

Influence of Boundary Conditions on the Sheared Edge Fracture Limits of a 3rd Generation Advanced High Strength Steel

by

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A thesis

presented to the University of Waterloo

in fulfillment of the

thesis requirement for the degree of

Doctor of Philosophy

in

Mechanical and Mechatronics Engineering

Waterloo, Ontario, Canada, 2025

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Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Statement of Contributions

This thesis is organized into eight chapters. Chapters 1 and 2 present the introduction and literature review, while Chapter 3 outlines the research scope, objectives, and tasks. Chapters 4 to 6 provide a concise summary of the research that has been published or prepared as manuscripts submitted for possible publication. Finally, Chapters 7 and 8 present the conclusions of the thesis and recommendations for future work, respectively.

The following co-authors and colleagues have contributed to the current work:

Professor Cliff Butcher supervised this PhD research, provided detailed guidance and constructive feedback on research direction, and thorough reviews of the manuscripts.

Ryan George (University of Waterloo) provided significant assistance and inputs on the design and fabrication of the in-plane bend test fixture.

Cameron Tolton (University of Waterloo) developed Python scripts to automate DIC data extraction for various constitutive and fracture characterization experiments.

The balance of the work is my own.

Abstract

A fundamental trade-off between strength and ductility exists in advanced high strength steels (AHSS), particularly for sheared edge splitting in automotive forming operations. The widely used ISO16630 conical hole expansion test for edge stretchability is known to be a poor representation of the in-plane deformation modes that are the primary source of edge splitting in stamping, leading to an overestimation of formability in virtual tryouts. Additionally, virtual experiments rely upon the input of a single fracture strain value to predict edge cracking in stamped parts, disregarding the effects of deformation mode and element size. An efficient and reliable modeling approach for edge failure is required without having to simulate the shear cutting process. The present work addresses some of these challenges through four interrelated tasks aimed at developing guidelines to efficiently characterize the anisotropic plasticity behavior and edge fracture limits, to support reliable experimental assessment and finite-element modelling of sheared edge fracture in practical forming applications.

There is a need to develop efficient strategies for anisotropic plasticity characterization of sheet materials to be able to accurately simulate the various tensile edge stretching modes ranging from splitting without necking to potential localization before fracture. To this end, the baseline plasticity characterization of four approximately pressure-independent aluminum alloys (AA) and steels with varying ductilities and anisotropy levels: AA5182-O, AA7075-T6, DC04, and 980GEN3 steels were performed using uniaxial tensile tests in multiple orientations. Using digital image correlation (DIC), the area strain at the neck center was monitored to measure the flow stress response to strain levels more than twice the uniform elongation, with the added advantage of probing anisotropic hardening effects. A hybrid inverse analysis procedure was further developed and applied to notch tensile tests to obtain the major stress under plane strain tension while constraining the minor-to-major principal stress ratio to remain near 1:2. Anisotropic yield functions were subsequently calibrated using data from a range of stress states with emphasis on plane strain tension. The calibrated yield functions and hardening responses were shown to accurately reproduce both the local and global behavior in flat punch hole expansion tests, which activate a wide range of tensile-dominated stress states. Flat punch hole expansion simulations using yield functions calibrated without plane strain data consistently deviated from the DIC in-plane strain magnitudes with absolute differences of up to 15% for DC04 steel. The proposed methods provide general guidelines for efficient calibration of anisotropic constitutive models for approximately pressure-independent materials that are accurate to large deformation levels.

Next, the mechanics of the conical hole expansion test were examined to assess the role of necking and anisotropy and to develop methodologies for fracture strain estimation. Finite-element (FE) models of the

test were created in LS-DYNA software for two AHSS grades with differing plastic strain anisotropies using hexahedral solid elements. An analysis of through-thickness stress and strain gradients from the numerical models revealed that localization is suppressed until a hole expansion ratio of 200%, with the outer hole edge exhibiting a proportional uniaxial tensile stress state. Any non-uniformity in hole shape or thickness around the circumference of the extruded hole was found to be a manifestation of the tensile plastic strain anisotropy distribution and not necking. The hole expansion ratio was found to be suboptimal for quantifying edge stretchability since the inner hole edge undergoes a non-linear strain path transitioning from compression to uniaxial tension. Furthermore, when using the HER as a fracture metric, the local outer hole edge element strains from FE simulations were underpredicted with absolute differences of up to 10%. An analytical technique was proposed to obtain the local major fracture strain from conical hole expansion using the outer hole diameter measured at the crack location, with the equivalent failure strain then obtained using plastic work equivalence. The strains obtained using the proposed method were in excellent agreement with the elemental strains from numerical models with a maximum difference of 4% for the highly anisotropic CP800, confirming its suitability for fracture strain measurement from the test.

Subsequently, a novel four-point fixture and specimen geometry that promotes failure under the deformation mode of in-plane bending was developed to characterize the uniaxial fracture limits of moderate ductility materials. The in-plane bending mode is also representative of edge splitting at peripheral regions of stamped parts. Techniques to detect the onset of fracture and accurately measure the edge strains from the in-plane bend tests were proposed that is applicable to a wide range of material ductilities. The uniaxial fracture strain measured in the in-plane bend test conducted with a machined edge was found to agree closely with the conical hole expansion true fracture strain of 0.68 for a 3rd generation 980GEN3 advanced high strength steel. The in-plane bend test also showed promise for plastic strain anisotropy characterization under uniaxial tension and compression to strain levels much larger than the material uniform elongation. A gauge height-to-thickness ratio of 4.0 or lower is recommended as a specimen design guideline to mitigate buckling based on a comprehensive experimental study conducted on multiple materials and thicknesses.

Finally, the influence of loading conditions on the sheared edge fracture limits of 980GEN3 steel punched with a 5.0 mm hole and 12% clearance was investigated using five different test methods that imposed different stress and strain gradients in the vicinity of the sheared edge. A convergent fracture strain value of approximately 0.30 was observed across the in-plane edge fracture tests, with the conical hole expansion test exhibiting a higher strain of 0.45 due to out-of-plane deformation and fracture being defined at through-thickness cracking. Differences in fracture strains between the in-plane tests were also magnified by the choice of DIC lengthscale or virtual strain gauge length, reflecting each test's varying sensitivity to DIC

strain averaging. Global stretchability metrics were proposed for each deformation mode, enabling edge crack assessment in industrial applications without the need for DIC. The global edge stretch metrics were also found to inform the appropriate choice of DIC lengthscale for design and FE modelling. Finally, FE simulations of the edge fracture tests were conducted using multiple mesh sizes, revealing that a boundary condition dependence can also manifest in simulations with the added influence of lengthscale sensitivity. The predicted major strains at the experimental fracture instant varied with mesh size, suggesting that a single strain value may be insufficient to describe edge fracture. The elemental thinning strain showed reduced dependence on mesh size, making it a more reliable parameter for assessment of edge fracture in simulations. Importantly, the simulations indicated that the edge fracture strain cannot be represented by a unique value but is rather a function of the imposed loading condition. The in-plane stretching mode exhibited the lowest engineering thinning strain limit of 8.8%, making it the critical deformation mode for edge crack initiation in 980GEN3 steel.

A key outcome of this work is the quantitative understanding of the effect of boundary conditions and lengthscale on the edge fracture limits. Prediction of sheared edge fracture must account for both the imposed loading and numerical lengthscale, with thinning strain offering a more robust metric for use in simulations. The developed methodologies provide practical and efficient guidelines that can be implemented in industrial environments for edge crack assessment and prediction in stamping simulations.

Acknowledgements

I'm incredibly grateful to my advisor, Prof. Cliff Butcher, for his guidance, encouragement, and support throughout my PhD. Beyond being a great mentor, he has been a true inspiration in shaping how I approach research. His extensive knowledge and sharp insights have added so much to this work, and I feel really lucky to have him as my advisor. The skills and experiences I gained under his guidance will help me not just in my career but throughout my life.

I'm also grateful to Prof. Michael Worswick for his mentorship, constructive feedback, and valuable discussions on sheared edge fracture. His insights have been key to shaping my understanding of this topic, and I am grateful to have been part of his projects.

A huge thanks to the research associates in the Forming and Crash Lab: Ryan George, Cameron Tolton, and Jose Imbert-Boyd for all their help during the experiments. I'm especially thankful to Ryan for his expertise and input during the design and fabrication phases of the in-plane bending fixture. Cameron was a tremendous help in troubleshooting issues with the Python scripts for automating DIC data extraction and resolving LS-DYNA simulation problems. I also really appreciated Jose's guidance and feedback on testing procedures. The support from Tom Gawel and Eckhard Budziarek in the lab was priceless, and it was great working with them.

I would like to express my gratitude to Prof. Jacqueline Noder and Dr. Armin Abedini for their mentorship and guidance. Sorry for constantly bugging you with so many emails and questions. Your patience and support meant a lot. I would also like to thank Dr. Pedram Samadian for all the conceptual discussions, both research related and otherwise, and for helping me debug LS-DYNA simulation problems. I'm also grateful to Dr. Mohammad Shojaee and Dr. Abdelbaset Midawi for their help with sample preparation for hardness testing, training me on flat tip indentation, and answering all my questions about steel microstructure.

To the students at the Forming and Crash Lab: Kenneth Cheong, Lucas Piloza-Hibbit, Berkay Yuksel, Matthew Hunt, Joseph Arciero, and Mahmoud Howeyze; thank you all for the engaging chats on various topics and for sharing your insights. I learned a lot from our discussions on research and everything else. A special thank you to my Team Steel teammates, Rhys Northcote and Patrick Cleary, for their immense support and for handling the bulk of experimental work on the SHTT project, allowing me to focus on writing my thesis.

I also want to acknowledge the financial support I received from the Steel Testing and Harmonization (STHT) and Stamping Teams of Auto/Steel Partnership, the Natural Sciences and Engineering Research Council of Canada, Hexagon Manufacturing Intelligence, and Men at Work GmbH during my PhD.

Finally, I'm deeply grateful to my wife, Preethi, and my family for their constant moral and emotional support. Their belief in me has been my greatest strength and motivation throughout this journey.

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List of Abbreviations

AA	Aluminum alloy
AHSS	Advanced high strength steel
ARM	Area reduction method
DIC	Digital image correlation
DP	Dual phase
EFTT	Edge fracture tensile test
FE	Finite-element
FEA	Finite-element analysis
FLC	Forming limit curve
GEN3	3 rd generation
HER	Hole expansion ratio
HF72	Hosford (1972) yield function
HF-C	Constrained form of anisotropic Hosford (1985) yield function
HTT	Hole tension test
HX	Hole expansion
IPB	In-plane bending
MS	Martensitic steel
PSC	Plane strain constraint
PST	Plane strain tension
RD, DD, TD	Rolling, diagonal, and transverse directions
SAZ	Shear affected zone
TE	Total elongation
UE	Uniform elongation
UTS	Ultimate tensile strength
VSG	Virtual strain gauge length
YS	0.2% offset yield strength

List of Symbols

d_0	Initial hole diameter
d_{inner}^{avg}	Average inner hole diameter at fracture
d_{outer}^{avg}	Average outer hole diameter at fracture
d_{outer}^{θ}	Local outer diameter at fracture location
$\epsilon_1^{TT\ Crack}, \epsilon_3^{TT\ Crack}$	Major and thickness strain corresponding to through-thickness cracking
$\epsilon_{convex}^{\theta}$	Major strain from arc length method
ϵ_{eq}^{MC}	Macroscopic strain
ϵ_{eq}^{WC}	Work conjugate equivalent plastic strain
η	Stress triaxiality
h	Gauge height of bend specimen
J_1, J_2, J_3	Maximum, intermediate, and minimum deviatoric stress invariants
L	Lode parameter
L_0	Initial gauge length for the arc length method
M_b	Bending moment
r_{convex}	Radius at the convex surface
R_0, R_{45}, R_{90}	Tensile R-values in rolling, diagonal, and transverse directions
ΔR	Planar anisotropy
s_1, s_2, s_3	Maximum, intermediate, and minimum deviatoric principal stresses
$\sigma_1, \sigma_2, \sigma_3$	Maximum, intermediate, and minimum principal stresses
σ_b	Nominal bending stress
$(\sigma_b)_{convex}$	Nominal bending stress at convex surface

σ_{eq}	Equivalent stress
$\bar{\sigma}_{HF72}$	Equivalent stress from Hosford (1972) yield function
σ_{θ}^{PS}	Major stress in plane strain tension at an angle of θ from rolling direction
t_0	Initial thickness
t	Instantaneous thickness
t_f	Final thickness
θ_{convex}	Included angle at the convex surface

1. Synopsis of Thesis

1.1. Introduction and Motivation

The increasing adoption of electric vehicles has driven the demand for advanced high strength steels in battery enclosures and structural components to offset the added mass of battery packs. These materials enable the use of thinner gauges while enhancing crashworthiness and fracture resistance. However, a trade-off exists between strength and formability, with AHSS exhibiting reduced ductility compared to conventional mild and stainless steels. To address this, 3rd generation AHSS (GEN3) with multiphase microstructures containing retained austenite have been developed, offering high strength along with improved global ductility (Hance, 2018). Despite these advantages, the ability of AHSS to undergo localized deformation modes such as bending or hole extrusion can be limited (Noder *et al.*, 2021a).

Mechanical punching, a subset of the shear cutting process, involves through-thickness shearing of sheet metal to create closed-line features that are commonly used in automotive components for structural joining or deep drawing. While laser cutting and CNC machining offer superior surface finish, mechanical punching remains the preferred method due to its high speed and cost efficiency. In the present work, the terms ‘punching’ and ‘shear cutting’ are used interchangeably, although trimming (open-line cutting) is another subset of shear cutting. The process of shear cutting generates four distinct edge zones: rollover, burnish, fracture zone, and burr. Additionally, it introduces a region of severe plastic deformation near the edge known as the shear affected zone (SAZ) (Levy and Van Tyne, 2008). The strains in the SAZ, along with the edge profile, can result in premature fracture during subsequent forming operations (Pathak *et al.*, 2016; Habibi *et al.*, 2019; Won *et al.*, 2020a), with some examples depicted in Figure 1. Accurate prediction of such failures during stamping requires knowledge of residual edge formability limits, which can be evaluated using coupon experiments for subsequent application to FE simulations.

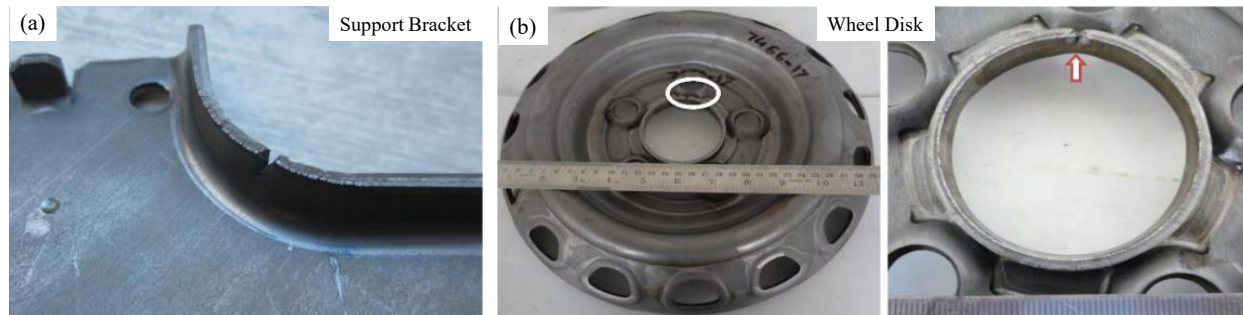


Figure 1 Examples of sheared edge cracking in automotive (a) support bracket and (b) wheel disk (Bae *et al.*, 2020).

The edge formability limits are commonly characterized at the coupon level through standardized test methods. The most widely used technique is the conical hole expansion (HX) test, which emulates a hole extrusion process by subjecting the sheared edge to an out-of-plane deformation. The resulting hole expansion ratio (HER) is calculated at the instant of through-thickness crack onset. However, HER is not a direct measure of fracture strain and cannot be used in FE simulations due to non-linear stress state at the hole edge. Additionally, the sheared edge can be subjected to several deformation modes such as in-plane uniaxial bending and stretching that are common in curved automotive structures (Thomas, 2013; Won *et al.*, 2020b) which may not be well represented by conical HX. To address this limitation, alternative test methods such as flat punch hole expansion (Siebel and Pomp, 1929), hole tension (Feistle *et al.*, 2019), and edge fracture tensile tests (Feistle *et al.*, 2015) have been proposed. These techniques aim to replicate in-plane uniaxial and axisymmetric loading conditions. However, limited work has been conducted on evaluating edge formability under in-plane bending due to potential for buckling in sheet metals. Naseem *et al.* (2020) developed a four-point in-plane bending fixture, which was later adapted by Khalilabad *et al.* (2022) to study edge fracture in DP800 steel with a trimmed edge. These prior studies of in-plane bending have only achieved true strain levels of 0.30 at the convex edge, which may not be sufficient to cause failure in more ductile materials or for machined edges.

Different test methods can produce varying edge fracture strains for the same material and cutting condition, despite the sheared edge being under uniaxial tension (Kesti *et al.*, 2022; Feistle *et al.*, 2022). These discrepancies may be influenced by differences in loading conditions away from the sheared edge and within the SAZ, which remain unexplored. Consequently, assuming a single failure limit within FE models may not be practical. Additionally, the lack of consistent reporting on experimental lengthscales used to quantify the fracture strains introduces further challenges when comparing test methods. Differences in reported local edge fracture strains may be misinterpreted as an advantage of one test method over another. Finally, the influence of boundary conditions on the observed deformation has not been investigated at the level of FE simulations. The deformation mode, as well as the numerical lengthscale dictated by the mesh size, can significantly impact elemental strain magnitudes and edge crack prediction.

The present thesis aims to address these gaps in the literature by experimentally and numerically investigating the influence of boundary conditions on the edge fracture limits for a 980GEN3 steel punched using a cutting clearance of 12% of sheet thickness. Emphasis is placed upon the development of analytical techniques to estimate the fracture strain from the conical HX, as well as the development of a modified in-plane bending fixture able to promote edge cracks in sheet metals having a true fracture strain of up to 0.70, enabling fracture characterization of a broader range of sheet materials. Subsequently, the influence of applied boundary conditions on the sheared edge fracture limits of AHSS is investigated using a 980GEN3

steel as the reference material. Various test methods that emulate in-plane stretching, bending, and out-of-plane extrusion of the sheared edge are considered to assess the influence of deformation mode on the fracture strains. Finally, the influence of experimental and numerical lengthscales on the edge fracture limits will be investigated. The choice of reference DIC lengthscale and mesh size will be analyzed for transferability to design and modelling of edge cracks on automotive stamped components. The outcome of this research will provide guidelines for establishing edge formability limits for failure under different deformation modes as well as the measures that can be employed to predict edge crack occurrence in industrial stamping simulations.

1.2. Thesis Outline

A manuscript-based style has been adopted in this thesis. A concise literature review that highlights current gaps is discussed in Chapter 2. The objectives of the thesis are subsequently established in Chapter 3. A synopsis of the research conducted to accomplish the immediate objectives outlined in Section 3.1 is presented in Chapters 4-6. The corresponding full-length articles are included in the appendices and comprise manuscripts that have been published in or submitted to peer-reviewed journals. Conclusions and recommendations for future work are provided in Chapters 7 and 8.

Appendix A: **Narayanan, A.**, Arciero, J., Tolton, C. and Butcher, C., 2025. Characterization and assessment of anisotropic constitutive models using the flat punch hole expansion test. *International Journal of Material Forming*, 18(2), p.50.

Appendix B: **Narayanan, A.**, Abedini, A., Khameneh, F. and Butcher, C., 2023. An experimental methodology to characterize the uniaxial fracture strain of sheet metals using the conical hole expansion test. *Journal of Materials Engineering and Performance*, 32(10), pp.4456-4482.

Appendix C: **Narayanan, A.**, George, R. and Butcher, C., 2025. Uniaxial fracture and plastic anisotropy characterization of sheet metals using the in-plane bending test. *Engineering Fracture Mechanics*, p.111333.

Appendix D: **Narayanan, A.**, Northcote, R., Cleary, P., and Butcher, C. On the Influence of Boundary Condition on Sheared Edge Fracture Limits: An Experimental and Numerical Study with Application to 980GEN3 Steel, *submitted for possible publication*, December 2025.

Additional journal and conference publications stemming from this research, which are not the primary focus of the thesis but are referenced throughout, are:

Abedini, A., **Narayanan, A.** and Butcher, C., 2022. An investigation into the characterization of the hardening response of sheet metals using tensile and shear tests with surface strain measurement. *Forces in Mechanics*, 7, p.100090.

Narayanan, A., Bourque, C., Fast-Irvine, C., Abedini, A., Anderson, D. and Butcher, C., 2022. Identification of the Plane Strain Yield Strength of Anisotropic Sheet Metals Using Inverse Analysis of Notch Tests (No. 2022-01-0241). SAE Technical Paper.

Abedini, A., **Narayanan, A.** and Butcher, C., 2022. A Deterministic Methodology to Calibrate Pressure-Independent Anisotropic Yield Criteria in Plane Strain Tension Using Finite-Element Analysis. *Applied Mechanics*, 3(3), pp.905-934.

Narayanan, A. and Butcher, C., 2023. Design of a Test Geometry to Characterize Sheared Edge Fracture in a Uniaxial Bending Mode (No. 2023-01-0730). SAE Technical Paper.

Abedini, A., **Narayanan, A.** and Butcher, C., 2024. On the flat punch hole expansion test of sheet metals: Mechanics of deformation and evaluation of anisotropic plasticity models. *Mechanics of Materials*, 191, p.104931.

2. Literature Review

This chapter provides an overview of the literature concerning the characterization of constitutive, fracture, and sheared edge formability limits of sheet metals. Section 2.1 addresses the key components and associated challenges with constitutive characterization that are critical for virtual tryouts. Section 2.2 presents a review of ductile fracture models and the experimental methods to determine the fracture strain under proportional uniaxial loading conditions. Sections 2.3 and 2.4 focus on the experimental assessment of sheared edge stretchability characterization and numerical modeling, respectively. The chapter concludes with a summary and discussion of current gaps in the literature.

2.1. Characterization of Anisotropic Plasticity

2.1.1. Calibration of Yield Functions

The introduction of new material grades often requires modifications to tooling and part designs to ensure their successful implementation in forming processes. In response, the automotive industry has adopted FE simulations to optimize stamping processes, replacing traditional trial-and-error methods to reduce iteration cycles and costs, and accelerate shop floor integration (Wang *et al.*, 2009; Andrade-Campos *et al.*, 2022). However, the reliability of these FE models heavily depends on the accuracy of input material data. This necessitates precise characterization and calibration of material behavior, followed by evaluation of the generated material cards before they can be applied. A key component of the material description is the yield function, which describes its behavior under various stress states and accounts for anisotropy. While simpler isotropic yield criteria, such as those proposed by Tresca (1864) and von Mises (1913), may be used for convenience and do not require any experimental data, they fail to capture the anisotropy characteristics of sheet metals. The distribution of plastic anisotropy can have a significant influence on the predicted thinning as well as earring profile during cup drawing (Hu *et al.*, 2001). To this end, many yield functions have been proposed that can incorporate material anisotropy to calibrate their respective parameters (*e.g.*, Barlat *et al.*, 1989; Barlat *et al.*, 2003; Yoshida *et al.*, 2013; Hu *et al.*, 2023). The coefficients of such anisotropic yield functions typically need to be calibrated using the data obtained from various coupon tests. Uniaxial tensile tests are typically performed in multiple orientations with respect to the rolling direction (RD) to obtain the R-values and the tensile stress ratios that can be used to determine the yield function coefficients. Complementary tests that impose the stress states of simple shear (Peirs *et al.*, 2012; Grolleau *et al.*, 2022), plane strain tension (Deng *et al.*, 2015; Ha *et al.*, 2020), or equibiaxial loading (Lăzărescu *et al.*, 2013; Rossi *et al.*, 2022) can also be conducted to aid in calibration of such yield functions. Choice of yield function and its calibration procedure can significantly influence the accuracy of predicted formability and failure behavior (Fourmeau *et al.*, 2011; Lin *et al.*, 2020). Figure 2 depicts the

yield surface shapes for a DC04 mild steel from calibration of Hill-48 (Hill, 1948), HF72 (Hosford, 1972), and Yld2000 (Barlat *et al.*, 2003) functions. The yield function shapes are only shown in the 1st or tensile quadrant for brevity. A significant difference can be observed in the shapes predicted by the three functions despite using the same experimental data, owing to the respective yield function definitions and number of calibration parameters.

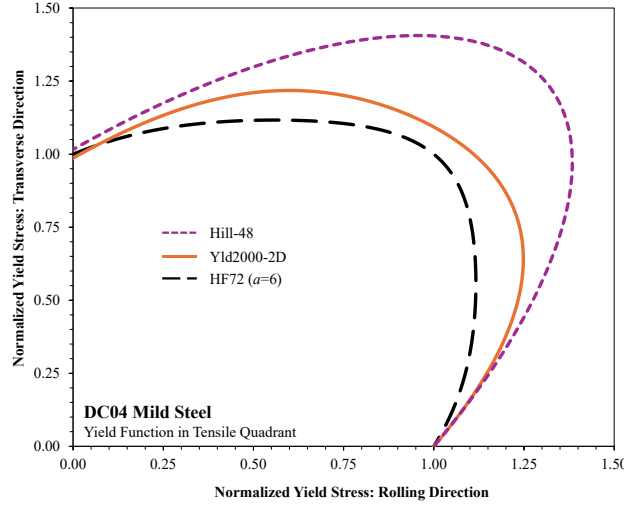


Figure 2 Yield surface contours for a DC04 mild steel obtained using Hill-48, HF72, and Yld2000 functions (Narayanan *et al.*, 2025).

Conventionally, the uniaxial tensile and biaxial test data are employed to calibrate anisotropic yield functions. The stress state of plane strain tension (PST) is often overlooked despite being a critical deformation mode for forming and fracture in sheet metals. The location and magnitude of PST on the yield surface are often left unconstrained as schematically illustrated in Figure 3 and rendered as a by-product of the calibration process. The incorporation of plane strain data into the yield function calibration can significantly improve the accuracy of analytical methods to estimate material formability (Vegter and van Den Boogaard, 2006). Several studies have also noted the importance of PST on the accuracy of FE simulations of hole expansion tests conducted using a cylindrical punch (Lee *et al.*, 2019; Park *et al.*, 2023). However, experimental measurement of plane strain data needs specialized equipment that can promote a stress-controlled multiaxial deformation such as the cruciform test. The maximum achievable deformation in such tests can also be limited due to the intricate sample shape and design. Butcher and Abedini (2019) demonstrated that the stress state of PST occurs near a principal stress ratio σ_2/σ_1 of 0.5 for materials that approximately follow pressure-independent plasticity (*i.e.* plastic deformation is governed by deviatoric stresses only). Fast-Irvine *et al.* (2021) compiled the data from cruciform tests conducted in the literature for various aluminum alloys and steels at a principal stress ratio of 1:2. The direction of the normal vectors approximately corresponded to a plane strain tensile stress state regardless of material anisotropy and

ductility, demonstrating strong experimental support for the plane strain constraint (PSC) of Butcher and Abedini (2019). Consequently, the location of PST can be constrained to occur at a principal stress ratio of 1:2 during anisotropic yield function calibration process. Recent studies have recognized the importance of plane strain tension and have consequently developed new yield functions with greater flexibility to capture data under PST (Hou *et al.*, 2023; Ren *et al.*, 2025).

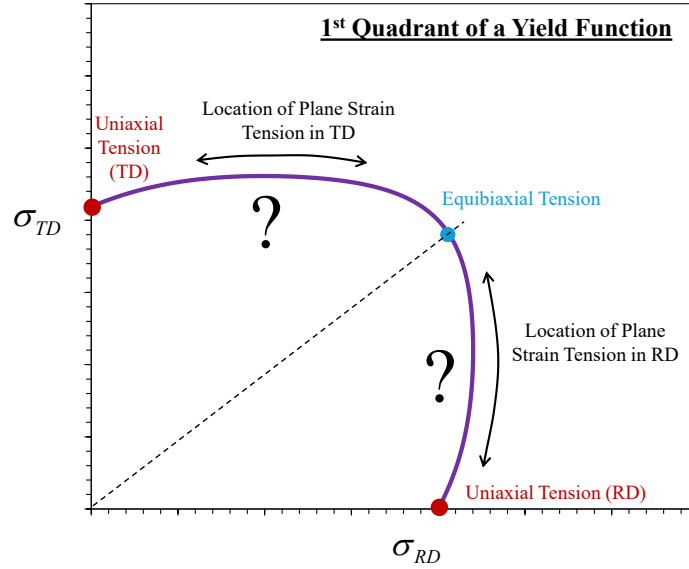


Figure 3 Schematic illustrating the first quadrant of the yield surface and the points corresponding to uniaxial and equibiaxial tension. The plane strain point can occur at any intermediate point between uniaxial and equibiaxial locations.

The magnitude of the plane strain yield stress needs to be determined in addition to its corresponding stress ratio to accurately calibrate the yield surface in the tensile quadrant. The shape of the yield surface can be significantly affected by the plane strain yield strength, as shown schematically in Figure 4(a) using the isotropic HF72 yield function that satisfies the PSC and is defined as:

$$\bar{\sigma}_{HF72} = \left(\frac{|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a}{2} \right)^{\frac{1}{a}} \quad (\text{Eq. 1})$$

The yield function exponent, a , is inversely related to the plane strain yield strength as seen in Figure 4(a). Since cruciform test data is rarely available, there is a need to explore alternative test techniques that can be exploited for PS data measurement. The notch tensile test induces a stress state of PST near its gauge center, offering potential for estimating the plane strain data. A stress-state gradient is observed near the edge with a transition towards uniaxial tension, as shown schematically in Figure 4(b). The global response from notch tensile tests can be used in conjunction with the tensile test data to calibrate yield surfaces, as

done by Aydin *et al.* (2011) and Ha *et al.* (2020). However, the plane strain constraint was not incorporated during the coefficient determination process.

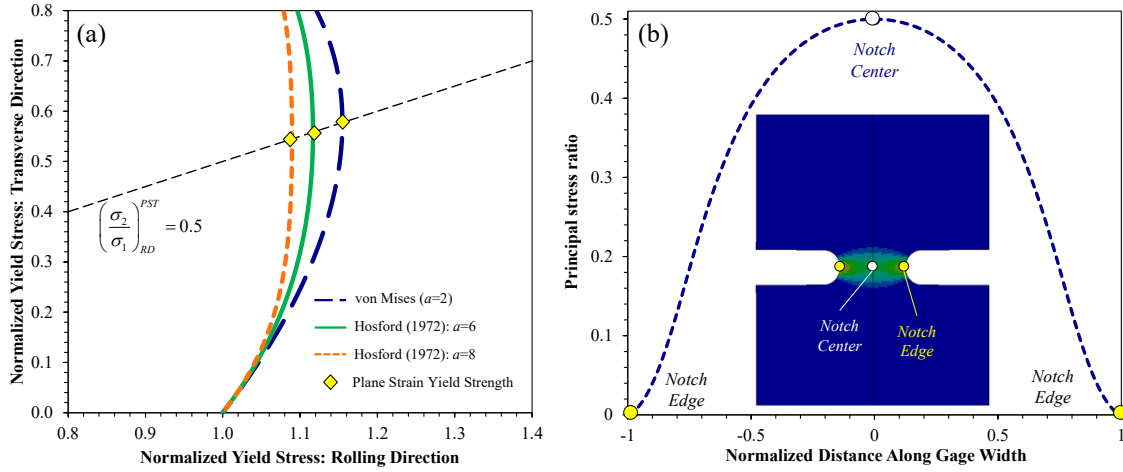


Figure 4 (a) Effect of the yield exponent, a , on the plane strain yield strength, and (b) stress state gradient across the gauge width in a notch tensile test (Narayanan *et al.*, 2025). The tensile R-value is unity.

The plane strain stress cannot be directly measured from the forces and the cross-sectional area in notch tests due to stress gradients inherent in the notch. Consequently, an inverse analysis approach is required to determine the plane strain yield strength. Fast-Irvine *et al.* (2021) proposed an experimental methodology that leverages the local DIC strain distribution across the gauge width of a notch specimen, combined with a modified anisotropic Hosford (1985) yield function (HF-C) incorporating the PSC. The measured strain state was converted into principal stresses using the HF-C formulation and flow stress data from tensile tests, which were then integrated to compute the total force at the minimum gauge width. The exponent of the HF-C function was iteratively adjusted until the numerically predicted force matched the experimental measurement. While the experimental approach is robust, it relies upon the availability of local DIC strain measurements. In the absence of DIC, a hybrid experimental–numerical approach is required to estimate the plane strain yield strength.

2.1.2. Evaluation of Hardening Response

To accurately mimic the deformation experienced by stamped parts, the hardening response should be characterized to large strains. Conventional uniaxial tensile tests are limited by diffuse necking, which occurs at much lower strains than those in other forming modes (Do *et al.*, 2017). This limitation forces the extrapolation of the stress-strain curve, introducing uncertainties that can reduce the accuracy of FE simulations and forming limit predictions (Pham *et al.*, 2018; Kim *et al.*, 2022). To obtain the stress response beyond necking, an inverse analysis procedure can be applied (see Figure 5). A common approach involves FE simulation of tensile tests with iterative adjustments performed to the hardening curve until the

predicted response matches measurements beyond the ultimate tensile strength (Do Kweon *et al.*, 2021; Pham *et al.*, 2021). The accuracy of this hybrid approach also depends on numerical parameters such as element size and material property inputs. Another option is the virtual fields method (Kim *et al.*, 2013; Oliveira *et al.*, 2025), but it is more resource-intensive since it requires full-field DIC strain data and the assumption of the yield surface.

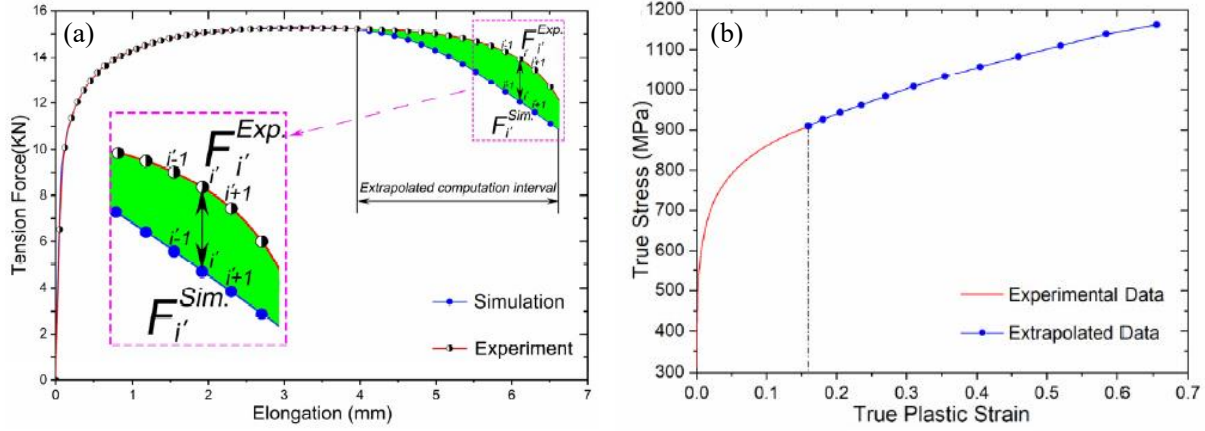


Figure 5 (a) Modifications performed to input flow curve to minimize difference between the experimental and predicted tensile test force using the inverse FE approach, and (b) extrapolated stress-strain curve obtained for a DP780 steel after optimization (Wang *et al.*, 2023).

Direct measurement of the hardening response is also possible through experiments that suppress or delay necking, eliminating the need for inverse analysis. For example, simple shear (see Figure 6) and equibiaxial stretching tests have been used to extend the strain range for flow stress determination (Abedini *et al.*, 2020; Grolleau *et al.*, 2022; Rossi *et al.*, 2022), although they generally assume isotropic hardening. A so-called area reduction method (ARM) was introduced by Abedini *et al.* (2022) to determine post-necking hardening behavior from tensile specimens. This method estimates equivalent stress from the minimum cross-sectional area after diffuse necking, applying empirical correction factors from Choung and Cho (2008) to account for stress and strain homogenization. The accuracy of the ARM was evaluated numerically across various hardening exponents and material anisotropies resulting in excellent agreement with the input flow stress. A tensile gauge width-to-thickness ratio of 4 or lower was recommended to promote a triaxial neck and facilitate ease of cross-sectional area measurement. A more intricate approach was used by Jordan *et al.* (2023), wherein a surround DIC setup with four synchronized systems was used to capture displacement fields on all free surfaces, enabling accurate necking area measurement of a shear band localization mode. The resulting hardening curves for a DP780 steel closely matched the hydraulic bulge test data. As the use of DIC has become increasingly common, ARM offers a practical means to evaluate anisotropic hardening effects in materials using standard tensile tests.

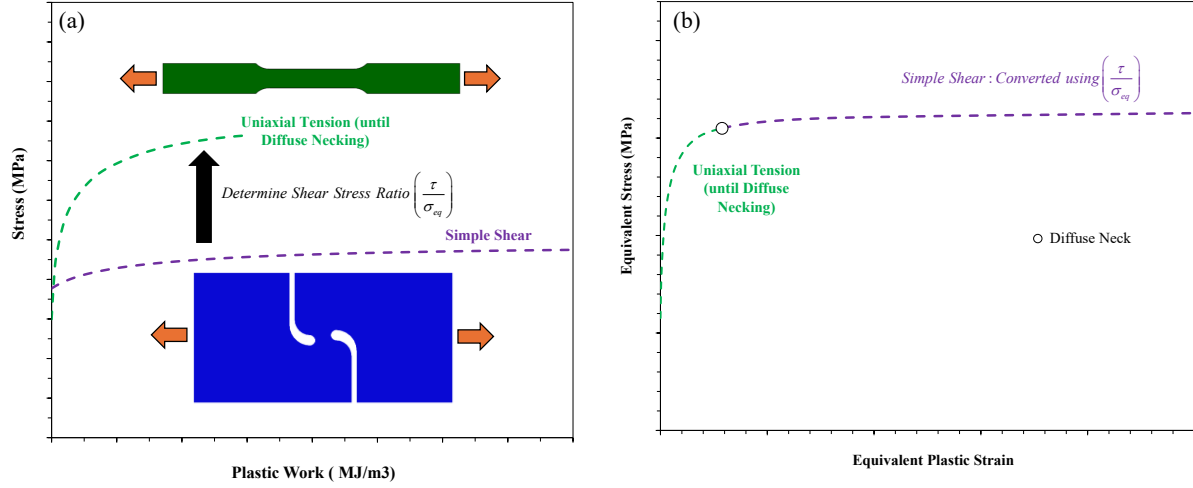


Figure 6 (a) Stress evolution from uniaxial tensile and simple shear tests with respect to plastic work and (b) hardening response obtained to large strain by converting the shear stress to an equivalent stress using the determined shear stress ratio. Further details regarding the methodology are provided in Abedini *et al.* (2020).

2.2. Characterization of Ductile Fracture Models

The forming limit curve (FLC) is used to define the formability limits of sheet materials under a range of stretching conditions from uniaxial to biaxial tension. The FLC is characterized using standardized Marciniak or Nakazima punch tests (ISO 12004, 2008) with the limit strains corresponding to the formation of an acute neck. The FLC is not a fracture criterion although for low ductility metals, the fracture strains can be comparable to the necking strains. The FLC does not account for conditions in which fracture occurs without necking, such as in stretch-bending, compression, and shear-dominated loading. In deformation modes that suppress necking, the fracture limit strains are the effective forming limits and must be characterized to supplement the FLC. Forming and crash simulations require accurate fracture criteria that account for the stress state and loading history. The primary source of fracture initiation in ductile metals is the nucleation, growth, and coalescence of micro-voids into cracks as illustrated in Figure 7. The rate of damage accumulation is specific to the microstructure and the imposed stress state. To uniquely define the stress state, knowledge of the six independent components of the stress tensor is required. For an isotropic material or a specific direction in an anisotropic material, three stress invariants, or two ratios involving them can be employed to define the stress state in a more intuitive manner.

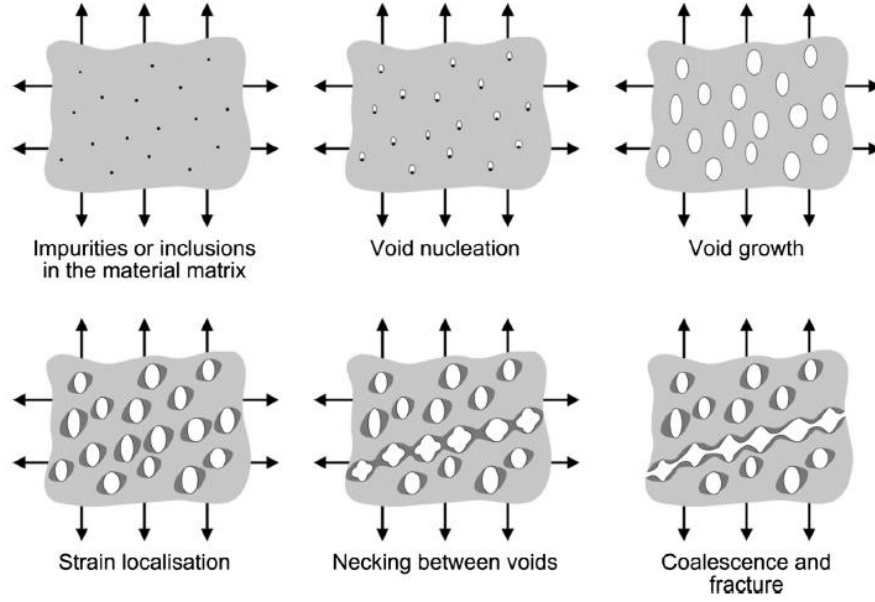


Figure 7 Void nucleation, growth, and coalescence to initiate ductile fracture (Gatea *et al.*, 2017).

The hydrostatic stress (or pressure) influence is of the second order on plasticity of metals but is critical for fracture prediction as it governs void growth and coalescence. The stress triaxiality, η , is defined as the ratio of hydrostatic to the von Mises equivalent stress. It can also be expressed as a function of the first stress invariant, I_1 , and the second deviatoric invariant J_2 .

$$\eta = \frac{\sigma_{hyd}}{\sigma_{eq}^{VM}} = \frac{I_1}{3\sqrt{3J_2}} ;$$

$$I_1 = tr(\boldsymbol{\sigma}) = 3\sigma_{hyd} = \sigma_1 + \sigma_2 + \sigma_3 ;$$

$$s_{ij} = \sigma_{ij} - \sigma_{hyd}\delta_{ij} ;$$

$$J_2 = \frac{s_{ij}s_{ji}}{2} \quad (\text{Eq. 2a-d})$$

where σ_i ($i=1,2,3$) are the principal stresses in descending order, σ_{hyd} is the hydrostatic or mean stress, and $\mathbf{s}_{ij}$ is the deviatoric stress tensor. To quantify the severity of shear loading, the third deviatoric invariant, J_3 , can be used or equivalently, the Lode parameter (Lode, 1926). The Lode parameter can be defined in terms of the ordered principal stresses as:

$$L = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{2s_2 - s_1 - s_3}{s_1 - s_3} \quad (\text{Eq. 3})$$

Alternate forms of the Lode parameter have been used in the literature (Bai and Wierzbicki, 2008; Kroon and Faleskog, 2013) for computational efficiency to avoid the principal stress calculations of Eq. (3). In the special case of plane stress, the triaxiality and Lode parameter are related with only one required to define the stress state.

Micromechanically motivated fracture criteria have been developed to incorporate the mechanisms of void nucleation, growth, and coalescence to describe ductile fracture (Johnson and Cook, 1985; Leblond *et al.*, 1994; Monchiet *et al.*, 2008). The Gurson model (Gurson, 1977) is one of the most widely used isotropic damage-based yield criteria, originally formulated from studies on nucleation and growth of a spherical void in a unit cell. It has since been extended to account for various fracture mechanisms, including stress- and strain-controlled nucleation (Chu and Needleman, 1980) and shear-dominated failure (Nahshon and Hutchinson, 2008; Nielsen and Tvergaard, 2010). Micromechanical damage models are attractive for understanding the mechanisms of fracture but consist of numerous parameters that need to be identified from detailed microstructural analysis. Consequently, micromechanical damage models have seen relatively little industrial application. Continuum models which contain fewer parameters are easier to calibrate and can predict fracture with as good or better accuracy (Dunand and Mohr, 2011; Bai and Wierzbicki, 2015).

Uncoupled continuum-level fracture criteria have seen widespread adoption over the past decade. These phenomenological fracture models are formulated in terms of stress triaxiality, Lode parameter, and the equivalent plastic strain. A fracture surface for general loading, or a locus in plane stress, is used to describe the variation of the fracture strain with stress state in proportional loading (see Figure 8). Numerous fracture criteria have been proposed in the literature that differ in their definition and the number of calibration parameters (Bai and Wierzbicki, 2008; Mohr and Marcadet, 2015; Rahmaan *et al.*, 2022). The functional form of the modified Mohr-Coulomb fracture criterion developed by Bai and Wierzbicki (2010) will be discussed here for brevity. The closed-form solution of the modified Mohr-Coulomb model in terms of the equivalent plastic strain, stress triaxiality, and Lode parameter is:

$$\bar{\epsilon}_f^p = \left\{ \frac{A}{c_2} \left[c_\theta^s + \frac{\sqrt{3}}{2 - \sqrt{3}} (1 - c_\theta^s) \left(\sec \left(\frac{\pi \bar{\theta}}{6} \right) - 1 \right) \right] \left[\sqrt{\frac{1 + c_1^2}{3}} \cos \left(\frac{\pi \bar{\theta}}{6} \right) + c_1 \left(\eta + \frac{1}{3} \sin \left(\frac{\pi \bar{\theta}}{6} \right) \right) \right] \right\}^{-\frac{1}{n}} \quad (\text{Eq. 4})$$

where $\bar{\theta}$ is an alternative representation of the Lode parameter that is defined as

$$\bar{\theta} = 1 - \frac{2}{\pi} \cos^{-1} \left(\frac{3\sqrt{3}J_3}{2J_2^{\frac{3}{2}}} \right); \quad -1 \leq \bar{\theta} \leq 1 \quad (\text{Eq. 5})$$

A total of five coefficients ($A, n, c_2, c_\theta^s, c_1$) need to be calibrated to completely define the fracture locus using the modified Mohr-Coulomb model.

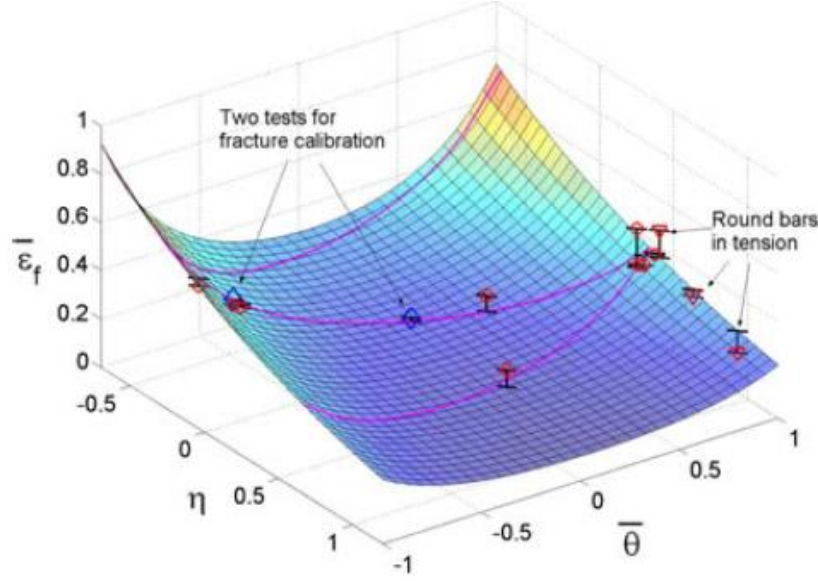


Figure 8 A 3-D fracture locus of an AA2024-T351 alloy (Bai and Wierzbicki, 2010).

The coefficients of the fracture model under proportional loading can be obtained using two methodologies. A hybrid experimental-numerical approach is commonly used and relies upon the local stress and strain histories from FE simulations of the coupon tests (Zhang *et al.*, 2022; Kacem *et al.*, 2022; Şen *et al.*, 2025). In tests that exhibit necking, such as the uniaxial or notch tensile tests, the DIC surface strains are not representative of the mid-thickness region where fracture initiates, necessitating the need for inverse finite-element analysis (FEA) to extract the local strain at the neck. The stress state is averaged during extraction of fracture strains from FE models of such tests, resulting in loss of information about loading path dependency leading to non-uniqueness in the observed fracture strains (Benzerga *et al.*, 2012). Figure 9 depicts the change in stress triaxiality at the neck center during a uniaxial tensile test. Another limitation of the hybrid approach to fracture characterization is its dependence upon the assumed plasticity and hardening model (Şen *et al.*, 2025), and simulation parameters such as element size and formulation.

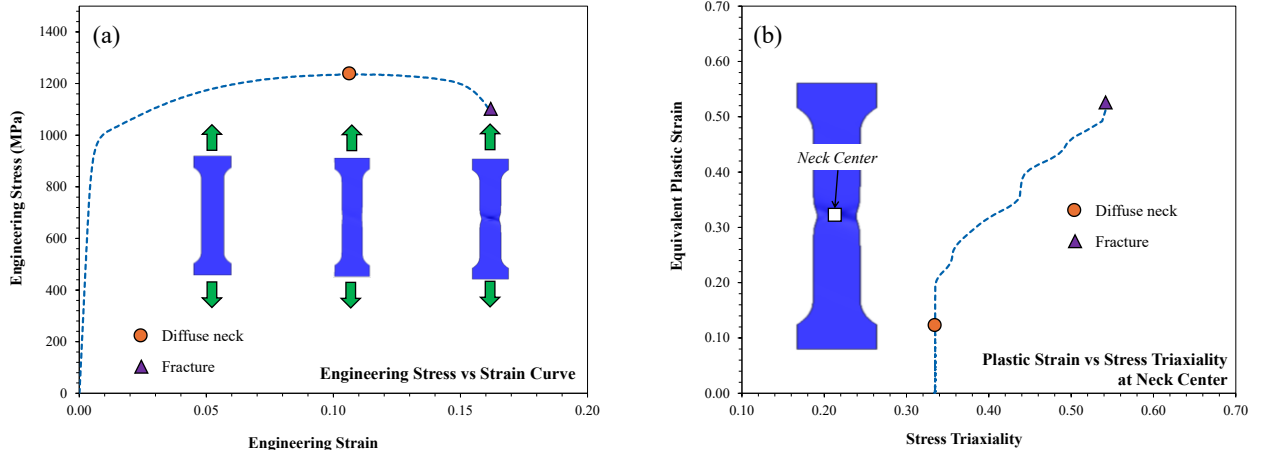


Figure 9 (a) Engineering stress-strain curve, and (b) local plastic strain vs stress triaxiality at neck center for an advanced high strength steel.

The second approach is purely experimental and relies upon selecting test and punch geometries that induce proportional deformation until fracture. Experiments that impose a bending or shear dominated loading to suppress necking are considered for fracture characterization such as the mini-shear specimen of Peirs *et al.* (2012) or its variants (Roth and Mohr, 2018), V-bending (Roth and Mohr, 2016), and conical hole expansion tests (Noder *et al.*, 2021a). The DIC strain path at the fracture location can be converted into equivalent plastic strain using an assumed yield function and used to optimize the fracture surface parameters, as done by Noder *et al.* (2021a) and Rahmaan *et al.* (2022) for a host of AHSS and aluminum alloys, respectively. The experimental approach is promising as it employs characterization tests that do not require inverse numerical analysis and exploits the advances in strain measurement using DIC. The experiments used to characterize the fracture strains under various stress states will be discussed in the subsequent paragraphs with emphasis on uniaxial tension due to its relevance to sheared edge failure.

2.2.1. Determination of Fracture Limits under Simple Shear

Multiple experimental setups and specimen geometries for fracture characterization in simple shear have been proposed that mitigate or circumvent edge fracture. Peirs *et al.* (2012) optimized the dimensions of a planar shear specimen using finite-element analysis and identified a so-called ‘mini-shear’ geometry suitable for testing at quasi-static and dynamic strain rates. Roth and Mohr (2018) noted that the mini-shear geometry was better suited for materials with moderate to high ductility. Three different shear specimens with different notch shapes and offsets were recommended, tailored to the material ductility levels based on numerical studies. A schematic of the high-ductility version of the shear specimen and the corresponding strain path at the gauge center for a 980GEN3 steel are shown in Figure 10(a, b). The mini-shear specimen and its variants do not require through-thickness machining and can be readily tested on a universal tensile frame. However, they are susceptible to premature edge fracture regardless of notch geometry due to the

lower uniaxial fracture strain at the free surface compared to that under simple shear conditions. Alternative shear tests that incorporate through-thickness machining, such as the butterfly shear specimen (Dunand and Mohr, 2011) and the in-plane torsion test (Yin *et al.*, 2015; Grolleau *et al.*, 2022), can be employed but require specialized test frames. Recently, Pilozo-Hibbit *et al.* (2024) introduced a shear specimen which features a through-thickness machined gauge section shaped like a peanut. The maximum major strain under simple shear in the peanut shear specimen was approximately 160% higher than the specimen without any through-thickness machining for a 2.0 mm thick mild steel. A key advantage of the peanut shear specimen over other machined geometries is its compatibility with standard universal testing machines.

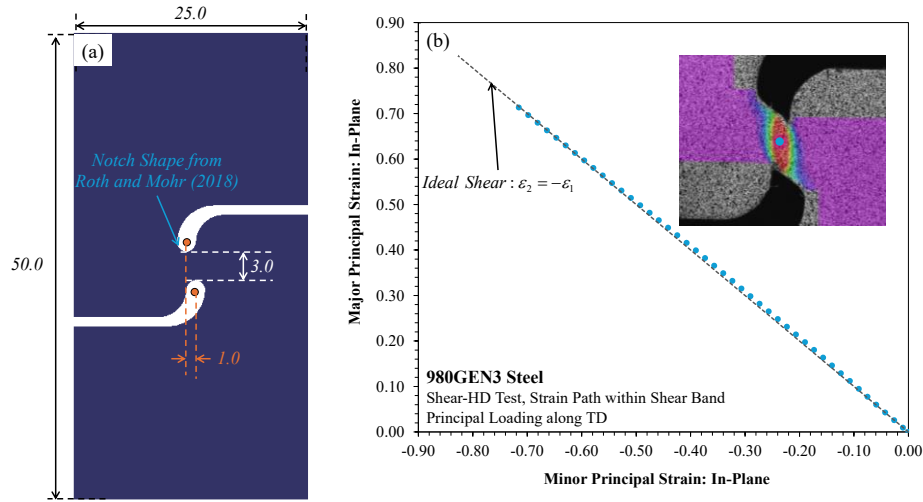


Figure 10 (a) Geometry of the high ductility variant of the mini-shear specimen (Roth and Mohr, 2018), and (b) strain path near the gage center until fracture for a 980GEN3 shear test.

2.2.2. Fracture Characterization under Plane Strain and Equibiaxial Stretching

Notched tensile specimens with different notch radii are commonly used for fracture characterization in stress states between uniaxial and plane strain tension (Ha *et al.* 2018; Sakaridis *et al.*, 2024). A hybrid experimental-numerical approach is required to obtain the fracture strains from the notch tension tests. Alternatively, tight radius bending tests can be employed to suppress the formation of tensile instability and promote fracture on the convex surface, where the strain can be measured using DIC. The VDA238-100 v-bending (Beerli *et al.*, 2022; Cheong *et al.*, 2023) and dihedral stretchbending tests (Grolleau *et al.*, 2019) have been proposed in recent years for plane strain fracture characterization. The dihedral punch test can be used for thin, ductile materials that fold over to form a complete bend without fracture during v-bending. For fracture characterization under equal biaxial stretching, Nakazima hemispherical punch or Marciniak tests used in formability studies can also be used but are limited by the occurrence of necking. Small radius punches can be used to impose a severe through-thickness stress gradient and foster a proportional strain path with fracture initiating on the convex surface in biaxial tension (Roth and Mohr, 2016; Cheong *et al.*,

2018). The influence of decreasing the hemispherical punch radius on the strain paths and the fracture limits for a DP980 steel can be seen in Figure 11.

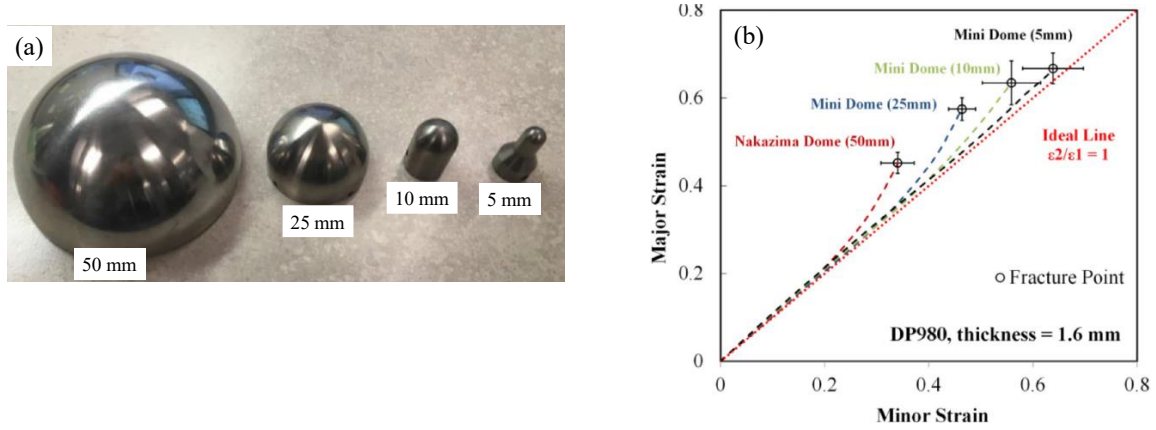


Figure 11 (a) Hemispherical punches with variable radii for fracture characterization under equal biaxial stretching, and (b) strain paths until fracture at the dome apex for a DP980 sheet (Cheong *et al.*, 2018).

2.2.3. Characterization of Uniaxial Fracture Strain

Experimental measurement of the uniaxial fracture strain is critical to assess the stretchability irrespective of whether the edge is machined, waterjet cut, laser cut or sheared. Conventional uniaxial tensile tests are attractive since plasticity and fracture data can be obtained from a single test. Unfortunately, the test is only uniaxial until the peak load, which is often only a small percentage of the local strain to failure in high strength steels and aluminum. Furthermore, fracture initiates from the center of the neck under a triaxial stress state requiring inverse analysis (Grüben *et al.*, 2011; Kasaei *et al.*, 2023). Alternative tests that maintain a constant uniaxial tensile stress state until fracture are required.

2.2.3.1. Hole Tension Test (HTT)

The central hole tension test has been widely adopted to calibrate ductile fracture models of the base metal in various studies (Lou and Huh, 2013; Erice *et al.*, 2018; Jia *et al.*, 2021). Pathak *et al.* (2017) conducted HTT on two AHSS grades having an ultimate tensile strength (UTS) of 800 MPa, with the hole processed by both reaming and punching to evaluate the edge stretchability. The fracture strains observed in HTT were found to be close to the values obtained from the conical hole expansion test for both edge conditions. The DIC major strain contour and stress state evolution near the hole edge are shown in Figure 12 (a, b) for a hole tension test conducted on a DC04 mild steel with a reamed edge. The ratio of the hole diameter to the ligament width can be adapted based upon material ductility and hardening rate to promote a favorable strain gradient and induce cracking at the edge as discussed by Roth and Mohr (2016).

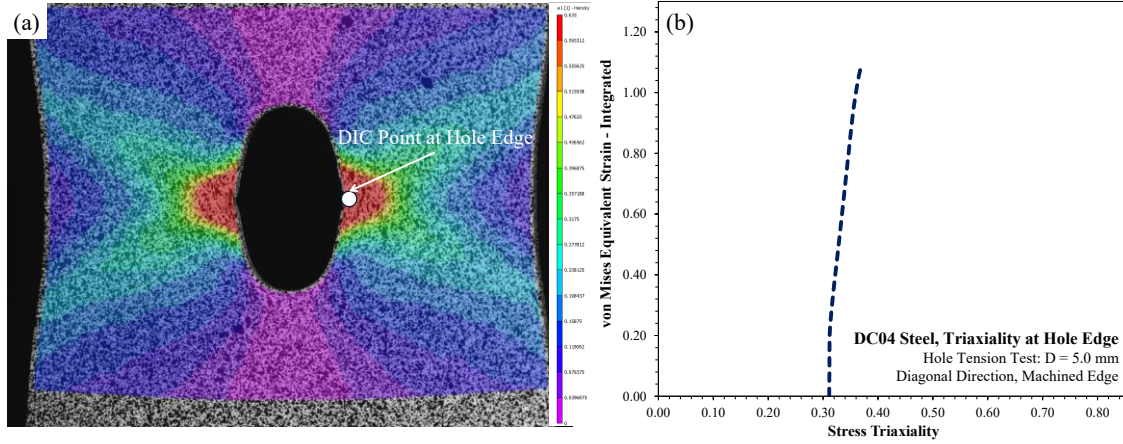


Figure 12 (a) Major strain contour from DIC before fracture onset, and (c) stress triaxiality vs plastic strain at the hole edge during the HTT of a DC04 mild steel conducted with a machined edge.

The HTT is compelling due to its geometric simplicity, ability to characterize fracture anisotropy, and ease of testing. However, issues of necking and fracture initiation behind the hole edge persist for an ideal edge condition as observed by Anderson *et al.* (2017) and Khameneh *et al.* (2024) for DP600 and DP1180 steels, respectively. An example of strain localization occurrence behind the hole edge for a DP600 steel is shown in Figure 13. The HTT may be better suited for sheared edge fracture characterization since the SAZ will promote fracture close to the edge at much lower strains than the machined edge tests.

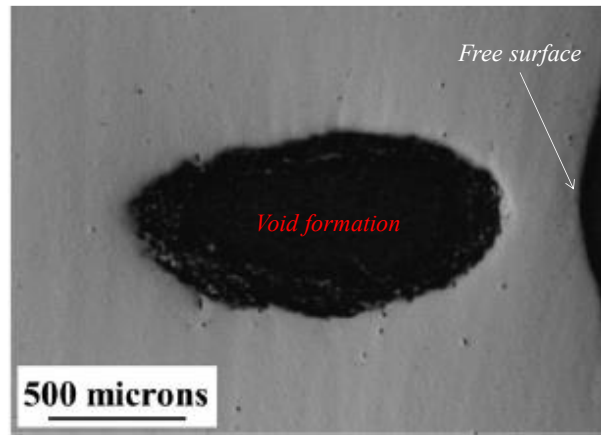


Figure 13 Strain localization behind the edge during hole tension test of DP600 sheet with machined hole (Anderson *et al.*, 2017).

2.2.3.2. Conical Hole Expansion Test

The mechanics of the hole expansion test vary with the punch geometry and may lead to different hole expansion ratios. The use of a cylindrical flat punch for hole expansion tests can be employed to validate calibrated plasticity models (Lee *et al.*, 2019; Park *et al.*, 2023). In general, the flat punch hole expansion test is not accurate for fracture characterization of the base material with a machined hole due to failure

initiating behind the edge in a necking mode. Hole expansion tests with a 100 mm diameter hemispherical punch induce a moderate curvature and out-of-plane bending on the sample. The measured hole expansion ratios with a hemispherical punch are generally higher than a cylindrical punch due to the imposed through-thickness stress gradients. However, failure may still occur away from the edge as reported by Paul (2020).

The conical HX produces strong through-thickness gradients during its initial stages that can reduce chances of necking and induce cracks under proportional loading. However, the viability of the test towards uniaxial fracture characterization must first be evaluated by assessing the stress history at the hole edge. The influence of material anisotropy and necking on the test is less understood, as a circumferential stretch is imposed in all directions that can cause localization and affect the hole shape as exemplified in Figure 14 for a CP590 steel.

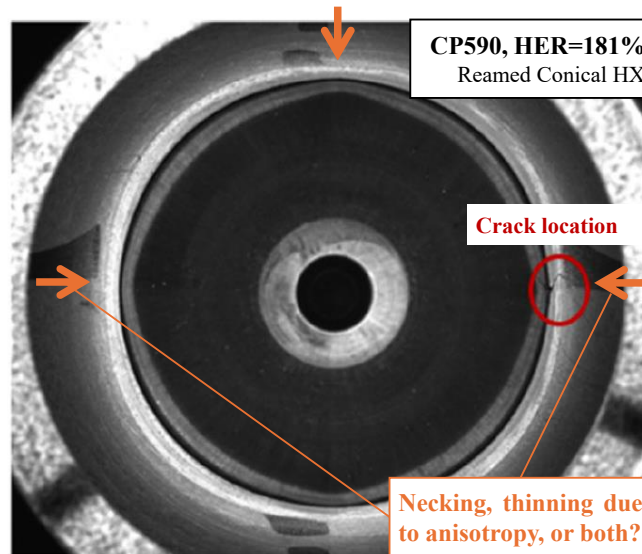


Figure 14 Image of a failed CP590 specimen after conical HX conducted on a drilled and reamed edge (Pathak *et al.*, 2016). The non-uniform thinning around the hole circumference could be due to necking, anisotropy, or a combination of both factors.

In the literature, the FE models used to analyze the conical HX have focused primarily upon AHSS with low-to moderate ductility and mild anisotropy. Such materials exhibit an approximately uniform hole shape and thickness distribution at fracture. Materials with high anisotropy and ductility can produce an elliptical shape at fracture with severe thinning at certain orientations. Apparent localization due to non-uniform thinning around the hole circumference was observed for mild steels and titanium by Johnson *et al.* (1977) and for complex-phase steels by Pathak *et al.* (2016). However, whether the observed thinning distribution and hole shape is a byproduct of necking, anisotropy, or a combination of both needs to be investigated for the test to be deemed suitable for experimental fracture characterization. The behavior of a sheared hole is expected to be similar to low ductility materials with crack initiation occurring at much lower strain levels

due to work-hardening and damage within the SAZ. As a result, the influence of anisotropy and potential necking is expected to be minimal with the sheared hole condition.

Several studies have numerically investigated the stress state and strain path near the hole edge during conical HX for near isotropic materials. A transition in stress state towards plane strain tension at the failure location indicates the presence of a tensile instability. If all through-thickness layers transit towards plane strain, then a neck has formed. The inner edge, mid-plane, and outer edges are shown in Figure 15. The outer edge is independent of any tool contact. The inner edge contacts the conical punch during the initial deformation stages.

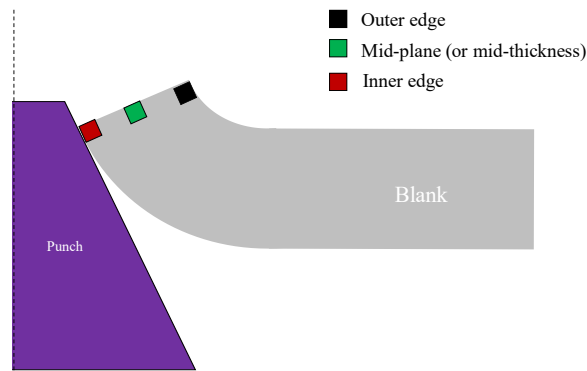


Figure 15 The various through-thickness layers of the blank during conical hole expansion.

Ko *et al.* (2007) performed conical HX simulations for a hot-rolled SAPH440 steel with an ideal edge condition using hexahedral solid elements assuming one-eighth symmetry and isotropic behavior. It was noted that the strain path at the hole periphery (outer edge) where cracking initiated approximately followed uniaxial tension. Butcher *et al.* (2013) evaluated the stress triaxiality and Lode parameter histories at the hole face using 2-D axisymmetric solid elements for a DP600 steel. Regardless of the hole processing method, the outer edge consistently experienced uniaxial tension until fracture (see Figure 16), while the inner edge transitioned from initial compression to uniaxial tension. Kim *et al.* (2025) observed similar behavior in simulations of conical HX for two steels with a nominal UTS of 980 MPa and hole expansion ratios near 100%. Paul (2020) reported localized neck formation in conical HX of an extra-deep drawing (EDD) steel, though no tensile instability was evident from the strain paths. Park *et al.* (2021) supported this finding through microscale simulations of a mildly anisotropic hot-rolled ferrite-bainite steel. However, the outer hole edge was found to exhibit a constant stress triaxiality corresponding to that of uniaxial tension in the macroscale simulations. These results appear inconsistent with the occurrence of strain localization during the test. It is possible that the observed non-uniform thinning around the circumference was a manifestation of the plastic strain anisotropy, given that the strain paths were found to be linear.

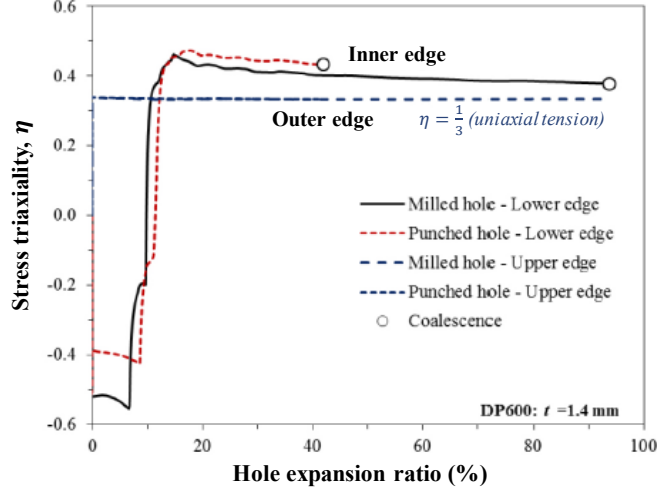


Figure 16 Stress triaxiality evolution of the hole edge elements during conical HX of a DP600 steel with machined and sheared edges (Butcher *et al.*, 2013).

There is consensus in the literature that fracture initiation in conical HX is expected to occur at the outer hole edge. Damage development at the inner edge is delayed due to initial contact with the conical punch that produces a transient compression mode. The outer edge experiences the highest tensile strain, promoting internal void nucleation and coalescence, causing fracture. Numerous studies have attempted to develop correlations between the conical HX performance and material properties such as the R-value, strain hardening rate, uniform and total elongation (Comstock *et al.*, 2006; Kim *et al.*, 2018; Plosila *et al.*, 2024). Larour *et al.* (2020) found that the tensile properties did not strongly correlate with the HER and instead proposed using the fracture strain from the minimum cross-section area in the tensile test. Hance (2017) proposed the ‘maximum edge stretch criterion’ to convert the HER into a true major fracture strain:

$$\varepsilon_1^f = \ln \left(1 + \frac{HER(\%)}{100} \right) \quad (\text{Eq. 6})$$

The HER employs the average inner hole diameter at fracture. However, the inner hole edge undergoes a non-linear strain history as seen in Figure 16. Instead, Pathak *et al.* (2017) proposed a ‘macroscopic’ fracture strain ($\varepsilon_{eq}^{macroscopic}$) combining the von Mises yield function with the average hoop (ε_θ) and thickness strains (ε_t) at experimental fracture.

$$\varepsilon_{eq}^{macroscopic} = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_\theta - \varepsilon_t)^2 + (\varepsilon_\theta + 2\varepsilon_t)^2 + (2\varepsilon_\theta + \varepsilon_t)^2} \quad (\text{Eq. 7})$$

$$\varepsilon_\theta = \ln \left(\frac{d_{inner}^{avg} + d_{outer}^{avg}}{2d_0} \right) \quad (\text{Eq. 8})$$

$$\varepsilon_t = \ln \left(\frac{t_f}{t_0} \right) \quad (\text{Eq. 9})$$

where d_{outer}^{avg} , t_0 , and t_f are the average outer hole diameter at fracture, initial thickness, and the final thickness at the rupture location, respectively. At present, the various approaches to estimate the local fracture strain in the conical HX have not considered the influence of anisotropy on thinning and potential necking. Additionally, a critical evaluation of the methods to estimate the fracture strain is required using FEA. The best-fit or average diameters are used to describe the hole geometry in Eq. (6) and Eq. (8), even though it will not remain circular for anisotropic materials with high ductility. An open question remains as to whether any of the proposed methods are sufficiently accurate to determine the uniaxial fracture strain for use in finite-element models of forming operations to predict edge splitting.

2.3. Characterization of Sheared Edge Fracture Limits

Sheared edges can split at much lower strains than edges prepared using other methods due to the surface quality and the strain distribution in the SAZ. While the sheared edge inherently experiences uniaxial tension as it is a free surface under stretch, the stress state within the shear affected zone can vary. Depending on the applied boundary conditions, a diffuse neck may form in the SAZ before edge cracking, affecting the reliability of tests used to characterize edge fracture. It is unclear whether there is a singular fracture limit for sheared edges for a given set of process parameters, or if the limit depends on the material and application. As a result, different tests may be needed to examine the effect of varying boundary conditions. Larour *et al.* (2016) showed that the use of fracture strain from conical HX may not necessarily lead to reliable prediction of edge cracks in multi-step forming simulations. This section discusses experimental methods used to evaluate edge stretchability and the strain and stress conditions that are imposed in its vicinity.

2.3.1. Sheared Edge Fracture under In-Plane Uniaxial and Circumferential Stretching

In multi-step forming operations in the automotive industry, the initial blank contour is typically produced through either open or closed-line cutting. During forming, the edges of the blank experience in-plane stretching while the central region is formed into the desired shape, as illustrated in a B-pillar forming simulation in Figure 17. Holes in complex geometries may also undergo similar in-plane deformation. Coupon-level tensile tests with sheared edges can replicate these conditions and are commonly used to evaluate edge fracture limits. Specimen design plays a critical role in such tests. Asymmetrical shapes like half-dogbones or rectangular strips with similar widths at the grip and gauge sections may introduce bending or cause early failure near the grips. The edge fracture tensile test (EFTT) developed by Feistle *et*

al. (2015) uses a symmetric dogbone geometry with the gauge region cut using a rectangular punch. This setup enables edge fracture characterization for both trimmed and pierced edges by modifying the initial blank shape used for cutting.

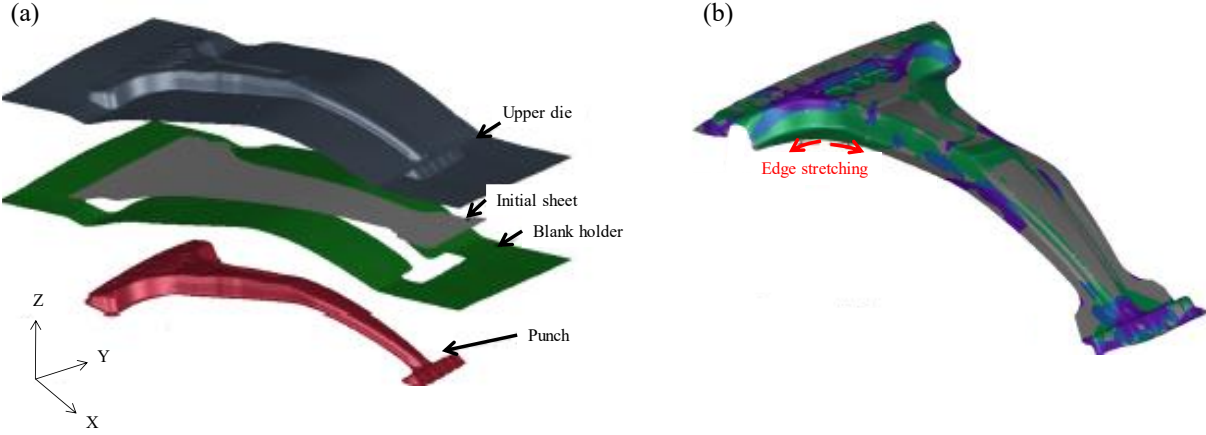


Figure 17 Example of in-plane edge stretching during a B-pillar forming simulation. (a) Tooling, and (b) stamped B-pillar component (Won *et al.*, 2020).

Figure 18 illustrates three distinct fracture modes that can occur in the EFTT due to competing failure mechanisms. In some high ductility materials, failure is governed by necking near the center of the gauge region, like a regular tensile test despite the presence of a sheared edge. In highly edge sensitive materials, edge cracks can form before diffuse necking begins. A third mode involves a combination of localization and edge cracking, where fracture initiates at the edge as the material approaches the acute necking strain. The EFTT is only representative of the forming condition at an edge of a part at strain levels below the UE. The supporting material behind the flange or in curved regions of stamped parts are not exposed to a uniform uniaxial tensile stress state to form a global neck. The strain gradient in these regions will constrain deformation and failure to occur near the sheared edge. Currently, there is no standardized approach for determining fracture strain in the EFTT. Some studies rely on total elongation to quantify edge sensitivity (Golovashchenko and Ilinich, 2005), while the local strain measured at the fracture initiation region can also be used (Ilinich *et al.*, 2011).

The hole tension test also represents a case of in-plane stretching and has been employed for sheared edge fracture characterization, as done by Gläsner *et al.* (2016) and Pathak *et al.* (2017). Gläsner *et al.* (2016) proposed measuring the local strains using an extensometer of 2 mm initial length placed 1 mm away from the hole edge at the instant of 5% drop in load from its peak value. Feistle *et al.* (2019) analyzed the strain rate using the method of Volk and Hora (2011) to identify the acute necking strains near the edge, while Pathak *et al.* (2016) used stereoscopic DIC to determine the highest major strain location near the edge at the onset of fracture. Feistle *et al.* (2022) reported that the sheared edge fracture strains obtained using HTT

were up to 10% higher than those obtained using the edge fracture tensile test for a DP980 steel. The differences can be attributed to the favorable strain gradient imposed during the HTT that can suppress or delay necking compared to the EFTT.

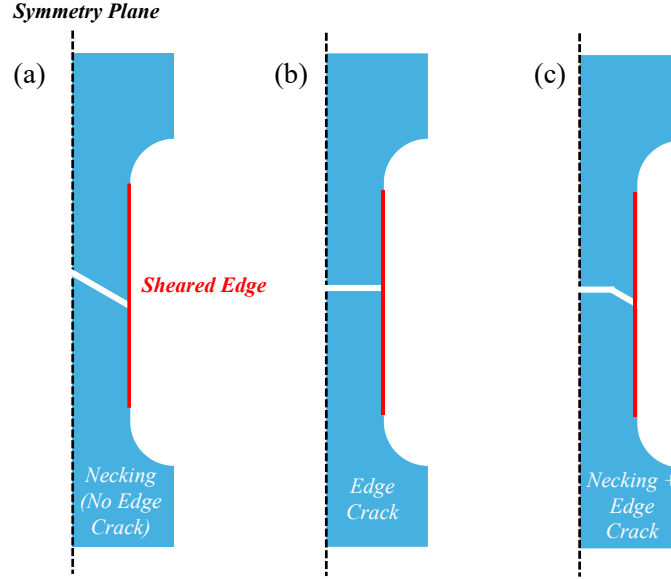


Figure 18 Observed fracture modes during a sheared edge tensile test. (a) Necking failure, (b) edge fracture, and (c) combined edge fracture and necking initiation.

The hole expansion test can also be performed using a cylindrical punch to characterize sheared edge fracture under in-plane circumferential stretching. However, failure in flat punch HX can occur away from the hole as observed during machined and sheared edge tests for a CP800 by Pathak *et al.* (2016), as well as by Barnwal *et al.* (2020) for a DP980 steel with wire cut edge. The presence of SAZ can promote cracks near the hole edge at significantly lower HER depending upon the material formability as observed by Wagner *et al.* (2025) during flat punch HX of DP780 and CP980 steels.

2.3.2. Edge Fracture Characterization in Out-of-Plane Loading

The conical HX discussed in Section 2.2.3.2 is the classical example of sheared edge fracture with out-of-plane deformation. A reduction in hole size or the conical punch tip angle can promote larger edge stretchability due to higher strain gradients (Krempaszky *et al.*, 2014). However, a widespread criticism of the conical hole expansion test is its tendency to produce large scatter in HER between repeats (Barlo *et al.*, 2022) and among test laboratories (Goncalves, 2018) for the same material due to variations in punching conditions, tool wear, and non-uniformity in clearances. As an alternative, V-bending or similar plane-strain fracture tests can be exploited for edge fracture limit assessment under out-of-plane loading provided the edges are fabricated using shear cutting. Tsoupis *et al.* (2014) investigated through-thickness bending of sheared edges using air bending for two high-strength low-alloy steels, focusing on the effects of material

anisotropy and burr orientation relative to the punch. Dong *et al.* (2016) utilized a camera system to directly monitor strains at the shear face in materials thicker than 3.0 mm.

Other out-of-plane tests include the half-specimen dome test of Shih and Shi (2011) and the so-called Diabolo test (Liewald and Gall, 2013; Falk, 2017). The half-specimen dome test employs the same tooling as the Nakazima formability tests (see Figure 19(a)). A rectangular blank is sheared along its longer side and aligned with the apex of the hemispherical punch. A layer of Teflon sheet is placed between the punch and blank to minimize friction. The blank is formed into a half-dome, following which the local fracture strains and the achievable dome height are measured. A limitation of this test is that the failure location cannot be controlled and can occur anywhere in the region between the die corner radius and the punch apex (see Figure 19(c)). Friction can influence the test results and induce inconsistencies in the limiting dome height and crack location. In contrast, the Diabolo punch features a double curvature and a recess at its center that forces the maximum strain to occur within a localized region during the test (see Figure 20). However, its design in terms of the curvature radii and ease of DIC strain measurement has not been discussed in detail within the literature. Overall, out-of-plane edge fracture characterization tests are less susceptible to the onset of necking than the in-plane stretching tests. The influence of burr orientation on edge fracture can also be readily assessed by performing tests with the sheared edge placed on the tooling such that the burr will be under compression or tension.

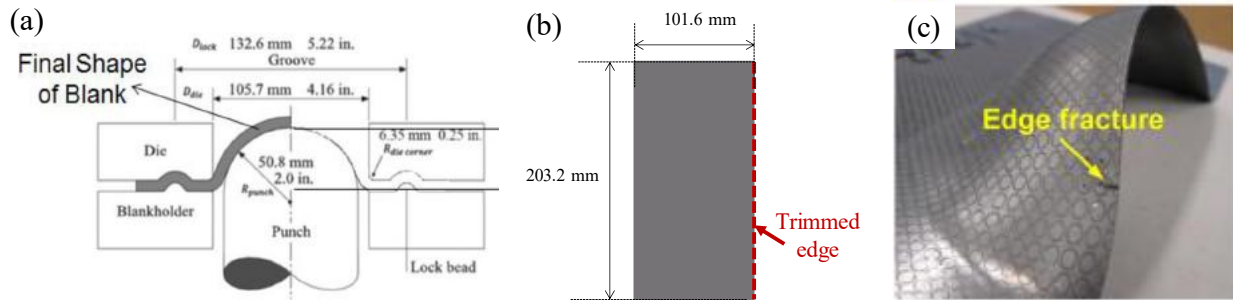


Figure 19 (a) Schematic of the half-dome test (Kim *et al.*, 2017), (b) initial blank outline, and (c) failed half-dome specimen (Shih *et al.*, 2014)

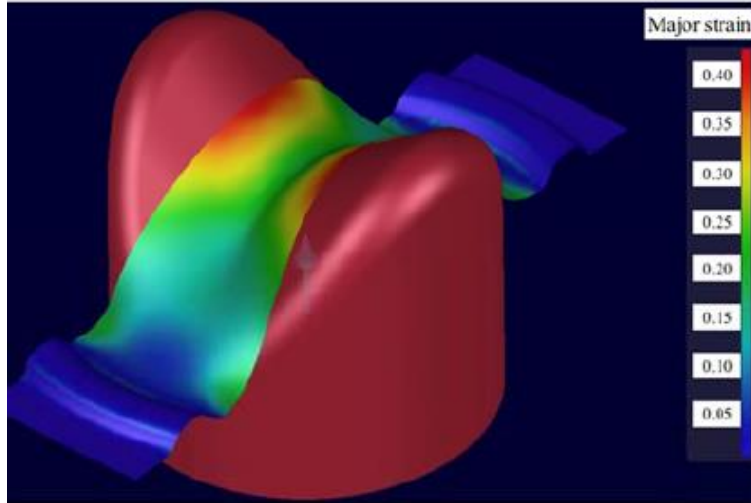


Figure 20 Strain distribution during the Diabolo test (Falk, 2017)

2.3.3. Sheared Edge Fracture under In-Plane Bending

Sheared edge fracture characterization under in-plane bending and superimposed bending and tension has received little attention in the literature due to experimental challenges but are important modes for industrial applications. The in-plane bending mode is more representative of deformation around curved regions such as the blank periphery of a B-pillar component (see Figure 17 for example), where edge splits can occur without necking. The stretch webs or connectors in a progressive die can also undergo in-plane bending during the transition phase between two consecutive forming steps, as illustrated schematically in Figure 21. The conical hole expansion test may be insufficient to describe the in-plane stretch-flangeability of a material as demonstrated by Hino *et al.* (2017) for two AHSS grades with tensile strengths of 590 MPa and further supported by Khalilabad *et al.* (2023) for a DP800 steel.

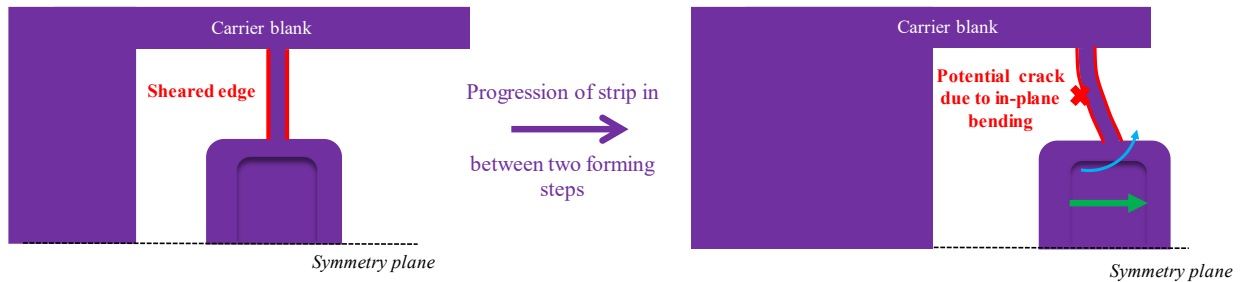


Figure 21 Potential for edge cracking during strip movement and vibration in a progressive die operation.

Bouaziz *et al.* (2010) introduced the double bending test to address the limitations of conical hole expansion and edge fracture tensile tests. In this method, the edge of a rectangular specimen is trimmed and then bent twice in perpendicular planes as shown in Figure 22. The first step involves flanging the edge by 90° using a bending brake or a wipe bending operation, followed by in-plane bending of the sheared edge in the

second stage using a punch with a 4 mm corner radius. A major concern with the double bending test is the risk of specimen buckling during the second bending stage. Its success further depends on the initial flange bending step, which is prone to springback. Another drawback is that fracture strain is calculated from edge thinning at the crack location (Dietsch *et al.*, 2017), which can overestimate the actual fracture strain because of straining during the rupture process. Sato *et al.* (2011) developed a so-called side bending fixture to quantify the stretch flangeability and fracture strain of a hot-rolled steel having an ultimate tensile strength of 590 MPa. The side bending process requires intricate bending fixtures to clamp the specimen and are driven by a hydraulic servo motor. Furthermore, the side bending test unavoidably induces an axial load on the sample since the clamps are not allowed to slide horizontally, resulting in a stretch-bending deformation. Matsuno *et al.* (2024) recently developed a 3-point in-plane bending fixture that integrates hydrogen charging to evaluate delayed fracture stress thresholds. However, the setup imposes fixed loads on the sample rather than using displacement control, which can complicate the precise identification of the fracture instant during sheared edge stretching.

