A Decentralised Transactive Energy Market Considering Physical System Constraints

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***Abstract*—Local energy markets leveraging direct Peer-to-Peer (P2P) trading have been proposed as a solution, which can increase prosumer participation in lower cost and more reliable supply of energy to consumers. Many approaches have been proposed to determine the optimal dispatch of distributed resources; however, existing approaches either provide inadequate representation of the physical layer or insufficient handling of infeasibilities for a continuous pricing mechanism. Hence, a gap remains in the research to date on efficiently allowing for prosumer decision autonomy while ensuring that the physical layer of the power system is considered. This paper addresses these issues with a proposed decentralised transactive solution that retains prosumer negotiation and decision autonomy, while using market determined prices to allocate limited system resources for a feasible system state. This is achieved through a transaction fee mechanism for prosumer interactions that considers the network characteristics such as topology and line congestion, and a congestion-clearing process to ensure efficient allocation of network resources . Previously reported distribution networks are used to compare the economic performance and transaction decisions of the proposed solution with existing approaches.**

***Index Terms*—Decentralised, energy trading, local energy market, network usage charge, peer-to-peer trading, physical network, prosumers, transactive energy.**

NOMENCLATURE

Indices

Ω^*	Set of matched peer trades
Ω_n	Set of trades for peer n
$b \in \mathcal{B}$	Set of buses
b_w/s_w	Indices of buying/selling peers for trade w
$l \in \mathcal{L}$	Set of feeders
$n \in \mathcal{N}$	Set of peers
$\mathcal{N}^b/\mathcal{N}^s$	Set of buying/selling peers
o_l/r_l	Indices of originating/receiving-end nodes of distributon feeder l
$w \in \Omega$	Set of all peer trades

Parameters

Γ_n	Proportion of P2P demand for peer n
$\Delta\rho$	Trade negotiation price increment [$/MWh]
Φ_{ij}^l	Power transfer distribution factor for feeder l corresponding to trade between peers $i,\ j$ [p.u.]
Ψ_l^n	Injection shift factor of feeder l for peer n
$\sigma_n^{u,f}$	Binary parameter = 1, if utility transactions are fixed, and 0 otherwise
σ_w	Binary parameter = 1, if trade is cleared, and 0 otherwise
ς_w	Binary parameter = 1, if trade is blocked, and 0 otherwise
B_b	Shunt susceptance of bus b [p.u.]
C^w	Wholesale electricity market price [$/MWh]
D_b^Q	Reactive power demand at bus b [MVAr]
$\underline{D}_n/\overline{D}_n$	Min/max demand by peer n [MW]
E^ϵ	Transaction fee increment [$/MWh]
E^Z	Network usage charge scalar [$/MWh]
G_b	Shunt conductance of bus b [p.u.]
$\underline{P}_n/\overline{P}_n$	Min/max generation by peer n [MW]
$\underline{P}_b^g/\overline{P}_b^g$	Min/max real power output limit of utility generator at bus b [MW]
$\overline{p}^w$	Standard peer-centric trade size [MW]
$\underline{Q}_b^g/\overline{Q}_b^g$	Min/max reactive power output limit of utility generator at bus b [MVAr]
R_l	Resistance of distribution feeder l [p.u.]
S_l	Apparent power flow limit of distribution feeder l [p.u.]
S_l^{res}	Reserved power flow limit of distribution feeder l [MW]
$\overline{S}_l$	Congestion clearing power flow limit of distribution feeder l [MW]
T_n^g	Feed in tariff rate for peer n [$/MWh]
T_n^d	Utility supply tariff rate for peer n [$/MWh]
$\underline{V}_b/\overline{V}_b$	Min/max limit on the voltage magnitude at bus b [p.u.]
X_l	Reactance of distribution feeder l [p.u.]
Z_{ij}^{th}	Thevenin impedance between peers $i,\ j$ [p.u.]

Variables

$\Delta\nu_b^{ij}$	Voltage sensitivity coefficient of bus b for a trade between peers $i,\ j$
$\epsilon_n^{u,g}/\epsilon_n^{u,d}$	Transaction fee for selling/buying to/from the utility for peer n [$/MWh]
ϵ_w	Transaction fee for trade w [$/MWh]
δ_b	Voltage angle of bus b [rads]
λ_b	Distributed locational marginal price at bus b [$/MWh]
$\rho_w^b/\rho_w^s/\rho_w^m$	Buying/selling/mutual price for trade w [$/MWh]
a_l	Squared current flow on distribution feeder l [p.u.]

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c_w	Network usage charge for trade w [$/MWh]
d_n	Total power bought by peer n [MW]
d_n^p	Power bought by peer n from peers [MW]
d_n^u	Power bought by peer n from the utilily [MW]
$d_n^{u,f}$	Fixed power bought from the utility by peer n [MW]
f_l^P / f_l^Q	Active/reactive power flow on distribution feeder l [p.u.]
p_n	Total power sold by peer n [MW]
p_n^p	Power sold by peer n to peers [MW]
p_n^u	Power sold by peer n to the utility [MW]
$p_n^{u,f}$	Fixed power sold to the utility by peer n [MW]
p_0^g	Active power purchased by the utility from the wholesale market at the root node [MW]
p_b^g / q_b^g	Active/reactive power output of the utility generator at bus b [MW/MVAr]
p_w^b / p_w^s	Trade size for buyer/seller of trade w, a linear variable for system-centric approaches, and a binary variable for peer-centric approaches = $\overline{p}^w$, if trade is selected, and 0 otherwise [MW]
q_n	Reactive power output of peer n [MVAr]
S_l^{sl}	Apparent power flow slack of distribution feeder l [p.u.]
V_b^{sl}	Slack on the voltage magnitude at bus b [p.u.]
v_b	Squared nodal voltage magnitude of bus b [p.u.]

Functions

$\Upsilon_n(\cdot)$	Value of surplus energy for peer n [$/MWh]
$C_b^u(\cdot)$	Cost of utility generator at bus b [$/MWh]
$C_n(\cdot)$	Cost function of selling peer n [$/MWh]
$U_n(\cdot)$	Utility function of buying peer n [$/MWh]

I. Introduction

THE electricity system is undergoing substantial changes with the increasing demand for clean electricity to support electrification and net-zero targets. Increasing penetration of variable renewables at the bulk power system level are challenging conventional operating practices and are requiring new services to ensure reliability and resiliency [1]. While the need for change is becoming increasingly obvious at the wholesale level, a similar transformation is projected to occur at the distribution system level with the increasing adoption of Distributed Energy Resources (DERs) to support decarbonisation [2].

DERs provide both a challenge and an opportunity for the electricity industry. Unchecked widespread adoption of DERs could quickly over-burden infrastructure and challenge existing distribution utility practices. Time sensitive peaks, reversed power flows, and resource visibility are just a few of the major issues that need to be addressed for cost-effective DER integration [3]. On the upside, DERs present an opportunity to unlock flexibility and value stacking from price responsive prosumers when efficient incentive mechanisms are available [4]. Significant questions remain regarding how prosumers may participate in the overall energy system and markets.

Many leading jurisdictions are exploring structures and entities that are best suited to supporting DER and net-zero targets, such as distribution system operators, load serving entities, local energy markets, and transactive energy [5], [6]. Local energy markets and transactive energy systems have been proposed as a solution to provide value stacking opportunities for DERs. These solutions can be used to support reliable system operation while maintaining transactive energy principles such as privacy through implementations leveraging blockchain [7]. Although general architectures are being pursued to understand value streams, a complete framework aligning peer transactions with network constraints is needed.

Transactive energy is a generalisation of Peer-to-Peer (P2P) trading, and as such can leverage existing approaches in the P2P literature. Transactive frameworks can be categorised into Peer-Centric (PC) and System-Centric (SC) solutions depending on the objective of the coordination mechanism [8]. The elements making up a framework can be further categorised within the virtual and physical layer. The virtual layer includes elements such as the market and pricing mechanism, and the physical layer includes the actual power flows and network limitations of the underlying system [9]. In proposing a feasible transactive framework, both the virtual and physical layers must be aligned.

Much of the existing literature on P2P markets relies upon an SC approach [9]. These methodologies attempt to maximise overall system welfare through a coordination mechanism. Centralised optimisation approaches have been proposed directly coupling virtual and physical layers through virtual power plant implementations and optimal power flow problems [10]. These approaches challenge the autonomy and privacy principles of transactive energy as they require access to private customer information such as utility or cost functions, as well as pose issues for scalability related to problem complexity as the number of DERs increase. Auctions with a central clearing mechanism ensuring physical constraints are respected have been proposed in, for example, [11] and [12]; however, these auction-based processes do not allow for product differentiation and direct negotiation between peers, since the bids and offers are generalized within the auction clearing process and assigned centrally to buyer-seller pairs. Rather than attempting to adapt centralized processes to address these challenges, many recent works have taken approaches which shift greater control and autonomy directly to prosumers.

