Assessing Climate Change Impacts and Associated Risks: Applications in Finance and Insurance

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The focus of this thesis is to investigate the application of sustainable and green investments in the finance and insurance industries, specifically addressing their relevance to climate change and resiliency. To develop a portfolio optimized for both financial returns and sustainability factors, Environmental, Social, and Governance (ESG) scores are integrated into the reward function of a conventional Reinforcement Learning model (a branch of Machine Learning models) in Chapter 2. This approach enables a more robust analysis of the impact of various ESG ratings published by major rating agencies on the coherence of investment strategies. The model addresses the widespread confusion arising from the high heterogeneity in published ESG ratings by treating it as a source of ambiguity (or Knightian uncertainty) and proposes four ESG ensemble strategies catering to investors with different risk and (smooth) ambiguity preference profiles. Additionally, a Double-Mean-Variance model is constructed to combine financial returns and ESG score objectives, define three investor types based on their ambiguity preferences, and develop novel ESG-modified Capital Asset Pricing Models to evaluate the resulting optimized portfolio performance.

In Chapter 3, the ESG score evaluation methods for large companies are expanded to assess sustainability of individual farmers' production in the context of climate change. We propose integrating agricultural sustainability factors into classical personal credit evaluation systems to create a sustainable credit score, called the Environmental, Social, Economics (ESE) score. This ESE score is then incorporated into theoretical joint liability models to derive optimal group size and individual-ESE score propositions. Moreover, the utility function of farmers is refined to a mean-variance form, accounting for the risk associated with expected profit. Simulation exercises are provided to examine the effects of different climatic conditions, offering a deeper understanding of the implications of incorporating ESE scores into the credit evaluation system.

Chapter 4 investigates strategic investments needed to mitigate transition risks, particularly focusing on sectors significantly impacted by the shift to a low-carbon economy. It emphasizes the importance of tailored sector-specific strategies and the role of government interventions, such as carbon taxes and subsidies, in shaping corporate behavior. In providing a multi-period framework, this chapter evaluates the economic and operational trade-offs companies face under four various decarbonization scenarios—immediate, quick, slow, and no transitions. The analysis provides practical insights for both policymakers and business leaders, demonstrating how regulatory frameworks and strategic investments can be aligned to manage transition risks while optimizing long-term sustainability effectively. The findings contribute to a deeper understanding of the economic impacts of regulatory

policies and offer a comprehensive framework to navigate the complexities of transitioning to a low-carbon economy.

Finally, Chapter 5 summarizes the thesis and outlines potential directions for further research.

Keywords: *Climate Risks, Sustainable Investment, ESG, Reinforcement Learning, Mean-Variance Utility, CAPM, Joint Liability, Transition Risk*

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Dedication

To my parents, Yabing Zhang and Qing Yan.

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Chapter 1

Introduction

1.1 Background and Literature Review

1.1.1 Climate Change and Sustainable Investing

Climate change is a phenomenon that refers to a long-term alteration of global temperatures, precipitation patterns, and sea levels. It is largely caused by human activities, such as burning fossil fuels and deforestation, which release greenhouse gases (GHG) into the atmosphere and trap heat, leading to a warming planet (Berrang-Ford et al. (2011)). Climate change is one of the most pressing issues of our time, with far-reaching impacts such as rising sea levels, extreme weather, biodiversity loss, and threats to human health. The urgency of the situation has sparked a global call for action to mitigate its potentially irreversible consequences. As a result, sustainable investments have become a focus of attention in both academic and industry circles. Broadly speaking, sustainable investments refer to investment strategies that prioritize environmental, social, and governance (ESG) factors, in addition to financial returns. Such investments are designed to support environmentally responsible companies, while divesting from industries with negative environmental impacts. The focus on sustainable investments is driven by the realization that climate change poses a significant threat to both the environment and the economy, and that investing in sustainable industries can help mitigate this risk. According to a recent study by the Global Sustainable Investment Alliance [1], the global sustainable investment market grew to $35.3 trillion in 2020, up 15% from 2018, indicating a growing interest in this type to investing.

[1] https://www.gsi-alliance.org/

An ESG score is used to assess how well a company manages its environmental and social impact, as well as how effective its governance practices are. ESG scores are typically based on a wide range of factors, including a company's carbon footprint, its labor practices, its board composition, and its policies on issues such as executive pay and political lobbying. The purpose of an ESG score is to provide investors with a way to evaluate a company's sustainability and social responsibility performance, and to identify potential risks and opportunities associated with these factors. Major ESG data providers, such as Bloomberg, FTSE, MSCI, Thomson Reuters, and Standard & Poor's, offer their own ESG score systems to evaluate companies' performance on ESG metrics. ESG scores have their roots in socially responsible investing, which emerged in the 1960s as a way for investors to align their investments with their own values. However, it was not until the early 2000s that ESG scores became more widely used by mainstream investors. Since then, the popularity of ESG investing has grown rapidly, with investors recognizing the potential for sustainable investments to deliver both financial returns and positive environmental and social impacts. Nowadays many investors consider ESG scores when making investment decisions to align their investments with their personal values and support companies that make positive contributions to society. ESG scores are now often used by investors and financial institutions to guide investment decisions and to determine which companies to include in sustainable investment portfolios. Numerous studies have shown a relationship between ESG and corporate financial performance (CFP). For instance, Friede et al. (2015a) reviewed around 2,200 papers and found that 90% of them showed a positive relationship between ESG and CFP. In a more recent meta-analysis of over 1,000 papers published between 2015 and 2020, Whelan et al. (2021) found that 58% of them showed a positive relationship, 8% showed a negative relationship, 13% showed no relationship, and 21% showed mixed results. However, various studies also have shown that different ESG rating agencies often disagree substantially on a company's ESG performance. According to another study by Friede et al. (2015b), there was a significant variation in ESG ratings across different agencies, with a correlation coefficient of only 0.61 between the two largest ESG rating agencies. Gibson Brandon et al. (2021) suggests that investors are increasingly using ESG ratings as a way to evaluate companies' sustainability performance, and disagreement in ESG ratings can lead to negative market outcomes for companies. One reason for the variations observed in ESG ratings among different agencies could be due to the different methodologies used by each agency, as discussed in Berg et al. (2022). Each agency may have its own unique approach to assessing ESG performance, which can lead to different ratings for the same company. For example, some agencies may place more emphasis on environmental factors, while others may focus more on social or governance factors. Another reason for the disagreement in ESG ratings could be due to differences in the quality and quantity of data used by each agency. Some agencies may rely on publicly

available data, while others may gather data directly from companies. This can lead to important differences in the accuracy and completeness of the data used by different agencies, which in turn can impact the ESG ratings to which they assign. Moreover, different ESG rating agencies may have different interpretations of what constitutes good or bad ESG performance. This can lead to different ratings for the same company, as the agencies may place different weights on various ESG factors. The disagreement in ESG ratings highlights an urgent need for greater transparency and standardization in ESG ratings, as well as increased awareness among investors of the potential implications of ESG rating disagreement.

In addition, numerous studies have been conducted on the application of ESG extensions in finance and in economics. Lydenberg (2013) describes in detail four different approaches currently adopted by institutional investors when considering ESG factors in their investment decisions. These approaches include the incorporation of internationally accepted ESG norms and standards, the use of industry-specific ESG ratings and rankings, the integration of ESG considerations into a stock valuation, and the identification of companies that address pressing societal needs, referred to as commonly impact investing. Henriksson et al. (2019) explore the incorporation of ESG factors into portfolio construction. The authors argue that ESG considerations are important for investors who wish to achieve sustainable and responsible long-term financial returns and that traditional financial analysis does not fully capture the impact of ESG factors on investment outcomes. García et al. (2019) propose a multi-objective approach for portfolio selection that allows investors to consider return and downside risk criteria, as well as ESG scores in their investment decision-making process. Pedersen et al. (2021) propose a theoretical framework in which a stock's ESG score plays a dual role in affecting investor preferences and providing information about firm's fundamentals. The authors use this framework to develop an ESG-efficient frontier that characterizes the highest attainable Sharpe ratio for each ESG level, leading to portfolios that satisfy the four-fund separation. Equilibrium asset prices are then determined by an ESG-adjusted capital asset pricing model. Using several large data sets, the authors compute the empirical ESG-efficient frontier and demonstrate the costs and benefits of responsible investing. Drempetic et al. (2020) use Thomson Reuters ASSET4 ESG ratings to examine the influence of firm size, resources available for providing ESG data, and the availability of a company's ESG data on sustainability performance. Yoo et al. (2021) study the effect of ESG performance on stock returns and volatility during the financial crisis resulting from the COVID-19 pandemic. The authors find that firms with higher sustainability activities performance experience lower volatility and higher returns than firms with lower sustainability activities performance. The study contributes to the literature by providing empirical evidence of the importance of sustainability activities

during financial crises such as the COVID-19 pandemic. Dai et al. (2022) investigate the incorporation of ESG considerations in investment decisions made by sovereign wealth funds (SWFs) in publicly listed corporations, and the subsequent evolution of target firms' ESG performance. Singhania and Saini (2022) conduct a cross-country benchmarking analysis to quantify ESG regulations, with a focus on the mandatory disclosure of non-financial information in European countries. Behl et al. (2022) examine the bidirectional causality and autoregression effects between ESG disclosures and firm value of Indian energy sector companies using a four-wave cross-lagged panel structural equation modeling.

Wang et al. (2022) provide references for ESG practice and sustainable corporate development in emerging countries. It contributes to the literature by providing evidence that Fintech development has a positive impact on corporate ESG performance, and this impact is more significant for firms with higher technological capabilities and greater financial development. The study emphasizes the importance of Fintech development in promoting sustainable finance and the need for policymakers to foster the development of Fintech in emerging markets. While these advancements highlight the growing role of Fintech in ESG performance, the integration of ESG factors into investment strategies presents its own set of challenges. Recent research has shown the potential of using machine learning techniques, such as deep learning and reinforcement learning (RL), to incorporate ESG scores into portfolio optimization strategies. However, there are still limitations to this approach, such as the lack of large-scale training data and the heterogeneity of ESG ratings. A study by Vo et al. (2019) used deep learning to incorporate ESG ratings into portfolio optimization for a small set of six companies. A more recent study by Maree and Omlin (2022) included financial returns, risk, and sustainability factors (via ESG scores) in their reward function and tested various RL agents. Nonetheless, all of these studies only use ESG scores from one rater, which fail to account for the uncertainty associated with the heterogeneity of ESG ratings. As such, there is still a need for further research to develop more robust and accurate models that can incorporate ESG factors into investment decisions.

Taken together, all of these studies highlight the growing importance of ESG considerations in investment decision-making and the impact of sustainable investments on financial performance of firms. As climate change continues to threaten our planet, investors have a crucial role to play in promoting sustainable and positive social and environmental impact through their investment decisions. Sustainable investment refers to the practice of investing in companies that prioritize sustainability and ESG principles. This approach aims at promoting positive environmental and social changes while also delivering financial returns for investors. The investments can be directed towards companies that develop eco-friendly products, prioritize renewable energy, and have lower carbon footprints, among other fac-

tors. This type of investment seeks not only to deliver financial returns but also to promote positive environmental and social changes. Clark et al. (2015) have shown that sustainable investment can lead to positive outcomes for both investors and society as a whole. In Eccles et al. (2014) and Alshehhi et al. (2018), companies that prioritize sustainability and ESG principles have been found to be more resilient in the face of environmental and social challenges, which can translate to better long-term financial performance. Additionally, investing in these companies can help to promote positive environmental and social changes. The need to consider sustainable investment under the pressure of climate change is critical. Climate change poses significant risks to both companies and investors, including physical risks such as damages incurred from extreme weather events, and transition risks such as potential impacts of new regulations and market changes. Investors who fail to consider sustainability and ESG factors may be exposed to these risks, while those who prioritize sustainability can potentially benefit from reduced risks and improved financial performance.

As a result, incorporating ESG scores into investment strategies has emerged as an important practice in sustainable investment. However, it is desirable to adopt a rigorous approach to the utilization of ESG scores and address inconsistencies arising from different ESG raters in order to establish an effective investment strategy. Therefore, soundly resolving the inconsistencies arising from ESG scores is a prerequisite for developing a sound investment strategy that aligns with sustainable investment principles. In Chapter 2, we will build a framework to provide some potential solutions to these inconsistencies and offer a structured approach to utilizing ESG scores effectively.

1.1.2 Sustainable Agriculture

Agriculture plays a crucial role in society by providing a dependable source of nourishment that is necessary for human survival. Moreover, it generates income and employment opportunities for rural communities and individuals through the food supply chain. However, as the demand for a higher agricultural productivity increases to feed a growing and more prosperous global population, it has led to various environmental and social challenges. For example, Campbell et al. (2017) mention that agriculture is a primary cause of deforestation, which refers to the large-scale removal of forests for land use, and biodiversity loss, which is the reduction in the variety of species within an ecosystem. Additionally, it accounts for around 90% of nitrogen and phosphorus inputs, as well as most of the pesticide chemical inputs, to the Earth's biogeochemical cycles from human activities. Moreover, it is responsible for 21% to 37% of anthropogenic GHG emissions and uses up 90% of the world's freshwater consumption, as pointed out in Change et al. (2007) and Giovannucci

et al. (2012). Besides these environmental concerns, many rural communities, who are heavily reliant on agriculture face social issues like poverty, malnutrition, and declining employment opportunities, even though agriculture as a sector has become increasingly productive over time, and hunger has decreased globally over time. As society's demand for food, fiber, and energy continues to rise, sustainable agriculture that is not only productive but also nutritionally sufficient, eco-friendly, and resilient is vital (Beltran-Peña et al. (2020)). As such, sustainable agriculture is explicitly included as one of the Sustainable Development Goals (SDGs), specifically as SDG 2.4.1, which all United Nations member countries ratified in 2015.

To ensure accountability for sustainable agriculture commitments and inform policy-making, consistent and transparent assessments are crucial. However, definitions of sustainable agriculture vary, and there are few quantitative assessments available in the literature. There is a growing consensus that sustainable agriculture should consider its impacts on the environmental, economic, and social pillars of sustainability, as pointed out by Zhang et al. (2021) and Rivera-Ferre (2008). Various frameworks and indicators have been developed to assess the sustainability of food systems, but few studies have focused on national-level impacts on all three pillars and established targets while analyzing synergies and trade-offs of achieving these targets in a comprehensive manner. Low data availability also affects many agriculture-related indicators, hindering effective trend assessments and the development of win-win solutions.

1.1.3 Agricultural Joint Liability Contracts

Joint liability is a concept where multiple parties share equal responsibility and liability for a particular debt or obligation. This means that if one party fails to fulfill its obligations, the other parties are responsible for fulfilling the debt or obligation. In agriculture, joint liability is often applied in the context of group lending, where a group of farmers jointly apply for a loan from a lender. In this arrangement, each member of the group is responsible for the entire loan amount, not just their individual portion of the loan. This creates an incentive for members of the group to ensure that everyone fulfills their loan obligations, as failure to do so can result in the entire group being held responsible for the debt.

The concept of joint liability has been used in various forms, including informal lending arrangements among friends and family. However, it was first formalized in the microfinance industry by the Grameen Bank in Bangladesh in the 1970s, mentioned in Yunus (2007). Joint liability has been proven to be effective in reducing default risk and promoting loan repayment rates. For instance, research conducted by Ghatak and Guinnane

(1999) demonstrates that joint liability groups experience significantly lower default rates than individual borrowers. Moreover, Khandker et al. (1998) found that group-based lending in Bangladesh increased loans given to poor borrowers and lowered the cost of credit delivery. The success of microfinance, which is an important tool for poverty reduction and technology adoption, is largely attributed to factors such as group lending. Joint liability contracts reduce the risk associated with lending to low-income individuals who may have limited collateral or credit history, by leveraging social networks and peer pressure to ensure repayment (Conning and Udry (2007)). Research on joint liability has explored various aspects of the contract, including its effectiveness in reducing default rates (Burgess and Pande (2005)), the optimal group size (Gine and Karlan (2006)), and the impact of group dynamics on repayment behavior (Karlan and Zinman (2009)). The use of joint liability in microfinance has been shown to increase credit access for low-income individuals and households, especially in developing countries (Armendáriz and Morduch (2010)). Joint liability group lending has been particularly promoted as a way of enhancing community welfare, reducing default rates, and addressing the drawbacks of conventional individual lending. However, more recent studies, such as Madajewicz (2011), question the superiority of joint liability group lending over individual lending. The size of the group, the nature of the borrowers, and the level of punishment strategies imposed by group members are important factors to consider when deciding between group and individual lending. Rezaei et al. (2017) develop a model to determine how group size affects optimal group-lending contracts. They find that joint liability contracts are feasible under a smaller set of parameter values than individual liability contracts. Joint liability has positive effects on the borrowers' repayment amount and welfare, and group size should increase with project risk. The authors also analyze the effects of partial joint liability, less severe punishment, and project correlation on the feasibility and characteristics of joint liability contracts. They find that a partial joint liability overall has a negative effect on the borrowers' repayment amount and welfare, but it can increase the loan ceiling of joint liability when collusion is not as likely, or when borrowers have high discount factors. A less severe punishment decreases the loan ceiling of joint liability, but larger groups can offset these negative effects.

After reviewing most of the existing literature on joint liability, it leaves a distinct impression to us that there are certain limitations in the studies conducted thus far. Firstly, a large proportion of studies tend to place a significant emphasis on the expected returns that can be obtained through this mechanism, often using them as the primary objective. However, there is frequently inadequate consideration given to the risks that are associated with joint liability. This shortcoming can result in a disproportionate focus on short-term gains, at the expense of longer-term consequences and potentially negative outcomes associated with this approach. This may result in an underestimation of risks,

leading to increased risk-taking behavior and potentially negative consequences for borrowers, lenders, and other stakeholders involved in the joint liability lending arrangement. Secondly, the literature on modeling a project success probability often fixes the probability parameter as a constant level throughout the analysis, such as the model setting in Rezaei et al. (2017). This practice does not account for potential variability or changes in the success probability that may arise due to shifts in the project's circumstances. As a result, this approach can lead to inaccurate conclusions about the project's actual likelihood of success. To address this limitation, our model in Chapter 3 will integrate a valuation score that considers the variability of project success probability. This modification will produce a more realistic and precise representation of the factors that can influence project outcomes, enabling us to more effectively evaluate the impact of various factors on the project's success. This enhanced analysis will help us make better-informed decisions regarding resource allocation and risk management. Furthermore, as the world focuses on sustainable development, sustainable agricultural practices have attracted much attention among policymakers, researchers, and industry stakeholders. This emphasis on sustainability is particularly relevant in the context of agriculture, where the potential for sustainable practices can have a significant impact on the future of agricultural projects. Therefore, Chapter 3 develops a joint liability framework that considers sustainability in the face of climate change, which will contribute to the ongoing discussion on the importance of sustainability in agriculture. This framework will provide insights to guide future agricultural policies and practices towards more sustainable outcomes. Specifically, by incorporating the concept of sustainability into the analysis of joint liability framework, we seek to better understand the role that joint liability can play in promoting sustainable agricultural practices.

1.1.4 Climate Risks

Climate risk includes physical and transition risks. Some studies also highlight liability risk as an additional aspect of this risk. Physical risks encompass both acute events, such as floods and hurricanes, and chronic shifts, like gradual temperature increases and sea-level rise. Venturini (2022) categorizes physical risks as those resulting in direct disruptions to operations, impacting infrastructure and supply chains. Empirical studies often employ top-down models that use macroeconomic data to simulate the broad impacts of these risks, such as the effects of coastal flooding on real estate values. Meanwhile, bottom-up models focus on firm-specific data, assessing the geographic and operational vulnerability of individual companies to climate events. For example, natural disaster datasets provide historical records of physical damages, allowing researchers to evaluate the impact of

extreme weather events on asset values. Forward-looking methodologies, including stress tests and scenario analysis, are increasingly integrated into models to predict long-term impacts. These approaches are crucial for assessing how chronic physical changes, such as rising temperatures, will influence economic outcomes over time (Campiglio et al. (2023)). However, translating these risks into financial models remains challenging to this date due to the unpredictable nature of extreme events and the difficulty in capturing their long-term economic effects. This highlights a need for improved models that incorporate geospatial data and firm-level exposure to better understand the financial implications of physical risks (Caselli and Figueira (2020)).

Transition risks focus on economic and structural transformations necessary to achieve a low-carbon economy. Such risks arise as governments implement policies, such as carbon taxes or renewable energy incentives, which impact businesses reliant on fossil fuels (Semieniuk et al. (2021)). Research shows that these risks are particularly significant for high-emission sectors, such as fossil fuel extraction, heavy manufacturing, and transportation. Companies in these industries face increasing pressure to innovate and adapt to evolving sustainability standards and consumer expectations, or risk losing competitiveness and market share (Lazard (2020)). Transition risks are typically modeled by using a combination of macroeconomic simulations and firm-level analyses. For instance, a scenario analysis allows researchers to project how various regulatory measures, such as emission caps, could affect firms' financial performance. These studies highlight the potential for a significant asset revaluation in carbon-intensive industries, particularly when policies are rapidly implemented (Caselli and Figueira (2020)). Moreover, sectors like steel production and aviation are particularly vulnerable as low-carbon alternatives are still emerging, and the required technology is not yet economically fully viable. These industries face substantial costs related to adopting new technologies or risk incurring penalties for non-compliance with emissions standards (Dietz et al. (2020)). The pace and unevenness of adaptation across industries are crucial to understanding transition risks. While some sectors, such as power generation, have made strides with renewable energy adoption, others are lagging due to technological and economic challenges. This disparity, combined with unpredictable policy developments, complicates long-term planning for companies (Holden et al. (2018)). Such policy uncertainty poses a risk to firms, as future regulations could impose unforeseen operational constraints, affecting financial stability and credit risk for high-emission sectors.

The systemic implications of transition risks are increasingly recognized in the literature. Rapid policy implementations, such as carbon taxes, can lead to devaluation of high-carbon assets and create financial instability. Campiglio et al. (2022) emphasize that, in the absence of effective adaptation strategies, such risks could propagate through the

financial system, potentially causing broader economic disruptions—referred to as "Green Swan" events. In this context, firms exposed to high regulatory uncertainty may face stranded assets, where stranded assets are defined as investments that have become obsolete or non-performing due to regulatory changes, market shifts, or environmental concerns, reduced market valuations, and financing challenges (Bolton and Kacperczyk (2021b)).

Despite the progress made in understanding these risks, existing models and methods still have important gaps. Most notably, the unpredictable nature of future regulations and the pace of technological advancements make it difficult to create accurate quantitative models for transition risks. Current models often lack the depth needed to accurately capture the complexities of these risks, especially given the unpredictable nature of regulatory changes and the multifaceted impacts of physical events. Building analytical frameworks that can integrate detailed data on policy scenarios and technological pathways of each sector is essential. Such frameworks would provide a more comprehensive assessment of the financial implications of climate risks, enabling firms, investors, and policymakers to make informed decisions and effectively manage these evolving challenges.

1.2 Objectives and Outline of the Thesis

The main objective of this thesis is to investigate the utilization of sustainable and environmentally friendly investments within the finance and insurance sectors, with a specific emphasis on climate change. This thesis firstly presents a portfolio selection approach that integrates financial returns and sustainability criteria (through an ESG score), by utilizing a RL method. At the same time, we also propose four strategies to deal with the heterogeneity problem of different ESG raters. Related to the insurance sector, we concentrate on sustainable joint liability agreements for individual agricultural borrowers to devise an assessment system that reflects the economic, social, and environmental features of these borrowers, and subsequently integrate this score into our analytical frameworks. Additionally, the thesis extends the research on climate risk by analyzing strategic investments aimed at mitigating transition risks. This part of the thesis evaluates the effectiveness of various policy and investment strategies, such as carbon taxes and technology subsidies, in a dynamic multi-period setting that accounts for different decarbonization scenarios. This approach offers insights into optimizing investment strategies to align with regulatory and technological changes, ultimately promoting a smooth and effective transition to a low-carbon economy.

Specifically, the objective of Chapter 2 is to propose an optimized portfolio that incorporates both financial returns and sustainability factors. To achieve this goal, we propose

explicit incorporation of ESG scores in the reward function of a conventional RL model. The model is used to analyze the effects of various ESG ratings published by major rating agencies on the coherence of investment strategies more robustly. Thus, this chapter addresses the widespread confusion arising from high heterogeneity in published ESG ratings by treating it as a source of ambiguity, and proposes four ESG ensemble strategies to cater to investors' different risk and ambiguity preference profiles. We also use a Double-Mean-Variance model to combine the objectives of financial returns and ESG scores. Three types of investors are defined with respect to their ambiguity preferences, and novel ESG-modified Capital Asset Pricing Models are constructed to evaluate the performance of the resulting optimized portfolio. Overall, this chapter proposes a new approach to creating an optimized portfolio that balances financial returns and sustainability factors. These findings can help investors make informed decisions by providing a more robust analysis of the impact of ESG scores on investment strategies. The proposed ESG ensemble strategies can cater to investors' different risk and ambiguity preferences, while the novel ESG-modified Capital Asset Pricing Models can be used to evaluate the performance of the optimized portfolio when the ESG factor is taken into account along with other traditional factors. In the future, empirical results to illustrate the efficacy of the ESG-modified CAPM will be provided and compared with those obtained from the traditional CAPM by using the appropriate set of data.

In Chapter 3, we explore the application of ESG score evaluation methods and ideas for large companies to a credit evaluation system of individual farmers, with the objective of assessing the sustainability of farmers' production in the presence of climate change. To achieve this objective, we propose the incorporation of factors that reflect agricultural sustainability into classical personal credit evaluation systems to form a sustainable credit score, called the Environmental, Social, Economics (ESE) score. Additionally, this chapter also seeks to incorporate this ESE score into theoretical joint liability models and derive a set of propositions about the optimal individual-ESE score from a group size of two to a group size of n. Furthermore, the utility function is refined explicitly to a mean-variance form, which incorporates risk associated with an expected profit into the utility function. In addition, this chapter also conducts a further analysis of mean-variance utility function of farmers in the context of joint liability models. A set of simulation exercises on the effects of different climatic conditions is provided. Lastly, this chapter provides a framework for assessing the sustainability of farmers' production in the context of climate change, which has important implications for the agricultural industry and the purpose of policy decision-making.

Chapter 4 examines strategic investments needed to mitigate transition risks, focusing on industries most affected by shifts to a low-carbon economy. The chapter emphasizes

the necessity of sector-specific strategies and the role of government interventions, such as carbon taxes and technology subsidies, in influencing firm's behavior and its investment decision-making processes. By developing a dynamic multi-period analytical framework, this chapter assesses the impact of different decarbonization scenarios—immediate, quick, slow, and no decarbonization—on firm profitability and financial performance. This analytical framework evaluates how firms balance immediate costs with long-term benefits, highlighting that aggressive early investments often yield the highest short-term gains for the firm but may not always be sustainable long-term without gradual and well-planned transitions. The results provide valuable insights into the trade-offs firms have to face when they navigate regulatory and technological shifts. The findings reported in this chapter suggest that while immediate decarbonization can generate early efficiency gains for the firm, slower and more deliberate approaches often lead to higher long-term profitability by spreading costs over time. This analysis offers a strategic framework for both policymakers and business leaders by demonstrating the importance of aligning investment strategies with regulatory environments to manage transition risks effectively. Moreover, the chapter also highlights an important point that a balanced mix of assertive taxation policies and targeted subsidies can enhance the effectiveness of government interventions by motivating firms to adopt sustainable practices while mitigating financial risks.

Finally, it is necessasry to note that Chapters 2 through 4 are designed to stand alone, as each chapter represents a core content of a separate but related theme on the subject. Although efforts have been made to ensure consistency in notation across the thesis, some variations may still be present. Readers are therefore advised to interpret the notations within each chapter individually.

Chapter 2

Navigating Uncertainty in ESG Investing

2.1 Introduction

The integration of Environmental, Social, and Governance (ESG) factors into capital markets represents a significant evolution in financial analysis and investment strategies. As global markets increasingly recognize the importance of sustainable practices, ESG factors have become critical indicators in evaluating the long-term viability and ethical positioning of enterprises. However, the assimilation of ESG criteria into capital markets is not without its complexities. A primary concern is the inconsistency and perceived unreliability of ESG data. Empirical evidence, such as that provided by Berg et al. (2022), indicates notable discrepancies among major ESG rating agencies, with sample correlation coefficients as low as 0.54 on average. This contrasts sharply with the high consistency found in credit ratings by established firms like Moody's and Standard & Poor's where sample autocorrelation coefficients are 0.92 on average. Such heterogeneity presents challenges for investors aiming at balancing both financial and ESG-centric return objectives. This prevailing heterogeneity around ESG ratings inevitably leads to a widespread confusion among investors who are striving to achieve both financial and ESG factor return objectives.

There are varying perspectives on how this confusion impacts investors. Some believe the ambiguity could allow investors to play a more active role in their asset decisions. Conversely, others posit that the absence of ambiguity could result in a complete integration of ESG factors into the market, potentially diminishing the profitability of trading strategies. Efforts to tackle this challenge are underway, exemplified by initiatives such as the ongoing

research conducted by the Aggregate Confusion Project within MIT's Sloan Sustainability Initiative.

The objective of this chapter is to develop methods for asset allocation for investors amidst the prevalent ESG rating confusion. Firstly, we incorporate the ESG scores explicitly in the reward function of a conventional reinforcement learning (RL) model to analyze the effect of widespread confusion caused by the heterogeneity in the ESG ratings on the coherence of investment strategies. In response to this confusion, the approach undertaken in this chapter does not rely on just one ESG rating system but introduces several ensemble strategies tailored to varying investor profiles regarding risk and ambiguity preferences.

Next, we present a Double-Mean-Variance (DMV) model to categorize investors based on their stance on ESG and their comfort level with rating uncertainties. The three types of investors are (i) those who are indifferent to ESG, (ii) those who prefer ESG but are not affected by uncertainty with respect to the ESG scores, and (iii) those who prefer ESG and are affected by uncertainty with respect to the ESG scores. In the calibration exercise, we propose a refined RL model by setting its reward function to be the specified DMV model under different sources of uncertainty associated with ESG ratings. Lastly, the study introduces an ESG-modified Capital Asset Pricing Model (CAPM) to assess the performance of the optimized portfolio for different types of ESG-focused investors.

The remaining parts of this chapter are organized as follows. Section 2.2 confirms the observed heterogeneity in ESG ratings, using samples from four prominent rating agencies. The implications of these discrepancies on portfolio selection are discussed, adopting an RL framework for calibration. In Section 2.3, instead of a singular focus on one agency's ESG ratings, we introduce strategies to amalgamate ratings from the four key agencies. This approach aims at accommodating investors' varying risk tolerance and perspectives on ESG rating ambiguity, more precisely, uncertainty. In Section 2.4, we showcase an ESG-focused portfolio model that integrates MV preferences at the same time. We detail a calibration exercise, where ESG scores are directly infused into the RL model's reward function, utilizing the outlined DMV model. Section 2.5 builds upon the DMV model's findings and designs ESG-adapted CAPMs. These cater to investors prioritizing ESG, considering both the presence and absence of uncertainties in ESG ratings. This section proposes a refined CAPM framework to evaluate the efficacy of the resulting optimized portfolios. Section 2.6 provides some concluding remarks for this chapter.

2.2 Heterogeneity in ESG ratings

To underscore the contribution of our analysis in this chapter, we begin by revisiting the results presented by Berg et al. (2022). The ESG data used in our study are acquired from four major rating agencies: SA (Sustainalytics), RobecoSAM (S&P Global), Asset4 (Refinitiv), and MSCI. The data from these rating agencies are downloaded from the Thomson Reuters Eikon database (for Asset4) and Bloomberg (for RobecoSAM, Sustainalytics, and MSCI scores) database respectively. Although the majority of our data originates from 2020, any unavailable updates for that year default to the 2019 data. Additionally, our analysis integrates ESG scores for the Dow Jones 30 stocks derived from the aforementioned raters.

ESG rating agencies can differ in terms of their sample coverage and rating scale. In particular, Asset4, Sustainalytics, and RobecoSAM apply a scale from 0 to 100, MSCI uses a seven-tier rating scale from the best (AAA) to the worst (CCC). For compatibility, MSCI's scale is transformed into a $0 - 100$ format by dividing the scale into seven equal intervals and assigning the average value of each segment to represent the respective grades.

Table 2.1 shows the sample correlation of ESG ratings based on Pearson Correlation Coefficients, echoing the findings of Berg et al. (2022). The highest sample correlation coefficient of the ESG ratings is only 0.504. There are even several pairs of ratings that have negative coefficients. This leads to the prevailing issue of ESG confusion. See Tyler et al. (2024) for the use of various correlation and distance measures on corporate ESG ratings and the use of clustering techniques to identify groups of firms in the sample with similar ESG profiles.

	RebecoSAM	**SA**	**MSCI**	**Asset4**
RebecoSAM	1.0000	-0.1591	0.4153	0.5041
SA	-0.1591	1.0000	-0.3387	0.1826
MSCI	0.4153	-0.3387	1.0000	0.3139
Asset4	0.5041	0.1826	0.3139	1.0000

Table 2.1: The Sample Correlation Matrix of the Four ESG Raters

2.2.1 Heterogeneity of ESG raters in portfolio selection

In order to have a better understanding of the impacts of the reported heterogeneity of the ESG ratings on investors' asset allocation, we calibrate a portfolio selection process by

adopting an explicitly RL framework.