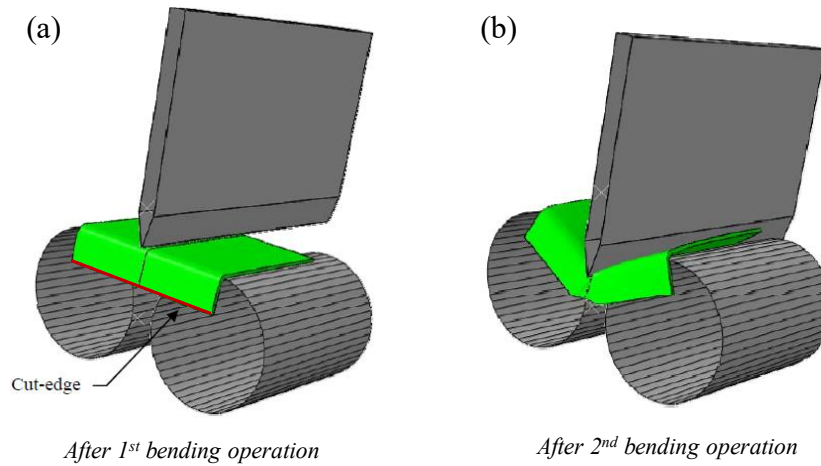


Figure 22 Schematic of the double bending test. (a) Sheared sample after the first and (b) second bending stages (Bouaziz *et al.*, 2010)

Naseem *et al.* (2020) recently developed a novel 4-point in-plane bending fixture to evaluate the flow stress response of AHSS with low uniform elongation to strains of up to 20%. The apparatus consists of spherical rod ends with ball bearings attached to linear bearing ball carriages that pin load a specimen with a reduced cross-section gauge area. A schematic of the test fixture is shown in Figure 23(a) and the boundary conditions in Figure 23(b). The linear bearing carriages slide along guide rails minimizing axial forces during bending. The fixture can be mounted on a tensile testing machine, enabling deformation measurement with DIC. The vertical motion of the upper crosshead subjects the specimen to a 4-point bend. Since the gauge area does not contact any components, frictional effects are avoided. Khalilabad *et al.* (2022) later modified the fixture design and specimen geometry to enable edge cracking in a 1.3 mm thick

DP800 steel with a trimmed edge exhibiting a fracture strain of about 0.30. The in-plane bending setup generates a stress state gradient across the gauge height with the convex surface in tension and the concave surface in compression. The presence of a stress gradient can potentially suppress necking, making the test also amenable to uniaxial fracture characterization of the base metal without the SAZ. However, the capabilities of the test for uniaxial fracture characterization need to be evaluated to larger strains. The specimen geometry needs to be optimized to promote larger deformation levels and trigger fracture in moderate and high ductility materials while simultaneously minimizing the risk of buckling. Furthermore, techniques to detect fracture onset and methods to measure the strain without relying solely on DIC also need to be developed to expand its applicability.

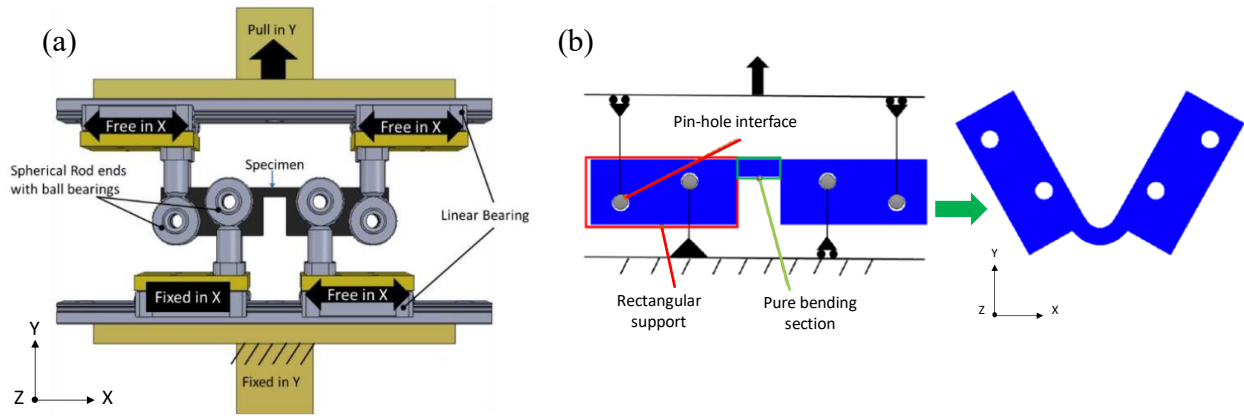


Figure 23 (a) Schematic of the in-plane bending test, and (b) boundary conditions imposed on the specimen during deformation (Naseem *et al.*, 2020).

2.4. Evaluation of Lengthscale Effects on Edge Stretchability Characterization

2.4.1. Influence of DIC Lengthscale on Edge Fracture Limits

In addition to the potential influence of boundary conditions on edge fracture limits, the choice of experimental lengthscale can also affect comparisons and measured edge strains. The DIC is inherently limited in that it cannot resolve the deformation exactly at the edge. Moreover, DIC strain measurement has an associated virtual strain gauge length (VSG) since it performs spatial averaging and smoothing of the data. The VSG amalgamates the camera resolution with the parameters used to process the captured images. The VSG can be defined following the guidelines of Jones and Iadicola (2018) as:

$$VSG(mm) = \left[\{ (Filter - 1) \times Step(pixel) \} + Subset(pixel) \right] \times Image\ Scale\ (mm / pixel) \quad (Eq. 10)$$

The image scale or resolution is determined by the camera optics and setup of the image acquisition system. The subset size refers to the pixel dimension of the square area that is tracked to obtain the displacement and strain fields and is typically dictated by the speckle pattern. Step and filter sizes correspond to the

number of pixels analyzed and the extent of spatial averaging used to reduce noise. Khameneh *et al.* (2021) and Pilozo-Hibbit *et al.* (2024) reported a strong sensitivity of DIC local strains in simple shear tests to the chosen lengthscale, with fracture strain values differing by more than 100% depending on material ductility. A strong sensitivity can also be observed in uniaxial tensile tests, where the local fracture strain depends on the VSG and the specimen geometry as shown in Figure 24 for a 980GEN3 steel. Despite its importance, the DIC lengthscale is often not reported in studies of edge fracture tests, which can complicate the comparison of experimental results since the various test methods may exhibit different VSG sensitivity. It is necessary to examine how the VSG length influences strain measurements in edge fracture tests to assess whether the observed trends related to boundary conditions remain consistent across different lengthscales.

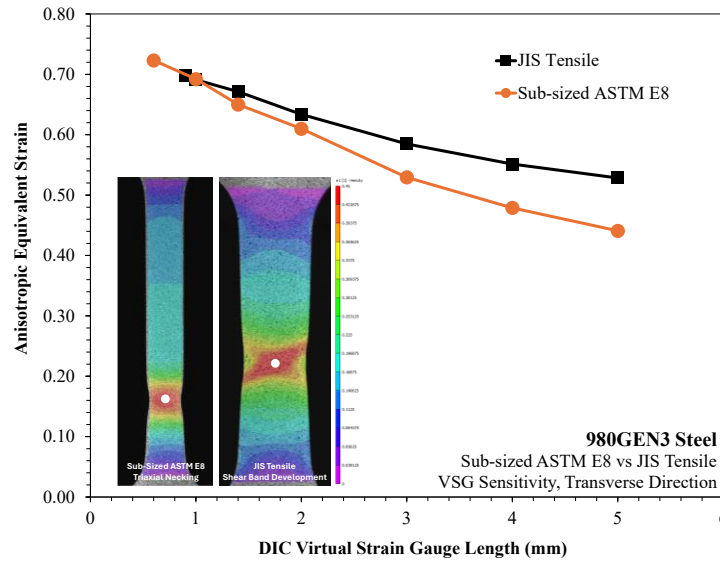


Figure 24 Influence of DIC virtual strain gauge length on the local surface strains at the neck center from uniaxial tensile tests conducted for a 980GEN3 steel.

In industrial applications, localized strain measurements may not be feasible due to the large sizes of formed components necessitating reliance upon thinning limits for edge crack assessment (Zhou *et al.*, 2010; Kim *et al.*, 2017). As a result, the edge fracture strains measured from DIC identified from the coupon tests for each loading scenario should be directly applicable for use to actual components. The overall change in length of the sheared edge up to the point of crack initiation can be used to mitigate DIC lengthscale or guide the appropriate choice of VSG. Extensometers have been widely used to quantify deformation for some of the in-plane stretching tests, such as hole and edge fracture tensile tests (Shih and Shi, 2011; Gläsner *et al.*, 2016). However, comparable global strain measures are also needed for other loading modes including in-plane and out-of-plane axisymmetric stretching as well as in-plane bending to closely assess the boundary condition effect. Additionally, the choice of optimal extensometer length can significantly

affect the measured global strain, particularly when a localized strain field is present (Xu *et al.*, 2021) making it essential to evaluate its effects.

2.4.2. Numerical Modelling of Sheared Edge Failure

Finite-element simulations can play a crucial role towards understanding the material performance under shear cutting and edge stretching processes and identifying areas of potential design improvements. A two-step finite-element analysis approach can be used to simulate the shear cutting and the subsequent edge stretching process (Qian *et al.*, 2021; Habibi *et al.*, 2024). In particular, modelling of the shear cutting process can be beneficial to understand the influence of process parameters like cutting clearance, punch speed and shape, and tool wear on the cut edge geometry, forces, or the SAZ strain distribution (Li *et al.* 2021; Kurra *et al.* 2021; Pätzold *et al.*, 2023). However, accurate modeling of the cutting process requires solid elements with very fine meshes, which increases the computational cost significantly. The results can also be sensitive to the choice of remeshing strategy, element deletion method, and fracture locus (Habibi *et al.*, 2023). As shown by Sandin *et al.* (2025) in simulations of CP1000 steel, mesh size can strongly affect the predicted strains and consequently the point at which cracks initiate. Shell elements can be used to model the edge stretching stage to manage the overall cost of the two-step approach (Wang *et al.*, 2015). In this case, the strain field must be transferred from the cutting simulation homogenized, and then transferred to the edge stretching model.

Alternatively, SAZ measurements from experiments can be integrated directly, avoiding the need to simulate the cutting process. Butcher *et al.* (2013) used flow line tilt measurements in DP600 to measure the equivalent plastic strain distribution within the SAZ. Subsequently, the average SAZ strain was input into a coupled damage model to characterize its damage and work hardening behavior, which was subsequently implemented into FE models of the conical hole expansion test. This approach led to an excellent agreement between the measured and predicted hole expansion ratios for both punched and drilled specimens. Pathak *et al.* (2019) estimated SAZ strain in DP780 and CP800 steels from Vickers hardness data, achieving good agreement with the flow line method despite differences in lengthscale. Li *et al.* (2022) used the ductile fracture criterion of Hu *et al.* (2017) calibrated using shear, hole tension, and notch experiments to simulate a flanging operation for a quench and partitioned (QP) steel with a nominal UTS of 980 MPa. The pre-damage due to the punching process was estimated using an analytical function that was dependent on the cutting clearance and the shear rake angle. The simulations were found to be in close agreement with the experiments. However, the analytical function should be tailored to the material of interest to accurately estimate the pre-damage due to cutting. Additionally, the influence of mesh size on the pre-damage value was not considered in their study.

While these modelling strategies show considerable potential, their application to stamping simulations with coarse shell meshes remains difficult. At present, shear cutting is carried out along a predefined contour within simulations without modelling the deformation that occurs during the process. Instead, a nominal fracture strain value can be assigned to the resulting edge elements to evaluate edge cracking during stretching (Gläsner *et al.*, 2016; Falk, 2017). However, a singular edge failure limit may not reliably predict fracture across all regions of a stamped part, as local boundary conditions can vary significantly. It is well known that the predicted strains are also influenced by the mesh size. Fracture models such as the generalized incremental stress state dependent fracture model (Neukamm *et al.*, 2008; Haufe, 2011) employ mesh regularization to scale the fracture locus and ensure consistent fracture displacements across different element sizes (Kim *et al.*, 2017). This highlights the importance of quantifying how numerical lengthscale affects local major and thinning strains under different loading conditions near the sheared edge, and to develop simple and efficient guidelines to define critical edge fracture limits for use in FE simulations.

2.5. Summary and Current Deficits in Literature

The accurate determination of sheared edge fracture limits has become increasingly important with the growing use of high strength materials such as 3rd generation advanced high strength steels. The traditional forming limit curve determined according to ISO 12004-2 is not a reliable indicator of edge formability and often requires the use of ad hoc safety factors. The conical hole expansion test is frequently used to evaluate edge stretchability limits, but the hole expansion ratio is not suitable for input as fracture strain in ductile fracture criteria due to homogenization of non-linear stress state at the inner edge. Several analytical and empirical methods have been proposed to estimate fracture strains from the conical hole expansion test. However, these approaches often neglect the influence of plastic anisotropy and vary in their treatment of strain localization during the test.

Although the conical hole expansion test provides a good indicator of sheared edge stretchability during hole extrusion processes, it may not be representative of other edge deformation modes during stamping such as in-plane stretching and bending. Uniaxial and hole tension tests with sheared edges can assess stretchability under in-plane stretching, but the behavior of sheared edges under in-plane bending remains largely unexplored despite its industrial relevance. Efforts to suppress out-of-plane geometric instability using anti-buckling plates can introduce contact and friction effects that can influence the results. There is a need for a practical and efficient test method that promotes edge cracking under in-plane uniaxial bending while reducing the likelihood of buckling.

Another unresolved issue is the influence of loading conditions on the residual edge stretchability. Differences in fracture strains between test methods are often misinterpreted as one method being superior

without considering the boundary condition or stress and strain gradients near the sheared edge. The through-thickness and in-plane stress gradients in the SAZ can influence fracture strains depending on the onset of necking. With increasing use of DIC to measure local deformation, the choice of virtual strain gauge length can significantly affect measured values. However, the lengthscale used to obtain the fracture strains is often not reported in the literature, which adds uncertainty when comparing different tests and deformation modes. The effect of loading conditions extends to finite-element simulations of edge stretching, but this is yet to be systematically investigated.

Although significant progress has been made in simulating the shear cutting process and subsequent sheared edge stretching, most existing numerical studies rely on solid elements with very fine meshes. However, they are not practical for large-scale industrial automotive simulations which predominantly employ shell elements with mesh sizes on the order of the sheet thickness. The small depth of the SAZ poses an additional challenge since the pre-strain and damage need to be mapped over to coarser shell meshes in FE models. A computationally efficient method is needed to evaluate the influence of boundary conditions on edge failure predictions without relying on the shear affected zone properties, allowing for efficient prediction of edge cracking across different deformation modes and mesh sizes.

The accuracy of simulations also depends on well calibrated material models. Anisotropic plasticity models must be calibrated using data from stress states such as uniaxial tension, plane strain, and equibiaxial tension to ensure reliable predictions. The calibrated yield function and hardening response should be evaluated using experiments and simulations of laboratory-scale tests that produce heterogeneous stress states before they are applied in full-scale models. Identification of material response under plane strain tension is critical since it is the dominant deformation mode during necking and fracture scenarios. However, experimental characterization of the plane strain yield stress remains difficult due to the complexity of the required test setups. A practical and efficient approach is needed to capture the PST behavior from tests routinely used for constitutive and fracture characterization to improve the quality of material models used in FE simulations.

3. Research Scope

3.1. Objectives

The primary aim of this research is to analyze the influence of the imposed boundary conditions on the sheared edge fracture limits of advanced high strength steels. The findings are expected to support reliable assessment of edge cracking in industrial applications and guidance for edge crack predictions in FE simulations of forming processes within commercial software using plane stress shell elements. A 980GEN3 advanced high strength steel will be considered as the primary material of interest to characterize the boundary condition effect on the edge stretchability. However, certain aluminum alloys, mild and dual phase (DP) steels will also be considered in selected tasks to evaluate the developed methodologies. This thesis has been structured around four primary objectives:

1. Develop and evaluate an efficient framework for calibration of anisotropic plasticity models for approximately pressure-independent materials, while incorporating the stress state of plane strain tension.
2. Establish an experimental methodology to characterize the uniaxial fracture strain using the conical hole expansion test that is valid for anisotropic and ductile materials.
3. Enable characterization of edge cracking under an in-plane uniaxial bending mode across a broad range of AHSS by developing a dedicated test fixture along with methodologies for crack detection and fracture strain measurement.
4. Quantify the effect of applied boundary conditions and measurement lengthscales on the experimental and numerical determination of sheared edge fracture limits in AHSS.

3.2. Tasks

Four tasks have been defined to address the objectives outlined in Section 3.1.

3.2.1. Task 1: Characterization and Evaluation of Anisotropic Constitutive Models

In this task, the anisotropic Yld2000 and Yld2004 yield functions are calibrated to satisfy the plane strain constraint (PSC) proposed by Butcher and Abedini (2019) for four materials with varying hardening behavior and anisotropy. The plane strain yield stress in different orientations is determined from notch tests using an inverse hybrid experimental–numerical approach. The data is then used for yield function calibration along with the PSC, with correlations proposed to estimate the plane strain yield strength using

the tensile R-value for aluminum and steels in the absence of experimental data. Next, the hardening response along the reference direction is characterized using the area of the neck measured from DIC surface measurements from tensile specimens that promote triaxial necking. The assumption of isotropic hardening is assessed to large strains using these tensile tests conducted in multiple orientations. Finally, the calibrated plasticity models are evaluated through FE simulations of flat punch HX having an initial 20 mm machined hole. The simulated radial and circumferential strain distributions are compared with DIC measurements conducted on flat punch HX experiments. Additional simulations are conducted using yield functions that either satisfy or violate the PSC to demonstrate the importance of plane strain data in yield function calibration.

3.2.2. Task 2: Characterization of Uniaxial Fracture Strain from the Conical Hole Expansion Test

Numerical models of conical HX are developed for DP1180 and CP800 steels, representing mild and high in-plane anisotropy, respectively, to evaluate the test's suitability for uniaxial fracture characterization. The FE models assume an ideal edge condition with zero pre-damage, enabling assessment of the test mechanics to large deformation levels. Stress and strain state evolution at various through-thickness locations along and away from the hole edge is examined to determine whether proportional loading conditions exist. The influence of necking on conical hole expansion is assessed using the strain rate method of Volk and Hora (2011) and the concave side rule of Tharrett and Stoughton (2003). An analytical technique is then proposed for determining uniaxial fracture strain from conical HX and compared with conventional approaches such as using the inner hole expansion ratio. The proposed method is further evaluated against elemental strains at the hole edge obtained from FE models for both materials. The developed methodology for determining uniaxial fracture strains from conical HX will be applied to sheared edge fracture characterization in Task 4.

3.2.3. Task 3: Characterization of Uniaxial Fracture Limits under In-Plane Bending

A modified in-plane bending fixture and specimen geometry are developed using the design of Naseem *et al.* (2020) as a baseline. The sample geometry and fixture are optimized to reach a true strain of approximately 0.70 at the edge without the occurrence of buckling. The capability of the in-plane bend test to characterize plastic strain anisotropy under uniaxial tension and compression is first explored. Subsequently, the accuracy of the peak load and bending moment criteria to detect crack onset is evaluated for 980GEN3 as well as a martensitic steel (MS) with a tensile strength of 1500 MPa. Alternate fracture detection methods such as the stress metric based on the study of Noder *et al.* (2021b) as well as the strain rate methodology of Volk and Hora (2011) are also evaluated against visual observation of edge cracking. A geometric method to measure the edge strains is proposed and compared against DIC which cannot capture data exactly at the edge. The uniaxial fracture strain obtained from the in-plane bend test for a

machined edge is compared against the conical HX. Finally, design guidelines for the gauge section of the specimen to suppress buckling are established through tests conducted on a range of AHSS with varying strengths and thicknesses.

3.2.4. Task 4: Influence of Boundary Conditions on Sheared Edge Fracture Limits

Five different experiments that subject the sheared edge to different deformation modes, such as conical and flat punch HX, hole tension, edge fracture tension, and in-plane bending tests, are performed on the 980GEN3 steel after hole punching with 12% clearance. A schematic illustrating the gradients promoted near the sheared edge during each test is depicted in Figure 25. The fracture strains are measured using DIC for the various tests except for the conical HX, for which the methodology developed in Task 2 is employed. The sensitivity of 980GEN3 steel to the applied boundary conditions is analyzed, following which the sensitivity of DIC-measured local strains to the virtual strain gauge length is evaluated. Measures to estimate a global edge stretch under each deformation mode are proposed to guide the choice of reference VSG for design and FE modelling purposes. Subsequently, FE simulations of the five test methods are then conducted in LS-DYNA using shell elements with multiple mesh sizes and the calibrated Yld2000 function in Task 1 without consideration of the SAZ. The FE strains for different element sizes were compared with the corresponding DIC values obtained using various VSGs to map the relationship between experimental and numerical lengthscales relevant to ductile fracture modelling practices. Finally, the influence of mesh size on the FE-predicted strains was evaluated across the five tests to highlight the role of deformation mode and to guide best practices for edge crack prediction in numerical modelling using shell elements.

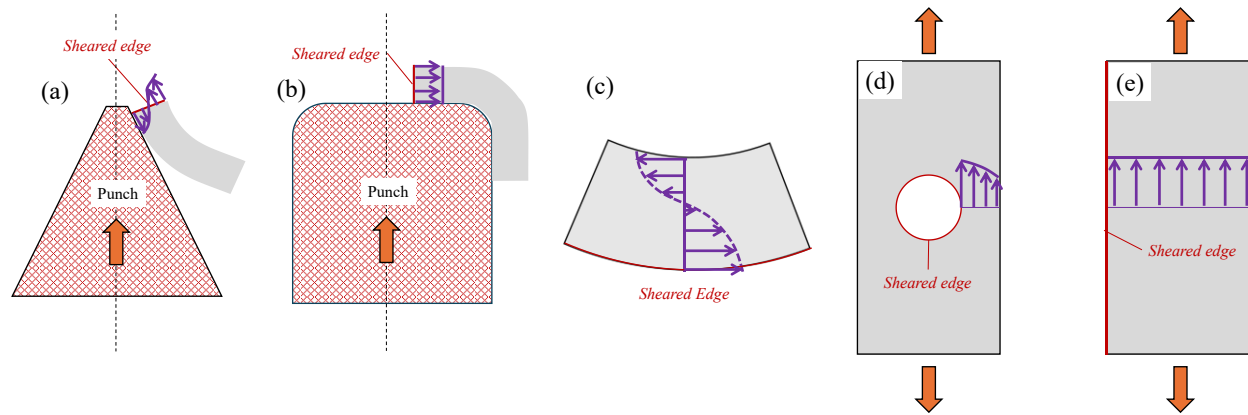


Figure 25 Deformation gradient during (a) initial stages of conical HX, (b) flat punch HX, (c) in-plane bending, (d) hole tension, and (e) edge fracture tensile tests.

4. Calibration and Evaluation of Anisotropic Constitutive Models

This chapter discusses specific results from Appendix A and focuses on the anisotropic plasticity characterization of materials that approximately follow deviatoric plasticity, emphasizing the importance of incorporating the plane strain data during yield function calibration. To evaluate the accuracy of the calibrated yield functions, the flat punch hole expansion is employed since it activates a spectrum of stress states ranging from uniaxial tension at the hole edge to plane strain and biaxial stretching modes away from the hole, as depicted in Figure 26.

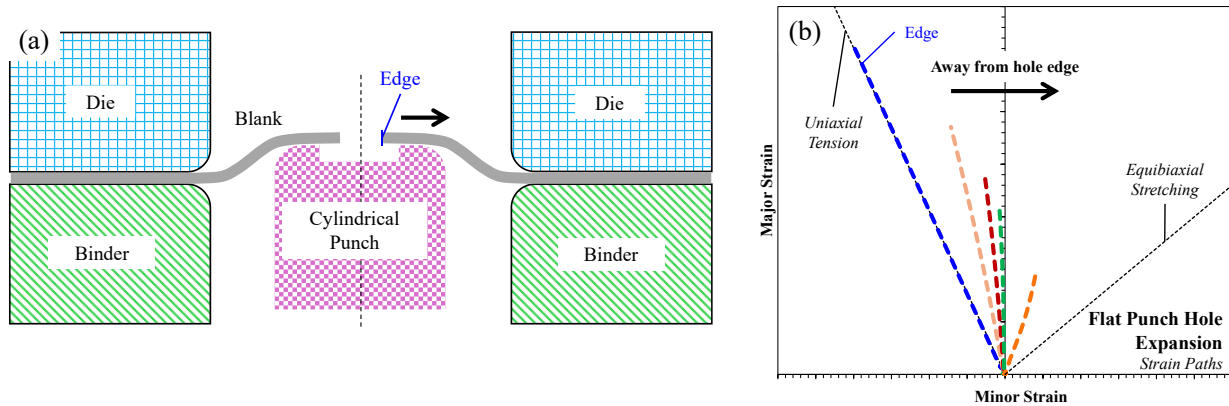


Figure 26 (a) Schematic of flat punch hole expansion, and (b) strain path variation away from the hole edge during flat punch HX.

4.1. Materials

Four automotive sheet metals were considered: an AA5182-O aluminum with dynamic strain ageing; a high strength AA7075-T6 aluminum alloy; a deep draw quality (DC04) mild steel with strong plastic anisotropy, and the 980GEN3 steel. The mechanical properties of the four materials in RD, DD, and TD are summarized in Table 1. The JIS Z2201 tensile geometry was used to evaluate the mechanical response for all materials except AA7075-T6, which was previously characterized by Rahmaan *et al.* (2020) using the sub-sized ASTM E8 specimen.

Table 1 Tensile properties of the materials with the standard deviation in brackets. The extensometer lengths used for the JIS and sub-sized ASTM E8 geometries were 50 mm and 25 mm, respectively. Five tests were conducted per orientation for all the materials.

Material	Thickness (mm)	Direction	Tensile Geometry	0.2% YS (MPa)	UTS (MPa)	UE (%)	TE (%)
AA5182-O Aluminum	0.85	RD	JIS Z2201	140 (1)	285 (3)	25.4 (2.6)	28.8 (0.9)
		DD		138 (1)	282 (3)	27.1 (1.1)	31.4 (0.3)
		TD		139 (1)	280 (2)	26.2 (2.2)	29.9 (0.8)
	2.00	RD		516 (2)	572 (2)	12.2 (0.2)	17.6 (0.6)

AA7075-T6 Aluminum		DD	Sub-sized ASTM E8	492 (2)	563 (2)	12.2 (0.2)	18.0 (0.4)
		TD		507 (2)	578 (2)	11.6 (0.2)	16.2 (0.8)
DC04 Steel	1.00	RD	JIS Z2201	159 (1)	296 (1)	27.2 (0.1)	47.8 (0.9)
		DD		170 (3)	307 (1)	24.8 (0.2)	44.0 (0.7)
		TD		164 (1)	291 (1)	25.4 (0.1)	45.1 (0.7)
980GEN3 Steel	1.40	RD	JIS Z2201	654 (2)	1006 (2)	19.5 (0.1)	27.7 (0.2)
		DD		633 (2)	1010 (4)	19.1 (0.3)	27.4 (0.1)
		TD		624 (5)	1012 (6)	18.7 (0.1)	28.0 (0.1)
YS – Yield Strength, UTS – Ultimate Tensile Strength, UE – Uniform Elongation, TE – Total Elongation							

4.2. Experiments for Constitutive Characterization and Evaluation

The specimen geometries used for constitutive characterization along with their dimensions are depicted in Figure 27. All tests were conducted at a nominal strain rate of 0.001 s^{-1} . In addition to the standard tensile geometries for mechanical property evaluation, a miniature tensile specimen with 3.0 mm gauge width was considered for application of the area reduction method to obtain hardening response to large strains (Abedini *et al.*, 2022). Tensile specimens that induce a width-to-thickness ratio of lower than 4.0 were found to induce triaxial necking ensuring ease of cross-sectional area measurement using DIC. The notch tensile test shown in Figure 27(d) was used to identify the plane strain yield strength of the materials with further discussion on the methodology in Section 4.3. The miniature and notch tensile tests were conducted along the rolling, diagonal (DD), and transverse (TD) directions. The flat punch hole expansion tests were performed using an MTS hydraulic system, with further details on the tooling and setup provided in Section A.2.2 of Appendix A. The square blank features a 20 mm diameter machined hole to promote failure behind the edge and critically evaluate the calibrated plasticity models. Stereo-DIC was employed for all the tests with the corresponding DIC processing parameters and data extraction tools discussed in Section A.2.3 and Section A.4 of Appendix A.

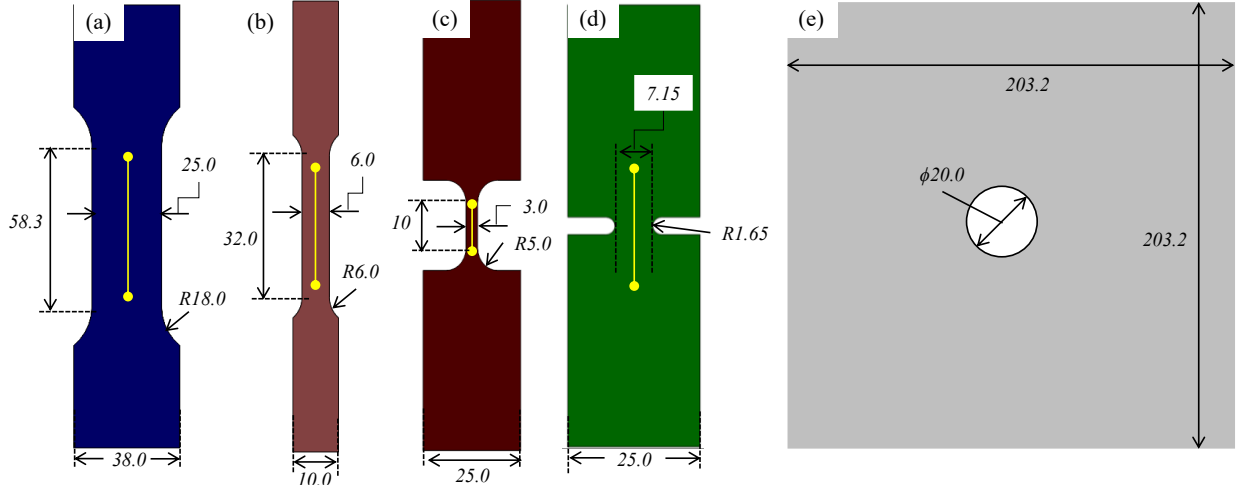


Figure 27 (a) JIS Z2201, (b) sub-sized ASTM E8, and (c) miniature tensile specimens, and geometries for (d) notch tensile and (e) flat punch HX. All dimensions are in mm.

4.3. Characterization of Plane Strain Response and Yield Function Calibration

The PSC of Butcher and Abedini (2019) constrains the stress state of plane strain tension to occur at an in-plane principal stress ratio of 0.5 ($s_2=0$) for pressure-independent materials. Deformation mechanisms like twinning can be influenced by the hydrostatic stress because of which the PSC does not apply to materials such as magnesium and titanium. An inverse finite-element procedure was adopted to estimate the plane strain yield strength in the three directions from the notch tests. A concise summary of the inverse analysis procedure is depicted in Figure 28, with further details regarding the FE model setup, adopted yield function and the determination of its coefficients provided in Section A.3.2 of Appendix A. The inverse analysis approach only requires the flow stress curve until the UE strain, and R-value from tensile tests conducted in the same material orientation as the notch test being simulated. Alternatively, the PSC-enforced form of Hosford (1985) yield function can be employed when using the hybrid approach with plane stress shell elements (Fast-Irvine *et al.*, 2021). Note that the simulations were terminated at the instant when the edge elements reached the UE strain.

The results of the inverse finite-element analysis (FEA) approach applied to DC04 and 980GEN3 steels along the RD are shown in Figure 29. Simulation conducted using a yield exponent of 3.0 exhibited the closest match to the representative experiment for DC04, underpredicting it by approximately 2% at an engineering strain of 0.05. The normalized plane strain yield strength of DC04 in RD exceeds the maximum value attainable with the Yld91 function for the specified R-value. Alternate yield criteria with larger flexibility (Vegter and van den Boogaard, 2006; Yoshida *et al.*, 2013) can be selected during the inverse analysis procedure to successfully capture the DC04 plane strain data. For the mildly anisotropic 980GEN3, an exponent value of 5.0 resulted in an excellent agreement between the FE and experimental force

response. The finalized exponent values are then used to calibrate the local arcs of the yield function from the uniaxial to plane strain tensile regions in the same direction as the notch test as shown in Figure 30 for the 980GEN3 steel.

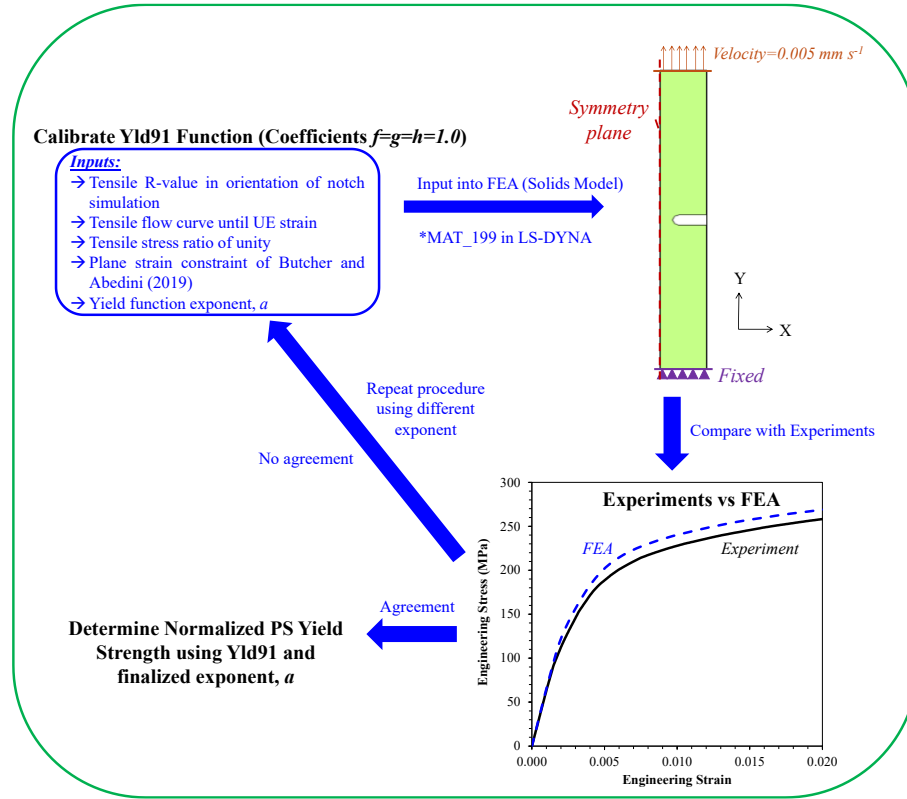


Figure 28 Inverse FEA approach for characterization of plane strain yield strength from notch tests. Note that the procedure is specific to solid elements.

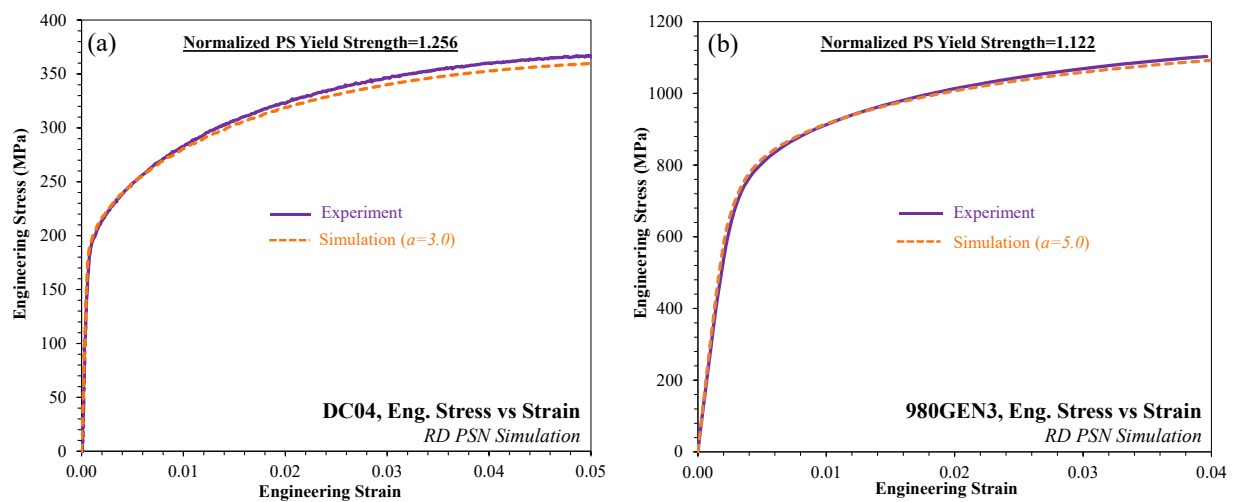


Figure 29 Comparison of engineering stress response from notch test simulation conducted with finalized yield exponent versus a representative experiment for (a) DC04 and (b) 980GEN3 steel in the RD.

The R-values and stress ratios from uniaxial tensile tests, as well as the plane strain data obtained in multiple orientations, were used to calibrate the anisotropic non-quadratic Yld2000 and Yld2004 surfaces for the four materials at the minimum plastic work corresponding to the UE strain. The DC04 mild steel was an exception for which 50% of the UE plastic work was used. The Yld2000 and Yld2004 functions require calibration of 8 and 18 parameters, respectively, with only 16 coefficients being unique for Yld2004 (van den Boogaard *et al.*, 2016). The measured R-values and stress ratios in various stress states, the calibrated coefficient values, and predictive accuracy of the yield functions can be found in Appendix A (see Section A.3.3).

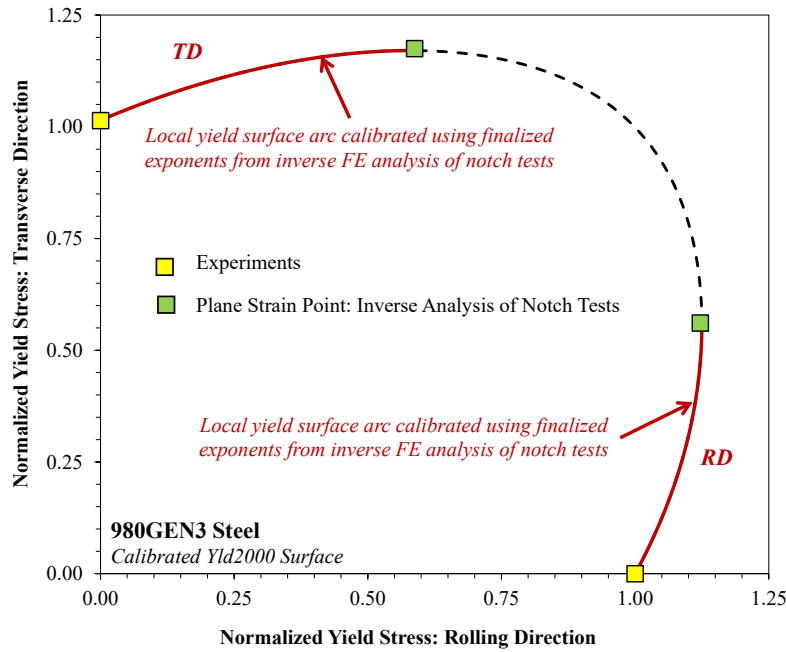


Figure 30 Calibrated Yld2000-2D surface of the 980GEN3 steel showing local arcs that were characterized using the proposed inverse FEA procedure of notch tests.

In the absence of notch tensile and cruciform test data, an estimate of the plane strain yield strength can be obtained using the tensile properties. A Dirac function that correlates the tensile R-value and the major stress under plane strain tension was first used by Abspoel *et al.* (2017). The form of the Dirac function is given by:

$$\frac{\sigma_{\theta}^{PS}}{\sigma_{\theta}^{UT}} = \left[\frac{F_{\min}^{PS}}{1 + f_{dirac}} + \frac{F_{\max}^{PS}}{1 + \frac{1}{f_{dirac}}} \right]; \quad f_{dirac} = \exp \left[a_{dirac} \left(R_{\theta} - F_{trans}^{PS} \right) \right] \quad (\text{Eq. 11a, b})$$

where σ_{θ}^{PS} , σ_{θ}^{UT} , and R_{θ} are the respective PS stress, uniaxial tensile stress, and R-value in the material orientation of interest (θ). The original Dirac fit proposed by Abspoel *et al.* (2017) was found to provide suboptimal predictions of plane strain data obtained from inverse FEA of notch tests, as well as cruciform tests reported in the literature at a stress ratio of 1:2. To address this limitation, the coefficients of the Dirac function were recalibrated for steels and aluminum alloys separately, with the updated values summarized in Table 2. The studies from which the cruciform test data were obtained can be found in Narayanan *et al.* (2022) and Section A.3.2 of Appendix A. A comparison between the proposed correlation and the experimental data is shown in Figure 31. The proposed Dirac correlations were able to capture the normalized plane strain yield stresses with first-order accuracy or better across a range of R-values and thus provide a practical alternative when notch or cruciform testing is not feasible.

Table 2 Dirac function parameter values for estimating plane strain yield strength in steel and aluminum

Dirac function	F_{min}^{PS}	F_{max}^{PS}	a_{dirac}	F_{trans}^{PS}
Steels	0.984	1.274	3.0	0.95
Aluminum	0.999	1.155	7.8	0.63

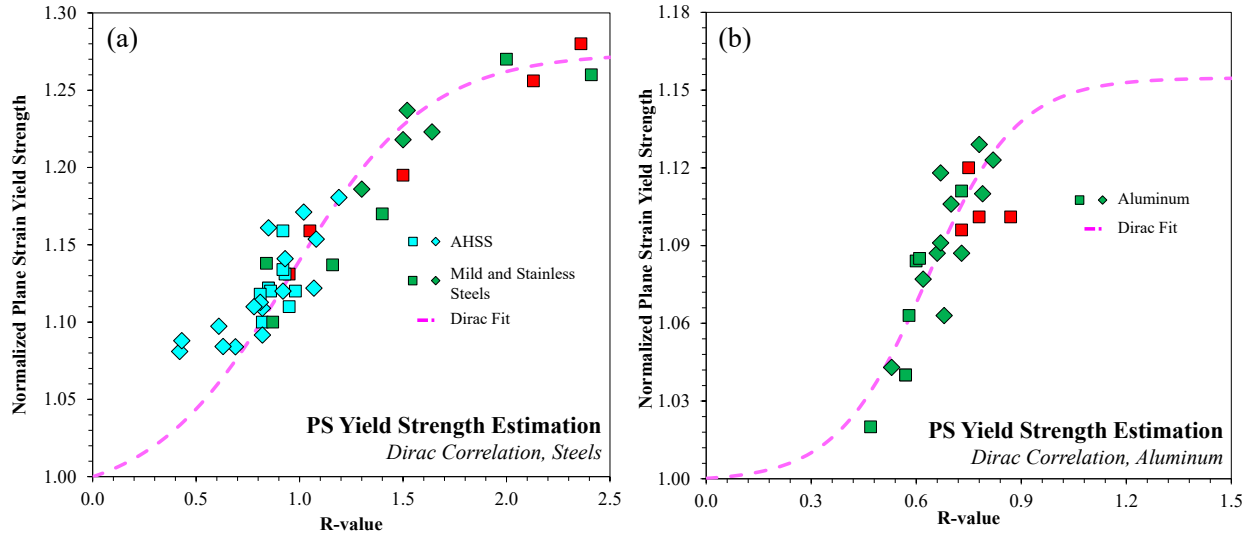


Figure 31 Dirac correlation predictions of plane strain yield stress versus R-value for (a) steels and (b) aluminum alloys. Data obtained using the proposed inverse analysis procedure are denoted by squares, while diamond markers represent the literature-based cruciform test results. Plane strain yield strengths for the materials studied in this work are highlighted as red squares.

4.4. Hardening Response Characterization and Evaluation of Anisotropic Hardening

The area reduction method was applied to miniature tensile tests to obtain the flow stress data and evaluate anisotropic hardening effects. The DIC Z-displacement field in the necked region was used to

estimate the minimum cross-sectional area and consequently, the average true stresses and strains. To account for stress gradients due to necking, the phenomenological correction factors of Choung and Cho (2008) were applied to stress and strain calculations with further details in Abedini *et al.* (2022). A constrained cubic spline that enforces a continuously decreasing hardening rate with plastic strain was fit to the equivalent stress-strain response. The Considère (1885) criterion for uniform elongation was enforced during the spline fit. The data beyond the terminal ARM strain was extrapolated using the terminal n -value to ensure that the flow stress curve and hardening rate remained continuous. The equivalent stress-strain curves for the four materials along the RD can be found in Appendix A (see Section A.3.4).

The calibrated Yld2000 and Yld2004 yield surfaces inherently assume isotropic hardening, meaning the stress ratios across directions and stress states remain constant with increasing plastic work. To assess the validity of this assumption, the equivalent stress vs strain curves along the DD and TD obtained from the ARM were compared with the predicted response from the RD hardening curve, scaled by the tensile stress ratios from the Yld2000 and Yld2004 functions (see Figure 32 to Figure 35). The assumption of isotropic hardening appears to be valid for the AA5182-O and AA7075-T6 alloys but not for DC04 at strain levels beyond the UE. The 980GEN3 steel exhibits mild anisotropic hardening beyond a plastic strain of approximately 0.40.

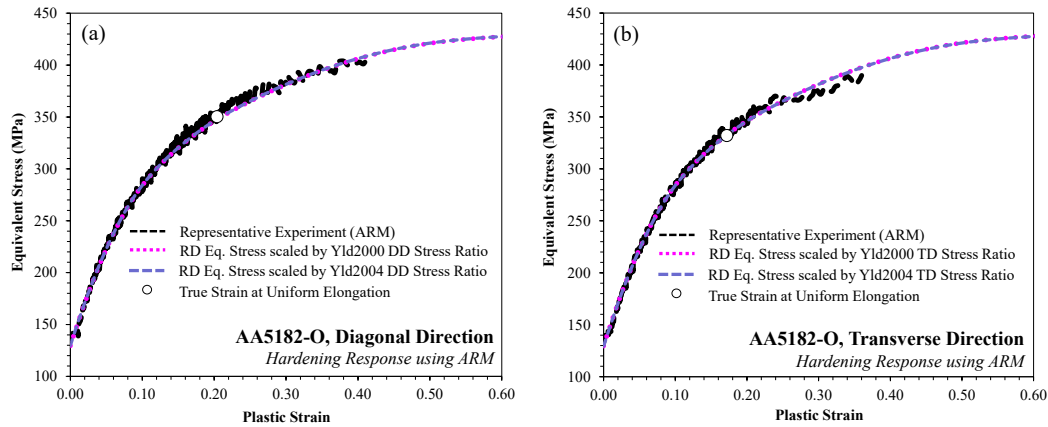


Figure 32 AA5182-O flow stress curves along (a) DD and (b) TD from ARM, superimposed with the predicted hardening responses from Yld2000 and Yld2004 using the RD flow stress scaled by the corresponding predicted tensile stress ratios.

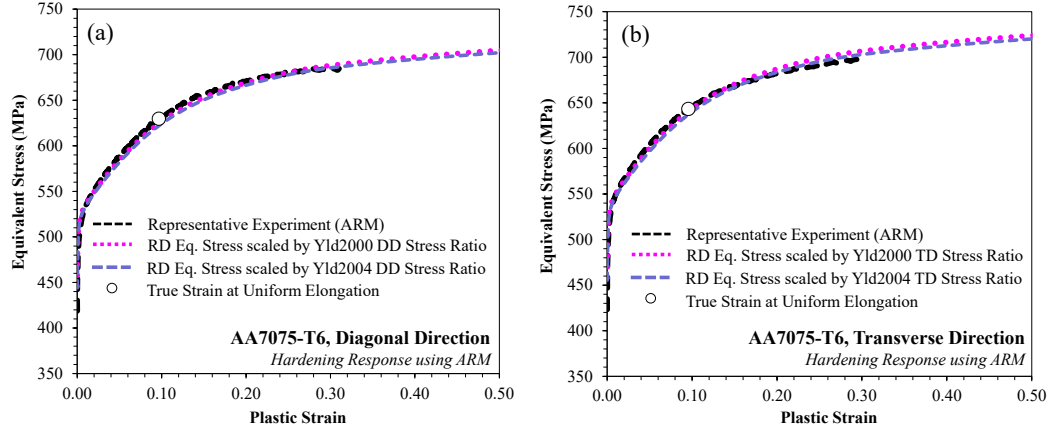


Figure 33 AA7075-T6 flow stress curves along (a) DD and (b) TD from ARM, superimposed with the predicted hardening responses from Yld2000 and Yld2004 using the RD flow stress scaled by the corresponding predicted tensile stress ratios.

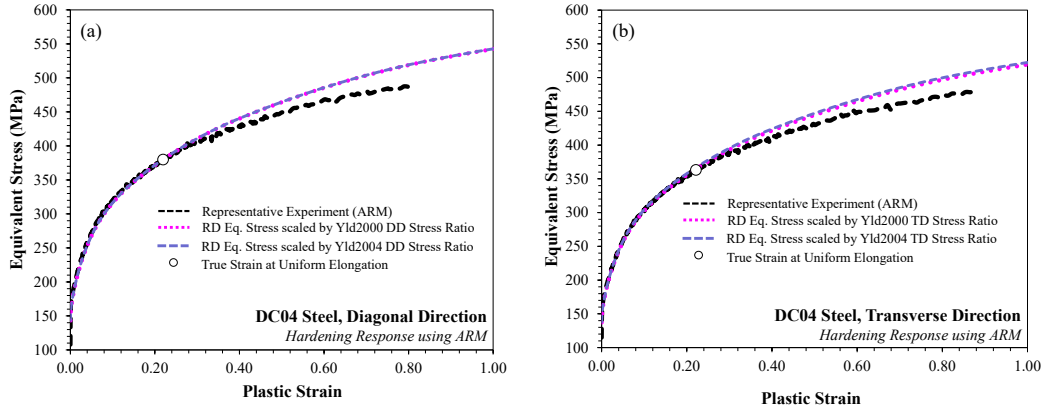


Figure 34 DC04 flow stress curves along (a) DD and (b) TD from ARM, superimposed with the predicted hardening responses from Yld2000 and Yld2004 using the RD flow stress scaled by the corresponding predicted tensile stress ratios.

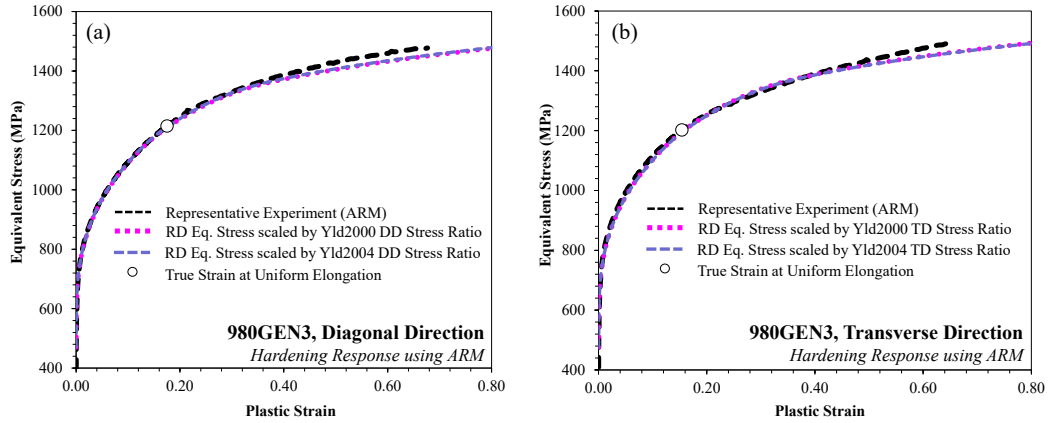


Figure 35 980GEN3 flow stress curves along (a) DD and (b) TD from ARM, superimposed with the predicted hardening responses from Yld2000 and Yld2004 using the RD flow stress scaled by the corresponding predicted tensile stress ratios.

4.5. Evaluation of Calibrated Constitutive Models using Flat Punch Hole Expansion Test

4.5.1. Global Response

To evaluate the accuracy of the calibrated constitutive models, the yield function coefficients and hardening response in the RD were input into FE models of flat punch HX in LS-DYNA software. The FE simulations were performed using both shell and solid elements for evaluation of Yld2000 and Yld2004 functions, respectively. Further details regarding the flat punch HX simulation setup and mesh pattern are discussed in Section A.5 of Appendix A. A comparison of punch force versus the out-of-plane motion (or Z-displacement) of the blank from a representative experiment and simulations is shown in Figure 36 for the DC04 and 980GEN3 steels. The predicted punch forces from the simulations are in good agreement with the experiments up to the peak force instant for the 980GEN3 steel, demonstrating the suitability of the ARM to characterize the material flow stress. Differences can be observed between the simulations and the experiments for the DC04 that are attributed to a combined effect of its plane strain yield strength being underestimated as well as anisotropic hardening. A comparison of the experimental and predicted punch forces for the AA5182-O and AA7075-T6 are discussed in Section A.6.1 of Appendix A.

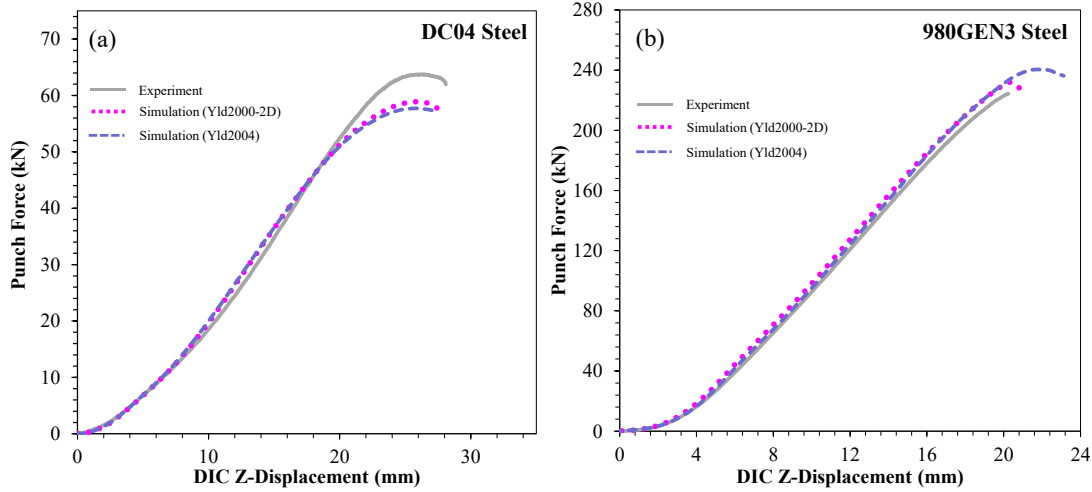


Figure 36 Experimental and simulated punch force vs Z-displacement curves for (a) DC04 and (d) 980GEN3 steels. Only the force response from a representative experiment is shown for convenience.

4.5.2. Predictions of Local Strain Distribution

The radial distribution of elemental strains from simulations was extracted along multiple orientations consistent with the experiments. The variation of principal strains with radial distance from the hole edge along the TD and RD lineslices is shown in Figure 37 and Figure 38, respectively. Due to circumferential nature of the test, the data from a given lineslice corresponds to principal stretching in the perpendicular orientation (*e.g.* TD lineslices correspond to RD principal stretching). Overall, the simulations predict the radial distribution of the local strains, as well as the location of plane strain tension (where $\varepsilon_2 = 0$) accurately. A notable exception is the DC04 steel along the TD lineslices that is attributed to the underprediction of its plane strain yield strength resulting in higher strain magnitudes in the simulations (see Figure 37(c)). Interestingly, an excellent agreement is observed between the DC04 experimental and predicted strain magnitudes along the RD lineslices, which is likely due to strong anisotropic hardening effects counteracting the underprediction of plane strain data. Predictions of radial strain distribution along the DD and localization directions, as well as circumferential strain variation, are discussed in Appendix A (see Section A.6.2).

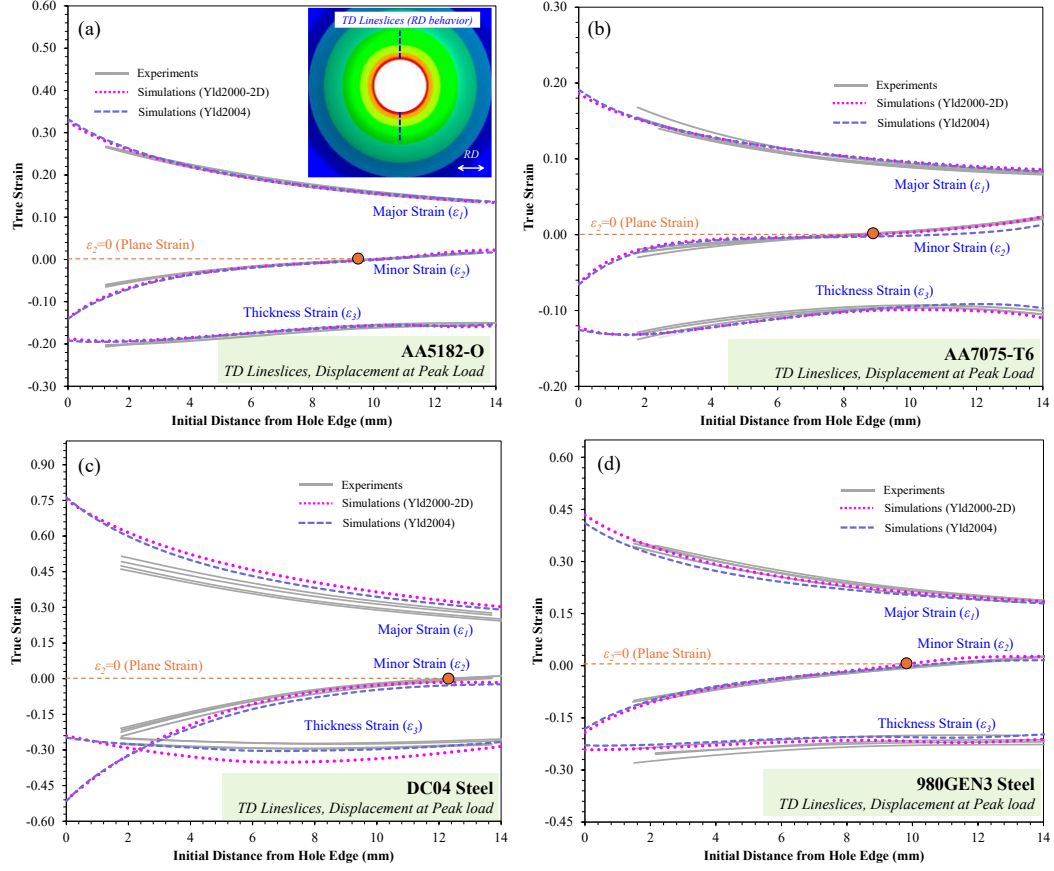


Figure 37 Variation of principal strains away from the hole edge in RD (*i.e.* TD lineslices) for (a) AA5182-O, (b) AA7075-T6, (c) DC04, and (d) 980GEN3 steels at the peak load displacement. The thickness strain was calculated using volume constancy ($\epsilon_3 = -\epsilon_1 - \epsilon_2$).

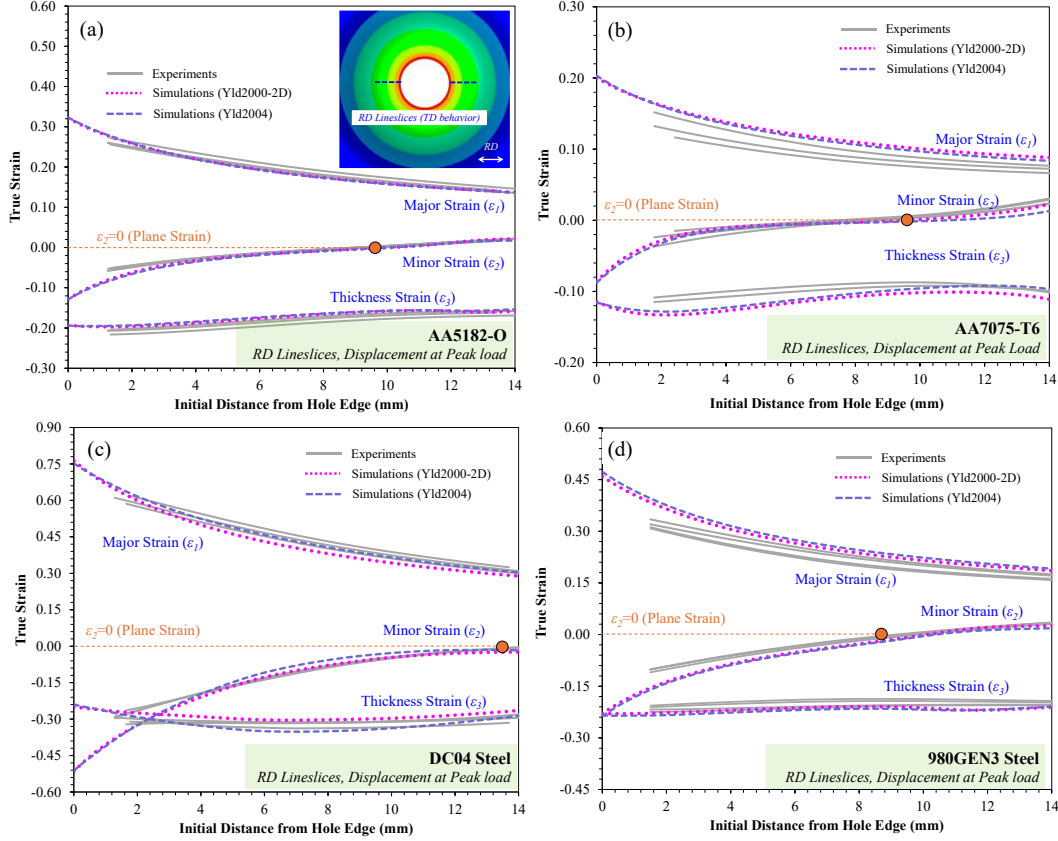


Figure 38 Variation of principal strains away from the hole edge in TD (*i.e.* RD lineslices) for (a) AA5182-O, (b) AA7075-T6, (c) DC04 and (d) 980GEN3 steels at the peak load displacement.

4.5.3. Importance of Plane Strain Constraint in Yield Function Calibration

To demonstrate the importance of including the PSC and the corresponding normalized plane strain major stress during yield function calibration, additional simulations were performed using the HF72 and the plane stress quadratic Hill-48 yield functions for the DC04 mild steel. The Hill-48 surface was calibrated using the tensile R-values along the RD, DD, and TD and is shown in Figure 39(a), alongside the HF72 and Yld2000 loci that enforce the PSC. The plane strain point is predicted to occur close to equibiaxial tension

at a principal stress ratio of $\left(\frac{\sigma_2}{\sigma_1}\right)_{PS} = 0.67$ using the Hill-48 function. Figure 39(b) compares the radial

distribution of principal strains away from the hole edge along the TD lineslices from simulations conducted using the three plane stress yield functions. The simulation conducted using the Hill-48 criterion consistently overestimates the magnitudes of the in-plane principal strains. Moreover, the condition of $\varepsilon_2 = 0$ is never achieved behind the edge due to the location of PST on the yield surface, which broadens the range of allowable stress states on the draw side that is characterized by a negative minor strain. The HF72 yield function correctly identifies the location of $\varepsilon_2 = 0$ since it satisfies the PSC, but the magnitude

of the strain components are not well predicted due to plastic anisotropy being neglected. The Yld2000 model best replicated the experiments due to its ability to describe anisotropy as well as satisfy the PSC. Surprisingly, the Hill-48 model estimated the thickness distributions for DC04 to good accuracy, primarily because the overestimation of in-plane strains negated each other when computing the thickness strain from volume conservation. Overall, enforcement of the PSC significantly improves the predictions of flat punch hole expansion tests for steel and aluminum alloys that can be idealized as being pressure-independent.

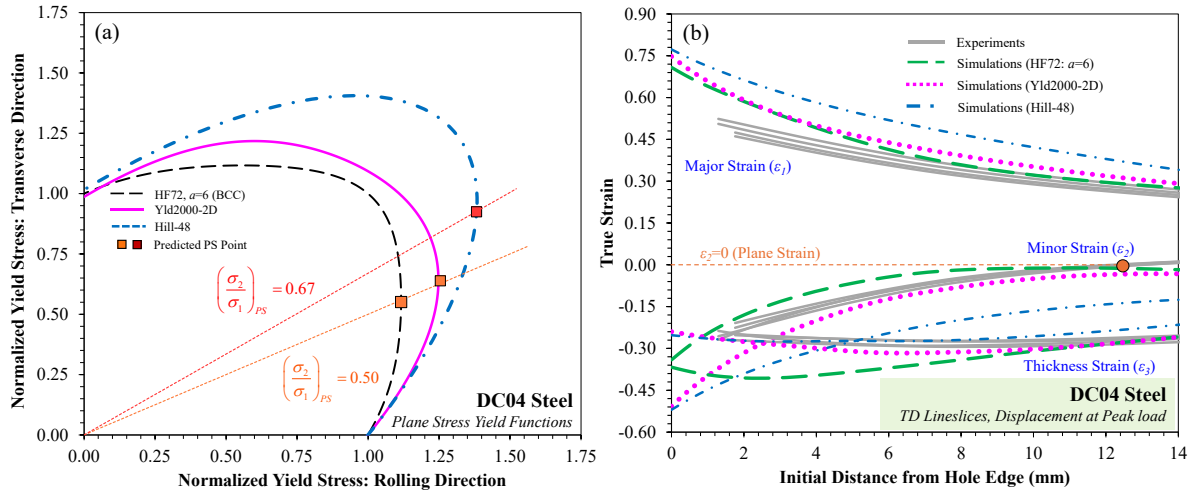


Figure 39 (a) Yield loci of Hill-48, HF72, and calibrated Yld2000 functions in the tensile quadrant for DC04 steel, and the (b) predicted principal strains in the RD obtained from TD lineslices.

5. Experimental Characterization of Uniaxial Fracture Limits

As discussed earlier, the uniaxial fracture strain for ductile materials cannot be directly determined from a tensile test due to the onset of localization, necessitating hybrid experimental–numerical approaches. The hole tension test can also exhibit failure behind the edge at an intermediate stress state closer to plane strain tension for certain materials, making the measured edge strains potentially conservative for uniaxial fracture. This chapter is divided into two sections. Section 5.1 presents the results from Appendix B and examines the mechanics of the ISO 16630 conical hole expansion test for uniaxial fracture characterization of the base metal, following which analytical methods to estimate the fracture strains from the test are proposed. Section 5.2 presents a novel four-point in-plane bend fixture and specimen geometry for uniaxial fracture characterization, with strains compared against the conical hole expansion test. A concise summary of the results from the in-plane bend test is provided here with further details in Appendix C.

5.1. Conical Hole Expansion Test for Uniaxial Fracture Characterization

5.1.1. Materials

To analyze the influence of anisotropy and potential localization in the conical hole expansion, two AHSS grades were considered: a DP1180 steel with mild anisotropy and moderate ductility, and the hot-rolled CP800 of Pathak *et al.* (2016) with strong anisotropy, low hardening rate, and relatively high local ductility. The mechanical properties of the two materials along the RD are tabulated in Table 3, with the data for CP800 also available in Pathak *et al.* (2016). The stress ratios under various stress states and the R-values in multiple directions can be found in Appendix B (see Section B.2.1 and Section B.2.3). The planar anisotropy, denoted by ΔR , is also provided in Table 3 and calculated as:

$$\Delta R = \frac{R_0 - 2R_{45} + R_{90}}{2} \quad (\text{Eq. 12})$$

At the time of this study, the experimental constitutive characterization of the 980GEN3 steel was not complete. Instead, the DP1180 steel was selected due to its comparable planar anisotropy (−0.05 for DP1180 vs 0.00 for 980GEN3). The conical hole expansion performance with a machined edge was also similar between the two materials (hole expansion ratios of 66% vs 68%). However, the edge fracture limits for a sheared hole are expected to diverge between the two materials owing to differences in their microstructures. In DP1180, strain partitioning between the softer ferrite and harder martensite can reduce the edge formability during subsequent stretching (Basu *et al.*, 2025). In contrast, the transformation of retained austenite into martensite during the shearing process is expected to reduce the subsequent edge stretchability of the 980GEN3.

Table 3 A summary of mechanical properties for the studied materials in the RD. The uniform and total elongation were identified based on a DIC extensometer length of 50 mm for DP1180 and 25 mm for CP800

Material	Thickness (mm)	YS (MPa)	UTS (MPa)	UE (%)	TE (%)	Planar Anisotropy, ΔR
DP1180	1.0	841	1193	7.1	12.5	-0.05
CP800	2.9	710	810	7.0	19.6	-0.51

5.1.2. Conical Hole Expansion Experiments

The conical HX experiments followed ISO 16630 guidelines, except the hole was prepared by drilling and reaming to a final diameter of 10 mm to evaluate the base metal. Further details on the test setup and dimensions are provided in Section B.3 of Appendix B. A 135×135 mm² square blank was used with the hole centrally located, and the conical punch had a 60° included angle as specified by ISO 16630 standard. For the DP1180 tests, deformation was recorded using two Gazelle Point Grey® 4.1 MP cameras (model GZL-CL-41C6M-C) with 35 mm Schneider Xenoplan® lenses. DIC was not employed due to significant out-of-plane motion during hole extrusion. The fracture instant was identified as the first recorded image showing a through-thickness crack in accordance with ISO 16630. The identified image was subsequently analyzed using Image-Pro Plus® software for hole geometry measurements. Figure 40(a) shows the image of a DP1180 specimen at through-thickness crack, while Figure 40(b) schematically illustrates the inner and outer hole diameter measurements conducted after the test. The average inner hole diameter can be used to estimate the hole expansion ratio as:

$$HER(\%) = \frac{d_{inner}^{avg} - d_0}{d_0} \times 100 \% \quad (\text{Eq. 13})$$

where d_0 and d_{inner}^{avg} are the initial and average inner hole diameter at fracture, respectively. Note that the initial hole diameter was equal to 10 mm for this study. The outer hole diameter was also measured at multiple orientations including the through-thickness crack orientation (θ). The HER for DP1180 and CP800 were found to be 66% and 190%, respectively, with crack initiation occurring along the RD, indicating that the TD is the limiting direction for failure. The measured values of the hole geometry can be found in Section B.3 of Appendix B.

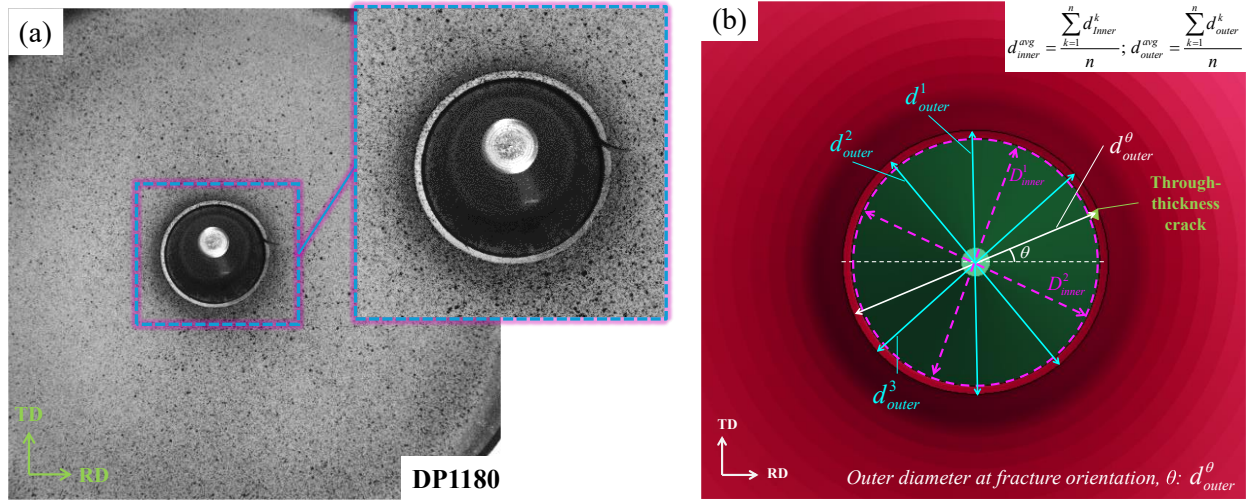


Figure 40 (a) Image of a DP1180 conical HX sample at the instant of through-thickness crack, and (b) schematic illustrating the inner and outer hole diameter measurements taken at fracture instant. The appearance of eccentricity between the punch and blank hole in the experiments is due to inclination of the cameras.

5.1.3. Finite-Element Modelling of Conical HX

FE models of conical HX were developed to evaluate the strain and stress states near the hole and evaluate the analytical techniques used to measure the fracture strains. A full model of the conical HX was created in LS-DYNA with solid elements employed for the blank due to contact of inner hole edge with the punch, inducing a 3-D stress state. Based on a convergence study of the force versus stroke response, a mesh size of approximately 10% of the sheet thickness was selected for the blank with coarser elements used away from the hole edge where crack initiation was not likely. Further details regarding the setup of the FE model, material inputs, and the imposed boundary conditions on the tools and blank are provided in Section B.4.1 of Appendix B. The FE models were developed assuming an ideal edge with no pre-strain to replicate the experiments conducted using a reamed hole, which does not exhibit a shear affected zone unlike the sheared hole. Furthermore, a fracture model was not incorporated into the simulations since the primary aim was to examine the mechanics of conical HX. The FE model accuracy was assessed by comparing the predicted and measured outer hole diameters along various orientations at the fracture HER. The predicted diameters in the RD, DD, and TD showed good agreement with experimental results as seen in Figure 41. Additionally, the equivalent plastic strain distribution along the hole edge revealed the highest strain in the RD element, indicating that it is most susceptible for edge cracking consistent with crack direction in the experiments (see Section B.4.2 of Appendix B). Therefore, it can be concluded that the FE models reliably reproduce the experimental behavior for these materials.

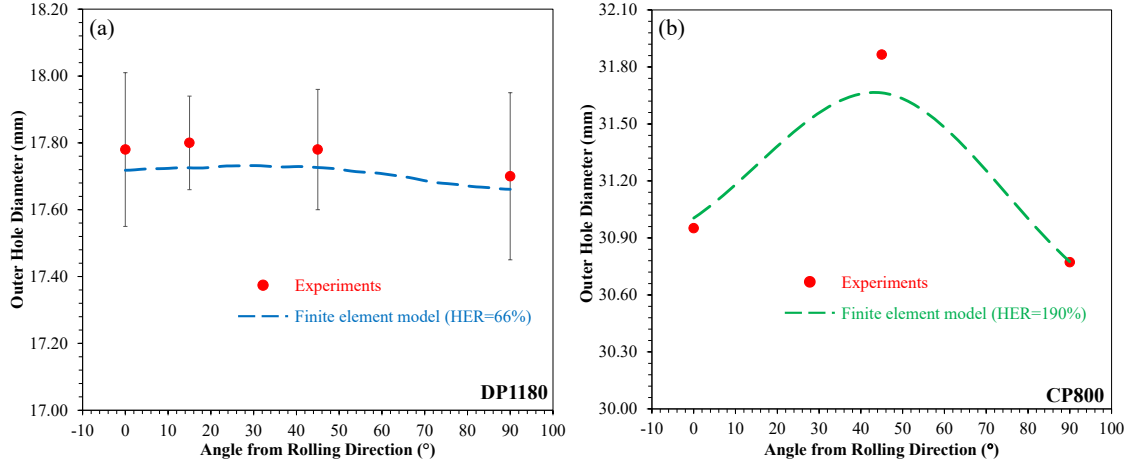


Figure 41 Comparison of the experimental and predicted outer hole diameters at various orientations at the experimental fracture HER for (a) DP1180 and (b) CP800. The experimental data for CP800 was obtained from Pathak *et al.* (2016) for which the spread in the measured diameters were not available.

5.1.4. Mechanics of Conical Hole Expansion Test

5.1.4.1. Through-Thickness Strain Gradients

The equivalent plastic strain at the through-thickness layers located on the hole edge was first analyzed to assess the critical location for fracture initiation. Three primary locations of interest along the thickness were considered as shown in Figure 42(a), with the elements oriented along the RD. Figure 42(b, c) depicts the local plastic strain evolution at these elements as a function of the HER for both materials. The strains accumulated at the inner edge are significantly higher than at the outer edge and mid-plane locations with an absolute difference of approximately 80% at the experimental fracture HER. These results suggest that fracture initiation would occur at the inner hole edge. However, fracture strains of sheet metals exhibit a stress-state dependence. For example, the fracture strain under compression dominated stress state is significantly higher than under uniaxial tension (Sebek *et al.*, 2018). Therefore, the 3-D stress state at these elements must be analyzed to shed light on the expected fracture initiation location.

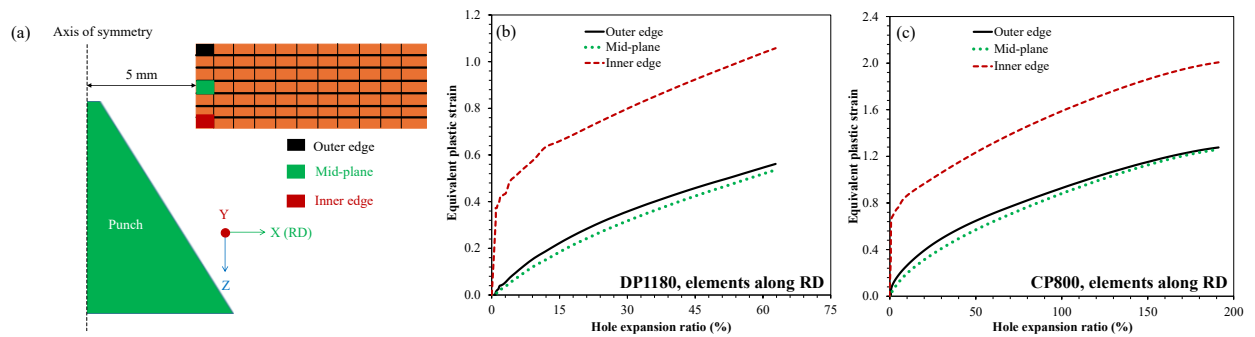


Figure 42 (a) Schematic showing the location of elements at the outer edge, mid-plane, and inner hole edge in the FE model of conical HX, and the local equivalent plastic strain history at these elements for (b) DP1180 and (c) CP800 steels.

5.1.4.2. Stress State at the Hole Edge

A proportional stress state of uniaxial tension is required at the hole edge for the conical HX to be suitable for experimental fracture characterization. To this end, the stress triaxiality and Lode parameter defined previously in Eq. (2) and Eq. (3) were analyzed at the three through-thickness locations. The theoretical stress triaxiality and Lode parameter values for uniaxial tension are $1/3$ and 1 , respectively. A compressive stress state is characterized by negative values of stress triaxiality and Lode parameter (*i.e.* $\eta < 0; L < 0$) whereas a plane strain tensile stress state is characterized by $\eta = \frac{1}{\sqrt{3}}$ and $L = 0$. The evolution of stress triaxiality and Lode parameter at the hole edge until the experimental fracture HER is shown in Figure 43 for the CP800 steel with results for DP1180 provided in Section B.4.4 of Appendix B. The outer edge element is always under uniaxial tension ($\eta = \frac{1}{3}, L = 1$), whereas the inner edge is under compression at the beginning and subsequently shifts towards uniaxial tension after loss of contact with the punch. Additionally, no evidence of localization can be seen at the hole edge (see Figure 43). Similar trends were also noted away from the hole edge (see Section B.4.5 of Appendix B). Thus, a proportional uniaxial loading condition exists at the outer hole edge. Within the phenomenological framework of fracture modeling, it is generally accepted that damage accumulates incrementally throughout deformation under non-proportional loading (Marcadet and Mohr, 2016). Since the fracture strains under compressive stress states are typically large, the initial compressive deformation of the inner edge is expected to result in a low damage accumulation rate and delay fracture initiation relative to the outer edge. As a result, fracture is expected to initiate at the outer edge followed by its propagation through-thickness to the inner edge.

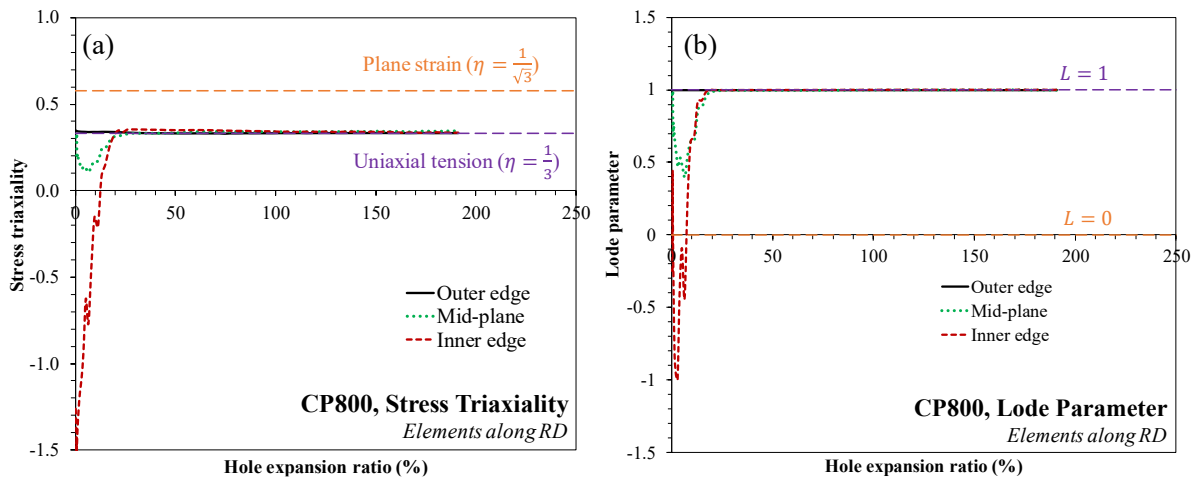


Figure 43 (a) Stress triaxiality and (b) Lode parameter evolution at the various through-thickness layers in conical HX simulation of CP800 steel.

5.1.4.3. Influence of Material Anisotropy on Hole Shape and Thickness Distribution

The high local ductility and strong in-plane anisotropy of CP800 produces a non-circular hole shape with non-uniform thickness distribution as seen in Figure 44(a) that is also predicted by the FE model (see Figure 44(b)). The fractured conical HX sample provides the misleading appearance of necking in the RD and TD due to significant thinning although the stress state indicated no localization (see Figure 43). Furthermore, the edge elements exhibited a near-constant strain rate (see Section B.4.6 in Appendix B), confirming the absence of any acute necking in accordance with the strain rate method of Volk and Hora (2011). Consequently, it is instructive to correlate the non-uniformity in the hole geometry with the plastic strain anisotropy of the material. The inhomogeneity in thickness and outer hole diameter distribution around the circumference from the FE models exhibited a strong correlation with the tensile R-value distribution, as seen in Figure 45. The mildly anisotropic DP1180 exhibited a more uniform hole shape and thickness at the experimental fracture HER and is not discussed here for convenience. A near-uniform hole geometry is also expected for the sheared hole due to edge cracks occurring at significantly lower strains as noted for the CP800 steel by Pathak *et al.* (2016).

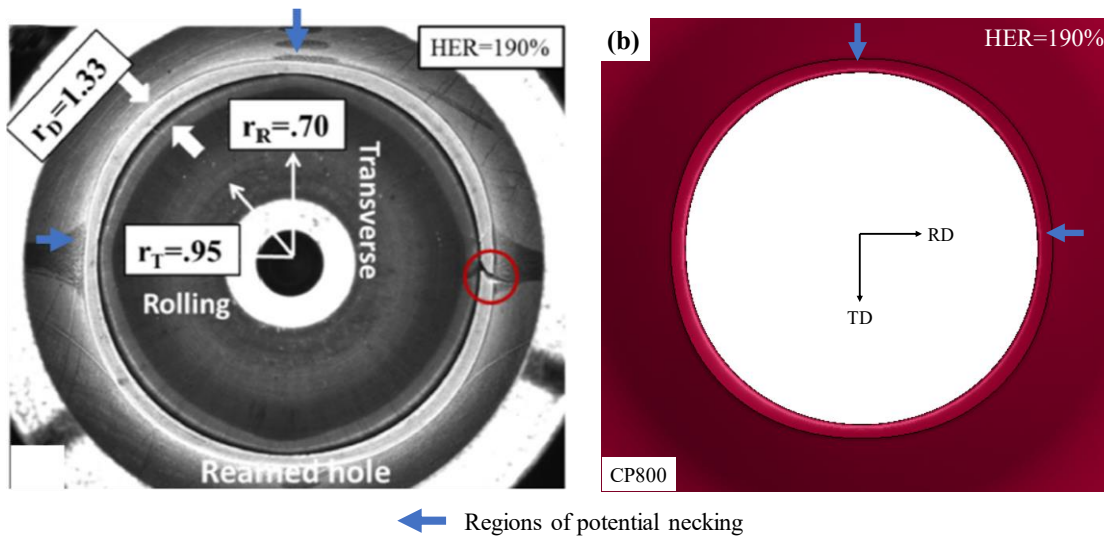


Figure 44 Top view of the deformed CP800 sample at an HER of 190% from (a) experiments (Pathak *et al.*, 2016) and (b) simulations.

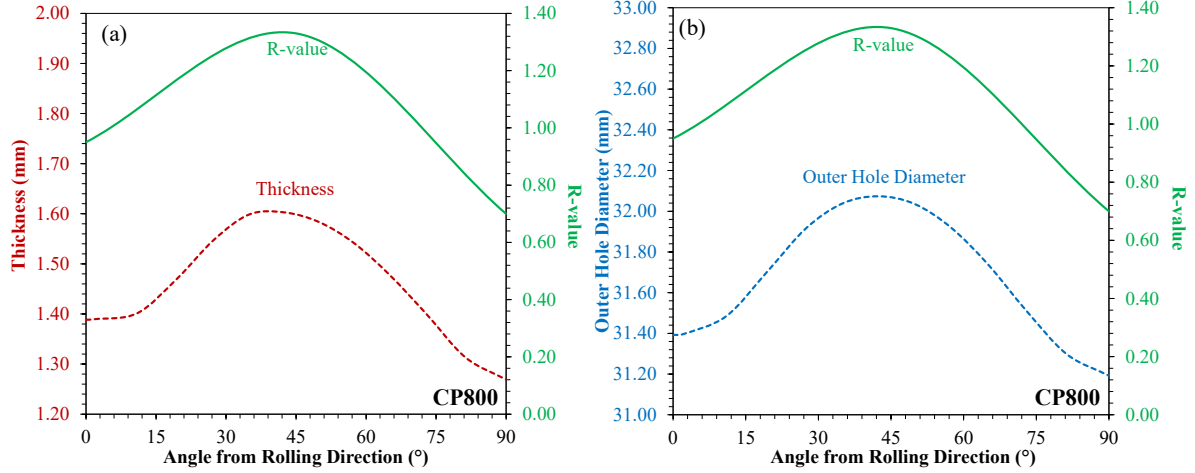


Figure 45 The predicted (a) blank thickness and (b) outer hole diameters around the circumference of the hole edge versus tensile R-value distribution for CP800 at the experimental fracture HER. A given angle from the RD corresponds to the tensile R-value obtained at 90° to that direction.

5.1.5. Methodology to Measure Fracture Strains from Conical HX

The hole expansion ratio can be converted to a true major strain using Eq. (6) of Hance (2017). However, the HER does not reflect the material response at the outer hole edge, which experiences proportional uniaxial tension and governs edge crack initiation. Consequently, elemental strains from FEA are consistently underpredicted by about 8% (see Figure 46). In contrast, the major strain obtained using the macroscopic strain method in Eq. (8) accurately reproduces the average FEA strains for both DP1180 and CP800 steels, as it incorporates the outer hole diameter. However, a notable difference in strain of roughly 4% is observed between local and average strain values for the CP800, indicating non-uniform strain distribution along the circumference. Therefore, a local method is needed to determine the fracture strain based solely on the outer hole geometry at the fracture location.