In a PC approach, peers retain decision making autonomy to select the optimal set of transactions for their own objectives and are not required to disclose sensitive utility and capability information. This process ensures the security of peer information, limits communication overhead, and allows for individual product differentiation. Several PC approaches relying on direct negotiation have been explored in the literature to pursue these objectives. For example, a decomposition of the SC P2P problem with a novel primal-dual gradient method for decentralised clearing is presented in [13]. A multi-leader, multi-follower Stackelberg game approach is considered in [14] for non-collaborative P2P trading. In [15], a bilateral contract network is introduced satisfying full substitutability for establishing a stable outcome, while [16] includes consideration of prosumer preferences for energy classes. In [17],

the authors propose a relaxed consensus-innovation approach adapted for bilateral trading, which allows for product and price differentiation. These references focus on the virtual layer without proper consideration of the physical layer.

Solutions utilising distributed optimisation and optimal power flow techniques have been proposed that introduce price signals considering network constraints in the physical layer, while also addressing concerns of scalability and privacy to arrive at a feasible, system optimal state. The authors of [18] establish a generalised Nash game and propose a sharing mechanism which ensures a near-optimal market equilibrium with minimized price discrimination, but only consider a dc power flow model for line flows without consideration of voltages, hindering the application to real power systems. Other approaches such as [19]–[21] use ac power flow representations and rely on dual variables to set trade prices while requiring only knowledge of local coupling variables for peer decision making. In [22], the approach considers a multi-energy network with a lower-level P2P trading problem and an upper-level network optimisation problem considering network reconfiguration and optional trading adjustments for feasibility; however, the forced trading adjustments may violate prosumer preferences in the case of an infeasible set of trades rather than utilising a softer price signal to influence beneficial behaviour. Although these approaches have been shown to be successful in achieving similar results to SC approaches and have been extensively discussed in the literature, alternative PC methods using direct price negotiation between peers and network derived prices for influencing system feasible behaviour have also shown promise.

Alternative mechanisms to those discussed also offer the potential to ensure a consistent coordination between the virtual peer transactions and physical layer limits. A decentralised clearing method is proposed in [23] that makes use of a network utilisation fee based on fixed electrical distances. For consideration of dynamic network usage, [24] explores various loss allocation methods with additional representation of system operators. In [25], the authors propose an efficient direct negotiation and peer matching process with a loss based transaction fee. A bilevel prosumer framework for decentralised bilateral trading is proposed in [26], where a Network Usage Charge (NUC) based on DLMPs and electrical distance elements is considered, but the handling of infeasible trade sets is not included. Reference [27] considers a bilateral negotiation approach consisting of an initial negotiation for active power trading without considering network constraints, followed by a coordinated decentralised optimisation of reactive power. This approach does not consider potential physical power flow violations or loss distribution, and the proposed infeasible set handling is inconsistent with the general trading mechanism. Paper [8] extends [15] with the addition of a trade NUC derived from DLMPs to consider physical system feasibility, introducing a penalty mechanism to handle infeasible trade sets; however, the proposed solution is inefficient and does not penalise trades in proportion to their system usage.

As seen from the literature review, SC approaches fail to provide a scalable solution respecting prosumer centric values of autonomy and privacy, raising concerns regarding long-term practicality. On the other hand, PC approaches have been proposed to ensure prosumer values are respected, including a number of approaches utilising distributed optimisation techniques; however, existing decentralised direct negotiation implementations do not efficiently account for the physical layer in guiding prosumer behaviour as part of a complete, consistent, transactive solution. Hence, this paper addresses some of the latter issues, contributing to the advancement of practical transactive energy frameworks as follows:

- Provide a generic transactive energy market formulation for real-time SC and PC decision making, extending the strictly P2P formulation to allow for local welfare maximization in selecting transactions with exposure to wholesale market prices.
- Develop a cohesive, consistent transaction fee mechanism for coupling of decentralised prosumer decision making with fixed and dynamic network considerations. The proposed mechanism provides an intuitive representation of network cost factors for effective coupling of virtual peer transactions with the physical layer, addresses continuity issues for infeasible trade sets, and satisfies the remuneration requirements for the compensation and balancing services provided by the utility.
- Propose a novel congestion clearing process maximising high priority feeder usage while maintaining prosumer decision autonomy. The proposed novel congestion clearing process achieves efficient management of stressed network conditions, while ensuring that the resulting network usage approximates the optimal SC solution.
- Present a complete, decentralised transactive energy market model that ensures an efficient and practical system feasible, PC solution, while maintaining transactive principles of privacy, decision autonomy, and direct negotiation.

The remainder of the paper is organised as follows: Section II provides an overview of market formulations for P2P networks, including a specific decentralised solution with physical system consideration. Section III details the novel contributions to the formulation of transactive market models, which includes a SC model as well as the novel sub-elements necessary to provide a complete decentralised PC transactive solution. The economic and network performance of the models are assessed and compared in Section IV using 15-bus, 33-bus, and 141-bus benchmark distribution grids. Section V highlights the main conclusions and contributions, and discusses possible future work.

II. Peer-to-Peer Energy Market Overview

P2P energy market implementations provide a foundation which can be extended to consider Peer-to-Utility (P2U) transactions as well. Overviews of the SC and PC P2P trading formulations proposed in [8] are presented next, which serve as the foundation for the proposed transactive approaches.

A. System-Centric Peer-to-Peer Trading

The SC model consists of two perspectives, the peer and the central coordinating entity. Peer welfare is determined

according to the utility of demand and cost of supply, while welfare of the central coordinator is determined by the cost of balancing the system. The first part of the model, which accounts for the peer welfare resulting from P2P transactions, is as follows [8]:

$$\max_{\Xi^{\text{Peer}}} \quad O^{\text{Peer}} := \sum_{n\in\mathcal{N}^b} U_n(d_n) - \sum_{n\in\mathcal{N}^s} C_n(p_n) \tag{1a}$$

$$\text{s.t.} \quad U_n(d_n) = \Upsilon_n(d_n - \underline{D}_n) \ \ \forall n \in \mathcal{N}^b \tag{1b}$$

$$\underline{P}_n \le p_n \le \overline{P}_n \ \ \forall n \in \mathcal{N}^s \tag{1c}$$

$$\underline{D}_n \le d_n \le \overline{D}_n \ \ \forall n \in \mathcal{N}^b \tag{1d}$$

$$p^p_n = \sum_{w\in\Omega_n} p^s_w \ \ \forall n \in \mathcal{N}^s \tag{1e}$$

$$d^p_n = \sum_{w\in\Omega_n} p^b_w \ \ \forall n \in \mathcal{N}^b \tag{1f}$$

$$p^p_n = p_n \ \ \forall n \in \mathcal{N}^b \tag{1g}$$

$$d^p_n = d_n\Gamma_n \ \ \forall n \in \mathcal{N}^b \tag{1h}$$

$$p^b_w = p^s_w \ \ \forall w \in \Omega \tag{1i}$$

where $\Xi^{\text{Peer}} = \{d_n, p_n, p^b_w, p^s_w\}$, with the parameters, variables, and functions being defined in the Nomenclature Section for this and other models presented in this paper. In this P2P trading formulation, peers are only able to sell energy directly to another peer, and cannot sell to the utility. The parameter Γ_n can be adjusted in a range of [0, 1] to modify the amount of demand supplied by P2P trading. This approach assumes a fixed proportion of demand is supplied through P2P trading, rather than determining the optimal proportion of P2P demand according to cost and utility functions. Constraint (1i) ensures coordination between the buyer and seller trade sizes for the SC optimisation.

