

What can detectors detect?

by

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A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Physics (Quantum Information)

Waterloo, Ontario, Canada, 2021

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Author's Declaration

This thesis consists of material all of which I authored or co-authored: see Statement of Contributions included in the thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Statement of contributions

This thesis is based on the following published and forthcoming articles.

- Chapter 3 is based on
 - **Laura J. Henderson** and Nicolas C. Menicucci. Bandlimited entanglement harvesting. *Phys. Rev. D*, 102:125026, Dec 2020
- Chapter 4 is based on
 - **Laura J. Henderson**, Alessio Belenchia, Esteban Castro-Ruiz, Costantino Budroni, Magdalena Zych, Časlav Brukner, and Robert B. Mann. Quantum temporal super-position: The case of quantum field theory. *Phys. Rev. Lett.*, 125:131602, Sep 2020.
- Chapter 5 is based on
 - **Laura J. Henderson**, Robie A. Hennigar, Robert B. Mann, Alexander R. H. Smith, and Jialin Zhang. Entangling detectors in anti-de sitter space. *Journal of High Energy Physics*, 2019(5):178, May 2019.
- Chapter 6 is based on
 - **Laura J Henderson**, Robie A Hennigar, Robert B Mann, Alexander R H Smith, and Jialin Zhang. Harvesting entanglement from the black hole vacuum. *Classical and Quantum Gravity*, 35(21):21LT02, Oct 2018.
- Chapter 7 is based on
 - Nadine Stritzelberger, **Laura J. Henderson**, Valentina Baccetti, Nicolas C. Menicucci, and Achim Kempf. Entanglement harvesting with coherently delocalized matter. *Phys. Rev. D*, 103:016007, Jan 2021.

Abstract

This thesis is focused on using a pair of Unruh-DeWitt detectors, which interact locally with a quantum field, to gain information about properties of the field and the underlying spacetime.

First, we consider a massless scalar field with a bandlimit, a hard ultraviolet cutoff. We show that when UDW detectors interact with a bandlimited field, they are able to become entangled over larger distances than would be expected in the case of no cutoff. The bandlimit introduces some non locality into the interaction, which boosts the nonlocal correlations between the detectors. This boost in entanglement could, in principle, be used to put a lower bound .

Then we study the affect of a quantum controlled switch to the entanglement harvesting protocol. This switch will control when the detectors interact with the quantum scalar field and can allow for them to each couple to the field in a superposition of two times. We show that when a detector interacts with a field at a superposition of times, its transition probability is reduced compared to what it interacted at a mixture of times. This in turn increases the entanglement harvested by a pair of detectors, which can even violate no-go theorems.

Next, we turn our attention to $(2+1)$ -dimensional AdS spacetime. We map out a large amount of the parameter space, tuning both detector and spacetime properties. Most interestingly, we find an “island of seprability” in the parameter space, which occurs at a relatively small Ads length. When the detectors’ properties fall in the “island of seprability”, they cannot become entangled through local interactions with the field; however for a good range of parameters around the island, they can.

We then apply the entanglement harvesting protocol to a black hole spacetime, specifically the BTZ black hole. We find an entanglement “death zone” which marks the point where entanglement harvesting is no longer possible as two detectors with fixed proper separation approach the horizon.

Finally, we make a change to the detector model itself. We consider the the center of mass degree of freedom of the detectors to be quantum, and so will it coherently delocalize over the course of the interaction with the quantum field. We find that this delocalization inhibits entanglement harvesting, and it is only in the limit of very heavy and very sharply localized detectors that we recover the case of a pair of UDW detectors classical center center of mass.

Acknowledgements

During my time at Waterloo, I have had the chance to meet so many so many amazing people, who have made my time here an enriching and enjoyable experience.

I first want to thank my supervisor Robert Mann, who has been incredibly supportive over the last four years. You always make time for your students and cook up many interesting projects and problems to solve, so I am always learning something new. I also want to thank you for facilitating the wonderful collaborations I have been a part of.

Next, I want to thank Nicolas Menicucci, who hosted me at RIMT in Australia and provided me with mentorship and a new perspective on my research. Working with you has taught me to keep pushing to find answers and to be flexible if one technique is not working.

I also want to thank my collaborators, colleagues and friends both at Waterloo and RMIT. You have all been a source of many interesting, funny, thought provoking and educational conversations, especially, Daniel Grimmer, Paulina Corona Ugalde, Robie Hennigar, Valentina Baccetti, Matthew Robbins, Jack Davis, Giacomo Pantaleoni, Fiona McCarthy, and Erickson Tjoa.

Thank you to my parents, Paul and Kathy Henderson, who have supported me through the stressful times and celebrated my successes.

Finally, I want to thank my boyfriend, Micheal Pennington, who has been my rock through this entire journey. I couldn't have done it without you by my side.

Dedication

Dedicated to my grandmother, Joyce Tomsett.

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Chapter 1

Introduction

Modern physics is made up of two main branches, general relativity and quantum field theory. On their own, each theory works incredibly well in its own regime, and have made predictions that have been verified to incredible degrees of accuracy. But despite the efforts of the best and the brightest over the past several decades, the two branches have not been made fully compatible with each other.

Not all is lost, and there have been numerous advances on various approximations. One of these advances is the theory of quantum fields in curved spacetime, that treats both the quantum field and gravitational effects of equal physical significance. However, the gravitational field is treated classically and the fields do not back-react on the spacetime [10, 95]. Despite that, this area of research has provided useful insights into black hole physics and the early Universe. These insights provide clues as to what can be expected from a bona fide theory of quantum gravity.

Detector models, such as the Unruh-DeWitt detector [89, 20], provide an operational approach to probing various properties of quantum fields including those that live on curved spacetimes. For example, it has been known for the past 40 years that uniformly accelerating particle detectors in flat space, that are coupled to a scalar field in the Minkowski vacuum state, detect a thermal bath of particles [89]. These models can provide notions of locality, since they are usually taken to be pointlike objects which following a classical spacetime trajectory and their first-quantized internal degrees of freedom interact with the quantum field at the position of the detector. After the interaction, the detector, or a pair of such detectors, can be measured. The state of the detector(s) will depend on properties of the quantum field, which in turn will depend on properties of the underlying spacetime. This allows us to not only probe properties of the field, such as local fluctuations

or correlations between two spacetime regions, but can even reveal properties of spacetime structure.

Entanglement is almost synonymous with quantum, making it an especially interesting property of a quantum field to study. It has a wide range of applications across various areas of physics, such as in the description of non-classical states of light within the field of quantum optics [47, 96], in the study of critical phenomena in condensed matter systems [57, 93, 3], and most interestingly, offering an explanation for the origin of black hole entropy [11, 12, 80].

Field entanglement was first discovered within the context of algebraic field theory. It was shown that the vacuum state of a free quantum field is entangled as seen by local observers in Minkowski space, even if the observers are in spacelike-separated regions [84, 83, 85]. This suggests that a pair of observers can violate a Bell-like inequality just by observing the local fluctuations in a quantum vacuum. In other words, the vacuum is a resource for entanglement.

It was later shown that this property of the electromagnetic vacuum can, in principle, be used to generate entanglement between a pair of atoms [91]. More specifically, if the two atoms were prepared in a separable state and each localized to a region that is spacelike separated from the other, and then interact with the field while remaining spacelike separated through the duration of the interaction, they will become entangled. Since the two atoms never had a chance to directly interact with each other, any resulting entanglement must have come from the quantum vacuum state. This was studied for pointlike localized two-level detectors (or qubits) [89, 20] that interact with a scalar field [65, 66] a number of years later.

Since then, extensive research has been carried out on the process of localized detectors extracting entanglement from the vacuum state of a quantum field [92, 54, 55, 35, 51, 48, 50, 61, 62, 43]. This process has been come to known as the entanglement harvesting protocol [74]. We note that in cases where the detectors are timelike or lightlike separated, the extracted entanglement is the result of both field mediated interactions and being swapped from the vacuum. However, regardless of the exact origin of the entanglement, its properties will still depend on properties of the field.

One interesting application of the entanglement harvesting protocol is to study how vacuum entanglement is effect by the underling spacetime structure, such as the expansion rate of the Universe [92]. Recently, there has been a huge amount of work showing that entanglement harvesting is sensitive to the number of spacetime dimensions [61], the curvature [92, 41, 53, 29] and topology [51] of the spacetime, and boundary conditions [30, 19].

There has also been very recent studies done on entanglement harvesting in $(1+1)$ dimensional Schwarzschild/Vaidya [87] and rotating BTZ [67] spacetimes.

The goal of this thesis is to use a pair of detectors initialized in a separable state which interact locally with a quantum scalar field, to gain information about various properties of the field and/or the underlying spacetime. In particular we are interested in the resulting entanglement of the two detectors, and under what circumstances they are able to become, or cannot become entangled. A secondary question focuses on the detectors themselves, and asks how can they be tuned to maximize the detection of a field of spacetime property, such as an ultraviolet cutoff or the spacetime curvature.

In chapter 2, we present a brief review of quantum field theory, the UDW model and measures of entanglement, with the goal to provide the background and context of the results presented in the rest of the thesis. The next four chapters apply the entangling harvesting protocol to various scenarios in order to explore what can be known about the field or background spacetime based on the resulting entanglement between the detectors, as well as their probability of becoming excited. In chapter 3, the quantum scalar field is bandlimited, so that modes with momentum higher than some cutoff are not allowed. In chapter 4, we condition the when the detectors interact with the field on the output of a quantum control qubit. Depending on the measurement of the control qubit, the detectors can be put in a superposition of interacting with the field at different times. This has a significant affect on entanglement harvesting. In chapter 5, we consider a scalar field in AdS_3 . Not only does this introduce curvature, but the detectors no longer have the same proper time. We analyze a wide parameter space which includes properties of both the spacetime and detectors. In chapter 6, we present results from the first application of the entanglement harvesting protocol in a black hole spacetime.

The final results are presented in chapter 7 where we turn our attention to the UDW detector itself, and upgrade its center of mass degree of freedom to a quantum variable. This effectively means that there is some uncertainty in the location of the particle. Not only that, but the wavefunction that describes the position of the detector will evolve in time, meaning that the detector will coherently delocalize over the course of the interaction with the field. We present the effects of delocalization on the entanglement harvesting protocol and show the specific limits that recover the “classical” UDW detector case.

Finally, chapter 8 presents the overall conclusions and potential directions for future work.

Chapter 2

Fields, Detectors and Entanglement

This chapter will briefly review the background theory and formalism that provide the building blocks of the entanglement harvesting protocol. First, we will discuss the basics of quantum field theory [39]. In the next section, we will discuss the Unruh-DeWitt detector model in both the perturbative and non-perturbative regimes. In the final section, we will review three ways of quantifying the entanglement between a pair of qubits: the negativity, the entanglement of formation and the concurrence.

2.1 Quantum field theory

Starting from the Klein-Gordon equation for a massless scalar field in natural units ($c = \hbar = 1$)

$$\left(\frac{\partial^2}{\partial t^2} - \Delta\right)\phi = 0, \quad (2.1)$$

we then promote $\phi(\mathbf{x}, t)$ and its canonically conjugate momentum $\pi(\mathbf{x}, t) = \dot{\phi}(\mathbf{x}, t)$ to operators, which obey the equal time quantization conditions

$$\begin{aligned} \left[\hat{\phi}(\mathbf{x}, t), \hat{\pi}(\mathbf{x}', t)\right] &= i\delta^{(n)}(\mathbf{x} - \mathbf{x}') \\ \left[\hat{\phi}(\mathbf{x}, t), \hat{\phi}(\mathbf{x}', t)\right] &= 0 \\ \left[\hat{\pi}(\mathbf{x}, t), \hat{\pi}(\mathbf{x}', t)\right] &= 0. \end{aligned} \quad (2.2)$$

The Hamiltonian for the second quantization is

$$\hat{H} = \int d^n \mathbf{x} \frac{1}{2} \hat{\pi}^2(\mathbf{x}, t) + \frac{1}{2} \hat{\phi}(\mathbf{x}, t) \Delta \hat{\phi}(\mathbf{x}, t). \quad (2.3)$$

We can expand the field operator in the particle basis in terms of mode functions $u_{\mathbf{k}}(\mathbf{x}, t)$

$$\hat{\phi}(\mathbf{x}, t) = \frac{1}{(2\pi)^{n/2}} \int d^n \mathbf{k} \left(u_{\mathbf{k}}(\mathbf{x}, t) \hat{a}_{\mathbf{k}} + u_{\mathbf{k}}^*(\mathbf{x}, t) \hat{a}_{\mathbf{k}}^\dagger \right) \quad (2.4)$$

where

$$u_{\mathbf{k}}(\mathbf{x}, t) = \frac{1}{\sqrt{2\omega_{\mathbf{k}}}} e^{-i(\omega_{\mathbf{k}} t - \mathbf{k} \cdot \mathbf{x})} \quad (2.5)$$

and $\omega_{\mathbf{k}} = |\mathbf{k}|$ is the dispersion relation. The mode function $u_{\mathbf{k}}(\mathbf{x}, t)$ is a positive energy solution of equation (2.1) with momentum $\mathbf{k}$. The time independent $\hat{a}_{\mathbf{k}}^\dagger$ and $\hat{a}_{\mathbf{k}}$ are the creation and annihilation operators respectively, and they inherent the canonical commutation relations (2.2)

$$\begin{aligned} [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] &= \delta^{(n)}(\mathbf{k} - \mathbf{k}') \\ [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] &= [\hat{a}_{\mathbf{k}}^\dagger, \hat{a}_{\mathbf{k}'}^\dagger] = 0. \end{aligned} \quad (2.6)$$

We can use the creation and annihilation operators to construct a Fock basis for the Hilbert space, starting with defining the vacuum state, $|0\rangle$, as the state that is annihilated by all operators $\hat{a}_{\mathbf{k}}$

$$\hat{a}_{\mathbf{k}} |0\rangle = 0. \quad (2.7)$$

From here, we can construct any state with successive applications of the creation operators to the vacuum. For example

$$|n_1, n_2, \dots\rangle = \prod_{\alpha} \frac{(\hat{a}_{\mathbf{k}_{\alpha}})^{n_{\alpha}}}{\sqrt{n_{\alpha}!}} |0\rangle \quad (2.8)$$

is a state with n_1 particles of momentum $\mathbf{k}_1$, n_2 particles of momentum $\mathbf{k}_2$, and so on.

Substituting (2.4) into (2.3) gives

$$\begin{aligned}\hat{H} &= \frac{1}{(2\pi)^{n/2}} \int \frac{d^n \mathbf{k}}{2} \omega_{\mathbf{k}} \left(\hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger + \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} \right) \\ &= \frac{1}{(2\pi)^{n/2}} \int d^n \mathbf{k} \omega_{\mathbf{k}} \left(\hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger + \frac{1}{2} (2\pi)^{n/2} \delta^{(n)}(0) \right)\end{aligned}\quad (2.9)$$

which is divergent due to the $\delta(0)$ term. This is analogous to the zero point energy of a quantum harmonic oscillator $\omega/2$; however, we model the quantum field as having an oscillator at every point in space, so it diverges with the volume of the system. Since we this energy is inaccessible, and we only ever care about energy differences, we can safely remove it. We have

$$\hat{H} = \int \frac{d^n \mathbf{k}}{(2\pi)^{n/2}} \omega_{\mathbf{k}} \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}. \quad (2.10)$$

Since $\hat{N}_{\mathbf{k}} := \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}$ is the number operator for mode $\mathbf{k}$, this set of modes diagonalizes the Hamiltonian.

In curved spacetime, the Klein-Gordon equation is

$$\left(\frac{1}{\sqrt{|g|}} \frac{\partial}{\partial x^\mu} g^{\mu\nu} \sqrt{|g|} \frac{\partial}{\partial x^\nu} \right) \hat{\phi}(x, t) = 0. \quad (2.11)$$

Again, we choose the ansatz

$$\hat{\phi}(x, t) = \sum_k u_k(x, t) \hat{a}_k + u_k^*(x, t) \hat{a}_k^\dagger \quad (2.12)$$

where the $\hat{a}_k$ and $\hat{a}_k^\dagger$ are constant operators, and the functions $u_k(x, t)$ are assumed to be a solution to (2.11).

Substituting (2.12) into the canonical commutation relations, (2.6) imposes the condition

$$\sqrt{|g|} g^{0\nu}(x') \sum_k \left(u_k(x, t) \frac{\partial}{\partial x'^\nu} u_k^*(x', t) - u_k^*(x, t) \frac{\partial}{\partial x'^\nu} u_k(x', t) \right) = i \delta^{(n)}(x - x') \quad (2.13)$$

on the $\{u_k(x, t)\}$. If the spacetime is globally hyperbolic, that is it possesses a foliation by Cauchy surfaces, then a set of mode function, $\{u_k(x, t)\}$, that solves (2.11) and obey (2.13)

exists. However, they are not unique. Therefore, there are multiple mode expansions for $\hat{\phi}(x, t)$, each with their own associated raising and lowering operators, and each with their own vacuum. In general, we will need to choose which vacuum to build on.

2.2 Unruh-DeWitt detectors

The Unruh-DeWitt (UDW) detector [89, 83, 20] serves as a local probe of the field. Not only do they provide a concrete notion of locality, but detectors also provide an operational definition of particle: “a particle is what a particle detector detects” [88]. UDW detectors are two level quantum systems, qubits, with ground and excited states given by $|g\rangle_D$ and $|e\rangle_D$, respectively, and separated by an energy gap of Ω_D . A pair of detectors couple locally to a quantum scalar field $\hat{\phi}(\mathbf{x}, t)$, with $D \in \{A, B\}$. The interaction of each detector is described, in the interaction picture, by the Hamiltonian

$$\hat{H}_D(\tau_D) = \lambda_D \chi_D(\tau_D) (e^{i\Omega_D \tau_D} \hat{\sigma}_D^+ + e^{-i\Omega_D \tau_D} \hat{\sigma}_D^-) \otimes \hat{\phi}(\mathbf{x}_D(t), t) \quad (2.14)$$

where λ is the coupling strength of the interaction, $\chi_D(\tau_D)$ is the switching function, which controls the interaction time, $\hat{\sigma}_D^+ := |1\rangle_D \langle 0|$ and $\hat{\sigma}_D^- := |0\rangle_D \langle 1|$ are the SU(2) ladder operators acting on the Hilbert space of detector D , and $\mathbf{x}_D(\tau_D)$ is the detector’s spacetime trajectory parameterized by its proper time, τ_D . This model captures most of the features of the light-matter interaction when no angular momentum is exchanged [49, 2, 62].

The time evolution of the detector-field system with respect to the time t is generated by the unitary operator

$$\hat{U} := \mathcal{T} \exp \left[-i \int dt \left(\frac{d\tau_A}{dt} \hat{H}_A[\tau_A(t)] + \frac{d\tau_B}{dt} \hat{H}_B[\tau_B(t)] \right) \right] \quad (2.15)$$

where $\mathcal{T}$ is the time ordering operator.

Following the entanglement harvesting protocol, we consider the detectors to be initially ($t \rightarrow -\infty$) in the ground state and the field initially in the vacuum, so the joint state of the system is

$$|\Psi_0\rangle = |0\rangle_A \otimes |0\rangle_B \otimes |0\rangle_\phi \quad (2.16)$$

and after the interaction ($t \rightarrow \infty$), the system is in the state

$$|\Psi_f\rangle = \hat{U} |\Psi_0\rangle = \sum_n \hat{U}^{(n)} |\Psi_0\rangle \quad (2.17)$$

where $\hat{U}^{(n)}$ is the n^{th} term in the Dyson expansion of the time evolution operator (2.15)

$$\hat{U}^{(n)} = (-i)^n \int_{t_n < \dots < t_1} dt_1 \dots dt_n \hat{H}(t_1) \dots \hat{H}(t_n) \quad (2.18)$$

and $\hat{H}(t) = \sum_{D \in \{A, B\}} \frac{d\tau_D}{dt} \hat{H}_D[\tau_D(t)]$.

We are only interested in the subsystem of the two detectors, so the Hilbert space of the field is traced out:

$$\hat{\rho}_{AB} := \text{Tr}_\phi [|\Psi_f\rangle \langle \Psi_f|] = \sum_{m, n} \text{Tr}_\phi [\hat{U}^{(m)} |\Psi_0\rangle \langle \Psi_0| (\hat{U}^{(n)})^\dagger]. \quad (2.19)$$

It can be seen from equations (2.14) and (2.18) that in the expression $\hat{U}^{(n)} |\Psi_0\rangle$, the field operator $\hat{\phi}(\mathbf{x}, t)$ will be applied to the vacuum n times, and so the only terms that survive the partial trace and contribute to the partial state are terms where m and n have the same parity. Additionally, in the final state, $\hat{U}^{(n)} |\Psi_0\rangle$, an even n results in both detectors being excited or both remaining in the ground state, and an odd n results in only one detector being excited. Since m and n must be both even or both odd, the reduced density matrix of the two detectors will be of the form

$$\hat{\rho}_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix} \quad (2.20)$$

in the basis $\{|0\rangle_A \otimes |0\rangle_B, |0\rangle_A \otimes |1\rangle_B, |1\rangle_A \otimes |0\rangle_B, |1\rangle_A \otimes |1\rangle_B\}$.

From here, there are generally two approaches to calculating the final state of the two detectors:

1. Use the Dyson series of the time evolution operator (2.18) to calculate $\hat{\rho}_{AB}$ to lowest order in the interaction strength.
2. Couple the detectors to the field using Dirac-delta switching and calculate the time evolution operator (2.15) exactly [77, 76]

2.2.1 Expansion of the density matrix

Substituting the interaction Hamiltonian given by equation (2.14) into the equation (2.19) and expanding to the lowest order in the interaction strength gives

$$\begin{aligned}
\hat{\rho}_{AB} &= \text{Tr}_\phi \left[|\Psi_0\rangle \langle \Psi_0| \right] + \text{Tr}_\phi \left[\mathbb{1} |\Psi_0\rangle \langle \Psi_0| \hat{U}^{(2)\dagger} \right] + \text{Tr}_\phi \left[\hat{U}^{(1)} |\Psi_0\rangle \langle \Psi_0| \hat{U}^{(1)\dagger} \right] \\
&\quad + \text{Tr}_\phi \left[\hat{U}^{(2)} |\Psi_0\rangle \langle \Psi_0| \mathbb{1} \right] + \mathcal{O}(\lambda^4) \\
&= |0\rangle_A \langle 0| \otimes |0\rangle_B \langle 0| + \hat{\rho}_{AB}^{(0,2)} + \hat{\rho}_{AB}^{(1,1)} + \hat{\rho}_{AB}^{(2,0)} + \mathcal{O}(\lambda^4)
\end{aligned} \tag{2.21}$$

where explicitly

$$\begin{aligned}
\hat{U}^{(1)} &= -i \int_{-\infty}^{\infty} dt \frac{d\tau_A}{dt} \hat{H}[\tau_A(t)] + \frac{d\tau_B}{dt} \hat{H}_B[\tau_B(t)] \\
\hat{U}^{(2)} &= - \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \left(\frac{d\tau_A}{dt} \hat{H}_A[\tau_A(t)] + \frac{d\tau_B}{dt} \hat{H}_B[\tau_B(t)] \right) \\
&\quad \times \left(\frac{d\tau_A}{dt'} \hat{H}_A[\tau_A(t')] + \frac{d\tau_B}{dt'} \hat{H}_B[\tau_B(t')] \right)
\end{aligned} \tag{2.22}$$

From equations (2.21) and (2.22), the reduced density matrix of the two detectors is

$$\hat{\rho}_{AB} = \begin{pmatrix} 1 - P_A - P_B & 0 & 0 & X^* \\ 0 & P_B & C^* & 0 \\ 0 & C & P_A & 0 \\ X & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4) \tag{2.23}$$

where

$$P_D := \lambda_D^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{d\tau_D}{dt} \frac{d\tau_D}{dt'} \chi_D[\tau_D(t)] \chi_D[\tau_D(t')] e^{i\Omega_D[\tau_D(t) - \tau_D(t')]} W(x_D(t), x_D(t')) \quad (2.24)$$

$$C := \lambda_A \lambda_B \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{d\tau_A}{dt} \frac{d\tau_B}{dt'} \chi_A[\tau_A(t)] \chi_B[\tau_B(t')] e^{i[\Omega_A \tau_A(t) - \Omega_B \tau_B(t')]} W(x_A(t), x_B(t')) \quad (2.25)$$

$$X := \lambda_A \lambda_B \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \left[\frac{d\tau_A}{dt} \frac{d\tau_B}{dt'} \chi_A[\tau_A(t)] \chi_B[\tau_B(t')] e^{i[\Omega_A \tau_A(t) + \Omega_B \tau_B(t')]} W(x_A(t), x_B(t')) \right. \\ \left. + \frac{d\tau_B}{dt} \frac{d\tau_A}{dt'} \chi_B[\tau_B(t)] \chi_A[\tau_A(t')] e^{i[\Omega_B \tau_B(t) + \Omega_A \tau_A(t')]} W(x_B(t), x_A(t')) \right] \quad (2.26)$$

for $D, D' \in \{A, B\}$.

$$W(x(t), x(t')) = \langle 0 \left| \hat{\phi}(\mathbf{x}(t), t) \hat{\phi}(\mathbf{x}(t'), t') \right| 0 \rangle \quad (2.27)$$

is the Wightman function, the two-point corrector of the field between spacetime points $x(t)$ and $x(t')$.

We note that the ρ_{44} term in the density matrix (2.23) is non-zero when expanded to higher orders in interaction strength. It is [79]

$$\rho_{44} = |X|^2 + |C|^2 + P_A P_B + \mathcal{O}(\lambda^6). \quad (2.28)$$

Tracing out either detector B or detector A gives the reduced density matrix for the remaining detector

$$\hat{\rho}_A = \text{Tr}_B [\hat{\rho}_{AB}] = \begin{pmatrix} 1 - P_A & 0 \\ 0 & P_A \end{pmatrix} \\ \hat{\rho}_B = \text{Tr}_A [\hat{\rho}_{AB}] = \begin{pmatrix} 1 - P_B & 0 \\ 0 & P_B \end{pmatrix} \quad (2.29)$$

respectively, so we can interpret P_D as the transition probability of detector D . To leading order, the term X is a coherence term and encodes the non-local correlations between the two detectors, and the term C encodes the total correlations [72].

In general, the coupling strength could be different for each detector; however, we will

take $\lambda_A = \lambda_B = \lambda$ in order to simplify the parameter space. This is a straightforward simplification since to lowest order in the coupling strength, measures of entanglement, such as negativity (2.42) and concurrence (2.53) are proportional to the product $\lambda_A \lambda_B$.

2.2.2 Dirac-delta switching

An alternative approach to using perturbation theory to compute the reduced density matrix of the two detectors is to implement δ -switching

$$\chi_D(t) = \eta_D \delta(t - T_D) \quad (2.30)$$

which can be related to the switching chosen used in the previous section by defining the Dirac delta as the weak limit of a normalised Gaussian,

$$\delta(x) = \lim_{a \rightarrow 0} \frac{e^{-x^2/(2a^2)}}{\sqrt{2\pi}a}. \quad (2.31)$$

Physically, we can interpret this as the time of interaction being much smaller than any other relevant scale in the problem.

