model the clayey soil with a proper thermo-mechanical constitutive relationship so that, the proper response of the pile that is embedded in clay can be obtained for thermal loading. The thermo-mechanical model is capable to capture both the linear and nonlinear elastic response of the clayey soil by using pressure (mean stress) dependent soil modulus. The effect of temperature on the pre-consolidation pressure of clay is explicitly considered in the model that governs the plastic response of the soil. The thermo-mechanical constitutive model has been developed using the hyperplasticity approach that satisfies the laws of thermodynamics. The return stress algorithm based on implicit Euler backward integration method with a Newton-Raphson iterative procedure is used to perform the stress integration and calculate the tangent stiffness tensor. In the stress integration scheme, the residuals are minimized iteratively to attain convergence. The constitutive model is based on the p - q space and the conversion of stress from true space to p - q space is done accordingly. The consistent tangent operator is used in the present semi-analytical framework to calculate the thermo-mechanical response of the energy pile.

The results obtained from the present analysis are compared with the experimental results available in the literature for different energy piles. The comparison between the results from the analytical method and the field shows that incorporation of the thermo-mechanical constitutive model produces better results for clayey soil. The analytical method produces pile response with good accuracy. Therefore, some parametric study will be conducted in the next chapter to observe the pile response for different loading conditions with different soil types which is important for the design of energy pile.

Chapter 5

Parametric study on the thermo-mechanical response of energy pile

5.1 Introduction

In this chapter, some parametric studies are conducted on the energy pile using the present analytical framework. The advantage of using an energy pile as an energy geo-structure is that it can be used as a ground heat exchanger apart from its primary purpose of carrying the superstructure load to the ground. Therefore, an energy pile is always subjected to mechanical loads and changes in temperature. The change in the axial response of the energy pile caused by the temperature change is an important aspect to be considered. The stress developed in any conventional pile under axial load is compressive in nature. However, the stress developed in a pile that undergoes temperature changes may be compressive or tensile in nature depending on the increase or decrease in temperature. Therefore, stresses developed in an energy pile under different loading conditions (both mechanical and thermal loads) are calculated in this study to differentiate the zones in the pile with compressive and tensile stress. This parametric study considers the pile to be embedded in single-layer and two-layer soils with different mechanical and thermal properties.

5.2 Calculation of equivalent loads

stresses in the energy pile are calculated for mechanical and combined loading (mechanical load + temperature change) conditions. A correlation between the applied mechanical load and the temperature change is established by defining a dimensionless parameter η that is described as,

$$\eta = \frac{\Delta T_p}{\Delta T_p^{(eq)}} \quad (5.1)$$

where ΔT_p is the temperature change in the pile as stated in Chapter 2. $\Delta T_p^{(eq)}$ is termed as the equivalent temperature change in the pile and is calculated as,

$$\Delta T_p^{(eq)} = \frac{Q_p}{E_p A_p \alpha_p} \quad (5.2)$$

where Q_p is the applied axial load at the head of the pile, E_p is Young's modulus of the pile material, A_p is the cross-sectional area of the pile, and α_p is the coefficient of thermal expansion of the pile

material as given in Chapter 2. $\Delta T_p^{(eq)}$ can be considered as the equivalent temperature change in a pile for which the axial load developed in the pile is Q_p when the whole thermal strain in the axial direction ($\alpha_p \Delta T_p$) is restrained. In this study, η is chosen in such a manner, so that the temperature changes in soil can be kept within the practical range of change in pile temperature.

5.3 Results

Vertical/axial stress distributions along the length of the pile are presented in Figure 5-1 to Figure 5-9. The stress along the length of the pile is normalized using the absolute value of the maximum stress at the head of the pile under applied mechanical load only ($|\sigma_{max,m}|$). The depth from the ground level is also normalized using the length of the pile (L_p). Stress profiles are plotted for different η values as mentioned in Figure 5-1 to Figure 5-9. $\eta = 0$ represents the pile subjected to only the applied axial load. In the figures, negative stress values represent the compressive stress in the pile. The dimensions and material properties of the pile that is used in the analysis are $L_p = 20$ m, $r_p = 0.5$ m, $E_p = 25$ GPa, $\kappa_p = 2.1$ W/m/°C, $\alpha_p = 10^{-5}$ /°C. Q_p is considered as 1000 kN, which gives the value of $\Delta T_p^{(eq)}$ as 5°C from equation (5.2).

5.3.1 Energy pile in single-layer soil

The normalized vertical stress profiles along the normalized depth of the pile in single-layer are shown in Figure 5-1(a) for mechanical and combined loading conditions represented by η . The temperature in the pile is increased with the increased η values (i.e. $\Delta T_p > 0$). It is observed from Figure 5-1(a) that the axial stress in the pile is compressive along the total length of the pile. The mechanical and thermal properties used in this particular analysis are $G_s = E_p/10^3$, $\nu_s = 0.3$, $\kappa_s = 2.0$ W/m/°C, $\alpha_s = 10^{-5}$ /°C. The vertical stress profiles for $\Delta T_p < 0$ are shown in Figure 5-1(b). It is observed from Figure 5-1(b) that for $\eta = 3$ and $\eta = 4$, tensile stress is developed at the bottom part of the pile. Therefore, the location of the tension zone for high values of temperature decrease can be present at the bottom part of the pile ranging for the distance of $0.5L_p$ to $0.25L_p$ from the tip of the pile. It is also observed that the increase in pile temperature increases the maximum compressive stress at the head of the pile (under combined loading) whereas for $\Delta T_p < 0$, the maximum compressive stress at the head of the pile almost remains unchanged.

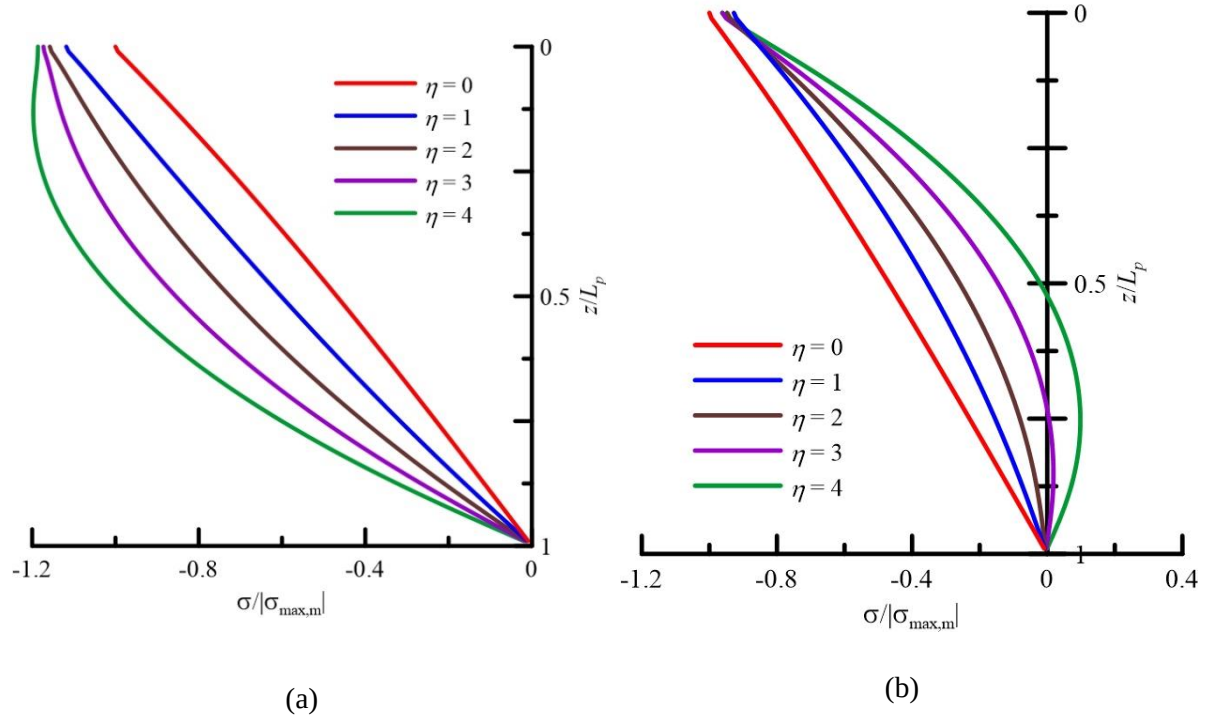


Figure 5-1 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads. a) $\Delta T_p > 0$, b) $\Delta T_p < 0$

5.3.2 Energy pile in two-layer soil

The effect of layering in the response of the energy pile in two-layer soil is taken into account by considering different mechanical and thermal properties of soils. The effect of different soil strength in different soil layers is considered by relating the shear modulus of the soil in two different layers with the term G_{s2}/G_{s1} where G_{s1} and G_{s2} are the shear modulus of soil in layer-1 and layer-2, respectively. The thermal properties of soils in different layers are considered in the parametric study by the terms α_{s2}/α_{s1} and κ_{s2}/κ_{s1} where α_{s1} , κ_{s1} ; and α_{s2} , κ_{s2} are thermal expansion coefficient and thermal conductivity of the soil in layer-1 and layer-2, respectively. The H_1/L_p value considered in this analysis is 0.5 where H_1 is the depth of top soil layer.

5.3.2.1 Effect of the thermal expansion coefficient (α_s) of soil

The effect of the thermal expansion coefficient of soil (α_s) on the vertical stress is shown in Figure 5-2(a) for $\alpha_{s2}/\alpha_{s1} = 0.1$ and Figure 5-2(b) for $\alpha_{s2}/\alpha_{s1} = 10$. Figure 5-2 represents the stress for $\Delta T_p > 0$. No tensile zone in the pile is present in this case for any temperature increase.

Figure 5-3(a)-(b) show the vertical stress profiles for $\alpha_{s2}/\alpha_{s1} = 0.05$ and $\alpha_{s2}/\alpha_{s1} = 0.5$, respectively when $\Delta T_p < 0$. In all the cases that are shown in Figure 5-2 and Figure 5-3 for different α_{s2}/α_{s1} values, no tension is developed in the pile for any thermal load.