The system component of the welfare optimisation considers the physical layer using the second-order-cone ac OPF model [28], as follows [8]:

$$\max_{\Xi^{\text{Dist}}} \quad O^{\text{Dist}} := \sum_{b\in\mathcal{B}}(-C^u_b(p^g_b)) - C^w p^g_0 \tag{2a}$$

$$\text{s.t.} \quad 0 = f^P_{l|o_l=b} - \sum_{l|r_l=b}(f^P_l - a_l R_l) - p^g_b - p_{n=b} + d_{n=b} + G_b v_b \ \ \forall b \in \mathcal{B} \tag{2b}$$

$$0 = f^Q_{l|o_l=b} - \sum_{l|r_l=b}(f^Q_l - a_l X_l) - q^g_b - q_{n=b} + D^q_b - B_b v_b \ \ \forall b \in \mathcal{B} \tag{2c}$$

$$S^2_l \ge (f^P_l)^2 + (f^Q_l)^2 \ \ \forall l \in \mathcal{L} \tag{2d}$$

$$S^2_l \ge (f^P_l - a_l R_l)^2 + (f^Q_l - a_l X_l)^2 \ \ \forall l \in \mathcal{L} \tag{2e}$$

$$v_{r_l} = v_{o_l} - 2(R_l f^P_l + X_l f^Q_l) + a_l(R^2_l + X^2_l) \ \ \forall l \in \mathcal{L} \tag{2f}$$

$$v_{o_l} \ge \frac{(f^P_l)^2 + (f^Q_l)^2}{a_l} \ \ \forall l \in \mathcal{L} \tag{2g}$$

$$\underline{P}^g_b \le p^g_b \le \overline{P}^g_b \ \ \forall b \in \mathcal{B} \tag{2h}$$

$$\underline{Q}^g_b \le q^g_b \le \overline{Q}^g_b \ \ \forall b \in \mathcal{B} \tag{2i}$$

$$\underline{V}^2_b \le v_b \le \overline{V}^2_b \ \ \forall b \in \mathcal{B} \tag{2j}$$

where $\Xi^{\text{Dist}} = \{p^g_b, q^g_b, p^g_0, q_n, v_b, a_l, f^P_l, f^Q_l\}$. Here, the welfare of the system utility considers the cost of utility generation $C^u_b(p^g_b)$ and the cost of energy purchased from the wholesale market $C^w p^g_0$. The model assumes that peer reactive power output q_n is controlled by the system operator to minimise balancing costs. In a transactive system, the utility could create a product to recognise the value of this service. The complete system model is described in detail in [8] and [28].

Considering both the peer and utility welfare components, the SC P2P trading formulation is:

$$\max \quad O^{\text{Peer}} + O^{\text{Dist}} \tag{3a}$$

$$\text{s.t.} \quad \text{Eqs. (1b)–(1i)} \quad \text{Peer constraints} \tag{3b}$$

$$\text{Eqs. (2b)–(2j)} \quad \text{Network constraints} \tag{3c}$$

B. Peer-Centric Peer-to-Peer Trading

The PC approach to P2P trading decouples peer and utility decision making. Ensuring coordination between the virtual and physical layers of the peer and utility requires the addition of a physical layer based NUC, described in Section II-B1. The revised peer decision making criteria with consideration of the NUC is presented in Section II-B2. The peer matching process enabling decentralised negotiation between peers is explained in Section II-B3. Finally, Section II-B4 describes the overall implementation of the PC approach.

1) System Derived Network Usage Charge: An important component of the proposed PC framework in [8] is the NUC c_w for trade w. This pricing mechanism links the virtual and physical layers of the P2P system by providing a price signal derived from dynamic system conditions. The NUC is derived using the DLMPs resulting from (2), and is calculated as follows:

$$c_w = \begin{cases} (\lambda_{b_w} - \lambda_{s_w})/2 \ \ \forall w \in \Omega^* & \text{if OPF is feasible} \\ c_w + \epsilon \ \ \forall w \in \Omega^* & \text{otherwise} \end{cases} \tag{4}$$

Recognising that peers may select an infeasible set of trades in which the DLMPs cannot be calculated, a small penalty increment ϵ is used. All trades included in the proposed infeasible optimal set Ω^* have their NUC increased by the same fixed increment. In the scenario that the optimal set is feasible, all trades in the optimal set have their NUC set to the default DLMP based value, and any prior penalty increments are removed. This approach introduces an approximate coupling mechanism between the virtual layer and physical layer to support convergence to a feasible solution, but can result in sub-optimal trade selections. Trades included in the infeasible set of trades are penalised regardless of their proportional contribution to system infeasibility. Although the virtual penalties are removed if a trade is included in a feasible set, the approach to penalisation can result in penalty values which prevent an otherwise optimal trade from being selected, resulting in a sub-optimal solution.

2) Peer-Centric Decision Formulation: In the PC formulation, peers select the subset of all possible trades which maximises local welfare. Peers must determine the optimal total power bought d_n, and determine the least cost set of trades to supply the P2P component of demand d^p_n. With the

addition of the NUC mechanism proposed in [8], peer decision making criteria for selection of the locally optimal set of trades is as follows:

$$\Omega_n^* = \begin{cases} \arg\max_{\Omega_n}\{\sum_{w\in\Omega_n}(\rho_w^s - c_w)p_w^s - C_n(p_n)\} \\ \quad \forall\, n \in \mathcal{N}^s \\ \arg\max_{\Omega_n}\{U_n(d_n) - \sum_{w\in\Omega_n}(\rho_w^b + c_w)p_w^b\} \\ \quad \forall\, n \in \mathcal{N}^b \end{cases} \tag{5a}$$

$$\text{s.t.} \quad p_w^b, p_w^s \in \{0, \overline{p}^w\} \;\; \forall\, w \in \Omega \tag{5b}$$

$$\text{Eqs. (1b)–(1h)} \quad \text{Peer constraints} \tag{5c}$$

The peer selection result is a locally optimal set of P2P trades Ω^n, which ensures $d_n^p = d_n\Gamma_n$. The trade specific NUC c_w is critical to ensuring peers consider the cost of network usage in their decision making, since the utility system optimisation is now completely decoupled. Similar to the SC formulation, [8] assumes a predetermined proportion of total power bought is supplied by P2P transactions rather than allowing peers to determine the welfare optimal proportion. Note the absence of the coordinating constraint Eq. (1i) since the optimal trades are locally determined, and the addition of Eq. (5b), which ensures p_w^b and p_w^s are binary variables equal to $\overline{p}^w$, if trade w is selected, and 0 otherwise for the PC scenario.

3) Peer Matching: The PC trading architecture enables peers to keep control of how they would like to best use their own resources; however, a process is needed to facilitate peer negotiation and ensure a stable solution is achieved. Thus, as proposed in [15] and further developed in [8], the peer matching process satisfies full substitutability for establishing a stable outcome with direct bilateral negotiation.

The P2P negotiation process is described in detail in [8] and [15], which begins with setting all trade prices to zero. Peers then select their optimal set of trades according to the current iteration trade prices and NUCs. Trade prices are incremented if a buyer selects a trade and a seller does not. This process is repeated until all trade prices are stable, indicating that all trades are either mutually accepted by buyer and seller, or rejected by the buyer.

4) Implementation: After determining the stable set of optimal trades Ω^*, (2) is solved separately by the utility for balancing and to assess the trade set feasibility. If the trade set is infeasible, NUCs are incremented, while if the trade set is feasible, the peer matching is checked to ensure the solution is stable after any prior NUC updates. If the matching is unchanged, a stable solution has been obtained, otherwise the NUCs are updated according to the system DLMPs. The complete PC P2P trading process proposed in [8] is illustrated in Fig. 1, where the functions are grouped by the entity responsible for their completion.

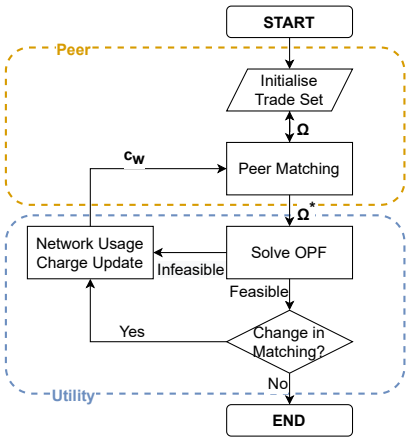


Fig. 1: PC P2P trading solution [8].

III. Proposed Transactive Energy Market Approach

P2P frameworks only consider potential transactions between peers; however, the option to transact with either peers or utilities should also be considered. In this context, Section III-A extends the above SC formulation to a general transactive model that considers possible P2U transactions. To ensure peers consider the system cost of transactions, a novel split pricing mechanism is also proposed in Sections III-B and III-C. Then, a congestion clearing mechanism is proposed in Section III-D, which coordinates with the fixed and dynamic transaction fees to maximise high priority feeder usage, and the criteria for decentralised PC decision making is provided in Section III-E. Lastly, a revised peer matching process is proposed in Section III-F as part of a complete decentralised transactive solution.