Brief introduction to RL

RL serves as a pivotal paradigm for training agents to make sequential decisions in dynamic environments, as explained in Sutton and Barto (2018) and recently reviewed in Hambly et al. (2023). Specifically, RL agents interact with their environment, receiving observations of the current state and rewards or penalties for their actions. The agents then learn to choose actions that maximize the expected cumulative reward, using trial-and-error exploration (of uncharted territory) and exploitation (of current knowledge), as highlighted in Kaelbling et al. (1996). This optimization process is crucial in scenarios where a sequential decision-making process takes place. As a result, RL algorithms can be used to explore different investment strategies and learn which ones are effective and which ones are not. RL learns optimal portfolio allocation strategies by continuously interacting with market data and making decisions over time. This approach accounts for the evolving nature of financial markets, where historical patterns may not always hold. RL's capacity to balance exploration and exploitation allows it to uncover potentially profitable investment opportunities while minimizing risk. Several prior studies have found that RL algorithms can outperform traditional investment strategies, particularly in volatile or complex markets. For example, Jiang et al. (2017) found that an RL algorithm was able to outperform traditional portfolio selection strategies with a high commission rate of 0.25% in the backtests. Li et al. (2021) present a FinRL-Podracer framework to accelerate the development pipeline of deep reinforcement learning (DRL)-driven trading strategy and to improve both trading performance and training efficiency. A foundational concept in RL is known as Markov Decision Process (MDP), which provides a framework for modeling decision-making in a stochastic environment. The value of each state within an MDP is ascertained through a Bellman equation, which is a fundamental equation that governs a trade-off between immediate rewards and future rewards. The Bellman equation is expressed as:

$$V(s) = \max_{a \in A(s)} \sum_{s'} P\left(s' \mid s, a\right) \left(R\left(s, a, s'\right) + \gamma V\left(s'\right)\right) \tag{2.1}$$

Here, the symbol $V(s)$ represents the value of state s, while $P(s'|s,a)$ denotes the probability of transitioning from state s to state s' when action a is taken. The term $R(s,a,s')$ corresponds to the reward obtained by taking action a in state s and transitioning to state s'. The parameter γ, residing in the interval $[0,1]$, represents the discount factor that modulates the impact of future rewards, balancing the level of exploration and exploitation in

decision-making processes. The objective of the agent is to maximize the cumulative reward, which is accomplished by selecting the action a that yields a maximum value within the sum. The set $A(s)$ denotes all possible actions that can be taken in state s.

While Equation (2.1) is a general representation applicable to stochastic MDPs, deterministic MDPs simplify the transition probability distribution $P(s'|s,a)$ to unity, signifying that there is only one possible next state for a given action and state. This mathematical foundation lays the groundwork for various RL algorithms, enabling agents to learn optimal policies and make informed decisions in complex and uncertain environments. Through the elegant interplay of mathematical principles and computational strategies, RL facilitates the development of intelligent agents capable of solving diverse real-world problems. In the field of quantitative finance, algorithmic trading involves making real-time decisions about where to trade, at what price, and how much to trade in a volatile and intricate financial market. Using various economic data, an RL trading system creates a multifaceted model for automated trading. This can be challenging for human traders to replicate. As a result, RL is seen as having a potential advantage as a tool in quantitative finance.

The unique flexibility of RL algorithms lies in their adaptability. They can be tailored to include multiple constraints or objectives. For example, an algorithm could be trained to maximize returns while also minimizing risk, or incorporating specific ESG criteria into investment decisions. This level of customization opens up a new opportunity in portfolio management, where sustainability and profitability go hand in hand. This is the feature that we exploit in this chapter.

However, the current literature on the convergence of RL and ESG in portfolio management is still relatively sparse. While some studies have begun to incorporate ESG ratings using deep learning algorithms, they often focus on a narrow set of companies and rely on ESG scores from a singular source. For instance, Vo et al. (2019) firstly leveraged deep learning and incorporated ESG ratings into portfolio optimization by considering only six companies. Maree and Omlin (2022) not only included financial returns and risk (via a Sharpe ratio performance metric), and sustainability (via an ESG factor consideration) into their reward function, but they also experimented with several state-of-the-art RL agents, such as multi-agent deep deterministic policy gradients (MADDPG). This approach is problematic due to the noted inconsistencies in ESG ratings across different agencies. Basing an RL model on one source could lead to suboptimal investment decisions, given the discrepancies and potential biases in ratings.

Therefore, in this chapter, we propose novel methods that can effectively incorporate ESG factors into investment decisions in the presence of widespread confusion among the investors with respect to the ESG ratings. Such research would be instrumental in ensuring

that investment decisions align well with principles of sustainable finance and promote long-term sustainability in the economy.

Overview of FinRL framework

Creating an RL trading strategy can be challenging. The programming part is prone to errors and involves meticulous debugging. The development process includes tasks such as preparing market data, establishing a training environment, handling trading states, and evaluating trading performance through backtesting. In our empirical research, we employ a FinRL framework, introduced by Liu et al. (2020), to address the complexities of financial decision-making within dynamic market environments. FinRL stands as a robust and adaptable platform specifically designed for applying reinforcement learning techniques to financial scenarios. The framework is uniquely tailored to cater to the intricacies of financial time-series data, offering a versatile foundation for developing and testing trading strategies, portfolio management techniques, and risk mitigation approaches.

The FinRL package provides built-in functionalities to facilitate feature engineering, ensuring that the state variables in the RL framework are appropriately designed. These state features include OHLC prices, trading volumes, and various technical indicators such as Moving Average Convergence Divergence (MACD) and Relative Strength Index (RSI). These features are engineered to summarize historical information effectively, encapsulating dependencies in the current state. This approach supports the Markov property by ensuring that the state representation contains all necessary information for future decision-making, independent of earlier states. The dynamic interactions between the agent and the environment are managed within the `PortfolioEnv` environment provided by the FinRL framework. The environment is designed to handle state transitions based on the Markov assumption, where the next state depends only on the immediately previous state and the chosen action. By defining the state variables to include both the technical indicators and the portfolio-specific details (e.g., the account balance and the stock holdings), the environment ensures that the state transitions remain Markovian. This design simplifies the reinforcement learning process while maintaining the fidelity of the financial decision-making process.

The FinRL framework is structured as a three-layer system:

1. **Environment Layer**: At the base, this layer emulates real financial markets using historical data. It includes essential metrics such as closing prices, stock quantities, trading volume, and technical indicators, creating a realistic environment for the agent's interaction.

2. **Agent Layer**: This middle layer consists of RL algorithms that drive the agent's actions. Reward functions within this layer guide the agent's behavior as it navigates the market environment, enabling learning and decision-making through defined state and action spaces.

3. **Application Layer**: At the top level, this layer represents practical applications of RL, including stock trading, portfolio allocation, and cryptocurrency trading. Here, the trading strategy is mapped into the RL framework by specifying key components like the state space, action space, and reward function.

In the context of our research in this chapter, we detail each layer as follows:

On the **application layer**, we map an algorithmic trading strategy into the RL framework by specifying the state space, action space, and reward function.

State Space: The state space describes how the agent perceives the environment. A trading agent observes many features to make sequential decisions in an interactive market environment. In our calibration exercise, the states are set as follows:

- Balance $b_t \in \mathbb{R}_+$: account balance at the current time step t.
- Shares $\boldsymbol{k}_t \in \mathbb{Z}_+^n$: current shares for each asset, where n represents the number of stocks in the portfolio.
- Open-high-low-close (OHLC) prices $\boldsymbol{o}_t, \boldsymbol{h}_t, \boldsymbol{l}_t, \boldsymbol{p}_t \in \mathbb{R}_+^n$.
- Trading volume $\boldsymbol{v}_t \in \mathbb{R}_+^n$.
- Technical indicators: including Moving Average Convergence Divergence (MACD), and Relative Strength Index (RSI), which are momentum indicators widely used in momentum trading.

Action Space: The feasible set for actions in this RL framework consists of adjustments to portfolio holdings for each asset at each timestep. Specifically, the action space is defined as $a_t \in [-k, k]^n$, where k represents a maximum allowable trading proportion (e.g., as a fraction of the current holding or the account balance) and n is the number of assets in the portfolio. Actions are constrained by practical factors such as transaction costs, leverage limits, and liquidity considerations, ensuring that the trading strategy remains realistic and implementable. For example:

- $a_t = -1$: Sell all units of an asset.

- $a_t = 0$: Hold the current position.
- $a_t = 1$: Buy the maximum allowable units of an asset.

For instance, 'Buy 10 shares of AAPL' or 'Sell 10 shares of AAPL' correspond to actions of 10 or -10, respectively.

Reward Function: We update the reward function by adding a weighted ESG score of the portfolio in the RL framework. It is assumed that the managing agent's reward is a linearly weighted function of the expected return and the mean ESG score for the portfolio.

$$\text{Reward} = \sum_{i=1}^{n} \omega_i r_{i,t} + \alpha \sum_{i=1}^{n} \omega_i \text{ESG}_i, \tag{2.2}$$

where $r_{i,t}$ is a standardized return of the i^{th} stock at time t, ESG_i is a standardized ESG score of the i^{th} stock, and $\sum_{i=1}^{n} \omega_i = 1, \omega_i \in [0, 1]$. The parameter α determines the relative importance of financial returns versus ESG considerations in the reward function. In this model, α is set to 1 under the assumption that both the returns and the ESG scores are standardized, ensuring that they are on comparable scales. The returns are standardized by subtracting the mean and dividing by the standard deviation of the historical returns, while the ESG scores are transformed and standardized across their respective ranges (e.g., 0-100 or categorical ratings converted to numerical equivalents). This ensures both components contribute equally, facilitating a balanced optimization process of financial and sustainability objectives. The choice of $\alpha = 1$ reflects an equal weighting, making it a suitable baseline for most investor profiles. Variations in α (e.g., higher for the ESG-focused investors or lower for the return-maximization) yield alternative portfolio preferences, suggesting its flexibility for tailoring the strategies.

Also, this reward function can change the form according to the different objective requirements, such as changing the reward to the Mean-Varianfe framework. This reward function is adaptable and can be reconfigured to accommodate different objective requirements. For example, it could be modified to align with a Mean-Variance framework, allowing the agent to optimize for both return and risk alongside ESG criteria.

On the **Agent layer**, FinRL allows users to plug in and play with the standard RL algorithm. In particular, we select a Deep Deterministic Policy Gradient (DDPG) algorithm as the agent as the main agent for our FinRL-based studies. DDPG is a notable advancement in RL that operates within a continuous action space. It seeks to optimize the performance of deterministic policies by maximizing the objective function $J(\theta) = E_{s\sim p^{\pi}, a\sim\pi(\theta)}[R]$, where

p^π denotes the distribution of states and $\pi(\theta)$ represents the policy dictated by the parameter set θ. DDPG operates by modeling the action selection process ($\mu_\theta : S \mapsto A$) through an actor-network, allowing it to predict an optimal action based on the current state. Concurrently, the algorithm approximates the reward function, represented by $Q^\mu(s, a)$, using a critic network. The critic network in DDPG is a neural network that estimates the value of a stateaction pair (s, a) by predicting the expected cumulative reward when action a is taken in state s and the policy μ is followed thereafter. This score allows the agent to evaluate the quality of actions and iteratively refine its policy for improved decision-making over time.

A key insight within DDPG lies in the formalization of the gradient of the objective function for deterministic policies, which can be represented as:

$$\nabla_\theta J(\theta) = E_s \left[\nabla_\theta \mu_\theta(a \mid s) \nabla_a Q^\mu(s, a)|_{a=\mu_\theta(s)} \right] \tag{2.3}$$

where the actor-network, denoted as $\mu_\theta : S \mapsto A$, serves as a function approximator guiding action selection based on the current state s, producing an anticipated action a as per the policy parameter θ. This mapping allows the agent to translate states into corresponding actions. The critic network, represented by $Q^\mu(s, a)$, approximates the value of state-action pairs (s, a) under the policy μ by estimating the cumulative reward when taking action a in state s and subsequently following policy μ. The gradient of the objective function, $\nabla_\theta J(\theta)$, indicates how changes in the policy parameter θ influence the goal of maximizing expected cumulative rewards. The expectation operator E_s accounts for the average value over states, as dictated by the current policy μ_θ. The gradient $\nabla_\theta \mu_\theta(a \mid s)$ represents the impact of policy parameter adjustments on the actions forecasted by the actor-network for a given state s. Similarly, $\nabla_a Q^\mu(s, a)$ characterizes the effect of action changes on the estimated state-action value provided by the critic network. The expression $\nabla_\theta \mu_\theta(a \mid s) \nabla_a Q^\mu(s, a)| \, a = \mu_\theta(s)$ captures the combined influence of policy parameter and action modifications on the overall objective. This equation outlines the mechanism for adjusting the policy's parameters to maximize cumulative rewards. It involves the computation of the gradient of the actor network's output with respect to the policy's parameters, multiplied by the gradient of the critic network's output with respect to the action, evaluated at the action predicted by the actor-network.

In the context of financial applications, integrating DDPG within the FinRL framework empowers us to train intelligent agents capable of making well-informed trading decisions, optimizing portfolio allocations, and navigating the intricacies of financial markets while considering continuous action spaces and the underlying uncertainties.

The **Environment layer** in FinRL is responsible for observing current market information and translating that information into states of the MDP problem. The state variables can be categorized into the state of an agent and the state of the market. In our study, the state of the market includes the open-high-low-close prices and volume (OHLCV) and technical indicators (MACD and RSI); the state of an agent includes the account balance and the shares for each stock.

The following is an algorithmic representation (See Algorithm 1) that provides a detailed breakdown of the steps involved in RL with the Deep Deterministic Policy Gradient (DDPG) agent.

Algorithm 1 RL with DDPG Agent for Portfolio Optimization

Input:

State variables: Portfolio data (holdings, cash balance), market data (OHLC prices, technical indicators).

Hyperparameters: Discount factor γ, learning rates, noise parameters.

Custom-designed reward function (e.g., portfolio returns and metrics like ESG scores).

Replay buffer size, batch size, and number of episodes.

Initialization:

Initialize actor network $\mu(s|\theta^\mu)$ and critic network $Q(s, a|\theta^Q)$ with random weights.

Initialize target networks μ' and Q' as copies of the actor and critic networks.

Initialize replay buffer $\mathcal{D}$ to store experience tuples $(s_t, a_t, r_t, s_{t+1}, \text{done})$.

Procedure:

for each episode **do**

Reset the environment and initialize s_0.

for each timestep **do**

Select action:$a_t = \mu(s_t|\theta^\mu) + \text{noise}$

Execute a_t in the environment.

Observe reward r_t, next state s_{t+1}, and done flag.

Store transition $(s_t, a_t, r_t, s_{t+1}, \text{done})$ in $\mathcal{D}$.

Sample mini-batch of N transitions from $\mathcal{D}$ and update:

Critic:

$$y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'}))$$

Minimize Bellman error: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$

Actor: Update using policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N}\sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu}\mu(s|\theta^\mu)|_{s_i}$$

Periodically update target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$

$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

where τ is the target update rate (e.g., $\tau \in [0.001, 0.01]$).

end for

end for

Output: Trained actor network $\mu(s|\theta^\mu)$ for optimal portfolio allocation.

The process of training through RL consists of monitoring price fluctuations, making decisions (selling, holding, or purchasing a specific quantity of stock), and computing a reward. By interacting with the environment, the agent updates iteratively and eventually obtains a trading strategy to maximize the expected return. To calibrate the portfolio selection process by using an RL framework, we download daily data for DOW 30 from 06/30/2007 to 06/30/2022 by accessing the Wharton Research Data Services (WRDS).

For concreteness, we introduce a general flow chart of RL under the context of our study in Figure 2.1. This approach compares several key components:

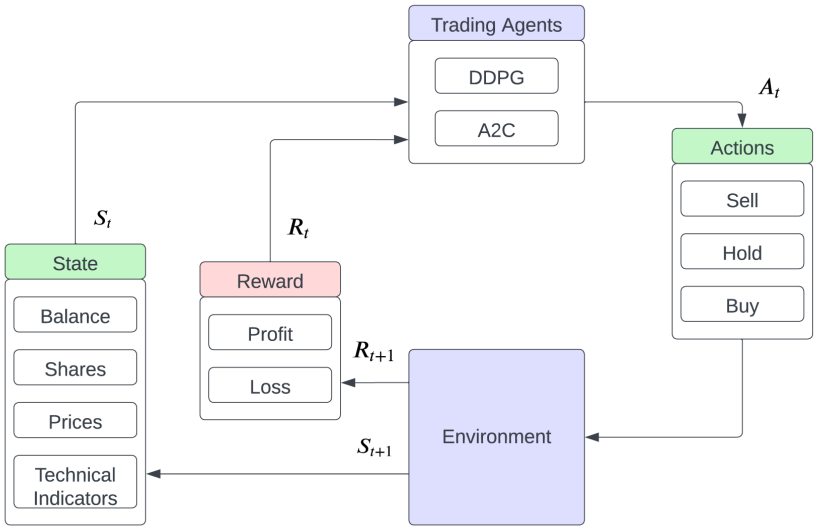


Figure 2.1: A general flow chart of FinRL applied in portfolio selection

Moving-window is commonly used for time-series data, and we choose the format of 3-year training data and 1-year testing data. Specifically, the first moving window covers 06/30/2007 to 06/30/2011, with 06/30/2007 to 06/30/2010 as the training set and 07/01/2010 to 06/30/2011 as the testing set. Subsequent moving windows follow the same rule. We explore a total of 12 distinct moving window combinations, referred to as period 1 to period 12, wherein RL frameworks utilizing different ESG raters are constructed.

Each period involves establishing the RL environment, determining states and actions, and modifying reward functions as previously described. Specifically, the reward function is updated through a linear combination of the portfolio return and the weighted ESG score from each ESG rater for the corresponding period. We compare the values of the reward

function, defined as a weighted combination of the portfolio market returns and the ESG scores, by varying the ESG ratings sourced from different raters over the same time period. The rankings of the reward function for the four raters across the 12 periods from 2011 to 2022 are visually depicted in Figure 2.2. The code is implemented in Python by utilizing the open-source FinRL framework created by Liu et al. (2020), which provides a well-calibrated set of hyperparameters optimized for financial market applications. As a primary objective of this research is to evaluate the incorporation of ESG factors into portfolio management, rather than to optimize the RL algorithm performance, no additional fine-tuning of hyperparameters is performed. The FinRL framework includes pre-tuned settings that have been tested extensively on various financial markets, ensuring robustness and efficiency for standard scenarios. These pre-configured parameters, along with the rolling-window approach used in training, are sufficient for the goals of this study.

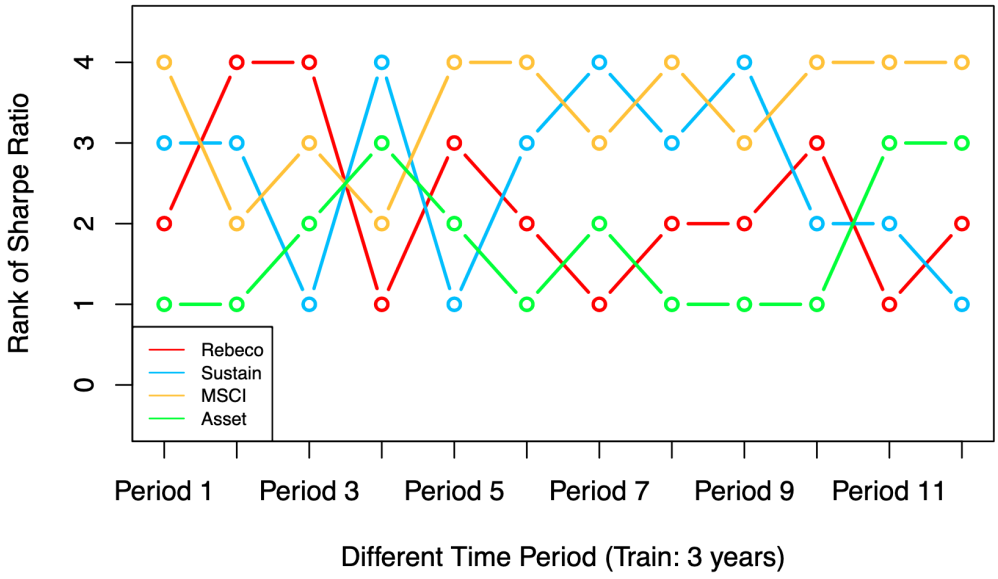


Figure 2.2: Ranking comparisons

The figure shows that there is no single rater that is capable of producing a portfolio that performs the best consistently over all of the 12 combinations of moving windows under consideration. Instead, we observe that the rankings appear to switch back and forth among the four raters for the time periods under study. This vividly illustrates the impacts of a widespread confusion among investors who are uncertain about which ESG rater to rely on to identify the best-performing portfolios from the ranked companies. Additionally, the

divergence in ESG ratings has a real potential to undermine the confidence of companies seeking to improve their ESG performance.

2.3 Addressing the heterogeneity of ESG raters

2.3.1 Data-driven ensemble strategies

ESG inconsistency poses substantial challenges, often stemming from issues related to data availability, quality, and bias. A key problem for this inconsistency is the lack of sufficient or accurate ESG metrics, as many datasets are incomplete or fail to cover the full spectrum of environmental, social, and governance factors. The quality of available data is another concern, with discrepancies arising from differences in reporting standards and methodologies across regions. Additionally, some ESG data may be published to meet specific objectives, potentially introducing bias in favor of certain agendas or stakeholders. For instance, ESG frameworks were introduced predominantly in developed countries, which may not adequately represent the challenges and priorities of emerging economies. These limitations can lead to inconsistent evaluations, and potentially reduce the reliability of ESG assessments.

In investigating factors for the reported divergence of ESG ratings, Berg et al. (2022) decompose the ratings into three sources: (1) 56% of the ranking heterogeneity can be traced back to purely measurement issues; (2) 38% of the ranking disagreement is attributed to scope, where different raters include different attributes; and (3) 6% of the rating differences is due to weight placed by different raters on individual components (E, S, and G) when they calculate their aggregate ESG scores for the companies. Furthermore, Berg et al. (2022) point out that there is even a divergence of ratings on verifiable facts from public records. Additionally, there exists a so-called 'rater effect', where a company rated highly on a particular attribute enjoys high ratings on its other attributes.

The question of interest to us is: *what are the main implications of high heterogeneity in ESG rankings of companies to investors in terms of their asset allocation?* It certainly does not imply that ESG ratings are so unreliable that they should not be used by investors as factors in forming their investment strategies. After all, a company's performance in general is known to be difficult to measure with any degree of accuracy. However, it does mean that investors need to acquire a better understanding of what the ESG ratings published by different rating agencies are measuring.

Ambiguity (also known as Knightian uncertainty in microeconomics literature) and risk are two related but distinct concepts in Decision Theory. Ambiguity refers to uncertainty

(or lack of clarity) with unknown odds, while risk refers to the potential for loss or failure with known odds. Investors can have multiple views when forming their investment strategies. In general, investors' views may vary in their level of optimism (or conservatism); as a result, these views may be associated with more or less favorable scenarios. However, it is impossible to know with certainty which views are more likely to be relatively more accurate, so investors must weigh the pros and cons of each view in order to arrive at their final decision on investment. In this chapter, ambiguity specifically refers to the uncertainty surrounding these different views.

In this chapter, investors are regarded to have incomplete information about ESG scores from different raters. They are not sure which one is right and what are the better weights of different ESG scores. In particular, each ESG rater can be regarded by investors as a 'view'. Hence, confusion about ESG ratings can be regarded as a type of ambiguity. In fact, each rater has its own evaluation system. It is hard for investors to figure out which ESG rater provides a more accurate view of the ESG score and it is also not easy for investors to determine the most effective way to combine or incorporate these different views.

Instead of attempting to identify an optimal universal ESG representative for any given time period, we take an agnostic approach in this chapter. We provide several strategies to ensemble the ESG scores based on investors' attitudes toward the ambiguity of ESG ratings. This involves considering methods that essentially form certain weights among different ESG raters. The assemble strategies that we propose in this chapter are:

Clustering centroid: The centroid of a group of points is a point that represents the center or average of the group. For example, in a data analysis context, the centroid of a cluster of data points might be used as a reference point to calculate the dissimilarity of each point to the centroid. For each company, we use Euclidean distance to measure a 'dissimilarity' between the ESG rater and the centroid. The centroid measures a minimum sum of the dissimilarity of the values in a dataset. After a trivial derivation, the centroid will reduce to the sample mean of the four raters. The centroid gives us an idea of where the 'center value' is located among the raters and this is an appropriate criterion if the investor takes all observations into consideration without losing any information. In other words, the centroid represents a decision-maker who places the same weight on those different views. However, the simple mean is known to be misleading when the data sample is skewed or contains outliers.

Median: The median is a measure of central tendency that is resistant to outliers or extreme values. Ambiguity-averse investors may prefer to use the median as their decision-making tool because it is a more stable and reliable measure of central tendency that is

less influenced by extreme views or opinions. This can help them deal with ambiguity and make more informed investment decisions. Thus, if the ratings of the investor's target company differ greatly among the rating agencies, the median is a more reliable metric in terms of its robustness.

Principal component analysis (PCA): PCA is a popular data-driven technique that is used to reduce the dimensionality of a dataset. It does this by transforming the data into a new coordinate system, in which the dimensions or variables are made to be orthogonal. The new dimensions are called principal components, and they are ordered by the amount of variation or information that they capture from the original data.

PCA is often used to decide on a representative from a set of choices because it can help to identify the underlying structure or patterns in the data. By reducing the dimensionality of the dataset, PCA can reveal the most important variables or dimensions that capture the majority of the variation in the data. This can be useful for identifying the key factors or characteristics that are driving the variation in the data, and for selecting a representative that captures the essence of the dataset as a whole.

Alpha-Maxmin: Ghirardato et al. (2004) characterize the ambiguity attitude by a linear combination of the worst view and the best view, which they term as an *α-maxmin expected utility* (α-MEU) form,

$$\alpha \inf_{\mathbb{Q}\in\mathcal{Q}} \mathbb{E}^{\mathbb{Q}}[U(X)] + (1-\alpha) \sup_{\mathbb{Q}\in\mathcal{Q}} \mathbb{E}^{\mathbb{Q}}[U(X)], \tag{2.4}$$

where U represents a Bernoulli utility function that captures the decision-maker's preferences over outcomes, typically reflecting risk attitudes such as risk aversion (e.g., $U(X) = \log(X)$ for a logarithmic utility or $U(X) = X^{1-\gamma}/(1-\gamma)$ for a constant relative risk aversion, where γ denotes a risk aversion parameter). The variable X denotes uncertain outcomes or payoffs, such as returns or wealth generated by a portfolio of assets, with its definition depending on the context (e.g., future cash flows or asset returns). The set $\mathcal{Q}$ is a collection of subjective probability measures or priors that represent the decision-maker's ambiguity about the true distribution of X, capturing a Knightian uncertainty (or ambiguity). This framework generalizes the α-maxmin rule of Hurwicz to settings of Knightian uncertainty, where a subjective perception of ambiguity is represented by $\mathcal{Q}$ and the parameter $\alpha \in [0, 1]$ captures the attitude toward ambiguity, balancing pessimism (worst-case scenarios) and optimism (best-case scenarios).

Unlike the first three metrics, which are purely data-driven, the Alpha-Maxmin metric combines a behavioral criterion on the part of the investors with a data-driven criterion. Also, unlike the median, Alpha-Maxmin takes into account the most extreme cases among

investors' ambiguous views. This means that Alpha-Maxmin considers not just the average or typical view or opinion of investors, but also the most optimistic and most pessimistic perspectives of investors. Therefore, investors who are more concerned about the likelihood of extreme outcomes may prefer to use Alpha-Mammin as their decision-making tool.

These four metrics will be used to form ensemble strategies by combining different investment styles to create a well-rounded portfolio that meets the investor's goals. This is acheived by incorporating investors' ambiguous views of ESG ratings in the RL framework given the prevalent heterogeneity of the ratings.

2.3.2 Steps to Improve ESG Measure Consistency

The high degree of inconsistencies in ESG measures provided by rating agencies like Bloomberg, Thomson Reuters, and Standard & Poor's can be attributed to their differences in scope, measurement methodologies, and the weight assigned by them to individual ESG factors. To address these issues and enhance the consistency of the ESG measures from the sources, several steps are possible:

1. Standardization of Metrics: Establishing a universally agreed-upon framework for ESG metrics is essential. Agencies should collaborate to define key indicators under Environmental, Social, and Governance categories. For example, standardized environmental metrics could include carbon emissions intensity, while social metrics could cover gender diversity rates. This would reduce variability caused by differing scopes across agencies.

2. Transparent Methodologies: ESG rating agencies should disclose their methodologies in greater detail, including how they collect data, weigh individual metrics, and calculate overall ESG scores. Increased transparency would allow investors to better understand and compare ratings across agencies.

3. Third-party Oversight: An independent body or consortium could be established to oversee and audit ESG rating methodologies. This body could provide certifications for agencies that adhere to standardized processes, similar to how financial auditing works under frameworks such as International Financial Reporting Standards (IFRS) or Generally Accepted Accounting Principles (GAAP).

4. Data Integration: Agencies could adopt ensemble techniques, as proposed in this study, to reconcile the divergent ESG scores. These techniques use a combination

of scores from multiple raters to create a more balanced and robust ESG rating, mitigating the impact of individual agency biases or methodological differences.

By implementing these steps, the financial industry can move closer to achieving greater consistency in ESG measures, fostering trust and facilitating informed decision-making by investors. Our findings further emphasize the need for such standardization, as the current inconsistencies not only create confusion but also limit the scalability of ESG investing.

2.4 ESG-related portfolio with mean-variance preference

In the previous section, we have treated the heterogeneity in ESG scores generated by different ESG raters as a form of ambiguity within the realm of sustainable investments. In this section, our initial focus is on a single category of ESG scores in the presence of a general notion of uncertainty, in order to allow us to explore the linkages between risk and uncertainty which was emphasized in recent studies such as Pástor et al. (2021) and Avramov et al. (2022).

We will return to the concept of ambiguity in the empirical analysis later on. Our current focus in this section is on unraveling distinct analytical ways in which the identical ESG score type influences portfolio performance through the lens of uncertainty in general. This approach underscores our commitment to providing a nuanced and in-depth understanding of the intricate dynamics at play within the realm of sustainable investments.

To frame our discussion, we start with a general analytical framework initially. Instead of formulating investors' preferences with an exponential utility (CARA) function as in Pástor et al. (2021) and Avramov et al. (2022), we use a mean-variance (MV) preference to formulate the utility function. Our analysis draws heavily from Maccheroni et al. (2013), which shows that the optimal investment strategy under the MV preference has an analytical closed-form expression, which provides a sharp separation for key factors of interest, such as expected return and risk. Most importantly, Maccheroni et al. (2013) show that for a von Neumann-Morgenstern expected utility maximizing investor with a utility function $u(\cdot)$ representing investor's attitude toward risk, and wealth a, who considers an investment project h, a linear Arrow-Pratt approximation of her certainty equivalent for an uncertain investment prospect is given by:

$$c\left(a+h,P\right)\approx a+\mathbb{E}_P(h)-\frac{1}{2}\gamma_u(a)\sigma_P^2(h) \tag{2.5}$$

where $c(\cdot)$ is a first-order certainty-equivalent measure and $\gamma_u(a) = -u''(a)/u'(a)$ is a parameter that links risk premium and volatility. In Equation (2.5), both $\mathbb{E}_P(\cdot)$ and $Var_P(\cdot)$ are taken with respect to a particular probabilistic model (denoted by P), which describes a stochastic nature of the investment problem.

This approximation leads to an MV model with a prospect f being evaluated at:

$$U(f) = \mathbb{E}(f) - \frac{\gamma_u(a)}{2}\sigma^2(f) \tag{2.6}$$

with $f = a + h$ and $\gamma = \gamma_u(a)$.

MV analysis is a crucially preliminary step in our investment strategy due to its direct relevance to the subsequent implementation of the Capital Asset Pricing Model (CAPM). By employing MV analysis, we aim at optimizing the risk-return profile of our investment portfolio. MV analysis allows us to evaluate the relationship between an asset's expected returns and its associated risk, thus enabling the identification of an efficient frontier of portfolios that offer the highest possible return for a given level of risk or the lowest possible risk for a targeted level of return. This efficient frontier forms the foundation for CAPM's principles, as it provides a range of diversified portfolios that theoretically maximize returns based on an investor's risk tolerance. Therefore, the judicious application of MV analysis not only refines the portfolio construction but also paves the way for an extension of the Modern Portfolio Theory (MPT), proposed by Markowitz (1952), through the subsequent utilization of CAPM's insights.

2.4.1 Model setting

With additional efforts, it is possible to derive our ensuing analytical results directly from the general framework set up above. However, we do not pursue this strategy in this chapter. Instead, we present a model of an ESG-related portfolio with MV preferences in the presence of general uncertainty in the sense of Pástor et al. (2021) and Avramov et al. (2022). More specifically, in a single-period economy, suppose that there is an agent who trades at time 0 and closes her position at time 1. Denote a random rate of return on the market portfolio as $\tilde{r}_M$, and a true and unobservable ESG score of the market portfolio as $\tilde{g}_M$. Let μ_f be a risk-free rate. We model the excess returns of the market and the ESG factor as follows:

$$Market: \quad \tilde{r}_M - \mu_f = \mu_M + \tilde{\epsilon}_M, \tag{2.7}$$

and

$$ESG: \quad \tilde{g}_M = \mu_g + \tilde{\varepsilon}_{g,M}, \tag{2.8}$$

where $\mathbb{E}[\tilde{r}_M] = \mu_M + \mu_f$ is an expected market return, $\mathbb{E}[\tilde{g}_M] = \mu_g$ is an expected value of the ESG-factor return, and $\tilde{\epsilon}_M$ and $\tilde{\epsilon}_{g,M}$ are zero-mean error terms. We also define the term $\sigma_M^2 = \boldsymbol{X}_M^T \boldsymbol{\Sigma}_M \boldsymbol{X}_M$ and $\sigma_g^2 = \boldsymbol{X}_M^T \boldsymbol{\Sigma}_{g,M} \boldsymbol{X}_M$ to respectively represent the variance of market returns and the variance of the ESG-factor returns respectively. Here, $\boldsymbol{X}_M$ represents a column vector of portfolio weights, and $\boldsymbol{\Sigma}_M$ and $\boldsymbol{\Sigma}_{g,M}$ represent the covariance matrix of market returns and covariance matrix of ESG-factor returns respectively.