For both implementations of a δ -switching function in this thesis, it is applied to a pair of stationary detectors located at $\mathbf{x} = \mathbf{x}_A$ and $\mathbf{x} = \mathbf{x}_B$ respectively in Minkowski space, and so we are able to simplify the interaction Hamiltonian to

$$\hat{H}_D(t) = \lambda_D \eta_D \delta(t - T_D) (e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^-) \otimes \int d^n \mathbf{x} F_D(\mathbf{x} - \mathbf{x}_D) \hat{\phi}(\mathbf{x}, t) \quad (2.32)$$

where

$$\hat{\phi}(\mathbf{x}, t) = \frac{1}{(2\pi)^{n/2}} \int \frac{d^n \mathbf{k}}{\sqrt{2|\mathbf{k}|}} \left(e^{i(|\mathbf{k}|t - \mathbf{k} \cdot \mathbf{x})} \hat{a}_{\mathbf{k}}^\dagger + \text{H.c.} \right) \quad (2.33)$$

and we introduce a spatial smearing profile $F_D(\mathbf{x})$ to avoid divergences in $\hat{\rho}_{AB}$.

The choice of a δ -switching function allows for the calculation of the time evolution operator (eq. (2.15)) exactly. Following the notation of Simidzija and Martín-Martínez [77, 76], we define detector A to be the one that switches first ($T_A \leq T_B$) and calculate the

time-evolution operator as

$$\begin{aligned}\hat{U}_\delta = & \left(\mathbb{1}_A \otimes \mathbb{1}_B \otimes \cosh(\hat{Y}_B) + \mathbb{1}_A \otimes \hat{\mu}_B(T_B) \otimes \sinh(\hat{Y}_B) \right) \\ & \times \left(\mathbb{1}_A \otimes \mathbb{1}_B \otimes \cosh(\hat{Y}_A) + \hat{\mu}_A(T_A) \otimes \mathbb{1}_B \otimes \sinh(\hat{Y}_A) \right)\end{aligned}\quad (2.34)$$

where

$$\hat{\mu}_D(t) = e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^- \quad (2.35)$$

and

$$\hat{Y}_D := -i\lambda_D \eta_D \sigma \int d^n \mathbf{x} F_D(\mathbf{x} - \mathbf{x}_D) \hat{\phi}(\mathbf{x}, T_D). \quad (2.36)$$

The detector-field system is initially in the state $|\Psi_0\rangle = |g\rangle_A \otimes |g\rangle_B \otimes |0\rangle_\phi$ and the final state of the two detector subsystem is

$$\hat{\rho}_{AB} = \text{Tr}_\phi \left[\hat{U}_\delta |\Psi_0\rangle \langle \Psi_0| \hat{U}_\delta^\dagger \right] = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix} \quad (2.37)$$

where

$$\rho_{11} = \frac{1}{4} \left(1 + f_A + f_B \cos(2\theta) + f_A f_B \cosh(\omega) \right) \quad (2.38a)$$

$$\rho_{14} = \frac{1}{4} e^{-i(\Omega_A T_A + \Omega_B T_B)} f_B \left(i \sin(2\theta) + f_A \sinh(\omega) \right) \quad (2.38b)$$

$$\rho_{22} = \frac{1}{4} \left(1 + f_A - f_B \cos(2\theta) - f_A f_B \cosh(\omega) \right) \quad (2.38c)$$

$$\rho_{23} = -\frac{1}{4} e^{-i(\Omega_A T_A - \Omega_B T_B)} f_B \left(i \sin(2\theta) + f_A \sinh(\omega) \right) \quad (2.38d)$$

$$\rho_{33} = \frac{1}{4} \left(1 - f_A + f_B \cos(2\theta) - f_A f_B \cosh(\omega) \right) \quad (2.38e)$$

$$\rho_{44} = \frac{1}{4} \left(1 - f_A - f_B \cos(2\theta) + f_A f_B \cosh(\omega) \right) \quad (2.38f)$$

and we have defined

$$f_D := \exp \left(-\lambda_D^2 \eta_D^2 \int \frac{d^n \mathbf{k}}{|\mathbf{k}|} \left| \tilde{F}_D(\mathbf{k}) \right|^2 \right) \quad (2.39a)$$

$$\begin{aligned} \theta &:= -i \left[\hat{Y}_A, \hat{Y}_B \right] \\ &= -\frac{i\lambda_A \lambda_B \eta_A \eta_B}{2} \int \frac{d^n \mathbf{k}}{|\mathbf{k}|} \left(\tilde{F}_A^*(\mathbf{k}) \tilde{F}_B(\mathbf{k}) e^{i|\mathbf{k}|(T_A - T_B)} e^{-i\mathbf{k} \cdot (\mathbf{x}_A - \mathbf{x}_B)} - \text{H.c.} \right) \end{aligned} \quad (2.39b)$$

$$\omega := \lambda_A \lambda_B \eta_A \eta_B \int \frac{d^n \mathbf{k}}{|\mathbf{k}|} \left(\tilde{F}_A^*(\mathbf{k}) \tilde{F}_B(\mathbf{k}) e^{i|\mathbf{k}|(T_A - T_B)} e^{-i\mathbf{k} \cdot (\mathbf{x}_A - \mathbf{x}_B)} + \text{H.c.} \right). \quad (2.39c)$$

After substituting equations (2.39) into equation (2.37) and tracing out either detector B or detector A gives the reduced density matrix for the remaining detector as

$$\begin{aligned} \hat{\rho}_A &= \text{Tr}_B [\hat{\rho}_{AB}] = \frac{1}{2} \begin{pmatrix} 1 + f_A & 0 \\ 0 & 1 - f_A \end{pmatrix} \\ \hat{\rho}_B &= \text{Tr}_A [\hat{\rho}_{AB}] = \frac{1}{2} \begin{pmatrix} 1 + f_B \cos(2\theta) & 0 \\ 0 & 1 - f_B \cos(2\theta) \end{pmatrix}, \end{aligned} \quad (2.40)$$

so we have the transition probability of each detector

$$P_A = \frac{1}{2}(1 - f_A), \quad P_B = \frac{1}{2}(1 - f_B \cos(2\theta)), \quad (2.41)$$

which are only equal when the field commutator, vanishes. Since the time evolution operator in equation (2.34) is calculated exactly rather than through a Dyson series expansion, this model allows for the back-reaction of the detectors on the field. If detector A is in causal contact with detector B , then B will be interacting with the field, which was modified by A . It is also worth noting that the transition probabilities are independent of the energy gap of the detector, and so they can be interpreted as both excitation and de-excitation probabilities.

Finally, we state the no-go theorem described in [77] states that:

A pair of UDW detectors with Dirac-delta switching functions and arbitrary spatial profiles and coupling strengths cannot harvest entanglement from a coherent state of a massless scalar field.

2.3 Quantifying entanglement

One advantage of considering the two level UDW detectors, is that the amount entanglement between a pair of qubits is a relatively straight forward to quantify. We will focus on two entanglement monotones, the negativity and the concurrence, which itself feeds into the entanglement of formation.

2.3.1 The negativity

One commonly used measure of entanglement is the negativity [94]. The negativity for a two qubit system (or a qubit-qutrit system) is an entanglement monotone that is only zero for separable states and is defined as

$$\mathcal{N}(\hat{\rho}_{AB}) := \frac{\|\hat{\rho}_{AB}^{\Gamma_A}\| - 1}{2} = \sum_{\lambda_i < 0} |\lambda_i| \quad (2.42)$$

where Γ_A denotes the partial transpose with respect to A , $|\cdot|$ is the trace norm and the sum is over the negative eigenvalues of $\hat{\rho}_{AB}^{\Gamma_A}$.

For a density matrix of the form of equation (2.20), the eigenvalues of $\hat{\rho}_{AB}^{\Gamma_A}$ are

$$\begin{aligned} \lambda_1 &= \frac{1}{2} \left(\rho_{22} + \rho_{33} - \sqrt{(\rho_{22} - \rho_{33})^2 + 4|\rho_{14}|^2} \right) \\ &= \frac{1}{2} \left(P_A + P_B - \sqrt{(P_A - P_B)^2 + 4|X|^2} \right) + \mathcal{O}(\lambda^4) \\ \lambda_2 &= \frac{1}{2} \left(\rho_{22} + \rho_{33} + \sqrt{(\rho_{22} - \rho_{33})^2 + 4|\rho_{14}|^2} \right) \\ &= \frac{1}{2} \left(P_A + P_B + \sqrt{(P_A - P_B)^2 + 4|X|^2} \right) + \mathcal{O}(\lambda^4) \\ \lambda_3 &= \frac{1}{2} \left(\rho_{11} + \rho_{44} - \sqrt{(\rho_{11} - \rho_{44})^2 + 4|\rho_{23}|^2} \right) \\ &= 0 + \mathcal{O}(\lambda^4) \\ \lambda_4 &= \frac{1}{2} \left(\rho_{11} + \rho_{44} + \sqrt{(\rho_{11} - \rho_{44})^2 + 4|\rho_{23}|^2} \right) \\ &= 1 - P_A - P_B + \mathcal{O}(\lambda^4) \end{aligned} \quad (2.43)$$

The only eigenvalue that can be zero to lowest order in the coupling strength, is λ_1 , so we calculate the negativity of the reduced density matrix (eq. (2.23)) as

$$\mathcal{N}(\hat{\rho}_{AB}) = \max \left[0, -\frac{1}{2} \left(P_A + P_B - \sqrt{(P_A - P_B)^2 + 4|X|^2} \right) \right], \quad (2.44)$$

which reduces to

$$\mathcal{N}(\hat{\rho}_{AB}) = \max[0, |X| - P_D] \quad (2.45)$$

when the transition probabilities of the two detectors are equal ($P_A = P_B = P_D$).

2.3.2 The entanglement of formation and concurrence

Given a density matrix $\hat{\rho}$ of a pair of subsystems A and B , the entanglement of formation, $E_f(\hat{\rho})$, is defined as [9]

$$E_f(\hat{\rho}) := \min \sum_i p_i E(\psi_i) \quad (2.46)$$

where the minimization is over all pure-state decompositions of $\hat{\rho}$ and $E(\psi_i)$ is the von Neumann entropy of either of the two subsystem

$$E(\psi_i) = -\text{Tr}(\hat{\rho}_A \log_2 \hat{\rho}_A) = -\text{Tr}(\hat{\rho}_B \log_2 \hat{\rho}_B) \quad (2.47)$$

and $\hat{\rho}_A$ and $\hat{\rho}_B$ are the partial traces of $|\psi_i\rangle \langle \psi_i|$ over subsystems B and A respectively. The entanglement of formation has the operational interpretation of the number of Bell states required to prepare the state ρ via local operations and classical communication.

For a pair of qubits, the solution to the minimization problem of the entanglement of formation is

$$E_f(\hat{\rho}) = h \left(\frac{1 + \sqrt{1 - \mathcal{C}(\hat{\rho})^2}}{2} \right) \quad (2.48)$$

where $h(x) := -x \log_2(x) - (1-x) \log_2(1-x)$ and $\mathcal{C}(\rho)$ is the concurrence

$$\mathcal{C}(\hat{\rho}) := \max[0, w_1 - w_2 - w_3 - w_4]. \quad (2.49)$$

The w_i 's are the square roots of the eigenvalues ($w_1 \geq w_2 \geq w_3 \geq w_4$) of the matrix

$$\hat{\rho}[(\hat{\sigma}_y \otimes \hat{\sigma}_y)\hat{\rho}^*(\hat{\sigma}_y \otimes \hat{\sigma}_y)] \quad (2.50)$$

and $\hat{\sigma}_y$ is the Pauli y matrix. Since the entanglement of formation is a monotonically increasing increasing function of $\mathcal{C}$ and ranges from 0 to 1 as $\mathcal{C}$ goes from 0 to 1, the concurrence is also a measure of entanglement.

For a density matrix of the form of equation (2.20), the (unordered) square roots of eigenvalues of equation (2.50) are

$$\begin{aligned} w_1 &= \sqrt{\rho_{11}\rho_{44}} + |\rho_{14}| \\ w_2 &= \sqrt{\rho_{22}\rho_{33}} + |\rho_{23}| \\ w_3 &= \sqrt{\rho_{11}\rho_{44}} - |\rho_{14}| \\ w_4 &= \sqrt{\rho_{22}\rho_{33}} - |\rho_{23}|, \end{aligned} \quad (2.51)$$

so the concurrence is

$$\mathcal{C}(\hat{\rho}_{AB}) = 2 \max [0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}]. \quad (2.52)$$

If ρ_{AB} is calculated to lowest order in the interaction strength (2.23), then, using equation (2.28), the we have

$$\begin{aligned} \mathcal{C}(\hat{\rho}_{AB}) &= 2 \max [0, |X| - \sqrt{P_A P_B}, |C| - \sqrt{|C|^2 + P_A P_B + |X|^2}] \\ &= 2 \max [0, |X| - \sqrt{P_A P_B}] \end{aligned} \quad (2.53)$$

which also uses the observation that

$$|C| - \sqrt{|C|^2 + P_A P_B + |X|^2} < 0. \quad (2.54)$$

Written in this form, we can see that the concurrence has a nice physical interpretation: the detectors are entangled if the nonlocal coherence term $|X|$ dominates the local noise of the detectors P_A and P_B respectively. Finally, we note, that when the transition probabilities of the detectors are equal $P_A = P_B = P_D$, the negativity is half the concurrence

$$\mathcal{N}(\hat{\rho}_{AB}) = \max[0, |X| - P_D] = \frac{1}{2}\mathcal{C}(\hat{\rho}_{AB}). \quad (2.55)$$

Throughout this thesis, we will generally use concurrence as our choice of entanglement measure. However in chapters 3 and 7, for reasons of convention, when the transition probabilities of the detectors are equal and (2.55) holds, we will use negativity.

With the field, the detectors and measures of entanglement laid out, we present our results.

Chapter 3

Bandlimited Entanglement Harvesting

One of the most important open problems of modern physics is the question of what is happening at the highest energy scales where quantum field theory and general relativity are expected to be incorporated into a larger theory of physics. These high energy scales correspond to the shortest length scales, where it is widely thought that, due to quantum fluctuations of the metric, the notion of space and time break down [34]. Currently, there are many approaches to consistent theories of quantum gravity that describe the physics at these length scales, including string theory, loop quantum gravity and many others [68, 14].

It has been shown that if there is a finite, minimum-length uncertainty in a quantum field, the field will obey the Shannon sampling theorem, meaning the continuous field can be reconstructed from a discrete set of sampling points [38]. This also implies the field will be bandlimited. The Shannon Sampling Theorem [75] is a theory in classical information that provides an equivalence between a continuous and a discrete representation of information. If a signal $f(t)$ is bandlimited, so that it has no frequencies higher than some Λ , then knowing the value of the signal $f(t_n)$ on a lattice of points $\{t_n\}$ is enough to reconstruct the continuous signal for all t provided the average spacing of the lattice is less than or equal to the Nyquist spacing, π/Λ . Unlike naively putting a QFT on a lattice, the resultant bandlimited QFT preserves local Euclidean symmetries, but is still not Lorentz invariant. However, the conventional momentum cutoff generalizes to a covariant cutoff, where modes with a wavelength smaller than the cutoff have very small bandwidth and are effectively frozen out [37, 15].

It has been previously shown [64] that applying a conventional bandlimit to a $(1+1)$ -

dimensional scalar QFT results in field degrees of freedom that occupy an incompressible spatial volume. As a result, the two-point correlations of the field and entanglement entropy are modified from the case of no cutoff, with the least modification occurring at distances of the Nyquist spacing.

However, questions of locality remain, specifically in trying to understand what happens at volumes smaller than the finite spatial volume of the discrete degrees of freedom. Detectors, especially when they are pointlike, are local objects, and can be used as a tool to explore this question. This is the purpose of this chapter and in doing so, we can also consider a detection of this bandlimit.

3.1 Implementing the bandlimit

The bandlimit, Λ , is a hard cutoff and is implemented by expanding the field operator in plane-wave modes and cutting off momenta where $|\mathbf{k}| > \Lambda$:

$$\hat{\phi}_\Lambda(\mathbf{x}, t) = \frac{1}{(2\pi)^{n/2}} \int_{|\mathbf{k}| < \Lambda} \frac{d^n \mathbf{k}}{\sqrt{2|\mathbf{k}|}} \left(e^{i(|\mathbf{k}|t - \mathbf{k} \cdot \mathbf{x})} \hat{a}_\mathbf{k}^\dagger + \text{H.c.} \right). \quad (3.1)$$

This implementation of the cutoff is not covariant but is chosen this way for ease of calculation and to follow on from the framework of [64]. Reproducing these calculations with a fully covariant cutoff [37] is left to future work. That said, we expect that similar result will hold in the case of a covariant cutoff, since our UDW detectors only couple to the field for a finite time due to the switching function, and modes with a small bandwidth would be further suppressed.

3.2 The density matrix $\hat{\rho}_{AB}$

With the field operator (3.1), the interaction Hamiltonian for a single stationary detector located at $\mathbf{x} = \mathbf{x}_D$ becomes

$$\begin{aligned} \hat{H}_D(t) &= \lambda \chi_D(t) \left(e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^- \right) \otimes \int d^n \mathbf{x} F_D(\mathbf{x} - \mathbf{x}_D) \hat{\phi}(\mathbf{x}, t) \\ &= \lambda \chi_D(t) \left(e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^- \right) \otimes \int_{|\mathbf{k}| < \Lambda} \frac{d^n \mathbf{k}}{\sqrt{2|\mathbf{k}|}} \left(\tilde{F}_D^*(\mathbf{k}) e^{i[|\mathbf{k}|t - \mathbf{k} \cdot \mathbf{x}_D]} \hat{a}_\mathbf{k}^\dagger + \text{H.c.} \right) \end{aligned} \quad (3.2)$$

where we have included a spatial smearing function for the detector $F_D(\mathbf{x})$ and

$$\tilde{F}_D(\mathbf{k}) = \frac{1}{(2\pi)^{n/2}} \int d^n \mathbf{x} F_D(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}} \quad (3.3)$$

is the Fourier transform of that spatial profile.

Mathematically, as was shown in [25], the hard momentum cutoff in the interaction Hamiltonian (3.2) can be absorbed into the definition of the spatial profile:

$$\begin{aligned} \hat{H}_D(t) &= \lambda \chi_D(t) (e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^-) \otimes \int \frac{d^n \mathbf{k}}{\sqrt{2|\mathbf{k}|}} \Pi\left(\frac{|\mathbf{k}|}{2\Lambda}\right) \left(\tilde{F}_D^*(\mathbf{k}) e^{i[\mathbf{k}|t - \mathbf{k}\cdot\mathbf{x}_D]} \hat{a}_{\mathbf{k}}^\dagger + \text{H.c.}\right) \\ &= \lambda \chi_D(t) (e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^-) \otimes \int \frac{d^n \mathbf{k}}{\sqrt{2|\mathbf{k}|}} \left(\tilde{G}_D^*(\mathbf{k}) e^{-i[\mathbf{k}|t - \mathbf{k}\cdot\mathbf{x}_D]} \hat{a}_{\mathbf{k}}^\dagger + \text{H.c.}\right) \end{aligned} \quad (3.4)$$

where the new spatial profile is

$$G(\mathbf{x}) = F(\mathbf{x}) * \frac{1}{(2\pi)^{n/2}} \int d^n \mathbf{k} \Pi\left(\frac{|\mathbf{k}|}{2\Lambda}\right) e^{-i\mathbf{k}\cdot\mathbf{x}} \quad (3.5)$$

and $\Pi(k)$ is the rectangle function

$$\Pi(k) := \begin{cases} 0, & |k| > \frac{1}{2} \\ 1/2, & |k| = \frac{1}{2} \\ 1, & |k| < \frac{1}{2} \end{cases} \quad (3.6)$$

In 3 dimensions, its inverse Fourier transform is

$$\mathcal{F}^{-1} \left[\Pi\left(\frac{|\mathbf{k}|}{2\Lambda}\right) \right] = \sqrt{\frac{2}{\pi}} \Lambda^2 \frac{j_1(\Lambda |\mathbf{x}|)}{|\mathbf{x}|} \quad (3.7)$$

where $j_1(r)$ is the order-1 spherical Bessel function of the first kind.

By writing the interaction Hamiltonian in this way, it is made clear that an alternative interpretation of the hard bandlimit $\mathbf{k} < \Lambda$ is the pair of detectors that interact with a non-bandlimited ($\Lambda \rightarrow \infty$) massless scalar field have the spatial profile given by equation (3.7). Since the rectangle function is compact in k -space, the modified spatial profile is not compact—even if the initial profile $F(\mathbf{x})$ was compact or even point-like.

We will now consider $(3 + 1)$ -dimensional Minkowski space and take the two UDW

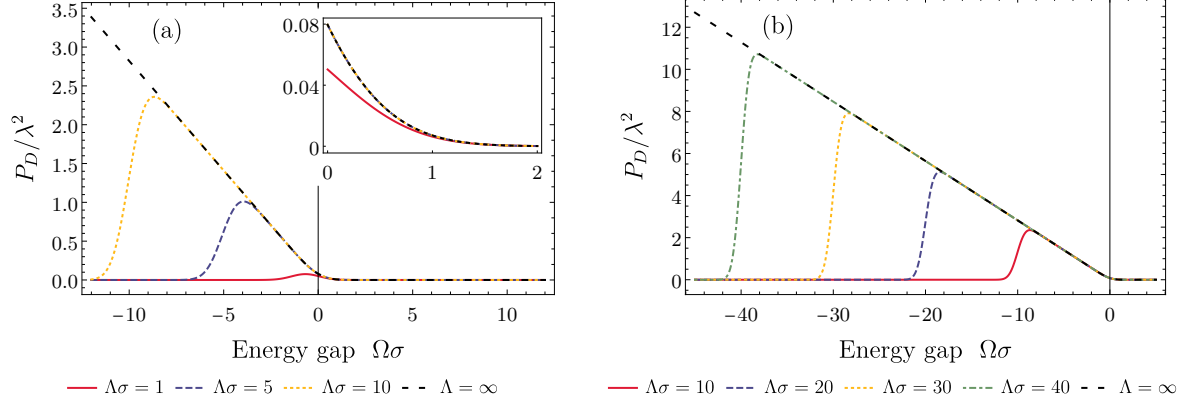


Figure 3.1: The transition probability P_D of a single point-like detector as a function of its energy gap Ω at various values of the bandlimit Λ of the field. A negative energy gap corresponds to an initially excited detector. When the energy gap of the detector is larger than the bandlimit, the transition probability of an initially excited detector rapidly falls off. When there is no bandlimit, the transition probability grows with energy gap.

detectors to be separated by a distance $S := |\mathbf{x}_A - \mathbf{x}_B|$. We will consider the detectors to be point-like

$$F_D(\mathbf{x} - \mathbf{x}_D) = \delta^{(3)}(\mathbf{x} - \mathbf{x}_D) \quad (3.8)$$

and have identical energy gaps $\Omega_A = \Omega_B = \Omega$, and we will take the switching function to be a Gaussian with characteristic width σ :

$$\chi_D(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right). \quad (3.9)$$

With the field (3.1), the Wightman function (2.27) becomes

$$\begin{aligned}
W(x(t), x(t')) &= \left\langle 0 \left| \hat{\phi}(\mathbf{x}, t) \hat{\phi}(\mathbf{x}', t') \right| 0 \right\rangle \\
&= \int_{|\mathbf{k}| < \Lambda} \frac{d^3 \mathbf{k}}{\sqrt{2|\mathbf{k}|}} \int_{|\mathbf{k}'| < \Lambda} \frac{d^3 \mathbf{k}'}{\sqrt{2|\mathbf{k}'|}} \left\langle 0 \left| \left(e^{i(|\mathbf{k}|t - \mathbf{k} \cdot \mathbf{x})} \hat{a}_{\mathbf{k}}^\dagger + e^{-i(|\mathbf{k}|t - \mathbf{k} \cdot \mathbf{x})} \hat{a}_{\mathbf{k}} \right) \right. \right. \\
&\quad \left. \left. \times \left(e^{i(|\mathbf{k}'|t' - \mathbf{k}' \cdot \mathbf{x}') } \hat{a}_{\mathbf{k}'}^\dagger + e^{-i(|\mathbf{k}'|t' - \mathbf{k}' \cdot \mathbf{x}') } \hat{a}_{\mathbf{k}'} \right) \right| 0 \right\rangle \\
&= \int_{|\mathbf{k}| < \Lambda} \frac{d^3 \mathbf{k}}{\sqrt{2|\mathbf{k}|}} \int_{|\mathbf{k}'| < \Lambda} \frac{d^3 \mathbf{k}'}{\sqrt{2|\mathbf{k}'|}} e^{-i(|\mathbf{k}|t - \mathbf{k} \cdot \mathbf{x})} e^{i(|\mathbf{k}'|t' - \mathbf{k}' \cdot \mathbf{x}') } \hat{a}_{\mathbf{k}'} \left\langle 0 \left| \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{k}'}^\dagger \right| 0 \right\rangle \\
&= \int_{|\mathbf{k}| < \Lambda} \frac{d^3 \mathbf{k}}{2|\mathbf{k}|} e^{-i|\mathbf{k}|(t-t')} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')} \tag{3.10}
\end{aligned}$$

Now we can calculate the matrix elements in equations (2.24) and (2.26) as

$$P_{A,\Lambda} = P_{B,\Lambda} = P_{D,\Lambda} = \frac{\lambda^2}{4\pi} \left(e^{-\sigma^2 \Omega^2} - e^{-\sigma^2 (\Omega + \Lambda)^2} + \sqrt{\pi} \sigma \Omega \left[\operatorname{erf}(\sigma \Omega) - \operatorname{erf}(\sigma(\Omega + \Lambda)) \right] \right) \tag{3.11}$$

$$\begin{aligned}
X_\Lambda &= \frac{\lambda^2 \sigma}{4\sqrt{\pi} S} e^{-\sigma^2 \Omega^2} e^{-S^2/(2\sigma)^2} \left[\operatorname{erfi}\left(\frac{S}{2\sigma}\right) - \operatorname{Re} \left(\operatorname{erfi}\left(\frac{S}{2\sigma} + i\sigma\Lambda\right) \right) \right] \\
&\quad + i \frac{\lambda^2 \sigma^2}{2\pi S} e^{-\sigma^2 \Omega^2} \int_0^\Lambda dk e^{-\sigma^2 k^2} \operatorname{erfi}(\sigma k) \sin(Sk) \tag{3.12}
\end{aligned}$$

where the subscript Λ is used as a reminder that the scalar field is bandlimited,

$$\operatorname{erf}(x) := \frac{2}{\sqrt{\pi}} \int_{-\infty}^x dt e^{-t^2} \tag{3.13}$$

is the error function, and $\operatorname{erfi}(x) := -i \operatorname{erf}(ix)$.

Using the large- x expansion of

$$\operatorname{erfi}(x) = \frac{2}{\sqrt{\pi}} \frac{e^{x^2}}{x} + \mathcal{O}\left(\frac{1}{x^3}\right) \tag{3.14}$$

the imaginary part of X_Λ can be approximated as

$$\begin{aligned}\text{Im}(X_\Lambda) &\approx \frac{\lambda^2 \sigma^2}{2\pi S} e^{-\sigma^2 \Omega^2} \left(\int_0^\infty dk e^{-\sigma^2 k^2} \text{erfi}(\sigma k) \sin(Sk) - \frac{1}{\sqrt{\pi}} \int_\Lambda^\infty dk \frac{\sin(Sk)}{\sigma k} \right) \\ &= \frac{\lambda^2 \sigma^2}{4\pi} \frac{e^{-\sigma^2 \Omega^2}}{|S|} \left(e^{-S^2/(2\sigma)^2} - 1 + \frac{2}{\pi} \text{Si}(|S| \Lambda) \right)\end{aligned}\quad (3.15)$$

where $\text{Si}(x)$ is the sine integral

$$\text{Si}(x) := \int_0^x dt \frac{\sin t}{t}$$

provided $\Lambda\sigma \gg 1$. We find good agreement between this approximation and numeric integration of (3.12) when $\Lambda\sigma > 5$.

