

Simulation of Multiple Hydraulically Driven Fractures

by

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A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Science
in
Civil Engineering

Waterloo, Ontario, Canada, 2015

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Authors declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Hydraulic fracturing is a process that is used to release and extract natural gas from the pores of shale rocks. The process involves drilling vertical and horizontal wells through shale formation beds. After which, a mixture of fluid and sand is pumped into the rock formation through the horizontal well, pressurizing the shale around the well, causing multiple permeable cracks to form. Studying hydraulic fracturing helps Oil and Gas companies to improve the efficiency and productivity of this process.

Because the hydraulic fracturing process takes place hundreds of meters below the ground's surface, its behavior is difficult to physically assessed. Computer modeling is an efficient and economical way to study and analyze the behavior of the process. Finite element modeling, as a numerical tool, can be used to solve such non-linear fracture problems.

In this thesis, finite element modeling is used to study two-dimensional, single, and multiple crack propagation problems that occur during fluid injection. The single fracture problem is compared with a well known analytic model (KGD model) in order to verify the efficiency of the numerical finite element model. The effects that rock and fluid material properties have on the fracture propagation, crack width, and fluid pressure is studied. As a result, the finite element numerical model is found to be in good agreement with the KGD analytical solution. Moreover, the analysis revealed that small changes in the material properties (e.g., rock elasticity modulus, permeability, and fluid viscosity) have significant effects on fracture propagation.

Multiple crack problems, using three parallel cracks, are also investigated. The effects of the fracture spacing and type of fluid control (flow rate or pressure control) are studied. Stress shadowing (induced stresses from the adjacent fracture) between multiple fractures is evaluated. For the edge cracks, it is found that as the fracture spacing decreases, the crack length increases. While, for the middle crack, as the fracture spacing decreases, the crack length decreases. It is shown that fluid flow controlled injection leads

to stable crack injection, while pressure control injection leads to unstable crack propagation.

In summary, this thesis finds that an optimal spacing for three crack hydraulic fracturing is between equal fracture spacing and two-third the distance between the middle and any of the edge cracks. It is recommended that future engineers extend this research to simulate a three-dimensional problem with randomly oriented fractures.

Acknowledgements

I would like to thank my supervisor, Dr. Robert Gracie for his guidance and support during my study at the University of Waterloo.

I would like to thank my father, my mother and my sister for their enormous love, support and encouragement.

I would like to thank my dear friends for being supportive and providing warm environment.

Dedication

This thesis is dedicated to my dear family, Hesham Khedr and Nahla Zaki and Reem Khedr.

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Nomenclature

English notations

b	Body forces	N
$C = C_b = C_t$	Leak-off coefficient	$m/\sqrt{s}$
$\mathbf{C}$	Fourth order elasticity tensor	Pa
C_m	Compressibility of porous material	m^2/N
C_s	Compressibility of matrix material	m^2/N
E	Modulus of elasticity	Pa
E'	Softening elastic modulus	Pa
$\mathbf{F}_p$	Nodal flux force	N
$\mathbf{F}_u$	External nodal force	N
g	Gravitational acceleration	m^2/s
G	Shear modulus	Pa
G_f	Fracture energy	$Pa - m$
h	Fracture height	m
$\mathbf{H}$	Permeability matrix	Pa
k	Permeability of material	mD
K	Bulk modulus	Pa
$\mathbf{K}$	Stiffness matrix	Pa
K_{IC}	Critical stress intensity factor	$Pa/\sqrt{m}$
K_s	Bulk modulus of solid material	Pa
l	Fracture length	m
$\mathbf{L}$	Differential operator	
$\mathbf{N}_I^u$	Displacement interpolation function	
$\mathbf{N}_I^p$	Pressure interpolation function	
p	Pressure	Pa
p_f	Pore pressure	Pa
$\mathbf{q}$	Fluid flow rate	m/s
Q	Total volume mass rate	m^3/s
$\mathbf{Q}$	Coupling matrix	Pa
t	Time	s
$\mathbf{T}$	Nominal traction	Pa

T_0	Tensile strength	Pa
T_n	Tangential traction	Pa
T_s	Shear traction	Pa
$\mathbf{u}$	Displacement	m
U_s	Strain energy	J
U_y	Surface energy	J
v	Fluid velocity	m/s
w	Fracture opening	m

Greek notations

α	Biot's coefficient	
ς	Increment of fluid content	$\frac{m^2s}{kg}$
δ_{ij}	Kroneker delta	
δ_0	Separation	m
δ_f	Final separation	m
ε	Strain	
γ	Specific weight	N/m
Γ	Boundary surface	
μ	Viscosity of fluid	$Pa.s$
ν	Poisson's ratio	
Ω	Body domain	
ϕ	Porosity of material	
ρ_s	Density of solid material	kg/m^3
$\boldsymbol{\sigma}$	Total stress	Pa
$\boldsymbol{\sigma}'$	Effective stress	Pa
$\boldsymbol{\sigma}^0$	Field Cauchy stress	Pa
σ_{Hmin}	Minimum horizontal stress	Pa
σ_{Hmax}	Maximum horizontal stress	Pa
σ_v	Vertical horizontal stress	Pa

Chapter 1

Introduction

1.1 Background

The hydraulic fracturing (HF) process is designed to facilitate the production of natural gas and oil in tight formations, such as shale. Natural gas is extracted by pumping in large amounts of fluid and proppants (i.e., sand or treated sand) into the shale formation forming fractures. The fractures increase the rock's permeability that ease the transport of trapped gas and oil.

The extraction of natural gas from shale formations has revolutionized the natural gas industry in the United States and Canada, because of the development of horizontal drilling and new HF practices [2]. In 2011, the domestic production of natural gas in the U.S increased from 65% to 95% [3, 4]. The U.S. shale gas production has increased 12-fold over the last decade and this trend is expected to continue until at least 2035. Also by 2035, it is estimated that shale gas production in the U.S will comprise 46% of the total global natural gas production [5]. In Canada, many companies are currently developing shale gas resources in Alberta, British Columbia, Quebec, and New Brunswick [6]. In 2012, Canada's shale gas accounted for

15% of the total natural gas production [7]. As of June 2015, the Canadian Association of Petroleum Producer (CAPP) reported that Canada was the world's fifth largest producer of natural gas [6].

Hydraulic fractures are usually located around a horizontal well where perforated guns are placed along the well shaft. The hydraulic fracturing process is done over several stages, each stage consists of generating multiple fractures. Multiple fractures play an important role in the maximization of natural gas production from shale formation. However, this process is hindered if the fractures are formed too close to one another. Dense fracture spacing induces compressive stresses between adjacent fractures preventing the cracks from opening. Therefore, simulating multiple hydraulic fractures and representing the fractures in a practical model are beneficial, to ensure that proper fracture distance will be attained.

Many researchers have made efforts to create models that simulate hydraulic fracturing. Early attempts focused on developing analytical solutions representing plane strain and axi-symmetric hydraulic fractures, e.g., Perkins-Kern- Norgren [8], Geertsma and de Klerk [9] and Nilson and Griffiths [10]. Other numerical models developed by Advani and Lee [11], and Vandamme and Curran [12] used dry crack fracture propagation algorithms based on linear elastic fracture mechanics. The previous models do not consider fracture tip nonlinearities. Fracture tip nonlinear behavior affects the equilibrium and fluid mass conservation equations. Recently researchers have developed a series of numerical models representing simple plane strain geometries with fluid flow injection into the crack, e.g., Adachi [13], Adachi and Detournay [14, 15], Bunger [16] and Bao [17].

The use of horizontal wells helps in developing many fractures along the well, resulting in producing shale gas in a more economically feasible manner. According to Miller [18], 30-40% of multistage hydraulic fractures do not produce natural gas. When simulating multiple driven hydraulic fractures, it is necessary to find the most efficient way to represent the process. Fracture

growth is affected by two main aspects. First, the spacing between the fractures, and second the rock material properties. Smaller fracture spacing exert stresses known as stress shadowing, which leads to fracture closure. Models that describe stress shadowing are explained by Gemanovich et al [19]. Moreover, material properties can change the behavior of fracture propagation from one zone to another, Baihly [20].

When modeling hydraulic fracturing, the boundary conditions for fluid injection also affect fracture growth. The models described above use a constant injection pressure into the rock formation. In order to propagate the fracture, the fluid must be pumped at a pressure that exceeds the sum of the minimum principal stress (perpendicular to the crack) and the tensile strength of the rock. Because of the considerable depths of the wells, it is difficult to estimate accurately the confining stresses and consequently the fluid pressure required for fracture propagation.

In this thesis, a numerical representation of multiple driven fractures is developed to reduce such inaccurate predictions. A finite element plane strain three-crack-model (using ABAQUS) is developed. The three crack model compares the application of different boundary conditions such as constant flow rate and constant pressure on the fractures. It also examines the effect of the cracks' spacing on fracture propagation. Studying these elements will help to provide a practical and efficient way to simulate hydraulic fracturing. In addition to that, studying fracture propagation helps in the optimization and performance of an effective hydraulic fracturing process.

1.2 Scope and objectives

The objective of this thesis is to study the effect that fracture spacing has on the fracture properties during a single stage hydraulic fracturing process. Achieving the thesis goal will help to simulate and provide a solution to a contemporary hydraulic fracturing problem. A two dimensional finite element model for a multiple hydraulic fracturing process is created in order

to address this problem. The objectives of this thesis are to:

1. Provide a literature review of hydraulic fracturing to account for the steps taken during the process of hydraulic fracturing, while providing a discussion of the mechanics of hydraulically driven fractures.
2. Develop and validate a non-linear finite element model of a single hydraulic fracture through shale formation. A comparison between the analytical Geertsma and de Klerk (KGD) [9] model and the finite element model is developed, to verify its accuracy.
3. Study the effect of changing the material properties (such as the rock elastic modulus, and fluid leak-off coefficients and viscosity) on the fracture propagation.
4. Develop a three-crack finite element model to access the optimal spacing of a hydraulic fracture process.
5. Explore the effect of stress shadowing for different fracture spacing on crack propagation.
6. Study the difference between fluid injection rate control and fluid pressure control boundary conditions (that represent the fluid pumped into the fractures) on fracture propagation.

1.3 Thesis organization

Chapter 2 provides an introduction and a literature review of the thesis related topics. An overview of the properties of shale and the steps by which hydraulic fracturing is being processed is addressed. Moreover, the mechanics of hydraulic fracturing and stress shadowing are investigated. Chapter 3

provides an overview of the governing equations used for modeling hydraulic fracturing, as well as the analytical model equations and finite element equations. Chapter 4 develops a finite element one crack model that is compared to the analytic KGD solution. The comparison helps in validating the results extracted from ABAQUS. Also, the effect of material properties on crack propagation is studied. Chapter 5 presents the finite element modeling for a three-crack hydraulically driven fractures. The effect of the boundary conditions is studied. Furthermore, the effect of fracture spacing on crack propagation is addressed. Chapter 6 summarizes the conclusions of the study and offers recommendations for future research topics.

Chapter 2

Literature review

2.1 Overview

This chapter covers extensive literature on topics related to hydraulic fracturing. First, the steps to perform hydraulic fracturing are discussed. Second, the history of hydraulic fracturing is reviewed, and the locations where hydraulic fracturing is most common, and where hydraulic fracturing is spreading to are stated. Third, the chapter includes an explanation of how shale gas was formed, providing the necessary background information to understand the formation's material properties, and how the mechanics of hydraulic fracturing affect the formation. Finally, this chapter gives a definition of stress shadowing and multi-fracturing. By explaining the stages of hydraulic fracturing it helps to generate a better understanding of how the model is constructed to represent this process.

2.2 Hydraulic fracturing

2.2.1 What is hydraulic fracturing?

Hydraulic fracturing starts starts after an oil or gas well has been drilled

(vertically and/or horizontally) into unconventional shale gas reservoirs (Figure 2.1). Shale formations are fractured by pumping a mixture of pressurized fluids (e.g. water), proppant (e.g. sand), and other additives into the formation to release the trapped natural gas or oil from the tiny pores in shale. Fractures created in the process act as ducts that ease production. In order to complete the process of hydraulic fracturing, the fluid pumped (generating pressure within the formation) should exceed the rock's strength, which creates openings and enlarges fractures in the rock.

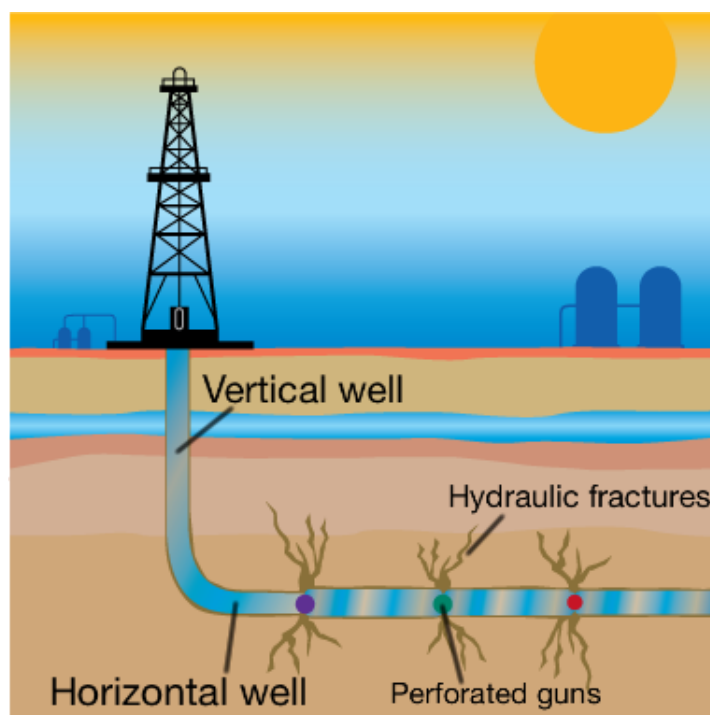


Figure 2.1: Multiple hydraulic fractures represented along a horizontal well [1].

As mentioned before, hydraulic fracturing can be performed vertically and/or horizontally. Horizontal drilling helps to overcome the low permeability of shale. The horizontal wellbore intersects with natural existing fractures in the reservoir, and natural fractures tend to have a higher de-

gree of permeability than the rest of the formation [21]. It should also be noted that horizontal drilling increases production when creating multiple fractures. Most of the wells that undergo hydraulic fracturing, located in the fields mentioned in the coming section (Section 2.2.3), use horizontal drilling for natural gas extraction. It takes a few months to construct a single well that gives 20 to 40 years of production. Horizontal drilling takes from 4 to 8 weeks to prepare the site, drill, and case the wells [22].

2.2.1.1 Formation of shale gas

As previously stated, natural gas is found in two forms of reservoirs, conventional and unconventional where classifications are made based on the ease and cost of extraction. Conventional gas is a natural gas that migrates, out of shale traveling upwards due to its low density, through the sediments until it gets trapped in cap rocks. Unconventional gas is considered a recoverable gas resource; for example, tight gas, coal bed methane, gas hydrate and shale gas are unconventional gas types. Also, unconventional gas requires more complicated techniques, such as hydraulic fracturing, to extract.

Shale gas is found in fine-grained sedimentary rock (shale). Shale is formed from the compaction of silt and clay minerals. Gas is tightly locked in very small pores within the reservoir rock. It should be taken into consideration that many unconventional reservoirs termed “shale” that produce natural gas, are not actually shale, but rather mudstone, such as the Montney Formations in Canada [23].

Shale gas was first formed 360-415 million years ago during the Devonian period. Shale rocks were formed from fine silt and clay particles that were relatively bounded by water beds [24]. Methane, trapped in shale, is formed from organic materials, such as plants (or algae) and the first amphibians. When these organic matters that were buried within the sediments and sandy rock layers are fed on by bacteria, it releases biogenic methane as its bi-product. And after millions of years the organic materials form conventional

gas that is easy to extract. On the other hand, some of these gases are locked in low permeable shale layers forming unconventional gas.

The main elements of shale are clay, silica, carbonates and organic materials. The shale beds may also contain sandstone, limestone or dolostone [25]. The presence of hard material such as silica (which breaks like glass) increases shale's ability to fracture. As an example, The Horn River Basin (which is a rich source of shale gas) is one area rich in silica [26]. On the other hand, clay soil strata usually absorb pressure and bend when fractured.

Shale permeability is very low, and these pores can hold a certain amount of water or gas but will not allow them to flow. Pores in shale are approximated to be a thousand times smaller than any other conventional sand stone. The permeability of shale ranges from tens to hundreds of nanodarcys [24].

2.2.1.2 Drilling stage

Unconventional well construction starts with the drilling process. The well is drilled to a depth of 300-2400 meters below the Earth's surface (Figure 2.2) [27]. Then, steel and cement casing are inserted along the vertical well. Casing the well helps to create a solid barrier between the well and groundwater formations (i.e. aquifers). Two or more casings are added at the groundwater level, to prevent leakage during the pumping stage and the flow-back stage. The number of these casing layers decreases as the well travels deeper. After casing the vertical well, horizontal drilling starts. The well extends hundreds of meters in the horizontal direction to maximize natural gas production. Casing is also embedded into the horizontal well, which makes it difficult for natural gas to travel through the wellbore when horizontal wells are cased. For this reason perforation guns are distributed over different stages of the horizontal well to form initial cracks. The perforation guns punch holes through the casing and into the formation forming fractures (1 to 2 meters in length) that act like ducts [7]. After that, initial cracks are

formed in order to prepare the well for the next step, which is to pump the pressurized fracturing fluid into the well.

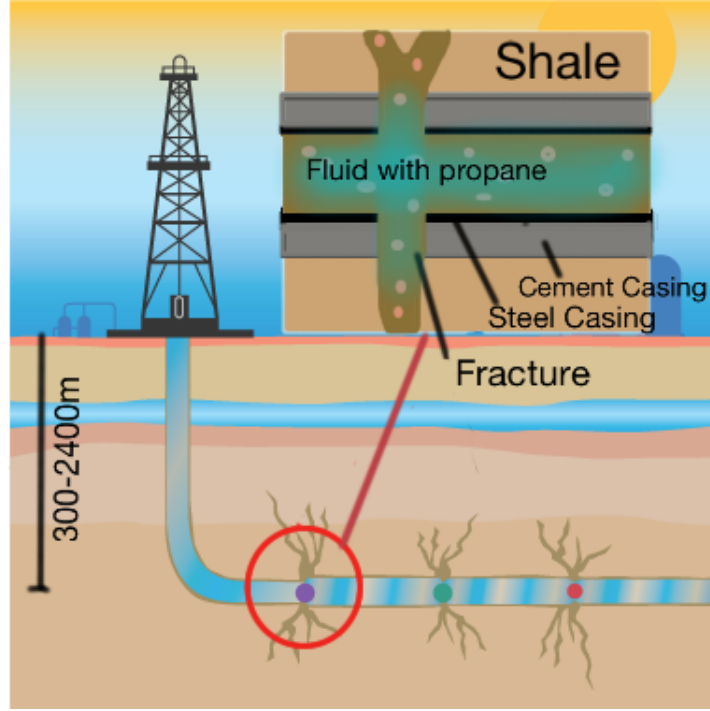


Figure 2.2: Components of hydraulic fracturing process.

2.2.1.3 Hydraulic Fracturing Stages

The first stage is known as spearhead stage, when the pressurized fluid and diluted acids (e.g., hydrochloric acid) are mixed together, pumped into the well and forced into the initial cracks creating longer fractures. The diluted acid helps to dissolve some of the rock material, which assists in cleaning out the rock pores and enable the fluid and gas to flow. Each fracture, on average, extends 100 meters from the horizontal leg (the maximum fracture length is approximately 300 meters) [28].

Secondly, the pad stage occurs when a batch of carrying fluid with no proppant is pumped into the formation. This initiates hydraulic fracturing

and forms the fractures.

Thirdly, the proppant stage is where the wellbore is fed with water and sand. In some cases, not all fractures lead to conductive pathways between the formations and wellbore, because the insitu stresses in the surrounding formations can close the newly formed fractures, preventing natural gas from traveling. The presence of sand (proppants) in the fracturing fluid helps to prevent the fracture from closing, as the sand grains get trapped inside the fracture opening, allowing water to flow back to the well. The proppant sand produces high permeability ducts that help natural gas to migrate to the well.

The final stage, is the flush stage, where fresh water is pumped down the well to clear any unneeded proppants from the fractures. Since the injection process increases the internal pressure of the rock, the pressure causes some of the injected fluid to flow back to the well (the fluid is known as flow back). Initially the flow-back rate is high, but it declines quickly as the saturation of formation is reduced. Only 20-40% of the water volume flows-back to the surface [29]. The flow-back water contains injected chemicals and naturally occurring minerals from the formation.

Usually flow back is stored on site in tanks or pits before treatment. It may also be injected into saline aquifers, but in this case, approvals for the disposal by injection must be taken. Some companies use this water for another cycle of hydraulic fracturing.

It takes 2-5 days to complete the entire multi-stage fracturing operation. A well and its surroundings usually do not exceed the size of a two-car parking area once all equipment used for the hydraulic fracturing process are removed.

2.2.1.4 Components of the hydraulic fracturing fluid

Water represents 90% of the fracturing mixture and sand (proppants) is about 9.5%. The rest (0.5-1%) is comprised of chemicals added to the mixture of water and sand. Some of the chemicals used are [30]:

- Acids, which help to dissolve minerals and initiate rock pre-fracture,
- Sodium chloride, which acts as a stabilizer and prevents clays from swelling,
- Polyacrylamide, which minimizes the friction between the fluid and pipe,
- Borate salts, which help maintain the fluid viscosity when the temperature increases,
- Guar gum, which increases the viscosity of water, and suspends the sand inside the fractures.

The amount of water used in hydraulic fracturing is massive, a single well consumes from 190 to 1300 cubic meters of water for fracturing a single well [31]. For example, in Barnett Shale, Texas 250 billion liters of fresh water were used in well fracturing between the years 2011 and 2012 [32]. But in some cases water is not necessary for completing the fracturing process. Instead, carbon dioxide, nitrogen, propane, and butane are used to complete the fracture process. In the Monterey formation in California, carbon dioxide is used as a fracturing fluid. Some wells can also be sensitive to water due to rock type (rocks that contain clay absorb water and swell), as in South-East Alberta and South- West Saskatchewan, where nitrogen or propane or butane are used instead [29].

2.2.2 History

Shale gas formations were discovered in the Appalachian Mountains (United States) in the 1800s, where natural gas was first extracted from shale formations [29]. However, production rates were negligible, due to the low

permeability of the shale formation and the lack of technology, which which led to unpopularity of shale gas extraction at that time. Also, the fact that conventional natural gas (free gas) and oil were available and easy to extract from the Persian Gulf and the Gulf of Mexico [33] added to it.