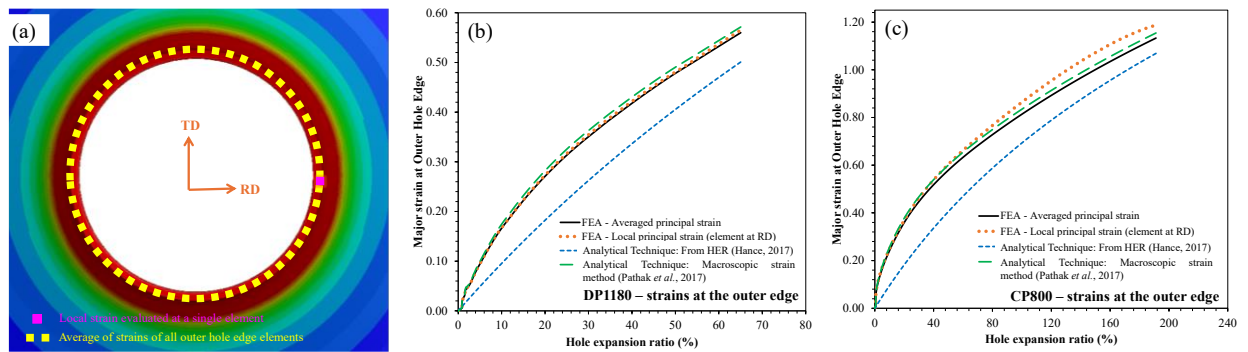


Figure 46 Evolution of average and local principal strains at the outer hole edge overlaid with the predictions of Eq. (6) developed by Hance (2017) and Eq. (8) of Pathak *et al.* (2017) for (a) DP1180 and (b) CP800.

An improved methodology to estimate the local equivalent strain can be proposed for anisotropic materials from the confirmation that the outer edge remains in uniaxial tension. The local in-plane major strain for an element on the outer hole edge at an angle of θ from the RD can be obtained using:

$$\varepsilon_{1,\theta} = \ln \left(\frac{d_{outer}^\theta}{d_0} \right) \quad (\text{Eq. 14})$$

The use of Eq. (14) enables the treatment of non-circular hole shapes as only the local radius or diameter at the location of interest is used. The major strain must be converted into an equivalent plastic strain for use in the calibration of ductile fracture models. The major and equivalent plastic strain are identical for isotropic materials due to similar tensile strengths across all material orientations. For anisotropic materials, the yield function and work conjugate equivalent plastic strains are defined with respect to a reference direction, usually the RD. The principle of work conjugacy must then be used to obtain the anisotropic equivalent plastic strain ($\varepsilon_{eq,\theta}^{WC}$) with respect to the RD:

$$\sigma_{eq} \varepsilon_{eq,\theta}^{WC} = \sigma_{1,\theta+\frac{\pi}{2}} \varepsilon_{1,\theta} + \sigma_{2,\theta+\frac{\pi}{2}} \varepsilon_{2,\theta} + \sigma_{3,\theta+\frac{\pi}{2}} \varepsilon_{3,\theta} \quad (\text{Eq. 15})$$

$$\Rightarrow \varepsilon_{eq,\theta}^{WC} = \left(\frac{\sigma_{1,\theta+\frac{\pi}{2}}}{\sigma_{eq}} \right) \varepsilon_{1,\theta} \quad (\text{Eq. 16})$$

where $\varepsilon_i (i=1,2,3)$ are the principal components of the strain tensor, σ_i are the corresponding principal stresses, and σ_{eq} is the equivalent stress along the RD. The mechanics of circumferential stretching for conical HX alongside the premise of a uniaxial tensile stress state on the outer hole edge (*i.e.* $\sigma_2 = \sigma_3 = 0$) were applied to obtain the work conjugate strain in Eq. (16).

Figure 47 compares the equivalent plastic strain history at the outer hole edge along the RD from the proposed work conjugate method with predictions from the macroscopic strain method in Eq. (7) and the FE model. The strain evolution is monitored with respect to the hole expansion ratio based on the outer hole diameter (OHER). For DP1180, both the macroscopic and proposed work conjugate strain methods accurately predict the RD elemental strains with an absolute difference of only 1% at the fracture OHER. For CP800, the macroscopic strain method agrees well with FEA up to an outer hole expansion ratio of about 50% after which a divergence occurs, with the local equivalent plastic strain underestimated by roughly 11% at an OHER of 215% (see Figure 47(b)). The macroscopic method incorporates the hole thickness in its calculations that homogenizes through-thickness stress gradients and is also sensitive to the

location of thickness measurement near the edge (see Section B.5.2 in Appendix B). In contrast, the work conjugate equivalent plastic strain provides a superior estimate of the local deformation for CP800 with the FE strain underpredicted by only 4% at the instant of experimental fracture. The strain measures developed in Eq. (14) and Eq. (16) can also be applied to characterize the fracture limits for punched holes.

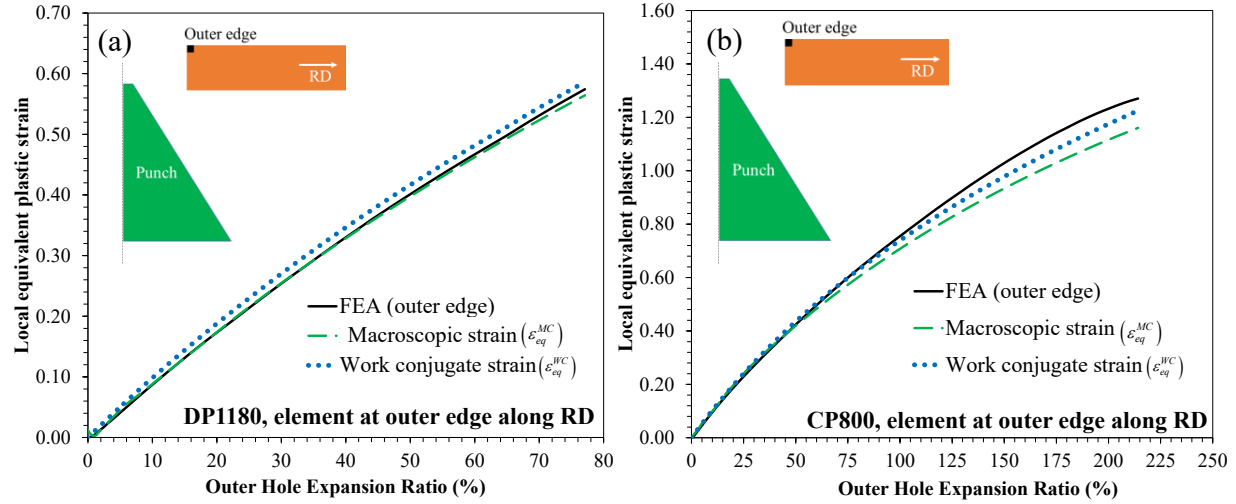


Figure 47 A comparison of local equivalent plastic strain histories from FEA and the analytical methods for (a) DP1180 and (b) CP800 steels.

5.2. In-Plane Bending Test for Uniaxial Fracture Characterization

The machined conical HX is suitable for uniaxial fracture characterization of a large range of sheet materials. However, cracks can consistently initiate in the direction that corresponds to circumferential loading along the limiting orientation (*e.g.*, cracks in the RD for most steels, corresponding to TD principal stretching). As a result, fracture anisotropy cannot be assessed with this test unlike in uniaxial and hole tension tests where specimen machining orientation can be modified accordingly. The importance of the in-plane bending mode for sheared edge fracture was discussed in Chapter 2. The mechanics of the process can be exploited to promote edge cracking in machined edges under proportional loading conditions, making it a promising and unexplored alternative to the conical HX. The ability to control specimen orientation also provides the added advantage of enabling characterization of fracture anisotropy. This section presents the design and application of an in-plane bend test for characterizing the uniaxial fracture limits of sheet metals. The fixture setup of Naseem *et al.* (2020) is used as a baseline with tooling and specimen design modified to promote larger strains, ensure robustness and enhance resistance to buckling.

5.2.1. Materials of Interest

The 980GEN3 steel with 1.4 mm thickness was selected as the primary material of interest with its mechanical properties listed in Table 1. A 1.4 mm thick martensitic MS1500 steel was also considered to

evaluate the bend test behavior for a material with a low hardening rate and uniform elongation. The mechanical properties of the MS1500 steel along the transverse direction from JIS Z2201 tensile geometry are summarized in Table 4. The tensile response of the MS1500 was characterized as part of a study on plane strain V-bending by Cheong *et al.* (2023). The transverse direction was found to be the limiting orientation for fracture initiation and was consequently selected for tensile response characterization.

Table 4 Mechanical properties of the MS1500 along the transverse direction from JIS Z2201 tensile specimen geometry from three test repeats.

Material	Thickness (mm)	YS (MPa)	UTS (MPa)	UE (%)	TE (%)	R-Value
MS1500	1.4	1377 (8)	1582 (4)	3.5 (0.3)	5.9 (0.7)	0.90 (0.02)

5.2.2. In-Plane Bending: Setup and Test Procedure

A schematic of the in-plane bend fixture design and its installation onto a 10 kN Shimadzu® AGS-X universal test frame is illustrated in Figure 48(a). The adapters that connect the fixture to the tensile frame were originally designed for use on a 50 kN Shimadzu® AGS-X tensile machine with the corresponding setup shown in Section C.3.1 of Appendix C. The linear ball bearing carriages slide on guide rails to move freely along the *X*-direction. One of the carriages on the lower mount was constrained in the *X*-direction using a hand brake during the tests, while the other three were free to slide. Clevis mounts with a pin hole diameter of 5.5 mm were selected to provide a double shear loading during the tests with enhanced stability against out-of-plane sample buckling. Preliminary tests with single shear mounting of the pins using spherical ball joint rod ends, similar to the design of Naseem *et al.* (2020) were observed to promote buckling after strain levels of approximately 0.40. Several rounds of design iterations were performed and a new fixture fabricated to address buckling and test repeatability. Sleeve bearings with an internal diameter of 4 mm were press-fit into the clevis joint pinholes to reduce friction with the dowel pins. Face plates were bolted onto the specimen grip section to further improve geometrical stiffness and minimize any out-of-plane motion that may trigger buckling. Details regarding the design of the clevis joints and face plates are provided in Section C.3.1 of Appendix C. Note that the gauge section of the specimen is not influenced by any contact or frictional effects.

The geometry of the in-plane bend specimen is shown in Figure 48(b) with the gauge length oriented along the TD for the two steels. Narayanan and Butcher (2023) performed a parametric study using FE simulations to assess the influence of gauge section dimensions on the edge strains. The gauge length, height, and the transition radius were varied until a tangential true strain of approximately 0.70 was observed at the convex edge that is sufficient to induce cracking across a broad range of AHSS. The optimal dimensions from that study are adopted here, except for a smaller transition radius of 1.0 mm, which was later found to further increase maximum edge strains. A symmetric specimen geometry was also found to improve buckling

stacked sheets to achieve a favorable aspect ratio for compression (Steglich *et al.*, 2014). In contrast, the in-plane bend test enables measurement of plastic strain anisotropy to strain levels well beyond the UE without introducing any contact or frictional effects.

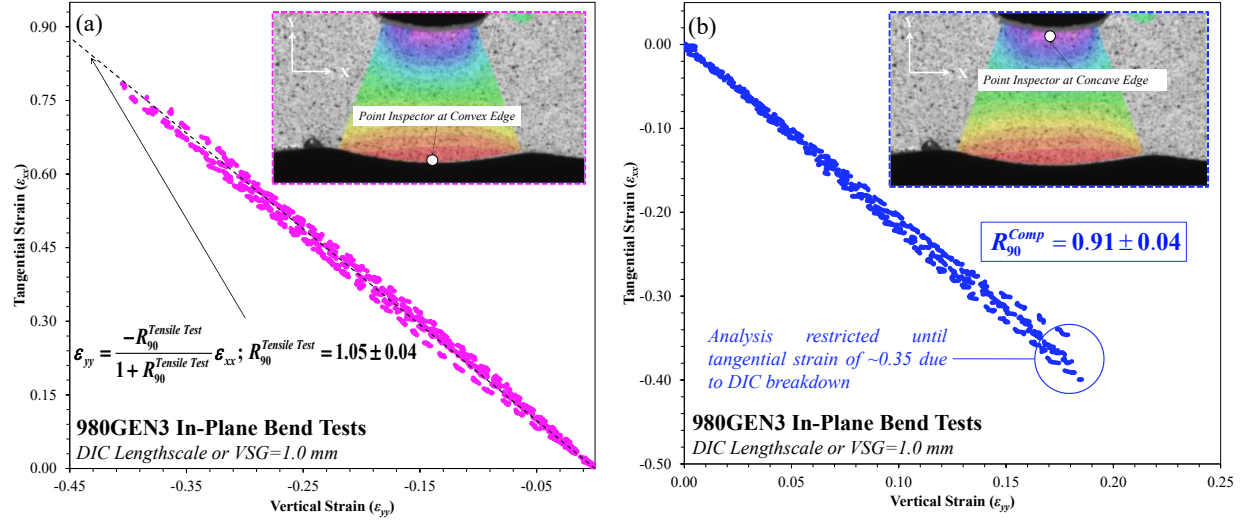


Figure 49 Strain paths at the convex and concave edges of the gauge region during 980GEN3 in-plane bend tests.

5.2.3.2. Detection of Fracture Onset during In-Plane Bending

Cracks initiated at the convex edge during the in-plane bend tests, as seen from the DIC image in Figure 50(a). Various criteria were investigated to identify the instant of crack onset during the tests since the peak force and bending moment thresholds were found to be reliable for select cases. To evaluate the accuracy of different methods for fracture initiation detection, an additional camera was placed underneath the bend test specimen facing the thickness side of the convex edge for selected tests as shown in Figure 50(b, c).

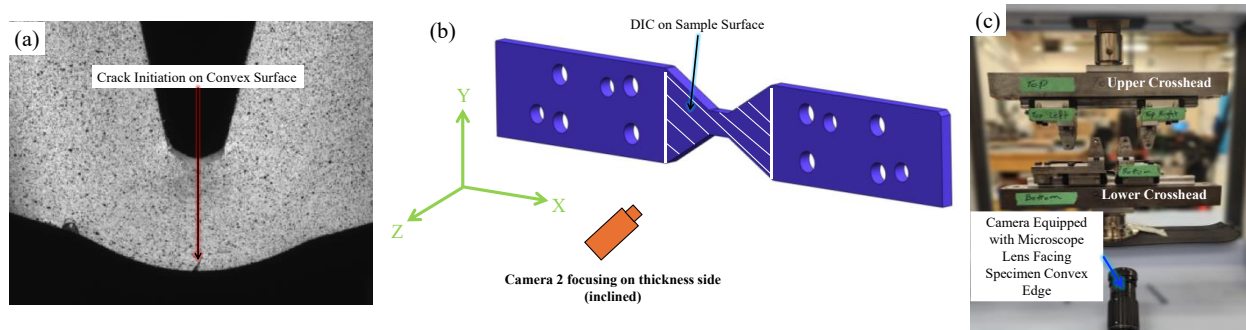


Figure 50 (a) DIC image of 980GEN3 bend test specimen showing crack formation at convex surface, (b) schematic and (c) setup of camera for crack initiation detection.

In addition to the peak force and bending moments, a nominal tensile bending stress metric was used like in the study of Noder *et al.* (2021b) for V-bending. The nominal bending stress at the convex surface can be calculated as:

$$\sigma_b = \frac{M_b y}{I_{zz}} \Rightarrow (\sigma_b)_{convex} = \frac{M_b \left(\frac{h}{2} \right)}{\frac{1}{12} t h^3} = \frac{6M_b}{t h^2} \quad (\text{Eq. 17})$$

where y is the distance of the convex edge from the neutral axis and is assumed to be equal to half of the instantaneous gauge height, h . The variables M_b and t denote the bending moment and instantaneous sample thickness, respectively. For simplicity, the nominal bending stress at the convex edge is calculated under the assumption that the unstretched fibre remains at the mid-plane throughout the test. This assumption generally does not hold during plastic bending as the location of unstretched layer shifts towards the edge in compression. Additionally, the stress computed in Eq. (17) should not be interpreted as representing the true material response at the outer fiber that requires a multi-layer analysis as done by Tan *et al.* (1995) for plane strain bending. The variation in overall gauge height and thickness was measured using a DIC box inspector of 5 x 5 mm² placed in Vic-3D[®] software (see Section C.3.4.1 in Appendix C for details). Fracture was predicted to occur at the instant corresponding to the peak bending stress. The strain rate method (Volk and Hora, 2011) was also considered, wherein a rapid increase in the thickness strain rate is used to detect the fracture onset. For conciseness, the details of the strain rate methodology are not discussed here.

The peak bending moment was found to be suboptimal for fracture detection in the 980GEN3 machined in-plane bend test as shown in Figure 51 (a, b), since no cracks were observed at the convex edge. In contrast, the force, nominal bending stress, and strain rate thresholds predicted similar sample rotation angles corresponding to fracture. The predicted fracture instants also corresponded well with visual observation of through-thickness cracks for the 980GEN3 steel. For the MS1500 bend tests, the force peaked prematurely at a sample rotation of approximately 7° due to low hardening rate of the material and was not found to be suitable for fracture detection. The maximum value of the nominal bending stress occurred at a much higher bend angle of 24°, but no cracks were evident (see Figure 51 (c, d)). The strain rate method was able to detect the instant of crack onset accurately, demonstrating its suitability for fracture detection across a range of material ductilities.

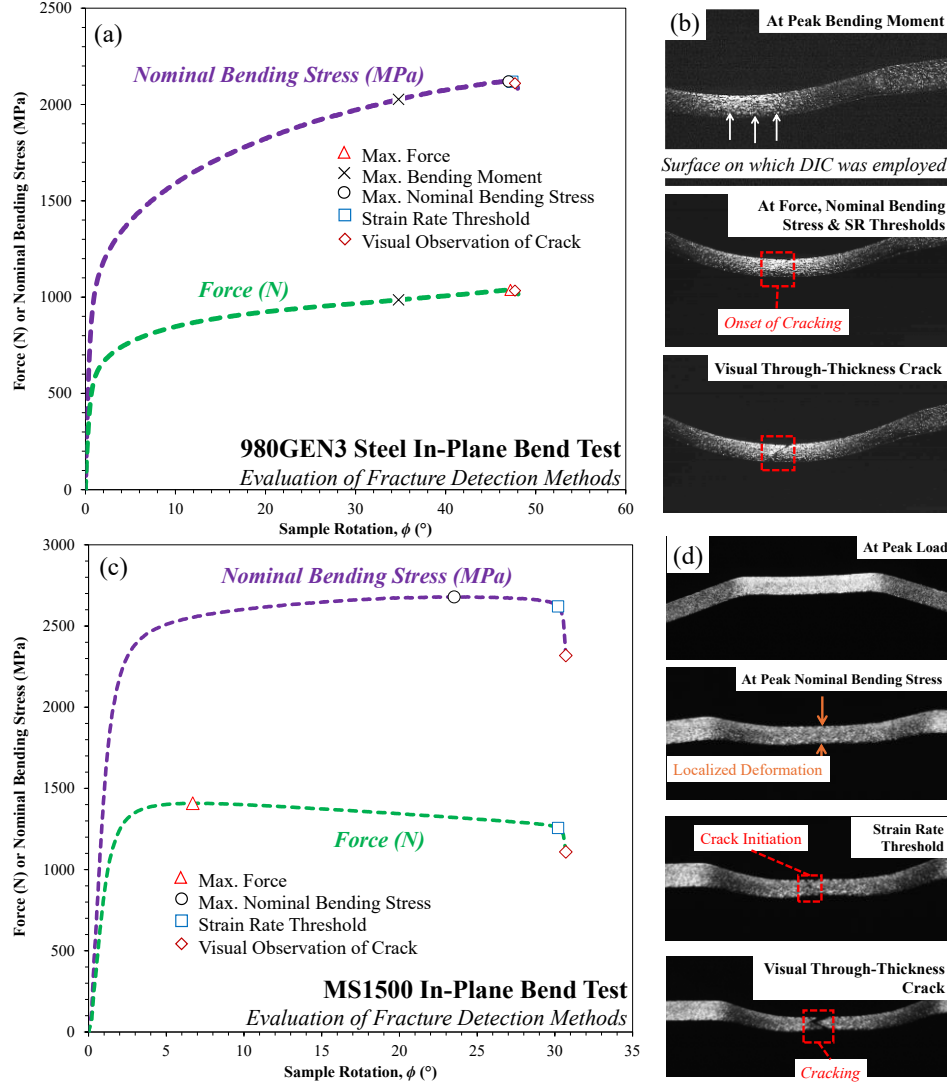


Figure 51 (a, c) Evolution of crosshead force and nominal bending stress for 980GEN3 and MS1500 bend tests with markers indicating the instants corresponding to the thresholds of the various detection methods, and (b, d) views of the convex edge at the various fracture thresholds.

5.2.3.3. Measurement of Fracture Strain and Comparison with Conical HX

The arc length technique was proposed by Narayanan and Butcher (2023) to measure strains near the convex surface in in-plane bend tests without relying on the spatial averaging in DIC strain calculations. The tangential strain from the arc length method can be determined directly from the radius, r_{convex} , and the included angle, θ_{convex} , at the convex surface as:

$$\varepsilon_{\theta}^{convex} = \ln \left(\frac{r_{convex} \theta_{convex}}{L_0} \right) \quad (\text{Eq. 18})$$

where L_0 is the initial arc length chosen to evaluate the bend radius and the included angle computed from three points along the arc on the convex edge using the DIC-measured coordinates and displacements. Figure 52 schematically depicts the measurements conducted in the initial and deformed configurations for application of the arc length method. An initial length of 1.0 mm was found to be appropriate for the arc length based on agreement with the DIC local strain. Additional discussion regarding the sensitivity of the DIC and arc length methods to the virtual strain gauge length and choice of appropriate VSG are discussed in Section C.5.3 of Appendix C.

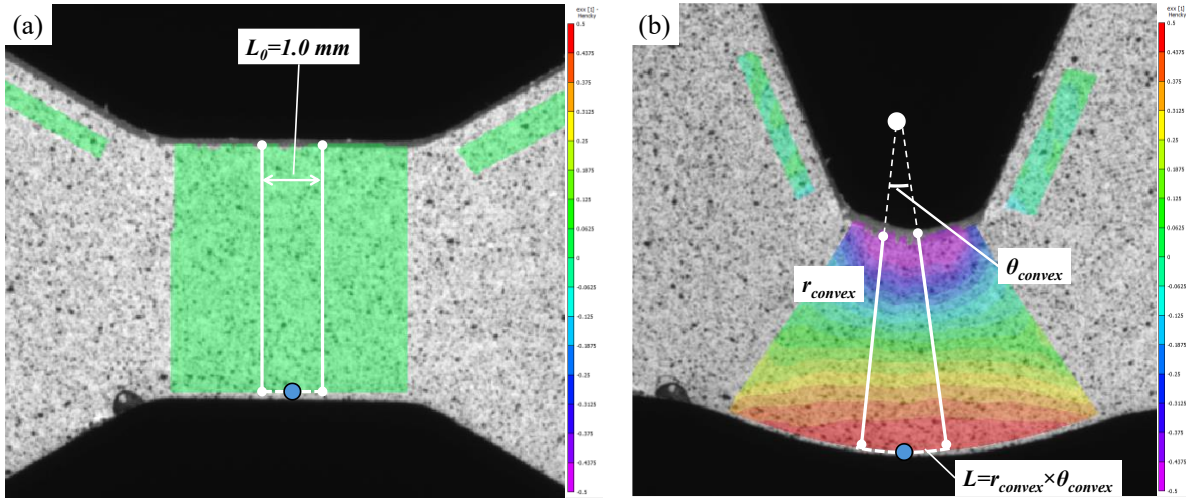


Figure 52 Arc length method for strain measurement in in-plane bend tests. (a) Initial and (b) deformed configurations at the convex edge during a 980GEN3 bend test. The blue circle marker denotes the apex point on the convex edge where fracture initiated.

A comparison of the uniaxial fracture strains for the 980GEN3 steel from machined in-plane bending and conical HX conducted using a 5.0 mm initial hole diameter is shown in Figure 53. A lower hole diameter was found to promote larger fracture strains in a companion study at UWaterloo by Khameneh *et al.* (2024) for a DP1180 steel, due to which the 5.0 mm hole diameter was chosen for conical HX. An excellent agreement in the uniaxial fracture strains can be observed between the conical HX and the in-plane bend tests for the 980GEN3 steel. Due to marginal in-plane anisotropy of the material, a close agreement between the fracture strains obtained using the outer hole expansion ratio and the local diameter methods discussed in Section 5.1.5 was also seen. The conventional HER underestimated the machined edge stretchability of the 980GEN3 steel consistent with the FE model results presented in Section 5.1.5. An absolute difference in true strain of approximately 17% was observed relative to the outer hole diameter-based techniques. Khalilabad *et al.* (2023) reported a similar trend for DP800 steel, where the conical HX fracture strain measured from the inner diameter was 25% lower than that from in-plane bending. Overall, the uniaxial

fracture limits for the 980GEN3 steel appear to be relatively insensitive to the loading condition for a machined edge.

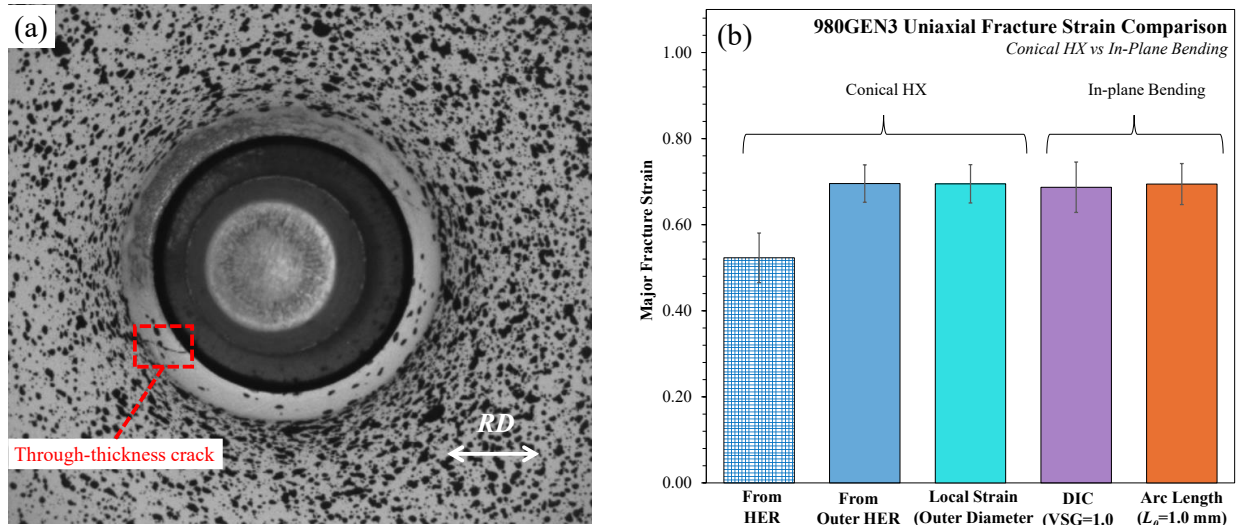


Figure 53 (a) Image of a 980GEN3 machined conical HX sample at through-thickness cracking, and (b) comparison of uniaxial fracture strain from in-plane bending and conical HX. Error bars denote the standard deviation.

5.2.4. Geometric Limitations of the In-Plane Bend Test

The sensitivity of the in-plane bend test geometry to buckling must be evaluated, as the test does not employ any anti-buckling devices. To this end, an experimental study was conducted on a range of steels with nominal UTS values from 270 MPa to 1500 MPa. For each nominal tensile strength, multiple sheet thicknesses between 1.0 mm and 2.0 mm were tested where available. Note that the sheets of different thicknesses correspond to materials from different suppliers. The bend specimen geometry was kept fixed (see Figure 48(b)) during the buckling study. The list of materials and thicknesses are provided in Section C.5.6 of Appendix C. The initial gauge height-to-sheet thickness ratio was found to be the governing factor for buckling initiation, with the material strength playing a secondary role, as seen in Figure 54. An aspect ratio of 4.0 or lower was found to inhibit buckling during the in-plane bend tests. In addition, the in-plane bend tests were not able to induce cracking in some high ductility materials such as mild steels and AHSS with tensile strengths below 780 MPa (see Section C.5.6 in Appendix C). Larger deformation levels can be achieved by reducing the gauge length or increasing the gauge height, provided that the recommended aspect ratio of 4.0 is maintained. Alternatively, pre-straining the samples in uniaxial tension prior to bending may also be a viable strategy.

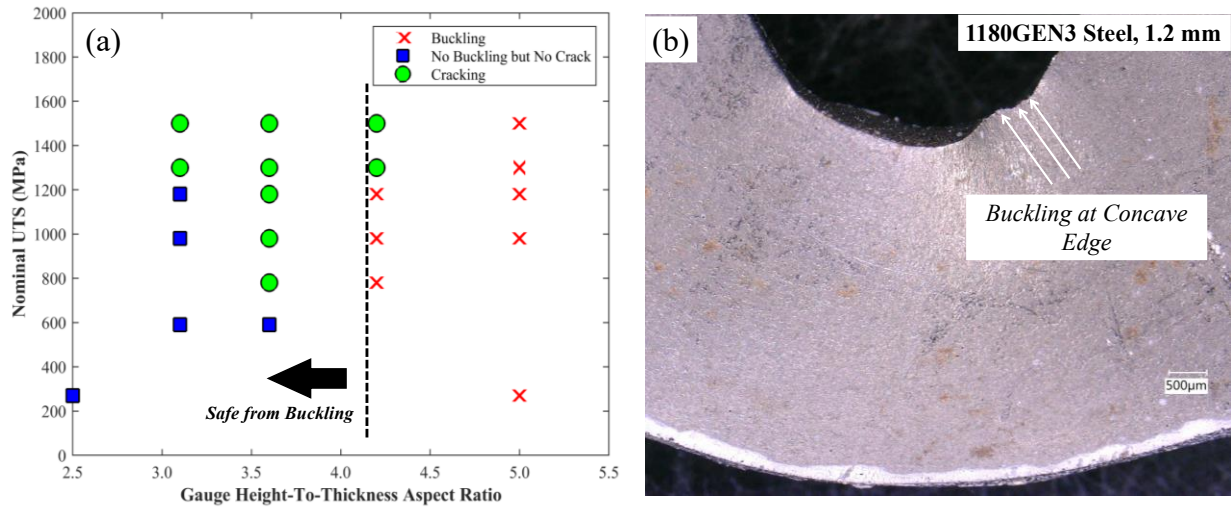


Figure 54 (a) Design chart summarizing the buckling study results from in-plane bend tests conducted on various steels, and (b) front view of a buckled 1180GEN3 bend test specimen having a thickness of 1.2 mm

6. Boundary Conditions and Lengthscale Effects on Experimental and Numerical Characterization of Sheared Edge Fracture Limits

This chapter focuses on analyzing the influence of the applied deformation mode on determination of sheared edge fracture strains in AHSS, both in the experiments and during FE modelling. Emphasis is placed upon the appropriate measurement of the fracture strains using DIC, the influence of the DIC virtual strain gauge length, and its transferability to FE lengthscales using shell elements. The 980GEN3 steel was considered as the reference material for analysis and to propose transferable guidelines for edge crack prediction and design in AHSS. The sheared edge stretching tests were performed on samples punched with 12% clearance with the five test techniques schematically illustrated in Figure 25. The first section discusses the experimental findings, highlighting the effects of boundary conditions and DIC lengthscale on the measured edge fracture limits. Global edge stretch metrics are proposed to guide the appropriate selection of VSG for each coupon test to ensure direct transferability of measured strains to both shop-floor fracture assessments and FE modelling. Subsequently, FE models of the edge fracture coupon tests are developed using shell elements incorporating the calibrated Yld2000-2D function and hardening response from Task 1 as the inputs. The influence of SAZ was not considered within the FE models to reflect simulations of large-scale stampings that adopt coarse shell elements, making its integration into edge elements challenging. The simulation results are used to assess how the deformation mode and mesh resolution can affect the elemental strain distributions at the experimental fracture instant. These findings provide insight into the numerical manifestation of boundary condition effects and inform best practices for modeling edge cracking in large-scale forming simulations. Specific results are discussed here with further details provided in Appendix D.

6.1. Experimental Procedure

Figure 55(a) shows a four-post hole punching tool mounted on a 120-ton hydraulic Macrodyne press. The cutting punch and die buttons were secured in their retainers using a ball-lock mechanism. Circular and square holes of 5.0 mm were punched into 203.2×203.2 mm² square blanks of 980GEN3 steel using a 12% cutting clearance. Monarch Klir-Cut 2 cutting oil was applied to the blank surface as a lubricant during punching. The blanks with circular cutouts were used directly for the flat and conical HX experiments with the tooling setups described in Section D.3.2.1 of Appendix D. For the hole tension specimens, the outline corresponding to the JIS Z2201 tensile geometry (see Figure 55(c)) was extracted by CNC machining while retaining the central sheared hole. For the EFTT, a sub-sized ASTM E8 specimen with a 6.0 mm gauge width was prepared (see Figure 55(d)), whereas the geometry in Figure 48(b) was used for the in-plane bend test with the sheared edge placed along the convex side. In both cases, one side of the punched square

hole aligned with the TD was kept intact, while the remaining specimen outlines were extracted by CNC machining.

A DIC virtual strain gauge length of 1.0 mm was selected for strain measurement from all tests except conical hole expansion for which the outer hole diameter-based technique proposed in Chapter 5 was used. A similar analytical approach was also applied to the sheared-edge flat punch hole expansion tests, with the exception that the inner and outer hole diameters were identical. Baseline tests with machined edges were also conducted for all five test techniques to assess their suitability for base metal fracture characterization. The edge fracture and hole tension tests represent cases of in-plane stretching of the edge with and without strain gradients, respectively, with the caveat that edge cracks occur before the UE strain in the EFTT. The hole expansion tests impose axisymmetric stretching of the hole, while the in-plane bend test subjects the edge to in-plane uniaxial bending. Additional details on test speeds, DIC process parameters, and the extensometer lengths used to measure engineering strain are provided in Section D.3 of Appendix D.

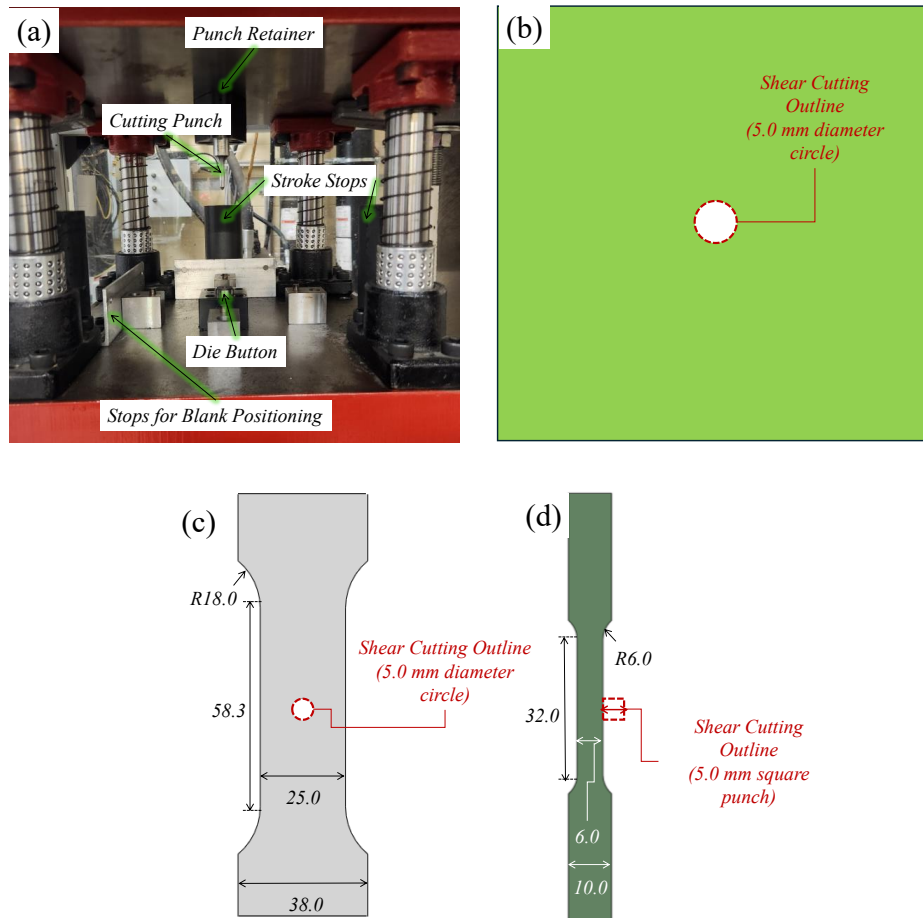


Figure 55 (a) Tool for hole punching, and geometries of (a) flat and conical HX, (c) hole tension, and (b) edge fracture tensile tests.

6.2. Results of Edge Fracture Characterization Experiments

6.2.1. Analysis of Failure Location and Local Fracture Strains

Figure 56 (a, b) shows the locations of through-thickness cracks during machined and sheared conical hole expansion tests conducted for the 980GEN3 steel. In the machined conical HX tests, through-thickness cracks consistently formed near the RD, while the orientations were randomized in the sheared specimens due to pre-strains introduced during punching. Several microcracks were also observed around the hole circumference in addition to the primary through-thickness crack in the punched conical HX tests. Regardless of hole preparation method, cracks initiated at the hole edge in conical HX due to mechanics of the test. Since the strain state gradient in flat punch HX can promote localization behind the edge (see Figure 26(b)), the radial distribution of thinning strains along the localization direction was analyzed. Localization occurred behind the hole edge at a distance of approximately 1.2 mm for the machined flat punch HX, while failure was driven by the SAZ damage, as evident from the maximum thinning strain occurring near the edge in Figure 56(d).

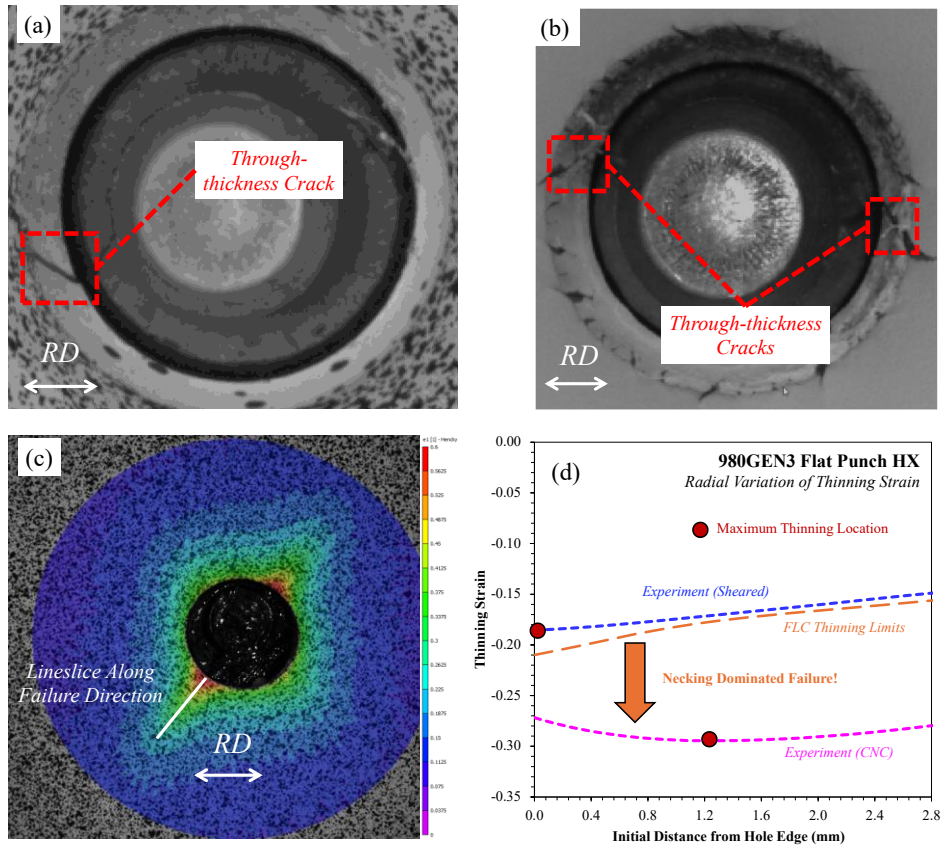


Figure 56 An image from a representative conical HX conducted on (a) machined and (b) sheared edge, (c) DIC strain contour at the image before fracture for a 980GEN3 flat punch HX repeat, and (d) location of maximum thinning during 980GEN3 flat punch HX.

Figure 57(a, b) shows the DIC strain contours at failure for representative HTT repeats on specimens with machined and sheared edges, respectively. Identifying the location of fracture initiation in HTT can be challenging, so the thinning distribution was examined under a Keyence® VHX-5000 microscope after through-thickness cracking. The maximum thinning for the machined hole occurred at approximately 0.3 mm away from the edge, as seen in Figure 57(a). In contrast, fracture initiated at the sheared edge for the punched HTT tests with the highest thinning at the edge as seen in Figure 57(b). The EFTT results also showed similar trends to the hole tension test with necking observed in the machined EFTT (see Figure 57(c)), resembling the response in a standard uniaxial tensile test. To assess whether the diffuse necking limit was reached during the sheared EFTT, its global extension was measured using a virtual extensometer of 25.0 mm initial length as per the ASTM E8 standard. The true extensometer strain from the sheared EFTT was found to be 0.15 ± 0.02 , which is marginally lower than the true UE strain of 0.17 for the 980GEN3 steel, confirming the absence of necking in the sheared EFTT. For the in-plane bend tests, cracks consistently initiated at the convex edge regardless of the edge condition (see Section D.3.2.3 of Appendix D for failure images). The local edge strain paths for HTT, EFTT, and in-plane bending are provided in Section D.5.3 and Section D.5.4 of Appendix D. For the machined edge condition, only the in-plane bending and conical HX are able to promote uniaxial fracture under proportional loading for the 980GEN3 steel. The other considered test methods are prone to localization or failure behind the hole edge, necessitating inverse analysis approaches.

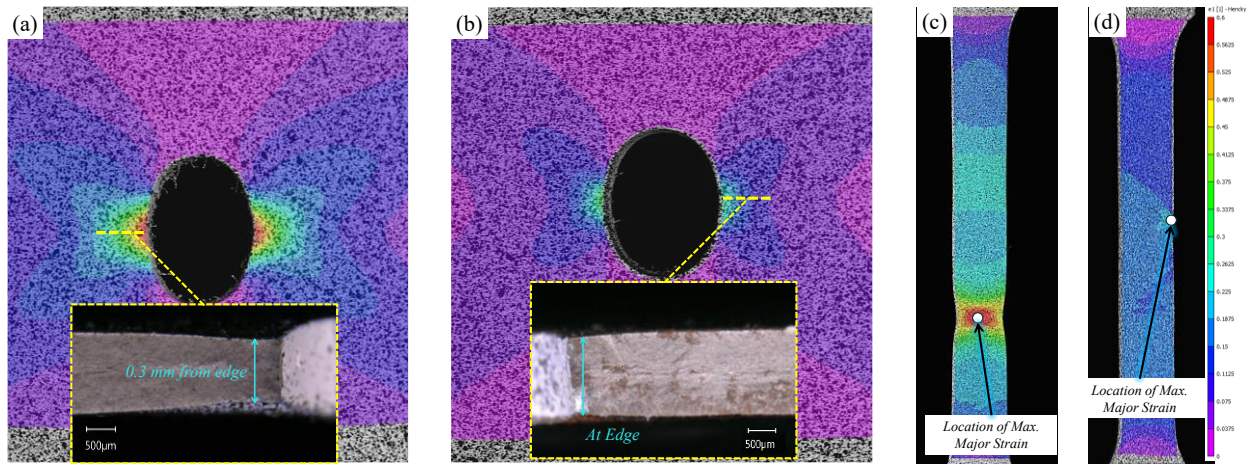


Figure 57 DIC strain contours at failure for 980GEN3 HTT conducted with (a) machined and (b) sheared edges, and EFTT with (c) machined and (d) sheared edges.

6.2.2. Comparison of Sheared Edge Fracture Limits: Boundary Condition Effect

The major fracture strains obtained from the various testing methods at a DIC VSG of 1.0 mm are summarized in Figure 58(a). Convergence was obtained between the in-plane sheared edge tests with local fracture strains of approximately 0.30 at 12% clearance. The exception was the axisymmetric out-of-plane

conical HX that reached a fracture strain of 0.45. This difference is attributed to conical HX failure being defined at the instant of through-thickness crack as per ISO16630 standard. In contrast, failure during the in-plane tests corresponded to onset of edge cracking rather than a through-thickness crack. Failure preferentially initiated from the burr side by virtue of the sheared edge damage resulting in conservative fracture predictions. To this end, the thinning strain from the hole tension and in-plane bend specimens were measured post through-thickness (TT) cracking and converted into a major strain using the tensile R -value as done by Khameneh *et al.* (2024):

$$\varepsilon_3^{TT \text{ Crack}} = \ln\left(\frac{t_f}{t_0}\right); \quad \varepsilon_1^{TT \text{ Crack}} = -(1 + R_{TD}) \times \varepsilon_3^{TT \text{ Crack}} \quad (\text{Eq. 19})$$

where t_0 and t_f are the initial and final thickness measured at through-thickness crack instant, respectively.

The sheared IPB and HTT strains obtained at through-thickness crack instant are in close agreement with the conical HX as seen in Figure 58(b). Overall, the 980GEN3 steel exhibits marginal sensitivity to boundary conditions when punched at 12% clearance. Note that the predicted fracture instant in the machined IPB closely corresponded to through-thickness cracking resulting in a close agreement in fracture strains with the conical HX.

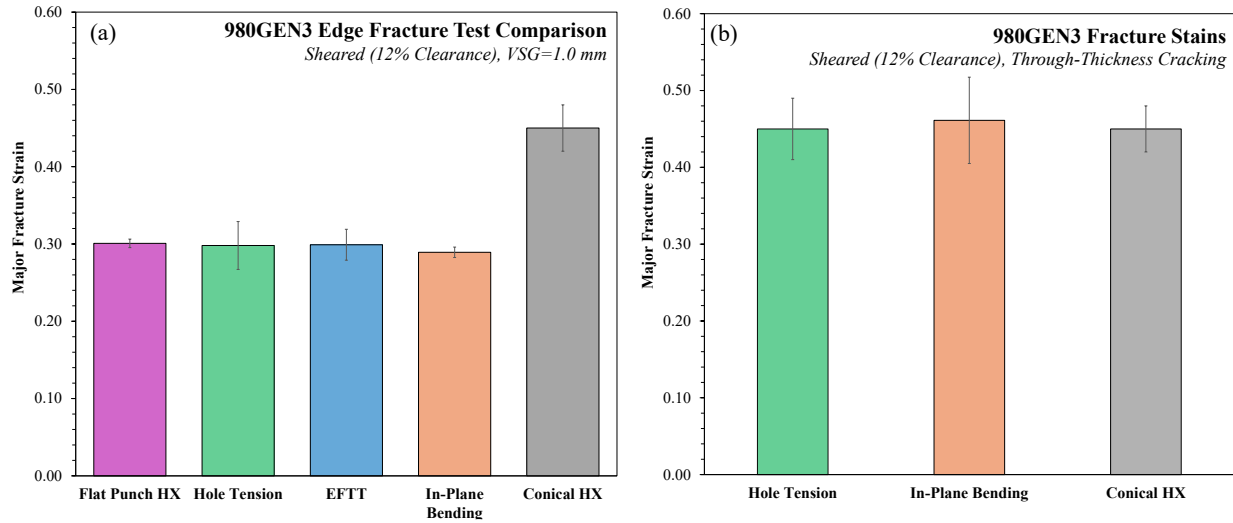


Figure 58 (a) Comparison of 980GEN3 edge fracture strains from the various test methods for 12% clearance, and (b) fracture strains at instant of through-thickness cracking from IPB, HTT, and conical HX.

6.2.3. Influence of DIC Lengthscale on Edge Fracture Limit Comparison

It is instructive to investigate the sensitivity of the DIC lengthscale for the in-plane fracture tests given the apparent convergence using $VSG = 1.0$ mm. Two representative repeats from each in-plane test were analyzed using multiple VSGs ranging from 0.6 mm to 4.0 mm. The corresponding DIC step and filter sizes

are summarized in Appendix D. Representative in-plane bending and HTT repeats conducted with a machined edge were also analyzed for the same range of VSGs to evaluate how the lengthscale effects manifest relative to the sheared edge. Figure 59 presents the effect of DIC lengthscale on the major fracture strains for the two edge conditions. The machined EFTT and flat punch HX were excluded since localization developed more than 1 mm away from the edge, making the measured strains not representative of edge fracture. The hole tension test was retained despite localization since fracture occurred close to the edge at a distance of 0.3 mm. The conical HX test did not exhibit lengthscale dependence since the fracture strains were obtained directly from geometry.

A convergence in the uniaxial fracture limits can be observed between the machined in-plane bending, hole tension, and conical hole expansion tests at a VSG of 1.0 mm for the 980GEN3 steel as seen in Figure 59(a). The machined HTT showed a strong sensitivity to the experimental lengthscale with the DIC hole edge strain dropping from 0.80 to 0.30 with an increase in VSG from 0.6 mm to 4.0 mm, leading to an apparent agreement with the other tests at a VSG of 1.0 mm. The deformation in HTT is constrained to a highly localized region near the hole as seen in Figure 59(a), which is attributed to the small initial hole radius of 2.5 mm. A larger hole radius could potentially promote a broader region of straining and reduce the sensitivity of strains in HTT to the VSG. Based on the convergence of results, a VSG of 1.0 mm appears to be an appropriate experimental lengthscale for the 980GEN3 steel. Furthermore, the experimental scatter in major strain measurements decreased systematically with increasing VSG, which is attributed to higher spatial averaging at larger lengthscales.

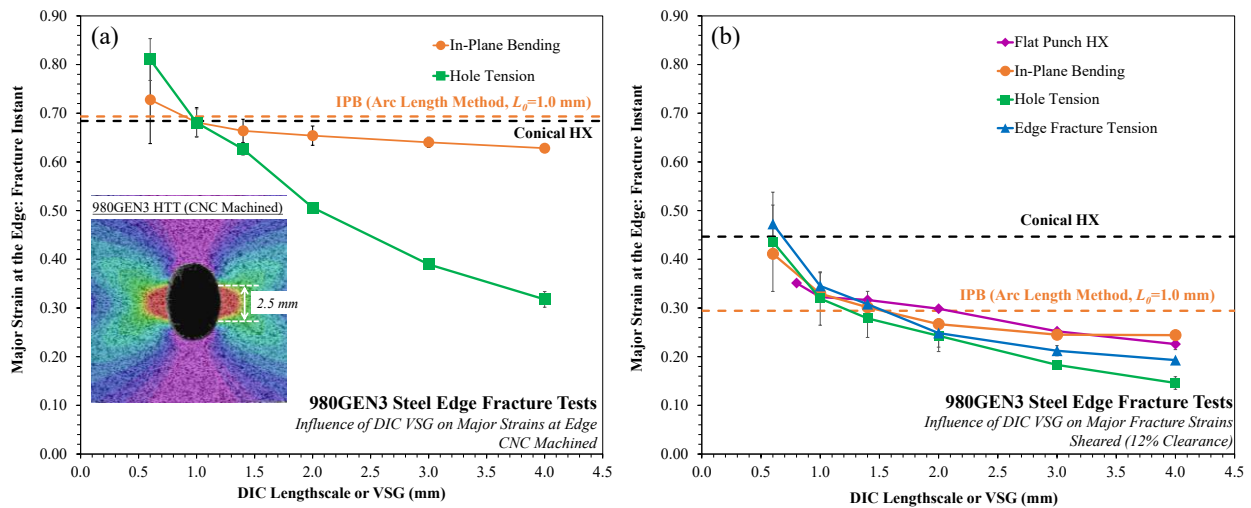


Figure 59 Effect of virtual strain gauge length on the local DIC major edge strains from the various edge characterization tests under (a) machined and (b) sheared conditions.

A larger region of VSGs promote convergence in local fracture strains for the sheared edge condition as seen in Figure 59(b). The best agreement was obtained for VSGs ranging from 0.6 to 1.4 mm. At larger

lengthscales, the sheared HTT fracture strains diverge from the other in-plane tests due to its strong VSG sensitivity. The deformation in flat punch HX and IPB is primarily governed by geometric changes resulting in local DIC strains exhibiting a low sensitivity to the VSG. An apparent sensitivity to the applied boundary condition can emerge because of the choice of VSG. Virtual strain gauge lengths greater than 2.0 mm can show a divergence in the fracture strains for the considered 980GEN3 with potential for divergence at VSGs smaller than 0.6 mm as well.

6.2.4. Global Edge Stretch and Thinning Limits under Various Deformation Modes

Edge thinning is a critical parameter during part design and thus it is worthwhile investigating its sensitivity to the VSG and applied loading condition. DIC is seldom amenable to practical stamping operations to measure deformation since the parts are enclosed within the toolset. As a result, micrometers are often used to measure the fracture limits near the edge split location, resulting in an averaged or global measurement. Therefore, it is useful to quantify the so-called global edge stretchability from the considered coupon tests that emulate the various loading scenarios encountered in formed parts and can be directly applied for assessment of edge cracks.

The global major strain in the flat punch and conical HX can be directly measured using the initial and respective average final hole diameters at fracture (note that the outer hole diameter is used for conical HX). For in-plane bending, the sheared edge transforms from a straight 5.0 mm segment to an arc which can be quantified using the arc length method of Eq. (18) with an initial span length or L_0 of 5.0 mm. Figure 60 presents the comparison between thinning strain from global edge stretch measures and local DIC data at various VSGs for sheared flat punch HX and IPB. For both the flat punch HX and IPB, VSGs of 2.0 mm and 3.0 mm exhibit convergence with the global edge stretchability measures, indicating that these DIC gauge lengths are more appropriate for direct application to part design. From a local fracture strain convergence perspective, smaller VSGs close to 1.0 mm appear to be suitable for the 980GEN3 steel as discussed in Section 6.2.3.

Under in-plane stretching, the global edge stretchability can be evaluated using an extensometer of 5.0 mm length positioned near the sheared edge in the EFTT. The choice of 5.0 mm extensometer was validated by agreement with DIC strains at larger VSGs. No global metric was defined for the HTT given the localized deformation that occurs during the test. Further discussion regarding the choice of the extensometer length and agreement with DIC local strains are provided in Section D.5.7 of Appendix D.

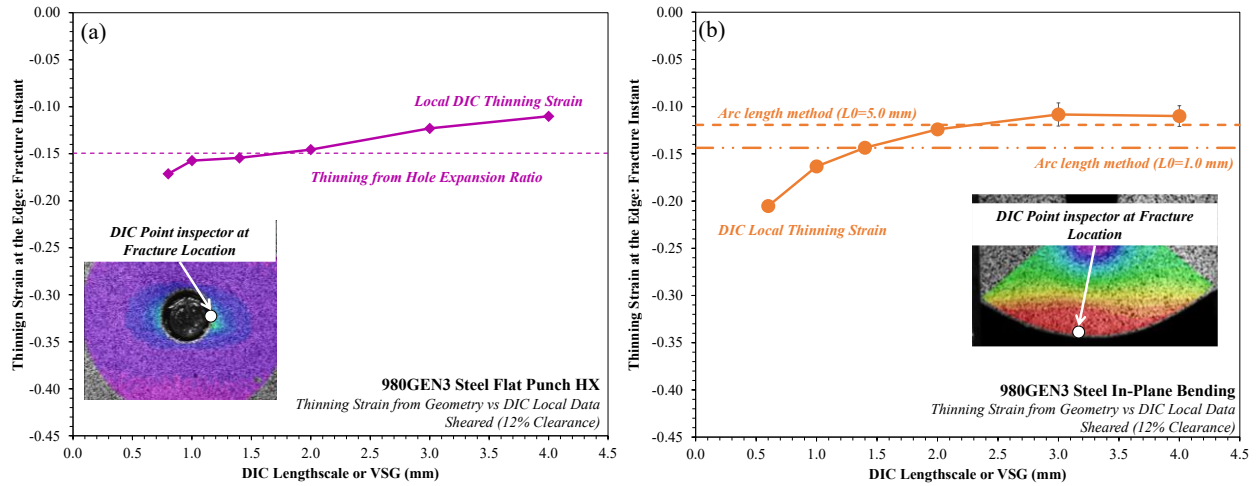


Figure 60 Comparison of global and local DIC thinning strains for a representative 980GEN3 sheared (a) flat punch hole expansion and (b) in-plane bending test at various VSGs.

Figure 61 shows the variation of 980GEN3 DIC edge thinning strains with VSG for the five test methods, along with the corresponding global edge stretch measures overlaid. The conical HX displays the highest global edge formability as expected due to out-of-plane deformation of the sheared edge. The flat punch HX has a lower global formability than the conical HX due to in-plane axisymmetric loading but still surpasses the in-plane bending and EFTT that impose a uniaxial stretch on the cut edge. The EFTT exhibits the lowest global thinning limit of 8.8% due to lack of any strain gradient until the onset of diffuse necking, making it the most critical deformation mode for edge crack initiation. Interestingly, the DIC thinning strain from the hole tension test converges toward the global EFTT strain at VSGs of 3.0 mm and 4.0 mm for the 980GEN3 steel. The global edge stretchability metrics tie the local DIC strain to geometric deformation and provide guidance for identification of appropriate DIC VSGs which can enable direct transferability of strains into design-relevant metrics. These findings underscore the importance of appropriate VSG selection that is tied to the underlying deformation mode. For 980GEN3 steel, VSGs in the range of 2.0 mm to 4.0 mm strike a balance between obtaining localized strain measurements and ensure transferability to large components. The VSGs for various test methods can be anchored to the global edge stretch measures to enhance the transferability of experimental strain data into design-relevant values for edge fracture prediction in industrial scenarios. However, different VSGs may be required for convergence of fracture strains, which was found to be near 1.0 mm for the 980GEN3 steel.

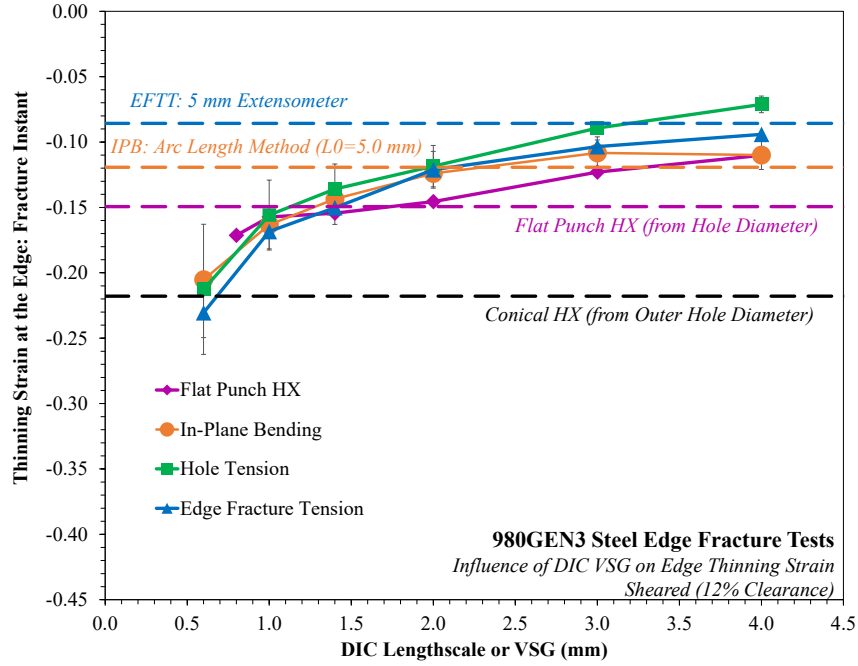


Figure 61 Sensitivity of edge thinning strain to DIC virtual strain gauge length in sheared edge tests conducted for the 980GEN3 steel.

6.3. Numerical Modelling of Edge Fracture Tests

6.3.1. Influence of Shear Affected Zone on Experimental Response

The SAZ introduced during hole punching can alter the force and local strain evolution in edge fracture tests, leading to deviations from the reference response obtained using machined specimens. To this end, the global and local responses from in-plane bending and hole tension tests were compared between specimens with machined and sheared edges, representing the two hole shapes considered in this study. The force versus bend angle evolution during in-plane bending was in good agreement between the two edge preparation techniques up to a bend angle of approximately 16° as seen in Figure 62(a), beyond which a rapid load drop was observed in the sheared edge specimens. Similarly, the convex edge strains evaluated using the DIC point inspector showed excellent agreement between the two edge conditions until near fracture when the sheared edge strains exhibit a rapid strain increase (see Figure 62(b)). Similarly, the engineering stress response in hole tension test was unaffected by the edge preparation method for the 980GEN3 steel as shown in Figure 62(c). The observations are in line with the study of Lingbeek *et al.* (2022) where the hole tension test forces were not influenced by the cutting process for a 1.6 mm thick DP780 steel. The strain evolution at hole edge was also largely consistent between the two edge fabrication methods as shown in Figure 62(d) with minor deviations noted near fracture in the sheared edge tests. The

SAZ had a secondary influence on the 980GEN3 local and global response for tests where crack was constrained to occur within a specific region.

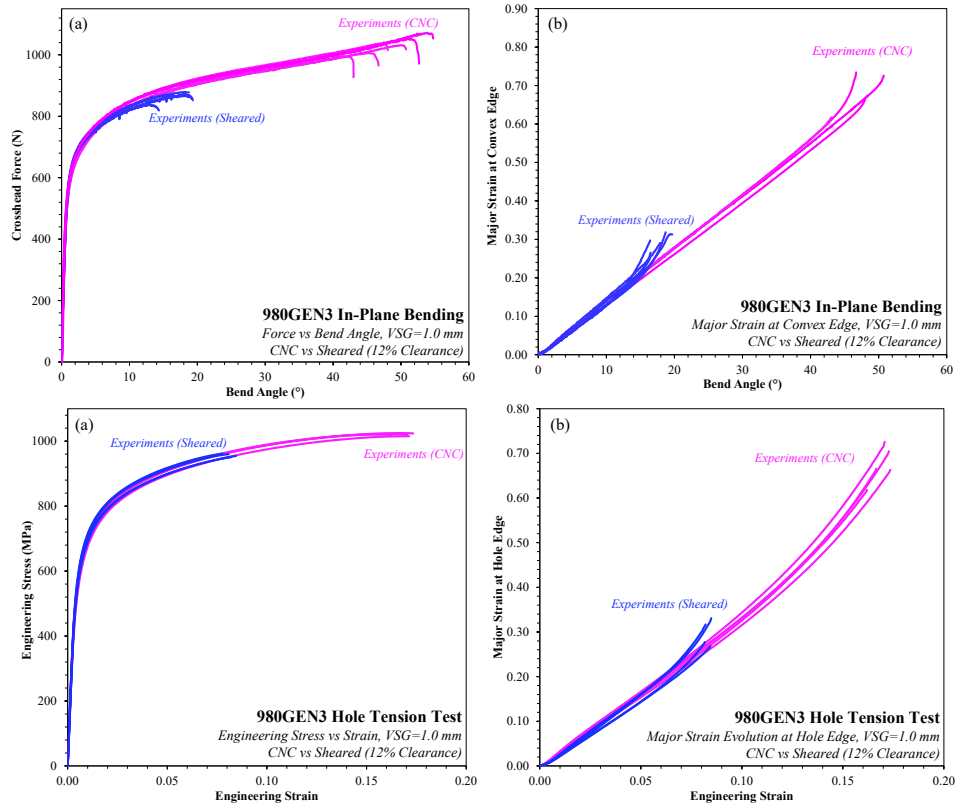


Figure 62 Influence of edge preparation technique on (a) force and (b) convex edge strain during 980GEN3 IPB, and (c) engineering stress and (d) edge strains in 980GEN3 hole tension tests.

The SAZ damage can influence the fracture location in necking-based tests depending upon the material's cut edge sensitivity. To this end, the mechanical response during EFTT was evaluated as it exhibited a shift in fracture location due to presence of SAZ. The engineering stress-strain responses for the machined and sheared edge specimens were in good agreement as seen in Figure 63(a) owing to the relatively large gauge width of 6.0 mm compared to the SAZ depth of 0.675 mm for the 980GEN3 steel (see Section D.5.1 in Appendix D). However, notable differences in local strain evolution at the fracture location were observed as shown in Figure 63(b). Therefore, incorporating SAZ properties into FE models of sheared edge stretching is important for accurate prediction of fracture location and local strains. Nevertheless, FE models of EFTT performed without the SAZ can still provide valuable insight into the influence of applied deformation mode on elemental strains for edge crack prediction even though the experimentally measured DIC strains are expected to be underpredicted.

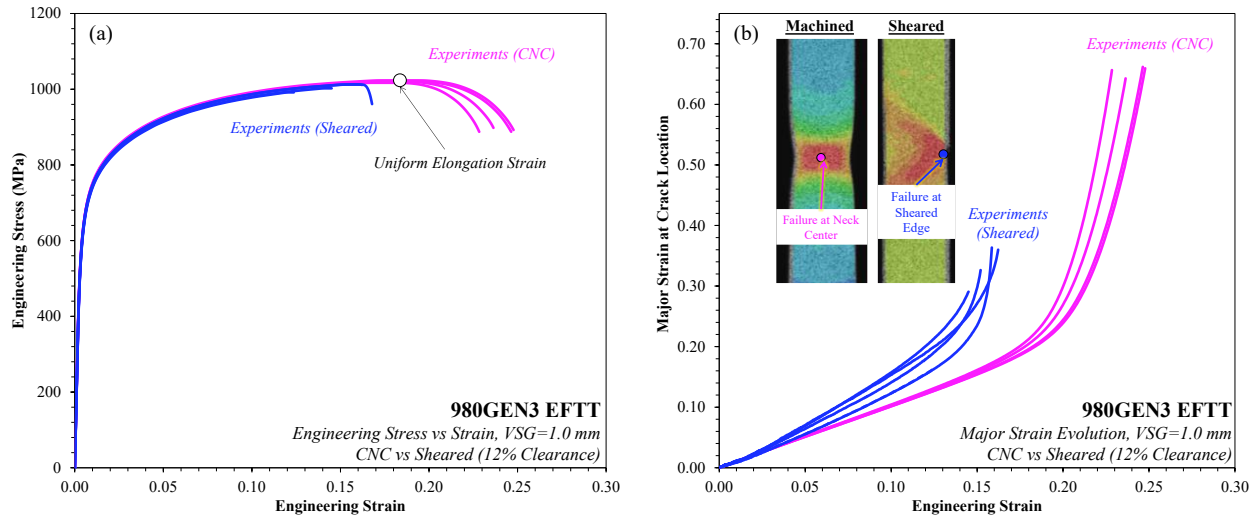


Figure 63 Comparison of (a) engineering stress response and (b) local strain evolution at the failure location in machined and sheared 980GEN3 EFTT.

6.3.2. Finite-Element Model Predictions of Global Response

FE models of the five edge fracture tests were created using shell elements in LS-DYNA software. Multiple mesh sizes in the range of 0.1 mm to 1.4 mm were considered to assess the influence of numerical lengthscale in addition to boundary condition effects. Further details regarding the FE model setup and the input material properties can be found in Section D.4 of Appendix D. Figure 64 shows the representative global response from the experiments overlaid with the predictions of the FE simulations for the in-plane edge fracture characterization tests. The numerical models accurately reproduce the experimental force across all evaluated element sizes in the flat punch HX and edge fracture tensile tests. For the HTT and IPB, the FE simulations also show excellent agreement with experimental data for mesh sizes between 0.1 mm and 1.0 mm. However, the coarsest mesh with a 1.4 mm element size overpredicts the hole tension response by 3% as seen in Figure 64(b) due to the inability to adequately capture the stress concentration around the hole periphery. Similarly, the IPB test response is overestimated by 4% (see Figure 64(d)) due to limitations in capturing the stress and strain gradients within the $5 \times 5 \text{ mm}^2$ gauge region. The simulations conducted using the smallest mesh sizes of 0.1 mm and 0.2 mm were unable to reach the fracture displacement for the machined HTT and flat punch HX due to premature localization. However, the simulation results for these mesh sizes are still suitable for assessment of elemental strain magnitudes at the experimental fracture displacement for the sheared edge tests. The local strain evolution comparison between experiments and FE simulations for the HTT and in-plane bending tests are discussed in Section D.6.1 of Appendix D.

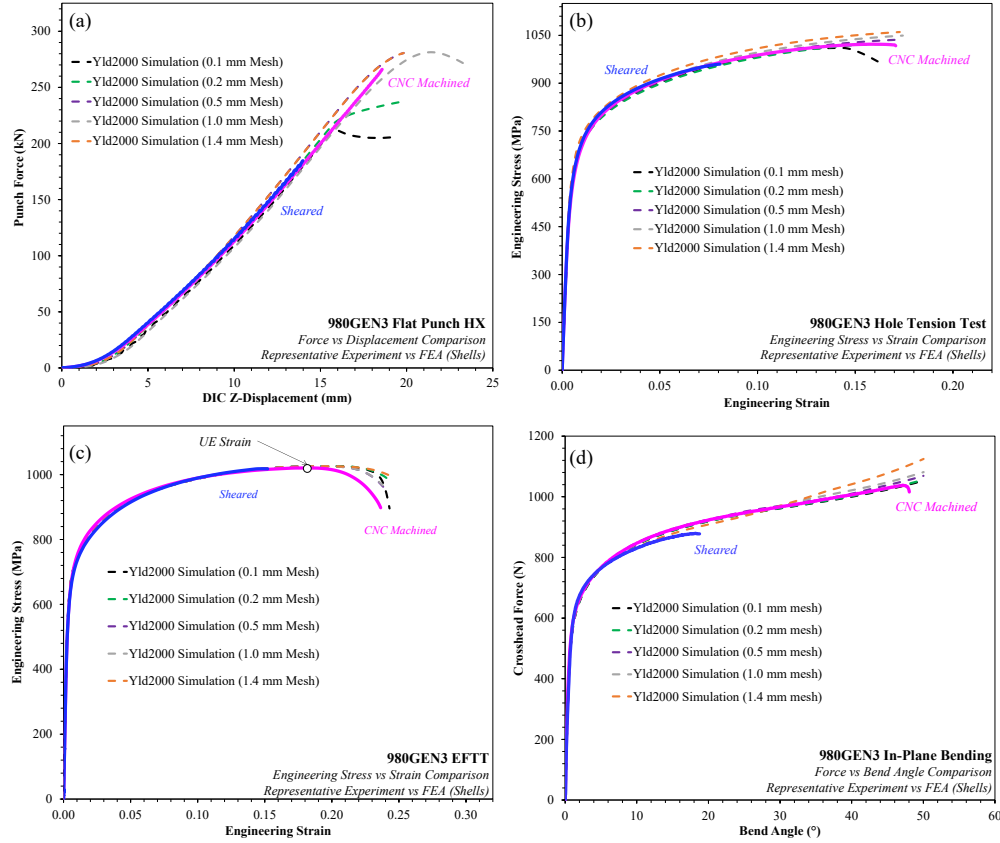


Figure 64 Comparison of global response from FE simulations with representative repeat for (a) flat punch HX, (b) HTT, (c) EFTT, and (d) IPB. Solid and dashed lines denote experiment and simulations, respectively.

6.3.3. Boundary Condition and Lengthscale Effects in FE Simulations of Edge Stretching

Figure 65(a) summarizes the elemental major strains from FE simulations of the edge fracture characterization tests at the instant of fracture in the corresponding experiments conducted on a punched edge. The FE simulations exhibit a pronounced sensitivity to the applied loading conditions regardless of the numerical lengthscale. Major strain differences exceeding 12% can be seen between the in-plane tests regardless of the mesh size. Interestingly, the mesh size effect is largely reduced upon the use of elemental thinning strains for all test methods as seen in Figure 65(b). The reduced sensitivity of the thinning values to the numerical lengthscales is attributed to the homogenization of strain paths away from the edge in the FE models that is not apparent in the experiments as seen in Figure 66 for the hole tension test.

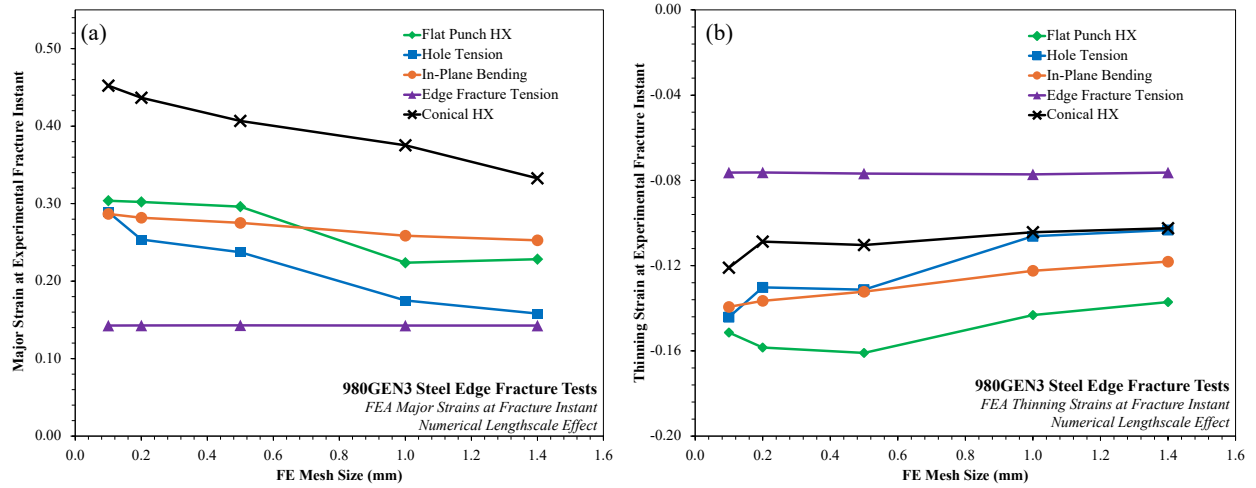


Figure 65 Influence of numerical lengthscale on (a) major and (b) thinning strains at the critical location for the 980GEN3 steel from FE simulations of the five edge fracture tests.

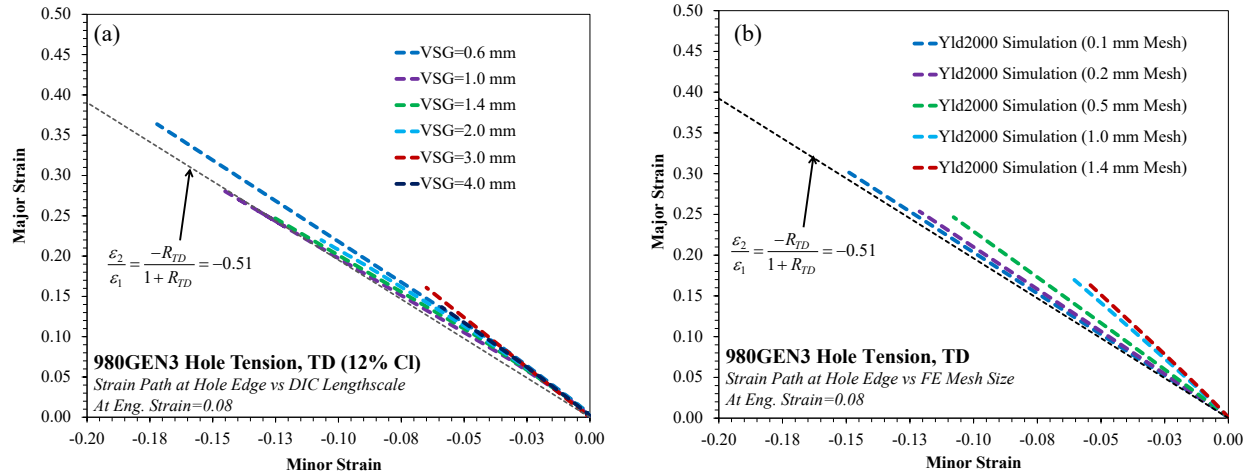


Figure 66 Variation of strain paths at the hole edge with (a) DIC and (b) finite-element model lengthscales in the 980GEN3 HTT.

The axisymmetric stretching modes exhibit the highest thinning limits in the simulations consistent with trends observed in the experiments and is followed by the IPB deformation mode. The conical HX demonstrates a comparatively lower thinning strain in the simulations despite showing the highest elemental major strain among all test methods. The presence of strong through-thickness strain gradients in the test due to contact of the inner edge with the punch counteracts the thickness reduction at the outer edge resulting in a higher sample thickness. The in-plane uniaxial stretching deformation mode appears to be critical for edge crack initiation for the 980GEN3 steel with a thinning limit of 8%, consistent with the experiments. Notably, the global edge thinning limits measured from the coupon tests for the 980GEN3 steel are also in good agreement with FE thinning strains as seen in Figure 67, with absolute differences of less than 1.5% across the tests. These trends demonstrate the suitability of global metrics for application to

FE modelling of edge cracking. Note that the comparison in Figure 67 was performed using the results for FE mesh size of 1.4 mm which corresponded to the lowest thinning magnitude in the in-plane edge fracture tests.

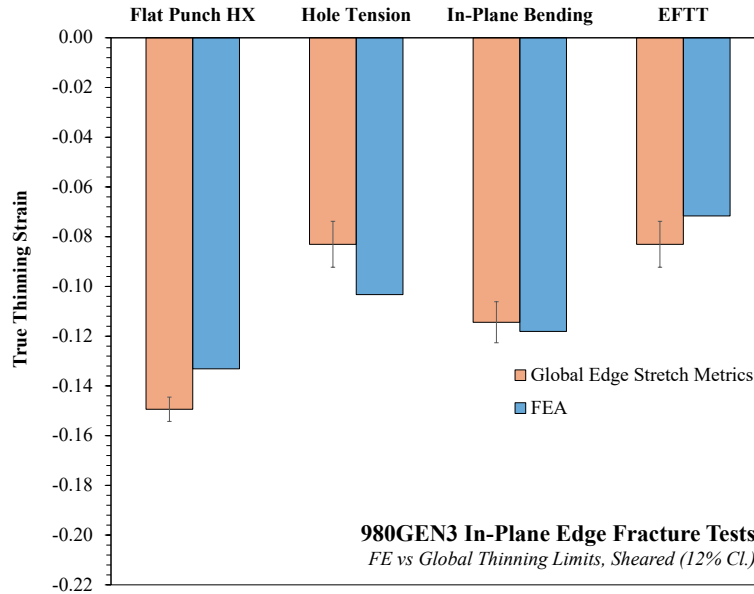


Figure 67 Comparison of true thinning strain predicted by FE simulations and the global edge stretchability measures at experimental fracture instants for the 980GEN3 in-plane edge fracture tests.

The numerical edge fracture strains can be highly sensitive to the imposed boundary conditions, despite convergence between test types in the experiments. The numerical lengthscale significantly influences local elemental major strains, potentially skewing edge crack predictions if they are used as failure criteria. Instead, the thinning strain offers a more robust alternative since it is less affected by the mesh size and is primarily governed by the applied boundary condition. A summary of the thinning fracture strains obtained from the various test methods using multiple DIC lengthscales, as well as FEA predictions, is provided in Table 5 for the 980GEN3 steel, along with an overview of some recommended practices in

Table 6. The proposed methods are expected to be applicable to other AHSS grades for experimental and numerical quantification of edge fracture thinning limits due to reliance on macroscale strain measurements using DIC and FEA. This approach also enables informed choices of reference DIC and simulation lengthscales for calibrating fracture models.

Table 5 Summary of determined fracture strains from experiments and shell numerical models at various lengthscales for the 980GEN3 after hole punching with 12% clearance. A VSG of 3.0 mm was considered to be

representative of the range of VSGs that exhibited approximate convergence with the global edge stretch measures.

Test Type	True Thinning Strain at Fracture Location, VSG=1.0 mm	True Thinning Strain at Fracture Location, VSG=3.0 mm	Global True Thinning Strain	FE Predicted Elemental Thinning Strain (Shells)
Flat Punch HX	-0.161	-0.123	-0.149	-0.133
IPB	-0.141	-0.108	-0.114	-0.118
HTT	-0.145	-0.089	-0.083	-0.103
EFTT	-0.146	-0.117	-0.083	-0.072

Table 6 Comparison of DIC and global edge stretch measures for fracture convergence and numerical prediction of edge cracking for the 980GEN3 steel.

	Fracture Strain Convergence between Tests	Transferability to FE Lengthscales (Shells)	Applicability for Edge Crack Prediction in FEA (Shells)	Recommendation
VSG=1.0 mm	✓	✗ (Needs fine mesh sizes)	✗	Suitable for fracture strain convergence
VSGs larger than 2.0 mm	✗	✓	✓	Suitable for transferability to FE lengthscales
Global Edge Stretch Measures	✗	✓	✓	Suitable for use in edge crack prediction in FEA

7. Conclusions

The present research provides insights into the influence of applied loading conditions on the experimental and numerical evaluation of sheared edge fracture limits with a 980GEN3 AHSS being selected as the primary material of interest. Anisotropic plasticity models were calibrated to incorporate various stress states in the tensile quadrant that are most relevant to sheared edge deformation. A hybrid experimental–numerical methodology was proposed to estimate plane strain data from notch tensile tests while the hardening response was characterized to large strains using the area reduction method. The critical role of accurate yield function calibration and hardening behavior was demonstrated through FE simulations of the flat punch HX.

The mechanics of the conical hole expansion test were subsequently investigated to assess its suitability for uniaxial fracture characterization. The role of necking and planar anisotropy on the test were investigated. Analytical techniques were developed to measure the local fracture strains from the test and were evaluated against FE simulations. Next, a novel specimen geometry and fixture were designed and applied to promote in-plane bending deformation in sheet metals, enabling proportional uniaxial fracture characterization of both base metal and sheared edges. The edge strain states were shown to be promising for characterizing plastic strain anisotropy under both tension and compression. Methods to detect fracture onset and measure the edge strains were proposed and evaluated.

Finally, the influence of edge stretching mode on the local fracture strains following hole punching was examined using various coupon tests. The study highlighted the importance of accounting for both boundary condition effects, as well as experimental and numerical lengthscale effects for fracture strain evaluation. The main outcome of this research is a set of efficient practices covering plasticity and sheared edge fracture characterization that can be adopted for reliable edge crack prediction in industrial simulations. A secondary outcome is the development of the in-plane bend fixture, which has potential for evaluating kinematic hardening in addition to fracture and plastic strain anisotropy.

The following conclusions can be drawn from this research and are organized based on the tasks performed:

Task 1: Characterization and Evaluation of Anisotropic Constitutive Models

- The location and magnitude of plane strain tension on the yield surface can significantly influence the predictive accuracy of FE simulations for tests that promote heterogeneous stress fields. Simulations conducted using the quadratic anisotropic Hill-48 yield function without considering the plane strain data consistently deviated from experimental in-plane strain magnitudes by up to 15% for the DC04

steel. Incorporating the plane strain yield strength data, in addition to the PSC of Butcher and Abedini (2019) significantly improved the ability of simulations to accurately predict the experimental response of flat punch hole expansion tests.

- An inverse hybrid experimental-numerical methodology can be employed to estimate the plane strain yield strength using notch tensile tests for approximately pressure-independent materials as an alternative to stress-controlled cruciform tests.
- The area reduction method offers a reliable technique to characterize the flow stress response of sheet metals beyond the uniform elongation strain using standard tensile tests. The ARM can be applied to tensile specimens using surface displacements obtained from stereo-DIC. It is recommended to use specimens having a gauge width-to-thickness ratio of 4.0 or below to promote triaxial necking. An important advantage of the ARM is that the mechanical properties as well as the flow stress to large strains can be obtained from a given tensile specimen.
- The ARM can also be applied to evaluate anisotropic hardening effects at large strain levels. The presence of anisotropic hardening, which can have a notable impact on the predicted local strain distribution in flat punch hole expansion tests.

Task 2: Characterization of Uniaxial Fracture Strain from the Conical Hole Expansion Test

- Accurate determination of fracture strains under proportional uniaxial tensile loading is essential for calibrating phenomenological ductile fracture models. Uniaxial or hole tension tests experience necking prior to fracture that can introduce data averaging or make the fracture strain dependent on numerical artefacts.
- The conical hole expansion test was demonstrated to be suitable for characterizing uniaxial fracture strain under proportional loading conditions. The presence of through-thickness stress and strain gradients can delay or suppress necking, providing a linear strain path until the onset of fracture.
- The outer hole edge of the conical HX specimen experiences a uniaxial tensile stress state throughout the test making it the appropriate location for measuring fracture limits. The inner edge follows a non-linear strain path due to punch contact causing initial compression that can make it unreliable for fracture strain measurements.

- The conventional hole expansion ratio is not suitable as a fracture metric due to the value representing a homogenization of the non-linear stress state at the inner hole edge. The local elemental strains in FE models of the conical HX were consistently underpredicted with absolute differences of up to 10%.
- The work-conjugate strain method was shown to be an effective approach for determining local fracture strain from conical HX tests without requiring any DIC or inverse finite-element analysis. This method only requires knowledge of the uniaxial tensile stress ratio in the principal stretching direction and the local outer hole diameter at the through-thickness crack location.

Task 3: Characterization of Uniaxial Fracture Limits under In-Plane Bending

- A novel 4-point planar bending fixture and specimen were designed to induce uniaxial fracture in sheet metals with true fracture strains approaching 0.70. This in-plane bending test provides an attractive alternative to the conical HX with the added benefits of enabling R-value characterization under compression, and ability to assess fracture anisotropy.
- The fracture strains from the in-plane bend test can be measured using the arc length method which tracks the geometric changes along the specimen edge during deformation. The arc length technique is less sensitive to the choice of DIC parameters used to process the experimental data. An initial span length of 1.0 mm is recommended to measure the local fracture strains from the in-plane bend test.
- The strain rate and nominal bending stress metrics were found to be effective for identifying the precise instant of fracture onset during in-plane bend tests. The peak force and bending moment were found to be suboptimal for identification of fracture initiation for low ductility materials, with the fracture bend angle underpredicted by approximately 20° for an MS1500 steel.

Task 4: Boundary Condition and Lengthscale Effects on Sheared Edge Fracture Limits

- The sheared edge fracture limits are influenced by both the applied deformation mode and the analysis lengthscale. For the 980GEN3 steel, convergence among the in-plane edge fracture tests was achieved when the DIC virtual strain gauge length approached the sheet thickness. However, at larger lengthscales, a divergence emerged between the tests due to differences in test kinematics, with the fracture strain magnitudes differing by up to 8% for the 980GEN3 steel. Consequently, differences in

VSG in coupon tests or comparisons of test data from different labs can lead to the appearance of a boundary condition effect, even if it is not present for the material.

- The global edge stretch measures enable quantification of the average extension of the sheared edge up to the onset of cracking. These measures are more representative of the thinning-based assessments typically conducted on the shop floor and effectively mimic an averaged fracture strain measurement.
- The appropriate DIC virtual strain gauge length for each deformation mode can be identified based on its convergence with the proposed global edge stretch metrics. For the 980GEN3 steel, VSGs in the range of 2.0 mm to 4.0 mm resulted in good agreement between local DIC and global edge thinning limits in the flat punch HX, IPB, and EFTT. This approach provides a rational basis for selecting VSGs that ensure direct transferability to FE lengthscales for ductile fracture model calibration and potential application in manufacturing environments.
- A boundary condition effect emerges as a natural consequence of the numerical modelling even when the experimental fracture strains show convergence in the different coupon tests.
- The use of thinning strain as a failure metric can mitigate the numerical lengthscale effects. Consequently, assigning a single major fracture strain value may be insufficient for reliable edge crack prediction across varying mesh sizes in shell simulations. The thinning strain can provide a more robust measure that is primarily governed by the applied deformation mode.
- The global edge thinning limits can provide good first-order estimates of the edge thinning strain for input into FE models for edge crack prediction across different mesh sizes. Further analysis across a broader range of AHSS grades with varying microstructural and mechanical behavior is required to validate this approach. Appropriate scale factors can also be introduced to ensure conservative and robust failure predictions.