Next, denote w as a weight of risky assets in a portfolio, i.e. the weight of the market portfolio. The parameters γ and θ represent the investor's attitude toward risk associated with the market return and uncertainty associated with the ESG-factor return respectively. The objective functions for the market and the ESG factor are expressed respectively as:

$$Market \quad Objective : w \cdot \mu_M + (1 - w)\mu_f - \frac{\gamma}{2} w^2 \sigma_M^2, \tag{2.9}$$

and

$$ESG \quad Objective : w \cdot \mu_g - \frac{\theta}{2} w^2 \sigma_g^2. \tag{2.10}$$

The total expected utility function leads to a linear combination of the expected MV utility function of the market return and the expected MV utility function of the ESG-factor return, which gives rise to a new model that we dub as a Double MV (DMV) model:

$$w \cdot \mu_M + (1 - w)\mu_f - \frac{\gamma}{2} w^2 \sigma_M^2 + b \left(w \cdot \mu_g - \frac{\theta}{2} w^2 \sigma_g^2 \right), \tag{2.11}$$

where $b > 0$ captures the relative taste of the investor between 'pecuniary' return and 'non-pecuniary' return.

Pecuniary return refers to a financial gain or loss from an investment, such as the change in the value of the investment or the income generated by the investment, such as dividends or interest. Pecuniary return is typically measured in monetary units, such as Dollars or Euros. Pecuniary return is an important consideration for investors because it represents the financial gain or loss from the investment. Investors typically want to maximize their pecuniary return in order to achieve their financial goals, such as building up wealth or generating income.

Non-pecuniary return, on the other hand, refers to non-monetary benefits or costs of an investment project, such as social, environmental, or ethical impacts of the investment. Non-pecuniary return is often referred to as 'externalities', as it represents the effects of the investment on parties that are not directly involved in the investment decision, which can also impact the overall performance and risk profile of the investment. For example, if

a company has poor environmental practices, it may face regulatory risks or reputational damages that could negatively impact its financial performance. Similarly, if a company has poor working conditions or engages in unethical practices, it may face reputational risks or consumer backlashes that could also impact its financial performance.

Therefore, by considering both pecuniary and non-pecuniary returns in the model, we allow the investor to make informed investment decisions that take into account a full range of potential risks, uncertainties, and rewards of the investment universe. It can also help investors align their investments better with their values and ethical principles. Next, we divide the value of b into three cases:

- If $b > 1$, the investors will place more emphasis on non-monetary gains.
- If $b < 1$, the investors will place more emphasis on monetary gains.
- If $b = 1$, the investors will be indifferent between monetary and non-monetary gains.

The optimal portfolio weights are obtained from the first necessary order condition of the underlying optimization program:

$$\mu_M - \mu_f - \gamma w \sigma_M^2 + b\left(\mu_g - \theta w \sigma_g^2\right) = 0 \tag{2.12}$$

as

$$w = \frac{\mu_M - \mu_f + b\mu_g}{\gamma\sigma_M^2 + b\theta\sigma_g^2} \tag{2.13}$$

This optimal portfolio weight can be rewritten as:

$$\begin{aligned}
\frac{\mu_M - \mu_f + b\mu_g}{\gamma\sigma_M^2 + b\theta\sigma_g^2} &= \frac{\mu_M - \mu_f + b\mu_g}{\gamma\sigma_M^2} \cdot \frac{\gamma\sigma_M^2}{\gamma\sigma_M^2 + b\theta\sigma_g^2} \\
&= \left[\frac{1}{\gamma}\frac{(\mu_M - \mu_f)}{\sigma_M^2} + \frac{1}{\gamma} b \frac{\mu_g}{\sigma_M^2}\right] \cdot \left[1 - \frac{b\theta\sigma_g^2}{\gamma\sigma_M^2 + b\theta\sigma_g^2}\right] \\
&= \frac{1}{\gamma}\frac{(\mu_M - \mu_f)}{\sigma_M^2} + \frac{1}{\gamma} b \frac{\mu_g}{\sigma_M^2} \\
&\quad - \frac{1}{\gamma}\frac{\mu_M - \mu_f + b\mu_g}{\sigma_M^2}\left(\frac{b\theta\sigma_g^2}{\gamma\sigma_M^2 + b\theta\sigma_g^2}\right)
\end{aligned} \tag{2.14}$$

From the last line of Equation (2.14), we obtain three terms:

- First Term $\frac{1}{\gamma}\frac{\left(\mu_M-\mu_f\right)}{\sigma_M^2}$: There is no parameter in this term which involves the ESG factor, and it serves as a benchmark case of ESG indifference.

- Second Term $\frac{1}{\gamma}b\frac{\mu_g}{\sigma_M^2}$: This term involves the return of the ESG factor, but it does not incorporate uncertainty with respect to the ESG rating. Thus, it serves as a benchmark case with ESG preferences when the ESG profile is known with certainty.

- Last Term $\frac{1}{\gamma}\frac{\mu_M-\mu_f+b\mu_g}{\sigma_M^2}\left(\frac{b\theta\sigma_g^2}{\gamma\sigma_M^2+b\theta\sigma_g^2}\right)$: This term captures the incremental effect of ESG uncertainty.

Next, we define three types of investors, which have ESG indifference (I), ESG preference with no uncertainty (N), and ESG preference with uncertainty (U).

- Type-I Investors: These investors do not consider ESG factors in their decision-making processes about their decisions. They are indifferent with respect to the impacts of their investments and focus solely on their profit considerations.

- Type-N Investors: These investors prioritize ESG factors and try to align their investments with their values. However, they do not face any uncertainty associated with the ESG scores. This implies that they solely factor in the expected portfolio ESG score and do not take the ESG score's variability into account.

- Type-U Investors: These investors also prioritize ESG factors, but they face uncertainty about the impacts of their investments on these factors. Consequently, these investors must grapple with the risk arising from variability in ESG scores. This complexity can render it more challenging for these investors to harmonize their investments with their values.

To derive a market premium, we set the first term in the above equation to 1 and derive the excess return of investors of type I. Similarly, letting the first two terms be 1 will lead to the formation of an excess return for investors of type N. Finally, the excess return for investors of type A is derived from the original weight form $\frac{\mu_m-\mu_f+b\mu_{g,m}}{\gamma\sigma_m^2+b\theta\sigma_g^2}=1$. After re-arranging terms in these formulas, the excess returns of the three types of investors can be expressed succinctly as:

- Type-I Investors:

$$\mu_M^I-\mu_f=\gamma\sigma_M^2 \tag{2.15}$$

- Type-N Investors:

$$\mu_M^N - \mu_f = \gamma {\sigma_M}^2 - b\mu_g \tag{2.16}$$

- Type-U Investors:

$$\mu_M^U - \mu_f = \gamma \sigma_M^2 + b\theta\sigma_g^2 - b\mu_g \tag{2.17}$$

Hence, the differences in expected returns between these three types of investors are given respectively by:

$$\mu_M^N - \mu_M^I = -b\mu_g < 0 \tag{2.18}$$

$$\mu_M^U - \mu_M^N = b\theta\sigma_g^2 > 0 \tag{2.19}$$

$$\mu_M^U - \mu_M^I = b\theta\sigma_g^2 - b\mu_{g,M} = b(\theta\sigma_g^2 - \mu_g) \tag{2.20}$$

If we assume that $b > 0$, then the no-uncertainty case is associated with a negative ESG incremental premium. Pástor et al. (2021) pointed out that sustainable investing produces a positive social impact by making firms greener and by shifting real investment toward green firms. In order to achieve the gains from ESG, investors are willing to sacrifice their returns. Green assets have low expected returns because investors enjoy holding them and also because green assets hedge climate risk.

In addition, it is evident from the second equation that the market premium increases with ESG uncertainty. The uncertainty of ESG scores contributes to additional risk for investors; so the required return should reflect the corresponding compensation for higher uncertainty. However, the sign of the incremental premium is indeterminate when we compare the ESG indifference case and the ESG preference with uncertainty case due to the conflicting forces emanating from the variance and the return of the ESG scores. This can be explained as follows: the investors who prefer non-monetary returns with ESG uncertainty will get compensation from the risks of the ESG uncertainty as well as take the loss from the non-monetary return.

To derive the portfolio variances for the three types of investors (who are ESG indifferent, without ESG uncertainty, and with ESG uncertainty), we substitute the respective expressions for the weights (w) into the general portfolio variance formula, $\sigma^2 = w^2 \cdot \sigma_M^2$, where w is the weight of the market portfolio and σ_M^2 is the variance of the market return. The weights for the three investor types are derived from the optimal portfolio weight decomposition in Equation (2.14):

- ESG Indifference (w^I): The weight corresponds to only the first term of Equation (2.14), $\frac{1}{\gamma}\frac{\mu_M-\mu_f}{\sigma_M^2}$. This reflects the benchmark case of ESG indifference, where the portfolio weight depends solely on the market return premium ($\mu_M - \mu_f$) relative to the market risk, with no consideration of any ESG factors.

- Without ESG Uncertainty $\left(w^N\right)$: The weight includes the first two terms of Equation (2.14):

$$\frac{1}{\gamma}\frac{\mu_M-\mu_f}{\sigma_M^2}+\frac{1}{\gamma}\frac{b\mu_g}{\sigma_M^2}.$$

  This formulation adds a deterministic ESG premium ($b\mu_g$) to the portfolio weight, reflecting an investor who accounts for the ESG factors but does not consider the uncertainty associated with them.

- With ESG Uncertainty $\left(w^U\right)$: The weight incorporates all three terms from Equation (2.14), or equivalently, uses the full form of Equation (2.13):

$$\frac{1}{\gamma}\frac{\mu_M-\mu_f+b\mu_g}{\sigma_M^2}\cdot\left[1-\frac{b\theta\sigma_g^2}{\gamma\sigma_M^2+b\theta\sigma_g^2}\right].$$

  This weight adjusts for both ESG premium and its associated uncertainty.

By substituting these weights into the variance formula, we obtain the portfolio variances for the three types of investors, reflecting their different attitudes toward ESG factors and uncertainty. Next, we derive the differences in the portfolio variances for each pair of the three investor types:

$$\sigma_M^N-\sigma_M^I=\frac{2\left(\mu_M-\mu_f\right)b\mu_g+b^2\mu_g^2}{\gamma^2\sigma_M^2}>0, \tag{2.21}$$

and

$$\sigma_M^U-\sigma_M^N=-\frac{\left(\mu_M-\mu_f+b\mu_g\right)^2}{\gamma^2{\sigma_M}^2}\cdot\left[\frac{2\gamma\sigma_M^2b\theta{\sigma_g}^2+b^2\theta^2\sigma_g^4}{\left(\gamma\sigma_M^2+b\theta\sigma_g^2\right)^2}\right]<0, \tag{2.22}$$

and

$$\sigma_M^U-\sigma_M^I=\frac{\left(\mu_M-\mu_f\right)^2\left[E_g-1\right]+\left[2\left(\mu_M-\mu_f\right)b\mu_g+b^2\mu_g^2\right]E_g}{\gamma^2\sigma_M^2}, \tag{2.23}$$

where $E_g=\frac{\left(\gamma\sigma_M^2\right)^2}{\left(\gamma\sigma_M^2+b\theta\sigma_g^2\right)^2}\in(0,1)$, which can be interpreted as a ratio of the risk of market returns to the overall risk after taking into account the degree of aversion.

The variance relationships between different investors are generally the reverse of the return relationships, with the exception of the indeterminate relationship between the ESG indifference case and the ESG preference with uncertainty case.

	Return	**Variance**	**Sharpe Ratio**
ESG Indifference	Large	Small	Large
Without Uncertainty	Small	Large	Small
With ESG Uncertainty	Around Large	Around Small	Around Large

Table 2.2: Ex-ante Relationships between Different Types of Investors

Thus, it is reasonable to expect the returns, variances, and Sharpe ratios for these three types of investors to have the following ex-ante relationships between the different types of investors (see Table 2.2). In Table 2.2, the phrases 'Around Large' and 'Around Small' are used with a slight abuse of notation to describe the nature of indeterminate relationships in expected returns between different investor types (Type-I and Type-U). For example, 'Around Large' refers to the situation where, due to ESG uncertainty, it is unclear whether Type-U investors will receive higher returns than Type-I investors. This indeterminacy arises from the earlier proof, which showed that the relationship between Type-I and Type-U investors depends on the compensation for uncertainty relative to the potential loss from ESG integration. The results in Table 2.2 reflect the trade-offs and uncertainties introduced by ESG considerations in portfolio decisions. For ESG-indifferent investors, the lack of constraints allows them to achieve high returns, low variance, and strong risk-adjusted performance (high Sharpe Ratios). In contrast, investors integrating ESG factors without uncertainty face reduced returns and diversification limitations, leading to higher variance and lower Sharpe Ratios. When ESG uncertainty is introduced, performance metrics become indeterminate: 'Around Large' and 'Around Small' highlight the fuzziness of the relationships among the return, variance, and Sharpe Ratio due to the ongoing trade-offs between potential compensation for managing uncertainty and the costs of ESG constraints. This also underscores the importance of addressing ESG rating inconsistencies in ensuring investors with better predictable outcomes.

2.4.2 Calibration Exercise

To assess the alignment between theoretical models and real-world portfolio selection outcomes, we conduct the following calibration exercises using the RL framework.

Data Source: The study utilizes a dataset covering the period from 06/30/2018 to 06/30/2021 for the training set and 07/01/2021 to 06/30/2022 for the testing set. In each calibration, we choose to use ESG data from the four major ESG raters separately. In addition, the four kinds of proposed ensemble ESG scores in Section 2.3 present an alternative ESG feature option for investors, and consequently, we conduct calibration exercises by incorporating the four suggested ensemble scores individually. This exploration allows us to comprehensively assess the viability and potential impact of these ensemble scores within our analytical framework. By considering these ensemble scores alongside individual ESG metrics, we endeavor to provide investors with a more nuanced understanding of their investment choices and their alignment with sustainable practices.

RL Framework: We maintain the configuration of the RL framework in accordance with the FinRL framework, as elaborated in Section 2.2. Our framework continues to comprise three distinct layers. Within the Agent layer, we still utilize the DDPG algorithm as the central agent for our FinRL-based investigations. In the bottom layer, the environment layer, we persist in simulating authentic financial markets, relying on historical data that encompasses various metrics, including closing prices, stock quantities, trading volume, and technical indicators.

Within the application layer, we integrate an algorithmic trading strategy into the RL framework by defining the state space, action space, and reward function. The state space remains inclusive of information such as account balance, current asset holdings, OHLC prices, trading volume, and technical indicators. The action space outlines the allowable actions through which the agent engages with the environment. Typically, an action is composed of three components: -1, 0, 1, where -1, 0, and 1 correspond to selling, holding, and purchasing one share, respectively. In terms of the reward function modification, the study explicitly incorporates the MV preference as a distinct reward function in the RL framework. This modified reward function is subsequently trained separately for the three types of investors we have specified earlier. To be specific, the reward function for each type of investor is set as:

- Type-I Investors:
$$w \cdot \mu_M + (1-w)\mu_f - \frac{\gamma}{2} w^2 \sigma_M^2 \tag{2.24}$$
- Type-N Investors:
$$w \cdot \mu_M + (1-w)\mu_f - \frac{\gamma}{2} w^2 \sigma_M^2 + bw \cdot \mu_g \tag{2.25}$$

	$b < 1 (b = 0.2)$		$b < 1 (b = 0.6)$		$b = 1$		$b > 1 (b = 1.4)$		$b > 1 (b = 1.8)$	
	Return	**SR**	**Return**	**SR**	**Return**	**SR**	**Return**	**SR**	**Return**	**SR**
ESG Indifference	8.63%	0.77	8.63%	0.77	8.63%	0.77	8.63%	0.77	8.63%	0.77
RebecoSAM (N)	9.52%	0.78	8.01%	0.70	7.99%	0.66	7.93%	0.65	7.25%	0.53
Sustain (N)	7.66%	0.63	7.63%	0.66	7.68%	0.60	7.63%	0.58	7.45%	0.54
MSCI (N)	8.04%	0.72	7.89%	0.69	7.21%	0.58	6.26%	0.53	7.00%	0.52
Asset4 (N)	9.64%	0.81	8.21%	0.72	8.23%	0.70	8.03%	0.67	7.45%	0.55
RebecoSAM (U)	9.56%	0.77	8.45%	0.74	7.61%	0.68	8.31%	0.72	7.38%	0.47
Sustain (U)	9.61%	0.80	8.78%	0.81	8.54%	0.77	7.73%	0.62	7.51%	0.53
MSCI (U)	9.99%	0.88	8.42%	0.74	7.87%	0.69	7.62%	0.62	7.40%	0.58
Asset4 (U)	9.23%	0.84	8.63%	0.75	8.33%	0.72	8.23%	0.64	7.33%	0.48

Table 2.3: Calibration Results for Four Individual ESG Raters

- Type-U Investors:

$$w \cdot \mu_M + (1-w)\mu_f - \frac{\gamma}{2} w^2 \sigma_M^2 + b\left(w \cdot \mu_g - \frac{\theta}{2} w^2 \sigma_g^2\right) \tag{2.26}$$

After constructing the layers, we proceed to train the process by using FinRL for each calibration iteration. This training involves monitoring price fluctuations, making decisions (such as selling, holding, or buying a specific quantity of stock), and determining rewards based on various investor profiles. The outcomes generated by the models, for the three types of investors, utilizing ESG scores sourced from four individual ESG raters and four ensemble methods, are presented in Table 2.3 and Table 2.4 below. The RL framework's configuration remains consistent across the creation of these two tables, with the only variation being the source of ESG scores or the ensemble methods applied. Both tables display the calibration results for three distinct investor types.

The results presented in Table 2.3 and Table 2.4 can be summarily stated as follows. In scenarios without uncertainty, investors exhibiting indifference towards ESG factors typically achieve higher expected returns compared to those who prioritize ESG considerations. Conversely, investors who account for ESG uncertainty tend to realize higher expected returns than their counterparts who do not factor in such uncertainties. It is noteworthy to point out, however, that the expected returns and corresponding Sharpe ratios of investors with ESG uncertainty do not consistently surpass those of ESG-indifferent investors. Moreover, an increase in the parameter b corresponds to a heightened emphasis on non-pecuniary returns, ultimately resulting in lower overall expected returns. Interestingly, the highest expected returns and Sharpe ratios are often attained by investors displaying indifference

	$b < 1 (b = 0.2)$		$b < 1 (b = 0.6)$		$b = 1$		$b > 1 (b = 1.4)$		$b > 1 (b = 1.8)$	
	Return	**SR**	**Return**	**SR**	**Return**	**SR**	**Return**	**SR**	**Return**	**SR**
ESG Indifference	8.63%	0.77	8.63%	0.77	8.63%	0.77	8.63%	0.77	8.63%	0.77
Centroid (N)	8.45%	0.68	8.12%	0.65	7.38%	0.64	7.05%	0.64	6.68%	0.60
Median (N)	7.66%	0.63	7.63%	0.66	7.68%	0.62	7.63%	0.64	7.45%	0.63
PCA (N)	8.04%	0.72	7.89%	0.69	7.21%	0.66	6.26%	0.63	7.00%	0.62
Alpha-Maxmin (N)	9.03%	0.74	8.34%	0.70	8.12%	0.68	7.88%	0.65	7.43%	0.63
Centroid (U)	9.29%	0.73	8.62%	0.69	8.03%	0.64	7.68%	0.59	7.12%	0.53
Median (U)	9.61%	0.80	8.78%	0.81	8.54%	0.77	7.73%	0.62	7.51%	0.53
PCA (U)	9.23%	0.84	8.63%	0.75	8.33%	0.72	8.23%	0.64	7.33%	0.48
Alpha-Maxmin (U)	9.99%	0.88	8.72%	0.78	8.63%	0.72	7.62%	0.62	7.40%	0.58

Table 2.4: Calibration Results for Four Ensemble ESG Scores

towards ESG factors or those considering ESG uncertainty. Notably, among the ensemble ESG score methodologies, Alpha-Maxmin consistently demonstrates superior performance in terms of expected portfolio return and Sharpe ratio when compared to other ensemble methods.

The results presented above are broadly in agreement with the theoretical expectations of the model. However, there are a few isolated instances, in which the empirical results deviate from the theoretical predictions. Such deviations may arise due to a number of reasons, such as overfitting the model, a limited amount of the data used for training the model, etc.

The calibration study reveals that incorporating ESG considerations into investment decisions generally leads to a lower short-term financial performance, as evidenced by reduced Sharpe Ratios in the one-year test set. This finding suggests that in the short term, the focus on ESG may come at the expense of financial returns. However, in the long term, companies with a strong ESG performance are better positioned to manage policy or sustainability risks, potentially enhancing their financial resilience and performance over time. Additionally, we observe that the performance difference between Type-I and Type-U investors remains ambiguous. This reflects the trade-off between the loss of returns due to ESG considerations and the compensation for uncertainty through different ESG ratings.

The results of our calibration study, combined with the findings reported in the existing literature, underscore the complex and context-dependent relationship between ESG performance and financial returns. While some studies suggest that higher ESG performance

can enhance financial outcomes, others argue that it may lead to reduced returns under specific conditions. For instance, Ademi and Klungseth (2022) demonstrate that strong ESG practices are associated with better financial performance and higher market valuations in the U.S. market, where regulatory frameworks and consumer preferences increasingly favor sustainability. Similarly, MSCI (2023) highlight the positive impact of ESG ratings on stock price performance, attributing this to reduced risk exposure and improved corporate resilience. Conversely, other research points to the variability in ESG's impact across different markets and contexts. For instance, Assael et al. (2023) show that ESG scores do not uniformly translate to higher financial returns across sectors or company sizes in the European equity market, suggesting that sector-specific dynamics and investor priorities influence outcomes. Additionally, ESG considerations may introduce reputational risks, as evidenced by Nicolas et al. (2024), where spikes in ESG-related social media discussions led to significant reductions in abnormal returns, particularly in the Social and Governance pillars of ESG.

The lack of consensus in reaching clear conclusions in the literature can be attributed to several factors in their findings. First, the time horizon of analysis plays a crucial role. As our calibration study shows, in the short term, integrating ESG factors can reduce financial returns, particularly when ESG investments are priced at a premium due to a high demand. However, over the long term, companies with a strong ESG performance are better positioned to manage sustainability risks and adapt to regulatory changes, potentially enhancing resilience and generating superior returns. Second, regional differences in ESG awareness and policy adoption are significant contributors to these discrepancies. For instance, countries with well-established regulatory frameworks and strong investor interest in ESG may exhibit stronger positive correlations between ESG and financial performance compared to regions where ESG is less prominently emphasized. These regional differences highlight how government policies and cultural attitudes towards sustainability can shape market outcomes. Third, variations in the time periods of the datasets used in these studies further explain the discrepancies. Research conducted in the early stages of ESG adoption, when awareness and integration were limited, may show weaker or even negative correlations between ESG and financial performance. In contrast, more recent studies, conducted at a time when ESG factors are more widely acknowledged and integrated into the investment decision-making process, tend to demonstrate stronger positive impacts. This shift reflects the growing market acceptance of ESG as a critical component of corporate performance. Finally, differences in the scope and weighting of ESG components across studies also contribute to varying results. For example, studies emphasizing environmental factors may show stronger correlations with financial performance in industries such as renewable energy, while studies focusing on social or governance factors might yield mixed

results.

These findings reinforce the need for investors to align their ESG preferences with their investment goals and risk tolerance. They also highlight the importance of addressing ESG rating heterogeneity and ensuring that long-term sustainability objectives are routinely factored into portfolio management strategies.

2.5 ESG-modified Capital Asset Pricing Model

For completeness of our analysis, we extend a Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964) to include the ESG scores and also by accounting for uncertainty by leveraging the results obtained earlier for the MV model. The CAPM has long been a cornerstone of modern finance, providing a theoretical framework for understanding the trade-off between risk and return. Traditionally, the CAPM assumes that all investors consider market risk as the sole factor affecting expected returns. However, with the growing emphasis on ESG considerations in investment decision-making, researchers have sought to expand the CAPM framework to account for these factors. Recent literature explores various methods of incorporating ESG constraints into the traditional model. For instance, some studies focus on adjusting the utility function to include ESG preferences, thereby redefining the efficient frontier to reflect sustainable investment priorities Pedersen et al. (2021). Other works examine the relationship between corporate social responsibility and the cost of equity, highlighting how strong ESG performance reduces risk premiums for firms Albuquerque et al. (2019). Additionally, meta-analyses and empirical research reveal that ESG factors are increasingly regarded as non-diversifiable risks that should be incorporated into asset pricing models Friede et al. (2015c).

Despite these advancements, the current literature often assumes that ESG factors are either deterministic or can be linearly incorporated into the model. These assumptions overlook the inherent uncertainty in the ESG ratings, which often vary significantly across the different rating agencies. Moreover, many existing models treat the ESG factors as static inputs, failing to address the dynamic nature of the ESG considerations in response to market conditions or investor preferences. The CAPM model proposed in this section seeks to address these limitations by introducing a novel Double Mean-Variance framework that incorporates ESG constraints while explicitly accounting for uncertainty. A key innovation of this framework is its ability to decouple the components of return, risk, and uncertainty aversion. This decomposition provides a more general formulation of the CAPM, allowing for a greater degree of flexibility in modeling investor behavior. For example, the framework separates the effects of the market return and the ESG return,

enabling independent adjustments of risk and ESG uncertainty. This separation is particularly advantageous for investors with diverse preferences or regulatory constraints, as it allows for tailored adjustments to portfolio weights based on specific aversion levels.

To build up an ESG-CAPM, firstly, we provide a list of variable declarations in Table 2.5.

Variable	Description	Dimension
$\boldsymbol{X}_i$	Agent i's portfolio weights for n stocks	$n \times 1$ vector
$\boldsymbol{X}_M$	Market portfolio vector	$n \times 1$ vector
$\boldsymbol{\mu}_r$	Market excess return vector	$n \times 1$ vector
$\mu_M = \boldsymbol{X}_M^T \boldsymbol{\mu}_r$	Market expected return	A number
$\boldsymbol{\mu}_{g,M}$	Expected ESG score vector	$n \times 1$ vector
$\mu_g = \boldsymbol{X}_M^T \boldsymbol{\mu}_{g,M}$	Market ESG score	A number
$\boldsymbol{\Sigma}_M$	Covariance matrix of returns	$n \times n$ matrix
$\boldsymbol{\Sigma}_{g,M}$	Covariance matrix of ESG ratings	$n \times n$ matrix
w_i	Weight of Agent i	A number
$\sigma_M^2 = \boldsymbol{X}_M^T \boldsymbol{\Sigma}_M \boldsymbol{X}_M$	Market return variance	A number
$\sigma_g^2 = \boldsymbol{X}_M^T \boldsymbol{\Sigma}_{g,M} \boldsymbol{X}_M$	Market ESG score variance	A number

Table 2.5: Description of Variables

The direct utility function for investor i is formulated as:

$$\boldsymbol{X}_i^T \boldsymbol{\mu}_r - \frac{\gamma_i}{2} \boldsymbol{X}_i^T \boldsymbol{\Sigma}_M \boldsymbol{X}_i + b_i \left(\boldsymbol{X}_i^T \boldsymbol{\mu}_{g,M} - \frac{\theta_i}{2} \boldsymbol{X}_i^T \boldsymbol{\Sigma}_{g,M} \boldsymbol{X}_i \right) \tag{2.27}$$

After taking the first-order condition, the agent i's portfolio weights for n stocks can be expressed as:

$$\begin{aligned}
\boldsymbol{X}_i &= \left(\gamma_i \boldsymbol{\Sigma}_M + b_i \theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} \left(\boldsymbol{\mu}_r + b_i \boldsymbol{\mu}_{g,M}\right) \\
&= \left(\gamma_i \boldsymbol{\Sigma}_M + b_i \theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} \cdot \gamma_i \boldsymbol{\Sigma}_M \left(\frac{1}{\gamma_i} {\boldsymbol{\Sigma}_M}^{-1} \right) \cdot \left(\boldsymbol{\mu}_r + b_i \boldsymbol{\mu}_{g,M}\right) \\
&= \left(\gamma_i \boldsymbol{\Sigma}_M + b_i \theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} \cdot \gamma_i \boldsymbol{\Sigma}_M \cdot \frac{1}{\gamma_i} \left[\boldsymbol{\Sigma}_M^{-1} \left(\boldsymbol{\mu}_r\right) + {\boldsymbol{\Sigma}_M}^{-1} b_i \boldsymbol{\mu}_{g,M} \right] \\
&= \left[\frac{1}{\gamma_i} \boldsymbol{\Sigma}_M^{-1} \left(\boldsymbol{\mu}_r\right) + \frac{1}{\gamma_i} {\boldsymbol{\Sigma}_M}^{-1} b_i \boldsymbol{\mu}_{g,M} \right] \\
&\quad - \left[\left(\gamma_i \boldsymbol{\Sigma}_M + b_i \theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} \cdot b_i \theta_i \boldsymbol{\Sigma}_{g,M} \right] \left[\frac{1}{\gamma_i} \boldsymbol{\Sigma}_M^{-1} \left(\boldsymbol{\mu}_r\right) + \frac{1}{\gamma_i} {\boldsymbol{\Sigma}_M}^{-1} b_i \boldsymbol{\mu}_{g,M} \right]
\end{aligned} \tag{2.28}$$

2.5.1 ESG-modified CAPM without uncertainty

In the absence of ESG uncertainty, we only take the first two terms of Equation (2.28) into consideration, and as a result, the market portfolio is given by:

$$\boldsymbol{X}_M = \sum_{i=1}^{I} w_i \left[\frac{1}{\gamma_i} \boldsymbol{\Sigma}_M^{-1} \left(\boldsymbol{\mu}_r \right) + \frac{1}{\gamma_i} b_i \cdot {\boldsymbol{\Sigma}_M}^{-1} \cdot \boldsymbol{\mu}_{g,M} \right] \tag{2.29}$$

Then,

$$\boldsymbol{\Sigma}_M \boldsymbol{X}_M = \sum_{i=1}^{I} \frac{w_i}{\gamma_i} \cdot \boldsymbol{\mu}_r + \sum_{i=1}^{I} \frac{w_i}{\gamma_i} \cdot b_i \cdot \boldsymbol{\mu}_{g,M} \tag{2.30}$$

Let $\gamma_M = \frac{1}{\sum_{i=1}^{I} w_i \cdot \frac{1}{\gamma_i}}$ and $b_M = \frac{\sum_{i=1}^{I} w_i \cdot \frac{b_i}{\gamma_i}}{\sum_{i=1}^{I} w_i \cdot \frac{1}{\gamma_i}} = \left(\sum_{i=1}^{I} w_i \cdot \frac{b_i}{\gamma_i} \right) \cdot \gamma_M$, then

$$\boldsymbol{\Sigma}_M \boldsymbol{X}_M = \frac{1}{\gamma_M} \cdot \boldsymbol{\mu}_r + \frac{b_M}{\gamma_M} \cdot \boldsymbol{\mu}_{g,M}, \tag{2.31}$$

$$\begin{aligned} \boldsymbol{\mu}_r &= \gamma_m \boldsymbol{\Sigma}_M \boldsymbol{X}_M - b_M \cdot \boldsymbol{\mu}_{g,M} \\ &= \frac{\boldsymbol{\Sigma}_M \boldsymbol{X}_M}{\sigma_M^2} \cdot \gamma_M \sigma_M^2 - b_M \cdot \boldsymbol{\mu}_{g,M} \end{aligned} \tag{2.32}$$

Next, we derive the market expected return μ_M by multiplying $\boldsymbol{X}_M^T$ on both sides of Equation (2.32).

$$\mu_M = \boldsymbol{X}_M^T \boldsymbol{\mu}_r = \gamma_M \boldsymbol{X}_M^T \boldsymbol{\Sigma}_M \boldsymbol{X}_M - \boldsymbol{X}_M^T \left(b_M \cdot \boldsymbol{\mu}_{g,M} \right) = \gamma_M \sigma_M^2 - \boldsymbol{X}_M^T \left(b_M \cdot \boldsymbol{\mu}_{g,M} \right) \tag{2.33}$$

From Equation (2.33), we know $\gamma_M \sigma_M^2 = \mu_M + \boldsymbol{X}_M^T \left(b_M \cdot \boldsymbol{\mu}_{g,M} \right)$. Then, we replace $\gamma_M \sigma_M^2$ by $\mu_M + \boldsymbol{X}_M^T \left(b_M \cdot \boldsymbol{\mu}_{g,M} \right)$ in Equation (2.32).

After re-arranging terms, an ESG-modified CAPM without uncertainty is shown in Equation (2.34), where $\boldsymbol{\beta} = \frac{\boldsymbol{\Sigma}_M \boldsymbol{X}_M}{\sigma_M^2}$. Here, Alpha ($\boldsymbol{\alpha}$) can be expressed as $b_M \left(\boldsymbol{\beta} \mu_g - \boldsymbol{\mu}_{g,M} \right)$.

$$\begin{aligned} \boldsymbol{\mu}_r &= \frac{\boldsymbol{\Sigma}_M \boldsymbol{X}_M}{\sigma_M^2} \cdot \mu_M + \frac{\boldsymbol{\Sigma}_M \boldsymbol{X}_M}{\sigma_M^2} {\boldsymbol{X}_M}^T \cdot b_M \cdot \boldsymbol{\mu}_{g,M} - b_M \cdot \boldsymbol{\mu}_{g,M} \\ &= \boldsymbol{\beta} \mu_M + b_M \left[\boldsymbol{\beta} \mu_g - \boldsymbol{\mu}_{g,M} \right] \end{aligned} \tag{2.34}$$

The expected excess return of investors is affected by two factors, which are a modified market premium (purely market features $\boldsymbol{\beta} \mu_M$) as well as an Alpha component (features

related to ESG scores $b_M \left[\boldsymbol{\beta}\mu_g - \boldsymbol{\mu}_{g,M}\right]$). The alpha value, which represents excess returns not explained by the stock's beta and the market return, is influenced by the difference between a company's own ESG score and the market ESG score, multiplied by the stock's beta.

To briefly illustrate this concept by means of a numerical example, consider a stock with a beta of 1.5 and a market ESG score of 2. If the stock's own ESG score is 2.5, it will have a positive alpha (+0.5). On the other hand, if the ESG score is greater than 3.0, the alpha will be negative. This also is in line with the previous conclusion, i.e., after considering the preference for the ESG factor, the expected return will decrease. A higher ESG requirement will lead to a greater loss of expected return.