It was not until the early 1970s that shale gas extraction became viable again. The oil embargo (the oil crisis) dwindled natural gas supplies, and extractors subsequently sought to maximize returns from existing wells, which lead to the implementation of hydraulic fracturing processes in vertical wells [34].

In the 1980s and early 1990s, George Mitchell, founder of Mitchell Energy and Development Corp., came up with the process of combining horizontal drilling with hydraulic fracturing [35]. This technique boosted shale gas production considerably. The Mitchell Energy Group developed more efficient techniques for use in the Barnett shale formations in Texas between the years 1981 and 2002. Horizontal drilling largely became popular as oil prices declined in the 1980s, and the cost of developing new downhole devices also increased. Thus, the need to reduce finding costs became important, and the need for existing well efficiency increased significantly [36]. Because of these economic conditions, the horizontal drilling technique and hydraulic fracturing became very popular in the United States and Canada. Subsequently, these processes gained worldwide popularity. In the USA and Canada, production has increased since the 1970s, but it still did not exceed 1.6% of gas production until 2000 [34]. Currently, 35,000 shale gas wells are produced each year worldwide [34].

2.2.3 Where is hydraulic fracturing conducted?

Hydraulic fracturing is used globally, but more specifically, it is employed in places such as the United States, Canada, China, India, England, and Saudi Arabia. Hydraulic fracturing is a significant component of the gas production process in each of these countries. For example, shale gas and

other forms of unconventional gas represent a major portion of North America's future of gas production. Unconventional gas represented 42% of the total US gas production in 2007 and is expected to increase to 64% in 2020 [37]. The Geological Survey of Canada estimated that Canada has approximately 140 trillion cubic meters of natural gas producing shale, making Canada the fifth largest shale deposit after China, Argentina, Algeria, and USA. If the current levels of natural gas production of Canada and the USA were combined, it could supply enough natural gas for the North American market for more than one hundred years, given that the consumption rates do not change [6]. Therefore, it is necessary to develop accurate models that allow the fracturing process to be completed efficiently, while maximizing the return on existing and new wells.

2.2.3.1 Production in Canada

The distribution of shale gas in Canada is spread out between almost every province across the country from Nova Scotia to British Columbia. In Ontario, shale gas deposits are distributed over three main major zones the Kettle Point Formation (Antrim Shale), the Collingwood/ Blue Mountain formation (Utica Shale), and the Marchellus Shale [38]. Utica shale, which also extends into Quebec, produces about 2800 cubic meters of natural gas per day, but it is extracted exclusively in the United States [5]. According to US Energy Information Administration, 590 billion cubic meters of natural gas were produced in 2013 from Utica shale alone [39]. The natural gas from the Frederick Brook Shale in New Brunswick recently started to be extracted from its first horizontal well in 2011 [39]. The major shale reserves in Canada are located in British Colombia and Alberta, however. In British Columbia, the Muskwa Shale in the Horn River basin contains 170 billion cubic meters of natural gas [40]. There are more than 13 trillion cubic meters of gas in Canada that can be extracted from shale formations [41]. The largest shale gas resource is located in Montney shale. The Montney reaches across

northwest Alberta and into northeast British Columbia where an estimated 1.5 trillion cubic meters of natural gas is trapped in this deposit [42]. In 2012 the Montney shale produced an average of 40 million cubic meters per day of natural gas (12% of the total 300 million cubic meters per day produced in all of Canada that year). In 2013, Wood Mackenzie analysts estimated that the production in the Montney would reach 100 million cubic meters of per day by 2018 [43]. The following map shows the distribution of shale gas basins, as well as gas and oil plays in Canada.

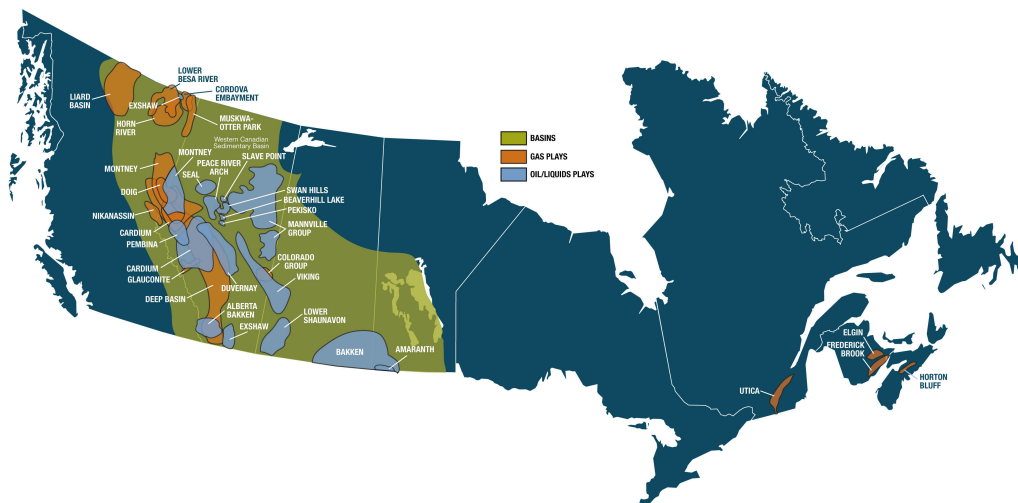


Figure 2.3: Shale gas basin distribution and production in Canada.
[44]

2.2.3.2 Production in the United States

Shale gas basins in the United States basins are distributed across the country, the main shale plays are located in the following areas: Barnett Shale in the Ft. Worth Basin in Texas, Fayetteville Shale in Arkoma Basin in Arkansas, Marcellus Shale in the Appalachian Basin, and Utica Shale the Appalachian Basin. According the the U.S Energy Information Agency (EIA), the currently discovered shale plays in the United States contain about 3 billion cubic meters of recoverable shale oil and 21 trillion cubic meters of

recoverable shale gas. EIA announced the reserve and production of shale gas grew between the years 2012 to 2013. The following tables (Table 2.1) and (Table 2.2) show a summary of the statistics announced by EIA. The tables show that there has been an increase in production by 10% from 2012 to 2013 and an increase in reserves of approximately 30% [45] .

Shale play	2012 reserves (tcm)	2012 production (tcm)
Marcellus	1.2	0.067
Barnett	0.67	0.059
Eagle Ford	0.45	0.028
Haynesville/Bossier	0.5	0.076
Woodford	0.35	0.016
Fayetteville	0.24	0.028
Subtotal	3.4	0.27
Other Shale Gas	0.18	0.016
All U.S. Shale Gas	3.6	0.29

Table 2.1: EIA announcements of the United States shale production and reserves year 2012 in trillion cubic meter [45]

Shale play	2013 reserves (tcm)	2013 production (tcm)
Marcellus	1.8	0.1
Barnett	0.73	0.05
Eagle Ford	0.49	0.04
Haynesville/Bossier	0.45	0.05
Woodford	0.35	0.02
Fayetteville	0.34	0.028
Subtotal	4.2	0.3
Other Shale Gas	0.3	0.01
All U.S. Shale Gas	4.5	0.31

Table 2.2: EIA announcements of the United States shale production and reserves year 2013 in trillion cubic meter [45]

2.2.3.3 Worldwide production

Shale gas formations are also spread worldwide. In 2013 by the EIA revealed that there are huge amounts of shale resources located in China, Argentina, Australia and South Africa. It is expected that these countries are to see an increase in its energy production. Also other countries such as Poland, Turkey, and Ukraine expect to see changes in their energy politics, as shale gas will reduce their dependence on conventional gas exporters from Russia and Iran. On the other hand, it is predicted that Russia, China, Argentina and Libya actually holds more than half of the world's shale oil resources [46]. The amount of the available oil sources in some European countries are being restricted, however, as countries such as France and the United Kingdom are minimizing the production of shale gas production due to environmental concerns.

2.3 Mechanics of hydraulic fracturing

This section introduces the mechanics and the conditions for a hydraulic fracture propagation. As previously mentioned, for a fracture (in hydraulic fracturing) to split and grow, the fluid pressure of the rock must exceed the summation of the minimum principal stress and the tensile strength of the rock.

Hydraulic fractures are formed in the direction perpendicular to the least principal stresses (direction of the least resistance). Depending on the location of the well, the path of least stress changes the fracture's direction. For instance, if the fracture is located in between the Earth's surfaces and 600 meters below the surface, the least principal stresses are considered to be the overburden pressure. In this case, the fractures will be oriented in the horizontal direction parallel to the bedding plane of the formation. For fractures located deeper than 600 meters below the surface, the overburden stress is the dominant stress and the horizontal confining stresses become the

least principal stresses. Therefore, fractures are more likely to propagate in the vertical direction [47] (Figure 2.4). Thus, fractures will change direction according to the depth at which the process is occurring.

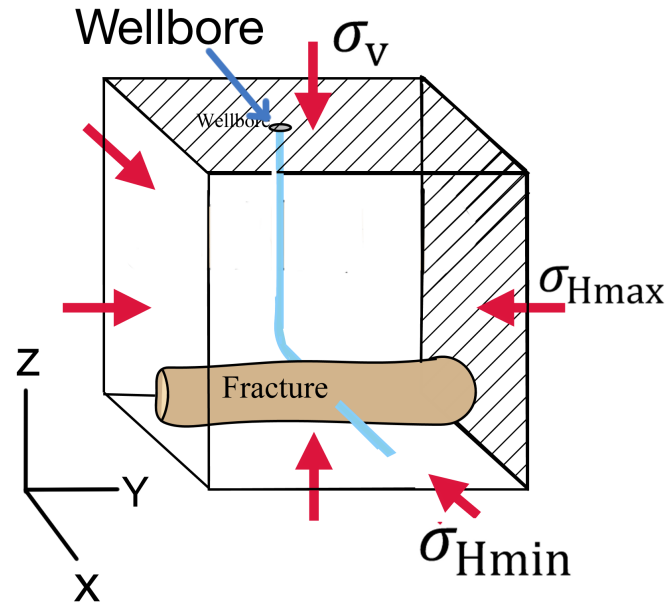
The confining stresses of the formation, the volume of the pumped fluid, the injection rate, shale and fluid properties, etc. control the fracture length. Also, propagation of the fracture is also influenced by the preexisting natural fractures or faults. Confinement in the formation limits the vertical growth of the fracture, due to the presence of the formations' high strength, which can contain the pressure of the injected fluid. As a result, the injected fluid volume can be insufficient to open cracks. Thus, it is important that the pressure caused by the fluid injection exceeds the maximum strength of the formation.

Hydraulic fracturing occurs because of tensile failure. In order to understand tensile failure, consider a macroscopic view for a plug inserted in a porous material as shown in Figure 2.5. The circles represent the grains of the porous material. The specimen is subjected to external stress, σ , from both sides. A fracture occurs when the tractions exceed the tensile strength T_0 . Stresses are positive in compression. The condition of a material to fail is expressed as:

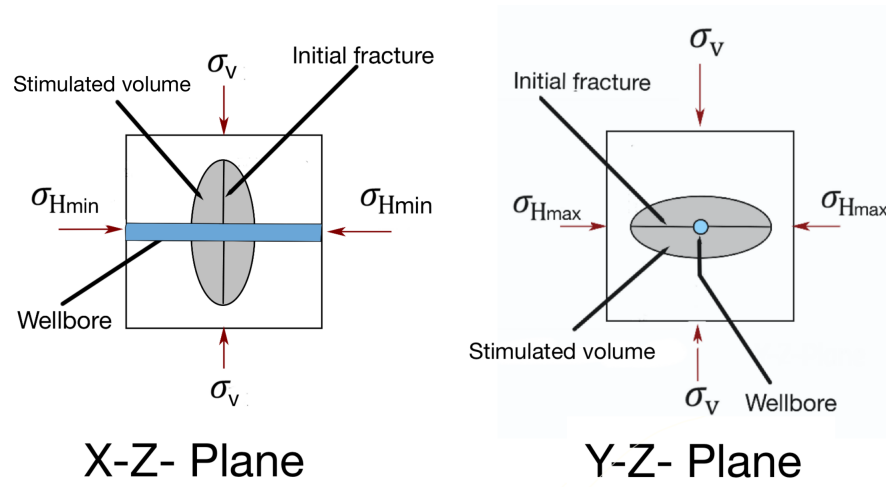
$$\sigma < -T_0 \quad (2.1)$$

For a porous material the total stress is replaced by the effective stress ($\sigma' = \sigma - p_f$), where p_f is varying pore pressure, Equation 2.1 becomes:

$$\sigma - p_f < -T_0 \quad (2.2)$$



(a) principal stress distribution in hydraulic fracturing.



(b) Cross sections from X-Z plane and Y-Z plane. σ_v is the vertical stress, σ_{Hmin} is the minimum horizontal stress and σ_{Hmax} is the maximum horizontal stress.

Figure 2.4: Stress distribution for hydraulic fracturing process.

When considering cemented grains forming strong bonds within these grains, the dashed line represents the breaking of the bond forming a closed fracture. The formula expressing the re-opening of the closed fracture phenomena is represented by Equation 2.3,

$$\sigma - p_f < 0 \quad (2.3)$$

Equation 2.3, concludes that the condition of fracture opening is reached by decreasing σ or by increasing p_f .

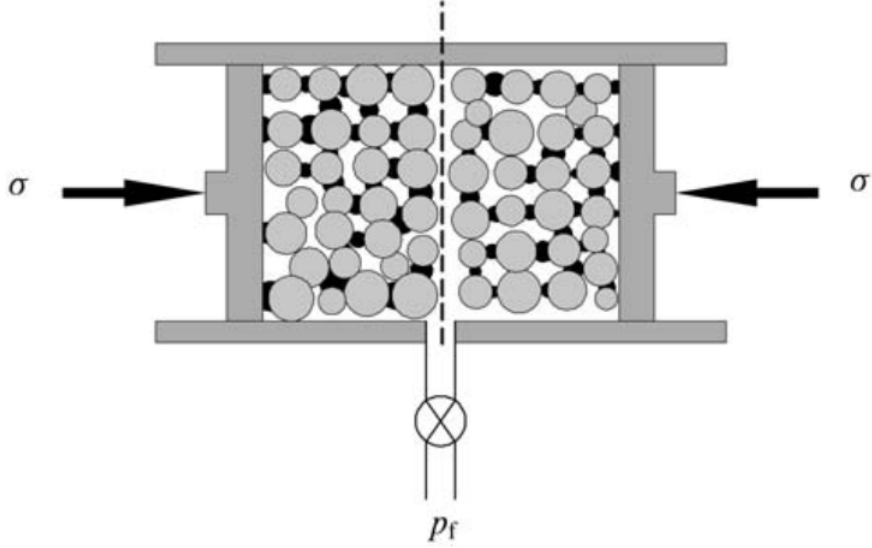


Figure 2.5: Experiment representing hydraulic fracturing process.
[48]

Figure 2.6 shows an idealized pore pressure plot of a fracture's initiation and propagation that occurs during the hydraulic fracturing of a formation. At first the formation experiences a linear elastic deformation. The fracture initiation occurs after the formation reaches its maximum strength. Then a divergence from the linear behavior occurs as shown in [48], where the pressure instantaneously drops at this point which implies an unstable fracture

growth. Meanwhile, the fracture volume is growing as the fluid injection rate goes higher. Finally, the fluid pressure becomes constant, which represents a stable fracture growth.

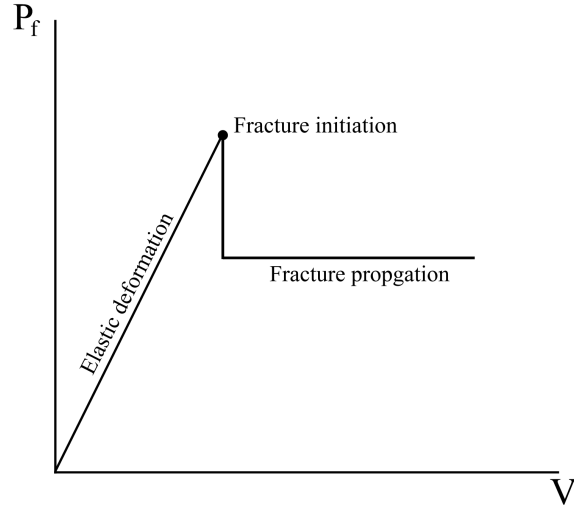


Figure 2.6: Ideal borehole pressure response during hydraulic fracturing versus the volume of fracture for two pressure cycles.

[48]

2.4 Linear fracture mechanics

Before the study of fracture mechanics, engineers usually made sure that the structure loads did not exceed the yield stress of a material. Such calculations led to many failures in many structures, leading engineers to develop the field of fracture mechanics. Fracture mechanics, specifically linear fracture mechanics, is not only applied on structural materials but on porous media as well [49].

In 1913, Inglis defined the toughness of the materials, as he showed the existence of local stresses around the corners and a hole of a stressed plate. The stresses are many times higher than the average applied stress. Inglis

showed that the degree of stress concentration is dependent on the radius of the curvature of the hole, which he defined by a stress concentration factor for an elliptical hole, k , as:

$$k = 1 + 2\sqrt{\frac{c}{r}} \quad (2.4)$$

where, c is the hole radius (Figure 2.7) and, r , the radius of the curvature of the tip of the hole. One drawback in the theory suggested by Inglis is that the stress increase at the tip of the crack is only dependent on the crack's geometrical shape.

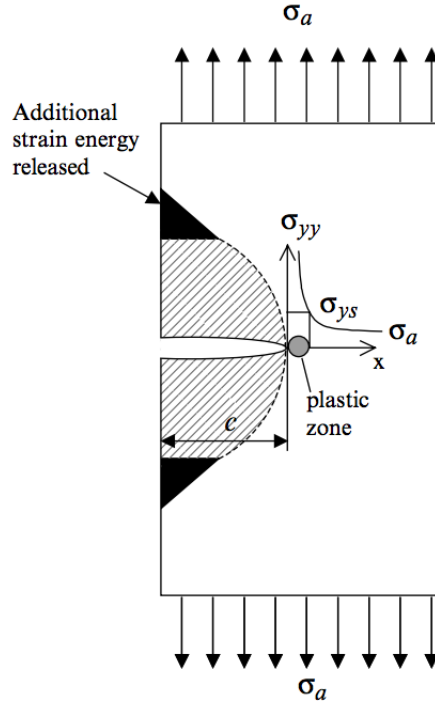


Figure 2.7: Semi-infinite plate under a uniformly applied stress with single-ended surface crack of half-length c .

[49]

Griffith (1920) proved that the stress intensity factor (i.e. the estimated

stress intensity near the crack tip caused by a far load or residual stresses) is dependent on the geometry and the fracture size, larger cracks propagate more easily than smaller cracks. Griffith based his analysis on the minimum potential energy principal, the strain energy induced from the formation of cracks is equal or greater than the increase in the surface energy required to form new cracks. He stated that cracks grow under two factors [50]: The first is that the stresses on the bonds between the formation at the crack tip must reach the point of failure. The stress is a function of stress intensity factor.

$$\sigma_{yy} = \frac{K_{Ic}}{e\sqrt{\pi l}} \quad (2.5)$$

where K_{Ic} the critical stress intensity factor, l is the fracture length and e is a geometrical parameter equal to 1 for edge cracks and less than 1 for other situations.

The second factor states that the strain energy released must be equal or greater than the surface energy of the two new crack faces, represented in the following equation:

$$\frac{dU_s}{dc} \geq \frac{dU_y}{dc} \quad (2.6)$$

where U_s is the strain energy, U_y is the surface energy, and dc is the crack length increment.

The strain energy defined by Iglis and Griffith for a narrow elliptical crack as follows

$$U_s = \frac{\pi \sigma^2 c^2}{E} \quad (2.7)$$

where, σ is the total stress. For plane strain the modulus of elasticity E is replaced with $E' = \frac{E}{1-\nu^2}$ (ν is Poisson's ratio). The fracture energy may involve an additional dissipative energy induced from micro-cracking, phase transformation and partial deformation [49].

The total surface energy for two surfaces with a unit width and a length of $2c$ is

$$U_y = 4\gamma c \quad (2.8)$$

Figure 2.8 shows Griffith energy balance criterion for crack growth. The figure shows that the crack will not grow until the strain energy released U_s becomes equal to the required surface energy U_y . After this point, more energy is released causing instability in the crack growth. A crack only grows when the stresses at the crack tip are sufficient enough to break the bonds between the crack surface. In other words, the stresses must reach the maximum tensile strength T_0 .

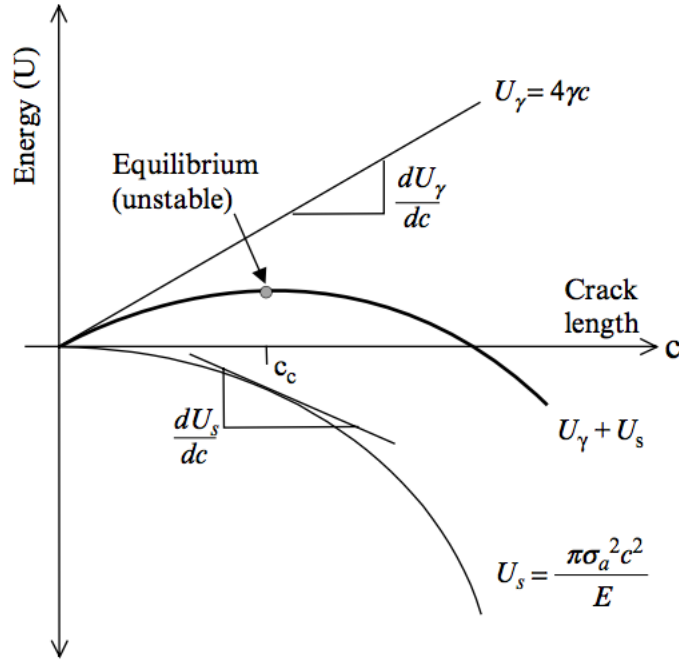


Figure 2.8: Griffith energy balance criterion for crack growth and the relation between the surface energy, strain energy and crack length.

[49]

In 1957, Irwin showed that the stress field $\sigma(r, \theta)$ goes to infinity for a sharp crack tip. Figure 2.7 illustrates a semi-finite plate under tensile uniform loading. The crack is located at one side with a crack half length c . The dark shaded area indicates additional release in strain energy due to the presence of the surface compared to a fully embedded crack in an infinite solid.

There are different modes of failure associated with fracture propagation. Common modes of failure are represented in Figure 2.9 as:

- (a) for tensile failure when a specimen is subjected to tensile forces (Mode I).
- (b) for shear failure for a specimen subjected to inside-out direction forces (Mode II).
- (c) for combined shear failure when a specimen is subjected to side ways direction forces (Mode III).

In this work only Mode I and Mode II will be considered.