8. Future Work

The following future work is proposed to expand the scope of this thesis:

- The stress and strain gradients induced by the various edge fracture tests should be quantified using numerical models for correlation with the observed local and global edge strains obtained in this study.
- An efficient approach should be developed to map the strain and damage distribution within the shear-affected zone into edge elements of FE models. Appropriate homogenization schemes must be adopted to account for element size effects and ensure robust transfer of deformation induced by the shearing process into simulations.
- A methodology to calibrate phenomenological fracture loci for sheared edges based on boundary condition-dependent fracture limits should be developed. The failure loci should be implemented into edge elements of commercial FE software such as LS-DYNA, and validated against experimental results. Regularization studies should also be performed to ensure that the fracture locus can be appropriately scaled according to mesh size, ensuring consistent predictions across different numerical lengthscales.
- The capabilities of a dedicated through-thickness shear cutting fixture that enables DIC strain measurement should be explored. Such a setup would allow for deformation measurement using DIC during cutting and facilitate investigation of lengthscale effects on strains developed during the shear cutting process.
- Edge fracture test geometries featuring larger hole sizes should be considered to accommodate higher mesh sizes during FE modelling to expand the range of numerical lengthscales for assessment of boundary condition effect. The majority of the edge fracture tests considered in this study had a gauge size of 5 mm, restricting the largest FE mesh size to 1.4 mm.
- The influence of process parameters such as clearance, tool wear, and open-line cutting in trim stages as this thesis focused upon piercing with a fixed clearance of 12% in closed line cutting.
- The results of this thesis are specific to 980GEN3, which may be an outlier material given its apparently low sensitivity to the boundary condition for sheared edge cracking. This may be due to the shear

affected zone and cut edge surface containing a large amount of martensite formed through deformation induced transformation of retained austenite. As a result, edge fracture may occur at a critical strain threshold regardless of the applied deformation mode. Future work should consider multiphase steels without any retained austenite, as well as grades with microstructural inhomogeneities (*e.g.*, centerline segregation or the presence of hard Ti-based inclusions) that are expected to produce a stronger boundary condition effect. In addition, the effect of burr orientation on the fracture strains should be explored to assess its role in edge crack formation. A study on the boundary condition dependence of edge fracture limits in multiphase steels is being conducted by two colleagues in the research group, Rhys Northcote and Patrick Cleary.

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Appendix A: Journal Publication #1

Narayanan, A., Arciero, J., Tolton, C. and Butcher, C., 2025. Characterization and assessment of anisotropic constitutive models using the flat punch hole expansion test. *International Journal of Material Forming*, 18(2), p.50.

“Accessible through the link below as well as the University of Waterloo’s Institutional Repository (UW Space)”

<https://link.springer.com/article/10.1007/s12289-025-01908-5>

Appendix B: Journal Publication #2

Narayanan, A., Abedini, A., Khameneh, F. and Butcher, C., 2023. An experimental methodology to characterize the uniaxial fracture strain of sheet metals using the conical hole expansion test. *Journal of Materials Engineering and Performance*, 32(10), pp.4456-4482.

“Accessible through the link below as well as the University of Waterloo’s Institutional Repository (UW Space)”

<https://link.springer.com/article/10.1007/s11665-022-07460-6>

Appendix C: Journal Publication #3

Narayanan, A., George, R. and Butcher, C., 2025. Uniaxial fracture and plastic anisotropy characterization of sheet metals using the in-plane bending test. *Engineering Fracture Mechanics*, p.111333.

“Accessible through the link below as well as the University of Waterloo’s Institutional Repository (UW Space)”

<https://www.sciencedirect.com/science/article/pii/S001379442500534X>

Appendix D: Journal Publication #4

Narayanan, A., Northcote, R., Cleary, P., and Butcher, C. On the Influence of Boundary Condition on Sheared Edge Fracture Limits: An Experimental and Numerical Study with Application to 980 MPa Steel, submitted for possible publication, December 2025.

“Accessible through the University of Waterloo’s Institutional Repository (UW Space)”

Appendix A: Characterization and Assessment of Anisotropic Constitutive Models using the Flat Punch Hole Expansion Test

A.1. Introduction

The automotive sector has embraced the use of finite-element simulations to conduct virtual tryouts and optimization of stamping processes [1, 2]. Differences in the selected yield function and hardening model along with their calibration procedure can significantly affect the formability and failure predictions [3, 4]. Therefore, the calibrated constitutive functions must first be evaluated using coupon tests and finite-element modelling before application to real-life stamping simulations. In recent years, the flat punch hole expansion test (HET) has emerged as a promising test to critically evaluate anisotropic plasticity models [5-7]. The flat punch HET activates a range of in-plane deformation modes from uniaxial tension to equibiaxial stretching across all directions as depicted in Figure 68. The test employs a cylindrical punch that stretches a blank having a circular hole and held between two dies. The circular cutout imposes a stress state gradient from uniaxial tension at the hole edge while stress states of plane strain tension (PST) and biaxial stretching are probed away from the edge.

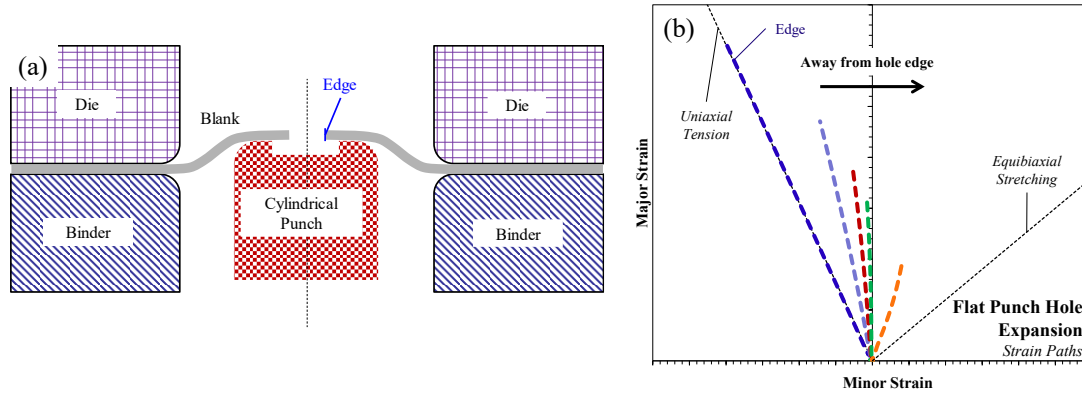


Figure 68 (a) Flat punch HET schematic, and (b) strain paths at various material points away from the hole edge

Finite-element studies of the flat punch HET in the literature have shown that the yield function choice and its calibration strategy can significantly influence the trends in the predicted radial distribution of thickness strains behind the hole edge [5, 6]. Simulation parameters such as the mesh size, friction coefficient of the contacting surfaces, and the element type (shells vs solids) were observed to have a second order effect on the HET results in studies by Lee *et al.* [7] and Park *et al.* [8]. Lee *et al.* [7] further demonstrated using a mild steel sheet that the conventional calibration strategies for the Yld2000 [9] criterion using uniaxial and biaxial test data did not necessarily lead to reliable thickness predictions behind the hole edge. An alternative calibration strategy incorporated the plastic strain ratios obtained in PST from cruciform tests conducted with an in-plane principal stress ratio of 1:2, corresponding to a minor deviatoric principal stress of zero. The thinning trends significantly improved with the minimum thickness location better predicted compared to the conventional calibration. Butcher and Abedini [10] reviewed the HET literature in the context of pressure-independent plasticity and argued that the inaccuracies in the predicted thinning for the various yield calibrations were directly related to lack of any constraint upon the stress state at which PST occurs. The inclusion of plane strain cruciform data of Lee *et al.* [7] effectively enforced a constraint upon the plastic potential consistent with pressure-independent plasticity. The numerical studies by Ha and Korkolis [11] and Park *et al.* [8] also highlighted the necessity of incorporating the plane strain tensile stress state during yield function calibration to accurately predict the minimum thickness location and strain magnitudes in HET simulations.

Unfortunately, stress-controlled cruciform test data for plane strain characterization is seldom available as it requires a specialized test frame with deformation generally limited to low strains due to failure of the sample arms [8, 12]. Without cruciform test data, assuming pressure-independent associated plasticity, as is typical in sheet metal forming, a generalized plane strain constraint (PSC) emerges for shear as well as plane strain tension and compression [10, 13]. For plane strain tension, the applied principal stress ratio is

$\sigma_2 / \sigma_1 = 0.5$ which corresponds to zero minor deviatoric stress ($s_2=0$) and consequently a zero plastic strain increment, $d\epsilon_2=0$, under associated flow. Associated yield functions generated using linear transformations such as the well-known Yld2000 model inconsistently applies the plane strain constraint. For example, it is always satisfied for pure shear ($\sigma_{12} \neq 0, \sigma_{11} = \sigma_{22} = \sigma_{33} = 0$) but a constraint needs to be imposed upon the parameter calibration to satisfy the other conditions that correspond to generalized plane strain loading. Recent advancements have seen the development of yield functions with enhanced flexibility that integrate these constraints for plane strain tension and shear and have been calibrated for both steel and aluminum alloys [14, 15]. Sener [16] demonstrated strong support for the generalized PSC during yield function calibration wherein the constraint for pure shear significantly improved the predicted earing profile in cup drawing simulations of an AA2090-T3 aluminum alloy. Similarly, the minimum thickness location in HET of a low carbon steel could be estimated accurately after enforcing the principal stress ratios for plane strain tension. However, the PSC does not apply to pressure-dependent materials like magnesium, which has an HCP crystal structure and exhibits slip and twinning deformation modes based on the stress state.

In addition to anisotropic yield function calibration, the hardening behavior and its anisotropy must be characterized to strains larger than the uniform elongation to be suitable for stamping simulations. Hydraulic bulge tests [17-19] and simple shear tests [20] can be used along with the tensile data but inherently assume isotropic hardening. Inverse finite-element analysis of tensile tests can be performed to iteratively modify the hardening curve to match the measured load-displacement data [21, 22] but is sensitive to the assumed yield function and element type. Khameneh *et al.* [23] also showed that matching the global response between experiments and simulations does not necessarily correspond to agreement in the local deformation and thinning in the neck. Recently, Abedini *et al.* [20] proposed a practical methodology to evaluate the hardening response of sheet metals using tensile tests by measurement of the cross-sectional area at the neck. The advent of stereo-digital image correlation (DIC) enables real-time measurement of cross-section dimensions during tensile tests, offering a valuable opportunity to apply the area reduction method (ARM) to characterize anisotropic hardening effects at large strain levels.

The advancements in anisotropic plasticity and hardening response characterization coupled with the growing consensus in the literature regarding the importance of plane strain tension during yield function calibration for accurate prediction of the flat punch HET provides the motivation for the current work. Plane strain tension is a particularly important stress state for practical forming operations as it governs thinning and fracture and can only be accurately predicted when incorporated into yield function calibration. The primary objective is to propose a practical and efficient calibration methodology for anisotropic yield functions of steels and aluminum alloys such as Yld2000 and Yld2004 that incorporates the plane strain data points. The area reduction method will be applied to tensile test experiments to determine flow stress

curves to large strains and evaluate anisotropic hardening. Four disparate automotive sheet metals with varying ductilities and anisotropy were considered for calibration and evaluation purposes to determine if the proposed methods are robust. The plasticity models will be critically evaluated by comparing the experimental and numerical results of flat punch hole expansion using the local strain distribution and the global punch force-stroke data.

A.2. Materials and Experiments for Anisotropic Plasticity Characterization and Evaluation

Four automotive sheet metals were considered: an AA5182-O aluminum used in closure panels that exhibits dynamic strain ageing; a high strength AA7075-T6 aluminum alloy with potential for structural applications; a deep draw quality (DC04) mild steel with relatively strong plastic anisotropy, and a cold rolled 980GEN3 third generation advanced high strength steel (AHSS) having mild anisotropy.

A.2.1. Experiments for Constitutive Characterization

Uniaxial and notch tensile tests were performed in multiple orientations at a nominal strain rate of 0.001 s^{-1} with the geometries depicted in Figure 69. The mechanical properties of the materials were evaluated using the JIS tensile geometry shown in Figure 69(a) with the exception of the AA7075-T6 that was previously characterized using the sub-sized ASTM E8 geometry by Rahmaan *et al.* [24] and shown in Figure 69(b). A miniature tensile specimen with a gauge width of 3.0 mm and shown in Figure 69(c) was also employed to evaluate hardening response to large strains in the rolling (RD), diagonal (DD), and the transverse (TD) directions using the ARM technique to promote triaxial necking instead of a shear band localization mode. The minimum cross-sectional area within the gauge region was estimated using the out-of-plane displacement fields obtained through stereoscopic DIC under the assumptions of a symmetrical necking profile about the specimen width, and the edges having no curvature. The notch tensile tests were performed in three directions (RD, DD, and TD) on a tensile frame at a crosshead speed of 0.005 mm/s using the geometry in Figure 69(d).

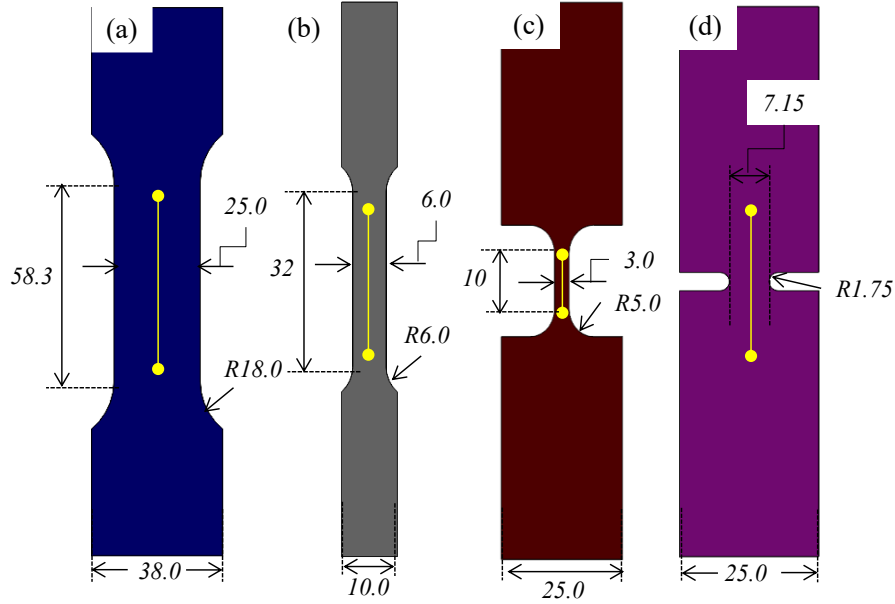


Figure 69 (a) JIS, (b) sub-sized ASTM E8, (c) miniature tensile, and (d) plane strain notch tensile geometries employed for plasticity model calibration. The yellow lines denote the extensometers. All units are in mm.

A.2.2. Flat Punch Hole Expansion Experiments

A double-action MTS hydraulic formability press was used to perform the flat punch HET experiments. The tooling and blank dimensions are shown in Figure 70. The 100 mm diameter punch had a 25 mm nose radius and a 32 mm recessed hole to minimize contact with the blank during hole expansion. The square blank was 200 x 200 mm² in size with a 20 mm diameter CNC machined central hole. A diameter of 20 mm or larger was recommended by Abedini *et al.* [25] to promote thinning and localization behind the edge. To minimize friction at the punch-blank interface, two PTFE sheet layers with petroleum jelly were applied to the head of the punch. The tests were conducted at a punch speed of 0.1 mm/s with a clamping force of 640 kN. Deformation was recorded at a frame rate of 5 fps using two Point Grey® research cameras (model: GRAS50S5MC), each fitted with a Schneider Xenoplan® 17 mm focal length lens.

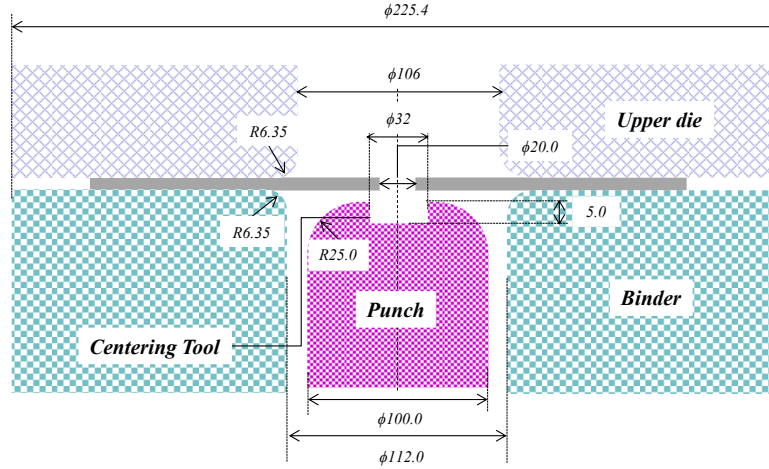


Figure 70 Schematic of the flat punch hole expansion tooling with dimensions

A.2.3. DIC Parameters for Analysis of Experiments

All tests were analyzed using Vic-3D® software provided by Correlated Solutions, Inc. The lengthscale used for DIC processing or the virtual strain gauge (VSG) length is defined by

$$VSG(mm) = \{ [Filter - 1] \times Step(pixel) + Subset(pixel) \} \times Resolution(mm / pixel) \quad (Eq. A1)$$

The VSG for each test and the corresponding DIC subset, step, and filter sizes are tabulated in Table 7.

Table 7 Selected DIC parameters and the corresponding virtual strain gauge length for the various experiments considered in the study.

Test	Image Resolution (mm/pixel)	Subset Size (pixel)	Step Size (pixel)	Filter Size	VSG (mm)
JIS Tensile	0.050	35	2	5	2.0
Sub-sized ASTM E8	0.018	35	5	5	1.0
Plane Strain Notch	0.025	31	2	5	1.0
Mini-Tensile	0.010	35	8	9	1.0
Flat Punch HET	0.050	27	7	23	10.0

A.3. Anisotropic Plasticity Characterization

A.3.1. Uniaxial Tension

The nominal thicknesses for the four materials and tensile properties obtained in RD, DD, and TD are tabulated in Table 1. The engineering stress-strain response of a representative RD test is shown in Figure 71.

Table 8 Tensile properties of the materials with the standard deviation in brackets. The extensometer lengths used for the JIS and sub-sized ASTM E8 geometries were 50 mm and 25 mm, respectively.

Material	Thickness (mm)	Direction	Tensile Geometry	0.2% YS (MPa)	UTS (MPa)	UE (%)	TE (%)
AA5182-O Aluminum	0.85	RD	JIS Z2201	140 (1)	285 (3)	25.4 (2.6)	28.8 (0.9)
		DD		138 (1)	282 (3)	27.1 (1.1)	31.4 (0.3)
		TD		139 (1)	280 (2)	26.2 (2.2)	29.9 (0.8)
AA7075-T6 Aluminum	2.00	RD	Sub-sized ASTM E8	516 (2)	572 (2)	12.2 (0.2)	17.6 (0.6)
		DD		492 (2)	563 (2)	12.2 (0.2)	18.0 (0.4)
		TD		507 (2)	578 (2)	11.6 (0.2)	16.2 (0.8)
DC04 Steel	1.00	RD	JIS Z2201	159 (1)	296 (1)	27.2 (0.1)	47.8 (0.9)
		DD		170 (3)	307 (1)	24.8 (0.2)	44.0 (0.7)
		TD		164 (1)	291 (1)	25.4 (0.1)	45.1 (0.7)
980GEN3 Steel	1.40	RD	JIS Z2201	654 (2)	1006 (2)	19.5 (0.1)	27.7 (0.2)
		DD		633 (2)	1010 (4)	19.1 (0.3)	27.4 (0.1)
		TD		624 (5)	1012 (6)	18.7 (0.1)	28.0 (0.1)

YS – Yield Strength, UTS – Ultimate Tensile Strength, UE – Uniform Elongation, TE – Total Elongation

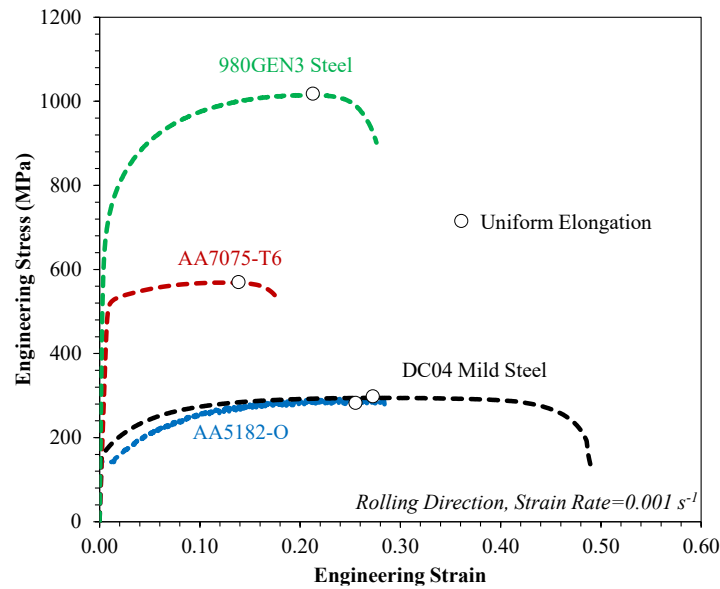


Figure 71 Engineering stress response of the materials along the rolling direction

A.3.2. Plane Strain Characterization using Notch Tensile Tests

Without stress-controlled cruciform tests to precisely determine the stress state for plane strain conditions, the stress state of PST can be fixed to occur at an in-plane principal stress ratio of 0.5 ($s_2=0$) for approximately pressure-independent materials in the yield function calibration stage. Subsequently, the plane strain yield strength must be determined to accurately characterize the yield surface in the tensile quadrant. The shape of the yield function in the first quadrant can be significantly influenced by the

determined plane strain yield strength as depicted in Figure 4(a) using the simple non-quadratic isotropic Hosford yield function [26] that enforces the PSC and is defined as:

$$\bar{\sigma}_{HF72} = \left(\frac{|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a}{2} \right)^{\frac{1}{a}} \quad (\text{Eq. A2})$$

Note that alternate yield functions can be selected and calibrated under these constraints that provide the same arc of the yield function as discussed in Fast-Irvine *et al.* [27]. The notch tensile test offers a viable alternative for plane strain characterization with a gradient in the stress state from uniaxial tension at the edge to plane strain tension at the middle of the gauge width (see Figure 4(b)). However, the plane strain data cannot be measured directly using the forces and the cross-section area in notch tests. An inverse finite-element analysis procedure is used to vary the yield exponent, thus modifying the local yield surface arc and by extension the plane strain yield strength until the simulated load response and strain fields match the experiment.

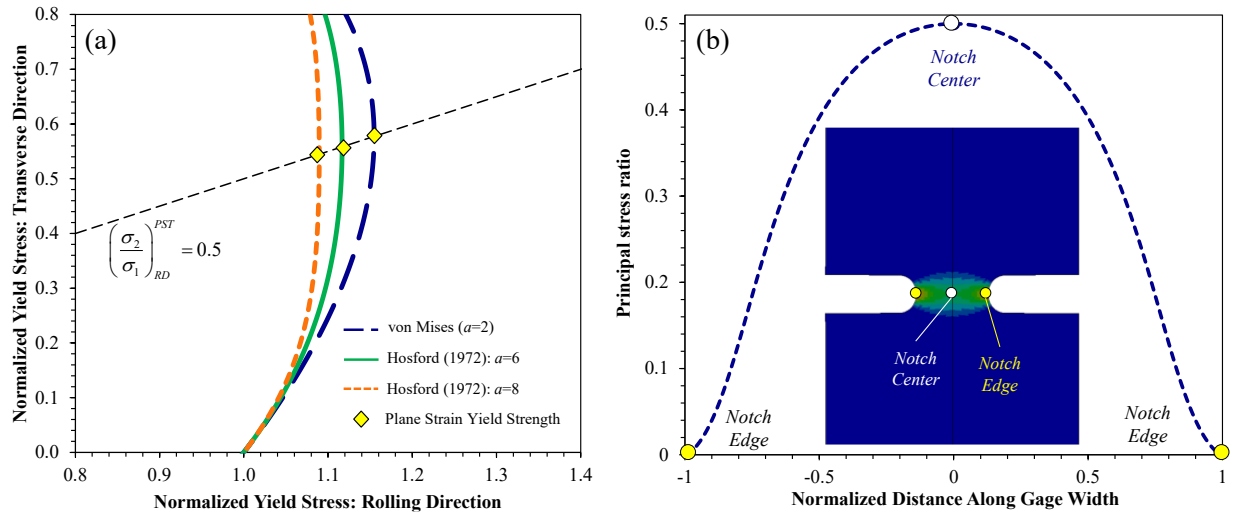


Figure 72 (a) Effect of the yield exponent (a) on the plane strain yield strength, and (b) stress state gradient across the gauge width in a notch tensile test

Numerical simulations of the notch test were conducted in LS-DYNA using solid elements and adopting symmetry in the width and thickness directions. The inverse analysis approach only requires the flow stress curve until the UE strain, and R-value from tensile tests conducted in the same material orientation as the notch test being simulated. Closed-form solutions for the plane strain yield strength are possible for plane stress conditions using the constrained form of Hosford function [28] as demonstrated in Fast-Irvine *et al.* [27] that can be adopted for shell element simulations. The constrained Hosford yield function [28] can be obtained from the Yld2000-2D with its parameters determined according to:

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \left(\frac{2R}{1+R} \right)^{\frac{1}{a}}, \alpha_5 = \alpha_6 = \left(\frac{2}{1+R} \right)^{\frac{1}{a}}, \alpha_7 = \alpha_8 = 1.0 \quad (\text{Eq. A3})$$

For solid elements, a 3-D yield function is necessary, so the non-quadratic Yld91 function [29] was implemented in LS-DYNA through the Yld2004 [30] material card by setting both transformation tensors to be identical.

$$\bar{\sigma}_{Yld91} = \left(\frac{|S_1 - S_2|^a + |S_2 - S_3|^a + |S_3 - S_1|^a}{2} \right)^{\frac{1}{a}} \quad (\text{Eq. A4})$$

where $S_i (i=1,2,3)$ are the principal values of the linearly transformed deviatoric stress tensor, $\tilde{\mathbf{s}}$, given by:

$$\tilde{\mathbf{s}} = \mathbf{C}\mathbf{s}; \quad \mathbf{C} = \frac{1}{3} \begin{bmatrix} 0 & b+2c & 2b+c & 0 & 0 & 0 \\ a+2c & 0 & 2a+c & 0 & 0 & 0 \\ a+2b & 2a+b & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3f & 0 & 0 \\ 0 & 0 & 0 & 0 & 3g & 0 \\ 0 & 0 & 0 & 0 & 0 & 3h \end{bmatrix} \quad (\text{Eq. A5a, b})$$

The coefficients f , g , and h are set to unity which was found to be admissible for the notch test inverse analysis by Narayanan *et al.* [31]. A Newton-Raphson scheme was employed to solve the system of equations to determine the three remaining coefficients as a function of the yield exponent during which the R-value, the PSC, and a tensile stress ratio of unity were enforced. The isotropic Hosford (HF72) yield surface is recovered from both the Yld2000 and Yld2004 models by setting all calibration coefficients to unity.

Figure 73 compares the experimental and simulated engineering stress-strain responses and strain paths at the notch center for DC04 and 980GEN3 steel notch tests conducted in the rolling direction. Simulations performed using a yield exponent of 5.0 replicated the force response and strain paths for the 980GEN3 steel. An exponent of 3.0 was selected for the DC04 steel despite the simulations under predicting the engineering stress by approximately 2% at an engineering strain of 0.05. This discrepancy arose because the RD normalized yield strength of DC04 in plane strain tension exceeds the maximum value attainable with the Yld91 function for the specified R-value. Alternate yield criteria that accommodate more flexibility such as the Vegter [32] or Drucker-Yoshida models [33] could be considered. The underestimated PS yield strength for the DC04 is expected to translate into higher predicted strains in the flat punch hole expansion

simulations similar to the study of Lee *et al.* [7]. The plane strain yield strengths for all materials in different orientations will be summarized in Section A.3.3.

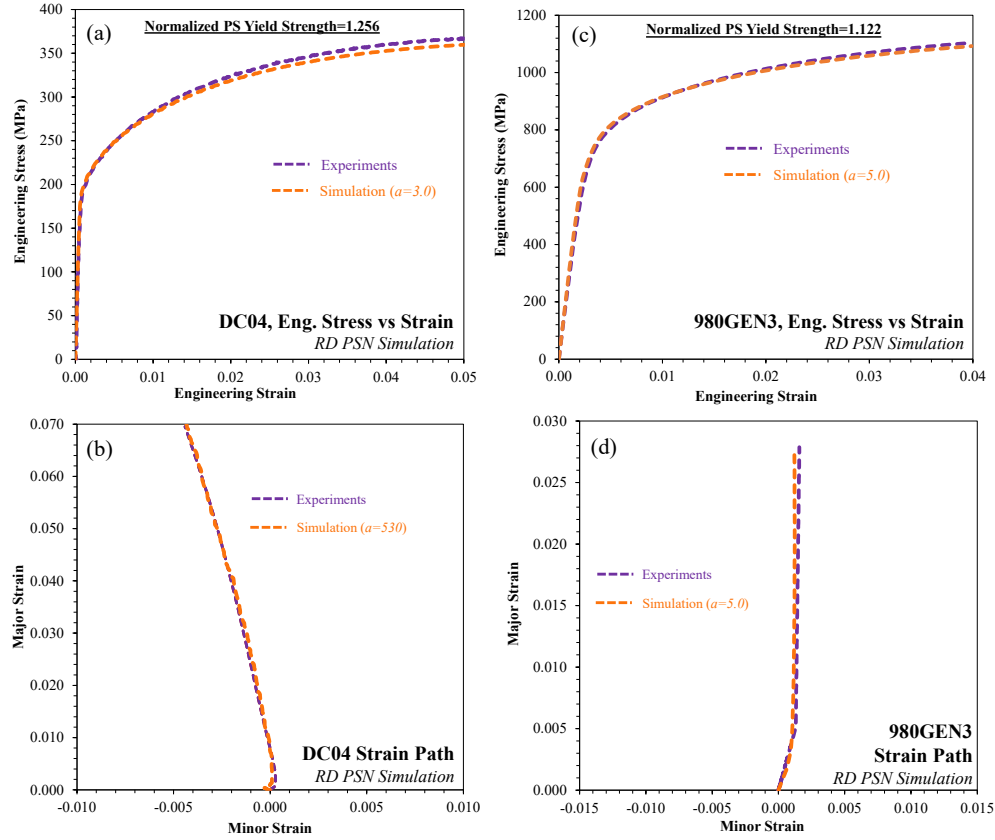


Figure 73 (a, c) Predicted engineering stress-strain responses and (b, d) strain paths at the gauge center from notch test simulations of DC04 and 980GEN3 steels in the rolling direction. Inverse analysis is terminated at the moment of UE strain occurrence at the notch edge element.

Narayanan *et al.* [31] proposed correlations to estimate the normalized plane strain yield strength of aluminum alloys and steels using only tensile properties. The functional form of the Dirac fit and coefficients for steel and aluminum in Table 2 are:

$$\frac{\sigma_{\theta}^{PS}}{\sigma_{\theta}^{UT}} = \left[\frac{F_{\min}^{PS}}{1 + f_{dirac}} + \frac{F_{\max}^{PS}}{1 + \frac{1}{f_{dirac}}} \right]; \quad f_{dirac} = \exp \left[a_{dirac} \left(R_{\theta} - F_{trans}^{PS} \right) \right] \quad (\text{Eq. A6a, b})$$

where σ_{θ}^{PS} , σ_{θ}^{UT} , and R_{θ} are the respective PS stress, uniaxial tensile stress and R-value in the material direction of interest (θ). Figure 31 shows the Dirac correlations against the current notch results and literature data from cruciform tests compiled by Narayanan *et al.* [31] for a principal stress ratio of 1:2. Additional cruciform test data for 440 MPa hot-rolled and DP980 steels from Nomura and Kuwabara [34]

and Park *et al.* [8] have also been included. Overall, the Dirac correlations are able to estimate the normalized plane strain yield stresses to first order accuracy or better across a range of R-values and can be used when notch or cruciform testing is not possible.

Table 9 Dirac function parameter values for estimating plane strain yield strength in steel and aluminum

Dirac function	$F_{\min}^{PS}$	$F_{\max}^{PS}$	a_{dirac}	F_{trans}^{PS}
Steels	0.984	1.274	3.0	0.95
Aluminum	0.999	1.155	7.8	0.63

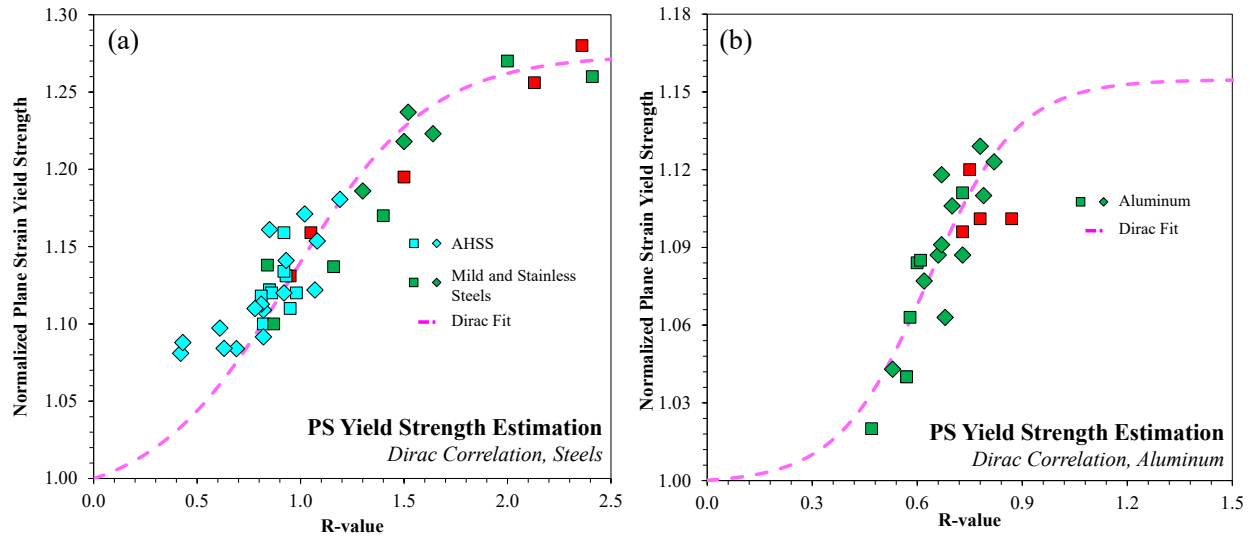


Figure 74 Dirac correlation predictions of plane strain yield stress versus R-value for (a) steels and (b) aluminum alloys. Experimental data points include inverse notch analysis (squares) and literature-based cruciform tests (diamonds). Red markers highlight the materials studied in this work.

A.3.3. Calibration of Anisotropic Yield Surfaces

Table 10 provides a summary of the R-values and yield stresses normalized by the tensile stress in the rolling direction. The correlations of Abspoel *et al.* [35] were used to estimate biaxial yield stress and R-values using uniaxial tension test data. The tensile R-values were obtained by performing a linear fit of the average width and thickness strains within the gauge area over a true axial strain range of 0.01 to 80% of the uniform elongation. The planar anisotropy, ΔR , was also computed as a measure of in-plane anisotropy in the materials. Table 10 also includes the plane strain yield strengths for aluminum and steel predicted by the Hosford yield function [26] with exponents of 6 and 8, respectively. The estimated PS yield strengths using HF72 yield function closely aligned with values from the inverse analysis procedure for AA5182-O and 980GEN3 in all three directions, due to minimal anisotropy in tensile stress ratios and negligible planar anisotropy. The estimated normalized plane strain yield stresses for AA7075-T6 also align closely with the data from inverse notch analysis with the exception of the RD which exhibits a deviation of up to 5%. The

HF72 function underestimates the PS yield strengths of the DC04 by up to 10% and does not appear to be suitable to characterize its anisotropic plasticity behavior.

Table 10 Normalized tensile and plane strain yield stresses relative to the RD tensile stress, along with R-values at various orientations for the materials. Stress ratios were determined at the minimum plastic work level corresponding to the onset of uniform elongation, except for DC04, where plastic work at 50% of the UE was used. Values in brackets indicate the standard deviation.

Material	AA5182-O	AA7075-T6	DC04	980GEN3
$w^P [MJ/m^3]$	54.5	30	30	175
σ_θ/σ_0	1.000 (0.007)	1.000 (0.004)	1.000 (0.001)	1.000 (0.005)
$\sigma_{22.5}/\sigma_0$	0.995 (0.007)	-	1.011 (0.001)	-
σ_{45}/σ_0	0.983 (0.007)	0.979 (0.005)	1.033 (0.002)	1.004 (0.007)
$\sigma_{67.5}/\sigma_0$	0.984 (0.007)	-	1.013 (0.001)	-
σ_{90}/σ_0	0.984 (0.007)	1.004 (0.005)	0.991 (0.002)	1.014 (0.008)
$(\sigma_{PS}/\sigma_\theta)_{RD}$	1.100 (0.007)	1.040 (0.004)	1.256 (0.001)	1.122 (0.005)
$(\sigma_{PS}/\sigma_\theta)_{DD}$	1.100 (0.007)	1.077 (0.005)	1.234 (0.001)	1.136 (0.007)
$(\sigma_{PS}/\sigma_\theta)_{TD}$	1.079 (0.007)	1.071 (0.005)	1.215 (0.002)	1.175 (0.008)
$(\sigma_{PS}/\sigma_\theta)_{Hosford}$	1.089	1.089	1.117	1.117
σ_b/σ_0	0.986 (0.009)*	1.002 (0.006)*	1.139 (0.003)*	1.023 (0.011)*
R_0	0.78 (0.02)	0.57 (0.01)	2.13 (0.06)	0.85 (0.03)
$R_{22.5}$	0.79 (0.02)	-	1.75 (0.05)	-
R_{45}	0.75 (0.01)	0.87 (0.01)	1.50 (0.04)	0.95 (0.02)
$R_{67.5}$	0.75 (0.01)	-	1.83 (0.04)	-
R_{90}	0.73 (0.01)	0.87 (0.01)	2.36 (0.05)	1.05 (0.04)
ΔR	0.005	-0.15	0.75	0.0
R_b	1.07 (0.02)*	0.65 (0.02)*	0.90 (0.08)*	0.81 (0.05)*
* Obtained from correlations proposed by Abspoel <i>et al.</i> (2017)				

The plane stress Yld2000 and 3D Yld2004 yield functions were calibrated using the test data in Table 10. The Yld2000 function can be written as follows:

$$\bar{\sigma}_{Yld2000} = \left[\frac{|S'_1 - S'_2|^a + |2S''_2 + S''_1|^a + |2S''_1 + S''_2|^a}{2} \right]^{\frac{1}{a}} \quad (\text{Eq. A7})$$

where a is the yield exponent. S'_i and S''_i ($i=1,2$) are the eigenvalues of the linearly transformed deviatoric stress tensors $\mathbf{S}'$ and $\mathbf{S}''$ that are functions of the eight calibration coefficients α_i ($i=1,\dots,8$) with additional details on the Yld2000 model provided in Barlat *et al.* [9]. Similarly, the Yld2004 function is given by:

$$\bar{\sigma}_{Yld2004} = \left\{ \frac{1}{4} \sum_{i,j=1}^3 \left(|X'_i - X'_j|^a \right) \right\}^{\frac{1}{a}} \quad (\text{Eq. A8})$$

where X'_i ($i=1,2,3$) and X''_j ($j=1,2,3$) are the principal values of the linearly transformed deviatoric stress tensors. The Yld2004 model has 18 calibration parameters with 16 being unique [36]. The coefficients for through-thickness shear were set to be equal to the in-plane shear coefficients.

The calibrated Yld2000 and Yld2004 surfaces for the materials are shown in Figure 75 to Figure 78 with the corresponding coefficients listed in Table 11 and Table 12, respectively. The Yld2000 and Yld2004 loci are similar and in good agreement with the calibration data. Overall, Yld2004 better captures the variation in the R-values and plane strain yield stresses due to its larger number of calibration parameters.

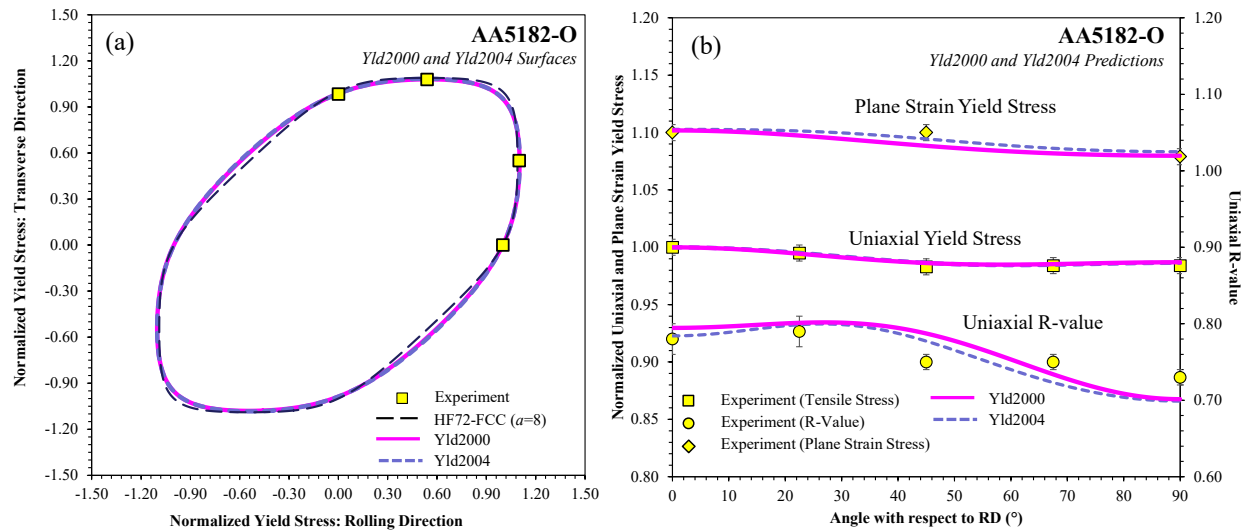


Figure 75 (a) Yld2000 and Yld2004 yield surfaces, and (b) predicted uniaxial and plane strain stress ratios, as well as tensile R-values for the AA5182-O aluminum alloy.

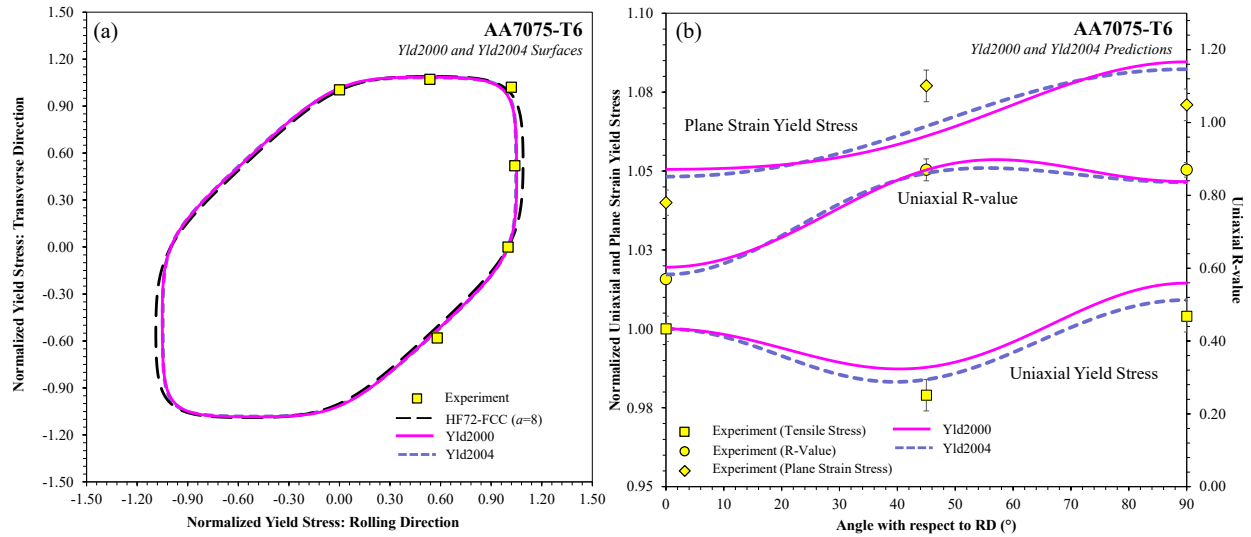


Figure 76 (a) Yld2000 and Yld2004 yield surfaces, and (b) predicted uniaxial and plane strain stress ratios, as well as tensile R-values for the AA7075-T6 alloy.

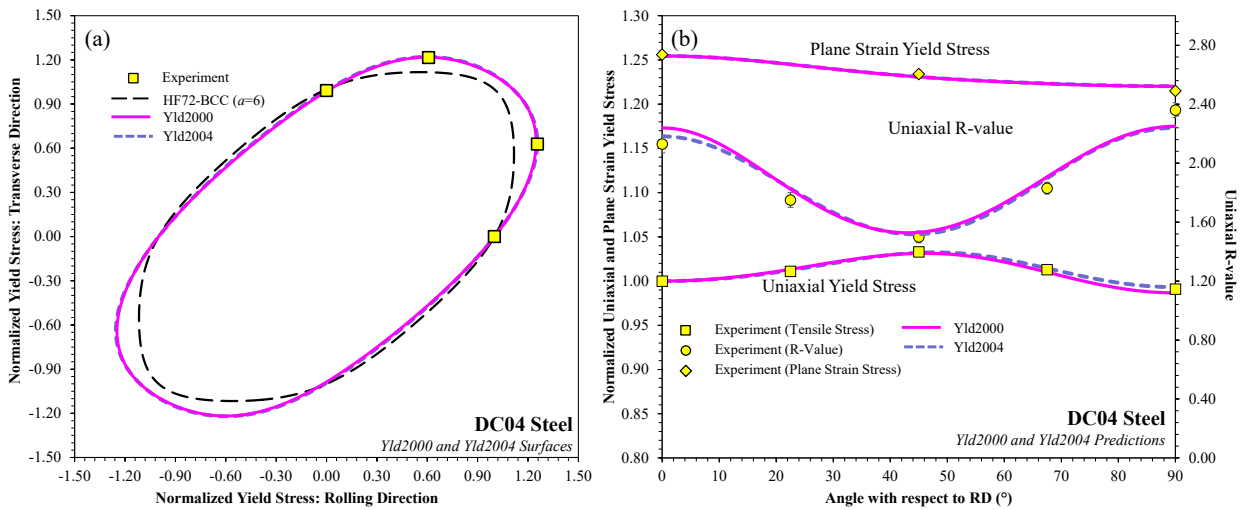


Figure 77 ((a) Yld2000 and Yld2004 yield surfaces, and (b) predicted uniaxial and plane strain stress ratios, as well as tensile R-values for the DC04 steel.

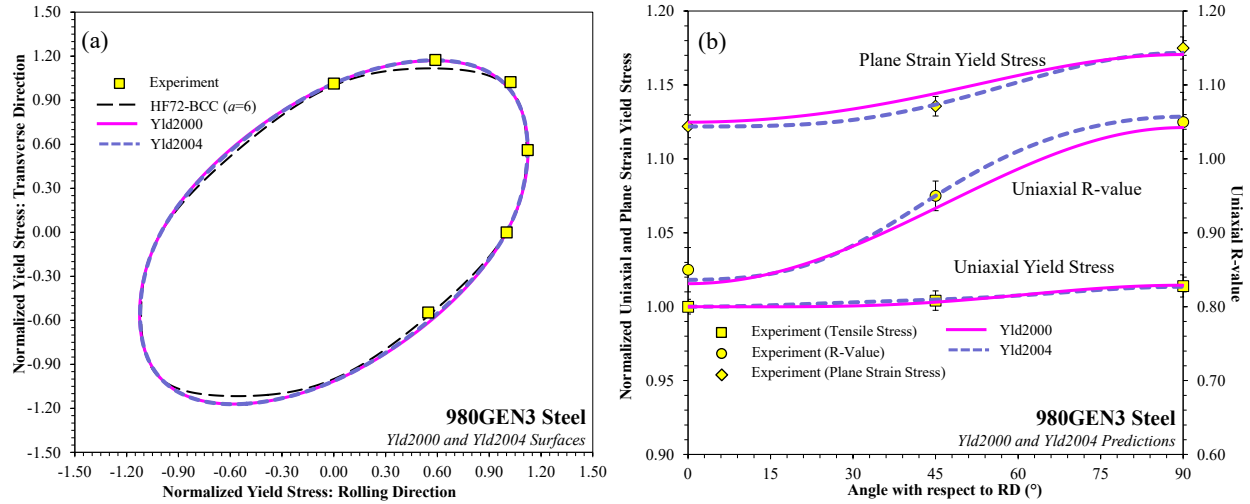


Figure 78 (a) Yld2000 and Yld2004 yield surfaces, and (b) predicted uniaxial and plane strain stress ratios, as well as tensile R-values for the 980GEN3 steel.

Table 11 Calibrated Yld2000 function coefficients for the materials

Coefficient	AA5182-O	AA7075-T6	DC04	980GEN3
a	5.8	9.0	4.5	4.4
α_1	0.9788	0.9612	0.8804	0.9514
α_2	0.9772	0.9847	1.2656	1.0170
α_3	1.0244	0.9993	1.3277	0.9740
α_4	1.0388	0.9953	0.9219	0.9755
α_5	1.0171	1.0278	0.8775	1.0243
α_6	1.0094	1.0368	0.5388	1.0239
α_7	0.9896	1.0034	1.0146	0.9876
α_8	1.04714	1.0275	0.9034	1.0119

Table 12 Coefficients of the Yld2004 function for the materials

Coefficient	AA5182-O	AA7075-T6	DC04	980GEN3
a	8.0	10.0	7.0	7.0
c'_{12}	0.7396	1.2613	1.3763	0.7573
c'_{13}	1.0777	0.9377	0.9124	0.8141
c'_{21}	0.6067	0.3839	1.5261	0.5102
c'_{23}	1.0425	0.9730	1.4188	1.0726
c'_{31}	1.1110	1.2152	1.1895	0.9970
c'_{32}	1.0697	1.4503	0.8829	1.0704
c'_{66}	0.6589	0.8890	1.4657	0.6522
c''_{12}	1.1939	1.3235	0.0974	1.1787
c''_{13}	0.7735	0.9749	0.6033	0.8897
c''_{21}	1.2181	0.7684	0.7764	1.2618

c_{23}''	1.0245	0.6068	0.8463	0.8626
c_{31}''	0.9519	0.3478	0.7796	1.1194
c_{32}''	0.8209	0.8815	0.3882	0.7169
c_{66}''	1.2415	1.0759	0.4214	1.2578

The strain (v_L) and stress-based Lode parameter (L) values from the calibrated anisotropic yield functions were monitored to evaluate how well the plane strain constraints of $\frac{\sigma_2}{\sigma_1} = 0.5$ ($s_2 = 0$) and $N_2 = 0$ were satisfied across all material directions. The Lode parameter definitions are given by:

$$v_L = \frac{3d\varepsilon_2}{d\varepsilon_1 - d\varepsilon_3}; L = \frac{2s_2 - s_1 - s_3}{s_1 - s_3} \quad (\text{Eq. A9, A10})$$

where $d\varepsilon_i$ ($i=1,2,3$) and s_i ($i=1,2,3$) are the ordered principal strain increments and deviatoric stresses, respectively. A plane strain stress state is characterized by strain and stress-based Lode parameters being equal to zero. The variation in the Lode parameters for Yld2000 and Yld2004 are shown in Figure 79 for the aluminum alloys and Figure 80 for the steels. The plane strain constraints are satisfied for the AA5182-O, DC04, and 980GEN3 steels with both the stress and strain-based Lode parameter values being lower than 0.05 in agreement with the values reported in literature review of cruciform data by Fast-Irvine *et al.* [27]. The PSC was not properly satisfied for the AA7075-T6 with a large deviation in the stress-based Lode parameter due to the relatively flat shape of the yield surface. The plane strain point is closer to uniaxial tension in the RD and TD, leading to a larger range of stress states that produce a biaxial stretch. Conversely, the plane strain point shifts toward equibiaxial tension in the DD, as indicated by the positive value of the stress-based Lode parameter, signifying a broader range of stress states for uniaxial stretching with a negative minor strain. The shifts in the plane strain point are expected to influence the predicted thinning behind the edge from the HET simulations.

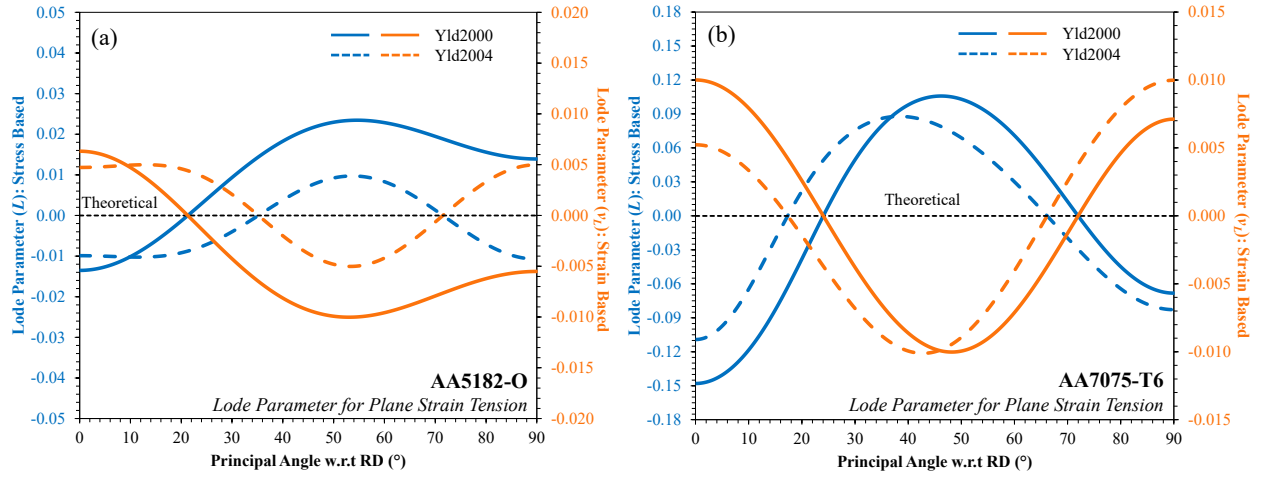


Figure 79 Variation of stress and strain-based Lode parameters with direction of principal loading in the Yld2000 and Yld2004 functions for (a) AA5182-O and (b) AA7075-T6 under plane strain tension

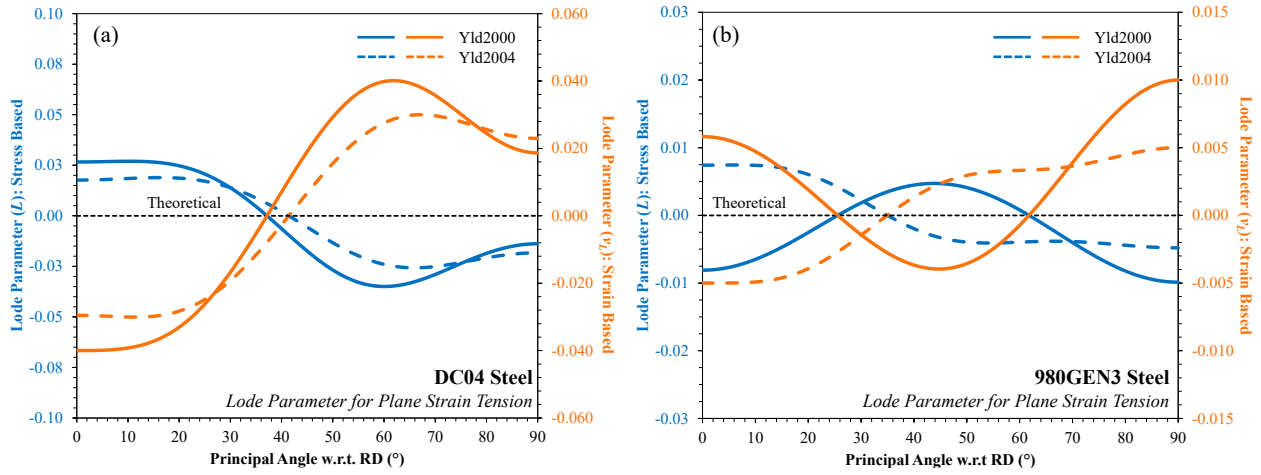


Figure 80 Variation of stress and strain-based Lode parameters with direction of principal loading in the Yld2000 and Yld2004 functions for (a) DC04 and (b) 980GEN3 steels under plane strain tension

A.3.4. Hardening Response Characterization

A.3.4.1. Area Reduction Method

The fundamentals of area reduction method applied to experimental test data are briefly discussed here with further details available in Abedini *et al.* [20] who developed the methodology using finite-element data. The hardening response was first characterized using the classical tensile analysis based upon the axial extensometer data until the true uniform elongation strain (ε_1^{UE}). A schematic depicting a deformed tensile sample with area measurement is shown in Figure 81.

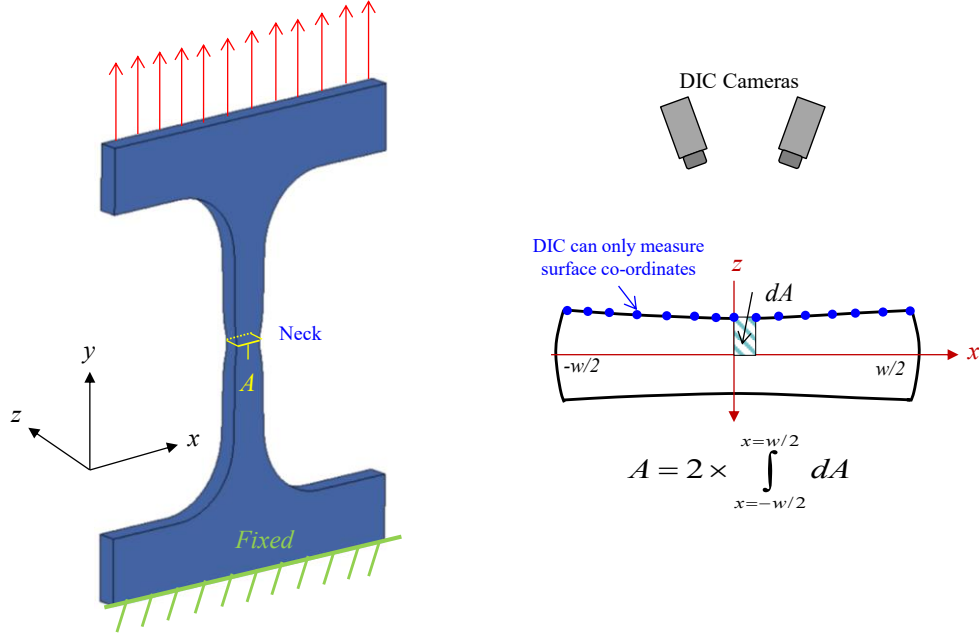


Figure 81 Schematic illustrating the neck cross-section and the surface locations of Z-displacement from DIC that are then integrated to measure the area

The correction factor, k , correlating the DIC measured area to the actual measurement is determined at the true strain at uniform elongation as:

$$k = \frac{A_0 \exp(-\varepsilon_1^{UE})}{A^{DIC}} \quad (\text{Eq. A11})$$

where A_0 is the measured cross-sectional area of the undeformed sample. The corrected area at any instant during deformation, $\bar{A}$, and the corresponding average plastic strain of the cross-section are determined as:

$$\bar{A} = kA^{DIC}; \quad \varepsilon_{avg}^p = \ln\left(\frac{A_0}{\bar{A}}\right) - \frac{\xi}{E}\left(\frac{F}{\bar{A}}\right) \quad (\text{Eq. A12, A13})$$

where ξ is the phenomenological correction factor introduced by Choung and Cho [37] to account for stress state changes during necking, similar to the parameter of Bridgman [38] for round specimens. The corrected true stress is:

$$\sigma_{corrected} = \xi \left(\frac{F}{\bar{A}} \right) \quad (\text{Eq. A14})$$

The correction factor, ξ , varies with plastic strain as follows:

$$\xi = \begin{cases} 1 & \varepsilon_{eq}^p < 1.4\varepsilon_1^{UE} \\ A(\varepsilon_{eq}^p)^2 + B\varepsilon_{eq}^p + C & \varepsilon_{eq}^p > 1.4\varepsilon_1^{UE} \end{cases} ; \quad \begin{aligned} A &= -0.0704n - 0.0275 \\ B &= 0.4550n - 0.2926 \\ C &= 0.1592n + 1.0240 \end{aligned} \quad (\text{Eq. A15a, b})$$

Note that the instantaneous n -value at the uniform elongation is used in Eq. (A15). A DIC contour plot depicting the location of minimum cross-section area during a miniature tensile test and the evolution of its average plastic strain calculated using Eq. (A13) can be seen in Figure 82. The average plastic strain measured at the minimum cross-section is significantly higher than the global extensometer strain for the DC04, owing to its high fracture resistance after diffuse necking. Similarly, fracture occurs at approximately twice the uniform elongation strain for the AA7075-T6 allowing the ARM to characterize the hardening response to strains of up to about 30%.

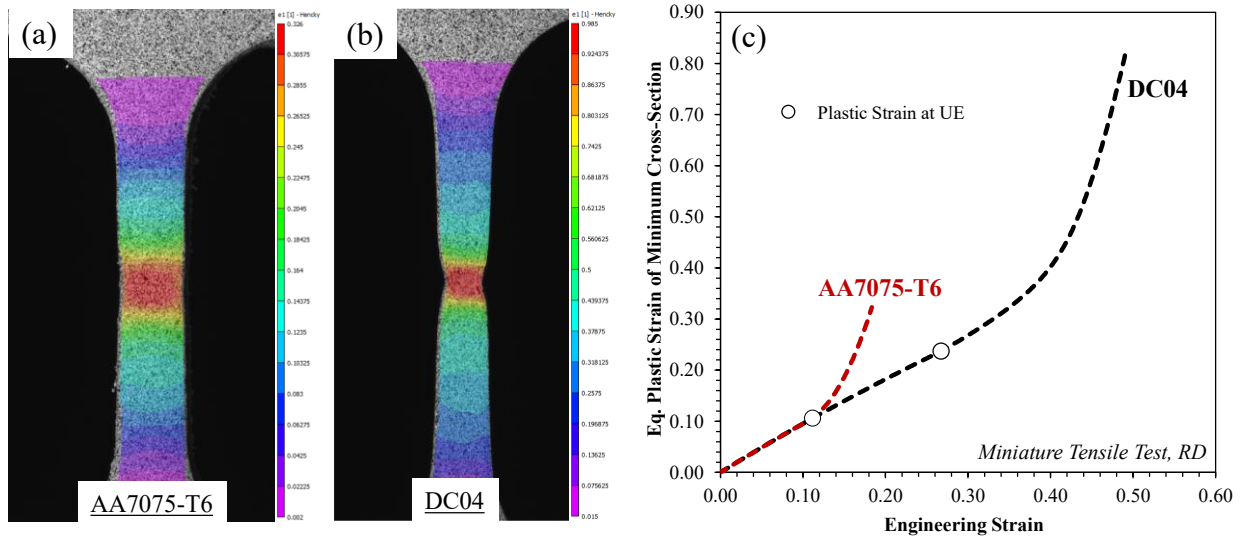


Figure 82 DIC major strain contours before fracture from representative (a) DC04 and (b) AA7075-T6 miniature tensile repeats in the RD and (c) evolution of equivalent plastic strain at the minimum cross-section calculated using Eq. (A13) and Eq. (A15).

A.3.4.2. Isotropic Hardening Response along the Rolling Direction

The hardening curves for the four materials in the rolling direction are shown in Figure 83 and Figure 84. A tabulated hardening model was used employing a constrained cubic spline that ensures a continuously decreasing hardening rate with plastic strain and satisfies the Considère criterion [39] for uniform elongation. The hardening response beyond the terminal ARM data was extrapolated to large strains using the terminal n -value (n_t) to ensure that the flow stress and hardening rate remain continuous.

$$n_t = \varepsilon_t \times \left(\frac{h_t}{\sigma_t} \right); \quad K_t = \frac{\sigma_t}{(\varepsilon_t)^{n_t}} \quad (\text{Eq. A16})$$

where ε_t , σ_t , and h_t are the terminal values of the ARM measured plastic strain, equivalent stress, and the hardening rate.

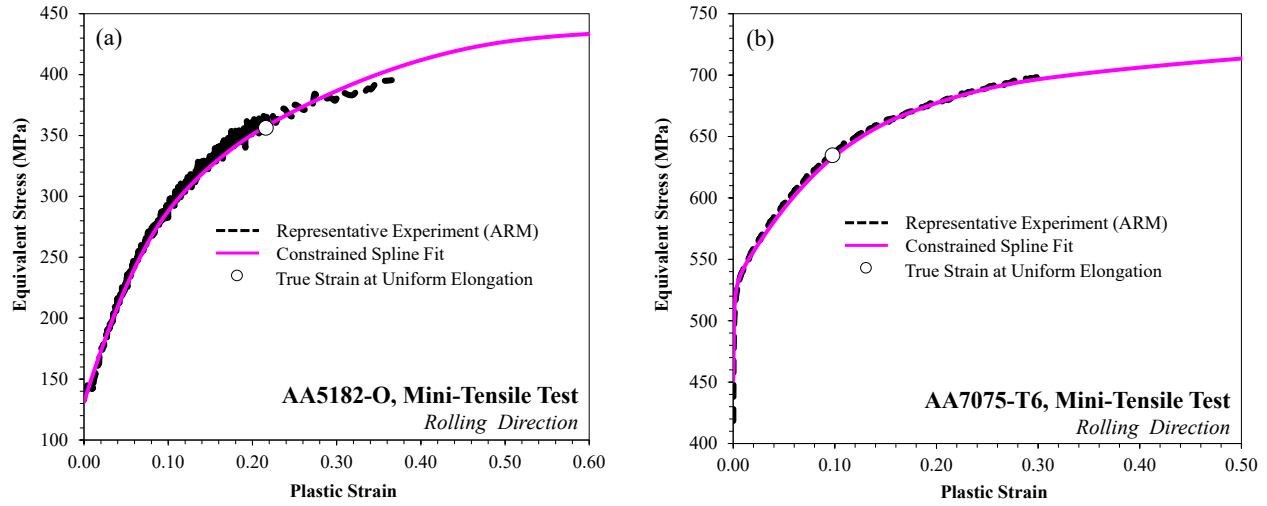


Figure 83 Equivalent stress-strain curves obtained along the RD for (a) AA5182-O and (b) AA7075-T6 using the area reduction method

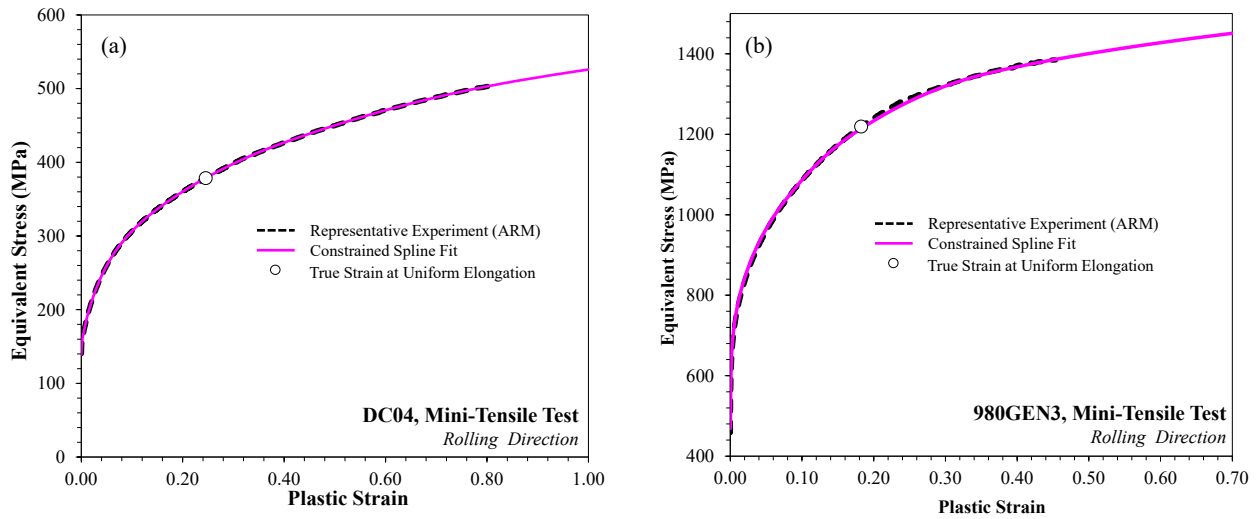


Figure 84 Equivalent stress-strain curves obtained along the RD for (a) DC04 and (d) 980GEN3 steels using the area reduction method

A.3.4.3. Evaluation of Anisotropic Hardening

The Yld2000 and Yld2004 models inherently assume isotropic hardening wherein the hardening curve in the RD is linearly scaled to obtain the hardening behavior in another orientation. To evaluate the assumption of isotropic hardening and accuracy of the calibrated yield functions, the equivalent stress-strain curves along the DD and TD obtained using the ARM method are depicted in Figure 32 to Figure 35 for the four materials. The predicted stress response from the RD hardening curve scaled by the tensile stress

ratios estimated by the Yld2000 and Yld2004 functions are overlaid for comparison. The assumption of isotropic hardening approximately holds for the AA5182-O and AA7075-T6 alloys but not for the DC04 at strain levels higher than the uniform elongation. Kuwabara and Sugawara [40] also reported anisotropic hardening of an interstitial-free steel with similar plastic anisotropy using multiaxial tube expansion tests. The 980GEN3 steel exhibits mild anisotropic hardening beyond a plastic strain of approximately 0.40.

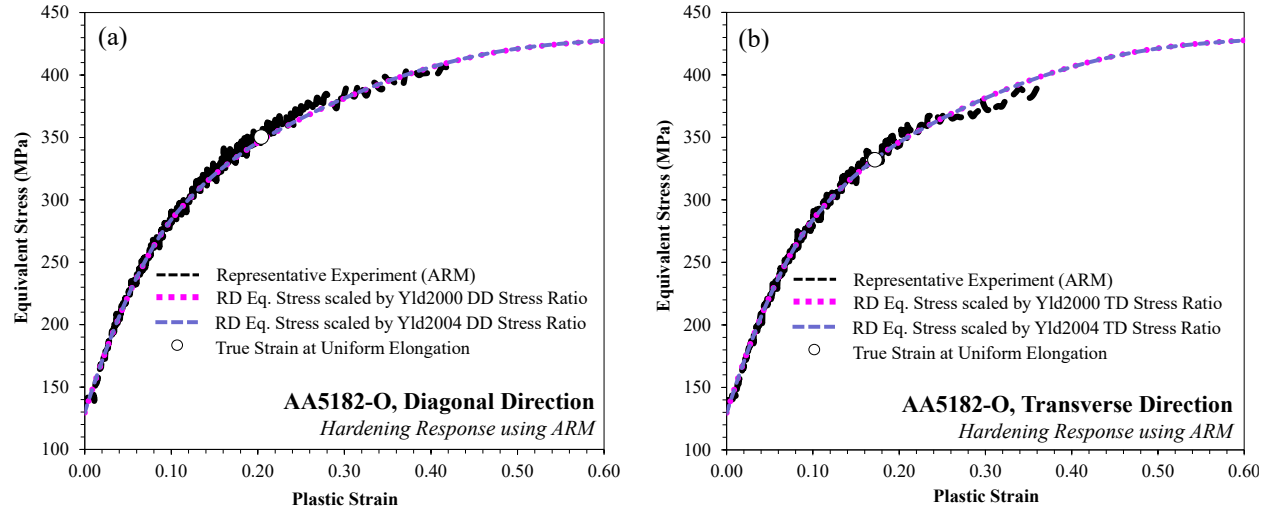


Figure 85 AA5182-O flow stress curves along (a) DD and (b) TD from ARM superimposed with the hardening response along the RD scaled by the predicted tensile stress ratios from Yld2000 and Yld2004.

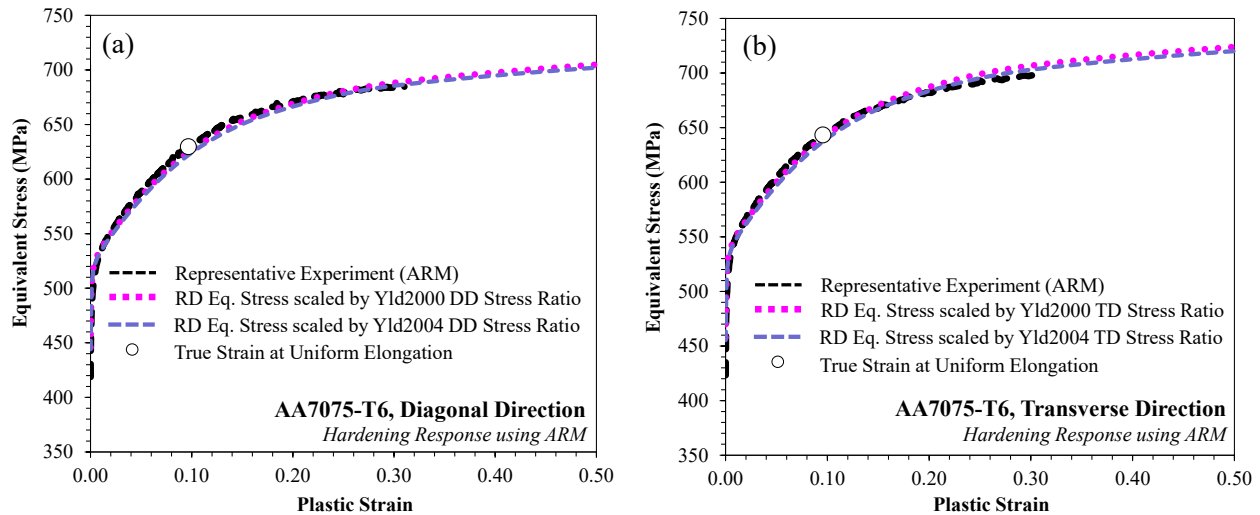


Figure 86 AA7075-T6 flow stress curves along (a) DD and (b) TD from ARM superimposed with the hardening response along the RD scaled by the predicted tensile stress ratios from Yld2000 and Yld2004.

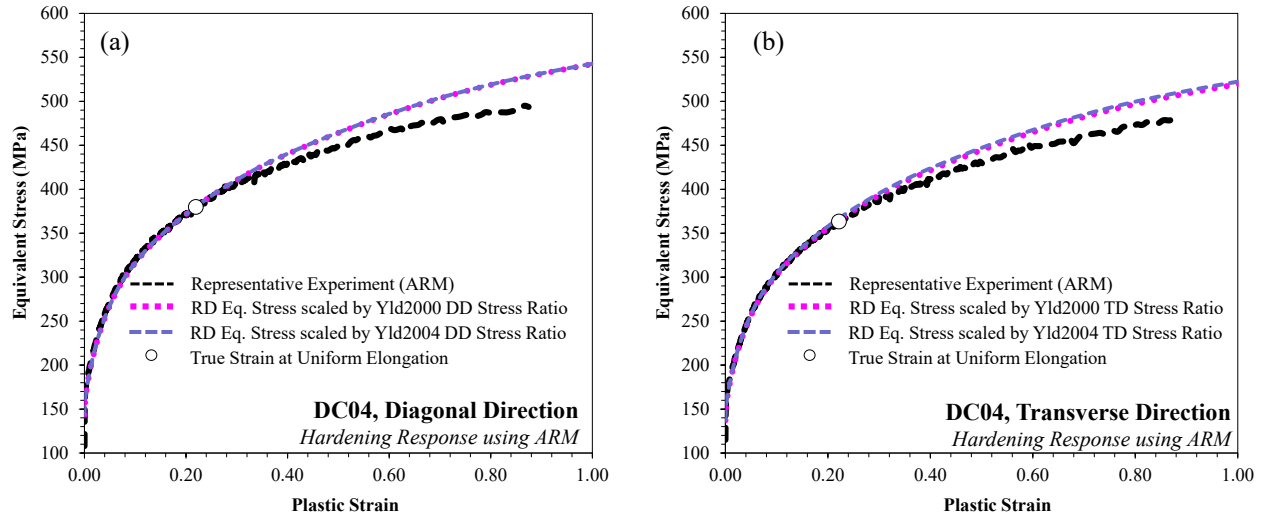


Figure 87 DC04 flow stress curves along (a) DD and (b) TD from ARM superimposed with the hardening response along the RD scaled by the predicted tensile stress ratios from Yld2000 and Yld2004.

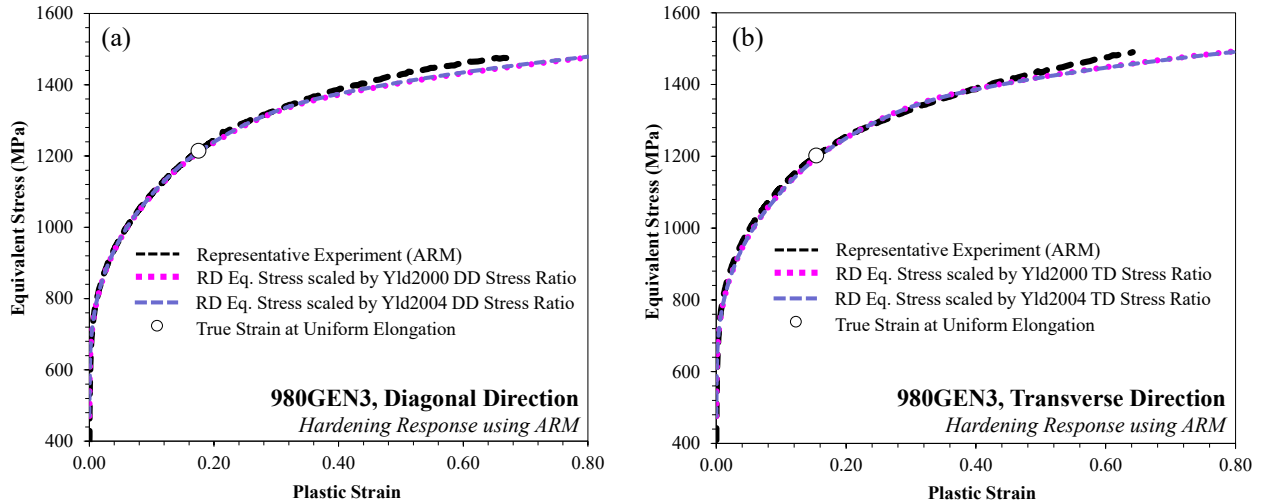


Figure 88 980GEN3 flow stress curves steel along (a) DD and (b) TD from ARM superimposed with the hardening response along the RD scaled by the predicted tensile stress ratios from Yld2000 and Yld2004.

A.4. Experimental Measurements from Flat Punch Hole Expansion Test

The DIC measured principal strains as a function of radial distance from the hole edge was extracted from the flat punch hole expansion experiments in the RD, DD and TD as illustrated in Figure 89(a). It's important to note that due to mechanics of the hole expansion test, the orientation of the radial lineslice corresponds to stretching perpendicular to that direction. For example, strains measured using the RD lineslice correspond to tensile stretching in the TD. The displacement corresponding to the peak load was considered to assess the predictive accuracy of the finite-element models at large strain levels. Data from multiple line slices corresponding to the same material orientation (e.g. RD line slices at 0° and 180°) were

considered for comparison against the numerical models to account for potential deformation asymmetry in the experiments. Note that the Z-displacement was measured using a point inspector tool placed near the hole edge in the Vic-3D[®] software as shown in Figure 89(a). An additional lineslice that corresponded to the fracture initiation direction in the experiments was also analyzed to assess the ability of the finite-element simulations to capture localization. Finally, the circumferential variation in thickness strain at the radius corresponding to the maximum thinning location behind the edge (see Figure 89(b)) was also measured for selected tests at the peak load displacement.

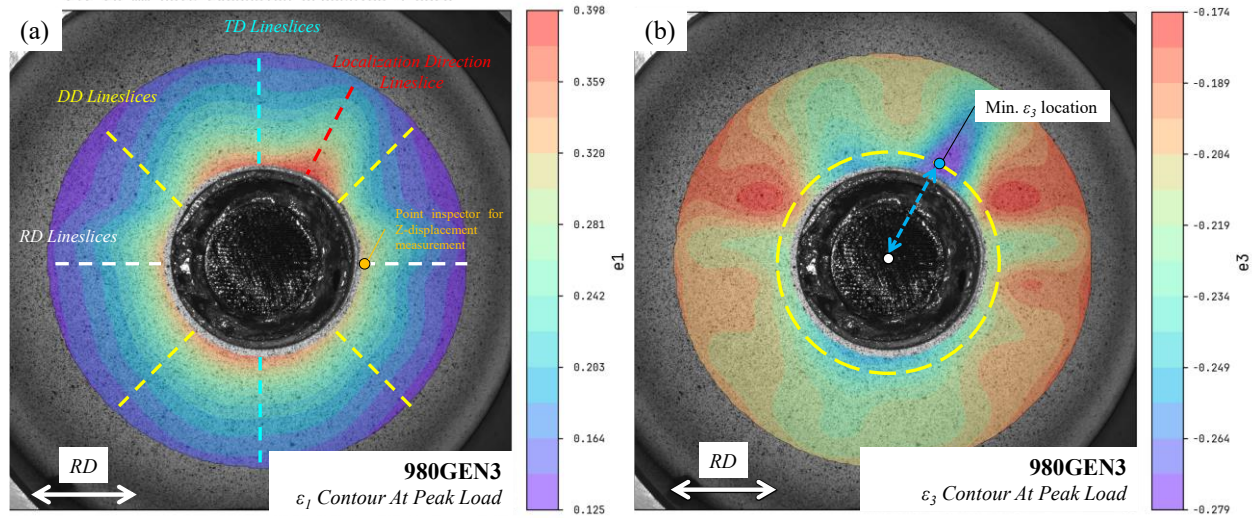


Figure 89 Material orientations along which (a) DIC lineslices and (b) circle inspector were extracted to compare principal strains against the finite-element models.

A.5. Finite-element Modelling of the Flat Punch HET

The hole expansion test simulations were conducted in LS-DYNA using both solid and shell elements. The blank was meshed using fully-integrated shell elements (ELFORM=16), featuring a mesh size of $0.5 \times 0.7 \text{ mm}^2$ near the hole edge followed by a transition in lengthscale to approximately 1 mm until a radial distance of 20 mm. The elements consisted of five through-thickness integration points based on the Gauss-Lobatto integration scheme. A sectional view of the flat punch HET model is shown in Figure 90. The coefficient of friction for PTFE film between the punch and the blank was 0.037, while a friction coefficient of 0.40 was applied between the blank and the dies to mimic the knurled surface condition of the tooling. The solid element models used the same mesh pattern as the shell models but employed quarter-symmetry to reduce the simulation run time.

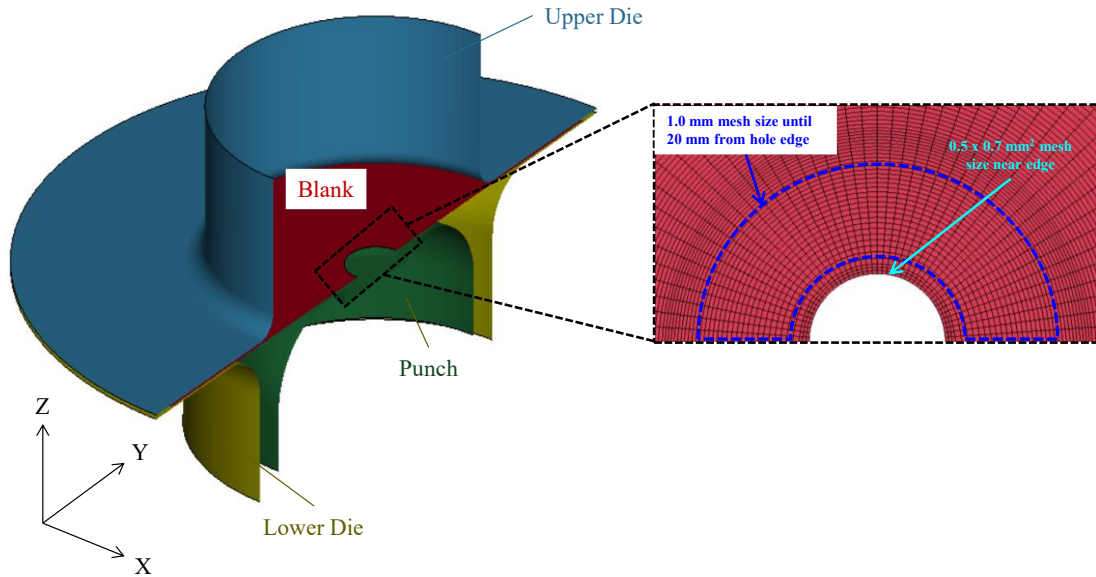


Figure 90 Sectional view of the flat punch HET finite-element model using shell elements

A.6. Results and Discussion

A.6.1. Punch Force-Displacement Curves

The punch force versus the Z-displacement of the blank from a representative experiment and the simulations are shown for the four materials in Figure 36. The predicted punch forces from the numerical models conducted using both shell and solid element formulations matched with the experiment until the peak force, supporting the use of the ARM to characterize the material flow stress behavior. The choice of the yield function was found to exhibit a second order influence on the predicted global force response. The DC04 steel was a notable exception since the HF72 yield surface significantly underpredicted its plane strain yield strength. Fracture occurred instantly after the peak load for all materials except for the high ductility DC04 that underwent gradual localization prior to fracture.

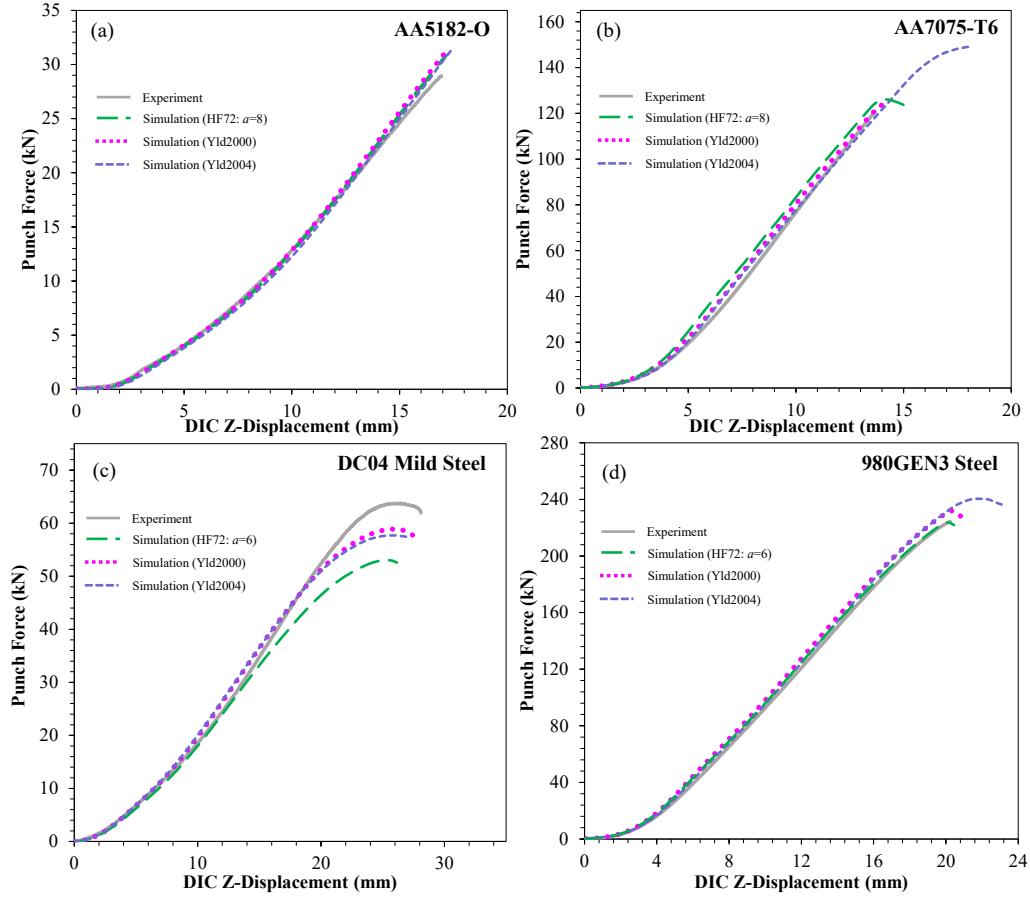


Figure 91 Experimental and simulated punch force vs Z-displacement curves for (a) AA5182-O, (b) AA7075-T6, (c) DC04, and (d) 980GEN3 steel. Only the force response from a representative experiment is shown for convenience.