3.3 Transition probability

First we consider the dependence on the transition probability of a single detector on its energy gap, Ω , and the bandlimit of the quantum field, Λ , which is shown in figure 3.1. When the detector is initially in the ground state, the transition probability falls off exponentially with increasing energy gap. Additionally, as shown in the inset of figure 3.1(a), a small bandlimit lowers the transition probability, and this effect is strongest for small values of the energy gap due to the exponential suppression. We also find that for values of the bandlimit larger than $\Lambda\sigma \approx 2$, the transition probability (from the ground to the excited state) is nearly indistinguishable from the non-bandlimited case regardless of the energy gap.

In order to better understand this behaviour, it is useful to write the transition probability in terms of the non-bandlimited case as

$$P_{D,\Lambda}(\Omega) = P_{D,\infty}(\Omega) - P_{D,\infty}(\Omega + \Lambda) - \frac{\lambda^2 \sigma \Lambda}{4\sqrt{\pi}} \text{erfc}(\sigma(\Omega + \Lambda)) \quad (3.16)$$

where $\text{erfc}(x) := 1 - \text{erf}(x)$. Since the bandlimit enters the expression like a modified energy gap, it is easy to see that for large values, both $P_{D,\infty}(\Omega + \Lambda)$ and $\text{erfc}(\sigma(\Omega + \Lambda))$ are exponentially suppressed and $P_{D,\Lambda}(\Omega) \approx P_{D,\infty}(\Omega)$.

If the detector was initially excited, which is mathematically equivalent to taking $\Omega < 0$, the transition probability increases linearly with increasing energy gap up to a maximum value after which it exponentially falls off, which is seen in the negative- Ω regions of

figure 3.1. As the bandlimit is increased, we find that the maximum value of de-excitation probability occurs at larger (more-negative) values of energy gap and that the falloff after is sharper.

We define Ω_{crit} to be the value of energy gap associated with the maximum de-excitation probability. Since the transition probability falls off so rapidly when $|\Omega| < |\Omega_{\text{crit}}|$, this energy gap can be interpreted as the largest energy gap for which excited detectors are able to decay into the bandlimited field (any larger and the de-excitation probability is exponentially suppressed). This is not surprising, since bandlimiting the field also puts limits on the frequency of modes that exist in the theory, and since we have only calculated the reduced density matrix to lowest order in the coupling constant, we only consider single-mode de-excitations. In order to decay into the ground state, the detector must emit a field quanta of greater energy than is allowed by the field theory.

The dependence of the Ω_{crit} and the bandlimit of the scalar field is plotted in figure 3.2, where the detector is assumed to be initially excited. Ω_{crit} grows with nearly the same rate as the bandlimit of the field. The most discrepancy occurs at small values of $\Omega\sigma$ where there is a high degree of quantum uncertainty in the effective energy gap of the detector. At large values of Ω_{crit} and Λ , the relationship approaches the linear relationship $\Omega_{\text{crit}} = \Lambda - 2/\sigma$.

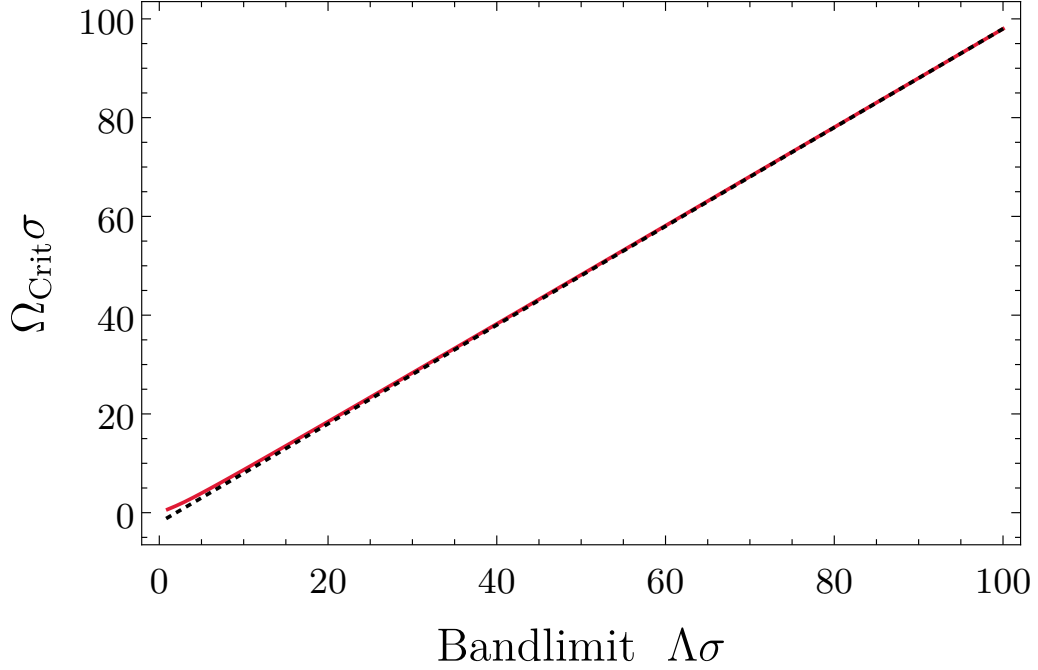


Figure 3.2: The energy gap of the detector corresponding to the maximum de-excitation probability as a function of the bandlimit (red solid line). The de-excitation probability rapidly falls off for detectors with energy gaps greater than this critical value. The black dotted line corresponds to a line of best fit $\Omega_{\text{Crit}} = \Lambda - 2/\sigma$.

3.4 Negativity

Now we consider the entanglement between two identical detectors A and B , which are initialized in the ground state and separated by a distance S . We have found that the dependence of the negativity on the energy gap of the detectors is qualitatively the same across a wide range of value of the bandlimit. The actual values of the negativity at given value of Ω are much lower than the non-bandlimited case for a small enough value of Λ , and as Λ increases, the negativity oscillates around the non-bandlimited value. For this reason we only consider detectors with an energy gap of $\Omega\sigma = 0.01$ throughout this section.

In figure 3.3 we plot the dependence of the entanglement harvested on the bandlimit and the separation of the detectors. When the detectors are held a fixed separation of $S\sigma \approx 1.5$, marked by the third red line, there are striking oscillations in the contours.

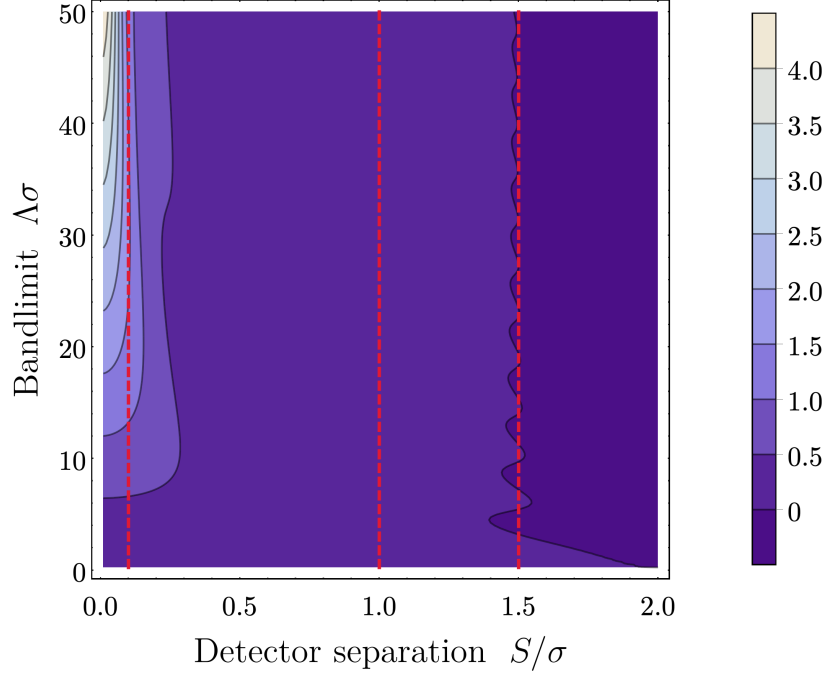


Figure 3.3: The negativity of the reduced density matrix for the detectors' internal states, $\mathcal{N}(\hat{\rho}_{AB})/\lambda^2$, as a function of detector separation S and the bandlimit Λ of the field when the detectors have an energy gap of $\Omega\sigma = 0.01$. The darkest region corresponds to a value of zero negativity when the two detectors remain in a separable state. The red dashed lines mark the constant separation slices shown in figure 3.4.

These oscillations also exist for any values of fixed detector, but are not visible due to the scale of the plot.

To highlight these oscillations, we plot the negativity as a function of bandlimit, for three values of fixed detector separation, in figure 3.4. For values of separation where entanglement harvesting is possible in the non-bandlimited case (figures 3.4(a) and 3.4(b)), we find the negativity increases with increasing values of the bandlimit up to some maximum value. As the bandlimit increases further, the negativity will exhibit damped oscillations around the non-bandlimited value. The frequency of these oscillations increases as the detector separation is increased.

These oscillations can also be present when separation of the detectors is too large for entanglement harvesting to be possible in the non-bandlimited case, which we show in figure 3.4(c). In this case, the damped oscillations that are still present in the function

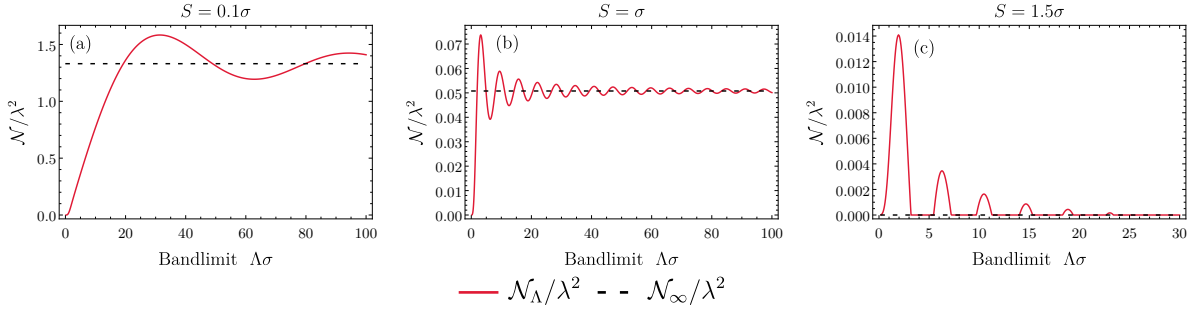


Figure 3.4: Red solid line: The negativity of the reduced density matrix, $\mathcal{N}(\hat{\rho}_{AB})/\lambda^2$, as a function of the bandlimit Λ . Black dashed line: The value of the negativity when there is no bandlimit ($\Lambda \rightarrow \infty$). The detector energy gap is set to $\Omega\sigma = 0.01$.

$|X| - P_D$ are cutoff for negative values, but remain when $|X| > P_D$. This results in values of the bandlimit where entanglement harvesting is possible, even if it not possible when there is no bandlimit.

In figure 3.5, we plot the reduced density matrix as a function of detector separation at a fixed bandlimit of $\Lambda = 50\sigma$ and $\Lambda \rightarrow \infty$. Similar to the plots in figure 3.4, we find the negativity exhibits damped oscillations around the non-bandlimited curve. Interestingly, we also find that these oscillations match up with the oscillations on the effective spatial profile (equation (3.7)), which implies that the oscillations in negativity may be a result of increased/decreased overlap of the detectors' effective spatial profiles.

Since neither the transition probability of the detectors nor the real part of the matrix element X_Λ depend significantly on the bandlimit when it is larger, the oscillatory behaviour in both figures 3.4 and 3.5 must be due the imaginary part of X , and can be approximated as resulting from the $\text{Si}(S\Lambda)$ term in equation (3.15). Therefore, the oscillations with respect to Λ in the imaginary part of X can be shown to be upper bounded by

$$\frac{\lambda^2\sigma}{2\pi^{3/2}} \frac{e^{-\sigma^2\Omega^2}}{S^2\Lambda} + \text{Im}(X_\infty). \quad (3.17)$$

Using this envelope, it is possible to tune the separation of a pair of detectors of fixed energy gap so that the entanglement between them is zero for all values of the bandlimit greater than some chosen threshold and non-zero for *some* values of the bandlimit less than that threshold. In other words, for a chosen value of $\Lambda_{\text{threshold}}$ and a given value of Ω

(or S), find a value of S (or Ω) so that

$$P_{D,\Lambda_{\text{threshold}}} = \left| \text{Re}(X_{\Lambda_{\text{threshold}}}) + i \left(\frac{\lambda^2 \sigma}{2\pi^{3/2}} \frac{e^{-\sigma^2 \Omega^2}}{S^2 \Lambda_{\text{threshold}}} + \text{Im}(X_{\infty}) \right) \right| \quad (3.18)$$

is satisfied. It is important to note that if only one pair is used, then there will be regions of zero negativity at some values of $\Lambda < \Lambda_{\text{threshold}}$. For example, consider a value of $\Lambda\sigma = 4$ in figure 3.4(c); the negativity $\mathcal{N}_{\Lambda} = 0$ for this particular choice of bandlimit, even if it is possible for the detectors to become entangled if the bandlimit were higher.

However, it is possible to build up an array of such pairs of detectors, each with different energy gaps and at different values of separation, so that the frequency of oscillations in negativity are different. This can be used to fill in the gaps of each individual curve and in principle test if a quantum field has a bandlimit $\Lambda < \Lambda_{\text{threshold}}$. An example of such an array is shown in figure 3.6. For all detector configurations, the (S, Ω) pairs, considered in this figure, entanglement harvesting is not possible for values of the bandlimit $\Lambda > \Lambda_{\text{threshold}} = 20/\sigma$. However, for values of bandlimit $\Lambda < \Lambda_{\text{threshold}}$, there is at least one detector configuration where the negativity is non-zero, indicating entanglement harvesting is possible. If at least one of the detector configurations is able to become entangled after interacting with the field, then it must be because $\Lambda < \Lambda_{\text{threshold}}$, and if none of the detector configurations become entangled, then it can be concluded that $\Lambda > \Lambda_{\text{threshold}}$.

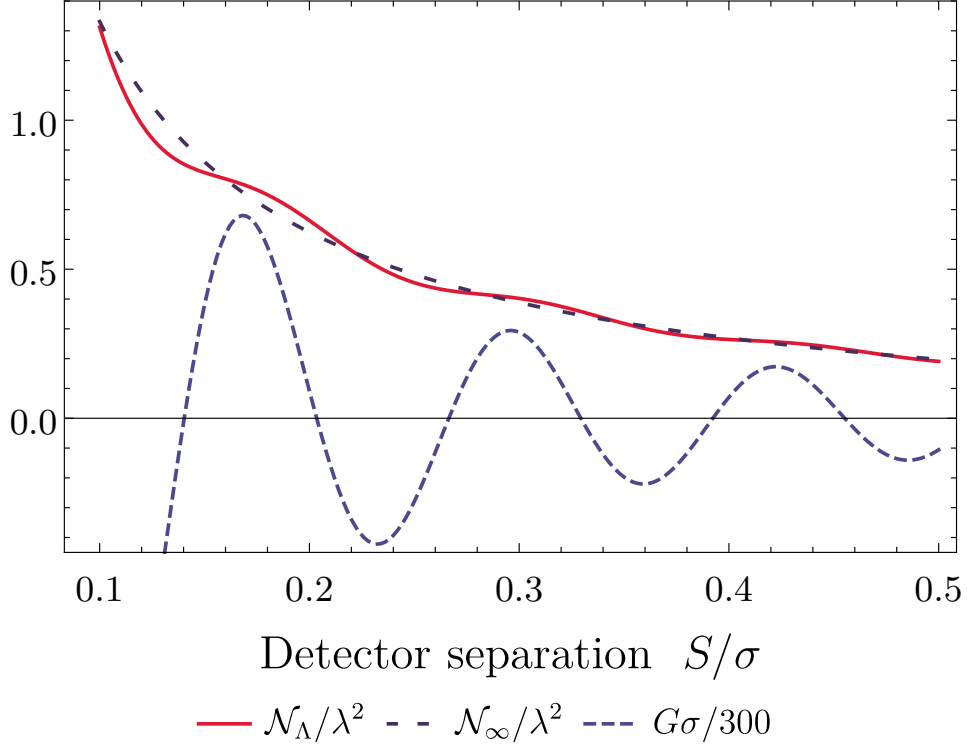


Figure 3.5: Red solid line: The negativity of the reduced density matrix, $\mathcal{N}(\hat{\rho}_{AB})/\lambda^2$, as a function of the detector separation S when the bandlimit is set to $\Lambda\sigma = 50$. Black dashed line: The negativity as a function of detector separation when there is no bandlimit ($\Lambda \rightarrow \infty$). Blue dashed line: A plot of the effective spatial profile $G = \sqrt{\frac{2}{\pi}}\Lambda^2 \frac{j_1(\Lambda S)}{S}$ for $\Lambda\sigma = 50$ scaled by a factor of $1/300$ for ease of plotting. The energy gap is set to $\Omega\sigma = 0.01$. The overall decay of the bandlimited plot matches the decay of the non-bandlimited one, and oscillations in the bandlimited plot match up with the oscillations in the effective spatial profile.

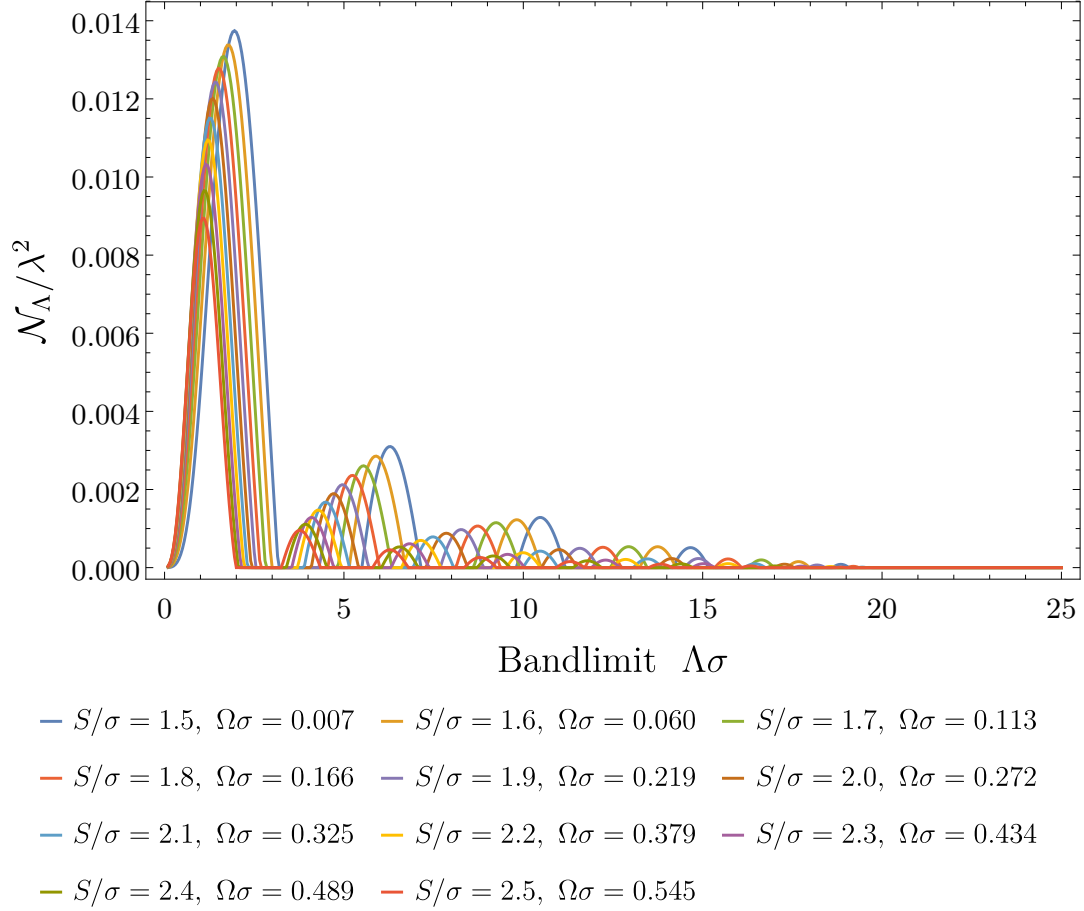


Figure 3.6: An array of detectors with properly tuned energy gaps are set at different separation distances so that equation (3.18) is satisfied at $\Lambda_{\text{threshold}}\sigma = 20$. If the negativity of any one of the pairs of detectors (each corresponding to a different experiment) is non-zero, then $\Lambda < \Lambda_{\text{threshold}}$, since this will indicate entanglement harvesting is possible for larger values of detector separation than predicted by a non-bandlimited theory.

3.5 Going beyond perturbation: δ -switching

Now we consider δ -switching for a non-perturbative approach. This construction of the time-evolution unitary (2.34) will allow for interactions involving more than one field quantum, and in particular allow for multi-mode de-excitations.

In order to compare to our perturbative results, we again consider $(3+1)$ -dimensional Minkowski space and take the UDW detectors to be identical ($\Omega_A = \Omega_B = \Omega$), at rest in a common reference frame, and separated by a distance S . To avoid divergences in θ and ω in the non-bandlimited limit, we set the spatial profile of the detectors to be Gaussian

$$F_D(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}a^3} \exp\left(-\frac{\mathbf{x}^2}{2a^2}\right) \quad (3.19)$$

rather than point like. The construction of the density matrix $\hat{\rho}_{AB}$ continues on from section 2.2.2. With the dimensionality and spatial profile specified, the matrix-element functions (2.39) become

$$f_A = f_B = f = \exp\left(-\frac{\lambda^2\sigma^2}{2\pi a^2}\left(1 - e^{-a^2\Lambda^2}\right)\right) \quad (3.20a)$$

$$\theta = \frac{\lambda^2\sigma^2}{4\sqrt{\pi}aS} \sum_{j=0}^1 (-1)^j \exp\left(-\frac{(S + (-1)^j T)^2}{4a^2}\right) \text{Im}\left[\text{erfi}\left(\frac{S + (-1)^j T}{2a} + ia\Lambda\right)\right] \quad (3.20b)$$

$$\begin{aligned} \omega = -\frac{\lambda^2\sigma^2}{2\sqrt{\pi}aS} \sum_{j=0}^1 \exp\left(-\frac{(S + (-1)^j T)^2}{4a^2}\right) & \left[\text{erfi}\left(\frac{S + (-1)^j T}{2a}\right) \right. \\ & \left. - \text{Re}\left(\text{erfi}\left(\frac{S + (-1)^j T}{2a} + ia\Lambda\right)\right) \right] \end{aligned} \quad (3.20c)$$

where $T := T_B - T_A$.

3.5.1 One detector

First, we consider the delta-switching interaction of a single detector with a bandlimited scalar field. Since the bandlimit acts as an effective spatial profile, we can safely take the $a \rightarrow 0$ limit of the equation (3.19) and again consider a point-like detector. In this

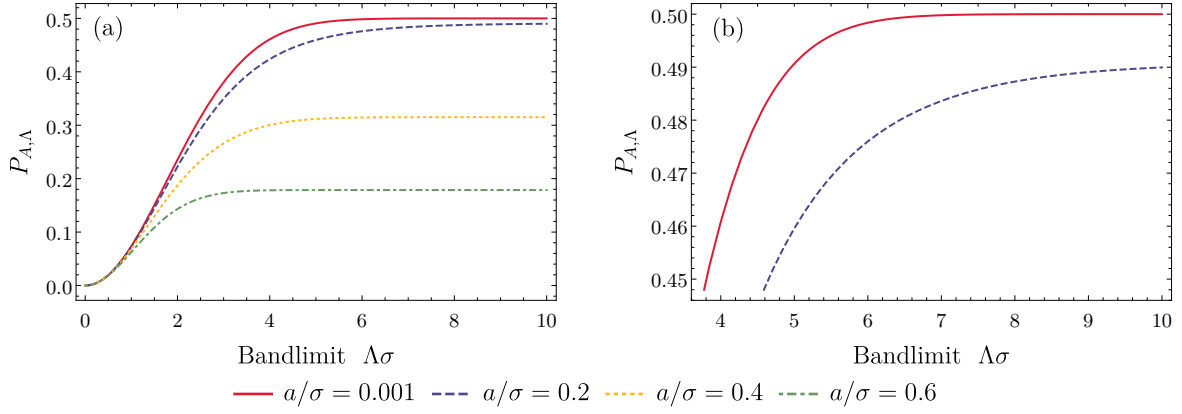


Figure 3.7: The transition probability of a single detector interacting with a bandlimited scalar field with delta switching as a function of the bandlimit Λ for various widths of the spatial profile. As the bandlimit increases, the transition probability quickly asymptotes to the non-bandlimited ($\Lambda \rightarrow \infty$) value. The right hand figure details the asymptotic behaviour of the narrow ($a = 0.001\sigma$ and $a = 0.2\sigma$) detectors. The interaction strength is set to $\lambda = 1$.

limit, the transition probability is

$$\lim_{a \rightarrow 0} P_{A,\Lambda} = \frac{1}{2} \left[1 - \exp \left(-\frac{\lambda^2 \sigma^2 \Lambda^2}{2\pi} \right) \right]. \quad (3.21)$$

We note that due to the exponential decay of f with increasing Λ the transition probability of a single point-like detector interacting with a non-bandlimited field limit of

$$\lim_{\Lambda \rightarrow \infty} \frac{1}{2} \left[1 - \exp \left(-\frac{\lambda^2 \sigma^2 \Lambda^2}{2\pi} \right) \right] = \frac{1}{2}. \quad (3.22)$$

Additionally, this limit is well defined; taking the limits in the other order (first $\Lambda \rightarrow \infty$ then $a \rightarrow 0$) produces the same result.

One does not need to take limits of equation (3.20a) to calculate the transition probability of a point-like ($\tilde{F}(\mathbf{k}) = (2\pi)^{-n/2}$) detector in a non-bandlimited field, and can directly consider equation (2.39a), which is

$$P_{A,\infty} = \frac{1}{2} \left[1 - \exp \left(-\frac{2\pi\lambda^2\sigma^2}{(2\pi)^n} \int \frac{d^n \mathbf{k}}{|\mathbf{k}|} \right) \right] = \frac{1}{2} \quad (3.23)$$

provided $n \geq 2$. In other words, if a two level detector is initialized in the ground (or excited) state and is coupled to the field at a single point in space and time, the result would be the completely mixed state.

This is still consistent with the conventional idea that the transition probability of point-like detector with delta switching will diverge [60] if one considers the perturbative calculation, since to lowest order in the coupling strength

$$P_{A,\infty} \approx \frac{2\pi\lambda^2\sigma^2}{(2\pi)^n} \int \frac{d^n \mathbf{k}}{|\mathbf{k}|} \rightarrow \infty. \quad (3.24)$$

This divergence is solely a mathematical artefact of using perturbation theory, however. The physical behaviour in this case (interaction at a single spacetime point) is in fact well defined, finite, and given by equation (3.23).

In figure 3.7, we plot the dependence of the transition probability of a single detector with various widths of its Gaussian spatial profile interacting with delta switching on the bandlimited scalar field. Since there is no dependence on the energy gap of the detector, this can be interpreted as either the probability of excitation or de-excitation. Regardless of the width of the detector, the transition probability increases with increasing value of the bandlimit and asymptotes to the non-bandlimited value.

We make comparisons to section 3.3 by considering the small- σ limit where the unitless energy gap $\Omega\sigma$ will also be close to zero. In this regime, the dependence of the transition probability on the bandlimit can be best seen in the inset of figure 3.1(b). Here, the same behaviour is seen: when the bandlimit is small, the transition probability of the detector is increased when the bandlimit is increased. But larger values of the bandlimit transition probability are indistinguishable from the non-bandlimited ($\Lambda \rightarrow \infty$) case.