2.5 Multiple fracturing and stress shadowing

The research on hydraulic fracturing was limited until the 1990s when horizontal wells became more popular and more reliable sources of natural gas extraction. Researchers attempted to find solutions to represent the hydraulic fracturing process and relate the design to more efficient user friendly models. According to Soliman [51, 52], the following criteria must be available before starting the hydraulic fracturing process:

1. The fracture orientation with respect to the wellbore, should be perpendicular to the least principal stress.
2. Data on the mechanical properties of the formation must be collected.
3. Leak-off characteristics of the formation should also be available, hence, the permeability of the formation.

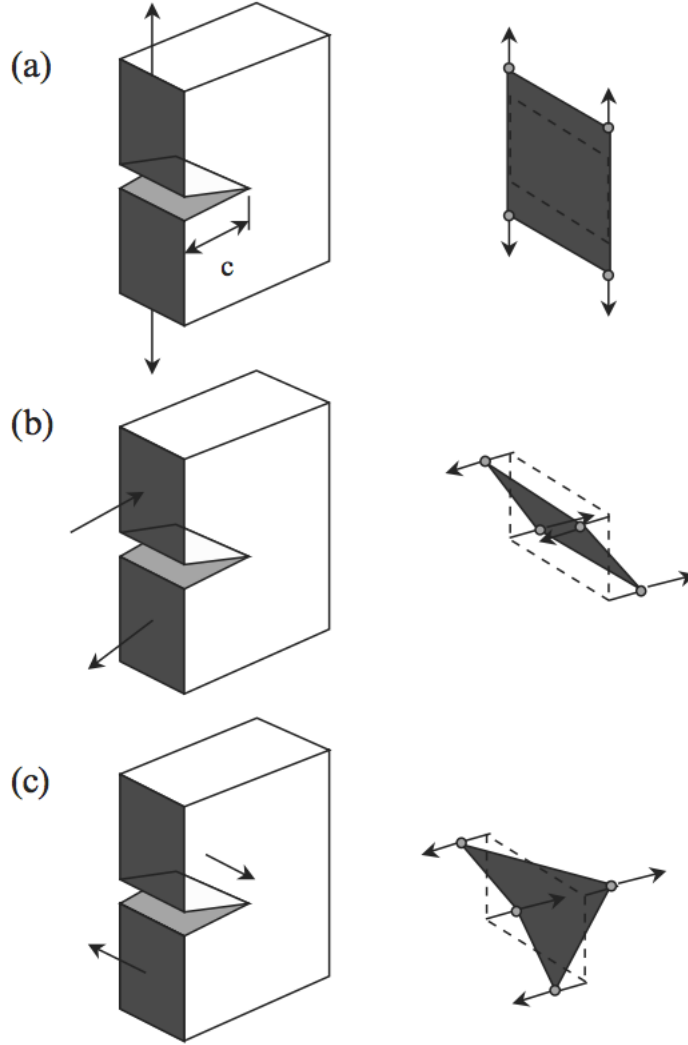


Figure 2.9: Modes of failure of a specimen a) Model I (tensile failure), b) Mode II (shear failure), c) Mode III (shear failure).

[49]

Changing the fracture spacing can lead to different fracture lengths. The longer the fracture length, the more optimum the solution is, and, thus, the higher the production [53]. In order to reach reliable results, the effect of stresses on one another must be studied during multiple stage fracturing. The

interference among stresses is known as stress shadowing, which occurs when extended fracturing causes compression in one or more dormant fractures [54]. The pressure increase starts when the active fracture gets close to the passive dormant fracture; the fracture gets larger when the shadow area (area of stress interference) increases. The pressure increases quickly towards the end of each stage, and some residual increases occur beyond the end of pumping due to the opening of new fractures. The shadowing effect is cumulative and increases as more fractures are created in an active well.

Daneshy [55] conducted an experimental study with four horizontal wells with a total of 68 fractures. The experiments detected stress shadowing at each fracture stage. He concluded that smaller spacing between fractures stages increases the cumulative production by 30-years, with higher recovery for very low permeability reservoirs.

Many numerical simulations have been conducted to investigate the same problem of stress shadowing. Cheng [56] studied the stress distribution among multiple parallel fractures using a boundary element model. He found that both the minimum and the maximum horizontal stress concentrations occur between any pair of fractures, and the increase in the minimum horizontal stress influence fracture propagation. This concludes that the geometries of the fractures (fracture length and opening) are different when several fractures are modeled. Fracture spacing influences the width of the crack. Cracks at the edges, for example, are not affected as much as central cracks. This may cause more closure. Finally, small spacing cracks might result in smaller width with greater fracture lengths.

Meyer [57, 58] introduced a Discrete Fracture Network numerical simulator. His methodology utilized a pseudo- three-dimensional ellipsoidal fracture with interaction between the mechanical and fluid equations, proppant transport, and planar fracture propagation on the principal planes. By studying multiple parallel fractures closely spaced, he concluded that these fractures caused a major impact on fracture aperture, especially on interior cracks

rather than edge cracks. Meyer’s models and Cheng’s models agreed on the same observation. Meyer also added that dominant fractures showed major implications for proppant transport and placement, assuming they remain within the dominant fractures.

2.6 Summary

Previous research focused on developing numerical models that simulate multiple hydraulic fractures, e.g., Denashy[55], Meyer [57, 58] and Cheng [56] (Section 2.5). Most of the models provided in the research assume that multiple fractures are created in ideal planar regular fracture geometries with homogeneous material properties. Moreover, it is common to assume that fracture spacing is equal. However, this is not the case when performing hydraulic fracturing, especially when a horizontal well is located at a depth of 600 meters or more it can be hard to extract data on the fractures’ behavior.

Despite the fact that the cost of extracting unconventional gas from very low permeable formations is relatively high, it has in fact enhanced the global economy and oil industry [59]. Yet, there is currently a lack of design criteria, as most oil and gas companies base their current well designs on experiments [60] and trial and error. For example:

1. Some companies and researchers tend to copy material properties from successful shale play rather than testing them [61]. The information on the material property of rocks located at such depth is hard to obtain. As well as, the material properties of the injected fluid to the well undergo changes from the surface to bottom due to change in temperature. In some research the material properties are assumed, which neglects the importance of the properties as they can change the results of the fracture propagation drastically.

2. The spacing between fractures is also unknown. The perforated guns, placed inside the horizontal well, may or may not generate initial fractures. Also the locations of the perforated guns can shift forming unequal initial fracture spacing [62, 63]. Little information is known on irregular fracture spacing. Research suggests that in case of equal spacing, reducing the spacing between the fractures will enhance the well production [64, 65]. On the other hand, it might not be the case when using unequal spacing, as the stress shadowing effect will behave differently.
3. Accurate boundary conditions for fluid flow injection are undefined. Gas companies use pressure control or fluid control pumps (e.g., reciprocating plunger pump, triplex or quintuplex pumps) [66]. The fluid injection rate or pressure occurring at the well depth is not the same as the one pumped from the surface. Researchers tend to use fixed flow rate or fixed pressure boundary conditions at the injection point depending on the scope of the research.

In summary, studying all these elements will help to provide a practical and effective way to simulate hydraulic fracturing. In addition to that, the study of multiple fracturing will assist oil and gas companies to optimize and perform an effective hydraulic fracturing process.

Chapter 3

Governing equations

3.1 Overview

Hydraulic fracture models are governed by equations that couple the following processes: the mechanical deformation induced by the in-situ stresses, and the fluid pressure that causes the fracture to propagate during fluid injection. Static equations are represented by linear and nonlinear elasticity, while fluid flow is defined by the conservation of mass. Fracture propagation can be modeled by linear fracture mechanics. Porous materials are inhomogeneous and nonlinear, and, therefore, produce complicated equations that require simplifications and assumptions in application.

This chapter discusses the equations that govern porous media while incorporating a fracture coupled with fluid flow. It also describes models used to represent hydraulic fracturing, such as the analytical and the numerical solutions.

3.2 Governing equations of a three-dimensional elastic body with a crack

Several hydraulic fracturing models are based on elasticity. The relevant equations are discussed below.

Consider an infinite body Ω with a crack Γ_c . Let x_i , $i = 1, 2, 3$, denote the direction vectors of the coordinate system. Let $\mathbf{x}$ denote a point in Ω such that $\mathbf{x} = x_1\vec{e}_1 + x_2\vec{e}_2 + x_3\vec{e}_3$. Figure 3.1 illustrates a fracture domain subjected to a far field Cauchy stress $\boldsymbol{\sigma}^0$, where

$$\boldsymbol{\sigma}^0 = \begin{bmatrix} \sigma_{11}^0 & \sigma_{12}^0 & \sigma_{13}^0 \\ \sigma_{21}^0 & \sigma_{22}^0 & \sigma_{23}^0 \\ \sigma_{31}^0 & \sigma_{32}^0 & \sigma_{33}^0 \end{bmatrix}; \quad \boldsymbol{\sigma}^0 = \boldsymbol{\sigma}^{0T}$$

Assuming the far field has no shear stresses:

$$\sigma_{12}^0 = \sigma_{13}^0 = \sigma_{23}^0 = \sigma_{21}^0 = \sigma_{31}^0 = \sigma_{32}^0 = 0$$

.

Pressure, p , is applied on the crack surfaces, Γ_c . The domain is assumed to be linear elastic and isotropic. Body forces acting on Ω are assumed to be zero, hence $\mathbf{b} = 0$.

3.2.1 Field equations

3.2.1.1 Strain displacement equation

Assume that the deformations are small, and strains are infinitesimal and elastic.

Let $\mathbf{u}(\mathbf{x})$ denoted the displacement at point $\mathbf{x}$. The strain-displacement equation defines the strain tensor $\boldsymbol{\varepsilon}$ as

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \nabla^s \mathbf{u} = \frac{1}{2}(\nabla^T \mathbf{u} + \nabla \mathbf{u}) \quad (3.1)$$

The strain displacement equation ensures that compatibility is satisfied

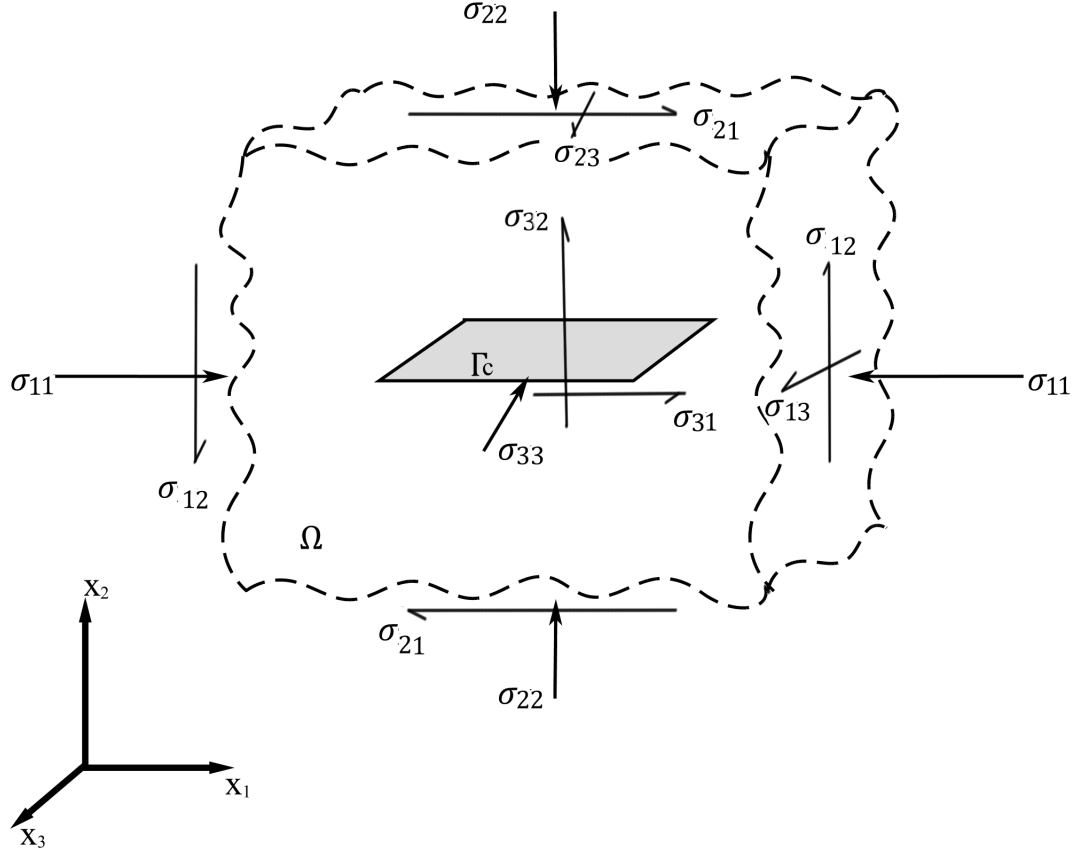


Figure 3.1: A linear elastic three-dimension infinite body with a finite crack subjected to compressive stresses.

3.2.1.2 Constitutive equation

Hooke's law relates the Cauchy stress tensor and the strain elastic tensor through a linear elastic relationship.

$$\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C} : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^0 \quad (3.2)$$

where $\boldsymbol{\sigma}$ is positive in tension, while $\boldsymbol{\sigma}^0$ is positive in compression. $\mathbf{C}$ is the

fourth order elasticity tensor. Even though shale is considered an anisotropic material it is often modeled as isotropic [67]. For an isotropic material $\mathbf{C}$ is symmetric, $C_{ijkl} = C_{ijlk} = C_{jkil} = C_{jilk}$, and hence

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + 2G(\delta_{ij} \delta_{jl} + \delta_{il} \delta_{jk}) \quad (3.3)$$

Where, λ and G are *Lamé* constant, also G is the shear modulus (in literature usually μ is used as *Lamé* constant instead of G but in this thesis μ is used for viscosity), and $\delta_{ij} = \delta_{kl} = \delta_{jl} = \delta_{jk}$ are the Kronecker delta, i.e:

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$

Young's modulus E and Poisson's ratio ν are the most common properties used to characterize elastic solids, but other measures are also used. For example, the shear modulus, bulk modulus K and *Lamé* modulus of an elastic solid are defined as follows:

$$K(E, \nu) = \frac{E}{3(1 - \nu)} \quad (3.4)$$

$$G(E, \nu) = \frac{E}{2(1 + \nu)} \quad (3.5)$$

$$\lambda(E, \nu) = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \quad (3.6)$$

3.2.1.3 Internal equilibrium equation

A material point $\mathbf{x}$ is in equilibrium when the internal forces are balanced by the external forces acting at $\mathbf{x}$. When body forces are applied the equilibrium equation is:

$$\nabla \cdot \boldsymbol{\sigma} + \rho_s \mathbf{g} = 0, \quad \forall \mathbf{x} \in \Omega \quad (3.7)$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, ρ_s is the density of the solid material and g is the gravitational acceleration.

For the fracture field Γ_c shown in Figure 3.2, equilibrium of tractions along the crack surface requires that:

$$p(\mathbf{x})\mathbf{n}^+ = \mathbf{t}^p = \boldsymbol{\sigma} \cdot \mathbf{n}^+, \quad \text{on } \Gamma_c^+ \quad (3.8)$$

$$p(\mathbf{x})\mathbf{n}^- = \mathbf{t}^p = \boldsymbol{\sigma} \cdot \mathbf{n}^-, \quad \text{on } \Gamma_c^- \quad (3.9)$$

$\mathbf{n}^+$ and $\mathbf{n}^-$ are the upper and lower normals to the crack boundary Γ_c respectively. Γ_c^+ and Γ_c^- are the upper and lower boundaries of the crack respectively.

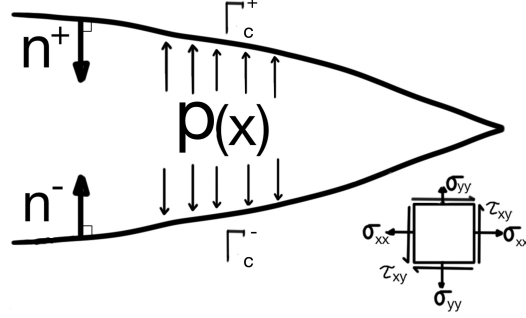


Figure 3.2: Representation of crack boundaries and stresses at point $\mathbf{x}$ under a pressure $p(x)$.

3.2.2 Effective stress

The effective stress is a portion of the total stress that is carried by the rock itself. Biot's effective stress[68] is defined as:

$$\sigma'_{ij} = \sigma_{ij} + \alpha p \delta_{ij} \quad (3.10)$$

where, σ'_{ij} is the effective stress, σ_{ij} is the total stress, α is Biot's co-efficient.

The Biot coefficient α is defined as:

$$\alpha = 1 - \frac{C_s}{C_m} = 1 - \frac{K}{K_s} \quad (3.11)$$

where C_m is the compressibility of the porous material, C_s is the compressibility of matrix material, K and K_s are the bulk modulus of the porous material and the bulk modulus of the solid material respectively. C_m is the inverse of the bulk compression modulus, i.e. $C_m = \frac{1}{K}$,

The fluid mass balance Biot's equation [69] is:

$$\varsigma = \alpha \nabla \cdot \mathbf{v} + \frac{p}{M} \quad (3.12)$$

where ς is the increment of fluid content. $\frac{1}{M}$ ($\frac{m^2s}{kg}$) is the specific storage coefficient at constant strain which relates the compressibility of the fluid phase to the solid phase, for a compressible fluid as follows:

$$\frac{1}{M} = \frac{\alpha - \phi}{K_s} + \frac{\phi}{K_f} \quad (3.13)$$

where ϕ is the porosity of the material and K_f is the bulk modulus of the fluid.

Substituting 3.2 and 3.10 into 3.7 and , assuming $\sigma_o = 0$, the equilibrium equation becomes:

$$\nabla \cdot (\mathbf{C} : \boldsymbol{\varepsilon}) - \alpha \nabla p + \rho_s g = 0 \quad (3.14)$$

3.3 Flow between parallel plates

The flow of fluid through a fracture during hydraulic fracturing can be represented by fluid flow between parallel plates [70].

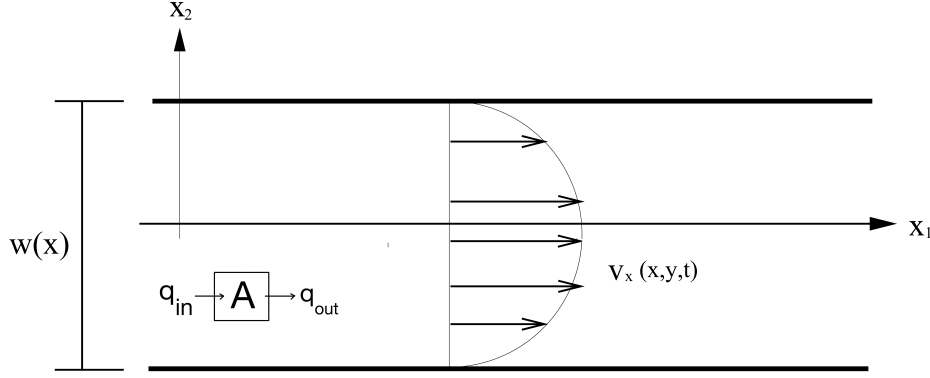


Figure 3.3: Fluid flow through parallel plates separated by a distance of $w(x)$.

Consider a mass flow in a two dimensional domain between two fixed parallel plates separated by a distance $w(x)$, as shown in Figure 3.3. Assume a unit thickness h in the x_3 -direction. The material particles are assumed to stick to the walls of the plate which leads to a zero fluid velocity at the walls. The velocity $\mathbf{v}(x_1, x_2, t)$ is a function of the spatial coordinates (x_1, x_2) and time t .

3.3.1 Conservation of mass

The conservation of mass states that at every point the mass must be conserved. Considering the mass flow through a controlled volume (A) (Figure 3.3) the mass entering per unit time must equal the sum of the mass leaving per unit time and the mass increase in (A) per unit time. Assuming there is no leakage through the plates, the mass conservation can be described as:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \quad \forall x_1, x_2, t \quad (3.15)$$

where ρ is the density of fluid per unit thickness h , $\frac{\partial \rho}{\partial t}$ is the change in density at $\mathbf{x}$ per h and $\nabla \cdot (\rho \mathbf{v})$ is the mass per unit volume leaving the material at point $\mathbf{x}$

The unknowns in this relation are $\rho(x_1, x_2, t)$, $v_x(x_1, x_2, t)$ and $v_y(x_1, x_2, t)$.

The flow is assumed to be in a steady state, so $\frac{\partial \rho}{\partial t} = 0$, and therefore, (3.15) becomes:

$$-\left[\frac{\partial(\rho v_1)}{\partial x_1} + \frac{\partial(\rho v_2)}{\partial x_2} + \frac{\partial(\rho v_3)}{\partial x_3}\right] = 0 \quad (3.16)$$

There is no fluid flow normal to x_2 -or the x_3 -directions, i.e., $v_2 = v_3 = 0$, and hence (3.16) becomes

$$\frac{\partial(\rho v_1)}{\partial x_1} = \frac{\partial \rho}{\partial x_1} v_1 + \frac{\partial v_1}{\partial x_1} \rho = 0 \quad (3.17)$$

The fluid is assumed to be incompressible; therefore $\nabla \rho = 0$ and (3.17) becomes:

$$\frac{\partial v_1}{\partial x_1} = v_{1,1} = 0 \quad (3.18)$$

3.3.2 Cubic law

The Navier Stokes equation for an incompressible, Newtonian fluid traveling through a channel [70] is written as:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = -\nabla p + \mu \nabla^2 \mathbf{v} \quad (3.19)$$

Equation (3.19) is nonlinear. Assuming that the velocity profile is constant along x_1 , flow is in steady state and the inertia terms $(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v})$ are negligible, a linear-Navier Stokes equation is developed as follows:

$$\nabla p = \mu \nabla^2 \mathbf{v} \quad (3.20)$$

Equation (3.20) is integrated twice with respect to x_2 . The velocity equation becomes

$$v_1 = \frac{dp}{dx_1} \frac{x_2^2}{2\mu} - c_1 x_2 - c_2 \quad (3.21)$$

where c_1 and c_2 are constants.

The velocity boundary conditions $v_1(x_2 = 0) = 0$ and $v_1(x_2 = w) = 0$ are applied, to solve for c_1 and c_2 as follows:

$$v_1(x_2) = \frac{1}{2\mu} \frac{dp}{dx_1} (x_2^2 - x_2 w) \quad (3.22)$$

The vertically averaged fluid flow rate, q , is determined by integrating the velocity over the height of the channel, as:

$$q = \int_0^w v_x(x_2) dy \quad (3.23)$$

Substituting (3.22) into (3.23) and integrating from 0 to w gives the cubic law as follows:

$$q = -\frac{w^3}{12\mu} \frac{dp}{dx_1} \quad (3.24)$$

where μ is the fluid viscosity, $\frac{dp}{dx_1}$ is the pressure gradient and μ is the viscosity of the fluid.