A. System-Centric Transactive Market

Transactive energy markets are an extension of the P2P concept, enabling not only P2P transactions but considering the complete relationship between prosumers and utility to derive maximum benefits from DERs through P2U transactions. Additionally, peers should not have a fixed proportion of P2P trading to satisfy demand and instead should be able to determine their welfare optimal mix of P2P and P2U transactions. The addition of P2U transactions changes the role of the utility from a passive entity responsible for ensuring a balanced system to facilitate internal P2P trades, to a new role which allows transacting directly with peers and thus introduce increased market opportunities by facilitating transactions derived from external wholesale market prices. The peer cost function $C_n(\cdot)$ and peer utility function $U_n(\cdot)$ determine the problem type, which in this work are considered to be non-linear in the general formulation. To extend the NLP model to being fully transactive, the following modifications to (1) are made:

$$\max_{\Xi^{\text{TPeer}}} \; O^{\text{TPeer}} := \sum_{n\in\mathcal{N}^b}(U_n(d_n) - T_n^d d_n^u) + \sum_{n\in\mathcal{N}^s}(T_n^g p_n^u - C_n(p_n)) \tag{6a}$$

$$\text{s.t.} \quad U_n(d_n) = \Upsilon_n(d_n - \underline{D}_n) \;\; \forall\, n \in \mathcal{N}^b \tag{6b}$$

$$p_n = p_n^p + p_n^u \;\; \forall\, n \in \mathcal{N}^s \tag{6c}$$

$$d_n = d_n^p + d_n^u \;\; \forall\, n \in \mathcal{N}^b \tag{6d}$$

$$\text{Eqs. (1b)–(1f)} \quad \text{Peer constraints} \tag{6e}$$

Eq. (1i) Coordination constraint (6f)

where $\Xi^{\text{TPeer}} = \{d_n, d_n^p, d_n^u, p_n, p_n^p, p_n^u, p_w^b, p_w^s\}$. To consider the option of P2U transactions, (6c) and (6d) are added. The objective function is modified to include T_n^g and T_n^d, which are the fixed P2U rates associated with selling energy to and buying energy from the utility. The utility may adjust these P2U fees to account for cost of supply or to incentivise specific behaviour through, for example, time-of-use pricing mechanisms. With these additions, the SC formulation is now generally transactive rather than limited solely to P2P.

The distribution utility welfare is modified to consider P2U trading as follows:

$$\max_{\Xi^{\text{Dist}}} \quad O^{\text{TDist}} := \sum_{b\in\mathcal{B}}(-C_b^u(p_b^g)) - C^w p_0^g + \sum_{n\in\mathcal{N}}(T_n^d d_n^u - T_n^g p_n^u) \tag{7a}$$

$$\text{s.t.} \quad \text{Eqs. (2b)–(2j)} \quad \text{Network constraints} \tag{7b}$$

where (7a) considers the revenue from P2U demand, and the cost of P2U generation. The addition of P2U transactions enables surplus local energy to be sold at the wholesale market price to increase overall system welfare. The revenue and cost terms from P2U transactions are included in (6a) and (7a) for clarity; however, they are internal and are therefore cancelled out in the following total system welfare maximisation:

$$\max \quad O^{TPeer} + O^{TDist} \tag{8a}$$

$$\text{s.t.} \quad \text{Eqs. (6b)–(6f)} \quad \text{Transactive peer constraints} \tag{8b}$$

$$\text{Eqs. (2b)–(2j)} \quad \text{Network constraints} \tag{8c}$$

B. Fixed Path Network Usage Charge

Coordination between the decoupled virtual and physical layers is necessary in the form of a pricing mechanism. Rather than a single price as proposed in [8], a split mechanism is proposed here consisting of a fixed cost element and dynamic cost element for greater pricing visibility. The fixed cost component is described in this section, while the dynamic cost component is detailed in Section III-C.

To consider the fixed network cost element of transactions, the unique path between buyer and seller node is used by the transacton platform to determine a buyer-seller pair specific NUC. These NUCs are determined using the distance based Thevenin Impedance Method proposed for P2P trades in [29]. The Thevenin impedance Z_{ij}^{th} between nodes i, j and associated NUC for trade w, as well as the node specific P2U trading prices T_n^g and T_n^d are calculated as follows:

$$Z_{ij}^{th} = Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji} \tag{9a}$$

$$c_w = Z_{s_w b_w}^{th} E^Z \tag{9b}$$

$$T_n^g = C^w - Z_{n1}^{th} E^Z \tag{9c}$$

$$T_n^d = C^w + Z_{n1}^{th} E^Z \tag{9d}$$

where Z_{ij} is the element in the i^{th} row and j^{th} column of the Z_{bus} matrix. The transaction platform is responsible for setting the constant scalar E^Z, which scales Z_{ij}^{th} to obtain the individual trade NUC. This method allows E^Z to be scaled as necessary to cover the fixed costs of network transactions while distributing those costs according to the electrical distance of the transaction paths, such as the costs of losses.

C. Dynamic System Transaction Fee

While the fixed network costs associated with transactions are covered by the NUC c_w, an additional mechanism is required to represent peer willingess to pay for dynamic network usage. The method proposed in [8] introduces an incremental penalty assigned to all individual trades included in an infeasible optimal set; hence, rather than the general approach of treating all trades in the infeasible set as equal, a novel approach is proposed here utilising derived network factors to introduce a price signal accounting for a peer's willingess to pay for network usage as described next.

1) Flow Constraints: The optimal set of trades may violate feeder flow limits due to decoupling of the virtual and physical layers. To ensure consideration of congestion in the PC trade selection, the dynamic pricing mechanism should introduce a transaction fee element proportional to an individual transaction's usage of limited network resources. The contributions to congestion for individual transactions are determined using the simplified Power Transfer Distribution Factors (PTDFs) proposed in [11], which are calculated as follows:

$$\Phi_{ij}^l = \Psi_l^i - \Psi_l^j, \tag{10}$$

where Ψ_l^i is the injection shift factor derived in [11].

2) Voltage Constraints: In the scenario that the proposed set of optimal transactions results in an infeasible system due to voltage constraint violations, a network sensitivity based mechanism is necessary to penalise trades contributing to that violation. The first step of deriving this sensitivity coefficient is to recover the voltage angles δ_b at each node as the second-order cone ac OPF model utilises an angle relaxation [28]. The centralised angle recovery algorithm described in [28] is used, which is guaranteed to ensure angle recovery for radial systems. Upon recovering voltage angles, Voltage Sensitivity Coefficients (VSCs) are computed using the Jacobian associated with the Newton Rhapson method for the relaxed system, although alternative methods may be used such as those proposed in [30]. The derived VSCs for each transaction are calculated as follows:

$$\Delta\nu_b^{ij} = \Delta\nu_b^i - \Delta\nu_b^j. \tag{11}$$

3) Dynamic Transaction Fee Assignment: The central coordinator, generally the utility, is responsible for assessing the feasibility of the optimal set of trades and updating proportional transaction fees using the above derived sensitivity coefficients. For asessment of the trade set feasibility, the conic relaxation of (2g) is changed to an equality constraint. This is necessary since, although [28] proves the conic relaxation OPF is always exact for radial networks when there are no upper bounds on the loads, the conic relaxation can result in a feasible solution of the OPF even when peer energy injections would physcially violate power flow constraints in an unrelaxed problem. A transaction fee assignment process is

proposed in Algorithm 1 to update individual transaction fees in the case of an infeasible set of locally optimal trades.

Algorithm 1 Transaction Fee Assignment Process

1: $status, feederState, nodeState =$ LRelaxedOPF()
2: **if** $status \neq feasible$ **then**
3: $\quad feederState, nodeState, v_b, \delta_b =$ LVRelaxedOPF()
4: $\quad \Delta\nu_b^{ij} =$ VSC(v_b, δ_b)
5: **for** $l \in feederState$ **do**
6: $\quad$ Update all $\epsilon_w, \epsilon_n^{u,g}, \epsilon_n^{u,d}$ with $E^\epsilon \Phi_{ij}^l feederState_l$
7: **for** $b \in nodeState$ **do**
8: $\quad$ Update all $\epsilon_w, \epsilon_n^{u,g}, \epsilon_n^{u,d}$ with $E^\epsilon \Delta\nu_b^{ij} nodeState_b$
9: Limit all changes of $\epsilon_w, \epsilon_n^{u,g}, \epsilon_n^{u,d}$ to magnitude E^ϵ
10: **return** ϵ

