

Comparative Analysis of MPC and Integrated Skyhook-LQR Controllers for a CDC Damper Suspension System in Passenger Cars

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The performance of vehicle suspension systems is critical for ride comfort, handling stability, and safety. Passive suspensions, while simple and reliable, lack adaptability to road conditions, whereas fully active suspensions provide superior performance but are impractical for passenger vehicles due to high energy consumption. Semi-active suspensions offer a balance by dynamically adjusting damping forces with minimal power requirements, but their effectiveness depends on the chosen control strategy.

This study evaluates two advanced control strategies: Model Predictive Control (MPC) and an integrated Skyhook-Linear Quadratic Regulator (LQR) controller. A seven-degree-of-freedom (7-DOF) full-vehicle model is developed using Lagrange's equations to analyze vehicle dynamics. MPC is formulated as a predictive controller that anticipates road disturbances, while the integrated Skyhook-LQR controller combines classical Skyhook damping with optimal state feedback for improved stability.

The controllers are implemented in a CarSim-Simulink simulation environment and tested under various road conditions. To assess real-world feasibility, experimental testing is conducted on a passenger vehicle equipped with semi-active CDC dampers using the Skyhook-LQR controller.

The comparative analysis highlights key trade-offs. MPC provides superior predictive control and optimizes suspension performance across multiple axes but requires high computational power, limiting real-time implementation. The integrated Skyhook-LQR controller, while computationally efficient and practical for embedded systems, has limitations in handling complex disturbances.

These findings underscore the importance of selecting control strategies based on application requirements. While MPC enhances ride comfort and stability, Skyhook-LQR remains a more feasible real-time solution. This study contributes to the advancement of semi-active suspension technology, offering insights for future vehicle suspension designs.

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Dedication

To Lele

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Chapter 1

Introduction

1.1 Problem Statement and Motivation

Vehicle suspension systems play an important role in ride comfort, handling stability, and overall safety. Traditional passive suspension systems are reliable and cost-effective, but their fixed damping characteristics have limitations and cannot adapt to changing road conditions. In contrast, fully active suspension systems significantly improve the ride quality and handling performance by dynamically adjusting suspension forces in real time. However, it is difficult to commercialize in commercial passenger vehicles due to high energy consumption, high cost, and control complexity.

To address this contradiction, a semi-active suspension system is developed as a compromise, which combines the adaptability of active suspension with the low energy consumption and high-reliability advantages of passive suspension. Continuous damping control (CDC) system stands out as its ability to adjust the damping force in real-time according to road conditions and vehicle dynamics [1]. However, the effectiveness of CDC systems depends on robust control strategies, which need to optimize suspension performance while considering the computational efficiency in real-time embedded automotive systems.

Recent research has explored various control strategies for semi-active suspension systems, ranging from classical Skyhook and Ground hook control to intelligent and hybrid controllers, such as Model Predictive Control (MPC), Linear Quadratic Regulator (LQR), and fuzzy logic controllers. While model predictive control (MPC)-based controllers excel in multi-objective optimization, their high computational requirements limit their application in automotive real-time systems. In contrast, canopy-LQR hybrid controllers lack

the predictive power of MPCs despite their high computational efficiency. Although significant progress has been made in semi-active suspension control technology, there are still research gaps in comparative performance studies, computational feasibility analysis, and commercial vehicle applicability validation for different controllers, which need to be systematically explored.

1.2 Overall Structure of the Research and Contributions

This research presents a detailed investigation into the mathematical modeling, simulation, and real-world validation of semi-active suspension control strategies for a Continuously Damping Control (CDC) damper system. The primary objective is to develop a high-fidelity 7-degree-of-freedom (7-DOF) full-vehicle suspension model using Lagrange's equations, capturing vertical, roll, and pitch dynamics while incorporating actuator dynamics and real-world damping force constraints.

Two advanced semi-active control strategies are implemented and compared: Model Predictive Control (MPC) and an integrated Skyhook-Linear Quadratic Regulator (LQR) controller. MPC is formulated as a constrained optimization-based approach that predicts future vehicle states and adjusts damping forces accordingly. The integrated Skyhook-LQR controller, on the other hand, is a computationally efficient control strategy that combines Skyhook damping principles with optimal state feedback control to improve stability and ride comfort.

In order to evaluate the effectiveness of different controllers, this study carries out performance tests through the CarSim-Simulink co-simulation platform, simulating typical road conditions such as speed bumps, lane changing conditions, rough road surfaces and emergency braking scenarios. The joint simulation method can reproduce the vehicle dynamics with high accuracy and provide technical support for analyzing the dynamic response of the suspension under real driving conditions.

Beyond simulation-based validation, real-world vehicle experiments are conducted on a Hongqi H9 equipped with CDC dampers to verify the practical feasibility of the MPC and Skyhook-LQR controllers. The experimental setup includes an instrumented vehicle with an embedded control system, enabling real-time suspension adjustments and data acquisition. The test vehicle undergoes various dynamic maneuvers to assess ride comfort, handling stability, and the ability of each control strategy to mitigate road disturbances.

A comparative analysis of the two controllers is performed, highlighting their respective strengths, limitations, and trade-offs regarding ride comfort, handling performance, computational efficiency, and real-time implementation. MPC demonstrates strong predictive control capabilities, optimizing all three suspension axes—vertical, roll, and pitch—but presents challenges in terms of computational complexity. In contrast, the integrated Skyhook-LQR controller provides a practical, real-time alternative, excelling primarily in vertical motion suppression while being more feasible for embedded automotive systems.

This study is not only limited to theoretical analysis but also fills the gap between simulation optimization of suspension systems and actual vehicle applications. The results of this study provide important theoretical support for the development of a new generation of semi-active suspension systems for commercial passenger cars and also lay the foundation for the development of more adaptive and intelligent suspension control strategies.

1.3 Thesis Outline

This thesis is structured as follows.

Chapter 1 introduces the problem statement, motivation, and contributions of this research, outlining the need for advanced control strategies in semi-active suspension systems.

Chapter 2 provides a systematic review of semi-active suspension systems and their control methods. This chapter analyzes the inherent defects of passive and fully active suspensions and explains the technical advantages of semi-active suspensions regarding energy consumption and performance balance. At the level of control strategies, classical methods such as canopy control and floor-shed control, as well as hybrid strategies such as canopy-floor-shed switching control, are systematically sorted out. The realization mechanisms of advanced algorithms such as model predictive control (MPC), linear quadratic regulator (LQR), fuzzy logic control, and neural network adaptive controller are also highlighted. In addition, this chapter also integrates the research results on the comparison of suspension control methods in recent years, evaluating the technical characteristics of different control strategies in terms of smoothness, maneuverability, and energy efficiency to provide a theoretical basis for subsequent controller design and optimization.

Chapter 3 develops the mathematical model of a 7-degree-of-freedom (7-DOF) full-vehicle suspension system using Lagrange’s equations, capturing vertical, roll, and pitch dynamics. The CDC damper is modeled as a first-order system with actuator lag considerations. The chapter formulates the MPC and Skyhook-LQR controllers in state-space

form and describes the implementation of constraints, optimization objectives, and stability considerations. The system’s controllability and observability are also analyzed.

Chapter 4 presents the simulation-based performance evaluation of the proposed control strategies using a CarSim-Simulink co-simulation framework. The MPC and Skyhook-LQR controllers are implemented, and their effectiveness is analyzed under various road conditions, including single and offset speed bumps, lane change maneuvers, and continuous rough roads. The impact of suspension tuning, actuator response time, and control constraints on ride quality, handling stability, and computational efficiency are discussed.

Chapter 5 concludes the results and contributions of this thesis. It provides recommendations for future work, including potential improvements in predictive control strategies, real-time embedded implementation of MPC, and integration of machine learning-based adaptive control approaches for semi-active suspension systems.

Chapter 2

Literature Review

2.1 Semi-Active Suspension Systems in Passenger Cars

Suspension systems are critical to vehicle dynamics, directly impacting ride comfort, handling, and overall safety. Passive suspension systems have traditionally been widely used due to their simplicity and reliability. However, their fixed damping characteristics limit adaptability to varying road conditions. In contrast, fully active suspension systems offer superior performance by continuously adjusting suspension forces. However, they require high energy consumption and complex control mechanisms, making them impractical for widespread use in passenger vehicles. Semi-active suspension systems have emerged as a practical middle ground, providing adjustable damping characteristics that enhance ride comfort and handling while maintaining relatively low power consumption [2].

Semi-active suspension systems adapt to driving conditions by adjusting the suspension damping force in real-time. The system's core uses adjustable dampers, such as magnetorheological (MR) and electro-thermal (ER) dampers, to replace the fixed damping elements of passive systems. Compared to traditional passive suspensions, semi-active systems significantly improve performance by dynamically adjusting the damping characteristics while maintaining the advantage of low energy consumption. Mainstream control strategies include canopy control, floor control, and hybrid control methods, which improve vehicle dynamics by optimizing vibration attenuation effects, thus easing the conflict between occupant comfort and tire grounding [2].

Research has revealed the advantages of semi-active suspensions over passive versus fully active systems through numerous comparative experiments. It has been shown that

semi-active suspensions require only a tiny fraction of the energy required by fully active systems while significantly improving vehicle smoothness ([3]). In order to verify the dynamic performance improvement effect, researchers commonly use quarter-body, half-body, and other simulation models to evaluate the effectiveness of different damping control algorithms through numerical simulation and bench tests ([4]), which can effectively analyze the dynamic response characteristics of the system under road excitation and driving condition changes.

In recent years, research focusing on semi-active suspension control technology has shifted to improving the adaptability and robustness of the strategy. Intelligent algorithms such as fuzzy logic control and neural network adaptive control are introduced into the system ([5]) to optimize the dynamic response characteristics. Meanwhile, the hybrid control method ([6]), which integrates multiple control laws, shows significant advantages in balancing comfort and maneuverability. These technological breakthroughs improve vehicle stability and vibration suppression and enhance the system's adaptability to complex driving environments.

The comparative performance of semi-active suspension systems has been widely studied, with results indicating their effectiveness in reducing vehicle vibrations and enhancing road-holding capabilities. Experimental results have shown that semi-active suspensions can significantly reduce acceleration levels experienced by passengers while maintaining adequate tire-road contact [7]. Additionally, the integration of predictive control techniques has further improved the efficiency of semi-active suspensions by enabling proactive adjustments based on anticipated road conditions.

In summary, the semi-active suspension system realizes the optimal balance of performance, energy consumption, and cost between traditional passive and fully active suspensions and shows significant engineering application value. With the innovation of control strategies and the advancement of material technology, this system is expected to become the standard configuration of modern passenger cars. Future research may focus on the deep optimization of control algorithms and the fusion of predictive models based on machine learning to further improve the system's real-time adaptability and dynamic performance.

2.2 Control Strategies for Semi-Active Suspension Systems

Semi-active suspension systems have gained significant attention due to their ability to enhance ride comfort and handling performance while maintaining low energy consumption. Unlike passive suspensions, which rely on fixed damping characteristics, semi-active systems allow real-time tuning of damping properties based on external road and vehicle conditions. Various control strategies have been developed to optimize performance, including classical control techniques, intelligent control methods, and hybrid approaches.

Classical control strategies for semi-active suspensions include Proportional-Integral-Derivative (PID) and Skyhook controllers. Skyhook control is a well-known technique that effectively reduces body acceleration, improving ride comfort, but it may compromise road holding [8]. Groundhook control, on the other hand, focuses on enhancing road contact and reducing tire deflections while slightly increasing body vibrations. Hybrid controllers that dynamically switch between these strategies, such as the supervisor-based switching controller, have been proposed to optimize ride comfort and handling [6].

Intelligent control approaches, such as fuzzy logic, neural networks, and model predictive control (MPC), have been introduced to enhance adaptability. Fuzzy logic controllers, for example, can dynamically adjust damping characteristics based on uncertain and non-linear inputs, improving robustness and adaptability in real-world scenarios [9]. Similarly, neural network-based controllers have demonstrated the ability to learn optimal damping characteristics from data, allowing for self-adaptive tuning in response to varying road conditions. Model predictive control (MPC), which incorporates future road conditions into control decisions, has also been explored. The predictive control approach enables semi-active suspension systems to anticipate road irregularities and adjust damping characteristics accordingly, leading to improved ride quality [10].

Hybrid control strategies have also been developed to integrate multiple control techniques, balancing performance trade-offs. A gain-scheduling control strategy has been introduced to adjust control gains based on real-time operating conditions dynamically, achieving semi-active suspension performance comparable to fully active systems [11]. Adaptive hybrid controllers, which can switch between skyhook and groundhook strategies based on predefined conditions, have been shown to offer superior ride comfort and road holding compared to traditional controllers [12].

Recent studies have also examined the effectiveness of semi-active suspension controllers in practical applications. A study evaluating semi-active suspension strategies on a complete vehicle model found that while many controllers were initially designed for

ride comfort, some also contributed to improved handling performance [13]. Additionally, experimental results from MATLAB/Simulink simulations and real-world tests have demonstrated that intelligent and hybrid control approaches significantly outperform traditional semi-active controllers in terms of adaptability and robustness to varying driving conditions.

In summary, semi-active suspension control strategies have evolved from classical approaches to more sophisticated intelligent and hybrid techniques. While traditional controllers such as Skyhook and PID remain widely used, intelligent control methods, including fuzzy logic and MPC, have enhanced suspension adaptability. Hybrid approaches have further improved overall system performance, balancing ride comfort and handling. Integrating machine learning and predictive control into semi-active suspension systems will lead to even more excellent vehicle ride quality and stability advancements as research continues.

2.3 Comparative Analysis of Semi-Active Suspension Controllers

The choice of control strategy significantly influences the performance of semi-active suspension systems. Various controllers have been developed and compared to optimize the trade-offs between ride comfort, handling stability, and suspension deflection. This section provides a comparative analysis of different semi-active suspension controllers based on recent research findings.

A comparative study of passive, active, and semi-active suspensions with various optimal control strategies was conducted using La Linear Quadratic Regulator (LQR) and road preview control. The study demonstrated that semi-active suspensions with road preview could achieve performance close to active suspensions, particularly when optimizing ride comfort and vehicle handling [14]. The results suggest that while active suspensions remain superior, semi-active systems offer a cost-effective compromise.

Several studies have evaluated different proportional-integral-derivative (PID) -based controllers for semi-active suspensions. A study using genetic algorithms to tune different PID variants found that integral-separation PID controllers provided better adaptive performance under varying road conditions [15]. Similarly, a comparative study of the skyhook, hybrid, and model predictive control (MPC) strategies identified sliding mode control as a highly effective approach for magnetorheological (MR) dampers [16].

On-off semi-active controllers have also been analyzed in embedded systems, with research highlighting the impact of real-time constraints and computing power on control performance [17]. Meanwhile, a classical control comparison study examined different semi-active strategies and found that predictive control methods, while effective, still pose challenges in terms of cost and implementation [18].

The stability of switched semi-active controllers has been investigated using Lyapunov stability analysis, revealing that switching effects can introduce non-intuitive design constraints. Despite these challenges, switched controllers such as the skyhook-groundhook hybrid continue to be widely used due to their robustness [19].

More recent approaches have explored fuzzy logic and type-II fuzzy controllers for semi-active suspensions. A comparative study demonstrated that type-II fuzzy controllers outperformed traditional fuzzy logic controllers by achieving superior adaptability and reducing damping coefficient variations [20]. Similarly, fractional-order PID (FOPID) controllers have been shown to outperform conventional PID controllers, reducing vehicle body acceleration significantly under varying road conditions [21].

Experimental validation of different semi-active controllers has demonstrated their practical applications. For instance, real-world tests comparing hybrid controllers, skyhook-ground hook systems, and frequency-based controllers confirmed the feasibility of semi-active suspensions in commercial vehicles [22]. Moreover, research into automotive semi-active MR suspensions for electric vehicles has explored how traction influences control effectiveness, highlighting the importance of vehicle-specific control tuning [23].

In summary, semi-active suspension controllers vary in performance depending on their application and control methodology. While classical controllers such as Skyhook and PID remain widely used, modern strategies incorporating fuzzy logic, model predictive control, and switched hybrid approaches have demonstrated superior adaptability and robustness. Future research should focus on integrating machine learning-based controllers and optimizing real-time adaptive control techniques for enhanced ride quality and vehicle stability.

Chapter 3

Mathematical Modeling and Control Strategies for CDC Damper Suspension System

In this chapter, the integrated framework for vehicle system modelling, control system design, and signal processing is presented. A simplified full-car model is developed to capture the essential dynamics of vehicle motion, ensuring accurate representation for control applications. The controllability and observability of the model are examined, followed by the actuator dynamics, which play a crucial role in force generation and system response.

Building on this foundation, a Model Predictive Control (MPC) system is introduced to optimize suspension performance, addressing multi-objective control constraints. Observers based on Kalman filtering techniques are utilized to estimate vertical, roll, and pitch motions, enhancing the system's ability to respond to varying driving conditions. Additionally, the integrated LQR-Skyhook control strategy is discussed as a benchmark for performance comparison.

To ensure the reliability of the control system, an RC low-pass filter is implemented to eliminate high-frequency noise from sensor data, thereby improving the accuracy of system responses. The subsequent sections delve into the mathematical formulations, control strategies, and filtering techniques that collectively contribute to the development of an advanced adaptive suspension system.

3.1 Vehicle System Modeling

In this section, the mathematical model of the full-vehicle system is formulated in the state-space representation. The equations of motion governing the vehicle dynamics are derived, incorporating the effects of the CDC damper.

3.1.1 State-Space Formulation of the Full-Vehicle Model

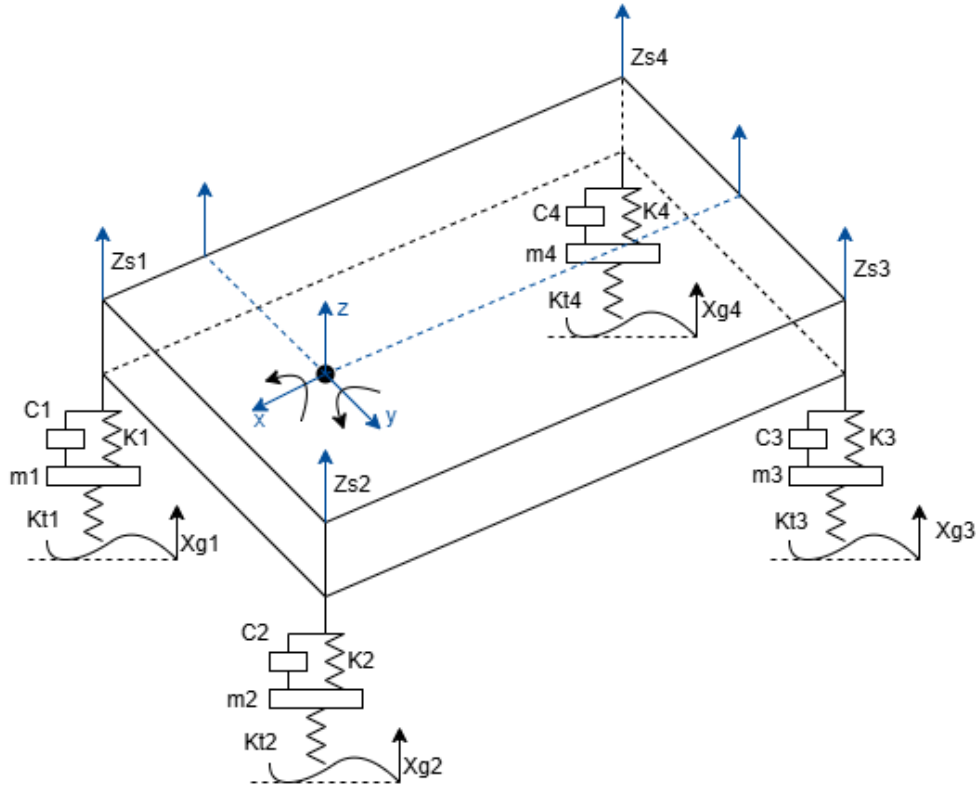


Figure 3.1: 7 Degree of Freedom (7DOF) Full-Vehicle Model

The right-handed Cartesian coordinate system defined in ISO-8855 [24] is used to describe the vehicle system. As illustrated in the updated schematic, the vehicle is modeled as a seven-degree-of-freedom (7-DOF) system, including the vertical, roll, and pitch motions of the sprung mass, as well as the independent vertical motion of each unsprung mass. The

system is formulated using Lagrange's equations, which provide an alternative approach to Newton's Second Law for deriving the equations of motion. Lagrange's method is particularly useful for complex, multi-degree-of-freedom systems, as it systematically accounts for energy distribution within the system rather than relying on direct force balances [25].