3.3.3 Flow in porous media

For slow fluid transport, flow is governed by Darcy's law [71]. Darcy's law assumes laminar flow of an incompressible fluid in a saturated porous media. Darcy's law is expressed as:

$$q = -\frac{\rho}{\mu} k (\nabla p - \rho g) \quad (3.25)$$

where k is the permeability of the material.

The relation between the fluid flow rate and the increment of fluid content is [71]:

$$\frac{\partial \varsigma}{\partial t} = -\frac{1}{\rho} \nabla \cdot q = \nabla \cdot \left(\frac{1}{\mu} k (\nabla p - \rho g) \right) \quad (3.26)$$

From Equation (3.12) and mass conservation (3.15), (3.26) becomes:

$$\left(\frac{\alpha - \phi}{K_s} + \frac{\phi}{K_f}\right) \frac{\partial p}{\partial t} + \alpha \nabla \cdot \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot \left(\frac{1}{\mu} k(-\nabla p + \rho g)\right) = 0 \quad (3.27)$$

The purpose of deriving Equation (3.27) is that it will be used in (Section 3.5) to obtain the finite element formation for a coupled porous fluid diffusion.

3.4 Constant height Quasi two dimensional plane strain models

Fracture propagation undergoes nonlinearity at the fracture tip due to the release of strain energy. The presence of the nonlinearity makes it difficult to represent fracture propagation during hydraulic fracturing. In order to develop a simple analytical hydraulic fracturing model, some approximations must be applied. Two examples for such models are the Perkins-Kern-Norgren (PKN) [8] and the Kristonovich-Geertsma-Daneshy (KGD) [9].

3.4.1 The Perkins-Kern-Nordgren (PKN) Model

Consider a fracture propagating in a straight line away from the well along x_1 (Figure 3.4), where the plane of fracture is perpendicular to the x_2 -direction. x_2 is assumed to be parallel to the minimum principal stress. The well lies along x_3 -direction. To construct the PKN model the following assumptions are made:

1. The vertical height of the fracture, h , is constant. The fracture lies in the $x_1 - x_3$ plane where $0 \leq x \leq l$, and the fracture length l , is $l \gg h$.
2. Plane strain condition is applied in the vertical plane ($\varepsilon_{11} = 0$)
3. Pore pressure variation on the vertical direction is neglected.

4. Fracture toughness is assumed to be zero, i.e., $K_{IC} = 0$.
5. The formation is assumed to be linear, isotropic and elastic.
6. Flow travels in a steady state between parallel plates along x_1 direction.
The fluid is Newtonian, incompressible.
7. The fracture cross section is elliptical in the $x_2 - x_3$ plane.

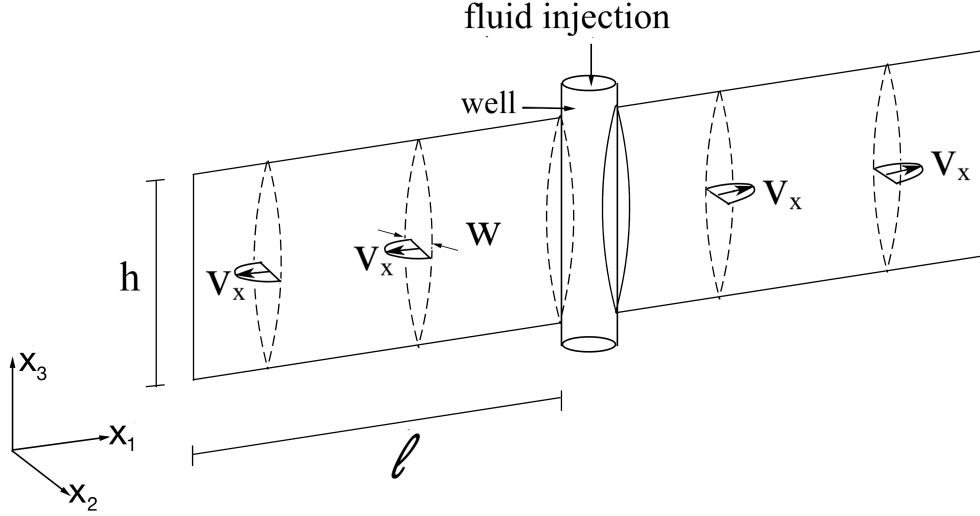


Figure 3.4: PKN model representation.

According to [72] the relationship between the maximum fracture opening $w(x_1, t)$ and the fluid pressure is:

$$w(\mathbf{x}_1, t) = (1 - \nu) \frac{hp}{G} \sqrt{1 - \left(\frac{\mathbf{x}_1}{l}\right)^2}, \quad 0 < x_1 < l \quad (3.28)$$

where p is the net pressure and x is the relative distance from the well along x_1 .

The mass conservation principle (3.15) is expressed by the continuity equation for PKN models assuming no leak-off as follows:

$$\frac{G}{64(1-\nu)\mu h} \frac{\partial^2 w}{\partial x_1^2} = \frac{\partial w}{\partial t} \quad (3.29)$$

where the left hand side term is the rate of fluid volume flowing into point $\mathbf{x}$ through a cross-section of fracture.

The initial boundary condition used to solve (3.29) is that the crack is initially closed, therefore:

$$w(\mathbf{x}, 0) = 0 \quad (3.30)$$

and the boundary condition defining constant injection rate ($m^3/s/m$) for the well is

$$q(0, t) = q_t \quad (3.31)$$

The crack width is always zero at the top of the crack ($x_3 = h$) and at the bottom ($x_3 = 0$).

Equation (3.29) is solved numerically with these boundary conditions to give the approximated solution of the PKN model [8].

When leak-off is neglected, the length of the fracture as a function of time is:

$$l(t) = 0.68 \left[\frac{Gq}{(1-\nu)\mu h^4} \right]^{1/5} t^{4/5} \quad (3.32)$$

The fracture width as a function of time at the wellbore is:

$$w(0, t) = 2.5 \left[\frac{(1-\nu)\mu q^2}{Gh} \right]^{1/5} t^{1/5} \quad (3.33)$$

The injection pressure as a function of time is:

$$p(0, t) = p_{inj} = 2.5 \left[\frac{G^4 \mu q^2}{(1-\nu)^4 h^6} \right]^{1/5} t^{1/5} \quad (3.34)$$

3.4.2 The Khristianovic-Geertsma-de Klerk (KGD) Model

Similar to the PKN model, in the KGD model the well lies along x_3 and the fracture is in the $x_1 - x_3$ -plane. The fracture length l , on x_1 and fracture width w , on x_2 where x_2 is parallel to the minimum principal stress. The following assumptions are made for the KGD model:

1. The formation is assumed to be linear elastic and isotropic.
2. Flow travels between parallel plates.
3. The fracture height h is constant along x_3 .
4. Plane strain conditions are applied in the $x_1 - x_2$ -plane, where $\varepsilon_{33} = 0$.
5. Fracture tip has a cusp-shape typical of cohesive cracks [73], that removes crack tip stress singularities found in elasticity analysis.

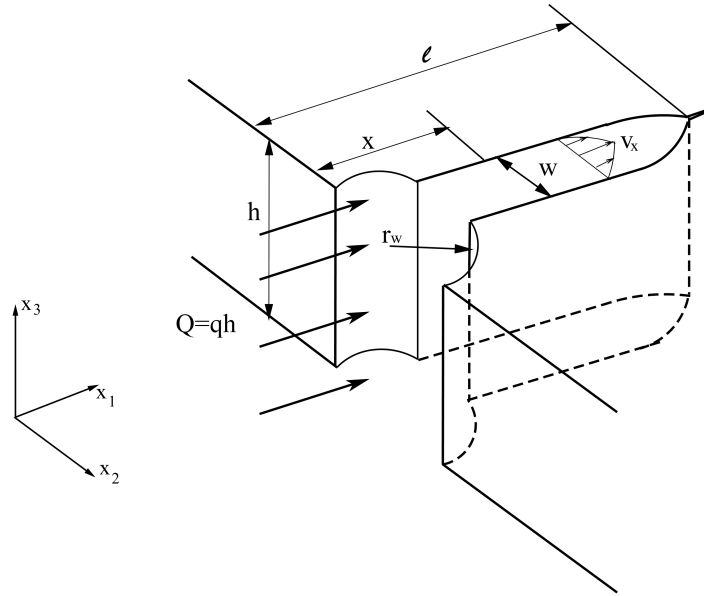


Figure 3.5: KGD model representation.

The pressure distribution for the flow of a viscous Newtonian fluid inside a fracture is derived from (3.24) assuming that the flow profile does not change along the height of the fracture. The total volume mass flow rate is:

$$Q = qh = -\frac{w^3 h}{12\mu} \frac{dp}{dx_1} \quad (3.35)$$

Equation (3.35) is integrated between x and r_w to give the pressure at $\mathbf{x}$ in terms of the injected pressure

$$p_w - p(x_1) = \frac{12\mu Q}{h} \int_{r_w}^{x_1} \frac{1}{w^3} dl \quad (3.36)$$

where p_w is the wellbore pressure, $p(x_1)$ is the local fluid pressure, and r_w is the wellbore radius.

The maximum fracture opening which occurs at the wellbore for a constant height and an infinite extension is given as [74]:

$$w = \frac{2(p_w - p(x_1))h(1 - \nu^2)}{E} \quad (3.37)$$

To solve equations (3.36) and (3.37) the following boundary condition or fracture tip condition is assumed:

$$\left(\frac{dw}{dx_1}\right) = 0, \text{ at } x_1 = l \quad (3.38)$$

This fracture tip condition is considered as a good approximation to solve for fracture properties. However, Equation (3.38) does not guarantee that $w(l, t) = 0$. The overall effect of using the previous boundary condition is considered small since there is mathematical inconsistency at the fracture tip due to singularities.

The KGD model calculates the fracture opening w and pressure p at the injection point, i.e $x = 0$. Assuming no leak-off the length of the fracture as a function of time is [9]:

$$l(t) = 0.68[\frac{8GQ^3}{(1-\nu)\mu}]^{1/6}t^{2/3} \quad (3.39)$$

The fracture width as a function of time at the wellbore is:

$$w(0, t) = 1.87[\frac{8(1-\nu)Q^3\mu}{G}]^{1/6}t^{1/3} \quad (3.40)$$

The injection pressure as a function of time is:

$$p(0, t) = p_w = \sigma_{min} + 0.96[\frac{2G^3Q\mu}{(1-\nu)^3l^2}]^{1/4} \quad (3.41)$$

3.4.3 Plane strain model application

This section presents a comparison between the PKN and KGD models assuming no leak-off. Comparing the PKN and KGD models assist in selecting the proper analytical solution that will be compared with the finite element solution (constructed in the next chapter). It will also show the impact of the different assumptions used in each model on the fracture solution. The fracture height is fixed to be 1 m. The fluid and formation properties required for the application of both models are described in (Table 3.1).

Table 3.1: Input data for PKN and KGD models verification

Parameter	Symbol	value	units
Fluid viscosity	μ	8.94E-10	MPa.s
Injection Discharge at Inlet	Q	2.00E-03	m ³ /s
Poisson's ratio	ν	0.2	
Shear Modulus	G	18.8	GPa
Injection Time	t	100	s

Using Equations (3.32)-(3.34) from the PKN model and Equations (3.39)-(3.41) for the KGD model, it can be observed from Figure 3.6-a that the

fracture length for the PKN model is nearly double the fracture length for the KGD. This implies that the existence of cohesive forces in the KGD model at the crack tip while the energy release rate is not equal to zero. Thus, crack propagation rate is expected to be lower in the KGD model than the PKN model. Since the PKN fracture is longer than the KGD and the ratio of the total mass to injected volume is the same, the crack opening displacement at the PKN model must be smaller than the KGD, as shown in Figure 3.6-b. Also, the differences between PKN and KGD are due to the difference in the assumed crack geometry, which are more difficult to quantify.

Figure 3.7 shows the fracture net pressure at the inlet. The presence of cohesive forces during fracture propagation results in the difference in pressure behavior between the PKN and KGD model. In KGD, the cohesive forces are higher, therefore high pressure is required to overcome these cohesive forces (i.e. fracture toughness) in order to achieve fracture propagation, thus resistance is increased. The concept of cohesive forces are further explained in Section 6.

3.4.4 Two-dimensional plane strain models limitation

There are many limitations accompanying the PKN and KGD model. These limitations are due to the assumptions made to simplify the analytic solution.

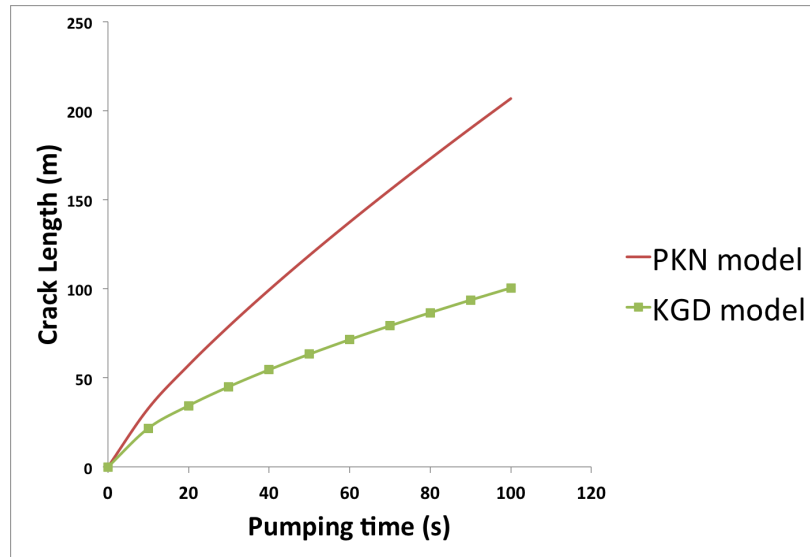
1. An assumption is made to fix the height of the fracture. One can draw attention to the fact that the PKN models are commonly used when the fracture length is much greater than the fracture height[75], while the KGD model is used when the fracture height is far greater than the fracture length. The fracture height is estimated by the user based on experience. If the fracture height is underestimated this can lead to overextending the fracture into the gas or water leg which can ruin the well design. In other words, for a given pump rate, as the fracture

height decreases, the fracture length increases depending on the leak-off. On the other hand, if the fracture height is shorter than estimated it can cause under estimations of the productivity of the well [7].

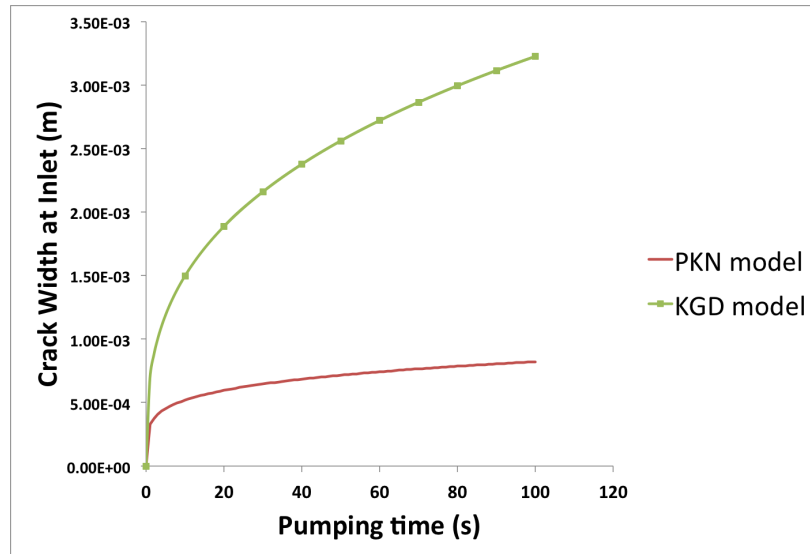
2. The simplification of the fracture shape is a problem (using elliptical or rectangular fractures), which is not the case when dealing with the actual hydraulic fracturing problems. Because the formation permeability is very low, inhomogeneous and multi-layered, it leads to the formation of irregular fracture shapes [48].
3. The two dimensional PKN and KGD models are best used when the fracture grows in a homogeneous stress formation with the same mechanical and material properties. This is not the case when dealing with actual shale formations.
4. Assuming viscosity of fractured fluid to be constant with time and space is problematic, as this is typically not the case, especially when dealing with deep wells due to the changes in temperature and pressure throughout the formation [48].

3.5 Finite element method

In order to simulate a hydraulic fracturing, a FE model of the porous media should include both solid and the fluid phases. From the equilibrium and continuity equations (3.7, 3.15), it is observed that the pore pressure and the strain in the solid matrix interact with each other. Any change in the pore pressure or in the fluid volume will affect the strain of the solid, or vice versa. This section develops a finite element solution for a poro-elastic medium based on Biot's equations.



(a) Fracture length



(b) Fracture opening

Figure 3.6: Plot for fracture properties over a 100 seconds pumping time.

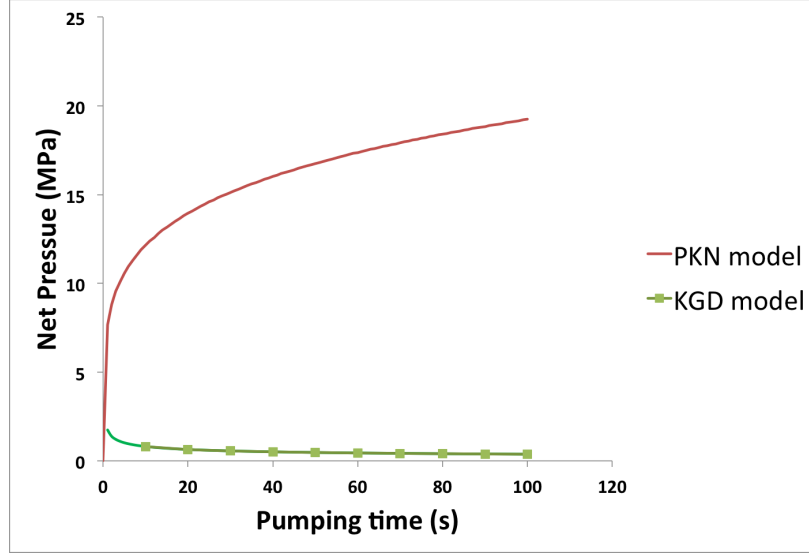


Figure 3.7: Plot for fracture pressure at inlet over a 100 second pumping time.

The Finite Element Method discretizes a complex geometry into pieces called elements: the finer the elements, the more accurate the solution. The problem might include different material properties within the same model. In two dimensional analysis each element is composed of 3 or more nodes in this thesis four noded elements are used. The set of elements that define a complex geometry is known as a mesh. The finite element method also captures local effects in each element. Each element is described in terms of global co-ordinate system (x-y coordinates) local co-ordinates (η - ζ coordinates).

Consider a four nodes element as shown in Figure 3.8. The displacements and pressures fields are approximated by:

$$\mathbf{u}(\mathbf{x}) = \sum_{I=1}^4 \mathbf{N}_I^e(\mathbf{x}) \mathbf{u}_I^e = \mathbf{N}^e \bar{\mathbf{u}}^e, \quad \mathbf{x} \in \Omega^e \quad (3.42)$$

$$\mathbf{p}(\mathbf{x}, t) = \sum_{I=1}^4 \mathbf{N}_I^e(\mathbf{x}) \mathbf{p}_I^e(t) = \mathbf{N}^e \bar{\mathbf{p}}^e, \quad \mathbf{x} \in \Omega^e \quad (3.43)$$

where Ω^e is the domain of the element e , I are the element and node number, respectively, $\mathbf{N}_I^e$ are the interpolation functions and $\mathbf{u}_I^e$ and $\mathbf{p}_I^e$ are the values of the nodal unknowns for displacement and pressure, respectively.

In the above equations the same interpolation functions are used for $\mathbf{u}$ and p .

The shape functions (interpolation functions) are defined in a parent (local) coordinate system (see Figure 3.8). For a four nodes element, the shape functions are:

$$N_1^e = \frac{1}{4}(1 - \eta)(1 - \zeta)$$

$$N_2^e = \frac{1}{4}(1 - \eta)(1 + \zeta)$$

$$N_3^e = \frac{1}{4}(1 + \eta)(1 + \zeta)$$

$$N_4^e = \frac{1}{4}(1 + \eta)(1 - \zeta)$$

where η and ζ are the local coordinates.

The displacement and pressure approximations $\mathbf{u}$ and p given by (3.42) and (3.43) can be rewritten as an approximation over the whole domain Ω as,

$$\mathbf{u}(\mathbf{x}) = \sum_{I=1}^{nn} \mathbf{N}_I(\mathbf{x}) \mathbf{u}_I = \mathbf{N} \bar{\mathbf{u}}, \quad \mathbf{x} \in \Omega \quad (3.44)$$

$$p(\mathbf{x}, t) = \sum_{I=1}^{nn} \mathbf{N}_I(\mathbf{x}) p_I(t) = \mathbf{N} \bar{\mathbf{p}}, \quad \mathbf{x} \in \Omega \quad (3.45)$$

where nn is the number of nodes in the mesh and $\mathbf{N}(\mathbf{x}) = \mathbf{N}^e(\mathbf{x})$, for $\mathbf{x} \in \Omega^e$

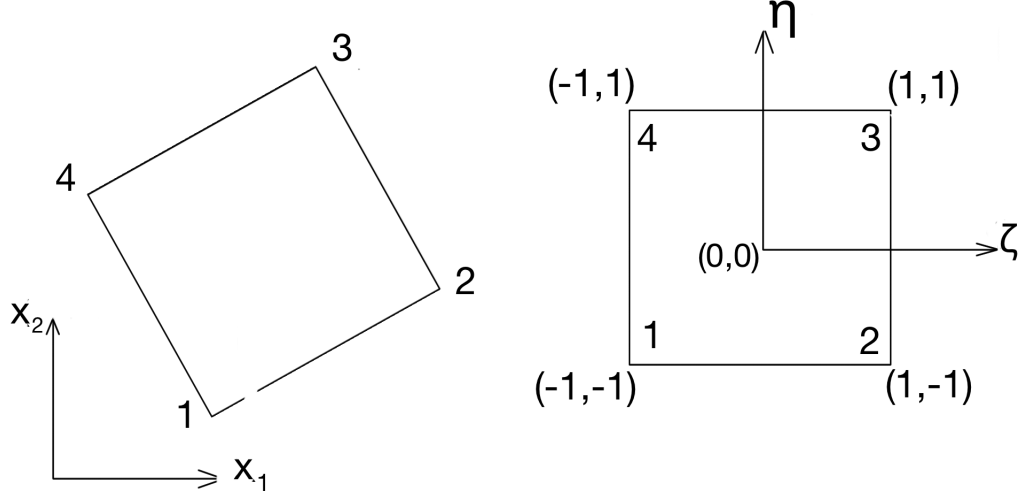


Figure 3.8: Illustration of physical coordinate system and element (local) coordinate system.