In the scenario that a set of negotiated transactions is infeasible, a series of relaxed OPFs are used to assess if constraint relaxation results in a feasible solution. This procedure assumes that a relaxed OPF solution exists, and it also assumes that the proposed transactions can be modified to meet system constraints. Firstly, the function LRelaxedOPF(·) in Algorithm 1 runs the modified OPF (2) where the feeder power flow limit constraints (2d) and (2e) become softened with the addition of the slack variable S_l^{sl}, which appears in the modified objective function and associated constraints:

$$\max_{\Xi^{\text{LDist}}} \quad O^{\text{LDist}} := O^{\text{Dist}} - \sum_{l \in \mathcal{L}} M(S_l^{sl}) \tag{12a}$$

$$\text{s.t.} \quad \text{Eqs. (2b)–(2c), (2f)–(2j)} \quad \text{Network constraints} \tag{12b}$$

$$S_l^2 + S_l^{sl} \geq (f_l^P)^2 + (f_l^Q)^2 \quad \forall\, l \in \mathcal{L} \tag{12c}$$

$$S_l^2 + S_l^{sl} \geq (f_l^P - a_l R_l)^2 + (f_l^Q - a_l X_l)^2 \quad \forall\, l \in \mathcal{L} \tag{12d}$$

where $\Xi^{\text{LDist}} = \{\Xi^{\text{Dist}}, S_l^{sl}\}$, and M is a large number. The function returns the status of the modified OPF and if feasible, the set of feeder and node index value pairs, $feederState$, $nodeState$, for which the proposed optimal set of trades violates their associated limits, and where values are 1 if the upper limit is violated, -1 if the lower limit is violated, and null otherwise. If LRelaxedOPF(·) returns an infeasible status, the function LVRelaxedOPF(·) is run, which in addition to relaxing the power flow limit constraints, also softens the nodal voltage constraints (2j) with addition of the slack variable V_b^{sl}, which appears in the modified objective function and associated constraints:

$$\max_{\Xi^{\text{LVDist}}} \quad O^{\text{LVDist}} := O^{\text{LDist}} - \sum_{b \in \mathcal{B}} M(V_b^{sl}) \tag{13a}$$

$$\text{s.t.} \quad \text{Eqs. (2b)–(2c), (2f)–(2i)} \quad \text{Network constraints} \tag{13b}$$

$$\text{Eqs. (12c)–(12d)} \quad \text{Feeder flow constraints} \tag{13c}$$

$$\underline{V}_b^2 \leq v_b + V_b^{sl} \leq \overline{V}_b^2 \quad \forall\, b \in \mathcal{B} \tag{13d}$$

where $\Xi^{\text{LVDist}} = \{\Xi^{\text{LDist}}, V_b^{sl}\}$, and M is a large number. LVRelaxedOPF(·) returns the resulting $feederState$, $nodeState$ in the same format as LRelaxedOPF(·). The introduction of softened constraints with slack variables ensures that the resulting solutions exceed regular bounds only when necessary for the sake of feasibility, thereby avoiding unnecessary changes to transaction fees.

Violated feeders and nodes are iterated through to update individual transaction fees. These fees represent the market price for the proportional contribution of each transaction to stresed network conditions, which for the proposed method includes congested lines and constrained nodal voltages. Each P2P trade w has an associated transaction fee ϵ_w, and each prosumer n has P2U transaction fees $\epsilon_n^{u,g}$ and $\epsilon_n^{u,d}$ for selling and buying respectively. These transactions fees are incremented by a fixed quantity E^ϵ, which is scaled by the relevant sensitivity coefficient. For all feeders with flow violations, transactions are incremented according to $E^\epsilon \Phi_{ij}^l feederState_l$ in Line 6 of Algorithm 1, where $feederState_l$ indicates the direction of the flow violation, and Φ_{ij}^l is used to calculate the approximate impact on the flow of feeder l for the buyer-seller pair of each transaction. Hence, if $feederState_l$ and Φ_{ij}^l are in the same direction, the violation is increased and the transaction fee is also increased; if in the opposite direction, the violation is reduced and the transaction fee is decreased, otherwise no change is made for a PTDF of zero. The same approach is used for calculating the transaction fee change associated with the VSC in the case of a voltage constraint violation, where the product of $\Delta\nu_b^{ij} nodeState_b$ will result in either an increase, decrease, or no change to each transaction fee for each node in $nodeState$.

The selection of E^ϵ is a market design decision considering the competing objectives of market efficiency and solution time, where a smaller E^ϵ may provide more precise adjustments to transaction fees at the cost of additional solution iterations. In cases where a trade reduces a constraint violation, applicable transaction fees are reduced and can become negative to incentivise constraint reducing trades. As a final step, total penalty increments for each transaction within a single assignment iteration are limited to the magnitude of E^ϵ. The introduction of transaction fees, proportional to sensitivity coefficients, ensures that the dynamic value of network usage is considered in the PC transaction selection process.

D. Congestion Clearing Process

While the above noted transaction fees introduce a price signal representing a peer's willingness to pay for limited network resources, an additional clearing mechanism is necessary to ensure that marginal congestion is cleared efficiently. Thus, transaction fees are incremented until congestion is relieved in previously impacted feeders. However, an issue might arise where as a result of incrementing the transaction fees for using a congested feeder, a block of trades could become suboptimal, resulting in under utilisation of the previously congested feeder and deadweight loss of welfare.

A novel congestion clearing process is proposed in Algorithm 2 to address the aforementioned issue. This process runs when the congestion on any upstream previously impacted feeder has been relieved, indicating that fees have been incremented such that a congested feeder, and any similarly impacted downstream feeders, has been resolved. Once the congestion clearing process is complete, all transactions implicating the relieved feeders are fixed at an optimal level and

must remain in that state for the remainder of the negotiation process. To achieve this, σ_w and ς_w are introduced as clearing and blocking variables respectively for P2P transactions. When σ_w is set to 1, this trade must be selected, and when ς_w is set to 1, this trade must not be selected. Similarly, $\sigma_n^{u,f}$, $p_n^{u,f}$, and $d_n^{u,f}$ are introduced for P2U trades. For P2U transactions that implicate the cleared feeder, $\sigma_n^{u,f}$ is set to 1, requiring that $p_n^u = p_n^{u,f}$ and $d_n = d_n^{u,f}$ for the remainder of the negotiation.

Algorithm 2 Congestion Clearing Process

1: $feederRel, feederCon, flow$ = FeedersRelieved()
2: $\overline{S}_l \leftarrow S_l - S_l^{res} \, \forall \, l \in feederCon$
3: **for** $w \in \Omega_{old}^*$ **do**
4: **if** $\sigma_w = 1$ **then**
5: Update all $flow_l$ with $\Phi^l_{s_w b_w} \overline{p}^w$
6: **else**
7: **if** w reduces congestion $\forall \, l \in feederRel$ **then**
8: Update $\epsilon_w, \sigma_w, flow$
9: **for** $n \in \mathcal{N}$ **do**
10: **if** $\sigma_n^{u,f} = 1$ **then**
11: Update all $flow_l$ with $\Phi^l_{n,0} p_n^{u,f} + \Phi^l_{0,n} d_n^{u,f}$
12: **else**
13: **if** $p^u_{n,old}, d^u_{n,old}$ reduce congestion $\forall \, l \in feederRel$ **then**
14: Update $\epsilon_n^{u,g}, \epsilon_n^{u,d}, \sigma_n^{u,f}, p_n^{u,f}, d_n^{u,f}, flow$
15: **for** $w \in \Omega_{old}^*$ **do**
16: **if** $\sigma_w \neq 1$ **then**
17: **if** w impacts $flow \, \forall \, l \in feederRel$ **then**
18: **if** w does not violate $\overline{S}_l \, \forall \, l \in feederCon$ **then**
19: Update $\epsilon_w, \sigma_w, flow$
20: **else**
21: Update ς_w
22: **for** $w \notin \Omega_{old}^*$ **do**
23: **if** w impacts $flow \, \forall \, l \in feederRel$ **then**
24: Update ς_w
25: CongestionClearingP2U()
26: **return** $\epsilon, \sigma, \varsigma, p^{u,f}, d^{u,f}$

The clearing process for transactions impacting relieved feeders is as follows: (1) function FeedersRelieved(·) establishes a set of the relieved feeders, a set of constrained feeders, and a set of flow variables tracking relieved feeder usage; (2) the congestion clearing limit $\overline{S}_l$ is set to $S_l - S_l^{res}$ considering a utility determined capacity S_l^{res} for the congestion clearing process to ensure the network is able to compensate any discrepancies between real and virtual power flows for congested lines; (3) all transactions reducing congestion in the relieved feeders are cleared; (4) flow variables are incremented with usage of Φ^l_{ij} to monitor the available feeder capacity; (5) transaction fees are reversed to the prior iteration amount with full feeder utilisation; (6) for P2P trades, σ_w is set to 1, while for P2U transactions, $\sigma_n^{u,f}$ is set to 1 and the respective $p_n^{u,f}$ and $d_n^{u,f}$ are set to the cleared value; (7) P2P transactions are sequentially cleared until the flow variable is at the limit S_l for congested feeder l; (8) any P2P trades impacting the relieved lines that are either in the optimal set but not cleared, or not in the optimal set, are blocked; and (9) any remaining capacity is allocated to P2U transactions as seen in Line 23 of Algorithm 2.

The order of clearing to determine which transactions are accepted and which are blocked is as follows: (1) P2P trades are cleared first, with remaining capacity allocated to P2U transactions; and (2) transactions within the P2P or P2U pool are cleared in ascending order of the associated transaction NUC c_w. This ordering prioritises P2P trades above P2U transactions, and further prioritises transactions according to the fixed NUC, which is proportional to the electrical distance of the transaction. The trade set Ω_{old}^* and set of peers $\mathcal{N}$ are assumed to be ordered according to this logic. This method seeks to maximise welfare; however, a fairness based approach could be implemented with an alternative allocation of the limited capacity. With this process, the congestested feeders are cleared, utilising all available capacity, with all transactions impacting those feeders fixed at a specified amount.