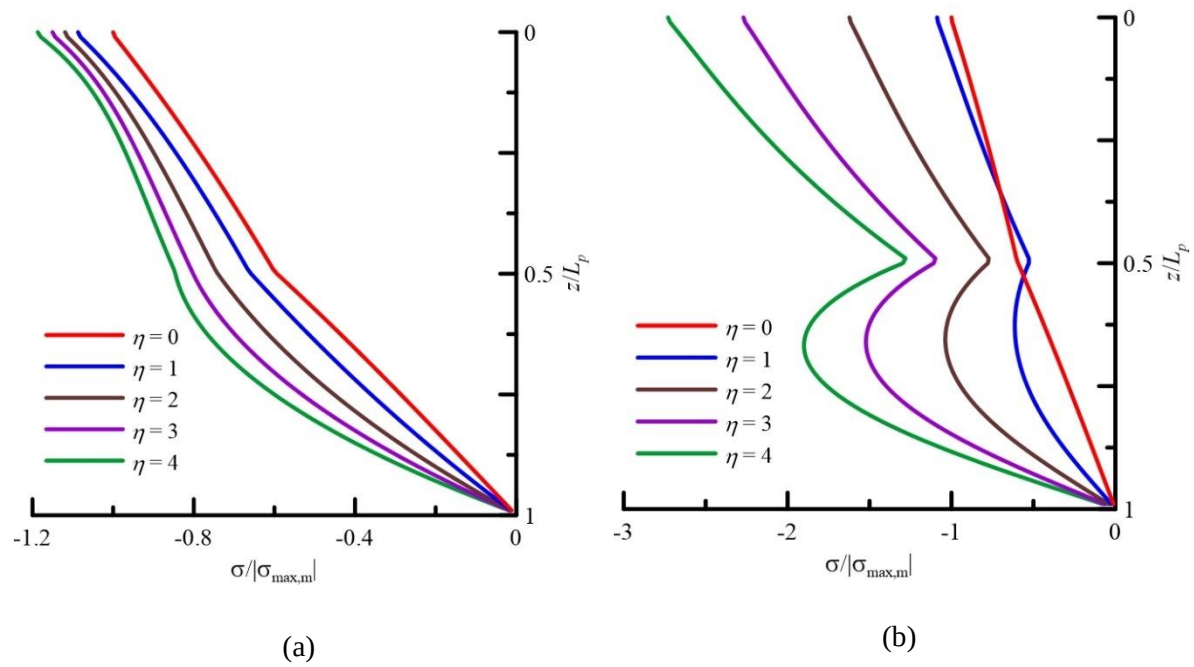


Figure 5-2 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p > 0$) for a) $\alpha_{s2}/\alpha_{s1} = 0.1$, and b) $\alpha_{s2}/\alpha_{s1} = 10$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\kappa_{s2}/\kappa_{s1} = 1$)

The variation of stress for different α_{s2}/α_{s1} are shown in Figure 5-4 for $\eta = 4$ when other parameters are kept constants as stated before. For $\Delta T_p < 0$, the maximum stress ratio decreases with the increase of α_{s2}/α_{s1} and vice-versa for $\Delta T_p > 0$.

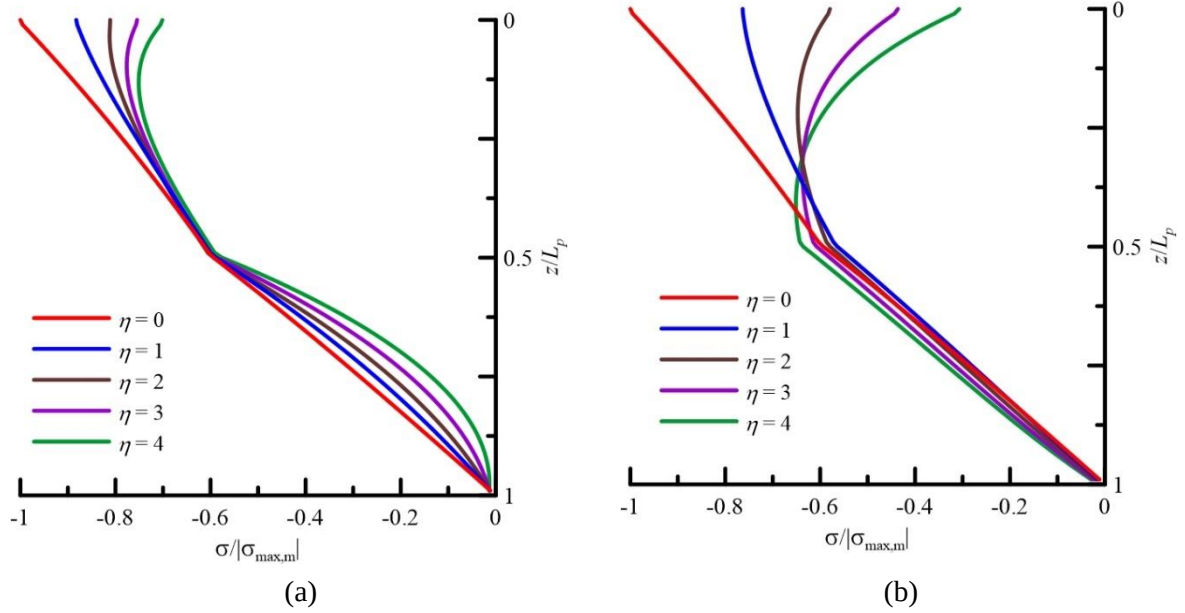


Figure 5-3 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p < 0$) for a) $\alpha_{s2}/\alpha_{s1} = 0.1$, and b) $\alpha_{s2}/\alpha_{s1} = 0.05$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\kappa_{s2}/\kappa_{s1} = 1$)

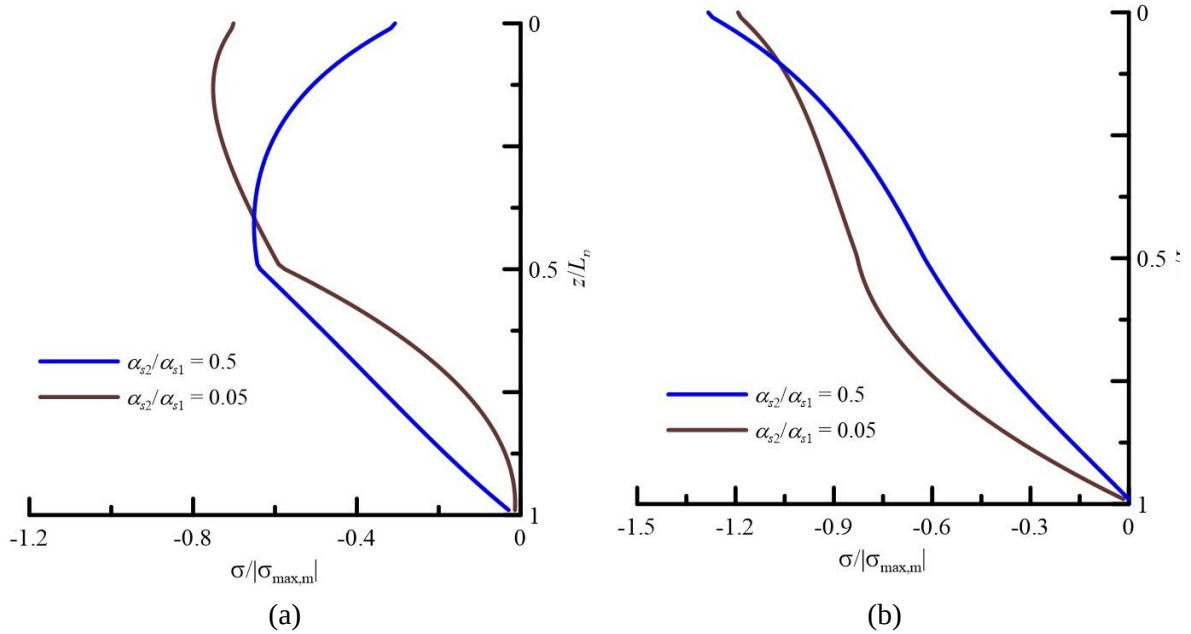


Figure 5-4 Normalized vertical stress profile plotted against the normalized pile length for an energy pile for different ratios of thermal expansion coefficient of soil for $\eta = 4$ for a) $\Delta T_p < 0$ b) $\Delta T_p > 0$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\kappa_{s2}/\kappa_{s1} = 1$)

5.3.2.2 Effect of the thermal conductivity (κ_s) of soil

The effects of the thermal conductivity of soil in different layers for a two-layer soil profile on the stress profiles of an energy pile under mechanical and thermal loads are shown in Figure 5-5 for $\Delta T_p > 0$ and Figure 5-6 for $\Delta T_p < 0$. Other soil properties are kept constant in this analysis. For the cases with $\Delta T_p > 0$, tensile stresses are developed under combined load ($\eta = 2, 3, 4$) at the bottom half of the pile for $\kappa_{s2}/\kappa_{s1} = 0.5$ (Figure 5-5(a)). For $\Delta T_p < 0$, the stress in the pile remains compressive in all the cases under different combined loads (Figure 5-5(b)).