Symbol	Definition
Z_s	Vertical displacement of the vehicle's center of gravity (CG)
ϕ_s, θ_s	Roll and pitch angles of the vehicle body
Z_{s_i}	Vertical displacement at each suspension attachment point
Z_{u_i}	Vertical displacement of the unsprung masses
X_{g_i}	Road input displacement
m_s, m_i	Sprung mass (m_s) and unsprung masses (m_i , where $i = 1, 2, 3, 4$)
K_i, C_i	Suspension stiffness (K_i) and damping coefficients (C_i)
Kt_i	Tire stiffness at each wheel
l	Half-track width of the vehicle
L_1, L_2	Distances from the CG to the front and rear axles

Table 3.1: Vehicle model parameters

The Lagrangian function is defined as:

$$L = T - V \quad (3.1)$$

where T is the total kinetic energy of the system, and V is the total potential energy. The equations of motion for generalized coordinates q_i are given by:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (3.2)$$

where Q_i represents the generalized external forces.

The kinetic energy of the system is expressed as:

$$T_s = \frac{1}{2}m_s\dot{Z}_s^2 + \frac{1}{2}I_x\dot{\phi}_s^2 + \frac{1}{2}I_y\dot{\theta}_s^2 \quad (3.3)$$

$$T_u = \sum_{i=1}^4 \frac{1}{2}m_i\dot{Z}_{u_i}^2 \quad (3.4)$$

The potential energy stored in the suspension springs and tires is given by:

$$V_s = \sum_{i=1}^4 \frac{1}{2} K_i (Z_{s_i} - Z_{u_i})^2 \quad (3.5)$$

$$V_t = \sum_{i=1}^4 \frac{1}{2} K t_i (Z_{u_i} - X_{g_i})^2 \quad (3.6)$$

Applying Lagrange's equations for the vertical, roll, and pitch motions leads to:

$$m_s \ddot{Z}_s = \sum_{i=1}^4 (K_i (Z_{u_i} - Z_{s_i}) + C_i (\dot{Z}_{u_i} - \dot{Z}_{s_i})) \quad (3.7)$$

$$I_x \ddot{\phi}_s = l \sum_{i=1}^4 (-1)^i \left(K_i (Z_{u_i} - Z_{s_i}) + C_i (\dot{Z}_{u_i} - \dot{Z}_{s_i}) \right) - (K_{\text{bar}_1} + K_{\text{bar}_2}) \phi_s \quad (3.8)$$

$$I_y \ddot{\theta}_s = \sum_{i=1}^4 (-1)^{\lfloor i/2 \rfloor} L_i \left(K_i (Z_{u_i} - Z_{s_i}) + C_i (\dot{Z}_{u_i} - \dot{Z}_{s_i}) \right) \quad (3.9)$$

For each unsprung mass:

$$m_i \ddot{Z}_{u_i} = -K_i (Z_{u_i} - Z_{s_i}) - C_i (\dot{Z}_{u_i} - \dot{Z}_{s_i}) + K t_i (X_{g_i} - Z_{u_i}) \quad (3.10)$$

Finally, the system is expressed in state-space form as:

$$\dot{X} = AX + B_u U + B_w W \quad (3.11)$$

$$Y = CX + D_u U + D_w W \quad (3.12)$$

in which the state vector X is:

$$x = [Z_s \ \phi_s \ \theta_s \ Z_{u1} \ Z_{u2} \ Z_{u3} \ Z_{u4} \ \dot{Z}_s \ \dot{\phi}_s \ \dot{\theta}_s \ \dot{Z}_{u1} \ \dot{Z}_{u2} \ \dot{Z}_{u3} \ \dot{Z}_{u4}]^T \quad (3.13)$$

Moreover, the state transition matrix A , the control input matrix B_u , the disturbance input matrix B_w , the output matrix C , the feedforward matrices D_u and D_w are given by:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-(k_1+k_2+k_3+k_4)}{m_s} & 0 & \frac{(k_1+k_2)L_1-(k_3+k_4)L_2}{m_s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-(k_1+k_2+k_3+k_4)L_1^2+k_{bar1}+k_{bar2}}{I_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{(k_1+k_2)L_1-(k_3+k_4)L_2}{I_y} & 0 & \frac{-(k_1+k_2)L_1^2+(k_3+k_4)L_2^2}{I_y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.14)$$

$$B_u = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m_s} & \frac{1}{m_s} & \frac{1}{m_s} & \frac{1}{m_s} \\ \frac{L_1}{I_x} & \frac{L_2}{I_x} & -\frac{L_1}{I_x} & -\frac{L_2}{I_x} \\ -\frac{L_1}{I_y} & \frac{L_2}{I_y} & -\frac{L_1}{I_y} & \frac{L_2}{I_y} \end{bmatrix} \quad (3.15)$$

$$B_w = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k_1}{m_s} & \frac{k_2}{m_s} & \frac{k_3}{m_s} & \frac{k_4}{m_s} & 0 & 0 \\ \frac{k_{bar1}}{2I_x} & \frac{k_2+k_{bar2}}{2I_x} & -\frac{k_3+k_{bar1}}{2I_x} & -\frac{k_4+k_{bar2}}{2I_x} & 0 & 0 \\ -\frac{k_1L_1}{I_y} & \frac{k_2L_2}{I_y} & -\frac{k_3L_1}{I_y} & \frac{k_4L_2}{I_y} & 0 & 0 \end{bmatrix} \quad (3.16)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-(k_1+k_2+k_3+k_4)}{m_s} & 0 & \frac{(k_1+k_2)L_1-(k_3+k_4)L_2}{m_s} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-(k_1+k_2+k_3+k_4)L_1^2+k_{bar1}+k_{bar2}}{I_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{(k_1+k_2)L_1-(k_3+k_4)L_2}{I_y} & 0 & \frac{-(k_1+k_2)L_1^2+(k_3+k_4)L_2^2}{I_y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.17)$$

$$D_u = B_u, \quad D_w = B_w \quad (3.18)$$

By employing Lagrange's method, the equations of motion have been systematically derived, ensuring a consistent energy-based approach to modeling the 7-DOF vehicle system. This formulation facilitates the development of advanced control strategies, which will be explored in subsequent chapters. The mathematical framework follows standard vehicle dynamics references such as Goodarzi and Khajepour [25] and state-space modeling principles from Friedland [26].

3.1.2 Controllability and Observability Analysis

The controllability and observability of the state-space model are evaluated using the controllability matrix $\mathcal{C}$ and the observability matrix $\mathcal{O}$.

The controllability matrix is given by:

$$\mathcal{C} = [B_u \quad AB_u \quad A^2B_u \quad \dots \quad A^{n-1}B_u] \quad (3.19)$$

where A is the state transition matrix, B_u is the control input matrix, and n is the number of states.

Similarly, the observability matrix is:

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad (3.20)$$

where C is the output matrix.

For controllability, the rank of $\mathcal{C}$ is computed:

$$\text{rank}(\mathcal{C}) = n \quad (3.21)$$

Matrix $\mathcal{C}$ is formed by computing $A^k B_u$ for $k = 0$ to $n - 1$ and stacking the resulting matrices.

For observability, the rank of $\mathcal{O}$ is evaluated:

$$\text{rank}(\mathcal{O}) = n \quad (3.22)$$

Matrix $\mathcal{O}$ is constructed by stacking CA^k for $k = 0$ to $n - 1$.

Since both $\mathcal{C}$ and $\mathcal{O}$ are full-rank, the system is fully controllable and observable, allowing all states to be influenced by control inputs and estimated from outputs.

3.1.3 Actuator Dynamics and CDC Damper Modeling

The semi-active Continuously Damping Control (CDC) damper is modeled as a first-order system, where the damping force is generated based on the applied current. The relationship between the commanded damping force F_d and the applied current I can be described as:

$$\dot{F}_d = KI_d + \tau F_d \quad (3.23)$$

Expanding this equation into matrix form for the four corners of the vehicle suspension system:

$$\begin{bmatrix} \dot{F}_1 \\ \dot{F}_2 \\ \dot{F}_3 \\ \dot{F}_4 \end{bmatrix} = K \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} + \tau \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \quad (3.24)$$

where:

- F_d is the damping force generated by the CDC damper,
- I_d is the applied control current,
- K is the actuator gain,
- τ is the time constant of the damper response.

The first-order dynamics describe the response time of the damper system and ensure that the commanded force does not instantly follow the input current. Instead, it has a lag characteristic defined by τ . This model is important in designing controllers that compensate for actuator lag and optimize vehicle ride comfort and stability.

3.2 Model Predictive Control (MPC)

This section presents the state-space formulation of the Hongqi H9 vehicle model as the foundation for MPC. The system dynamics, including vehicle body, suspension, and wheel models, are structured into a predictive control framework to optimize ride comfort and handling performance. The controller predicts future system behavior over a defined horizon and determines an optimal sequence of control actions by solving a constrained optimization problem.

3.2.1 Formulation of MPC Based on the State-Space Vehicle Model

Model Predictive Control (MPC) is based on the discretized state space representation of the vehicle suspension system. The formulation includes system discretization, state and output prediction, cost function optimization, and limit enforcement to ensure system stability and performance.

The continuous-time state-space model is discretized using the zero-order-hold (ZOH) method with a sampling period T_s . The system dynamics are approximated by assuming that the inputs remain constant over the sampling period. The continuous-time system is given as:

$$\dot{X}(t) = AX(t) + B_u U(t) + B_w W(t), \quad Y(t) = CX(t) + D_u U(t) + D_w W(t) \quad (3.25)$$

To obtain the discrete-time equivalent, the system is transformed using matrix exponentials and integral calculations:

$$X(k+1) = A_d X(k) + B_d U(k) + B_w^d W(k), \quad Y(k) = C_d X(k) + D_d U(k) + D_w^d W(k) \quad (3.26)$$

where the discrete-time system matrices are defined as:

$$A_d = e^{AT_s}, \quad B_d = \int_0^{T_s} e^{A\tau} B_u d\tau, \quad B_w^d = \int_0^{T_s} e^{A\tau} B_w d\tau \quad (3.27)$$

Once the system is discretized, the future state evolution over a prediction horizon N_p can be computed recursively. The state propagation is expressed as:

$$X(k+1) = A_d X(k) + B_d U(k) \quad (3.28)$$

By expanding this equation over multiple time steps, the state predictions over the prediction horizon are obtained as:

$$\begin{bmatrix} X(k+1) \\ X(k+2) \\ \vdots \\ X(k+N_p) \end{bmatrix} = \begin{bmatrix} A_d \\ A_d^2 \\ \vdots \\ A_d^{N_p} \end{bmatrix} X(k) + \mathcal{T}_u \begin{bmatrix} U(k) \\ U(k+1) \\ \vdots \\ U(k+N_p-1) \end{bmatrix} \quad (3.29)$$

where $\mathcal{T}_u$ is a lower triangular Toeplitz matrix containing the control influence through B_d . This allows prediction of future states based on initial conditions and applied control inputs.

Similarly, the output predictions are formulated in terms of the state and control sequences. The predicted outputs are expressed as:

$$\begin{bmatrix} Y(k+1) \\ Y(k+2) \\ \vdots \\ Y(k+N_p) \end{bmatrix} = \mathcal{T}_y X(k) + \mathcal{T}_u U_{seq} \quad (3.30)$$

where $\mathcal{T}_y$ incorporates powers of A_d , ensuring that the output is correctly represented over the prediction horizon.

The control objective is to minimize the vertical, roll, and pitch accelerations of the sprung mass while also limiting excessive control effort. This is achieved by formulating a quadratic cost function:

$$J = \sum_{k=0}^{N_p} (Y(k)^T Q Y(k) + U(k)^T R U(k)) \quad (3.31)$$

where Q penalizes deviations in system output, and R penalizes aggressive control inputs to maintain system efficiency and stability.

To simplify the optimization problem, the objective function is rewritten in matrix form as:

$$J = X(k)^T \mathcal{Q}X(k) + U_{seq}^T \mathcal{H}U_{seq} + 2X(k)^T \mathcal{L}U_{seq} \quad (3.32)$$

where $\mathcal{H}$, $\mathcal{Q}$, and $\mathcal{L}$ are block matrices constructed using Q and R over the prediction horizon. This reformulation ensures the problem is expressed in a standard quadratic programming (QP) form, which can be efficiently solved by solving a quadratic optimization problem at each time step while ensuring constraints on states and control inputs. The optimization problem is formulated as:

$$\min_{U_{seq}} X(k)^T \mathcal{Q}X(k) + U_{seq}^T \mathcal{H}U_{seq} + 2X(k)^T \mathcal{L}U_{seq} \quad (3.33)$$

where the constraints can be formulated as linear combination of the optimization variables and also by upper and lower bounds on the optimization variables.

$$X_{min} \leq X(k) \leq X_{max} \quad (3.34)$$

$$U_{min} \leq U(k) \leq U_{max} \quad (3.35)$$

$$U(k) \geq 0 \quad \text{if} \quad C_v X(k) > 0 \quad (3.36)$$

$$U(k) \leq 0 \quad \text{if} \quad C_v X(k) < 0 \quad (3.37)$$

It can be observed that the constraints on the damping forces generated by the semi-active dampers depend on the direction of suspension deflection velocities, which are components of the system states. At each time step, the previously computed optimal control signal $U^*(k-1)$ is retrieved and utilized for updating system states. To simplify the dissipativity constraint problem, it is assumed that the damping forces remain constant over both the prediction and control horizons. Under this assumption, subsequent vehicle states can be determined, which are then used to compute the bounds of the optimal control signals $U^*(k)$ for the next time step. Based on this description, the proposed MPC control algorithm is summarized as follows:

Algorithm 1 MPC Control Algorithm

Input: Initial state vector $X(0)$, Prediction horizon N_p , control horizon N_c , weight matrices Q, R ,

Output: Optimal control sequence U_{seq}

Initialize: Set initial state $X(k)$, define cost function J

```
1: while system is running do
2:   Predict system evolution over  $N_p$  steps:
3:   for  $i = 1$  to  $N_p$  do
4:      $X_{\text{pred}}(i + 1) = A_d X_{\text{pred}}(i) + B_d U(i)$ 
5:      $Y_{\text{pred}}(i) = C_d X_{\text{pred}}(i)$ 
6:   end for
7:   Define quadratic cost function:
8:    $J = \sum (Y_{\text{pred}}(i)^T Q Y_{\text{pred}}(i) + U(i)^T R U(i))$ 
9:   Subject to constraints:
10:   $X_{\min} \leq X_{\text{pred}}(i) \leq X_{\max}$ 
11:   $U_{\min} \leq U(i) \leq U_{\max}$ 
12:  Dissipativity constraint:  $U(i) \cdot C_v X_{\text{pred}}(i) \geq 0$ 
13:  Solve Quadratic Program (QP) to get optimal  $U_{\text{seq}}$ 
14:  Apply first control input:  $U(k) = U_{\text{seq}}(0)$ 
15:  Update system state:
16:   $X(k + 1) = A_d X(k) + B_d U(k)$ 
17: end while
```

3.2.2 Observer-Based Compensation for Model Uncertainties in MPC

In order to ensure the effectiveness of Model Predictive Control (MPC) in semi-active suspension systems, reliable state estimation and direct sensor measurements are essential. However, they are often subject to noise, bias, or drift, which can affect the accuracy of control decisions. Furthermore, some states required for control, such as vertical displacement or roll and pitch angles, cannot always be measured directly. In order to overcome these challenges, an observer structure is implemented that uses a Kalman filter to estimate unmeasured states and to mitigate the effects of sensor errors. By integrating sensor readings with a model-based state space representation, the observer improves the robustness of MPC in the presence of uncertainties.

The estimation process is based on the Kalman filter framework, which minimizes the estimation error by optimally weighting model predictions and sensor data. The Kalman gain is computed as:

$$G_k = S_k D_o^T (D_o S_k D_o^T + R_y)^{-1}, \quad (3.38)$$

$$S_k = (I - G_k D_o) S_{k-1}. \quad (3.39)$$

Here, G_k represents the Kalman gain, which determines the relative influence of measurement updates versus model predictions. The term S_k is the error covariance matrix, reflecting the uncertainty in the state estimates. The matrix D_o is the output matrix of the observer, which relates the measured outputs to the system states, and R_y is the measurement noise covariance, quantifying the level of uncertainty associated with sensor readings.

The state update equation for the observer is given as:

$$X_o(k+1) = M_o X_o(k) + N_o U(k) + G_k [y_m(k) - D_o X_o(k)], \quad (3.40)$$

$$S_{k+1} = M_o (I - G_k D_o) S_k M_o^T + W_y. \quad (3.41)$$

where $X_o(k)$ is the estimated state vector at time k , and M_o is the discrete-time state transition matrix of the observer. The term N_o represents the discrete-time input matrix, and $y_m(k)$ is the actual measured output. The term W_y is the process noise covariance, which accounts for modeling uncertainties.

Since vertical displacement is not directly measured, a vertical motion observer is designed to estimate it using the available vertical acceleration measurements. Direct integration of acceleration to obtain velocity and displacement is affected by drift due to sensor bias and noise. To mitigate this problem, an additional state is introduced to capture the accumulated drift error. The observer structure for estimating vertical motion is given by:

$$\begin{bmatrix} \dot{h}_s \\ \dot{f} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} h_s \\ f \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_z. \quad (3.42)$$

where h_s is the vertical displacement of the sprung mass, f represents the accumulated integration drift error, and v_z is the measured vertical acceleration. This formulation allows

the observer to compensate for slow variations in sensor bias, improving the long-term accuracy of displacement estimation.

The estimated velocity is obtained through integration:

$$y_m = \int v_z = h_s + f. \quad (3.43)$$

Similarly, the roll and pitch angles of the vehicle body are also key states required for suspension control. However, they are not always directly measurable. Instead, they are estimated using relative suspension movement and angular acceleration data. The observer structure for roll and pitch estimation is formulated as:

$$\begin{bmatrix} \dot{\gamma}_s \\ \dot{\beta}_s \\ \dot{\gamma}_s \\ \dot{\beta}_s \end{bmatrix} = E_o \begin{bmatrix} \gamma_s \\ \beta_s \\ \dot{\gamma}_s \\ \dot{\beta}_s \end{bmatrix} + F_o \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ h_{s1} - g_{u1} \\ h_{s2} - g_{u2} \\ h_{s3} - g_{u3} \\ h_{s4} - g_{u4} \\ v_x \\ v_y \end{bmatrix}. \quad (3.44)$$

where γ_s and β_s are the roll and pitch angles of the sprung mass, respectively, while $\dot{\gamma}_s$ and $\dot{\beta}_s$ denote their angular rates. The system matrix E_o governs the evolution of roll and pitch states, while F_o maps external forces and sensor measurements to state variations. The input variables include suspension forces (T_1, T_2, T_3, T_4) , relative suspension displacements at each wheel $(h_{si} - g_{ui})$, and inertial accelerations (v_x, v_y) .

This observer formulation ensures that the roll and pitch motion estimates remain accurate regardless of model uncertainties and sensor noise. The control system can achieve improved performance in maintaining vehicle stability and ride comfort by incorporating these estimated states into the MPC.

3.3 Integrated Skyhook-Linear Quadratic Regulator (LQR) Controller

Model Predictive Control (MPC) provides excellent performance for suspension control but has significant computational requirements that make real-time implementation difficult. An alternative approach is the Linear Quadratic Regulator (LQR), which provides an optimal control strategy with lower computational complexity. While LQR is well adapted to multi-objective control problems, it lacks direct damping control. To overcome this limitation, the Skyhook control strategy is integrated with LQR to control suspension damping effectively. In this chapter, a hybrid control approach integrating skyhooks and LQR is developed for semi-active suspensions, and the theoretical background and practical implementation are explained.