The width of the detector determines how quickly the transition probability approaches its corresponding non-bandlimited values, which can be seen in figure 3.7. When the spatial profile of the detector is relatively wide and the width is increased, the transition probability flattens out and approaches its asymptotic value at lower values of the bandlimit. However, when the spatial profile is very narrow, as seen in figure 3.7(b), the transition probability flattens for the detector with a width of $a = 0.001\sigma$ at a smaller value of the bandlimit than the detector with a width of $a = 0.2\sigma$.

This observation is made more precise in figure 3.8, where we plot the bandlimit, denoted Λ_{Max} , for which the absolute difference between the transition probability and its corresponding non-bandlimited value is equal to a chosen tolerance as a function of the width of the spatial profile. This quantity was chosen for two reasons. First, it provides a

concrete measure of determining that the transition probability is “close” to its asymptotic value. Second, from an operational perspective the tolerance can be interpreted as the resolution of some experiment where UDW detectors are used to determine the bandlimit of the field.

We find that for all three values of the chosen tolerance, the behaviour of the absolute difference is the same. When the spatial profile is very narrow, increasing the width of the profile actually increases the value of $\Lambda_{\max}$ up to a maximum value that occurs at $a \approx 0.2\sigma$. As the width is increased further, the value of $\Lambda_{\max}$ rapidly decreases.

The region where increasing width decreases $\Lambda_{\max}$ is easily explained by naively looking at equation (3.5). When the bandlimit is large enough such that $\frac{1}{\Lambda} \ll a$, the effective profile,

$$G(\mathbf{x}) = \frac{e^{-a^2\Lambda^2/2}}{(2\pi)^{3/2}a^3} \left[e^{(a^4\Lambda^2 - \mathbf{x}^2)/(2a^2)} \operatorname{Re} \left(\operatorname{erf} \left(\frac{a^2\Lambda + i|\mathbf{x}|}{\sqrt{2}a} \right) \right) - \frac{\sqrt{2}a\Lambda}{\sqrt{\pi}} \operatorname{sinc}(\Lambda|\mathbf{x}|) \right], \quad (3.25)$$

will be dominated by the spatial profile of the detector, and the transition probability will be nearly the same as the non-bandlimited case. However, when the width of the spatial profile is very small, the complicated relationship between the width and the bandlimit in the effective spatial profile leads to the increase in sensitivity with increasing width. The takeaway message is this: When comparing detectors with a Gaussian profile and delta switching, the detector most sensitive to the bandlimit is not the one with the narrowest profile, as one may naively think, but rather one with a width of approximately $a \approx 0.2\sigma$

3.5.2 Two detectors

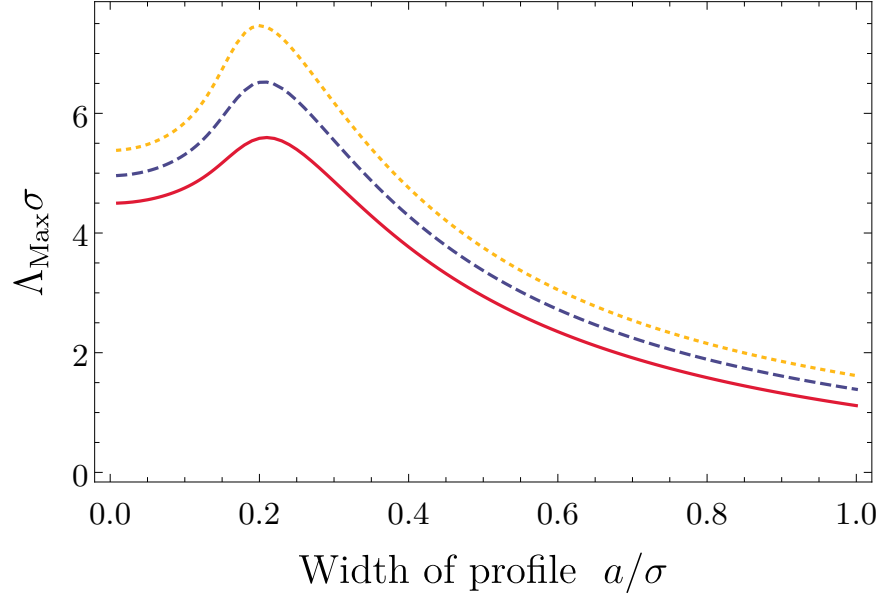
The no-go theorem described in for entanglement harvesting [77, 76] described in section 2.2.2 holds for any arbitrary spatial profile. Therefore, the bandlimit—which is mathematically equivalent to a spatial profile—will not make a difference on the entanglement harvested (none).

One can still look at the transition probability of the second detector, detector B , which depends on the effective field commutator through the parameter θ . In figure 3.9(a), it is shown that θ , when plotted as a function of the bandlimit exhibits damped oscillations with frequencies at $(S \pm T)$. By studying the parameter space, we have found that when the spatial profile of the detector is made smaller, the oscillations decay more slowly, and in the limit of point-like detectors, $a \rightarrow 0$, the oscillations do not decay. In this limit, θ reduces to

$$\lim_{a \rightarrow 0} \theta = \frac{\lambda^2 \sigma^2 \Lambda}{2\pi S} [\operatorname{sinc}(\Lambda(T + S)) - \operatorname{sinc}(\Lambda(T - S))] \quad (3.26)$$

which is simply the difference of the one-dimensional Fourier transforms of the rectangle function $\Pi(k/(2\Lambda))$ evaluated at $(T + S)$ and $(T - S)$.

The oscillations in θ are observable in the transition probability of the second detector when the bandlimit is small, and as shown in the inset of figure 3.9(b), and these can result in a transition probability greater than $1/2$, the largest transition probability of a single detector. However, when the bandlimit is increased, the transition probability exponentially approaches its non-bandlimited quantity at the same rate as a single detector, and the oscillations resulting from θ are exponentially suppressed. If the bandlimit is small, then a second detector with instantaneous switching can be used to give some insight into the commutator between the two detectors, but if the bandlimit is large, then a second detector will not provide any advantage.



— $|P_{A,\Lambda} - P_{A,\infty}| = 0.02$
 - - - $|P_{A,\Lambda} - P_{A,\infty}| = 0.01$
 . . . $|P_{A,\Lambda} - P_{A,\infty}| = 0.005$

Figure 3.8: The value of the maximum bandlimit Λ_{Max} such that the difference between the transition probability in the bandlimited field and non-bandlimited field ($|P_{A,\Lambda_{\text{Max}}} - P_{A,\infty}|$) is equal to a specified (arbitrary) tolerance (0.02, 0.01, 0.005) as a function of the width of the spatial profile of the detector. When the bandlimit is larger than Λ_{Max} , the difference will be less than the tolerance. For all three values, a maximum occurs near at width of $a \approx 0.2\sigma$, indicating the transition probability of a detector with this width is more sensitive to the bandlimit than a wider or narrower detector. The interaction strength is set to $\lambda = 1$.

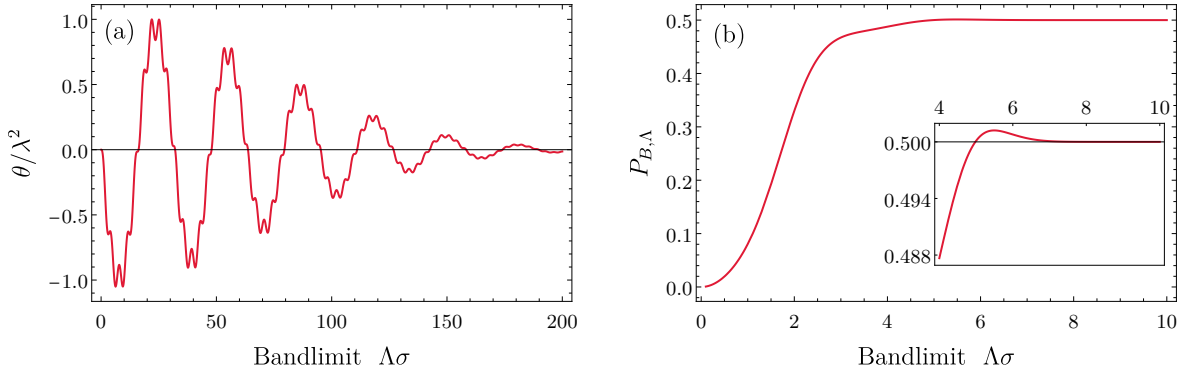


Figure 3.9: A plot of (a) the parameter θ , which is proportional to the effective field commutator between detectors A and B , and (b) the transition probability of detector B as a function of the bandlimit Λ . The negative values in the parameter θ result in values of $P_B > \frac{1}{2}$. Although θ shows dependency on the bandlimit over a large range, we find that $P_{B,\Lambda}$ is only sensitive to the bandlimit when it is small ($\Lambda < 7\sigma$). The width of the spatial profile is set to $a = 0.01\sigma$, the detectors have a time delay of $T = \sigma$ and a separation of $S = 0.8\sigma$, and the interaction strength is set to $\lambda = 1$.

3.6 Summary of results

We have studied the entanglement harvesting protocol and response of two UDW detectors interacting with a conventionally bandlimited $(3+1)$ -dimensional scalar field, where modes with $|\mathbf{k}| \geq \Lambda$ do not exist. We find that the application of this cutoff is equivalent to modifying the spatial profile of the UDW detector by convolving it with the dimensionally appropriate Fourier transform of the rectangle function. This interpretation results in a non-local detector interacting with a non-bandlimited scalar field. On the other hand, if the detectors were interpreted as local, then the degrees of freedom of the field are non-local. The two perspectives are entirely equivalent.

When a point-like detector couples to the bandlimited field with Gaussian switching, we find to lowest order in the coupling strength that the probability of excitation is reduced compared to the non-bandlimited value. The effect only becomes pronounced, however, when the bandlimit is very small. We find, for all values of the bandlimit, when the detector is prepared in the excited state, the de-excitation probability increases linearly with increasing values of the energy gap of the detector, until the energy gap is slightly smaller than the bandlimit of the field. For larger energy gaps, the de-excitation probability exponentially falls off to nearly zero. The interpretation of this is that, to lowest order, the detector must emit a single phonon in order to decay, but if the energy gap is larger than the bandlimit, the emitted photon cannot propagate in the field.

We also note that it is possible for a pair of point-like UDW detectors, coupling with Gaussian switching, to harvest entanglement from the vacuum of the bandlimited scalar field at larger separation distances than in the ordinary case of a field without a bandlimit. The enhancement results from oscillations in the imaginary part of the coherence term of the two detectors, which can be interpreted as resulting from the overlap of the non-local effective spatial profile of the detectors. By taking advantage of the enhancement in entanglement harvesting, one can in principle take an array of pairs of detectors, with specific energy gaps and separations, and put bounds on the bandlimit of a quantum field.

Finally, we use a Dirac-delta switching function to couple two detectors with Gaussian spatial profiles to the field at a single instance in time, which allows for non-perturbative solutions. Again, we find that the transition probability of the first detector is lower when the field is bandlimited, and the effect is much more apparent when the bandlimit is small. Surprisingly, the transition probability of a detector is most sensitive to the bandlimit when the detector's width is small—but not too small. We also find that the transition probability of the second detector has a similar dependence on the bandlimit: any changes are due to the effective field commutator between the two detectors.

Chapter 4

Indefinite Temporal Ordering

The relationship between effects and causes is foundational to our everyday experience and is rooted in the basic laws of physics as we presently understand them. Both in classical and quantum physics, events happen in a fixed causal order.

One of the more intriguing proposals put forward in recent years, is that quantum physics may admit nonclassical causal structures where the order of events is indefinite [28, 17, 18, 56]. Apart from advantages afforded in computation [5, 16] and communication [23, 26, 21, 73, 27], the incorporation of this idea into the foundations of physics may provide new insights toward realizing a quantum theory of gravity [28, 98, 59]. This idea has become known as “Indefinite Causal Order”, or ICO.

Recently a scenario featuring an indefinite causal order of events has been experimentally realized in the form of a quantum switch [63, 69, 24]. In this setting the order in which two quantum operations A and B are performed on some target is coherently controlled by a quantum system, placing the order of operations in temporal superposition [1]. An ordinary quantum circuit using the same number of operations cannot reproduce the effects of the quantum switch [17, 5].

Little is known about how ICO affects quantum entanglement, particularly entanglement of quantum fields. The vacuum state of a free quantum field has long been known to contain correlations between timelike and spacelike separated regions [84, 83], and it is of foundational interest to understand how such correlations are affected by ICO. We will use UDW detectors to explore whether ICO can provide an advantage to the amount of entanglement generated by the detectors interacting locally with a quantum scalar field.

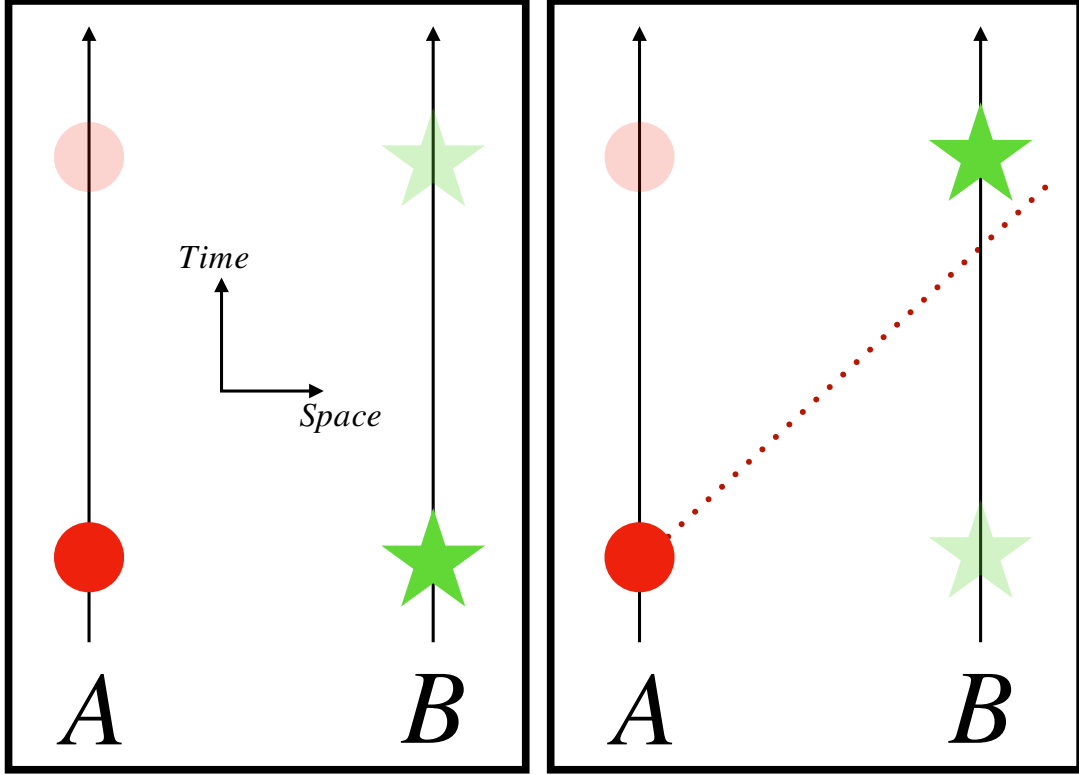


Figure 4.1: Left: scenario (i) in which the two detectors are in a superposition of being active either both in the past or both in the future, controlled by the state of the control qubit $|0\rangle$ or $|1\rangle$ respectively. In this case the spacetime activation regions for the two detectors are spacelike in both branches of the quantum superposition. Right: scenario (ii) with indefinite causal order. In this case the two detectors are active when timelike related. The quantum superposition between “ A before B ” and “ B before A ” is controlled by the state of the control qubit $|0\rangle$ or $|1\rangle$ respectively. Note that in the main text, scenario (ii) also encompasses the case of space-like separated detectors where A and B are placed at different slices of constant time.

4.1 Quantum control of two UDW detectors.

We consider a composite system formed by two pointlike UDW detectors, which are locally coupled to a real scalar field, whose Hamiltonian is

$$\hat{H}_I(t) = \sum_{D=A,B} \sum_{i=0,1} \lambda \chi_{D,i}(t) (e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^-) \otimes \hat{\phi}(\mathbf{x}_D, t) \otimes |i\rangle_C \langle i| \quad (4.1)$$

where $i = 0, 1$ is determined by the computational basis states $\{|0\rangle, |1\rangle\}$ of a control qubit (system C henceforward) that governs the switching functions, $\chi_D(t)$, of the detectors A and B . Placing the initial state of the qubit in a coherent superposition of the computational basis states will thus give rise to a superposition of different switching times of the two detectors. We shall focus on two scenarios, depicted in Fig. 4.1, which allow us to probe two distinct physical situations. Without loss of generality in all cases considered we assume a fixed but arbitrary foliation of flat spacetime into space-like slices (hypersurfaces), each associated with a different time coordinate. Furthermore, in all scenarios each detector is switched on exactly once.

In the first scenario (i) detectors A and B are switched on jointly on a common space-like slice, but in a quantum-controlled superposition of two slices associated with different times. We call this scenario a *past-future* (PF) superposition. The terms ‘past’ and ‘future’ refer to the relation between the superposed switching times. This case allows us to probe how entanglement between space-like regions of the field is affected by quantum delocalization of these regions in time.

In the second scenario (ii) detector A is switched on on the past slice and B on the future slice, in superposition with A and B interchanged. We call this scenario a *cause-effect* (CE) superposition, since for time-like separation between the involved regions the relation between A and B is causally indefinite – we have a superposition of A in the past causal cone of B and vice versa.

One might think that scenario (ii) when A and B are space-like separated is equivalent to scenario (i), where A and B are switched on simultaneously – since for space-like separated regions one can find coordinates in which the switching is simultaneous. Despite this fact, there is no equivalence between these scenarios: in (i) the switching is simultaneous in each superposed amplitude (albeit at different times) whereas choosing coordinates in scenario (ii) so that A and B are simultaneous for one amplitude will result in A being before or after B in the other amplitude, illustrated in figure 4.2, left panel. In other words, A, B are simultaneous in superposition with A earlier than B (or B earlier than A). An analogous difference arises if we try to change coordinates in scenario (i) so as to map it

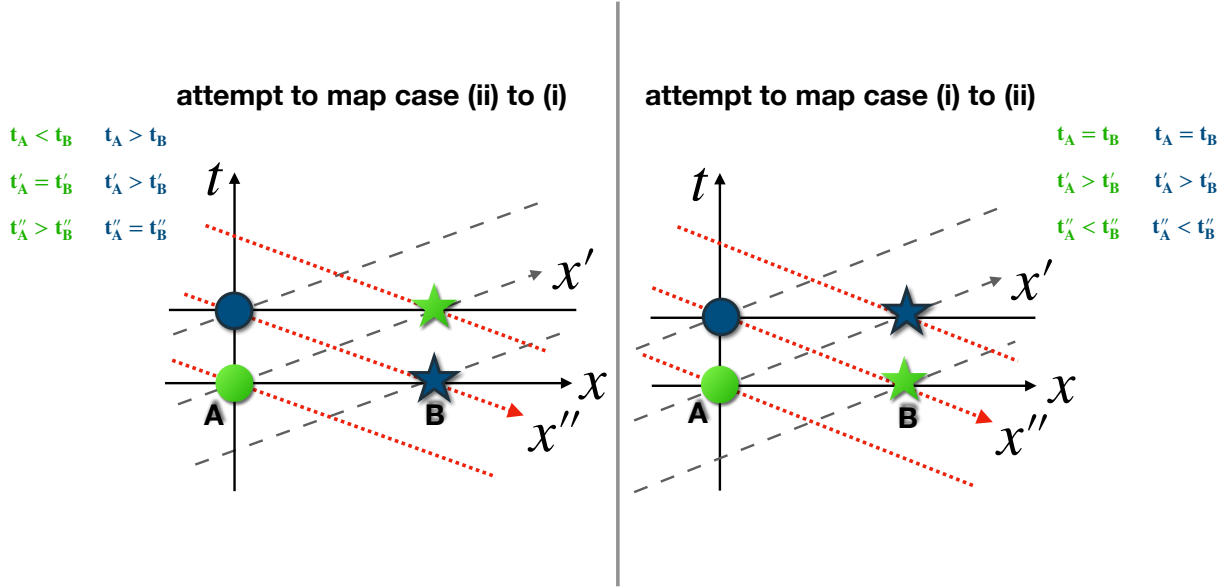


Figure 4.2: It is not possible to map scenarios (ii) and (i) to each other. Left panel: when trying to map scenario (ii) to (i) we can make make A and B simultaneous only in one of the superposed amplitudes – rather than in each of them, as required in scenario (i). Right panel: when trying to map scenario (i) to (ii) we can make make A from one amplitude simultaneous B from the other amplitude, but we do not end up with a superposition of A earlier than B and B than A – in each amplitude A will be switched on earlier than B (or in each amplitude B will be earlier than A). In both panels, x coordinates (black solid lines) make past A and past B simultaneous and future A and future B simultaneous, the x' coordinates (grey dashed lines) make past A simultaneous with future B and the x'' coordinates (red dotted lines) make future A simultaneous with past B .

to scenario (ii). Using coordinates in which A from one amplitude is simultaneous with B from the other amplitude – as it is in case (ii) – actually yields a different scenario: where in each amplitude A is switched on before B (or B before A – depending which amplitude is chosen to define simultaneity) – see also figure 4.2, right panel. In other words, we end up with A before B in superposition with A before B but both switched on some time later (or B before A in each superposed amplitude). Recall, however that in scenario (ii) we have a superposition of A switched on before B and B before A . Thus, even in case of space-like separation between the relevant regions, the two scenarios (i) and (ii) are not equivalent – there is no single coordinate transformation that allows one to map them onto each other.

We will take the background space time to be Minkowski and proceed following the standard entanglement harvesting protocol. First we take the initial state of ABC and field that we chose as

$$\hat{\rho}_0 = \frac{1}{2} \sum_{i,j} |g\rangle_A \langle g| \otimes |g\rangle_B \langle g| \otimes |0\rangle_\phi \langle 0| \otimes |i\rangle_C \langle j|, \quad (4.2)$$

that is, both detectors in the ground state, field in its vacuum state, and the control qubit in the $|+\rangle$ state. Next the system evolves through the time evolution operator given by (2.15) which we expand in a Dyson series (2.18). The next step is to trace over the field degrees of freedom and obtain the reduced density matrix of the 3 qubit subsystem to lowest order in interaction strength.

$$\begin{aligned} \hat{\rho}_{ABC} &= \text{Tr}_\phi \left[\hat{U} \hat{\rho}_0 \hat{U}^\dagger \right] \\ &= \text{Tr}_\phi [\hat{\rho}_0] + \text{Tr}_\phi \left[\hat{\rho}_0 \hat{U}^{(2)\dagger} \right] + \text{Tr}_F \left[\hat{U}^{(1)} \hat{\rho}_0 \hat{U}^{(1)\dagger} \right] + \text{Tr}_F \left[\hat{U}^{(2)} \hat{\rho}_0 \right] + \mathcal{O}(\lambda_D^4), \end{aligned} \quad (4.3)$$

which takes the form

$$\hat{\rho}_{ABC} = \frac{1}{2} \sum_{i,j} \begin{pmatrix} 1 + Y_{ii} + Y_{jj}^* & 0 & 0 & X_{jj}^* \\ 0 & P_{B,ij} & C_{ji}^* & 0 \\ 0 & C_{ij} & P_{A,ij} & 0 \\ X_{ii} & 0 & 0 & 0 \end{pmatrix} \otimes |i\rangle_C \langle j| + \mathcal{O}(\lambda_D^4) \quad (4.4)$$

where for $D = A, B$

$$Y_{ii} = -\lambda^2 \sum_D \int \frac{d^n \mathbf{k}}{2|\mathbf{k}|} \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \frac{\chi_{D,i}(t) \chi_{D,i}(t')}{(2\pi)^n} e^{-i\Omega_D(t-t')} e^{-i|\mathbf{k}|(t-t')} e^{i\mathbf{k} \cdot (\mathbf{x}_D(t) - \mathbf{x}_D(t'))} \quad (4.5)$$

$$P_{D,ij} = \lambda^2 \int \frac{d^n \mathbf{k}}{2|\mathbf{k}|} \left[\left(\int_{-\infty}^{\infty} dt \frac{\chi_{D,i}(t)}{\sqrt{(2\pi)^n}} e^{it(\Omega_D + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_D(t)} \right) \left(\int_{-\infty}^{\infty} dt' \frac{\chi_{D,j}(t')}{\sqrt{(2\pi)^n}} e^{it'(\Omega_D + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_D(t')} \right)^* \right] \quad (4.6)$$

$$C_{ij} = \lambda^2 \int \frac{d^n \mathbf{k}}{2|\mathbf{k}|} \left[\left(\int_{-\infty}^{\infty} dt \frac{\chi_{A,i}(t)}{\sqrt{(2\pi)^n}} e^{it(\Omega_A + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_A(t)} \right) \left(\int_{-\infty}^{\infty} dt' \frac{\chi_{B,j}(t')}{\sqrt{(2\pi)^n}} e^{it'(\Omega_B + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_B(t')} \right)^* \right] \quad (4.7)$$

$$X_{ii} = -\lambda^2 \sum_{D \neq D'} \int \frac{d^n \mathbf{k}}{2|\mathbf{k}|} \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \frac{\chi_{D,i}(t) \chi_{D',i}(t')}{(2\pi)^n} e^{-it(|\mathbf{k}| - \Omega_D)} e^{i\mathbf{k} \cdot \mathbf{x}_D(t)} e^{it'(|\mathbf{k}| + \Omega_{D'})} e^{-i\mathbf{k} \cdot \mathbf{x}_{D'}(t')} \quad (4.8)$$

in the $\{ |0\rangle_A |0\rangle_B, |0\rangle_A |1\rangle_B, |1\rangle_A |0\rangle_B, |1\rangle_A |1\rangle_B \}$ basis and the Y_{ii} terms satisfy the identity that $Y_{ii} + Y_{ii}^* = -(P_{A,ii} + P_{B,ii})$.

It is worth pointing out that even if the control qubit were measured in the $|\pm\rangle$ basis, the matrix elements X and Y will not contain cross terms as a consequence of $\hat{U}^{(2)}$ not containing cross terms

$$\begin{aligned} \hat{U}^{(2)} &= - \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' H_I(t) H_I(t') \\ &= - \sum_{D,D'} \sum_{i,j} \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' (H_{D,i}(t) \otimes |i\rangle_C \langle i|) (H_{j,D'}(t') \otimes |j\rangle_C \langle j|) \\ &= - \sum_{D,D'} \sum_i \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' H_{D,i}(t) H_{D',i}(t') \otimes |i\rangle_C \langle i|. \end{aligned} \quad (4.9)$$

Directly applying $\hat{U}^{(2)}$ to $\hat{\rho}_0$ and tracing out the field gives

$$\begin{aligned}
\text{Tr}_\phi [\hat{U}^{(2)} \hat{\rho}_0] &= \text{Tr}_\phi \left[- \sum_{D,D'} \sum_{ijk} \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' (H_{D,i}(t) H_{D',i}(t') \otimes |i\rangle_C \langle i|) (|\Psi_0\rangle \langle \Psi_0| \otimes |j\rangle_C \langle k|) \right] \\
&= - \sum_{ik} \text{Tr}_\phi \left[\sum_{D,D'} \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' H_{D,i}(t) H_{D',i}(t') |\Psi_0\rangle \langle \Psi_0| \right] \otimes |i\rangle_C \langle k| \\
&= - \sum_{ik} \left\{ \text{Tr}_\phi \left[\sum_D \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' H_{D,i}(t) H_{D,i}(t') |\Psi_0\rangle \langle \Psi_0| \right] \right. \\
&\quad \left. + \text{Tr}_\phi \left[\sum_{D \neq D'} \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' H_{D,i}(t) H_{D',i}(t') |\Psi_0\rangle \langle \Psi_0| \right] \right\} \otimes |i\rangle_C \langle k| \\
&= \sum_{ik} \left(Y_{ii} (|0\rangle_A \langle 0| \otimes |0\rangle_B \langle 0|) + X_{ii} (|1\rangle_A \langle 0| \otimes |1\rangle_B \langle 0|) \right) \otimes |i\rangle_C \langle k| \quad (4.10)
\end{aligned}$$

This seems to imply that non-local correlations are not affected by the causal ordering to lowest order in the interaction strength.