The finite element formation starts with (3.14) and (3.27) derived in Sections 3.2.2 and 3.3.3. The nodal displacements, $\bar{\mathbf{u}}$, and pore pressure, $\bar{p}$, are the unknowns. The stress and strain in matrix form are written as:

$$\{\boldsymbol{\sigma}\} = \{\sigma_{11}, \sigma_{22}, \sigma_{12}\}^T \quad (3.46)$$

$$\{\boldsymbol{\varepsilon}\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}\}^T \quad (3.47)$$

The divergence of the stress tensor is expressed by introducing a differential operator $\mathbf{L}$ [76] as:

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{L}^T \{\boldsymbol{\sigma}\} \quad (3.48)$$

where

$$\mathbf{L} = \begin{Bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{Bmatrix} \quad (3.49)$$

Strain in Voigt form is similarly expressed as:

$$\boldsymbol{\varepsilon} = \mathbf{L}\{\mathbf{u}\} \quad (3.50)$$

From Biot's theory the effective stress in Voigt form is expressed as:

$$\{\boldsymbol{\sigma}\} = \mathbf{C}(\mathbf{L}\mathbf{u}) - \alpha \mathbf{m}p \quad (3.51)$$

where $\mathbf{m}^T = \{1, 1, 0\}$ and $\mathbf{C}$ is the elasticity matrix.

Galerkin's method [77] is applied to the governing equations (3.7) and (3.15) yielding a system of semi-discrete equation which govern a poroelastic media with an incompressible fluid,

$$\begin{bmatrix} 0 & 0 \\ \mathbf{Q}^T & \mathbf{S} \end{bmatrix} \frac{d}{dt} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{p} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{Q} \\ 0 & \mathbf{H} \end{bmatrix} \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_u \\ \mathbf{F}_p \end{Bmatrix} \quad (3.52)$$

where $\mathbf{K}$ is the stiffness matrix of the solid phase

$$\mathbf{K} = \int_{\Omega} \mathbf{B}^T \mathbf{C} \mathbf{B} d\Omega \quad (3.53)$$

$$\mathbf{B} = \mathbf{L} \mathbf{N}_u \quad (3.54)$$

$\mathbf{Q}$ is the coupling matrix related to Biot's constant α

$$\mathbf{Q} = \int_{\Omega} \mathbf{B}^T \alpha \mathbf{m} \mathbf{N}_p d\Omega \quad (3.55)$$

$\mathbf{S}$ is the compressibility matrix

$$\mathbf{S} = \int_{\Omega} \mathbf{N}_p^T \left(\frac{\alpha - \phi}{K_s} + \frac{\phi}{K_f} \right) \mathbf{N}_p d\Omega \quad (3.56)$$

$\mathbf{H}$ is the permeability matrix

$$\mathbf{H} = \int_{\Omega} (\nabla \mathbf{N}_p)^T \frac{k}{\mu} (\nabla \mathbf{N}_p) d\Omega \quad (3.57)$$

$\mathbf{F}_u$ and $\mathbf{F}_p$ are the external nodal forces and fluxes, respectively, due to gravity g for external traction $\mathbf{t}$, and fluid mass injected into the system q .

$$\mathbf{F}_u = \int_{\Omega} \mathbf{N}_u^T \rho g d\Omega + \int_{\Gamma} \mathbf{N}_u^T \mathbf{t} d\Gamma \quad (3.58)$$

$$\mathbf{F}_p = \int_{\Omega} (\nabla \mathbf{N}_p)^T \frac{k}{\mu} \rho g d\Omega + \int_{\Gamma} \mathbf{N}_p^T \frac{q}{\rho} d\Gamma \quad (3.59)$$

3.6 Cohesive elements

Cohesive elements models the crack tip process zone, where material damage and softening occurs before fracture. Cohesive elements are placed between continuum elements (porous medium in this case) and does not represent any physical material, but describe the cohesive forces that occur when a material elements are being pulled apart. Cohesive elements are placed along the expected crack path as shown in Figure 3.9. This presumes that the crack path is known apriori, which is not always the case.

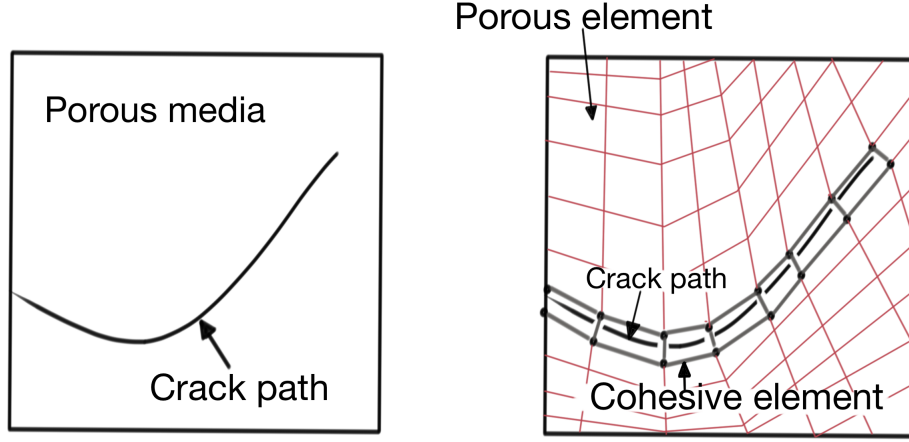


Figure 3.9: Representation of mesh for a porous media with a fracture represented with embedded cohesive element.

Cohesive elements act as nonlinear springs that represent the resisting cohesive traction and open when the stresses pass the yield strength σ_y of the material. This is illustrated in Figure 3.10, Figure 3.10-a illustrates an elastic crack loaded by internal pressure p . As expected in a linear model there is no crack tip process zone and the normal stresses at the crack tip tend to infinity. Figure 3.10-b shows a cohesive crack, with cohesive traction acting at the crack tip. Due to the presence of the cohesive tractions the maximum stress at the crack tip is the yield stress. The zone where the traction T are applied is the cohesive zone, also called the crack tip process zone.

In two dimensions, cohesive elements are defined by a nominal traction field $\mathbf{T}$, with normal $\mathbf{T}_n$ and tangential $\mathbf{T}_s$ components to the cohesive interface. The description of the failure behavior is defined by traction-separation laws. The traction-separation law describes the relationship between tractions as a function of separations. As shown in Figure 3.11, the values, $\mathbf{T}_0^N$

and $\mathbf{T}_0^S$ are the maximum normal and shear tractions, respectively, and the values, δ_0^N and δ_0^S are the corresponding separation values where the maximum traction occurs. Before the initiation of a crack, the model is assumed to undergo a linear elastic phase. The plastic phase /damage starts at the yield point σ_y once the stresses pass the yield strength the material starts deforming and the deformation is irreversible. The cohesive strength, $\mathbf{T}_0^N$, is assumed to be equal to the yield strength. When the medium starts cracking the tractions decrease as a function of separations. This phase is known as the strain softening [78, 79, 80]. There are a number of factors that play an important role in the resulting failure behavior, such as, the area under the traction separation curve that corresponds with the fracture energy, G_f . Also, the initial stiffness of the cohesive zone model has a large influence on the overall elastic deformation.

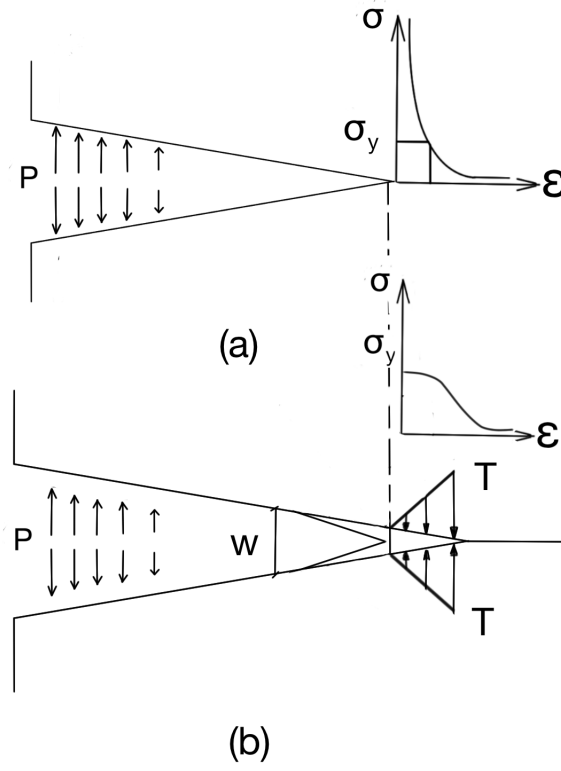


Figure 3.10: Representation of cohesive traction applied at the crack tip.

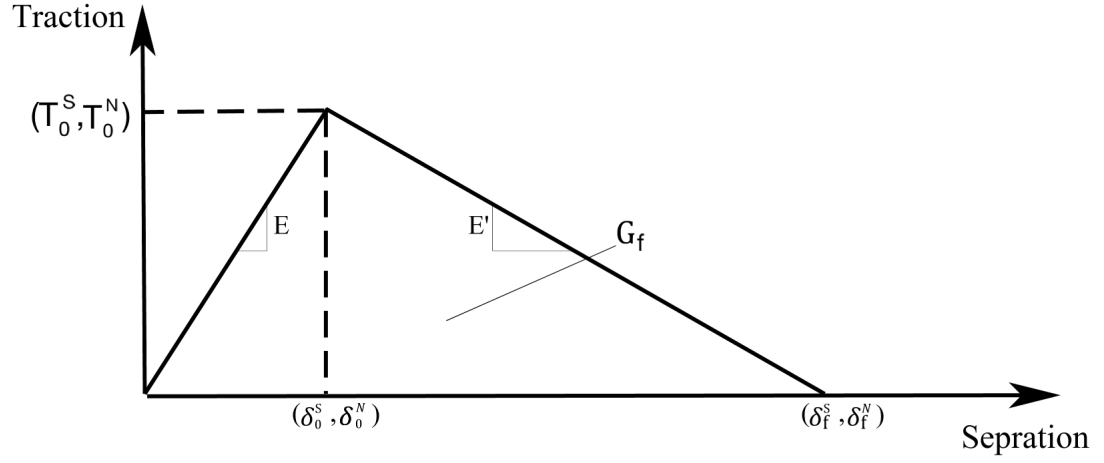


Figure 3.11: Cohesive element traction separation law showing the relation between traction and separation.

Pore fluid properties inside the cohesive elements are divided into two components as follows (Figure 3.12):

1. Tangential flow, which is governed by the cubic law (3.24),
2. Normal flow, which describes leak-off of fluid into the matrix.

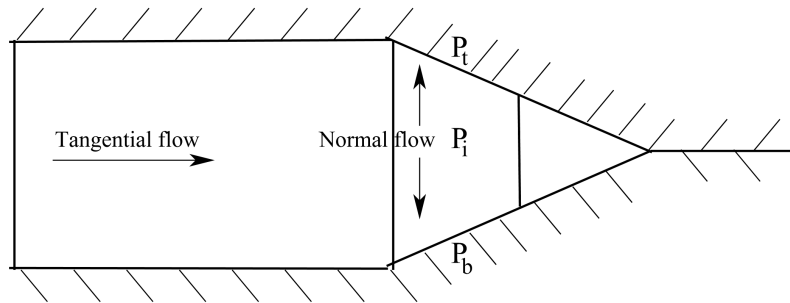


Figure 3.12: Normal and tangential flow representation in a fracture.

Assuming an incompressible fluid traveling between two parallel plates. The normal flow is defined as [60]:

$$\mathbf{q}_t = C_t(\mathbf{p}_i - \mathbf{p}_t) \quad (3.60)$$

$$\mathbf{q}_b = C_b(\mathbf{p}_i - \mathbf{p}_b) \quad (3.61)$$

where $\mathbf{q}_t$ is the flow rate at the top surface, $\mathbf{q}_b$ is the flow rate at the bottom surface, $\mathbf{p}_i$ is the mid-face pressure and $\mathbf{p}_t$ and $\mathbf{p}_b$ are the pressure at the top and bottom surfaces respectively. C_t and C_b are the top and bottom leak-off coefficient, respectively.

Leak-off reflects the resistance of fluid flow from entering from the fracture to the formation. The value of leak-off coefficient is often uncertain but can be critical to the system behavior. A study on the effect of leak-off coefficient and its effect on fracture propagation is given in Chapter 4.

Chapter 4

Single fracture model

4.1 Overview

In this chapter, a finite element model is developed for a single-crack that propagates through a shale formation. The FEM model is based on a zero-thickness cohesive element surrounded by a poro-elastic medium. The results are compared with the KGD solution (Section 3.4.3) in terms of fracture geometry (crack length and opening) and pressure. The comparison will verify the level of accuracy of the cohesive elements used in ABAQUS. Finally, a parametric study is done to analyze the effect of changing the mechanical properties such as: the elastic modulus, permeability, and viscosity on crack propagation.

4.2 FE model implementation and convergence study

4.2.1 Model description

A finite element model was developed using ABAQUS that simulates a two dimensional plane strain porous medium with a vertical fracture normal to a horizontal well (Figure 4.1). The model lies on the $x_1 - x_2$ -plane, while

the fracture lies on the x_2 -direction. Consider a datum applied at the centroid of the model, the dimensions of the model are 150m x 150m, split into two halves vertically, where the fracture is defined. It is assumed that:

1. The cohesive zone at the fracture has a zero thickness and follows the cohesive model defined in Section 3.6.
2. The model is defined as plane strain and follows Biot's theory for estimating the effective stress (Section 3.2.2).
3. The horizontal well is assumed to be symmetrical, only one-half of the model is considered (Figure 4.2), cutting from the wellbore (only the upper side of the wellbore).
4. A Newtonian fluid is injected at a constant flow rate. The fluid is incompressible with constant viscosity and density.
5. Fluid flow has one component, tangential as described in (Section 3.6), in other words the leak-off is zero.

4.2.2 Boundary conditions

The finite element model should be restrained from both sides and at the bottom boundary (the location of the horizontal wellbore). This is not the case in the model described in Figure 4.2. In order to compare between the KGD model and the finite element model, the same boundary conditions should be used as described in Equation (3.38). The KGD model only considers the flow inside the fracture. The top of the crack is restrained in the x -direction only, to allow the movement along the crack and to keep symmetry at both sides of the porous media. This approximation is acceptable for validation purposes.

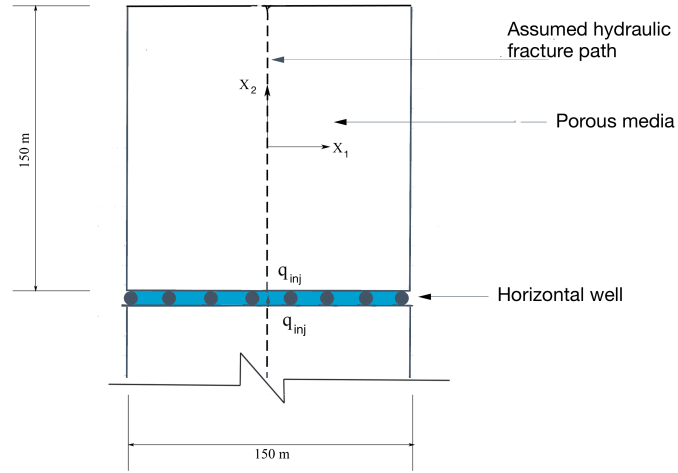


Figure 4.1: Illustration of a one crack injected with fluid through a horizontal well.

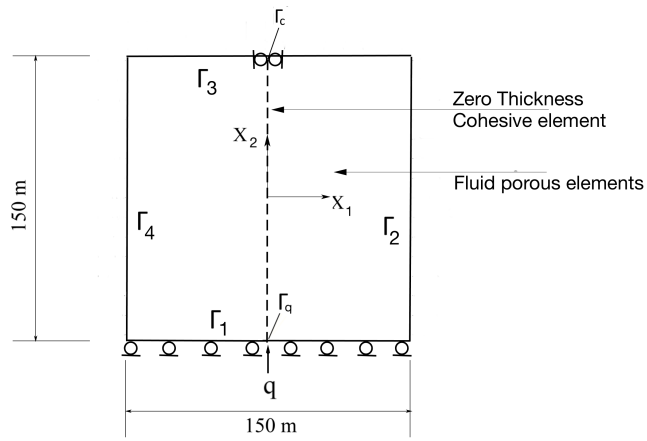


Figure 4.2: Loading and boundary conditions for FEM porous material model with a cohesive fracture.

However, when modeling a hydraulic fracture that addresses an actual hydraulic fracture problem, different boundary conditions should be used (described in Chapter 5). Considering the datum in Figure 4.2, the boundary conditions are:

$$u_2(\mathbf{x}, t) = 0 \quad \mathbf{x} \in \Gamma_1, \forall t$$

$$u_1(\mathbf{x}, t) = 0 \quad \mathbf{x} \in \Gamma_c, \forall t$$

$$q(x, t) = \bar{q} \quad \mathbf{x} \in \Gamma_q, \forall t$$

$$p(\mathbf{x}, t) = 0 \quad \mathbf{x} \in \Gamma_2, \Gamma_3, \Gamma_4, \forall t$$

where Γ_c is the top crack boundary, Γ_q is the bottom crack boundary and $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 are the model boundaries.

while the initial condition is:

$$u(\mathbf{x}, 0) = 0 \quad , \mathbf{x} \in \Omega$$

$$p(\mathbf{x}, 0) = 0 \quad , \mathbf{x} \in \Omega$$

$$q(\mathbf{x}, 0) = 0 \quad \mathbf{x} \in \Omega$$

where $\mathbf{u}$ is the displacement and p is the pore pressure boundary conditions, Ω is the domain of the model.

The fluid is injected with a constant fluid flow rate at the wellbore as defined in the KGD model.

4.2.3 Applied elements and element orientation

To simulate a porous material in ABAQUS a coupled porous fluid/stress diffusion (CPE4P) model is used, while cohesive elements with coupled pore fluid diffusion/stress (COH2D4P) analysis is used to model the fracture.

In ABAQUS, porous elements consist of four nodes, each node consists of three degrees of freedom, two for displacement and one for pore pressure (Figure 4.3). While for cohesive elements two pore pressure nodes are generated in addition to the four nodes applied to the porous element. These additional nodes must be distributed in the same direction as the fracture growth [Figure 4.3]. In order to add cohesive elements in a model, the fracture path must be determined apriori.

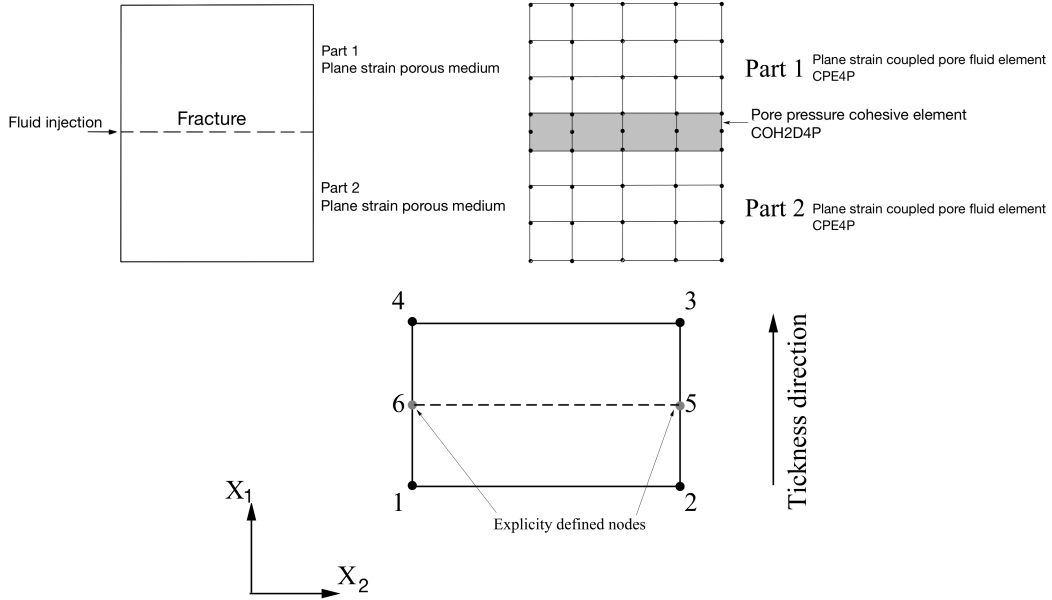


Figure 4.3: Mesh generation for the FEM model, and cohesive element node distribution and orientation.

4.2.4 Material properties

The material properties applied to the porous medium and cohesive FEM model are shown in Table 4.1 and Table 4.2, respectively.

Table 4.1: Material properties for porous media

Parameter	Symbol	Value	units	Reference
Modulus of elasticity	E	45	GPa	[81]
Poisson's ratio	ν	0.2		[81]
Permeability	k	0.0001	m/s	[82]
Porosity		0.3		[81]
Specific weight	γ	1000	N/m^3	

Table 4.2: Material properties for cohesive element

Parameter	Symbol	Value	units	Reference
Modulus of elasticity	E	45	GPa	[81]
Shear modulus	G	18.8	GPa	[81]
Failure stress	$T_0^N = T_0^S$	10	MPa	[81]
Fracture energy	G_f	1	$Mpa \cdot s$	[83]
Final separation	δ_f	5E-3	mm	
Fluid viscosity at temp (25 C)	μ	8.94e-3	Pa.s	[84]
Fluid leak off coefficient	C	5.879e-13	$m^3/Pa.s$	

The leak-off coefficient, C , is chosen as a small value to achieve a zero leak-off as in the KGD model (Table 4.2). The matrix cohesive fracture properties are chosen to be consistent with shale and the fluid properties of water.

4.2.5 Mesh Convergence study

Strategic construction of the model is essential to guarantee an accurate solution, while minimizing processing time. The density of a mesh needs to be condensed in the areas where stresses or the gradient of stresses are high.

In this case, the areas around the fracture are considered to have high change in stress values. The mesh density decreases further from the crack, starting with an initial mesh size of 10m x 10m around the crack. Other iterations are done by decreasing the mesh size until it reaches 1.25m x 1.25m. Figure 4.4 shows the convergence study of the different mesh size applied to the model. Readings of stresses are taken at a distance $x= 5\text{ m}$ $y= 0\text{ m}$ (refer to Figure 4.2). The plot shows values of the fracture height (mesh size) versus the horizontal stress at the defined point ($x= 5\text{ m}$, $y= 0\text{ m}$) for a pumping time of hundred seconds. The solutions starts to converge at a mesh size of 2.5m x 2.5m, which is sufficient for the analysis.