The schematic in Figure 3.2 shows an integrated skyhook-LQR control strategy for an adaptive suspension system. It combines the benefits of Skyhook and Linear Quadratic Regulator (LQR) control. The Skyhook controller calculates a damping force F_{sky} to minimize vertical body oscillations. This is done by simulating a virtual damper linked to an inertial reference. In parallel, the LQR controller optimally calculates an additional force correction ΔF to minimize a cost function that balances ride comfort, road handling and steering effort. The total control force is applied through adaptive dampers. These dynamically adjust their damping characteristics in response to real-time measurements. The feedback loop ensures continuous adaptation, using the Skyhook for improved comfort and the LQR for optimum system performance and stability.

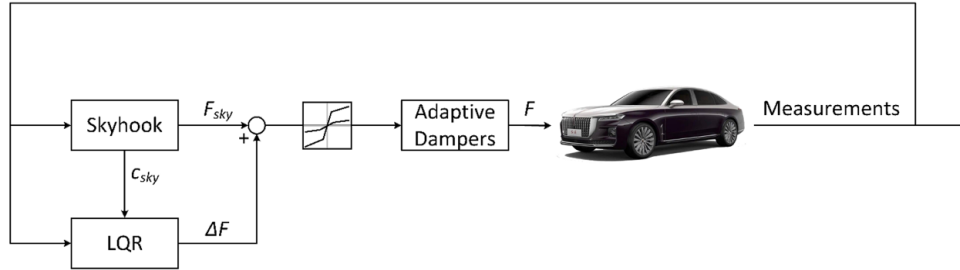


Figure 3.2: Integrated Skyhook-LQR Control

3.3.1 Skyhook Control Strategy

Skyhook control reduces vertical oscillations by connecting the sprung mass (m_s) to an imaginary fixed reference ("sky") via a variable damper (C_{sky}). Unlike conventional dampers, which link the sprung mass to the unsprung mass (m_f), this approach allows for selective energy dissipation based on velocity conditions. The suspension stiffness is represented by the spring constant (K_s), which provides a restoring force between the two masses. The damping coefficient dynamically adjusts to optimize vibration attenuation. The vertical displacements of the sprung and unsprung masses are denoted as (Z_s) and (Z_f), respectively. By continuously tuning (C_{sky}) in real-time, Skyhook control effectively reduces cabin acceleration, improving ride comfort while maintaining road-holding stability.

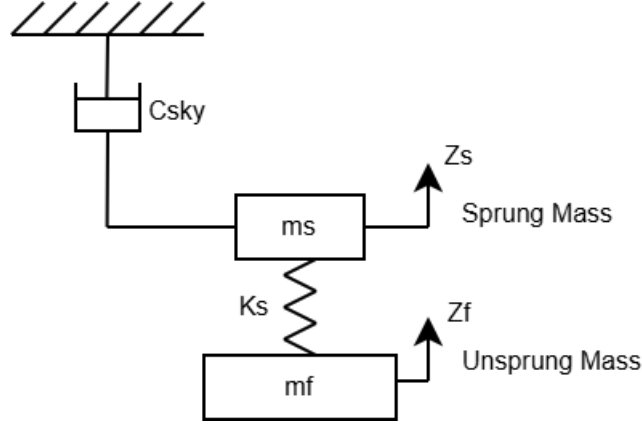


Figure 3.3: Integrated Skyhook-LQR Control

The system consists of two damping states: high damping when the relative velocity suggests energy should be dissipated and low damping otherwise. The switching logic for the damping coefficient is given by:

$$d_{sh} = \begin{cases} d_{max}, & \text{if } \dot{s}_m(\dot{s}_m - \dot{u}_m) \geq 0 \\ d_{min}, & \text{if } \dot{s}_m(\dot{s}_m - \dot{u}_m) < 0 \end{cases} \quad (3.45)$$

where d_{sh} represents the Skyhook damping coefficient, $\dot{s}_m$ is the velocity of the sprung mass, and $\dot{u}_m$ is the velocity of the unsprung mass. The control force applied by the Skyhook damper is then:

$$F_{sh} = -d_{sh}\dot{s}_m. \quad (3.46)$$

This approach minimizes absolute vertical acceleration of the cabin, improving ride comfort. Hence, the Skyhook approach has been widely used in minimizing the vertical vibrations [Reference]. However, classic Skyhook control does not optimize pitch and roll dynamics. To address this, LQR is integrated to adjust the Skyhook control force dynamically.

3.3.2 Linear Quadratic Regulator (LQR)

The Linear Quadratic Regulator (LQR) is formulated to optimize the suspension control by minimizing a quadratic cost function, ensuring a balance between ride comfort and control effort. The primary objective of the LQR design is to minimize the pitch, roll, and vertical accelerations of the vehicle's cabin while keeping control inputs within reasonable limits. The cost function J is expressed as:

$$J = \int_0^\infty (Y^T Q_y Y + U^T R_y U) dt \quad (3.47)$$

where $Q_y \geq 0$ is the state weighting matrix, ensuring that the key system states, such as vertical acceleration, pitch rate, and roll rate, are penalized appropriately. $R_y > 0$ is the control effort weighting matrix, preventing excessive control inputs. Y is the output vector, which consists of measured and estimated states influencing the ride comfort and handling stability.

The output vector is defined as:

$$Y = \dot{X} = [\dot{z}_s, \dot{\theta}_s, \dot{\phi}_s, \dot{z}_f, \dot{\theta}_f, \dot{\phi}_f]^T \quad (3.48)$$

where $\dot{z}_s$ and $\dot{z}_f$ represent the vertical velocities of the sprung and unsprung masses, respectively. $\dot{\theta}_s$ and $\dot{\phi}_s$ denote the pitch and roll angular velocities of the cabin. $\dot{\theta}_f$ and $\dot{\phi}_f$ are the pitch and roll angular velocities of the unsprung mass.

To derive an optimal control law, the cost function must be rewritten in terms of the system's state-space representation. Given the system dynamics:

$$\dot{X} = AX + BU \quad (3.49)$$

Substituting $Y = CX$ into the original cost function:

$$J = \int_0^\infty (X^T C^T Q_y C X + U^T R_y U) dt \quad (3.50)$$

Additionally, considering the control effect in the cost function transformation, the term U can be substituted using the system input mapping:

$$J = \int_0^\infty (X^T Q X + U^T (R_y + D^T Q_y D) U) dt \quad (3.51)$$

where $Q = C^T Q_y C$ represents the transformed state weighting matrix, ensuring that system states are penalized correctly. $R = R_y + D^T Q_y D$ is the modified control effort weighting, adjusted to account for how inputs affect the system.

The optimal control law that minimizes the cost function follows the standard linear feedback control strategy:

$$U = -KX \quad (3.52)$$

where K is the optimal feedback gain matrix, which is computed using the algebraic Riccati equation (ARE):

$$K = R^{-1} B^T P \quad (3.53)$$

The matrix P satisfies the continuous-time algebraic Riccati equation (ARE):

$$A^T P + P A - P B R^{-1} B^T P + Q = 0 \quad (3.54)$$

This equation ensures that the closed-loop system remains stable and the cost function is minimized over an infinite time horizon.

In the Skyhook-LQR hybrid approach, the Skyhook controller provides the initial damping force, and the LQR fine-tunes this force for multi-objective optimization. The final control force is given by:

$$F_i = F_{sky} + \Delta F_i \quad (3.55)$$

where ΔF_i represents the correction term computed by LQR. This integration allows the system to achieve both vertical vibration attenuation and improved pitch-roll stability.

3.4 Summary

This chapter presents the mathematical modeling and control strategies for a damper suspension system with continuous damping control (CDC) in a commercial vehicle. A full vehicle model with seven degrees of freedom (7-DOF) is developed in state space form to capture the essential dynamics of the vertical, roll, and pitch motions. The controllability and observability of the system are analyzed to ensure the feasibility of the control design. Actuator dynamics are incorporated to model the response characteristics of the CDC dampers.

Two control strategies are introduced: Model Predictive Control (MPC) and an integrated Skyhook-Linear Quadratic Regulator (LQR) controller. MPC is formulated using a predictive optimization framework, where a cost function is minimized under system constraints. The state-space model is discretized, and an observer-based compensation using a Kalman filter is implemented to estimate unmeasured states and mitigate sensor noise. The controller optimally adjusts damping forces over a prediction horizon to enhance ride comfort and stability.

The integrated Skyhook-LQR controller is presented as a computationally efficient alternative to MPC. The Skyhook strategy reduces vertical acceleration by simulating a virtual damper connected to an inertial reference, improving ride comfort. The LQR component optimally adjusts damping forces to balance ride comfort, handling, and control effort. The two strategies are combined, where Skyhook provides baseline damping, and LQR fine-tunes the force distribution for enhanced stability.

Chapter 4

Simulation and Experimental Studies of Coordinated Semi-Active Suspensions

This chapter presents the simulation and experimental evaluation of coordinated semi-active suspension control strategies. A Model Predictive Control (MPC) approach is assessed through a co-simulation framework between Simulink and CarSim, while an LQR-Skyhook controller is tested on an actual vehicle. Both methods are analyzed under various road conditions to quantify their impact on ride comfort and handling.

Performance evaluation is based on frequency-weighted acceleration root-mean-square (RMS) values, as defined by ISO 2631-1, to objectively assess ride comfort. This standard quantifies human body vibrations, providing a metric for comparing different suspension control strategies. The results offer insights into the trade-offs between predictive optimization and real-time adaptive damping, highlighting the effectiveness and practical feasibility of each approach for coordinated semi-active suspension control.

4.1 Vibration Assessment Based on ISO 2631-1

ISO 2631-1 evaluates human exposure to vehicle vibrations through frequency-weighted root-mean-square (RMS) acceleration:

$$a_{w,\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T a_w^2(t) dt} \quad (4.1)$$

where $a_w(t)$ is the frequency-weighted acceleration, measured in different axes:

- a_z : Vertical acceleration, affecting ride comfort.
- a_y : Lateral acceleration, influencing stability.
- a_x : Longitudinal acceleration, relevant for braking and acceleration.
- a_ϕ : Roll acceleration, impacting vehicle cornering.
- a_θ : Pitch acceleration, important in road undulations.

Among these, vertical acceleration (a_z) is the most critical for comfort, as the human body is highly sensitive to oscillations in the 0.5–12 Hz range. Lateral and longitudinal accelerations (a_y , a_x) influence stability, with discomfort peaking between 0.5–2 Hz. Roll (a_ϕ) and pitch (a_θ) contribute to motion sickness and handling perception, primarily affecting frequencies below 1 Hz.

To obtain frequency-weighted acceleration, the raw acceleration signal is transformed using the ISO-defined weighting function:

$$A_w(s) = W(s)A(s) \quad (4.2)$$

where $A(s)$ represents the unweighted acceleration in the Laplace domain, and $W(s)$ is the corresponding weighting function.

For ride comfort assessment, these values prioritize vertical acceleration, reflecting human sensitivity to vertical oscillations, the standard weighting coefficients are:

$$k_x = k_y = 1.4, \quad k_z = 1.0 \quad (4.3)$$

For controllability assessment, lateral acceleration has a stronger influence, leading to an increase in k_y :

$$k_y > 1.4 \quad (4.4)$$

For stability evaluation, particularly in braking and acceleration, longitudinal acceleration plays a key role, resulting in a higher k_x :

$$k_x > 1.4 \quad (4.5)$$

When horizontal vibrations are significant, such as during rough road driving or sharp lane changes, total whole-body vibration (WBV) is assessed using vector summation:

$$a_{WBV} = \sqrt{k_x a_{wx,RMS}^2 + k_y a_{wy,RMS}^2 + k_z a_{wz,RMS}^2} \quad (4.6)$$

In simulation (CarSim-Simulink), unweighted accelerations are filtered through $W(s)$, and RMS values are computed. In experimental testing, IMU data from the cabin undergoes the same filtering process to ensure consistency. Vertical RMS acceleration ($a_{wz,RMS}$) remains the dominant metric for comfort evaluation, while lateral and longitudinal accelerations are critical in handling assessments.

In experimental vehicle testing, IMU data from the cabin or seat reference frame are processed using these weighting functions to ensure consistency with simulation-based assessments. Vertical RMS acceleration $a_{wz,RMS}$ remains the primary metric for evaluating ride comfort, while lateral, longitudinal, roll, and pitch components are considered in stability and handling evaluations. For aggressive driving conditions such as rough terrain or sharp steering maneuvers, total whole-body vibration (WBV) exposure is assessed using vector summation:

4.2 Co-Simulation Between CarSim and Simulink

The Simulink-CarSim co-simulation integrates a Model Predictive Control (MPC)-based active suspension system with a high-fidelity vehicle dynamics model. CarSim simulates real-time vehicle responses, while Simulink executes the MPC algorithm to compute optimal suspension forces for the semi-active dampers. This setup enables real-time bidirectional data exchange, ensuring improved ride comfort and vehicle stability under various road conditions.

The co-simulation framework consists of multiple subsystems. The MPC controller receives vehicle states such as vertical acceleration, roll rate, pitch rate, longitudinal and lateral accelerations, and relative suspension displacement. It then optimizes suspension forces to minimize unwanted vibrations. The actuator dynamics model applies a discrete transfer function to filter the MPC force command before passing it to CarSim. The Kalman Filter (KF) subsystem estimates unmeasured states from sensor inputs, while the data collection unit records simulation results, including vehicle accelerations, suspension deflections, and control efforts.

4.2.1 MPC Implementation in Simulink

The Simulink implementation of the Model Predictive Control-based semi-active suspension system is structured to process real-time vehicle states, estimate unmeasured variables, and compute optimal suspension forces to minimize unwanted vibrations. The MPC algorithm operates within a discrete-time state-space framework, solving a constrained optimization problem at each control step. The implementation consists of structured subsystems:

- **MPC Controller:** Computes optimal suspension forces based on predicted vehicle states.
- **State Estimation:** Uses Kalman filtering to refine sensor data and estimate unmeasured states.
- **Actuator Dynamics:** Filters force commands before application.
- **Reference Inputs:** Accepts road disturbances and driver inputs.
- **Data Collection:** Logs results for analysis and tuning.

The system dynamics are defined as:

$$X_{k+1} = A_d X_k + B_{ud} U_k + B_{wd} W_k \quad (4.7)$$

where X_k is the state vector, U_k is the control input, and W_k represents road disturbances. The discretized matrices (A_d, B_{ud}, B_{wd}) are obtained using a Zero-Order Hold (ZOH) method with a sampling time of 0.05 s.

The parameters and gains used in this study are listed in Table 4.1, and the cost function optimized by the MPC controller is:

$$J = \sum_{k=0}^{N_p} (X_k^T Q X_k + U_k^T R U_k) \quad (4.8)$$

Model Predictive Control (MPC)	
Prediction horizon N_p	5
Control horizon N_c	5
Output penalty matrix Q	$\text{diag}(0, 0, 0, 1000, 1000, 1000)$
Control penalty matrix R	$0.001 \cdot \text{diag}(1, 1, 1, 1)$

Table 4.1: MPC control parameters

which prioritize minimizing vertical acceleration (z), roll rate (ϕ), and pitch rate (θ) while limiting excessive control effort.

The Kalman Filter (KF) is integrated in Simulink for state estimation, compensating for sensor limitations. The estimated states $\hat{X}_k$ are updated using:

$$\hat{X}_{k+1} = A_d \hat{X}_k + B_{ud} U_k + G(Y_k - C \hat{X}_k) \quad (4.9)$$

where Y_k represents measured sensor data and G is the Kalman gain. The filter reconstructs unmeasured states such as vertical displacement and roll-pitch motion, improving control accuracy.

The actuator dynamics model applies a first-order discrete transfer function to process the MPC force command before it is sent to the suspension actuators, where τ represents actuator delay, ensuring smooth force application.

$$F_{MPC}(s) = \frac{K}{\tau s + 1} U(s) \quad (4.10)$$

Further tuning of Q and R matrices, actuator delay parameters, and Kalman filter gains is required to enhance robustness. The Simulink-based MPC framework ensures predictive suspension control, dynamically optimizing damping forces for improved ride quality and stability.

4.2.2 High-fidelity vehicle model in CarSim

The vehicle model in CarSim is configured based on the Hongqi H9 specifications to ensure accurate simulation of real-world dynamics. Table 4.2 summarizes the key parameters of the test vehicle.

Table 4.2: Test vehicle specifications.

Specifications	Details
Curb weight (kg)	1875
Gross mass (kg)	2325
Dimensions (mm)	$5137 \times 1904 \times 1493$
Ground clearance (mm)	188 - 248
Track width (mm)	1633 (front); 1629 (rear)
Wheelbase (mm)	3060
Suspension type	Double wishbone (front); Multi-link (rear)
Tire	245/45 R19
Engine	2.0T
Drive Type	Rear drive

The suspension configuration replicates the real-world setup, featuring a double-wishbone front suspension for precise steering control and a multi-link rear suspension to enhance ride comfort. Suspension forces are defined by:

$$F_s = K_s(\Delta Z) + C_s(\Delta \dot{Z}) \quad (4.11)$$

where K_s is the suspension stiffness, and C_s is the damping coefficient.

Figure 4.2 and Figure 4.3 below illustrate the finalized CarSim configuration for the Hongqi H9 vehicle model. The parameters, such as track width, wheelbase, suspension type, and mass moments of inertia, are carefully defined to reflect the physical characteristics of the Hongqi H9, ensuring fidelity in dynamic response when subjected to various road conditions and control inputs.

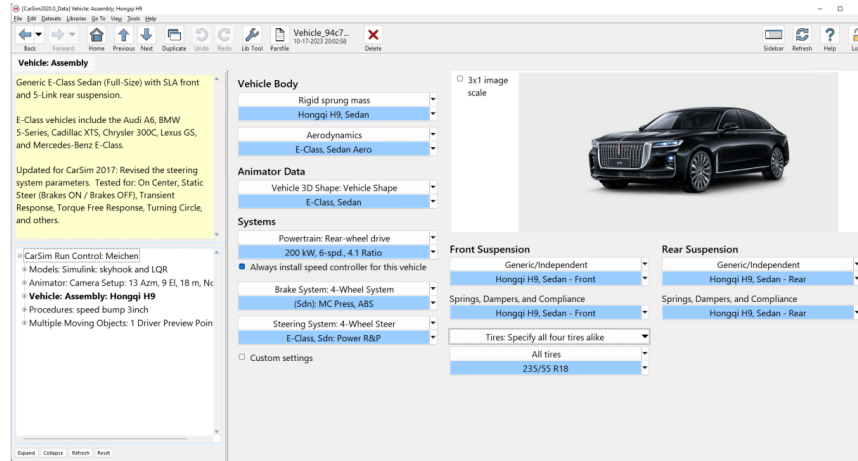


Figure 4.1: CarSim Hongqi H9 vehicle configurations

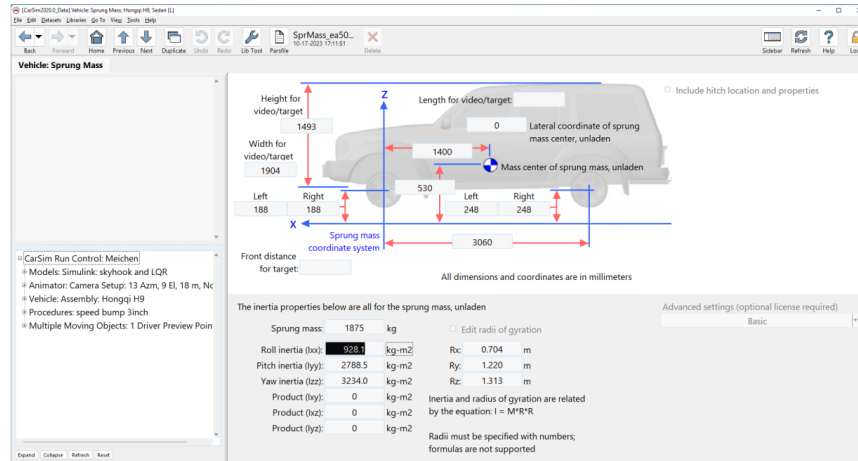


Figure 4.2: CarSim Hongqi H9 sprung mass main configuration

To evaluate the semi-active suspension system, three distinct road profiles are considered, each presenting unique challenges in ride comfort, stability, and handling.