4.1.1 Measuring the control

The state of the control qubit (C) governs the choice of scenario. Consider first scenario (i) illustrated in the left panel in Fig. 4.1. We see that $\text{Supp}(\chi_{A,i}) = \text{Supp}(\chi_{B,i})$ for $i = 0, 1$. Contrast this with the right panel of Fig. 4.1 depicting scenario (ii), where now $\text{Supp}(\chi_{A,i}) = \text{Supp}(\chi_{B,j})$ for $i \neq j$. We eliminate the control qubit by either tracing it out or measuring it in the basis $\{|+\rangle, |-\rangle\}$. Our notation applies to both scenario by just a different labelling of the switching functions.

Tracing out the control qubit yields

$$\hat{\rho}_{AB}^{(\text{Tr})} = \begin{pmatrix} 1 - (P_A^{(\text{Tr})} + P_B^{(\text{Tr})}) & 0 & 0 & X^{(\text{Tr})*} \\ 0 & P_B^{(\text{Tr})} & C^{(\text{Tr})*} & 0 \\ 0 & C^{(\text{Tr})} & P_A^{(\text{Tr})} & 0 \\ X^{(\text{Tr})} & 0 & 0 & 0 \end{pmatrix}, \quad (4.11)$$

where

$$P_D^{(\text{Tr})} = \frac{1}{2}(P_{D,00} + P_{D,11}) \quad (4.12)$$

$$C^{(\text{Tr})} = \frac{1}{2}(C_{00} + C_{11}) \quad (4.13)$$

$$X^{(\text{Tr})} = \frac{1}{2}(X_{00} + X_{11}). \quad (4.14)$$

The results is a classical (probabilistic) mixture of *two detectors active only once*, either spacelike related (i), or timelike related (ii), since the reduced state in Eq. (4.11) is the mixture (with equal weights) of the classical case in which both detectors are active only once. This should have been expected. Indeed, borrowing some intuition from the process matrix formalism [56], the situation is similar to the *quantum switch* in which an indefinite causal order is present only if the control qubit is allowed to quantum interfere [18, 63, 69, 24]. Tracing out the control qubit corresponds to losing the coherence of the superposition, effectively yielding ‘which-when’ information about detector activation and thus remaining with a classical mixture.

Conversely, measuring the control qubit in the $\{|+\rangle, |-\rangle\}$ basis and retaining, e.g., the $|+\rangle$ results, allows the control qubit to interfere, yielding

$$\hat{\rho}_{AB}^{(+)} = \begin{pmatrix} 1 - (P_A^{(+)} + P_B^{(+)}) & 0 & 0 & X^{(\text{Tr})*} \\ 0 & P_B^{(+)} & C^{(+)*} & 0 \\ 0 & C^{(+)} & P_A^{(+)} & 0 \\ X^{(\text{Tr})} & 0 & 0 & 0 \end{pmatrix} \quad (4.15)$$

where now

$$P_D^{(+)} = \frac{1}{4}(P_{D,00} + P_{D,01} + P_{D,10} + P_{D,11}) \quad (4.16)$$

$$C^{(+)} = \frac{1}{4}(C_{00} + C_{11} + C_{01} + C_{10}). \quad (4.17)$$

We see that, while the X terms are the same whether we trace out the control qubit or let it interfere, the other terms in the density matrix differ in the two cases. In particular, in $\hat{\rho}_{AB}^{(+)}$ there appear terms $P_{D,i \neq j}$ and $C_{i \neq j}$ that contain two-point correlation functions of the field between two different spacetime regions in which the same detector or both the detectors are activated in superposition. These quantum interference terms are signatory of the coherence of the superposition. It should be noted that, without the controlled

superposition the field correlations contained in the interference terms would not be accessible with a single use of each detector. By placing the detectors in quantum temporal superposition, we read out correlations between space time regions which can be accessed in the standard schemes with no superposition only if the detectors are used more than once (in more than one space-time region).

This is our primary result: quantum controlled temporal superpositions allow UDW detectors to access field correlations between spacetime regions pertaining to the two different branches in a quantum superposition. As we show in the following, this has relevant consequences for information-theoretic quantities, such as the extractable entanglement from the quantum vacuum.

4.1.2 The double-switch scenario

Here we consider the case in which each detector is active twice and no quantum control is present. This type of switching is chosen to contrast the quantum controlled switching where the detectors switched once in each branch of the superposition.

The interaction Hamiltonian is given by

$$\hat{H}_I(t) = \sum_{D=A,B} \lambda \frac{\chi_{D,0}(t) + \chi_{D,1}(t)}{2} (e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^-) \otimes \hat{\phi}(\mathbf{x}_D, t), \quad (4.18)$$

where the switching functions is a combination of the switching functions used in the quantum superposition case (4.1). Once the field is traced out, the density matrix of AB at the final time is given by

$$\hat{\rho}_{AB}^{(DS)} = \begin{pmatrix} 1 - (P_A^{(+)} + P_B^{(+)}) & 0 & 0 & X^{(DS)*} \\ 0 & P_B^{(+)} & C^{(+)*} & 0 \\ 0 & C^{(+)} & P_A^{(+)} & 0 \\ X^{(DS)} & 0 & 0 & 0 \end{pmatrix}, \quad (4.19)$$

where

$$X^{(DS)} = \frac{1}{4}(X_{00} + X_{11} + X_{01} + X_{10}) \quad (4.20)$$

We see that (i) the density matrix contains the field correlations between all the four spacetime regions in which the detectors are active, analogously to the case in which the

control qubit is measured in $|+\rangle$. However, in this case this is due to the fact that the detectors are actually active twice each, in contrast to the quantum case in which each detector (in a quantum superposition) is used only once. (ii) The $(1, 4), (4, 1)$ terms of the density matrix are different from the corresponding ones in Eq.(4.15). Thus, the double-switch scenario is distinguishable from the quantum coherent one already at the perturbative level.

If a particular case of superposition is chosen (say CE), then it can be shown that if the $(1, 4), (4, 1)$ terms in the quantum case are equal to $2(X_{00} + X_{11})$ in the double-switch scenario, then the remaining cross terms, $2(X_{01} + X_{10})$, will be equal to the $(1, 4), (4, 1)$ terms in the other case of superposition (in this case PF) with the same time delay. Mathematically we have

$$\hat{\rho}_{AB}^{(\text{DS})} = \frac{1}{2} \left(\hat{\rho}_{AB}^{(+\text{PF})} + \hat{\rho}_{AB}^{(+\text{CE})} \right) \quad (4.21)$$

indicating that the double-switch scenario can be decomposed into the two quantum cases we considered.

4.2 Entanglement harvesting

Let us now look at instances in which the additional correlations in (4.15), stemming from the quantum interference, have relevant effects. From now on we consider $(3 + 1)$ -dimensional Minkowski space and take the two UDW detectors to be separated by a distance of $S := |\mathbf{x}_A - \mathbf{x}_B|$

4.2.1 Cosine switching

First, we consider a cosine switching function

$$\chi_D(t) = \begin{cases} \cos\left(\frac{2(t-T_D)}{\eta}\right), & -\frac{\pi}{4} \leq \frac{t-T_D}{\eta} \leq \frac{\pi}{4} \\ 0, & \text{otherwise} \end{cases} \quad (4.22)$$

where η characterizes the width of the function. We choose a function with compact support so that we can easily determine when the detectors are in causal contact. With

this choice of switching, the matrix elements become

$$\begin{aligned}
P_{D,ij} &= \frac{\lambda^2 \eta^2}{4\pi^2} \int_0^\infty dk \, k \frac{e^{i(k+\Omega_D)(T_{D,i}-T_{D,j})}}{(1 - (\eta/2)^2(k + \Omega_D)^2)^2} \cos^2\left(\frac{\pi\eta}{4}(k + \Omega_D)\right) \\
C_{ij} &= \frac{\lambda^2 \eta^2}{4\pi^2} e^{i(\Omega_A T_{A,i} - \Omega_B T_{B,j})} \int_0^\infty dk \frac{e^{ik(T_{A,i} - T_{B,j})}}{(1 - (\eta/2)^2(k + \Omega_A)^2)(1 - (\eta/2)^2(k + \Omega_B)^2)} \\
&\quad \times \frac{\sin(kS)}{S} \cos\left(\frac{\pi\eta}{4}(k + \Omega_A)\right) \cos\left(\frac{\pi\eta}{4}(k + \Omega_B)\right) \\
X_{ij} &= \int_0^\infty dk \, M_{AB,ij}(k) + M_{BA,ij}(k)
\end{aligned} \tag{4.23}$$

Due to the time ordering, expression for $M_{DE,ij}(k)$ depends on the time delay between detector D and detector E (for $D = A, B$ and $E = A, B$ and $D \neq E$). Explicit expressions are provided in appendix A.1.

In Fig. 4.3, we plot the concurrence of the density matrix $\hat{\rho}_{AB}$ as a function of detector separation and temporal separation for four different spacetime configurations. We find that the detectors can harvest entanglement when they are spacelike separated in the case of the quantum controlled CE superposition (upper left). The lower left plot in Fig. 4.3 corresponds to a classical mixture of a classical mixture of A before B and B before A , and has non-zero concurrence over a much smaller range of the parameter space. This is a striking demonstration of how quantum temporal interference effects can affect the ability of UDW detectors to harvest entanglement.

Since $\mathcal{C} = |X| - \sqrt{P_A P_B}$, and based on the form of the density matrices (4.11) and (4.15), we conclude that $P^{(\text{Tr})} \geq P^{(+)}$. Indeed, this is always true regardless of the switching function (see appendix A.2 for a proof). The interference terms $P_{D,ij}$ lower the transition probability of the detector.

The upper right plot in Fig. 4.3 plots the concurrence for a pair of detectors in the case of the quantum controlled PE superposition. This type of quantum control does not allow for spacelike entanglement harvesting in the parameter space we have chosen. However, there are regions where concurrence is higher than in the case of the CE superposition. The detectors in these two cases have identical transition probabilities; therefore this difference is solely due to differences in the coherence term $|X|$.

The lower right plot in Fig. 4.3 plots the concurrence of a pair of detectors which couple

to the field through the double switching function

$$\chi_D(t) = \frac{1}{2}(\chi_{D,0}(t) + \chi_{D,1}(t)) \quad (4.24)$$

where the $\chi_{D,i}(t)$ are the switching functions used in the quantum superposition case. Spacelike entanglement harvesting is not possible for this choice of switching. We also note based on the look of this plot as compared to the other three in the figure, that from the perspective of entanglement harvesting, double switching is distinct from the quantum controlled switching.

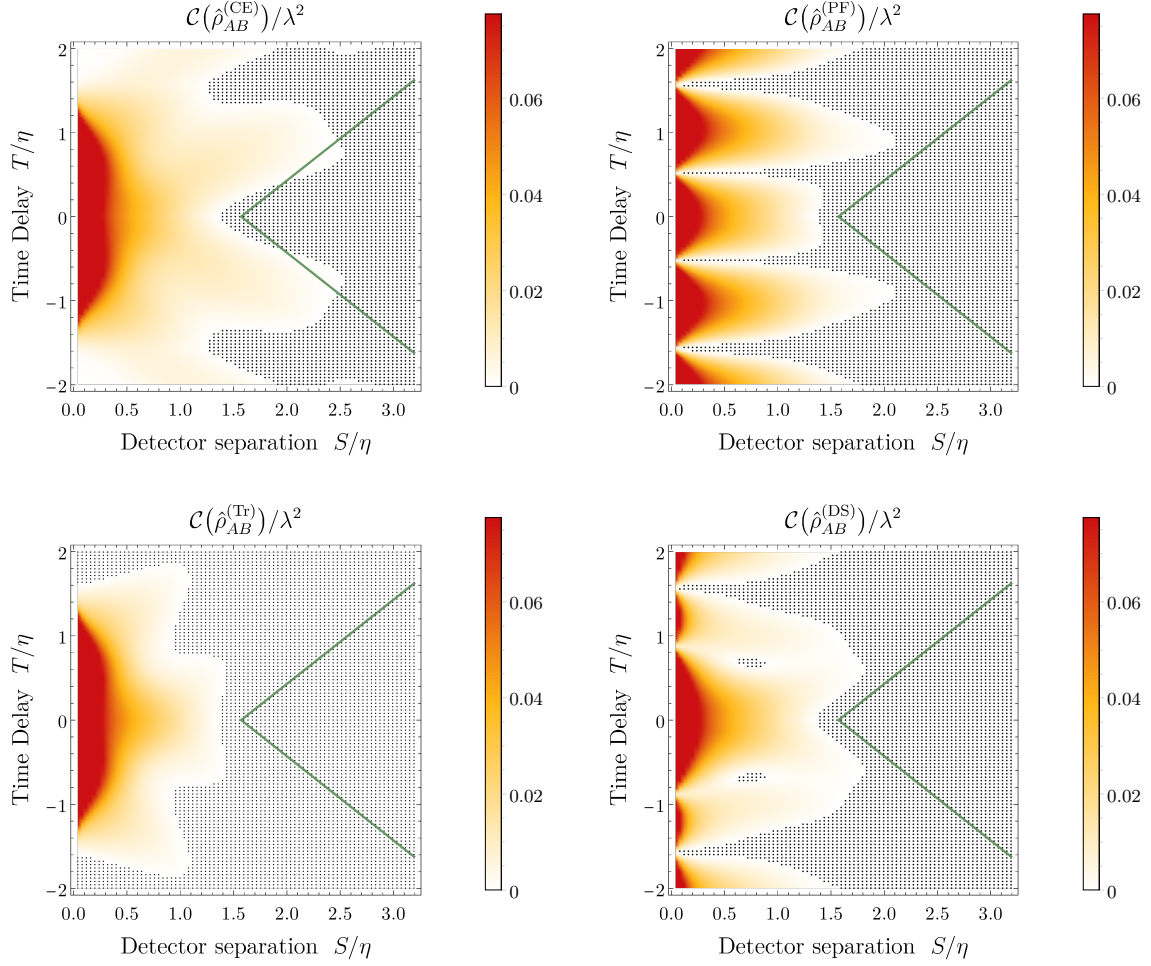


Figure 4.3: The concurrence of the perturbative reduced density matrix of two point-like detectors that couple to the field with a cosine switching function in a (*upper left*) quantum controlled CE superposition, (*upper right*) quantum controlled PE superposition, (*lower left*) a classical mixture of A before B and B before A and (*lower right*) a double switching function (4.24) plotted as a function of the spatial separation S and the temporal separation $T := (T_A - T_B)$ of the detectors. The green lines mark the region of spacelike separation of the detectors and the black dots represent the regions where no entanglement can be harvested. For the chosen parameters, spacelike entanglement harvesting is possible only when the detectors are in the CE superposition. The energy gaps of the detectors are set equal $\Omega_A\eta = \Omega_B\eta = 3$. The elements of the reduced density matrix have been numerically calculated to 15 significant figures.

4.2.2 Cheating the no-go theorem

Now we consider a δ -switching function given by

$$\chi_{D,i}(t) = \eta \delta(t - T_{D,i}) \quad (4.25)$$

which allows us to calculate the final form of the detector-control system without the need of a perturbative expansion by following the method of [77, 76] and briefly outlined in section 2.2.2. Time ordering requires that the switching order between detectors *on both* branches of the superposition be fixed for the calculation, so we must consider the PF case separate from the CE case.

For convenience we define

$$\hat{H}_{D,i}(t) = \lambda \chi_{D,i}(t) (e^{i\Omega_D t} \hat{\sigma}_D^+ + e^{-i\Omega_D t} \hat{\sigma}_D^-) \otimes \int d^3 \mathbf{x} F_D(\mathbf{x} - \mathbf{x}_D) \hat{\phi}(\mathbf{x}, t) \quad (4.26)$$

so that

$$\hat{H}_I(t) = \sum_{D \in \{A, B\}} \sum_{i \in \{0, 1\}} \hat{H}_{D,i}(t) \otimes |i\rangle_C \langle i| \quad (4.27)$$

Notice that a spatial smearing function $F_D(\mathbf{x})$ has been added to avoid any divergences in $\hat{\rho}_{ABC}$.

First, we consider the PF case under the assumption that $T_{A,0} \leq T_{B,0} \leq T_{A,1} \leq T_{B,1}$. The time evolution operator can be simplified to

$$\begin{aligned} \hat{U} &= \exp \left(-i \hat{H}_{B,1}(T_{B,1}) \otimes |1\rangle_C \langle 1| \right) \exp \left(-i \hat{H}_{A,1}(T_{A,1}) \otimes |1\rangle_C \langle 1| \right) \\ &\quad \times \exp \left(-i \hat{H}_{B,0}(T_{B,0}) \otimes |0\rangle_C \langle 0| \right) \exp \left(-i \hat{H}_{A,0}(T_{A,0}) \otimes |0\rangle_C \langle 0| \right) \\ &= U_0 \otimes |0\rangle_C \langle 0| + U_1 \otimes |1\rangle_C \langle 1| \end{aligned} \quad (4.28)$$

where

$$\begin{aligned} \hat{U}_i &= \left(\mathbb{1}_A \otimes \mathbb{1}_B \otimes \cosh(\hat{X}_{B,i}) \cosh(\hat{X}_{A,i}) \right) + \left(\mathbb{1}_A \otimes \hat{\mu}_B(T_{B,i}) \otimes \sinh(\hat{X}_{B,i}) \cosh(\hat{X}_{A,i}) \right) \\ &\quad + \left(\hat{\mu}_A(T_{A,i}) \otimes \mathbb{1}_B \otimes \cosh(\hat{X}_{B,i}) \sinh(\hat{X}_{A,i}) \right) \\ &\quad + \left(\hat{\mu}_A(T_{A,i}) \otimes \hat{\mu}_B(T_{B,i}) \otimes \sinh(\hat{X}_{B,i}) \sinh(\hat{X}_{A,i}) \right) \end{aligned} \quad (4.29)$$

and

$$\hat{X}_{D,i} = -i\lambda\eta \int d^3\mathbf{x} F_D(\mathbf{x} - \mathbf{x}_D) \hat{\phi}(\mathbf{x}, T_{D,i}) \quad (4.30)$$

$$\hat{\mu}_D(t) = (e^{i\Omega_D t} \hat{\sigma}_+ + e^{-i\Omega_D t} \hat{\sigma}_-). \quad (4.31)$$

We consider the same initial state as Eqs.(4.2), and the final state is again $\hat{\rho}_f = \hat{U} \hat{\rho}_0 \hat{U}^\dagger$. Tracing out the field degrees of freedom gives

$$\begin{aligned} \hat{\rho}_{ABC}^{(\text{PF})} = \frac{1}{32} \sum_{i,j} \left\{ \sum_{w,x,y,z} \left[e^{\frac{i}{2}\Omega_A[(1-z)T_{A,i} - (1-x)T_{A,j}]} e^{\frac{i}{2}\Omega_B[(1-y)T_{B,i} - (1-w)T_{B,j}]} \right. \right. \\ \times f_{A,i} f_{A,j} f_{B,i} f_{B,j} \sum_{p,q,r,s} \left(x^{(1+p)/2} w^{(1+q)/2} y^{(1-r)/2} z^{(1-s)/2} \right. \\ \times e^{pq(\frac{i}{2}\theta_{A,j;B,j} + \omega_{A,j;B,j})} e^{pr(\frac{i}{2}\theta_{A,j;B,i} + \omega_{A,j;B,i})} e^{ps(\frac{i}{2}\theta_{A,j;A,i} + \omega_{A,j;A,i})} \\ \times e^{qr(\frac{i}{2}\theta_{B,j;B,i} + \omega_{B,j;B,i})} e^{-qs(\frac{i}{2}\theta_{A,i;B,j} - \omega_{A,i;B,j})} e^{-rs(\frac{i}{2}\theta_{A,i;B,i} - \omega_{A,i;B,i})} \Big) \\ \left. \times \left| \frac{1-z}{2} \right\rangle_A \left\langle \frac{1-x}{2} \right| \otimes \left| \frac{1-y}{2} \right\rangle_B \left\langle \frac{1-w}{2} \right| \right] \otimes |i\rangle_C \langle j| \Big\} \end{aligned} \quad (4.32)$$

where $i, j \in \{0, 1\}$ and $p, q, r, s, w, x, y, z \in \{+1, -1\}$.

$$f_{D,i} = \exp \left(-\frac{1}{2} \int d^3\mathbf{k} |\beta_{D,i}(\mathbf{k})|^2 \right) \quad (4.33)$$

$$\theta_{D,i;E,j} = i \int d^3\mathbf{k} \left(\beta_{D,i}^*(\mathbf{k}) \beta_{E,j}(\mathbf{k}) - \beta_{D,i}(\mathbf{k}) \beta_{E,j}^*(\mathbf{k}) \right) \quad (4.34)$$

$$\omega_{D,i;E,j} = -\frac{1}{2} \int d^3\mathbf{k} \left(\beta_{D,i}^*(\mathbf{k}) \beta_{E,j}(\mathbf{k}) + \beta_{D,i}(\mathbf{k}) \beta_{E,j}^*(\mathbf{k}) \right) \quad (4.35)$$

and

$$\beta_{D,i}(\mathbf{k}) = \frac{i\lambda\eta}{\sqrt{2|\mathbf{k}|}} \left(\int d^3\mathbf{x} F_D(\mathbf{x}) e^{i\mathbf{x} \cdot \mathbf{k}} \right) e^{-i(|\mathbf{k}|T_{D,i} - \mathbf{k} \cdot \mathbf{x}_D)} \quad (4.36)$$

for $D, E \in \{A, B\}$.

Next, we consider the CE scenario. The time evolution operator is constructed in a similar way to the PF case, this time using the assumption that $T_{A,0} \leq T_{B,1} \leq T_{A,1} \leq T_{B,0}$. The resulting detector-control density matrix for is

$$\begin{aligned}
\hat{\rho}_{ABC}^{(\text{CE})} = & \frac{1}{32} \sum_{i,j} \left\{ \sum_{w,x,y,z} \left[e^{\frac{i}{2}\Omega_A[(1-iy-(1-i)z)T_{A,i}-(1-jw-(1-j)x)T_{A,j}]} \right. \right. \\
& \times e^{\frac{i}{2}\Omega_B[(1-(1-i)y-iz)T_{B,i}-(1-(1-j)w-jx)T_{B,j}]} f_{D,j} f_{E,j} f_{F,i} f_{G,i} \\
& \times \sum_{p,q,r,s} \left(x^{(1+p)/2} w^{(1+q)/2} y^{(1-r)/2} z^{(1-s)/2} \right. \\
& \times e^{pq(\frac{i}{2}\theta_{E,j;D,j}+\omega_{E,j;D,j})} e^{pr(\frac{i}{2}\theta_{E,j;F,i}+\omega_{E,j;F,i})} e^{ps(\frac{i}{2}(\theta_{E,j;G,i}+\omega_{E,j;G,i}))} \\
& \times e^{qr(\frac{i}{2}\theta_{D,j;F,i}+\omega_{D,j;F,i})} e^{qs(\frac{i}{2}\theta_{D,j;G,i}+\omega_{D,j;G,i})} e^{rs(\frac{i}{2}\theta_{F,i;G,i}+\omega_{F,i;G,i})} \Big) \\
& \times \left| \frac{1-iy-(1-i)z}{2} \right\rangle_A \left\langle \frac{1-jw-(1-j)x}{2} \right| \\
& \otimes \left| \frac{1-(1-i)y-iz}{2} \right\rangle_B \left\langle \frac{1-(1-j)w-jx}{2} \right| \Big] \otimes |i\rangle_C \langle j| \Big\}. \tag{4.37}
\end{aligned}$$

The labels D, E, F, G depend on the values of i and j and are summarized in Eq. (4.38) below.

$$\begin{aligned}
D &= \begin{cases} B, & j = 0 \\ A, & j = 1 \end{cases} & E &= \begin{cases} A, & j = 0 \\ B, & j = 1 \end{cases} \\
F &= \begin{cases} B, & i = 0 \\ A, & i = 1 \end{cases} & G &= \begin{cases} A, & i = 0 \\ B, & i = 1 \end{cases} \tag{4.38}
\end{aligned}$$

Once the control qubit is measured (or traced out), the density matrices given in Eqs. (4.37) and (4.32) will take the form of Eq. (2.37) and are then used to compute the entanglement between detectors A and B shown in Fig. 4.4.

We find that when the detectors couple to the field with a δ -switching in quantum controlled superposition, they are able to become entangled, appearing to violate the no-go theorem. This is shown in Fig. 4.4. Physically, this is likely in part due to reduced local noise, similar to what was seen in section 4.2.1. Although the detectors only switch once in each branch of the superposition, the two branches do interact, which likely plays a

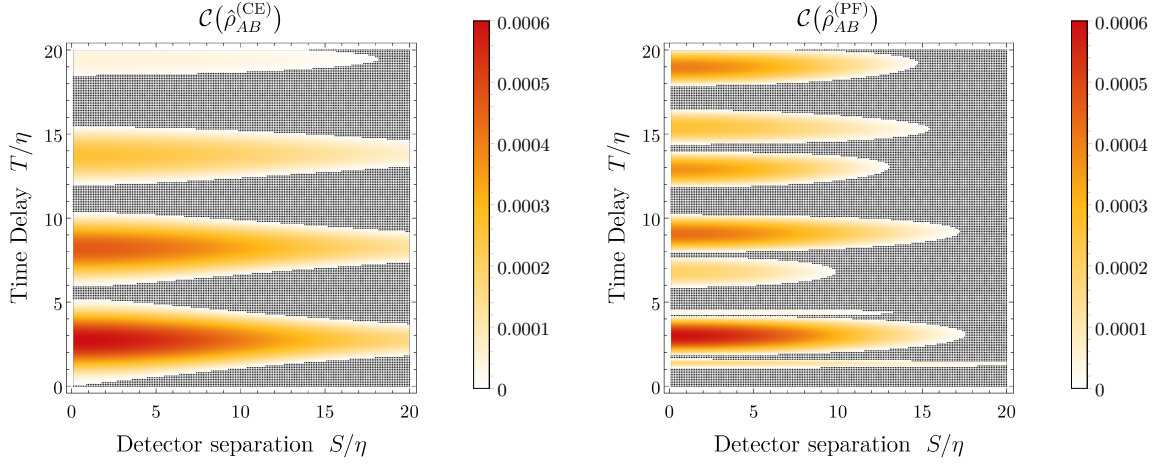


Figure 4.4: Apparent violation of the no-go theorem for δ -switched detectors with (*left*) CE superposition and (*right*) PF superposition. The black dots represent the regions where no entanglement can be harvested. The detectors have a spatial profile of $F(\mathbf{x}) = \frac{\exp[-(\mathbf{x}/9)^2]}{(9\sqrt{\pi})^3}$, identical energy gaps of $\Omega_A\eta = \Omega_B\eta = 1$ and couple to the field with a strength of $\lambda = 1$. The elements of the reduced density matrix have been numerically calculated to 15 significant figures.

role in boosting non-local correlations of the detectors. Mathematically, the interaction between branches can be easily seen in the time ordering requirements of the time evolution operator (4.28) which depends on the ordering across both branches of the superposition. As a result, the unitary describing the detectors' interaction is complex enough to allow for quantum controlled switching to get around the no-go theorem [64].