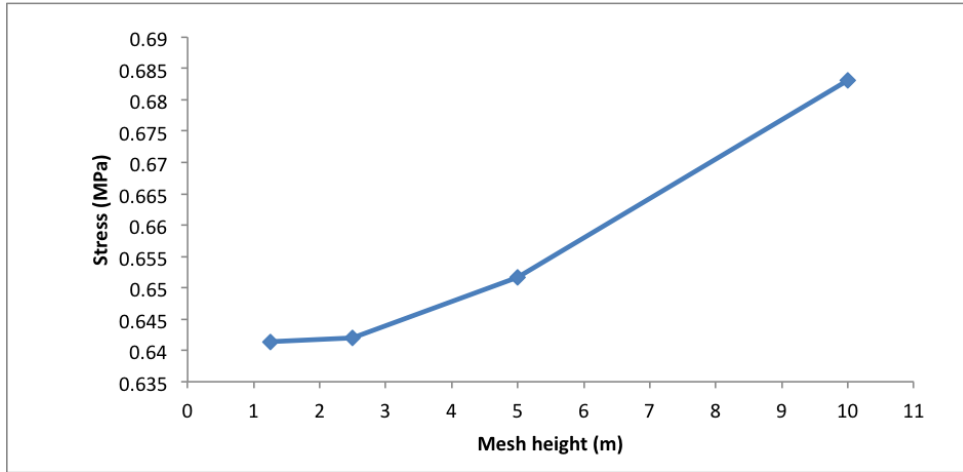


Figure 4.4: Convergence study for a vertical hydraulic fracture simulation plot.

4.2.6 Results

Figure 4.5 shows the horizontal stress contours S11 (i.e., $S11 = \sigma_{11}$) for the one crack finite element model at different time intervals. The total pumping time is 100 seconds. It is noticed that the region around the fracture tip has the highest values of stress and the fracture is under tension (Figure 4.6). In ABAQUS values for the horizontal stresses inside the crack are not showing

any results, which indicates that the stresses in this area have exceeded the maximum tensile strength of the material as defined in Table 4.2.

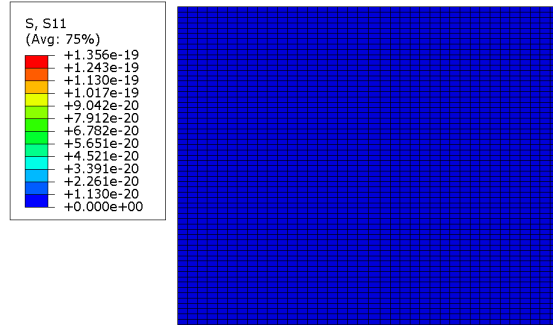
4.3 Comparison between the FE and KGD models

This section compares the FE model with the KGD model. Readings for the fracture aperture and pressure are taken at the crack inlet (wellbore).

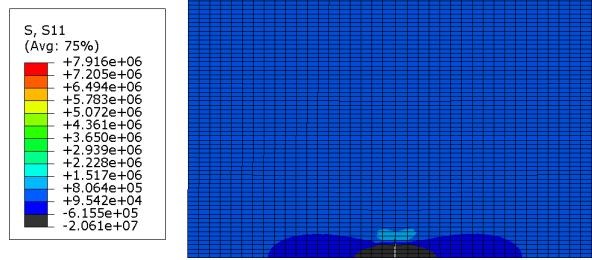
In Figure 4.7, the fracture length for both KGD and FE models shows a deviation of 5% in the results. It is expected that the two models would not exactly agree, though it is expected that they would show similar trends. The crack opening value drops as the fracture length increases. The finite element solution has higher values for the fracture length and lower values for the fracture opening than the KGD model.

As shown in Figure 4.8 the FE model starts with zero pressure at time equal to zero then suddenly increases. Meanwhile, while the KGD model starts with a pressure value of 2 MPa at time equal to zero. This behavior occurs because the FE model initial fluid flow rate is defined as zero then suddenly increases to $2.00\text{E-}03 \text{ m}^3/\text{s}$, which remains constant until the end of the simulation as opposed to the KGD model where the fluid flow rate is defined constant at all pumping time.

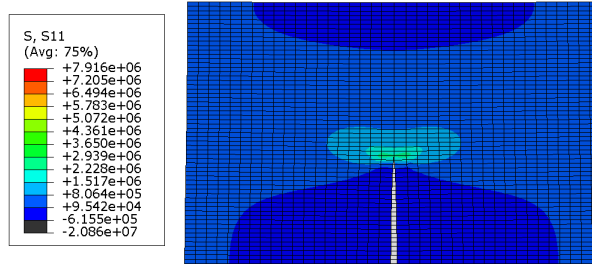
The initial value of pressure in the FE model is higher than the KGD model (Figure 4.8). The FE model at time approximately equal to zero is 5 MPa while the KGD model reads 2 MPa. This behavior occurred since the KGD model has no fracture toughness defined as opposed to the FE model. The presence of fracture toughness increases the resistance for the initial fracture to open and thus the pressure increases.



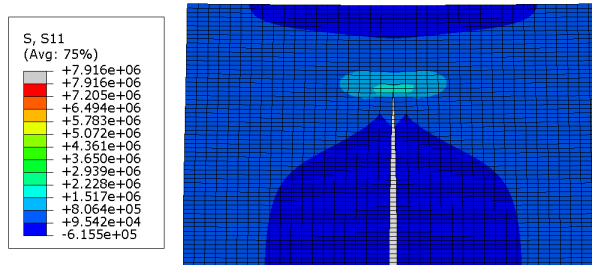
(a) S11 at initial time step



(b) S11 at time step= 2 seconds

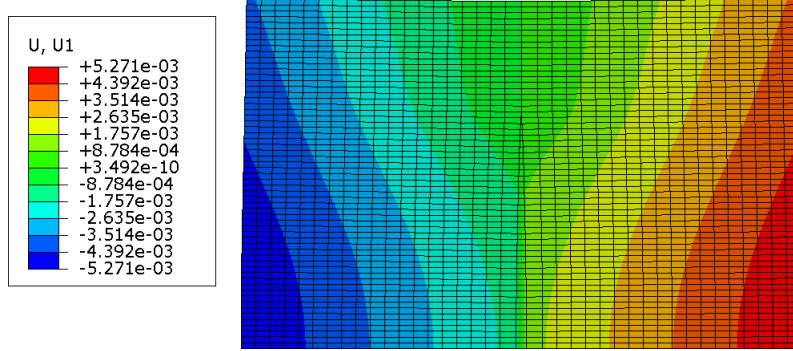


(c) S11 at time step= 50 seconds

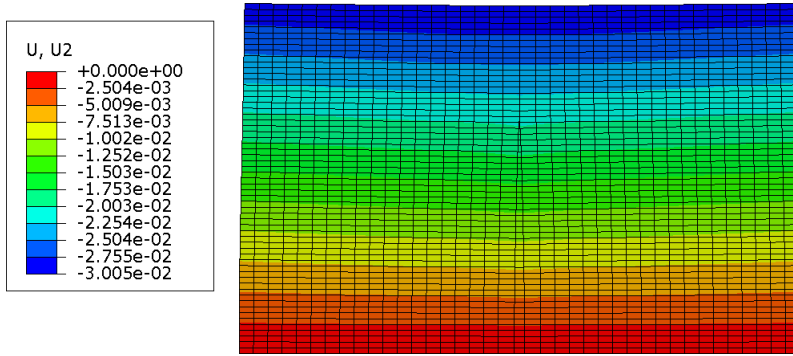


(d) S11 at at time step= 100 seconds

Figure 4.5: Stress contours S11 in Pascals at different time steps and displacements. Magnification multiplier = 1000 times.



(a) Displacement contours in the x_1 -direction



(b) Displacement contours in the x_2 -direction

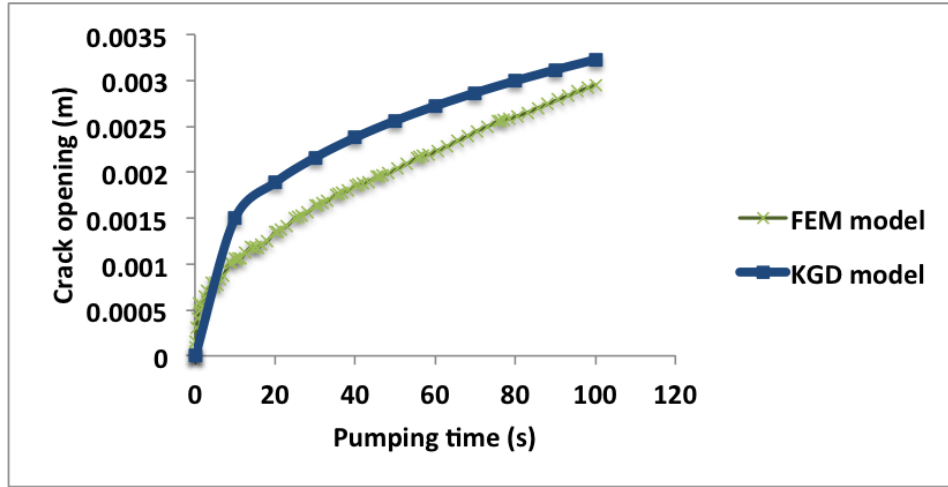
Figure 4.6: Displacement contours in meters for a vertically hydraulically fractures into porous media in u_1 and u_2 at time = 100s, Magnification factor = 1000 times.

4.4 Parametric study: the effect of material properties on fracture properties.

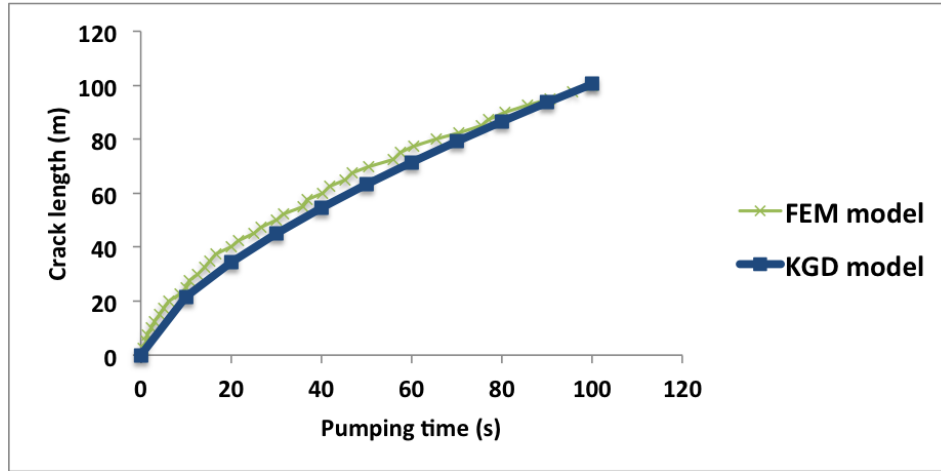
Figure 4.9 shows the effect of the formation's modulus of elasticity on the fracture propagation. Generally, as the material stiffness increases (higher elastic modulus), the strains decrease. It is observed from the crack opening plot that the increase in elastic modulus shows a decrease in fracture opening (displacements), which implies an increase in fracture length, and decrease

in the injection pressure.

Figure 4.10 illustrates the relation between the leak-off coefficient and crack propagation. In general, when injecting a fluid, some of the fluid goes



(a) Crack opening versus time at fracture inlet



(b) Crack length versus time

Figure 4.7: The crack properties for the KGD and FEM models at $t=100$ s.

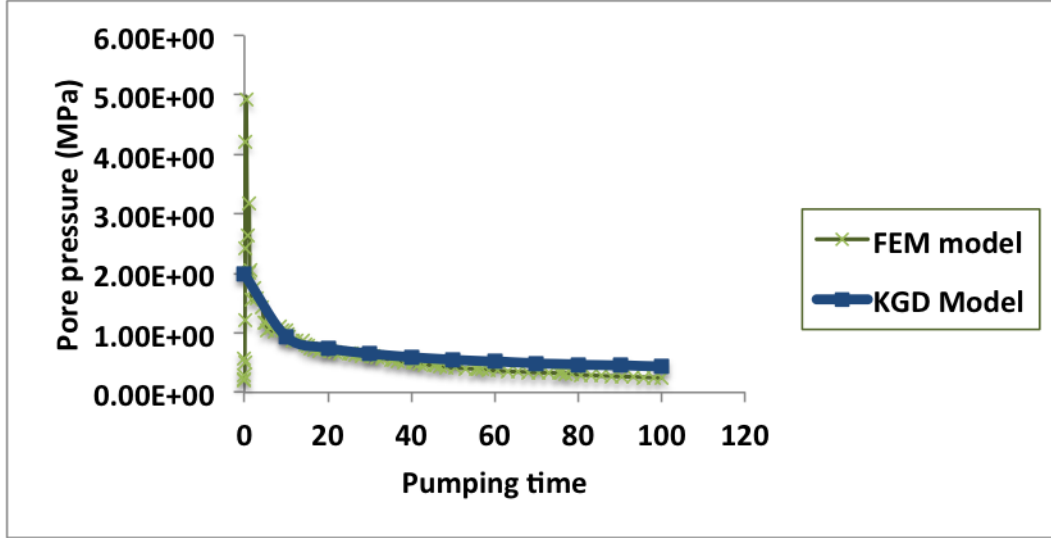


Figure 4.8: Plot for pressure at the fracture inlet showing KGD model versus FEM model for $t=100s$.

to opening and propagating the crack, while the rest flows to the surrounding porous material. The leak-off coefficient acts as a membrane that acts as a resistance for the fluid to transfer from the fracture to the surrounding material and its behavior is not well understood. From (3.60) and (3.61) the relation between leak-off and pore pressure is inversely proportional. As the leak-off coefficient increases the dissipation to the surrounding formation increases.

The plots shown in Figure 4.10 have two regimes, which can be explained as follows:

- For the higher values of leak-off $5.8 \times 10^{-11} \text{ m}^3/\text{Pa.s}$, $7.5 \times 10^{-11} \text{ m}^3/\text{Pa.s}$ and $9.5 \times 10^{-11} \text{ m}^3/\text{Pa.s}$, since the injected fluid increases suddenly, most of the fluid is lost to the surrounding area. The pressure difference between the crack and the porous material increases, and thus, the crack length increases gradually until the pressure difference is stable. After the pressure becomes stable (at 60 seconds), most

of the fluid injected opens the fracture. Therefore, a drastic increase in the fracture length occurs.

- In the case of smaller leak-off coefficient, most of the fluid injected goes into opening the fracture. The pressure difference between the crack and the porous media is small since pumping of the fluid starts. In this case, the crack width decreases with the increase of leak-off coefficient, and the crack length increases.

Figure 4.11 shows the derivative of fracture length with respect to time. The derivative of fracture length illustrates the rate of change of fracture propagation through pumping time. It is noticed that the slope of the curve does not change for the lower values of leak-off $5.8 \times 10^{-13} \text{ m}^3/\text{Pa.s}$, $5.8 \times 10^{-12} \text{ m}^3/\text{Pa.s}$ and $1 \times 10^{-11} \text{ m}^3/\text{Pa.s}$. On the other hand, for leak-off coefficient $5.8 \times 10^{-11} \leq C \leq 9.5 \times 10^{-11}$ show a change in their slope. Specifically, an increase in crack propagation rate is observed between time $20 \leq t \leq 60 \text{ m}^3/\text{Pa.s}$.

From the previous discussion it is noticed that the a small change in leak-off, from $1 \times 10^{-11} \text{ m}^3/\text{Pa.s}$ to $5.8 \times 10^{-11} \text{ m}^3/\text{Pa.s}$ leads to a large change in the fracture behavior.

Next, the effect of viscosity was studied. Any increase in viscosity will cause more friction between the fracture and the surrounding formation. Friction leads to more resistance for a fracture to propagate. Thus, fracture opening increase, fracture length decreases, and injection pressure increases as the viscosity of fluid increases as shown in (Figure 4.12).

In summary, it is very important to identify the actual material properties of a well, a small change in material property (i.e. $E = 25\text{GPA}$ to 125GPA which accounts as 20% change, and $\nu = 8.9 \times 10^{-4}$ to 8.1×10^{-4} which accounts as 0.1% change) will increase or decrease the fracture area up to 80% and thus impacts well production.

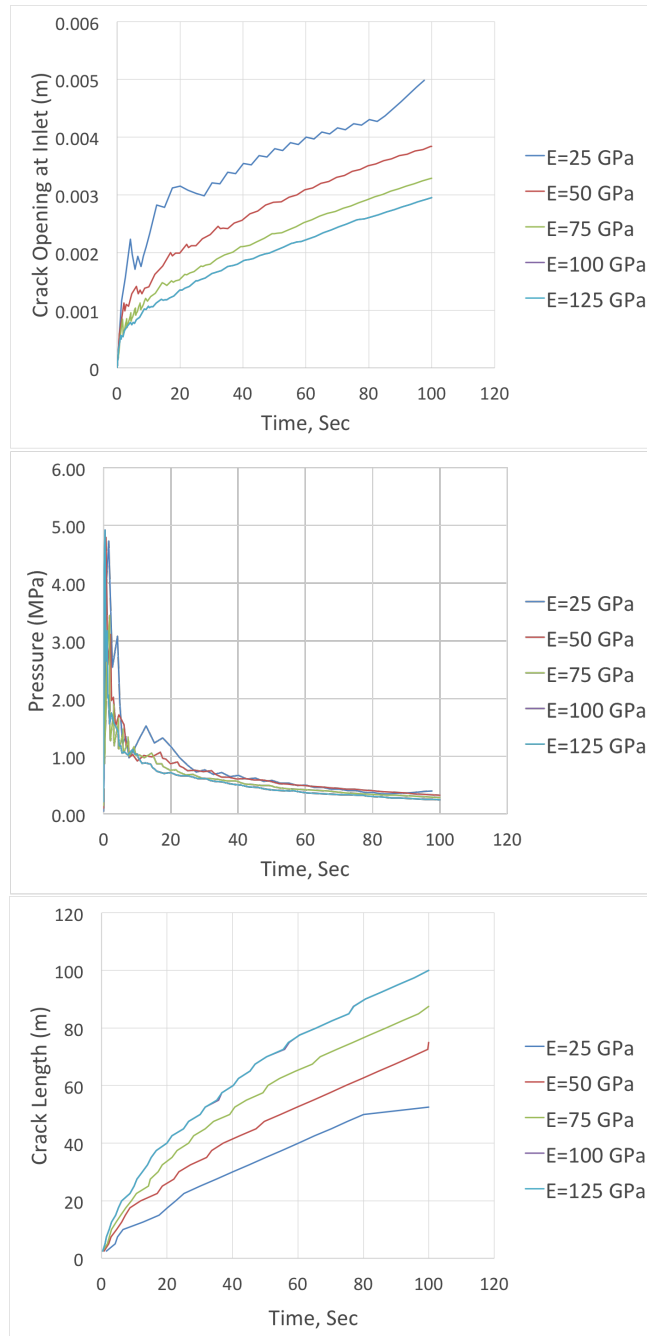


Figure 4.9: Effect of modulus of elasticity on FE model results

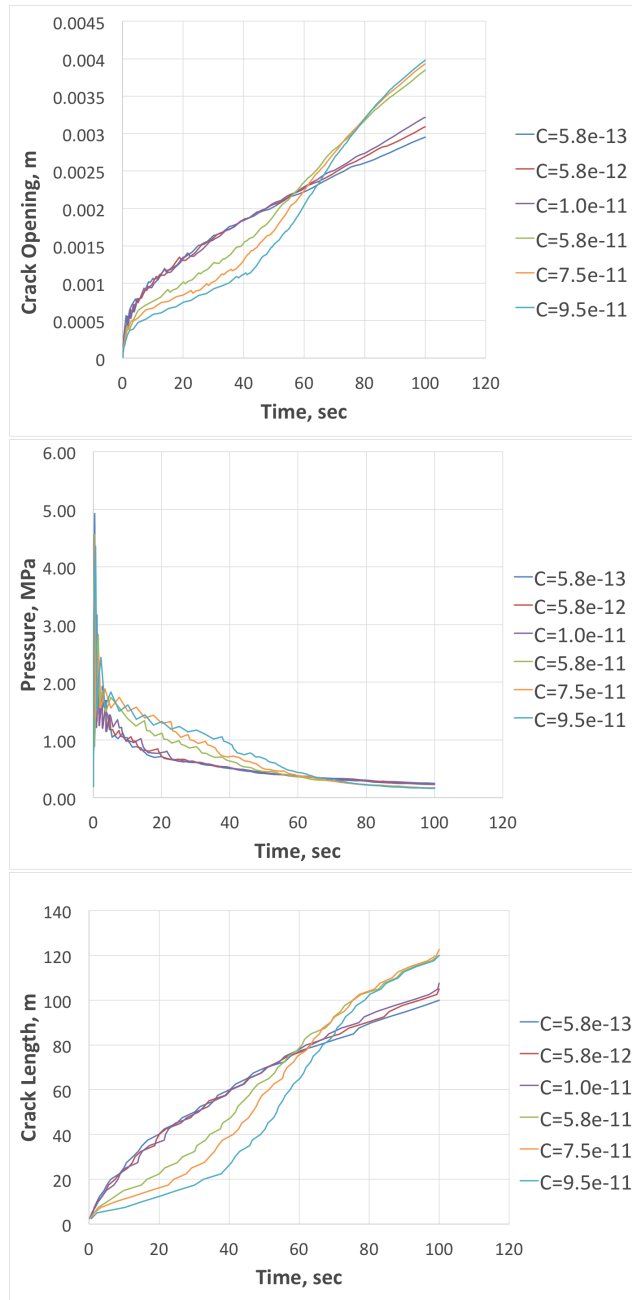


Figure 4.10: Effect of leak off coefficient and permeability on FE model.