- **Single Speed Bump:** As shown in figure 4.3. A 3-inch bump introduces a sharp vertical disturbance, testing the suspension's ability to reduce peak vertical acceleration (a_z) and suspension deflection (Z_s). An effective system minimizes peak acceleration while controlling overshoot in suspension travel.

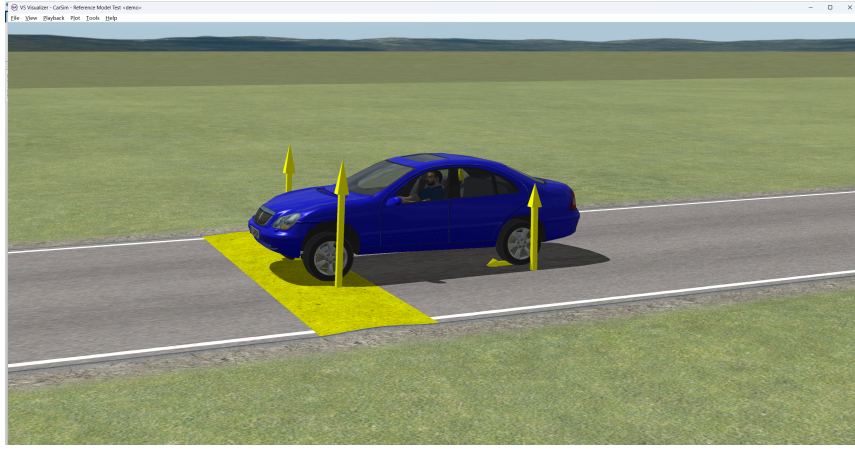


Figure 4.3: Single Speed Bump

- **Offset Speed Bump:** An asymmetric disturbance that affects one side of the vehicle before the other, primarily evaluating roll response ($\dot{\phi}$). The effectiveness of the roll-damping strategy is assessed based on the magnitude and duration of induced roll motion.
- **Double Lane Change on Paved Road with ISO Standard Roughness:** Moderate roughness with mid-frequency disturbances, assessing the suspension's ability to filter vibrations.

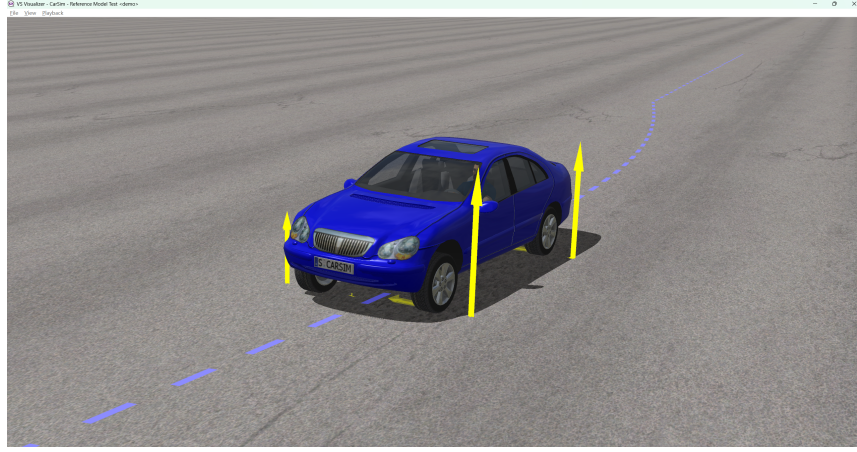


Figure 4.4: Paved ISO Standard Roughness Road

The co-simulation between CarSim and Simulink enables real-time bidirectional data exchange for closed-loop suspension control. CarSim computes vehicle responses under road excitations, exporting key states to Simulink, where the MPC controller optimizes suspension forces, which are then applied back to CarSim.

CarSim exports the vehicle states:

$$Y_k = [a_z \quad V_z \quad Z_{cg} \quad A_x \quad A_y \quad \dot{\phi} \quad \dot{\theta}]^T \quad (4.12)$$

where a_z is vertical acceleration, V_z is vertical velocity, Z_{cg} is CG displacement, A_x , A_y are longitudinal and lateral accelerations, and $\dot{\phi}$, $\dot{\theta}$ are roll and pitch rates. These states serve as MPC inputs.

Simulink computes the optimal suspension forces:

$$U_k = [F_{D,L1} \quad F_{D,L2} \quad F_{D,R1} \quad F_{D,R2}]^T \quad (4.13)$$

which are sent to CarSim for real-time damping adjustments.

Synchronization between CarSim and Simulink requires a fixed sampling interval Δt , ensuring:

$$\Delta t \leq \frac{1}{f_s} \quad (4.14)$$

where f_s is the update frequency. A balance is needed between computational efficiency and system responsiveness.

The control cost function:

$$J = \sum_{k=0}^{N_p} (X_k^T Q X_k + U_k^T R U_k) \quad (4.15)$$

requires careful tuning of Q and R . Larger Q prioritizes minimizing body motion, while higher R limits excessive control input.

Actuator dynamics introduce delays modeled as:

$$F_D(t) = \frac{K_a}{\tau_a s + 1} U(t) \quad (4.16)$$

where K_a is actuator gain and τ_a is response time. Delay compensation is necessary to prevent phase lag in damping adjustments.

State estimation using a Kalman filter improves robustness against sensor noise and model uncertainties:

$$\hat{Y}_{k+1} = A\hat{Y}_k + BU_k + G(Y_k - C\hat{Y}_k) \quad (4.17)$$

where G is the Kalman gain matrix.

Optimizing prediction horizon N_p and control horizon N_c is critical for computational feasibility. The optimal N_p minimizes:

$$N_p^* = \arg \min_{N_p} (J + C_{\text{comp}}(N_p)) \quad (4.18)$$

where $C_{\text{comp}}(N_p)$ represents computational cost.

Precise tuning of control parameters, synchronization, and estimation ensures effective suspension control across diverse road conditions.

4.3 Simulation Results and Discussion

In this section, the simulation results for the semi-active suspension system are presented and analyzed across three distinct road conditions: single speed bump, offset speed bump, and paved road with ISO standard roughness. These road profiles introduce different excitation characteristics, enabling a comprehensive assessment of the controller’s ability to enhance ride comfort and vehicle stability. The evaluation is conducted based on the root-mean-square (RMS) values of vertical, roll, and pitch accelerations, which serve as objective metrics for quantifying suspension performance. By comparing these acceleration responses, the effectiveness of the proposed control strategies in mitigating vibrations and improving dynamic stability is systematically examined.

The output penalty matrix Q and the control signal penalty matrix P for the MPC controller are constant values regardless of the driving conditions. The following gains were used in the co-simulations:

Model Predictive Control (MPC)	
Prediction horizon N_p	5
Control horizon N_c	5
Output penalty matrix Q	$\text{diag}(0, 0, 0, 1000, 1000, 1000)$
Control penalty matrix R	$0.001 \cdot \text{diag}(1, 1, 1, 1)$

Table 4.3: MPC control parameters

4.3.1 Single Speed Bump at 25kph

The single speed bump test evaluates the suspension’s response to a sharp vertical disturbance, modeling real-world road irregularities such as speed bumps or sudden potholes. In CarSim, the speed bump is defined as a 3-inch high obstacle with a short wavelength, inducing rapid changes in wheel vertical displacement. The test challenges the suspension system’s ability to mitigate peak vertical acceleration (a_z) and suspension deflection (Z_s) while preventing excessive oscillations. An effective control strategy should minimize the peak acceleration response and regulate suspension travel to enhance ride comfort and vehicle stability. The root-mean-square (RMS) analysis of a_z provides a quantitative measure of the system’s shock absorption capability.

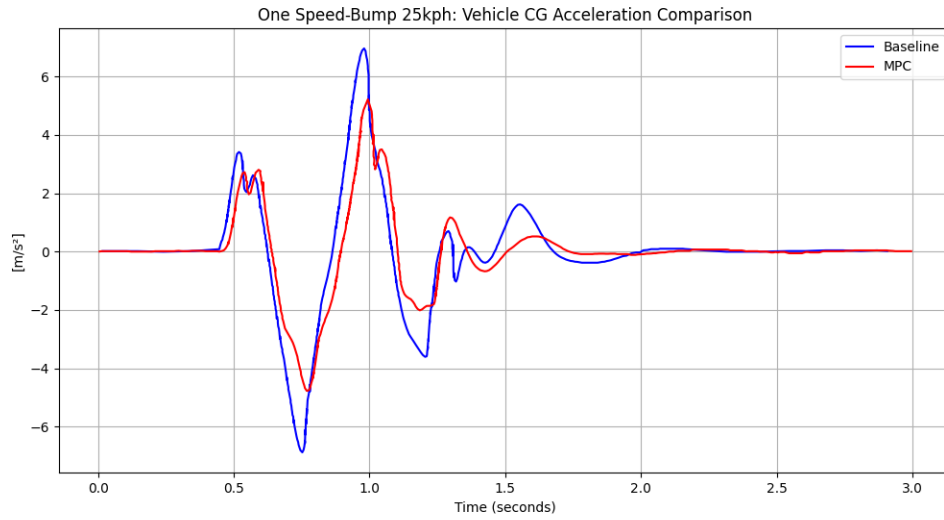


Figure 4.5: CarSim simulation for single bump vertical acceleration at 25 km/h

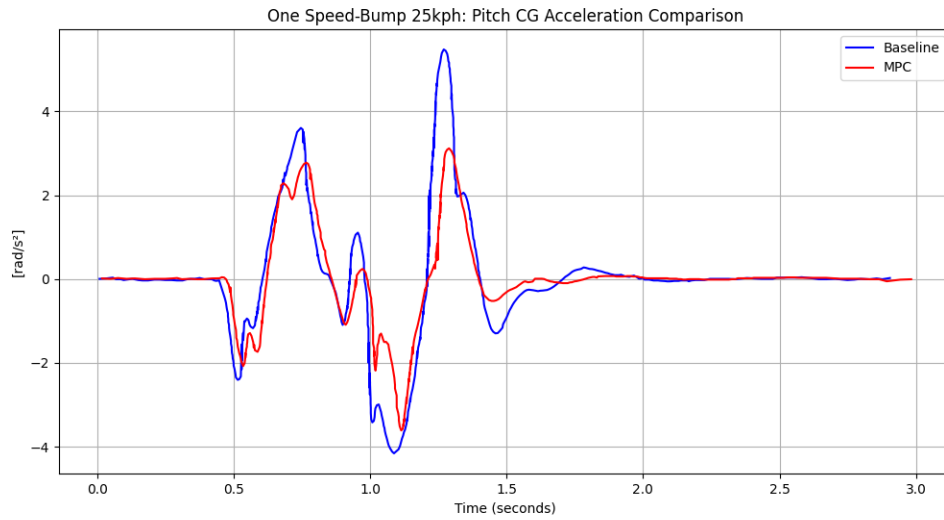


Figure 4.6: CarSim simulation for single bump pitch acceleration at 25 km/h

Table 4.4: Comparison of MPC and Passive Suspension for Single Bump

	Passive	MPC	Percentage Change
$\bar{a}_z$: Vertical RMS acceleration-Weighted (m/s ²)	1.3851	1.1357	-18.4%
$\bar{a}_\theta$: Pitch RMS acceleration-Weighted (rad/s ²)	0.8651	0.6488	-25.2%
a_{w-c} : Overall weighted RMS acceleration (m/s ²)	2.2028	1.7146	-22.2%

The simulation results for the single speed bump at 25 kph demonstrate the effectiveness of the MPC-based suspension system in reducing peak acceleration responses and improving ride comfort. As shown in Figure 4.5, the vertical acceleration of the vehicle’s center of gravity exhibits significantly lower peak values when controlled by MPC compared to the passive suspension, indicating improved shock absorption. Similarly, Figure 4.6 illustrates that pitch acceleration is mitigated more effectively with MPC, reducing excessive oscillations following the bump impact.

Quantitatively, Table 4.6 confirms these improvements, with the MPC controller reducing vertical RMS acceleration ($\bar{a}_z$) by 18.4%, pitch RMS acceleration ($\bar{a}_\theta$) by 25.2%, and overall weighted RMS acceleration ($\bar{a}_{w-c}$) by 22.2%. These reductions highlight the MPC’s capability to enhance both ride comfort and vehicle stability by dynamically adjusting the suspension force in response to road disturbances.

4.3.2 Offset Speed Bump at 18kph

The offset speed bump introduces an asymmetric excitation, where one side of the vehicle encounters the disturbance before the other, generating both vertical motion and roll dynamics. This scenario is critical for evaluating the suspension’s roll damping effectiveness and lateral stability. In CarSim, the offset bump profile induces differential wheel displacement, leading to roll motion (ϕ) that tests the suspension’s ability to counteract uneven excitations. The control strategy should reduce excessive roll motion to maintain vehicle stability and passenger comfort. The effectiveness of the suspension system is assessed using the RMS roll acceleration, which provides insight into the mitigation of motion sickness and the overall handling stability of the vehicle.

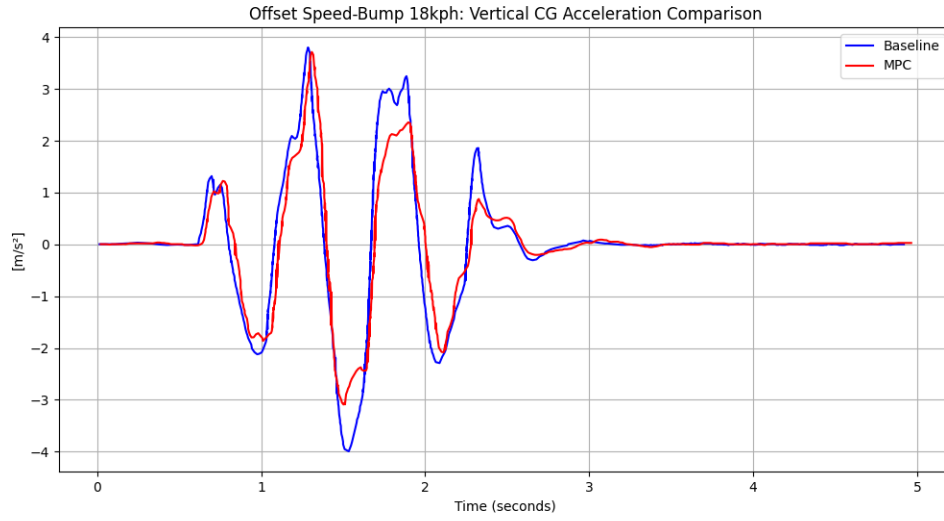


Figure 4.7: CarSim offset bump vertical rate at 18 km/h

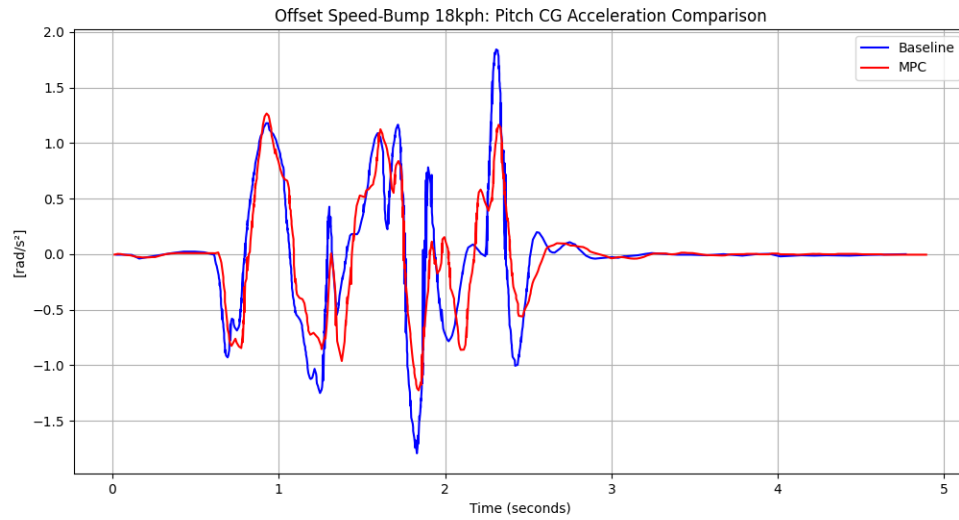


Figure 4.8: CarSim offset bump pitch rate at 18 km/h

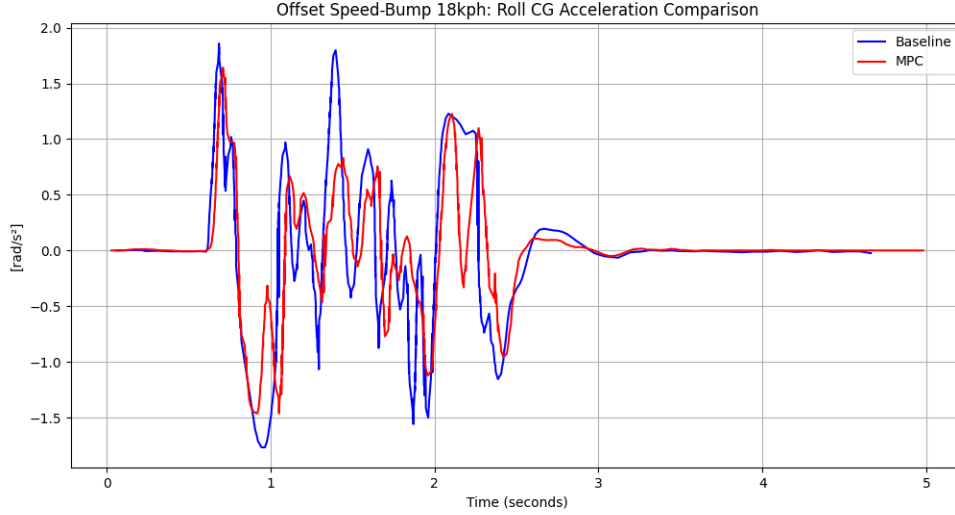


Figure 4.9: CarSim offset bump roll rate at 18 km/h

Table 4.5: Comparison of MPC and Passive Suspension for Offset Bump

	Passive	MPC	Percentage Change
$\overline{a_z}$: Vertical RMS acceleration-Weighted (m/s ²)	0.6573	0.5784	-12.1%
$\overline{a_\phi}$: Roll RMS acceleration-Weighted (rad/s ²)	0.3477	0.2678	-23.68%
$\overline{a_\theta}$: Pitch RMS acceleration-Weighted (rad/s ²)	0.3354	0.3186	-5.10%
$a_{\nu-c}$: Overall weighted RMS acceleration (m/s ²)	0.9431	0.8210	-12.95%

The results indicate that the MPC-based suspension effectively reduces vertical, roll, and pitch accelerations compared to the passive system, particularly in roll dynamics. The largest improvement is observed in roll acceleration, highlighting the MPC controller's ability to mitigate asymmetric disturbances. This is attributed to the predictive nature of MPC, which anticipates roll-induced imbalances and optimally adjusts damping forces at each wheel.

The reduction in overall weighted RMS acceleration confirms that the controller improves ride comfort while maintaining stability. The relatively smaller reduction in pitch acceleration suggests that pitch dynamics are less influenced by the control strategy under this road condition, likely due to the short-duration nature of the disturbance. The vertical acceleration reduction further reinforces that MPC successfully regulates suspension forces to minimize peak responses and improve passenger comfort. These results validate the controller's effectiveness in handling unbalanced excitations by dynamically adjusting suspension forces in real time.

4.3.3 Paved ISO Road at 60kph

The paved ISO road test represents smooth highway conditions, classified as ISO Class A, with minimal road surface irregularities. This scenario is essential for evaluating steady-state damping efficiency, where the suspension system must suppress minor excitations without introducing unnecessary oscillations. The primary focus is on vertical acceleration (a_z) and pitch motion ($\dot{\theta}$), which influence ride comfort and vehicle stability. A well-tuned suspension system should provide a smooth ride while maintaining optimal road contact. The RMS values of a_z and $\dot{\theta}$ are analyzed to assess the controller's performance in maintaining low-frequency vibration attenuation and long-term driving comfort.