2.5.2 ESG-modified CAPM with uncertainty

For the case with ESG uncertainty, we should consider the full form of Equation (2.28) as the agent i's portfolio weights for n stocks. Thus, the market portfolio is given by:

$$\begin{aligned} \boldsymbol{X}_M &= \sum_{i=1}^{I} w_i \cdot \boldsymbol{X}_i \\ &= \sum_{i=1}^{I} w_i \cdot \left(\gamma_i \boldsymbol{\Sigma}_M + b_i\theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} \cdot \boldsymbol{\Sigma}_M^{-1} \left(\boldsymbol{\mu}_r + b_i \boldsymbol{\mu}_{g,M}\right) \end{aligned} \tag{2.35}$$

$$\begin{aligned} \boldsymbol{\Sigma}_M \boldsymbol{X}_M &= \sum_{i=1}^{I} w_i \left(\gamma_i \boldsymbol{\Sigma}_M + b_i\theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} \left(\boldsymbol{\mu}_r + b_i \boldsymbol{\mu}_{g,M}\right) \\ &= \sum_{i=1}^{I} w_i \left(\gamma_i \boldsymbol{\Sigma}_M + b_i\theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} \boldsymbol{\mu}_r + \sum_{i=1}^{I} w_i \left(\gamma_i \boldsymbol{\Sigma}_M + b_i\theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} b_i \cdot \boldsymbol{\mu}_{g,M} \\ &= {\boldsymbol{\Gamma}_{M,U}}^{-1} \cdot \boldsymbol{\mu}_r + \boldsymbol{B}_{M,U} {\boldsymbol{\Gamma}_{M,U}}^{-1} \cdot \boldsymbol{\mu}_{g,M} \end{aligned} \tag{2.36}$$

where $\boldsymbol{\Gamma}_{M,U} = \left(\sum_{i=1}^{I} w_i \left(\gamma_i \boldsymbol{\Sigma}_M + b_i\theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1}\right)^{-1}$, and

$$\begin{aligned} \boldsymbol{B}_{M,U} &= \left(\sum_{i=1}^{I} w_i \left(\gamma_i \boldsymbol{\Sigma}_M + b_i\theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} b_i\right) \cdot \left(\sum_{i=1}^{I} w_i \left(\gamma_i \boldsymbol{\Sigma}_M + b_i\theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1}\right)^{-1} \\ &= \boldsymbol{\Gamma}_{M,U} \left(\sum_{i=1}^{I} w_i \left(\gamma_i \boldsymbol{\Sigma}_M + b_i\theta_i \boldsymbol{\Sigma}_{g,M}\right)^{-1} b_i\right). \end{aligned} \tag{2.37}$$

Equation (2.36) has the same form as Equation (2.31). The following derivation process should be the same as the case without ESG uncertainty. Thus, the CAPM for Type-U investors is given by:

$$\begin{aligned}\boldsymbol{\mu}_r &= \frac{\boldsymbol{\Sigma}_M \boldsymbol{X}_M}{\sigma_M^2} \cdot \mu_M + \frac{\boldsymbol{\Sigma}_M \boldsymbol{X}_M}{\sigma_M^2} \boldsymbol{X}_M{}^T \cdot \boldsymbol{B}_{M,U} \cdot \boldsymbol{\mu}_{g,M} - \boldsymbol{B}_{M,U} \cdot \boldsymbol{\mu}_{g,M} \\ &= \boldsymbol{\beta}\mu_M + \boldsymbol{B}_{M,U} \left(\boldsymbol{\beta}\mu_g - \boldsymbol{\mu}_{g,M}\right),\end{aligned} \tag{2.38}$$

where $\boldsymbol{\beta} = \frac{\boldsymbol{\Sigma}_M \boldsymbol{X}_M}{\sigma_M^2}$ and the Alpha ($\boldsymbol{\alpha}$) can be expressed as $\boldsymbol{B}_{M,U} \left(\boldsymbol{\beta}\mu_g - \boldsymbol{\mu}_{g,M}\right)$.

The final form for Type-U investors is the same as the case for the type-N investors, except that the $\boldsymbol{B}_{M,U}$ is defined with the incorporation of $\boldsymbol{\Sigma}_{g,M}$ to express the features of ESG Uncertainty. Similarly, when dealing with situations that encompass uncertainty, the Alpha-value retains its sensitivity to the disparity between a company's specific ESG score and the market's ESG score, multiplied by the stock's beta. Employing the same example as previously demonstrated, envision a stock with a beta of 1.5 and a market ESG score of 2. In this context, if the stock's individual ESG score remains at 2.5, a positive alpha (+0.5) will be generated. Conversely, should the ESG score exceed 3.0, the resultant alpha will shift to a negative value.

In principle, a full calibration exercise can be carried out to illustrate the efficacy of the ESG-modified CAPM with and without uncertainty and the results can then be compared with those obtained from the traditional CAPM. This is beyond the scope of the present study and left for a future study.

2.6 Conclusions

Modern investors prioritize ESG scores, aligning their investments with companies that champion environmental responsibility, social awareness, and robust governance. The landscape of ESG data providers is diversified and encompasses well-established players. The existing literature tends to focus on the beneficial role of ESG integration in generating expected excess returns or risk premia.

With the abundance of variety comes the inevitability of heterogeneity. It is not uncommon for the rating agencies to disagree with their published ESG ratings for companies in their samples. This divergence in the ESG ratings affects both the investors, who wish to achieve the financial and ESG-factor return objectives, and the companies, who seek to improve their ESG performances.

The heterogeneity of ESG ratings by major rating agencies is treated in this chapter as a source of ambiguity. In response to this ambiguity, we propose four ESG ensemble strategies that cater to different risk and ambiguity preference profiles, rather than providing a single, optimal ESG scoring system. Our findings underscore the urgent need for standardizing ESG metrics to address the inconsistencies highlighted in this chapter. By revealing the challenges posed by divergent ESG measures, this study contributes to the growing recognition that standardization and transparency are critical for the effective adoption of ESG investing. The proposed ensemble strategies can serve as a bridge until a generally acceptable framework is developed and embraced. Furthermore, the steps outlined earlier, including metric standardization, methodological transparency, and independent oversight, offer actionable pathways to achieve greater consistency in ESG measures. These efforts are not only feasible but also necessary to enhance the reliability of ESG metrics and their integration into financial decision-making processes.

Next, in our portfolio optimization strategy in Section 2.2, we explicitly incorporated ESG scores in the reward function of the RL model to analyze the effect of widespread confusion caused by the heterogeneity in ESG ratings on the coherence of investment strategies.

To combine the goals of achieving both financial market return (pecuniary return) and ESG-factor return (non-pecuniary return) objectives, we presented a DMV model, a linear combination of the two objectives with a preference ratio parameter. In this model, we defined three types of investors: those who are indifferent to ESG, those who prefer ESG but are not affected by uncertainty with respect to the ESG ratings, and those who prefer ESG and are affected by uncertainty with respect to the ESG ratings. Our analysis of the optimal portfolio weights for these investors can be summarized as follows: (i) the no-uncertainty case is associated with a negative ESG incremental premium; (ii) uncertainty with respect to the ESG ratings increases the risk for the investors, and the required return reflects this added risk; and (iii) the sign of the incremental premium is indeterminate when comparing the ESG indifference case and the ESG preference with uncertainty case due to conflicting forces at work between the risk and the return of the ESG factor.

Additionally, the relationships for the variances of the investors are reversed to those for the expected mean relationships. When incorporating ESG ratings with uncertainty in the model, its variance is larger than that without incorporating ESG ratings. However, the difference between the ESG indifference case and the ESG preference with uncertainty case is indeterminate. To verify the practicality of the proposed theoretical framework, we use an RL model by setting the reward function to be the specified DMV model.

To complete our analysis, we leverage the results obtained for the DMV model to con-

struct ESG-modified CAPMs for ESG preference investors with and without uncertainty with respect to ESG ratings, in order to evaluate the performance of the resulting optimized portfolio.

Both models are shown to have a similar structure and divide the expected return into two components, which are the modified market premium (based on the purely market factors) and the Alpha component (based on the ESG factor). Our analysis showed that the Alpha value is affected by the difference between a company's individual ESG ratings and the market ESG ratings, multiplied by the stock's beta. This information can be used to determine a threshold for positive Alpha in a sustainable investment approach, aiming at shedding light on these intricacies, and guiding both investors and company managers towards better-informed sustainable choices. In principle, a full-fledged empirical study can be performed by using the analytical framework developed in this subsection based on real data. This is left for a future study.

Chapter 3

Joint Liability Model with Adaptation to Climate Change

3.1 Introduction

In recent years, the importance of incorporating sustainability factors into financial decision-making has gained significant traction across various sectors of the economy, particularly within agriculture and finance. While Environmental, Social, and Governance (ESG) metrics have become a cornerstone for evaluating corporate sustainability, a notable gap persists in assessing individual borrowers—particularly farmers seeking credit to finance their agricultural operations. To address this gap, this chapter introduces a novel concept: a scoring system coined as Environmental, Sustainability, and Economics (ESE). The ESE score adapts the ESG framework to the agricultural sector, emphasizing economic considerations alongside environmental and social aspects as critical components of sustainable farming practices. This approach reflects the unique realities of individual farmers, where governance metrics are less applicable, and financial sustainability is paramount. For a more detailed discussion and analysis of this individual-focused ESE concept, refer to Appendix A.

The ESE scoring system is designed to address the needs of multiple stakeholders who stand to benefit from such a framework. Policymakers and regulators can utilize ESE scores to design and implement targeted subsidies or tax incentives that promote sustainable farming practices. Financial institutions, such as banks and insurers, could leverage ESE metrics to assess the creditworthiness and risk profiles of individual farmers, while aligning their lending portfolios with sustainability goals. Similarly, agribusiness

corporations can use ESE scores to evaluate their suppliers' sustainability performance, ensuring greater transparency and alignment with their ESG-related commitments. Most importantly, farmers and cooperatives can benefit directly from the ESE scoring system, as it provides a structured framework to benchmark their practices, improve access to sustainable financing, and signal their commitment to sustainability to potential partners and stakeholders.

Although the ESE score draws inspiration from the ESG framework, there are both shared characteristics and critical differences between the two concepts. Both ESE and ESG scores emphasize sustainability and evaluate entities based on environmental and social metrics, such as resource use efficiency, emissions reduction, and biodiversity impacts. However, while ESG scores are primarily designed for corporations, investors, and financial markets, ESE scores are tailored specifically for the agricultural sector, focusing on individual farmers. A key distinction lies in the explicit inclusion of economic metrics in the ESE score, addressing the financial realities of farming—such as profitability and cost-efficiency—which are not standalone components in ESG. Additionally, ESG scores encompass broader social and governance considerations, such as diversity, labor practices, and organizational governance. In contrast, the ESE score narrows its scope to factors directly relevant to sustainable agriculture and individual creditworthiness. This granularity makes ESE scores a more specialized tool for supporting agricultural stakeholders while complementing broader ESG frameworks.

In this chapter, we also propose a novel link between individual ESE scores and a joint liability model, providing lenders and borrowers (where lenders are insurance companies and borrowers are farmers) with optimal scores and corresponding optimal group sizes that maximize the total utility of a group of individual borrowers. We begin our analysis with the simplest case of two borrowers in a group and later generalize it to a group of n individual borrowers. Incorporating the individual ESE score mechanism into the joint liability model offers benefits to both lenders and borrowers. For lenders, this mechanism provides a better understanding of the borrower's risk profile, enabling informed lending decisions and ensuring more sustainable joint liability contracts. Additionally, it offers insights into optimal group size for lending purposes, enhancing the efficiency of group-based borrowing. For borrowers, this mechanism offers a clearer picture of their ranking relative to peers, allowing them to identify areas for improvement and enhance their individual ESE scores. It also helps borrowers determine the ideal number of peers needed to form a group for borrowing purposes, improving group success rates and access to credit. Therefore, integrating the individual ESE score mechanism into credit evaluation processes can result in better-informed lending decisions, improved credit access, and enhanced sustainable development outcomes.

The remaining parts of this chapter are organized as follows. In Section 3.2, we introduce the set-up and assumptions of the model and derive a set of results from the models consisting of a group size of two and a group size of n. In Section 3.3, we refine the utility functional form of the model to that of a mean-variance one, which incorporates both the expected value of return and risk simultaneously in the face of climate change. As the resulting model does not have cleanly closed-form analytical expressions, a set of simulation exercises is provided in this chapter based on a set of climate scenarios. Section 3.4 concludes this chapter.

3.2 The Framework for an Individual ESE-Joint Liability Model

We are interested in examining a microcredit lending scenario in which a benevolent lender extends loans to a group of self-selected micro-entrepreneur borrowers who are jointly liable for repayment of the loan. Although each member of the group is given an individual loan, the group is collectively responsible for repaying the loans, and if the total loan is repaid, the entire group can qualify for a subsequent loan. The loans are then invested in a number of projects, which have an equal chance of success and can be either disjoint from, or correlated with, each other. It is assumed that the group members know the realized output of each project, while the lender does not. Both the lender and the borrowers have to make decisions in this setup. The lender has to decide on the optimal contract structure, the degree of joint liability, and the level of punishment in the event of a strategic default, while the borrowers have to decide whether to repay or default, the loan.

There are two prior studies which have inspired us in particular to write this chapter. First Rezaei et al. (2017) show that a group size can affect lending efficiency. Larger groups are more reliable in repaying loans for riskier projects, but the group size cannot become too large. It is shown that joint liability has a positive effect on borrowers' repayment amount and welfare compared to individual liability, provided that group members impose strong punishment strategies against any strategically defaulting member. In addition, Rezaei et al. (2017) also examine the possibility of relaxing some of the simplifying assumptions made in the basic model of joint liability. Second, in the context of climate change, Castaing (2021) investigates the impact of risk–sharing networks on the adoption of weather shock management strategies in Burkina Faso; particularly, she studies how the principle of forced solidarity may reduce efforts to adopt techniques that mitigate exposure to climate change. The study finds that a system based on mutual assistance between farmers may reduce their incentives to adopt risk–mitigating strategies, both incrementally and transformationally.

The study highlights the need for complementary mechanisms, such as insurance, to help farmers reduce their vulnerability to climate change.

Following the aforementioned two studies, this chapter pursues two primary objectives. The first one is to determine an optimal group size that maximizes the benefit for the borrowers, while ensuring that the lender breaks even. A larger group size can have a positive effect on the expected repayment amount, as more individuals are liable for repaying defaulted payments, thus ensuring a higher rate of repayment. However, a larger group size can also pose a threat to repaying members, as they must repay all defaulting peers' repayments, when everyone else in the group defaults on repayment. Therefore this paper proposes a method to determine the optimal group size that enables joint liability to achieve its maximum loan ceiling. A second objective of this chapter is to derive an optimal individual–ESE score. The importance of such a scoring system for both lenders and borrowers cannot be overstated. For lending companies, having access to a specific scoring mechanism for individual borrowers can provide a better understanding of their creditworthiness and repayment prospects. This information is essential for lending companies in determining the risk associated with lending to a particular borrower. Additionally, the optimal group size of borrowers can be determined using the scoring system, which enables the lending company to mitigate risk by lending to a well-diversified portfolio. For borrowers, having a scoring system can equip them with a clearer picture of their ranking and eligibility for loans. This information is critical for farmers as they search for an ideal group for borrowing purposes since this can help them determine the number of peers needed in a group to secure a loan and, in the process, improve their chances of obtaining funding. Therefore, both lenders and borrowers can benefit from the implementation of such a comprehensive scoring system.

3.2.1 Model Assumptions

Below we state a number of assumptions underlying the proposed model.

ASSUMPTION 1. The borrowers (who are the individual farmers) produce an agricultural product. For each borrower, the probability of high production (i.e., a successful project) is the same as given by e and the probability of low production (i.e., a failed project) is given by $1 - e$ respectively.

ASSUMPTION 2. A higher individual–ESE score means that the farmer has a stronger ability to succeed in their project due to a high-quality comprehensive evaluation. For simplicity, the probability of high yield is assumed to be linearly related to the borrower's ESE score; i.e., we assume that there is a linear relationship between e and ESE: $e = k \cdot$

$ESE+b$. The parameter k represents the weight or strength of the impact of the borrower's ESE score on the probability of high yield. A higher value of k indicates that a higher ESE score leads to a more significant positive impact on the probability of success. This aligns well with the idea that a borrower's positive ESE practices contribute to a project's success. In other words, higher ESE scores indicate a better alignment with responsibly sustainable practices. The constant b represents a baseline/constant contribution of factors beyond an individual's control, such as climatic and geographical conditions. Even if a borrower has a lower ESE score, this constant term acknowledges that external factors can still influence the project's success. Therefore it is essential to account for such factors to avoid unfairly penalizing borrowers for circumstances beyond their own control.

The selection of the linear expression $e = k \cdot ESE + b$ is motivated by both the characteristics of the variables involved and its practical applicability. Notably, the non-negativity of k, b, and ESE ensures that the resulting probability, denoted as e, maintains its intrinsic non-negative nature. As ESE encompasses a continuous range from 0 to 100, it inherently aligns with this non-negative property. It is important to stress that the technical requirement for e, being a probability, to lie between 0 and 1, is conveniently met by setting $b = 0$ and $k = \frac{1}{100}$. In addition, this specification also accommodates real-world situations by allowing for a non-zero baseline influence ($b > 0$), often linked to exogenous factors. To this end, a modest value such as $b = 0.3$ can be accommodated without violating the aforementioned probability constraint. Thus this adjustment necessitates setting $k = \frac{0.7}{100}$, thereby guaranteeing that e remains within the bounds of a probability as required. While, at the same time, it also acknowledges that e cannot span the interval from 0 to 0.3; this constraint is justified by underscoring that e is a representation of a probability of a high yield, which is inherently a positive entity in practical settings. In summary, the rationale underlying the proposed equation effectively marries mathematical coherence with nuanced demands from real-world applications.

At first sight, the use of a sigmoid function to model e in the model as $e = \frac{1}{1+\exp^{-ESE}}$ or $e = \frac{1}{1+\exp^{-(k \cdot ESE+b)}}$ appears to be an ideal choice. However, upon a closer examination, a critical observation related specifically to our study emerges; i.e., under the assumption that k, b, and ESE are non-negative, the use of the sigmoid function would result in a constraint where e precludes the values below 1/2. This deviation from the alignment with realistic scenarios and the entire spectrum of probability values was deemed to be undesirable in the context of our study. Thus, while the use of the sigmoid function to model e may possess certain merits in other contexts, its inherent restriction on precluding the probability values above 1/2 in our model is at odds with the overarching goal of accurately representing the probability of high yield across its entire spectrum. As a result, a decision was made to retain the linear equation $e = k \cdot ESE + b$, which both adheres to

the established criteria and provides the desired flexibility to represent a plausible range of probability values.

ASSUMPTION 3. The amount of a loan secured by each borrower is denoted as L, while the amount for this borrower to be repaid is denoted as w. The unit selling price of the product is denoted as p, and the total income of a high production is denoted as $p\overline{Y}$, while the total income of a low production is denoted as $p\underline{Y}$.

ASSUMPTION 4. High-yield borrowers repay their loans and low-yield borrowers pay back all of the income they earn. As long as this group repays the total loans, this group can continue to secure subsequent loans. In the event that all borrowers are unsuccessful in their projects, the group will not be eligible for any future loans. This may result in a negative credit history for all of the members in the group, which could lead to further restrictions placed on them, such as being unable to use public transportation or purchasing high-cost items. Therefore, if all borrowers fail in their projects, they may have to use all of their earnings to avoid receiving negative credit reports, even though their total earnings will be less than the total repayments.

ASSUMPTION 5. To increase the successful probability e, the borrower incurs additional costs, such as purchasing green pesticides, advanced tools, etc. A convex disutility function, denoted by $C(e)$, is included in the specification of the total utility function in the proposed model.

3.2.2 Joint Liability Contracts with a Group Size of Two

Under a joint liability agreement with a group size of two ('Farmer A' and 'Farmer B'), from the view of 'Farmer A', his/her ex-ante expected profit π_i can be written as

$$\pi_i = e^2[p\overline{Y} - w] + e(1-e)[p\overline{Y} + p\underline{Y} - 2w] - C(e) \tag{3.1}$$

We emphasize that, in this model, both farmers have a chance of achieving a successful harvest $\overline{Y}$ with a probability of e^2, which gives them a profit of $p\overline{Y} - w$. The probability of one farmer defaulting is given by $e(1-e)$, which means that farmer i would receive his/her own surplus from a successful harvest minus the other farmer's deficit, $p\overline{Y} + p\underline{Y} - 2w$. If both farmers fail in their respective projects at the same time, they will pay all of their incomes to the lenders, which means that their profit will be zero. The detailed profit distributions are summarized conveniently in Table 3.1. The probabilities and profit outcomes are calculated entirely from the perspective of Farmer A, reflecting the cooperative repayment structure under the assumption that high-yield borrowers (successful farmers) repay their

loans in full, while low-yield borrowers (unsuccessful farmers) repay only the income they earn ($p\underline{Y}$). When Farmer A succeeds ($p\overline{Y}$) and Farmer B fails, Farmer A must not only repay their own loan obligation (w) but also cover the shortfall left by Farmer B, who repays only $p\underline{Y}$. This results in Farmer A's profit being $p\bar{Y} - w - (w - p\underline{Y}) = p\overline{Y} + p\underline{Y} - 2w$. Conversely, when Farmer A fails and Farmer B succeeds, Farmer A's repayment is limited to their income ($p\underline{Y}$), with Farmer B's success ensuring the cooperative repayment obligation is met, leading to a profit for Farmer A of $p\underline{Y} - p\underline{Y} = 0$. The asymmetry in profits arises from this risk-sharing mechanism, where successful farmers are expected to shoulder additional responsibility to maintain the system's financial integrity, while unsuccessful farmers contribute as much as their income allows them to do so, shielding them from excessive financial burdens.

Case Description	**Probability**	**Profit=Earning-Payment**
'Farmer A' succeeds, 'Farmer B' succeeds too.	e^2	$p\overline{Y} - w$
'Farmer A' succeeds, 'Farmer B' fails.	$e(1-e)$	$p\overline{Y} + p\underline{Y} - 2w$
'Farmer A' fails, 'Farmer B' succeeds.	$(1-e)e$	$p\underline{Y} - p\underline{Y} = 0$
'Farmer A' fails, 'Farmer B' fails.	$(1-e)^2$	$p\underline{Y} - p\underline{Y} = 0$

Table 3.1: Profit Distributions

Before setting the total objective (utility) function for the whole group in the model, we first list a number of exogenous constraints which we introduce to the optimization program.

The constraints used here are motivated by the commonly accepted assumptions in the literature on joint liability models. For example, Rezaei et al. (2017) introduced similar constraints in their analysis of the optimal group size in the joint liability contracts to ensure the feasibility and sustainability of the contract. These constraints are based on expected returns and the repayment strategies of group members. While our study adopts a similar structure for these constraints, the detailed settings of the expected profit and the underlying model assumptions differ from their study. Moreover, our model explicitly incorporates sustainability features, addressing dynamic adjustments and long-term stability of joint liability contracts under varying economic conditions and group compositions.

CONSTRAINT 1. If the group wants a subsequent loan from the lender, the group's total repayment amount must be affordable even in the worst–case scenario when everyone else's project has failed except for him/hers:

$$\text{Constraint 1: } 2w \leqslant p\overline{Y} + p\underline{Y} \tag{3.2}$$

CONSTRAINT 2. Strategic default in a joint liability contract occurs when one party (or more parties) intentionally chooses (or choose) to default on his/her obligation (or their obligations), even though he/she has (or they have) the ability to pay back the loan. This is often done in order to gain a strategic advantage, such as to force the other parties to pay a greater share of the debt or renegotiate the terms of the contract. To avoid a strategic default for each successful borrower, the payoff of strategically defaulting cannot be larger than the payoff of repaying and being refinanced.

Assuming that the other borrower also succeeds in his/her project, this borrower will only be responsible for his/her own payment. In this situation, the earnings plus the discounted cost of the next loan would at least be equal to the earnings of strategically defaulting.

$$\text{Constraint 2: } p\overline{Y} \leqslant p\overline{Y} - w + \delta L \tag{3.3}$$

CONSTRAINT 3. If the other borrower fails in his/her project, then this borrower needs to pay for himself/herself as well as part of the other borrower's repayment amount. Now, the earnings plus the discounted cost of the next loan would also at least equal the earnings of strategically defaulting.

$$\text{Constraint 3: } p\overline{Y} \leqslant p\overline{Y} + p\underline{Y} - 2w + \delta L \tag{3.4}$$

CONSTRAINT 4. The lender must be able to sustain the lending game, or at least break even, over the periods. Thus the expected repayment amount of each borrower has to be at least as large as $L(1+\epsilon)$, where ϵ is a risk-free rate. It is important to note that given that all constraints have been established under the assumption that the group will successfully repay the total loans to ensure their eligibility for subsequent loan rounds, we have not factored in the case, in which all borrowers within the group experience failure in their projects, on the left-hand side of Constraint 4.

$$\text{Constraint 4: } w\left[1-(1-e)^2\right] \geqslant L(1+\varepsilon) \tag{3.5}$$

Since $p\underline{Y} - w < 0$, it is clear that Constraint 3 is tighter than Constraint 2. In the ensuing analysis, we will only consider Constraint 1, Constraint 3, and Constraint 4. Firstly π_i and w have a strictly negative relationship as can be observed from Equation (3.1). So, to maximize π_i, w has to be set at a minimum value. From Constraint 4, the minimum value of w should be set at

$$w = \frac{L(1+\varepsilon)}{1-(1-e)^2} \tag{3.6}$$

.

Next we substitute $w = \frac{L(1+\varepsilon)}{1-(1-e)^2}$ into Constraint 1, and rewrite the equation as

$$2 \cdot \frac{L(1+\varepsilon)}{1-(1-e)^2} \leqslant p\overline{Y} + p\underline{Y}, \tag{3.7}$$

and the first loan ceiling bound can be expressed in Equation (3.8) as

$$L_1 \leqslant \frac{p\overline{Y} + p\underline{Y}}{2(1+\varepsilon)} \left[1-(1-e)^2\right]. \tag{3.8}$$

After substituting $w = \frac{L(1+\varepsilon)}{1-(1-e)^2}$ into Constraint 3, we obtain

$$p\overline{Y} \leqslant p\overline{Y} + p\underline{Y} - 2\frac{L(1+\varepsilon)}{1-(1-e)^2} + \delta L, \tag{3.9}$$

$$\left(\frac{2(1+\varepsilon)}{1-(1-e)^2} - \delta\right) L \leqslant P\underline{Y}, \tag{3.10}$$

This allows us to obtain another bound for the loan ceiling as in Equation (3.11)

$$L_2 \leqslant \frac{p\underline{Y}}{\frac{2(1+\varepsilon)}{1-(1-e)^2} - \delta} \tag{3.11}$$

Next we substitute $w = \frac{L(1+\varepsilon)}{1-(1-e)^2}$ into the objective function (which is Equation (3.1) and replace $e = k \cdot E + b$. The ex-ante expected profit function of an individual borrower can now be rewritten as

$$\begin{aligned}
&\max_E e^2(p\overline{Y} - w) + e(1-e)[p\overline{Y} + p\underline{Y} - 2w] - C(e) \\
&= e^2 p\overline{Y} + e(1-e)(p\overline{Y} + p\underline{Y}) - \left(2e - e^2\right) w - C(e) \\
&= e^2 p\overline{Y} + e(1-e)(p\overline{Y} + p\underline{Y}) - L(1+\varepsilon) - C(e) \\
&= (kE+b)^2 \cdot p\overline{Y} + (kE+b)(1-kE-b)(p\overline{Y} + p\underline{Y}) - L(1+\varepsilon) - C(e)
\end{aligned} \tag{3.12}$$

To simplify the problem even further, we set $C(e) = \frac{1}{2}ce^2$ (where the cost parameter c is used to scale the total cost) as the disutility function. The disutility function $C(e) = \frac{1}{2}ce^2$ is a commonly used function in Economics to represent the cost or disutility associated

with effort or work. See e.g. Clark and Oswald (1996) and Kahneman et al. (1997). The function is chosen to be quadratic so that it can accommodate the following relationship: as an effort increases, the cost or disutility of that effort increases at an increasing rate. In the context of an individual agricultural farmer, it is reasonable to use a disutility function to acknowledge that while some effort may be necessary, there is a point at which additional effort becomes more and more unpleasant or burdensome. For instance, a farmer may be willing to put in extra effort to cultivate crops or tend to livestock, but as the required effort increases, the farmer may experience fatigue, stress, and reduced productivity, which can negatively impact their overall well–being. Furthermore, the quadratic form of the disutility function is also supported by empirical evidence on the relationship between effort and well-being. It is reasonable to assume that farmers will experience a phenomenon known as a diminishing marginal utility of effort, which implies that the additional unit of well-being gained by exerting more effort decreases as the level of effort increases.

3.2.3 Main Results

In this subsection, we state a number of key results expressed conveniently in the form of propositions. These propositions provide useful insights into the optimal individual–ESE scores, optimal group size, and loan ceiling constraints.

PROPOSITION 1 (Optimal individual–ESE score). Finding an optimal ESE score for both the lenders and borrowers in an individual ESE–joint liability model is critical because it can help address the challenge of promoting sustainability in agricultural production while ensuring access to credit for small-scale farmers. By incorporating environmental and social factors into credit evaluation systems, it can incentivize farmers to adopt sustainable practices that can lead to positive environmental outcomes and long-term economic benefits. At the same time, lenders can mitigate the risks associated with lending to farmers by incorporating sustainability factors into their credit evaluation process.

To find the optimal value of individual–ESE score, E, we take a first-order partial derivative with respect to E of the ex-ante expected payoff function to obtain the following result

$$\begin{aligned}\frac{\partial \pi}{\partial E} &= 2(kE+b)\cdot k(p\overline{Y}) + (k - 2(kE+b)\cdot k)\cdot[p\overline{Y} - p\underline{Y}] - k\cdot\frac{\partial C(e)}{\partial e} \\ &= kp\overline{Y} - [k - 2(kE+b)\cdot k]p\underline{Y} - kc\cdot(kE+b) = 0.\end{aligned} \tag{3.13}$$

After rearranging terms, we obtain an optimal individual-ESE score (where $0 \leqslant E \leqslant 100$) as

$$E = \frac{p\overline{Y} + (2b-1)p\underline{Y} - cb}{ck - 2kp\underline{Y}} \tag{3.14}$$

PROPOSITION 2 (Collapsing two constraints on loan ceiling to a single constraint). The two loan ceiling constraints, Constraint 1 and Constraint 3, originate from analyzing different worst-case scenarios to ensure the affordability and sustainability of the group's repayment structure. However, most of the previous studies in this area have not explored the connection between these two constraints. Notably, in Rezaei et al. (2017), they found that a feasible range of loan ceilings depends on the borrower's discount rate δ. Nevertheless, it is possible to reduce these two constraints to a single constraint in our model.

Recall the two constraints on the loan ceiling in Equation (3.8) and Equation (3.11), labelled respectively as L_1 and L_2. Next, we show that L_1 is always larger than L_2, which means that we can only use L_2 as a final loan ceiling bound. It simplifies the problem and potentially makes it easier to solve the model. In addition, L_2 takes δ into consideration, which may include more information and additional key factors. This result is illustrated in Figure 3.1.

Proof:

If $L_1 \leqslant L_2$, then

$$\frac{p\overline{Y} + p\underline{Y}}{2(1+\varepsilon)}\left[1-(1-e)^2\right] \leqslant \frac{p\underline{Y}}{\frac{2(1+\varepsilon)}{1-(1-e)^2} - \delta} \tag{3.15}$$

After rearranging terms, we obtain

$$\delta \geqslant \frac{\overline{Y} \cdot 2(1+\varepsilon)}{\left[1-(1-e)^2\right](\overline{Y} + \underline{Y})} > 1 \tag{3.16}$$

From the above results, if $L_1 \leqslant L_2$, the discount factor δ is always greater than 1, which leads to a contradiction. This leads us to conclude that $L_1 > L_2$.

PROPOSITION 3 (A higher individual–ESE score leads to a higher loan ceiling). Next, we examine the relationship between the individual–ESE score and the loan

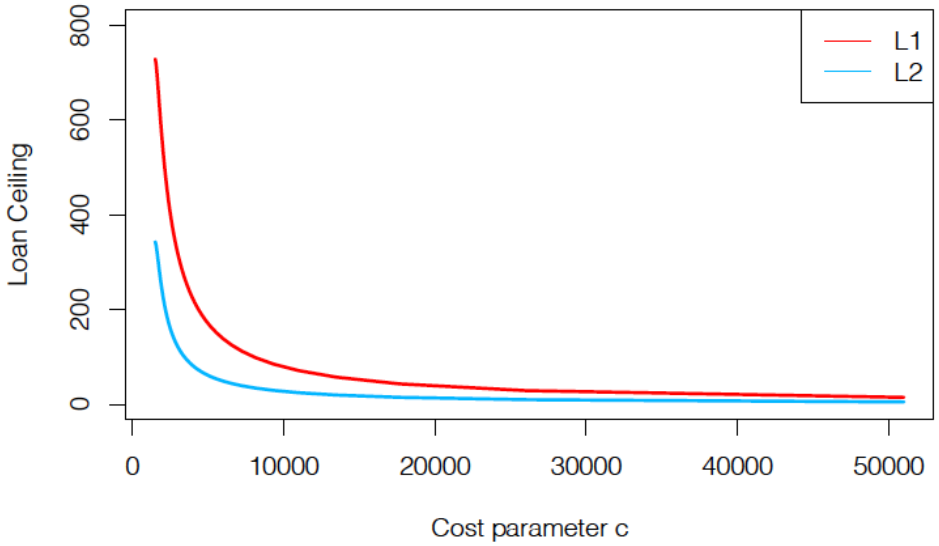


Figure 3.1: The Relationship between Two Loan Ceiling Constraints

ceiling. Intuitively a borrower with a high-quality ESE score should be eligible for a higher loan amount, and therefore the loan ceiling should also be higher.