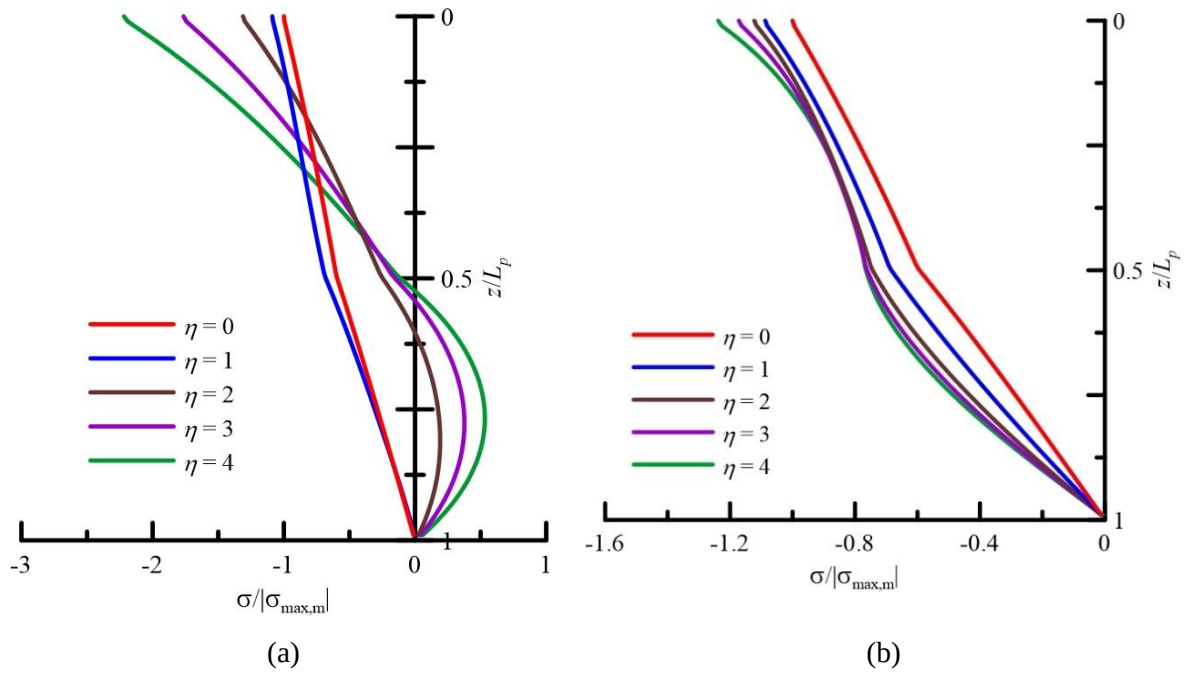


Figure 5-5 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p > 0$) for a) $\kappa_{s2}/\kappa_{s1} = 0.5$, and b) $\kappa_{s2}/\kappa_{s1} = 2$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\alpha_{s2}/\alpha_{s1} = 1$)

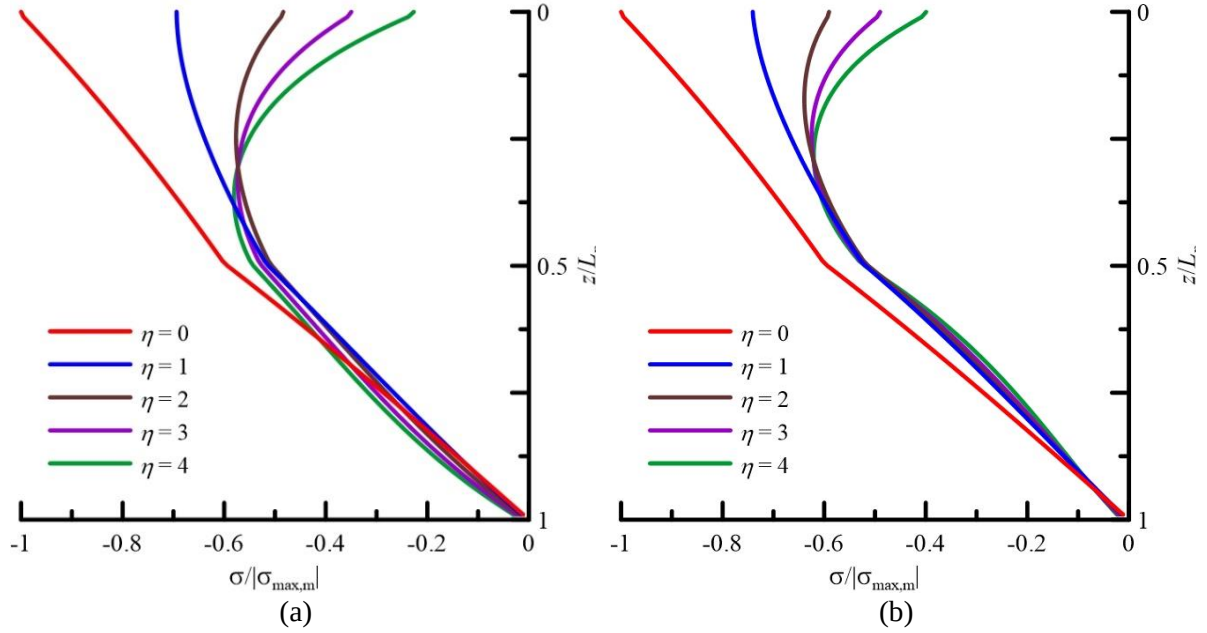


Figure 5-6 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p < 0$) for a) $\kappa_{s2}/\kappa_{s1} = 1.5$, and b) $\kappa_{s2}/\kappa_{s1} = 2$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\alpha_{s2}/\alpha_{s1} = 1$)

The effect of the thermal conductivity of soil on the normalized stress in the pile is shown in Figure 5-7 where different normalized stress profiles for different soil thermal conductivity values are plotted against the normalized depth (other soil properties are kept constant). For $\Delta T_p < 0$, stress at the head of the pile increases as κ_{s2}/κ_{s1} increases. The stress in the bottom half of the pile does not depend much on κ_{s2}/κ_{s1} . For $\Delta T_p > 0$, the normalized stress profile is almost similar for all κ_{s2}/κ_{s1} .

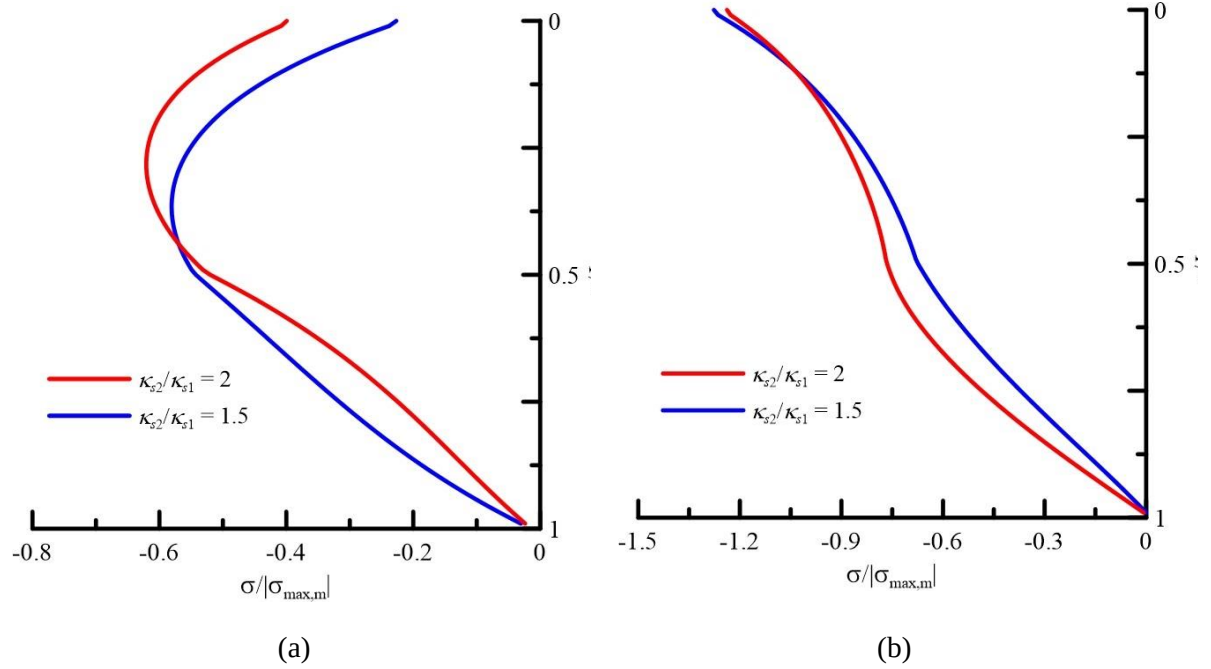


Figure 5-7 Normalized vertical stress profile plotted against the normalized pile length for an energy pile for different ratios of thermal conductivity of soil for $\eta = 4$ for a) $\Delta T_p < 0$ b) $\Delta T_p > 0$ when other parameters are kept constants, ($G_{s2}/G_{s1} = 2$, $\alpha_{s2}/\alpha_{s1} = 1$)

5.3.2.3 Effect of the shear modulus (G_s) of soil

The strength of soil in different layers influences the vertical stress developed in the energy pile under mechanical and thermal loads. The term G_{s2}/G_{s1} is considered in the analysis for different soil strengths in two different layers. The normalized vertical stress distribution in the pile (including the effect of different soil strengths through the term G_{s2}/G_{s1}) along its normalized strength is shown in Figure 5-8 and Figure 5-9 for $\Delta T_p > 0$, and $\Delta T_p < 0$, respectively. Other soil properties are kept constant as mentioned in Figure 5-8 and Figure 5-9. For $\Delta T_p > 0$, when the bottom soil layer is much stiffer than the top layer, tension develops in the bottom quarter length of the energy pile under high thermal loads.

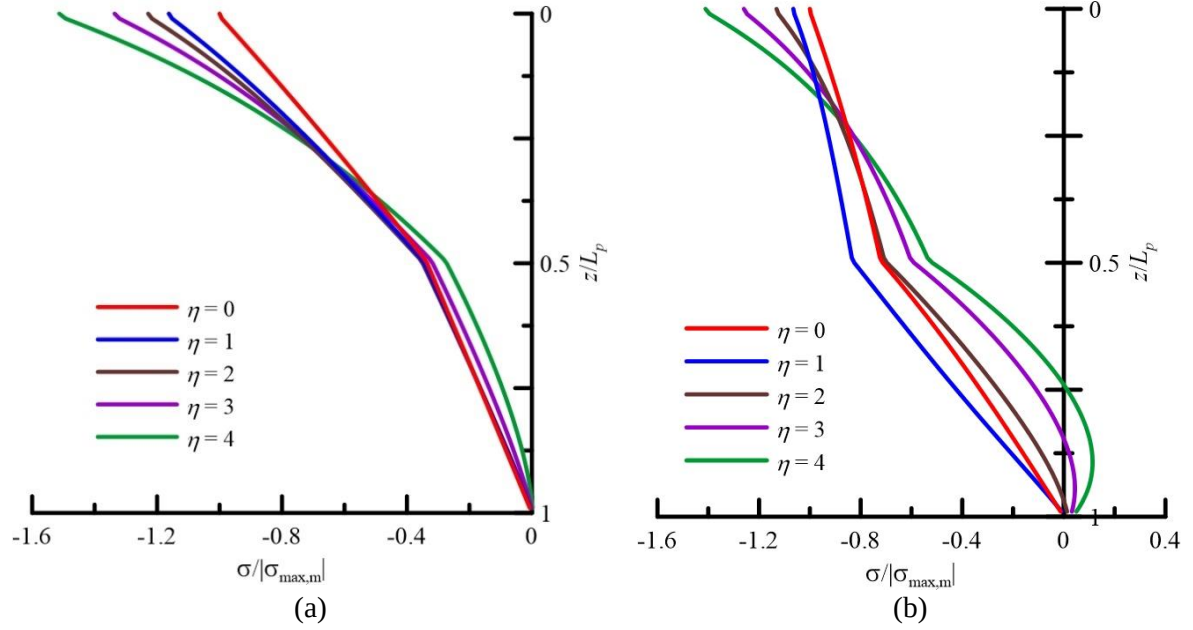


Figure 5-8 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p > 0$) for a) $G_{s2}/G_{s1} = 0.5$, and b) $G_{s2}/G_{s1} = 5$ when other parameters are kept constants, ($\alpha_{s2}/\alpha_{s1} = 1$, $\kappa_{s2}/\kappa_{s1} = 1$)