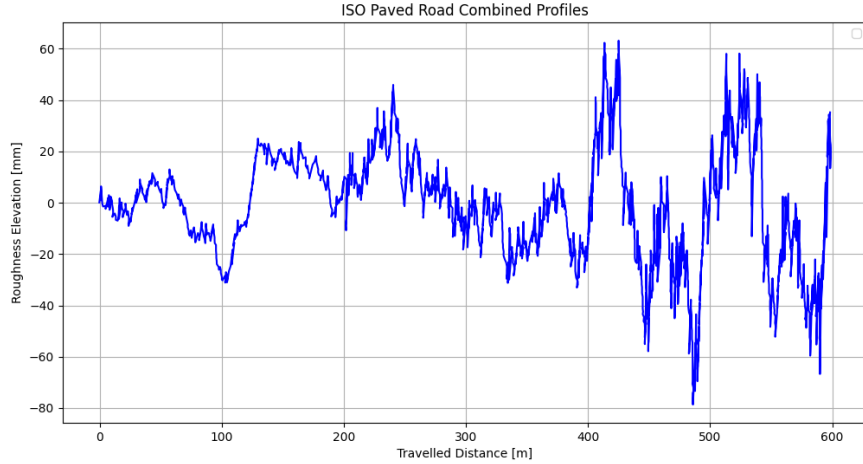


Figure 4.10: ISO Paved Road Combined Profiles

Table 4.6: Comparison of MPC and Passive Suspension for ISO Paved Road

Metric	Passive	MPC	Percentage Change
a_z : Vertical RMS acceleration-Weighted (m/s^2)	0.6573	0.5784	-12.00%
a_ϕ : Roll RMS acceleration-Weighted (rad/s^2)	0.3477	0.2678	-23.00%
a_θ : Pitch RMS acceleration-Weighted (rad/s^2)	0.3354	0.3186	-5.08%
a_{v-c} : Overall weighted RMS acceleration (m/s^2)	0.9431	0.8210	-12.95%

Road profile shown in Figure 4.10 consists of 200 meters ISO-A road, 200 meters ISO-B road, and 200 meters ISO-C road, which covers most of the paved road conditions. The results highlight the statistical effectiveness of the MPC controller in mitigating continuous road disturbances. The reduction in overall weighted RMS acceleration by 12.95% suggests a significant improvement in ride comfort across the entire driving scenario. The roll RMS acceleration exhibits the largest percentage reduction 23.00%, indicating that the controller effectively counteracts lateral oscillations caused by road unevenness, which is a primary concern in continuous irregularities. The vertical RMS acceleration decreases by 12.00%, reinforcing the MPC's capability in reducing sustained vertical vibrations. However, the pitch RMS acceleration shows a relatively lower improvement of 5.08%, suggesting that the frequency content of the disturbances in this scenario is less influential on pitch dynamics compared to roll and vertical motions. This trend suggests that MPC prioritizes attenuating roll motion over pitch, likely due to the greater sensitivity of lateral oscillations to suspension damping adjustments. The statistical consistency in the improvements across different acceleration components indicates that the controller effectively optimizes damping forces over prolonged irregularities, contributing to a measurable enhancement in ride comfort and vehicle stability.

4.4 Vehicle Testing Setup

This section presents the experimental evaluation of the integrated Skyhook-LQR controller using a modified Hongqi H9 sedan. The testing procedure follows the same assessment methodology as the MPC-based simulation, ensuring consistency in performance comparison. Initially, baseline data was collected using the vehicle’s passive suspension. Subsequently, the vehicle was equipped with a custom semi-active suspension system, consisting of CDC dampers, height sensors, an IMU, and an electric control unit (ECU). The same road profiles and driving conditions were used for both setups, enabling a direct comparison of vertical, roll, and pitch acceleration responses.

4.4.1 Challenges in Deploying MPC Controller on ECU

Deploying the suspension Model Predictive Control (MPC) system on a resource-constrained ECU, operating at 120 MHz with only 64 KB of SRAM and a strict 10 ms control loop requirement, presented several critical challenges.

First, the computational complexity of solving the Quadratic Programming (QP) problem at every control step posed a major challenge in deploying the suspension MPC controller on an embedded Electronic Control Unit (ECU) with a 120 MHz processor and a strict 10 ms deadline per control iteration. The MPC controller requires solving the following cost function:

$$J = \sum_{k=0}^{N_p} (X_k^T Q X_k + U_k^T R U_k), \quad (4.19)$$

where $N_p = 5$ is the prediction horizon, $N_c = 5$ is the control horizon, $X_k \in \mathbb{R}^6$ represents the state vector, and $U_k \in \mathbb{R}^4$ represents the control input. The weight matrices are defined as:

$$Q = \text{diag}(0, 0, 0, 1000, 1000, 1000), \quad R = 0.001 \cdot \text{diag}(1, 1, 1, 1). \quad (4.20)$$

To optimize the control input sequence U , the QP solver must handle a decision vector of size:

$$4 \times N_c = 4 \times 5 = 20. \quad (4.21)$$

The resulting cost function matrix H in the QP formulation has dimensions:

$$H \in \mathbb{R}^{20 \times 20}, \quad (4.22)$$

along with a constraint matrix of similar size. At each control step, the MPC computation involves:

- Predicting future states over the horizon $N_p = 5$.
- Constructing prediction matrices Φ and Γ .
- Solving the QP of the form:

$$\min_U \quad \frac{1}{2} U^T H U + f^T U, \quad \text{s.t.} \quad A U \leq b. \quad (4.23)$$

The execution time is significantly constrained by the 120 MHz processor, which allows approximately:

$$\frac{120 \times 10^6 \text{ cycles/sec} \times 10^{-2} \text{ sec}}{1} = 1.2 \times 10^6 \text{ cycles} \quad (4.24)$$

for all computations per control step. However, even a simple matrix multiplication, such as 6×6 floating-point operations, can consume thousands of cycles. A full QP solve typically requires hundreds of thousands of cycles, making it extremely difficult to execute within the given 10 ms deadline.

Even efficient solvers such as CVXGEN and FORCES require 10–30 ms to solve QPs of this scale on a desktop CPU, which is significantly faster than the embedded ECU. Given these constraints, it is highly unlikely that the solver can meet the real-time requirements without hardware acceleration (e.g., FPGA, DSP) or significant algorithmic optimizations, such as reducing the prediction horizon or using a custom fixed-point implementation.

The second major limitation was memory availability. The 64 KB SRAM had to accommodate the state-space matrices, Kalman filter matrices, MPC workspace, logging, diagnostics, and program variables. The memory footprint for the QP solver alone could reach 50 KB, leaving little room for other essential computations. Each state vector, prediction matrix, and associated constraints contributed to a rapidly growing memory demand. Without extreme memory optimization, such as aggressive pruning of unused variables and

conversion to fixed-point arithmetic, the MPC framework exceeded the available memory, making real-time execution impractical.

Lastly, the Kalman filter complexity introduced an additional computational burden. The filter was essential for state estimation, especially for unmeasured variables such as vertical displacement and roll/pitch motion. However, updating the state estimate required matrix multiplications and gain updates, both of which introduced significant overhead. In particular, the matrix-vector operations involved in computing the state transition and measurement update had an $\mathcal{O}(n^2)$ complexity, which further strained the limited CPU resources. Additionally, if floating-point arithmetic was used, the computational latency became even more significant. Given the combined constraints of computational load, memory limitations, and estimation overhead, achieving a 10 ms control loop required major simplifications, such as reducing prediction horizons, lowering state dimensions, and switching to fixed-point arithmetic where possible.

4.4.2 Simulink Implementation

The Simulink implementation focuses on sensor data processing, error computation, the integration of Skyhook and LQR control algorithms, and actuator control. These key subsystems achieve real-time data processing and system optimization.

The data acquisition and processing subsystem utilizes the NXP Model-Based Design Toolbox to interface with various sensors via hardware peripherals such as Controller Area Network, Pulse-Width Modulation Input Capture, Analog-to-Digital Converters, and Serial Peripheral Interface. These interfaces enable efficient data communication between the vehicle hardware and Simulink, ensuring reliable acquisition of signals such as vehicle corner height, velocity, steering angle, and angular orientation.

Raw sensor data is first calibrated to eliminate bias and align measurements with physical units. Calibration blocks as shown in figure 4.11, such as `H1_Calib` and `angFL_Calib`, compensate for sensor-specific offsets and gain errors. The calibrated signals are then filtered using low-pass filters to suppress high-frequency noise and retain the relevant dynamic content required by the control algorithms.

Height sensors located at each suspension corner provide ride height measurements, which are also used to estimate the vehicle's roll and pitch angles. An Inertial Measurement Unit (IMU) supplies accelerations along the x - (longitudinal), y - (lateral), and z - (vertical)

The Simulink implementation of the integrated Skyhook-LQR controller combines two control strategies to optimize the performance of the suspension system. The Linear Quadratic Regulator (LQR) ensures optimal control force allocation by minimizing a quadratic cost function, while the Skyhook controller dynamically adjusts the damping force based on the relative velocity between the suspension and the vehicle body. The Simulink model is structured such that real-time sensor data, including the vehicle state vector X , relative suspension velocity v_{rel} , and road disturbances L_1, L_2 , etc., feed directly into both controllers. The LQR controller processes the full-state feedback and generates optimal control forces, while the Skyhook controller refines the damping characteristics based on real-time suspension behavior.

The LQR controller is designed based on a discrete-time state-space representation of the suspension system, defined as:

$$X_{k+1} = AX_k + BU_k \quad (4.25)$$

where X_k is the state vector consisting of suspension displacement, velocity, pitch, and roll states, while U_k represents the control input applied to the actuators. The optimal control force is computed using the precomputed gain matrix K :

$$U_k = -KX_k. \quad (4.26)$$

In Simulink, the system is modeled using a State-Space block that takes the input matrices A , B , C , and D to compute the state evolution. The LQR gain matrix K is calculated offline and loaded into the model. The ‘Matrix Multiply’ block then computes the control force as $U = -KX$, which is passed to the actuator model.

The Skyhook controller modifies the damping force by evaluating the relative velocity between the suspension and the vehicle body. The damping force is computed using:

$$F_{\text{sky}} = c_{\text{sky}}v_{\text{rel}} \quad (4.27)$$

where c_{sky} is the damping coefficient that varies based on system states, and v_{rel} is the relative suspension velocity. The Skyhook logic is implemented in a Simulink function block (‘fcn’), which processes the suspension velocities and dynamically adjusts the damping coefficient. The relative suspension velocities for the four corners of the vehicle are computed as:

$$\dot{z}s_1 = v_s + l_1\omega_r - L_1\omega_p, \quad (4.28)$$

$$\dot{z}s_2 = v_s + l_1\omega_r + L_2\omega_p, \quad (4.29)$$

$$\dot{z}s_3 = v_s - l_1\omega_r - L_1\omega_p, \quad (4.30)$$

$$\dot{z}s_4 = v_s - l_1\omega_r + L_2\omega_p. \quad (4.31)$$

The function block evaluates whether the relative velocity and the sprung mass velocity have the same sign to determine the appropriate damping coefficient. If they have the same sign, indicating that the sprung mass is moving in phase with the suspension, the damping coefficient is set to c_{nom} . Otherwise, it is reduced to near zero:

$$c_{\text{sky},i} = \begin{cases} c_{\text{nom}}, & \text{if } v_{\text{rel},i} \cdot z s_{;i} \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.32)$$

This logic is implemented in the function block as follows:

```
for i = 1:4
    delta_v = rel_susp_vel(i);
    zsV = zs_dot(i);

    if delta_v * zsV >= 0    % High damping state
        c_sky(i) = c_nom;
    else    % Low damping state
        c_sky(i) = 0;
    end

    F_sky(i) = c_sky(i) * zs_dot(i);
    c_controller(i) = F_sky(i) / (delta_v + eps);
end
```

After computing the Skyhook and LQR forces, the final damping force applied to the suspension actuators is given by:

$$F_{\text{final}} = F_{\text{LQR}} + F_{\text{Skyhook}}. \quad (4.33)$$

This force is then processed through the actuator dynamics block in Simulink, which consists of a transfer function that models the physical actuator response. The force command is converted into a Pulse Width Modulation (PWM) signal to drive the controllable damper, ensuring real-time adjustment of damping levels.

The actuator control subsystem translates the computed damping force into control signals for the CDC dampers. The output force is converted into a continuous current using PWM signals generated by the S32K microcontroller. The PWM duty cycle is mapped to the required damping force using calibration functions, ensuring accurate force application. The generated control signals are sent to each suspension corner, adjusting damping in real time.

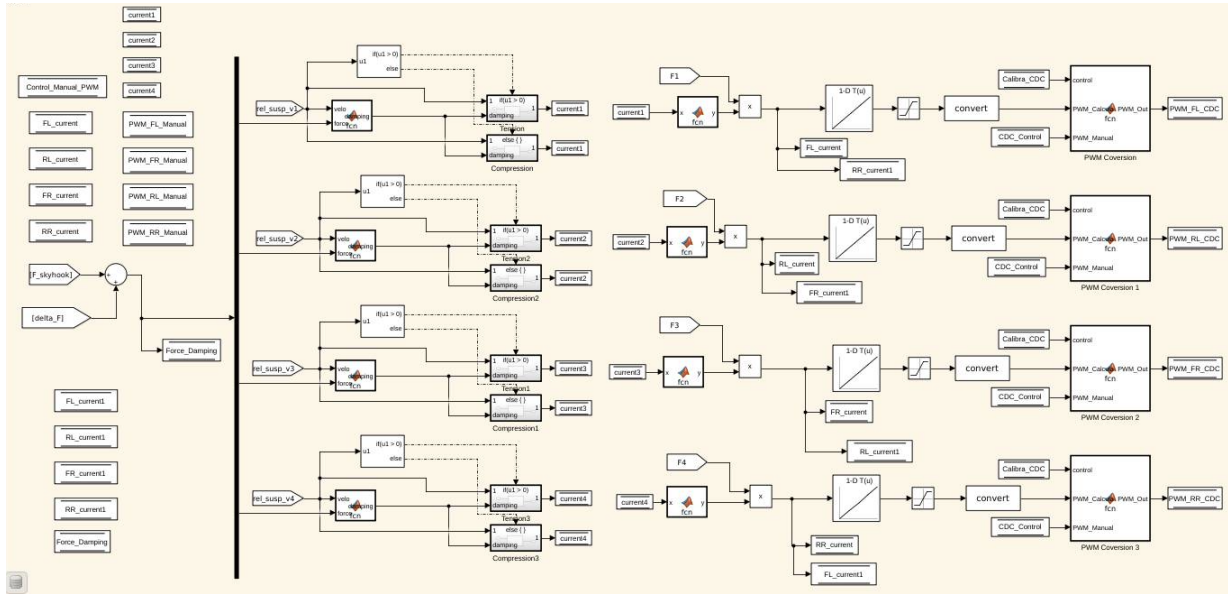


Figure 4.12: Damper Control SubSystem

4.4.3 Experimental Setup

The in-vehicle setup includes the following components:

- **Rapid Prototyping ECU** – The FlexCase ECU is used for real-time execution of the Skyhook-LQR control strategy in the Hongqi H9 suspension test. The equipped S32K344 MCU ensures low-latency control processing. Enriched CAN FD, analog inputs, and PWM outputs ensure enough IOs to integrate with CDC dampers, IMU, height sensors, and vehicle CAN. The ECU’s rapid prototyping capability allows direct deployment of the Simulink-based control logic developed in the simulation phase, minimizing discrepancies between simulated and real-world performance.

Normal Operating Voltage: 8V–36V
Functional Safety: ASIL D
Microcontroller (MCU): NXP S32K344
Microprocessor (MPU): Rockchip RK3566
Input/Outputs:
- 5 CAN FD channels support CAN wake-up
- 2 LIN channels support LIN wake-up
- 12 digital inputs support PWM, Quadrature
- 30 analog inputs with 12-bit ADC 0-10V
- 18 continuous current 3A GPO/PWM outputs, peak current 7A

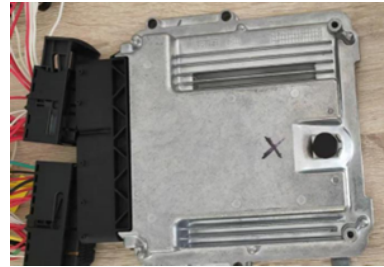


Figure 4.13: FlexCase ECU Specifications

- **Inertial Measurement Unit (IMU)** – The LSM6DSV16BX IMU is 6-axis sensing, high sampling rate (7.68 kHz), and low-latency SPI/I2C interface ensure precise acceleration and angular velocity measurements for suspension optimization. The IMU is installed near the vehicle’s center of gravity (CG) to accurately capture vertical, pitch, and roll dynamics, minimizing localized disturbances. Placement at non-central locations could introduce asymmetrical noise, affecting control accuracy. With a wide dynamic range ($\pm 16g$, ± 4000 dps) and low-power operation, it enables accurate feedback control, improving ride comfort and stability across varying road conditions.

Safety Level: ISO26262 ASIL-B
Measurement Range: 8g, 60Hz
Operating Voltage: $3.3 \pm 0.3V$
Operating Current: $<10mA$
Communication Protocol: SPI, CAN
Operating Temperature: $-40^{\circ}C$ to $125^{\circ}C$



Figure 4.14: Sensor Specifications

- Height Sensors – The height sensors provide real-time measurements of suspension displacement and body movement. Installed at each wheel corner, these sensors track variations in ride height, enabling precise damping adjustments. Their placement ensures accurate detection of both individual wheel motion and overall body displacement, preventing localized errors. Improper positioning could lead to misinterpretation of suspension travel, reducing control effectiveness.

Angle Range: $-60^{\circ} - 60^{\circ}$
Operating Temperature: $-40^{\circ}C - 125^{\circ}C$
Operating Voltage: 5V
Operating Current: $<10mA$
Output Type: PWM
Lever Length: 50mm



Figure 4.15: Sensor Specifications

- CDC Dampers – The Continuously Damping Control (CDC) dampers play a crucial role in the semi-active suspension system of the Hongqi H9 test vehicle. These dampers are electronically adjustable and regulate damping force in real time based on control inputs from the integrated Skyhook-LQR controller. The dampers are installed at all four corners of the suspension system as shown in figure 4.16, and are actuated by an electric current supplied by the ECU.

By leveraging the CDC damper's continuously variable damping capability, the controller fine-tunes the suspension response to different road conditions. This en-

sure minimal vibration transmission in comfort-focused scenarios while providing increased stiffness for stability during aggressive maneuvers.

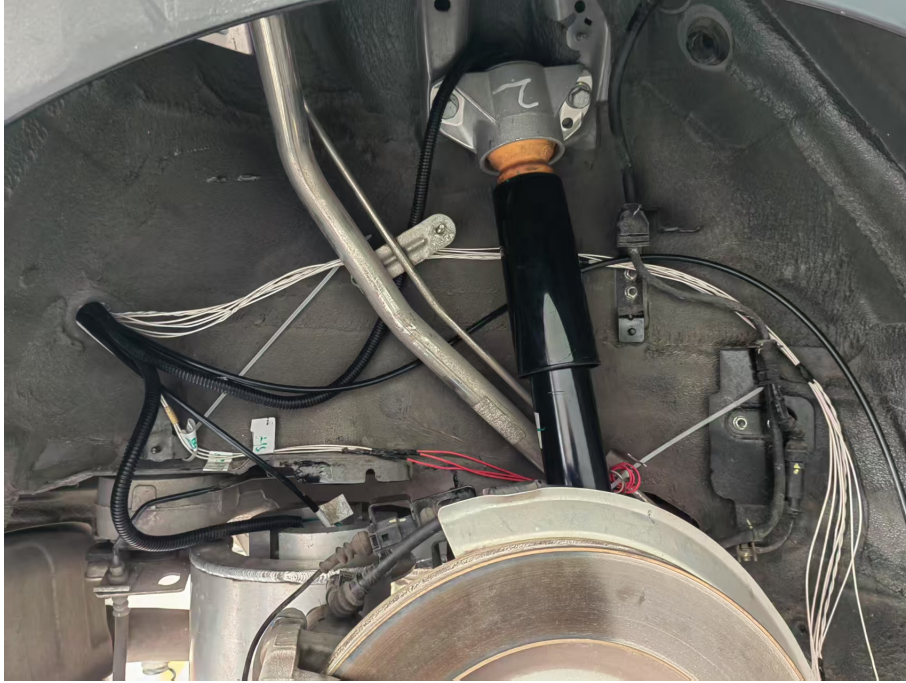


Figure 4.16: Installation of CDC dampers in the test vehicle

The damping force characteristics of the CDC dampers are defined by their rebound and compression curves, which indicate the relationship between current input and damping force. The front left and right dampers share identical properties, as do the rear left and right dampers. As shown in the figure 4.17 to figure 4.20, increasing the current reduces the damping force, allowing for adaptable suspension stiffness. The Skyhook-LQR controller dynamically adjusts this current based on vehicle state feedback, including vertical acceleration, pitch rate, and roll rate. The ECU continuously processes sensor inputs and modulates the current to each damper to achieve optimal ride comfort and stability.