Since L_1 is always greater than L_2, we will use L_2 to determine the loan ceiling, which defines the maximum value of L_2 to be the loan ceiling.

$$\begin{aligned} L_2 &= \frac{p\underline{Y}}{\frac{2(1+\varepsilon)}{1-(1-e)^2} - \delta} \\ &= \frac{p\underline{Y}\left[1-(1-e)^2\right]}{2(1+\varepsilon) - \delta\left[1-(1-e)^2\right]} \\ &= \frac{p\underline{Y}\left(2e-e^2\right)}{2(1+\varepsilon) - \delta\left(2e-e^2\right)} \end{aligned} \tag{3.17}$$

Then we take a partial derivative of L_2 with respect to E to obtain

$$\begin{aligned} \frac{\partial L_2}{\partial E} &= \frac{\partial L_2}{\partial e} \cdot \frac{\partial e}{\partial E} \\ &= \frac{p\underline{Y}(2-2e)\cdot\left[2(1+\varepsilon) - \delta\left(2e-e^2\right)\right] - p\underline{Y}\left(2e-e^2\right)(-\delta)(2-2e)}{\left[2(1+\varepsilon) - \delta\left(2e-e^2\right)\right]^2} \times k \end{aligned} \tag{3.18}$$

It is clear that the denominator is greater than 0. The numerator is also greater than 0, which can be illustrated more clearly by writing out the following expression

$$\begin{aligned} \text{Numerator} &= kp\underline{Y}(2-2e) \times \left[2(1+\varepsilon) - \delta\left(2e-e^2\right) + \delta\left(2e-e^2\right)\right] \\ &= kp\underline{Y}(2-2e)[2(1+\varepsilon)] > 0 \end{aligned} \tag{3.19}$$

A borrower with a higher optimal ESE score, which means that he/she has a better overall performance, will be perceived as less risky by the lender. As a result the borrower will be offered a higher loan ceiling, which is the maximum amount of loan that a lender is willing to provide to a borrower. In turn, if a borrower wants to obtain a larger loan amount, he/she may be motivated to improve his/her ESE performance.

3.2.4 Joint Liability Contracts with a Group Size of n

In this subsection, we present an individual ESE–joint liability model which formalizes the assumption that the group of borrowers is collectively responsible for the entire group loan (i.e., nw). The expected utility of a borrower i who adopts a repayment strategy during any period when they receive financing is specified as

$$\begin{aligned} \max_{E} \ \pi_i &= \sum_{k=0}^{n-1} \left(\begin{array}{c} n-1 \\ k \end{array} \right) \cdot \left[e^{n-k}(1-e)^k\right] \cdot \left[p\overline{Y} - w - \frac{k(w-p\underline{Y})}{n-k}\right] - C(e) \\ &= e(p\overline{Y}-w) - (w-p\underline{Y}) \cdot \sum_{k=0}^{n-1} \left(\begin{array}{c} n-1 \\ k \end{array} \right) \cdot \left[e^{n-k}(1-e)^k\right] \cdot \frac{k}{n-k} - C(e) \\ &= e(p\overline{Y}-w) - (w-p\underline{Y}) \cdot ((1-e) - (1-e)^n) - C(e) \\ &= ep\overline{Y} - w(1-(1-e)^n) + p\underline{Y}\left((1-e) - (1-e)^n\right) - C(e) \\ &= (kE+b)p\overline{Y} - w\left(1-(1-kE-b)^n\right) + p\underline{Y}\left[(1-kE-b) - (1-kE-b)^n\right] - C(e) \end{aligned} \tag{3.20}$$

Constraints similar to those in the case of a group size of two are as follows (where g stands for 'general').

CONSTRAINT 1g. There is at least one borrower who is successful at his/her project and capable of paying the entire group's repayment amount.

$$nw \leqslant p\overline{Y} + (n-1)p\underline{Y} \tag{3.21}$$

CONSTRAINT 2g. Since Constraint 3 is more restrictive than Constraint 2g when the group size is two, we will omit Constraint 2g from further consideration for a group size of n.

CONSTRAINT 3g. For each successful borrower, the payoff of strategically defaulting cannot be larger than the payoff of repaying and being refinanced.

$$p\overline{Y} \leqslant p\overline{Y} + (n-1)p\underline{Y} - nw + \delta L \tag{3.22}$$

CONSTRAINT 4g. The lender must be able to sustain the lending game, or at least break even, over the periods. Thus the expected repayment amount of each borrower has to be at least as large as $L(1+\varepsilon)$, where ε is the risk-free rate. As in the two-borrowers case, it is important to note that, given that all of the constraints have been established under the presumption that the group will successfully repay the total loans to ensure their eligibility for subsequent loan rounds, we have not factored in the case, where all borrowers within the group experience failure, on the left-hand side of this constraint.

$$w\left[1-(1-e)^n\right] \geqslant L(1+\varepsilon) \tag{3.23}$$

Similarly we obtain $w \geqslant \frac{L(1+\varepsilon)}{1-(1-e)^n}$. Thus the value of w should be set as $\frac{L(1+\varepsilon)}{1-(1-e)^n}$ due to the negative relationship between w and π_i.

After substituting $w = \frac{L(1+\varepsilon)}{1-(1-e)^n}$ in the objective function (which is Equation (3.20)), the ex-ante expected payoff function for the borrower i in a size n joint liability group is represented as

$$\max_E \pi_i = (kE+b)p\overline{Y} - L(1+\varepsilon) + p\underline{Y}\left[(1-kE-b) - (1-kE-b)^n\right] - C(e) \tag{3.24}$$

PROPOSITION 4 (Optimal individual–ESE score and group size n have a negative relationship). A higher ESE score indicates a lower default risk, which means that the borrower is more creditworthy and can obtain a loan with a smaller group size. On the other hand, each member of the group is jointly responsible for the repayment of the loan, and if one member defaults, the other members will have to repay the outstanding amount. Thus, a low-quality borrower may choose to loan with a larger group to reduce the probability of default and minimize the potential burden on any one individual. Next, we verify this intuition based on the following derivations.

We take a first-order partial derivative of the expected payoff with respect to E to obtain an optimal individual–ESE score for a borrower as below

$$\frac{\partial \pi}{\partial E} = kp\overline{Y} + p\underline{Y}k\left[n(1-kE-b)^{n-1}-1\right] - k\frac{\partial C(e)}{\partial e} = 0, \tag{3.25}$$

where the cost is $C(e) = \frac{1}{2}ce^2$.

Since it is hard to find an explicitly analytical form of n and E, we adopt an implicit function derivation below. Let $E(n)$ replace E and take the partial derivative with respect to n on both sides of $kp\overline{Y} + p\underline{Y}\left[n(1-kE-b)^{n-1}-1\right] - kc(kE+b) = 0$. Then we obtain the following result

$$p\underline{Y}\left\{(1-kE(n)-b)^{n-1} + n\cdot\frac{\partial\left[e^{(n-1)\log(1-kE(n)-b)}\right]}{\partial n}\right\} - c\left[e\frac{\partial E(n)}{\partial n}\right] = 0 \tag{3.26}$$

where,

$$\begin{aligned}\frac{\partial\left[e^{(n-1)\log(1-kE(n)-b)}\right]}{\partial n} &= e^{(n-1)\log(1-kE(n)-b)}\cdot\left[\frac{(n-1)\cdot(-k)\frac{\partial E(n)}{\partial n}}{1-k\cdot E(n)-b} + \log(1-kE(n)-b)\right] \\ &= (1-k(n)-b)^{n-1}\cdot\left[\frac{(n-1)k}{1-kE(n)-b}\cdot\left(-\frac{\partial E(n)}{\partial n}\right) + \log(1-kE(n)-b)\right]\end{aligned} \tag{3.27}$$

After reorganizing the formula, we can express the partial derivative of $E(n)$ with respect to n as

$$\frac{\partial E(n)}{\partial n} = \frac{p\underline{Y}\left\{(1-e)^{n-1}[1+n\cdot\log(1-e)]\right\}}{p\underline{Y}\left[n\cdot(1-e)^{n-1}\cdot\frac{(n-1)k}{1-e}\right] + ce} \tag{3.28}$$

It is clear that the denominator in Equation (3.28) is greater than 0. When $n \geqslant 1$ and $e \in (0,1)$, then $n\log(1-e) < -1$, and $(1-e)^{n-1}[1+n\cdot\log(1-e)] < 0$. Thus, the numerator is less than 0. As a result $\frac{\partial E(n)}{\partial n} < 0$, which shows that the optimal individual–ESE score and group size n have a negative relationship.

In the light of the negative correlation between the optimal individual–ESE scores and the group size, it is recommended that insurance companies prioritize lenders with higher ESE scores when the group size is smaller to mitigate potential risks. Borrowers with high scores are typically perceived to exhibit positive ESE performance, which leads to higher

expectations for sustained success and repayment probability over the long term. Despite the relatively small group size, lenders may still have heightened confidence in the ability of high-scoring borrowers to demonstrate honesty and success and to exhibit willingness to repay their loans. As a result, lenders may consider reducing group size requirements for high-scoring borrower groups. Conversely, as the group size increases, the requirement for lenders' ESE scores may potentially decrease, as a larger group may possess a greater risk mitigation capacity and an increased likelihood of successfully repaying the group loan. Consequently, lenders may opt to reduce the score requirements in such cases.

PROPOSITION 5 (Optimal individual–ESE score converges to a constant level for a large enough group size). Equation (3.28) tells us that $\lim_{n\to\infty} \frac{\partial E(n)}{\partial n} = 0$, which means that the optimal individual–ESE score converges to a constant level when the group size reaches a certain large enough number. To find the limit of the optimal score value, we recall the first–order condition of the ex-ante expected payoff, which is

$$\frac{\partial \pi}{\partial E} = kp\overline{Y} + p\underline{Y}k\left[n(1-kE-b)^{n-1}-1\right] - k\frac{\partial C(e)}{\partial e}. \tag{3.29}$$

Similarly we set $C(e) = \frac{1}{2}ce^2$, then

$$\frac{\partial C(e)}{\partial e} = ce = c(kE+b). \tag{3.30}$$

When $n \to \infty$, $n(1-kE-b)^{n-1} \to 0$, then Equation (3.29) approaches $kp(\overline{Y}-\underline{Y}) - kc\cdot(kE+b)$. Let $kp(\overline{Y}-\underline{Y}) - kc\cdot(kE+b) = 0$, the limit of the optimal ESE score will be expressed as a constant

$$E \to \left[\frac{p(\overline{Y}-\underline{Y})}{c} - b\right]/k. \tag{3.31}$$

To illustrate this proposition, we provide a numerical example, which is shown in Figure 3.2. We set the unit price $p = 1$, the successful probability $e = k \times E + b = 1/100 \times E + 0$, high yield amount $\overline{Y} = 1000$, low yield amount $\underline{Y} = 500$, and perform a simulation under a different group size $n \in [1, 100]$.

In situations where the group size is small, insurance companies may seek to attract borrowers with high ESE scores in order to mitigate risk. However, as the group size expands, the marginal benefit of an individual effort on group outcomes diminishes, resulting in a reduction of an individual farmer's optimal effort level. This can create incentives for

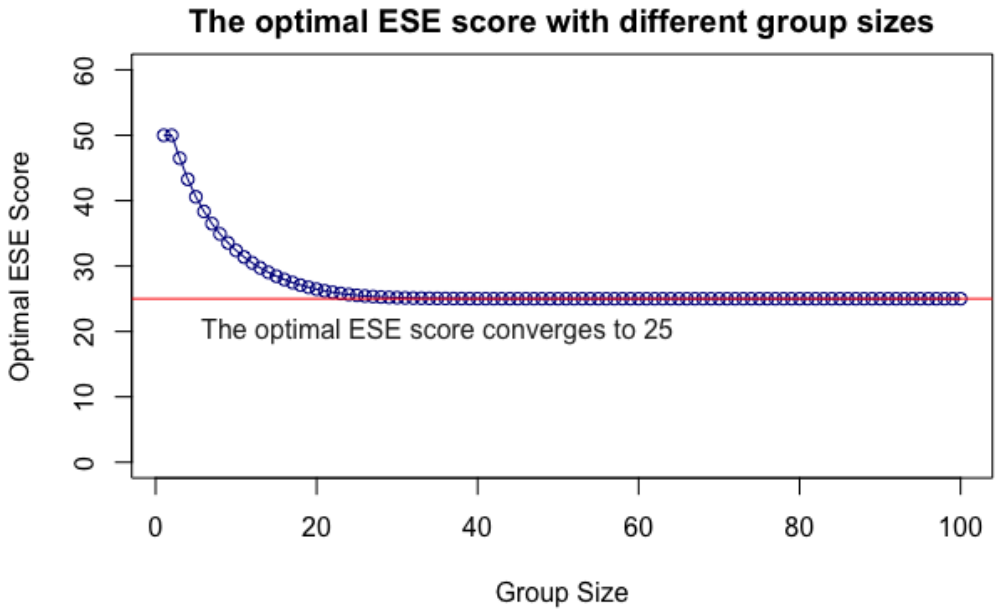


Figure 3.2: Simulation Results of A Numerical Example

farmers to contribute less effort, leading to a 'free-rider' problem and decreased profits for the group. Therefore the threshold value of the optimal ESE score can serve as a reference point for both the lenders and borrowers, enabling them to avoid a sub-optimal resource allocation and a 'free-rider' problem.

In the analysis above, $n \to \infty$ serves as a theoretical construct to explore the asymptotic behavior of the optimal ESE score, E. However, in practical applications, n, representing the group size, is bounded by real-world constraints. Factors such as coordination challenges, logistical limitations, and the effectiveness of group dynamics impose natural upper limits on the group size. From the results of the simulation plot, it is evident that the optimal ESE score converges once n exceeds a certain threshold. Beyond this point, increasing the group size further does not contribute meaningfully to improving the ESE score and may instead exacerbate issues such as the 'free-rider' problem. Consequently, the results derived for large n should be interpreted as indicative of the system's behavior in scenarios where n approaches the upper bounds of realistic group sizes. This bounded nature of n ensures the applicability and relevance of the model to real-world contexts.

3.3 Joint Liability Contracts under a Mean–Variance Utility Framework

When borrowers assess potential projects, they would like to know how likely it is that they will be able to earn a high return or experience a substantial loss. The variability of profits, or the range of possible returns, is an important consideration for these borrowers. If the potential variability of returns is too high or the risk of substantial losses is too large, the borrower may decide not to invest in the venture. To the best of our knowledge, the current literature on joint liability models focuses only on the expected return or expected profit. This means that they concentrate solely on the central point of an individual's profit without taking into account the distribution of potential profits or the variability of these returns. However, focusing only on the expected return can be limiting, as it does not account for the risks. In this paper, we incorporate the risk (proxied by variance) associated with expected profit into our utility function and conduct further analysis of mean-variance utility in the context of the individual ESE–joint liability model. Specifically, the mean-variance utility function for an individual farmer can be incorporated in his/her ex–ante expected profit function in Equation (3.32), where P (a random variable) represents the expected profit and γ represents a risk aversion parameter of the farmer. The choice to limit the group size to two in this section is intended only to serve as a stylized example to illustrate the key concepts of the mean-variance utility framework. Extending the analysis to larger group sizes introduces complexity further, as it requires simultaneously accounting for both the return and risk components across multiple group members. While such an extension would provide a more general analysis, it is beyond the scope of this study and is left for future work.

$$
\begin{aligned}
\max_{E} \pi_i &= \mathbb{E}(P) - \frac{\gamma}{2}\mathrm{Var}(P) - C(e) \\
&= e^2(p\overline{Y} - w) + e(1-e)[p\overline{Y} + p\underline{Y} - 2w] - C(e) \\
&- \frac{\gamma}{2}\left\{e^2(p\overline{Y} - w)^2 + e(1-e)(p\overline{Y} + p\underline{Y} - 2w)^2\right\} \\
&- \frac{\gamma}{2}\left\{-\left[e^2(p\overline{Y} - w) + e(1-e)(p\overline{Y} + p\underline{Y} - 2w)\right]^2\right\} \\
&= e[p\overline{Y} + p\underline{Y} - 2w] - e^2(p\underline{Y} - w) - C(e) \\
&- \frac{\gamma}{2}\left\{\left(e^2 - e^4\right)(p\overline{Y} - w)^2 + \left[e - 2e^2 + 2e^3 - e^4\right](p\overline{Y} + p\underline{Y} - 2w)^2\right\} \\
&+ \frac{\gamma}{2}\left\{2\left(e^3 - e^4\right)(p\overline{Y} - w)(p\overline{Y} + p\underline{Y} - 2w)\right\} \qquad (3.32)
\end{aligned}
$$

In order to derive the required optimal individual–ESE score in this setting, we take a first-order partial derivative of the ex-ante profit with respect to the ESE score

$$\begin{aligned}\frac{\partial \pi_i}{\partial E} =& k(p\overline{Y} + p\underline{Y} - 2w) - 2ek(p\underline{Y} - w) - k\frac{\partial C(e)}{\partial e} \\ & -\frac{\gamma k}{2}\left\{\left(2e - 4e^3\right)(p\overline{Y} - w)^2 + \left(1 - 4e + 6e^2 - 4e^3\right)(p\overline{Y} + p\underline{Y} - 2w)^2\right\} \\ & -\frac{\gamma k}{2}\{2\left(3e^2 - 4e^3\right)(p\overline{Y} - w)(p\overline{Y} + p\underline{Y} - 2w)\} \\ =& 0\end{aligned} \tag{3.33}$$

Unfortunately, the resulting analytical solution of the ESE score involves a rather complex expression and is difficult to interpret intuitively; so we resort to simulation studies to reveal several insightful patterns.

Recall that the probability of achieving success in a project is given by $e = k \times ESE + b$, wherein the parameters k and b are predetermined. Subsequently, the probability may be disaggregated into two constituent components. Specifically, the product of k and ESE represents a contribution of a personal impact towards a project's success, whereas b represents a constant contribution of exogenous factors beyond individual control. The value of k is utilized to calibrate the impact of ESE on a project's success, while b pertains to the influence of exogenous factors, including climatic and geographical conditions.

To determine the value of b, an Actuaries Climate Index (ACI) for each region can in principle be utilized as a reference. Briefly, the ACI is based on an analysis of six climate indicators, including high and low temperatures, heavy precipitation, drought, and sea level rise. The index measures changes in these indicators over time and across regions, providing an indication of how the climate is changing in different parts of a region.

It is commonly understood that favorable climate conditions have a positive effect on crop yields, which, in turn, increases the probability of project success. Conversely, unfavorable climate conditions have an adverse impact on the likelihood of success. Hence, we delve deeper into the impact of varying climate conditions caused by climate change on ESE score requirements. By changing the farmer's degree of risk aversion from 0 to 1 under different cost parameters, a series of simulations are conducted for a high success probability project under favorable climate conditions (where $b = 0.7$ in Figure 3.3), a neutral probability project under neutral climate conditions (where $b = 0.5$ in Figure 3.4), and a relatively low probability project under unfavorable climate conditions (where $b = 0.3$ in Figure 3.5). After observing and comparing these three figures, we conclude

our analysis with three findings below. In general, any factor that can increase the success probability of the project and the repayment probability can directly or indirectly reduce the requirements of the ESE score.

By examining Figure 3.3, Figure 3.4, and Figure 3.5, we notice that, as the probability of success decreases, the ESE score requirement continues to increase if the control cost remains constant; i.e., the probability of a project's success is inversely proportional to the required ESE score. This is because a higher success rate leads to an increased farmers' ability to repay their loans, thus lowering the risk of default for the insurance company. As a result, the insurance company can lower its ESE score requirements when lending to farmers.

From inspecting any one of the three following figures, we see that, as the degree of risk aversion increases, the required ESE score decreases monotonically. This is because an increase in farmers' risk aversion score leads to a more cautious contract behavior, reducing the probability of a strategic default and increasing the probability of repayment. In other words, a higher degree of risk aversion on the part of the farmers implies a lower level of risk for the insurance company, leading to a lower ESE score requirement.

Again, from inspecting any one of the three figures, we observe that an increase in the cost parameter will result in a higher requirement for the ESE score. An increase in the cost parameter raises the production expenditure of farmers and burdens their economic resources, thus reducing their profits. Therefore, to avoid a high default probability associated with high costs, the corresponding ESE score requirement should increase. Hence, as the cost parameter increases, the insurance company should raise its ESE score requirement to reduce the risk of default among farmers.

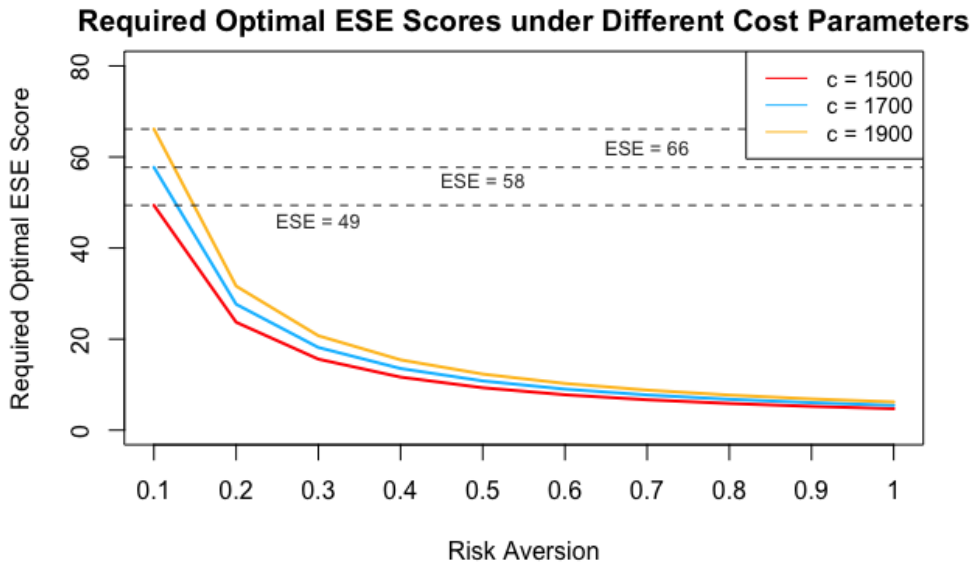


Figure 3.3: High Successful Probability (Favorable Climate Conditions)

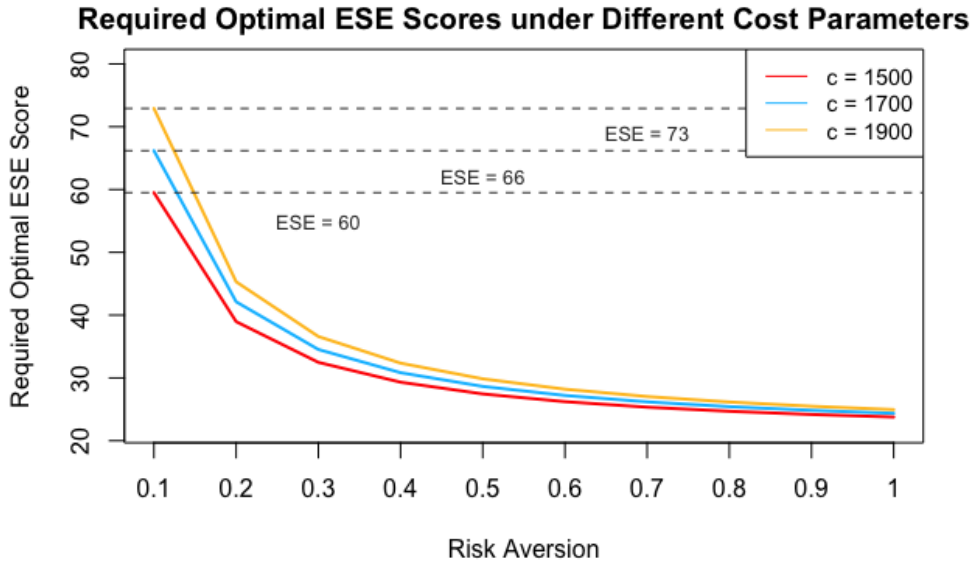


Figure 3.4: Neutral Successful Probability (Neutral Climate Conditions)

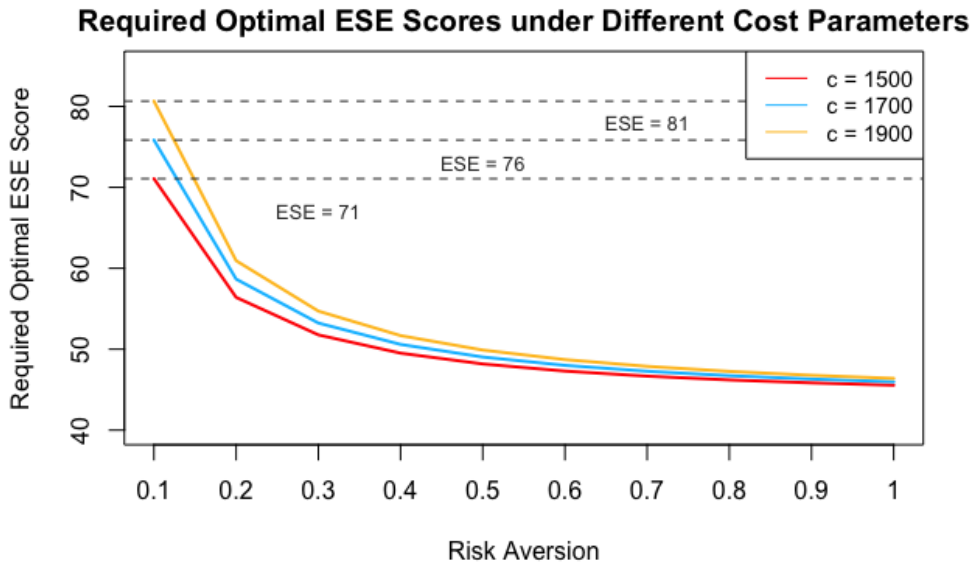


Figure 3.5: Low Successful Probability (Unfavorable Climate Conditions)

Finally, we arrive at the question of how changes in crop yields impact the required ESE score. To answer this question, we vary the values of yields and observe the effect on the required ESE score. Specifically we consider two scenarios: a 'Low Yield Case' where $\overline{Y} = 1,000$ and $\underline{Y} = 500$, and a 'High Yield Case' where $\overline{Y} = 600$ and $\underline{Y} = 300$". The results of our simulation in Figure 3.6 show that there is a negative correlation between crop yields and the required ESE score, i.e., as crop yields increase, the required ESE score decreases. Conversely, as crop yields decrease, the required ESE score increases. When a borrower has lower yields, this can be taken as an indication of a lower income and a potentially higher risk of default; in other words, a higher requirement for borrowers with lower yields is a reflection of the lender's risk management strategy. By setting stricter requirements for borrowers with lower yields, the lender is attempting to minimize their exposure to financial risk and ensure that the borrowers are able to repay their loans on time and in full.

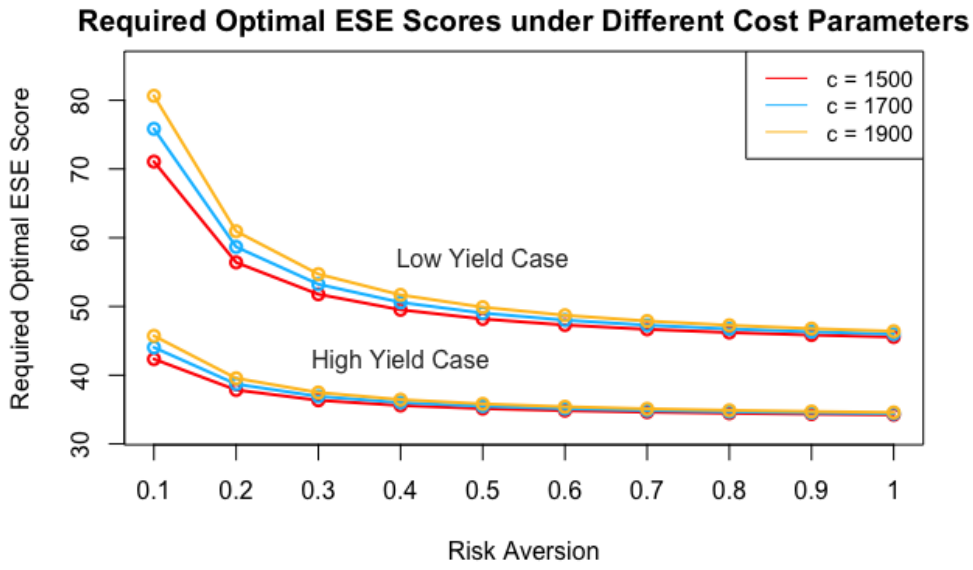


Figure 3.6: High Yields v.s. Low Yields

3.4 Conclusions

In this chapter, we have incorporated a novel concept coined as an individual–ESE score into a joint liability model for the purpose of evaluating the sustainability and creditworthiness of individual farmers acting as borrowers in the model in the face of climate change. To achieve this objective we initially considered the expected profit of an individual borrower resulting from the specification of his/her utility function. Our primary goal was to determine the optimal individual–ESE score and to investigate its relationship with key factors such as group size and loan ceiling. By understanding these relationships, we aimed at providing useful insights that benefit both the lending companies and the farmers acting as borrowers. The optimal individual–ESE score serves as a valuable tool for lenders by offering a better understanding of borrowers' risk profiles and informing decisions about appropriate group sizes for lending. For farmers, it can facilitate improved access to credit, thereby fostering greater financial inclusion. Integrating this score into credit evaluation processes not only supports better informed lending decisions but also promotes sustainable development outcomes by aligning financial practices with environmental and social considerations. We listed the main results of this paper conveniently in five propositions. Proposition 1 focused on finding an optimal individual-ESE score. Proposition 2 aimed at

collapsing two constraints on the loan ceiling into a single constraint. Proposition 3 demonstrated that a higher individual-ESE score leads to a higher loan ceiling. Proposition 4 revealed that the optimal individual-ESE score and group size n have a negative relationship. Finally, Proposition 5 showed that the optimal individual–ESE score converges to a constant level when the group size reaches a large enough number.

Our study went on to incorporate the risk associated with an expected profit into the utility function of the farmers and analyzed the resulting mean-variance utility framework in the context of the individual ESE–joint liability model. The study additionally disentangled the probability of achieving success in a project into two constituent components, namely the impact of personal factors (represented by $k \times ESE$) and exogenous factors beyond individual farmer's control (represented by b), such as climatic and geographical conditions. Our findings reveal that the probability of a project's success is inversely proportional to the required ESE score. This implies that a higher likelihood of a project's success leads to an increased farmers' ability to repay their loans, thereby lowering the risk of default for the insurance company. As a result, the insurance company can lower its ESE score requirements when lending to high-quality farmers. We also found that an increase in farmers' risk aversion score leads to a more cautious contract behavior, reducing the probability of a strategic default and increasing the probability of repayment. Hence, as the risk–aversion score increases, the corresponding ESE score requirement decreases. Finally, an increase in the cost parameter raises the production expenditure of the farmers and places a burden on their economic resources which reduces their profits. Therefore, to avoid a high default probability associated with high costs, the corresponding requirement of an ESE score should be increased.

Lastly, we explored the effects of climate change on the required ESE score in order to ensure a successful project implementation. Our simulation results demonstrated that a higher likelihood of project success, driven by high yields resulting from favorable climate conditions, leads to a lower ESE score requirement in the proposed individual ESE–joint liability model. This suggests that incorporating climate scenarios into the evaluation of creditworthiness can result in better-informed lending decisions and ultimately contribute to more favorable sustainable development outcomes.

As an avenue for future research, this chapter paves the way for an exploration of a partially joint liability legal framework that distributes shared responsibility among parties for specific obligations. Unlike a complete joint liability framework, a partially joint liability framework assigns a proportionate debt share to each party. Moreover, the concept of a flexible joint liability introduces aspects of innovation. Participants begin with a collaborative lending similar to a standard joint liability contract but, in strategic default scenarios, opt for a punitive phase rather than immediate maximum penalties. After

a defined period (T as previously mentioned), defaulters are reintegrated, which offers a path for forgiveness and adherence to the flexible joint liability contract. The diverse joint liability arrangements highlight the need for a thorough and separate analysis. Future research could explore the application of various joint liability frameworks to enhance understanding and implementation in this area.

Furthermore, to extend the model in Chapter 3 to appeal to the actuarial domain, it can incorporate key actuarial principles such as risk pooling, premium calculation, and correlated risks. Introducing loss distributions and reinsurance mechanisms would align the model with actuarial practices for assessing extreme losses and managing shared liabilities. The model can also include dynamic economic factors like interest rate and income to capture long-term liability risks, while stochastic optimization techniques could be employed to balance group composition and heterogeneity. Additionally, actuarial approaches for valuing contingent liabilities, setting capital requirements, and integrating ESG factors would make the model more robust and applicable to real-world financial and risk management scenarios. These enhancements would provide practical tools for actuaries to assess, mitigate, and optimize joint liability frameworks effectively.

Chapter 4

Strategic Investment to Mitigate Transition Risks

4.1 Introduction

4.1.1 Climate Risks

Mitigating climate change requires rapid and extensive decarbonization of the global economy. Climate change poses severe risks, such as altering extreme weather patterns and threatening ecosystems. Nonlinear climate dynamics can lead to sudden, irreversible changes, such as a polar ice melt or an ecosystem collapse, representing critical tipping points in Earth's systems (Lenton et al. (2008)). The 2015 Paris Agreement underscores the urgency of limiting a global temperature rise to well below 2°C to prevent catastrophic outcomes, with net-zero carbon emissions needed by mid-century to meet this goal (Holden et al. (2018)).

Achieving decarbonization demands transformative changes across industries. While sectors like a power generation have made strides toward a renewable energy adoption, industries such as a steel and aviation lag due to underdeveloped low-carbon alternatives (Kober et al. (2020), Dietz et al. (2020)). These challenges highlight the importance of understanding climate risks, which fall into two categories: physical and transition risks. Physical risks encompass direct impacts of climate change, such as extreme weather events, while transition risks arise from shifting to a low-carbon economy driven by regulations, technologies, and societal demands (Carney (2015)). This chapter prioritizes transition

risks, as they offer more scope for a strategic intervention and remain underexplored in quantitative modeling (Cisagara (2024)).

4.1.2 Understanding Transition Risk

Transition risks result from structural changes in industries due to the shift toward a low-carbon economy. High-emission sectors, such as steel, aviation, and energy-intensive manufacturing, face challenges adapting to stricter regulations and cleaner technologies. Policy uncertainty adds complexity, creating financial risks for firms and investors (Holden et al. (2018)). These risks include stranded assets, increased credit risk, and volatility in asset returns (Semieniuk et al. (2021)).