A.6.2. Comparison of Local Strains and Thinning behind the Hole Edge

The isotropic Hosford, and anisotropic Yld2000 and Yld2004 simulations well described the strain magnitude and gradients for AA5182-O and AA7075-T6 along the RD, DD, and TD at the peak load moment as seen in Figure 92. The distances from the hole edge at which $\varepsilon_2=0$ occurred were accurately predicted by the three yield functions, demonstrating the importance of enforcing the PSC on yield calibration.

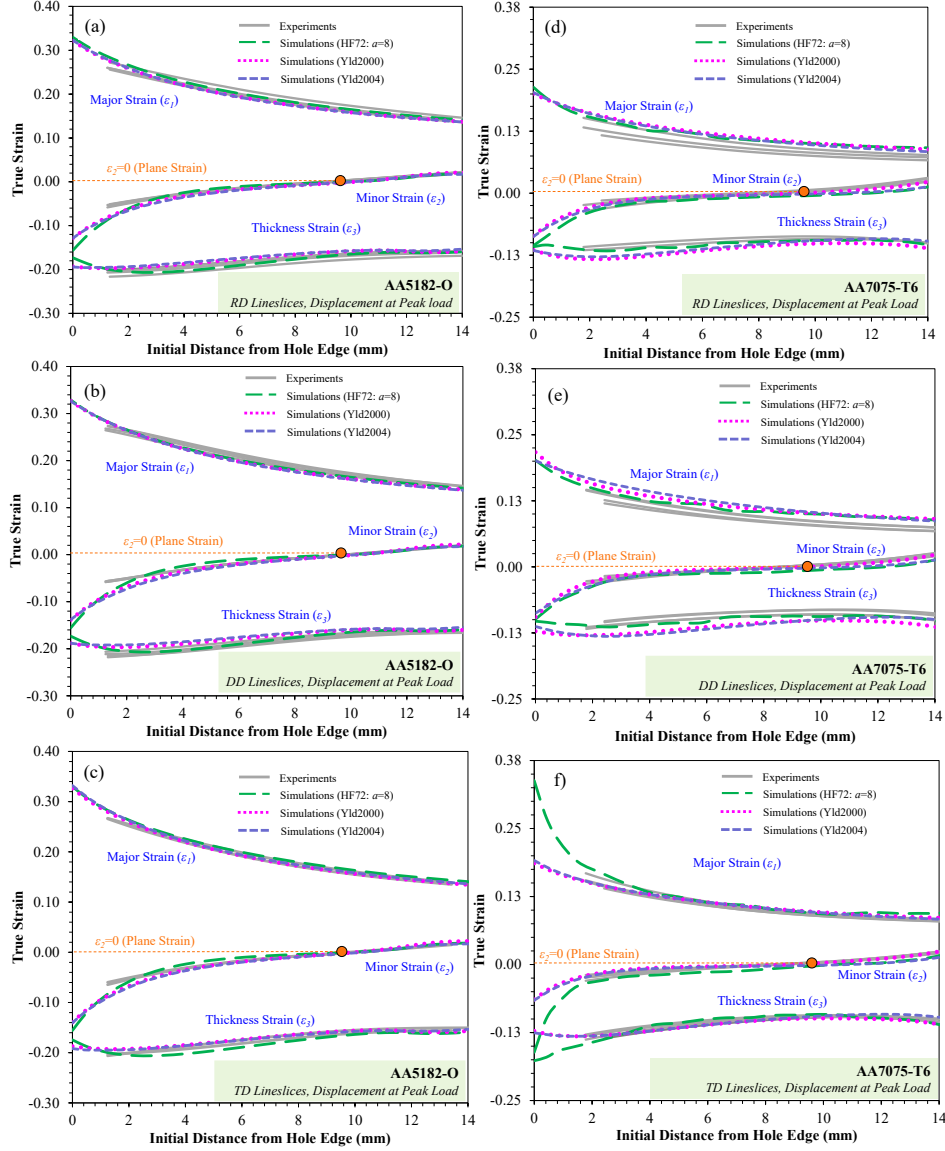


Figure 92 Variation of principal strains away from the hole edge in TD, DD, and RD (RD, DD, and TD lineslices) for (a-c) AA5182-O and (d-f) AA7075-T6 aluminum alloys at the displacement corresponding to peak load. The thickness strain was calculated using volume constancy ($\epsilon_3 = -\epsilon_1 - \epsilon_2$)

The variation of principal strains with radial distance from the hole edge for the DC04 and 980GEN3 along the RD, DD, and TD are shown in Figure 37. The simulations overpredicted the thinning magnitudes along the RD for DC04 (see Figure 37(c)) as expected due to underprediction of its plane strain yield strength. However, an excellent agreement is observed in the strain magnitudes in the DD and TD that is likely due to the strong anisotropic hardening effects counteracting the underprediction of the plane strain data. Both Yld2000 and Yld2004 accurately captured the magnitude and location of maximum thinning, with Yld2004 performing slightly better along the RD (i.e. TD lineslices). The Hosford yield function overpredicts the thinning strain magnitude of the DC04 by up to 10% across the three material orientations due to its inability

to account for anisotropy. The simulated principal strains also closely matched the experimental results for the 980GEN3 steel. However, the HF72 yield function overestimated thickness strains by up to 3% in the RD lineslices. The locations of plane strain tension and maximum thinning were accurately reproduced by the yield functions enforcing the PSC. Minor differences in the magnitude of in-plane principal strains between the simulations and the experiments for the two steels is attributed to DIC lengthscale effects with a sensitivity study on the influence of VSG on the DIC strains provided in Appendix A1.

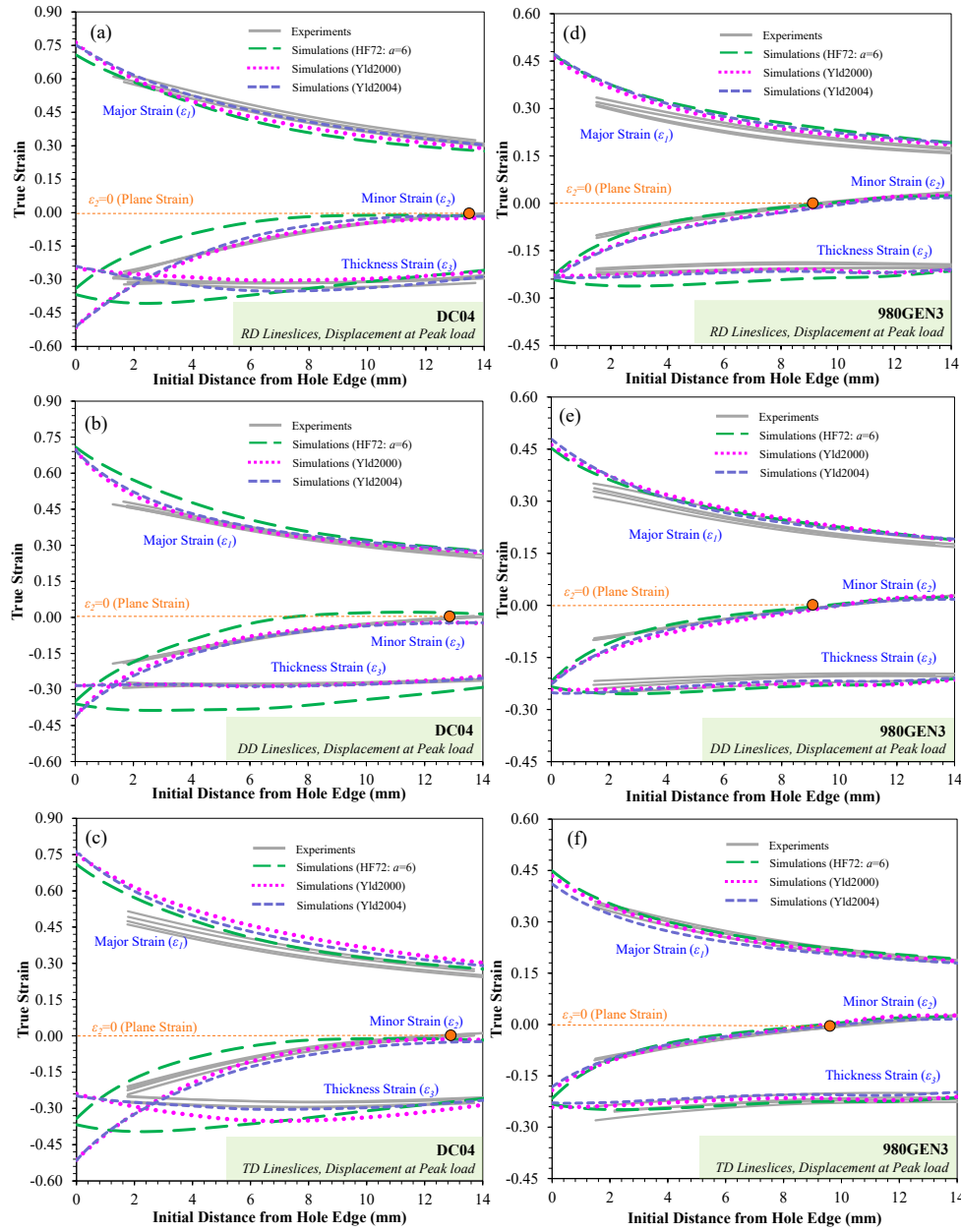


Figure 93 Variation of principal strains away from the hole edge in TD, DD, and RD (i.e. RD, DD, and TD lineslices) for (a-c) DC04 and (d-f) 980GEN3 steels at the displacement corresponding to peak load. The thickness strain was calculated using volume constancy ($\epsilon_3 = -\epsilon_1 - \epsilon_2$)

It is now instructive to assess if the orientation of strain localization in the flat punch HET experiments is captured with sufficient accuracy by the numerical models. Figure 94 shows a comparison of major strain contours at peak load for a representative DC04 test. The anisotropic Yld2000 and Yld2004 simulations replicate the observed strain localization behavior along the RD.

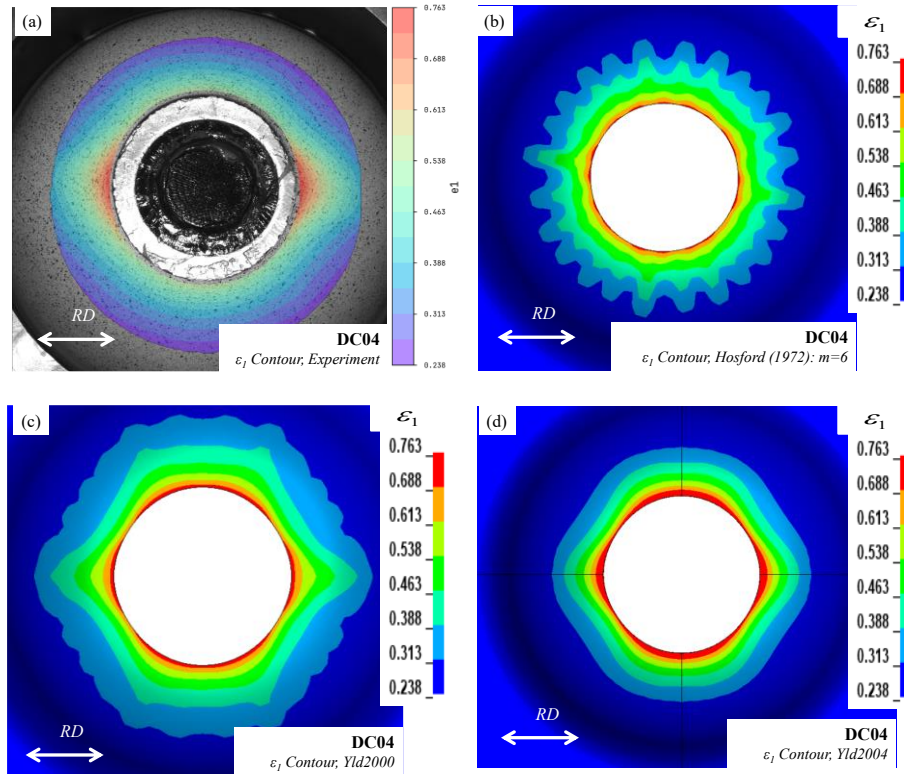


Figure 94 Major strain contours at peak load showing the onset of localization for (a) a representative experiment and simulations conducted using (b) HF72, (c) Yld2000, and (d) Yld2004 functions for DC04 steel. The Yld2004 strain contour was mirrored to display as a full model.

The location of maximum thinning for the AA7075-T6, DC04, and the 980GEN3 steels was observed at radial distances of approximately 3 mm, 5 mm, and 4 mm from the hole edge. A comparison of the circumferential thinning distribution between a representative experiment and the simulations for these three materials are shown in Figure 28. The circumferential thinning strain distributions of the AA5182-O tests were not shown because cracks initiated at the edge. Hotspots with maximum thinning occurred approximately at angles of 67.5° and 112.5° with respect to the RD for the AA7075-T6. The DC04 and 980GEN3 steels exhibited the highest thinning at angles of approximately 0° (RD) and 67.5° with respect to the RD, respectively. The Yld2000 and Yld2004 models were able to reproduce the trends in thinning accurately, with deviations being due to material localization occurring along a preferred direction in the experiments. Absolute differences in thinning strain magnitudes of up to 4% exist locally between the experiments and the simulations for DC04 and 980GEN3 steels by virtue of asymmetric deformation in the

experiments and the employed DIC lengthscale. The discrepancy in the predicted thinning values for the AA7075-T6 is attributed to the PSC not being enforced to high accuracy.

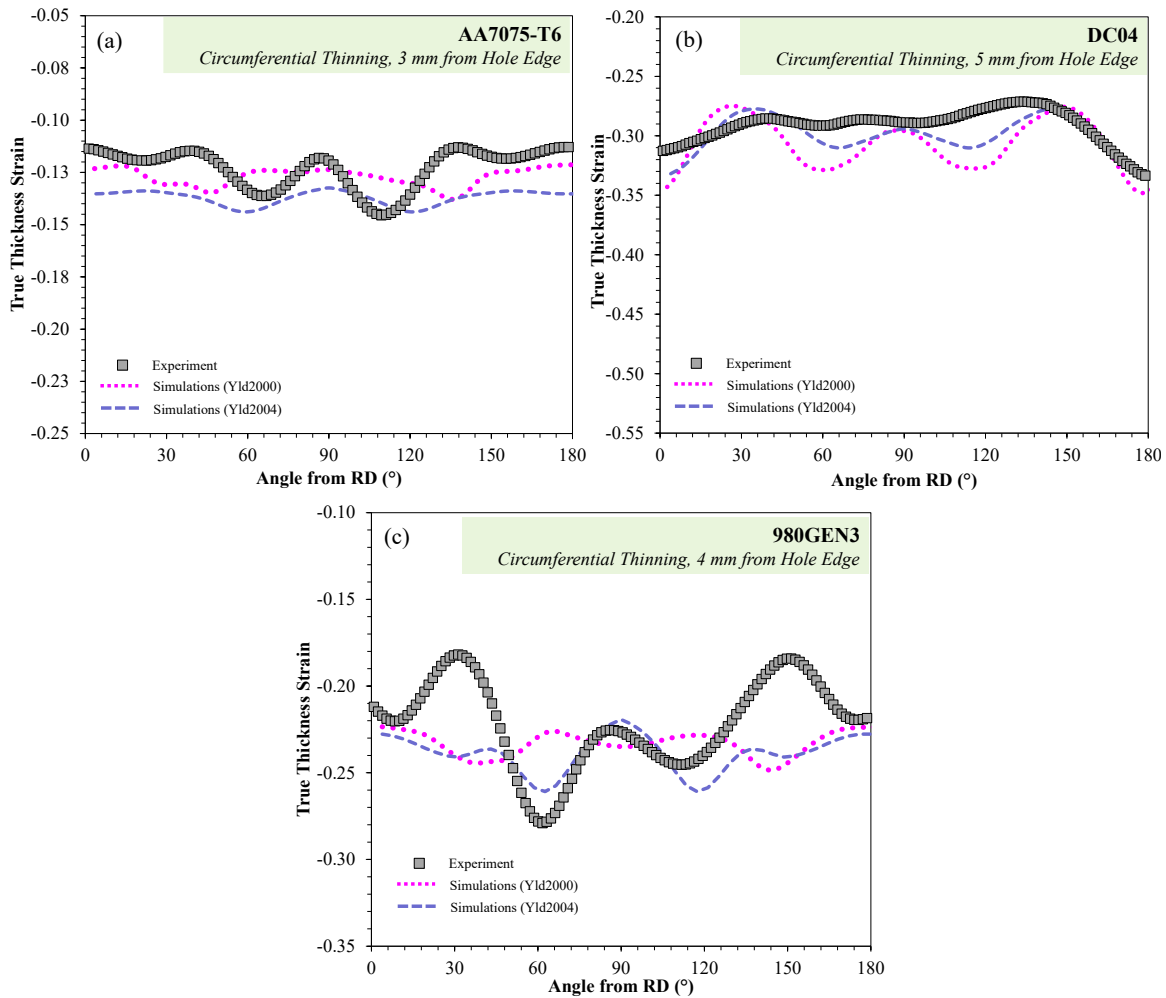


Figure 95 Comparison of thinning strain variation with respect to the material orientation between a representative experiment and numerical models for (a) AA7075-T6, (b) DC04 and (c) 980GEN3 steels at the distance corresponding to minimum thinning location

Finally, the principal strain variation as a function of the distance from the hole edge along the failure direction was assessed for the DC04 and 980GEN3 steels as shown in Figure 96 due to a stronger measure of strain localization observed in these materials than in the aluminum alloys as evident from the thinning strains in Figure 95. The HF72 model was unable to capture the maximum thinning location for both materials. The Yld2000 and Yld2004 simulations were able to capture the trends in principal strain and thinning distributions. However, the Yld2004 performed slightly better due to its superior calibration and because of the solid element's ability to preserve through-thickness stresses.

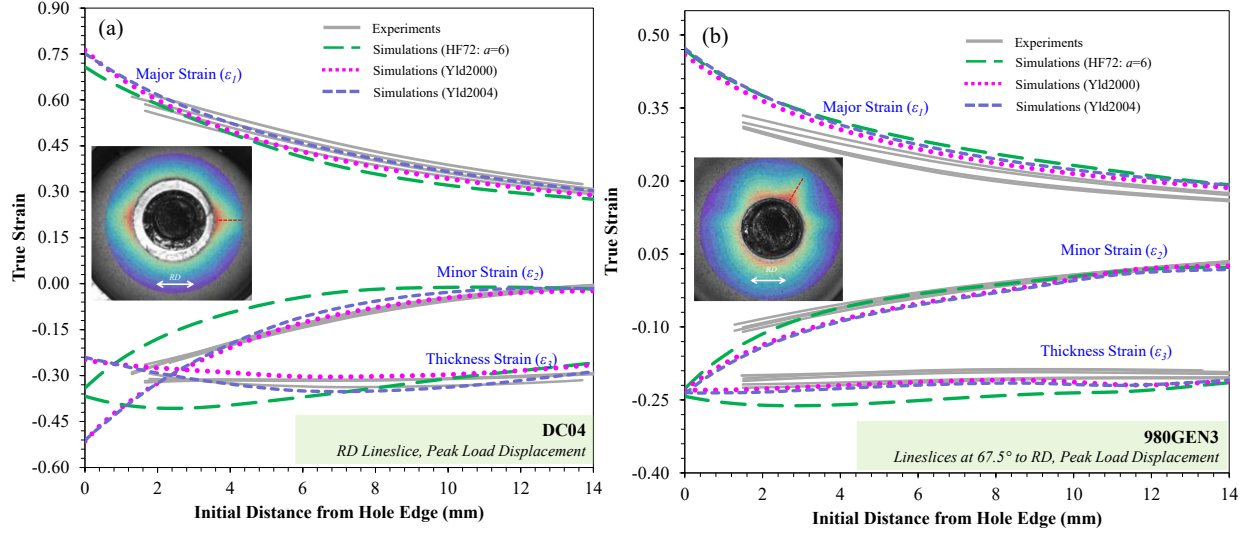


Figure 96 Comparison of local DIC strains and simulation predictions at the peak load displacement for (a) DC04 and (b) 980GEN3 steels. Lineslice considered along the respective localization directions.

A.6.3. Importance of PSC in Yield Function Calibration

Finally, an additional simulation was performed for the DC04 mild steel using the plane stress quadratic yield function of Hill [41] to highlight the significance of the plane strain constraint. The Hill-48 surface was calibrated using the tensile R-values along the RD, DD, and TD summarized in Table 1 and is shown in Figure 39(a) alongside the HF72 and Yld2000 loci that enforce the PSC. The plane strain point is predicted to occur close to equibiaxial tension at a principal stress ratio of $\left(\frac{\sigma_2}{\sigma_1}\right)_{PS} = 0.67$ using the Hill-48

function. Figure 39(b) compares the radial strain distribution away from the hole edge along the TD lineslices from simulations conducted using the three plane stress yield functions. The simulation conducted using the Hill-48 criterion consistently overestimates the magnitudes of the in-plane principal strains. Moreover, the condition of $\varepsilon_2 = 0$ is never achieved behind the edge in the Hill-48 simulation due to the location of PST on the yield surface, which broadens the range of allowable stress states on the draw side that is characterized by a negative minor strain. The observations are in line with the numerical results of Abedini *et al.* [25] when the location of PST is shifted towards equibiaxial tension. The HF72 yield function does not accurately predict the magnitude of the strain components for DC04 but correctly identifies the location of $\varepsilon_2 = 0$ since it exactly satisfies the PSC. The Yld2000 model is able to best replicate the experiments due to its ability to resolve the PS yield strength accurately as well as satisfaction of the PSC. Surprisingly, the Hill-48 model estimated the thickness distributions for DC04 with good accuracy, primarily because the overestimation of in-plane strains negated each other when computing the thickness strain from volume conservation. A similar finding was reported by Kuwabara and Ichikawa [42], where

the Hill-48 model provided reasonable estimates of thickness distribution during flat punch hole expansion of a low-carbon steel sheet, despite its suboptimal prediction of the experimental stress ratios. Overall, the PSC enforcement significantly improves the predictions of flat punch hole expansion tests for steel and aluminum alloys that can be idealized as being pressure-independent.

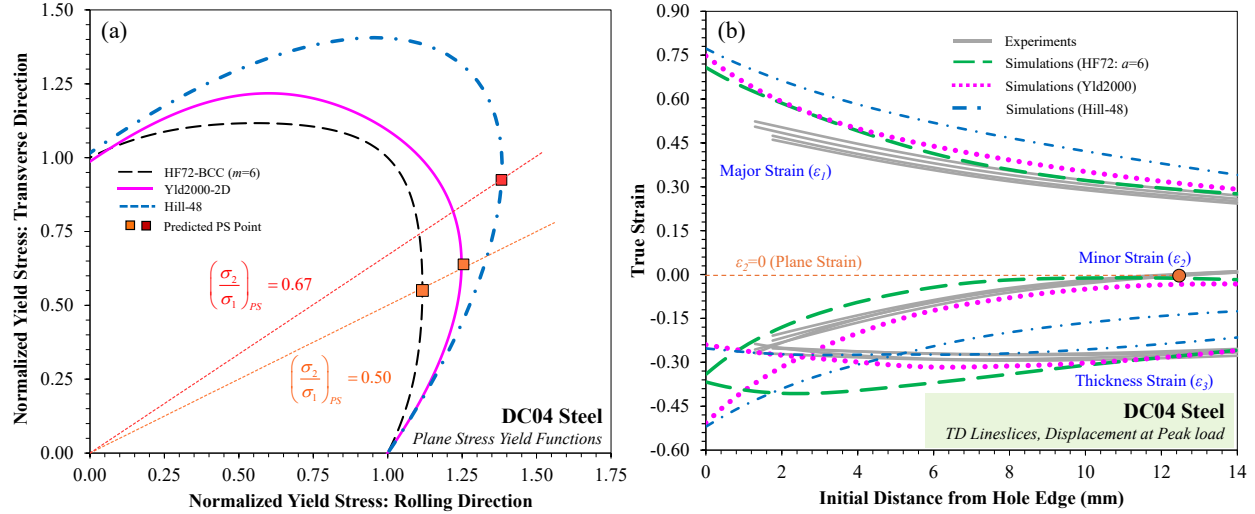


Figure 97 (a) Yield locus of Hill (1948) function in the tensile quadrant for DC04 steel calibrated using its tensile R-values and overlaid with the HF72 and Yld2000 surfaces, and the **(b)** predicted principal strains in the RD obtained from TD lineslices.

A.7. Summary & Conclusions

The current work investigated the role of yield function calibration with particular emphasis placed on the stress state of PST and its magnitude, on the predicted local strains and punch forces in the flat punch HET. The primary outcomes of this study are:

- An efficient and accurate methodology was developed to calibrate pressure-independent anisotropic yield functions that accounts for plane strain data and the generalized PSC.
- Enforcing the plane strain constraint to yield functions during the calibration stage led to accurate predictions of local strain distributions away from the hole edge in flat punch HET simulations. The finite-element models successfully captured the $\epsilon_2 = 0$ location determined using DIC for all four materials across the three evaluated material directions. These findings align with recent literature emphasizing the importance of accurately resolving the plane strain point in anisotropic yield function calibration for hole expansion simulations.

- An inverse hybrid experimental-numerical technique can be adopted for analyzing notch tensile tests to estimate the plane strain yield stress while simultaneously enforcing the PSC of Butcher and Abedini [10] when cruciform test data corresponding to a principal stress ratio of 1:2 is not available for steel and aluminum. The empirical correlation of Narayanan *et al.* [31] provided first order accuracy or better for predicting the plane strain yield strength in terms of the Lankford coefficient or R-value.
- The area reduction method can be employed to characterize the hardening response for materials with varying anisotropies and strengths to strains higher than the uniform elongation. The AA5182 and AA7075-T6 exhibited nearly isotropic hardening behavior. The 980GEN3 steel exhibited some anisotropic hardening effects at plastic strains beyond 0.40 while the DC04 mild steel exhibited significant anisotropic hardening after the tensile uniform elongation.
- The radial distribution of principal strains and the anisotropy in thinning strain along the circumferential direction could be accurately estimated by the anisotropic Yld2000 and Yld2004 models. The DC04 steel posed the greatest challenge for modelling, particularly in the RD, due to its anisotropic hardening and the normalized plane strain stress exceeding 1.20.

A.8. Acknowledgments

Technical and financial support for this work was provided by the Steel Testing and Harmonization (STHT) and Weld/HAZ Modeling project teams of the Auto/Steel Partnership (A/SP), Natural Sciences and Engineering Research Council of Canada (NSERC), Hexagon Manufacturing Intelligence, and Men at Work GmbH. Special thanks are extended to Kenneth Cheong, Dr. Armin Abedini, and Professor Michael Worswick from the University of Waterloo for their valuable insights and discussions on the mechanics of the flat punch hole expansion test, which significantly contributed to the results of this work.

A.9. Declaration of Competing Interest

The authors confirm that they have no conflict of interest.

A.10. Data Availability Statement

The data generated and analyzed during this study are not publicly available due to confidentiality reasons, the ongoing nature of the project, and the involvement of industry partners.

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Appendix B: An Experimental Methodology to Characterize the Uniaxial Fracture Strain of Sheet Metals using the Conical Hole Expansion Test

B.1. Introduction

The conical hole expansion test (HET) of ISO16630 (ISO16630, 2017) is extensively used throughout the automotive industry to evaluate the edge stretching limits of sheet materials with sheared edges. The test involves punching a 10 mm diameter hole at a prescribed 12% clearance, clamping the blank in a press, and then extruding the hole with a 60° conical punch until a through-thickness crack initiates at the edge. The inner hole diameter at fracture is then measured and reported as the hole expansion ratio (HER). The mechanics of the test are best described as a hole extrusion process as noted by Levy and van Tyne (2008) despite being referred to as a hole expansion test within the ISO standard and throughout the literature. It is important to differentiate the conical HET from tests that use a cylindrical punch to expand the hole in-plane and are also commonly referred to as hole expansion tests. The mechanics of the cylindrical or ‘flat punch’ hole expansion test are primarily governed by plane strain tensile instability that occurs behind the edge (Barnwal *et al.*, 2020). The flat punch hole expansion test is useful for evaluating anisotropic plasticity models to predict the thinning distribution (*e.g.* Kuwabara *et al.*, 2017; Amaishi *et al.*, 2020; Ha and Korkolis, 2021) as the edge is in uniaxial tension, while the stress state transitions to in-plane biaxial stretching behind the edge as reported by Pathak *et al.* (2016) and Ha *et al.* (2020).

The conical hole expansion test deforms the specimen out-of-plane and has a through-thickness strain gradient due to punch contact with the inner edge such that the tensile strains are concentrated at the outer edge to initiate fracture. Figure 1 shows a schematic of the conical hole expansion test and the through-thickness strain gradients at the hole edge. The conical hole expansion test can produce vastly larger hole expansion ratios than in-plane hole expansion with flat punches as documented by Pathak *et al.* (2016) because it provides an estimate of the uniaxial fracture strain, while the in-plane hole expansion test is limited by the plane strain tension formability limit. The differences between the tests are most apparent for a machined hole condition compared to a sheared or punched hole. The severity of the residual strains in the so-called shear-affected zone (SAZ) generated near the edge after punching can dominate the behavior for some materials like DP780 steel and provide similar hole expansion ratios for both conical and flat punches (*e.g.* Chiriac, 2010; Pathak *et al.*, 2016). The present study focuses upon the fracture characterization of the base material in uniaxial tension and will only consider the conical hole expansion test with a machined edge to omit the influence of the SAZ.

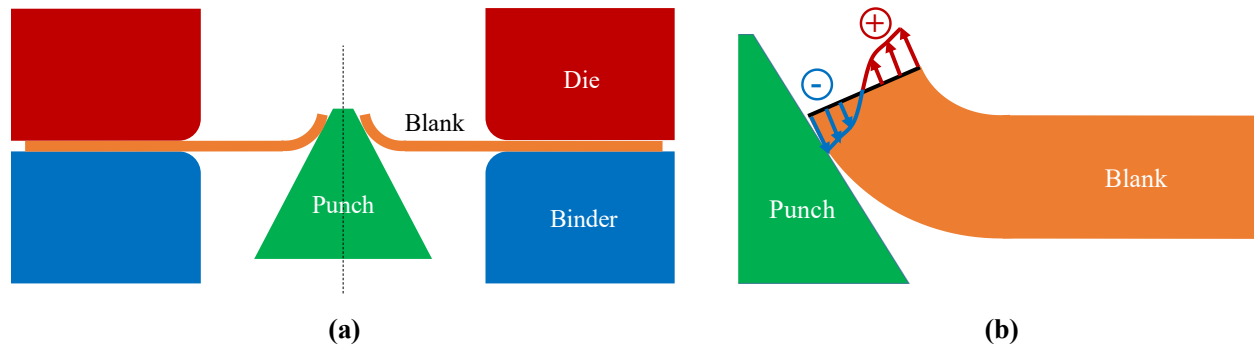


Figure 98 A schematic of the conical hole expansion test: (a) deformed configuration and (b) through-thickness strain gradient due to out-of-plane bending of the blank during the early stages of the test. The material initially in compression due to punch contact will also transition to uniaxial tension at larger deformation levels.

Advances in the experimental fracture characterization of automotive sheet metals over the past decade have been driven by the adoption of stress-state dependent fracture models for use in forming and crash simulations (*e.g.* Neukamm *et al.*, 2009; Bai and Wierzbicki, 2010; Chen *et al.*, 2011; Wang *et al.*, 2014). To construct a plane stress fracture locus under proportional loading, fracture strains are needed in the four primary stress states of uniaxial, plane strain and biaxial tension along with simple shear. Great strides have been made towards characterizing the fracture limits in plane strain tension and equibiaxial stretching by adopting a small radius punch to suppress through-thickness necking (Roth and Mohr, 2016; Cheong *et al.*, 2018). In-plane simple shear tests do not have tensile instability but are prone to premature edge fracture due to tensile stresses at the notch boundaries. The development of novel simple shear test geometries is an active area of research with many promising geometries proposed (Peirs *et al.*, 2012; Roth and Mohr, 2018; Traphöner *et al.*, 2019). Fracture characterization in uniaxial tension has proven challenging as necking generally precedes fracture and produces a non-linear strain path and evolving triaxial stress state. The local strains that develop within the center of the neck in standard or notched tensile tests cannot be measured experimentally as digital image correlation (DIC) techniques can only measure surface strains. Consequently, inverse finite-element analysis (FEA) is widely used to determine the fracture strain at the critical location for constructing a fracture locus with a pre-assumed damage model (*e.g.* Wang *et al.*, 2014; Lou *et al.*, 2017). The methodology, although attractive, is closely tied to the assumed plasticity and damage models, element size and element type (shells or solids).

The so-called central hole tension test or hole tension test has emerged as an alternative to the classic tensile test for uniaxial fracture characterization as the free surface of the hole remains in uniaxial tension. The strain gradient that develops across the gauge width ensures the largest strains occur at the hole edge as shown in Dunand and Mohr (2010). The hole tension test is better suited for inverse FEA than the standard tensile test since strain localization occurs consistently at the same location. However, tensile instability

and significant void formation may occur behind the hole edge in the hole tension test as observed for a DP600 by Anderson *et al.* (2017). The uniaxial fracture strain will be underestimated if failure occurs behind the edge where the stress state is between uniaxial and plane strain tension. The hole tension test geometry can be tailored for a given material to promote fracture in uniaxial tension at the edge. Roth and Mohr (2016) proposed a hole tension test geometry with a hole radius of 2 mm with a gauge width of 20 mm for a 1 mm thick sheet based upon a comprehensive numerical analysis. Fracture in hole tension tests will eventually start from the mid-thickness plane irrespective of the specimen geometry and cannot be measured directly using DIC techniques.

Butcher *et al.* (2013) recognized that the favorable mechanics of the conical hole expansion test could be exploited to obtain an experimental estimate of the fracture strain in uniaxial tension. The outer hole edge remains in uniaxial tension, similar to the hole tension test but is supplemented by a favorable through-thickness strain gradient which mitigates tensile instability. Fracture in conical hole expansion tests initiates at the outer edge (or potentially just behind the edge) and the crack propagates through the thickness so the average uniaxial fracture strain can be estimated from the outer diameter. The conical HET can be used to characterize the uniaxial fracture limit provided that the hole is machined rather than punched as in the ISO16630 standard. An ever-present concern in both the hole tension test and conical hole expansion test is the quality of the machined edge condition that could cause premature fracture due to machining defects. Roth and Mohr (2016) noted that the fracture strains from hole tension tests are very sensitive to the cutting process and recommended using CNC milling due to a small size of the machining affected area of less than 2 μm . A limitation of the conical hole expansion test compared to the hole tension test is that fracture anisotropy cannot be investigated as the weakest or limiting direction of the material will always fail first.

Pathak *et al.* (2017) experimentally observed that the uniaxial fracture strains evaluated from conical hole expansion tests of CP800 and DP780 steels were in close agreement with local fracture strains measured near the hole edge in hole tension tests using stereoscopic DIC techniques. Although promising from an experimental perspective, questions remain in the underlying analysis employed to evaluate the hole expansion test since a non-circular hole shape and non-uniform thinning around the hole edge were observed for CP800 by Pathak *et al.* (2016). The equivalent fracture strain was estimated by Pathak *et al.* (2017) by assuming the von Mises yield function to combine the circumferential strain from the average of inner and outer diameters with the thickness measured at the fracture location. This approach is reasonable as a first-order estimate of the local fracture strain but neglects the potential for necking and the influence of plastic anisotropy. The use of a best-fit or average diameter to measure the fracture strain also appears problematic as it does not reflect the local conditions at the fracture location if the hole becomes non-circular. The inclusion of the thickness strain at the edge in the fracture strain calculation to account for the

local behavior is reasonable but it averages the non-uniform through-thickness stress and strain fields. Furthermore, it remains unclear if non-uniform thinning around the hole circumference is due to diffuse necking, a manifestation of anisotropy, or a combination of both effects. A general methodology is required to interpret the hole expansion test results considering a wide range of automotive materials with different anisotropy and ductility levels for it to have practical value. If the hole measurements cannot be directly related to the local stress and strain fields at the fracture location, inverse analysis is required. In this case, the hole tension test would be preferable due to its simpler finite-element modeling.

The objective of the present study is to analyze the influence of anisotropy and potential localization within the conical HET for developing a methodology to relate the test measurements to the uniaxial fracture strain without inverse analysis. Two automotive advanced high strength steel (AHSS) grades are considered: a DP1180 steel with mild anisotropy and moderate ductility, and the hot-rolled CP800 of Pathak *et al.* (2016) with strong anisotropy, a low hardening rate, and relatively high local ductility. Finite-element simulations of the conical hole expansion test are performed to determine the local strain and stress history at the hole edge in ductile anisotropic materials to identify whether necking occurs. Finally, a novel yet simple methodology is proposed and evaluated to estimate the uniaxial fracture strain for anisotropic materials.

B.2. Materials

B.2.1. Constitutive Characterization

Two AHSS grades were considered to provide insight into the mechanics of the conical hole expansion test for materials with different local ductility levels. An uncoated DP1180 automotive structural steel sheet with a 1.0 mm nominal thickness and mild anisotropy is the primary material of interest. The hot-rolled CP800 of Pathak *et al.* (2016) with 2.9 mm thickness is also revisited to consider a thicker, more ductile steel with stronger anisotropy. The constitutive characterization for the same lot of DP1180 and CP800 in different directions was conducted and published in Abedini *et al.* (2020) and Pathak *et al.* (2020), respectively. A summary of the mechanical properties of the materials in the rolling direction (RD) is provided in Table 1. The r -values and normalized yield stresses in various directions are reported in Table 2. The planar anisotropy, denoted by Δr , is also provided in Table 2 and calculated as:

$$\Delta r = \frac{r_0 - 2r_{45} + r_{90}}{2} \quad (\text{Eq. B1})$$

The values of Δr for DP1180 and CP800 were found to be -0.05 and -0.51, respectively.

Table 13 A summary of mechanical properties for the studied materials in the rolling direction. The uniform and total elongation were identified based on a DIC extensometer length of 50 mm for DP1180 and 25 mm for CP800

Material/thickness	Yield strength 0.2% offset (MPa)	Ultimate tensile strength (MPa)	Uniform elongation (%)	Total elongation (%)
DP1180/1.0 mm	841	1193	7.1	12.5
CP800/2.9 mm	710	810	7.0	19.6

Table 14 Uniaxial r-values and normalized yield stresses in uniaxial tension and shear. w^p corresponds to the plastic work per unit volume at the point of uniform elongation in the tensile tests of the limiting direction for DP1180 (transverse or TD). The plastic work level for CP800 corresponds to when the ultimate tensile strength (UTS) is reached in the rolling direction (see Pathak et al., 2020). Tensile and shear test data at intermediate angles was not available for CP800.

Material	w^p [MJ/ m^3]	σ_0/σ_0	σ_{15}/σ_0	σ_{30}/σ_0	σ_{45}/σ_0	σ_{60}/σ_0	σ_{75}/σ_0	σ_{90}/σ_0	τ_0/σ_0	$\tau_{22.5}/\sigma_0$
DP1180	61.11	1.000	0.995	0.996	1.004	1.008	1.013	1.025	0.600	0.600
CP800	55.20	1.000	-	-	0.980	-	-	1.080	-	-
Material	τ_{45}/σ_0	r_0	r_{15}	r_{30}	r_{45}	r_{60}	r_{75}	r_{90}	r_b	Δr
DP1180	0.612	0.82	0.84	0.90	0.95	0.98	1.00	0.98	0.94	-0.05
CP800	0.630	0.70	-	-	1.33	-	-	0.95	-	-0.51

B.2.2. Isotropic Hardening Response

The flow stress curves of the two materials in the RD were obtained by combining the data from tensile and shear tests as shown in Abedini *et al.* (2020) and Pathak *et al.* (2020). The hardening response of DP1180 was captured using the modified Hockett-Sherby (MHS) model (Eq. 2) while a Swift law (Eq. 3) was found to be sufficient for CP800 by Pathak *et al.* (2020). The hardening functions are defined as

$$\sigma = B - (B - A) \exp\left(-C \{\varepsilon^p\}^D\right) + E \sqrt{\varepsilon^p} \quad (\text{Eq. B2})$$

$$\sigma = K \left(\varepsilon_0 + \varepsilon^p\right)^n \quad (\text{Eq. B3})$$

where ε^p is the equivalent plastic strain and σ is the equivalent stress. The coefficients of the hardening models are presented in Table 15 while the flow stress curves for the two materials are depicted in Figure 99.

Table 15 Hardening model coefficients for the two materials in the rolling direction obtained from uniaxial tensile and shear test data. The hardening model coefficients for DP1180 and CP800 are published in Abedini et al. (2020) and Pathak et al. (2020), respectively

Coefficients	A (MPa)	B (MPa)	C	D	E (MPa)	K (MPa)	ϵ_0	n
DP1180	665.41	1092.85	10.17	0.50	310.13	-	-	-
CP800	-	-	-	-	-	1024	0.0067	0.07

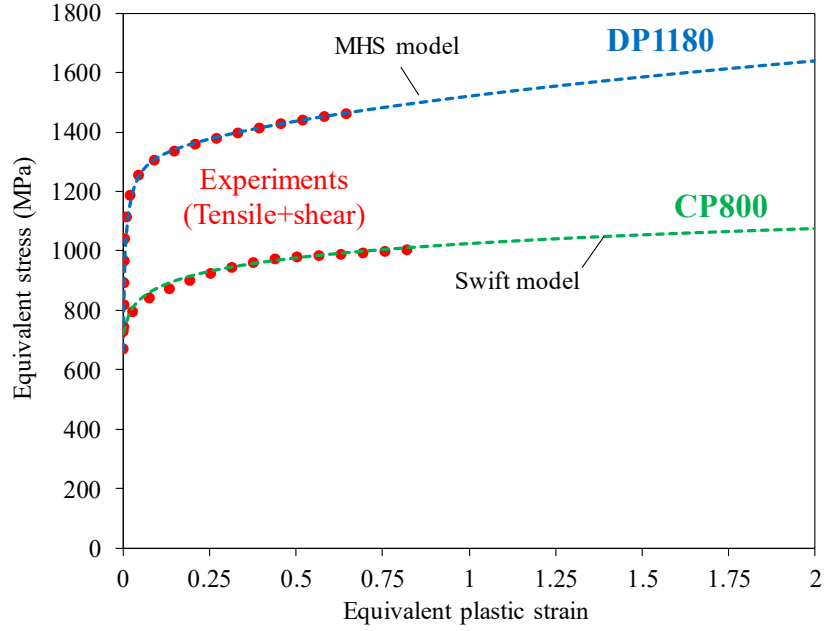


Figure 99 Flow stress curves for the studied materials along the RD overlaid with the predictions from the hardening models

B.2.3. Yield Function Calibration

The non-quadratic anisotropic Yld2004 criterion (Barlat *et al.*, 2005) was selected in the present study as it is capable of accounting for 3-D stress states that materials may experience during a hole expansion test. The yield function can be written as:

$$\begin{aligned}
 4\sigma^a = & \left| \tilde{S}'_1 - \tilde{S}''_1 \right|^a + \left| \tilde{S}'_1 - \tilde{S}''_2 \right|^a + \left| \tilde{S}'_1 - \tilde{S}''_3 \right|^a + \left| \tilde{S}'_2 - \tilde{S}''_1 \right|^a + \left| \tilde{S}'_2 - \tilde{S}''_2 \right|^a + \left| \tilde{S}'_2 - \tilde{S}''_3 \right|^a \\
 & + \left| \tilde{S}'_3 - \tilde{S}''_1 \right|^a + \left| \tilde{S}'_3 - \tilde{S}''_2 \right|^a + \left| \tilde{S}'_3 - \tilde{S}''_3 \right|^a
 \end{aligned} \quad (\text{Eq. B4})$$

where $\tilde{S}'_i$ ($i = 1,2,3$) and $\tilde{S}''_j$ ($j = 1,2,3$) are the principal values of the linearly transformed deviatoric stress tensors $\tilde{\mathbf{s}}'$ and $\tilde{\mathbf{s}}''$, respectively, a is the yield function exponent with typical values of 6 and 8 for BCC and FCC materials, respectively. The linearly transformed stress deviator tensors are derived by:

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3} \text{tr}(\boldsymbol{\sigma}) \mathbf{I} \quad (\text{Eq. B5})$$

$$\tilde{\mathbf{s}}' = \mathbf{C}' \mathbf{s} \quad \text{and} \quad \tilde{\mathbf{s}}'' = \mathbf{C}'' \mathbf{s} \quad (\text{Eq. B6})$$

The transformation tensors $\mathbf{C}'$ and $\mathbf{C}''$ are given by

$$\mathbf{C}' = \begin{bmatrix} 0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\ -c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\ -c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{66} \end{bmatrix}$$

$$\mathbf{C}'' = \begin{bmatrix} 0 & -c''_{12} & -c''_{13} & 0 & 0 & 0 \\ -c''_{21} & 0 & -c''_{23} & 0 & 0 & 0 \\ -c''_{31} & -c''_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c''_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c''_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c''_{66} \end{bmatrix} \quad (\text{Eq. B7})$$

Using the transformation tensors given in Eq. (B7), a total of 18 coefficients have to be calibrated to completely define the Yld2004 criterion. The experimental data from various constitutive characterization tests (*e.g.* uniaxial tension, simple shear, plane strain, and equibiaxial tension tests) can be used to calibrate the coefficients of the model. The coefficients c'_{44} , c'_{55} , c''_{44} and c''_{55} correspond to through-thickness shear properties of the materials. Due to experimental difficulties in characterizing the through-thickness shear properties of sheet metals, it was assumed that $c'_{44} = c'_{55} = c'_{66}$ and $c''_{44} = c''_{55} = c''_{66}$.

The yield function coefficients of DP1180 were calibrated using the experimental tensile and shear data reported in Table 2. Additional data points not measured directly from the experiments such as the estimated equibiaxial yield stress ($\sigma_b = \frac{\sigma_0 + 2\sigma_{45} + \sigma_{90}}{4}$), and the estimated plane strain tensile yield stress reported by Fast-Irvine *et al.* (2021) using notch tensile tests of the same batch of material were also considered and are reported in Table 16. The generalized plane strain constraints of Butcher and Abedini (2019) were enforced in the calibration of the yield function.

Only seven experimental data points were available for CP800 (see Table 2). No notch tensile results were available to estimate the plane strain tensile yield strength of the material using the DIC-based methodology

of Fast-Irvine *et al.* (2021). Besides employing the estimated equibiaxial tensile yield stress, plane strain tensile yield stresses of the material in the rolling, diagonal (DD), and transverse directions were estimated using the Hosford (1985) yield criterion with an exponent of $a=6$. The corresponding values reported in Table 16 were utilized in the calibration of the Yld2004 model. The plane strain constraint for deviatoric plasticity was also used to generate the normal vectors for plane strain tensile loading in the RD, DD, and TD for providing sufficient data points for calibration. The yield function calibration for both materials was focused on the tensile quadrant to be more appropriate for the conical hole expansion test simulations since uniaxial tensile stress is expected to prevail at the outer edge.

Table 16 The estimated values of yield stresses under plane strain and equibiaxial tension normalized by the uniaxial tensile yield stress in the rolling direction. The plane strain tensile yield stresses for CP800 have been estimated using the Hosford (1985) yield function with a flow exponent of 6 while the yield stresses in equibiaxial tension were estimated from the uniaxial tensile stresses for both materials.

Material	DP1180	CP800
σ_0^{PS}/σ_0	1.099*	1.117**
$\sigma_{45}^{PS}/\sigma_0$	1.119*	1.094**
$\sigma_{90}^{PS}/\sigma_0$	1.163*	1.206**
σ_b/σ_0	1.008***	1.010***
* Plane strain tensile yield stresses of DP1180 determined from DIC analysis of notched tensile tests performed by Fast-Irvine <i>et al.</i> (2021)		
**Plane strain tensile yield stresses of CP800 estimated from Hosford (1985) yield function with $a = 6$		
***Equibiaxial tensile yield stresses estimated using $\sigma_b = \frac{\sigma_0 + 2\sigma_{45} + \sigma_{90}}{4}$		

The calibrated Yld2004 model coefficients for the two materials are listed in Table 17. The calibrated yield locus and its prediction of the experimental yield stresses and r-values in various directions are shown in Figure 100 and Figure 101, respectively.

Table 17 Yld2004 coefficients for the materials calibrated using the experimental data

Material	c'_{12}	c'_{13}	c'_{21}	c'_{23}	c'_{31}	c'_{32}	c'_{44}	c'_{55}	c'_{66}
DP1180	0.9121	0.9047	0.9840	0.9274	0.9078	0.9288	0.8628	0.8628	0.8628
CP800	1.1621	0.4634	0.9543	0.0111	1.2512	1.1628	1.4157	1.4157	1.4157
Material	c''_{12}	c''_{13}	c''_{21}	c''_{23}	c''_{31}	c''_{32}	c''_{44}	c''_{55}	c''_{66}
DP1180	0.9954	1.0430	0.9924	1.0923	1.1545	0.9682	1.1200	1.1200	1.1200
CP800	0.8587	0.7918	0.6390	0.8698	0.4378	0.5363	0.5560	0.5560	0.5560

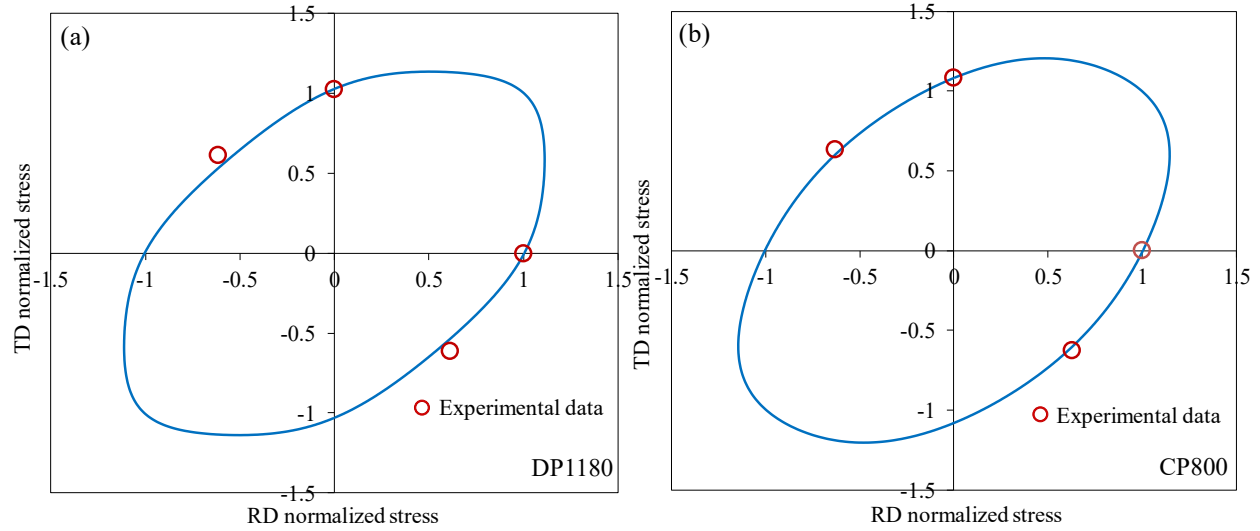


Figure 100 The calibrated Yld2004 loci of (a) DP1180 and (b) CP800. The solid blue curve is the calibrated yield function defined by Eq. (B4) – Eq. (B7) with the corresponding coefficients provided in Table 17.

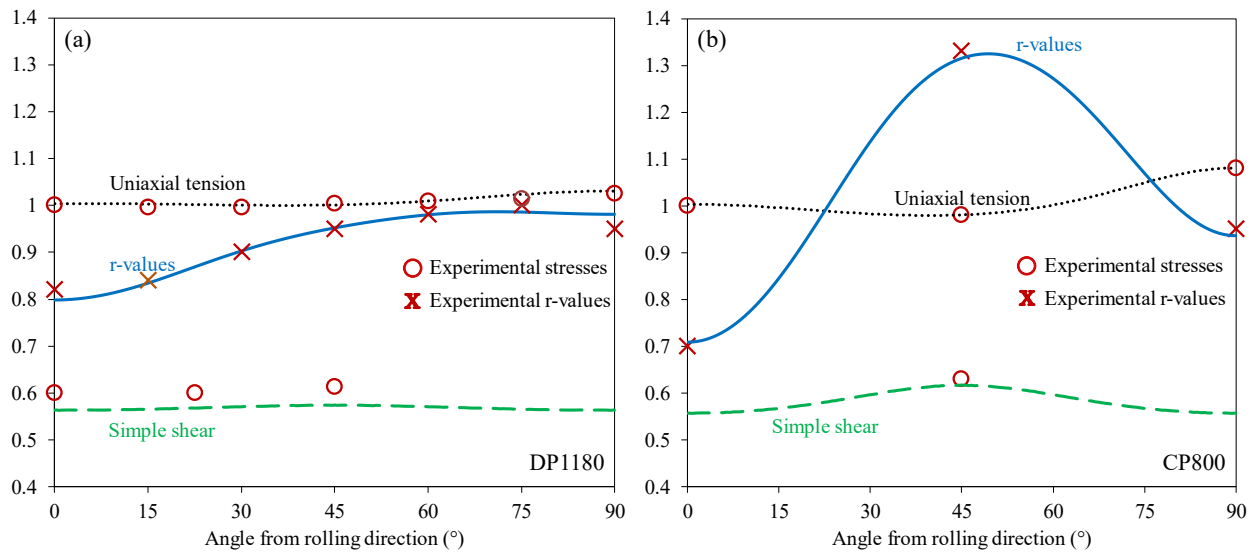


Figure 101 Predictions of the calibrated Yld2004 function for (a) DP1180 and (b) CP800. All stresses were normalized with respect to the uniaxial tensile yield stress in the rolling direction. The experimental data is denoted by red markers while the yield function predictions are depicted as solid and dotted curves.

B.3. Conical Hole Expansion Tests

The conical hole expansion experiments were conducted according to the ISO 16630 standard but with the hole processed by drilling followed by reaming to the final desired hole diameter of 10 mm. The hole edge was carefully sanded using 800 and 1000 grit sandpapers after reaming to remove any machining burrs and minimize surface defects during hole processing. The initial hole diameter was measured at four different orientations after polishing with the average diameter reported in Table 18. The standard deviation

of the initial hole diameter was up to 0.02 mm for the nominal 10 mm hole size. A two dimensional (2-D) schematic of the setup and dimensions for the tests are shown in Figure 102. A square blank with a dimension of $135 \times 135 \text{ mm}^2$ was considered with the hole at the center of the blank. The conical punch had a maximum diameter of 67 mm with an included angle of 60° set as per the ISO 16630 guidelines. The stationary die had an internal diameter of 105.6 mm with a die entry radius of 6.35 mm. The

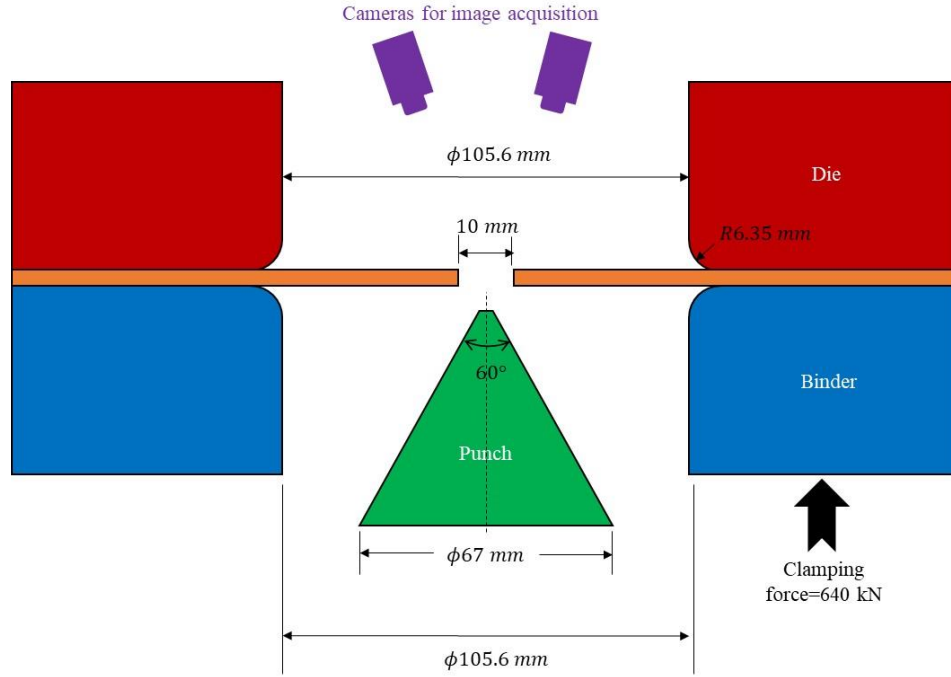


Figure 102 A schematic of the hole expansion test setup

The punch speed was set to be constant at 1 mm/s during the experiments, while the hydraulic binder imposed a high clamping force of 640 kN to prevent blank draw-in during the test. A total of 10 repeats were conducted per material to ensure the repeatability of test data. The hole expansion tests for DP1180 were recorded using Gazelle Point Grey® 4.1 MP cameras (model GZL-CL-41C6M-C) and two Schneider Xenoplan® lenses with a 35 mm focal length and a frame rate of 7 frames per second. The test was stopped as soon as a through-thickness crack was visually detected in the monitoring system. The first image that showed the onset of a through-thickness crack was chosen from the captured data, and the hole geometry was measured using Image-Pro Plus® software. An image of a DP1180 specimen at the instant of appearance of a through-thickness crack is shown in Figure 103. The hole geometry was evaluated from both cameras and subsequently the measured hole diameters were averaged to correct for any measurement errors induced due to inclination of cameras with respect to the blank surface. The differences between the measured diameters from the two complementary images were less than 0.10 mm so an additional correction was not employed.

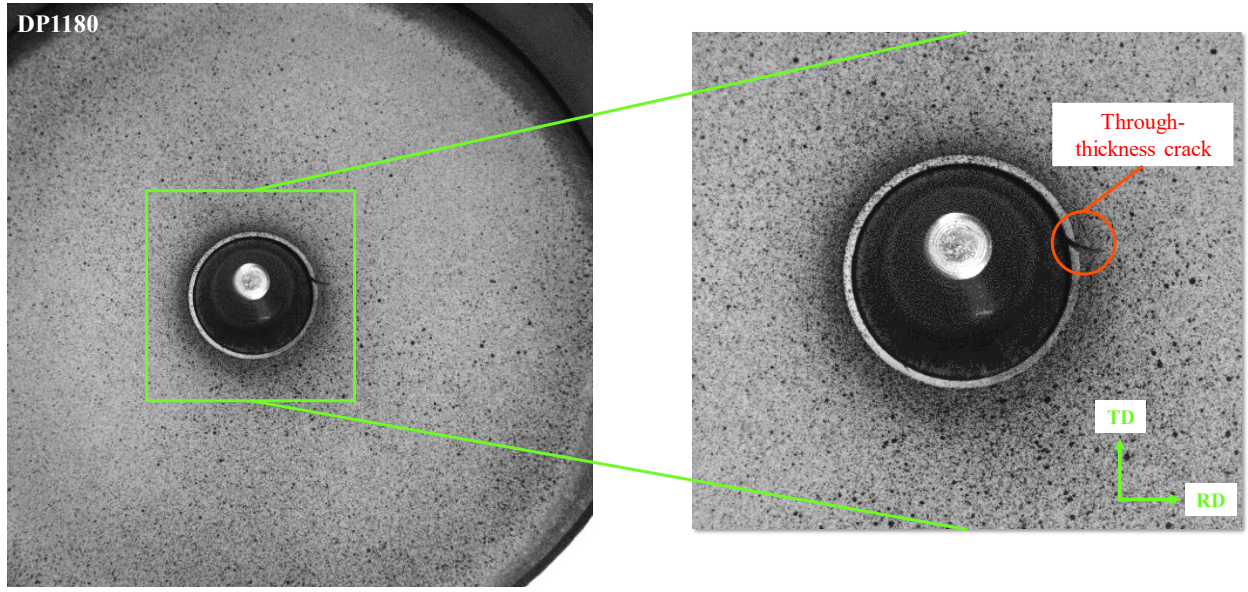


Figure 103 An image of a DP1180 test sample when a through-thickness crack is first observed. The appearance of eccentricity between the punch and the blank hole is due to camera inclination for stereoscopic DIC measurements. The red circle denotes the location of through-thickness crack

The hole geometry measurements for the two materials at the instant of fracture are tabulated in Table 18. Measurements of the inner and outer hole diameters were conducted at various locations and their corresponding best-fit values were evaluated by:

$$D_{inner}^{BF} = \frac{\left(\sum_{k=1}^n D_{inner}^k \right)}{n} ; D_{outer}^{BF} = \frac{\left(\sum_{k=1}^n D_{outer}^k \right)}{n} \quad (\text{Eq. 8a,b})$$

where D_{inner}^{BF} and D_{outer}^{BF} are the best-fit inner and outer hole diameters respectively, n is the number of measurements, D_{inner}^k ($k = 1 \dots n$) and D_{outer}^k ($k = 1, \dots n$) are the individual inner and outer hole diameter measurements. The outer hole diameter at the failure location (D_{outer}^F) was also measured to assess the local equivalent plastic strain at fracture. The correlation between D_{outer}^F and the local fracture strain will be discussed in Section B.6. The value of D_{outer}^F is specific to the crack location whereas D_{outer}^{BF} is the outer diameter averaged from measurements conducted at multiple locations. The distinction between the two quantities is further elaborated schematically in Figure 104.

The best-fit hole expansion ratio is calculated as:

$$HER(\%) = \frac{D_{inner}^{BF} - D_0}{D_0} \times 100 \quad (\text{Eq. B9})$$

where D_0 is the initial measured hole diameter. The HER can be sensitive to measurement resolution of the initial and final diameters. The absolute error in HER due to the initial and final hole diameter measurements was on the order of 1%, which is lower than the observed variation in HER between repeated tests.

Table 18 Measurements of the hole geometry at the fracture image using Image-Pro Plus® software. The values in the brackets denote the standard deviation.

Material	Fracture location	Initial hole diameter (mm)	Best-fit inner hole diameter, D_{inner}^{BF} (mm)	Best-fit outer hole diameter, D_{outer}^{BF} (mm)	Outer hole diameter at fracture location, D_{outer}^F (mm)	HER (%)	$\Delta D = D_{outer}^{BF} - D_{outer}^F$ (mm)
DP1180	RD	9.97 (0.01)	16.57 (0.13)	17.91 (0.20)	17.75 (0.23)	66.00 (1.30)	0.16 (0.20)
CP800	RD	10.00 (N/A*)	29.00 (0.73)	31.19 (N/A*)	30.95 (N/A*)	190.00 (7.30)	0.92 (N/A*)

* Except for the HER, the standard deviations were not reported in Pathak *et al.* (2016)

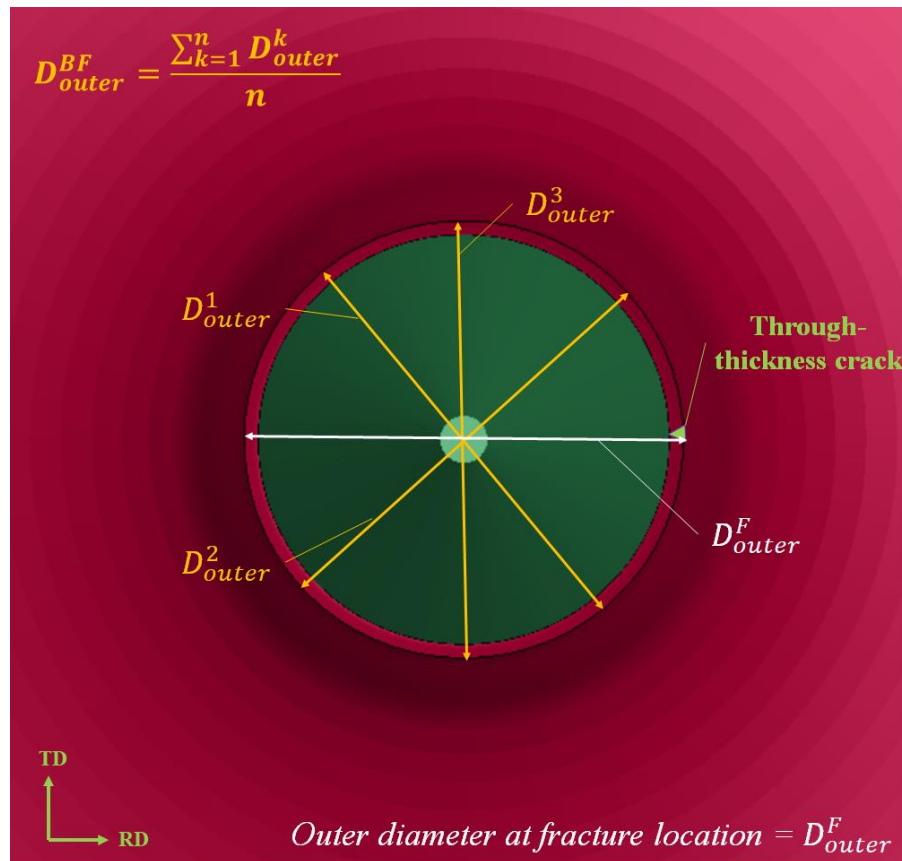


Figure 104 A schematic illustrating the difference between the best-fit and the local outer hole diameters measured at the image when a through-thickness crack is first seen.

The experimental values of ΔD ($=D_{outer}^{BF} - D_{outer}^F$) for DP1180 and CP800 were 0.16 mm and 0.92 mm, respectively, which are in line with their corresponding plastic anisotropy indicating that the divergence of the hole shape from a circle is more severe for CP800. A through-thickness crack was always observed in the RD for DP1180 and CP800 as the TD is generally the limiting direction for fracture in AHSS. The crack direction in the hole expansion test is always perpendicular to the limiting direction that is subjected to tensile loading. Uniaxial tensile stress at a given location on the outer hole edge acts perpendicular to the radius vector from the origin to that location. This is shown schematically in Figure 105 where the radius vectors from the center of the hole to the location of interest (R_0 , R_{45} , and R_{90}) are perpendicular to the stress directions at those locations (*i.e.* σ_{90} , σ_{45} , and σ_0 , respectively). The stresses acting at 45° and 135° angles from the rolling direction are equivalent due to assumed material symmetry.

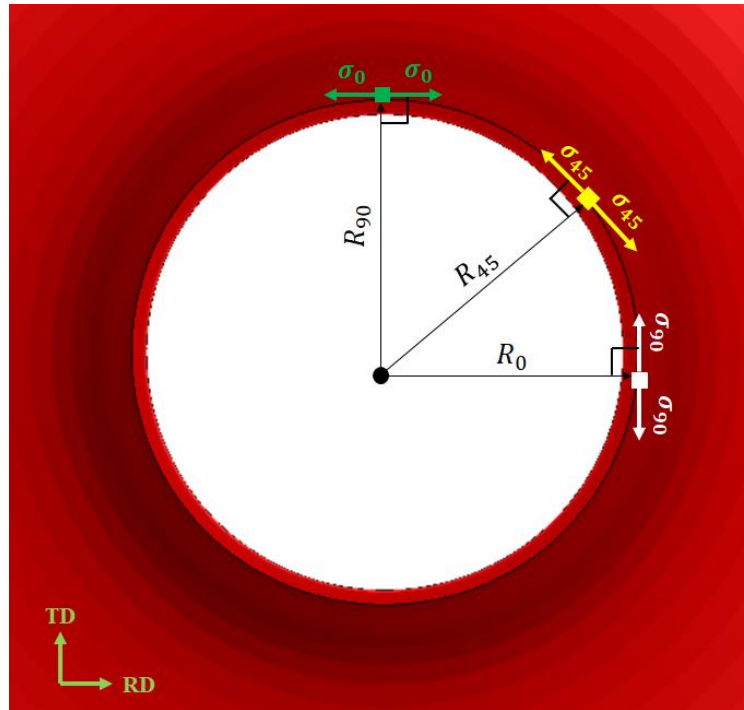


Figure 105 A schematic illustrating the nature of circumferential stretching during conical HET. The stress direction at a given location on the outer hole edge is perpendicular to the radius vector from the hole center to that location. ‘R’ denotes the radius vector from the origin and the subscripts indicate the angles of the locations with respect to the rolling direction

B.4. Finite-Element Modeling of the Conical Hole Expansion Test

B.4.1. Development of the Finite-Element Model

Finite-element models of the conical HET were developed to analyze the influence of plastic anisotropy and necking on the material behavior during deformation and to obtain the local stress and strain history near the hole edge. A full model of the HET geometry was created in LS-DYNA with a one-fourth

view of the mesh shown in Figure 106. Preliminary simulations using a reduced model with quarter symmetry was found to negatively affect the thickness measurements of the hole edge at the symmetry planes so the full model was adopted. The steel die, binder, and punch were modeled as rigid surfaces and meshed with shell elements. The coefficient of friction between the contact surfaces of the blank and the tools was assumed to be 0.10. A preliminary study on the effect of coefficient of friction indicated that the elements located on the outer hole edge are not affected by friction as they are not in contact with the punch.

The blank was meshed with hexahedral solid elements with a size of approximately 10% of the sheet thickness near the hole edge. Shell elements were not used for the blank due to its inability to account for three-dimensional (3-D) stress state in the material. A mesh convergence study was performed for DP1180, and it was found that the use of smaller elements did not affect the force *versus* stroke curve. The mesh transitioned to a coarser size away from the hole edge since the area near the edge is critical during the conical hole expansion test. A fully integrated selectively-reduced element formulation was used for the elements of the blank (element type of -1 in LS-DYNA) with hourglass control using the formulation of Puso (2000) and a coefficient of 0.15. The die was kept stationary while the binder moved upwards to exert a constant clamping force of 640 kN on the blank to mimic the experiment. The punch was then engaged with a constant velocity of 1 mm/s. The developed model is based upon an ideal edge condition without any residual strain from the hole fabrication process. A fracture model was not incorporated into the simulations as the focus is to evaluate the stress state and material behavior during the conical HET. The Invariant Node Numbering (INN) option was activated in LS-DYNA as it provides a more robust treatment of the rotation of the material coordinate system for anisotropic materials and was found to affect the local equivalent plastic strains at hole expansion ratios greater than 150%.

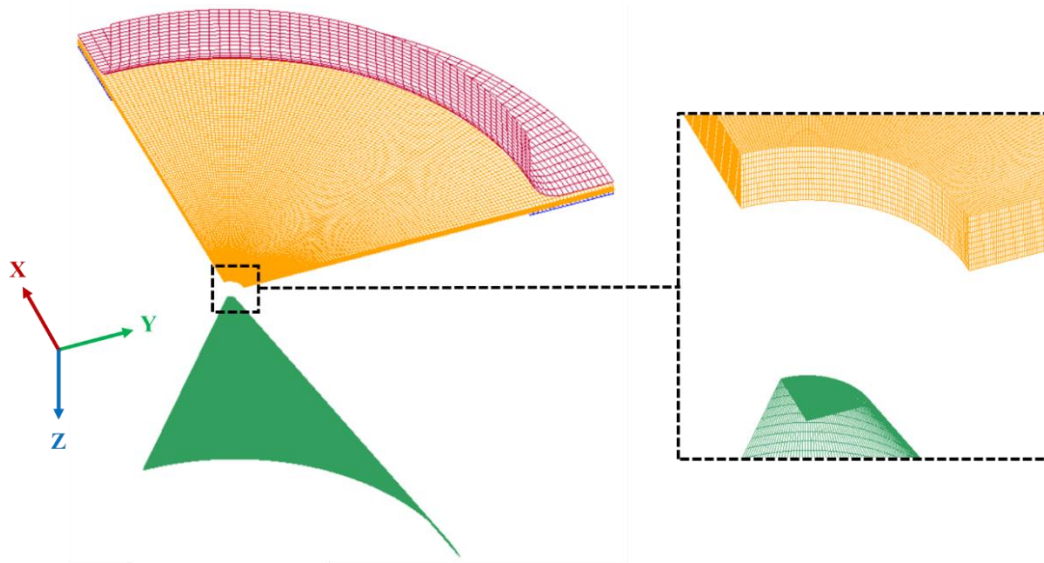


Figure 106 Finite-element mesh of the conical HET. No symmetry was invoked in the model. A one-quarter view is provided here for visualization of the mesh.

B.4.2. Evaluation of the Finite-Element Model

The simulations were terminated at the hole expansion ratio when a through-thickness crack first occurred in the experiments (listed in Table 18). The outer hole diameters at various locations from the experiments and simulations were compared as shown in Figure 41 for both materials to make an initial assessment of the model setup and calibrated plasticity model. The simulations predict the experimental outer hole diameters for DP1180 accurately with the predictions lying within the experimental variations as seen in Figure 41 (a). The model predictions are within 0.5% of the reported data for CP800 as well as shown in Figure 41 (b). The results of multiple tests were not reported by Pathak *et al.* (2016) to provide the variation in the diameter measurements.

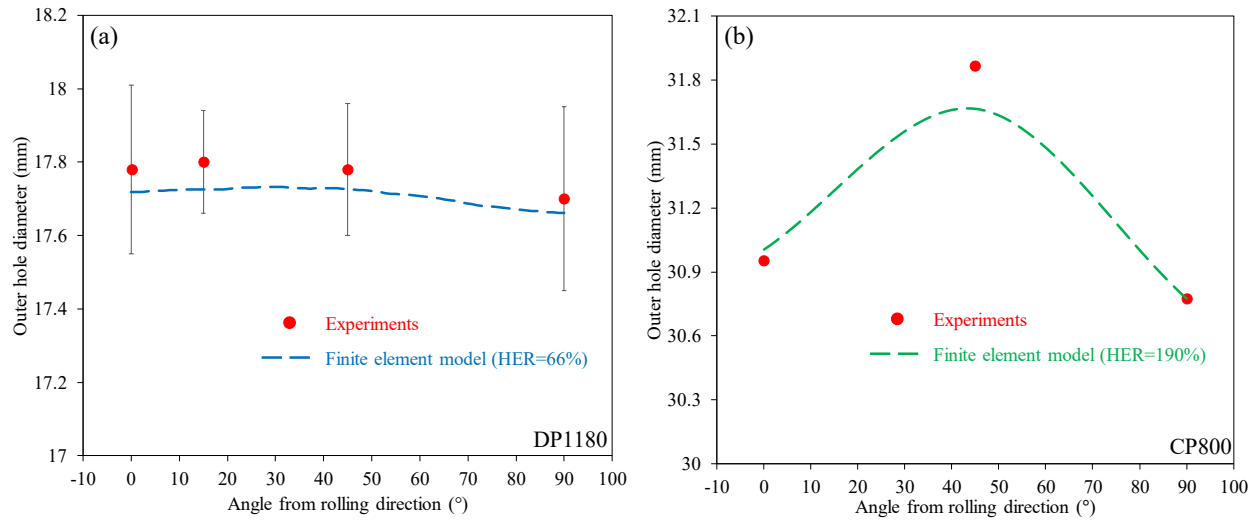


Figure 107 A comparison of the experimental and predicted outer hole diameters in various orientations at the instant when fracture occurred in the experiments for (a) DP1180 and (b) CP800. The experimental data for CP800 was obtained from Pathak et al. (2016) where error bars were not available.

The largest local equivalent plastic strains were predicted to occur in the RD in the finite-element models as shown in Figure 108 and is consistent with the fracture location in the experiments.

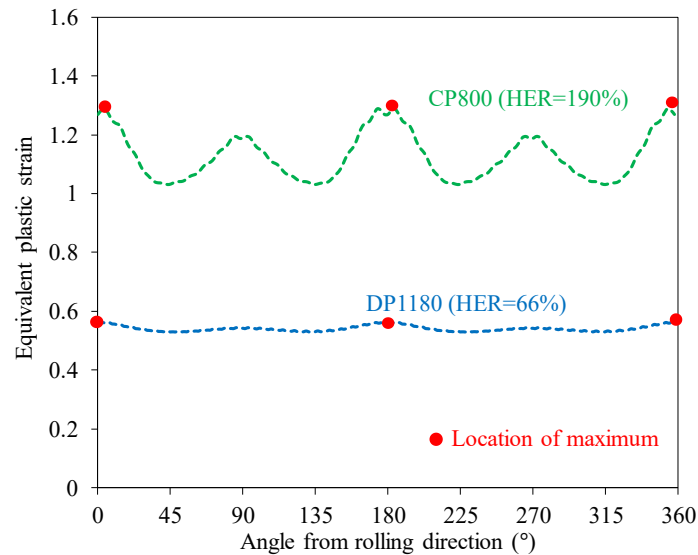


Figure 108 Predicted equivalent plastic strain distribution around the circumference of the outer hole edge in the finite-element model at the timestep when the experimental fracture HER for CP800 and DP1180 is reached

B.4.3. Through-Thickness Strain Gradients at the Hole Edge

The through-thickness stress and strain gradients and the influence of anisotropy on the hole expansion test must first be investigated to determine its suitability for fracture characterization under uniaxial tension. Three primary locations of interest along the thickness will be considered as shown in Figure 42. The outer

edge refers to the element on the free surface of the blank. The mid-plane and inner edge are the elements at mid-thickness and in initial contact with the punch, respectively.

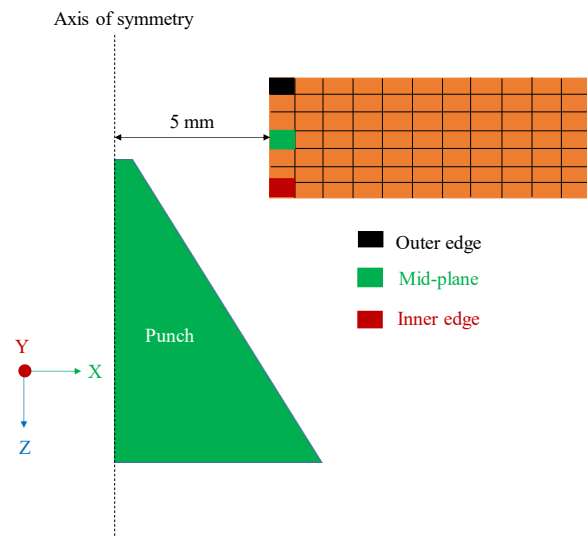


Figure 109 A 2-D schematic showing location of elements at the outer edge, mid-plane, and inner edge of the hole in the finite-element model of the conical hole expansion test.

Figure 110 and Figure 111 depict the local plastic strain history as a function of the HER for both materials with the elements oriented along the RD and TD, respectively. The strains accumulated at the inner edge are significantly higher than those at the outer edge and mid-plane with a difference of approximately 100% when the experimental fracture HER is reached. An analysis focused only upon the magnitude of the strains shown in Figure 110 and Figure 111 would conclude that fracture would initiate at the inner edge. However, the fracture strain of metals is stress-state dependent. For example, under compressive stress states, the fracture strain is expected to be significantly higher than uniaxial tension (for instance, see Sebek *et al.*, 2018). Thus the 3-D stress state at these elements must be analyzed to shed light on the expected fracture initiation location.

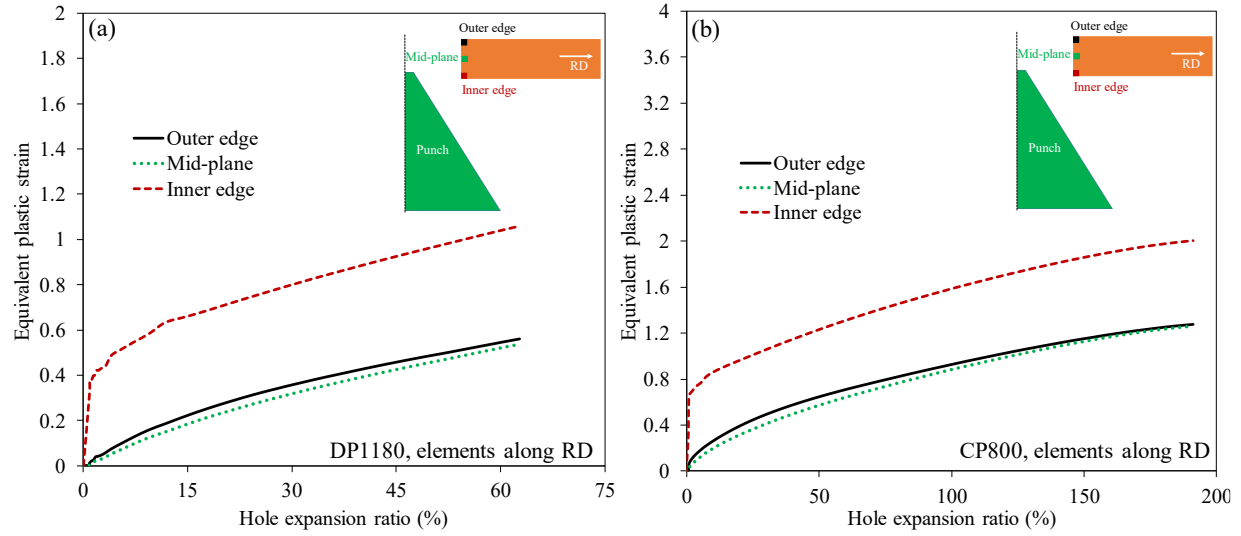


Figure 110 The local equivalent plastic strain history for elements at the hole edge located along the RD for (a) DP1180, and (b) CP800.

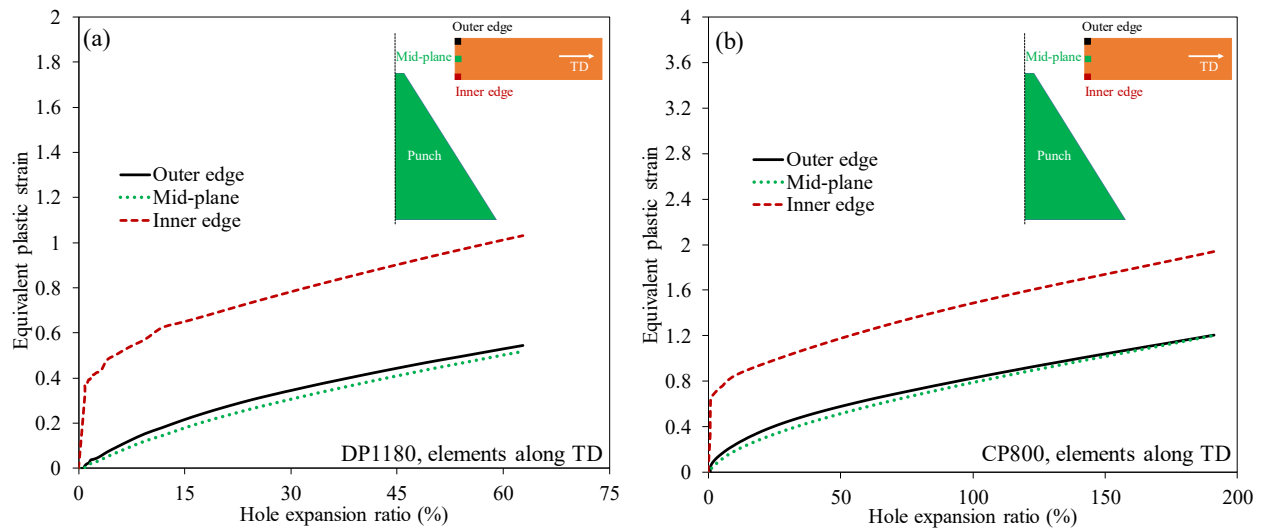


Figure 111 The local equivalent plastic strain history for elements at the hole edge located along the TD for (a) DP1180, and (b) CP800.

The strain gradient along the length of the deformed sample was monitored as depicted in Figure 112 for the studied materials with elements oriented along the RD. The largest plastic strain is accumulated at the hole edge followed by a continuous decrease in strains with increasing distance away from the edge. Due to punch contact, a steeper strain gradient along the length was observed at the inner surface compared to the outer surface. Similar behavior was observed at different element orientations with respect to the RD and is not shown for brevity.

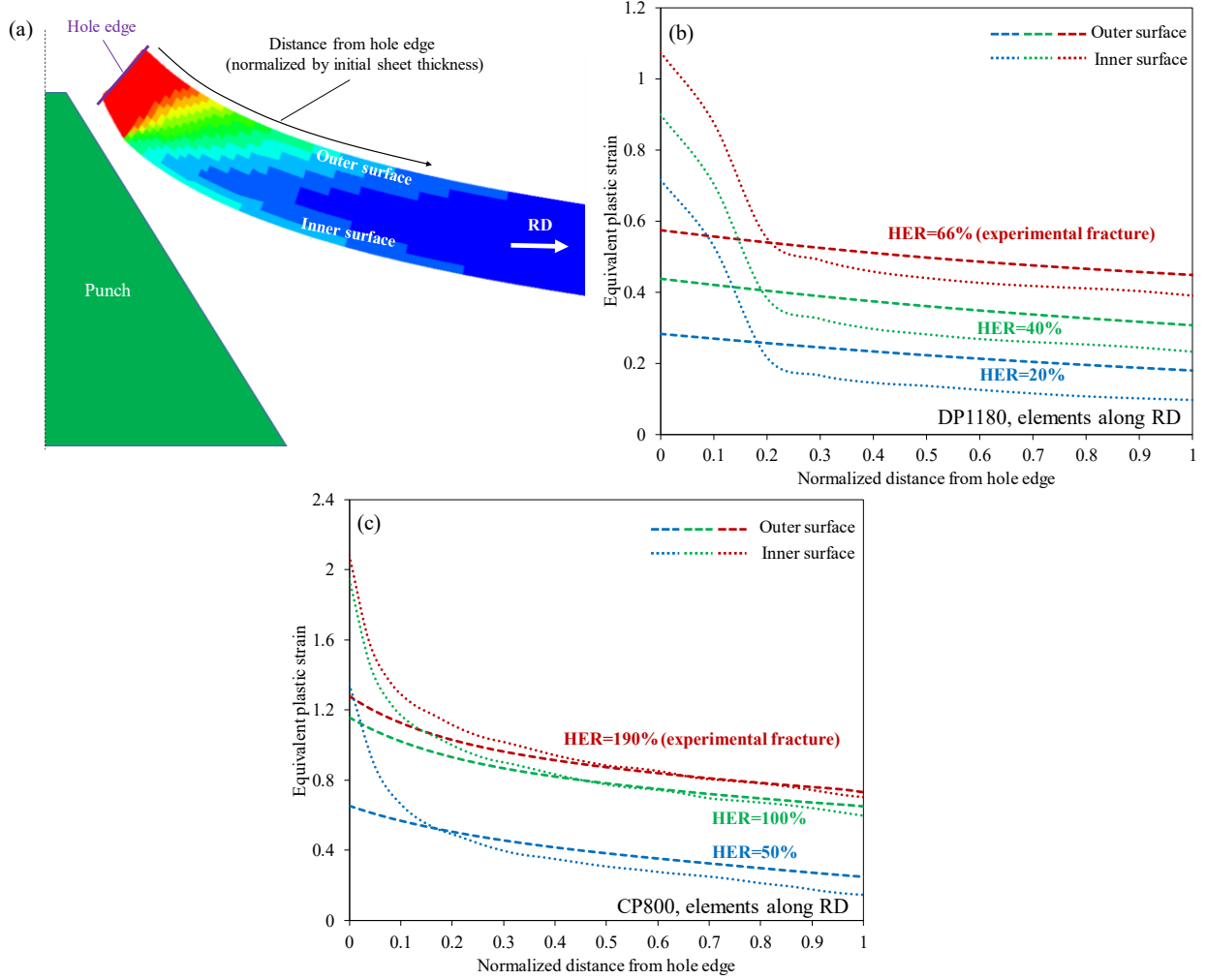


Figure 112 The variation of plastic strain with distance from the hole edge for the studied materials during conical hole expansion. (a) A schematic showing the location and methodology followed to calculate the normalized distance from the hole edge along the outer surface, (b) strain gradient along the length for DP1180, and (c) CP800.

B.4.4. Stress History at the Hole Edge

A proportional stress state of uniaxial tension is required for the test to be suitable for experimental fracture characterization without inverse analysis. The stress state is characterized using the stress triaxiality, η , and the Lode parameter, L , defined as

$$\eta = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3\sqrt{3}J_2} \quad (\text{Eq. B10})$$

$$L = \frac{3\sqrt{3}}{2} \left(\frac{J_3}{J_2^{3/2}} \right) \quad (\text{Eq. B11})$$

where J_2 and J_3 are the respective second and third deviatoric stress invariants. The theoretical stress triaxiality and Lode parameter values for uniaxial tension are $1/3$ and 1 , respectively. A compressive stress state is characterized by negative values of stress triaxiality and Lode parameter (*i.e.* $\eta < 0$; $L < 0$) whereas a plane strain tension stress state is characterized by $\eta = \frac{1}{\sqrt{3}}$ and $L = 0$. The onset of diffuse necking and localization is specified by a shift in stress state towards plane strain tension.