E. Peer-Centric Transactive Decision Formulation

Similar changes required for a transactive SC model are required in the PC decision formulation. In addition to considering P2U transactions and welfare based determination of P2P share, the fixed NUC, dynamic transaction fees, and congestion variables must be considered in selecting a peer's optimal set of trades. The following is the MINLP transactive PC decision criteria:

$$\Omega_n^* = \begin{cases} \arg\max_{\Omega_n} \{ \sum\limits_{w \in \Omega_n} (\rho_w^s - c_w) p_w^s & \forall \, n \in \mathcal{N}^s \\ \quad + (T_n^g - \epsilon_n^{u,g}) p_n^u - C_n(p_n) \} & \\ \arg\max_{\Omega_n} \{ U_n(d_n) - (T_n^d + \epsilon_n^{u,d}) d_n^u & \\ \quad - \sum\limits_{w \in \Omega_n} (\rho_w^b + c_w + \epsilon_w) p_w^b \} & \forall \, n \in \mathcal{N}^b \end{cases} \tag{14a}$$

$$\text{s.t.} \quad p_w^b \wedge p_w^s = \overline{p}^w \;\; \forall \, w | \, \sigma_w = 1 \tag{14b}$$

$$p_w^b \wedge p_w^s = 0 \;\; \forall \, w | \, \varsigma_w = 1 \tag{14c}$$

$$p_n^u = p_n^{u,f} \;\; \forall \, n | \, \sigma_n^{u,f} = 1 \tag{14d}$$

$$d_n^u = d_n^{u,f} \;\; \forall \, n | \, \sigma_n^{u,f} = 1 \tag{14e}$$

$$p_w^b, p_w^s \in \{0, \overline{p}^w\} \;\; \forall \, w \in \Omega \tag{14f}$$

$$\text{Eqs. (6b)–(6e)} \quad \text{Peer constraints} \tag{14g}$$

The optimal set of trades for peer n now fully considers the fixed NUC c_w, the dynamic transaction fees ϵ_w, $\epsilon_n^{u,g}$, and $\epsilon_n^{u,d}$, and the congestion clearing variables σ_w, ς_w, $\sigma_n^{u,f}$, $p_n^{u,f}$, and $d_n^{u,f}$. Additionally, both buyers and sellers consider the public mutual trade price ρ_w^m for final settlement rather than the private buyer and seller prices ρ_w^s and ρ_w^b, needed by changes made to the transaction matching process discussed in Section III-F next. It is of note that the transaction fee ϵ_w for trade w is fully paid by the buyer b_w, and as such does not appear in the selection criteria of seller s_w.

F. Comprehensive Decentralised Peer-Centric Framework

The matching process is the most significant computational component of the solution methodology, requiring multiple iterations as individual trade prices increase until a stable set of trades is reached. The matching process proposed in [8] requires a complete price renegotiation each time the NUCs are updated, since the set of trade prices is reset to zero for all trades. Thus, the matching process proposed in [8] is modified in Algorithm 3, accounting for the peer optimal set criteria of

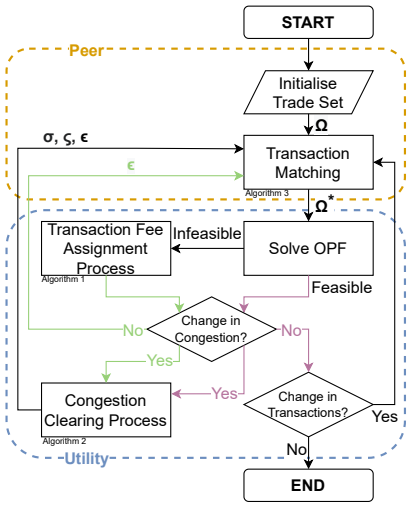


Fig. 2: Proposed PC transactive solution process.

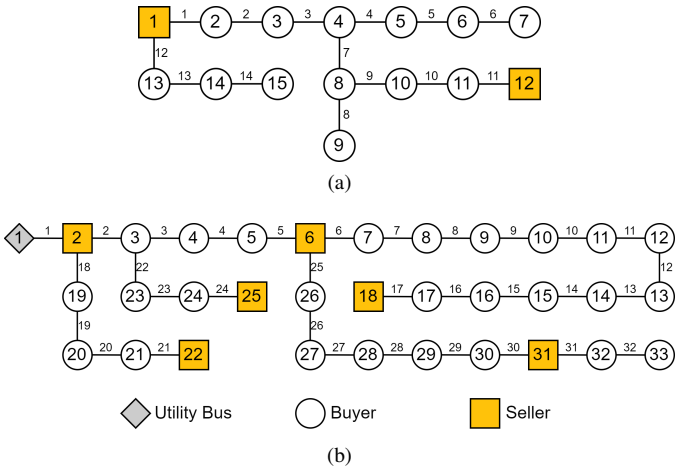


Fig. 3: Distribution test networks with peers: (a) 15-bus P2P grid from [32], and (b) 33-bus grid from [21] and [33].

(14), and allowing for trade price decrements in the scenario that a seller selects a trade but a buyer does not. To ensure coordination between the prices paid and received by buyers and sellers for mutually agreed stable trades, the variable ρ_w^m is introduced to track the last public mutually agreed trade price. This price is used for market clearing rather than the private buyer and seller prices ρ_w^s and ρ_w^b, which may differ locally from the public mutual price. These modifications reduce the computational needs of negotiating from a trade price of zero after any transaction fee change and also enable a more complete negotiation process where both buyers and sellers are able to directly influence the transaction prices. The complete decentralised PC transactive energy market structure accounting for all sub-processes is shown in Fig. 2, and the corresponding full implementation is provided in [31].

Algorithm 3 Transaction Matching Process

$\Lambda_w := \{\rho_w^b, \rho_w^s\},\ \forall w \in \Omega$
do
  $\Lambda_w^{old} \leftarrow \Lambda_w,\ \forall w \in \Omega$
  for $n \in \mathcal{N}$ **do**
    $\Omega_n^* \leftarrow \Omega_n^*$ (14)
  for $w \in \Omega$ **do**
    if $p_w^b = \overline{p}^w$ and $p_w^s \neq \overline{p}^w$ **then**
      if $\rho_w^b > \rho_w^s$ **then**
        $\rho_w^s \leftarrow \rho_w^s + \Delta\rho$
        $\rho_w^m \leftarrow \rho_w^b$
      else
        $\rho_w^b \leftarrow \rho_w^b + \Delta\rho$
    if $p_w^s = \overline{p}^w$ and $p_w^b \neq \overline{p}^w$ **then**
      if $\rho_w^s < \rho_w^b$ **then**
        $\rho_w^b \leftarrow \rho_w^b - \Delta\rho$
        $\rho_w^m \leftarrow \rho_w^s$
      else
        $\rho_w^s \leftarrow \rho_w^s - \Delta\rho$
while $\Lambda_w^{old} \neq \Lambda_w,\ \forall w \in \Omega$
return $\Omega^* := \bigcup_{w\in\Omega}(w \mid p_w^b = p_w^s = \overline{p}^w)$

IV. Results

In this section, the performance of the proposed PC method is compared against the SC approach and the PC approach in [8], as well as the proposed SC method. Results are obtained for the 15-bus network from [32] and a modified 33-bus network from [21] and [33], and are illustrated in Fig. 3. Solution performance is compared using the 141-bus network from [34]. For all proposed PC results, $E^Z = 1$, $E^\epsilon = 1$, and $S_l^{res} = 0.001$ unless stated otherwise.

A. 15-Bus Peer-to-Peer Trading Results

To compare the performance of the proposed PC approach with the existing methods in [8], the network in Fig. 3a is used, where the system parameters are described in [8]. P2U transactions in the proposed implementation are restricted to ensure comparable results with the existing P2P approaches, and Γ_n is set to 1 for all peers in the existing models to demonstrate the extreme scenario of full P2P. Note that even in full P2P, the utility maintains responsibility for ensuring the system is balanced and adequate according to (2).