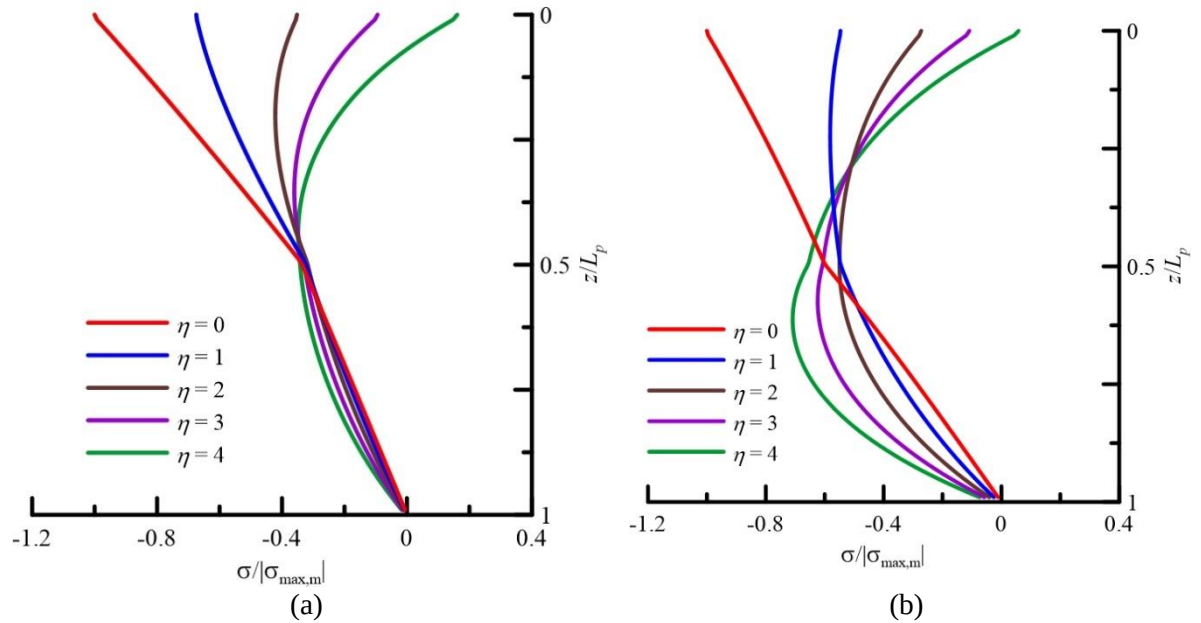


Figure 5-9 Normalized vertical stress profile plotted against the normalized pile length for an energy pile subjected to a mechanical load and different thermal loads ($\Delta T_p < 0$) for a) $G_{s2}/G_{s1} = 0.5$, and b) $G_{s2}/G_{s1} = 5$ when other parameters are kept constants, ($\alpha_{s2}/\alpha_{s1} = 1$, $\kappa_{s2}/\kappa_{s1} = 1$)

The effect of the shear modulus of soil on the normalized stress in the pile is shown in Figure 5-10 where different normalized stress profiles for different shear modulus ratios of soil (G_{s2}/G_{s1}) are plotted against the normalized depth (other soil properties are kept constant). For $\Delta T_p < 0$, stress at the head of the pile remains unaffected by the values of G_{s2}/G_{s1} where the maximum stress at the mid-span of the pile increases with the increase of G_{s2}/G_{s1} . For $\Delta T_p > 0$, the normalized stress profile is almost similar for all G_{s2}/G_{s1} except that at high thermal load tension develops at the bottom part of the pile.

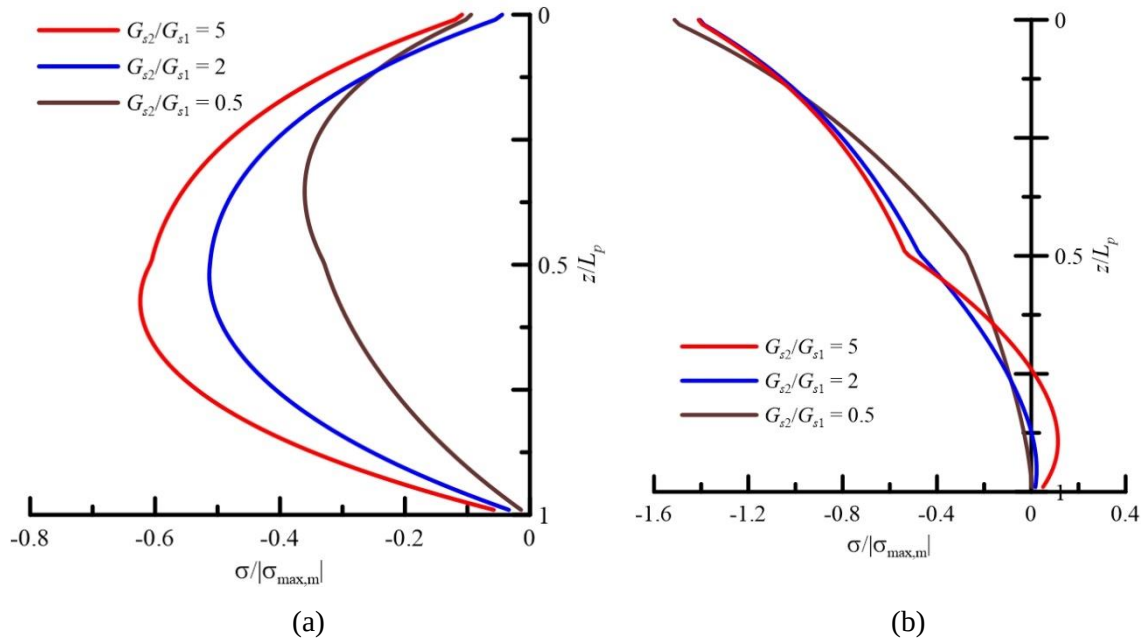


Figure 5-10 Normalized vertical stress profile plotted against the normalized pile length for an energy pile for different ratios of the shear modulus of soil for $\eta = 4$ for a) $\Delta T_p < 0$ b) $\Delta T_p > 0$ when other parameters are kept constants, ($\alpha_{s2}/\alpha_{s1} = 1$, $\kappa_{s2}/\kappa_{s1} = 1$)

5.4 Summary

A parametric study on an energy pile embedded in single-layer and two-layer soils and subjected to mechanical and thermal load is conducted using the present analytical framework. The vertical/axial stress which is mostly affected by the temperature change in the pile is observed in the study for different thermal loads and different soil properties. The temperature change and applied axial load at the head of the pile are correlated with a dimensionless parameter η . The stress in an energy

pile is different than that of a conventional pile which is always under compression because of the superstructure load. It is observed that a tension zone can develop in the energy pile under different circumstances with different thermal loads and soil properties (both mechanical and thermal). For single-layer soil, a tension zone can develop in the range of $0.5L_p$ - $0.75L_p$ at the bottom part of the pile for different values of temperature decrease. Also, for two-layer soil when the thermal conductivity of the bottom layer is half of the top layer or when the shear modulus of the bottom layer is much higher than the shear modulus of the top layer, the tensile zone of the same length as mentioned earlier can develop in the pile. Therefore, for single-layer and two-layer soil with these conditions, the development of tensile stresses needs to be considered, and the pile should be designed accordingly. Also, it is observed that, in the case of a temperature increase in the pile, the axial stress at the pile head increases, and in the case of a temperature decrease the axial stress at the pile head decreases or remains constant.

Chapter 6

Summary and Recommendations

6.1 Summary

In this thesis, a continuum-based semi-analytical framework for the analysis of the energy pile that takes into account the three-dimensional pile-structure interaction is proposed. In Chapter 1, the literature review on the thermo-mechanical response of energy piles with research gaps and objectives are presented. In Chapter 2, a semi-analytical framework for an energy pile that is embedded in multi-layered soil and subjected to mechanical axial load and thermal load is developed using the variational principle by minimizing the potential energy of the pile-soil continuum where both the pile material and soil are considered to behave as a linear elastic material. The present analytical framework produces accurate results with less computational time. In Chapter 3, a semi-analytical framework for the same energy pile is developed where the soil is modeled as an elastoplastic material. The analysis framework for the energy pile in elastoplastic soil is developed from the laws of thermodynamics with the energy potential and dissipation function where the plastic behavior of soil is taken into account through the dissipation potential. The derived analytical framework is used for elastoplastic soil response with the Drucker-Prager constitutive model. The results from the present analysis are verified with available test results from the literature and with the results from the numerical analysis. The Drucker-Prager constitutive model does not take into account the effect of temperature on the stress-strain response of the soil. As a result, this constitutive model is most suitable to model the thermo-mechanical behavior of sand and an energy pile in sand can be analyzed using the present framework with the Drucker-Prager constitutive model. In Chapter 4 of the thesis, the present analytical framework for energy pile in elastoplastic soil is used to analyze energy pile in clay with a thermo-mechanical constitutive model that takes into account the change in the mechanical response of clay due to temperature change because the thermo-mechanical behavior of clay is highly dependent on the temperature. The thermo-mechanical model has been developed using the hyperplasticity formalism that satisfies the laws of thermodynamics. The results from the present framework with elastoplastic clay that is represented by the thermo-mechanical constitutive model is verified with available experimental and numerical results from the literature. In Chapter 5, parametric studies are conducted on the developed vertical stresses in

an energy pile in single and two-layer soils under different loads to calculate the nature of stresses along the length of the pile for different soil and loading conditions.