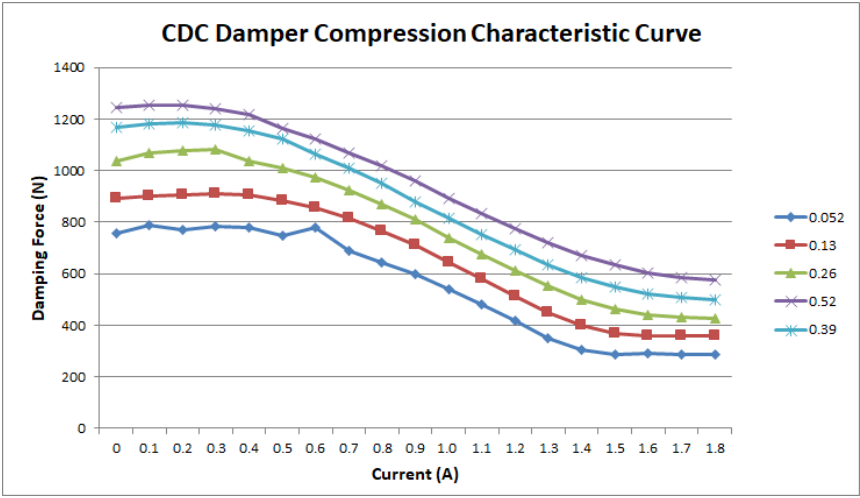


Figure 4.17: Front CDC Damper Compression

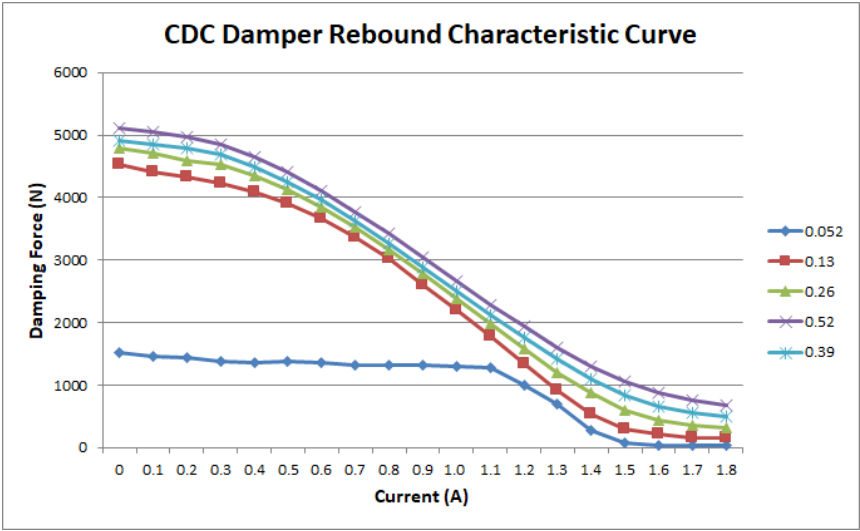


Figure 4.18: Front CDC Damper Rebound

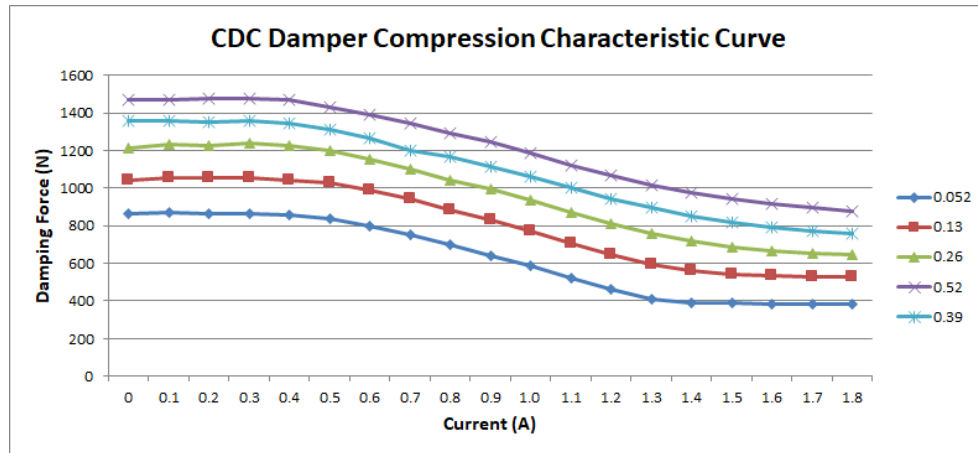


Figure 4.19: Rear CDC Damper Compression

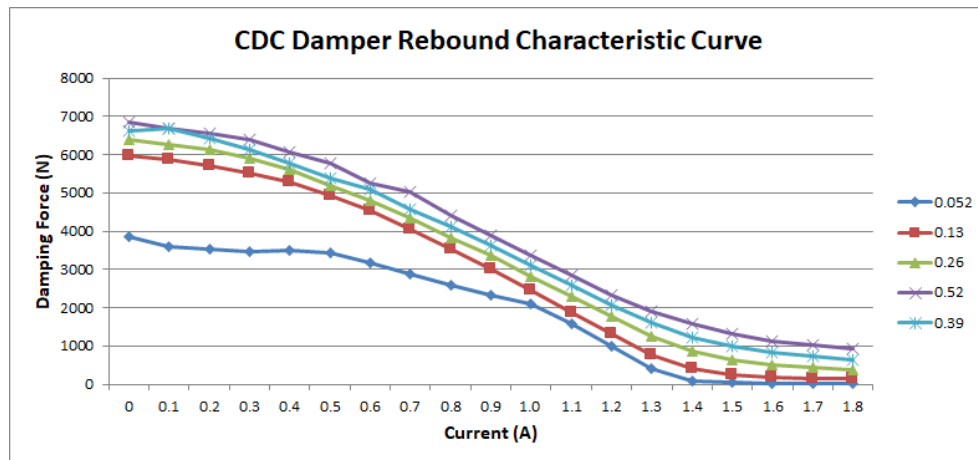


Figure 4.20: Rear CDC Damper Rebound

4.4.4 Road Testing Setup

In real-world driving conditions, suspension performance is influenced by various dynamic factors that cannot be fully replicated in simulation. To comprehensively evaluate the effectiveness of the integrated Skyhook-LQR controller, a series of vehicle tests were conducted under five distinct road and driving scenarios. Unlike the controlled environment in CarSim, real-world disturbances introduce variability in road excitations, sensor feedback, and actuator response, providing a more rigorous validation of the control strategy. Additionally, comparisons with simulated road profiles allow for cross-validation of the controller's adaptability and stability across different testing conditions.

- **Multi-Cross Speed Bump:** This test evaluates the suspension's ability to mitigate repetitive vertical disturbances at varying speeds. Unlike the single-speed bump used in CarSim simulations, which primarily assesses peak vertical acceleration response, the multi-cross speed bump provides insight into how well the controller handles successive impacts, ensuring ride comfort while maintaining suspension travel efficiency.



Figure 4.21: Multiple Sets of Cross Speed Bumps

- **Continuous Sharp S-Curve Lane Change:** This maneuver examines the suspension's roll control capabilities and overall lateral stability. Compared to the offset speed bump in CarSim, which introduces asymmetric disturbances, the sharp S-curve lane change applies continuous lateral forces, making it a more rigorous test of the controller's ability to regulate roll oscillations while preserving handling performance.



Figure 4.22: Multi-S turns test for evaluating roll stability

- **Gravel Road Performance:** Real-world disturbances from an uneven gravel road introduce high-frequency vibrations and irregular external forces that are difficult to replicate in simulation. This test helps assess the adaptability of the suspension

system in unpredictable terrain, particularly in filtering noise from IMU signals and adjusting damping in response to non-uniform road excitations, aspects that the ISO paved road profiles in CarSim could not fully capture.



Figure 4.23: Vehicle performance during off-road testing

- **Sudden Acceleration:** The effect of rapid acceleration on pitch dynamics is examined to determine how well the Skyhook-LQR controller minimizes excessive pitch motion. Unlike the longitudinal acceleration disturbances studied in CarSim, this test isolates acceleration-induced pitch instability, making it critical for evaluating suspension control in high-power driving conditions.
- **Emergency Braking Performance:** Hard braking generates significant forward weight transfer, leading to pronounced pitch oscillations. This test is essential for assessing the suspension's ability to stabilize the vehicle under extreme deceleration, providing a direct real-world counterpart to the simulated braking response observed in CarSim. By comparing results, the effectiveness of active damping in reducing pitch motion can be quantified.

4.5 Vehicle Testing Results and Discussion

The experimental results are analyzed across five different road conditions to compare the real-world performance of the Integrated Skyhook-LQR controller. The Skyhook-LQR controller was implemented with a constant output penalty matrix $Q = \text{diag}(0, 0, 0, 0, 100, 100)$ and control signal penalty matrix $R = 10^{-3}\text{diag}(1, 1, 1, 1)$ across all test scenarios. Adjusting the weighting factors in matrix Q allows control emphasis to be shifted between vertical, roll, and pitch motions. Increasing the pitch and roll weightings enhances stability but may compromise ride comfort. Similarly, modifying C_{sky} alters the damping response, with higher values improving vibration isolation but potentially reducing road-holding capability. The chosen parameters balance ride comfort and handling stability, ensuring effective suspension performance under varying road conditions.

4.5.1 Multi-Cross Speed Bump at 30kph

The multiple cross-speed bump test at 30 kph introduces sustained high-frequency disturbances, requiring the suspension to adapt continuously. The vertical acceleration RMS values in Table 4.7 show that the integrated Skyhook-LQR controller significantly outperforms passive suspension, with the highest damping setting ($C = 10000, Q = 75W$) reducing vertical RMS by 25%. This confirms that real-time damping adjustments effectively mitigate impact forces.

However, pitch and roll responses exhibit complex trends. High damping ($C = 10000, Q = 75W$) increases pitch RMS by 6%, suggesting that excessive stiffness limits suspension travel but transfers disturbances to the vehicle body. In contrast, moderate damping ($C = 5000, Q = 25W$) reduces pitch RMS by up to 14%, balancing compliance and control. Roll RMS reductions peak at 13% for ($C = 5000, Q = 25W$), indicating that lateral load transfer dynamics limit the controller's influence on roll stability.

The observed variations stem from how the integrated LQR-Skyhook controller distributes damping forces. Increasing C enhances high-frequency damping, reducing vertical acceleration but amplifying pitch oscillations due to stiffness. Adjusting Q modifies the LQR cost function, affecting the trade-off between ride comfort and control effort. Fine-tuning these parameters is essential to achieving optimal performance across all axes.

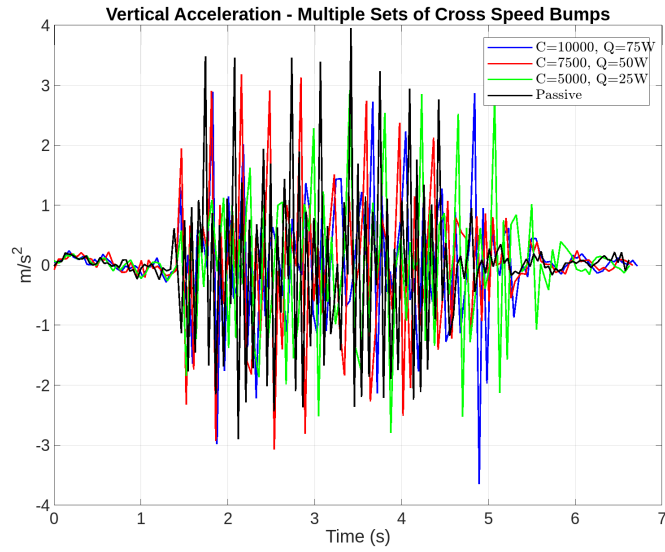


Figure 4.24: Vertical acceleration of the vehicle during an offset bump test at 30kph

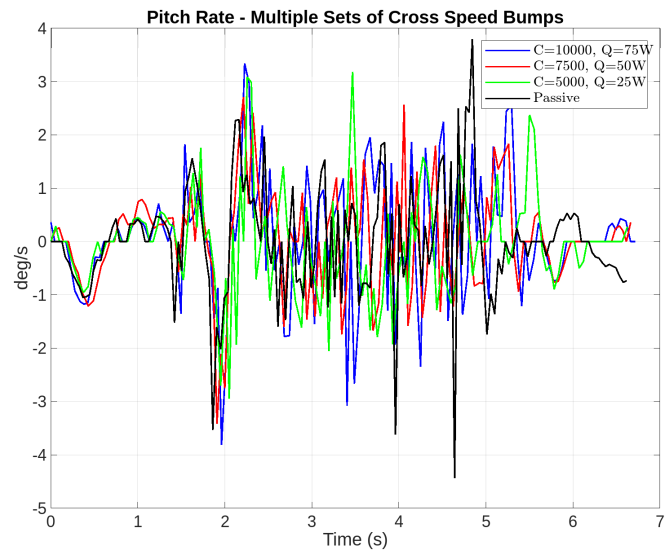


Figure 4.25: Pitch rate response during an offset bump test at 30kph

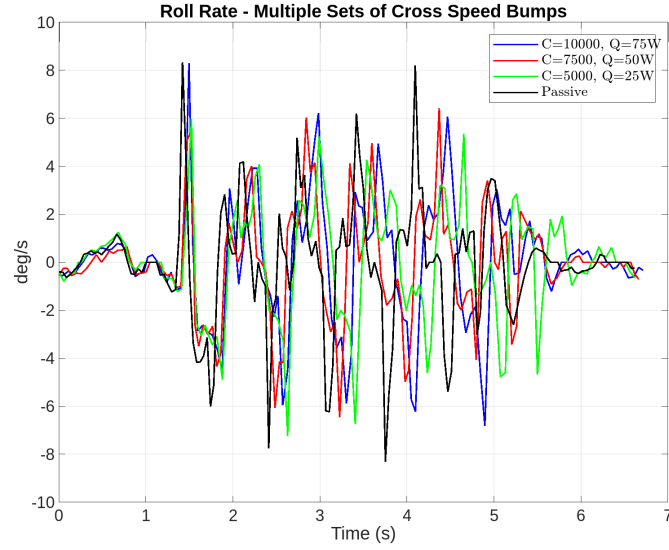


Figure 4.26: Roll rate response during an offset bump test at 30kph

Table 4.7: RMS Assessment for Cross Set Bumps at 30kph

	Passive	C=10000, Q=75W	C=7500, Q=50W	C=5000, Q=25W
Vertical acceleration RMS value	1.603	1.199 (-25%)	1.371 (-14%)	1.268 (-21%)
Pitch angular velocity RMS value	1.339	1.326 (-1%)	1.178 (-12%)	1.147 (-14%)
Roll angular velocity RMS value	3.154	3.123 (-1%)	2.835 (-10%)	2.723 (-13%)

4.5.2 Continuous Sharp S-Curve Lane Change at 30kph

The sharp S-curve lane change test at 30 kph evaluates the integrated LQR-Skyhook controller's ability to regulate roll and pitch dynamics under aggressive steering. As shown in Table 4.8, variations in C and Q significantly impact roll and pitch oscillations, with Table 4.8 quantifying their effects.

Compared to passive suspension, the controller effectively reduces roll angular velocity RMS by up to 17.2% ($C = 7500, Q = 50W$), demonstrating improved lateral stability. However, reducing C to 5000 increases roll RMS by 9.1%, indicating insufficient damping force to counteract lateral load transfers. Conversely, pitch RMS increases by 11.6% with $C = 5000$, revealing that lower damping compromises pitch suppression. This occurs because LQR optimizes control forces based on state feedback, and reducing C weakens the Skyhook effect, making pitch more susceptible to transient disturbances.

Vertical acceleration RMS remains relatively stable, as the primary challenge in this maneuver is lateral stability rather than ride comfort. The results confirm that C and Q tuning directly influences the trade-off between roll and pitch control. Higher C enhances stability but may over-dampen responses, while lower C improves responsiveness at the cost of increased oscillations. This underscores the need for careful parameter selection to balance handling and ride quality.

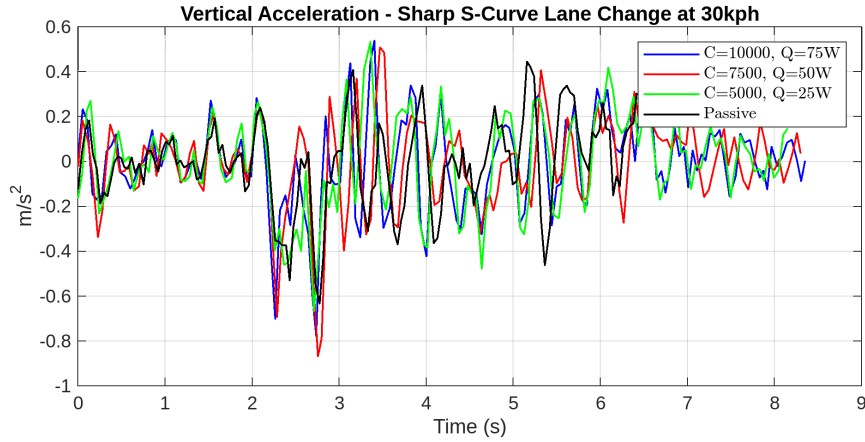


Figure 4.27: Vertical acceleration during Sharp S-Curve at 30kph

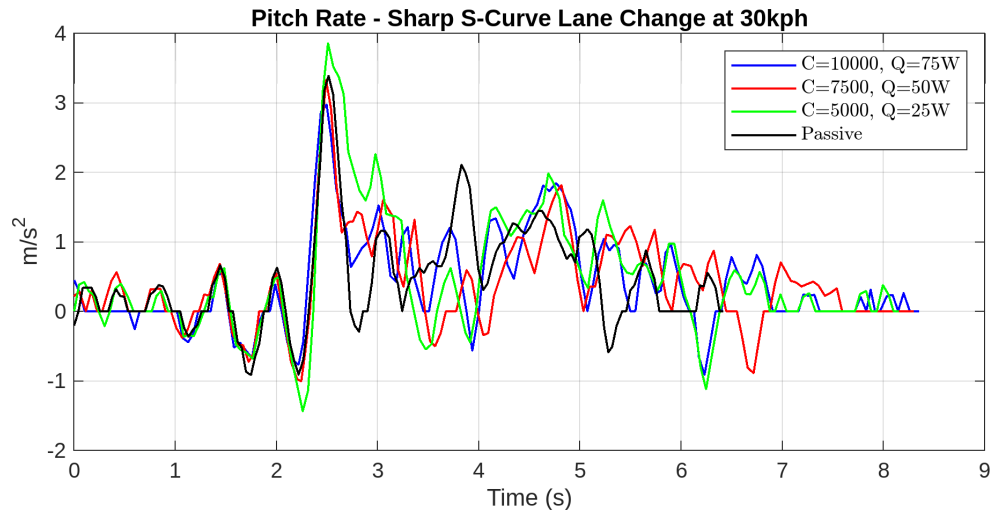


Figure 4.28: Pitch rate during Sharp S-Curve at 30kph

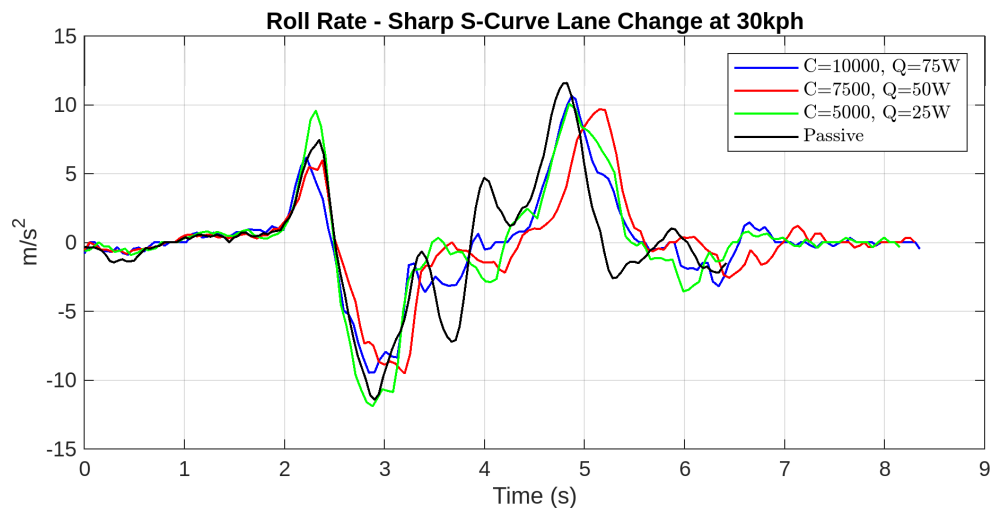


Figure 4.29: Roll rate during Sharp S-Curve at 30kph

Table 4.8: RMS Assessment for Sharp S-Curve Lane Change at 30kph

	Passive	$C=10000$, $Q=75W$	$C=7500$, $Q=50W$	$C=5000$, $Q=25W$
Vertical acceleration RMS value	0.225	0.219 (-2.4%)	0.231 (3.1%)	0.244 (8.1%)
Pitch angular velocity RMS value	1.018	1.000 (-2.1%)	0.991 (-3.2%)	1.141 (11.6%)
Roll angular velocity RMS value	5.1378	4.308 (-16.4%)	4.266 (-17.2%)	4.991 (-1.9%)

4.5.3 Broken Gravel Road at 45kph

The broken gravel road test at 45 kph subjects the suspension to continuous high-frequency disturbances and uneven pits, challenging the system’s ability to maintain stability. As shown in Table 4.9, pitch angular velocity RMS exhibits the most significant improvement, with a reduction of 21.3% at ($C = 10000, Q = 75W$), while vertical acceleration and roll angular velocity remain relatively unchanged across different control settings. This trend indicates that the integrated Skyhook-LQR controller prioritizes pitch stabilization over vertical and roll control in such road conditions.