4.3 Summary of results

Indefinite casual order, or more generally, a quantum controlled superposition of detector activation times, affect the entanglement harvesting capabilities of UDW detectors locally coupled to quantum fields. This happens under a variety of scenarios, producing the striking result that spacelike entanglement harvesting becomes possible in situations not possible for classical mixtures. We note that the additional entanglement does not come directly from the control qubit. If there were no field (i.e. $\hat{\phi} \rightarrow \mathbb{I}$) in Eq. (4.1), then after the interaction, the detectors would remain separable.

It is crucial to note that, the same field correlations entering Eq. (4.16) would also

be present if both detectors were active at two distinct times (and in the same spacetime regions as for the quantum superposition case), a scenario we call ‘double switching’. However, the reduced state of AB would differ from $\hat{\rho}_{AB}^{(+)}$. Perturbatively, the difference would be in the X terms, meaning that the entanglement harvesting would be significantly different. Additionally, for double switching violations of the no-go theorem are trivially possible, since the theorem requires each detector being active only once [76]. Quantum temporal superposition is therefore in principle distinguishable from the case in which each detector is twice activated (at least in certain parameter ranges), allowing for a clear way to unambiguously state that the detectors in the superposition are activated only once.

Chapter 5

Entangling Detectors in Anti-de Sitter Space

In this chapter we present a detailed study of entanglement harvesting in anti-de Sitter (AdS) space. Quantum fields on AdS space are of particular interest because of the rapid development of the anti-de Sitter/conformal field theory correspondence which posits a connection between conformal field theories and the bulk geometry of AdS [46]. We note that entanglement harvesting in the bulk may have implications for entanglement entropy in the context of the AdS/CFT correspondence conjecture. If one considers quantum corrections to the well-known Ryu-Takayanagi prescription [70] then the entanglement of quantum fields in the bulk across the Ryu-Takayanagi surface contributes to the entanglement entropy of the boundary CFT [22]. Understanding the nature of entanglement in the bulk is likely to be important for understanding the quantum corrections to the entanglement in the boundary.

We shall investigate how entanglement harvesting depends on the AdS length and the influence of the boundary conditions satisfied by the field at infinity, comparing our results to the flat spacetime counterpart. These results will also serve as a baseline for results obtained for the BTZ black hole presented in chapter 6; it for this reason we shall consider $(2 + 1)$ -dimensional AdS spacetime. This lower-dimensional setting has the advantage of yielding insight into the significant physical processes with relative computational ease and efficiency.

Throughout this chapter we adopt the mostly positive convention for the metric signature $(-1, 1, 1)$ and use natural units $\hbar = c = 8G = 1$. We have also introduced the dimensional coupling strength $\tilde{\lambda} := \lambda\sqrt{\sigma}$ for plotting convenience.

5.1 Entanglement harvesting in flat spacetime

To facilitate comparison between the entanglement harvesting protocol for static detectors in AdS_3 with the flat space limit $\ell \rightarrow \infty$, we evaluate both the transition probability of a single detector and the matrix element X appearing in the joint state of two detectors interacting with a real massless scalar field in (2+1)-dimensional Minkowski space.

For static detectors and with Gaussian switching functions with no time delay,

$$\chi_D(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right) \quad (5.1)$$

we can evaluate the transition probability of a single detector and matrix element X directly from their definition in Eqs. (2.24) and (2.26), with the result

$$P_D = \frac{\lambda^2 \sigma \sqrt{\pi}}{4} (1 - \text{erf}(\sigma\Omega)), \quad (5.2)$$

$$X = -\frac{\sigma \lambda^2}{4\sqrt{\pi}} \exp\left[-\frac{S^2}{8\sigma^2} - \sigma^2 \Omega^2\right] \left(\pi I_0(S^2/(8\sigma^2)) - i K_0(S^2/(8\sigma^2))\right) \quad (5.3)$$

where $S := |R_B - R_A|$ and K_0 and I_0 are zeroth order modified Bessel functions of the first and second kind, respectively. These expressions were in producing the flat space limits depicted in Figs. 5.4 and 5.5.

5.2 The AdS_3 Wightman function

AdS space is a maximally symmetric spacetime of constant negative curvature that solves Einstein's equations with negative cosmological constant $\Lambda := -1/\ell^2$. The (2+1)-dimensional AdS (AdS_3) geometry can be described by a 3-dimensional hyperboloid

$$X_1^2 + X_2^2 - T_1^2 - T_2^2 = -\ell^2, \quad (5.4)$$

embedded in a flat 4-dimensional geometry

$$dS^2 = dX_1^2 + dX_2^2 - dT_1^2 - dT_2^2, \quad (5.5)$$

with coordinates (X_1, X_2, T_1, T_2) [13].

Global coordinates $(\bar{\tau}, \varrho, \phi)$ covering AdS_3 are obtained via the transformation

$$\begin{aligned} X_1 &= \ell \sinh \varrho \sin \phi, & X_2 &= \ell \sinh \varrho \cos \phi, \\ T_1 &= \ell \cosh \varrho \cos \bar{\tau}, & T_2 &= \ell \cosh \varrho \sin \bar{\tau}, \end{aligned} \quad (5.6)$$

in which the induced metric on AdS_3 becomes

$$ds^2 = \ell^2 \left(-\cosh^2 \varrho d\bar{\tau}^2 + d\varrho^2 + \sinh^2 \varrho d\phi^2 \right), \quad (5.7)$$

where $\bar{\tau} \in \mathbb{R}$, $\varrho \in \mathbb{R}^+$, and $\phi \in [0, 2\pi)$ ¹. Under the coordinate transformation $t := \ell \bar{\tau}$ and $r := \ell \sinh \varrho$, the AdS_3 metric may be cast in the familiar form

$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right) dt^2 + \frac{dr^2}{1 + r^2/\ell^2} + r^2 d\phi^2. \quad (5.8)$$

It will be useful for what follows to introduce the function $d(R_1, R_2)$ denoting the proper distance between the spacetime points $x_1 = (t, R_1, \Phi)$ and $x_2 = (t, R_2, \Phi)$,

$$d(R_1, R_2) := \ell \ln \left[\frac{R_2 + \sqrt{R_2^2 + \ell^2}}{R_1 + \sqrt{R_1^2 + \ell^2}} \right]. \quad (5.9)$$

We shall consider a massless conformally coupled real scalar field on AdS_3 , since its vacuum Wightman function takes the particularly simple form [13]

$$W_{\text{AdS}}^{(\zeta)}(x, x') = \frac{1}{4\pi\ell\sqrt{2}} \left(\frac{1}{\sqrt{\sigma(x, x')}} - \frac{\zeta}{\sqrt{\sigma(x, x') + 2}} \right), \quad (5.10)$$

where

$$\sigma(x, x') := \frac{1}{2\ell^2} \left[(X_1 - X'_1)^2 - (T_1 - T'_1)^2 + (X_2 - X'_2)^2 - (T_2 - T'_2)^2 \right] \quad (5.11)$$

is the squared geodesic distance between X and X' in the embedding space $\mathbb{R}^{2,2}$ and the parameter $\zeta \in \{1, 0, -1\}$ specifies either Dirichlet ($\zeta = 1$), transparent ($\zeta = 0$), or Neumann ($\zeta = -1$) boundary conditions satisfied by the field at spatial infinity [6]. Furthermore, it is the vacuum state described by this Wightman function from which the

¹Strictly speaking, when obtained from this embedding picture, the time coordinate $\bar{\tau}$ is periodic with period 2π . By allowing $\bar{\tau}$ to take on all real values we are implicitly moving to the universal cover of the space. It is this universal cover that is customarily (and also in this work) referred to as AdS .

Hartle-Hawking vacuum on the BTZ black hole may be constructed [42]; the BTZ black hole being asymptotically AdS space.

Although the general method for computing Wightman functions is to carry out a mode sum over basis functions, the structure of the Wightman function in AdS₃ is simple enough that a combination of analytic and numerical integration is possible. Our methods are complementary to a study of entanglement harvesting in AdS₄ [53], in which mode sums were carried out to evaluate the relevant quantities of interest.

5.3 The detectors

Suppose detectors A and B are kept at distinct fixed positions R_A and R_B , respectively, along a common radial direction. The spacetime trajectories describing such detectors parameterized by their proper time τ_D is

$$x_D(\tau_D) := \left\{ t = \frac{\tau_D}{\gamma_D}, \ r = R_D, \ \phi = \Phi_0 \right\}, \quad (5.12)$$

$$\gamma_D := \sqrt{(R_D/\ell)^2 + 1} \quad (5.13)$$

We take the switching functions of the detectors to be Gaussians described by

$$\chi_A(\tau_A) = \exp\left(-\frac{(\tau_A + \gamma_A t_0/2)^2}{2\sigma^2}\right) \quad \chi_B(\tau_B) = \exp\left(-\frac{(\tau_B - \gamma_B t_0/2)^2}{2\sigma^2}\right). \quad (5.14)$$

where $t_0 > 0$ corresponds to detector A interacting with the field before detector B and vice versa for $t_0 < 0$, where the time delay t_0 is defined in the field frame. The width, σ , quantifies the amount of time the detector effectively interacts with the field.

To compute the transition probability P_D and coherence term, X we evaluate the Wightman function in Eq. (5.10) along the detectors' trajectories, substitute the resulting expression into Eqs. (2.24) and (2.26), and express the result in a form that lends itself to being evaluated numerically, all of which is done explicitly in Appendix B. The resulting

expression for the transition probability is

$$P_D = \frac{\lambda^2 \sigma}{2\sqrt{2\pi}} \left(-\text{PV} \int_0^\infty dy \frac{e^{-a_D y^2} \sin(\beta_D y)}{\sqrt{2} \sin(y/2)} + \frac{\pi}{\sqrt{2}} \sum_{n \in \mathbb{Z}} (-1)^n \cos(2n\pi\beta_D) e^{-4n^2\pi^2 a_D} \right. \\ \left. - \zeta \text{Re} \sum_{n \geq 0} (-1)^n \int_{\theta_{D_n}}^{\theta_{D_{n+1}}} dy \frac{e^{-a_D y^2} e^{-i\beta_D y}}{\sqrt{\cos y + \alpha_D^+}} \right), \quad (5.15)$$

where PV denotes that the principal value of the integral should be taken and we have defined

$$a_D := \gamma_D^2 \ell^2 / 4\sigma^2 \quad \beta_D := \gamma_D \ell \Omega \\ \alpha_D^+ := [-(R_D/\ell)^2 + 1] / \gamma_D^2 \\ \theta_{D_n} := \max[0, \arccos \alpha_D^+ + (2n-1)\pi]. \quad (5.16)$$

The resulting expression for the matrix element X is

$$X = -\frac{\lambda^2 \sigma}{2\sqrt{\pi}} K_X \sum_{n \geq 0} (-1)^n \left(\int_{\theta_{X_n}^-}^{\theta_{X_{n+1}}^-} dy \frac{e^{-a_X y^2} \cosh((\Delta_T + i\beta_X)y)}{\sqrt{\cos y + \alpha_X^-}} \right. \\ \left. - \zeta \int_{\theta_{X_n}^+}^{\theta_{X_{n+1}}^+} dy \frac{e^{-a_X y^2} \cosh((\Delta_T + i\beta_X)y)}{\sqrt{\cos y + \alpha_X^+}} \right) \quad (5.17)$$

where

$$K_X := \sqrt{\frac{\gamma_A \gamma_B}{\gamma_A^2 + \gamma_B^2}} \exp\left(-\frac{\Omega^2 \sigma^2 (\gamma_A + \gamma_B)^2}{2(\gamma_A^2 + \gamma_B^2)}\right) \exp\left(-\frac{t_0^2}{2\sigma^2} \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} + i \frac{\Omega t_0}{2} \frac{(\gamma_A + \gamma_B)^2 (\gamma_A - \gamma_B)}{\gamma_A^2 + \gamma_B^2}\right) \\ a_X := \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} \frac{\ell^2}{2\sigma^2} \quad \Delta_T := -\frac{t_0 \ell}{\sigma^2} \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} \\ \beta_X := \frac{\gamma_A \gamma_B (\gamma_A - \gamma_B)}{\gamma_A^2 + \gamma_B^2} \ell \Omega \quad \alpha_X^\pm := \frac{1}{\gamma_A \gamma_B} \left(-\frac{R_A R_B}{\ell^2} \pm 1 \right) \\ \theta_{X_n}^\pm := \max[0, \arccos \alpha_X^\pm + (2n-1)\pi]. \quad (5.18)$$

In what follows, both the transition probability and matrix element X given above were evaluated numerically in Mathematica to a precision of 10^{-17} using the “double exponential” and “double exponential oscillatory” integration methods.

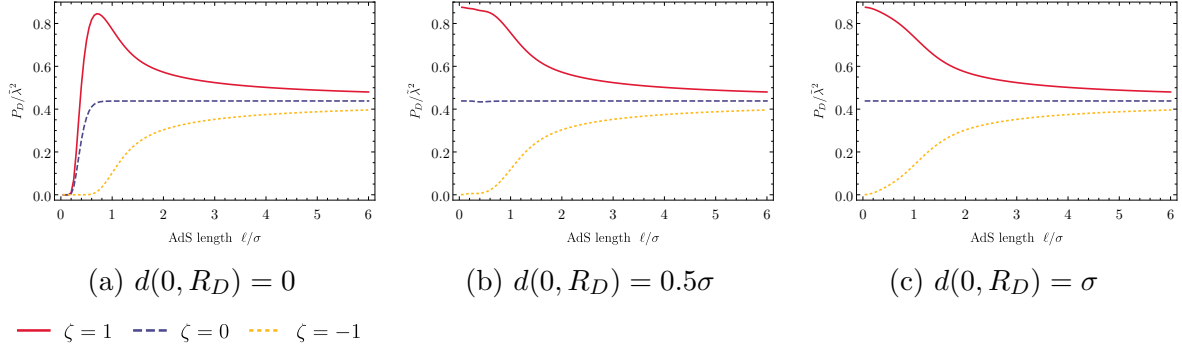


Figure 5.1: The transition probability P_D of a detector with energy gap $\Omega\sigma = 1/100$ is plotted as a function of the AdS length ℓ/σ for each of the three boundary conditions $\zeta = \{1, 0, -1\}$. Plots (a), (b), and (c) correspond to detectors located at different proper distances $d(0, R_D)/\sigma$ away from the origin.

5.4 The transition probability

We now evaluate the transition probability of a detector, given in Eq. (5.15), for a wide range of scenarios. We begin by noting that the transition probability in Eq. (5.15) can be evaluated perturbatively in σ/ℓ . We find that the leading contributions take a very simple form

$$\begin{aligned} \frac{P_D}{\sigma\lambda^2} = & \frac{\sqrt{\pi}}{4} (1 - \text{erf}(\sigma\Omega)) - \zeta \frac{e^{-\sigma^2\Omega^2}}{4} \left(\frac{\sigma}{\ell}\right) - \frac{\sigma\Omega e^{-\sigma^2\Omega^2}}{24} \left(\frac{\sigma}{\ell}\right)^2 - \zeta \frac{e^{-\sigma^2\Omega^2} (1 - 2\sigma^2\Omega^2)}{16} \left(\frac{\sigma}{\ell}\right)^3 \\ & + \frac{\sigma\Omega e^{-\sigma^2\Omega^2}}{2880} \left[120 \frac{d(0, R_D)^2}{\sigma^2} + 14\sigma^2\Omega^2 - 21 \right] \left(\frac{\sigma}{\ell}\right)^4 + \mathcal{O}\left(\left(\frac{\sigma}{\ell}\right)^5\right). \end{aligned} \quad (5.19)$$

The first term appearing above is simply the transition probability in (2+1)-dimensional Minkowski space (flat space). The proper distance $d(0, R_D)$ first enters at order $(\sigma/\ell)^4$ and its appearance can be understood as the manifestation of redshift (time dilation) and non-vanishing acceleration due to the detector being on a static trajectory.

The perturbative expression in Eq. (5.19) is particularly useful for understanding how the transition probability in AdS_3 deviates from the flat space result. We see that the leading order correction to the flat space result is proportional to ζ . This means that the most important factor in the deviation from flat spacetime is due to the boundary conditions satisfied by the field. A small amount of negative curvature will result in the detector clicking more (less) than it would in flat spacetime if the field satisfies Neumann

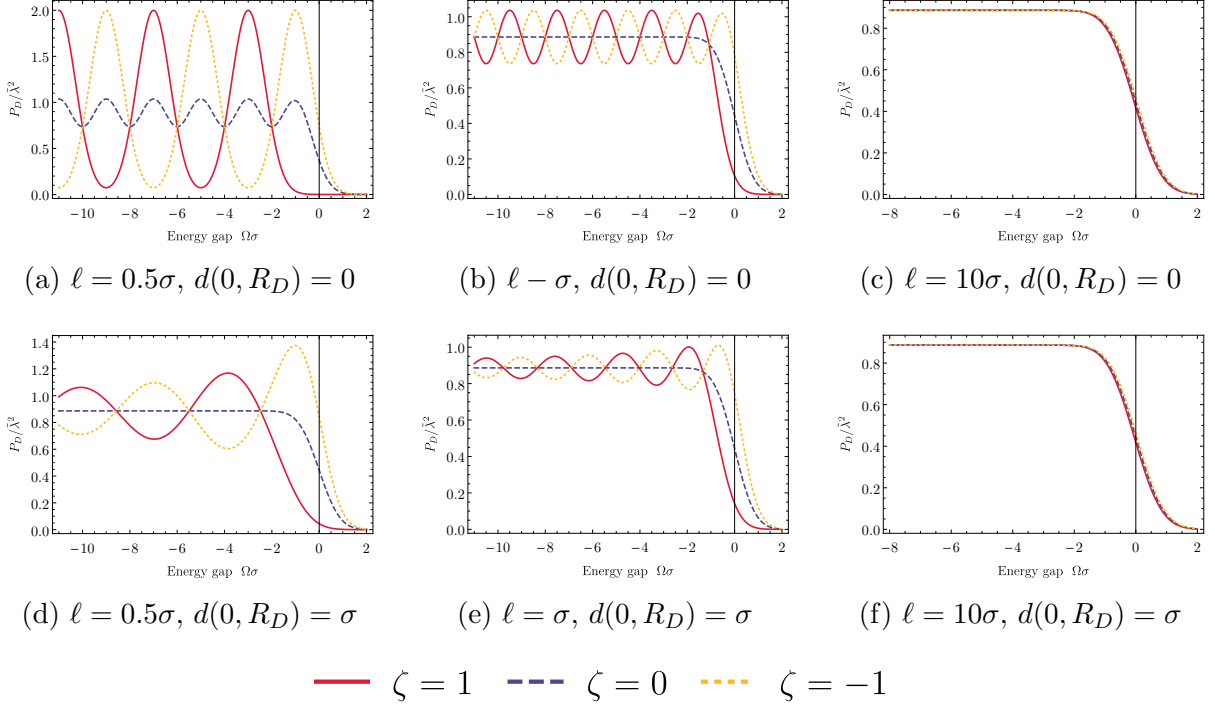


Figure 5.2: The transition probability P_D of a detector is plotted as a function of its energy gap $\Omega\sigma$ for all boundary conditions $\zeta = \{1, 0, -1\}$ and various values of the AdS length ℓ/σ . In plots (a), (b), and (c) the detector is located at the origin, while in plots (d), (e), and (f) the detector is located a proper distance $d(0, R_D) = \sigma$ away from the origin. Negative energy gaps $\Omega\sigma < 0$ correspond to a detector that is prepared in its excited state $|e\rangle_D$ prior to interacting with the field.

(Dirichlet) boundary conditions. If the field satisfies transparent boundary conditions then there is no correction to the flat space result at order σ/ℓ .

The next order correction is also quite interesting. It is independent of the boundary conditions satisfied by the field and so can be thought of as a “universal” contribution due purely to negative curvature. Since this term in the perturbative expansion is always negative for detectors starting in their ground state, we can interpret this as the statement that negative curvature tends to decrease the transition probability of a detector. Conversely, since this correction will always be positive for detectors that are initially excited ($\Omega < 0$), this means excited detectors in negatively curved spaces are more likely to relax to their ground state than they would in flat spacetime.

The perturbative expansion, while insightful, cannot be used when σ/ℓ is large, and there are many interesting phenomena in this regime. To explore this regime we must resort to a full numerical evaluation of the integral in Eq. (5.15) which is used in producing Figs. 5.1-5.3.

As seen upon comparison of the plots in Fig. 5.1, for all boundary conditions, the transition probability P_D is only sensitive to the proper distance $d(0, R_D)$ the detector is from the origin when the switching parameter σ is greater than or comparable to the AdS length ℓ . As ℓ/σ grows, regardless of the boundary conditions, P_D approaches its corresponding value in Minkowski space, indicated by the dashed line. Furthermore, for all boundary conditions, the transition probability of a detector located at the origin vanishes as $\ell/\sigma \rightarrow 0$, whereas for a detector positioned some proper distance away from the origin, the transition probability remains almost the same for any ℓ . The most notable property of a detector located at the origin is that for a field satisfying Dirichlet boundary conditions ($\zeta = 1$), its transition probability reaches a maximum near $\ell \approx 0.7\sigma$.

In Fig. 5.2 the transition probability is plotted as a function of the detector’s energy gap $\Omega\sigma$ for fixed AdS length $\ell = 0.5\sigma$, where $\Omega < 0$ corresponds to the detector initially prepared in its excited state. We observe that for $\Omega > 0$, the transition probability decays to zero exponentially with increasing $\Omega\sigma$ without oscillation, regardless of the boundary condition. Moreover, the decay is much faster for the Dirichlet condition ($\zeta = 1$) than for the other boundary conditions. However for $\Omega < 0$, the transition probability oscillates as a function of $\Omega\sigma$, this oscillation being suppressed by both the transparent boundary condition and for detectors located at large distances $d(0, R_D)$ from the origin. This feature is evidently dependent on dimensionality, and *not* on the general structure of AdS space-time, since the analogous graph in $(3 + 1)$ -dimensions exhibits a transition rate (modulo oscillations) that is roughly proportional to $|\Omega|$ for large negative energy gap [53]. It is well-known that the detector response exhibits different dependence on Ω as a function of

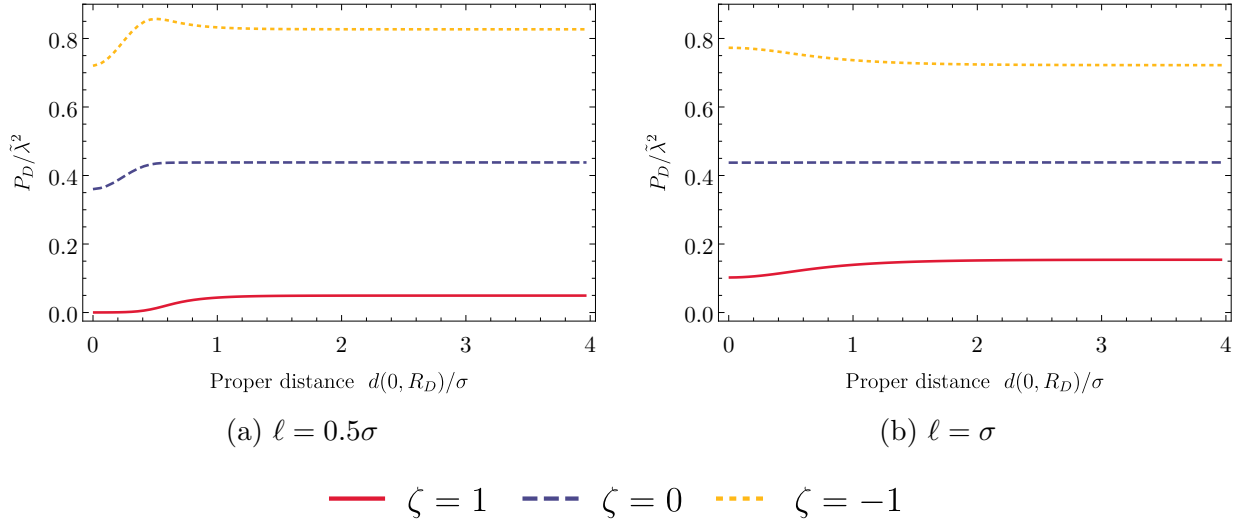


Figure 5.3: The transition probability P_D of a detector with energy gap $\Omega\sigma = 1/100$ is plotted as function of its proper distance from the origin for an AdS length of (a) $\ell = 0.5\sigma$ and (b) $\ell = \sigma$ for all boundary conditions $\zeta = \{1, 0, -1\}$.

spacetime dimension, and so this difference is not surprising. The dimension dependence arises due to the short distance behaviour of the Wightman function [32] or, equivalently, due to the energy scaling of the density of states of the field quanta sensed by the detector which goes as Ω^{d-3} (see, e.g., [86]).

In Fig. 5.3, we illustrate the behaviour of the transition probability for a detector located at different spatial positions in AdS_3 . The transition probability, regardless of the boundary conditions, is hardly influenced by the change of position if ℓ/σ is not vanishingly small. This is to be expected: for large AdS lengths the detector is highly localized in spacetime and so any change in the AdS curvature negligibly affects the detector. It is only when the AdS length is comparable to the width of the switching function that we see a dependence of transition probability of detector B on its proper distance from the origin.

5.5 Entanglement harvesting

We now consider harvesting entanglement with detector A at the origin and detector B placed at various fixed proper distances from A , and for now we assume no relative time delay in the detectors' switching functions ($t_0 = 0$). The transition probabilities P_A

and P_B and matrix element X can be obtained numerically using Eqs. (5.15) and (5.17), after which the concurrence given in Eq. (2.53) can be easily evaluated as a measure of the resulting entanglement between the detectors. We find that the effects of curvature, spatial separation, and detector energy gap are notably more dramatic for entanglement harvesting as compared to the transition probability, which we shall now demonstrate.

We first consider how the concurrence depends on the AdS length ℓ/σ . From Fig. 5.4 we see that the concurrence vanishes for all boundary conditions as $\ell/\sigma \rightarrow 0$. The concurrence attains its maximum value in the interval $0.5 \lesssim \ell/\sigma \lesssim 1$, and this maximum is largest if the field satisfies Dirichlet boundary conditions ($\zeta = 1$). We also note that as ℓ grows, the concurrence asymptotes to the flat space value for all boundary conditions, as expected.

A similar behaviour occurs when the energy gap is changed for fixed AdS length. In Fig. 5.5 we observe a peak in concurrence for positive $\Omega\sigma$ for each boundary condition, the peak again being largest for the Dirichlet boundary condition for the value of ℓ/σ considered.

For initially excited detectors ($\Omega < 0$) the decrease in concurrence with increasing $|\Omega|$ is much more rapid than for detectors initially in their ground states. Hence, similar to what we found for $(2+1)$ -dimensional flat space (see section 5.1), it is easier to perform entanglement harvesting using detectors in their ground-state instead of their excited-state.

Next, we analyze the dependence of the concurrence on the proper distance $d(0, R_A)$ detector A is from the origin for a fixed proper separation $d(R_A, R_B)$ of the detectors. Figure 5.6 shows that the large $d(0, R_A)$ behaviour changes depending on the proper separation of the detectors. When the detectors are close together, the concurrence approaches a constant value, but for larger separations, the concurrence rapidly falls to zero with increasing distance from the origin. We also note that the concurrence is maximum when the detectors are close to the origin. In this region they will experience a smaller acceleration and a lower transition probability, which likely leads to the larger value of the concurrence as seen from Eq. (2.53).

Finally, we consider the behaviour of the concurrence for different proper separations $d(R_A, R_B)$ between the two detectors. From the plots shown in Fig. 5.6 we see that that the larger the value of $d(R_A, R_B)$, the less the concurrence, commensurate with analogous results in flat spacetime and consistent with ones expectations from the fall off of the Wightman function in Eq. (5.10) as the spatial distance between x and x' grows. For Dirichlet boundary conditions ($\zeta = 1$) the decay rate is slowest and the complete elimination of entanglement (sudden death) occurs at the largest proper separation, with the opposite true for Neumann boundary conditions ($\zeta = -1$).