While low-carbon policies, such as carbon taxes and emissions trading schemes, are essential for meeting climate objecitves, their implementation introduces non-trivial transitional challenges (Bank of England (2018)). Companies are now evaluated not only on their financial metrics but also on their resilience to climate risks (Schulte and Hallstedt (2018)). Addressing these risks effectively requires targeted strategies and robust quantitative models.

4.1.3 Low-Carbon Transformation: Balancing Benefits and Challenges

The shift toward a low-carbon economy offers opportunities, such as cost savings, market leadership, and innovation, alongside significant challenges. Companies like Tesla and BP have successfully aligned with low-carbon strategies, reaping benefits such as new revenue streams and enhanced competitiveness. However, this effort of transition is capital-intensive, with high upfront costs and regulatory uncertainty posing risks to companies (NGFS (2019), Schulte and Hallstedt (2018)). Additionally, the rapid pace of technological advancements can render current investments obsolete, further complicating long-term planning.

Policymakers and businesses must balance a climate action with an economic stability. In the next section, we introduce a stylized model to guide low-carbon transition strategies, helping industries navigate uncertainties and optimize their transformation efforts.

4.2 Preliminaries and Model Building

4.2.1 Motivations

Sector-Specific Strategies and Investments in the Low-Carbon Transition

Different industries face unique challenges and investment needs in the low-carbon transition. Key sectors include:

- Energy: Transitioning to renewable energy requires upgrading infrastructure and advancing storage technologies (Boyd (2013)).
- Transportation: Decarbonization efforts focus on electric vehicles, hydrogen fuel, and charging networks (Tyfield (2013)).
- Manufacturing: Industries such as steel and chemical processing are exploring low-carbon processes like hydrogen-based production and carbon capture technologies.
- Agriculture: Investments in regenerative practices and ecosystem preservation are critical for reducing emissions (van Veelen (2021)).

Transition costs are substantial. Estimates suggest global spending on net-zero initiatives could reach $275 trillion by 2050, with varying investment levels across sectors (McKinsey (2021b)). Tailored, sector-specific strategies are crucial to balancing costs and achieving decarbonization goals effectively.

Balancing Costs and Benefits of Low-Carbon Transition

Investing in the low-carbon transition confers great benefits across various industries by driving long-term sustainability, reducing regulatory risks, and enhancing competitive positioning. For energy-intensive industries such as manufacturing and transportation, reducing carbon emissions can lead to cost savings through improved energy efficiency and reduced fuel expenses. In the financial sector, supporting low-carbon initiatives can mitigate portfolio risks linked to climate change, while at the same time also opening new investment opportunities in green technologies and sustainable businesses. Industries such as real estate and construction benefit by developing energy-efficient buildings that attract environmentally conscious tenants, boosting property value. Additionally, adopting low-carbon strategies can enhance the brand reputation for corporate responsibility and

environmental stewardship, meet evolving consumer preferences for sustainability, and ensure compliance with future climate regulations, positioning companies for growth in a low-carbon economy. However, the upfront costs of adopting cleaner technologies can strain businesses, particularly if the returns on investment are not immediate. For example, investing in renewable energy infrastructure may tie up capital that could otherwise be used for business expansion or other needs.

Companies must carefully balance the costs and benefits of low-carbon investments. This involves determining the optimal investment ratio, which refers to the proportion of financial resources allocated to low-carbon initiatives relative to total investments. Companies that can effectively manage financial risks and seize the long-term benefits of low-carbon investments will likely be able to strengthen their competitiveness and resilience in a rapidly evolving global economy.

Given these considerations, this chapter proposes a stylized analytical framework to determine the optimal low-carbon transition strategy across different sectors. The framework aims at capturing the unique underlying dynamics and investment needs of each sector, helping industries navigate the complexities of transitioning toward a sustainable, low-carbon future.

4.2.2 Variables Descriptions

The parameters and variables in this framework are introduced with the aim of replicating the dynamics of low-carbon transition investments and their effects on various industry operations, offering insights into investment strategies. Below, we provide a detailed description of each variable and parameter.

Selling price per unit (p): This parameter represents the price at which the product of interest is sold in the market. Taking the selling price into consideration is to describe and quantify the revenue. In the context of low-carbon transition, businesses may find opportunities to increase prices for eco-friendly products due to an increased consumer demand for sustainability. For instance, Tesla's Model 3 and other EVs are often priced higher than traditional internal combustion engine vehicles (ICEs), yet some consumers are willing to pay the premiums for the environmental advantages.

Production Cost per unit (c): Production cost per unit includes all expenses incurred in manufacturing each unit, such as raw materials, labor, and energy. Analyzing production costs is essential to evaluating profitability. A study by Siddique and Sultana (2018) found that switching to sustainable sourcing practices initially raised production

costs due to the adoption of cleaner and more expensive technologies, but long-term benefits such as energy savings and brand value will outweigh the initial investment.

Selling Profit per unit $(p - c)$: This parameter reflects the profit earned from each unit sold, determined by subtracting production costs from the selling price. It captures the direct profitability of the product after accounting for costs. Profit is the primary goal for a company when making decisions, as it ensures financial viability and supports long-term growth, influencing how much of the company's revenue can be allocated toward investments, including those for sustainable transitions.

Total Asset (A): Total asset represents all financial resources available to a company, encompassing both current and long-term assets. This metric is essential for evaluating the company's capacity to invest in production and growth opportunities, providing insight into the scale and potential of the business. Research indicates that firms with larger asset bases are more inclined to invest significantly in sustainability initiatives, as they can support longer investment horizons and absorb higher upfront costs associated with such projects (Eccles et al. (2014)). For example, Amazon has invested $2 billion in renewable energy through its Climate Pledge Fund, leveraging its extensive financial resources to lead in corporate sustainability (Amazon (2020)). Consequently, total assets not only reflect a company's financial strength but also its commitment to sustainable practices and long-term value creation.

Transition Investment Ratio (α): The Transition Investment Ratio is a critical metric in our model, measuring the proportion of total assets allocated to low-carbon initiatives. This ratio is essential as it reflects a company's commitment to sustainability and its responsiveness to market dynamics and regulatory pressures. By strategically adjusting this ratio, businesses can enhance their reputation and mitigate risks associated with climate-related regulations and consumer preferences. Our primary goal with this framework is to identify the optimal transition investment ratio that balances financial performance with sustainability objectives, ensuring that companies thrive economically while still contributing positively to the environment.

Practically, the Transition Investment Ratio offers insights into how effectively a company aligns its financial resources with sustainability goals. A higher ratio signifies a greater investment in low-carbon initiatives, leading to long-term cost savings and competitive advantages. For instance, IKEA's allocation of over 3 billion pounds toward renewable energy infrastructure exemplifies the benefits of a strategic adjustment in this ratio (IKEA (2020)). By investing in solar and wind energy projects, IKEA not only reduced its operational reliance on fossil fuels but also achieved improved operational efficiency and lower carbon emissions. Improved operational efficiency can manifest in reduced energy costs,

as renewable energy sources often lead to lower utility bills. IKEA's renewable energy infrastructure allows it to produce a substantial portion of its energy needs in a sustainable manner, ultimately lowering the company's operational costs and enhancing its energy independence. Additionally, the transition to renewable energy directly impacts carbon emissions, as utilizing solar and wind energy decreases IKEA's greenhouse gas footprint. This shift aligns with regulatory expectations and resonates with consumers who favor environmentally responsible brands. Thus, the Transition Investment Ratio is pivotal in our model, encapsulating the strategic allocation of resources toward sustainability and driving both operational efficiency and lower carbon emissions.

As previously mentioned, undergoing a low-carbon transition and investing in this transition can decrease production costs and increase selling prices due to improved operational efficiency, stronger brand value, and an alignment with consumer preferences. However, in the current framework, we assume both the production costs and the selling prices to be constant for simplicity. This assumption allows us to isolate the decision-making process regarding the resource allocation and emphasize the role of the Transition Investment Ratio in balancing financial performance and sustainability objectives without introducing additional complexities. Future work could build on this framework by incorporating functional relationships between these parameters—such as production costs and selling prices—and the Transition Investment Ratio.

Enterprise Low-carbon Production Efficiency Coefficient (k): This coefficient measures the improvement in production efficiency of the firm resulting from low-carbon initiatives, with a value greater than 1 indicating a gain in efficiency due to cleaner technologies. In our framework, we assume a higher value of k signifies greater production efficiency of the firm after investing in the transition to sustainable practices. For instance, Siemens reported a remarkable 20% increase in production efficiency after transitioning from traditional fossil fuel-based energy to renewable energy sources and incorporating energy-efficient manufacturing technologies (AG (2019)). This transition not only reduced their carbon footprint but also streamlined their operations, demonstrating that investments in sustainability can yield great returns in efficiency.

Original Productivity Coefficient (β): The value of this coefficient ranges between 0 and 1. A lower value indicates that, given a fixed amount of raw materials, the firm's production efficiency is relatively low, resulting in fewer products being produced per unit of input, which reflects suboptimal resource utilization. Conversely, a value closer to 1 suggests higher efficiency in material utilization, allowing the enterprise to produce more output with the same raw material input, indicating both optimized resource allocation and production process efficiency. Changes in this coefficient can thus reflect the current production efficiency of the enterprise and offer insights into potential improvements un-

der low-carbon transition strategies. There is evidence to suggest that many firms that start with below-average efficiency have successfully turned their operations around after investing in cleaner technologies. Research in Weng et al. (2015) indicates that companies in the regions with lower initial productivity often experience the most significant gains when transitioning to more efficient, low-carbon systems. By enhancing productivity through sustainable practices, these firms not only improve their operational performance but also position themselves for greater profitability in the long run. Thus, the firms with lower β values should prioritize upgrades to enhance productivity and better align with sustainability goals.

Carbon Price (B): The carbon price signifies the cost of emissions imposed by regulatory frameworks and is essential to a company's profitability. This price is sensitive to regulatory or policy changes, with high carbon prices creating significant financial pressure on firms to reduce emissions. When the market carbon price is elevated, it can substantially diminish a company's profit margins, compelling businesses to reassess their operational strategies. In response to these challenges, companies are incentivized to invest in cleaner technologies and adopt sustainable practices to mitigate their carbon liabilities. As firms strive to maintain profitability while adhering to environmental regulations, understanding and managing the implications of carbon pricing becomes crucial for their long-term success.

Carbon Intensity (I): Carbon intensity refers to the amount of carbon dioxide (CO2) emissions produced per unit of output, typically measured in terms of emissions per product manufactured or per unit of energy consumed. It serves as a critical metric for companies transitioning to low-carbon production, as reducing carbon intensity is essential for minimizing environmental impact while maintaining operational efficiency.

In the literature, carbon intensity is usually defined as a measure of the environmental performance of a company's production processes. For instance, the IPCC emphasizes that carbon intensity is a key indicator of how effectively an organization utilizes resources and minimizes emissions in its operations (Change et al. (2014)). Similarly, the World Resources Institute (WRI) defines carbon intensity as a means to assess the relationship between output and emissions, allowing for comparisons across industries and over time (Institute (2015)). In addition, carbon intensity is defined more specifically as the ratio of expected annual emissions to expected revenue in Fang et al. (2019). This formulation highlights the firm's contribution to climate change per unit of revenue, thus linking environmental impact directly to economic output and enabling the pricing of emissions-related risks in financial evaluations.

Flow and Interaction Between Variables: The interaction between these vari-

ables provides a comprehensive framework for assessing the financial, operational, and environmental trade-offs involved in a low-carbon transition. For example, increasing the transition investment ratio (α) may initially raise firm's production costs (c), but this can lead to greater efficiency (k) and reduced carbon intensity (I) for the firm over time. Firms in markets with high carbon prices (B) are more likely to invest in low-carbon technologies, seeing long-term benefits from both regulatory compliance and enhanced profitability through premium pricing and consumer demand for sustainable products. By considering these variables, companies can strategically navigate the low-carbon transition, balancing short-term financial considerations with long-term environmental and competitive advantages. This chapter will build the analytical framework based on these parameters and variables in the following parts.

In summary, the variables and associated parameters of the analytical framework are described in Table 4.1.

Notation	Description
p	Selling Price per unit
c	Production Cost per unit
$p - c$	Selling Profit per unit
A	Total Asset
α	Transition Investment Ratio
k	Enterprise Low-carbon Production Efficiency Coefficient
β	Original Productivity Coefficient
B	Carbon Price
I	Carbon Intensity

Table 4.1: Notations Description Table

4.2.3 Optimal Transition Investment Ratio without Government Intervention

The transition towards a low-carbon economy presents a complex challenge for enterprises, acting as a double-edged sword with respect to both arising opportunities and costs. While embracing low-carbon transformation holds the promise of environmental sustainability and enhanced competitiveness, it also entails a substantial amount of financial investments and operational adjustments. For many enterprises, particularly those operating in carbon-intensive industries, the costs associated with transitioning to cleaner technologies and sustainable practices can be daunting. These costs may include investments in renewable

energy infrastructure, efficiency upgrades, and emission reduction measures, all of which require a substantial amount of capital expenditure and major operational restructuring in order to embark on the transition path.

As a result, some companies may hesitate to fully commit to low-carbon transformation, fearing the resulting financial burden and potential disruptions to their existing business models. This reluctant proclivity to adopt green practices could undermine firm's progress towards sustainability goals and exacerbate environmental challenges. In such cases, the role of government intervention becomes relevant in incentivizing and regulating enterprises' low-carbon transformation efforts.

Governments possess the ability to influence corporate behavior through various policy instruments, including subsidies, tax incentives, and regulatory measures. By introducing targeted subsidies or tax breaks for low-carbon investments, governments can help alleviate the financial barriers faced by enterprises and encourage them to pursue sustainable practices. These financial incentives can offset the upfront costs of transitioning to cleaner technologies and provide companies with the necessary resources to invest in green initiatives.

Furthermore, governments can enact regulatory policies that mandate emission reductions, promote energy efficiency, and set emissions targets for industries. By establishing clear and enforceable regulations, governments create a framework that incentivizes companies to prioritize low-carbon transformation and align their business strategies with broader environmental objectives. Regulatory measures can also level the playing field for all companies by imposing costs on carbon emissions and internalizing the externalities associated with pollution, thereby encouraging companies to internalize the true costs of their environmental impact.

Hence, this section aims at analyzing and delineating the optimal low-carbon transition investment strategies for companies, considering both scenarios with and without external government intervention. By comparing these two approaches separately, we hope to elucidate the efficacy and implications of different pathways toward sustainable transformation within the corporate landscape and to provide valuable insights and suggestions for policymakers, such as government entities, to better support and incentivize effective low-carbon strategies in the corporate sector.

Model Building

Maximizing profit holds a paramount position among the objectives of any company. This strategic action emanates from the foundational principle that profit arises from the differ-

ence between total revenue and total costs. Therefore, to effectively navigate and optimize profit margins, it becomes indispensable to dissect the constituents of both total revenue and total costs.

In our framework, total revenue is represented by the unit selling price per unit of the product (denoted as p) multiplied by the total production volume. The total production volume is represented by $2 \cdot \text{Sigmoid}(\alpha k) \cdot [(1-\alpha) \cdot A]^{\beta}$, where $\text{Sigmoid}(\alpha k) = \frac{1}{1+e^{-\alpha k}}$. This formulation integrates several key parameters: the transition investment ratio (α), indicating the allocation of total assets towards low-carbon initiatives; the enterprise low-carbon production efficiency coefficient (k), denoting the effectiveness of transitioning to cleaner technologies; and the original productivity coefficient (β), reflecting the baseline production efficiency. The sigmoid function encapsulates the joint impact of transition investments and the firm's production efficiency on production volume, ensuring a realistic representation of the firm's saturation effects. Meanwhile, the term $(1-\alpha) \cdot A$ captures the portion of the firm's total assets available for production activities, adjusted by the original productivity coefficient β to reflect the company's inherent efficiency before investing in the low-carbon transition. Thus, the term $[(1-\alpha) \cdot A]^{\beta}$ represents the firm's total production volume before investing in a low-carbon transition. Simultaneously, the firm allocates a portion α of its total assets A towards the low-carbon transition investment. This investment is expected to enhance production efficiency, leading to an overall increase in production volume by a factor of $2 \cdot \text{Sigmoid}(\alpha k)$. By combining these various elements, the formula provides a greater understanding of how strategic decisions regarding transition investments and production efficiency influence the overall production capacity of the company.

The design of this production function originates from a Cobb-Douglas production function, a widely used model in microeconomics that represents the relationship between inputs (typically capital and labor) and output in a production process. The general form of the Cobb-Douglas function is $Q = A \cdot K^{\beta} \cdot L^{1-\beta}$, where Q is the total output, A is a scaling or efficiency parameter, K is capital input, L is labor input, and β is the elasticity of output with respect to capital. In our framework, the term $[(1-\alpha) \cdot A]^{\beta}$ corresponds to K^{β}, where K represents capital. The sigmoid function, multiplied by 2, acts as an adaptive transformation of the efficiency parameter A in Cobb-Douglas, reflecting the transition investment's impact on production efficiency. Unlike the traditional Cobb-Douglas function, which often considers both capital and labor inputs, our model excludes the labor component due to the specific context of low-carbon investment decisions. This adaptation ensures that the production process incorporates diminishing marginal returns to transition investments (α) and captures the saturation effects as efficiency reaches its practical limits. Notably, we introduce a scaling factor of '2' at the beginning of the

sigmoid function. This adjustment is made primarily to ensure that the sigmoid's baseline value remains at least 0.5 when the input (αk) exceeds 0 in our model. By multiplying the sigmoid by '2', we normalize its baseline rate to 1, resulting in a more pronounced increase above 1 as the values of α (Transition Investment Ratio) or k (Enterprise Low-carbon Production Efficiency Coefficient) rise. This modification aligns with our prior expectations about this relationship.

Regarding robustness, our results are likely to remain valid with alternative production functions such as Cobb-Douglas or CES (Constant Elasticity of Substitution), as these also exhibit concavity and diminishing marginal returns, which are key economic properties captured by our sigmoid-based framework. The trade-offs between transition investments and production efficiency, as well as the insights on profitability, depend on these shared characteristics and are not restricted to the functional form that we use for our study. However, CES functions, which allow for variable elasticity of substitution between inputs, could add an additional degree of flexibility and provide further insights into how substitutability between capital and efficiency affects production outcomes. While closed-form general solutions may be challenging with alternative production functions due to their increased complexity, numerical simulations could be in principle applied to extend our model to other functional forms. This flexibility demonstrates the potential for generalizing our framework while still maintaining the principle notions of diminishing returns and efficient allocation of low-carbon investments.

Based on the descriptions provided above, we can formulate the expression for 'Revenue' in the framework as follows:

$$\begin{aligned} \text{Revenue} &= \text{Selling Price per unit} \times \text{Unit Amount} \\ &= p \times \left\{ 2 \cdot \text{ Sigmoid } (\alpha k) \cdot [(1-\alpha) \cdot A]^{\beta} \right\} \\ &= p \times \left\{ \left(\frac{2}{1+e^{-\alpha k}} \right) \cdot [(1-\alpha) \cdot A]^{\beta} \right\} \end{aligned} \tag{4.1}$$

Next, we address the 'Cost' component of the framework, which comprises both the 'Production Cost' and the 'Carbon Cost'. Denote 'c' as the production cost per unit. The production cost can then be expressed as 'c' multiplied by the unit amount (equivalent to the unit amount in the revenue part). Additionally, we incorporate the carbon cost in our framework, which is calculated as the product of the 'Carbon Intensity', the unit amount, and the 'Carbon Price'. Here, the 'Carbon Intensity' is defined as $\left(2 - \frac{2}{1+e^{-\alpha k}}\right)$, a value ranging from 0 to 1. When αk is zero (signifying no transition investment), the carbon intensity reaches its highest level of 1. Conversely, as transition investment increases, the

carbon intensity decreases, indicating a reduction in emissions due to greater transition efforts. As mentioned, we define 'Carbon Intensity' as the 'Carbon Emission' divided by the total production volume, aligning with the traditional understanding of 'Carbon Intensity' discussed earlier. Based on these considerations, we can formulate the 'Cost' component in the framework as follows:

$$\begin{aligned}
\text{Cost} &= \text{Production Cost} + \text{Carbon Cost} \\
&= \text{Production Cost Per Unit} \times \text{Unit} + \underbrace{\text{Carbon Intensity} \times \text{Unit}}_{\text{Carbon Emission}} \times \text{Carbon Price} \\
&= c \cdot \left\{ \left(\frac{2}{1+e^{-\alpha k}} \right) \cdot [(1-\alpha) \cdot A]^{\beta} \right\} \\
&\quad + \left(2 - \frac{2}{1+e^{-\alpha k}} \right) \cdot \left\{ \left(\frac{2}{1+e^{-\alpha k}} \right) \cdot [(1-\alpha) \cdot A]^{\beta} \right\} \cdot B
\end{aligned} \tag{4.2}$$

Thus, the total profit π is calculated as:

$$\begin{aligned}
\pi &= \text{Revenue} - \text{Cost} \\
&= \left[p - c - \left(\frac{2e^{-\alpha k}}{1+e^{-\alpha k}} \right) \times B \right] \times \left[\frac{2}{1+e^{-\alpha k}} \times [(1-\alpha) \times A]^{\beta} \right]
\end{aligned} \tag{4.3}$$

To determine the optimal value of α under various conditions, we begin by taking the first-order derivative of the total profit with respect to α. This yields a necessary first-order condition (FOC) as follows:

$$\frac{2((1-\alpha)A)^{\beta} \left(-\frac{2Be^{\alpha(-k)}}{e^{\alpha(-k)}+1} - c + p \right)}{e^{\alpha(-k)}+1} = 0 \tag{4.4}$$

This equation helps us identify potential critical points of maximizing the Equation (4.3). To further analyze these points, specifically to assess their convexity and determine whether they are minima, maxima, or saddle points arising from the formulation of the model, we compute the second-order derivative with respect to α. The resulting second-order condition (SOC) is used to evaluate the nature of these critical points.

$$\begin{aligned}
&\frac{1}{(\alpha-1)^2 \left(e^{\alpha k}+1\right)^3} 2\alpha e^{\alpha k} (A - \alpha A)^{\beta} \left(-(\alpha-1)^2 k^2 \left(e^{\alpha k}-1\right) \right. \\
&\left. +(\beta-1)\beta \left(e^{\alpha k}+1\right)^2 + 2(\alpha-1)\beta k \left(e^{\alpha k}+1\right) \right)
\end{aligned} \tag{4.5}$$

From the analysis of the second derivative, we observe that the value is less than 0 when α and β are both within the range of (0, 1). Consequently, the optimal value of α maximizes profit, aligning with the expectations set by the problem.

Our first objective is to understand how changes in specific parameters affect the optimal transition investment ratio. For instance, we aim at analyzing how the optimal value of α adjusts when the carbon price (B) increases from 0 to 2. This analysis will elucidate the impact of carbon pricing on the optimal transition investment ratio for the firm. However, explicitly expressing α in terms of other parameters analytically is challenging based on the FOC due to several inherent complexities. The equation features exponential terms such as $e^{\alpha(-k)}$, which complicate the isolation of α since these terms do not simplify easily into a linear or polynomial form. Additionally, the equation includes a combination of linear and non-linear terms involving multiple parameters (A, B, c, p, β, and k), further complicating the process. The presence of α in both the numerator and the denominator within the fractional expressions adds another layer of difficulty. Therefore, we will resort to numerical methods to demonstrate how the optimal transition investment ratio of the firm varies in response to incremental parameter changes.

Numerical Study

In this subsection, we conduct a detailed numerical analysis of each critical parameter affecting the optimal transition investment ratio (α). Recall the optimal transition investment ratio represents the proportion of firm's financial resources allocated towards its low-carbon initiatives relative to its total investment. A key variable is a specific variable within our framework that we systematically vary to observe its impact on the optimal α. By focusing on one variable at a time, we can isolate its effect and gain deeper insights into its role within the overall system. This is known in Economics as a comparative static exercise.

While holding all other variables constant, we determine the optimal value of α for a range of values of a specific key variable. We then graphically represent the relationship between this key variable and the optimal α. Additionally, we will present the maximum profit achievable for different values of the key variable. This approach allows us to visualize how changes in one variable influence both the optimal investment decision and the profitability of the company.

The baseline values of the variables in the framework are provided in Table 4.2. These values are set within a reasonable range for illustrative purposes and can be adjusted by readers interested in exploring scenarios across different industries. Building on this

foundation, we will systematically vary one variable at a time to different values and analyze the resultant changes in the value of optimal α.

Variable Name	Description	Baseline Value
p	Selling Price per unit	3.6
c	Production Cost per unit	1.6
$p-c$	Selling Profit per unit	2
A	Total Asset	100
k	Enterprise Low-carbon Production Efficiency Coefficient	2
β	Original Productivity Coefficient	0.8
B	Carbon Price	1

Table 4.2: Baseline Parameters for Analysis

From Figure 4.1 to Figure 4.4, the left plot illustrates how changes in the key variable affect the optimal transition investment ratio, aiming to maximize profit under each scenario. The right plot shows the corresponding maximum profit achieved under the optimal transition investment as the key variable changes. We will explain each figure in detail below.

Relationship Between Profit Margin ($p-c$) and Optimal Alpha (Figure 4.1)

Figure 4.1 reveals a distinctive pattern where the optimal value of alpha for the firm remains high and constant initially, even when the selling profit per unit is very low or negative, suggesting a strategic focus on substantial investments in machinery upgrades or efficiency improvements during periods of low profitability. This high investment level, indicated by a stable high value of alpha, which is likely to reflect a counterintuitive but proactive approach: companies invest heavily during financially challenging times to enhance production efficiency and reduce operational costs, aiming at setting the groundwork for future competitiveness and sustainability.

As the selling profit per unit begins to increase for the firm, the optimal value of alpha sharply declines, indicating that further heavy investments in transformation by the firm are no longer necessary. The decrease in the value of alpha suggests that the earlier investments by the firm have begun to yield improved operational efficiencies and higher profits, reducing the need for continued high expenditure in transformations. This strategic shift emphasizes a dynamic financial strategy where initial heavy investments by the firm during low-profit periods are scaled back as profitability improves, aligning the investment levels of the firm with the diminishing marginal benefits of further transformations.

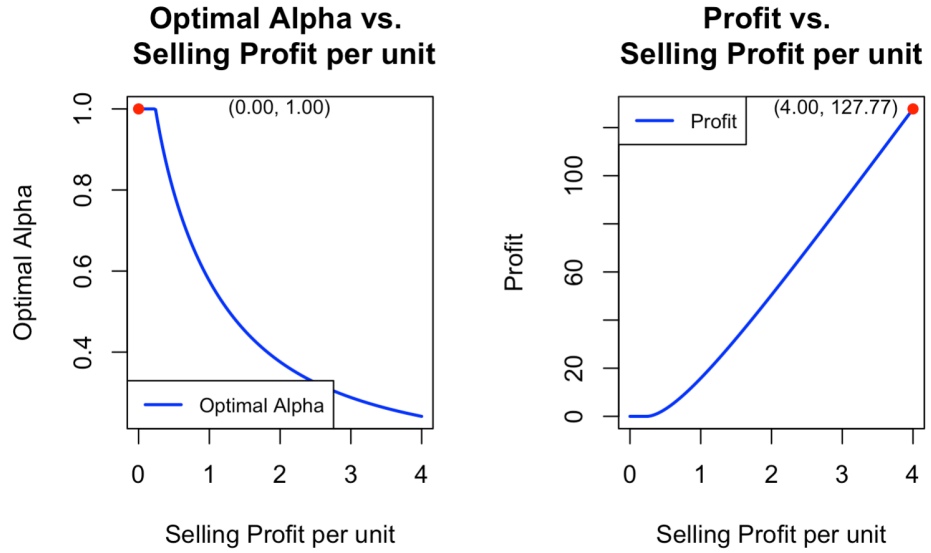


Figure 4.1: Optimal Level of Alpha and Profit Graph w.r.t. $p - c$

Effect of Increasing Carbon Prices (B) on Optimal Alpha (Figure 4.2)

As shown in Figure 4.2, when the carbon pricing variable B increases, it raises operational costs for the firm due to higher carbon taxes. In response, the optimal investment ratio α also increases, as shown in the left plot. This adjustment indicates a strategic move by the firm to allocate more resources towards low-carbon transition initiatives, which enhances production efficiency and reduces carbon emissions, thereby helping to offset the financial burden imposed by the carbon tax.

However, the right plot shows that despite these adjustments, a higher carbon tax still leads to a reduction in overall profit. Even with an optimal increase in the low-carbon investment ratio α, the profit trend declines as B increases. This underscores the financial impact of rising carbon costs on the firm. Nonetheless, by investing strategically in low-carbon transition, the firm can mitigate the extent of this profit decrease, cushioning some of the adverse effects of carbon pricing. In essence, while higher carbon taxes inevitably compress profits, allocating resources towards reducing the carbon footprint can help alleviate the downward pressure on profitability, offering a partial buffer against the increasing tax burden.

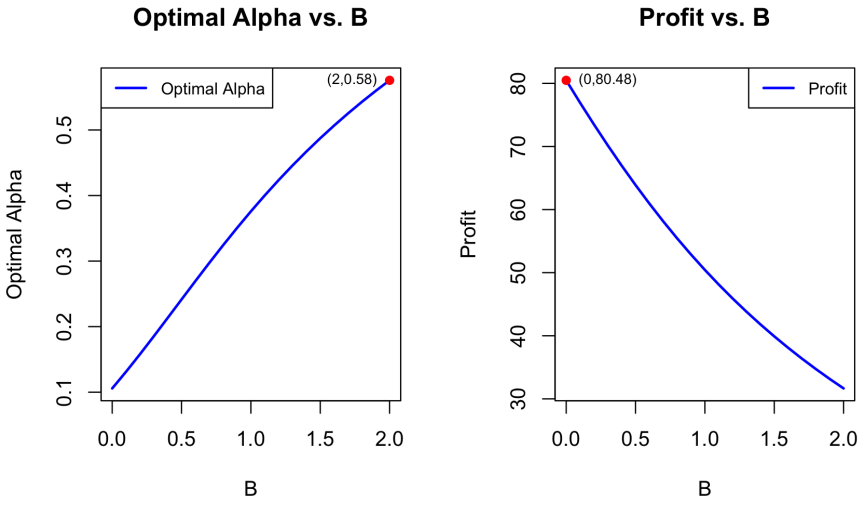


Figure 4.2: Optimal Level of Alpha and Profit Graph w.r.t. B

Impact of Efficiency Parameter (k) on Optimal Alpha and Profit (Figure 4.3)

Combining insights from both plots in Figure 4.3, we can observe that as the efficiency parameter k increases, profit consistently rises, while the optimal investment ratio α peaks around $k \approx 3.1$ and then gradually declines. This suggests that although higher k values are beneficial for the company's profitability, it does not require continuously increasing the investment ratio in low-carbon initiatives. Initially, when k is low, the marginal benefit from efficiency gains is high, encouraging a higher allocation to low-carbon investments. However, as k grows, the marginal efficiency gains diminish, making it less necessary to maintain a high α. At higher k values, profits continue to increase due to the enhanced production efficiency achieved by prior investments, allowing the company to reduce the investment ratio α while still benefiting from increased profits.

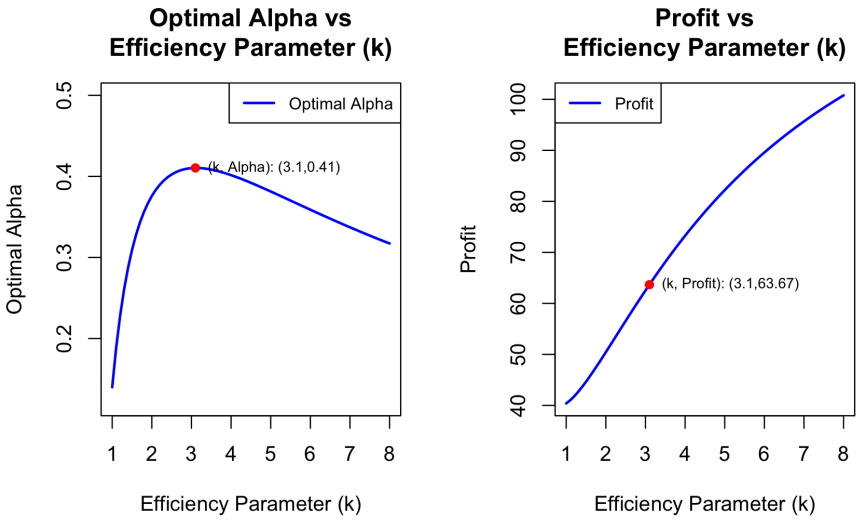


Figure 4.3: Optimal Level of Alpha and Profit Graph w.r.t. k

Influence of Original Productivity Coefficient (β) on Optimal Alpha and Profit (Figure 4.4)

The plots in Figure 4.4 demonstrate that as the Original Productivity Coefficient (β) increases, profit grows exponentially while the optimal transition investment ratio (α) declines. A higher β indicates strong baseline productivity, reducing the need for extensive low-carbon investments since the firm's production efficiency is already high. This makes the additional benefits from low-carbon investments less significant, allowing the firm to allocate fewer resources to α. The exponential growth in profit with rising β highlights the significant impact of inherent productivity on profitability, as firms with high baseline efficiency can achieve substantial profits without heavy investment in low-carbon initiatives. The decrease in optimal α reflects diminishing marginal returns on low-carbon investments for high-β firms, where the benefits of further investment in α are minimal.