6.2 Conclusions

The major conclusions are

- 1) The framework developed in Chapter 2 for energy piles in single or multi-layered elastic soils can predict the vertical response of the pile in terms of vertical settlement, strain, and stress under mechanical and thermal loads with acceptable accuracy in a fast and efficient manner considering the three-dimensional pile-structure interaction. A modification in the soil shear modulus needs to be made in this analysis.
- 2) The analysis method with elastoplastic soil behavior developed in Chapter 3 produces the vertical response of an energy pile in different soils under all the loading conditions quite accurately incorporating the nonlinear mechanical behavior of soil. The modification in the shear modulus of soil required in Chapter 2 is not required in the improved elastoplastic framework. This elastoplastic framework is further used to include the thermo-mechanical behavior of clay by using a temperature-dependent clay constitutive model in Chapter 4 that predicts the experimental results available in the literature quite accurately. Therefore, the semi-analytical models developed in the thesis can assess the thermo-mechanical response of energy piles accurately and is also computationally fast.
- 3) The parametric study in Chapter 5 shows that depending on the soil layering and loading conditions tensile zone may develop in the pile where the combined axial stress is tensile. This needs to be considered during the structural design of the energy pile.

6.3 Recommendations for future work

Based on the limitations of the current research work, the possible extensions of the present research work are as follows:

- 1) The present research work assumes no slippage or separation between the pile and the soil. However, slippage may influence the nonlinear response of the energy pile foundation. Therefore, the analysis can be modified considering the slippage between the pile and the soil.
- 2) The pile-soil interface behavior may be affected by the differential temperature change in the pile and the soil. That can be added in the present analytical framework.

- 3) The study includes the thermo-mechanical constitutive behavior of clayey soil. The water that is present in the voids between the soil particles can transfer the heat by convection. Therefore, the analysis can be repeated with a thermo-hydro-mechanical soil constitutive model with additional heat transfer mechanisms.
- 4) In the present study, the temperature change in the soil is calculated with the formula given by Zeng et al., (2002) which was developed for a finite-length heat source in homogeneous soil. A more advanced heat transfer model can be used in the present framework to calculate the temperature change in the soil which can be added as a subroutine in the existing algorithm.

References

- Abbo, A. J. (2005). Finite element algorithms for elastoplasticity and consolidation. *Engineering*, 1997(February 1997), 285.
- Abbo, A. J., & Sloan, S. W. (1995). A SMOOTH HYPERBOLIC APPROXIMATION TO THE MOHR-COULOMB YIELD CRITERION. *Computers & Structures*, 54(3), 427–441.
- Abuel-Naga, H. M., Bergado, D. T., & Bouazza, A. (2007). Thermally induced volume change and excess pore water pressure of soft Bangkok clay. *Engineering Geology*, 89(1), 144–154. <https://doi.org/https://doi.org/10.1016/j.enggeo.2006.10.002>
- Abuel-Naga, H. M., Bergado, D. T., Bouazza, A., & Pender, M. (2009). Thermomechanical model for saturated clays. *Geotechnique*, 59(3), 273–278. <https://doi.org/10.1680/geot.2009.59.3.273>
- Abuel-Naga, H. M., Bergado, D. T., & Lim, B. F. (2007). Effect of Temperature on Shear Strength and Yielding Behavior of Soft Bangkok Clay. *Soils and Foundations*, 47(3), 423–436. <https://doi.org/https://doi.org/10.3208/sandf.47.423>
- Abuel-Naga, H. M., Bergado, D. T., Ramana, G. V., Grino, L., Rujivipat, P., & Thet, Y. (2006). Experimental Evaluation of Engineering Behavior of Soft Bangkok Clay under Elevated Temperature. *Journal of Geotechnical and Geoenvironmental Engineering*, 132(7), 902–910. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2006\)132:7\(902\)](https://doi.org/10.1061/(ASCE)1090-0241(2006)132:7(902))
- Amatya, B. L., Soga, K., Bourne-Webb, P. J., Amis, T., & Laloui, L. (2012). Thermo-mechanical behaviour of energy piles. *Geotechnique*, 62(6), 503–519. <https://doi.org/10.1680/geot.10.P.116>
- Basu, D., Salgado, R., & Prezzi, M. (2009). A continuum-based model for analysis of laterally loaded piles in layered soils. *Geotechnique*, 59(2), 127–140. <https://doi.org/10.1680/geot.2007.00011>
- Berardi, R., & Bovolenta, R. (2005). Pile-settlement evaluation using field stiffness non-linearity. *Geotechnical Engineering*, 158(1), 35–44. <https://doi.org/10.1680/geng.158.1.35.56647>
- Bourne-Webb, P. J., Amatya, B., & Soga, K. (2013). A framework for understanding energy pile behaviour. *Proceedings of the Institution of Civil Engineers: Geotechnical Engineering*, 166(2), 170–177. <https://doi.org/10.1680/geng.10.00098>

- Bourne-Webb, P. J., Amatya, B., Soga, K., Amis, T., Davidson, C., & Payne, P. (2009). Energy pile test at lambeth college, London: Geotechnical and thermodynamic aspects of pile response to heat cycles. *Geotechnique*, 59(3), 237–248. <https://doi.org/10.1680/geot.2009.59.3.237>
- Bowles, J. E. (1996). *Foundation Analysis and Design* (5th ed.). McGraw-Hill.
- Brandl, H. (2006). Energy foundations and other thermo-active ground structures. *Géotechnique*, 56(2), 81–122. <https://doi.org/10.1680/geot.2006.56.2.81>
- Campanella, R. G., & Mitchell, J. K. (1968). Influence of Temperature Variations on Soil Behavior. *Journal of the Soil Mechanics and Foundations Division*, 94(3), 709–734. <https://doi.org/10.1061/JSFEAQ.0001136>
- Cekerevac, C., & Laloui, L. (2004). Experimental study of thermal effects on the mechanical behaviour of a clay. *International Journal for Numerical and Analytical Methods in Geomechanics*, 28(3), 209–228. <https://doi.org/https://doi.org/10.1002/nag.332>
- Chen, D., & McCartney, J. S. (2017). Parameters for Load Transfer Analysis of Energy Piles in Uniform Nonplastic Soils. *International Journal of Geomechanics*, 17(7), 04016159. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0000873](https://doi.org/10.1061/(asce)gm.1943-5622.0000873)
- Coccia, C. J. R., & McCartney, J. S. (2016). Thermal volume change of poorly draining soils II: Model development and experimental validation. *Computers and Geotechnics*, 80, 16–25. <https://doi.org/10.1016/j.compgeo.2016.06.010>
- Cui, W., Potts, D. M., Zdravković, L., Gawęcka, K. A., & Taborda, D. M. G. (2018). An alternative coupled thermo-hydro-mechanical finite element formulation for fully saturated soils. *Computers and Geotechnics*, 94, 22–30. <https://doi.org/10.1016/j.compgeo.2017.08.011>
- Cui, Y. J., Sultan, N., & Delage, P. (2000). A thermomechanical model for saturated clays. *Canadian Geotechnical Journal*, 37(3), 607–620. <https://doi.org/10.1139/cgj-37-3-607>
- Dauncey, G. (2001). *Stormy weather : 101 solutions to global climate change*.
- Di Donna, A., & Laloui, L. (2015a). Numerical analysis of the geotechnical behaviour of energy piles. *International Journal for Numerical and Analytical Methods in Geomechanics*, 39(8), 861–888. <https://doi.org/10.1002/nag.2341>

- Di Donna, A., & Laloui, L. (2015b). Response of soil subjected to thermal cyclic loading: Experimental and constitutive study. *Engineering Geology*, 190, 65–76.
<https://doi.org/10.1016/j.enggeo.2015.03.003>
- Einav, I., & Puzrin, A. M. (2004). Pressure-Dependent Elasticity and Energy Conservation in Elastoplastic Models for Soils. *Journal of Geotechnical and Geoenvironmental Engineering*, 130(1), 81–92. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2004\)130:1\(81\)](https://doi.org/10.1061/(ASCE)1090-0241(2004)130:1(81))
- Fathalikhani, M., Graham, J., Kurz, D., & Maghoul, P. (2022). Investigation and Modification of a CSSM-Based Elastic–Thermoviscoplastic Model for Clay. *International Journal of Geomechanics*, 22(10), 1–14. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0002480](https://doi.org/10.1061/(asce)gm.1943-5622.0002480)
- Frank, R., & Zhao, S. R. (1982). *Estimation par les paramètres pressiométriques de l'enfoncement sous charge axiale de pieux forés dans des sols fins*. 119.
- Gasparre, A. (2005). *Advanced Laboratory Characterisation of London Clay*.
- Gawecka, K. A., Taborda, D. M. G., Potts, D. M., Cui, W., Zdravković, L., & Haji Kasri, M. S. (2016). Numerical modelling of thermo-active piles in London Clay. *Proceedings of the Institution of Civil Engineers - Geotechnical Engineering*, 170(3), 201–219.
<https://doi.org/10.1680/jgeen.16.00096>
- Ghayoomi, M., Jarast, P., Mirshekari, M., & Borghei, A. (2018). Application of cone penetrometer for unsaturated soil inside geotechnical centrifuge. *7th International Conference on Unsaturated Soils 2018 (Hong Kong)*.
- Golchin, A. (2023). *Thermo-mechanics of energy piles: fine-grained soils, cycles, and interfaces*. Delft University of Technology.
- Golchin, A., Vardon, P. J., & Hicks, M. A. (2022a). A thermodynamically consistent two surface/bubble thermo-mechanical model considering thermal and mechanical cyclic behaviour of fine-grained soils. *International Journal of Solids and Structures*, 254–255(May), 111847.
<https://doi.org/10.1016/j.ijsolstr.2022.111847>
- Golchin, A., Vardon, P. J., & Hicks, M. A. (2022b). A thermo-mechanical constitutive model for fine-grained soils based on thermodynamics. *International Journal of Engineering Science*, 174(March), 103579. <https://doi.org/10.1016/j.ijengsci.2021.103579>