Clearly the concurrence exhibits interesting behaviour as a function of AdS length ℓ ,

the detectors’ energy gap Ω , and their proper separation $d(R_A, R_B)$. To illustrate this more fully, we provide density plots of the concurrence as a function of the detectors’ energy gap Ω and proper separation for different boundary conditions in Fig. 5.7. The shaded areas in each plot are regions in the parameter space where no entanglement harvesting is possible. We do not plot values of $\ell > 20\sigma$ as we have found no appreciable quantitative changes for these large values of AdS length.

We see from Fig. 5.7 several common features for all boundary conditions over the range of ℓ/σ . Regions of large concurrence are always in the lower-left corner (smaller energy gap and smaller detector separation), and regions of zero concurrence are in the lower-right corner (smaller energy gap and larger separation). We note also that entanglement harvesting is generally possible at any detector separation for sufficiently large energy gap, albeit in minuscule amounts compared to smaller energy gaps and detector separation. We also note that the boundary of entanglement in the $\{d(R_A, R_B)/\sigma, \Omega\sigma\}$ parameter space is approximately a straight line for large enough ℓ , a feature observed in $(3 + 1)$ -dimensions [53], and also in flat space in all spatial dimensionalities $D \leq 3$, noted in [61], and which we have observed in $(2 + 1)$ -dimensions (though we have not displayed the graph).

Distinct features arise when we consider the boundary of the zero-entanglement region (shaded region in Fig. 5.7). The intersection point of the boundary of this region with a horizontal line at $\Omega = 0$ slowly moves to smaller values of $d(R_A, R_B)/\sigma$ as ℓ/σ gets larger. More interestingly is the behaviour of this boundary as a function of ℓ/σ . We see for intermediate values of ℓ/σ that the shape of this region changes significantly for all boundary conditions at large detector separations.

Most intriguingly is the emergence of an “island” of no entanglement harvesting for intermediate values of $d(R_A, R_B)/\sigma$ and positive values of the detectors energy gap $\Omega\sigma$ for all boundary conditions (see plots (b), (e), and (h) in Fig. 5.7); we will refer to this region as a separability island. In order to see this separability island clearly, we provide a close-up view of this region for Dirichlet boundary conditions ($\zeta = 1$) in Fig. 5.8; other boundary conditions yield qualitatively similar plots. We see that the island is in the region $3.4 \lesssim \Omega\sigma \lesssim 3.85$.

When the value of ℓ/σ is increased from $\ell/\sigma \approx 1$, we find that for all boundary conditions, a “peninsula” of separability forms. A further increase of the AdS length causes the peninsula to stretch out and eventually break off, forming the island of separability. This behaviour is qualitatively the same for all boundary conditions, and we illustrate this behaviour for transparent boundary conditions ($\zeta = 0$) in Fig. 5.9.

Mathematically, the formation of these islands and peninsulas of separability is due to

the behaviour of the matrix element X , which we have plotted along with concurrence in Fig. 5.8b. Over the region of the island, we find that as $d(R_A, R_B)$ increases, X decreases to a minimum and then begins to increase again, while P_B remains nearly constant throughout. The local minimum is the result of two competing effects. When the separation of the detectors is small relative to the AdS length, then we find that the redshift factor γ_B approaches $\gamma_A = 1$ and the two detectors can be approximated to be in $(2 + 1)$ Minkowski space. In this regime, the coherence term falloff exponentially with increasing detector separation. As the detector separation increases further and $\gamma_B > 1$, the value of the coherence term begin to increase again due to the behaviour of K_X , which, provided $|\Omega\sigma| > 1$, has a local maximum at a value of $\gamma_B = \Omega^2\sigma^2 + \sqrt{\Omega^4\sigma^4 - 1}$. Of course, γ_B can equally well be expressed in terms of the detector separation. When considered from this perspective, the detector separation corresponding to the local maximum of K_X increases linearly with the AdS length. The combination of these effects leads to the existence of the island for moderate values of ℓ/σ . If the value of ℓ/σ was too small, then the maximum of K_X would occur when the detectors are very close together. The non-local correlations would then still be much larger than the local noise, leading to a positive concurrence. If the value of ℓ/σ was too large, the increase in K_X could not dominate over the exponential decay, and $|X|$ remains smaller than the noise and the concurrence remains zero.

Summarizing, the shape of the separability island in the $\{d(R_A, R_B)/\sigma, \Omega\sigma\}$ parameter space depends sensitively on the AdS length ℓ , and vanishes for values of $|\ell/\sigma - 2.5| \gtrsim 1$. Outside this region entanglement harvesting is possible because of the influence of K_X and a_X on the matrix element X . We also note that a similar island has been observed in the same general region of parameter space in $(3 + 1)$ -dimensions [53]. It is clear that this phenomenon merits further study.

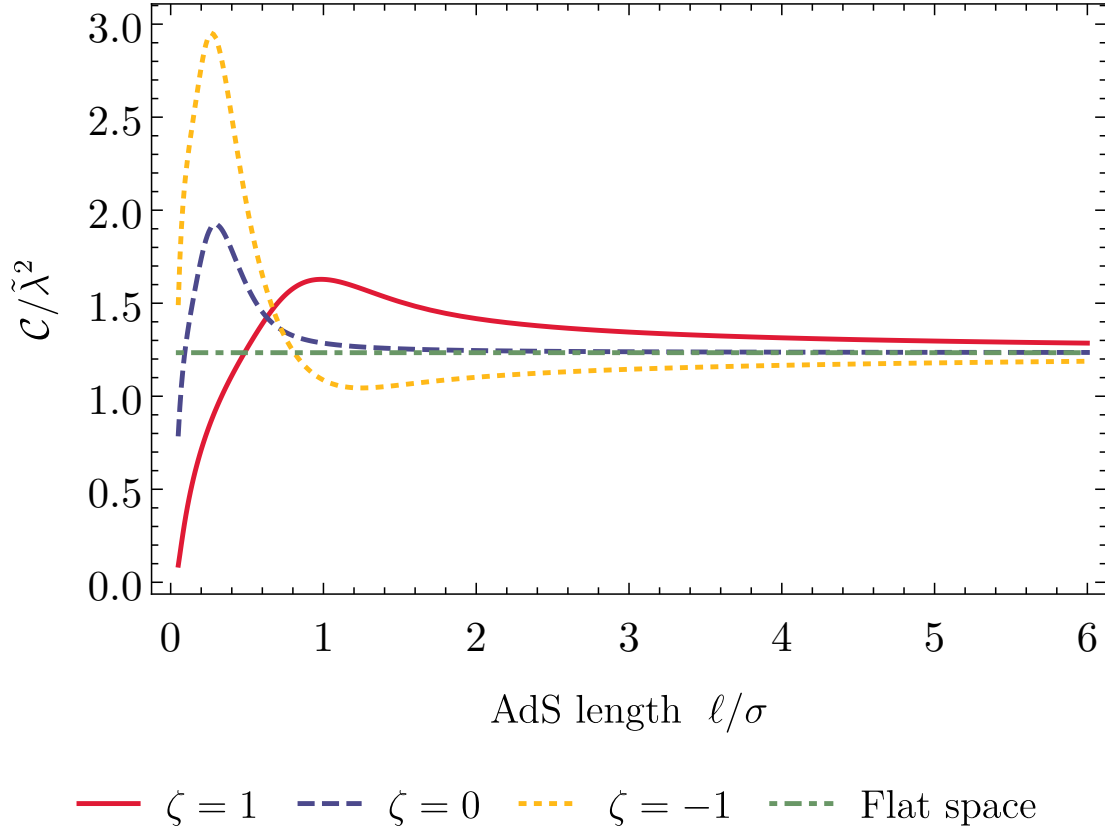


Figure 5.4: The concurrence $\mathcal{C}/\tilde{\lambda}^2$ associated with the state $\hat{\rho}_{AB}$ describing two detectors is plotted as a function of the AdS length ℓ/σ for all boundary conditions $\zeta = \{1, 0, -1\}$. The proper separation of the detectors is $d(R_A, R_B) = 0.1\sigma$, their energy gap is $\Omega\sigma = 1/100$, and detector A is located at the origin.

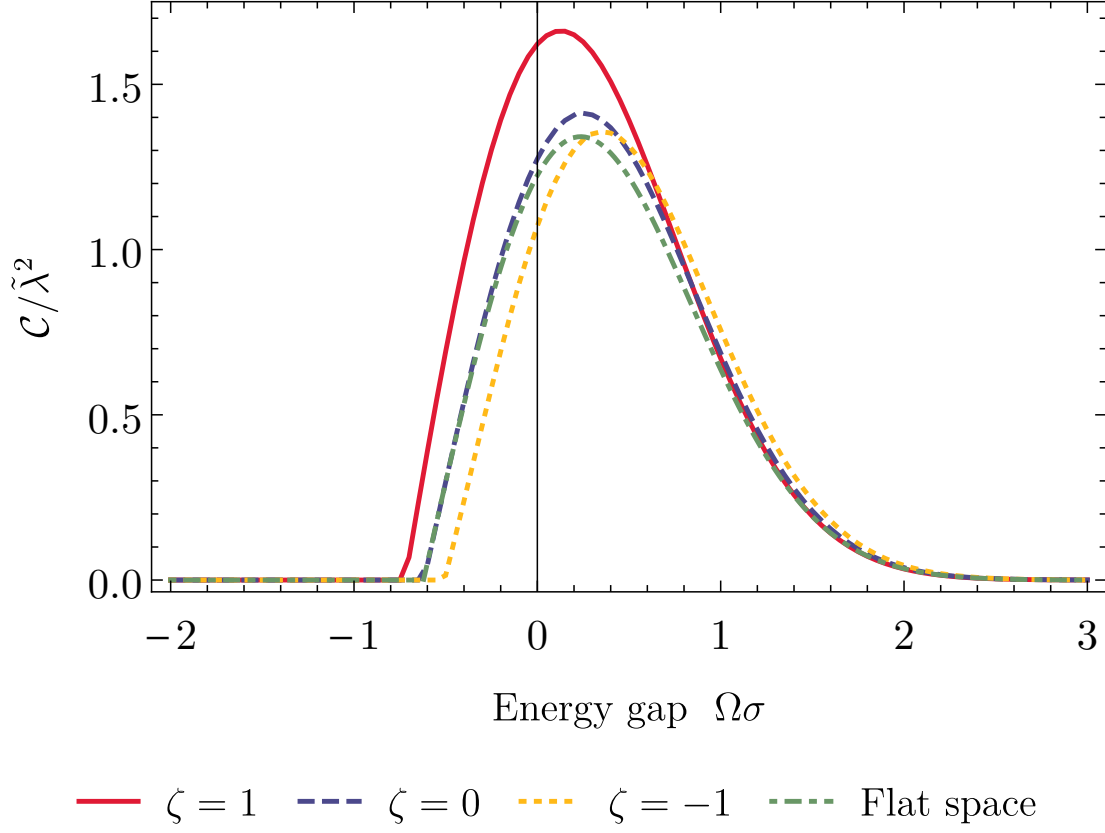


Figure 5.5: The concurrence $\mathcal{C}/\tilde{\lambda}^2$ associated with the state $\hat{\rho}_{AB}$ describing two detectors is plotted as a function of the detectors' energy gap $\Omega\sigma$ for all boundary conditions $\zeta = \{1, 0, -1\}$. The AdS length is chosen to be $\ell = \sigma$, the detectors are separated by a proper distance of $d(R_A, R_B) = 0.1\sigma$, and detector A is located at the origin.

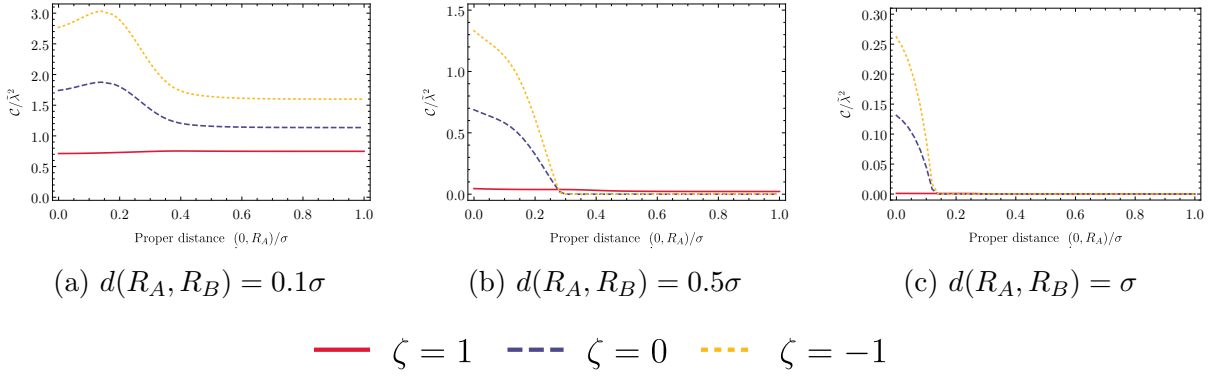


Figure 5.6: The concurrence $\mathcal{C}/\tilde{\lambda}^2$ associated with the state $\hat{\rho}_{AB}$ describing two detectors is plotted as a function of the proper distance $d(0, R_A)$ detector A is from the origin, for different proper separations $d(R_A, R_B)$ between the detectors; all boundary conditions $\zeta = \{1, 0, -1\}$ are shown. The AdS length is chosen to be $\ell = 0.2\sigma$ and the energy gap of the detectors is $\Omega\sigma = 1/100$.

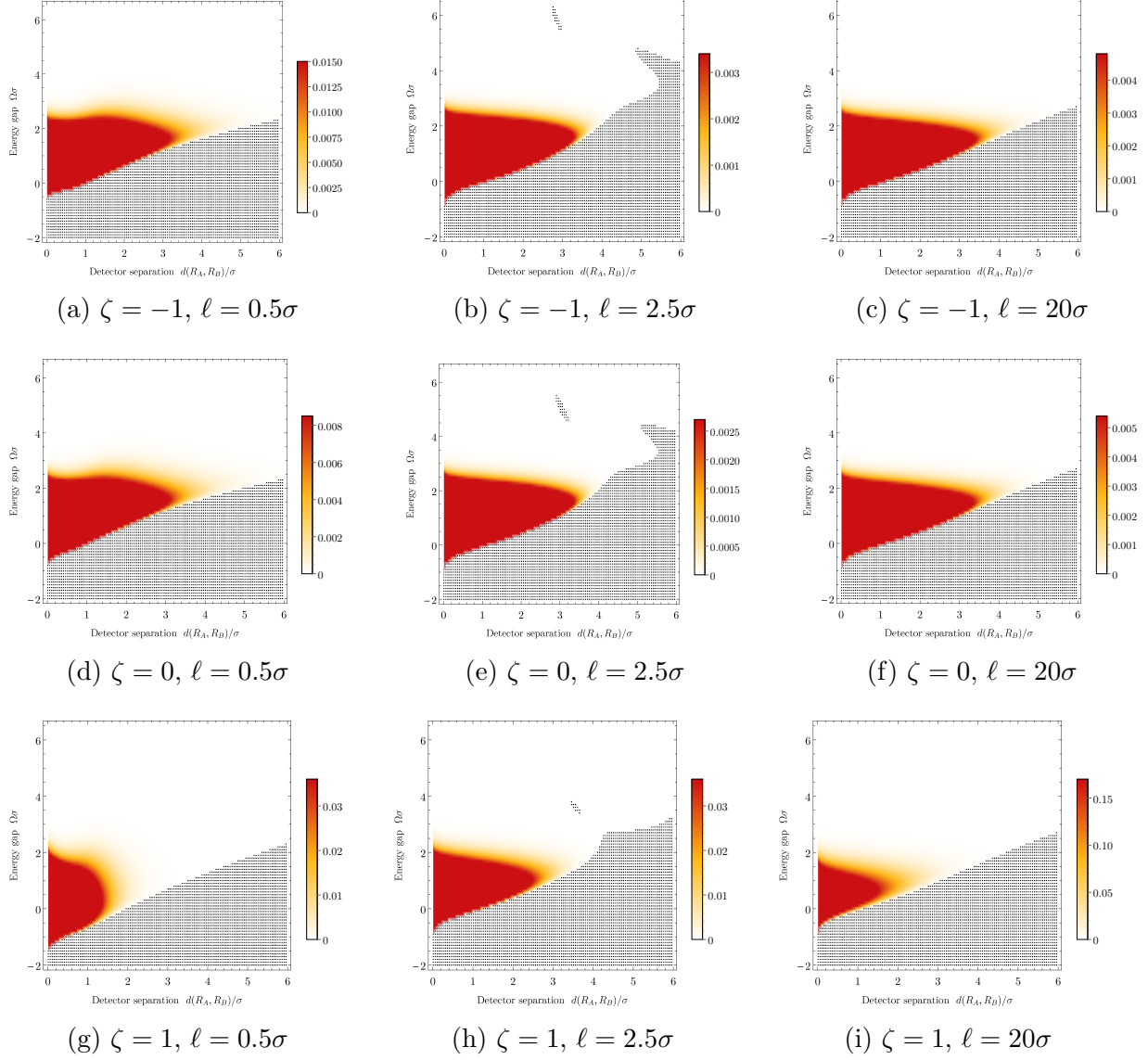
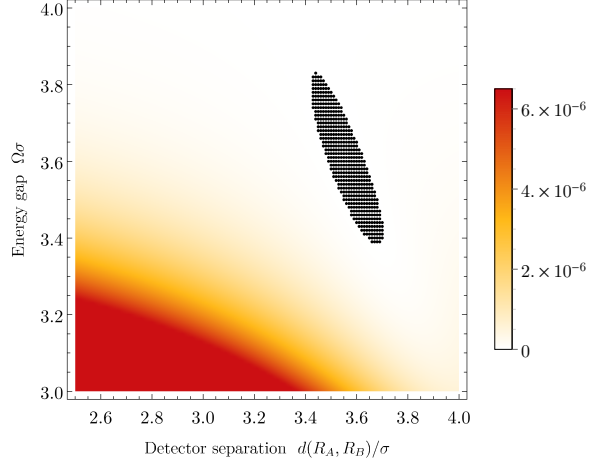
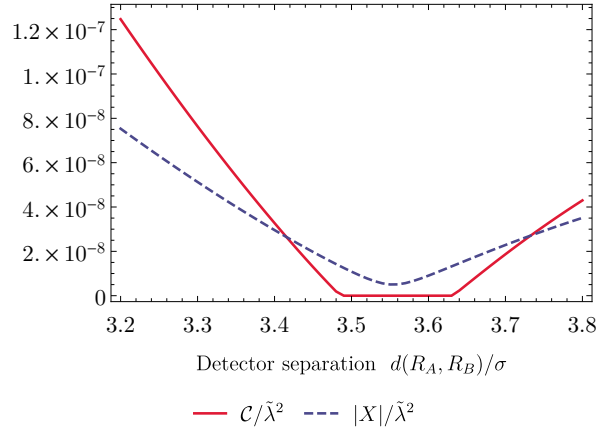


Figure 5.7: The concurrence $\mathcal{C}/\tilde{\lambda}^2$ associated with the state $\hat{\rho}_{AB}$ describing two detectors is plotted as a function of their proper separation $d(R_A, R_B)/\sigma$ and energy gap $\Omega\sigma$ for all boundary conditions $\zeta = \{-1, 0, 1\}$ and different values of the AdS length ℓ/σ . Detector A is located at the origin. The area filled with black dots represents the region where the concurrence vanishes and thus no entanglement harvesting is possible.



(a)



(b)

Figure 5.8: For Dirichlet boundary conditions ($\zeta = 1$) and an AdS length of $\ell = 2.5\sigma$, plot (a) depicts a close-up of the separability island in the parameter space spanned by proper detector separation $d(R_A, R_B)/\sigma$ (with A at the origin) and energy gap $\Omega\sigma$ in which no entanglement harvesting is possible; and in (b) the concurrence $\mathcal{C}/\tilde{\lambda}^2$ and the absolute value of the matrix element X are plotted as a function of the detectors proper separation $d(R_A, R_B)/\sigma$ for detectors with an energy gap of $\Omega\sigma = 3.6$. It is easy to see that the vanishing region of concurrence is located about the local minimum of $|X|$.

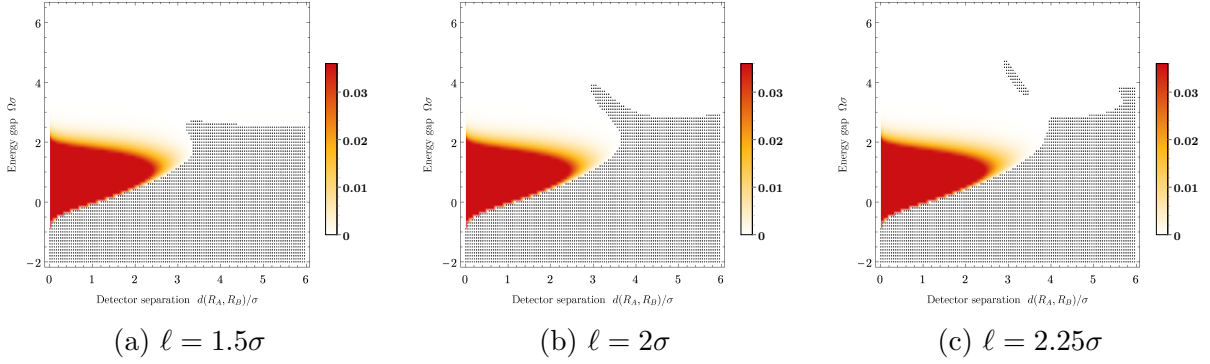


Figure 5.9: The concurrence is plotted as a function of a pair of detectors' proper separation $d(R_A, R_B)/\sigma$ and energy gap $\Omega\sigma$ for transparent boundary conditions $\zeta = 0$ satisfied by the field. As the AdS length ℓ/σ increases, moving from plot (a) to (c), an island of separability emerges. In all plots, detector A is located at the origin. The area filled with black dots represents the region where the concurrence vanishes and thus no entanglement harvesting is possible.

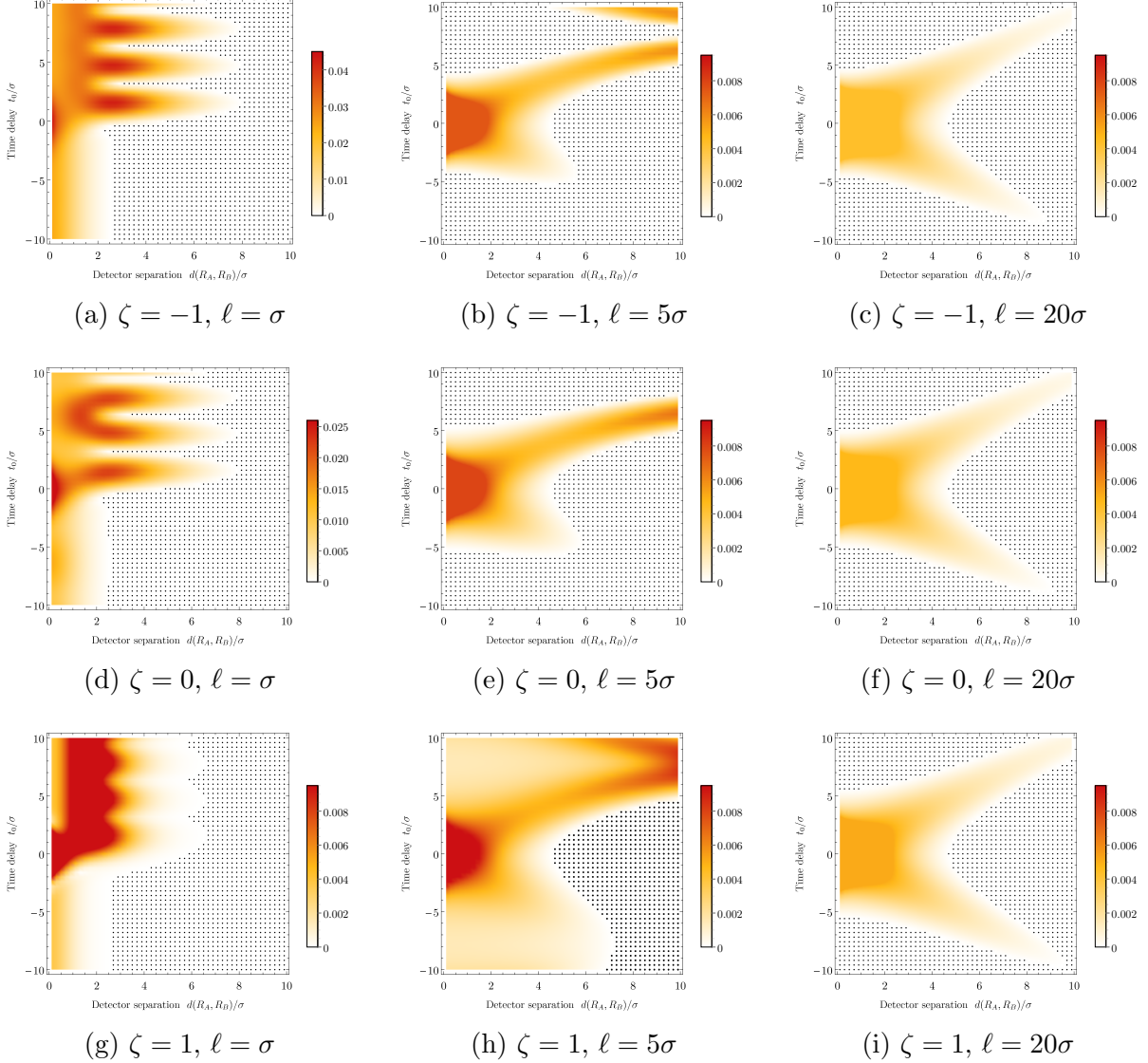


Figure 5.10: The concurrence $\mathcal{C}/\tilde{\lambda}^2$ associated with the state $|\hat{\rho}_{AB}\rangle$ describing two detectors is plotted as a function of their proper separation $d(R_A, R_B)/\sigma$ and the relative time delay in their switching functions t_0/σ for all boundary conditions $\zeta = -1, 0, 1$ and different value of the AdS length ℓ/σ . A negative t_0 means that detector B switches before detector A . Detector A is located at the origin, and the energy gap of the detectors is $\Omega\sigma = 2$. The area filled with black dots represents the region where the concurrence vanishes and thus no entanglement harvesting is possible.

5.6 Entanglement harvesting with a time delay

We now consider the case when the switching functions of the two detectors are offset by some $t_0 \neq 0$ in the coordinate time t . It is clear from the definition of X in Eq. (5.17) that the concurrence will not be symmetric under the transformation $t_0 \rightarrow -t_0$; in other words, the amount of entanglement harvested will depend on whether detector A or B interacts with the field first. Indeed, this can be seen in Fig. 5.10 where we consider the dependence of concurrence on detector separation and switching delay when the detectors' gaps are equal to $\Omega\sigma = 2$. The concurrence is non-zero for greater detector separation when detector A switches first. This effect is most noticeable in the case of Dirichlet ($\zeta = 1$) boundary conditions.

At moderate AdS length ($\ell = 5\sigma$) we note the formation of two “peninsulas” in the $\{d(R_A, R_B)/\sigma, t_0/\sigma\}$ parameter space, which is largest around the region of small time delay and detector separation. When the AdS length is large ($\ell = 20\sigma$), the peninsulas are nearly symmetric about $t_0 = 0$, look very similar for all three boundary conditions, and approach the flat space behaviour. In the limit of large ℓ , β_X vanishes as ℓ^{-1} so that $(\Delta_T + i\beta_X) \rightarrow \Delta_T$. Thus in the limit $\ell \rightarrow \infty$, X will be even in t_0 , recovering the Minkowski result.