The SC and PC set of selected trades are shown in Fig. 4. The existing SC set of optimal trades for the 15-bus network utilises 100% of the capacity of Feeder 11, since the cost of supply from Peer 12 is lower than Peer 1, as shown in Fig. 4a, and resulting in high DLMP based NUCs for trades with Peer 12. As discussed in Section II-B1, the existing PC implementation's consideration of the physical layer uses non-proportional virtual penalties, which results in an under utilisation of Peer 12 with only 228 kW of a possible 255 kW of supply and substantially reduced NUCs for selected trades compared to the SC results. The addition of the dynamic transaction fee and the congestion clearing process in the proposed PC method ensures maximal utilisation of priority feeders, and therefore Peer 12 optimises output. In addition to increasing feeder utilisation, the proposed transaction fee incentivises trading with least electrical distance peers as seen in Fig. 4c. Thus, the capacity of Peer 12 is traded with the least distance peers, rather than trading across the entire network as seen in Fig. 4b. The proposed PC approach also results in individual transaction fees and NUCs which closely approximate those from the SC approach, whereas the DLMP

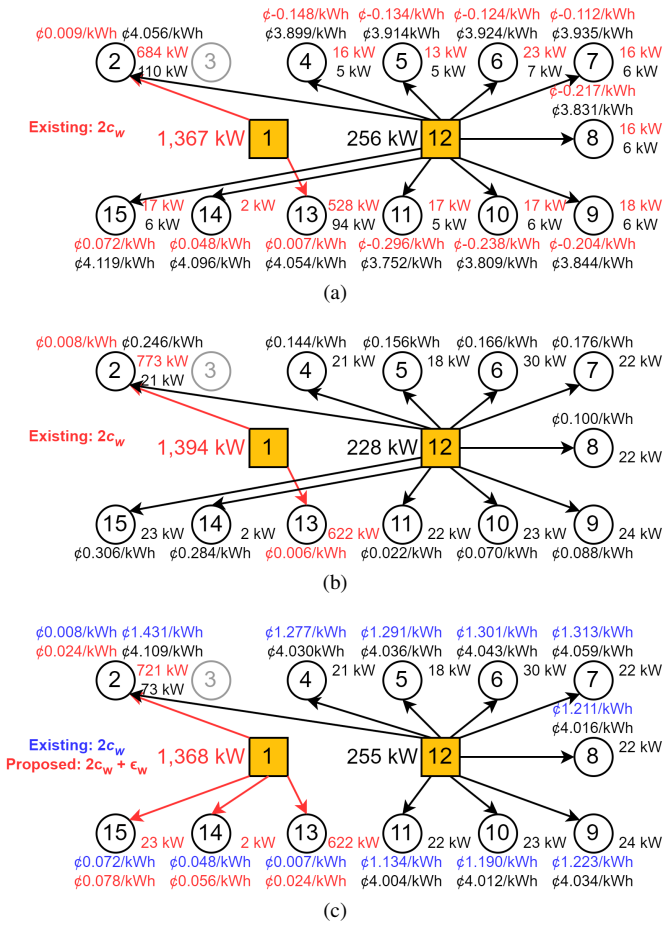


Fig. 4: P2P transactions for the 15-bus system: (a) existing SC, (b) existing PC, and (c) proposed PC.

TABLE I: 15-bus network economic performance in [$/h].

Model	Buyer costs	Seller revenues	Transaction fees	Cost of generation
SC [8]	80.94	70.8	– ‖ 10.13	71.25
PC [8]	79.46	79.01	– ‖ 0.45	72.26
New PC	83.01	72.33	2.68 ‖ 10.67	71.27

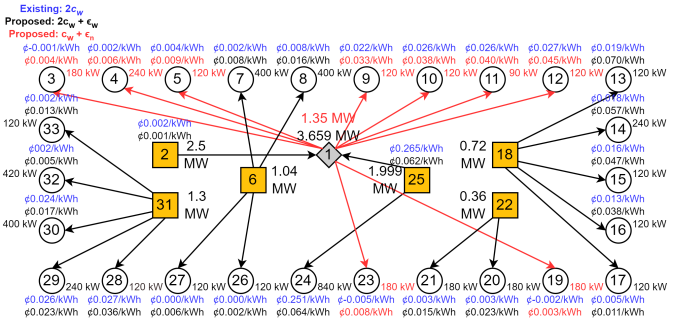


Fig. 5: Transactions for the 33-bus system using the proposed PC method.

derived NUCs in Fig. 4c are lower. This difference is due to the requirement for a fixed trade increment which reduces the DLMP discrepancy for constrained peers, and since, while the SC approach considers the peer decisions as variables, the DLMPs derived from (2) for the PC approach take the peer demand and production as fixed values and are therefore not directly equivalent.

Table I compares the economic performance of the models. The cost of buyers is calculated as $\sum_{w\in\Omega^*}(\rho^m_w + c_w + \epsilon_w)p^b_w$, the revenue of sellers as $\sum_{w\in\Omega^*}(\rho^m_w - c_w)p^s_w$, and the transaction fees as $\sum_{w\in\Omega^*}(2c_w) \parallel \sum_{w\in\Omega^*}(2c_w + \epsilon_w)$, where the left value is using the existing NUC approach and the right value is the proposed approach. Since marginal utility above minimum demand is not considered in this network, the welfare is captured in the cost of generation, which includes both peer owned generators and the cost of balancing by the utility. Sub-optimal usage of Peer 12 results in the highest cost of generation for the existing PC solution, whereas the proposed PC solution is almost identical in cost of generation to the SC solution in [8], with total reductions in welfare of 1.42% and 0.03% respectively compared to the SC maximum welfare. Transaction fees in the proposed PC solution are closer to the DLMP based fees in the existing SC approach, indicating a more accurate approximation of the network usage costs for P2P transactions than in the existing PC approach. The discrepancy in seller revenue between the proposed and existing PC approaches is due to the difference in transaction fees, where the penalty mechanism in the existing PC approach distorts the competitive market determination of trade prices and does not accurately account for the cost of network usage. The value of E^Z needs to be adjusted to ensure sufficient recovery of dynamic operational costs for a balanced budget, where the value of $E^Z = 1$ results in total NUCs of 0.47 $/h, which exceeds the utility compensation cost of 0.32 $/h and is therefore in a budget surplus.

B. 33-Bus Transactive Results

The proposed transactive models extend the standard P2P problem by including the option of P2U transactions. Performance of the proposed SC and PC models are assessed using a modified version of the 33-bus network from [21], which includes 6 selling peers and 26 buying peers participating in the transactive market. The generator cost functions from [21] are adjusted to increase competition [31].

The resulting transactions and transaction fees of the proposed PC solution are shown in Fig. 5. The optimal PC transactions display significant diversity, with all selling peers involved in either P2P or P2U transactions. Peers 25 and 31 are limited in their ability to transact due to congestion on Feeders 24 and 30. Peers prioritise least electrical distance transactions first, confirming the effectiveness of the proposed transaction fee. For example, Peer 6 prefers selling to Peers 7 and 8 over Peers 9 through 12, since the closer peers are able to offer higher trading energy prices with lower transaction fees. Peers 9 through 12 must satisfy their demand through P2U transactions as they represent the least cost remaining option.

Table II shows the economic results of the proposed solutions. Considering both P2P and P2U transactions, for the PC approach, the buyer costs are $\sum_{n\in\mathcal{N}^b}(T^d_n +$

TABLE II: 33-bus network economic performance in [$/h].

Model	Buyer costs	Seller revenues	Transaction fees	Cost of generation	System cost
New SC	42.73	63.98	– ‖ 1.87	62.94	40.41
New PC	43.49	58.94	5.70 ‖ 2.22	57.64	40.56

$\epsilon_n^{u,d})d_n^u + \sum_{w\in\Omega^*}(\rho_w^m + c_w + \epsilon_w)p_w^b$, the seller revenues are $\sum_{w\in\Omega^*}(\rho_w^m - c_w)p_w^s + \sum_{n\in\mathcal{N}^s}(T_n^g - \epsilon_n^{u,g})p_n^u$, and the transaction fees are $\sum_{w\in\Omega^*}(2c_w) \parallel \sum_{w\in\Omega^*}(2c_w + \epsilon_w)p_w^b + \sum_{n\in\mathcal{N}^b}(\epsilon_n^{u,d} + Z_{n1}^{th}E^Z)d_n^u + \sum_{n\in\mathcal{N}^s}(\epsilon_n^{u,g} + Z_{n1}^{th}E^Z)p_n^u$, where the left value is using the existing NUC approach and the right value is the proposed approach. Cost of generation only includes the cost of generation for peers, whereas the system cost includes net injection at the utility bus at wholesale price C^w and is thus the net cost of the system as a whole. Note that in the proposed SC optimal solution, peer costs and revenues are determined directly from the optimisation solution considering net injection at the nodal DLMPs, and transaction fees are calculated using the method described in [8].