The evolution of the stress triaxiality and Lode parameter at the hole edge until experimental fracture HER are shown in Figure 113 and Figure 114 respectively for both materials. The element at the outer edge is always under uniaxial tension ($\eta = \frac{1}{3}, L = 1$), whereas the inner edge is under compression at the beginning and subsequently shifts towards uniaxial tension after the loss of contact with the punch. A similar stress state distribution was also observed by Park *et al.* (2021) at the hole edge in their macroscale finite-element models of conical HET for an anisotropic hot-rolled ferrite-bainite steel. Within the phenomenological framework of fracture modeling, it is generally accepted that damage accumulates incrementally throughout deformation under non-proportional loading (Marcadet and Mohr, 2016). Since the fracture strains under compressive states are typically large, the initial compressive deformation of the inner edge is expected to result in a low damage accumulation rate and consequently provides a less favorable path to fracture initiation. Fracture in the tests initiate at the outer edge as a result and propagate through the thickness to the inner edge.

To assess through-thickness neck formation at the hole edge, the concave-side rule (CSR) developed by Tharrett and Stoughton (2003) was considered. The CSR postulates that each through-thickness layer of a material must reach its limit strain, based upon its loading history, for a neck to develop under combined bending and tension. If the CSR is satisfied, then all layers would exhibit a transition in the Lode parameter towards zero for plane strain tension loading which was not observed for the materials considered in this study as seen in Figure 114. Instead, the Lode parameter of the layers converge to uniaxial tension with $L = 1$ after a hole expansion ratio of 20% for both materials and then remains approximately constant. The stress triaxiality of all through-thickness layers also stabilize to $\eta = \frac{1}{3}$ for both materials during the test. Similar trends were observed in the diagonal and transverse directions, which are not shown for brevity.

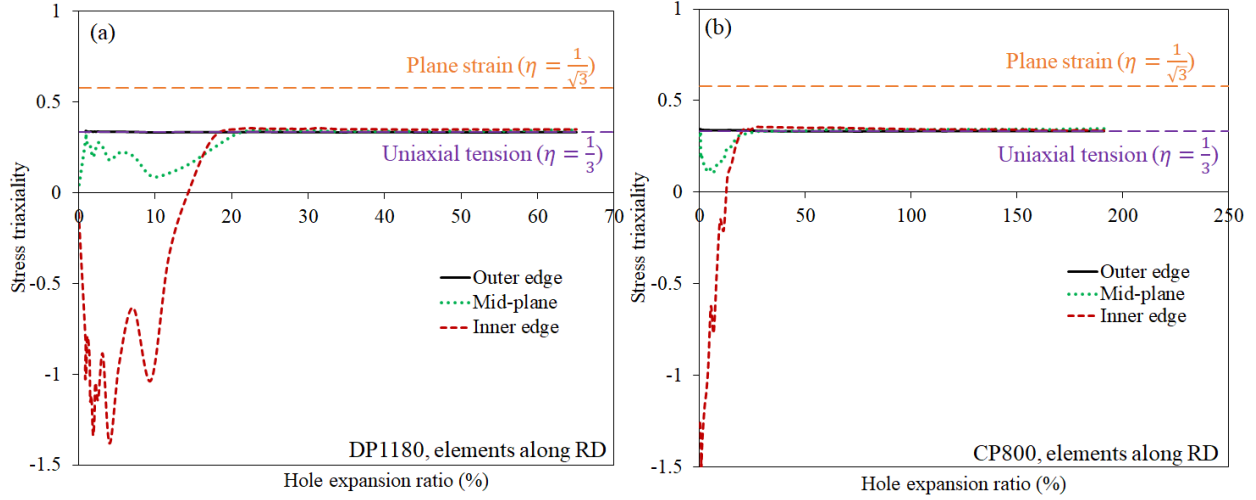


Figure 113 The evolution of stress triaxiality of the elements at the hole edge located along the RD during hole expansion of (a) DP1180 and (b) CP800 along with the theoretical triaxiality values of uniaxial ($\eta = \frac{1}{3}$) and plane strain tension ($\eta = \frac{1}{\sqrt{3}}$)

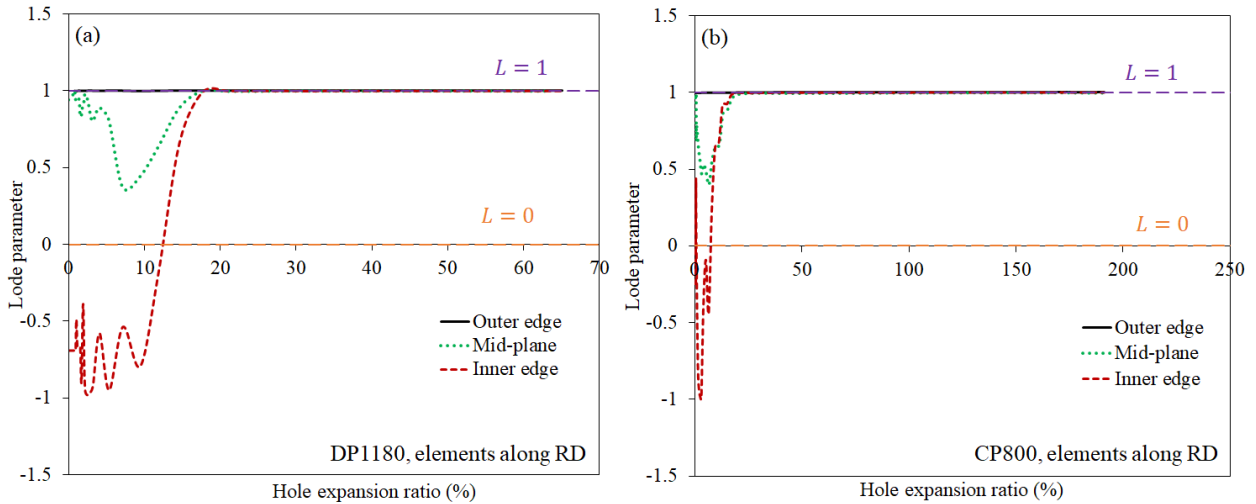


Figure 114 The evolution of Lode parameter of the elements at the hole edge located along the RD for hole expansion of (a) DP1180 and (b) CP800 along with the theoretical Lode parameter values of uniaxial ($L = 1$) and plane strain tension ($L = 0$)

To determine if a neck may form at some distance behind the edge, the variations in stress triaxiality and Lode parameter with distance away from the hole edge are shown in Figure 115 and Figure 116, respectively at inner and outer surfaces for CP800. Although the variation in the stress state is non-linear in general, the Lode parameter of the outer and inner surfaces do not move towards plane strain in concert but approach $L = 1$ for uniaxial tension as the hole expansion ratio increases. According to the employed anisotropic finite element model, it does not seem that a through-thickness neck forms behind the hole edge in the conical hole expansion test. A similar trend was observed for DP1180.

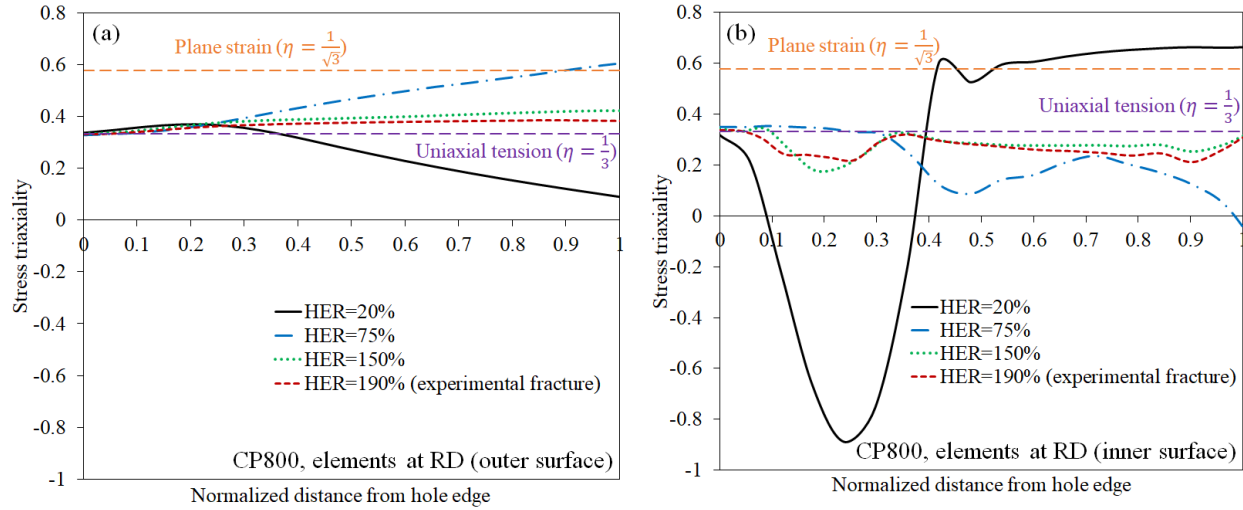


Figure 115 The variation of stress triaxiality along the length of the sample for various deformation levels at (a) outer and (b) inner surfaces of CP800. All elements are oriented along the RD. The distance from the hole edge was normalized with respect to the initial sheet thickness. See Figure 112 for description regarding the outer and inner surfaces.

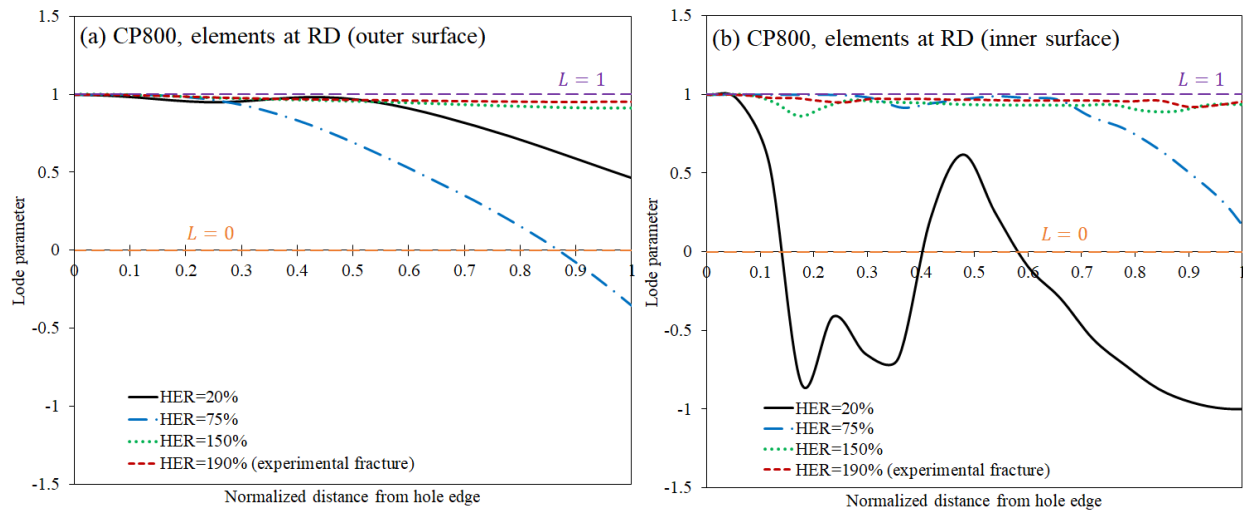


Figure 116 The variation of Lode parameter along the length of the sample for various deformation levels at (a) outer and (b) inner surfaces of CP800. All elements are oriented along the RD. The distance from the hole edge was normalized with respect to the initial sheet thickness. See Figure 15 for description regarding outer and inner surfaces.

B.4.5. Strain Path Near the Hole Edge

The analysis of 3-D stress state at the hole edge confirms that the outer edge is in approximately proportional uniaxial loading while the other material layers undergo non-linear loading before converging to uniaxial tension. An analysis of the strain path can also provide insight into the loading conditions as the ideal strain path for uniaxial tension can be computed using the tensile r -value.

$$\varepsilon_{2,\theta} = -\frac{r_{\theta+90}}{1+r_{\theta+90}} \varepsilon_{1,\theta} \quad (\text{Eq. B12})$$

where θ is the orientation of the element in degrees with respect to the RD. The ratio of minor-to-major strains uses the r-value perpendicular to the element location due to circumferential stretching.

The in-plane principal strain paths of the elements at the hole edge located along the RD are shown in Figure 117 for both materials. The element at the outer edge follows an approximately linear strain path for both materials and is in good agreement with the theoretical trend as shown in Figure 117. The element at mid-plane demonstrates a non-linear strain path that initially lies in between shear and uniaxial tension before transitioning to uniaxial tension. The inner edge initially undergoes a plane strain compressive stress state due to interaction with the punch but gradually shifts towards uniaxial tension as it loses contact with the punch. The strain paths of the three layers considered do not exhibit a transition towards plane strain tension associated with the formation of a through-thickness neck.

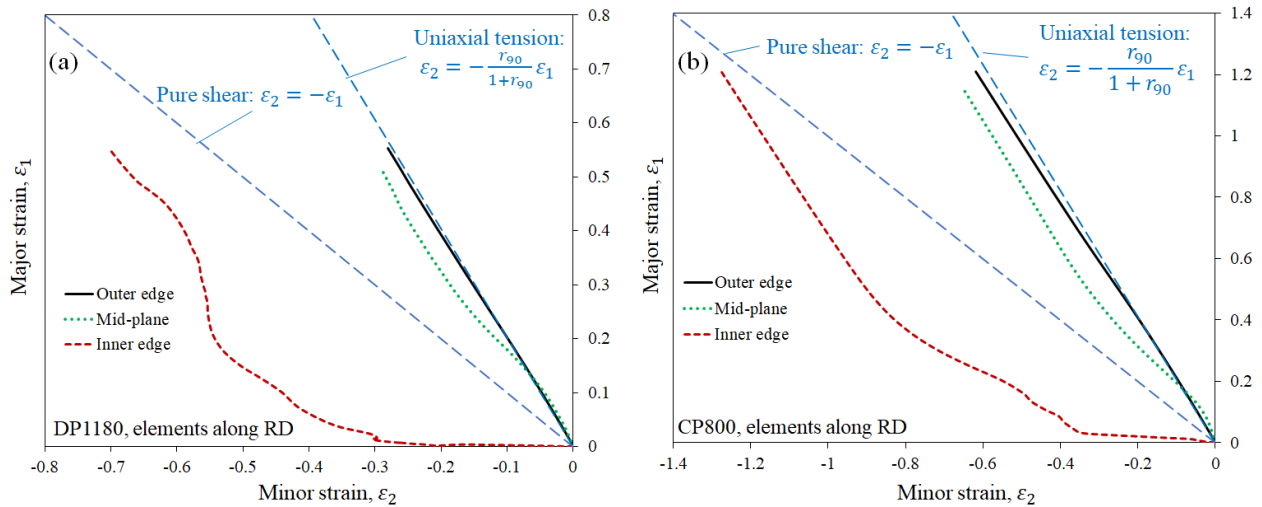


Figure 117 Strain paths of various elements at the hole edge along the RD for (a) DP1180 and (b) CP800

The strain paths of the elements away from the outer hole edge were also assessed for any neck formation. Six locations on the outer surface of the blank covering a total distance of approximately 3 mm from the hole edge were considered (see Figure 118a) and the strain paths at these locations for DP1180 are depicted in Figure 118(b). A transition in the strain path toward plane strain tension corresponding to necking was not observed. In fact, the elements located up to a distance of 2.0 mm from the hole edge have a strain path close to uniaxial tension.

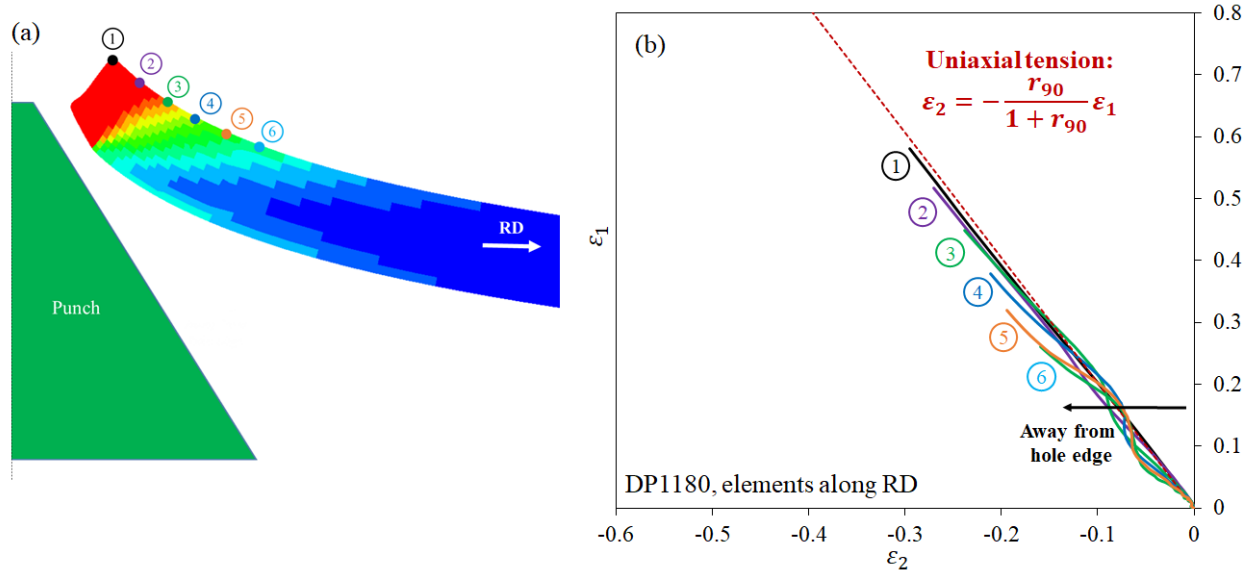


Figure 118 (a) A schematic of the locations away from the outer hole edge where the strain paths are assessed and **(b)** strain paths at the locations of interest for DP1180. Only the results for elements oriented along the RD are shown.

B.4.6. Evaluation of Thickness Distribution and Influence of Anisotropy

The predicted hole shape of DP1180 is approximately circular with a less pronounced thickness variation at the hole edge as shown in Figure 119. In contrast, the higher local ductility and stronger in-plane anisotropy of CP800 produces a less circular hole shape with a non-uniform thickness distribution as seen in Figure 44 and reflected in the FE model as depicted in Figure 121. Visually, the CP800 appears to exhibit necking in the RD and TD due to significant thinning but the stress and strain state analysis in Sections B.4.4 and B.4.5 indicated that plastic instability should not occur.

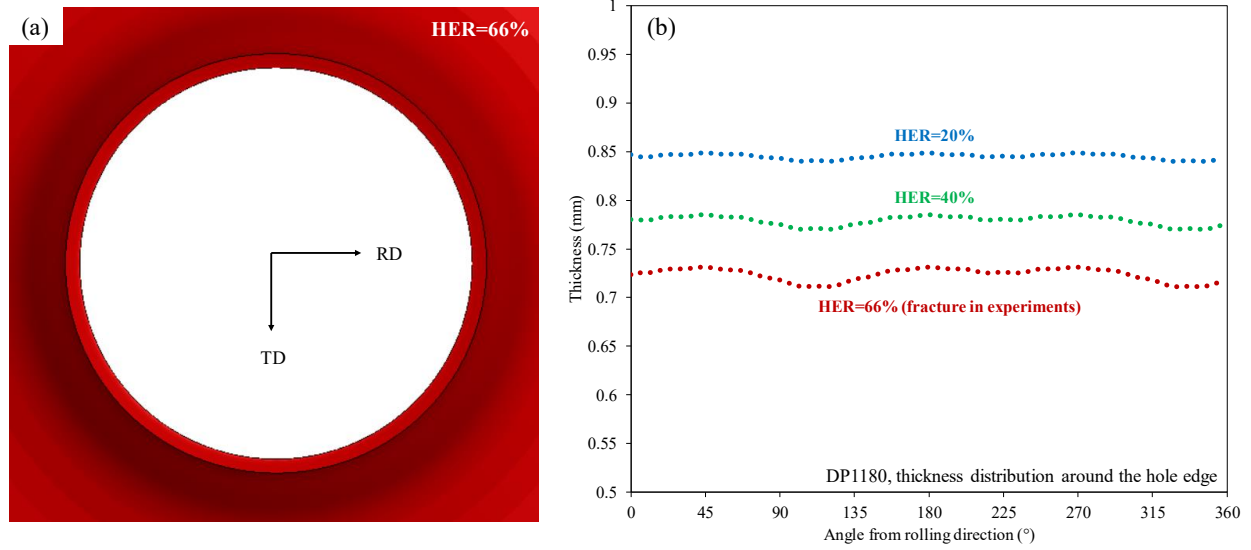


Figure 119 (a) Top view of the deformed DP1180 sample from simulations at an HER of 66%, and (b) predicted thickness distribution around the circumference of the hole edge

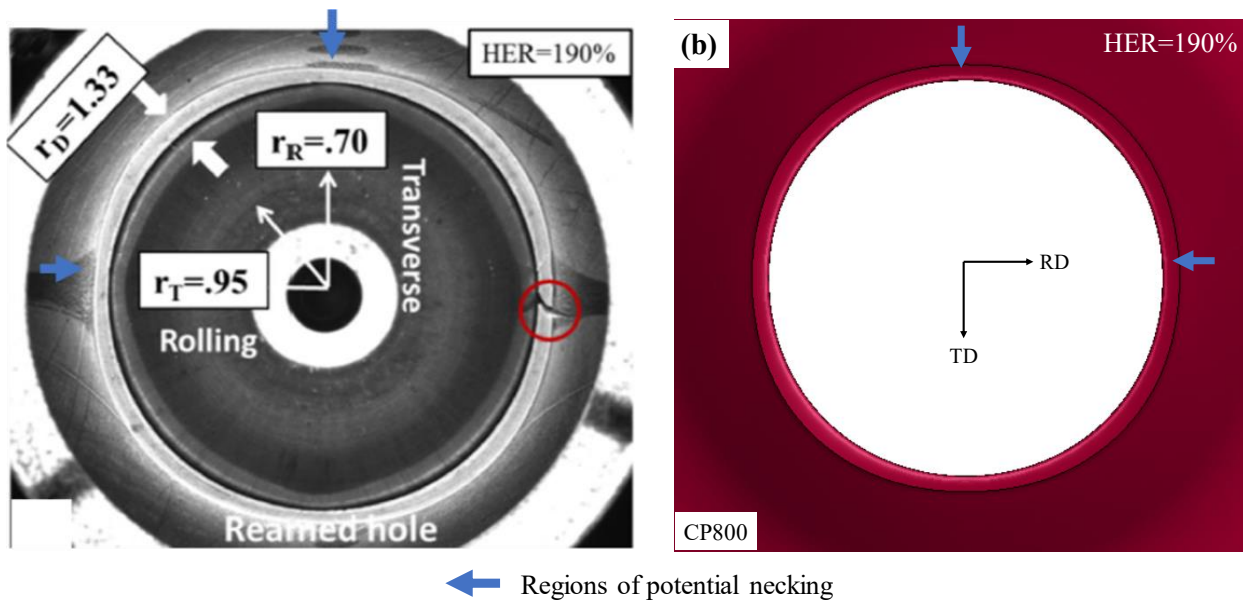


Figure 120 Top view of the deformed CP800 sample at an HER of 190% from (a) experiments (Pathak et al., 2016) and (b) simulations

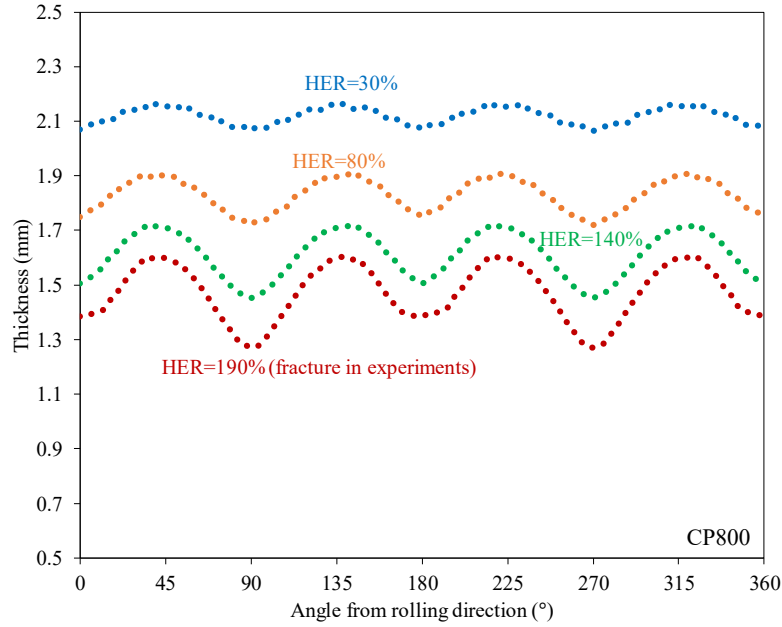


Figure 121 The predicted thickness distribution around the circumference of the hole edge for CP800 at various deformation levels

To further investigate if a localized or acute neck is formed in the simulations, the strain-rate based necking detection algorithm of Volk and Hora (2011) was considered. An acute neck is characterized by a rapid increase in local thinning and is generally represented by the limiting strains in the forming limit curve (FLC). The method of Volk and Hora (2011) detects an acute neck when a bifurcation in the strain rate evolution is observed with its rapid increase in a near exponential manner until fracture as shown schematically in Figure 122.

The major strain rates of elements at the hole edge located along the RD and TD for CP800 are shown in Figure 123. The major strain rate is almost constant throughout the deformation and does not increase abruptly which provides additional evidence that localization has not occurred at the hole edge even for a material with strong anisotropy and high local ductility.

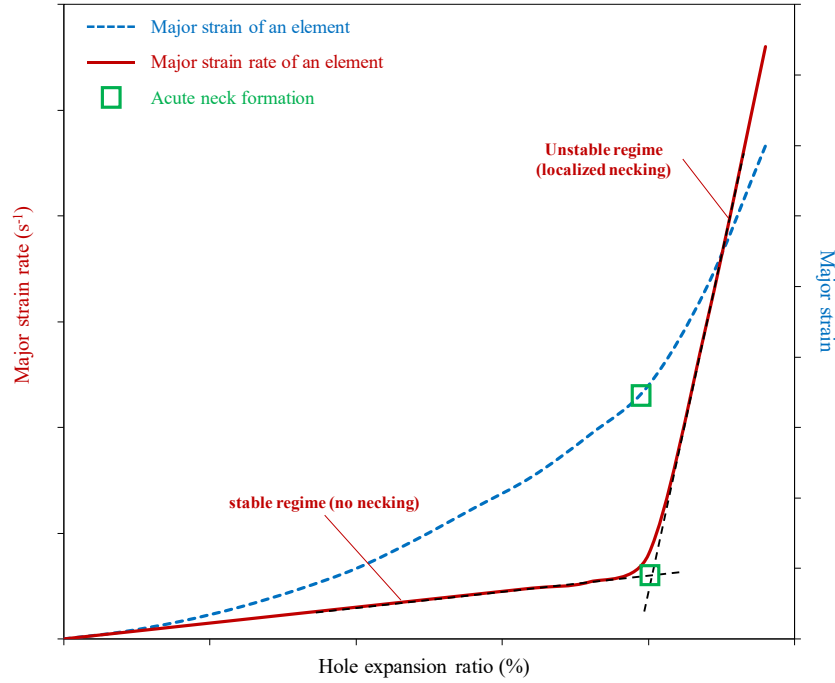


Figure 122 A schematic illustrating a bifurcation in strain rate expected when an acute neck develops according to the Volk and Hora (2011) methodology. The instant of a localized neck is recognized by the intersection of the trend lines for stable and unstable deformation. An instability does not manifest if the strain rate throughout the deformation is constant or is decreasing.

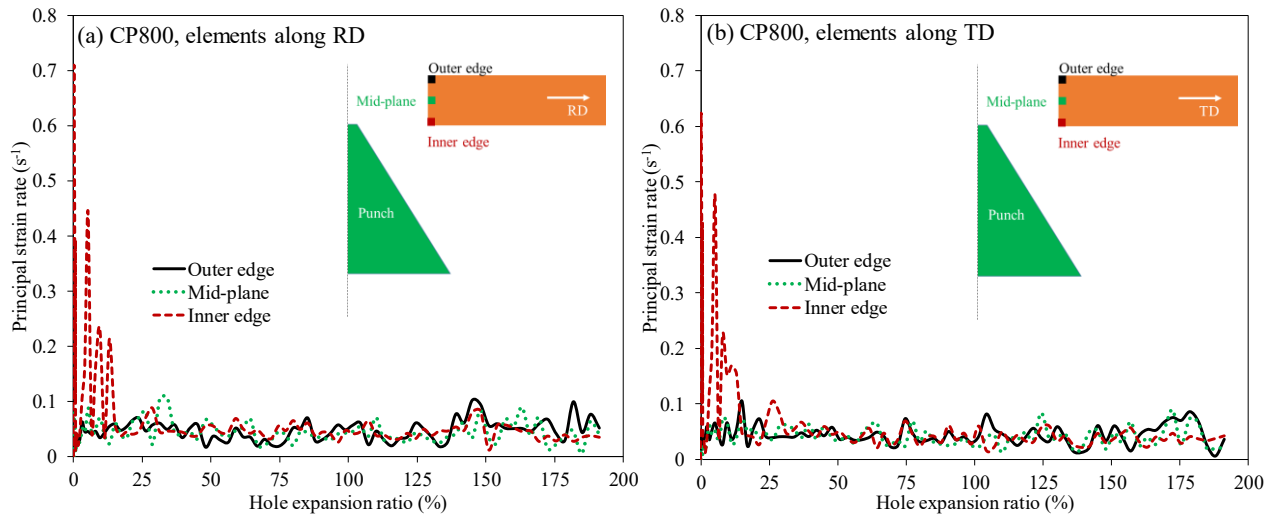


Figure 123 The major strain rate history for elements located along the (a) RD and (b) TD for CP800 till experimental fracture

Consequently, the inhomogeneous thickness distribution and non-circular hole shape are attributed to material anisotropy. The resemblance of the predicted distributions of thickness and outer hole diameter in the circumference of the hole edge to the tensile r-value distribution is displayed in Figure 45 and Figure 125 for the two materials. A material with milder anisotropy and lower ductility such as DP1180

demonstrates a more uniform thickness distribution and hole shape as illustrated by Figure 45(a) and Figure 125(a), respectively. The same behavior is expected to apply to a material with a sheared edge as edge cracking would be prompted at significantly lower strains and the influence of anisotropy would be suppressed. Pathak *et al.* (2016) noted that the hole shape and thickness distribution of CP800 was more uniform with a sheared edge condition. It is expected that tensile stress anisotropy also contributes to the thickness profile but it appears to be dominated by tensile plastic anisotropy for the two AHSS considered.

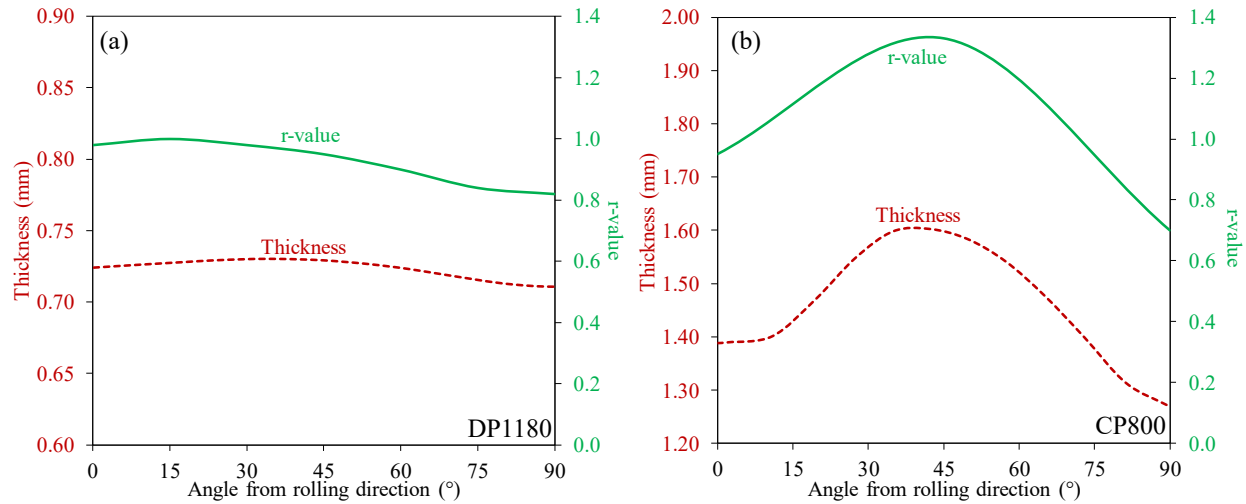


Figure 124 The predicted blank thickness around the circumference of the hole edge versus tensile r-value distribution for (a) DP1180 and (b) CP800 when the experimental HER at fracture is reached. Note that due to circumferential stretching, the angle from the rolling direction corresponds to the tensile r-value obtained at 90 degrees to that direction.

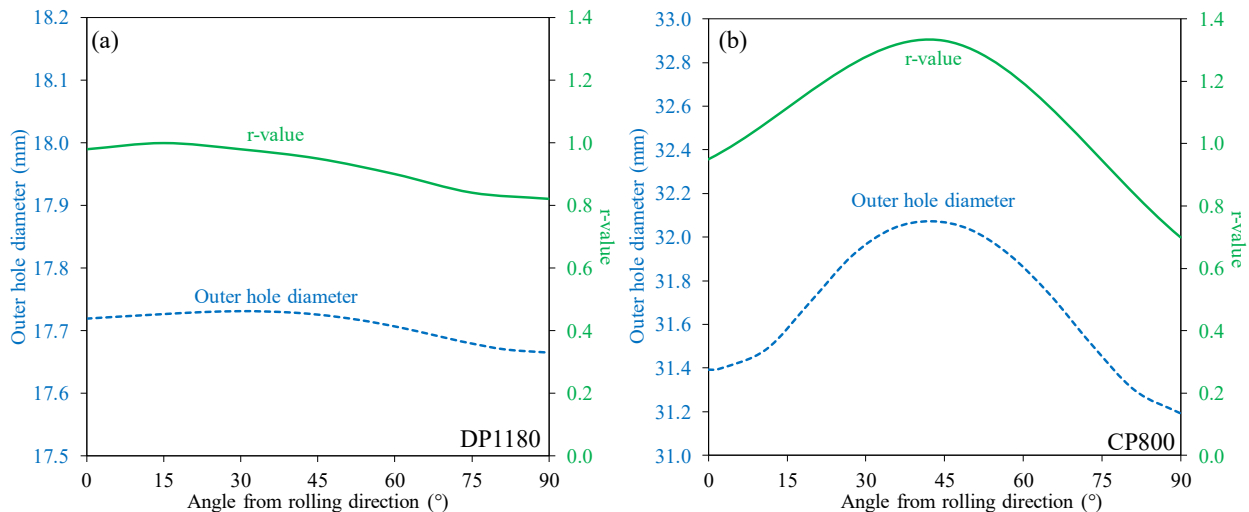


Figure 125 The predicted outer hole diameter around the circumference of the hole edge versus tensile r-value distribution for (a) DP1180 and (b) CP800 at the instant when the experimental HER at fracture is reached. Note that due to circumferential stretching, the angle from the rolling direction corresponds to the tensile r-value obtained at 90 degrees to that direction.

The conical hole expansion test appears to be suitable for uniaxial fracture strain determination with fracture expected to initiate from the outer hole edge. However, fracture strains still need to be estimated from the hole geometry since DIC cannot measure the strains at the hole edge accurately. The existing methods to experimentally characterize strains in the conical HET will be discussed in the subsequent section and a new methodology will be proposed and evaluated to determine the local equivalent plastic strain.

B.5. Existing Methodologies for Estimating Fracture Strain in Conical Hole Expansion Tests

B.5.1. ‘Maximum Edge Stretch’ Criterion by Hance (2017)

To incorporate the HER into finite-element models for predicting edge cracking, Hance (2017) proposed a ‘maximum edge stretch’ criterion wherein the hole expansion ratio is correlated to the true major strain at the hole edge

$$\varepsilon_1 = \ln \left(1 + \frac{HER}{100} \right) \quad (\text{Eq. B13})$$

Eq. (B13) can be used as a first-order estimate of the fracture strain but the HER metric itself is problematic for the following reasons:

- i. The HER is measured after the onset of a through-thickness crack in the material and not when a surface crack first appears on the hole edge. Therefore, the fracture strain under uniaxial tension could be overestimated.
- ii. The calculation of HER is based on the measurements of the inner hole diameter, which undergoes a non-linear strain path. The outer hole edge is independent of punch contact and is in a uniaxial tensile stress state throughout the test. Using the HER as a fracture metric homogenizes the stress state at the inner edge where failure is not initiating.
- iii. The calculation of HER is challenging for materials that produce a non-circular hole shape at fracture such as CP800 and is sensitive to the locations at which the inner diameters are measured.

An alternative metric based on the average outer hole diameter or outer hole expansion ratio (OHER) can be defined as:

$$OHER(\%) = \frac{D_{outer}^{BF} - D_0}{D_0} \times 100 \quad (\text{Eq. B14})$$

where D_{outer}^{BF} is the best-fit outer hole diameter at the instant of through-thickness crack. The use of image processing to measure the HER is on the rise as it reduces uncertainty with post-mortem fracture

measurements and enables the OHER to be readily measured alongside the HER. The OHER reflects deformation that is consistent with the assumption of a uniaxial stress state and is measured at the outer hole edge where fracture typically initiates.

B.5.2. Estimation of the Average Equivalent Fracture Strain Using the Method by Pathak *et al.* (2017)

Pathak *et al.* (2017) proposed a method to estimate the average equivalent strain at fracture assuming von Mises plasticity using the averaged value of the best-fit inner and outer hole diameters and thickness, henceforth referred to as macroscopic strain or ε_{eq}^{MC} . The macroscopic strain can be modified and written using only the outer diameter or OHER and thickness at the fracture location, t_f , as:

$$\varepsilon_{eq}^{MC} = \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_c - \varepsilon_t)^2 + (\varepsilon_c + 2\varepsilon_t)^2 + (2\varepsilon_c + \varepsilon_t)^2} \quad (\text{Eq. B15})$$

$$\varepsilon_c = \ln \left(\frac{D_{outer}^{BF}}{D_0} \right) = \ln \left(1 + \frac{OHER}{100} \right) \quad (\text{Eq. B16})$$

$$\varepsilon_t = \ln \left(\frac{t_f}{t_0} \right) \quad (\text{Eq. B17})$$

The major strain estimate developed by Pathak *et al.* (2017) in Eq. (B16) is similar to Eq. (B13) by Hance (2017) but uses the outer hole expansion ratio. The two analytical estimates are compared with the major strain evolution at the outer edge from the simulations of the current study in Figure 127 for both materials. The analytical strains are compared with the ‘averaged’ strain of all elements on the circumference of the outer hole edge to be consistent with the assumptions of Hance (2017) and Pathak *et al.* (2017). A comparison with the local strain of the element where fracture initiated in the experiment was also performed. The distinction between the local and averaged strain is illustrated schematically in Figure 126.

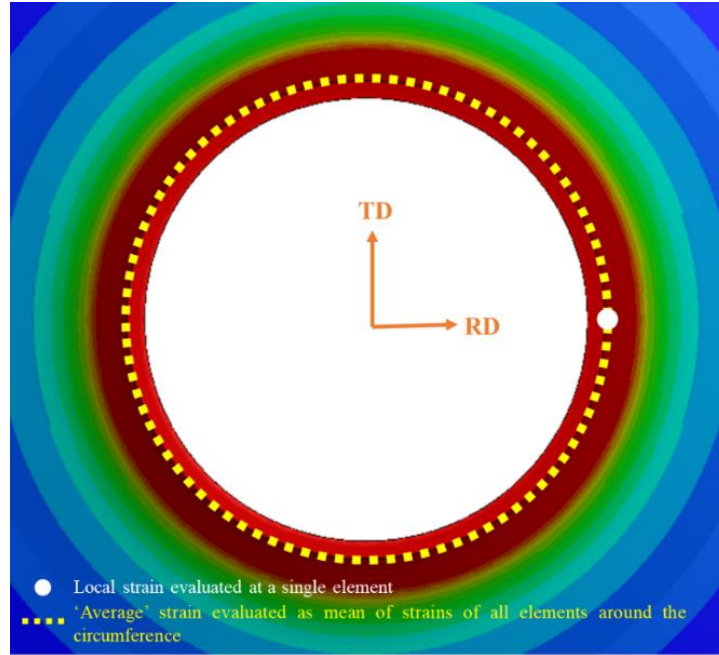


Figure 126 A schematic depicting how ‘average’ and ‘local’ strains are defined

The prediction of Eq. (B13) by Hance (2017) underestimates the average major strain by about 8% and 7% for DP1180 and CP800, respectively. The estimate by Pathak *et al.* (2017) in Eq. (B16) closely predicts the average strain at the outer hole edge with up to 1% error for both materials when fracture occurs in the experiments. In summary, the outer hole expansion ratio can be mapped to the major strain at the outer hole edge and be considered as a measure of the average fracture strain during the conical HET. The OHER is recommended as a fracture metric over the conventional HER.

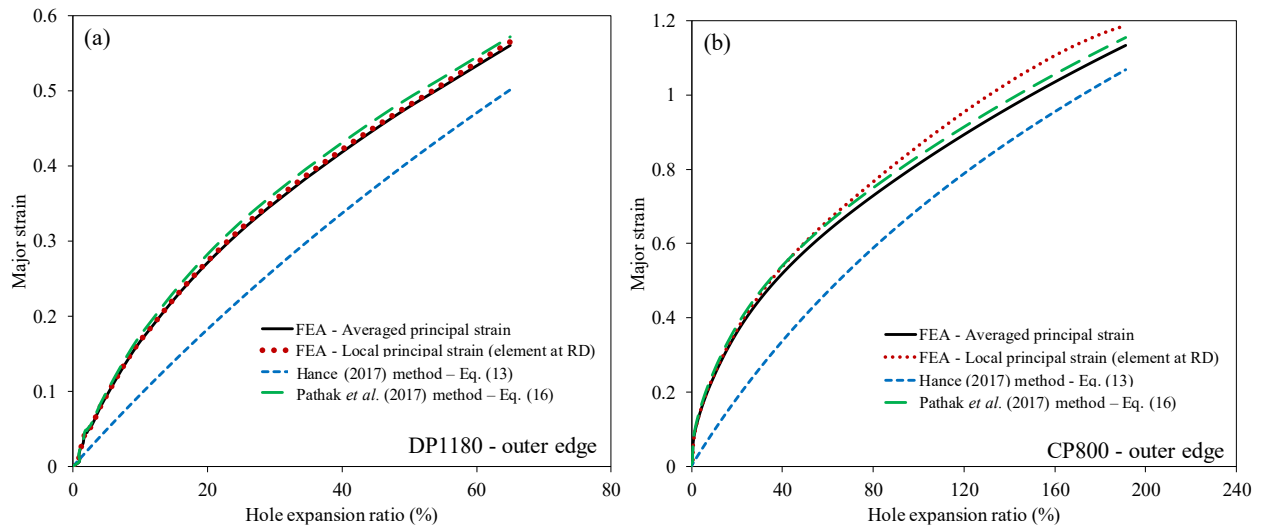


Figure 127 Evolution of averaged and local principal strains at the outer hole edge overlaid with the predictions of Eq. (B13) developed by Hance (2017) and Eq. (B16) by Pathak *et al.* (2017) for (a) DP1180 and (b) CP800

The average and local principal strain at the fracture location are almost equivalent for DP1180 with differences of only 0.3% which can be correlated to its circular hole shape (see Figure 119) and low in-planar anisotropy. However, notable differences between the local and average strain measures (approximately 4% strain difference) are observed for CP800 with a non-circular or oval hole shape at fracture indicating non-uniform strain along the circumference. The major strain from Eq. (B16) therefore serves as an accurate measure of the local fracture strain under uniaxial tension only for mildly anisotropic materials.

The primary advantage of the Pathak et al. (2017) methodology is that a uniaxial stress-state is not assumed a priori. However, the need for post-mortem thickness measurements makes it less desirable for use since the effect of through-thickness strain gradients and anisotropy are homogenized and combined into an average strain. Furthermore, the thickness measurement is sensitive to the distance from the hole edge at which the measurement is conducted as demonstrated in Figure 128 and Table 19 for five different distances from the hole edge. The error in macroscopic strain calculation was computed with respect to the anisotropic finite element model for different measurement locations as follows:

$$Error(\%) = \frac{\varepsilon_{eq}^{FEA} - \varepsilon_{eq}^{MC}(t)}{\varepsilon_{eq}^{FEA}} \times 100 \quad (\text{Eq. B18})$$

where ε_{eq}^{FEA} is the local equivalent plastic strain of the critical element at the outer edge (*i.e.* along the RD) from the numerical model at experimental fracture HER, and $\varepsilon_{eq}^{MC}(t)$ is the macroscopic strain calculated from Eq. (B15) – Eq. (B17) using the thickness $t_i (i = 1, 2, \dots, 5)$ measured at a given location from the finite-element model.

Significant differences are observed in the calculated thinning strains with change in measurement location resulting in a macroscopic strain calculation error of up to 10% with respect to the FE model for CP800 as shown in Figure 129. Differences are relatively minor for the thinner DP1180 (up to 3% error). Errors in macroscopic strain calculation can also be induced due to the lag between identification of a crack and stopping the test since the thickness measurements are conducted post-mortem. The micrograph from experiments and section view from FEA in Figure 128 could also give a false indication of necking occurring in the test when it is actually a byproduct of compression and curling of the inner hole edge due to contact with the punch as the hole is extruded out of plane.

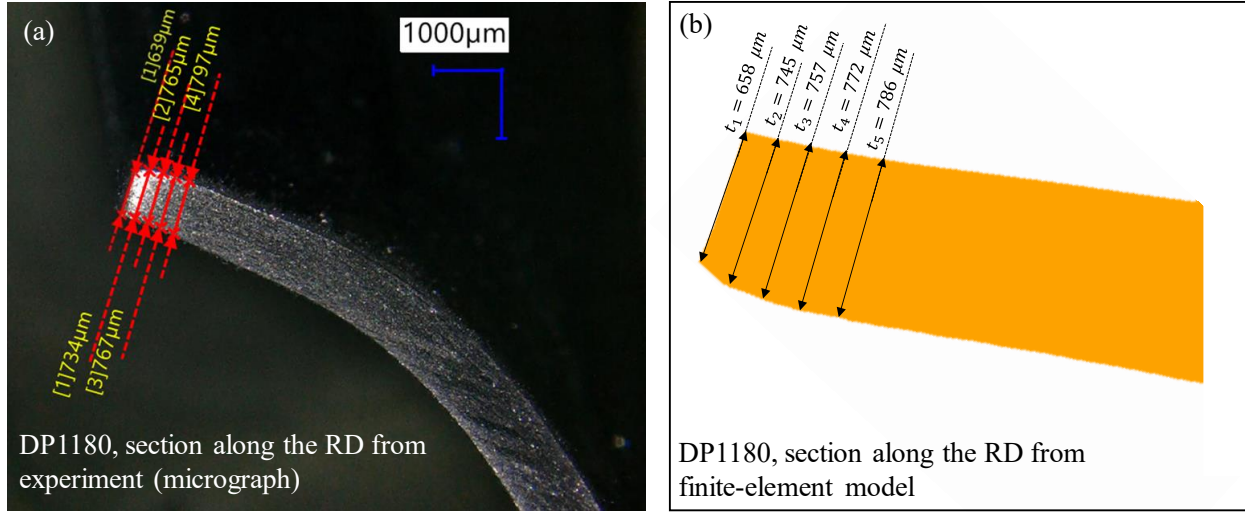


Figure 128 (a) Micrograph from experiments, and (b) cut section from finite-element model of DP1180 extracted at the instant of failure in experiments

Table 19 Sensitivity of thickness and macroscopic strains to the location of measurement

	Distance from hole edge (mm)	Thickness from micrograph (mm)	Thickness from FEA (mm)	$\varepsilon_t = \ln\left(\frac{t_f}{t_0}\right)$ from micrograph	$\varepsilon_t = \ln\left(\frac{t_f}{t_0}\right)$ from FEA	$\varepsilon_{eq}^{MC}(t)$ from FEA
t_1	0	0.639	0.658	-0.45	-0.42	1.20
t_2	0.1	0.734	0.745	-0.31	-0.29	1.16
t_3	0.2	0.765	0.757	-0.27	-0.28	1.16
t_4	0.3	0.767	0.772	-0.27	-0.26	1.15
t_5	0.4	0.797	0.786	-0.22	-0.24	1.15

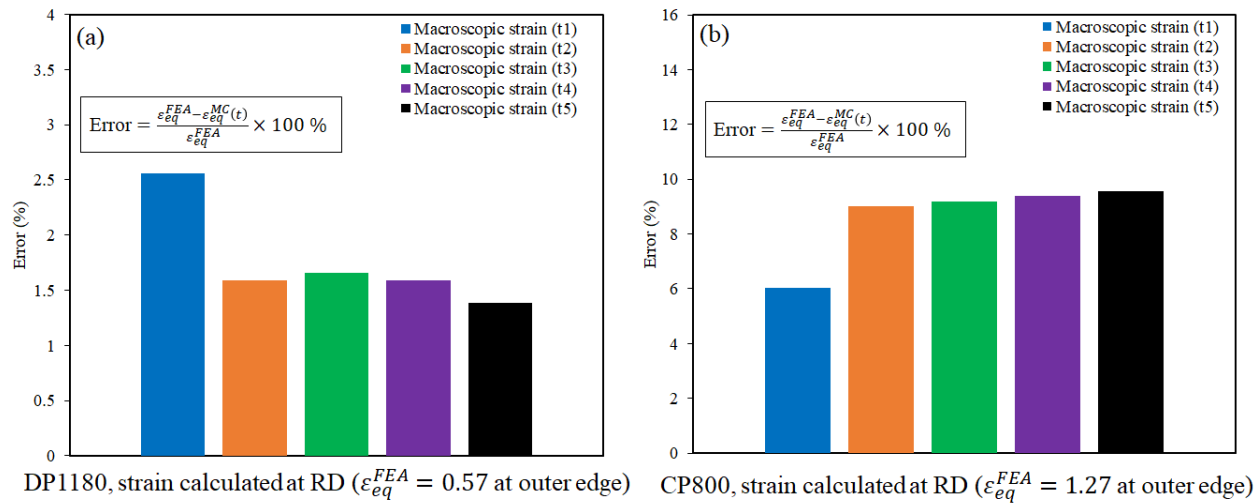


Figure 129 The sensitivity of macroscopic strain to the location of thickness measurement for (a) DP1180 and (b) CP800. The thickness measurements at various locations were extracted from the finite-element model.

B.6. A New Methodology to Measure the Local Equivalent Strain at the Hole Edge

An improved methodology to estimate the local equivalent strain for anisotropic materials can be proposed from the confirmation that the outer edge remains in uniaxial tension. The local in-plane major strain for an element on the outer hole edge at an angle of θ from the rolling direction is given by:

$$\varepsilon_{1,\theta} = \ln \left(\frac{D_{outer}^\theta}{D_0} \right) \quad (\text{Eq. B19})$$

where D_{outer}^θ is the measured outer hole diameter at the location of interest. The in-plane minor strain can be obtained from the r-values using Eq. (B12) with the major strain from Eq. (B19). The use of Eq. (B19) enables the treatment of non-circular hole shapes as only the local radius or diameter to the location of interest is used.

For isotropic materials, the major and equivalent strain are identical at the material direction where fracture occurs since the outer edge is under uniaxial tension. However, the yield function and work conjugate equivalent strains are typically defined with respect to the reference or rolling direction within the framework of anisotropic plasticity. The equivalent strain must therefore be determined with respect to the reference direction using the principal strains of Eq. (B19) for the direction where fracture is initiated. The anisotropic equivalent strain with respect to the reference direction, ε_{eq}^{WC} , is obtained from plastic work equivalence given by:

$$\sigma_{eq} \varepsilon_{eq}^{WC} = \sigma : \varepsilon^P = \sigma_1 \varepsilon_1^P + \sigma_2 \varepsilon_2^P + \sigma_3 \varepsilon_3^P \quad (\text{Eq. B20})$$

where ε_i^P ($i = 1,2,3$) are the principal components of the plastic strain tensor, σ_i ($i = 1,2,3$) are the principal stresses, and σ_{eq} is the equivalent stress. Applying the mechanics of circumferential stretching in the conical HET (see Figure 105 for more details) alongside the premise of a uniaxial tensile stress state on the outer hole edge (*i.e.* $\sigma_2 = \sigma_3 = 0$), the anisotropic equivalent strain becomes

$$\sigma_{eq} \varepsilon_{eq,\theta}^{WC} = \sigma_{\theta+\pi/2} \varepsilon_{1,\theta} \Rightarrow \varepsilon_{eq,\theta}^{WC} = \frac{\sigma_{\theta+\pi/2}}{\sigma_{eq}} \varepsilon_{1,\theta} \quad (\text{Eq. B21})$$

where the subscript θ denotes the angular orientation of the location of interest at the outer hole edge from the reference direction. If the RD is taken as the reference direction and fracture occurred in the transverse direction, then $\sigma_{eq} = \sigma_0$ and the major strain of Eq. (B19) is the equivalent strain. Since fracture occurs in

the RD for both steels which corresponds to uniaxial loading along the TD, the work conjugate equivalent plastic strain is:

$$\varepsilon_{eq}^{WC} = \left(\frac{\sigma_{90}}{\sigma_0} \right) \varepsilon_{1,0} \quad (\text{Eq. B22})$$

It is clear that the uniaxial stress ratios in various directions from the standard tensile tests are required to evaluate the fracture strain using the proposed method based on plastic work equivalence. The macroscopic strain from the analysis of Pathak *et al.* (2017) and the new methodology are compared with the local plastic strain of the element at the outer hole edge along the RD in Figure 130 and Figure 131 for DP1180 and CP800, respectively. The strain evolution in these plots is monitored with respect to the outer hole expansion ratio based upon its advantages over the conventional hole expansion ratio as demonstrated earlier. It is evident that both methodologies work equally well in estimating the local strains of DP1180 with a difference of only 1% in strains at the instant of experimental fracture.

The macroscopic strain is in excellent agreement with the finite-element model until outer hole expansion ratios of up to 50% for CP800. Differences increase thereafter with the local equivalent strain underestimated by roughly 11% at an OHER value of 215% as shown in Figure 131. The work conjugate equivalent strain is a superior estimate of the local strains throughout the deformation for CP800 with the local strains under predicted by only 4%. The work conjugate equivalent strain incorporating the local outer hole diameter and stress anisotropy of the material shows a notable improvement over the macroscopic strain for estimating the local fracture strain. The importance of accounting for anisotropy is further demonstrated by performing an additional simulation assuming isotropic behavior for CP800 as shown in Figure 131. A difference of approximately 17% in plastic strain was observed between the isotropic and anisotropic FEA at the instant of experimental fracture.

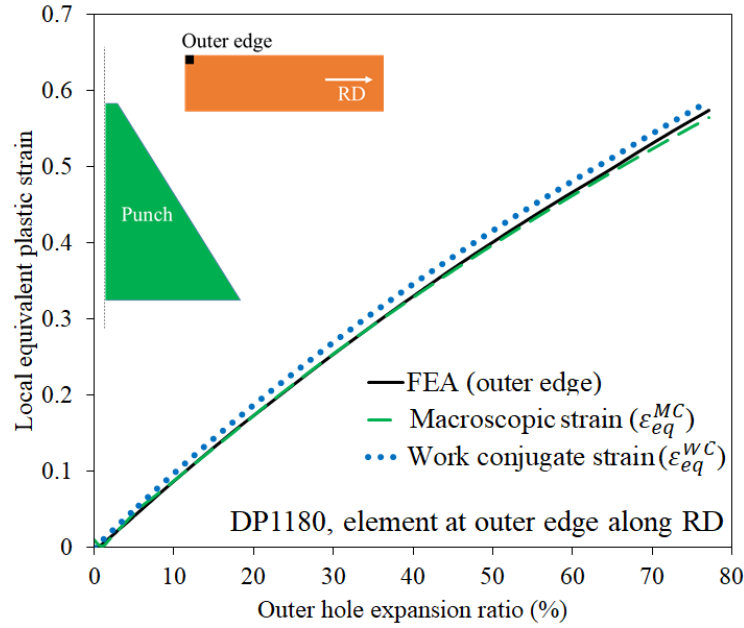


Figure 130 A comparison of local equivalent plastic strain histories from FEA and the analytical methods in conical hole expansion of DP1180. An OHER value of 77% corresponds to the average outer hole diameter when fracture occurred in the experiments for DP1180.

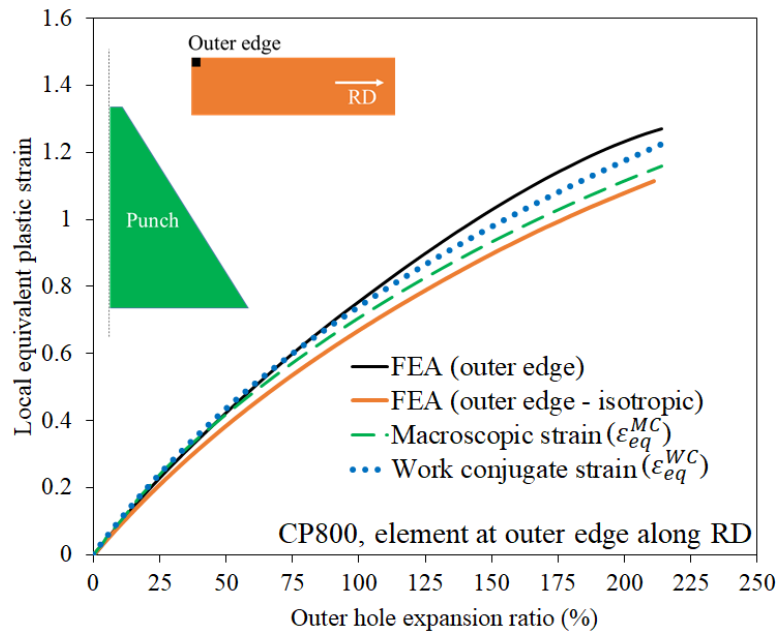


Figure 131 A comparison of local equivalent plastic strain histories from FEA and the analytical methods in conical hole expansion of CP800. An OHER value of 215% corresponds to the average outer hole diameter when fracture occurred in the experiments for CP800.

B.7. Conclusions

The use of conical hole expansion test for uniaxial fracture characterization of DP1180 and CP800 rolled sheets has been advocated in this study based on its favorable mechanics of out-of-plane bending and through-thickness gradients that mitigate the formation of a tensile instability as confirmed with detailed finite-element analysis. Analytical methods for predicting the local strain at the failure location using the hole geometry were proposed and their accuracy was discussed for the two different materials with strong contrasts in local ductility and plastic anisotropy. The main conclusions of the study are:

- The outer hole edge experiences a uniaxial tensile stress state throughout the conical hole expansion test. The inner edge undergoes non-proportional loading with a compressive stress state at the beginning that subsequently transitions to uniaxial tension. The mid-plane is subjected to a stress state in between shear and uniaxial tension before shifting towards uniaxial tension. The entire hole edge is under uniaxial tension after a HER of approximately 20% for the two materials considered. The regions away from the hole edge undergo non-proportional loading but the strain levels are much lower than the hole edge.
- The element at the outer hole edge follows an approximately linear strain path of ideal uniaxial tension with the slope of the path dictated by the r -value of the corresponding tensile direction.
- The thinning distribution along the circumference of the hole edge is a manifestation of the plastic anisotropy characterized by the variation of r -values. Inhomogeneous thinning might be interpreted as the diffuse or localized necking from inspection of the hole at fracture; however, necking was not observed in the anisotropic finite-element models from investigations of the local stress state, strain path, and strain rate.
- The use of hole expansion ratio based on the outer hole diameter (OHER) over the conventional HER is recommended as it is an average measure of the major strain accumulated at the outer hole edge where the crack is expected to initiate. Any strain metric using HER homogenizes the stress state at the inner edge and should not be used to calibrate stress-state dependent fracture models.
- The macroscopic strain method of Pathak *et al.* (2017) to estimate the uniaxial fracture strain is not recommended for general use since it is based on the isotropic von Mises criterion and is sensitive to uncertainty in post-mortem thickness measurements. The error in the uniaxial fracture strain estimation with respect to the FEA was about 10% for CP800 and 3% for DP1180.

- The work conjugate method can be employed to determine the local uniaxial fracture strain from the conical hole expansion test directly from experiments without needing inverse FEA. Only the normalized yield stress ratios from standard uniaxial tensile tests in different directions are required in addition to the hole expansion experiments to estimate the fracture strain.
- The new methodology proposed to estimate the uniaxial fracture strain is applicable to anisotropic materials that develop a non-uniform thickness distribution and non-circular hole shape. For the 1 mm thick DP1180 with its relatively low ductility and mild anisotropy, the difference in predicted strain with respect to FEA at experimental fracture HER was about 1%. For the 2.9 mm thick CP800 with pronounced anisotropy and high ductility, the prediction error for the equivalent strain was approximately 4%.

B.8. Acknowledgements

The authors would like to thank the members of the Nonlinear Strain Path Project Team of the Auto/Steel Partnership (A/SP) for technical and financial support. Financial support was also provided by the Natural Sciences and Engineering Research Council of Canada (NSERC). The authors would also like to acknowledge Dr. Thomas Stoughton of General Motors, Dr. Andrey Ilinich of the Ford Motor Company, Dr. Gang Huang of ArcelorMittal, Dr. Patrick Larour of Voestalpine Stahl GmbH, and Dr. Nikky Pathak for their valuable insights and discussions on conical hole expansion testing over the past few years which have greatly informed this study.

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Appendix C: Uniaxial Fracture and Plastic Anisotropy Characterization of Sheet Metals using the In-Plane Bending Test

C.1. Introduction

The evaluation of manufacturing feasibility and failure in stamped parts using numerical simulations requires accurate fracture models that can describe the failure strains in terms of the stress state [1] such as the Modified Mohr Coulomb model [2] or its variants [3, 4]. These phenomenological ductile fracture criteria are calibrated using coupon tests that promote fracture in proportional loading under various stress states. Despite experimental advances in fracture characterization under simple shear [5, 6], plane strain tension [7, 8] and biaxial stretching [9], uniaxial tension has remained challenging due to necking that alters the stress state at the mid-thickness of the neck to a triaxial plane strain condition that cannot be captured using optical methods such as digital image correlation (DIC). Numerical modelling can be coupled with the experiments to extract the stress history and failure strain, but its accuracy is tied to the input material data, assumed constitutive model, element formulation and lengthscale. An experimental technique is needed for direct measurement of the uniaxial fracture strain in proportional loading without the need for inverse finite-element modelling.

Test specimens that induce heterogeneous deformation fields have been proposed with fracture intended to occur at a specific location in approximately proportional loading. The hole tension test creates an in-plane strain gradient to induce fracture to initiate at the hole edge, or very close to it, under uniaxial tensile conditions [10, 11]. The hole tension test is appealing due to the simplicity of its geometry, ease of testing on a tensile frame, and ability to consider fracture anisotropy [4]. The test can be effective for certain materials, but cracks can still initiate behind the hole edge in a stress state that deviates from uniaxial tension as reported in studies of Lou and Huh [12] and Khameneh *et al.* [13]. By virtue of being a test that induces in-plane stretching, diffuse necking is difficult to avoid with crack initiation occurring at the mid-thickness plane [14]. Depending upon the material, a hybrid approach may still be necessary to evaluate the local failure strains from the hole tension test [15].

The conical hole expansion (HX) test also features a central hole but is subjected to an out-of-plane extrusion process until cracks develop at the outer edge that experiences circumferential uniaxial tension [16, 17]. A recent in-depth experimental and numerical study conducted by Khameneh *et al.* [13] for a DP1180 steel compared numerous uniaxial fracture geometries and found that only the conical hole expansion test provided proportional loading and thus reached the largest fracture strain. The HX test is not amenable to DIC strain measurement due to its inability to resolve edge deformation. A simple

methodology was developed by Narayanan *et al.* [18] using finite-element analysis to demonstrate that the radius of the outer hole edge at the crack location can be directly converted to an accurate estimate of the local fracture strain. However, failure under an out-of-plane loading condition may not necessarily correspond to a material's in-plane ductility as observed by Hino *et al.* [19] for two high strength steels having a nominal tensile strength of 590 MPa. Furthermore, the conical HX test cannot be used to investigate fracture anisotropy with respect to material orientation, as cracks will initiate along the limiting direction.

Tests that promote in-plane uniaxial bending have received considerably less attention in the literature and can be important for industrial application. Combined in-plane bending and stretching manifests prominently at curved regions of parts such as at the blank periphery of an automotive B-pillar component. The development of laboratory-scale tests that promote in-plane bending is challenging due to potential for buckling in sheet metals. Kuwabara *et al.* [20] developed a pure bending device that consisted of a dogbone-shaped specimen whose ends were fixed to two blocks. The blocks were attached to sliding guide rails that minimized any axial forces during bending. The pure bending fixture was used to evaluate tension-compression asymmetry in a phosphor bronze copper sheet and a 6xxx-series alloy but the strain level of approximately 3% was not sufficient to induce fracture. Bouaziz *et al.* [21] proposed a double bending test to characterize the edge stretchability wherein a square sample is first flanged by 90° parallel to the direction of the edge to be tested. The flanged portion is then subjected a three-point bend using a 4.0 mm radius punch to stretch the edge in-plane and promote failure. The double bending test was used to evaluate the fracture limit for three advanced high strength steels (AHSS) by Dietsch *et al.* [22]. The post-mortem thickness strain at the crack location was converted to an equivalent fracture strain since DIC strain measurement was not possible. Measuring the thickness strain post-fracture can also introduce uncertainties due to additional straining during the rupture process [13]. Another less addressed issue with the double bending test is its sensitivity to various parameters such as the flange height, horizontal gap between the dies, and likelihood of out-of-plane sample buckling. Sato *et al.* [23] developed a fixture that promotes planar bending to quantify the stretch flangeability and fracture strain of a hot rolled steel having an ultimate tensile strength of 590 MPa. The side bending process requires intricate bending fixtures to clamp the specimen and are driven by a hydraulic servo motor. Furthermore, the side bending test unavoidably induces an axial load on the sample since the clamps are not allowed to slide horizontally, resulting in a stretch-bending deformation.

Recently, a novel design for a 4-point in-plane bending test was proposed by Naseem *et al.* [24] to characterize the stress response of high strength steels having low uniform elongation to strains of up to 20%. The in-plane bending apparatus features modular components that load the specimen having a reduced

cross-section gauge area via dowel pins. The pin loaded connections are mounted to linear ball bearing carriages that move smoothly along guide rails to minimize any axial forces during bending. The bending fixture can be attached to a tensile testing machine, allowing the use of DIC to capture deformation. Furthermore, the gauge area of the specimen is not influenced by any contact or frictional effects. Khalilabad *et al.* [25] adapted the in-plane bend fixture design and the specimen geometry to promote edge cracking in a 1.3 mm thick DP800 steel having a trimmed edge with a fracture strain of approximately 0.30. The in-plane bend test induces a stress state gradient across the sample gauge height such that its convex surface undergoes tensile deformation, whereas the concave edge is under compression.

The in-plane bending apparatus appears promising for determining the uniaxial fracture strains as well as for constitutive characterization. To date, the test has only been evaluated to edge strains of up to 0.30, which may be sufficient to promote cracking of sheared edges but not for machined edges to determine the fracture strain of the base material. The geometry of the reduced cross-section can be altered to increase the achievable edge strain to promote uniaxial fracture as done in the numerical study of Narayanan and Butcher [26]. The objective of the current work is to evaluate the suitability of in-plane bending test towards uniaxial fracture characterization of AHSS, develop robust fracture detection criteria, and evaluate its potential for direct measurement of the R-value in both uniaxial tension and compression. A modified version of the in-plane bend fixture is developed alongside a new specimen design and applied to a third-generation advanced high strength steel with a nominal ultimate tensile strength (UTS) of 980 MPa and a fracture strain of approximately 0.70. The ability to determine both the tensile and compressive R-values in the bend test is explored experimentally and numerically. In contrast to current practices in which uniaxial compression tests are conducted using anti-buckling devices that introduce friction and restrict material flow, or stacked compression tests with layers of adhesive and friction at the interfaces, the in-plane bending test can directly measure the plastic strain anisotropy. Two metrics are proposed to detect the onset of fracture that are based upon the global response and the local deformation, respectively. An analytical technique to determine the tensile fracture strain at the convex edge was considered and compared against the strain measured using DIC, which unavoidably performs spatial averaging. The local fracture strains obtained from the in-plane bend tests were then compared against the conical HX results that corresponds to out-of-plane deformation with circumferential stretching. Finally, the limitations of the test in terms of geometric instability and the ability to characterize the fracture limits is assessed across a range of material strengths and thicknesses.

C.2. Materials and Mechanical Properties

A 980GEN3 steel having a thickness of 1.4 mm was selected to be representative of a material with relatively high work hardening, moderate global formability and marginal plastic anisotropy. Additionally,

a 1.4 mm thick martensitic MS1500 steel with a relatively low hardening rate and uniform elongation was also considered to contrast its fracture behavior with the 980GEN3 steel. The quasi-static tensile properties of the two steels along the transverse direction (TD) and their engineering stress-strain response are shown in Table 20 and Figure 132, respectively. The tensile properties were measured using a JIS Z2201 No. 5 tensile geometry having a reduced cross-section width of 25 mm. The strains were obtained from stereo-DIC with the elongations reported using a virtual extensometer of 50 mm length. The R-value was determined by performing a linear fit over a range of axial true strain from 0.01 until 80% of the uniform elongation. The tensile characterization of the MS1500 was already performed in the study of Cheong *et al.* [27] on plane strain v-bending. The transverse direction was found to be limiting for fracture initiation in the two steels and selected for tensile characterization.

Table 20 Material properties for 980GEN3 and MS1500 steels in the transverse direction

Material	0.2% Yield Strength YS (MPa)	Ultimate Tensile Strength UTS (MPa)	Uniform Elongation UE (%)	Total Elongation TE (%)	Average R-value (true strain of 0.01 to 80% of UE)
980GEN3	624 ± 5	1012 ± 6	18.7 ± 0.1	28.0 ± 0.1	1.05 ± 0.04
MS1500	1377 ± 8	1582 ± 4	3.5 ± 0.3	5.9 ± 0.7	0.90 ± 0.02

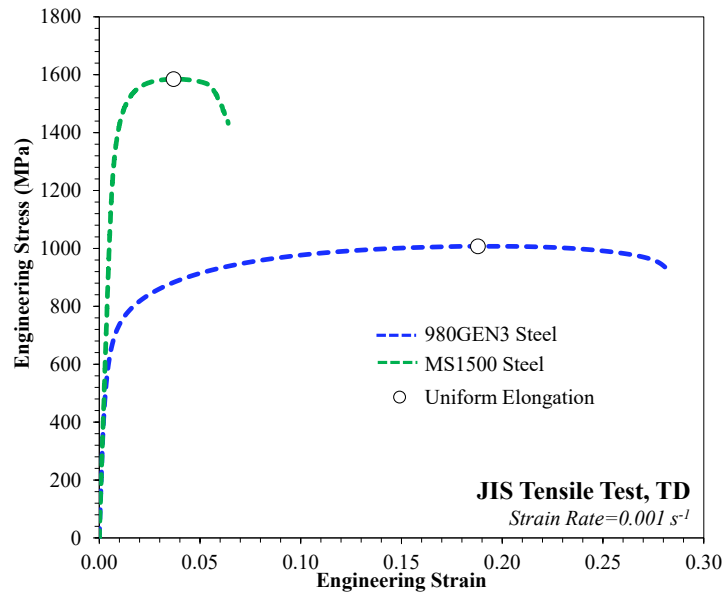


Figure 132 Engineering stress response for the 980GEN3 and MS1500 steels along the transverse direction. Only the response from a representative repeat is shown for each material.

C.3. In-Plane Bending Test: Design & Analysis Methods

C.3.1. Fixture Design

A schematic of the in-plane bend fixture design and its installation onto a 50 kN Shimadzu® AGS-X universal test frame is illustrated in Figure 48(a). A 30 kN Kistler® piezoelectric load cell and amplifier was included into the upper test mount to improve the measurement resolution of the applied force. The linear ball bearing carriages slide onto guide rails to move freely along the X-direction. One of the carriages on the lower mount was fixed using a hand brake while the other three were free to slide during the tests. Clevis mounts with a pin hole diameter of 5.5 mm were selected to provide a double shear loading and enhanced stability against buckling. Preliminary tests with single shear mounting of the pins similar to the design of Naseem *et al.* [24] were observed to promote buckling after strain levels of approximately 0.40. Sleeve bearings with an internal diameter of 4 mm were press-fit into the clevis joint pinholes to reduce friction. Face plates were bolted onto the specimen grip section to further improve the geometrical stiffness and minimize any tendency for out-of-plane deformation that can trigger buckling. A dedicated fixture was employed to apply the face plates to ensure that the sample remained planar. The face plates and the mounting fixture improved repeatability as any out-of-plane deviation or twist in the sample could lead to buckling. The design of the clevis mounts and the face plates are shown in Figure 48(b). The gauge section of the specimen is not influenced by any contact or frictional effects.

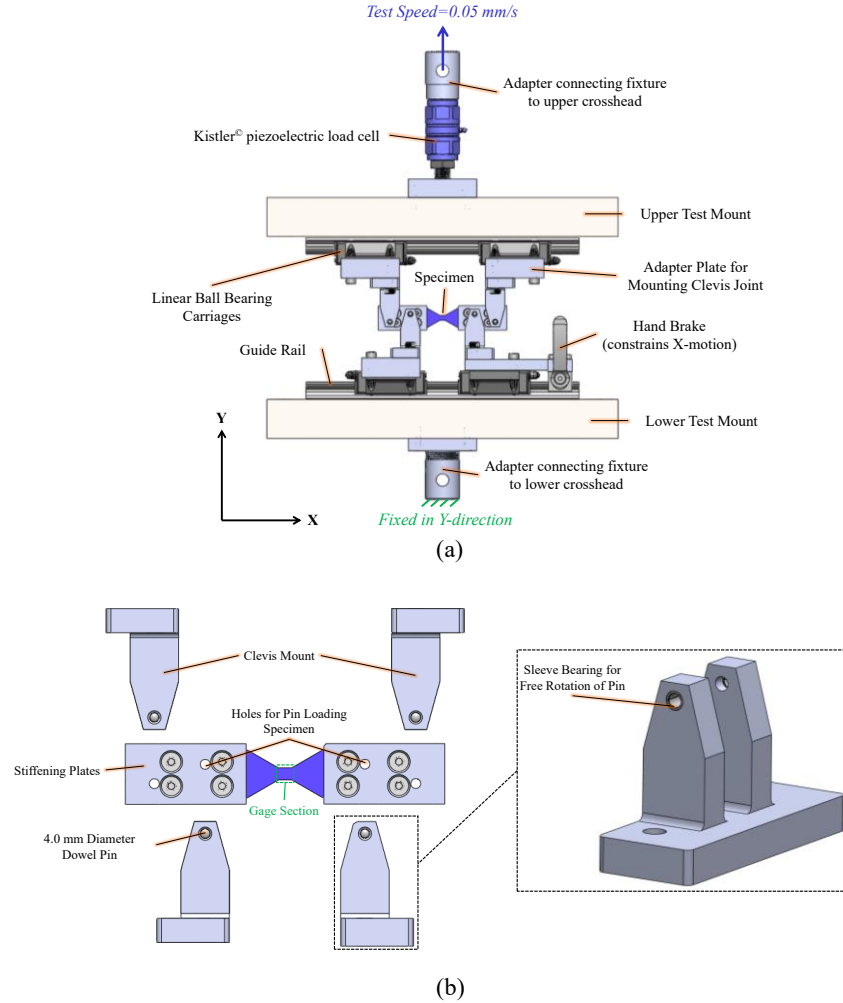


Figure 133 Schematic of the in-plane bending fixture. (a) Front view of the tooling and (b) the specimen clamping and pin loading mechanisms.

C.3.2. Specimen Geometry and Test Procedure

Narayanan and Butcher [26] performed a parametric study using finite-element simulations to assess the influence of gauge section dimensions on the edge strains. The gauge length, height, and the transition radius were varied until a tangential strain of 70% was observed at the convex edge. The same gauge dimensions are used in the present work except for a smaller transition radius of 1.0 mm that was later found to increase the maximum edge strains. The geometry of the in-plane bend sample is shown in Figure 134. The reduced cross-section area was fabricated using CNC machining following which the edges were manually polished using 600 and 800 grit sandpapers. Finally, the sample and clevis pin holes were lubricated using a dry film PolyTetraFluoroEthylene (PTFE) lubricant before each test. An additional camera equipped with a microscope lens was placed facing the convex side of the bend specimen for selected tests as shown in Figure 50 (a, b) to evaluate the fracture detection methods. A displacement-

controlled boundary condition with a velocity of 0.05 mm/s was applied at the upper crosshead during the tests.

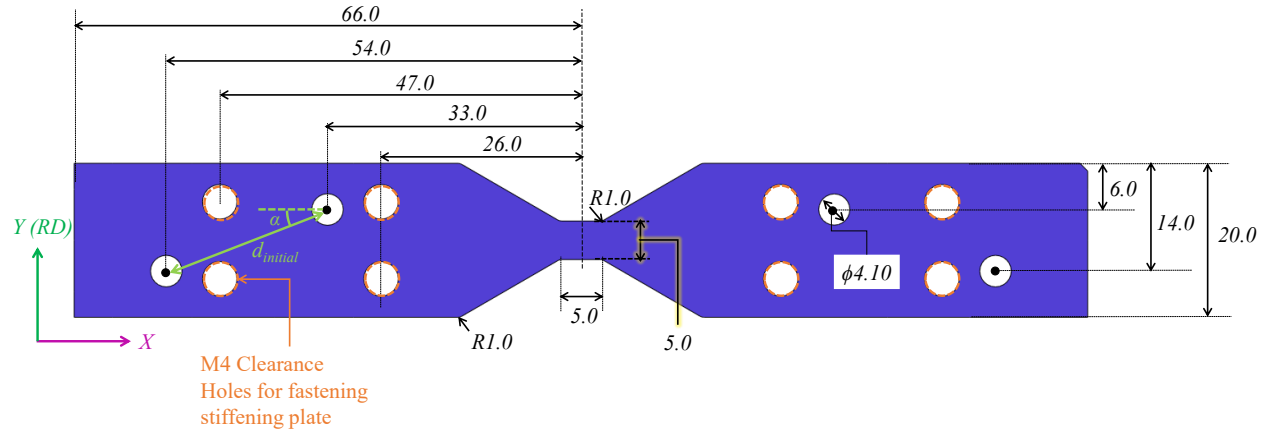
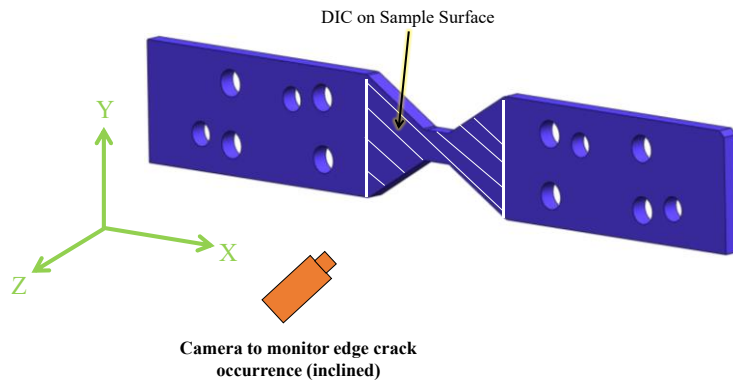
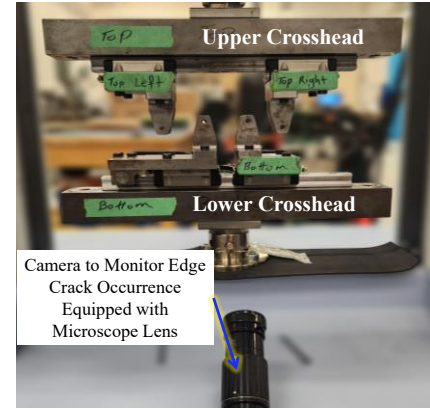


Figure 134 Schematic with dimensions of the in-plane bending specimen.



(a)



(b)

Figure 135 Camera setup for crack initiation detection. (a) Schematic and (b) setup of the bend test tooling with an additional camera to monitor crack initiation at the convex edge of the in-plane bend sample.

Stereoscopic DIC was used to measure the displacement and strain field in the gauge region with the cameras positioned approximately 150 mm apart. The sample was first painted with Rust-Oleum® white primer paint followed by application of black speckles using an airbrush. Calibration was performed using a dot grid target from Correlated Solutions, Inc. target featuring a 14×10 pattern with 3.0 mm spacing between dots (serial H95-00-01). A total of 30 calibration images were captured to achieve a residual error of approximately 0.050 pixels. Additionally, the noise floor in the major strain measurement was estimated to be around 0.0025, which is significantly lower than the anticipated strain levels of approximately 0.70 observed during in-plane bending. A summary of the DIC hardware and analysis parameters recommended

for reporting by Jones and Iadicola [28] is provided in Table 21. The virtual strain gauge length (VSG) that integrates the various DIC processing parameters was adopted [28] and is defined as:

$$VSG(mm) = \{step\ size(pixel) \times [filter\ size - 1] + subset\ (pixel)\} \times resolution(mm / pixel) \quad (Eq. C1)$$

The stereo-DIC parameters were chosen to correspond to a VSG of 1.0 mm with additional details on the selection of the VSG provided in Section C.5.2. For brevity, the terms DIC and stereo-DIC will be used interchangeably throughout this work.

Table 21 Summary of DIC hardware and analysis parameters

DIC Hardware Parameters		DIC Analysis Parameters	
Camera Manufacturer	FLIR	DIC Software	Vic-3D 8, Correlated Solutions, Inc.
Camera Model	Grasshopper®3 GS3-U3-123S6M	Subset Size and Shape Function	17 pixels, Affine
Resolution	4096 x 3000 pixels	Step Size	3 pixels (additional values considered in Table 22)
Lens Manufacturer	Nikon	Filter Size	13 (additional values considered in Table 22)
Lens Model and Focal Length	AF-S Micro-NIKKOR 60 mm f/2.8G ED, 60 mm	VSG	1.0 mm (additional values considered in Table 22)
Stand-off Distance	425 mm	Subset Weights	Gaussian
Field of View	70 mm x 55 mm	Strain Filter Weights	Gaussian—90% center-weighted
Stereo-Angle	20°	Noise Floor in Major Strain	2.5×10^{-3}
Patterning Technique	Airbrush	Tensor Type	Hencky (Logarithmic)
Average Pattern Feature Size	~0.02 mm (measured using ImageJ)	Interpolation	Optimized 8-tap splines
Image Scale (Average of Two Cameras)	0.018 mm/pixel	Radial Distortion Correction Model and Parameters	Seidel (Order=1), $k_1=0.065$ and 0.039 (two cameras)
Image Acquisition Rate	2 Hz	Image Center (both Cameras)	2048 x 1500 pixels

C.3.3. Calculation of Sample Rotation Angle and Bending Moment

The bend or sample rotation angle, ϕ , was tracked using two areas of interest (AOI) placed on either side of the gauge region and closer to the grip section as depicted in Figure 136(a, b) where there was

marginal deformation (see Figure 136(c)). The sample rotation measured on either side of the gauge section was found to be similar as shown in Figure 136(d) indicating a symmetric bend during the test (i.e. $\phi_L = \phi_R$). The bending moment of the cross-section, M_b , is determined from the force ($F_{crosshead}$) and moment arm, d_{arm} , as:

$$M_b = \frac{F_{crosshead}}{2} \cdot d_{arm} = \frac{F_{crosshead}}{2} d_{initial} \cos(\alpha - \phi) \quad (\text{Eq. C2})$$

where $d_{initial}$ and α are the initial center-to-center distance and the angle between the specimen pinholes as shown in Figure 134.

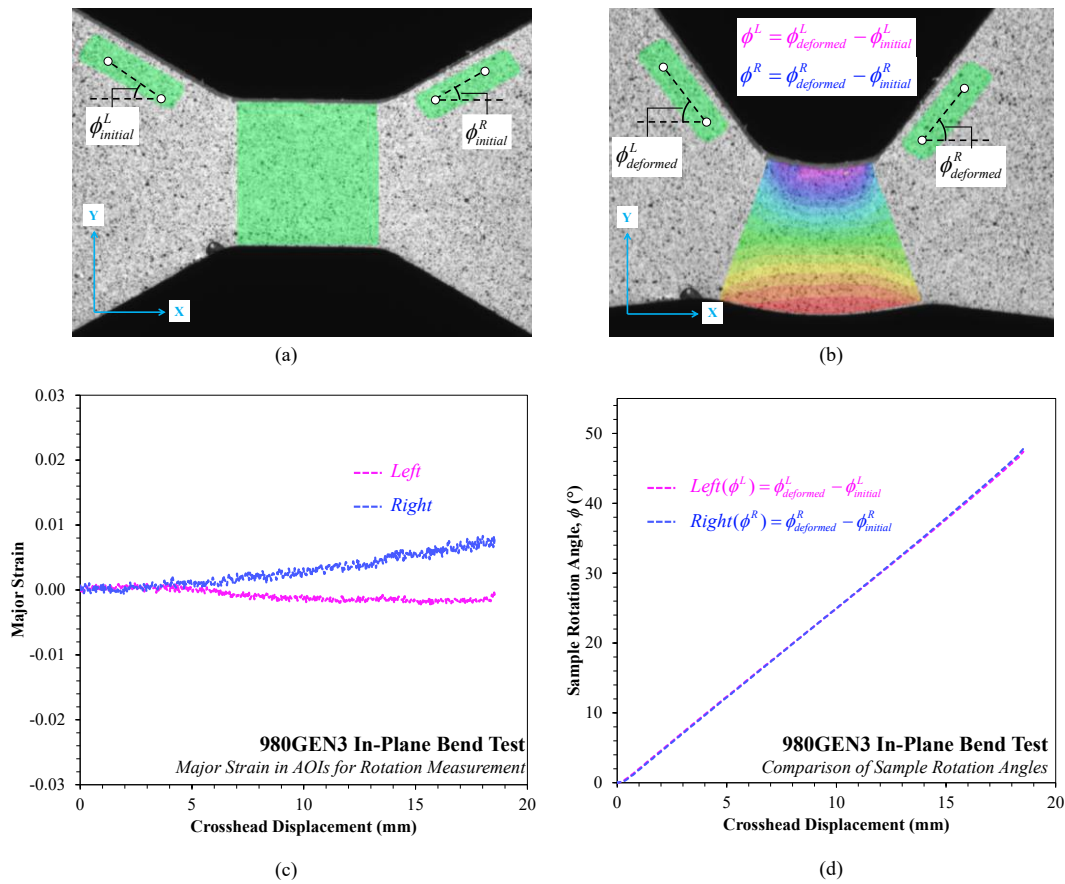


Figure 136 Measurement of sample rotation during in-plane bending. (a) Areas of interest on either side of the gauge section, (b) calculation of sample rotation angle, (c) strains experienced, and (d) comparison of bend angles measured using the two AOIs.

C.3.4. Methodologies to Detect Onset of Edge Cracking

The simplest criteria to determine the onset of fracture are the peak values of the force or bending moment. The VDA 238-100 v-bend test uses a force-based threshold for fracture detection. Troive [29] recommended monitoring the bending moment to avoid the drop in the force that occurs at large bend angles

when the v-bend test transitions to a four-point bend. Noder *et al.* [30] later demonstrated that while both the force and bending moments can be accurate for specific materials, neither was accurate across a range of low and high hardening materials. Instead, a bending stress metric that accounted for the moment in conjunction with geometry changes was successful, as well as a local method that tracked the strain rate using DIC. A recent study by Jeong *et al.* [31] tracked the real-time surface profile of v-bend specimens during deformation using a laser line scanner and found that the microcracks initiated before reaching the VDA load threshold. The choice of a suitable fracture metric for uniaxial in-plane bending remains unclear. Therefore, the peak force and bending moments, along with metrics that analyze the nominal bending stress and strain rate evolution are considered in the present study.