- Goode, J. C., & McCartney, J. S. (2015). Centrifuge Modeling of End-Restraint Effects in Energy Foundations. *Journal of Geotechnical and Geoenvironmental Engineering*, 141(8), 04015034. [https://doi.org/10.1061/\(ASCE\)GT.1943-5606.0001333](https://doi.org/10.1061/(ASCE)GT.1943-5606.0001333)
- Graham, J., Tanaka, N., Crilly, T., & Alfaro, M. (2001). Modified Cam-Clay modelling of temperature effects in clays. *Canadian Geotechnical Journal*, 38(3), 608–621. <https://doi.org/10.1139/cgj-38-3-608>
- Guo, W. D. (2000). Vertically Loaded Single Piles in Gibson Soil. *Journal of Geotechnical and Geoenvironmental Engineering*, 126(2), 189–193. [https://doi.org/10.1061/\(ASCE\)1090-0241\(2000\)126:2\(189\)](https://doi.org/10.1061/(ASCE)1090-0241(2000)126:2(189))
- Guo, W. D., & Lee, F. H. (2001). Load transfer approach for laterally loaded piles. *International Journal for Numerical and Analytical Methods in Geomechanics*, 25(11), 1101–1129. <https://doi.org/https://doi.org/10.1002/nag.169>
- Halder, S., & Sivakumar Babu, G. L. (2008). Load Resistance Factor Design of Axially Loaded Pile Based on Load Test Results. *Journal of Geotechnical and Geoenvironmental Engineering*, 134(8), 1106–1117. [https://doi.org/10.1061/\(asce\)1090-0241\(2008\)134:8\(1106\)](https://doi.org/10.1061/(asce)1090-0241(2008)134:8(1106))
- Hong, P. Y., Pereira, J. M., Cui, Y. J., & Tang, A. M. (2016). A two-surface thermomechanical model for saturated clays. *International Journal for Numerical and Analytical Methods in Geomechanics*, 40(7), 1059–1080. <https://doi.org/10.1002/nag.2474>
- Houlsby, G., & Puzrin, A. (2007). Principles of Hyperplasticity, An Approach to Plasticity Theory Based on Thermodynamic Principles. *Principles of Hyperplasticity: An Approach to Plasticity Theory Based on Thermodynamic Principles*, 1–351. <https://doi.org/10.1007/978-1-84628-240-9>
- Houlsby, G. T., Amorosi, A., & Rojas, E. (2005). Elastic moduli of soils dependent on pressure: a hyperelastic formulation. *Géotechnique*, 55(5), 383–392. <https://doi.org/10.1680/geot.2005.55.5.383>
- Hueckel, T., & Baldi, G. (1990). Thermoplasticity of Saturated Clays: Experimental Constitutive Study. *Journal of Geotechnical Engineering*, 116(12), 1778–1796. [https://doi.org/10.1061/\(ASCE\)0733-9410\(1990\)116:12\(1778\)](https://doi.org/10.1061/(ASCE)0733-9410(1990)116:12(1778))

- Hueckel, T., & Borsetto, M. (1990). Thermoplasticity of saturated soils and shales: Constitutive equations. *Journal of Geotechnical Engineering*, 116(12), 1765–1777.
[https://doi.org/10.1061/\(ASCE\)0733-9410\(1990\)116:12\(1765\)](https://doi.org/10.1061/(ASCE)0733-9410(1990)116:12(1765))
- International Energy Agency, I. E. A. (2016). *Energy and Air Pollution*.
- Iodice, C., Di Laora, R., & Mandolini, A. (2020a). Analytical Solutions for Ultimate Limit State Design of Thermal Piles. *Journal of Geotechnical and Geoenvironmental Engineering*, 146(5), 04020016. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0002204](https://doi.org/10.1061/(asce)gt.1943-5606.0002204)
- Iodice, C., Di Laora, R., & Mandolini, A. (2020b). Finite element analyses of energy piles using different constitutive models. *E3S Web of Conferences*, 205, 1–6.
<https://doi.org/10.1051/e3sconf/202020505013>
- Khalili, N., Uchaipichat, A., & Javadi, A. A. (2010). Skeletal thermal expansion coefficient and thermo-hydro-mechanical constitutive relations for saturated homogeneous porous media. *Mechanics of Materials*, 42(6), 593–598.
<https://doi.org/https://doi.org/10.1016/j.mechmat.2010.04.001>
- Knellwolf, C., Peron, H., & Laloui, L. (2011). Geotechnical Analysis of Heat Exchanger Piles. *Journal of Geotechnical and Geoenvironmental Engineering*, 137(10), 890–902.
[https://doi.org/10.1061/\(asce\)gt.1943-5606.0000513](https://doi.org/10.1061/(asce)gt.1943-5606.0000513)
- Kurz, D., Sharma, J., Alfaro, M., & Graham, J. (2016). Semi-empirical elastic-thermoviscoplastic model for clay. *Canadian Geotechnical Journal*, 53(10), 1583–1599.
<https://doi.org/10.1139/cgj-2015-0598>
- Laloui, L. (2001). Thermo-mechanical behaviour of soils. *Revue Française de Génie Civil*, 5(6), 809–843. <https://doi.org/10.1080/12795119.2001.9692328>
- Laloui, L., & Cekerevac, C. (2003). Thermo-plasticity of clays: An isotropic yield mechanism. *Computers and Geotechnics*, 30(8), 649–660. <https://doi.org/10.1016/j.compgeo.2003.09.001>
- Laloui, L., & François, B. (2009). ACMEG-T: Soil Thermoplasticity Model. *Journal of Engineering Mechanics*, 135(9), 932–944. [https://doi.org/10.1061/\(asce\)em.1943-7889.0000011](https://doi.org/10.1061/(asce)em.1943-7889.0000011)

- Laloui, L., Nuth, M., & Vulliet, L. (2006). Experimental and numerical investigations of the behaviour of a heat exchanger pile. *International Journal for Numerical and Analytical Methods in Geomechanics*, 30(8), 763–781. <https://doi.org/10.1002/nag.499>
- Laloui, L., & Rotta Loria, A. F. (2020). *Analysis and Design of Energy Geostructures*. Elsevier. <https://doi.org/10.1016/C2017-0-04166-6>
- Mašín, D., & Khalili, N. (2012). A thermo-mechanical model for variably saturated soils based on hypoplasticity. *International Journal for Numerical and Analytical Methods in Geomechanics*, 36(12), 1461–1485. <https://doi.org/10.1002/nag.1058>
- McCartney, J. S., & Rosenberg, J. E. (2011). *Impact of Heat Exchange on Side Shear in Thermo-Active Foundations*. 488–498. [https://doi.org/10.1061/41165\(397\)51](https://doi.org/10.1061/41165(397)51)
- Misra, A., & Roberts, L. A. (2009). Service limit state resistance factors for drilled shafts. *Geotechnique*, 59(1), 53–61. <https://doi.org/10.1680/geot.2008.3605>
- Modaressi, H., & Laloui, L. (1997). A thermo-viscoplastic constitutive model for clays. *International Journal for Numerical and Analytical Methods in Geomechanics*, 21(5), 313–335. [https://doi.org/10.1002/\(sici\)1096-9853\(199705\)21:5<313::aid-nag872>3.0.co;2-5](https://doi.org/10.1002/(sici)1096-9853(199705)21:5<313::aid-nag872>3.0.co;2-5)
- Najma, A., & Sharma, J. (2021). Incremental load transfer analysis of an energy pile under arbitrary mechanical and thermal loads. *Geomechanics for Energy and the Environment*, 28, 100243. <https://doi.org/10.1016/j.gete.2021.100243>
- Ng, C. W. W., Cheng, Q., Zhou, C., & Alonso, E. E. (2016). Volume changes of an unsaturated clay during heating and cooling. *Géotechnique Letters*, 6(3), 192–198. <https://doi.org/10.1680/jgele.16.00059>
- Ng, C. W. W., Wang, S. H., & Zhou, C. (2016). Volume change behaviour of saturated sand under thermal cycles. *Géotechnique Letters*, 6(2), 124–131. <https://doi.org/10.1680/jgele.15.00148>
- Ng, C. W. W., Zhang, C., Farivar, A., & Gomaa, S. M. M. H. (2019). Scaling effects on the centrifuge modelling of energy piles in saturated sand. *Geotechnique Letters*, 9(4), 1–6. <https://doi.org/10.1680/jgele.19.00051>

- Olia, A. S. R., & Perić, D. (2021). Thermomechanical Soil–Structure Interaction in Single Energy Piles Exhibiting Reversible Interface Behavior. *International Journal of Geomechanics*, 21(5), 04021065. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0002014](https://doi.org/10.1061/(asce)gm.1943-5622.0002014)
- Pasten, C., & Santamarina, J. C. (2014). Thermally Induced Long-Term Displacement of Thermoactive Piles. *Journal of Geotechnical and Geoenvironmental Engineering*, 140(5), 06014003. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0001092](https://doi.org/10.1061/(asce)gt.1943-5606.0001092)
- Perić, D., Cossel, A. E., & Sarna, S. A. (2020). Analytical Solutions for Thermomechanical Soil Structure Interaction in End-Bearing Energy Piles. *Journal of Geotechnical and Geoenvironmental Engineering*, 146(7), 04020047. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0002269](https://doi.org/10.1061/(asce)gt.1943-5606.0002269)
- Potts, D. M. (1999). *Finite element analysis in geotechnical engineering*. Thomas Telford Publishing. <https://doi.org/doi:10.1680/feaiget.27534>
- Reddy, J. N. (2002). *Energy principles and variational methods in applied mechanics*.
- Rendon-Herrero, O. (1980). Universal Compression Index Equation. *Journal of the Geotechnical Engineering Division*, 106(11), 1179–1200. <https://doi.org/10.1061/AJGEB6.0001058>
- Rotta Loria, A. F., Donna, A. Di, & Laloui, L. (2015). Numerical Study on the Suitability of Centrifuge Testing for Capturing the Thermal-Induced Mechanical Behavior of Energy Piles. *Journal of Geotechnical and Geoenvironmental Engineering*, 141(10), 04015042. [https://doi.org/10.1061/\(asce\)gt.1943-5606.0001318](https://doi.org/10.1061/(asce)gt.1943-5606.0001318)
- Rotta Loria, A. F., Gunawan, A., Shi, C., Laloui, L., & Ng, C. W. W. (2015). Numerical modelling of energy piles in saturated sand subjected to thermo-mechanical loads. *Geomechanics for Energy and the Environment*, 1, 1–15. <https://doi.org/10.1016/j.gete.2015.03.002>
- Rotta Loria, A. F., & Laloui, L. (2019). Thermo-mechanical schemes for energy piles. In *Springer Series in Geomechanics and Geoengineering* (Vol. 0, Issue 217729). Springer International Publishing. https://doi.org/10.1007/978-3-319-99670-7_28
- Saggu, R., & Chakraborty, T. (2015a). Cyclic Thermo-Mechanical Analysis of Energy Piles in Sand. *Geotechnical and Geological Engineering*, 33(2), 321–342. <https://doi.org/10.1007/s10706-014-9798-8>