Both approaches result in similar buyer costs, transaction fees, and system costs, with a slight increase compared to the optimal SC solution due to the approximations utilised by the transaction fee mechanism in influencing peer decisions, representing a reduction in welfare of 0.37%. The proposed transaction fees differ substantially from the DLMP based NUCs for the resulting transactions, and more closely approximate the SC values since the PC DLMPs are not directly comparable with the SC DLMPs as the peer decisions become fixed inputs to (2) for the PC approach rather than central decision variables. While seller revenues are higher in the proposed SC approach due to a direct consideration of system export value through DLMPs, the net profit of sellers after accounting for the peer cost of generation is actually higher in the proposed PC approach, with net seller profits of 1.30 and 1.04 for the PC and SC approaches, respectively. Comparable net system costs demonstrate that the proposed PC solution is still economically efficient while maintaining transactive and decentralised principles. The utility is in a budget surplus from the resulting transactions, as the total revenues are 10.33 $/h for P2U energy sold, 0.50 $/h for NUCs associated with P2U transactions, 0.84 $/h for NUCs associated with P2P transactions, and 17.08 $/h from net wholesale transactions, whereas the total costs are 27.99 $/h for P2U energy purchases. This results in a net budget surplus of 0.76 $/h.

C. 33-Bus Transactive Constrained Results

Simulation results for a constrained 33-bus network are included to further demonstrate the performance of the proposed method in addressing stressed network conditions. To accomplish this, the general network topology remains the same as in Fig. 3b; however, additional modifications are made to feeder power flow limits and cost curves of the selling peers to create a more constrained example. Most notably, apparent power flow limits for Feeders 1 through 5 are reduced to 4 MVA such that the network is export limited; the apparent power flow limit for Feeder 6 is reduced to 1 MVA to simulate an

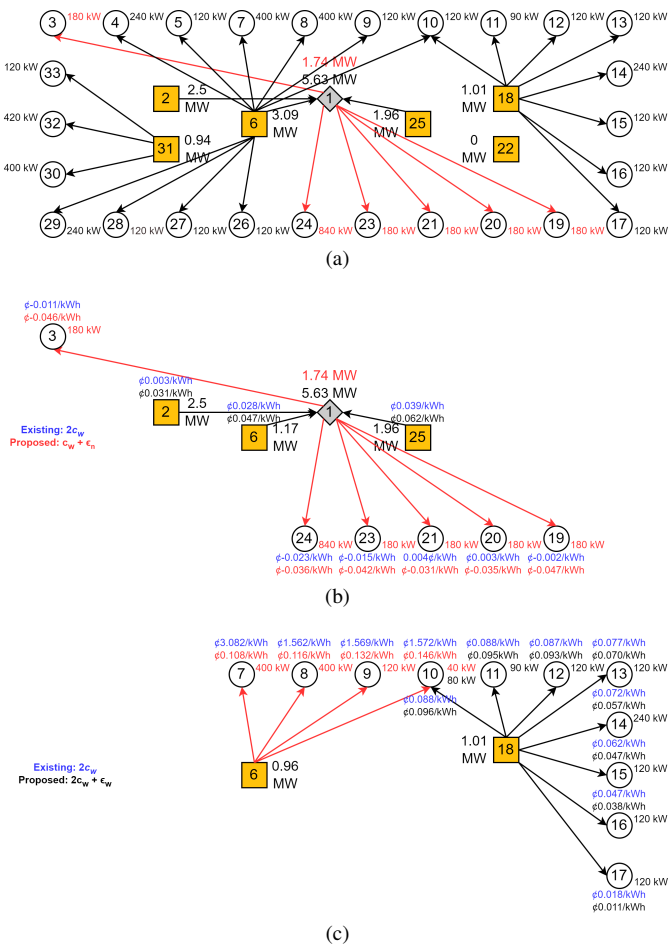


Fig. 6: Results for the 33-bus constrained system using the proposed PC method: (a) network transactions, (b) Feeder 1 transactions, and (c) Feeder 6 transactions.

operational line limit reduction; and the cost curves of Peers 6 and 18 are swapped such that Peer 6 is now one of the cheapest sellers and Peer 18 is the most expensive. For the simulation results, S_l^{res} is set to 40 kW to ensure the utility can compensate as needed using congested lines, which was chosen based on the observed network capacity deficiencies when relaxing line limits to address infeasible solutions.

The resulting PC transactions are shown in Fig. 6a, where the implemented changes result in increased production for Peers 6 and 18, reduced production for Peers 22 and 31, and relatively unchanged production for Peers 2 and 25. The reduced production for Peer 31 is a result of the improved economics of Peer 6, with Peers 28 and 29 now buying from Peer 6 rather than Peer 31, and the congestion on Feeder 30 is no longer present.

The system becomes export limited with congestion clearing completed for Feeder 1 and resulting in all P2U transactions being fixed and shown in Fig. 6b. Prior to congestion clearing, the respective optimal P2U selling for Peers 2, 6, and 26 is 2.5, 1.17, and 2.9 MW. With the increment of transactions fees for transactions impacting Feeder 1, the constraint is relieved and congestion clearing process is initiated. With the specified S_l^{res} of 40 kW, the congestion clearing process seeks to clear 3.96

MW of net export, with inclusion of 1.74 MW of imports for P2U buyers, and up to 5.7 MW of P2U selling. The congestion clearing seeks this objective successfully while ensuring no constrained lines are violated. Peers 2 and 6 have their entire optimal P2U selling cleared, and Peer 25 is restricted below the theoretical remaining capacity of Feeder 1, since clearing this value of 2.03 MW would violate the clearing limit of Feeder 24, thereby restricting Peer 25 P2U selling to 1.96 MW and achieving a net transaction export of 3.89 MW. The resulting physical export is 3.77 MW, which is less than the virtual transaction export when system compensation for losses is accounted for, and less than the SC optimal export of 3.99 MW. It is also noteworthy to observe that in the constrained network generation of Peer 22 is reduced to 0 MW and those peers originally buying from Peer 22 now choose P2U transactions. This occurs as the P2U transaction fees for buying become negative with export congestion, resulting in favourable economics for P2U rather than the closer P2P option.

The operational limit reduction of Feeder 6 is also a major constraint on the network, resulting in forced utilisation of the more expensive Peer 18 over Peer 6. The directional capacity of 0.96 MW for Feeder 6 is fully utilized in the supply of Peers 7 through 10 by Peer 6, requiring the remainder of the Peers downstream from the congestion to be supplied by Peer 18. Although the virtual flow on Feeder 6 is only 0.96 MW, the real flow resulting from the final transaction is 0.995 MW, demonstrating the importance of the reserved power flow capacity such that the utility can feasibly balance the network.

D. Scalability of the Proposed Solution

To assess the scalability of the proposed PC solution, the performance of the 141-bus network from [34] with generator parameters described in [8] is compared with the 15-bus and 33-bus networks. The results shown in Table III were obtained on a server with Intel® Xeon® Silver 4214 CPU implemented in Python 3.9 and using the Gurobi 10.0.1 optimisation solver. The time is calculated assuming that the peers or agents solve their sets of trades in parallel. The time for each iteration consists of the longest individual peer or agent decision time, and the process time associated with utility responsibilities such as solving the OPF, transaction fee assignment, and congestion clearing. The number of iterations is largely dependent on the discrepancy between initial transaction prices and the final clearing price rather than the system size. Convergence performance is also generally influenced by market design parameters including $\Delta\rho$, $\overline{p}^w$, E^Z, and E^ϵ which balance overall market efficiency and solution performance, along with how constrained the system is in facilitating locally optimal transactions. Considering the time per iteration, it is shown that the normalised performance is approximately linear. For real-time applications, division of the network into localised sub-networks with an upper coordination layer may be necessary.

TABLE III: Comparison of PC solution performance.

Network	Price iterations	Outer iterations	Time [s]	Time per iteration [s]
15-bus	406	43	153.23	0.38
33-bus	111	8	303.09	2.73
141-bus	227	13	4244.13	18.70

V. Conclusion

This paper extended existing peer-to-peer market formulations to develop a general transactive market design for both system-centric and peer-centric objectives. In addition, a decentralised peer-centric energy market was proposed consisting of a peer negotiation process, and new transaction fees and a congestion clearing mechanism to ensure a stable, feasible solution. The proposed transactive fee mechanism introduced a consistent and comprehensive price signal considering the fixed and dynamic network usage cost for individual transactions, ensuring coupling of the physical and virtual layers. The congestion clearing process ensured that network resources were allocated in an efficient manner for their optimal usage. The simulated results confirmed the effectiveness of the approach, yielding similar overall system costs as optimal SC solutions. Solution time per iteration was found to be approximately linear to network size, identifying the need for localised sub-networks in real-time applications.

Potential areas for future work include the addition of marginal utility for peers; expanding the solution horizon to multiple time periods, in which case consideration of peer behavioural uncertainty is necessary; introducing intertemporal considerations for resources such as battery storage or electric vehicles; and developing additional products rewarding beneficial peer behaviour such as reactive power provision.

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