The lack of improvement in vertical acceleration can be attributed to the nature of the terrain. Broken gravel roads induce rapid, unpredictable disturbances where tire compliance and unsprung mass characteristics play a dominant role in vertical acceleration response. Since Skyhook control primarily influences the damping forces of the sprung mass, its effect on high-frequency vertical disturbances remains limited. Similarly, roll dynamics exhibit minimal variation due to the stochastic lateral excitations of the gravel surface, where vehicle inertial properties and road-induced lateral forces outweigh the impact of suspension control.

The superior performance at ($C = 10000, Q = 75W$) stems from the balance between damping responsiveness and control authority. A high damping coefficient C enhances resistance to sudden pitch oscillations, preventing excessive forward and backward body motion over pits and bumps. Simultaneously, the high weighting factor Q reinforces the controller’s priority on pitch suppression, reducing unnecessary control actions that could destabilize other degrees of freedom. The data suggests that in broken gravel scenarios, the Skyhook-LQR controller is most effective when configured to emphasize pitch stabilization while allowing passive compliance for vertical and lateral dynamics.

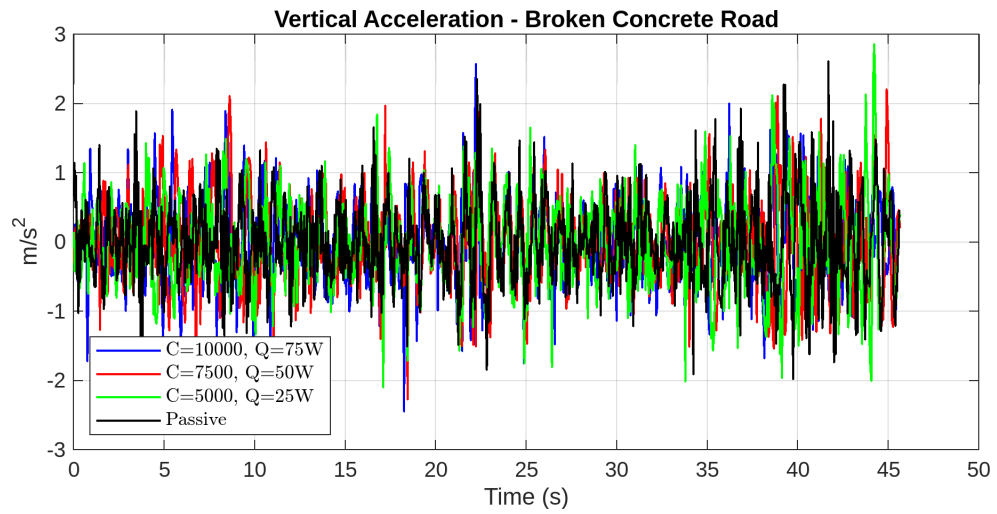


Figure 4.30: Vertical acceleration of Broken Gravel Road at 45kph

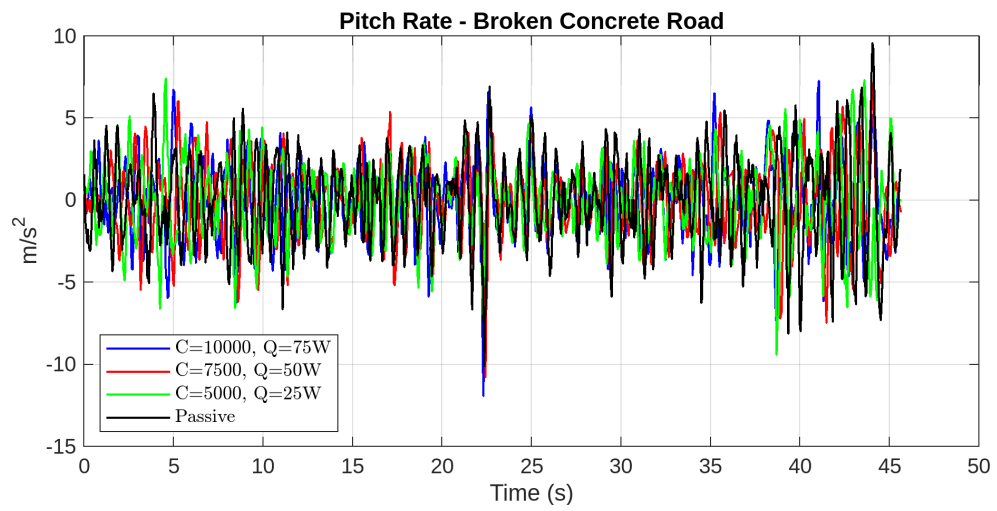


Figure 4.31: Pitch rate of Broken Gravel Road at 45kph

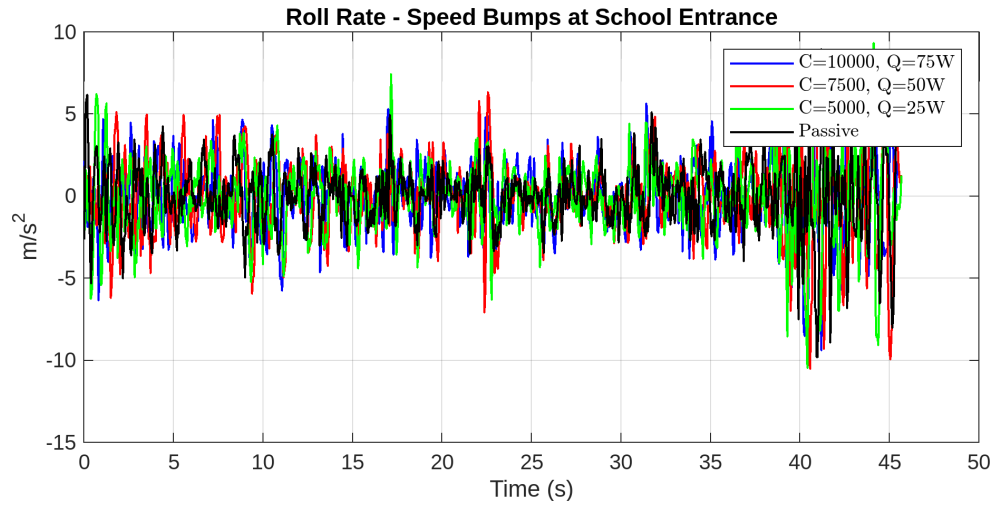


Figure 4.32: Roll rate of Broken Gravel Road at 45kph

Table 4.9: RMS Assessment for Broken Gravel Road

	Passive	C=10000, Q=75W	C=7500, Q=50W	C=5000, Q=25W
Vertical acceleration RMS value	0.623	0.648 (4.2%)	0.639 (3.6%)	0.64 (4.1%)
Pitch angular velocity RMS value	2.672	2.126 (-21.3%)	2.205 (-18.2%)	2.333 (-13.4%)
Roll angular velocity RMS value	2.075	2.129 (4.8%)	2.031 (-1.4%)	2.103 (3.5%)

4.5.4 Sudden Acceleration to 60kph

The sudden acceleration to 60 kph testing evaluates the suspension's response to rapid load transfer. Table 4.10 shows a significant reduction in vertical acceleration RMS, with ($C = 7500, Q = 50W$) achieving the best performance (-21%). This indicates that the integrated Skyhook-LQR controller effectively minimizes vertical oscillations by dynamically adjusting damping in response to acceleration-induced disturbances.

Roll RMS also improves, albeit less significantly (-14%), suggesting that while the controller mitigates lateral load transfer, roll inertia limits further enhancement. In contrast, pitch RMS remains nearly unchanged, implying that the controller has limited influence on pitch dynamics during acceleration, as pitch is largely governed by vehicle inertia.

The data for ($C = 10000, Q = 75W$) deviates from expected trends, likely due to excessive damping restricting necessary suspension movement. This highlights the importance of tuning C and Q to balance stiffness and adaptability. The results confirm that the integrated Skyhook-LQR improves ride comfort and stability, with moderate damping settings yielding optimal performance.

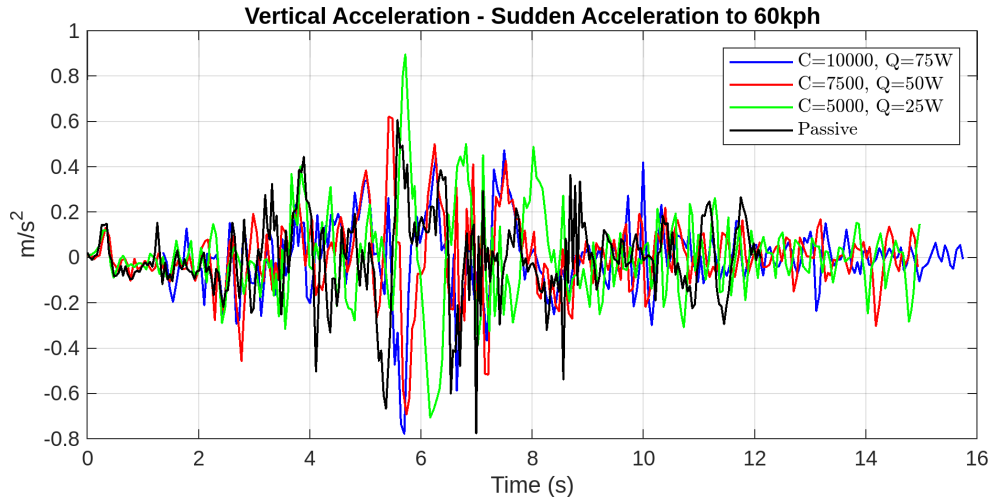


Figure 4.33: Vertical acceleration of Sudden Acceleration to 60kph

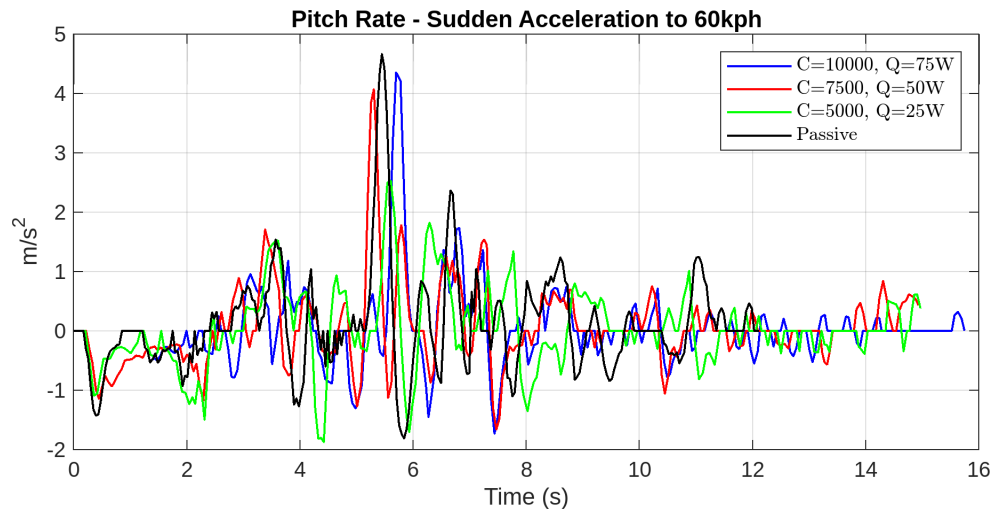


Figure 4.34: Pitch rate of Sudden Acceleration to 60kph

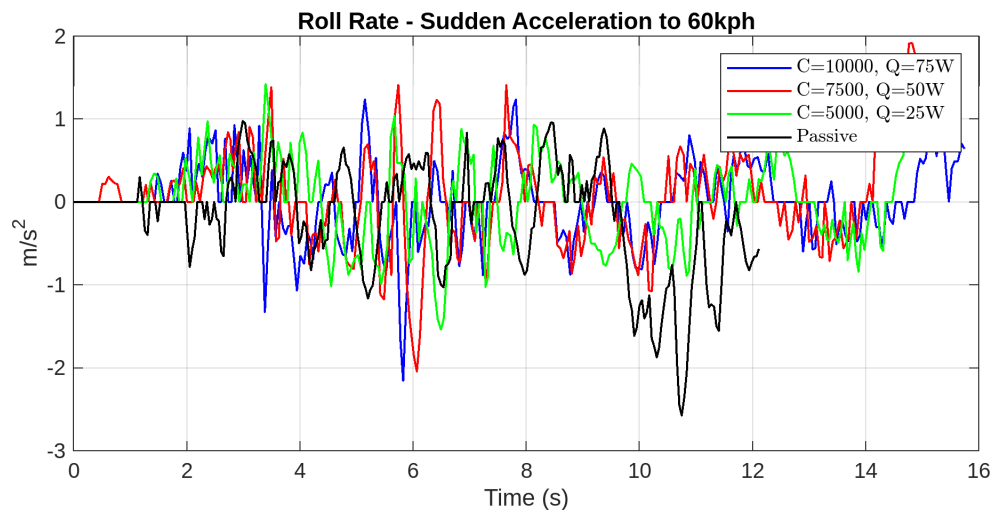


Figure 4.35: Roll rate of Sudden Acceleration to 60kph

Table 4.10: RMS Assessment for Sudden Acceleration to 60kph

	Passive	$C=10000,$ $Q=75W$	$C=7500,$ $Q=50W$	$C=5000,$ $Q=25W$
Vertical acceleration RMS value	0.173	0.211 (16.4%)	0.136 (-21%)	0.135 (-22%)
Pitch angular velocity RMS value	0.710	0.981 (38%)	0.756 (6%)	0.719 (1%)
Roll angular velocity RMS value	0.416	0.594 (42%)	0.400 (-4%)	0.367 (-14%)

4.5.5 Emergency Braking at 30kph

The results from the sudden braking test demonstrate the effectiveness of the integrated Skyhook-LQR controller in reducing vertical and pitch oscillations while showing an increase in roll motion compared to passive suspension.

Vertical acceleration exhibits the most significant reduction, with RMS values decreasing by up to 45.1% for $C = 10000, Q = 75W$, as shown in Table 4.11. This improvement is attributed to the Skyhook control mechanism effectively damping high-frequency vertical disturbances by dynamically adjusting the damping force. The LQR component further refines control by optimizing energy dissipation, enhancing ride comfort.

Pitch motion also experiences notable reductions, with a maximum decrease of 24.6% observed at $C = 5000, Q = 25W$. Since sudden braking induces a strong forward weight transfer, pitch oscillations are primarily influenced by the front suspension. A well-tuned damping coefficient effectively mitigates excessive nose-diving, stabilizing the vehicle during deceleration.

Conversely, roll motion shows an increase in RMS values across all control settings compared to passive suspension, with the highest increase of 29.7% at $C = 10000, Q = 75W$. This occurs because braking primarily affects the longitudinal axis, with minimal lateral excitation. The integrated controller prioritizes vertical and pitch stability, potentially redistributing some disturbances into roll motion. Moreover, since roll oscillations are inherently low under braking conditions, the controller's influence on roll suppression is limited.

Overall, the results confirm that tuning C and Q significantly enhances vertical and pitch stability during braking. However, carefully selecting these parameters is necessary to balance ride comfort and prevent unintended roll amplification.