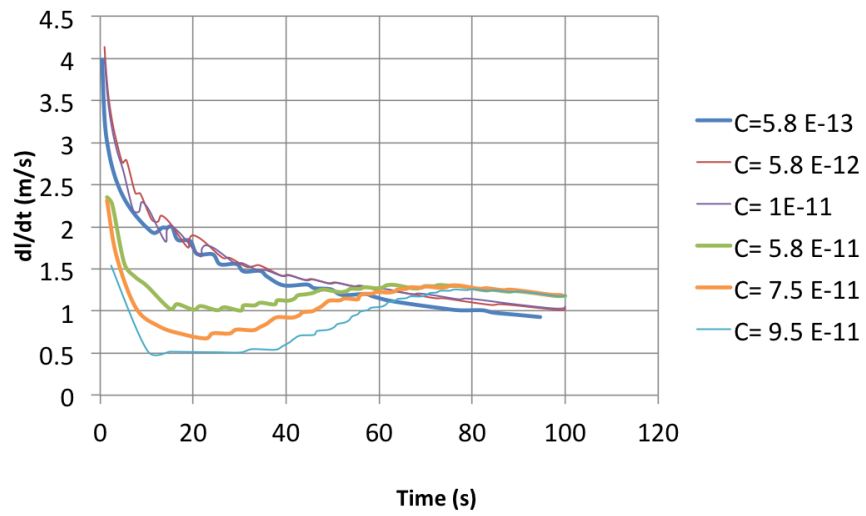


Figure 4.11: Plot of derivative of crack length with respect to time for different leak-off coefficients

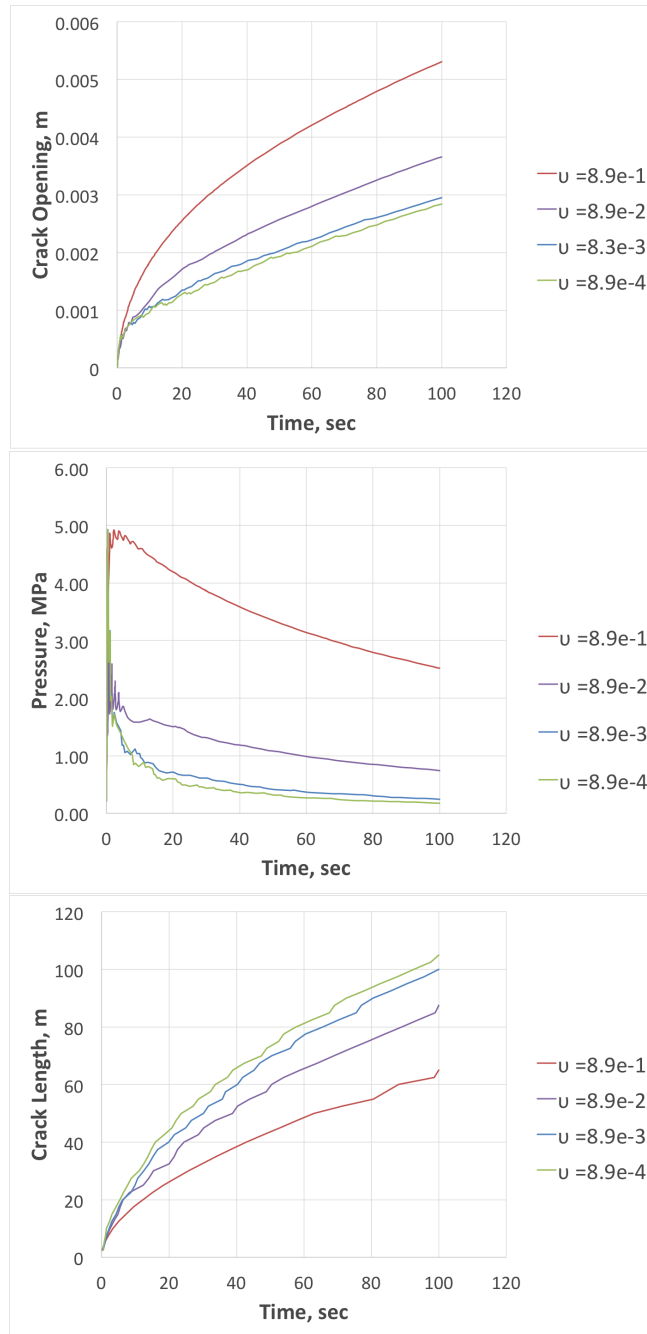


Figure 4.12: Effect of viscosity on FE model results.

Chapter 5

Three fracture model

5.1 Overview

The purpose of this chapter is to perform a practical simulation of multiple hydraulically driven fractures using finite element analysis. In order to achieve this purpose, first the effect of boundary conditions on fracture propagation (e.g., displacement and loading boundary conditions) is performed. Then, the effect of stress shadowing is addressed. Last, the effect of applying unequal fracture spacing between the cracks is studied.

5.2 Model description

Consider a 1000m deep horizontal well (Figure 5.1). The horizontal well is located along the x_1 -direction. The height of the model is 300m, which is equal to half the average height of Montney shale formation [85]. Three parallel cracks are placed as in Figure 5.2. In some hydraulic fracture fields the perforated guns are usually separated by a distance of 50m to 100m [55, 56]. For this reason, the middle crack is placed at the center of the model and the two edge cracks are initially separated by 50 m distance (i.e. $x_f = 50m$) from the center crack.

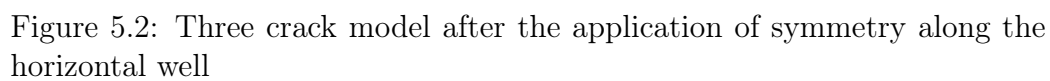
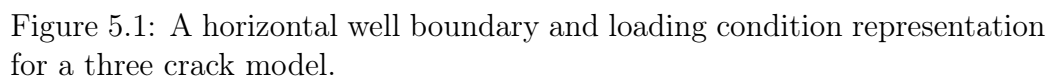
In addition to the assumptions applied in Section 4.2.1, the cracks are assumed to be parallel to each other and perpendicular to the horizontal well. Also, all cracks are pumped at the same time and rate. In the field the fluid may preferentially flow into a specific fracture this effect is called fluid partitioning and is neglected in this study [16]. The time in which the fluid reaches one initial fracture is different from the adjacent fracture. In this model the leak-off term is considered to be equal to the permeability of the formation. The same FE modeling technique used in Chapter 4 is applied to the multiple crack model.

Let S_1 , S_2 , and S_3 denote the left, center, and right crack, as illustrated in Figure 5.2. Let x_f be the distance between S_2 and S_1 . so the distance between S_2 and S_3 is $L - x_f$ where $L = 700m$.

Several case studies are conducted, the middle crack (S_2) moves towards the left crack (S_1). These increments of the crack spacings are applied as in Table 5.1.

Case Study	X_f	$L - X_f$
A	10 m	90 m
B	20 m	80 m
C	30 m	70 m
D	40 m	60 m
E	50 m	50 m

Table 5.1: Case studies applied to compare controlled pressure and fluid flow injection models



5.2.1 Boundary and loading conditions

Boundary conditions are shown in Figure 5.2. Overburden pressure is applied on the top of the domain Γ_4 , in addition to gravity loads on the whole domain Ω . Since the model is confined by the applied boundary conditions, initial stresses are subsequently added to the model, horizontal stresses on the sides are applied on Γ_2 and Γ_3 . The initial stresses are calculated as shown below.

The vertical stresses at a depth of 1000m below the Earth's surface, for a formation shale with density equal to 2500 kg/m^3 [81] is:

$$\sigma_{22}^0 = \rho h g = 25 \text{ MPa}$$

Horizontal stresses for a plane strain condition are:

$$\sigma_{11}^0 = \sigma_{33}^0 = \frac{\nu}{1 - \nu} \sigma_{22} = 0.25 \sigma_{22}$$

Boundary conditions are applied as follows:

$$u_2(\mathbf{x}, \hat{t}) = 0 \quad \mathbf{x} \in \Gamma_1, \forall t$$

$$u_1(\mathbf{x}, \hat{t}) = 0 \quad \mathbf{x} \in (\Gamma_2, \Gamma_3), \forall t$$

$$p(\mathbf{x}, \hat{t}) = 0 \quad \mathbf{x} \in (\Gamma_2, \Gamma_3, \Gamma_4), \forall t$$

$$\sigma_{22}(\mathbf{x}, \hat{t}) = \sigma_{22}^0 \quad \mathbf{x} \in \Gamma_4, \forall t$$

where u is the displacement and p is the pore pressure, Ω is the domain, Γ_i represents the boundary, where $i = 1, 2, 3, 4$.

The initial conditions are :

$$p(\mathbf{x}, 0) = 0 \quad \mathbf{x} \in \Omega$$

$$\sigma_{11}(\mathbf{x}, 0) = \sigma_{11}^0 \quad \mathbf{x} \in \Omega$$

$$\sigma_{22}(\mathbf{x}, 0) = \sigma_{22}^0 \quad \mathbf{x} \in \Omega$$

$$\sigma_{33}(\mathbf{x}, 0) = \sigma_{33}^0 \quad \mathbf{x} \in \Omega$$

Loading conditions for a constant fluid flow rate model is:

$$q(\mathbf{x}, \hat{t}) = \bar{q} \quad \hat{x} \in (S_1, S_2, S_3), t > 0$$

Loading conditions for a constant pressure model is:

$$p(\mathbf{x}, \hat{t}) = \bar{p} \quad \hat{x} \in (S_1, S_2, S_3), t > 0$$

where S_1 , S_s and S_3 are the injection points for the left edge, middle and right cracks respectively at the wellbore.

5.2.2 Material properties

Material properties represent the data collected from the Monterey shale [86], which are summarized in Table 5.2.

The final separation δ_f of the crack was calculated from shale's fracture energy $G_f = 1MPa \cdot m$, which is defined as the area under the curve (refer to in Figure 3.11) [83]. The maximum separation is calculated from the failure stress and the slope (modulus of elasticity) of the linear elastic line assumed in the traction separation curve (Section 3.6). The hydraulic fracture process occurs at significant depths and thus, the temperatures are high, which leads to changes in fluid viscosity. For simplicity it is assumed that the fluid has the

same viscosity as water at room temperature (25°C). The value of viscosity in this thesis is considered lower than the fluid's viscosity used in a hydraulic fracturing process.

Table 5.2: Material properties for three cracks FE model

Parameter	Symbol	Value	units	Reference
Modulus of elasticity	E	45	GPa	[81]
Shear modulus	G	18.8	GPa	[81]
Failure stress	$T_0^N = T_0^S$	10	MPa	[81]
Final separation	δ_f	5	mm	
Fluid viscosity at temp (25 C)	μ	0.001	Pa.s	[84]
Fluid leak off coefficient	C	3E-9	$m^3/Pa.s$	
Permeability	k	10,000	mD	[82]
Porosity	ϕ	0.3		[81]
Specific weight	γ	1000	N/m^3	

5.3 Mesh convergence study and model verification

In this section, a mesh convergence study is conducted, as in Chapter 4, Section 4.2.1), denser mesh is applied around the crack areas and the mesh gets larger further from the fracture. Starting with a mesh size of 10m x 10m around the crack, subsequent iterations are made by decreasing the mesh size until it reaches 1m x 1m. The following chart (Figure 5.3) shows the convergence study at each mesh size. Data reading are taken at a point 80 m away from the horizontal well and 10m away from the middle crack. The plot shows the height (mesh size) versus the horizontal stress at the defined point. At a mesh size of 2m x 2m the model starts to converge. In this case, using a mesh size no smaller than 2m x 2m decreases the model run time while maintaining the accuracy of the results. It is noted that the accuracy at the crack tip is likely to be less than that at the point of investigation,

since the strain gradients are higher at the crack tip.

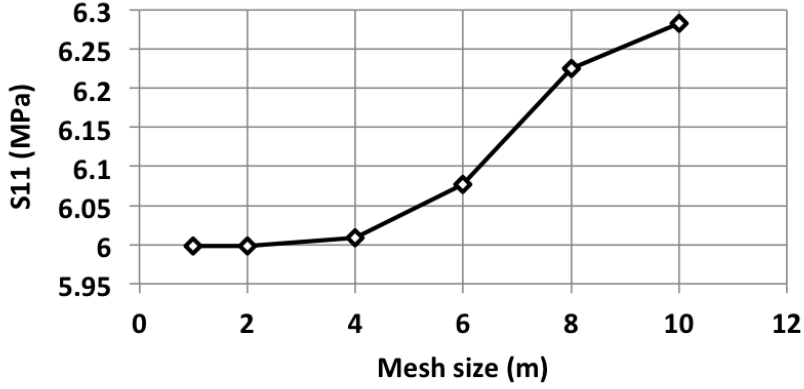


Figure 5.3: Convergence study for multiple fracture simulation.

5.4 Effect of model size

Next, the effect of model size is studied on fracture propagation. Smaller model size are faster to run while larger models are more accurate. For this reason a study is conducted to find an accurate and a fast solution. In addition to that, the boundary location could cause the model to be confined and could even prevent a fracture from propagating. Three cases with different model widths (600m, 700m and 900m) are studied. The model width for each case study is plotted against the fracture properties (fracture length and width). Case E from Table 5.1 is applied to all three models.

From Figure 5.4a it is noticed that the crack length of the outer fracture for the 600m width model has increased by 3% versus the 700m and 900m models, which shows that the smaller models add confinement to the fracture openings, since the injection rate was fixed, more confinement led to less fracture opening and so longer fractures. The 700m and 900m models show no difference in their crack length. The middle crack was not affected by the change in the model width.

The fracture width displayed in Figure 5.4b shows a 0.5% change between the 600m and 700m models for the middle crack, and 2.25% between the 700m and 900m models. The middle crack shows a 0.5% change between the 600m and 700m model and another 0.5% between the 700m and 900m models.

Hence, using a 700m model width does not affect the results or the fracture propagation, but it does decrease the model's run time.

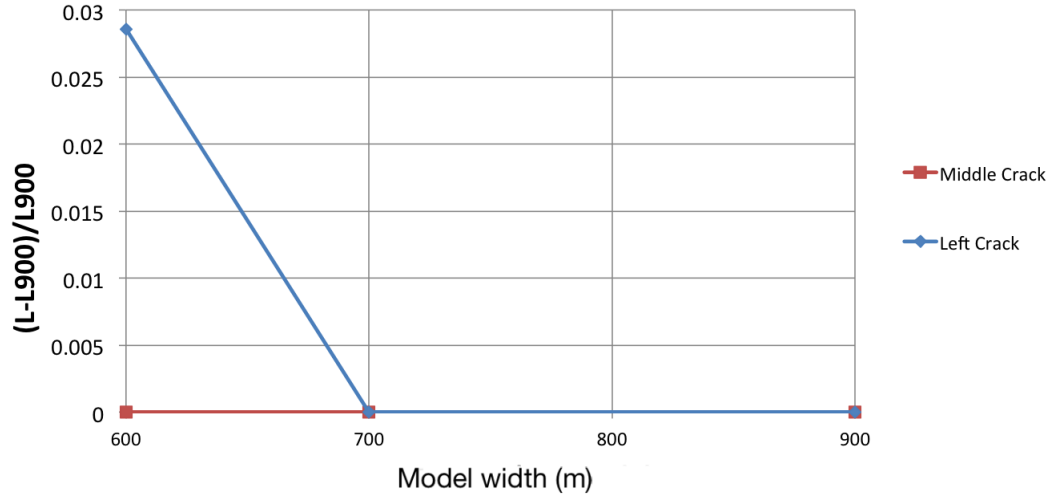
5.5 Results

5.5.1 Stress shadowing

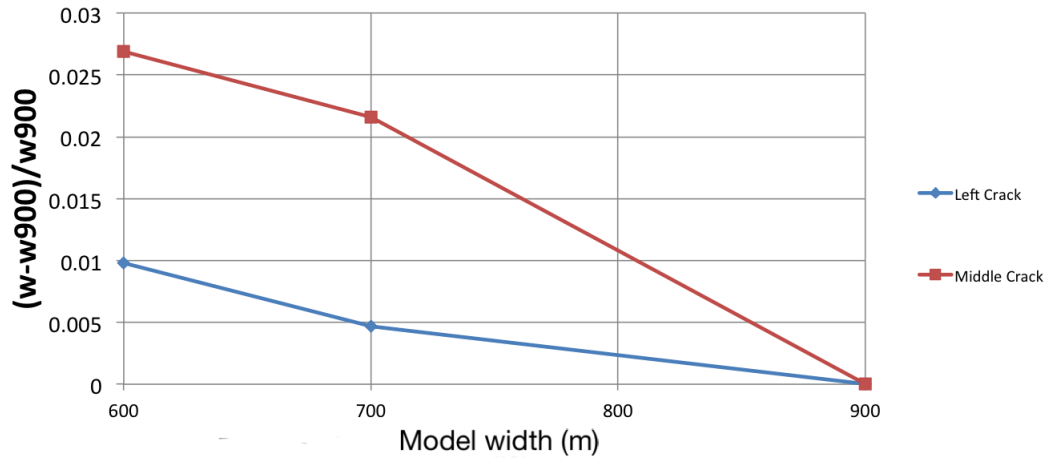
This section discusses the effect of stress shadowing on the modeled fractures. The measurements for the horizontal stresses, σ_{11} , are taken 5 meters away from the fracture inlet along the horizontal direction of the model at 1000 seconds of pumping time. Figure 5.5 compares the horizontal stresses for the different case studies described in Table 5.1.

When the fracture spacing is 50m, a peak appears at the center crack, and crack tip plasticity zone becomes smaller for the left and right cracks. This implies more brittle-like behavior and is caused by stress shadowing, the stresses introduced from the neighboring fracture (middle crack) interfere with the edge cracks. For other case studies ($x_f = 40m$ and $x_f = 20m$) the stress shadow area increases and stress shadowing effects become more obvious at the left and middle cracks, while the right crack is not affected because it becomes far from the plasticity zone of a neighboring fracture. The less fracture spacing there is, the more compressive forces that are applied to each neighboring crack. However, when the fracture spacing is reduced to 10m, it is noticed that there are only two peaks throughout the horizontal well that could be confused with a model of two fractures only. Hence, the fracture spacing becomes very small and both fractures act as one (shown as one peak). The fracture length and opening will be further studied in Section 5.5.2.1.

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(a) Crack length



(b) Crack opening

Figure 5.4: Fracture propagation properties for three different domain sized (600 m, 700 m, and 900 m), where L_{900} is the fracture length for the 900 m model at $t= 1000$ s and w_{900} is the fracture opening at the wellbore for the 900 m model at $t= 1000$ s.

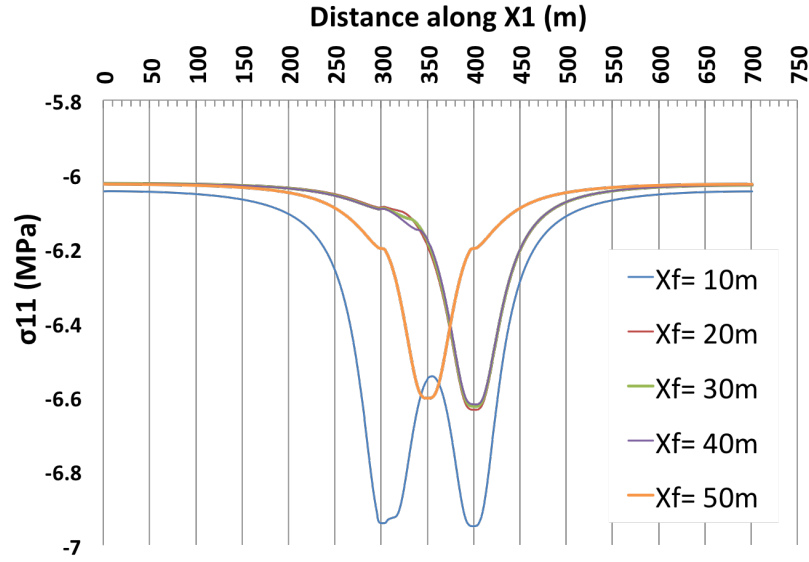
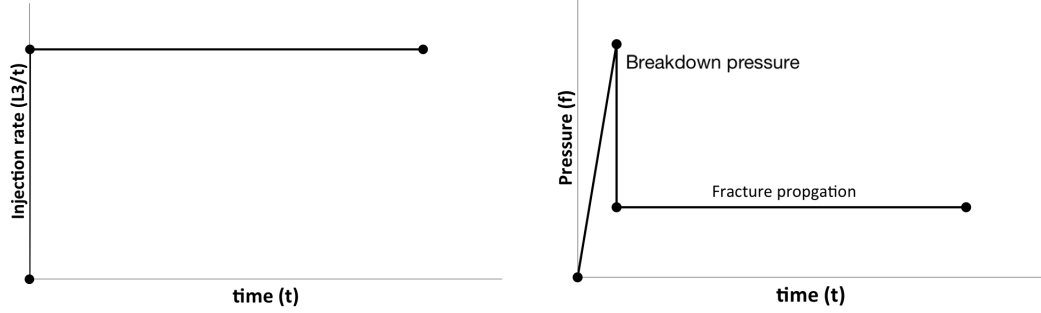


Figure 5.5: Stress shadowing effect for all combined case studies for a model width of 700m.

5.5.2 Flow versus pressure controlled injection

In hydraulic fracturing before pumping starts the fluid flow rate is zero. As soon as the fluid is pumped into the perforated cracks there is a sudden increase in fluid flow rate as in Figure 5.6a, after which it is assumed to remain constant throughout the duration of the pumping process. Figure 5.6b shows the corresponding idealized fluid pressure, required for crack propagation.

ABAQUS calculates the pressure for the amount of fluid injected at each time step. As the amount of fluid entering a point increases, the pressure increases, giving the linear relation shown in Figure 5.6b. When the pressure reaches the ultimate strength of the material the crack starts to open and propagates. It is not possible to maintain injection pressure at or above the breakdown pressure and maintain stable crack propagation and system behavior.



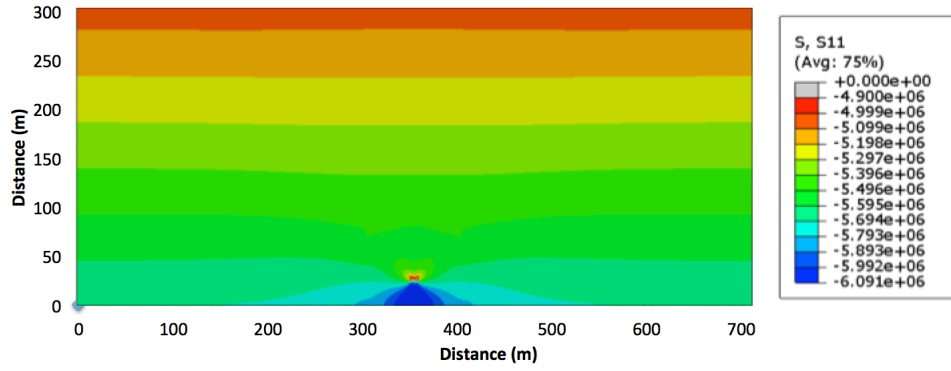
(a) Constant fluid flow injection for fracture propagation (b) Idealized fluid pressure for fracture propagation

Figure 5.6: Controlled fluid injection during hydraulic fracturing.