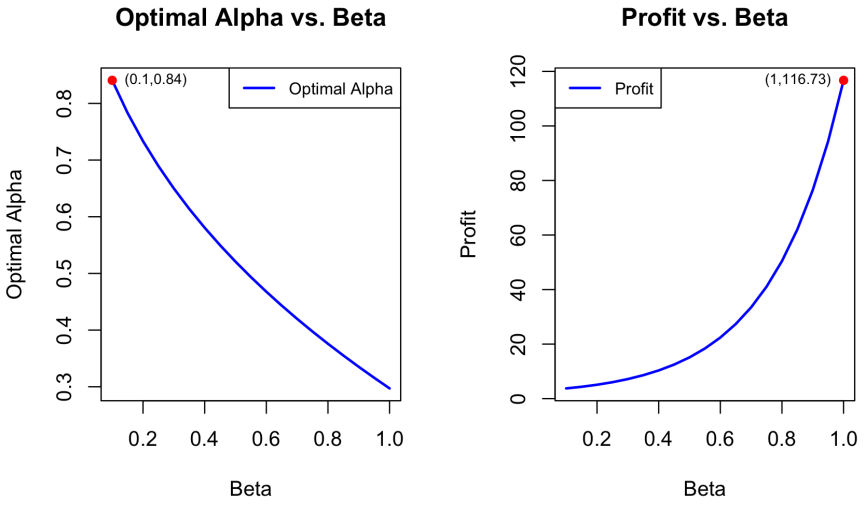


Figure 4.4: Optimal Level of Alpha and Profit Graph w.r.t. β

Sensitivity Analysis

The sensitivity analysis was conducted to evaluate the impact of variations in critical parameters on the optimal alpha, maximum objective value, and profit change rate. The parameters scrutinized include the enterprise low-carbon production efficiency coefficient (k), production cost per unit (c), original productivity coefficient (β), and carbon price per unit (B). Adjustments for each parameter were performed at $\pm 10\%$ and $\pm 50\%$ based on the baseline values in Table 4.2, except for β, where the upper limit was constrained to 1.00 due to inherent limitations. The results are concluded in the Table 4.3 and the analysis of each parameter is presented below.

k (Enterprise Low-carbon Production Efficiency Coefficient)					
Parameter Value	1.0	1.8	2.0	2.2	3.0
Optimal Alpha	0.1399250	0.3560002	0.3758660	0.3896782	0.4102122
Max Objective Value	40.38998	48.03570	50.43508	52.88084	62.52014
Alpha Change Rate (%)	-62.77263	-5.285352	0.000000	3.674763	9.137887
Profit Change Rate (%)	-19.916887	-4.757367	0.000000	4.849314	23.961613
c (Production Cost Per Unit)					
Parameter Value	0.80	1.44	1.60	1.76	2.40
Optimal Alpha	0.3011413	0.3571930	0.3758660	0.3970913	0.5191683
Max Objective Value	80.82201	56.40787	50.43508	44.53826	22.16245
Alpha Change Rate (%)	-19.880685	-4.967987	0.000000	5.647027	38.125899
Profit Change Rate (%)	60.249582	11.842530	0.000000	-11.69191	-56.05747
β (Original Productivity Coefficient)					
Parameter Value	0.40	0.72	0.80	0.88	1.00
Optimal Alpha	0.5806144	0.4107206	0.3758660	0.3430713	0.2971025
Max Objective Value	10.33973	36.31521	50.43508	70.34946	116.72607
Alpha Change Rate (%)	54.473780	9.273146	0.000000	-8.725114	-20.955217
Profit Change Rate (%)	-79.49894	-27.99613	0.000000	39.48516	131.43825
B (Carbon Price Per Unit)					
Parameter Value	0.5	0.9	1.0	1.1	1.5
Optimal Alpha	0.2414888	0.3505336	0.3758660	0.4002088	0.4874011
Max Objective Value	63.88578	52.87325	50.43508	48.11505	39.90827
Alpha Change Rate (%)	-35.751365	-6.739751	0.000000	6.476445	29.674174
Profit Change Rate (%)	26.669326	4.834262	0.000000	-4.600030	-20.871997

Table 4.3: Sensitivity Analysis Results

Enterprise Low-Carbon Production Efficiency Coefficient (k) :

The sensitivity analysis for k reveals that as k increases from 1.0 to 3.0, the optimal alpha rises from 0.14 to 0.41, which indicates that improving a firm's low-carbon production efficiency encourages a larger investment by the firm in low-carbon technologies. This trend is further reflected in the maximum objective value, which grows from 40.39 to 62.52, showing that better firm's efficiency leads to higher profitability for the firm. These results highlight a strong connection between firm's low-carbon production efficiency and its profitability.

Moreover, the alpha change rate starts with a large negative value (-62.77%) when $k =$

1.0 and makes a transition to a moderately positive change of 9.14% by $k = 3.0$. Similarly, the firm's profit change rate shifts from a notable loss (-19.92%) at $k = 1.0$ to a relatively large gain (23.96%) at $k = 3.0$. This indicates that as efficiency improves, firms not only increase their levels of investment in cleaner technologies but also experience greater profitability. These results underline the importance of improving low-carbon production efficiency, as it can serve as a key driver for both firm's optimal investment and its profit maximization.

Production Cost Per Unit (c):

As c increases from 0.80 to 2.40, the optimal alpha rises from 0.30 to 0.52, suggesting that firms must invest more to counterbalance higher production costs. However, this comes at the cost of its profitability, as the maximum objective value drops sharply from 80.82 to 22.16, showing that rising production costs significantly undermine a company's profitability potential.

The alpha change rate starts at -19.88% for $c = 0.80$, meaning that lower production costs lead to less investment in low-carbon technology. As c rises, the alpha change rate becomes positive (38.13%), showing that companies need to increase their investments to cope with higher costs. Profit change rates, meanwhile, vary drastically, reaching a peak at 60.25% for $c = 0.80$, followed by a steep decline to -56.06% at $c = 2.40$. This result illustrates the sensitivity of firm's profitability to changes in its production costs and highlights that controlling production costs is crucial for firms whose goal is to sustain profits while investing in low-carbon solutions. If its production costs rise unchecked, the firm may struggle to maintain its profitability, even with optimal investments.

Original Productivity Coefficient (β) :

When it comes to the value of β, the sensitivity analysis overall shows a unique pattern: as β increases from 0.40 to 1.00, the optimal alpha decreases from 0.58 to 0.30. This indicates that as its productivity improves, the firm requires less investment in low-carbon technologies to maintain profitability. Simultaneously, the maximum objective value surges from 10.34 to 116.73, highlighting the substantial impact of productivity on overall profitability of the firm. This result emphasizes that improving productivity not only reduces the required investment by the firm but also dramatically boosts the firm's profit potential.

The change rate of alpha starts at a high level (54.47%) when $\beta = 0.40$ and drops to -20.96% at $\beta = 1.00$, showing that better productivity substantially reduces the need for the firm to increase the level of investment. In terms of profitability, the profit change rate for the firm ranges widely, from -79.50% to a remarkable 131.44%. This further illustrates how vital firm's productivity improvements are for enhancing its profitability. Essentially, better productivity allows firms to generate substantially higher profits with a reduced

level of investment, making this an essential factor in optimizing their financial strategies of the firm.

Carbon Price Per Unit (B) :

Finally, the analysis of B shows that as carbon prices increase from 0.50 to 1.50, the optimal alpha rises from 0.24 to 0.49, suggesting that the firm needs to allocate more resources to low-carbon production as carbon prices rise. However, this comes at a cost, as the maximum objective value declines from 63.89 to 39.91, showing that higher carbon prices can lead to a reduced level of profitability.

The change rate of alpha is consistently negative, starting at -35.75% for $B = 0.50$ and improving slightly to -6.74% at $B = 1.10$, indicating that while increased carbon pricing necessitates the firm to invest more, its negative impact on profitability is relatively moderate. The change rate of the profit peaks at 26.67% for $B = 0.50$ and drops to -20.87% at $B = 1.50$. This suggests that while carbon pricing does affect firm's profitability, the sensitivity is not as severe for the firm compared to other variables. Companies can mitigate the impact of higher carbon prices through improved efficiency and strategic investments in cleaner technologies, ensuring that their profitability does not drastically diminish as carbon prices rise.

In comparing the variables in the framework, it is clear that the original productivity coefficient (β) is by far the most sensitive. Changes in the value of β lead to substantial shifts in the optimal value of alpha, maximum objective value, and the change rate of the profit. The strong relationship between productivity and profitability suggests that firms should prioritize improving productivity to maximize financial outcomes.

Conversely, the carbon price per unit (B) is the least sensitive variable in the framework. While changes in B do affect both the optimal alpha and the profitability, the overall impact for the firm is moderate compared to the other factors. Firms can manage the effects of carbon pricing through strategic investments in cleaner technologies, making it a more controllable factor in its financial performance.

In conclusion, firms should focus heavily on productivity improvements (β) to maximize their profitability while carefully managing their production costs (c) and their low-carbon production efficiency (k) to optimize investments. Carbon pricing (B), while important, presents a more manageable challenge to firms relative to the other factors.

Expanding the One-Period Model to a Multi-Period Model

In our initial analysis, we have focused on a static one-period model to evaluate the impact of transition investment on enterprise profitability. This framework provided basic

but useful insights into the immediate effects of various variables on a company's profit. However, real-world scenarios are dynamic in nature and involve changes over multiple periods. Therefore, as a next logical step, we are expanding our previous static framework to a multi-period setting to capture the evolving nature of the variables in the framework and their long-term impacts.

To achieve this, we revise the profit function to account for time dependency, reflecting the changes in key variables in the framework over multiple different periods. The time-dependent profit function is defined as follows:

$$\pi_t = \left[p - c_t - \left(\frac{2e^{-\alpha k_t}}{1 + e^{-\alpha k_t}}\right) \times B_t\right] \times \left[\frac{2}{1 + e^{-\alpha k_t}} \times [(1 - \alpha) \cdot A_t]^{\beta_t}\right] \tag{4.6}$$

This formulation captures the dynamic nature of transition investments, where p is the selling price per unit (assume p is a constant), c_t is the production cost per unit at time t, α is the transition investment ratio, k_t is the enterprise low-carbon production efficiency coefficient at time t, B_t is the carbon price at time t, and β_t is the original productivity coefficient at time t. By incorporating these time-dependent variables, we can more accurately model the impact of transition investments over multiple periods.

The resulting multi-period objective function can then be expressed as follows:

$$\max \left[\pi_t + g\pi_{t+1} + g^2\pi_{t+2} + \ldots + g^{n-1}\pi_{t+n}\right] \tag{4.7}$$

where g is the discount factor. This objective function accounts for firm's profits in the current period (π_t) and its discounted future profits (π_{t+1}, π_{t+2}, and so on), providing a more complete view of the impact of transition investments for the firm. The inclusion of future profits, discounted to the present time by the factor g, acknowledges the time value of money, opportunity costs, and inherent risks faced by the company. The discounting factor also acknowledges the uncertainties associated with long-term projections, ensuring a realistic assessment of the enterprise's financial trajectory.

By using this multi-period framework, we aim at finding the optimal value of α that maximizes the overall profit of the firm across multiple periods. The multi-period model is particularly useful for understanding the long-term implications of transition investments for the firm. Transitional investments typically incur immediate costs but generate substantial future benefits for the firm. For example, investments in low-carbon technologies might initially increase production costs for the firm due to the capital expenditure required for new equipment or processes. However, over time, these investments can lead to substantial efficiency gains, cost reductions, and improved regulatory compliance, which enhance future profits of the company.

Short-Term vs. Long-Term Observation:

Our goal is to find the optimal value of α to achieve the maximum objective function value for the firm over different observation periods. For simplicity, we will compare a shorter-term observation (of 3 years) with a longer-term observation (of 6 years).

- Short-Term Observation (3 years): This period allows us to understand the immediate and near-future impacts of transition investments for the firm. It is suitable for enterprises looking for quick wins and immediate adjustments in their strategies.
- Long-Term Observation (6 years): This extended period captures the long-term benefits and potential drawbacks of transition investments faced by the company. It helps the enterprises' plan strategically for sustained growth and ensures compliance with evolving regulations and market conditions.

By comparing these two observation periods, we can evaluate how the optimal value of α varies based on the length of the planning horizon for the company. This comparison will provide important insights into the trade-offs between short-term gains and long-term sustainability, guiding enterprises in making informed decisions about their transitional investments. The numerical exercise will be presented in the next subsection to illustrate these findings.

4.2.4 Decarbonization Scenario Analysis

Introduction to Decarbonization Scenarios

To illustrate the practical application of our multi-period framework, we examine four decarbonization scenarios by means of a numerical method: Immediate, Quick, Slow, and No Decarbonization. Decarbonization scenarios are strategic frameworks used to explore and analyze the pathways by which industries, economies, and societies can make a transition from high-carbon to low-carbon or carbon-neutral operations. These scenarios are designed to address the urgent need to mitigate climate change by reducing GHG emissions, particularly carbon dioxide (CO2), which is viewed as a major contributor to human-induced global warming. Each decarbonization scenario represents a different approach, pace, and scale of implementing low-carbon technologies, policies, and practices.

These scenarios draw inspiration from the pathways outlined in IPCC's Representative Concentration Pathways (RCPs) and Shared Socioeconomic Pathways (SSPs) (IPCC

(2018, 2022)), which provide a framework for modeling emissions trajectories under varying socio-economic, policy, and technological conditions. For example, the Immediate Decarbonization scenario aligns with the stringent measures seen in RCP1.9/SSP1-1.9, where immediate, large-scale investments aim to limit global temperature rise to 1.5°C (IPCC (2018)). Conversely, the No Decarbonization scenario parallels RCP8.5/SSP5-8.5, representing a "business-as-usual" trajectory with minimal climate action and escalating carbon emissions (IPCC (2023)).

By analyzing these different scenarios, we can better understand how decarbonization strategies impact enterprise profitability over multiple periods and identify the optimal transition investment ratio (α) for each strategy.

Immediate Decarbonization:

In this scenario, significant and immediate investments are made in low-carbon technologies and practices. The goal is to achieve rapid reductions in carbon emissions as quickly as possible. This approach mirrors the aggressive pathways described in RCP1.9/SSP1-1.9, where rapid decarbonization is pursued through stringent regulatory measures, substantial financial incentives, and rapid deployment of renewable energy and energy efficiency technologies (IPCC (2018)). Immediate decarbonization is typically driven by strong political will and societal urgency to address climate change impacts swiftly. While effective in achieving fast emissions reductions, it may entail higher short-term costs and risks of over-investment.

Quick Decarbonization:

Quick Decarbonization involves a fast-paced yet balanced transition. Investments in low-carbon technologies are substantial but spread out over a few years rather than all at once. This scenario resembles the pathways described in RCP2.6/SSP1-2.6, where emissions reductions are significant but distributed over time to balance economic feasibility and technical constraints (IPCC (2022); Agency (2022)). Quick decarbonization includes strategies such as accelerated deployment of renewable energy, electrification of transportation, and enhanced energy efficiency measures. It offers a cost-effective and pragmatic approach to achieving climate goals while reducing the economic and logistical burden of an immediate overhaul.

Slow Decarbonization:

The Slow Decarbonization scenario adopts a more gradual approach to transitioning to a low-carbon economy. Investments and regulatory changes are implemented over a longer period of time, allowing for a smoother transition with less immediate economic disruption. This approach corresponds to RCP4.5/SSP2-4.5, a "middle-of-the-road" pathway

where moderate policies and socio-economic factors lead to incremental emissions reductions over time (IPCC (2023)). While less disruptive, this scenario risks delaying critical action, potentially increasing long-term mitigation costs and reliance on future technological breakthroughs.

No Decarbonization:

In the No Decarbonization scenario, no substantial efforts are made to reduce carbon emissions. This approach parallels the "business-as-usual" trajectory represented by RCP8.5/SSP5-8.5, where industries and societies continue with their current high-carbon practices (IPCC (2018)). Continued reliance on fossil fuels results in escalating carbon emissions, environmental degradation, and increased climate risks (Agency (2022); IPCC (2023)). This scenario is widely regarded as unsustainable, as it amplifies the adverse economic, environmental, and social impacts of climate change over time.

Variable Changes and Rationale for Each Decarbonization Scenario

To effectively analyze the aforementioned impacts of different decarbonization strategies, it is essential to understand how key variables in the framework evolve under each scenario and the rationale behind these changes. This part delves into the specific modifications of variables such as the enterprise low-carbon production efficiency coefficient (k_t), production cost per unit (c_t), original productivity coefficient (β_t), and carbon price (B_t) for each of the decarbonization scenarios: Immediate, Quick, Slow, and No Decarbonization. By doing so, we can better comprehend the short-term and long-term effects of the transition on firm's profitability and identify the optimal transition investment ratio (α) for the firm in each case.

Immediate Decarbonization:

In the Immediate Decarbonization scenario, enterprises make substantial investments in low-carbon technologies immediately. This aggressive approach aims at achieving rapid decarbonization and substantial reductions in carbon emissions in the shortest possible time. As a result, the enterprise's low-carbon production efficiency coefficient (k_t) starts at a very high level due to a substantial initial level of investment, reflecting major improvements in production efficiency of the firm. Over time, as the immediate benefits are realized, k_t decreases gradually. The production cost per unit (c_t) of the firm decreases quickly as economies of scale and technological efficiencies are realized from the new low-carbon technologies. The original productivity coefficient (β_t) increases initially due to enhanced production capabilities of the firm but stabilizes over time as the effects of initial

investments are more fully integrated. The carbon price (B_t) remains at a very high level due to stricter regulations and market adjustments that favor low-carbon outputs.

Quick Decarbonization:

The Quick Decarbonization scenario involves rapid but slightly less aggressive investments compared to the Immediate scenario. Enterprises aim at achieving substantial decarbonization within a relatively short time frame but spread their investments over a few periods of time to balance costs and benefits. Consequently, the enterprise's low-carbon production efficiency coefficient (k_t) increases quickly but not as sharply as in the case of the Immediate scenario. The efficiency gains are substantial for the firm but spread over a slightly longer period of time. The production cost per unit (c_t) decreases rapidly but not as dramatically, reflecting more moderate cost reductions. The original productivity coefficient (β_t) shows a steady increase due to technological improvements and better resource allocation by the firm. The carbon price (B_t) keeps at a relatively high level, driven by increasing regulatory pressures and market responses.

Slow Decarbonization:

In the Slow Decarbonization scenario, enterprises adopt a gradual approach to transitioning to low-carbon technologies. This approach spreads investments over a longer period of time, allowing enterprises to manage costs and mitigate risks while steadily reducing carbon emissions. As such, the enterprise's low-carbon production efficiency coefficient (k_t) increases slowly, reflecting incremental improvements in production efficiency for the firm as investments are made gradually. The production cost per unit (c_t) decreases steadily but slowly, indicating progressive cost reductions over time. The original productivity coefficient (β_t) shows a gradual increase, correlating with incremental enhancements in productivity as new technologies are adopted by the firm over time. The carbon price (B_t) is set as a relatively low level, aligning with gradual regulatory changes and market adjustments.

No Decarbonization:

The No Decarbonization scenario assumes that enterprises do not make any major investments in low-carbon technologies. Instead, they continue with their current practices, resulting in minimal changes to carbon emissions and production efficiency. Consequently, there is no need to find the optimal transitional investment ratio under this scenario, as the transitional investment ratio, α, should always be zero.

Numerical Study

By analyzing these different scenarios, we can observe how different levels of investment and urgency in transitioning to low-carbon technologies affect key variables of the framework over time. This analysis helps us understand the trade-offs between immediate costs and long-term benefits, guiding enterprises in making informed decisions about their decarbonization strategies. The goal is to determine the optimal value of α that maximizes the overall profit across multiple periods, ensuring both short-term (3-year observation period) profitability and long-term (6-year observation period) sustainability. We will not consider the scenario of 'No Decarbonization' in this part, as there is no need to determine the optimal transitional investment ratio α under this scenario.

Year	**1**	**2**	**3**	**4**	**5**	**6**
k (Enterprise Low-carbon Production Efficiency Coefficient)						
Immediate	5	4.5	4	3.5	3	2.5
Quick	1.5	5	4.5	4	3.5	3
Slow	2	3	4	5	4.5	4
c (Production Cost Per Unit)						
Immediate	1.6	1.7	1.8	2	2.2	2.4
Quick	2.4	1.6	1.7	1.8	2	2.2
Slow	2.2	1.9	1.6	1.7	1.8	2
β (Original Productivity Coefficient)						
Immediate	0.9	0.85	0.8	0.75	0.72	0.7
Quick	0.6	0.9	0.85	0.8	0.75	0.72
Slow	0.7	0.8	0.9	0.85	0.8	0.75
B (Carbon Price Per Unit)						
Immediate	4	4	4	4	4	4
Quick	3	3	3	3	3	3
Slow	2	2	2	2	2	2

Table 4.4: Results for Scenario Analysis

Table 4.4 presents various variables under different scenarios of low-carbon transition for enterprises, focusing on production efficiency, costs, productivity, and carbon pricing.

In the Immediate Transition scenario, enterprises rapidly adopt low-carbon technologies, leading to a substantial increase in firm's production efficiency initially. This efficiency gain diminishes over time due to the fast adaptation rate and potential initial

over-investment by the firm, as reflected in the decreasing values of the enterprise's low-carbon production efficiency coefficient k. Production costs per unit rise initially due to high investments in new technologies, reaching their peak in later years as the benefits of the new technologies become more apparent and grounded. The original productivity coefficient β sees a rapid decrease due to the initial disruption and learning curve of the firm associated with adopting new technologies, eventually stabilizing at a lower level over time. High carbon pricing (B) is a crucial factor in this scenario, as it works to incentivize the rapid adoption of low-carbon technologies to mitigate the associated costs.

In the Quick Transition scenario, enterprises adopt low-carbon technologies at a moderately fast pace. This approach aims at achieving a rapid improvement in production efficiency, though at a slightly slower rate than in the Immediate scenario. As a result, the initial efficiency gains are significant but not as drastic, allowing the enterprise to avoid potential issues related to over-investment. Given the initially low value of k (starting at 1.5), there is an urgent need to enhance production efficiency quickly. To address this, the enterprise makes a transition investment before the second year, resulting in a sharp increase in k to 5 by the second year. This value then gradually declines each subsequent year, reflecting the natural efficiency losses due to wear and tear. Production costs per unit begin at a relatively high level (2.4), creating a strong need for transition investment to reduce costs. With the Quick Decarbonization approach, c drops significantly to 1.6 in the second year. However, it then rises slightly in the following years, likely due to normal depreciation and the ongoing costs associated with maintaining the new technology. Similarly, the original productivity coefficient β starts at a lower level (0.6), emphasizing the necessity for improvement. Due to the transition investment, β reaches a peak of 0.9 in the second year, reflecting increased productivity. Over time, it gradually decreases, aligning with the adaptation phase and the natural reduction in productivity gains. Carbon pricing B is maintained at a high but stable level (3) in the Quick Decarbonization scenario. This provides a continuous incentive for adopting low-carbon technologies, without imposing excessive financial burdens on the firm.

In the Slow Transition scenario, enterprises adopt low-carbon technologies at a gradual pace, resulting in a steady increase in production efficiency without abrupt changes. The Enterprise Low-carbon Production Efficiency Coefficient (k) begins at a modest value of 2 and gradually rises to 5 by the fourth year before leveling off slightly, indicating a more controlled and gradual improvement. This slower adoption rate minimizes the impact of efficiency losses due to wear and tear over time. The Production Cost Per Unit (c) starts relatively high at 2.2 but decreases progressively as the benefits of low-carbon technology adoption become more apparent, reaching a stable level at 1.8 by the final year. Such a pattern reflects slower capital outlays and reduced urgency in operational adjustments,

leading to consistent long-term savings. The Original Productivity Coefficient (β) also starts at a low value of 0.7 and improves steadily to a peak of 0.9 in the third year, maintaining stability thereafter. This steady incline suggests a smoother, less disruptive learning curve associated with gradual integration of new technologies. Finally, the Carbon Price Per Unit (B) remains low at 2 throughout the period, applying gentle pressure on firms to adopt low-carbon technologies while allowing them to adjust at a manageable pace, balancing the transition costs with sustained incentives.

The simulation results in Table 4.5 reveal an intriguing insight into the relationship between decarbonization strategies and firm profitability over different time horizons. In the short term, the Immediate decarbonization scenario yields the highest profits for the firm, while in the long term, profits are maximized under the Slow decarbonization strategy. The economic intuition behind this pattern lies in the interplay between the upfront investment requirements, efficiency gains, and financial planning associated under the different decarbonization strategies.

	Short-Period		**Long-Period**	
	Optimal α	Max Profit	Optimal α	Max Profit
Immediate	0.5692	173.0291	0.6216	212.0262
Quick	0.541	158.3299	0.5678	235.1407
Slow	0.4768	131.7251	0.4606	261.1729

Table 4.5: Optimal Investment Ratio and Max Profit for Short-Period and Long-Period Scenarios

In the short term, Immediate decarbonization leads to high profits because it allows the firm to achieve early efficiency gains and cost savings from the rapid adoption of low-carbon technologies. By quickly reducing energy and resource inefficiencies, the firm can capitalize on immediate cost reductions, enhanced operational performance, and potentially favorable regulatory incentives for early movers in decarbonization. These benefits outweigh the initial financial burden of the transition, resulting in the highest short-term profitability.

However, in the long term, the economic dynamics inevitably shift. The substantial upfront costs associated with Immediate decarbonization, including significant investments in low-carbon technologies and potential disruptions to existing operations, begin to erode long-term profitability. These high initial costs reduce the firm's capacity to sustain investments and adapt to evolving market and regulatory conditions over time. In contrast, the Slow decarbonization scenario allows the firm to distribute its transition costs more evenly over a longer period. This gradual approach minimizes financial strain, enhances

the firm's ability to adapt incrementally, and provides a greater degree of flexibility to optimize investments as low-carbon technologies become matured and more cost-effective. As a result, the Slow strategy enables the firm to achieve a higher long-term profitability. The Quick scenario represents a middle ground between Immediate and Slow strategies. In the short term, its profitability is lower than Immediate decarbonization due to moderate initial costs and slower realization of efficiency gains. However, in the long term, Quick decarbonization surpasses the Immediate scenario in profitability, as it balances higher levels of initial investments with a sufficient degree of flexibility to adapt and sustain low-carbon operations.

Overall, the results highlight the importance of strategic planning in decarbonization efforts. Firms must weigh the trade-offs between short-term efficiency gains and long-term financial stability when determining their optimal transitional investment ratios. While Immediate decarbonization may deliver immediate benefits, a gradual approach such as the Slow strategy offers a more sustainable path to maximizing long-term profits and resilience. This underscores a critical role of proactive, well-paced investment planning in achieving both financial and sustainability goals.

4.2.5 Optimal Transition Investment Ratio with Government Intervention

Based on the conclusions drawn from the previous subsections, it is evident that in certain scenarios, companies may exhibit a deficiency in transformational investments. In such instances, it becomes especially relevant for the government to undertake steps of market adjustments at both macro and micro levels, employing tools such as subsidies or tax penalties.

Subsidies can serve as a powerful incentive for companies by lowering the financial burden associated with making a transition to low-carbon technologies. These subsidies can take various forms, such as direct financial assistance, tax credits, or grants for research and development in green technologies. By reducing the initial costs, subsidies make low-carbon investments more attractive to companies, thereby accelerating the adoption of sustainable practices. For example, a company might receive a tax credit for every unit of renewable energy produced, which would directly offset its production costs and enhance profitability.

On the other hand, tax penalties act as a deterrent against environmentally harmful practices. By imposing higher taxes on carbon emissions or non-compliant activities, governments can create the risk of a financial disincentive for companies to continue with

high-carbon operations. This approach compels businesses to internalize the environmental costs of their actions, thereby encouraging them to seek out more sustainable alternatives. For instance, a carbon tax that increases with the level of emissions will drive companies to innovate and invest in cleaner technologies to minimize their tax liabilities.

Governments have a vested interest in incentivizing companies to undertake low-carbon transitions for several important reasons. Firstly, fostering a transition to low-carbon practices aligns with broader environmental and climate goals, mitigating the adverse effects of climate change and reducing overall carbon emissions in the country. Secondly, by encouraging businesses to adopt sustainable practices, governments contribute to the development of a more resilient and environmentally conscious economy. Additionally, promoting low-carbon transitions enhances a country's global competitiveness by positioning it as a leader in clean and sustainable technologies. This, in turn, attracts environmentally conscious investments, boosts innovation, and creates green jobs, fostering economic growth. Furthermore, by mitigating environmental risks and promoting sustainability, governments aim at improving public health and reduce the strain on healthcare systems in the country. Incentivizing low-carbon transitions therefore reflects a strategic approach to building a more sustainable and prosperous future, where economic development is harmonized with environmental stewardship.

Model Building

In the extended framework below, we categorize government regulatory tools into two groups: one from a macro-economic perspective and the other from a micro-economic perspective. The government acts as an 'invisible hand'. We assume that from a macro-economic perspective, the government subsidizes carbon prices that are too high in the market and taxes carbon prices that are too low in the market. On a micro-economic level, if a company surpasses a specified threshold for carbon emissions, the government levies taxes. Conversely, if a company's total emissions fall below the threshold, the government offers rewards and subsidies to companies, fostering increased enthusiasm for low-carbon transformation initiatives. To introduce the extension model of the the previous framework, additional variables and parameters are introduced to be described in Table 4.6.

Based on these variables and parameters, the expected total profit of the company after government subsidy or tax can be expressed as:

Notation	Description
s_1	Subsidy Rate for High Carbon Price
s_2	Subsidy Rate for Low Carbon Emission
q_1	Tax Rate for Low Carbon Price
q_2	Tax Rate for High Carbon Emission
$pr_1 = Pr(\text{ Carbon Price}(B) \geqslant cp)$	Probability of Subsidy for High Carbon Price
$pr_2 = Pr(\text{ Carbon Emission} \leqslant ce)$	Probability of Subsidy for Low Carbon Emission

Table 4.6: Variable and Parameter Description for the Extended Framework

$$
\begin{aligned}
\pi' &= \pi + s_1 \cdot (\text{ Total Unit Amount }) \cdot pr_1 - q_1 \cdot (\text{ Total Unit Amount }) \cdot (1 - pr_1) \\
&\quad + s_2 \cdot (\text{ Carbon Emission }) \cdot pr_2 - q_2 \cdot (\text{Carbon Emission }) \cdot (1 - pr_2) \\
&= \pi + s_1 \cdot \left\{ \left(\frac{2}{1 + e^{-\alpha k}} \right) \cdot [(1 - \alpha) \cdot A]^{\beta} \right\} \cdot pr_1 - q_1 \cdot \left\{ \left(\frac{2}{1 + e^{-\alpha k}} \right) \cdot [(1 - \alpha) \cdot A]^{\beta} \right\} \cdot (1 - pr_1) \\
&\quad + s_2 \cdot \left(2 - \frac{2}{1 + e^{-\alpha k}} \right) \cdot \left\{ \left(\frac{2}{1 + e^{-\alpha k}} \right) \cdot [(1 - \alpha) \cdot A]^{\beta} \right\} \cdot pr_2 \\
&\quad - q_2 \cdot \left(2 - \frac{2}{1 + e^{-\alpha k}} \right) \cdot \left\{ \left(\frac{2}{1 + e^{-\alpha k}} \right) \cdot [(1 - \alpha) \cdot A]^{\beta} \right\} \cdot (1 - pr_2)
\end{aligned}
\tag{4.8}
$$

where, the total profit π, excluding government intervention, is calculated as previously shown in Equation (4.3).

Numerical Study

In this subsection, we expand our analysis to explore the impact of government intervention on the optimal transition investment ratio (α) and profitability of the firm, through a revised objective function that incorporates both subsidies and taxes linked to carbon pricing and emissions. The new objective function, π', integrates additional terms representing financial incentives and penalties for the firm based on carbon-related metrics.

Our revised framework considers the following variables and parameters: subsidy rates (s_1 and s_2) for high carbon prices and low carbon emissions, respectively; tax rates (q_1 and q_2) for low carbon prices and high carbon emissions, respectively; and the probabilities (pr_1 and pr_2) of these rates being applied based on carbon price and emission thresholds. These factors are critical for determining the cost-benefit landscape of environmental compliance

and sustainable investment, as they directly influence the incentives and penalties faced by firms.

The baseline variables from our previous studies are maintained in this study for consistency, but now we add more variables and parameters to the framework. The baseline values for these new variables are shown in Table 4.7. Then we systematically vary each of the new variables and parameters individually to observe their effects on the optimal value of α and overall profitability for the firm. Graphical representations will be used to illustrate the relationship between these variables and the resulting economic outcomes, providing clear insights into the dynamics at play for the firm under different policy scenarios.

Notation	Description	Baseline Value
s_1	Subsidy Rate for High Carbon Price	0.8
s_2	Subsidy Rate for Low Carbon Emission	0.8
q_1	Tax Rate for Low Carbon Price	0.6
q_2	Tax Rate for High Carbon Emission	0.6
pr_1	Probability of Subsidy for High Carbon Price	0.5
pr_2	Probability of Subsidy for Low Carbon Emission	0.5

Table 4.7: New Baseline Values for Analysis

From Figure 4.5 to Figure 4.10, the left plot illustrates how changes in the key variable affect the optimal transition investment ratio, aiming to maximize profit under each scenario. The right plot shows the corresponding maximum profit achieved under the optimal transition investment as the key variable changes. We will explain each figure in detail below.

Impact of Probability of High Carbon Price Subsidy (pr_1) on Optimal Alpha and Profit (Figure 4.5)

As the probability of receiving a subsidy at higher carbon prices (pr_1) increases, the optimal investment ratio (α) for the firm decreases. This suggests that businesses, when expecting financial support through subsidies, feel less pressure to invest heavily in low-carbon technologies. The security provided by expected subsidies reduces the need for extensive investment by the firm in hedging against future regulatory changes or market conditions that favor lower emissions. This also highlights a strategic point for businesses: relying too heavily on subsidies might provide a short-term relief but could risk an insufficient preparation for a future with stricter environmental regulations.

Profit increases alongside higher subsidy probabilities, reinforcing the positive impact of government incentives on financial performance of the firm. For investors, this underscores the appeal of firms with strong expectations of a subsidy support, as subsidies directly enhance profitability of the firm. However, it's essential to remain cautious—companies that over-rely on subsidies may be less motivated to invest in long-term carbon reduction strategies, posing potential risks to the firm as market dynamics or regulations shift.