C.3.4.1. Nominal Bending Stress-Based Metric

The nominal tensile bending stress at the convex surface can be calculated as:

$$\sigma_b = \frac{M_b y}{I_{zz}} \Rightarrow (\sigma_b)_{convex} = \frac{M_b \left(\frac{h}{2} \right)}{\frac{1}{12} t h^3} = \frac{6 M_b}{t h^2} \quad (\text{Eq. C3})$$

where y is the distance of the convex edge from the neutral axis and is assumed to be equal to half of the instantaneous gauge height, h . For simplicity, the nominal bending stress at the convex edge is calculated under the assumption that the neutral axis remains at the mid-plane throughout the test. This assumption does not hold in general during plastic bending as the neutral axis shifts towards the edge in compression. Consequently, the stress computed in Eq. (C3) should not be interpreted as representing the true material response at the outer fibre. Accurate estimation of the local stress at any fibre under plastic bending should be determined using a multi-layer analysis that enforces equilibrium conditions as done by Tan *et al.* [32] for plane strain bending. The nominal bending stress defined in Eq. (C3) is used solely for the purpose of detecting the fracture onset.

If the gauge height and thickness are assumed to remain constant throughout the bending process, then the variation of the nominal bending stress from Eq. (C3) is equivalent to the bending moment for fracture detection. The use of DIC allows for estimation of any changes to the cross-sectional height and the thickness. The rectangular box inspector tool of 5 x 5 mm² size in Vic-3D[®] software shown in Figure 137(a) corresponds to the initial gauge region of the bend specimen. The average strains measured using the box inspector are depicted in Figure 137(b). The average shear strain within the gauge specimen is approximately zero. The average thickness change (ε_{zz}^{Box}) can be directly computed from the directional

strain components by assuming plastic volume conservation. Note that fracture is predicted to occur when the peak value of nominal bending stress is reached.

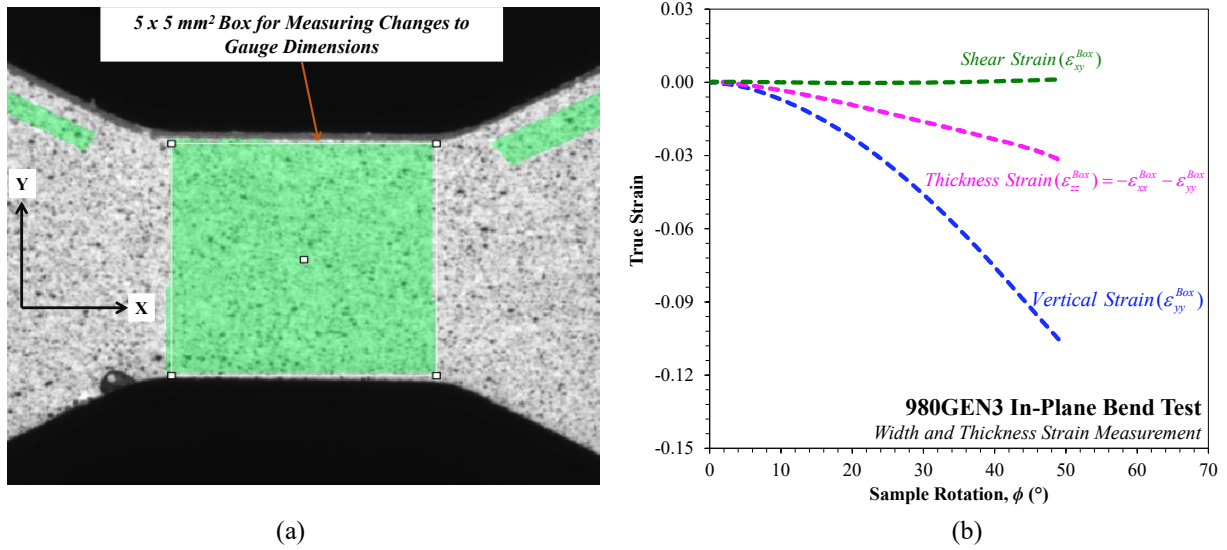


Figure 137 Measurement of gauge cross-section dimensions for nominal bending stress. (a) DIC box inspector for measuring gauge section dimension change and (b) average strains along the height and thickness directions for a 980GEN3 bend test.

C.3.4.2. Local Strain Rate Based Criterion

Noder *et al.* [30] applied the local strain rate method of Volk and Hora [33] for formability testing to detect the onset of cracking in v-bending. The Volk and Hora [33] method defines instability as a bifurcation in the thinning strain rate identified by linear fits of the stable and unstable regions of the strain rate history. In this work, the evolution of thinning with respect to the rotation angle, ϕ , is adopted instead of with respect to time, since it provides a more intuitive measure of change between local and global deformation in the in-plane bend tests. A similar pseudo strain rate was employed by Slota *et al.* [34] to detect the onset of localization in hemispherical punch dome tests using the thinning strain and punch displacement.

The absolute thickness strain was derived from the in-plane principal strain components at the crack initiation location (see Figure 138(a)) assuming plastic volume conservation and neglecting the elastic strains. Figure 138(b) shows the evolution of the absolute thinning strain rate during a representative 980GEN3 bend test. A constant thinning rate of approximately $0.006^{\circ-1}$ is seen at the convex edge for a large part of the before a rapid increase prior to fracture. The sample rotation angle at fracture was identified by the intersection of best-fit lines that characterize the stable and unstable deformation regimes, respectively. Noder *et al.* [30] estimated the starting point of the stable deformation regime for creating the best-fit line in v-bend tests by determining the approximate major strain at the convex surface that

corresponded to through-thickness sample yielding. The solutions of Hill [35] were employed to estimate a cut-off major strain value of 0.06 which is also utilized in this study.

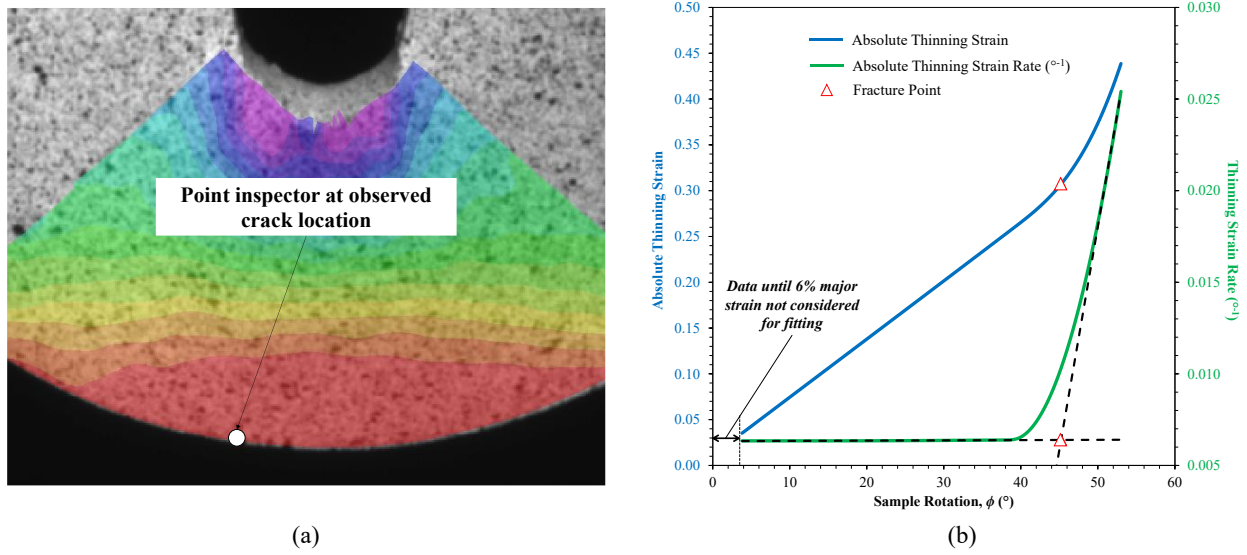


Figure 138 Strain rate method for fracture detection during in-plane bending. (a) Point inspector at observed crack initiation location and (b) evolution of absolute value of thinning strain and its derivative with respect to bend angle.

C.3.5. Analytical Technique to Evaluate Local Fracture Strain

The numerical study of Narayanan and Butcher [26] proposed the arc length technique to measure the deformation near the convex surface during in-plane bend tests using the radius, r_{convex} , and the included angle, θ_{convex} . The purpose of the arc length method was to enable strain measurement without the spatial averaging of DIC that cannot resolve strains at the edge. The tangential strain from the arc length method is determined directly from the edge geometry as:

$$\varepsilon_{\theta}^{convex} = \ln \left(\frac{r_{convex} \theta_{convex}}{L_0} \right) \quad (\text{Eq. C4})$$

where L_0 is the initial arc length chosen to evaluate the bend radius and the included angle that are computed using three points located on the convex edge. Figure 52 illustrates the macroscopic measurements performed at initial and deformed configurations for a representative 980GEN3 in-plane bend test repeat to calculate the convex edge strain. The choice of appropriate value of L_0 for using the arc length method will be discussed in Section C.5.3.

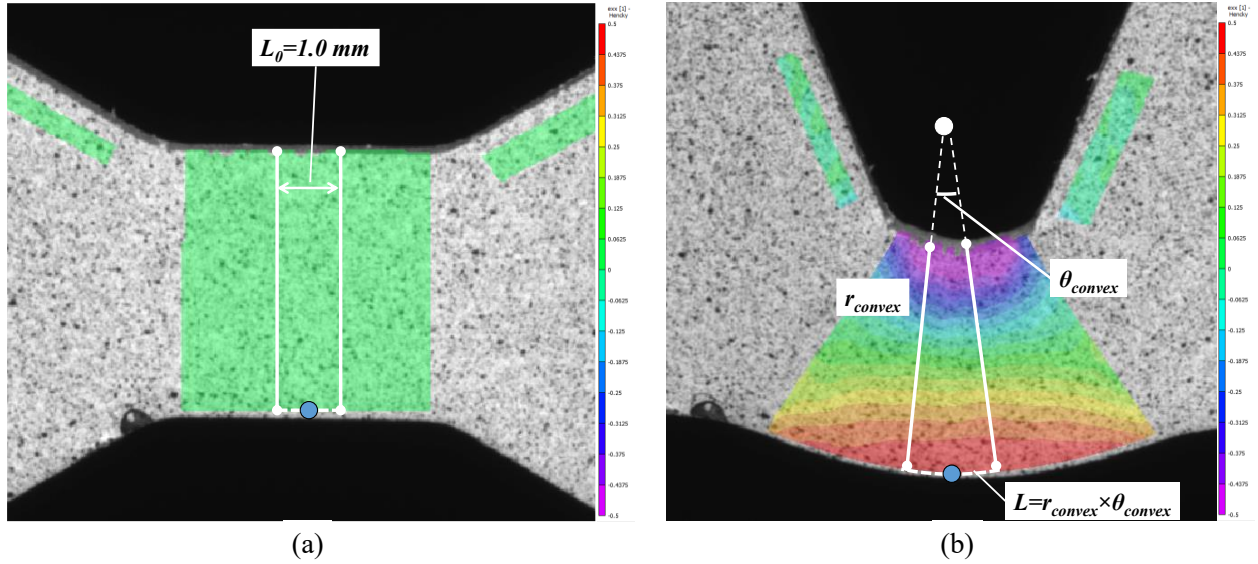


Figure 139 Arc length method for strain measurement in in-plane bend tests. (a) Initial and (b) deformed configurations at the convex edge during a 980GEN3 bend test. The circle marker denotes the apex point on the convex edge where fracture initiated.

C.3.6. Suitability of In-Plane Bend Test towards R-Value Measurement

The in-plane bend test simultaneously activates tensile and compressive deformation states so it is worthwhile to investigate if the strain states near the edges can be exploited to characterize plastic strain anisotropy. Uniaxial compression testing of sheet metals conventionally requires anti-buckling support plates [36] or stacking of multiple sheets together with adhesive layers to obtain a favorable aspect ratio for compression [37]. Finite-element (FE) simulations of the in-plane bend test were conducted for a commercial Ti-6Al-4V titanium alloy having a hexagonal closed pack (HCP) crystal structure. The titanium alloy was selected to evaluate the bend test in the severe case of a material having significantly different tensile and compressive R-values unlike AHSS. The Ti-6Al-4V was characterized by Odenberger *et al.* [38] and used to calibrate the asymmetric yield criterion of Cazacu *et al.* [39] with an exponent, a , of 2.0. The anisotropy coefficients of the Cazacu model were incorporated via the *MAT_233 card in LS-DYNA software. The hardening law of Swift [40] was considered to model the flow stress response:

$$\sigma_y (MPa) = 1500(0.002 + \varepsilon_{eq}^p)^{0.15} \quad (\text{Eq. C5})$$

A full-scale FE model of the in-plane bend test was created in LS-DYNA with the blank modeled using solid elements assuming a 1.4 mm sheet thickness with 5 elements through the thickness of the sheet. The mesh size within the gauge region was kept at 0.1 mm as shown in Figure 140(a) while a coarser lengthscale was adopted near the grip regions since they experience minimal deformation. The tools were modelled as rigid solid pins with a velocity of 0.05 mm/s along the Y-direction applied to the pins located furthest away

from the gauge region. For reference, uniaxial tensile and compression simulations were performed on a unit cube having 10 elements per side as shown in Figure 140(b). The principal stretching direction was the RD for both the unit cube and bend test simulations.

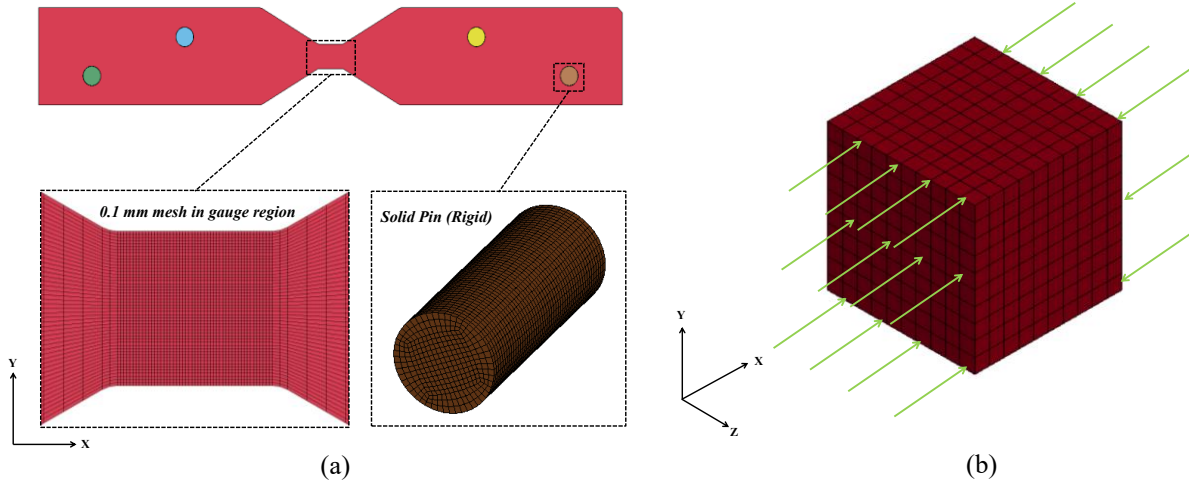


Figure 140 Finite element models for evaluating R-values from in-plane bending and uniaxial deformation. (a) Setup of the in-plane bend test models showing the mesh of the blank gauge region and the pin, and (b) mesh and applied loading for the uniaxial compression simulation performed on a unit cube.

The strain paths obtained from the unit cube and bend test simulations are shown in Figure 141. The Lankford coefficients under tension and compression can be evaluated from the in-plane bend test using the tangential (ϵ_{xx}) and vertical strain (ϵ_{yy}) components as follows:

$$\left(\frac{\epsilon_{yy}}{\epsilon_{xx}} \right)_{convex} = - \frac{R_{\beta}^{Tension}}{1 + R_{\beta}^{Tension}} ; \quad \left(\frac{\epsilon_{yy}}{\epsilon_{xx}} \right)_{concave} = - \frac{R_{\beta}^{Comp}}{1 + R_{\beta}^{Comp}} \quad (\text{Eq. C6})$$

The respective uniaxial tensile and compressive R-values of 0.62 and 0.37 in the RD from the yield function were accurately reproduced by the in-plane bend simulation, highlighting its suitability for plastic strain anisotropy characterization.

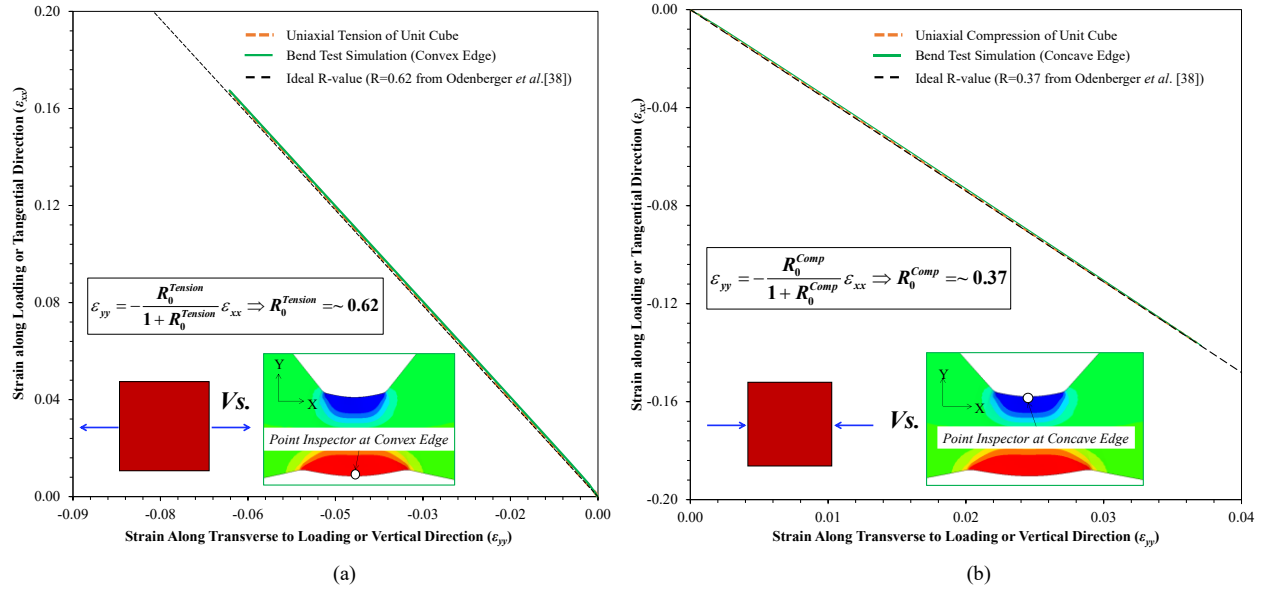


Figure 141 Strain paths from FE models of in-plane bending and deformation of a unit cube. (a) Uniaxial tension of unit cube versus in-plane bending (convex edge element) and (b) uniaxial compression of unit cube versus in-plane bending (concave edge).

C.4. Conical Hole Expansion Tests

Conical HX tests were performed on blanks having an initial CNC machined hole to assess the fracture strains under out-of-plane loading conditions relative to in-plane bending. An initial hole size (D_0) of 5.0 mm was selected to match the 5 mm gauge used in the bend specimen. The test setup with dimensions is shown in Figure 142. The machined hole was sanded with 600 and 800 grit sandpapers. The punch velocity was 0.25 mm/s while the hydraulic binder applied a force of 640 kN.

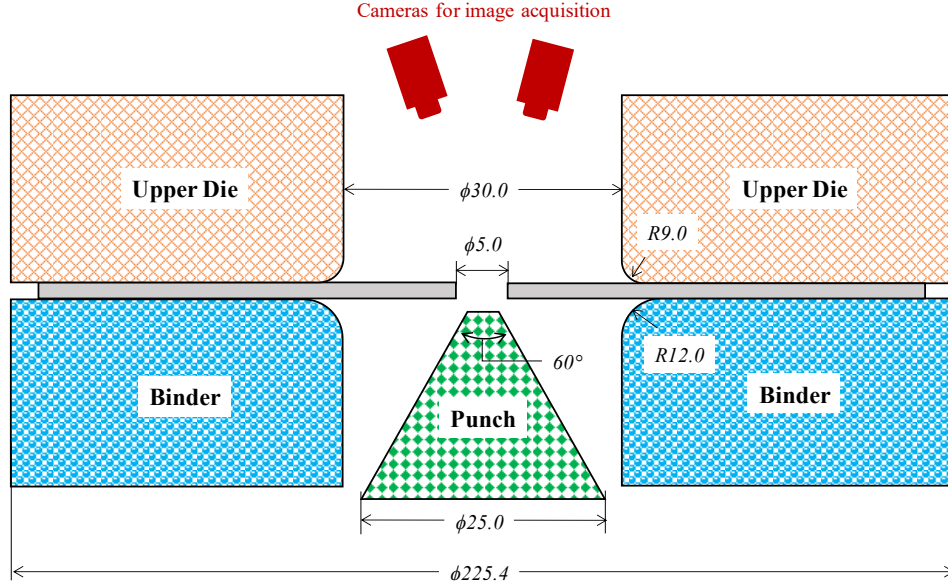


Figure 142 Conical hole expansion test tooling with dimensions.

The first image that showed a through-thickness crack was identified for measuring the hole geometry using ImageJ[®] image processing software. The inner and outer hole diameters were measured at the through-thickness crack image in four different directions and were subsequently averaged to estimate the hole expansion ratio based on the inner (HER) and the outer hole diameters (OHER).

$$HER = \left(\frac{D_{inner}^f - D_0}{D_0} \right) \times 100\% ; OHER = \left(\frac{D_{outer}^f - D_0}{D_0} \right) \times 100\% \quad (\text{Eq. C7})$$

where D_{inner}^f and D_{outer}^f are the average hole diameters at inner and outer edges, respectively. The HER is defined as per the ISO16630 standard but is flawed as a fracture metric since the inner diameter experiences severe compression followed by tension as argued by Butcher *et al.* [16]. The OHER is more representative since fracture initiates at the outer edge that does not contact with the punch. A disadvantage of the OHER is that it loses accuracy if the hole shape does not remain circular. Instead, the strain value based upon the outer hole diameter measured at the through-thickness crack region should be used:

$$\varepsilon_{1,\beta} = \ln \left(\frac{D_{outer}^\beta}{D_0} \right) \quad (\text{Eq. C8})$$

where D_{outer}^β is the local outer diameter measured at an angle of β with respect to the RD. Fracture in the 980GEN3 initiated close to the RD as shown in Figure 143, confirming that TD is the limiting direction for tensile fracture. The major strain in Eq. (C8) was found to be a close estimate of the local strains determined using finite-element simulations conducted on a mildly anisotropic DP1180 and a CP800 steel exhibiting

high planar anisotropy. The reader is referred to Narayanan *et al.* [18] for discussion regarding the effect of planar anisotropy and necking on conical HX test for HER values up to 200%.

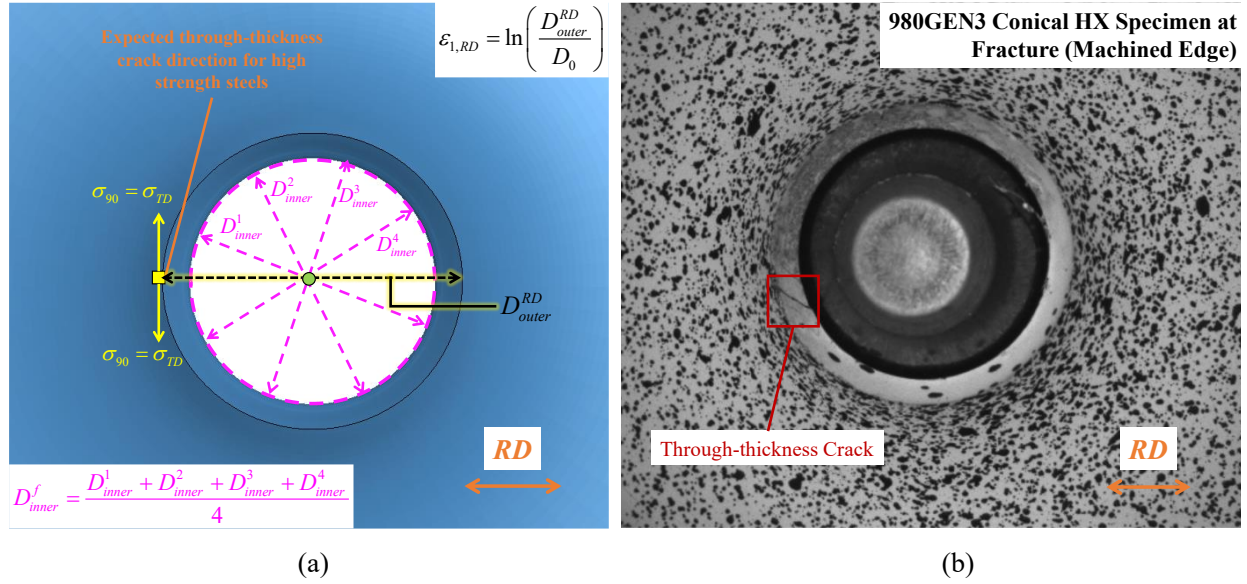


Figure 143 Measurement of local fracture strain from conical hole expansion. (a) Schematic illustrating the procedure employed to measure the hole geometry and local fracture strains from conical HX and (b) image at onset of through-thickness crack during a representative 980GEN3 conical HX repeat.

C.5. Results & Discussion

C.5.1. Local Strain History during In-Plane Bending

The strain path was assessed at several locations along the gauge height of the in-plane bend test in increments of approximately $0.13h$ (see Figure 144(a)) and shown in Figure 144(b). Proportional loading conditions exist near the convex and concave edges in uniaxial tension and compression, consistent with the virtual experiments in Section C.3.6. A loss of focus in the DIC system occurred at the concave edge for tangential strain magnitudes larger than 0.50. This is attributed to loss of DIC correlation due to local sample thickening out-of-plane and that the DIC cameras were focused for best resolution at the convex edge. Consequently, the strain path at the concave edge (see Figure 144(b)) is only shown until a strain magnitude of 0.50 for brevity. At layers located close to the mid-plane such as at $0.63h$ and until a depth of $0.88h$, a non-linear strain path was observed due to shifting of the neutral axis as compressive layers are eventually subjected to tension [35]. The mechanics of the in-plane bend test in conjunction with DIC represents its potential for future applications to characterize Bauschinger effects and calibration of kinematic hardening models.

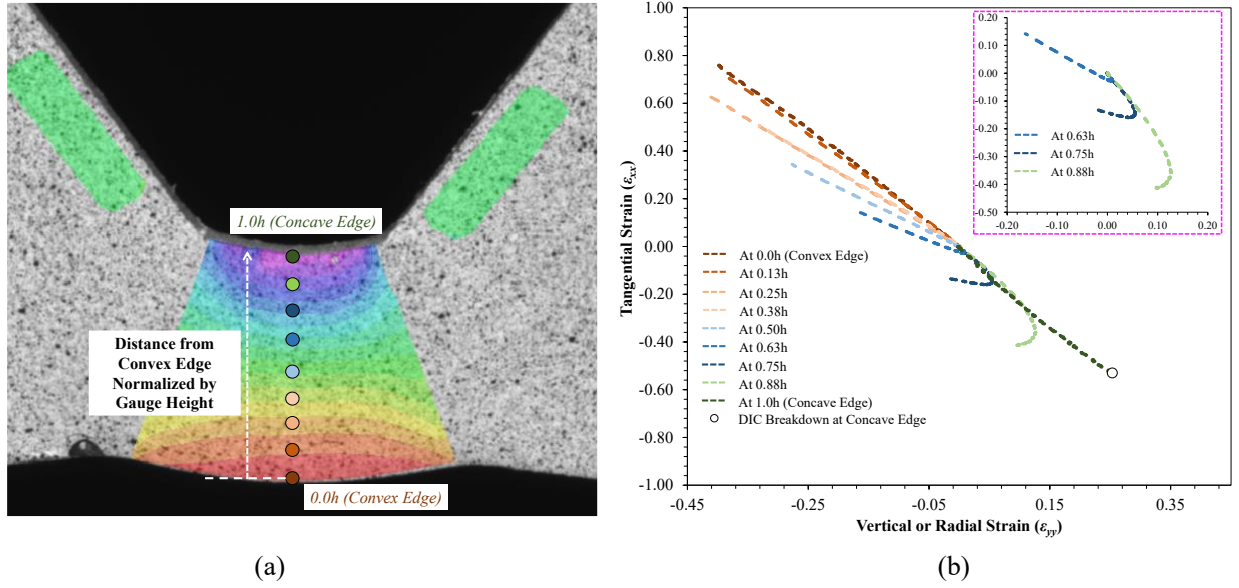


Figure 144 Strain state during in-plane bending. (a) Locations of interest across the gauge height and the corresponding (b) strain paths during a representative 980GEN3 in-plane bend test.

C.5.2. Measurement of Plastic Strain Anisotropy and Influence of DIC Lengthscale

The strain state at the edges were exploited to evaluate the R-values for the 980GEN3 steel as shown in Figure 49. As expected, the strain state at the convex surface is in close agreement with the strain path predicted by Eq. (C6) using the tensile test R-value of 1.05. Similarly, the estimated compressive R-value using the concave edge data until a tangential strain of approximately 0.40 was 0.91 ± 0.04 , showing a degree of tension-compression asymmetry in the 980GEN3 steel. By virtue of suppressing diffuse necking, the in-plane bend test enables characterization of the tensile R-value to strains much larger than the uniform elongation. The tensile R-value for the MS1500 steel from the convex edge data was found to be 0.92 ± 0.01 which is also in agreement with its TD tensile test results summarized in Table 20, while the compressive R-value was determined to be 0.88 ± 0.01 . The edge strain paths for the MS1500 bend tests are not shown here for brevity.

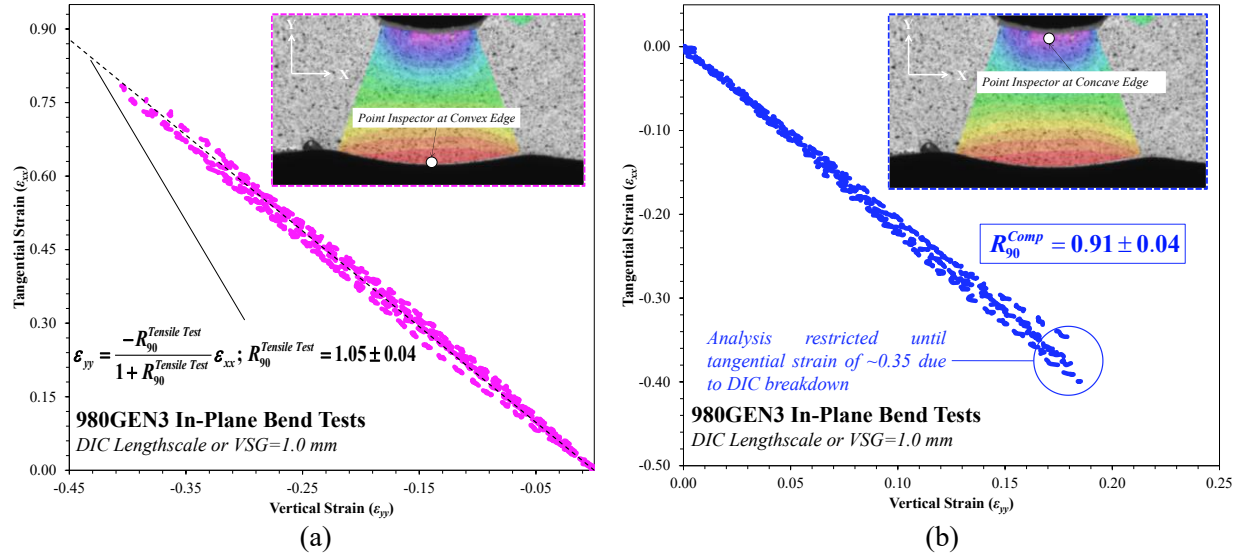


Figure 145 Evaluation of R-values under uniaxial tension and compression from in-plane bending. (a) Strain path at convex and (b) concave edges for the 980GEN3 steel

It is instructive to assess the effect of DIC parameters on strain measurements at the edge. A higher VSG reduces noise at the consequence of losing measurement accuracy near the convex and concave edges. Multiple VSGs were selected for the 980GEN3 bend tests in addition to the default value of 1.0 mm. The corresponding DIC processing parameters for the various lengthscales are summarized in Table 22. The subset size and the image resolution were fixed at 17 pixels and 0.018 mm/pixel, respectively. The resolution is a function of the optical measurement system while the appropriate subset size is selected such that it is large enough to cover approximately 3 to 4 speckles as per the recommendations in the DIC Good Practices Guide of Jones and Iadicola [28].

Table 22 Virtual strain gauge lengths and the corresponding processing parameters selected for performing the DIC lengthscale study on a representative 980GEN3 in-plane bend test.

Approximate DIC Lengthscale or VSG (mm)	Image Resolution (mm/pixel)	Subset Size (pixel)	Step Size (pixel)	Filter Size
0.6	0.018	17	3	7
1.0			3	11
2.0			6	17
3.0			6	25
4.0			10	21
5.0			9	29

The influence of VSG on the strain path at the convex and concave edges reported using a point inspector is depicted in Figure 146(a, b). All VSGs showed a proportional strain state at the edges, but the strain ratios were moderately affected with the predicted tensile R-values ranging from 0.95 to 1.06 compared to the measured R-value of 1.05 ± 0.04 . The concave edge exhibits a similar variation in the R-value in the range of 0.92 to 1.08. Overall, a VSG of 1.0 mm is recommended for analyzing the in-plane bend test as the edge

strain paths align closely with the tensile R-value. Larger stereo-DIC lengthscales show deviations from the ideal strain state that is attributed to increased data averaging near the bend apex. Conversely, the smallest VSG of 0.6 mm introduces a slight non-linearity in the edge strains due to noise that affects the measured strain ratio.

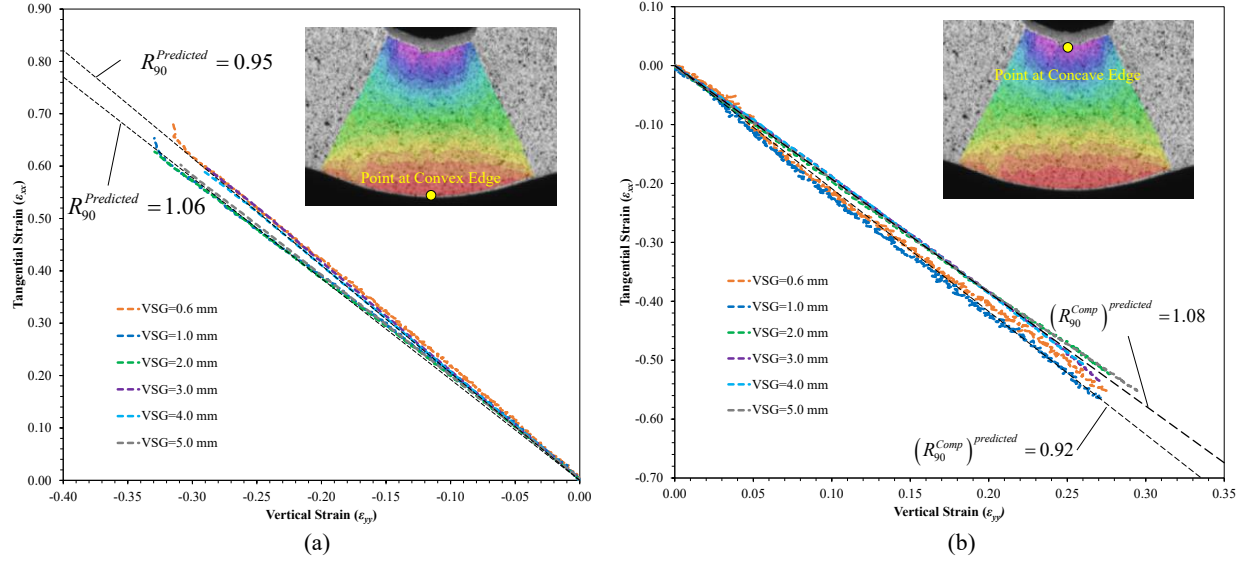


Figure 146 DIC lengthscales influence on edge strain states. (a) Strain path at convex and (b) concave edges for the various VSGs considered in the study.

C.5.3. Evaluation of the Arc Length Method and Identification of Initial Length (L_0)

A tangential strain gradient exists along the arc length of the convex edge with the peak value occurring at, or near the bend apex as depicted in Figure 147(a). To evaluate the edge strains calculated with the arc length method of Eq. (C4), different initial lengths of 0.1 mm, 0.2 mm, 0.5 mm, 1.0 mm, 2.0 mm, 3.0 mm, and 4.0 mm were considered and compared against the DIC point data in Figure 147(b). The tangential strain evolution measured using an initial span length of 1.0 mm was found to be in close agreement with the DIC point data and was chosen to be consistent with the VSG of 1.0 mm chosen for the in-plane bend test. The tangential strain computed using smaller L_0 values such as 0.1 mm and 0.2 mm are expected to produce higher variability and less reliable strain measurements due to significant overlap of the area over which the DIC displacement fields are calculated at the three edge points. The measured deformation for L_0 of 3.0 mm and 4.0 mm can be considered as a global fracture strain value from the in-plane bend test due to averaging of the strain gradient along the arc.

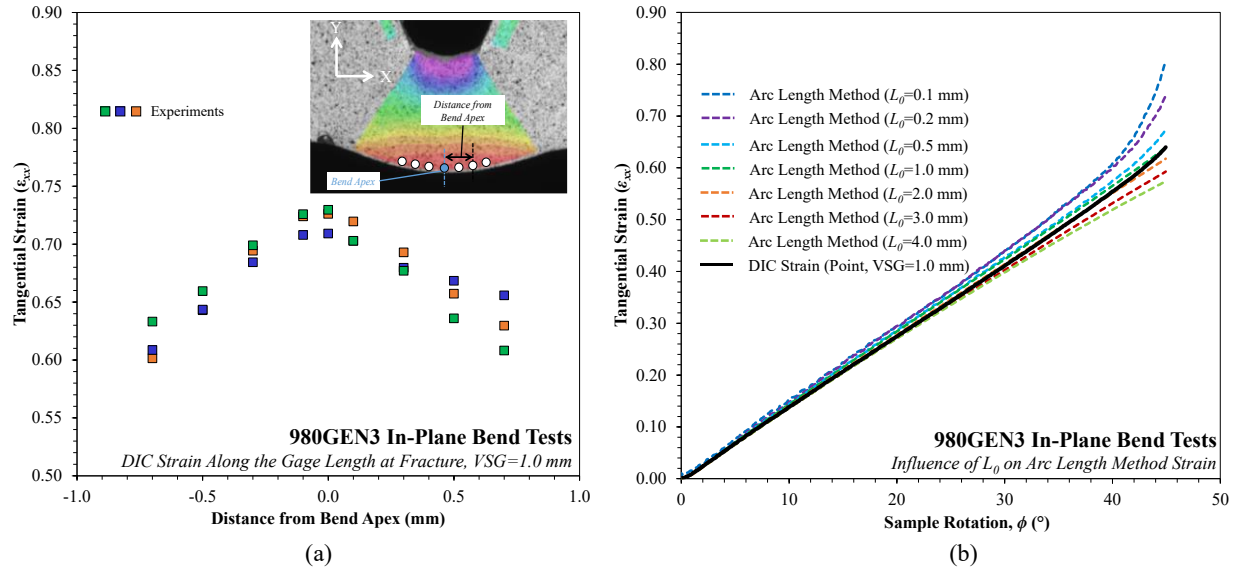


Figure 147 Determination of appropriate span for arc length method. (a) Variation in tangential strain away from the bend apex and along the convex surface in 980GEN3 bend tests, and (b) comparison of deformation obtained from the arc length method using different initial span lengths with the DIC strain obtained using a VSG of 1.0 mm.

Figure 148(a) illustrates the effect of VSG on both the local DIC point inspector and arc length method strains near the convex edge for the 980GEN3 steel at fracture. To investigate the influence of the selected material and sheet thickness, three additional steels from the buckling sensitivity study of Section C.5.6 were considered: a 1.4 mm thick DP780 steel and two 1180GEN3 steels with differing chemistries and initial thicknesses of 1.4 mm and 1.6 mm, respectively. Absolute differences in fracture strain ranging from 10% to 20% were observed between the smallest and largest VSG of 0.6 mm and 5.0 mm, respectively with the variation depending on the material (see Figure 148). The arc length method exhibited less sensitivity to the VSG across all materials as it relies directly on displacement data that is less influenced by the processing parameters. The fracture strain from the in-plane bend test is primarily influenced by the geometric or radius change at the convex edge making the measured deformation from the arc length method less sensitive to the optical lengthscale. The DIC strains are more sensitive to the VSG since they are derivatives of the displacement field and then subjected to further spatial averaging and filtering. The choice of VSG for analysis of in-plane bend tests can vary slightly depending on the material and its gauge. Regardless, a VSG in the range of 1.0 mm to the sheet thickness provides good agreement between the DIC local strains and the arc length method.

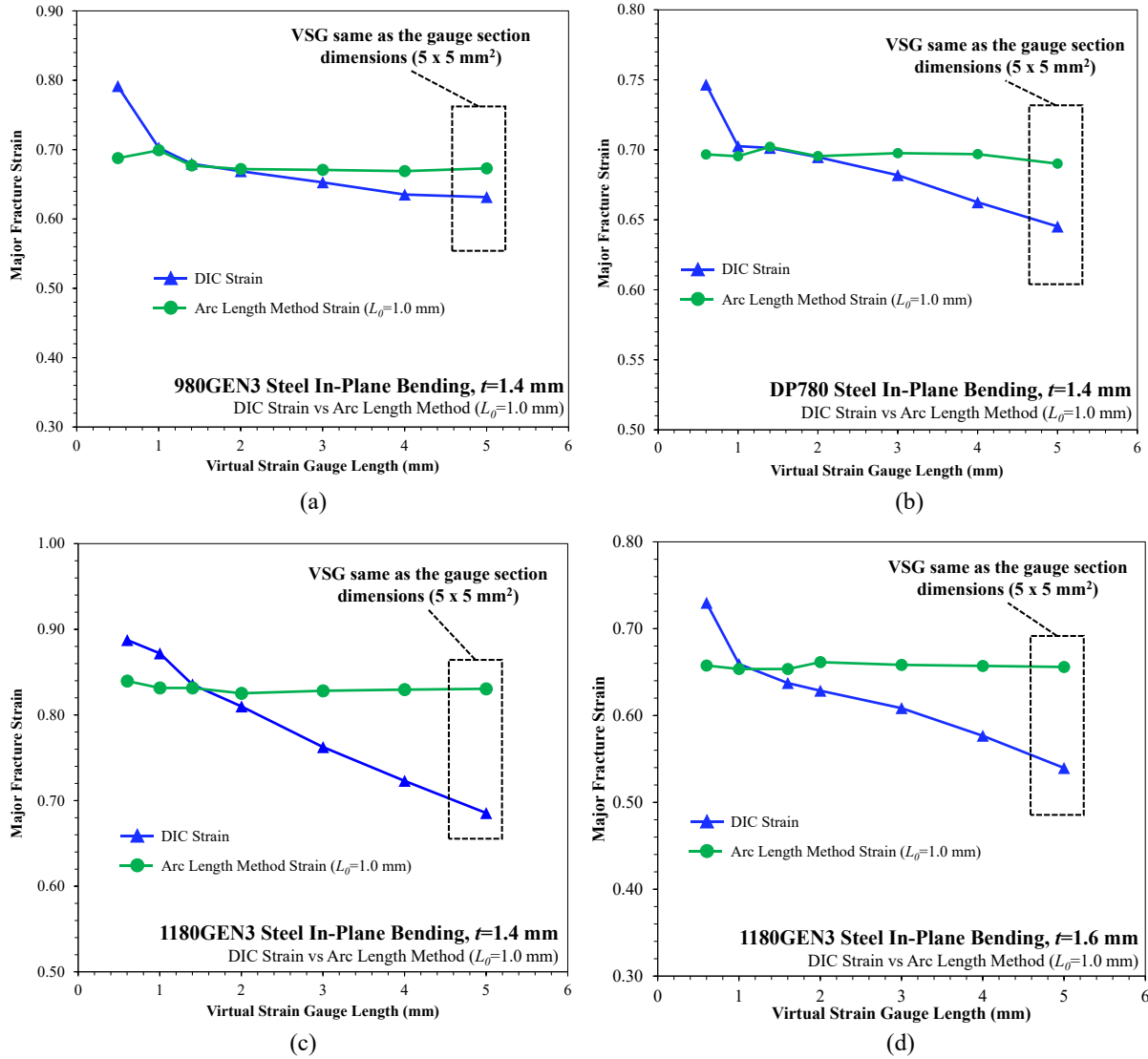


Figure 148 Sensitivity of convex edge strains measured using DIC point inspector and the arc length method to the DIC lengthscale. Results for (a) 980GEN3, (b) a 1.4 mm thick DP780, and two 1180GEN3 steels having a gauge of (c) 1.4 mm and (d) 1.6 mm, respectively.

C.5.4. Detection of Fracture during In-Plane Bending

The evolution of the applied force with respect to the crosshead displacement during the 980GEN3 in-plane bend tests are shown in Figure 149(a). A continuous load increase can be seen throughout the bending process followed by an abrupt drop at fracture. The bending moment computed using Eq. (C2) is shown in Figure 149(b) as a function of sample rotation. Contrary to the force, the bending moment peaks much earlier at a bend angle of approximately 35° following a gradual reduction until fracture. Cracks could be seen on the convex surface at the peak load instant as seen from the DIC image in Figure 149(c) but no visible cracking occurred at the peak bending moment (see Figure 149(d)).

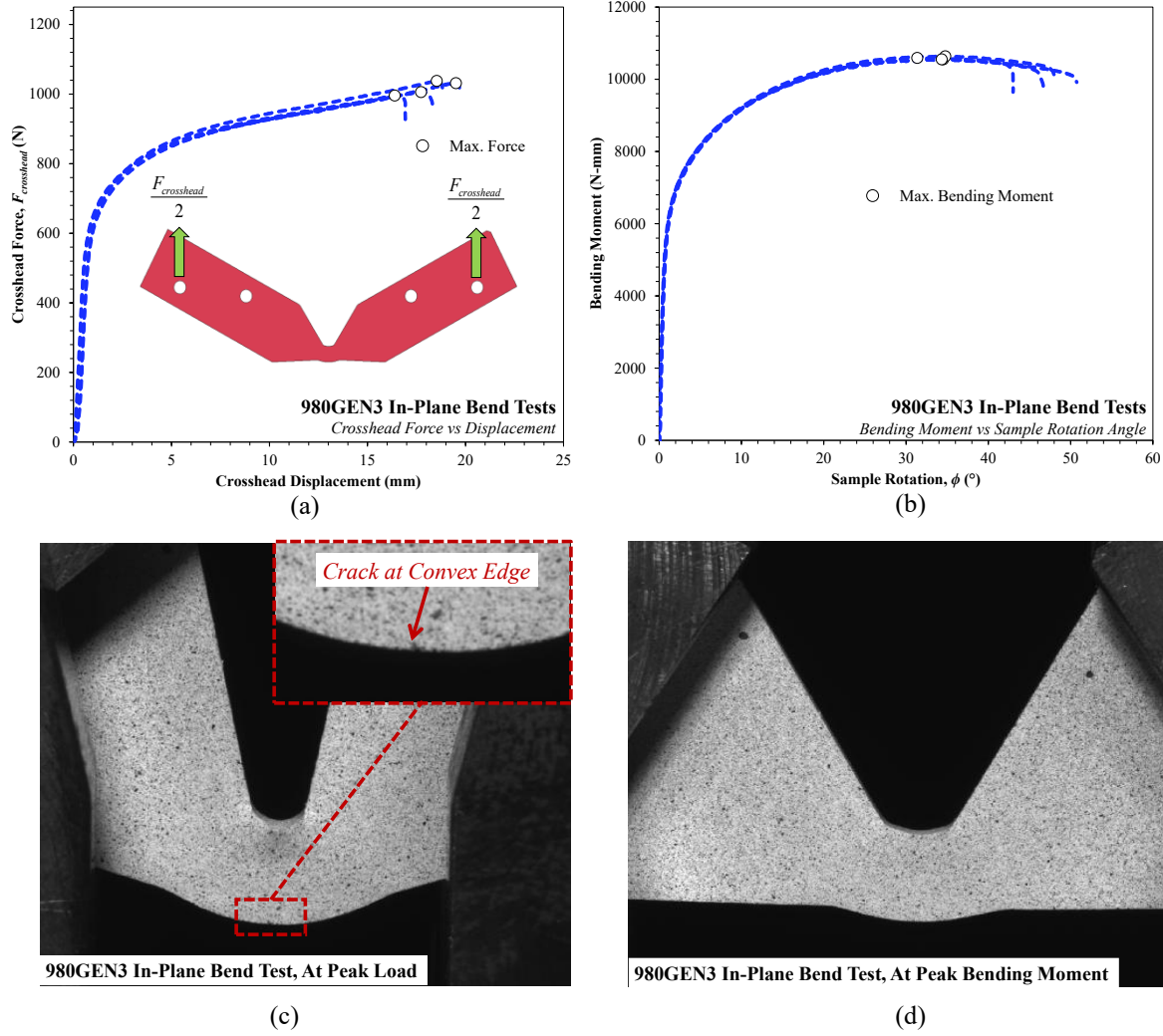


Figure 149 Force and bending moment evolution in the 980GEN3 in-plane bend tests. (a) Measured crosshead force vs displacement, (b) the bending moment vs sample rotation angle, and the corresponding images at (c) peak load and (d) maximum bending moment for a representative repeat.

The accuracy of peak force and the proposed fracture detection methods was visually confirmed from the views of the convex edge for a representative 980GEN3 bend test in Figure 51(a-d). No cracks were seen during the peak bending moment instant. Cracks started to develop at the instant corresponding to the thresholds of the force, nominal bending stress, and the strain rate criteria as shown in Figure 51(c) resulting in similar fracture strains (see Figure 51(e)).

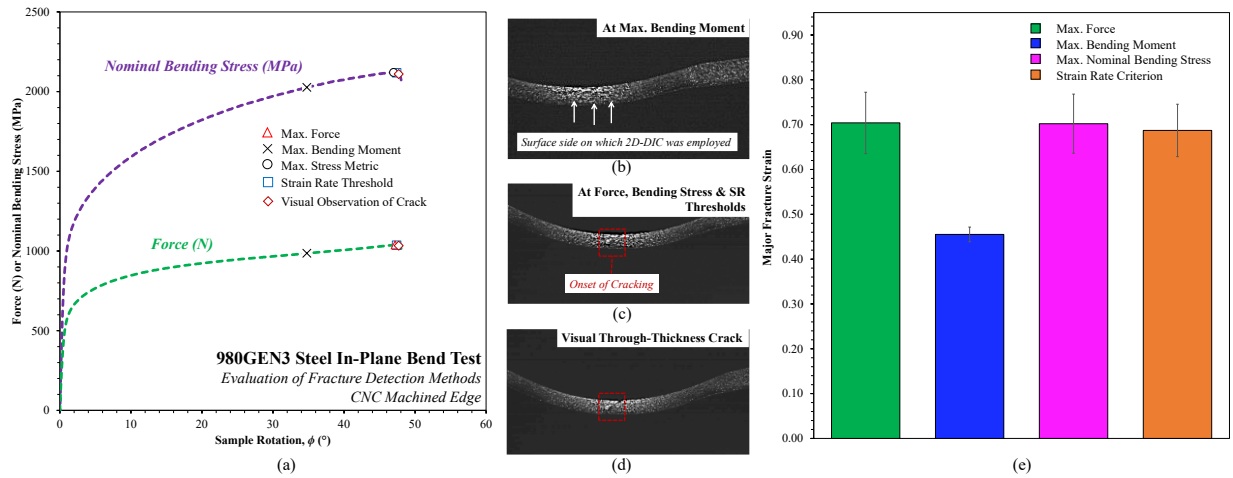


Figure 150 Comparison of fracture detection methods. (a) Evolution of force and nominal bending stress for a representative 980GEN3 bend test overlaid with the instants corresponding to the thresholds of the various detection methods, the view of the convex edge at (b) peak bending moment (c) peak force, nominal bending stress and strain rate thresholds, and (d) through-thickness crack, and (e) comparison of the major fracture strain predicted from the various methods.

A significant discrepancy in the resulting failure limits between the detection methods is possible in materials with lower hardening rates. The force and bending stress progression for the MS1500 bend test is shown in Figure 151(a). The bending moment evolution was comparable to the force and is not shown for brevity. The global force peaked at a sample rotation of approximately 7° followed by a gradual drop until about 30° when through-thickness cracks formed at the convex edge. The bending stress peaked at a much higher specimen rotation of 24° because it integrates the local and global responses, but no cracks were seen at this angle as shown in Figure 151(b). Only the strain rate criterion was able to successfully detect fracture initiation consistent with visual identification as seen in Figure 151(b). A comparison of the fracture strains predicted using the various detection methods for the MS1500 steel is presented in Figure 152(a). Overall, only the strain rate method was found to be suitable for both low and high hardening materials with the nominal bending stress being accurate for the 980GEN3 but conservative for the MS1500.

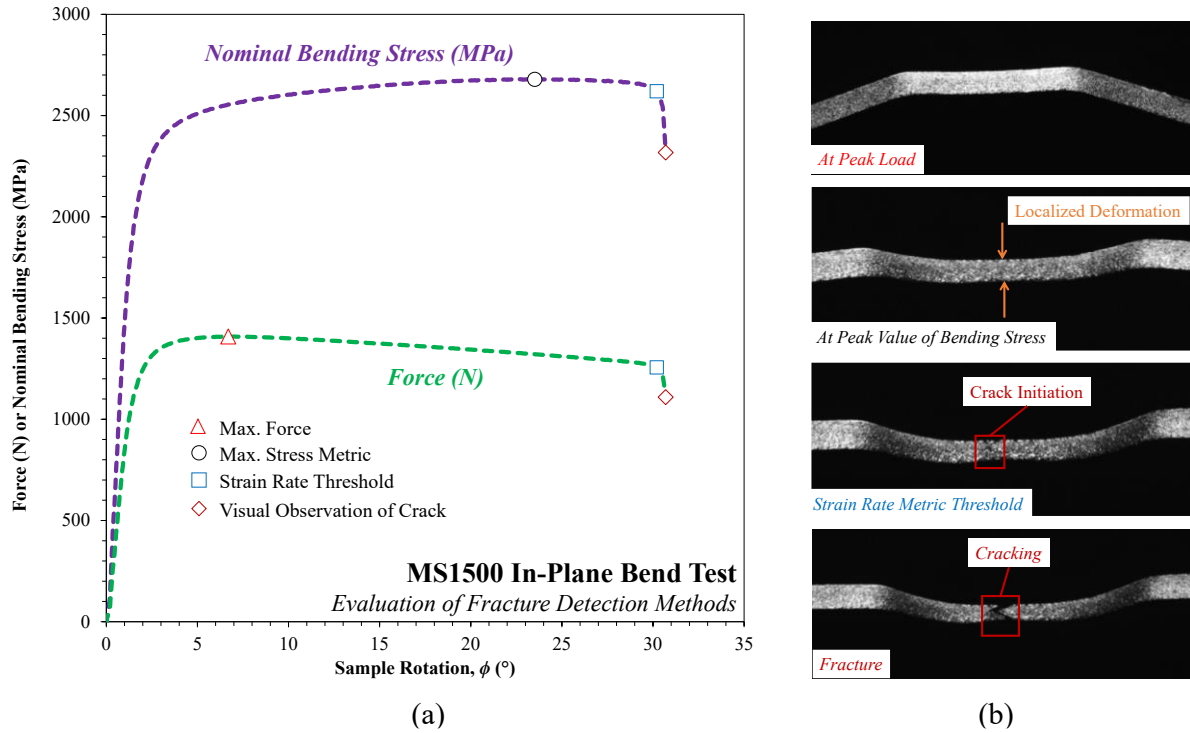


Figure 151 Fracture detection techniques applied to an MS1500 bend test. (a) Force and nominal bending stress evolution for an MS1500 bend test overlaid with the predicted fracture initiation angles from the various methods, and (b) thickness views of the convex edge at the corresponding instants.

The superior performance of the strain rate method for the low hardening MS1500 is because of the appearance of localization as evident from the DIC strain contour in Figure 152(b) and the flattening of the bend radius. This result suggests diffuse necking may be possible during in-plane bending at strain levels that far exceed the uniform elongation. For the MS1500, which has a uniform elongation on the order of 5%, the bending test reached tensile strains that are approximately 16 times higher. For the 980GEN3, in which diffuse necking was not observed, the fracture strain was only about four times higher than the uniform elongation. It is interesting to note that the so-called concave side rule (CSR) proposed by Tharrett and Stoughton [41] for through-thickness bending predicts that fracture will occur without necking when the sheet thickness to bend radius ratio was greater than 0.50. Applying the CSR to the in-plane bend test using the ratio of the gauge height to the radius at the concave surface led to bend ratios of 4.2 and 3.2 for the 980GEN3 and MS1500, respectively. The CSR appears to be generally applicable for the in-plane bend test in that necking is suppressed until fracture, with the low hardening MS1500 exhibiting that some localization may be possible. The mid-plane rule (MPR) can be a good indicator of localization before fracture under through-thickness bending for thickness to bend radius ratios in the range of 0.5 to 0.8 [42]. The ratio of the gauge height to radius at the mid-plane was 1.4 and 1.3 for the 980GEN3 and MS1500, respectively which is suggestive of no acute necking in the considered materials.

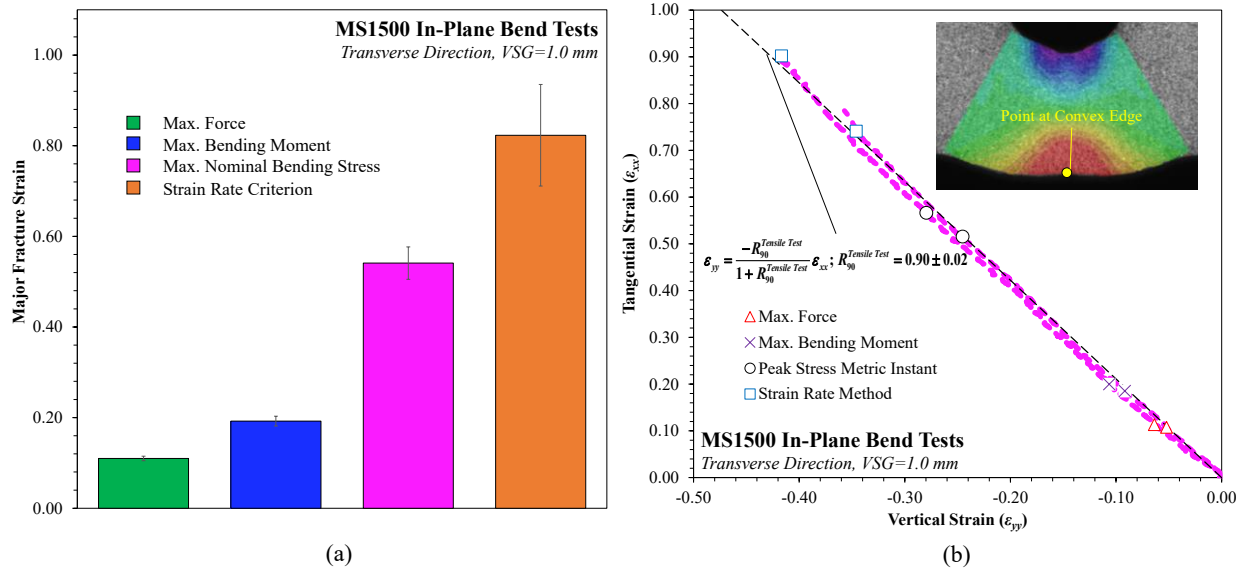


Figure 152 Evaluation of the fracture detection methods for MS1500 steel. (a) Comparison of major fracture strains obtained using different methods, and (b) strain path at the convex edge of the MS1500 steel.

C.5.5. Comparison of Fracture Strains under In-Plane vs Out-of-Plane Bending

Excellent agreement in the uniaxial fracture strains was observed between the conical HX and the in-plane bend tests as shown in Figure 153(a). The 980GEN3 steel exhibited a marginal in-plane anisotropy resulting in close agreement between the fracture strains obtained using the OHER and local diameter method of Eq. (C8). As expected, the conventional HER underestimates the machined edge stretchability of the 980GEN3 steel with an absolute difference in true strain of approximately 17% relative to the outer hole diameter-based techniques. Khalilabad *et al.* [25] also reported a conical HX fracture strain that was 25% lower than in-plane bending for a DP800 steel when using the inner diameter. Overall, the uniaxial fracture limits for the 980GEN3 steel appears to be relatively insensitive to the loading condition for a machined edge. It is expected that a sheared edge condition would show a stronger sensitivity to the boundary condition [43].

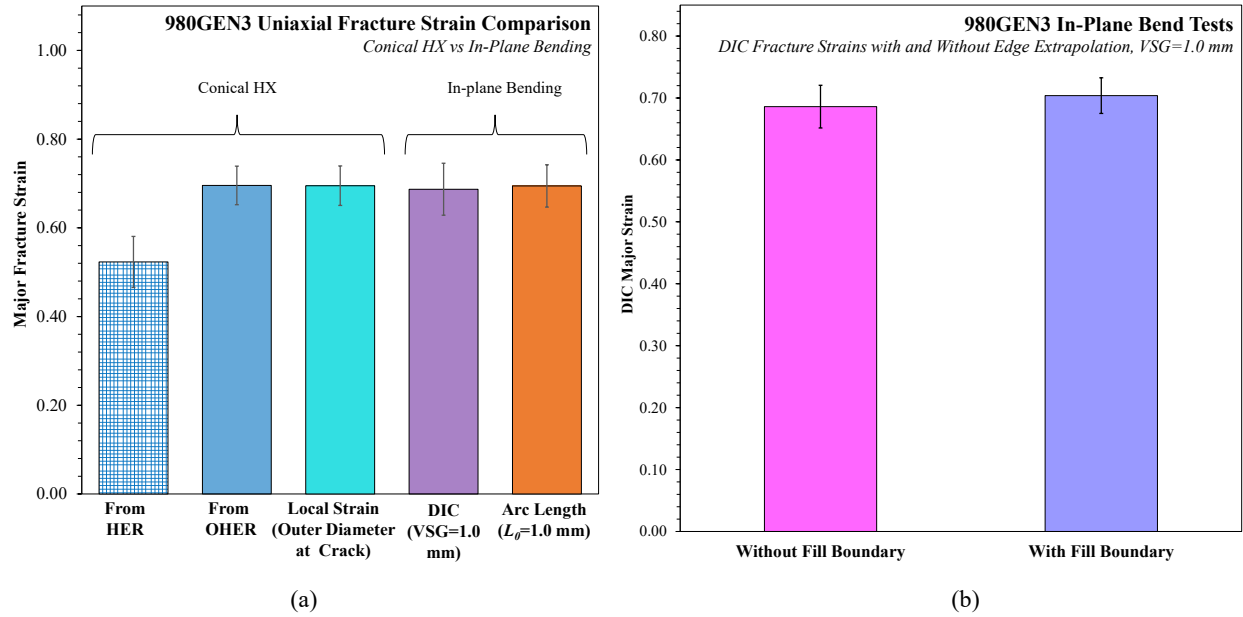


Figure 153 980GEN3 uniaxial fracture strain comparison. (a) Comparison of fracture strains from conical HX and in-plane bending, and (b) influence of DIC edge filling on the uniaxial fracture strain from bend tests

It is important to note that DIC-based strain measurement can have limitations in capturing deformation exactly at the convex edge. While extrapolation techniques, such as those available in DIC software, can extend the strain field to the edges, they may introduce some uncertainty in the measured strain magnitudes. To this end, the ‘Fill Boundary’ option in Vic-3D software was used for three representative 980GEN3 bend tests and the resulting fracture strains at the edge were compared to the data obtained close to the convex surface without using edge extrapolation. The tangential strains at the fracture instant were found to be statistically similar between the two methods as shown in Figure 153(b). Consequently, the reported DIC fracture strains in this work were obtained without using edge extrapolation techniques.

C.5.6. Influence of Material Strength and Thickness on Fracture and Buckling Propensity

The sensitivity of the in-plane bend test geometry to buckling needs to be evaluated over a large range of steels since the test does not include an anti-buckling device. The propensity to buckle depends upon the hardening rate, sheet thickness and gauge height. The gauge height can also influence the maximum achievable strains at the convex surface and by extension the potential for uniaxial fracture characterization. To this end, an experimental study was conducted for several steels encompassing a UTS in the range of 270 MPa to 1500 MPa with multiple thicknesses between 1.0 mm to 2.0 mm. The nominal material strength and thickness information for the steels considered are shown in Table 23.

Table 23 Materials evaluated in the buckling study, along with the uniaxial fracture limits for those that exhibited cracking during in-plane bend tests. Materials that buckled are indicated by a red cross while the blue circle indicates no buckling but no fracture.

Material	Nominal UTS (MPa)	Thickness (mm)	Gauge height-to-thickness aspect ratio	Test Direction	Uniaxial Fracture Strain
DC04	270	1.0	5.0	DD	X
DDQ		2.0	2.5		O
DP590	590	1.4	3.6	TD	O
		1.6	3.1		O
DP780	780	1.2	4.2	TD	X
		1.4	3.6		0.75
MP980	980	1.0	5.0	TD	X
		1.2	4.2		X
980GEN3	980	1.4	3.6	TD	0.70
		1.6	3.1		O
DP1180	1180	1.0	5.0	TD	X
1180GEN3	1180	1.2	4.2	TD	X
		1.4	3.6		0.77
		1.6	3.1		0.66
PHS1300	1300	1.0	5.0	TD	X
		1.2	4.2		0.79
		1.4	3.6		0.87
		1.6	3.1		0.94
MS1500	1500	1.0	5.0	TD	X
		1.2	4.2		0.81
		1.4	3.6		0.82
		1.6	3.1		0.74

The results of the buckling study are summarized in a design chart in terms of the UTS and the aspect ratio in Figure 54(a). Materials that exhibited buckling are denoted by a red cross, while a circle marker was assigned for the cases that were successful. An aspect ratio of lower than 4.0 inhibited buckling during the in-plane bend tests regardless of the material strength. A front-view of a buckled test specimen is depicted in Figure 54(b). The material strength has a second order effect on the limiting aspect ratio. For example, the limiting aspect ratio for the MS1300 and MS1500 steels was 4.2, while the steels having a tensile strength of 980 MPa or lower exhibited buckling before fracture for aspect ratios higher than 3.6. The in-plane bend tests were not able to induce fracture in deep draw quality (DDQ) and some AHSS with strengths below 780 MPa (see Table 23). It may be possible to reach larger strains with a reduction in the gauge length or an increased gauge height although larger gauge heights will also promote buckling. For high ductility materials, pre-straining of the bend test samples in uniaxial tension prior to bending may also be a viable option. The hardening behavior would be affected by Bauschinger effects, but fracture would initiate at the convex edge in tension.

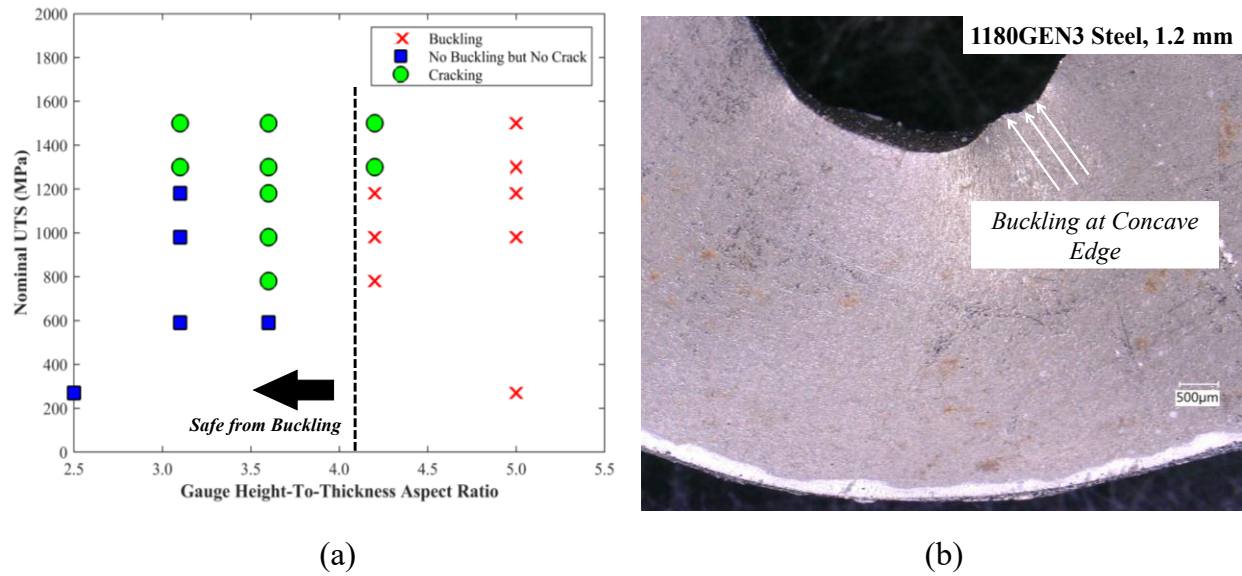


Figure 154 Results of the buckling study. (a) Design chart summarizing the in-plane bend tests conducted on various steels and (b) front view of a buckled 1180GEN3 bend test specimen having a thickness of 1.2 mm

C.6. Conclusions

The in-plane bend test is promising for uniaxial fracture strain measurement in sheet metals that can reach true fracture strains on the order of 70%. The mechanics of the in-plane bend test were evaluated using DIC with particular emphasis placed upon the strain state evolution in the gauge section for R-value measurement, and techniques to compute the edge strains using the change in the arc length. The influence of stereo-DIC analysis settings on the strain state and methodologies to detect fracture initiation during the tests using the global force or the local DIC strains were evaluated. Subsequently, the uniaxial fracture strain of a 3rd generation AHSS having a nominal tensile strength of 980 MPa was characterized using in-plane bending and compared against the results of the conical hole expansion test. Finally, an experimental study was performed to gain insights into the sensitivity of the in-plane bend test to buckling and fracture for a wide range of steels with varying ductilities and thicknesses. The key conclusions of the study are:

- Proportional loading conditions in uniaxial tension and compression were achieved during the in-plane bend tests until fracture with a reversal in strain path measured near the mid-plane due to a shift in the neutral axis.
- The R-values in uniaxial tension and compression can be determined to strain levels significantly larger than the uniform elongation using the in-plane bend test without need for an anti-buckling device. The tensile and compressive R-values for the 980GEN3 were 1.05 and 0.91, respectively. The MS1500

exhibited comparable plastic strain anisotropy under uniaxial tension and compression, with R-values of 0.92 and 0.88, respectively.

- Fracture detection based on peak crosshead force or bending moments may be effective for certain materials but proved unreliable for those with low hardening rates. The proposed metric based on nominal bending stress at the convex edge that accounts for evolution of the gauge region dimensions successfully detected fracture onset in the 980GEN3 but yielded conservative results for the MS1500 steel. The onset of fracture was successfully detected for both materials using the local strain rate based method that monitors the thinning strain near the convex edge.
- The DIC-measured tensile fracture limit for various AHSS from the in-plane bend test was sensitive to the VSG with a relative decrease in strain of approximately 10% to 20% with increase in VSG from 0.6 mm to 5.0 mm. The fracture strain measured near the edge using stereo-DIC converged towards the values determined using the arc length method for a VSG ranging from 1.0 mm up to the sheet thickness.
- The arc length method to determine the fracture strain avoids data averaging involved in stereo-DIC strain measurements and can determine the edge strain directly using the DIC displacement data or conventional image processing techniques. The strains computed using the arc length method was found to be nearly independent of the stereo-DIC lengthscale.
- The uniaxial fracture limit of the 980GEN3 steel determined from the conical hole expansion tests using the local outer hole diameter at fracture was in excellent agreement with the fracture strains measured from the in-plane bend tests. The method of strain measurement from the conical HX is important for estimation of the uniaxial fracture limits, with the use of inner hole diameter resulting in significant underestimation of the edge stretchability.
- To minimize the risk of buckling, a gauge height-to-thickness aspect ratio of 4 or lower is recommended. For ductile steels having ultimate tensile strengths lower than 780 MPa, fracture may not occur using the current test geometry that can attain a maximum strain of about 0.70. It is recommended to increase the gauge height, decrease the gauge length, or subject the bend specimen to a uniaxial pre-strain to promote tensile fracture.

C.7. Acknowledgements

We would like to express our gratitude to the members of the Steel Testing Harmonization Team (STHT) of the Auto/Steel Partnership (A/SP), particularly Eric McCarty and Jonathan Smith. Financial support was provided by A/SP and the Natural Sciences and Engineering Research Council of Canada (NSERC). We are grateful to Prof. Ton van den Boogaard from the University of Twente whose development of the in-plane bend test served as a foundation for this work. Finally, we would like to acknowledge Cameron Tolton of the University of Waterloo and Prof. Jacqueline Noder of the University of British Columbia for their valuable insights and discussions on bending mechanics and DIC analysis.

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Appendix D: On the Influence of Boundary Condition on Sheared Edge Fracture Limits: An Experimental and Numerical Study with Application to 980GEN3 Steel

D.1. Introduction

Mechanical cutting of sheet steels by the action of a punch and a stationary die is ubiquitous during large-scale manufacturing of automotive components regardless of the material strength and thickness. The shearing process induces extensive deformation and damage near the edge in the so-called shear affected zone (SAZ) as defined by Levy and Van Tyne (2008). The work hardened SAZ contains nucleated voids and microcracks that promote edge cracks during subsequent forming as noted by Karelova *et al.* (2009) in DP800 and CP800 steels. The issue of sheared edge failure is magnified in 3rd generation advanced high strength steels (AHSS) that possess a multiphase microstructure consisting of ferrite, martensite, and retained austenite. The retained austenite transforms into martensite via the TRIP-effect which improves the work hardening rate and global formability. However, the edge formability can be negatively affected relative to dual phase steels as observed by Li *et al.* (2023) during comparison of DP1000 and QP1000 steels. Efforts to improve the edge quality have focused on optimizing the cutting process parameters such as cutting clearance (Matsuno *et al.*, 2015) and punch geometry (Gürün, 2023). Secondary shaving operations and warm blanking can also reduce the depth and severity of the SAZ as noted by Pätzold *et al.* (2023) and Zheng *et al.* (2020), respectively.

The shear cutting process introduces complexities in the form of non-linear strain paths, punch contact, and severe material deformation that can be difficult to characterize in experiments and incorporate into finite-element (FE) simulations. Industrial simulations typically use shell elements with simplified fracture criteria to predict edge cracking while bypassing the modelling of shear cutting. One such fracture criterion is the traditional forming limit curve (FLC) that defines global formability in terms of an acute neck formation. However, FLCs can give poor predictions of edge crack susceptibility as observed by Silva *et al.* (2018) for a dual phase steel. Since the FLC is generally available to designers, it is still often used to predict edge cracking by applying scale factors to compensate for damage near the sheared edge. The failure mode of acute necking associated with FLC is also inconsistent with edge cracking where splitting can occur without any significant necking.

The sheared edge can undergo different deformation modes such as stretching or bending in the plane of the sheet metal, or out-of-plane deformation to form extrusions or a flange. The ISO16630 (2017) conical hole expansion (HX) test is the most widely used test to characterize the edge crack sensitivity of sheet

metals (ISO16630, 2017). The conical HX extrudes a hole punched at 12% clearance out-of-plane using a conical punch with 60° frustum angle with fracture being defined at the instant of through-thickness cracking. However, the conical HX loading scenario is not representative of the various in-plane edge deformation modes that a stamped component may experience as schematically shown in Figure 155. To this end, a variety of laboratory-scale test methods have been proposed in the literature that emulate such processes to promote edge cracks. Regardless of the loading condition, the surface of the sheared edge is always under uniaxial tension by virtue of it being a free boundary. However, the interaction of the SAZ with the stress and strain gradients imposed by the applied boundary condition can lead to different failure limits.

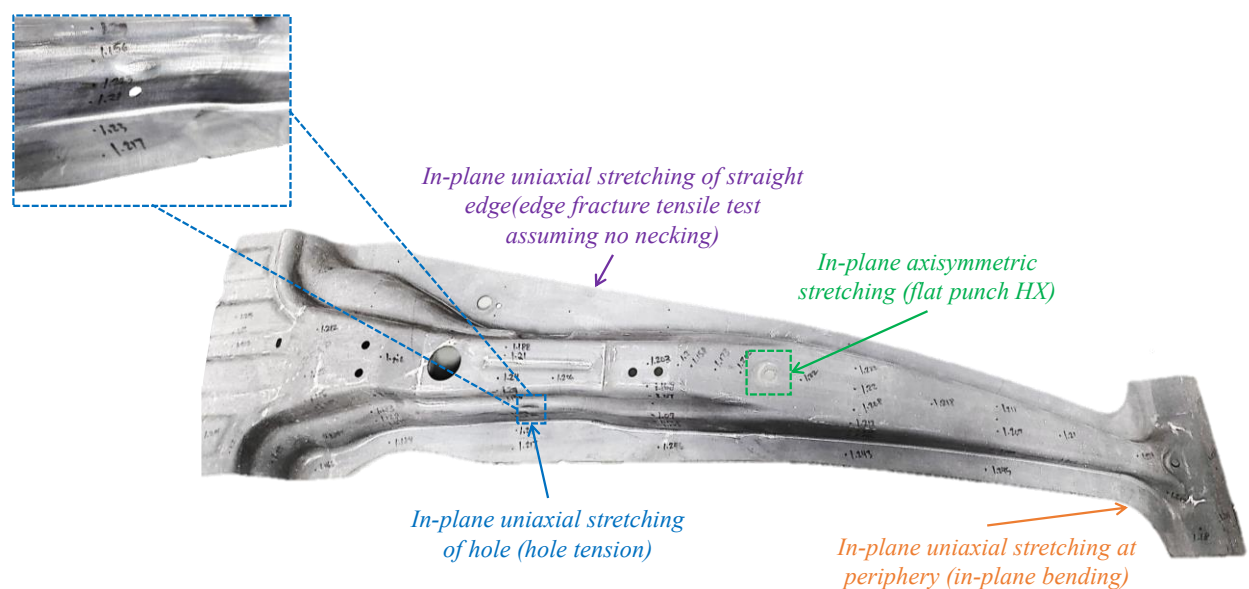


Figure 155 Potential in-plane deformation modes that can be experienced by the sheared edge for a structural B-pillar component. In a crash scenario, the holes that are pierced after stamping could be failure locations.

The hole expansion test can also be performed using a cylindrical punch to characterize sheared edge fracture of high strength steels under in-plane circumferential stretching. However, failure in flat punch HX can occur away from the hole as observed during machined and sheared edge tests for a CP800 by Pathak *et al.* (2016), as well as by Barnwal *et al.* (2020) for a DP980 steel with wire cut edge. The presence of SAZ can promote cracks near the hole edge at significantly lower HER depending upon the material formability as observed by Wagner *et al.* (2025) during flat punch HX of DP780 of CP980 steels.

In contrast to the hole expansion tests, the hole tension test (HTT) promotes unidirectional in-plane stretching of the hole and was first employed by Watanabe *et al.* (2006). The hole provides a stress concentration to trigger failure near the cut edge. The design of the HTT specimen governs the strain distribution near the hole edge as shown by Roth and Mohr (2016) and Feistle *et al.* (2019). The machined

HTT is routinely used to obtain the uniaxial fracture limits of sheet metals for fracture model calibration as done by Dai *et al.* (2023) and Shang *et al.* (2023). However, failure can initiate away from the hole edge as noted for a DP980 steel by Lou and Huh (2013) resulting in conservative fracture strains.

The edge fracture tensile test (EFTT) of Ilinich *et al.* (2011) and Feistle *et al.* (2016) also imposes a uniaxial stretch like the HTT but without a strain gradient across the gauge width at strain levels below the uniform elongation (UE). Beyond the UE, the center of the specimen also begins to neck to produce stress and strain gradients that can make interpretation of the test challenging. Failure in EFTT can be dominated by the occurrence of necking for materials with moderate to high ductility as noted by Larour *et al.* (2021) for DC04 and CP800 steels. The EFTT fracture limits can be quantified using the total elongation or the local strains near the failure location as done by Ilinich *et al.* (2011) for a DP500 steel. Feistle *et al.* (2022) also proposed a so-called edge crack sensitivity factor to analyze the EFTT that normalizes the average major strain at localized necking for a sheared edge to the corresponding value obtained for a machined tensile test. The EFTT is only representative of the forming condition at an edge of a part at strain levels below the UE. The supporting material behind the flange or in curved regions of stamped parts are not exposed to a uniform uniaxial tensile stress state to form a global neck. The strain gradient in these regions will constrain the deformation and failure to occur near the sheared edge.

The so-called in-plane bend test (IPB) first proposed by Naseem *et al.* (2020) allows for formation of a localized neck at the edge in tension without the occurrence of a global neck due to stress and strain gradients imposed by bending. The IPB performs a 4-point bending of the sheared edge that allows for DIC strain measurement and is free of frictional and tool contact effects. Narayanan *et al.* (2025a) further modified the fixture design of Naseem *et al.* (2020) to enable uniaxial fracture characterization of sheet materials having true fracture strains on the order of 70%. The flexibility in sample shape allows for characterization of sheared edge fracture limits using the in-plane bending test. Khalilabad *et al.* (2022) adapted the specimen design to have a straight section that can be fabricated using trimming. The influence of cutting parameters and material orientation on sheared edge fracture strain was investigated and compared against the results of the sheared EFTT and the conical HX tests. The local sheared edge strain from the IPB was found to be higher than conical HX and EFTT by approximately 30% for a DP800 steel, highlighting the potential influence of applied loading on the sheared edge stretchability. Recently, Matsuno *et al.* (2024) also developed a fixture to impose a 3-point bend on a notched specimen to assess the influence of punching and hydrogen charging on the fracture stress threshold.

In addition to the differences in the measured fracture limits due to boundary condition influence, how the fracture strains are measured using DIC and applied to FE models are additional sources of complexity.

Phenomenological fracture criteria are typically employed to model failure during forming and crash simulations and are calibrated using fracture experiments conducted in various stress states (Chen *et al.*, 2018; Li *et al.*, 2022). The fracture curve is calibrated for a specific reference mesh size in simulations which depends on the material and test methodology used to obtain the failure limits. Therefore, selecting appropriate reference DIC and simulation lengthscales becomes essential to ensure transferability of fracture strains. The choice of DIC virtual strain gauge length can significantly influence the reported strain values as shown by Khameneh *et al.* (2021) for simple shear tests. However, the virtual strain gauge length is often not reported during edge fracture test comparison which can complicate the interpretation of the measured strains. Additionally, boundary condition and lengthscale dependence can also arise in FE models of edge stretching due to the mesh size, element formulation and material model. Practical stamping simulations for industrial application employ plane stress shell elements with sizes varying from 1 to 5 mm while the SAZ depth is on the order of 30% of the sheet thickness that may only be a fraction of the element lengthscale. Detailed information on the SAZ is typically not available due to complexities of modelling the shear cutting process. Approaches such as the two-step simulation of shear cutting and stretching as done by Habibi *et al.* (2024), or incorporation of SAZ deformation from experiments into FE models of edge stretching by Park *et al.* (2022) have been proposed but have been limited to coupon level application. A nominal fracture strain value based on the cutting process can be prescribed in commercial FE software as done by Falk (2017) in AutoForm. However, this approach may not be feasible across various deformation modes and mesh sizes. A systematic investigation needs to be conducted to analyze the influence of numerical lengthscale and boundary condition effects for sheared edge fracture.

The objective of the present study is to perform a comprehensive investigation of the influence of applied boundary conditions on the experimental and numerical determination of sheared edge fracture limits. Emphasis is placed upon appropriate measurement of fracture strains using DIC, the influence of its lengthscale, and transferability to shell elements to support industrial application. Five coupon tests encompassing in-plane and out-of-plane deformation modes are evaluated to compare edge stretchability of a 1.4 mm thick 3rd generation 980 MPa steel: IPB, edge fracture tensile, hole tension, flat punch and conical hole expansion tests. For each test, a DIC virtual strain gauge length analysis is performed to examine the sensitivity of local strain measurements to the experimental lengthscale. Deformation measures that are based upon the global extension of the sheared edge are proposed to define the edge fracture limits and guide the choice of appropriate DIC gauge length. Finally, FE simulations of these coupon tests are conducted using various mesh sizes to evaluate how boundary condition effects can manifest numerically and identify best practices to account for lengthscale effects. The results of this work will help inform edge

thinning limits for design of stamped components, and for edge crack prediction using shell element simulations.

D.2. Material & Mechanical Properties

A commercially available 1.4 mm thick 3rd generation advanced high strength steel having a nominal tensile strength of 980 MPa (980GEN3) was selected as the material of interest. The 980GEN3 steel has a higher global formability than the corresponding DP grades due to the TRIP effect provided by presence of retained austenite. However, the transformation of retained austenite into martensite during the shearing process can negatively affect its subsequent stretchability leading to high edge sensitivity (Chang *et al.*, 2018). The mechanical properties and plastic strain anisotropy of the 980GEN3 steel at a strain rate of 0.001 s⁻¹ were characterized by Narayanan *et al.* (2025b) and are summarized in Table 20 and Figure 132, respectively.

Table 24 Mechanical properties of the 980GEN3 steel from JISZ2201 tensile tests conducted along the RD, DD, and TD (Narayanan et al., 2025b). A virtual extensometer of 50.0 mm length was used to evaluate the elongation

Direction	Yield Strength (MPa)	Ultimate Tensile Strength (MPa)	Uniform Elongation (%)	Total Elongation (%)	Average R-value
RD	654 ± 2	1006 ± 2	19.5 ± 0.1	27.7 ± 0.1	0.85 ± 0.02
DD	633 ± 2	1010 ± 2	19.0 ± 0.1	27.3 ± 0.1	0.95 ± 0.02
TD	624 ± 5	1012 ± 6	18.7 ± 0.1	28.0 ± 0.1	1.05 ± 0.04

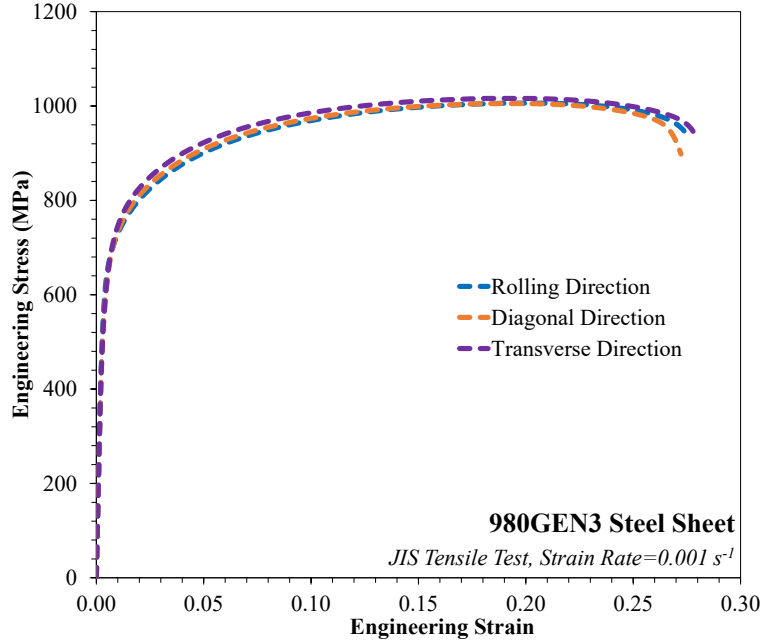


Figure 156 The representative engineering stress-strain response for the 980GEN3 steel along the RD, DD, and the TD.