- Saggu, R., & Chakraborty, T. (2015b). Thermal analysis of energy piles in sand. *Geomechanics and Geoengineering*, 10(1), 10–29. <https://doi.org/10.1080/17486025.2014.923586>
- Saggu, R., & Chakraborty, T. (2017). Thermomechanical Analysis and Parametric Study of Geothermal Energy Piles in Sand. *International Journal of Geomechanics*, 17(9), 04017076. [https://doi.org/10.1061/\(asce\)gm.1943-5622.0000962](https://doi.org/10.1061/(asce)gm.1943-5622.0000962)
- Salgado, R. (2008). *The Engineering of Foundations*. McGraw Hill, 2008.
- Seo, H., Basu, D., Prezzi, M., & Salgado, R. (2009). Load-Settlement Response of Rectangular and Circular Piles in Multilayered Soil. *Journal of Geotechnical and Geoenvironmental Engineering*, 135(3), 420–430. [https://doi.org/10.1061/\(asce\)1090-0241\(2009\)135:3\(420\)](https://doi.org/10.1061/(asce)1090-0241(2009)135:3(420))
- Stewart, M. A. (2010). *CENTRIFUGE MODELING OF STRAIN DISTRIBUTIONS IN ENERGY FOUNDATIONS*. University of Colorado.
- Sultan, N., Delage, P., & Cui, Y. J. (2002). Temperature effects on the volume change behaviour of Boom clay. *Engineering Geology*, 64(2), 135–145. [https://doi.org/https://doi.org/10.1016/S0013-7952\(01\)00143-0](https://doi.org/https://doi.org/10.1016/S0013-7952(01)00143-0)
- Suryatriyastuti, M. (2013). *Numerical study of the thermoactive piles behavior in cohesionless soils*. Université Lille1 Sciences et Technologies.
- Suryatriyastuti, M. E., Mroueh, H., & Burlon, S. (2014). A load transfer approach for studying the cyclic behavior of thermo-active piles. *Computers and Geotechnics*, 55, 378–391. <https://doi.org/10.1016/j.compgeo.2013.09.021>
- Towhata, I., Kuntiwattanaku, P., Seko, I., & Ohishi, K. (1993). Volume Change of Clays Induced by Heating as Observed in Consolidation Tests. *Soils and Foundations*, 33(4), 170–183. https://doi.org/https://doi.org/10.3208/sandf1972.33.4_170
- Uchaipichat, A., & Khalili, N. (2009). Experimental investigation of thermo-hydro-mechanical behaviour of an unsaturated silt. *Géotechnique*, 59(4), 339–353. <https://doi.org/10.1680/geot.2009.59.4.339>
- Wang, B., Bouazza, A., Singh, R. M., Haberfield, C., Barry-Macaulay, D., & Baycan, S. (2015). Posttemperature Effects on Shaft Capacity of a Full-Scale Geothermal Energy Pile. *Journal of*

- Geotechnical and Geoenvironmental Engineering*, 141(4), 1–12.
[https://doi.org/10.1061/\(asce\)gt.1943-5606.0001266](https://doi.org/10.1061/(asce)gt.1943-5606.0001266)
- Wood, D. M. (1991). *Soil Behaviour and Critical State Soil Mechanics*. Cambridge University Press.
[https://doi.org/DOI: 10.1017/CBO9781139878272](https://doi.org/DOI:10.1017/CBO9781139878272)
- Xiao, S., & Suleiman, M. T. (2015). Investigation of Thermo-Mechanical Load Transfer (t-z Curves) Behavior of Soil-Energy Pile Interface Using Modified Borehole Shear Tests. *IFCEE 2015*, 1658–1667. <https://doi.org/10.1061/9780784479087.150>
- You, S., Cheng, X., Guo, H., & Yao, Z. (2016). Experimental study on structural response of CFG energy piles. *Applied Thermal Engineering*, 96, 640–651.
<https://doi.org/10.1016/j.applthermaleng.2015.11.127>
- Yu, H. S. (1998). CASM: a unified state parameter model for clay and sand. *International Journal for Numerical and Analytical Methods in Geomechanics*, 22(8), 621–653.
[https://doi.org/https://doi.org/10.1002/\(SICI\)1096-9853\(199808\)22:8<621::AID-NAG937>3.0.CO;2-8](https://doi.org/https://doi.org/10.1002/(SICI)1096-9853(199808)22:8<621::AID-NAG937>3.0.CO;2-8)
- Yu, H.-S. (2006). Plasticity and Geotechnics. In *Springer* (Vol. 13, Issue 07). Springer US.
<https://doi.org/10.1007/978-0-387-33599-5>
- Zeng, H. Y., Diao, N. R., & Fang, Z. H. (2002). A finite line-source model for boreholes in geothermal heat exchangers. *Heat Transfer - Asian Research*, 31(7), 558–567.
<https://doi.org/10.1002/htj.10057>
- Zhou, C., & Ng, C. W. W. (2015). A thermomechanical model for saturated soil at small and large strains. *Canadian Geotechnical Journal*, 52(8), 1101–1110. <https://doi.org/10.1139/cgj-2014-0229>
- Zhou, C., Xu, J., & Ng, C. W. W. (2015). Effects of temperature and suction on secant shear modulus of unsaturated soil. *Géotechnique Letters*, 5(3), 123–128. <https://doi.org/10.1680/jgele.14.00096>
- ZIEGLER, H. (1983). *AN INTRODUCTION TO THERMOMECHANICS* (Second, re). NORTH-HOLLAND PUBLISHING COMPANY - AMSTERDAM · NEW YORK · OXFORD.

Appendix A

A1. Residuals:

A1.1 R_1 :

The residual equation for the state variable (strain) is given by equation (4.7) as,

$$\mathbf{R}_1 = -\frac{\partial g_1(\boldsymbol{\sigma}_{n+1})}{\partial \boldsymbol{\sigma}_{n+1}} + \Delta \Lambda \mathbf{r}(\boldsymbol{\sigma}_{n+1}, p_{cT}^{n+1}) - \boldsymbol{\varepsilon}^{\text{e,trial}} \quad (\text{A.1})$$

In equation (A.1), the Gibbs free energy (g_1) potential is given by (Einav & Puzrin, 2004)

$$g_1 = -\frac{p^{(2-m_s)} - (2-m_s) p p_0^{(2-m_s)}}{\bar{K}_s (2-m_s)(1-m_s) p_{ref}^{(1-m_s)}} - \frac{q^2}{6\bar{G}_s p_{ref}^{(1-n_s)} p^{n_s}} + \frac{q_0 (2q p_0 - n q_0 p)}{6\bar{G}_s p_{ref}^{(1-n_s)} p_0^{(1+n_s)}} \quad (\text{A.2})$$

where p_0 and q_0 are the initial mean and deviatoric stress, respectively; p_{ref} is the reference mean effective stress which is generally considered 1kPa. The derivative of g_1 with respect to stress ($\boldsymbol{\sigma}_{n+1}$) is a tensor that is obtained by applying the chain rule for derivative calculation as,

$$\frac{\partial g_1}{\partial \boldsymbol{\sigma}_{n+1}} = \frac{\partial g_1}{\partial p} \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} + \frac{\partial g_1}{\partial q} \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \quad (\text{A.3})$$

where

$$\frac{\partial g_1}{\partial p} = -\frac{p^{(1-m_s)} - p_0^{(1-m_s)}}{\bar{K}_s (1-m_s) p_{ref}^{(1-m_s)}} - \frac{n_s}{6\bar{G}_s p_{ref}^{(1-n_s)}} \left\{ \frac{q_0^2}{p_0^{(1+n_s)}} - \frac{q^2}{p^{(1+n_s)}} \right\} \quad (\text{A.3a})$$

$$\frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} = \frac{1}{3} \mathbf{I} \quad (\text{A.3b})$$

$$\frac{\partial g_1}{\partial q} = -\frac{2}{6\bar{G}_s p_{ref}^{(1-n_s)}} \left\{ \frac{q^2}{p^{n_s}} - \frac{q_0}{p_0^{n_s}} \right\} \quad (\text{A.3c})$$

$$\frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} = \frac{1}{2} \sqrt{\frac{3}{J_2}} \mathbf{s} \quad (\text{A.3d})$$

where

$$\mathbf{s} = \boldsymbol{\sigma} - p\mathbf{I} \quad (\text{A.4})$$

$$J_2 = \frac{1}{2} \mathbf{s} : \mathbf{s} \quad (\text{A.5})$$

$$p = \frac{1}{3} \text{tr}(\boldsymbol{\sigma}) \quad (\text{A.6})$$

$$q = \sqrt{3J_2} \quad (\text{A.7})$$

The flow rule ($\mathbf{r}$) can be described in true stress space as,

$$\mathbf{r} = r_p \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} + r_q \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \quad (\text{A.8})$$

A1.1 R_2 :

The residual equation for the hardening variable (pre-consolidation pressure) is given by equation (4.9) as,

$$R_2 = \frac{p_{cT}^{n+1}}{p_{cT}^n} - \exp\left(\frac{1+e_0}{\lambda-\kappa} \Delta \varepsilon_v^p - \mu \Delta T\right) \quad (\text{A.13})$$

where

$$\Delta \varepsilon_v^p = \Delta \Lambda r_p \quad (\text{A.13a})$$

A2. Components of J matrix:

A2.1 J_{11} :

The $\mathbf{J}_{11}$ tensor can be described as

$$\mathbf{J}_{11} = \frac{\partial \mathbf{R}_1}{\partial \boldsymbol{\sigma}_{n+1}} = -\frac{\partial^2 g_1}{\partial \boldsymbol{\sigma}_{n+1} \partial \boldsymbol{\sigma}_{n+1}} + \Delta \Lambda \frac{\partial \mathbf{r}}{\partial \boldsymbol{\sigma}_{n+1}} \quad (\text{A.14})$$