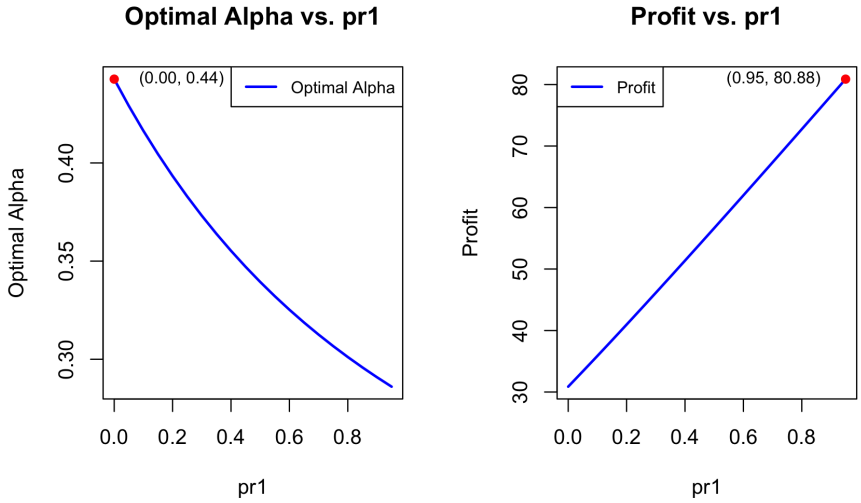


Figure 4.5: Optimal Level of Alpha and Profit Graph w.r.t. pr_1

Impact of Probability of Low Carbon Emission Subsidy (pr_2) on Optimal Alpha and Profit (Figure 4.6)

As pr_2 rises, it reflects a higher tolerance from the government for carbon emissions, effectively lowering the threshold required for firms to qualify for subsidies. This shift reduces the incentive for companies to invest in low-carbon transitions, as they can now receive financial support with less stringent emissions standards. Consequently, the optimal value of α, which represents the proportion of resources allocated to low-carbon investments, declines. This trend shows that firms are less motivated to prioritize sustainable transitions when subsidies become more accessible without rigorous emission targets.

On the profitability side, a higher pr_2 initially appears to be advantageous. With reduced pressure to invest in transition efforts, firms can allocate fewer resources to low-carbon initiatives, which boosts short-term profits. However, this profit increase may be

deceptive, as it comes at the expense of long-term resilience. Firms that minimize transition investments now could face significant financial risks if stricter carbon policies are introduced in the future. A sudden regulatory shift would leave them underprepared to adapt, potentially leading to substantial losses. This underscores the trade-off between short-term profit gains and the uncertain long-term stability that comes with underinvesting in sustainable practices.

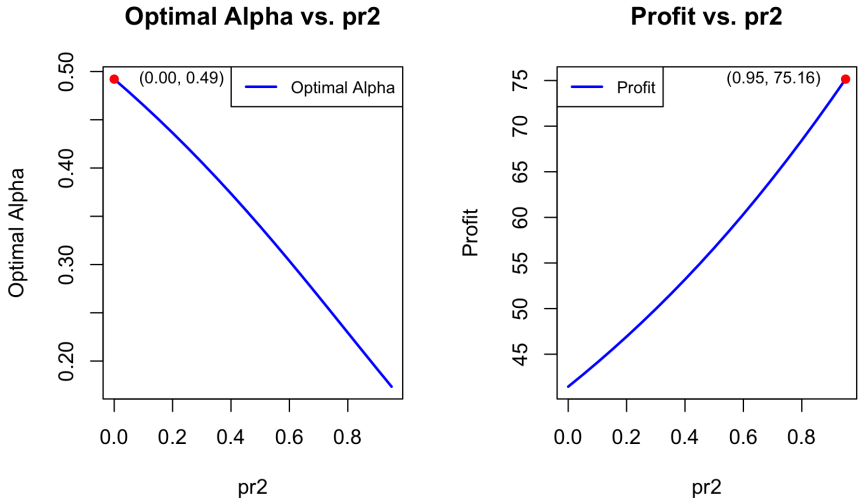


Figure 4.6: Optimal Level of Alpha and Profit Graph w.r.t. pr_2

Impact of Tax Rates for Low and High Carbon Emissions (q_1 and q_2) on Optimal Alpha and Profit (Figure 4.7 & Figure 4.8)

Higher tax rates on low carbon prices (q_1) or high carbon emissions (q_2) push the optimal value of α higher, indicating that businesses are incentivized to adopt cleaner technologies more aggressively when facing heavier tax burdens. This reflects the practical use of taxes as a tool for driving environmental sustainability—businesses react to them by increasing investments in green technologies to mitigate the financial impact of taxes. However, businesses must be strategic at the same time: while aggressive adoption of low-carbon technologies can reduce future tax liabilities for the firm, over-investing without aligning with long-term sustainability goals could lead to suboptimal resource allocation.

On the flip side, profits decrease as taxes rise, which makes high-emission activities less financially attractive for the firm. This highlights the financial risks for companies

slow to adopt cleaner practices, reinforcing the need for a more proactive sustainability planning. Investors should be mindful of firms exposed to significant carbon taxes, as their profitability may suffer without adequate mitigation strategies.

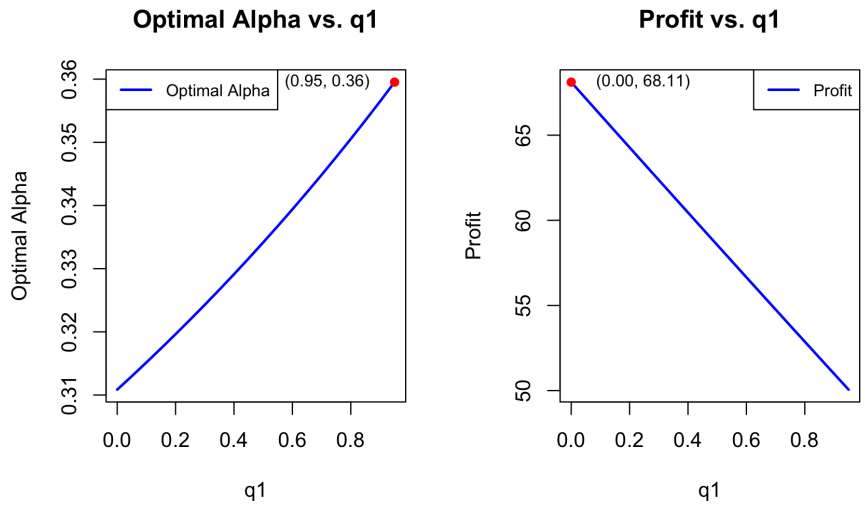


Figure 4.7: Optimal Level of Alpha and Profit Graph w.r.t. q_1

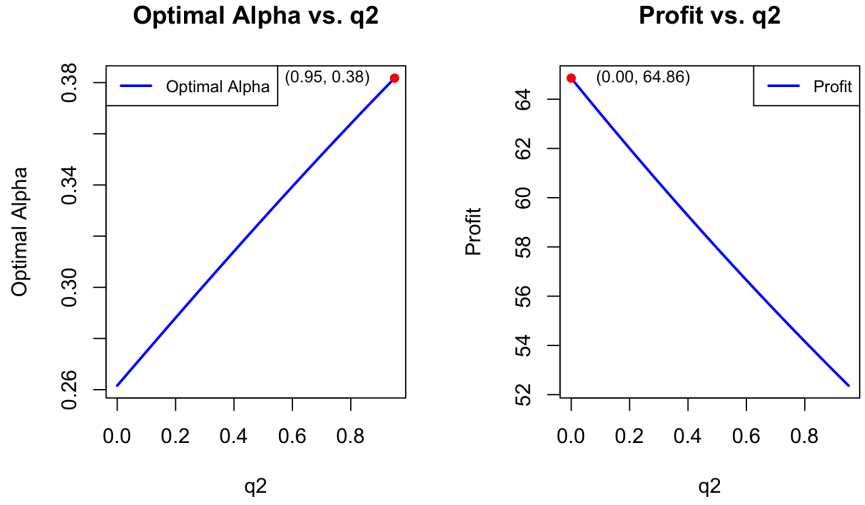


Figure 4.8: Optimal Level of Alpha and Profit Graph w.r.t. q_2

Impact of Subsidy Rates for High and Low Carbon Scenarios (s_1 and s_2) on Optimal Alpha and Profit (Figure 4.9 & Figure 4.10)

An increase in subsidy rates for high carbon price (s_1) or low carbon emission (s_2) scenarios results in a lower optimal value of α, showing that businesses feel less urgency to invest heavily in green technologies when financial incentives make it easier to comply with emission standards. While this may seem beneficial for companies in the short term, there is a risk that such subsidies could slow down the transition to greener technologies if businesses rely too heavily on them without making the necessary levels of investment in long-term sustainability.

Profit increases with higher subsidies, emphasizing the role of financial support in boosting corporate earnings. From an investment perspective, firms with access to a substantial amount of subsidies for meeting environmental standards are likely to be more profitable in the short term, making them more appealing to investors. However, investors should also assess whether these companies are using the subsidy windfall to fund long-term sustainable transformations, as over-reliance on temporary fiscal incentives could pose challenges for the firm in the face of evolving market or regulatory conditions.

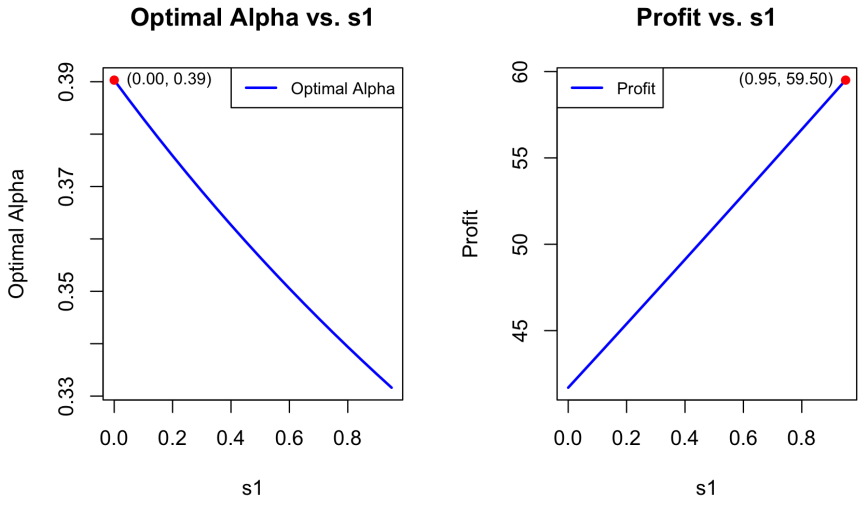


Figure 4.9: Optimal Level of Alpha and Profit Graph w.r.t. s_1

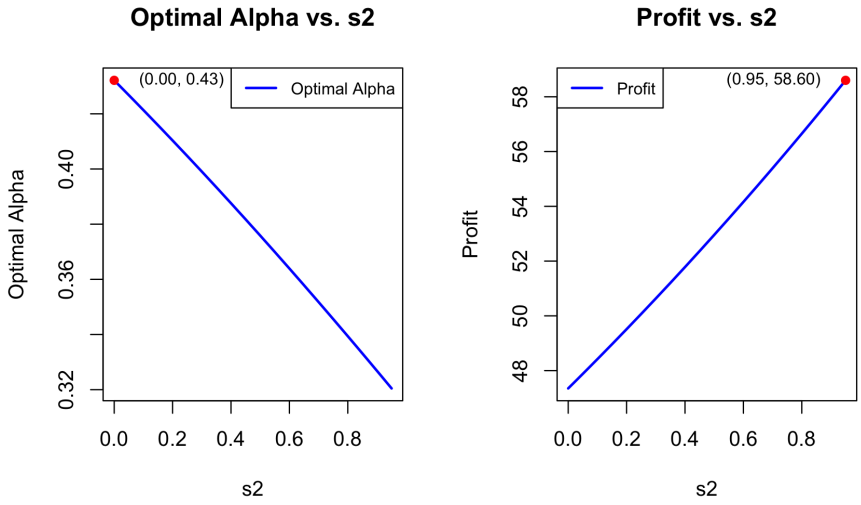


Figure 4.10: Optimal Level of Alpha and Profit Graph w.r.t. s_2

Policy Recommendations for Effective Low-Carbon Transitions

Encouraging effective low-carbon transitions requires a sensitive balance between assertive tax policies and a more targeted, limited subsidy approach. The above numerical analysis suggests several important and practical policy recommendations. While subsidies boost short-term profitability, they do not appear to incentivize long-term investment in low-carbon technologies adequately. This is evident from the decreasing trend in the optimal investment ratio (α) as subsidy probabilities increase. On the other hand, higher emissions taxes consistently drive higher values of α, indicating that tax policies play a more effective role in promoting robust investments in sustainable technologies.

- Expanding on Tax Strategies: A more structured and progressive carbon tax regime could enhance low-carbon transitions more effectively. By gradually increasing the cost of higher emissions, such a policy would not only penalize environmentally harmful practices but also provide businesses with the level of predictability they need in order to plan long-term investments in sustainability. This tax strategy could be further refined by introducing tax incentives for specific technologies that deliver substantial carbon reductions. For businesses, the clear financial penalties for high emissions and rewards for investing in sustainable solutions would create a strong motivation to prioritize clean technology adoption, ensuring a smoother and more predictable transition.

- Reassessing Subsidy Schemes: A more passive and targeted approach to subsidies

is recommended. Direct financial incentives should be limited to sectors of the economy where low-carbon alternatives are not yet economically viable or where early-stage technologies need support to scale. This approach minimizes the risk of companies relying on subsidies for profitability without making a genuine progress toward sustainability. It also encourages firms to seek out real, long-term solutions rather than temporary financial relief. Companies should anticipate stricter regulatory standards in the future and take a proactive approach to sustainability. Rather than depending on subsidies alone, businesses are advised to prioritize investments in scalable, low-carbon technologies that align with long-term industry trends. Building internal capabilities for sustainable practices and integrating environmental risk assessments into financial planning can help ensure resilience against future regulatory shifts. Additionally, companies should explore partnerships with other industry players or research institutions to advance low-carbon innovations, as these collaborations can reduce costs and accelerate progress toward sustainability goals.

In summary, integrating fiscal policies, such as carbon taxes and subsidies, with broader regulatory measures can amplify their effectiveness with respect to promoting long-term sustainability and reducing emissions. For instance, mandatory carbon emission disclosures and stricter compliance requirements would not only improve transparency but also make tax and subsidy regimes more impactful. Businesses would be incentivized to adopt greener practices both to avoid penalties and to benefit from tax reliefs. This would create a feedback loop where fiscal and regulatory measures reinforce each other, driving a greater amount of corporate investments in sustainability.

Additionally, corporate leaders should adopt a forward-thinking approach, moving beyond short-term financial benefits emanating from current subsidies and tax reliefs. Strategic investments in low-carbon technologies today will prepare businesses for future regulatory shifts, reducing the risk of compliance costs down the line. Companies should conduct detailed cost-benefit analyses that balance immediate economic advantages with future regulatory expectations. Proactively enhancing sustainability measures will not only align businesses with global environmental trends but also provide a competitive edge in an increasingly eco-conscious market. This approach places companies in a better position against stricter regulations in the future and boosts their appeal to environmentally aware consumers.

Finally, investors should closely monitor governmental shifts in environmental policies, as these will impact the profitability and strategic positioning of companies in their portfolios. There is a clear advantage in diversifying investments to include companies with strong environmental strategies or those investing heavily in green technologies. Firms that are better prepared for stringent environmental regulations are more likely to thrive, benefiting from enhanced market reputation and consumer loyalty. Additionally, investors

have the potential to influence corporate behavior by advocating for stronger sustainability practices within the companies they support. Active stewardship not only aligns investments with broader environmental goals but also helps mitigate the risks associated with regulatory changes, securing long-term profitability while promoting global sustainability objectives.

4.3 Conclusions

This chapter explores the dynamics of firm's strategic investment decisions that support the transition to a low-carbon economy, focusing on the interplay between government policies and corporate strategies. It reveals how corporations adjust their investment strategies based on anticipated returns and environmental impacts, with a cautious approach prevailing in the absence of government incentives. The analysis highlights that, while long-term sustainability goals are beneficial, they may impose significant short-term financial burdens on companies. When government intervention, through subsidies and taxes, is introduced, corporate behavior shifts dramatically. The findings in this chapter show that assertive tax policies on carbon emissions effectively drive firm's investment in sustainable technologies, aligning corporate actions with environmental targets. However, while subsidies enhance short-term profitability of firms, they can lead to a greater reliance on government aid and do not necessarily promote sustained innovation in green technologies.

The study suggests a balanced policy approach—where taxes are used assertively to stimulate investment, and subsidies are applied selectively to avoid dependency. This would encourage corporations to take proactive steps toward sustainability, aligning with long-term environmental goals rather than relying solely on government incentives. For stakeholders, particularly investors, the research underscores the importance of identifying companies that are actively adapting to evolving fiscal and regulatory landscapes. These companies are better positioned to meet future regulatory demands and capitalize on the growing market emphasis on sustainability, offering better long-term returns.

Extending the analytical framework to a multi-period setting under different decarbonization scenarios further reveals how the optimal investment ratio (α) evolves over time. While aggressive early decarbonization yields the highest short-term profits for the firm, slower approaches offer better long-term profitability due to the lower upfront costs. The analysis also shows that sustainable investment in low-carbon technologies is beneficial in the long run, particularly in scenarios involving gradual transitions. In conclusion, this study provides practical insights for policymakers and business leaders on how to promote

investments in low-carbon technologies. By implementing a balanced mix of taxes and targeted subsidies, governments can effectively encourage corporate sustainability efforts.

While the numerical examples presented in this chapter provide valuable insights into the dynamics of decarbonization strategies, there is a noticeably potential to extend the model into a more general framework that relies on abstract variables rather than specific parameter values. One direction for this generalization is to define the production function, cost structure, and investment dynamics in terms of general parameters (e.g., $f(\alpha, k)$ for production efficiency, $g(A)$ for asset allocation) without relying on the sigmoid function or fixed coefficients such as β and k. By introducing generalized mathematical forms, the model could capture a wider variety of industry-specific or region-specific decarbonization scenarios, making it applicable to diverse contexts.

Another avenue for extension lies in relaxing some of the simplifying assumptions made in this chapter. For example:

1. The framework could incorporate multi-factor production functions, such as CES, that allow for varying degrees of substitutability between capital and efficiency. This would provide more flexibility in modeling how firms adapt to changing technological and regulatory environments.

2. Introducing uncertainty variables to account for fluctuations in energy prices, regulatory costs, or technological advancements could make the framework more robust to real-world complexities.

3. Expanding the time horizon to include dynamic decision-making models would allow firms to adjust their investment strategies iteratively in response to external changes.

4. Making a transition from a partial equilibrium framework to a general equilibrium framework to capture first-order welfare effects that are important for drawing policy recommendations.

Finally, transitioning to a more generalized theoretical framework would facilitate the development of closed-form solutions or approximate analytical results for key metrics, such as optimal investment ratios or profit trajectories, under a range of assumptions. This would reduce reliance on numerical simulations while retaining the core insights of the model. These extensions would not only enhance the versatility of the framework but also deepen its relevance for policymakers, firms, and researchers exploring sustainable transitions in diverse industries.

Chapter 5

Conclusion and Future Works

5.1 Summary of the Thesis Work

This thesis investigates the multifaceted impacts of climate change and its associated risks within the financial and insurance sectors. Chapter 1 provides the background and introduction for each topic, outlining the existing research in the literature and identifying current gaps that this thesis aims at addressing. Chapters 2 through 4 present self-contained studies, each contributing uniquely to the overall theme of the thesis. While these chapters do not build upon one another directly, they collectively offer a comprehensive analysis of climate risks by developing models and frameworks to integrate these risks into financial decision-making and insurance practices.

Chapter 2 examines the implications of the heterogeneity of ESG ratings in the financial market. Given the inconsistency among major rating agencies, this chapter highlights how the ambiguity in ESG scores affects investors who seek to incorporate ESG factors into their portfolios. The research proposes ensemble strategies that cater to different risk profiles and ambiguity preferences of investors, moving beyond the existing literature that often assumes uniformity in ESG data. By developing a portfolio optimization strategy based on reinforcement learning (RL), this chapter also provides insights into how investors can adjust their strategies when they are faced with uncertain ESG ratings, contributing to a more nuanced understanding of climate-related financial risks.

Chapter 3 extends the focus from the financial market to the insurance sector, introducing a joint liability model that incorporates an individual Environmental Sustainability Evaluation (ESE) score. This model evaluates the creditworthiness and sustainability

of borrowers, particularly farmers, in the light of climate risks. The analysis identifies the optimal ESE score to balance the interests of both lenders and borrowers, ensuring profitability while reducing the risk of default. The chapter further integrates climate scenarios to assess how environmental changes influence lending outcomes. The findings in this chapter suggest that considering these scenarios in the credit evaluation process can lead to better-informed and more sustainable lending decisions, thus contributing to the overall stability of the insurance sector.

Chapter 4 addresses the challenges posed by transition risk, a difficult-to-measure aspect of climate risk due to its complexity and the scarcity of existing models in the literature. This chapter investigates how corporations respond to government policies aimed at promoting decarbonization and a low-carbon economy. The study emphasizes the dynamic interplay between corporate strategies and policy measures, showing that aggressive tax policies can effectively drive investments in sustainable technologies. It further explores the effectiveness of different government interventions, including targeted subsidies and taxes, revealing the conditions under which these policies lead to a substantial amount of corporate investment in green initiatives. The chapter's multi-period analysis under various decarbonization scenarios provides a strategic framework for both policymakers and businesses, offering a practical approach to managing transition risk.

Collectively, these chapters offer a comprehensive analysis of climate change impacts and their associated risks across different sectors of the economy. By bridging the gap between ESG integration in the financial market, insurance risk modeling, and transition risk analysis, the thesis hopes to provide a multifaceted perspective on managing climate risks. The findings reported in these chapters offer actionable insights for investors, insurers, and policymakers alike, emphasizing the need for adaptive strategies that account for the uncertainties inherent in climate change scenarios.

5.2 Future Research Avenues

Building on the research work presented in this thesis, several promising directions for future research emerge, offering opportunities to deepen our understanding of climate risks and enhance the integration of sustainability into financial decision-making. In this section, we outline potential research avenues for Chapters 2 through 4.

Chapter 2: Navigating Uncertainty in ESG Investing

Future research could build upon the findings of this study by incorporating a Fama-French three-factor model into the analysis while keeping stock return as the dependent

variable. This approach would involve integrating traditional factors like size, value, and market risk premium alongside ESG factors, and further extending the model to examine the individual contributions of the three ESG pillars (Environmental, Social, and Governance). Such an extension would enable a more comprehensive evaluation of how these traditional financial factors interact with each pillar of ESG in influencing stock performance, distinguishing genuine risk factors from anomalies.

Chapter 3: Joint Liability Model with Adaptation to Climate Change

One of the future research directions is to develop sustainable models that take into account a specific features and requirements of different joint liability contracts. This enables us to design more effective and tailored joint liability contracts that are better suited to the specific needs and circumstances of different groups. Another potential area of future research could be to incorporate collateral into the model's construction. Our study did not consider the role of collateral agreements in the joint liability model, but collateral is a critical factor in many lending decisions. Collateral can provide lenders with additional security and increase the likelihood of loan repayment, but it can also introduce complexities and potential inequalities in the lending process. By examining the role of collateral in joint liability models and how it interacts with other borrower characteristics and environmental factors, future research could provide a more nuanced understanding of creditworthiness and sustainable lending practices. Overall, the potential for future research in the field of sustainable lending is vast, and continued investigations are necessary in order to improve our understanding of creditworthiness and promote sustainable development outcomes.

Chapter 4: Strategic Investment to Mitigate Transition Risks

Future research for this chapter could delve into a more detailed analysis by examining the low-carbon transition needs of different sectors, such as utilities and agriculture, which face unique challenges and opportunities in decarbonization. By breaking down the model into sections for specific industries, researchers can fine-tune model coefficients to reflect sector-specific factors more accurately. For instance, the utility sector may prioritize large-scale renewable energy infrastructure, while agriculture might focus on reducing emissions through sustainable farming practices. Additionally, incorporating real-world data into the model could further enhance accuracy, allowing for a more realistic representation of the varying impacts of decarbonization efforts across sectors.

Another promising direction is to investigate the role of fiscal policies tailored to different industries. Future research should consider how sector-specific tax incentives, subsidies, or penalties influence firms' investment strategies in distinct ways. This approach would highlight the varying effectiveness of fiscal policies across sectors, helping policymakers design targeted incentives that drive meaningful progress toward sustainability.

Finally, the model could be enhanced by establishing functional relationships between currently independent parameters, such as the selling price (p), and the optimal investment ratio (α). For instance, companies investing in low-carbon transition often produce greener products, which could justify a higher selling price, as seen in sectors like electric vehicles where green technology typically commands a premium. By linking parameters like selling price with low-carbon investment levels, future models could better capture the economic benefits associated with sustainable practices, creating a more holistic and dynamic framework.

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APPENDICES

Appendix A

The Concept of individual-ESE scores

In order to maintain accountability for sustainable agriculture commitments and provide valuable insights for policymaking, conducting regular and transparent assessments is of paramount importance. However, there are a number of variations in the definitions of sustainable agriculture, and also a limited number of quantitative assessments of agricultural sustainability that are currently available, according to Agovino et al. (2018). Although some experts view sustainable agriculture as a collection of management strategies, others see it as an ideology or a specific set of goals. Despite this, there is an increasing consensus that sustainable agriculture should be evaluated based on its environmental, economic, and social impacts, as emphasized by Zhang et al. (2021).

Several frameworks and indicators have been developed in the literature. Existing sustainable agriculture indicators mainly assess the environmental impacts of agriculture production, and many have limited data availability. Although various organizations have called for monitoring agriculture globally, actual datasets to enable trend assessments are still currently lacking.

The absence of a consistent quantification of agricultural sustainability across multiple dimensions impedes the successful identification of undesirable trade-offs and the development of win-win solutions. Given these challenges, relying on literature-based metrics is a reasonable starting point in developing a sustainable agricultural credit scoring system. Literature-based metrics are metrics that have been previously used and validated in other studies or publications and are therefore based on a larger sample size and a more diverse set of variables than any single data collection effort.

Having literature-based metrics about agricultural credit scoring systems allows us to apply them when we obtain real data or use them to create a scoring system by using

synthetic data. By compiling these synthetic metrics and adjusting them to fit the specific needs of the agricultural sector, we can create a credit scoring system that is more comprehensive and accurate than any single data collection effort.

The objective of this research agenda is to eventually establish a comprehensive and sustainable agricultural credit scoring system at the individual level, which encompasses three critical pillars: Economic, Social, and Environmental. This system will therefore be coined as an individual–ESE score.

While traditional credit scoring systems focus mainly on borrowers' economic and financial status, our system will take into account a broader range of factors that contribute to sustainability. We recognize the importance of economic metrics and acknowledge that they are crucial to the classical credit scoring system. Therefore we will use some of the commonly used economic metrics in the related literature as a foundation for our economic metrics.

Presented below are synopses of selected academic papers that have conducted research on the agricultural credit scoring system.

1. Römer and Mußhoff (2018) investigated the use of weather data in agricultural credit scoring for microfinance institutions, finding that a simple correlation analysis between weather events and loan repayment is insufficient to forecast future repayment behavior.

2. Turvey (1991) reviewed the use of credit scoring for agricultural loans and its application to agricultural finance, identifying the importance of accurate data and the need for specialized credit scoring models for agricultural lending.

3. Lufburrow et al. (1984) developed a credit scoring model for farm loan pricing, utilizing statistical methods to determine the significant factors affecting loan repayment and default rates.

4. Limsombunchai et al. (2005) analyzed credit scoring for agricultural loans in Thailand, identifying the factors that affect credit scores and their impact on loan approval and interest rates.

5. Bumacov et al. (2014) examined the use of credit scoring in microfinance institutions, reporting that it can improve the outreach and efficiency of microfinance operations, but also highlighting the need for effective risk management strategies.

6. Mutamuliza et al. (2021) investigated the determinants of smallholder farmers' participation in microfinance markets in Rwanda, identifying factors such as gender, education, and access to information that can affect farmers' ability to access credit.

We provide a summary of the key metrics employed in their credit scoring models as the foundations of economic metrics in Table A.1. More recently Zhang et al. (2021)

presented a collection of useful metrics for the environmental and social pillars. With additional information provided in Fuglie et al. (2016) and Movilla-Pateiro et al. (2021), we compile a more comprehensive list of potential metrics for these pillars, along with detailed descriptions and methods of measurement. The metrics are shown in Table A.2.

Metric	Description	Used as a Metric in the Scoring System
Age	Age of the applicant in years	(1), (4)
Gender	Male, Female, Others	(1), (6)
Education	Education level of the applicant	(6)
Collateral	Collateral in thousands	(1), (3), (4)
Assets	Assets in thousands	(1), (2), (3), (4), (5)
Debt	Debt to other banks in thousands	(1), (2), (3), (4), (5)
Deposit	Deposit in the bank account in thousands	(1), (2), (5)
Income	Monthly business and household income in thousands	(1), (2), (3), (4), (6)
Marital status	Single, Married, Divorced, Others	(1)
No. family members	Number of family members	(1)
Loan Size	Loan amount in thousands	(1), (4), (6)
Loan duration	Loan Duration in Months	(4)
Repayment ability	Projected net cash flow plus projected inventory divided by a total line of credit	(1), (2), (3)
Repayment history	The average of loan principal repaid divided by principal due over the past three years	(1), (3)
Tenure	The ratio of acres owned and farmed to total acres farmed	(3), (6)
Resident	1 if applicant is a resident; 0 otherwise	(1),
Working experience	Working experience in current profession in months	(1),
Region and Country	Region and Country of the Agricultural Business	(1), (2), (4), (6)
Distance	Distance to the microfinance institution in km	(6)

Table A.1: A List of Economic (E) Metrics

Developing a comprehensive agricultural credit scoring system requires careful consideration of the various metrics that influence creditworthiness. Below is a scheme for generating a composite score based on multiple metrics, including how to assign weights

to each metric. We aim at eventually developing a comprehensive ESE scoring system by either obtaining real data or generating synthetic data according to this scheme in the future. We anticipate that this endeavor will involve the following steps:

STEP 1. Identify and gather relevant metrics: Begin by identifying the metrics that are most critical to assessing creditworthiness in agricultural lending, such as past repayment history, crop yields, farm management practices, and market conditions.

STEP 2. Determine the relative importance of each metric. this is done by assigning weights to each metric based on its perceived importance in predicting creditworthiness. This can be done through a combination of expert judgment and statistical analysis of historical data. Weights may also be assigned based on the level of risk exposure associated with each ESE factor, with higher weights assigned to metrics that pose a greater risk to a farmer's performance. Notable, the overall Thomson Reuters ESG Score is calculated by applying an automated, factual logic that determines the weight of each category. Each category consists of a different number of measures. The count of measures per category determines the weight of the respective category.

STEP 3. Normalize the metrics to ensure that each has the same scale, range, and units of measurement. This can be achieved through methods such as standardization or normalization.

STEP 4. Once the weights and normalized values of each metric have been determined, calculate the composite score by multiplying the weight of each metric by its normalized value and summing the products.

STEP 5. Validate the composite score by comparing it to actual default rates or repayment history. Refine the weights and metrics as needed based on the validation results.

STEP 6. Continuously monitor and update the credit scoring system by incorporating new data and adjusting weights and metrics as needed to improve the accuracy and predictive power of the model.

Following the above steps, a comprehensive agricultural credit scoring system can in principle be developed. This system will consider multiple metrics as highlighted in Table A.2 and assign weights based on their relative importance in predicting creditworthiness.

Our review of the literature has revealed that while traditional credit metrics concentrate on financial and economic factors, assessing sustainability requires that we take into account environmental and social factors as well. Therefore, in this appendix, we have suggested the concept of an individual–ESE score that integrates economic, social, and environmental considerations to offer an all–encompassing appraisal of the farming sector. A potentially fruitful avenue for future research is to eventually develop a credit

scoring system that addresses the unique requirements of the agricultural sector by applying literature-based metrics for agricultural credit scoring systems to the construction of the scores. By compiling and adjusting these metrics, we hope to eventually be able to create a comprehensive credit scoring system with adaptation to climate change for the agricultural sector that addresses limitations of the ESG scores when they are applied to the agricultural sector.

Sector	Metric	How to Measure
E	Soil health	Measuring the quality and health of the soil, including factors such as nutrient levels, organic matter content, and erosion rates. Soil testing kits could be used to measure.
E	Water Consumption	Modelled evapotranspiration using climate data (adjusted for crop type and water availability). Water usage tracking software could be used as a tool to measure.
E	Water Quality	N & P concentration (measured in stream, or modeled using land cover for landscapes)
E	Habitat Conversion	Remotely sensed land use (% of study area covered by natural habitat)
E	Habitat Composition	Remotely sensed land use (no. of land cover classes, indicating habitat diversity)
E	Habitat Connectivity	Remotely sensed land use (calculated connectivity score)
E	Species Richness	Field samples (richness relative to reference natural landscape)
E	Species Abundance	Field samples (abundance relative to reference natural landscape, i.e. the most undisturbed nearby similar habitat)
E	Yield / Area	Harvested mass of crop per unit area over a 5-year rolling average, perhaps using geometric mean to penalize variation in yield.
E	Use of pesticides and chemicals	Liters of pesticide or chemicals used per hectare
E	Carbon Footprint	The carbon footprint for different crop types are shown in Sah and Devakumar (2018). Measuring the amount of greenhouse gas emissions generated from the farmer's activities, including fertilizer use, transportation, and energy use.
S	Health & Safety Issues	Whether the applicant ever had safety issues while participating in agricultural activities (0 if no, 1 if yes).
S	Insuring Health	Whether the applicant has personal accident insurance or commercial insurance (0 if no, 1 if yes).
S	Labor Practices	Measuring the fairness and safety of labor practices on the farm, including minimum wage compliance, safety regulations, and worker satisfaction.
S	Community Engagement	Measuring the level of engagement and support the farmer provides to the local community, including economic benefits, charitable contributions, and community outreach.
S	Ethical Business Practices	Measuring the farmer's adherence to ethical business practices, including transparency, honesty, and accountability.
S	Compliance	Measuring the farmer's compliance with local and national regulations, including environmental, labor, and health and safety regulations.

Table A.2: A List of Social (S) and Environment (E) Metrics