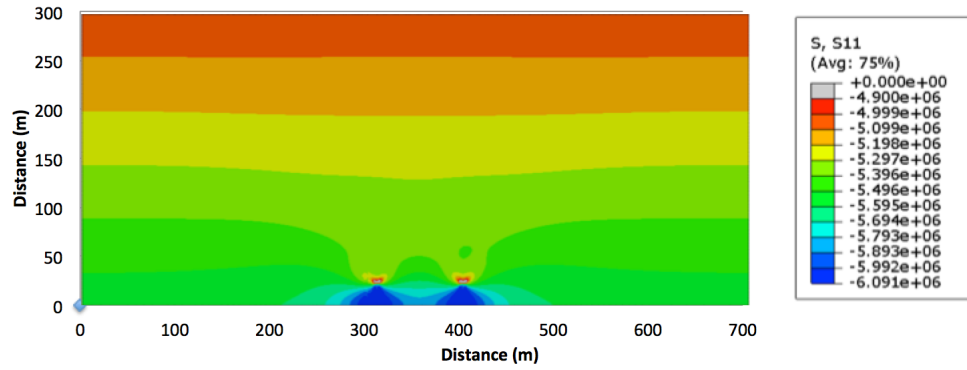
A comparison between simulations with constant injection flow rate and constant injection pressure using finite element modeling will be discussed in the coming section

5.5.2.1 Model 1: Fixed flow rate

For a constant fluid flow rate, Figures 5.7a and 5.7b show the horizontal stress and pore pressure contours for the cases of 10m and 50m fracture spacings, respectively. Pumping time is equal to 1000 seconds. The 50m spacing model shows symmetry in terms of stress contours around the fractures. It is noticed that the middle crack experiences an increase in stress around the fracture tip as previously explained and shown in Figure 5.5 while the edge cracks undergoes stress shadowing effect. The stresses increase for the $x_f = 10\text{m}$ model around the left edge crack and middle crack together and the edge right crack. The stress concentration for the left and middle crack where explained in Figure 5.5. The spacing between cracks is small causing more resistance for each crack to propagate, which leads to an increase in the stress required for each fracture to open.



(a) 10m fracture spacing



(b) 50m fracture spacing

Figure 5.7: Horizontal stresses contours for constant fluid flow injection rate.

Figure 5.8 shows plots for fracture properties versus time for left crack considering case studies A, C and E (Table 5.2). As the fracture spacing decreases, the fracture length increases. As previously explained the presence of the stress shadowing adds more compressive forces, causing less fracture opening. Since the same fluid mass is injected, most of the fluid goes into increasing the fracture length instead of opening the fractures.

In Figure 5.8b the curves show fracture opening. All cracks experience fracture closure due to the presence of compressive forces coming from one crack to the other. In the case of 50m crack spacing there is more delay

before fracture closure than those $x_f = 30\text{m}$ and 10m (Figure 5.2).

Figure 5.9 shows the same quantities but for the middle crack. In the case of 50m spacing, the middle crack has less crack opening than for the case of $x_f = 30\text{m}$. The middle crack is influenced by both edge cracks causing the fracture to open less also experiencing fracture closure (Figure 5.9b) and thus propagate longer. While, for $x_f = 10\text{m}$ spacing the middle crack is too close to the left crack which makes the cracks highly compressed and decreases the fracture length.

Figure 5.8c shows the actual pressure-time behavior for a constant injection rate, which was illustrated in Figure 5.6b. It can be seen that the breakdown pressure for the left most crack increases as the center crack is moved closer to the left crack. A similar trend is shown in Figure 5.9c for the middle crack, where the breakdown pressure is the smallest when S_2 is located in the middle of S_1 and S_3 and increases as the distance between S_2 and S_1 decreases.

5.5.2.2 Model 2: Pressure control injection

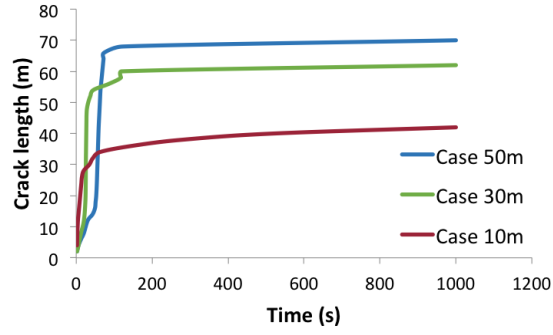
The simulation is conducted by increasing the pressure linearly with time at the injection points S_1 , S_2 and S_3 . Constant Pressure boundary condition caused stability problems. The fractures do not propagate until a certain time step when the injection pressure reaches the breakdown pressure after which an unstable fracture propagation occurs causing non-convergence of the FE model. The contours for the controlled pressure models (Figure 5.10) show longer fracture propagation than those for constant flow rate. However, the model diverges after 800 seconds, and solutions from these model are suspect.

All plots with fixed pressure boundary conditions show a delay in fracture propagation as the stresses in the formation keeps on increasing. When the formation reaches its ultimate strength, sudden unstable fracture propagation occurs and both fracture length and opening go to infinity (Figures 5.11 and 5.12) causing the model to diverge. The point of instability is reached

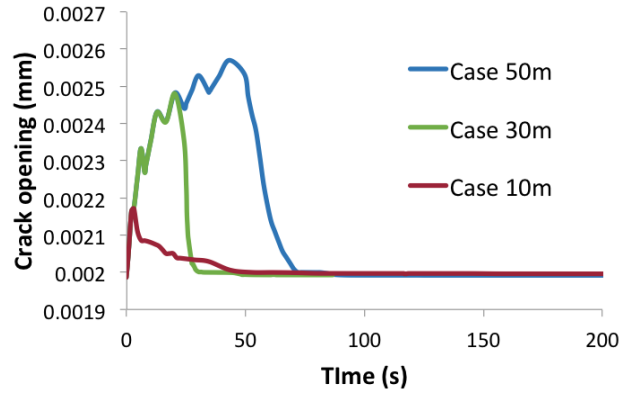
earlier when the center crack is closer to the left most crack. Therefore pressure control simulation do not produce useful results. This result is anticipated because physically we expect the injection pressure to decrease and breakdown occurs.

5.6 Summary of role of fracture spacing

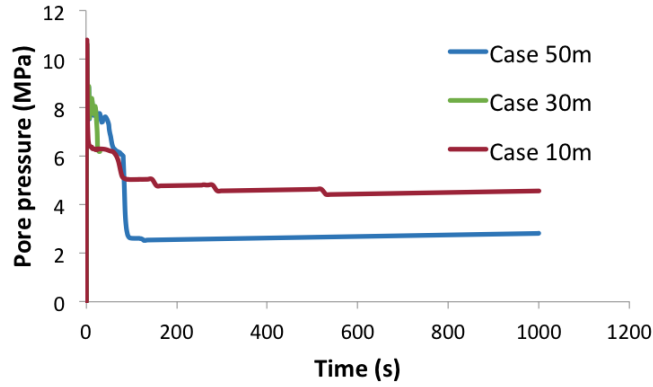
Total fracture length maybe used as a proxy for future well production since production is related to how much of the reservoir is stimulated. All of the case studies are plotted for fracture length versus the relative distance x_f . Figure 5.13 shows the results when using a fixed flow rate injection. It is observed that the total fracture length remains almost the same for all cases except for the two cases 20m and 10m fracture spacing. For those two cases the left and the middle cracks undergo a decrease in crack length due to the small spacing and high compressive stress causing the fracture to open less.



(a) Crack length versus time

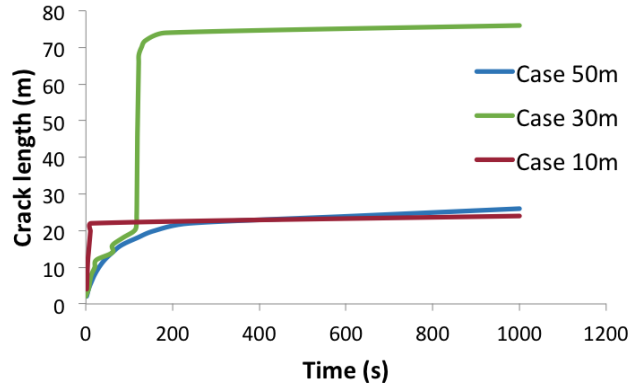


(b) Crack opening versus time at the wellbore

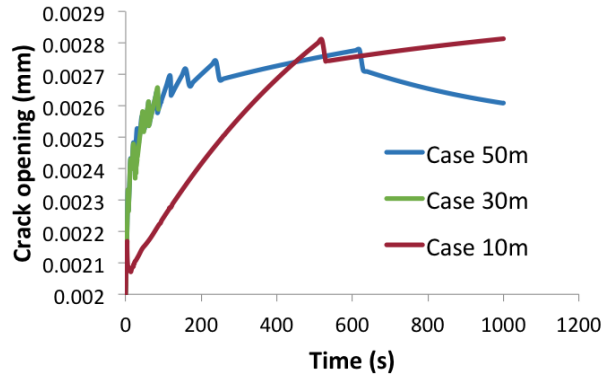


(c) Pore pressure versus time at the wellbore

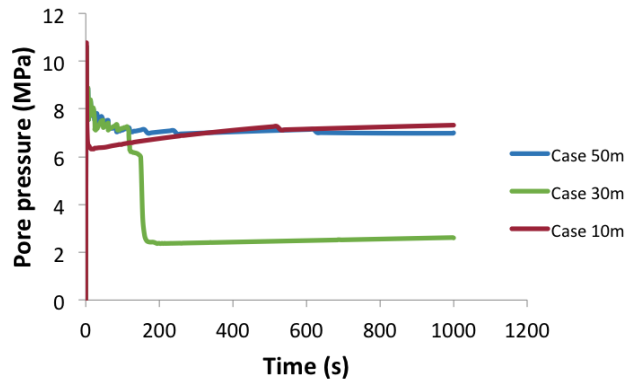
Figure 5.8: Fracture properties for the edge crack (S1) at constant flow rate.



(a) Crack length versus time

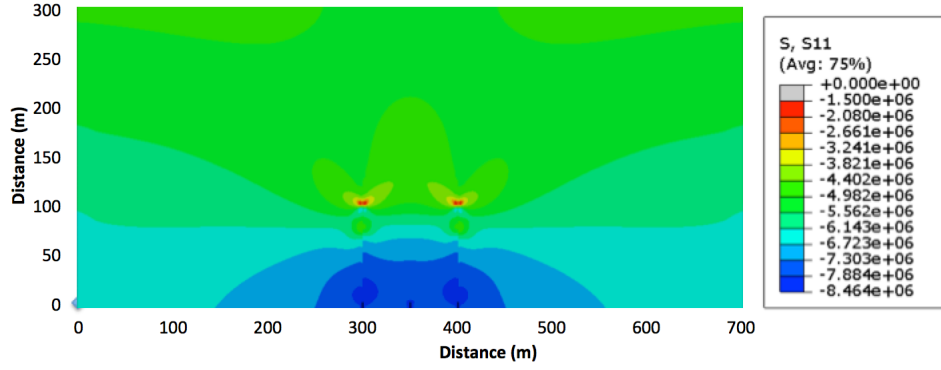


(b) Crack opening versus time at the wellbore

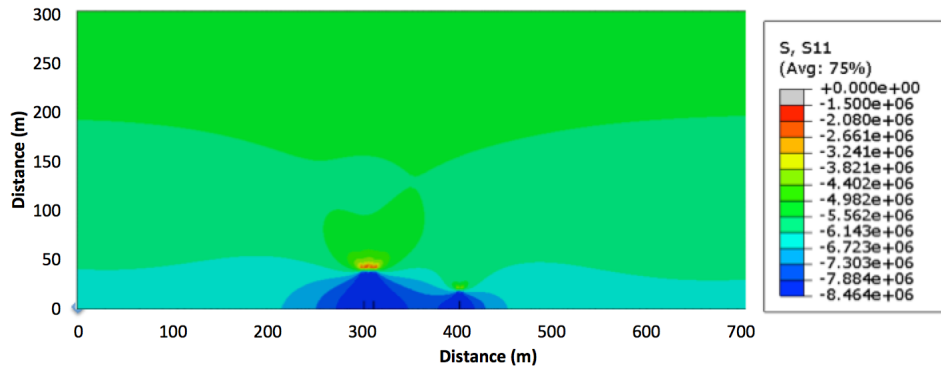


(c) Pore pressure versus time at the wellbore

Figure 5.9: Fracture propagation properties for mid crack (S2) at constant flow rate.

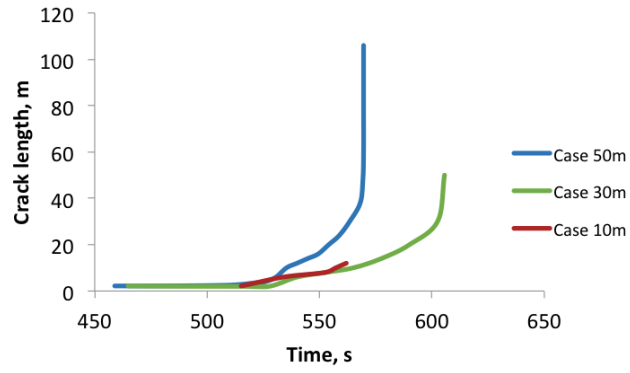


(a) 50m fracture spacing

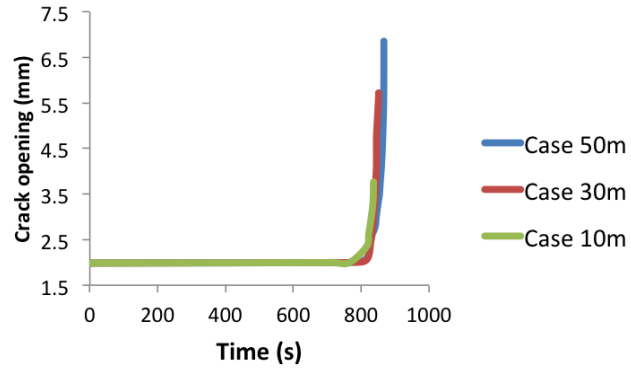


(b) 10m fracture spacing

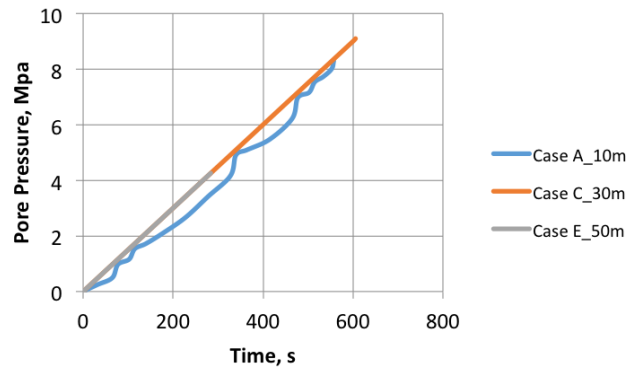
Figure 5.10: Horizontal stress contours for constant pressure injection at $t=800$ s.



(a) crack length versus time

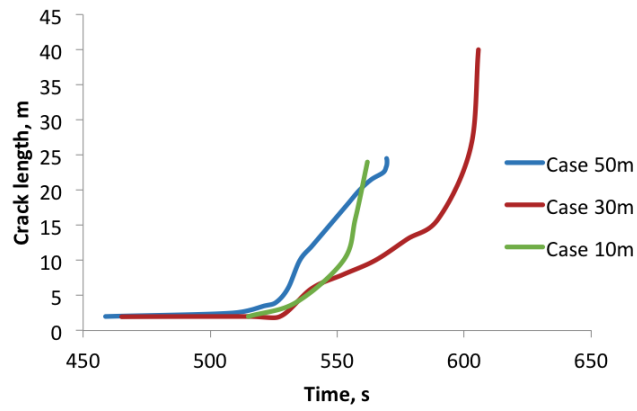


(b) crack opening versus time at wellbore

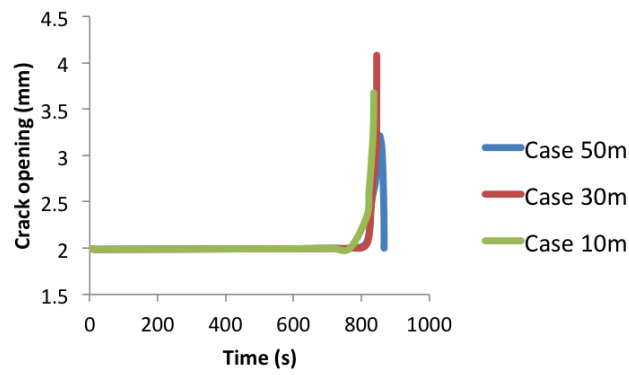


(c) Pore pressure versus time

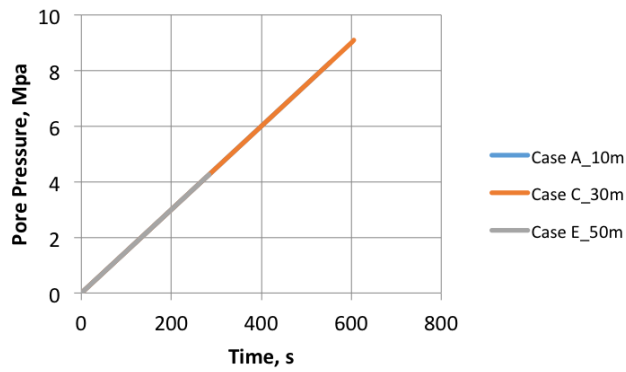
Figure 5.11: Fracture properties for constant pressure injection for edge crack (S1).



(a) crack length versus time



(b) crack opening versus time at wellbore



(c) Pore pressure versus time

Figure 5.12: Fracture properties for constant pressure injection for middle crack (S2).

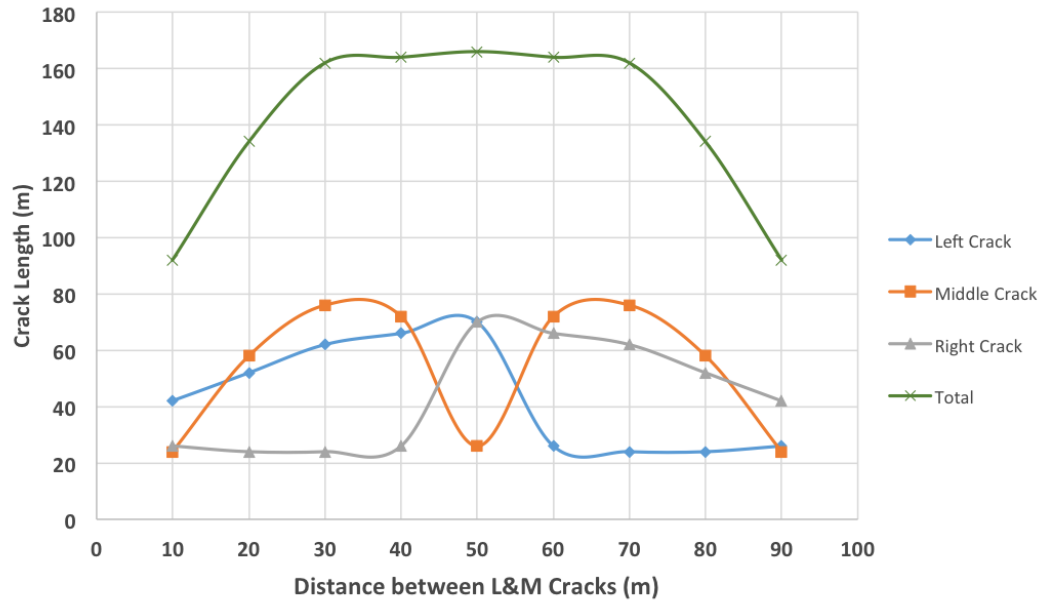


Figure 5.13: Fracture length versus fracture spacing for a constant flow rate.

Based on Figure 5.13 fracture spacing of $x_f = 30$ m or greater have little effect on total fracture length. The use of smaller fracture spacing may reduce the effectiveness of hydraulic fracturing to increase productivity.

Chapter 6

Conclusion and future direction

6.1 Conclusion

This chapter presents the summary and the conclusions of the research. Future research directions for multiple hydraulic driven fracture simulations are also provided. Simulation of multiple hydraulic fractures depends on different variables, such as: the material properties, the fracture spacing, the fluid properties, etc. In order to simulate hydraulic fracturing accurately, all variables should be taken into account. Considering all of the variables will help to provide a better understanding of fracture behaviour, allowing the production of the well to be enhanced.

The main results and accomplishments of the research presented in this thesis are:

1. A one crack hydraulic fracture finite element model was developed. The model represented a porous media being injected with fluid at a constant flow rate in order to initiate fracture propagation. The main objective of this model was to validate the cohesive elements used in ABAQUS with the KGD model. Both the analytical and numerical

model's plots showed the same trends in terms of crack properties. The fracture opening for the KGD model showed higher values than the FE model (by about 5%). The presence of cohesive tractions, and thus fracture toughness, applied near the fracture tip in the finite element model prevented the crack from opening as quickly as the KGD model. On the other hand, the fracture length for the KGD model was less than that of the FE solution. Ultimately, the FE model was deemed to be valid.

2. A parametric study was done on the one crack model to study the effect of material properties (e.g., modulus of elasticity of the formation, leak-off coefficient, and viscosity of the fluid) on the fracture propagation. It was noticed that the increase in the modulus of elasticity caused a decrease in strain and, consequently, in the fracture opening. As for the leak-off, there were two main regimes in the fracture behavior that largely depended on the pressure stability between the fracture and the surrounding material. Finally, the effect of viscosity was studied as well. As the viscosity of the fluid increased, the fracture width increased.
3. A three hydraulic fracture model was developed, which gave a practical simulation for a hydraulic fracturing process. Three basic components were studied in this model:
 - First, the effect of stress shadowing on fracture propagation: stress shadowing was found to affect the fracture behavior for multiple fractures with unequal spacing. As the fracture spacing decreases, the fracture length decreases for the edge cracks. The results for the middle crack were different. The fracture length was lower for a spacing of 10m and 50m than for a spacing of 30m. This is due to the effects of high compressive stress between the left and middle crack in the 10m model and the compressive stress coming

from both edge cracks to the middle crack for the 50m spacing model.

- Second, the effect of using constant fluid flow rate and constant pressure injection: The use of constant pressure injection caused divergence in the FE solution and did not lead to useful results. Therefore constant flow rate simulation should be used, to simulate stable crack propagation.
- Finally, the optimization of multiple hydraulic driven fractures: using equal spacing for a three crack hydraulic fractures resulted in optimizing well production in terms of crack length. Results suggest that decreasing the fracture spacing between an edge crack and middle crack by five times (keeping the third crack in place) will decrease the well production more than 50%. There is a range (e.g., approx. 10m-30m or 3-5 times the distance between the edge and middle crack) where the fracture spacing will not affect well production and will give the same amount of crack length as using equal spacing. The current research suggests a value of two-thirds the distance from the middle crack to the edge cracks.

6.2 Recommendation for future development

Simulations of fracture spacing during multiple stages of hydraulic fracturing along a horizontal well can be extended in several aspects:

1. The effects of fluid partitioning on fracture propagation and production should be studied.
2. The effects of temperature on the viscosity of the injected fluid, which was neglected in the simulations should be studied.

3. The effect of proppents on fracture openings should be studied.
4. Understanding the near fracture behavior to better quantify the leak-off coefficient, C , should be enhanced. It should be emphasized that the leak-off coefficient should be thoroughly studied as it has an effect on the results.
5. Lastly, a three-dimensional model should be developed.

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