D.3. Experiments and Procedure

D.3.1. Hole Processing

Figure 48(a) shows a four-post hole punching tool installed on a 120-ton hydraulic Macrodyne press. The cutting punch and die buttons can be readily seated into their respective retainers using a ball-lock mechanism. Circular and square holes of 5.0 mm size were punched on square blanks having a size of 203.2 x 203.2 mm² with a cutting clearance of 12%. Monarch Klir-Cut 2 cutting oil was applied to the surface of the blank for lubrication during hole punching. Blanks with circular holes were used for fabrication of hole tension, conical and flat punch hole expansion samples. For the edge fracture tensile and in-plane bending samples, one side of the punched square hole aligned along the TD was kept intact while remainder of the specimen outlines were extracted using CNC machining as depicted in Figure 48(b). The transverse direction was found to be limiting for fracture initiation and was considered as the principal stretching direction, where applicable.

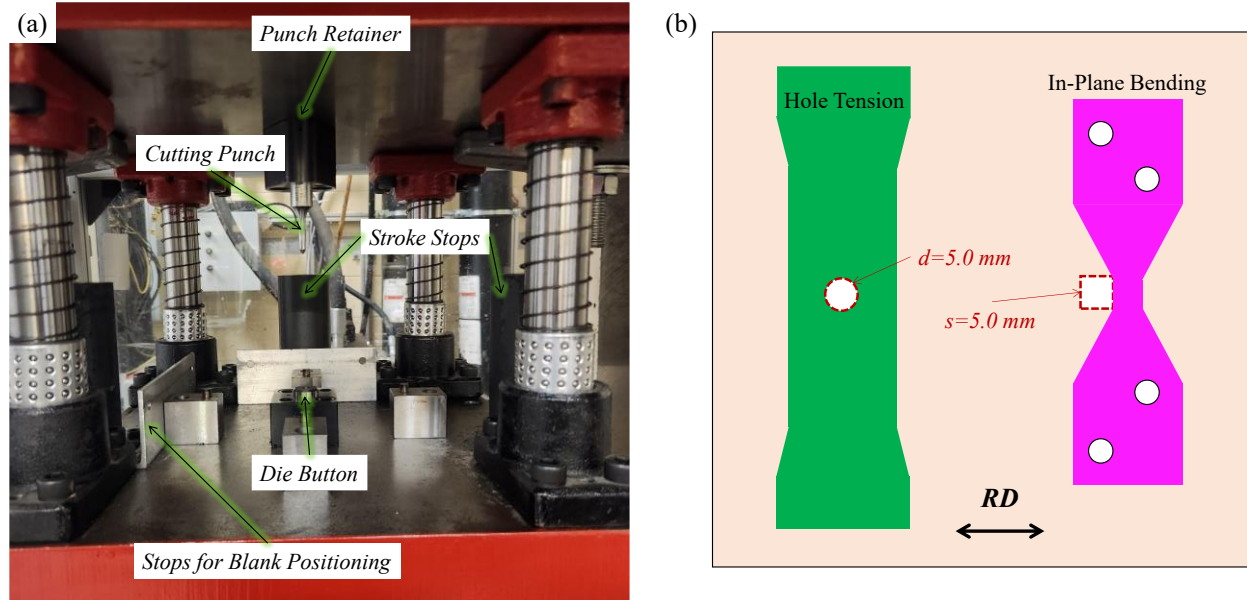


Figure 157 (a) Hole punching tool, and (b) schematic illustrating extraction of hole tension and in-plane bend samples along the transverse direction from circular and square sheared holes, respectively.

D.3.2. Tests for Edge Fracture Characterization

Five different experimental techniques, each producing distinct boundary conditions away from the periphery were considered to evaluate the machined and sheared edge stretchability of the 980GEN3 steel. The gradients induced by each of these tests are schematically illustrated in Figure 158. It is important to note that the conical HX gradients shown in Figure 158(a) occurs only during initial stages of the test due to contact of blank inner edge with the punch before transitioning to uniaxial tension at hole expansion ratios above 20% as shown by Narayanan *et al.* (2023). The strain gradients also evolve for the edge fracture tensile test if edge cracking does not occur prior to reaching the uniform elongation as the entire cross-section will form a neck.

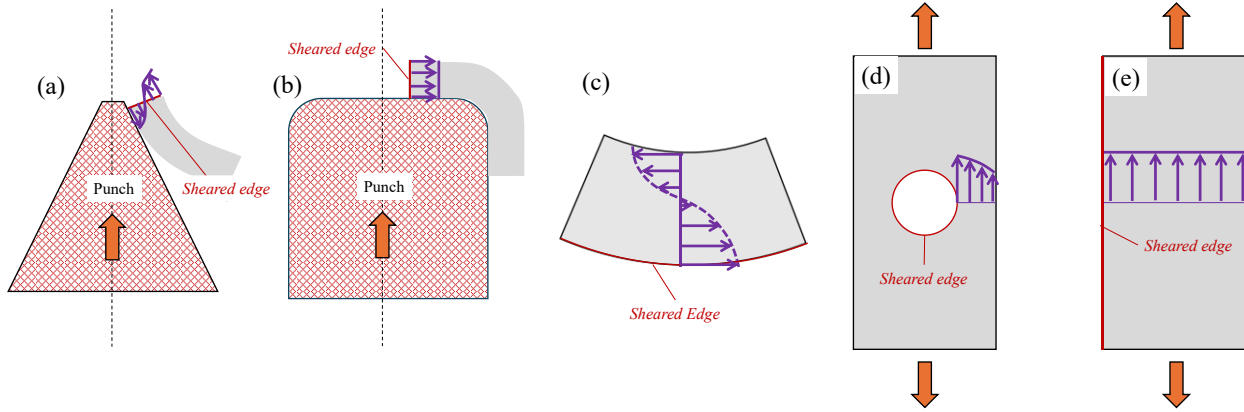


Figure 158 The applied deformation modes during (a) initial stages of conical hole expansion, (b) flat punch hole expansion, (c) in-plane bending, (d) hole tension, and (e) edge fracture tensile test before onset of diffuse necking.

D.3.2.1. Conical and Flat Punch Hole Expansion Tests

Figure 142 (a, b) shows the test setup with dimensions for the conical and flat punch hole expansion tests that were conducted with the burr oriented away from the punch. The conical HX punch had a 60° included angle consistent with ISO16630 standard and a 3.0 mm minimum diameter to precisely center the blank hole before testing. The upper die and binder had an internal diameter of 30.0 mm. The flat punch test used a 100 mm diameter Marciniak punch with a 32 mm recess to minimize friction near the hole, while the upper die had a diameter of 106 mm. To reduce friction in the flat punch HX, three layers of Teflon sheets coated with petroleum jelly were placed between the punch and specimen. Both tests were conducted on an MTS hydraulic press with a velocity of 0.25 mm/s specified for the punch while a clamping force of 640 kN was applied to constrain the blank motion. The conical HX tests were stopped as soon as a through-thickness crack occurred, while the flat punch hole expansion tests were terminated upon an abrupt reduction in the punch force that coincided with visible cracking.

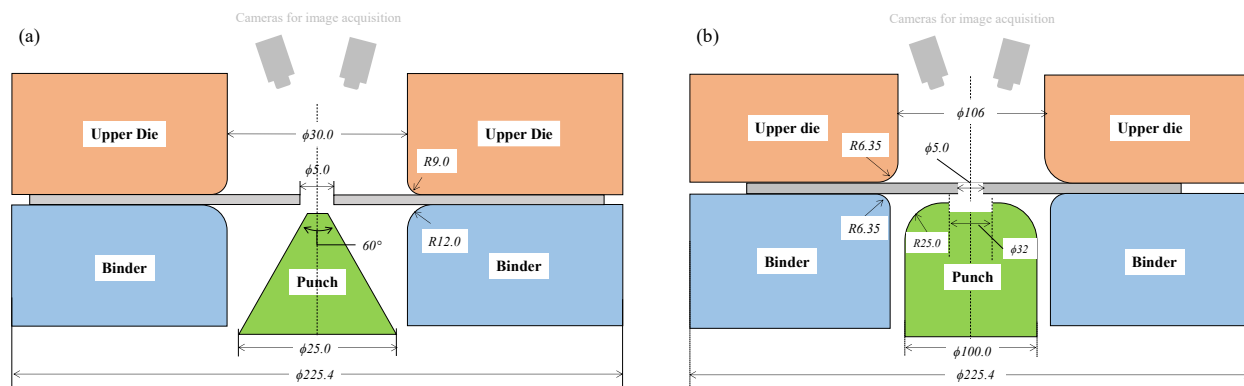


Figure 159 (a) Conical and (b) flat punch hole expansion test setups with dimensions.

Two cameras having an included angle of approximately 25° between them were used to record the deformation with a 35 mm Schneider® Xenoplan lens fitted to each camera. Stereo-DIC was utilized to measure the strain field during flat punch HX, with a black speckle pattern applied on a white background. The hole expansion ratio (HER) is typically used as a fracture metric from the conical HX and is measured using the average inner diameter at fracture to quantify the edge stretching limit. The HER is problematic to define fracture due to non-linear deformation of the inner diameter in compression from punch contact prior to tensile loading as shown in Butcher *et al.* (2013) and Qian *et al.* (2021). For measurement of fracture strains from the conical hole expansion test, the hole geometry was analyzed at the first image showing a through-thickness crack using ImageJ® software. The methodology of Narayanan *et al.* (2023) that uses the local outer hole diameter at the crack initiation location was employed to obtain the major fracture strain:

$$\epsilon_{1,\beta}^{CHX} = \ln \left(\frac{d_{outer}^{\beta}}{d_0} \right) \quad (\text{Eq. D1})$$

where d_{outer}^{β} is the local outer diameter measured along the crack orientation. Detailed discussion regarding the stress states across the different through-thickness layers during conical HX and evaluation of the strain metric in Eq. (D1) can be found in Narayanan *et al.* (2023). A similar methodology can also be applied to flat punch HX to estimate the local strain based on the diameter at the crack location (d^{β}) assuming failure initiates at the hole edge:

$$\epsilon_{1,\beta}^{FPHX} = \ln \left(\frac{d^{\beta}}{d_0} \right) \quad (\text{Eq. D2})$$

D.3.2.2. Edge Fracture and Hole Tension Tests

The hole tension test was conducted using a JISZ2201 tensile geometry featuring a 5.0 mm diameter central hole as illustrated in Figure 160(a) to be consistent with the guidelines of Roth and Mohr (2016). The test was performed on a Shimadzu AGS-X tensile frame with a 50 kN force capacity. A test speed of 0.005 mm/s was specified at the upper crosshead to achieve a local strain rate of 0.001 s⁻¹ near the hole edge. The sub-sized ASTM E8 specimen with a 6.0 mm gauge width was used for the EFTT as shown in Figure 160(b) with an applied velocity of 0.032 mm/s. In both tests, failure was accompanied by a sudden drop in the crosshead force. The local strain measurements were obtained using stereoscopic DIC. The hole tension test deformation was captured using two FLIR Grasshopper® 3 series cameras equipped with Sigma® 180 mm lenses. Sigma® 105 mm lenses were used to capture the deformation across the entire gauge length in the EFTT. The details regarding the DIC analysis parameters and the corresponding virtual strain gauge length are discussed in Section D.3.3. Virtual extensometer lengths of 10 mm and 25 mm were

adopted to analyze the global stress-strain responses from HTT and EFTT, respectively. Note that the tests were conducted with the burr side facing the DIC.

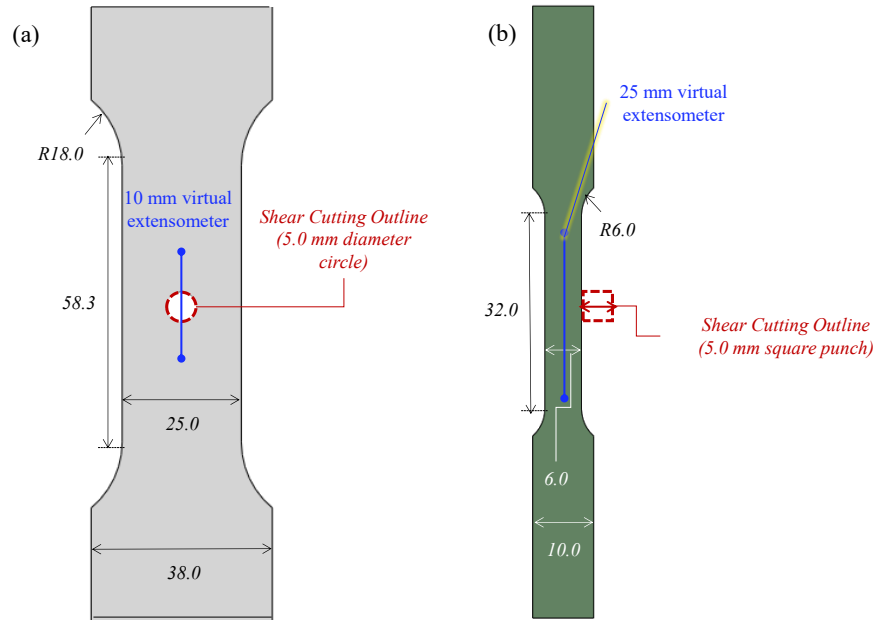


Figure 160 Geometries of (a) hole and (b) edge fracture tension tests. All dimensions are in mm (schematics not to scale). Both specimens were oriented such that principal stretching occurs along the TD.

D.3.2.3. In-Plane Bending Test

The in-plane bending fixture and sample geometry of Narayanan *et al.* (2025a) are shown in Figure 161(a). Clevis joints mounted to linear ball bearing carriages load the specimen having dowel holes using 4.0 mm diameter pins. A 4-point bending operation occurs with the gauge section free of any tool contact and subjected to minimal horizontal forces due to free motion of the carriages. The fixture was installed on a 10 kN Shimadzu AGS-X frame. A velocity of 0.05 mm/s was applied at the upper crosshead. A Sigma[®] 180 mm lens was fitted to a FLIR Grasshopper[®] 3 camera to monitor the deformation at the bend specimen surface using 2D-DIC. The sheared edge was placed on the convex side under uniaxial tension with the burr side facing the DIC. The geometry of the test specimen is depicted in Figure 161(b) while Figure 161(c, d) show the initial and crack onset images of a representative 980GEN3 sheared edge bend test. An additional camera equipped with microscope lens (see Figure 161(a)) was positioned underneath the convex edge to monitor fracture initiation and through-thickness crack formation during the tests.

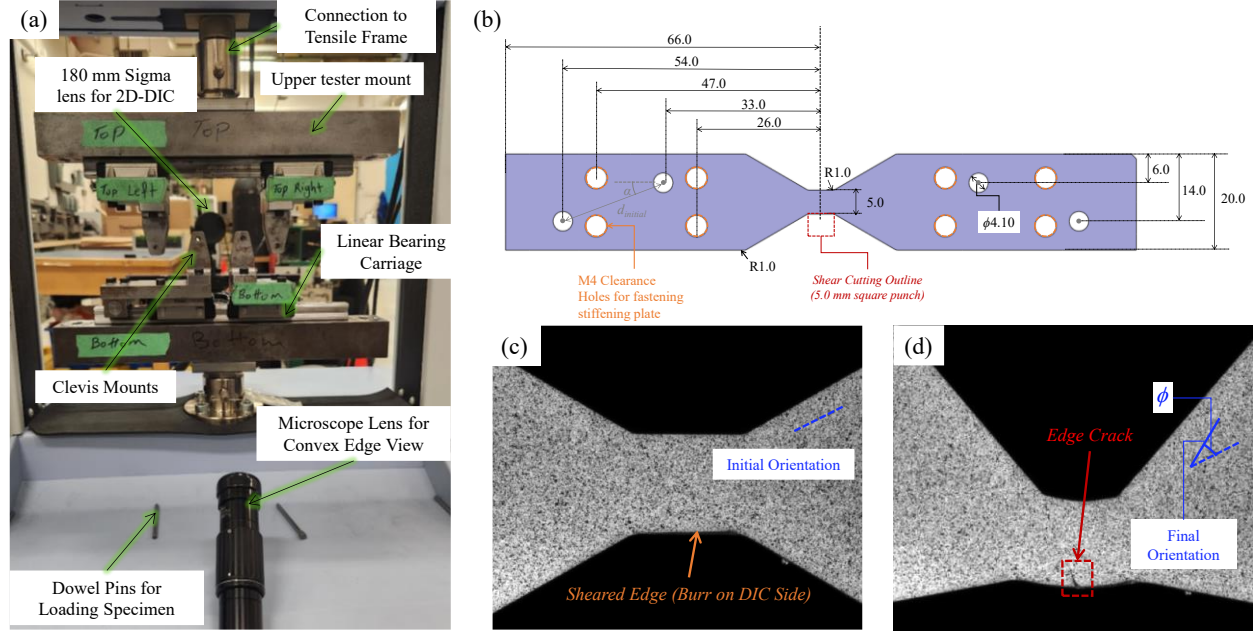


Figure 161 (a) In-plane bend fixture setup and components, (b) specimen geometry, and DIC images of a representative 980GEN3 sheared edge bend specimen at (c) start of test and (d) onset of edge cracking.

In addition to the DIC measured deformation, the so-called arc length method (Narayanan *et al.*, 2025a) was applied to determine the tangential strain at the convex edge:

$$\varepsilon_{\theta}^{\text{convex}} = \ln \left(\frac{r_{\text{convex}} \theta_{\text{convex}}}{L_0} \right) \quad (\text{Eq. D3})$$

where r_{convex} and θ_{convex} are the radius and the included angle at the convex edge, while L_0 is the initial span length and equal to 1.0 mm. Further details regarding the choice of optimal arc length and its sensitivity to DIC processing parameters are provided in Narayanan *et al.* (2025a) for multiple steels with a machined edge condition. The thinning strain rate criterion of Volk and Hora (2011) was employed to detect the onset of fracture initiation during in the bend tests. Figure 162(a) shows the crosshead force evolution versus the bend angle, ϕ , for a representative 980GEN3 bend test conducted with the punched edge. The strain rate metric better corresponds to the onset of edge cracking than the peak load instant as seen in Figure 162(b, c).

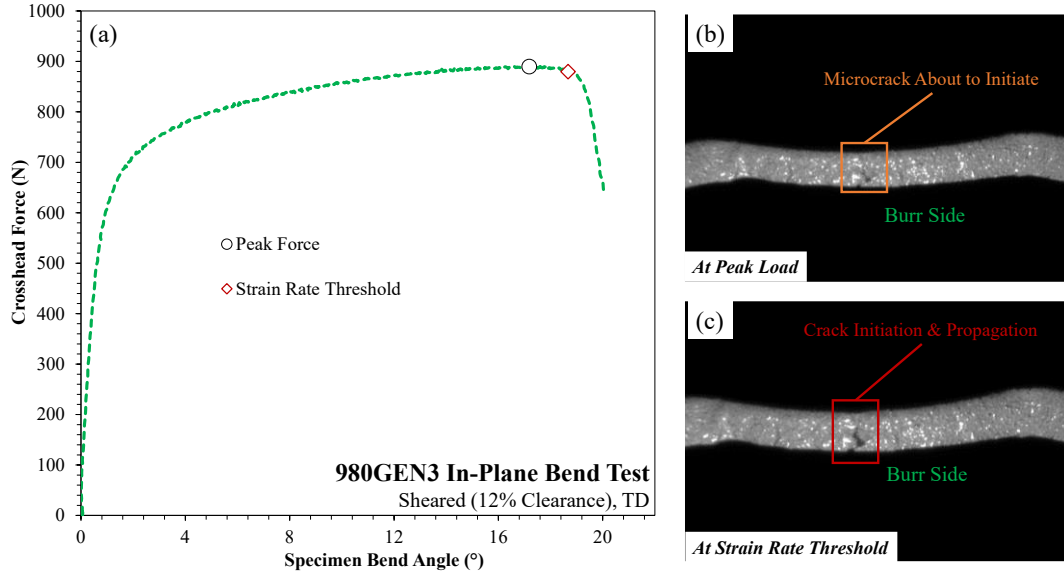


Figure 162 (a) Crosshead force evolution during a representative 980GEN3 sheared edge bend test, and the convex edge views corresponding to (b) peak force and (c) strain rate thresholds.

D.3.3. DIC Lengthscale for Edge Fracture Characterization Tests

A consistent DIC lengthscale is required to ensure a direct comparison of the experimental fracture strains. The virtual strain gauge (VSG) length as defined by Jones and Iadicola (2018) is:

$$VSG(mm) = \left[\left\{ (Filter - 1) \times Step(pixel) \right\} + Subset(pixel) \right] \times Image\ Scale\ (mm / pixel) \quad (Eq. D4)$$

A VSG of 1.0 mm was chosen as the reference for all tests. The associated DIC subset, step, and filter sizes are listed in Table 25. Additional VSGs were also considered for performing lengthscale analyses to assess its sensitivity on the local strains.

Table 25 DIC processing parameters selected for the various edge fracture test techniques corresponding to a VSG of 1.0 mm.

Test	Image Resolution (mm/pixel)	Subset Size (pixel)	Step Size (pixel)	Filter Size	VSG (mm)
Flat Punch Hole Expansion	0.025	25	3	7	1.0
Hole Tension	0.007	39	10	11	
Edge Fracture Tension	0.016	25	6	7	
In-Plane Bending	0.018	17	3	11	

D.4. Finite-Element Modelling of Edge Fracture Tests

FE simulations were conducted for the five edge fracture characterization tests to evaluate how the numerical lengthscale or mesh size can influence the predicted local elemental strains at the experimental fracture instant for various loading conditions. Numerical models of large-scale automotive components are typically conducted using plane stress shell elements to reduce computational cost which can pose challenges with regards to integration of SAZ properties and its homogenization to larger element sizes. As a result, the influence of the SAZ was not considered in this work during the modelling of coupon-level tests. The setup of the FE models in LS-DYNA along with the mesh configuration are depicted in Figure 163. Fully integrated shell elements (ELFORM=16) with five through-thickness integration points were used for the blank in all tests. No symmetry was adopted in the models with mesh sizes of 0.1 mm, 0.2 mm, 0.5 mm, 1.0 mm, and 1.4 mm considered for the blank. The largest mesh size of 1.4 mm was selected based upon the restrictions imposed by the hole or gauge size of 5.0 mm in all the tests. The conical HX induces through-thickness stress and strain gradients during the initial stages of deformation which is better captured using solid elements (Narayanan *et al.*, 2023). However, the outer hole edge remains unaffected by punch contact and bending as a result of which the shell element strain at the corresponding integration point is expected to accurately represent the material response.

The plane stress anisotropic Yld2000 (*MAT_133) model of Barlat *et al.* (2003) was previously calibrated by Narayanan *et al.* (2025b) for the 980GEN3 steel to predict thinning in the flat punch HX with a machined edge. The reader is referred to Narayanan *et al.* (2025b) for details regarding the 980GEN3 yield function coefficients and hardening response. The hole tension, in-plane bending, and edge fracture tensile simulations were conducted with principal stretching occurring along the TD to replicate the tested directions in the experiments.

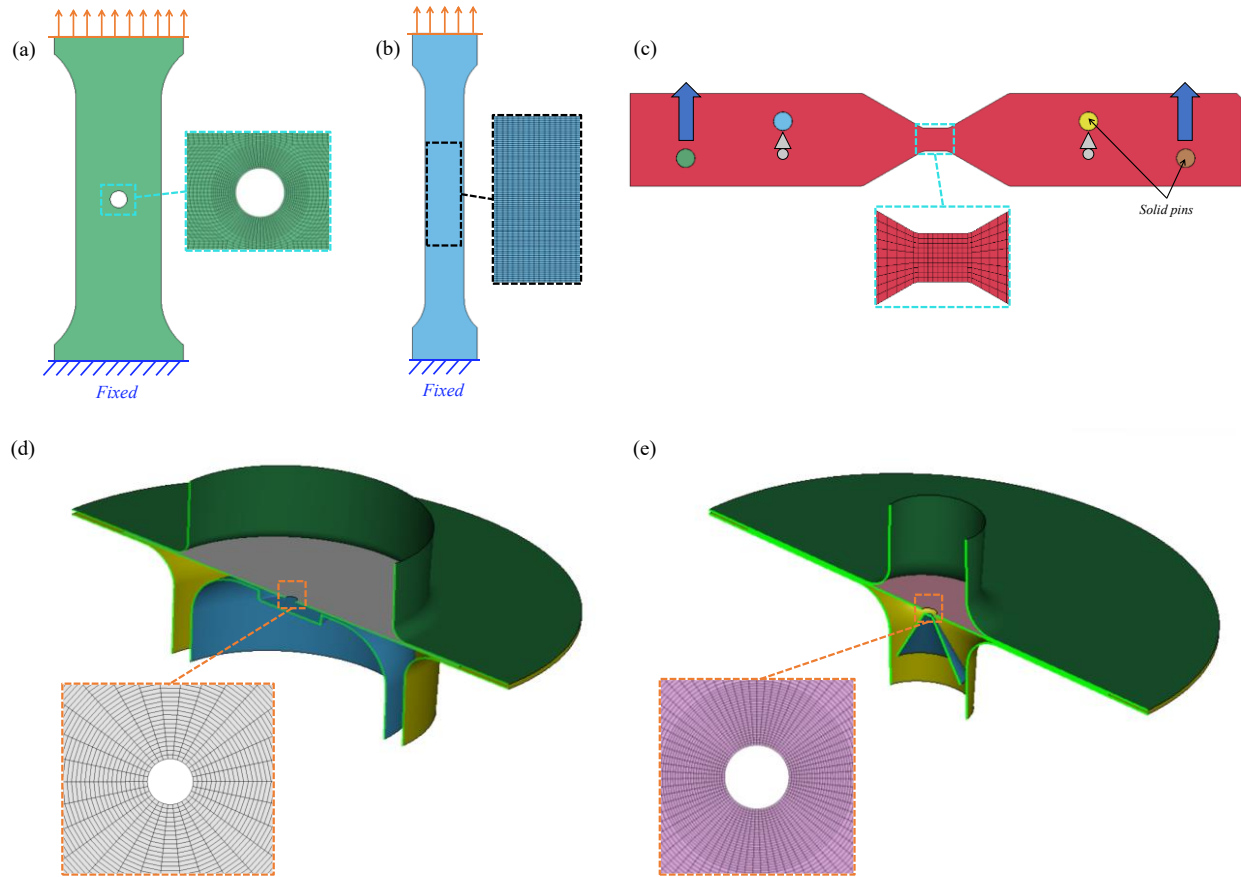


Figure 163 Finite-element model setup of (a) HTT, (b) EFTT, (c) IPB, (d) flat punch and (e) conical HX tests. Sectional views of the hole expansion tests are shown for brevity

D.5. Results of Edge Fracture Characterization Experiments

D.5.1. Edge Profile and Depth of Shear Affected Zone after Hole Punching

Figure 164(a) presents the edge profile of the 980GEN3 steel following circular hole punching with 12% clearance and subsequent sectioning along the rolling direction. A smooth edge profile with minimal burr formation was observed after the shearing process. The edge profile primarily consists of burnish and fracture zones as shown in Figure 164(b). Vickers microhardness measurements were conducted using a pyramidal diamond indenter with an applied load of 500 gf and a dwell time of 10 seconds to quantify the SAZ. Three rows of indents were created at depths of approximately 25%, 50%, and 75% of the sheet thickness from the top surface with eight indents per row. The indents were spaced 0.150 mm apart from each other with the first indent starting at a distance of 0.225 mm from the sheared edge. The hardness distribution away from the edge remained relatively consistent across all the rows of indents as shown in Figure 164(c). The depth of the SAZ was determined to be approximately 0.675 mm, which subsequently informed the selection of the smallest VSG of 0.6 mm used for the DIC lengthscale study in Section D.5.6.

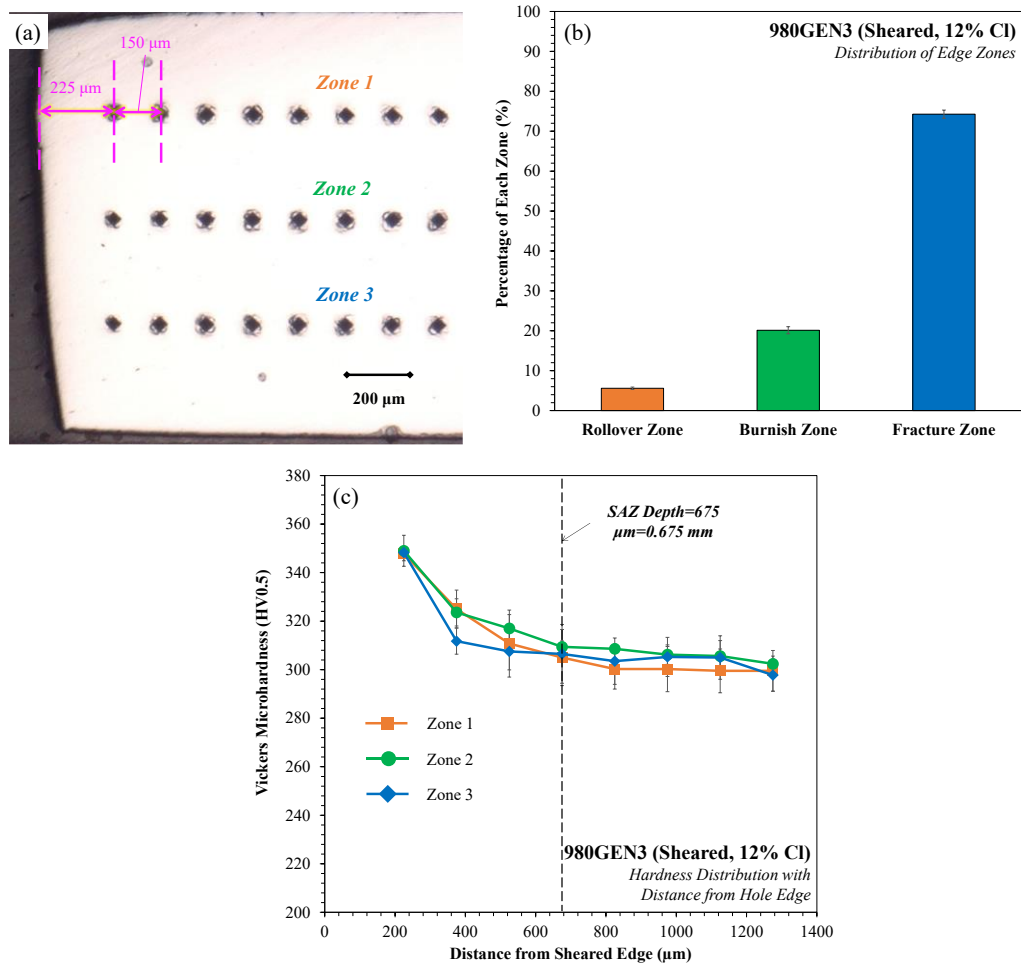


Figure 164 (a) 980GEN3 sheared edge after punching with 12% clearance showing locations of hardness indents, (b) distribution of cut edge zones, and (c) Vickers microhardness distribution away from the cut edge.

D.5.2. Edge Stretchability during Hole Expansion Tests

Figure 165(a, b) show representative images of through-thickness cracks in 980GEN3 conical HX tests for machined and sheared edges, respectively. In the machined samples, through-thickness failure occurred along the RD which corresponds to TD principal stretching. In contrast, the sheared edge samples exhibited multiple microcracks around the circumference in addition to the through-thickness crack. Figure 165(c) shows the hole expansion ratios and fracture strain measured using the local outer diameter. The HER for the punched hole dropped significantly to approximately 30% of the hole expansion of the machined condition.

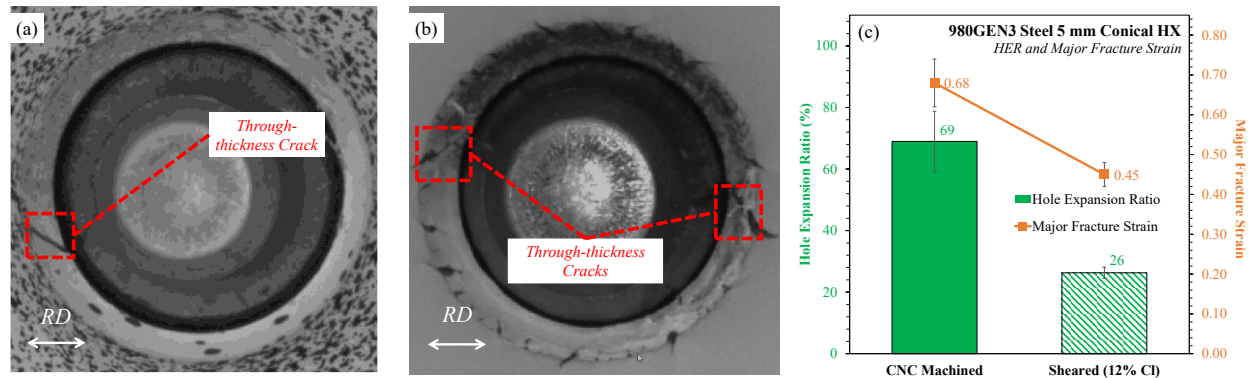


Figure 165 Image at through-thickness crack instant for a representative (a) machined and (b) sheared edge tests, and (c) hole expansion ratios and fracture strains from 980GEN3 conical hole expansion tests.

The DIC major strain contours at the image before fracture for a representative machined and sheared flat punch HX test are shown in Figure 166(a) and Figure 166(b), respectively. As previously observed by Pathak *et al.* (2016) for a CP800 steel with a sheared edge, failure during flat punch HX can occur away from the hole under a stress state in between uniaxial and plane strain tension. To this end, the thinning strain distribution in the direction of strain localization was analyzed with the values determined from the assumption of volume constancy. The maximum thickness reduction for machined flat punch HX occurred at approximately 1.2 mm away from the edge in the DD as seen in Figure 166(c). In contrast, maximum thinning occurred near the edge for the punched edge condition. The geometric method to determine the fracture strain in Eq. (D2) gives comparable fracture strains to the DIC for the sheared edge that exhibits a 50% reduction in the HER compared to the machined edge. The sheared edge fracture strain from flat punch HX was lower than conical HX by 0.15, highlighting the sensitivity of 980GEN3's edge formability to the nature of axisymmetric stretching (in-plane vs. out-of-plane) for the same cutting condition.

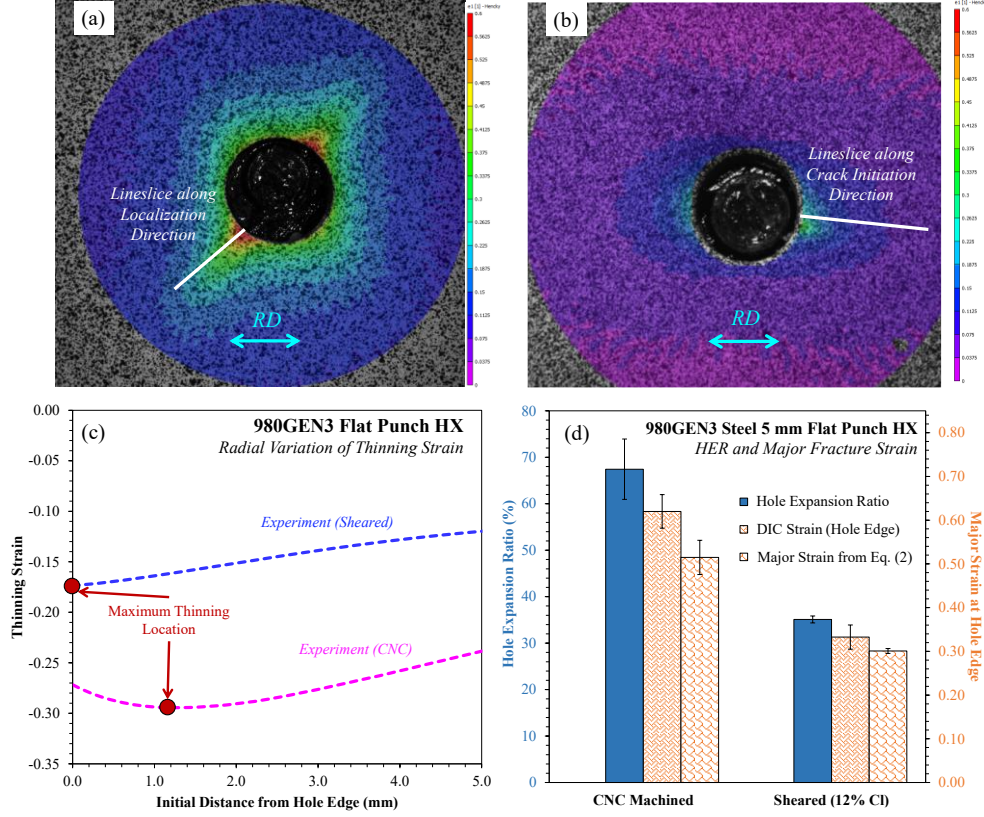


Figure 166 DIC major strain contours at fracture from a representative (a) machined and (b) sheared edge flat punch HX test, (c) variation of thinning strain with radial distance from the hole edge, and (d) summary of HER and fracture strains for the 980GEN3 steel.

D.5.3. Hole and Edge Fracture Tension Tests

Figure 167(a, b) show the DIC strain contours at failure for representative HTT samples with machined and sheared edges, respectively. Identifying the location of fracture initiation in HTT can be challenging, so the thinning distribution was examined under a Keyence® VHX-5000 microscope after through-thickness cracking. The maximum thinning for the machined hole occurred at approximately 0.3 mm away from the edge as seen in Figure 167(a). In contrast, fracture initiated at the sheared edge for the punched HTT with the highest thinning at the edge as seen in Figure 167(b). Figure 167(c, d) compare the strain paths and hole edge strains at fracture instant for the two fabrication techniques. A reduction in DIC edge strain of approximately 55% is observed in sheared specimens relative to the machined tests. Since fracture initiated at the hole edge, the thinning strain measured post through-thickness (TT) cracking can be converted to a major strain using the tensile R-value as done by Khameneh *et al.* (2024):

$$\varepsilon_3^{HTT, TT \text{ Crack}} = \ln \left(\frac{t_f^{HTT}}{t_0} \right); \quad \varepsilon_1^{HTT, TT \text{ Crack}} = -(1 + R_{TD}) \times \varepsilon_3^{HTT, TT \text{ Crack}} \quad (\text{Eq. D5})$$

where t_0 and t_f^{HTT} are the initial and final thicknesses measured near the through-thickness crack, respectively. The strain measured after through-thickness cracking was approximately 50% higher relative to the DIC measured surface strain, highlighting significant strain localization during the rupture process.

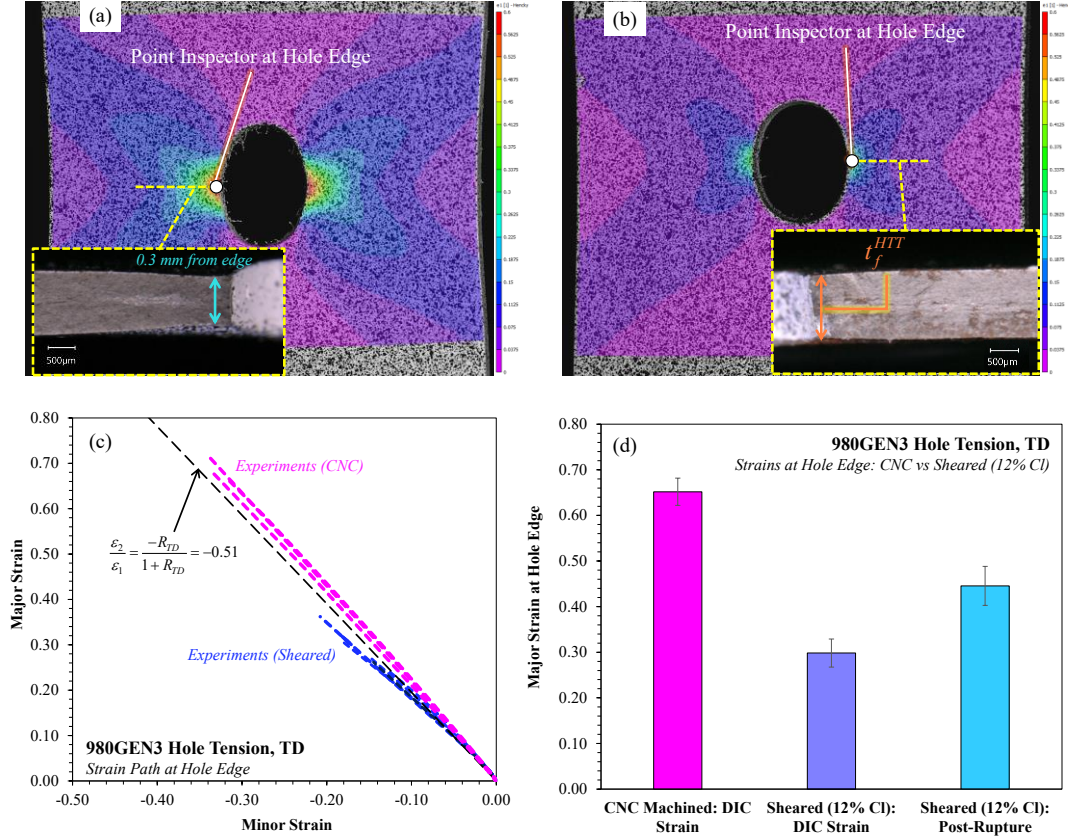


Figure 167 DIC major strain contours at fracture instant for representative (a) machined and (b) sheared samples, (c) strain paths at the hole edge, and (d) major fracture strains from 980GEN3 hole tension tests.

Figure 168(a, b) depict the DIC strain contours from machined and sheared EFTT at the instant of fracture onset. The classical triaxial necking mode was observed in the 980GEN3 machined EFTT specimens due to the width-to-thickness aspect ratio of approximately 4.3 that resulted in a transition in strain path towards plane strain tension as observed in Figure 168(c). In contrast, a proportional strain state was observed near the edge in the punched EFTT suggesting that failure was dominated by edge cracking without the occurrence of any localization. To assess whether the diffuse necking limit was reached during the sheared EFTT, its global extension was measured using a virtual extensometer of 25.0 mm initial length as per the ASTM E8 standard. The true extensometer strain was 0.15 ± 0.02 , which is marginally lower than the true uniform elongation strain of 0.17 for the 980GEN3, confirming that edge fracture initiated before diffuse necking of the cross-section.

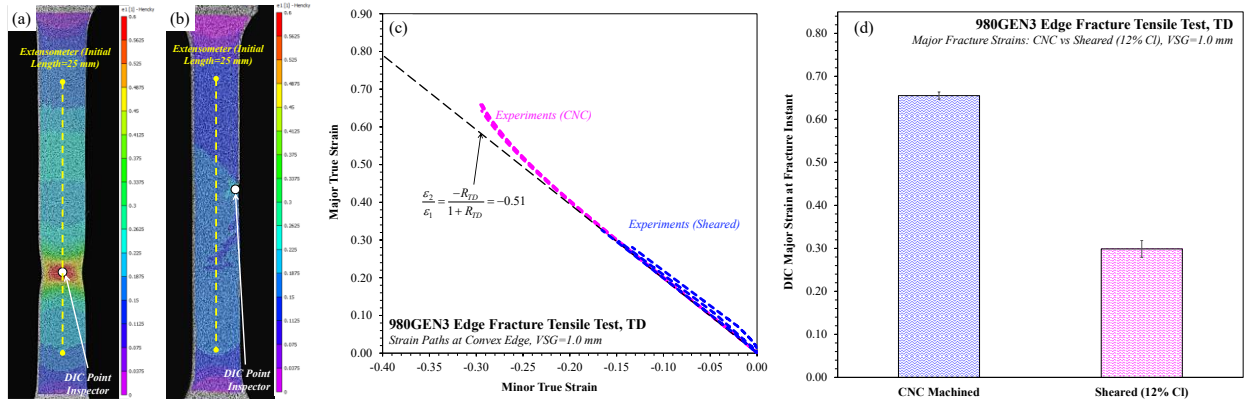


Figure 168 DIC major strain contours at fracture for a representative (a) machined and (b) sheared EFTTs, (c) strain paths and (d) major fracture strains for 980GEN3 EFTT.

A comparison of DIC local fracture strains from the machined and sheared edge EFTT at the respective failure locations are shown in Figure 168(d). A significant drop in the in-plane stretchability was observed with the DIC failure strain of the punched edge being approximately 55% lower than the reference edge. Furthermore, the local DIC strain for the sheared EFTT is in close agreement with the HTT, suggesting comparable in-plane edge stretching limits for the same DIC lengthscale when failure occurs at the edge.

D.5.4. Edge Formability under In-Plane Bending

The DIC strain paths obtained using a point inspector at the convex edge where fracture initiated during the 980GEN3 IPB are presented in Figure 169(a) for both machined and sheared edge conditions. A proportional strain state is observed until fracture that closely follows the strain path for uniaxial tension predicted by the TD R-value. Figure 169(b) compares the major fracture strains for the machined and sheared IPB tests obtained using DIC and the arc length methods. A strong agreement was observed between the local DIC strain and the arc length method regardless of the edge condition at the convex surface. Thickness measurements were also conducted at the convex edge post through-thickness cracking similar to HTT and converted into a major strain value using Eq. (D5). The rupture strains are similar to the DIC and arc length methods for the machined IPB tests as seen in in Figure 169(b). A larger difference in strain of approximately 13% can be observed for the sheared edge since a complete through-thickness crack was not observed at the strain rate threshold in these tests as seen in Figure 162(c). This difference is attributed to edge cracks preferentially initiating from the burr side that faces the DIC camera, resulting in a lower bound fracture strain estimate.

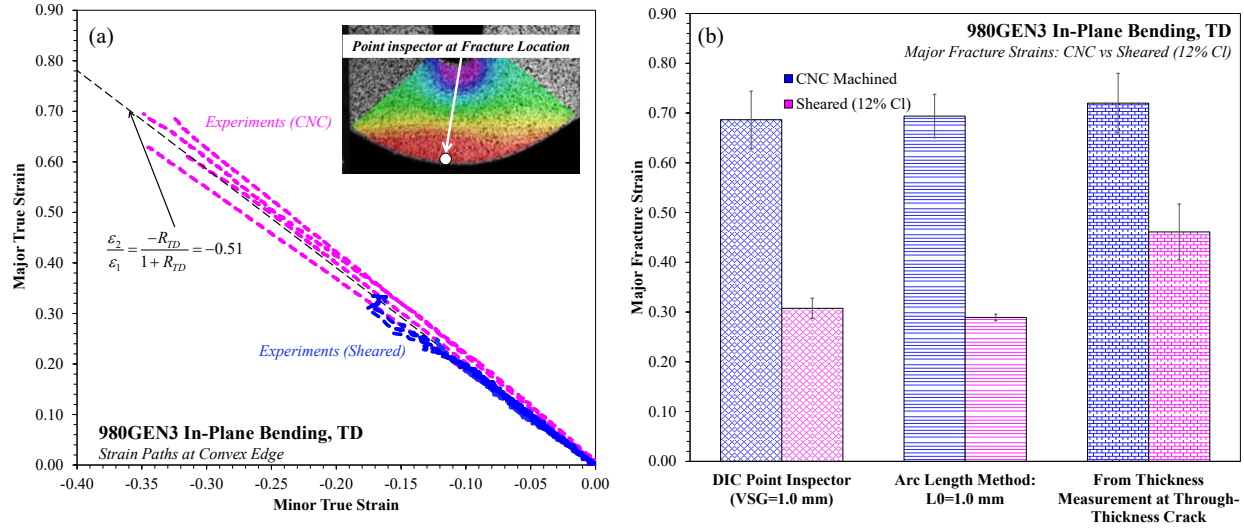


Figure 169 (a) Strain paths at the fracture location on the convex edge and (b) strains at fracture from machined and sheared IPB tests conducted on the 980GEN3 steel.

D.5.5. Comparison of Sheared Edge Fracture Limits

The major fracture strains obtained from the various testing methods at a DIC VSG of 1.0 mm are summarized in Figure 170. Convergence was obtained between the in-plane sheared edge tests with a local fracture strain of approximately 0.30 at 12% clearance. The exception was the axisymmetric out-of-plane conical HX that reached a fracture strain of 0.45. This difference is attributed to conical HX failure being defined at the instant of through-thickness crack as per ISO16630 standard. In contrast, failure during the in-plane tests corresponds to onset of edge cracking as shown in Figure 162(c) for the IPB that is attributed to the influence of sheared edge as well as the fracture detection criterion. The sheared IPB and HTT strains obtained at through-thickness crack instant are in close agreement with the conical HX. For the machined edge condition, the detected fracture instant closely corresponded to through-thickness cracking due to which the IPB and conical HX fracture strains were in good agreement (Narayanan *et al.*, 2025a). Overall, the 980GEN3 steel exhibits marginal sensitivity to boundary conditions when punched at 12% clearance. The influence of test type is expected to be more pronounced in materials with banded microstructures, as noted by Narayanan *et al.* (2025c) for two hot-rolled MP800 steels exhibiting centerline segregation.

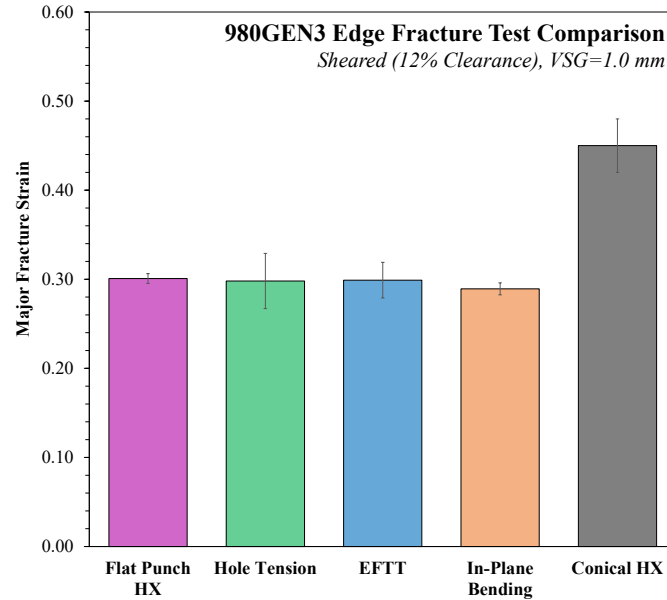


Figure 170 Comparison of 980GEN3 edge fracture strains from the various test methods after hole punching with 12% clearance.

D.5.6. Influence of DIC Lengthscale on Fracture Strains

It is instructive to investigate the sensitivity of the DIC lengthscale for the in-plane fracture tests given the apparent convergence using a VSG of 1.0 mm. Two representative repeats from each in-plane test were analyzed using multiple VSGs ranging from 0.6 mm to 4.0 mm. The subset size and image resolution remained constant as they are a function of the applied speckle pattern and the imaging system, respectively. Representative machined edge repeats were also analyzed for the same range of VSGs to evaluate how the lengthscale effects manifest relative to the sheared edge. Figure 59 presents the effect of DIC lengthscale on the major fracture strains for the two edge conditions. The machined EFTT and flat punch HX were excluded since localization developed at a distance of more than 1 mm away from the edge, making the measured strains not representative of edge fracture. The hole tension test was retained despite localization since fracture occurred close to the edge at a distance of 0.3 mm. The conical HX test did not exhibit lengthscale dependence since the fracture strains were obtained directly from geometry.

A convergence in the uniaxial fracture limits can be observed between the machined in-plane bending, hole tension, and conical hole expansion tests at a VSG of 1.0 mm for the 980GEN3 steel as seen in Figure 59(a). The machined HTT showed a strong sensitivity to the experimental lengthscale. The DIC hole edge strain dropped from 0.80 to 0.30 with increase in VSG from 0.6 mm to 4.0 mm, leading to an apparent agreement with the in-plane bending and conical HX at a VSG of 1.0 mm. The deformation in HTT is constrained to a highly localized region near the hole as seen in Figure 59(a) that is attributed to the small

initial hole radius of 2.5 mm. A VSG of 1.0 mm appears to be the optimal choice for local fracture strain measurement based on the convergence between the test methods.

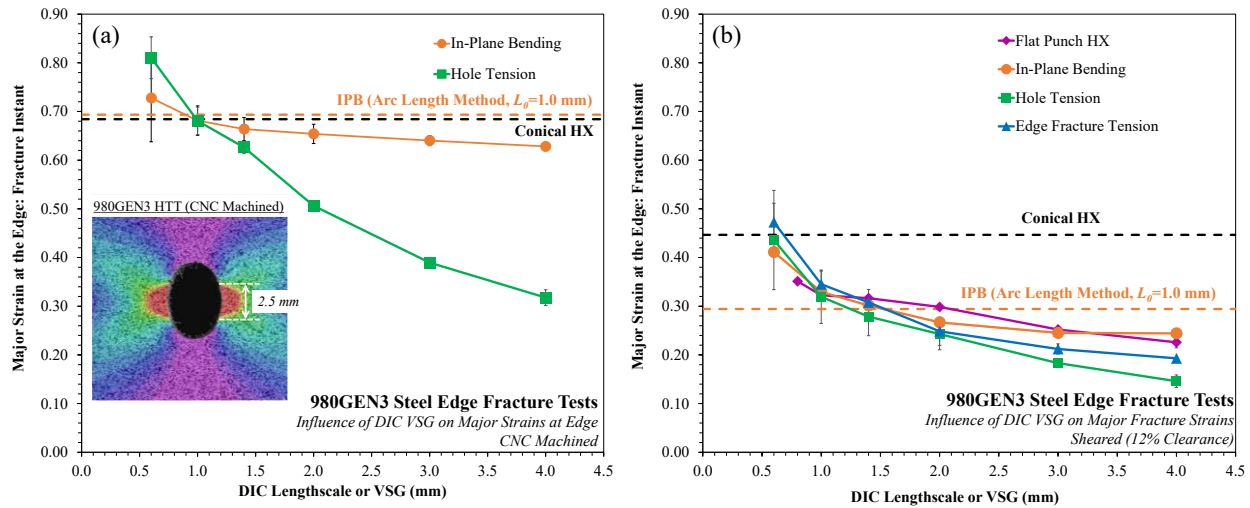


Figure 171 Effect of virtual strain gauge length on the local DIC major strains at the edge from the various fracture tests for (a) machined and (b) sheared conditions.

A larger region of VSGs from 0.6 to 1.4 mm provides approximate convergence in local fracture strains for the sheared edge condition as seen in Figure 59(b). At larger lengthscales, the sheared HTT fracture strains diverge from the other in-plane tests due to its strong VSG sensitivity. The deformation in flat punch HX and IPB is primarily governed by geometric changes resulting in local DIC strains exhibiting a low sensitivity to the VSG. It is interesting that an apparent sensitivity to the applied boundary condition can emerge as a consequence of the choice of VSG. Virtual strain gauge lengths greater than 2.0 mm can show a divergence in the fracture strains for the considered 980GEN3 steel, with potential for divergence at VSGs smaller than 0.6 mm as well.

D.5.7. Evaluation of Global Edge Stretchability and Thinning Limits

Edge thinning is a critical parameter during part design, and thus it is worthwhile investigating its sensitivity to the VSG and applied loading condition. DIC is seldom amenable to practical stamping operations to measure deformation since the parts are enclosed within the toolset. As a result, micrometers are often used to measure the thinning strain near the edge split location. Therefore, it is useful to quantify the so-called global edge stretchability from the considered coupon tests that emulate the various loading scenarios encountered in formed parts. The conical and flat punch HX tests apply out-of-plane and in-plane axisymmetric stretching to the sheared edge, respectively. The global edge stretch is thus the major strain measured using the outer hole diameter in Eq. (D1) and Eq. (D2) that is then converted to a thinning strain using the R-value along the transverse direction. The global thinning limit for the IPB test can be

determined from the major strain obtained using the geometric arc length method in Eq. (D3), but with an initial length of 5.0 mm that corresponds to the initial length of the sheared edge produced by the square punch.

Figure 60 presents the comparison between global thinning derived from geometric changes and local DIC data at various VSGs for sheared flat punch HX and IPB. For both the flat punch HX and IPB, VSGs of 2.0 mm and 3.0 mm exhibit convergence with the global edge stretchability measures, indicating that these DIC gauge lengths are more appropriate for direct application to part design. From a local fracture strain convergence perspective, smaller VSGs close to 1.0 mm appear to be suitable for the 980GEN3 steel as discussed in Section D.5.6.

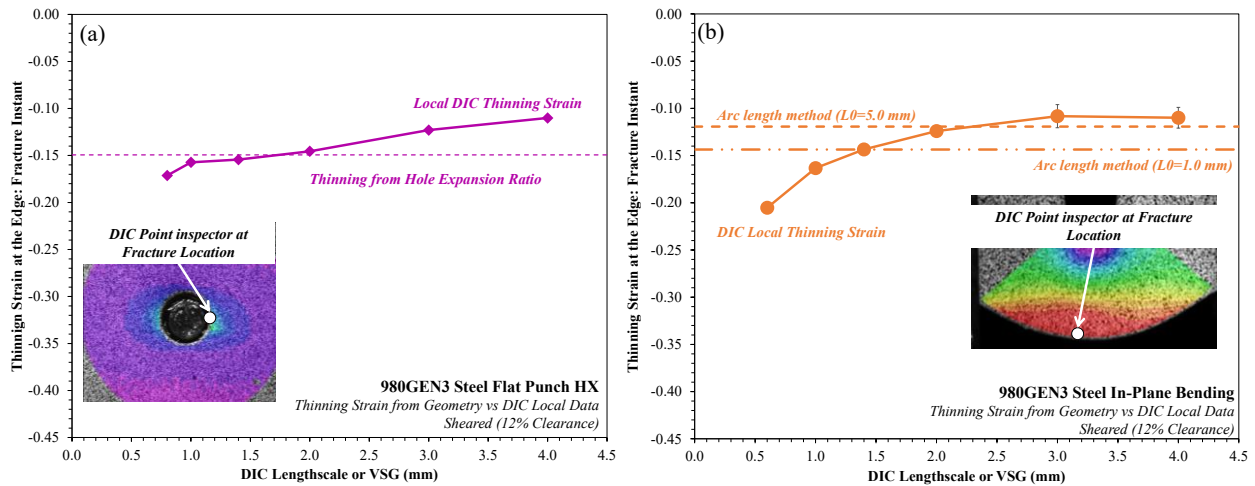


Figure 172 Comparison of global and local DIC thinning strains at various VSGs for a representative 980GEN3 sheared (a) flat punch HX and (b) in-plane bending test.

To evaluate the global fracture strain from the EFTT, virtual extensometers of various lengths were placed near the sheared edge as shown in Figure 173(a). The choice of initial extensometer length governs the global strain magnitude as seen in Figure 173(b). An extensometer length of 5.0 mm is recommended since it corresponds to the initial length of the sheared edge similar to the IPB. The 5.0 mm extensometer in the EFTT also corresponds well with the local DIC data at VSGs of 3 mm and 4.0 mm as seen in Figure 173(b). The HTT does not have a clear global metric due to highly localized region of deformation and was excluded from global strain evaluation as a result.

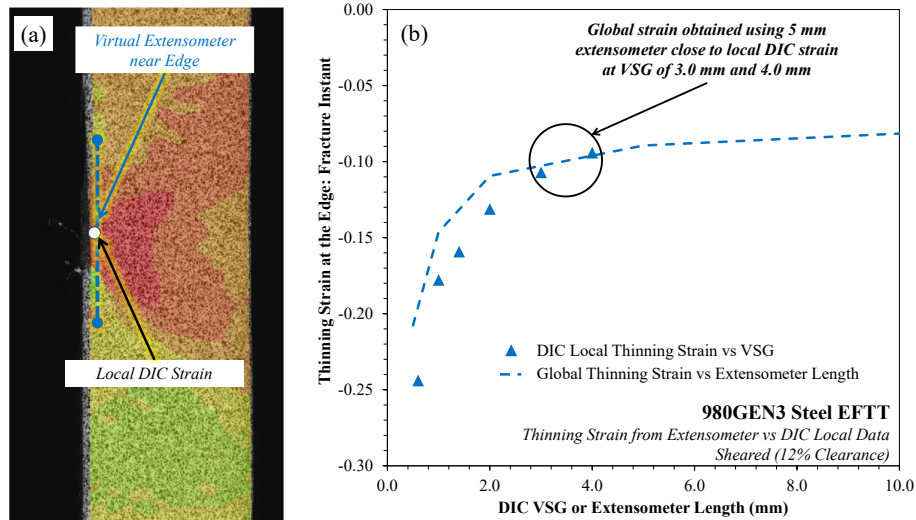


Figure 173 (a) Placement of virtual extensometer on the sheared EFTT specimen, and (b) comparison of 980GEN3 EFTT local DIC thinning strain as a function of VSG, overlaid with global strain from extensometers of various lengths.

Figure 61 overlays the variation of 980GEN3 local DIC edge thinning strains with VSG for the five test methods, along with the corresponding global edge stretch measures. As expected, the conical HX displays the highest global edge formability due to out-of-plane deformation of the sheared edge. The flat punch HX has a lower global formability than the conical HX due to in-plane axisymmetric loading, but still surpasses the in-plane bending and EFTT that impose a uniaxial stretch on the cut edge. The EFTT exhibits the lowest global true thinning strain of 8.3% due to lack of any strain gradient until the onset of diffuse necking, making it the most critical deformation mode for edge crack initiation. Interestingly, the DIC thinning strain from the hole tension test converges toward the global EFTT strain for the 980GEN3 steel. The global edge stretchability metrics tie the geometric deformation to local DIC strains. They provide guidance for identification of appropriate DIC VSGs for which the local strain data can be directly translated into design-relevant metrics. For the 980GEN3 steel, the local strains from the in-plane tests obtained using VSGs in the range of 2.0 mm to 4.0 mm appear more appropriate for use in design applications.

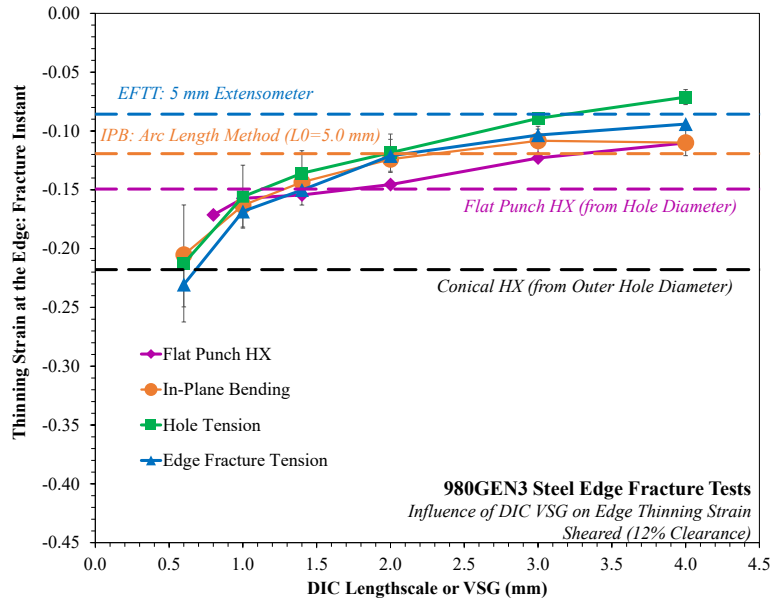


Figure 174 Sensitivity of edge thinning strain to DIC virtual strain gauge length in sheared edge tests conducted for the 980GEN3 steel.

D.6. Boundary Condition Effects During Numerical Modelling of Edge Stretching

D.6.1. Finite-Element Model Predictions of Global and Local Response

Figure 64 shows the representative global response from the experiments overlaid with the predictions of the FE simulations conducted using shell elements for the in-plane edge fracture characterization tests. The numerical models accurately reproduce the experimental force across all evaluated element sizes in the flat punch HX and edge fracture tensile tests. For the HTT and IPB, the FE simulations also show excellent agreement with experimental data for mesh sizes between 0.1 mm and 1.0 mm. However, the coarsest mesh with a 1.4 mm element size overpredicts the hole tension response by 3% as seen in Figure 64(b) due to the inability to resolve the strain concentration around the hole periphery. Similarly, the IPB forces are also overpredicted by 4% (see Figure 64(d)) by the coarse 1.4 mm mesh size model since the gauge region is $5 \times 5 \text{ mm}^2$. The opposite problem occurred for the smallest mesh sizes of 0.1 mm and 0.2 mm in the machined HTT and flat punch HX simulations with occurrence of premature localization. However, the simulation results for these mesh sizes were still suitable for the sheared edge HTT and flat punch HX due to lower displacements to fracture that avoided excessive localization.

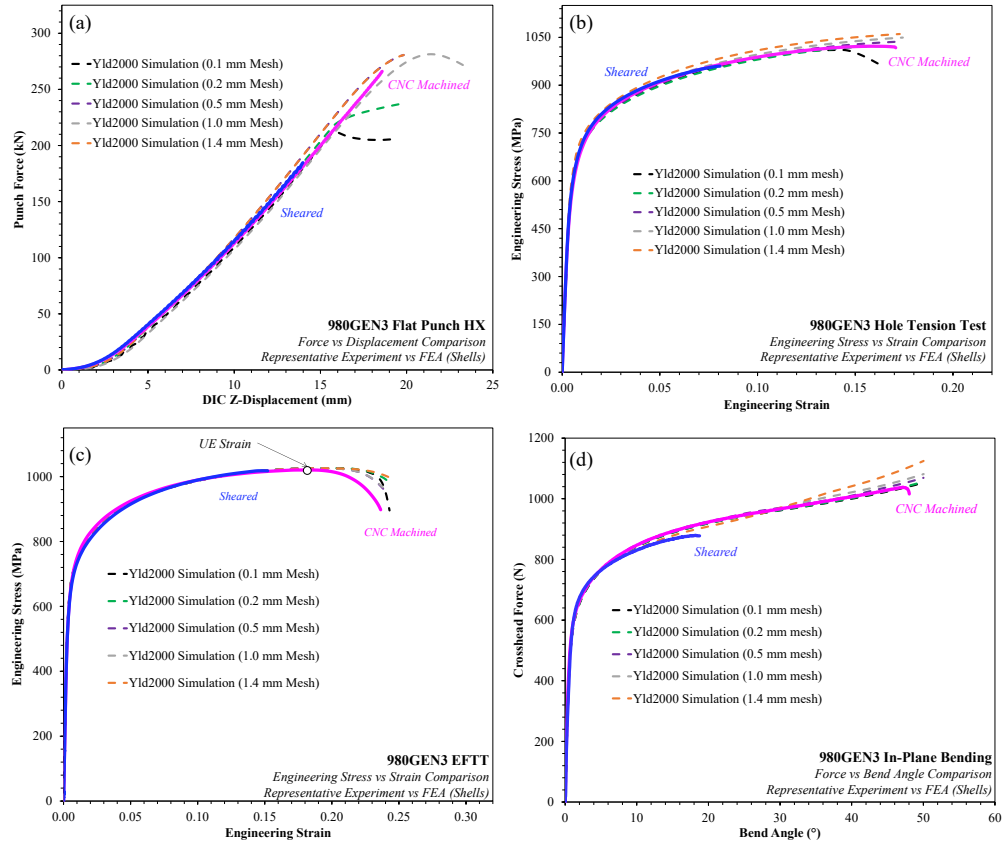


Figure 175 Comparison of global response from FE simulations conducted using shell elements with representative repeat for (a) flat punch HX, (b) HTT, (c) EFTT, and (d) IPB tests. Solid and dashed lines denote experiment and simulations, respectively.

Having compared the global responses from experiments and simulations, it is now instructive to evaluate the local strain evolution trends. Figure 176(a) shows the local edge strain evolution from HTT simulations compared to representative machined and sheared experiments. The local strain evolution trends exhibit a strong dependency to numerical lengthscale in the HTT with the 0.2 mm mesh providing the best agreement with the experiment at a VSG of 1.0 mm for 980GEN3 steel. For the IPB, the convex edge major strain evolution for a VSG of 1.0 mm was accurately reproduced across all element sizes except the 1.4 mm mesh until 90% of the fracture bend angle (see Figure 176(b)). The rapid localization in the sheared IPB tests prior to fracture was not captured and attributed to the omission of the SAZ properties in the models.

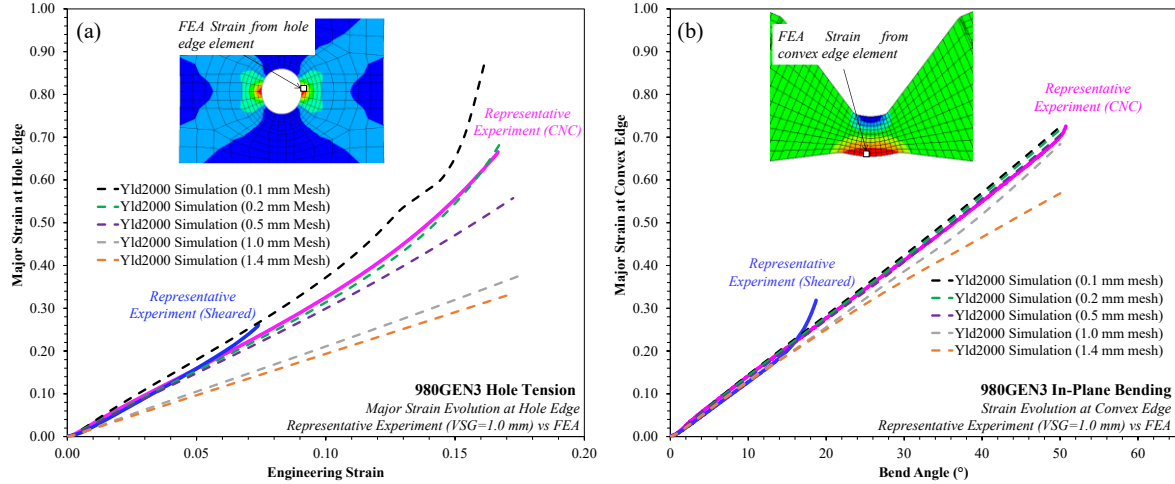


Figure 176 Comparison of local major strain evolution at the edge from representative (a) hole tension and (b) in-plane bending tests and corresponding simulations with various mesh sizes for 980GEN3 steel.

D.6.2. Correlation between Experimental and Simulation Lengthscales

Figure 177(a, b) compares the elemental edge strain for various FE mesh sizes against the DIC local strains obtained at VSGs ranging from 0.6 mm to 4.0 mm at the instant of fracture in the machined HTT and IPB tests. An FE mesh size of 0.2 mm corresponds well with a reference VSG of 1.0 mm for the 980GEN3 steel across the two tests. However, the DIC local strain at VSGs of 3.0 mm and 4.0 mm as recommended for design applications using the global edge stretch metrics, are best captured by FE mesh sizes of 1.0 mm and 1.4 mm. The predicted elemental strains for these mesh sizes lie within the experimental variation of the global edge stretch strains for the two test methods. The global edge stretch measures help identify DIC VSGs that are more suitable for calibrating phenomenological fracture models to mesh sizes typically used in FE simulations. Note that the global edge stretch from the machined EFTT, measured using a 5.0 mm extensometer was used as a reference for the HTT results in Figure 177(a). Figure 177(c) compares the edge elemental FE strain from conical HX at the integration point corresponding to the outer hole edge oriented along the RD with the experimental measurement obtained using Eq. (D1). The conical HX also exhibits a strong sensitivity to mesh size in simulations despite having a single uniaxial fracture strain value in the experiments. The smallest element sizes of 0.1 mm and 0.2 mm accurately capture the experimental deformation, owing to the proximity of the integration points to the free edge.

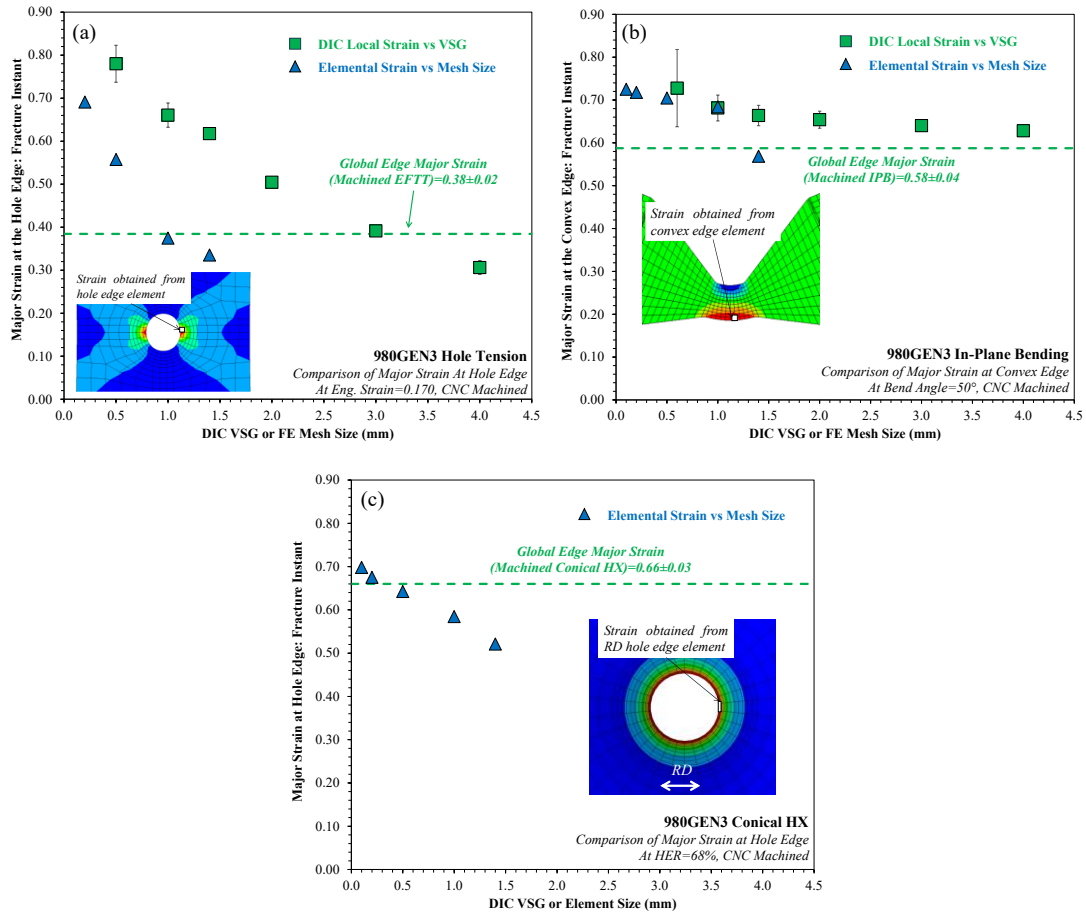


Figure 177 Lengthscale effect on experimental and numerical edge strains for machined 980GEN3 (a) HTT, (b) IPB, and (c) conical HX tests at the experimental fracture instant.

The DIC and simulation local strains at various lengthscales for the fracture tests conducted with the sheared edge are shown in Figure 178 for the axisymmetric tests at their corresponding experimental failure instants. FE mesh sizes of 0.5 mm and lower better capture the global edge stretch of the sheared edge in the conical and flat punch HX. Larger lengthscales exhibit a divergence that is attributed to the inability of the elements to adequately resolve the stress concentration near the hole, as well as the strong strain gradients in these tests. For tests which impose a uniaxial stretch on the edge, FE strains for mesh sizes of 1.0 mm and 1.4 mm correspond well with the global edge stretch as well as the local strains obtained using DIC VSGs in the range of 2.0 mm to 4.0 mm (see Figure 179). In the case of the EFTT, the elemental strains are unaffected by the mesh size since the SAZ is not modelled in addition to the experimental fracture displacement being below the diffuse necking limit. As a result, the strain localization at the edge is not captured. Future work will address characterization and mapping of the SAZ into the FE models.

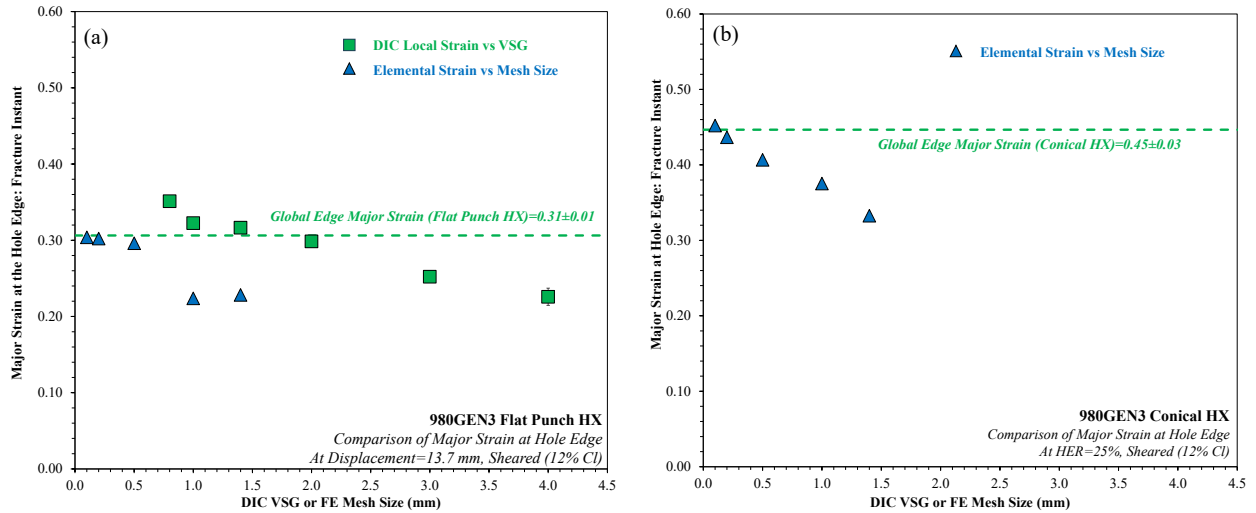


Figure 178 Lengthscale effect on experimental and numerical edge strains for sheared 980GEN3 (a) flat punch and (b) conical HX tests at the experimental fracture instant

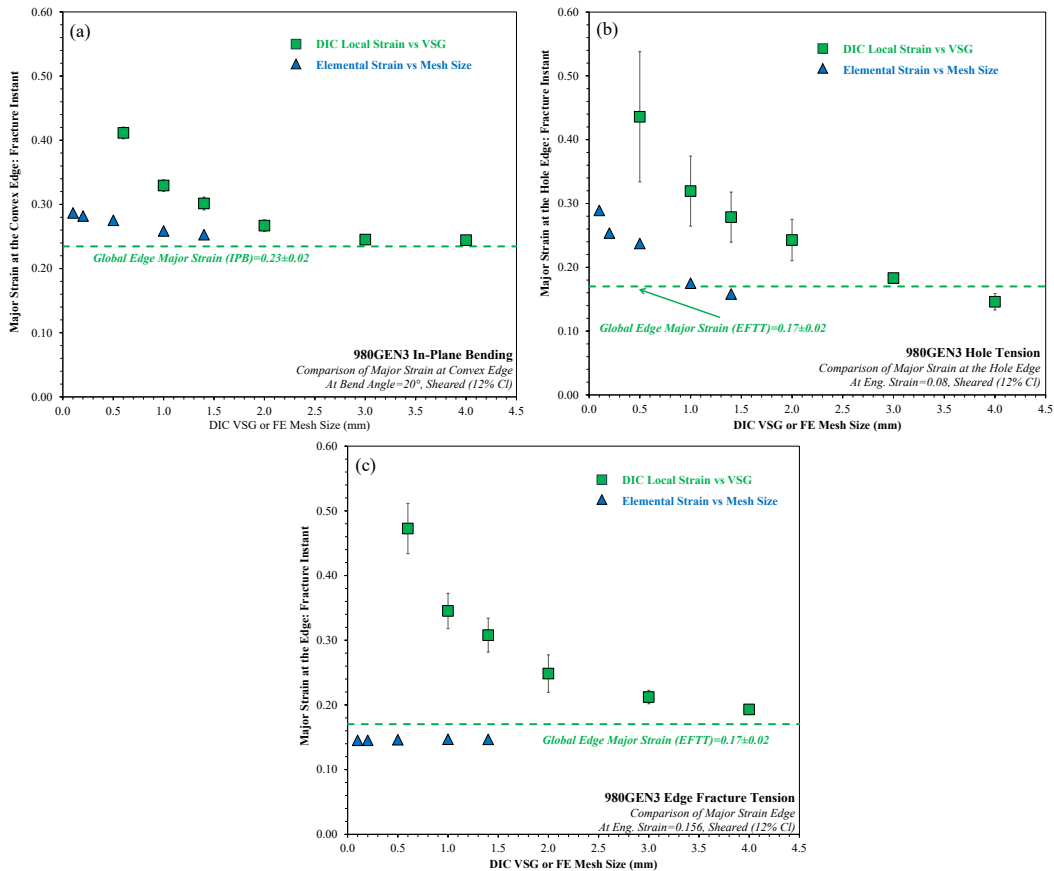


Figure 179 Influence of lengthscale on experimental and numerical edge strains for sheared 980GEN3 (a) IPB, (b) HTT, and (c) EFTT at the experimental fracture instant

Figure 65(a) summarizes the elemental major strains from FE simulations of the edge fracture tests at the instant of experimental fracture for the punched edge tests. The FE simulations exhibit a pronounced

sensitivity to the applied loading conditions regardless of the numerical lengthscale. Major strain differences exceeding 12% can be seen between the in-plane tests regardless of the mesh size. Interestingly, the mesh size effect is largely mitigated upon the use of elemental thinning strains for all test methods as seen in Figure 65(b). The relative consistency in thinning values across the different numerical lengthscales is attributed to the homogenization of strain paths away from the edge in the FE models. Notably, the global edge thinning limits measured from the coupon tests for the 980GEN3 steel are also in alignment with the FE thinning strains, further highlighting the transferability of these global metrics to FE modelling applications. The axisymmetric stretching mode exhibits the highest thinning limits in the simulations consistent with trends observed in the experiments and is followed by the IPB deformation mode. The in-plane uniaxial stretching deformation mode appears to be critical for edge crack prediction in the FE models for the 980GEN3 steel, with a predicted thinning limit of 7%. The conical HX exhibits a lower FE thinning limit than most deformation modes despite exhibiting the highest major strain due to through-thickness stress gradients in the test (Narayanan *et al.*, 2023).

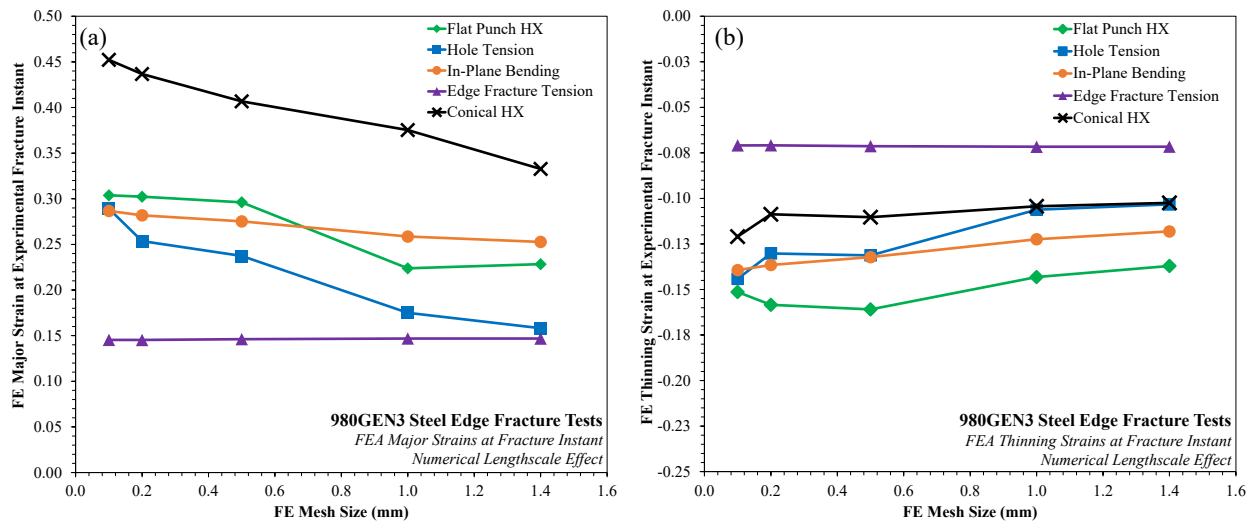


Figure 180 Influence of numerical lengthscale on the predicted (a) major and (b) thinning strains at the critical location for the 980GEN3 steel from FE simulations of the five edge fracture tests

The numerical edge fracture strains can be highly sensitive to the imposed boundary conditions, despite convergence between test types in the experiments. The numerical lengthscale significantly influences local elemental major strains, potentially skewing edge crack predictions if they are used as failure criteria. Instead, the thinning strain offers a more robust alternative since it is less affected by the mesh size and is primarily governed by the applied boundary condition. The proposed methods can be applied to any AHSS for experimental and numerical quantification of edge fracture thinning limits, enabling their use in industrial FE simulations without needing to incorporate the SAZ. This approach also enables informed choices of reference DIC and simulation lengthscales for calibrating fracture models. Future work will

focus on quantifying the strain and stress gradients associated with the various test methods and correlate to the observed differences in fracture strains.

D.7. Conclusions

This study investigated the effect of applied loading condition on the sheared edge stretchability of a 3rd generation 980GEN3 steel after piercing with 12% clearance. The corresponding fracture strains were measured using digital image correlation and through analytical methods. The influence of different DIC virtual strain gauge lengths on the measured deformation was explored for the sheared edge and contrasted against the behavior of the machined hole that served as the reference case. Finally, numerical simulations of the edge fracture tests were carried out using shell elements in LS-DYNA using different mesh sizes to assess the role of numerical lengthscale on the elemental strains at the instant of experimental fracture. Key conclusions from this work are as follows:

- The 980GEN3 steel did not exhibit a strong boundary condition effect with convergence observed between the in-plane tests with a major fracture strain of 0.30 at a VSG of 1.0 mm. An exception was the conical HX that reached larger fracture strains due to axisymmetric out-of-plane deformation.
- A pronounced sheared edge boundary condition effect can manifest in numerical simulations of edge stretching with the added influence of lengthscale, even if the experimental data is independent of the boundary condition. A single major strain value can be insufficient to describe sheared edge failure in FE simulations.
- The in-plane bending and conical hole expansion tests can provide a consistent measure of the uniaxial fracture limit for the base metal without a sheared edge. The flat punch hole expansion and edge fracture tensile tests are prone to strain localization behind the edge and at the neck center, respectively, while failure in hole tension tests can initiate slightly behind the hole edge.
- Measures of global edge stretchability can be derived using changes to the sheared edge length from the considered edge fracture tests except the HTT which exhibits a localized deformation. These metrics are representative of the average deformation experienced by the sheared edge prior to onset of edge cracking and can be used to guide the design of stamped components. For the 980GEN3 steel, the in-plane uniaxial tensile stretching mode exhibited the lowest global thinning limit and is thus expected to be the critical deformation mode for edge fracture.

- Global metrics can directly help guide the identification of optimal DIC lengthscales in the edge fracture tests to ensure direct transferability to design and FE modelling. For the 980GEN3 steel, local DIC strains obtained using VSGs in the range of 2.0 mm to 4.0 mm converged with the corresponding global edge stretch across the in-plane tests.
- A strict one-to-one correlation does not exist between the FE mesh size and DIC lengthscale. A lengthscale study must be performed to assess the reference lengthscales for calibration of fracture models. For the 980GEN3 steel, CAE mesh sizes of 1.0 mm and 1.4 mm consistently showed good correlation with the VSGs of 3.0 mm and 4.0 mm for the in-plane uniaxial stretching tests. Lower FE mesh sizes near 0.5 mm were found to replicate the experimental deformation better for the axisymmetric test methods.
- The use of elemental thinning strain can substantially reduce the lengthscale sensitivity to improve reliability of numerical predictions. These findings must be considered during industrial FEA of stamping operations to apply the correct thinning limit depending on the loading condition.

D.8. Acknowledgements

We would like to express our gratitude to the Steel Testing and Harmonization (STHT) and Stamping Teams of the Auto/Steel Partnership (A/SP) for their technical and financial support. We are also grateful to the A/SP project managers, Eric McCarty and Jonathan Smith, and the A/SP industrial partners with special recognition to Dean Kanelos of Nucor, Miguel Quinones of Metalsa, and Vince Millioto of MartinRea for sharing their extensive knowledge and insights on stamping operations. Additional financial support was provided by the Natural Sciences and Engineering Research Council of Canada (NSERC) Discovery Grant program.

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