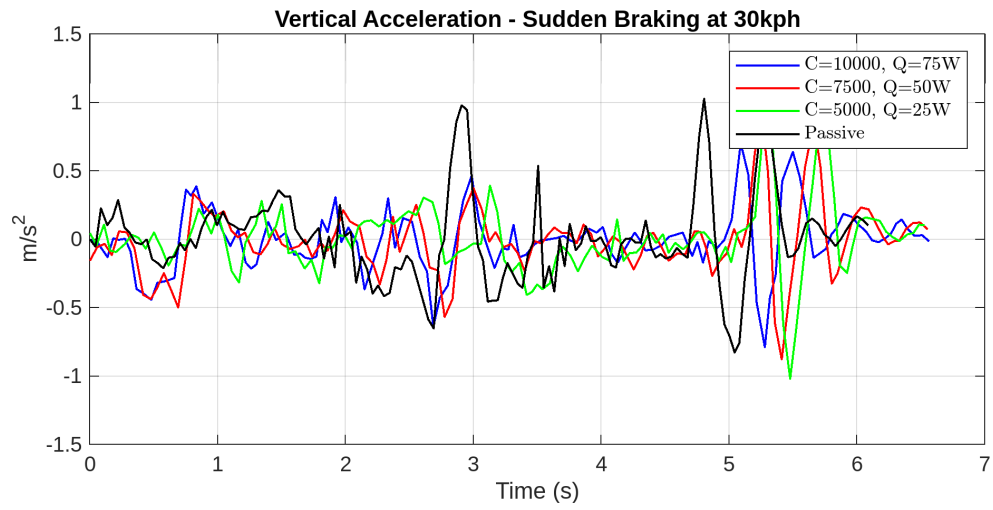


Figure 4.36: Vertical acceleration of Sudden Acceleration to 60kph

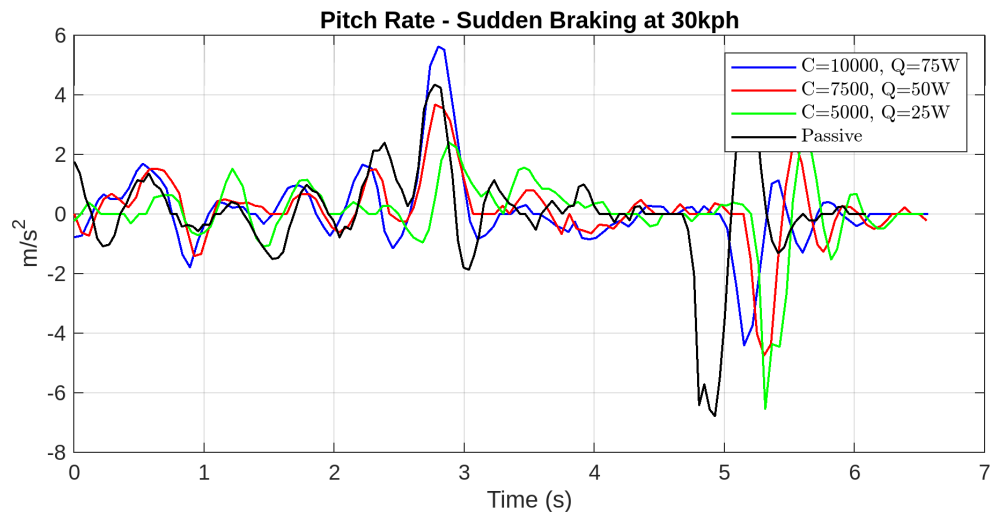


Figure 4.37: Pitch rate of Sudden Acceleration to 60kph

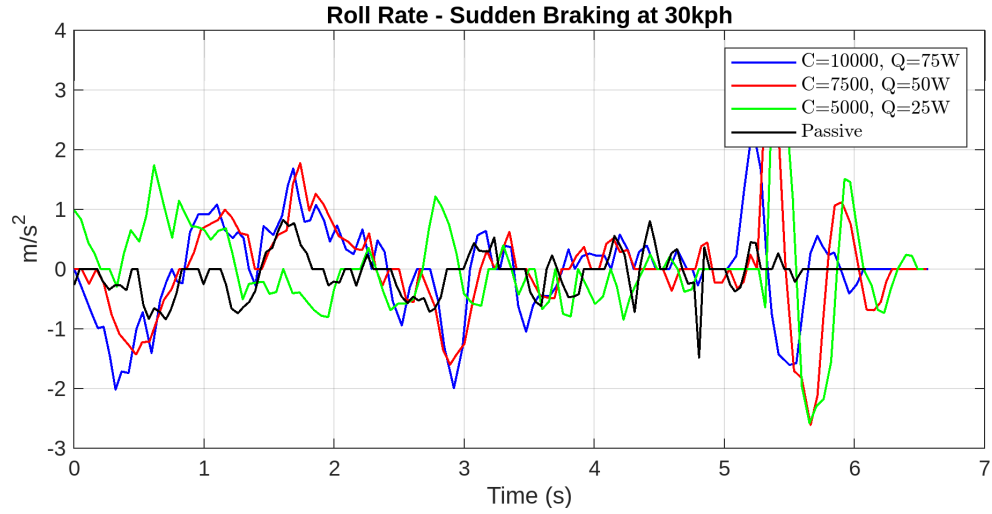


Figure 4.38: Roll rate of Sudden Acceleration to 60kph

Table 4.11: RMS Assessment for Sudden Braking at 30kph

	Passive	C=10000, Q=75W	C=7500, Q=50W	C=5000, Q=25W
Vertical acceleration RMS value	0.376	0.206 (-45.1%)	0.238 (-37.3%)	0.253 (-33.5%)
Pitch angular velocity RMS value	1.726	1.675 (-3.2%)	1.328 (-20.4%)	1.347 (-24.6%)
Roll angular velocity RMS value	0.416	0.548 (29.7%)	0.606 (23.4%)	0.419 (4.1%)

4.6 Comparison of MPC and Integrated Skyhook-LQR in Suspension Control System

The effectiveness of a semi-active suspension control system depends on its ability to enhance ride comfort, stability, and overall vehicle handling under varying road conditions. This section presents a comparative analysis between the Model Predictive Control (MPC) and the Integrated Skyhook-LQR controller, evaluating their respective advantages and limitations in real-time suspension control. The comparison is conducted across four key aspects:

1. **Multi-Directional Suspension Control:** The capability of each controller to mitigate vertical, pitch, and roll disturbances effectively across different driving conditions.
2. **Handling of Damping Constraints:** How well each control strategy enforces the physical constraints of the Continuously Damping Control (CDC) damper to prevent saturation and maintain optimal damping force.
3. **Control Performance:** A quantitative assessment based on root mean square (RMS) acceleration metrics, evaluating the controllers' ability to minimize vibration and improve ride quality.

By examining these factors, this section aims to provide a comprehensive understanding of the trade-offs between MPC and Integrated Skyhook-LQR, guiding the selection of an optimal control strategy for real-time semi-active suspension systems.

4.6.1 Multi-Directional Suspension Control

The effectiveness of a suspension control strategy in handling multi-directional dynamics—vertical, roll, and pitch—is critical for maintaining ride comfort, stability, and handling performance. Both Model Predictive Control (MPC) and the Integrated Skyhook-LQR controller attempt to optimize these dynamics, but their underlying mechanisms lead to distinct differences in performance.

MPC excels in multi-directional control by simultaneously optimizing damping forces for vertical, roll, and pitch motions. By incorporating a predictive model, MPC ensures optimal force distribution across all suspension actuators. This allows it to proactively manage load transfer, reducing excessive oscillations in all three axes. Its ability to enforce constraints on damping forces further prevents excessive control efforts while maintaining system stability. However, the computational cost is significant, as an optimization problem must be solved at every control step, limiting real-time feasibility.

In contrast, the Integrated Skyhook-LQR controller primarily focuses on vertical motion control through Skyhook damping while addressing roll and pitch stability using LQR feedback control. Although effective in reducing vertical acceleration, its influence on roll and pitch is limited due to the reactive nature of LQR, which lacks explicit prediction of future disturbances. The absence of a predictive element means that roll and pitch responses rely on real-time feedback rather than optimized force distribution over time, making the controller less effective in rapidly changing conditions.

A key distinction is how each controller manages road disturbances. MPC, by predicting disturbances over a horizon, can balance damping across different motion axes, achieving better multi-directional performance. This results in superior suppression of roll and pitch oscillations, particularly in transient maneuvers such as sudden lane changes or aggressive braking. In contrast, the Integrated Skyhook-LQR controller reacts to disturbances as they occur, leading to delayed correction of roll and pitch motions, making it less suitable for scenarios with high lateral or longitudinal forces.

The computational complexity of MPC remains its primary drawback, making it impractical for some real-time applications, particularly in embedded vehicle controllers with limited processing power. The Integrated Skyhook-LQR, by contrast, is computationally efficient and easier to implement in real-time systems, as it does not require solving optimization problems at each time step.

A comparative summary of these two controllers in the context of multi-directional suspension control is presented in Table [4.14](#).

Table 4.12: Comparison of MPC and Integrated Skyhook-LQR in Multi-Directional Suspension Control

Feature	MPC	Integrated Skyhook-LQR
Computational Cost	High (solves optimization problem at each step)	Low (direct control law)
Multi-Directional Control	Excellent (optimizes vertical, roll, and pitch)	Moderate (strong in vertical, limited in roll and pitch)
Predictive Capability	Yes (anticipates disturbances and adjusts proactively)	No (reacts to real-time states)
Real-Time Feasibility	Limited (high computational burden)	High (efficient for embedded systems)
Handling and Stability	Excellent (adaptive to dynamic conditions)	Moderate (focus on comfort over precision)

4.6.2 Handling of Damping Constraints

Effective damping constraint management is critical in semi-active suspension systems to ensure compliance with actuator limits, dissipativity conditions, and response constraints. This section compares the constraint-handling mechanisms of Model Predictive Control (MPC) and the integrated skyhook-LQR controller, focusing on their implementation and effectiveness.

MPC enforces constraints through quadratic programming (QP) optimization, ensuring strict adherence to actuator limits, dissipativity conditions, and damping rate constraints. The optimization is defined as:

$$J = \sum_{k=0}^{N_p} (X_k^T Q X_k + U_k^T R U_k) \quad (4.34)$$

subject to:

$$U_{\min} \leq U_k \leq U_{\max}, \quad U_k \cdot C_v X_k \geq 0, \quad \Delta U_k \leq \tau \quad (4.35)$$

This formulation guarantees that damping forces remain within operational limits, preventing excessive actuation and maintaining stability even under aggressive driving conditions. However, solving the QP problem at each control step introduces a high computational cost, limiting real-time feasibility.

In contrast, the Integrated Skyhook-LQR controller does not impose constraints explicitly but instead relies on force saturation and logical switching to enforce limits. The Skyhook control law adjusts damping based on velocity conditions:

$$F_{sh} = \min(\max(-d_{sh}\dot{Z}_s, U_{\min}), U_{\max}) \quad (4.36)$$

where the Skyhook damping coefficient is selected as:

$$d_{sh} = \begin{cases} d_{\max}, & \text{if } \dot{Z}_s(\dot{Z}_s - \dot{Z}_u) \geq 0 \\ d_{\min}, & \text{otherwise} \end{cases} \quad (4.37)$$

For LQR-based roll and pitch control, constraints are applied using a simple saturation function:

$$F_{LQR} = \min(\max(KX, U_{\min}), U_{\max}) \quad (4.38)$$

where K is the LQR gain from the Algebraic Riccati Equation. The final control force is:

$$F_{\text{total}} = \min(\max(F_{\text{Skyhook}} + F_{\text{LQR}}, U_{\min}), U_{\max}) \quad (4.39)$$

Unlike MPC, which proactively enforces constraints within its optimization framework, Skyhook-LQR applies constraints reactively, adjusting forces only after violations occur. This makes it computationally efficient but less optimal in strict constraint management. A comparative analysis of constraint-handling capabilities is summarized in Table 4.13.

In Summary, MPC provides precise and predictive constraint handling, making it ideal for applications requiring strict force regulation, such as high-performance vehicles and adaptive autonomous systems. However, its computational burden makes real-time implementation challenging. In contrast, the Integrated Skyhook-LQR controller offers a computationally efficient alternative, ensuring practical constraint enforcement while being easily deployable in real-time vehicle suspension systems. Nonetheless, its reactive nature makes it less effective in enforcing damping constraints optimally, particularly in highly dynamic driving conditions.

Table 4.13: Comparison of MPC and Integrated Skyhook-LQR in Handling Damping Constraints

Feature	MPC	Integrated Skyhook-LQR
Physical Constraints (Actuator Limits)	Enforced via QP optimization	Applied using force saturation
Dissipativity Constraint	Strictly enforced as an inequality	Ensured via velocity-based switching
Rate Limit Constraints	Directly imposed in optimization	Enforced via low-pass filtering
Predictive Handling	Yes (anticipates violations)	No (reactive adjustment only)
Computational Cost	High (QP solver per step)	Low (direct algebraic computation)
Real-Time Feasibility	Limited (high CPU demand)	High (efficient for embedded systems)

4.6.3 Control Performance

Control performance in semi-active suspension is evaluated by minimizing vertical, roll, and pitch accelerations while maintaining stability. Table 4.14 summarizes the performance of MPC and Integrated Skyhook-LQR across different driving conditions.

MPC optimizes damping forces by solving a quadratic programming problem at each step, ensuring predictive disturbance rejection. By penalizing unwanted motion while enforcing constraints. This enables MPC to anticipate disturbances and apply corrective damping, leading to superior roll and pitch suppression. In contrast, Skyhook-LQR operates reactively, combining Skyhook damping for vertical vibration and LQR-based pitch and roll stabilization:

$$F_{sh} = -d_{sh}\dot{Z}_s \quad (4.40)$$

$$F_{LQR} = -KX \quad (4.41)$$

Table 4.14: Comparison of MPC and Integrated LQR-Skyhook Controller

Metric	MPC	LQR-Skyhook ($C = 5000, Q = 25W$)
Single Bump		
Vertical RMS Acceleration (m/s^2)	1.1357 (-18.4%)	–
Pitch RMS Acceleration (rad/s^2)	0.6488 (-25.2%)	–
Roll RMS Acceleration (rad/s^2)	–	–
Cross Set Bump		
Vertical RMS Acceleration (m/s^2)	0.5784 (-12.1%)	1.268 (-21%)
Pitch RMS Acceleration (rad/s^2)	0.3186 (-5.1%)	1.147 (-14%)
Roll RMS Acceleration (rad/s^2)	0.2678 (-23.7%)	2.723 (-13%)
ISO Paved Road vs Gravel Road		
Vertical RMS Acceleration (m/s^2)	0.5784 (-12%)	0.64 (+4.1%)
Pitch RMS Acceleration (rad/s^2)	0.3186 (-5.1%)	2.333 (-13.4%)
Roll RMS Acceleration (rad/s^2)	0.2678 (-23.0%)	2.103 (+3.5%)
Sudden Acceleration		
Vertical RMS Acceleration (m/s^2)	–	0.135 (-22%)
Pitch RMS Acceleration (rad/s^2)	–	0.719 (+1%)
Roll RMS Acceleration (rad/s^2)	–	0.367 (-14%)
Sudden Braking		
Vertical RMS Acceleration (m/s^2)	–	0.253 (-33.5%)
Pitch RMS Acceleration (rad/s^2)	–	1.347 (-24.6%)
Roll RMS Acceleration (rad/s^2)	–	0.419 (+4.1%)

MPC exhibits superior roll suppression, reducing roll acceleration by 23.7% on cross bumps, demonstrating its predictive optimization’s effectiveness in mitigating lateral load transfers. In contrast, Skyhook-LQR struggles with roll stability, particularly during braking, where roll RMS increases due to its reactive nature.

Skyhook-LQR provides strong vertical vibration damping, achieving a 33.5% reduction in vertical RMS during sudden braking and 21% on cross bumps, aligning with Skyhook’s effectiveness in absorbing high-frequency road disturbances. However, on smoother road conditions, vertical acceleration increases (+4.1%), indicating less adaptability to continuous disturbances.

Pitch control varies significantly. MPC achieves moderate but consistent pitch suppression, reducing pitch RMS by 5.1% on paved roads and 25.2% on single bumps. Skyhook-LQR performs best under braking conditions, achieving 24.6% pitch RMS reduction during sudden braking, making it more effective at longitudinal weight transfer management, whereas MPC ensures balanced pitch stability across all conditions.

Table 4.15: Summary of Control Performance Trade-offs

Feature	MPC	Integrated LQR Skyhook-
Vertical Acceleration	Strong on impact absorption, moderate on high-frequency damping	Strong in high-frequency damping, less adaptable to smooth roads
Roll Suppression	Strong (23.7% reduction)	Weak (roll RMS increases in braking)
Pitch Stability	Consistent across conditions	Strong in braking scenarios (24.6% reduction)
Predictive Capability	Proactive disturbance rejection	Reactive, lacks predictive control
Overall Ride Comfort	Balanced across conditions	Best for transient disturbances

MPC provides robust and predictive control across multiple driving conditions, excelling in roll and pitch suppression but at a higher computational cost. Skyhook-LQR is effective for sudden disturbances, particularly braking, but lacks predictive adaptability, leading to weaker roll suppression and inconsistent vertical control on smooth roads. The choice between these controllers depends on whether computational resources allow for real-time MPC execution or if a lower-cost, real-time deployable alternative is preferred.

4.7 Summary

This chapter presents a comprehensive evaluation of semi-active suspension control strategies through both simulation and real-vehicle experiments, focusing on the Integrated Skyhook-LQR controller and Model Predictive Control (MPC). The study systematically examines their performance under various road conditions, including single bumps, cross-set bumps, gravel roads, and sudden acceleration and braking scenarios.

The simulation study validates the effectiveness of both controllers in optimizing vertical, roll, and pitch motion, with MPC demonstrating superior multi-directional control by predicting and adjusting damping forces proactively. The real-vehicle experiments further assess the feasibility and performance of Integrated Skyhook-LQR, confirming its ability to mitigate vertical acceleration while exhibiting limitations in roll suppression and transient pitch stabilization.

The handling of damping constraints is a key comparison point, where MPC enforces actuator limits, dissipativity conditions, and rate constraints explicitly through QP optimization, ensuring strict compliance and optimal force distribution. In contrast, Integrated Skyhook-LQR relies on force saturation and velocity-based switching, offering computational efficiency but limited constraint enforcement.

The control performance analysis reveals that MPC provides a well-balanced improvement across all dynamic responses, particularly excelling in roll suppression (23.7%) and proactive disturbance rejection. The Integrated Skyhook-LQR controller is more effective in reducing high-frequency vertical oscillations but results in increased roll motion during braking due to its reactive nature.

Overall, MPC emerges as the optimal solution for multi-directional control, balancing ride comfort, handling stability, and constraint enforcement at the cost of computational complexity. Integrated Skyhook-LQR remains a practical, low-cost alternative for real-time applications, particularly for ride comfort optimization in embedded systems.

Chapter 5

Conclusions and Future Work

This research provided a comprehensive evaluation of semi-active suspension systems, emphasizing their advantages over traditional passive and fully active suspensions. The study explored the mathematical modelling and control strategies for continuously damping control (CDC) damper suspension systems and validated these methodologies through extensive simulation and real-world vehicle testing.

5.1 Conclusion

To validate the controller performance on passenger cars. A seven-degree-of-freedom (7-DOF) full-vehicle model was developed using Lagrange's equations, enabling a detailed representation of the vehicle's vertical, pitch, and roll dynamics. Two control strategies, Model Predictive Control (MPC) and an integrated Skyhook-Linear Quadratic Regulator (LQR) controller, were implemented and evaluated through high-fidelity simulations and real-world vehicle testing.

Simulation results demonstrated that MPC significantly improved ride comfort and stability by proactively adjusting damping forces, reducing vertical acceleration by 18.4% and roll acceleration by 23.7%. However, its high computational demand limited its real-time applicability. The integrated Skyhook-LQR controller, tested in a real-world vehicle experiment, effectively suppressed vertical oscillations, achieving a 33.5% reduction in vertical acceleration during braking and improving roll stability by 17.2% in lane-change maneuvers. Despite its lower computational complexity, Skyhook-LQR showed limitations in controlling pitch and roll compared to MPC.

Comparative analysis revealed that while MPC provides superior predictive control, ensuring better performance in all suspension axes, its real-time implementation remains a challenge. Conversely, Skyhook-LQR offers a computationally efficient solution that is well-suited for embedded systems, though with trade-offs in handling asymmetric disturbances. The findings highlight the necessity of balancing computational efficiency with control performance in semi-active suspension design, providing insights for future development.

5.2 Future Work

To further enhance semi-active suspension performance and bridge the gap between simulation and real-world implementation, several improvements and new research directions are proposed.

One key area for future work is the simulation implementation of an Integrated Skyhook Control Strategy and its comparison with MPC and passive suspension. The existing study evaluated only the integrated Skyhook-LQR control in real-world tests, and its simulation performance remains unexplored. Implementing this control strategy in a simulation environment using a CarSim-Simulink co-simulation framework will allow a fair comparison under identical test conditions. The evaluation will include standard road disturbances such as single-bump, offset-bump, and continuous rough road scenarios, analyzing metrics such as root mean square (RMS) acceleration, suspension deflection, and road-holding ability. Further, this will enable a direct comparison between simulation and experimental results, identifying discrepancies and refining the vehicle model to improve real-world prediction accuracy.

Another significant aspect is the real-world vehicle testing of MPC with enhanced computational resources. The high computational demand of MPC was a limiting factor in this study, restricting its real-time application. To address this, a high-performance ECU with a multi-core processor (e.g., an automotive-grade NVIDIA Jetson or a DSP-based real-time controller) will be used to evaluate the feasibility of real-time MPC deployment. A key enhancement in this implementation will be reducing computational delays by optimizing the quadratic programming (QP) solver and implementing parallel computing techniques. The real-world testing will follow the same experimental protocols used for Skyhook-LQR testing, including speed bump traversal, lane-change maneuvers, emergency braking, and rough road tests. The results will be compared against the Skyhook-LQR controller, providing a cross-comparison between real-world MPC and Skyhook-LQR performance in ride comfort, handling, and stability.

A crucial challenge encountered in this study was sensor data fluctuation, which introduced noise into control inputs, affecting suspension response accuracy. To address this, future designs should integrate signal conditioning techniques, such as RC low-pass filters or Kalman filtering, to improve data quality before feeding it into the control algorithm. The implementation will involve designing hardware-based analog filters with optimized cutoff frequencies to suppress high-frequency noise while preserving useful signal information. Additionally, a sensor fusion algorithm combining inertial measurement unit (IMU) and wheel height sensor data will be explored to enhance state estimation accuracy. This will be tested in both simulation and real-world vehicle experiments, quantifying improvements in control precision and response time.

By implementing these advancements, future research can significantly improve the real-time feasibility, adaptability, and computational efficiency of semi-active suspension control strategies. These developments will contribute to next-generation vehicle suspension systems, optimizing ride comfort, vehicle stability, and road handling across various driving conditions.

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