The calculations for different coefficients of $\mathbf{J}_{11}$ are shown below;

$$\begin{aligned} \frac{\partial^2 g_1}{\partial \boldsymbol{\sigma}_{n+1} \partial \boldsymbol{\sigma}_{n+1}} &= \frac{\partial^2 g_1}{\partial p^2} \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} + \frac{\partial^2 g_1}{\partial p \partial q} \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} + \frac{\partial^2 g_1}{\partial q \partial p} \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} + \frac{\partial^2 g_1}{\partial q^2} \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \\ &\quad + \frac{\partial g_1}{\partial q} \frac{\partial}{\partial \boldsymbol{\sigma}_{n+1}} \left(\frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \right) \end{aligned} \quad (\text{A.15})$$

$$\frac{\partial^2 g_1}{\partial p^2} = -\frac{p^{(-m_s)}}{\bar{K}_s p_{ref}^{(1-m_s)}} - \frac{n_s (n_s + 1) q^2}{6 \bar{G}_s p_{ref}^{(1-n_s)} p^{(n_s+2)}} \quad (\text{A.15a})$$

$$\frac{\partial^2 g_1}{\partial p \partial q} = \frac{\partial^2 g_1}{\partial q \partial p} = \frac{2 n_s q}{6 \bar{G}_s p_{ref}^{(1-n_s)} p^{(n_s+1)}} \quad (\text{A.15b})$$

$$\frac{\partial^2 g_1}{\partial q^2} = \frac{2}{6 \bar{G}_s p_{ref}^{(1-n_s)} p^{n_s}} \quad (\text{A.15c})$$

$$\frac{\partial}{\partial \boldsymbol{\sigma}_{n+1}} \left(\frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \right) = \frac{\sqrt{3}}{2 \sqrt{J_2}} \left\{ \left(\boldsymbol{\Pi}^T - \frac{1}{3} \boldsymbol{\Pi}^T \right) - \frac{1}{2 J_2^3} \mathbf{ss}^T \right\} \quad (\text{A.15d})$$

$$\frac{\partial \mathbf{r}}{\partial \boldsymbol{\sigma}_{n+1}} = \frac{\partial r_p}{\partial \boldsymbol{\sigma}_{n+1}} \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} + \frac{\partial r_q}{\partial \boldsymbol{\sigma}_{n+1}} \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} + r_q \frac{\partial}{\partial \boldsymbol{\sigma}_{n+1}} \left(\frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \right) \quad (\text{A.16})$$

$$\frac{\partial r_p}{\partial \boldsymbol{\sigma}_{n+1}} = \frac{\partial r_p}{\partial p} \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} + \frac{\partial r_p}{\partial q} \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \quad (\text{A.17})$$

$$\frac{\partial r_p}{\partial p} = \frac{\partial r_p}{\partial A} \frac{\partial A}{\partial p} + \frac{\partial r_p}{\partial B} \frac{\partial B}{\partial p} + (2B^2 + 2\beta^2 A^2) \quad (\text{A.17a})$$

$$\frac{\partial r_p}{\partial A} = -4\beta A(q - \beta p) \quad (\text{A.17b})$$

$$\frac{\partial A}{\partial p} = 1 - \gamma \quad (\text{A.17c})$$

$$\frac{\partial r_p}{\partial B} = 4B(p - C) \quad (\text{A.17d})$$

$$\frac{\partial B}{\partial p} = M(1 - \alpha) \quad (\text{A.17e})$$

$$\frac{\partial r_p}{\partial q} = -2\beta A^2 \quad (\text{A.17f})$$

$$\frac{\partial r_q}{\partial \sigma_{n+1}} = \frac{\partial r_q}{\partial p} \frac{\partial p}{\partial \sigma_{n+1}} + \frac{\partial r_q}{\partial q} \frac{\partial q}{\partial \sigma_{n+1}} \quad (\text{A.18})$$

$$\frac{\partial r_q}{\partial p} = \frac{\partial r_q}{\partial A} \frac{\partial A}{\partial p} - 2A^2 \beta \quad (\text{A.18a})$$

$$\frac{\partial r_q}{\partial A} = 4A(q - \beta p) \quad (\text{A.18b})$$

$$\frac{\partial r_q}{\partial q} = 2A^2 \quad (\text{A.18c})$$

A2.2 J₁₂:

The $\mathbf{J}_{12}$ which is a tensor can be described as

$$\mathbf{J}_{12} = \frac{\partial \mathbf{R}_1}{\partial p_{cT}^{n+1}} = \Delta \Lambda \frac{\partial \mathbf{r}}{\partial p_{cT}^{n+1}} \quad (\text{A.19})$$

The calculations for $\mathbf{J}_{12}$ are shown below;

$$\frac{\partial \mathbf{r}}{\partial p_{cT}^{n+1}} = \frac{\partial r_p}{\partial p_{cT}^{n+1}} \frac{\partial p}{\partial \sigma_{n+1}} + \frac{\partial r_q}{\partial p_{cT}^{n+1}} \frac{\partial q}{\partial \sigma_{n+1}} \quad (\text{A.20})$$

$$\frac{\partial r_p}{\partial p_{cT}^{n+1}} = \frac{\partial r_p}{\partial A} \frac{\partial A}{\partial p_{cT}^{n+1}} + \frac{\partial r_p}{\partial B} \frac{\partial B}{\partial p_{cT}^{n+1}} + \frac{\partial r_p}{\partial C} \frac{\partial C}{\partial p_{cT}^{n+1}} \quad (\text{A.21})$$

$$\frac{\partial A}{\partial p_{cT}^{n+1}} = \frac{\gamma}{2} \quad (\text{A.21a})$$

$$\frac{\partial B}{\partial p_{cT}^{n+1}} = \frac{M\alpha\gamma}{2} \quad (\text{A.21b})$$

$$\frac{\partial r}{\partial C} = -2B^2 \quad (\text{A.21c})$$

$$\frac{\partial C}{\partial p_{cT}^{n+1}} = \frac{\gamma}{2} \quad (\text{A.21d})$$

$$\frac{\partial r_q}{\partial p_{cT}^{n+1}} = \frac{\partial r_p}{\partial A} \frac{\partial A}{\partial p_{cT}^{n+1}} \quad (\text{A.22})$$

A2.3 J₁₃:

The $\mathbf{J}_{13}$ tensor can be described as

$$\mathbf{J}_{13} = \frac{\partial \mathbf{R}_1}{\partial (\Delta \Lambda)} = \mathbf{r} \quad (\text{A.23})$$

A2.4 J₂₁:

The $\mathbf{J}_{21}$ tensor can be described as

$$\mathbf{J}_{21} = \frac{\partial \mathbf{R}_2}{\partial \sigma_{n+1}} = -\exp\left(\frac{1+e_0}{\lambda-\kappa} \Delta \Lambda r_p - \mu \Delta T\right) \left(\frac{1+e_0}{\lambda-\kappa}\right) \Delta \Lambda \frac{\partial r_p}{\partial \sigma_{n+1}} \quad (\text{A.24})$$

A2.5 J₂₂:

The component, J_{22} can be described as

$$J_{22} = \frac{\partial R_2}{\partial p_{cT}^{n+1}} = \frac{1}{p_{cT}^n} - \exp\left(\frac{1+e_0}{\lambda-\kappa} \Delta \Lambda r_p - \mu \Delta T\right) \left(\frac{1+e_0}{\lambda-\kappa}\right) \Delta \Lambda \frac{\partial r_p}{\partial p_{cT}^{n+1}} \quad (\text{A.25})$$

A2.6 J₂₃:

The component, J_{23} tensor can be described as

$$J_{23} = \frac{\partial R_2}{\partial (\Delta \Lambda)} = -\exp\left(\frac{1+e_0}{\lambda-\kappa} \Delta \Lambda r_p - \mu \Delta T\right) \left(\frac{1+e_0}{\lambda-\kappa}\right) r_p \quad (\text{A.26})$$

A2.7 J₃₁:

The $\mathbf{J}_{31}$ tensor can be described as

$$\mathbf{J}_{31} = \frac{\partial R_3}{\partial \boldsymbol{\sigma}_{n+1}} = \frac{\partial y}{\partial \boldsymbol{\sigma}_{n+1}} \quad (\text{A.27})$$

$$\frac{\partial y}{\partial \boldsymbol{\sigma}_{n+1}} = \frac{\partial y}{\partial p} \frac{\partial p}{\partial \boldsymbol{\sigma}_{n+1}} + \frac{\partial y}{\partial q} \frac{\partial q}{\partial \boldsymbol{\sigma}_{n+1}} \quad (\text{A.28})$$

$$\frac{\partial y}{\partial p} = 2B^2(p - C) - 2A^2\beta(q - \beta p) + \frac{\partial y}{\partial A} \frac{\partial A}{\partial p} + \frac{\partial y}{\partial B} \frac{\partial B}{\partial p} \quad (\text{A.28a})$$

$$\frac{\partial y}{\partial A} = 2A(q - \beta p)^2 - 2AB^2 \quad (\text{A.28b})$$

$$\frac{\partial y}{\partial B} = 2B(p - C)^2 - 2A^2B \quad (\text{A.28c})$$

$$\frac{\partial y}{\partial q} = 2A^2(q - \beta p) \quad (\text{A.29})$$

A2.8 $\mathbf{J}_{32}$:

The component, J_{32} can be described as

$$J_{32} = \frac{\partial R_3}{\partial p_{cT}^{n+1}} = \frac{\partial y}{\partial p_{cT}^{n+1}} \quad (\text{A.30})$$

$$\frac{\partial y}{\partial p_{cT}^{n+1}} = \frac{\partial y}{\partial A} \frac{\partial A}{\partial p_{cT}^{n+1}} + \frac{\partial y}{\partial B} \frac{\partial B}{\partial p_{cT}^{n+1}} + \frac{\partial y}{\partial C} \frac{\partial C}{\partial p_{cT}^{n+1}} \quad (\text{A.31})$$

$$\frac{\partial y}{\partial C} = -2B^2(p - C) \quad (\text{A.32})$$

A2.9 $\mathbf{J}_{33}$:

The component, J_{33} can be described as

$$J_{33} = \frac{\partial R_3}{\partial (\Delta\Lambda)} = \frac{\partial y}{\partial (\Delta\Lambda)} = 0 \quad (\text{A.33})$$