For small AdS length and small detector separation, the concurrence is also largest around the region of small time delay, but for moderate detector separation, the concurrence is much larger when detector A interacts with the field first. Again, this effect is seen for all three boundary conditions, but is most exaggerated for Dirichlet ($\zeta = 1$) boundary conditions. Additionally, in this regime, and when $t_0 > 0$, we find oscillations dependent on time delay in the concurrence that occur for all three boundary conditions, although they are much more pronounced for Neumann ($\zeta = -1$) and transparent ($\zeta = 0$) boundary conditions where the concurrence goes to zero in the troughs for some parameters. In the case of Neumann boundary conditions ($\zeta = -1$), these oscillations can be seen more clearly in Fig. 5.11.

It can be seen from the definition of X in Eq. (5.17) that it will be invariant under the transformation $t_0 \rightarrow -t_0$ and $\Omega \rightarrow -\Omega$. However, as depicted in Fig. 5.11, the concurrence is not. As illustrated in Fig. 5.2, the transition probability of a detector initialized in its excited state ($\Omega < 0$) is much higher than when the detector is initialized in its ground state, meaning in the former case, the term $\sqrt{P_A P_B}$ dominates over $|X|$ in Eq. (2.53) and the concurrence is zero.

Finally, in Fig. 5.12 we consider small energy gaps, and find that the concurrence is nearly symmetric about $t_0 = 0$ for all three boundary conditions. Here, β_X is small so

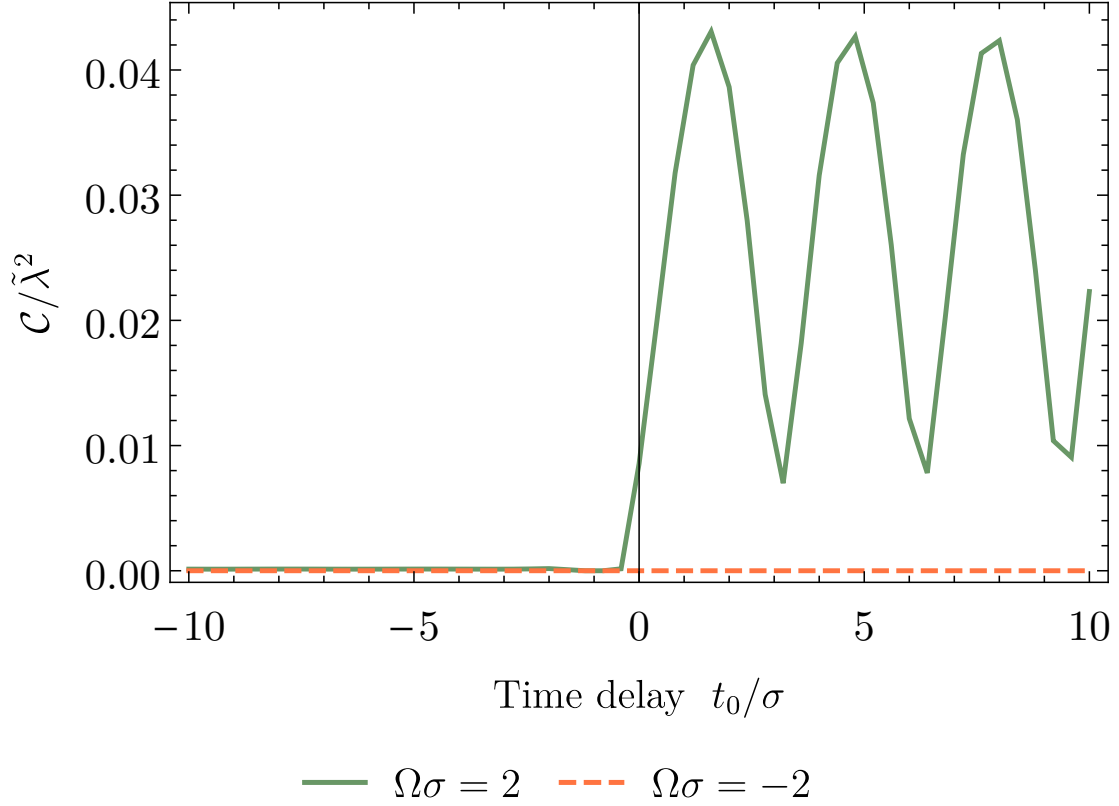


Figure 5.11: The concurrence $\mathcal{C}/\tilde{\lambda}^2$ associated with the state $\hat{\rho}_{AB}$ of the two detectors as a function of the relative time delay in their switching functions t_0/σ for an AdS length of $\ell = \sigma$, a detector separation of $d(R_A, R_B) = 2.5\sigma$, Neumann boundary conditions ($\zeta = -1$) and an energy gap of $\Omega\sigma = 2$ (green solid line) and $\Omega\sigma = -2$ (orange dashed line).

X is approximately even in t_0 (the large y behaviour of the integrand in Eq. (5.17) is exponentially suppressed by the Gaussian factor). Unlike in Fig. 5.10, there are no time delay dependent oscillations, nor is entanglement harvesting possible for moderate or large detector separation regardless of which detector interacts with the field first. However, the concurrence is maximum around the region of small time delay for all boundary conditions. Outside of this region, there are notable differences in behaviour for each boundary condition. For Neumann boundary conditions ($\zeta = -1$) there is little variation in the concurrence as $|t_0|$ increases. However, for Dirichlet boundary conditions ($\zeta = 1$) there are two regions around $|t_0| = 3\sigma$ where no entanglement harvesting is possible when the

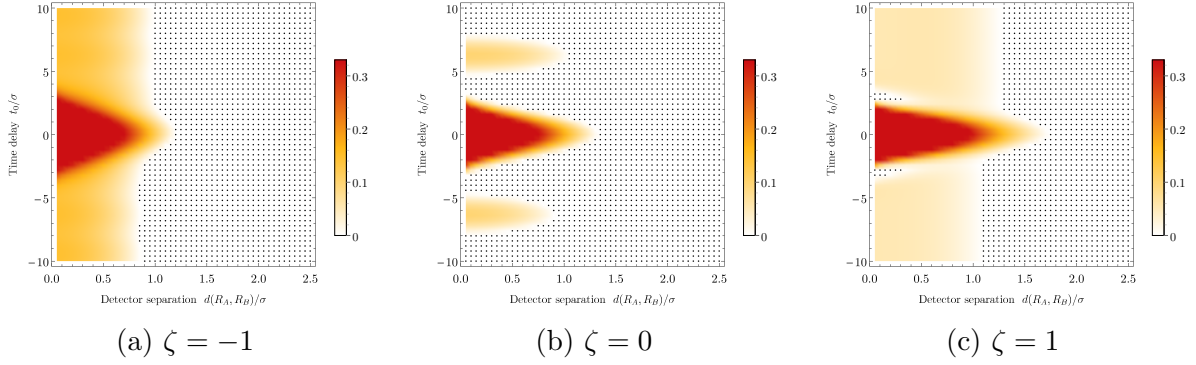


Figure 5.12: The concurrence $\mathcal{C}/\tilde{\lambda}^2$ associated with the state $\hat{\rho}_{AB}$ describing two detectors is plotted as a function of their proper separation $d(R_A, R_B)/\sigma$ and the relative time delay in their switching functions t_0/σ for all boundary conditions $\zeta = -1, 0, 1$ and an AdS length of $\ell = \sigma$. A negative t_0 means that detector B switches before detector A . Detector A is located at the origin, and the energy gap of the detectors is $\Omega\sigma = 0.1$. The area filled with black dots represents the region where the concurrence vanishes and thus no entanglement harvesting is possible.

detectors are very close together, but possible again when their separation increases. In the case of transparent boundary conditions ($\zeta = 0$), entanglement harvesting is only possible for three small regions of the parameter space.

Let us emphasize that the time delay asymmetries observed in the harvested entanglement do not, strictly speaking, reflect something specific to $(2+1)$ -AdS spacetime. Instead these asymmetries provide another example of the effects of a relative red shift (or relative time dilation) on the ability of atoms to harvest entanglement from the quantum vacuum. Note that the asymmetries can all be traced back to the non-vanishing of β_X , which is proportional to $\gamma_A - \gamma_B$. If one traces the origin of the β_X term, it arises from the term $\exp(-i\Omega_A\tau_A - i\Omega_B\tau_B)$ in the expression for X (2.26) which does not explicitly depend on any properties of the Wightman function. As a result, we expect that similar asymmetries should arise whenever two detectors have a relative redshift factor when compared to the frame in which the time delay is measured. Characterizing the differences in these asymmetries between equivalent detector setups in different spacetimes could then provide a probe of the underlying spacetime geometry, topology, etc.

When a time delay is present, the switching functions are modified, yielding a non-zero Δ_T . This is symmetric in t_0 , but the presence of a differential redshift breaks this symmetry. If $\gamma_A < \gamma_B$ and $t_0 > 0$, then entanglement can be harvested. However if

$t_0 < 0$, a simple transformation of the integrand in Eq. (2.26) converts the problem into one with $\Omega < 0$ —the problem effectively becomes that of harvesting entanglement from initially excited detectors, for which the $\sqrt{P_A P_B}$ term appearing in Eq. (2.52) is large and the harvested entanglement suppressed. For the case at hand $\gamma_A = 1 < \gamma_B$ and so entanglement harvesting is suppressed for $t_0 < 0$, the effect diminishing as the AdS length increases, causing $\gamma_B \rightarrow 1$. It also diminishes for small gap (Fig. 5.12), which likewise suppresses the asymmetry.

This asymmetry has also been observed in AdS₄ [53], though the effect is considerably less pronounced. We attribute this to Huygens’ principle being operative in this case, which has the effect of suppressing harvested entanglement within the light cones of the detectors.

5.7 Summary of results

We found that P_D is most sensitive to changes in parameters at small values of the AdS length ℓ/σ . For Dirichlet boundary conditions ($\zeta = 1$), the transition probability of a detector at any position approaches zero as $\ell/\sigma \rightarrow 0$, whereas for Neumann boundary conditions ($\zeta = -1$), the transition probability likewise decreases to zero for a detector fixed at the origin, but increases a little with decreasing ℓ/σ if the detector is located elsewhere. For transparent boundary conditions ($\zeta = 0$), the transition probability of a detector positioned far away from the origin remains constant as a function of ℓ/σ .

For entanglement harvesting, we found that for static detectors there is an optimal AdS length ℓ/σ and detector energy gap $\Omega\sigma$ at which the concurrence reaches a maximum value. One unexpected phenomenon is the appearance of “separability islands” for a range of small but finite $\ell/\sigma \approx 2.5$. In this region the detector transition probability P_D remains approximately constant as the proper separation of the detectors changes, but the coherence $|X|$ attains a local minimum whose origin remains to be understood.

We also observed a strong effect on the efficacy of entanglement harvesting if the detectors switch on at different times (in other words, the peak of their switching functions is offset by a time t_0). For $t_0 < 0$ entanglement harvesting is significantly suppressed compared to $t_0 > 0$, an effect due to locally different detector gaps induced by differing location-dependent redshifts. A similar effect is seen in AdS₄ [53] and we expect it to be present whenever the proper times of the detectors differ and a time delay is present in the switching functions.

Chapter 6

Harvesting Entanglement from the Black Hole Vacuum

In 2011, Hu and Yu, used the framework of open quantum systems show that while a pair of qubits located far from a Schwarzschild black hole could become entangled by the vacuum (Unruh, Hartle-Hawking and Boulware) of a massless scalar quantum field, they cannot if they are at the event horizon [36]. The two qubits were taken to be located at the same point in space. The entanglement harvesting protocol allows for a natural generalization of this setup, since in the interaction Hamiltonian (Eq. (2.14)) the field couples to the detector at the detector's position, the detectors can have independent trajectories. In this section, we present the first study of the entanglement harvesting protocol in a black hole background, and explore entanglement generation in UDW detectors at distinct positions outside of a BTZ black hole.

The $(2+1)$ -dimensional setting of the BTZ black hole provides a few advantages. First, quantities in the reduced density matrix of the detectors can be carried out using a sum over images instead of a mode sum, yielding significant computational efficiency when compared to the Schwarzschild black hole in $(3+1)$. Second, $(2+1)$ dimensional quantum gravity is much better understood than its $(3+1)$ counterpart, and so our work will provide an opportunity to study vacuum entanglement in that setting. In particular, we anticipate that the methods presented here can be used to investigate other problems in black hole physics such as the origin of black hole entropy and the information loss problem [40].

Throughout this chapter we adopt the mostly positive convention for the metric signature $(-1, 1, 1)$ and employ natural units $\hbar = c = 8G = 1$. We have also introduced the dimensional coupling strength $\tilde{\lambda} := \lambda\sqrt{\sigma}$ for plotting convenience.

6.1 The BTZ black hole

The BTZ black hole is a solution the Einstein equations with a negative cosmological constant $\Lambda = -1/\ell$ in $(2+1)$ -dimensions [8, 7], written here in Schwarzschild-like coordinates, $t \in (-\infty, \infty)$, $r \in (0, \infty)$, and $\phi \in (0, 2\pi)$

$$ds^2 = - \left(\frac{r^2 - r_h^2}{\ell^2} \right) dt^2 + \left(\frac{\ell^2}{r^2 - r_h^2} \right) dr^2 + r^2 d\phi^2. \quad (6.1)$$

This solution is asymptotically anti-de Sitter (AdS_3) and has a horizon located at $r_h := \ell\sqrt{M}$, where M is the black hole mass.

We consider the BTZ spacetime [33, 31, 78] because the Wightman functions for a conformally coupled scalar field (in the Hartle-Hawking vacuum) are known analytically [42, 13]. Since the BTZ spacetime can be constructed by a topological identification Γ of AdS_3 , the Wightman function associated with the Hartle-Hawking vacuum in the BTZ spacetime may be expressed as the image sum

$$W_{\text{BTZ}}(x, x') = \sum_{n=-\infty}^{\infty} W_{\text{AdS}_3}(x, \Gamma^n x'), \quad (6.2)$$

where $W_{\text{AdS}_3}(x, x')$ is the vacuum Wightman function associated with a massless conformally coupled scalar field in AdS_3 and $\Gamma x'$ takes $(t, r, \phi) \rightarrow (t, r, \phi + 2\pi)$. Explicitly, for two spacetime points x and x' outside the black hole horizon,

$$W_{\text{BTZ}}(x, x') = \frac{1}{4\pi\sqrt{2}\ell} \sum_{n=-\infty}^{\infty} \left[\frac{1}{\sqrt{\sigma_\epsilon(x, \Gamma^n x')}} - \frac{\zeta}{\sqrt{\sigma_\epsilon(x, \Gamma^n x') + 2}} \right], \quad (6.3)$$

where $\zeta \in \{-1, 0, 1\}$ specifies either Neumann ($\zeta = -1$), transparent ($\zeta = 0$), or Dirichlet ($\zeta = 1$) boundary conditions satisfied by the field at spatial infinity (below we take $\zeta = 1$), and

$$\begin{aligned} \sigma_\epsilon(x, \Gamma^n x') := & \frac{rr'}{r_h^2} \cosh \left[\frac{r_h}{\ell} (\Delta\phi - 2\pi n) \right] - 1 - \\ & \frac{\sqrt{(r^2 - r_h^2)(r'^2 - r_h^2)}}{r_h^2} \cosh \left[\frac{r_h}{\ell^2} t - t' - i\epsilon \right], \end{aligned} \quad (6.4)$$

where $\Delta\phi := \phi - \phi'$.

Consider a pair of detectors, A and B , which are located at distances R_A and R_B outside the horizon of the BTZ black hole, $R_A, R_B > r_h$. Their spacetime trajectories are

$$x_D(\tau_D) := \left\{ t = \frac{\tau_D}{\gamma_D}, r = R_D, \phi = \Phi_D \right\}, \quad (6.5)$$

where

$$\gamma_D := \frac{\sqrt{R_D^2 - r_h^2}}{\ell} \quad (6.6)$$

is the redshift factor for $D \in \{A, B\}$. Without loss of generality we will set $R_A < R_B$.

We will choose the switching functions of the detectors to be Gaussian,

$$\chi_D(\tau_D) = \exp\left(-\frac{\tau_D^2}{2\sigma^2}\right), \quad (6.7)$$

defined in the proper time τ_D of each detector, and centered around the spacelike hypersurface $t = 0$. We will also consider the detectors to have identical energy gaps $\Omega_A = \Omega_B = \Omega$, which are also defined in the detector's frame.

Having specified the detectors' trajectories, energy gaps and switching functions, we can compute the transition probabilities of the detectors and the coherence term, X .

Starting with the transition probability: we first define

$$\rho_n^-(x, x') := \sqrt{\sigma_\epsilon(x, x')}, \quad \rho_n^+(x, x') := \sqrt{\sigma_\epsilon(x, x') + 2} \quad (6.8)$$

for convenience, so transition probability can be written as

$$\begin{aligned} P_D &= \lambda^2 \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-\frac{\tau^2}{2\sigma_D^2}} e^{-\frac{\tau'^2}{2\sigma_D^2}} e^{-i\Omega_D(\tau-\tau')} W_{BTZ}(x_D(\tau), x_D(\tau')) \\ &= \frac{\lambda^2}{4\sqrt{2}\pi\ell} \sum_{n=-\infty}^{\infty} \left[\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \frac{e^{-\frac{\tau^2}{2\sigma_D^2}} e^{-\frac{\tau'^2}{2\sigma_D^2}} e^{-i\Omega_D(\tau-\tau')}}{\rho_n^-(x_D(\tau), \Gamma^n x_D(\tau'))} \right. \\ &\quad \left. - \zeta \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \frac{e^{-\frac{\tau^2}{2\sigma_D^2}} e^{-\frac{\tau'^2}{2\sigma_D^2}} e^{-i\Omega_D(\tau-\tau')}}{\rho_n^+(x_D(\tau), \Gamma^n x_D(\tau'))} \right], \end{aligned} \quad (6.9)$$

where

$$\begin{aligned}\rho_n^\pm(x_D(\tau), \Gamma^n x_D(\tau')) &= \left\{ \frac{R_D^2}{r_h^2} \cosh\left(\frac{r_h}{\ell} 2\pi n\right) \mp 1 - \frac{\ell^2 \gamma_D^2}{r_h^2} \cosh\left(\frac{r_h}{\ell^2 \gamma_D} (\tau - \tau') - i\epsilon\right) \right\}^{1/2} \\ &= \frac{\ell \gamma_D}{r_h} \left\{ \cosh(\alpha_{D,n}^\pm) - \cosh\left(\frac{r_h}{\ell^2 \gamma_D} (\tau - \tau') - i\epsilon\right) \right\}^{1/2}\end{aligned}\quad (6.10)$$

and we have also defined

$$\alpha_{D,n}^\pm := \operatorname{arccosh} \left[\frac{r_h^2}{\ell^2 \gamma_D^2} \left(\frac{R_D^2}{r_h^2} \cosh\left(\frac{r_h}{\ell} \pi n\right) \mp 1 \right) \right]. \quad (6.11)$$

We note that $\alpha_{D,n}^\pm \geq 0$ with the equality only occurring for $\alpha_{D,0}^-$ and is symmetric in $n \rightarrow -n$. Substituting equation (6.10) into equation (6.9) and integrating, the transition probability becomes

$$\begin{aligned}P_D &= \sum_{n=-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \frac{\lambda^2 \sigma_D}{4\sqrt{2\pi}} \int_{-\infty}^{\infty} dy \frac{e^{-a_D y^2} e^{-i\beta_D y}}{\sqrt{\cosh(\alpha_{D,n}^-) - \cosh(y - i\epsilon)}} \right. \\ &\quad \left. - \zeta \frac{\lambda^2 \sigma_D}{4\sqrt{2\pi}} \int_{-\infty}^{\infty} dy \frac{e^{-a_D y^2} e^{-i\beta_D y}}{\sqrt{\cosh(\alpha_{D,n}^+) - \cosh(y - i\epsilon)}} \right],\end{aligned}\quad (6.12)$$

where

$$a_D := \frac{\gamma_D^2 \ell^4}{4\sigma^2 r_h^2}, \quad (6.13)$$

$$\beta_D := \frac{\gamma_D \Omega \ell^2}{r_h} \quad (6.14)$$

When $\alpha_{D,n}^\pm \neq 0$, the singularity at $y = \alpha_{D,n}^\pm$ is not a pole; it is integrable, so we can safely drop the $i\epsilon$ regulator. However, when $\alpha_{D,n}^\pm = 0$, the singularity is a simple pole and

must be handled differently. We can re-write this image term as

$$\begin{aligned} & \frac{\lambda^2 \sigma_D}{4\sqrt{2\pi}} \int_{-\infty}^{\infty} dy \frac{e^{-a_D y^2} e^{-i\beta_D y}}{\sqrt{1 - \cosh(y - i\epsilon)}} \\ &= \frac{\lambda^2 \sigma_D^2}{4\sqrt{2\pi}} \frac{r_h}{\ell^2 \gamma_D} \int_{-\infty}^{\infty} d\omega e^{-\sigma_D^2 (\Omega_D - \omega)^2} \left(\int_{-\infty}^{\infty} du \frac{e^{-i\omega u}}{\sqrt{1 - \cosh(2\pi T u - i\epsilon)}} \right) \end{aligned} \quad (6.15)$$

where

$$T_D := \frac{r_h}{2\pi \ell^2 \gamma_D} = \frac{r_h}{2\pi \ell \sqrt{R_D^2 - r_h^2}}. \quad (6.16)$$

The u integral has been computed in [42] to be

$$\int_{-\infty}^{\infty} du \frac{e^{-i\omega u}}{\sqrt{1 - \cosh(2\pi T u - i\epsilon)}} = \frac{\sqrt{2}}{T} \frac{1}{e^{\omega/T} + 1}. \quad (6.17)$$

After a few more simplifications, we write the transition probability as

$$\begin{aligned} P_D &= \frac{\lambda^2 \sigma^2}{2} \int_{-\infty}^{\infty} dy \frac{\exp[-\sigma^2(y - \Omega)^2]}{\exp(y/T_D) + 1} \\ &\quad - \zeta \frac{\lambda^2 \sigma}{2\sqrt{2\pi}} \int_0^{\infty} dy \operatorname{Re} \left[\frac{\exp(-a_D y^2) \exp(-i\beta_D y)}{\sqrt{\cosh(\alpha_{D,0}^+) - \cosh(y)}} \right] \\ &\quad + \frac{\lambda^2 \sigma}{\sqrt{2\pi}} \sum_{n=1}^{\infty} \left\{ \int_0^{\infty} dy \operatorname{Re} \left[\frac{\exp(-a_D y^2) \exp(-i\beta_D y)}{\sqrt{\cosh(\alpha_{D,n}^-) - \cosh(y)}} \right] \right. \\ &\quad \left. - \zeta \int_0^{\infty} dy \operatorname{Re} \left[\frac{\exp(-a_D y^2) \exp(-i\beta_D y)}{\sqrt{\cosh(\alpha_{D,n}^+) - \cosh(y)}} \right] \right\}. \end{aligned} \quad (6.18)$$

The coherence term is calculated in the same way. Starting from

$$\begin{aligned}
X &= -\lambda^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \gamma_A \gamma_B \left[\left(e^{-\frac{\gamma_B t^2}{2\sigma^2}} e^{-\frac{\gamma_A t'^2}{2\sigma^2}} e^{-i\Omega \gamma_B t} e^{-i\Omega \gamma_A t'} \right. \right. \\
&\quad \left. \left. \times W_{BTZ}(x_A(t'), x_B(t)) \right) + A \leftrightarrow B \right] \\
&= -\frac{\lambda^2 \gamma_A \gamma_B}{4\sqrt{2}\pi\ell} \sum_{n=-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \left[\left(\frac{e^{-\frac{\gamma_B t^2}{2\sigma^2}} e^{-\frac{\gamma_A t'^2}{2\sigma^2}} e^{-i\Omega \gamma_B t} e^{-i\Omega \gamma_A t'}}{\rho_n^-(x_A(t'), \Gamma^n x_B(t))} \right. \right. \right. \\
&\quad \left. \left. \left. - \zeta \frac{e^{-\frac{\gamma_B t^2}{2\sigma^2}} e^{-\frac{\gamma_A t'^2}{2\sigma^2}} e^{-i\Omega \gamma_B t} e^{-i\Omega \gamma_A t'}}{\rho_n^+(x_A(t'), \Gamma^n x_B(t))} \right) + A \leftrightarrow B \right] \right\} \quad (6.19)
\end{aligned}$$

where

$$\begin{aligned}
\rho_n^{\mp}(x_A(t'), \Gamma^n x_B(t)) &= \left\{ \frac{R_A R_B}{r_h^2} \cosh \left[\frac{r_h}{\ell} (\Delta\phi + 2\pi n) \right] \mp 1 - \frac{\gamma_A \gamma_B \ell^2}{r_h^2} \cosh \left[\frac{r_h}{\ell^2} (t' - t) - i\epsilon \right] \right\}^{1/2} \\
&= \sqrt{\gamma_A \gamma_B} \frac{\ell}{r_h} \left\{ \cosh(\alpha_{X,n}^{\mp}) - \cosh \left(\frac{r_h}{\ell^2} (t' - t) - i\epsilon \right) \right\}^{1/2} \quad (6.20)
\end{aligned}$$

and

$$\alpha_{X,n}^{\pm} := \operatorname{arccosh} \left[\frac{r_h^2}{\gamma_A \gamma_B \ell^2} \left(\frac{R_A R_B}{r_h^2} \cosh \left(\frac{r_h}{\ell} (\Delta\phi + 2\pi n) \right) \pm 1 \right) \right]. \quad (6.21)$$

and $\Delta\phi := \Phi_A - \Phi_B$. We again note that $\alpha_{X,n}^{\pm} > 0$ and is invariant when $A \leftrightarrow B$ and $n \rightarrow -n$ simultaneously. In particular, we find that $\alpha_{X,n}^{\pm} \neq 0$ provided the two detectors do not occupy the same point in space. Substituting equation (6.20) into equation (6.22) and integrating, the coherence term becomes after some algebraic simplifications:

$$X = K_X \sum_{n=-\infty}^{\infty} \eta^n \left[\int_0^{\infty} \frac{e^{-a_X y^2} \cos(\beta_X y)}{\sqrt{\cosh(\alpha_n^-) - \cosh(y)}} - \zeta \int_0^{\infty} \frac{e^{-a_X y^2} \cos(\beta_X y)}{\sqrt{\cosh(\alpha_n^+) - \cosh(y)}} \right] \quad (6.22)$$

where

$$K_X := \frac{\lambda^2 \sigma}{2\sqrt{\pi}} \sqrt{\frac{\gamma_A \gamma_B}{\gamma_A^2 + \gamma_B^2}} \exp\left(-\frac{\sigma^2 \Omega^2 (\gamma_A + \gamma_B)^2}{2(\gamma_A^2 + \gamma_B^2)}\right), \quad (6.23)$$

$$a_X := \frac{1}{2\sigma^2} \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} \frac{\ell^4}{r_h^2}, \quad (6.24)$$

$$\beta_X := \Omega \frac{\gamma_A \gamma_B (\gamma_A - \gamma_B)}{\gamma_A^2 + \gamma_B^2} \frac{\ell^2}{r_h}, \quad (6.25)$$

and we dropped the $i\epsilon$ regulator since the singularity at $y = \alpha_{X,n}^\pm$ is integrable.

6.2 Entanglement harvesting

$$d(R_1, R_2) = \ell \ln \left[\frac{R_2 + \sqrt{R_2^2 - r_h^2}}{R_1 + \sqrt{R_1^2 - r_h^2}} \right], \quad (6.26)$$

where $R_2 \geq R_1 \geq r_h$.

In Fig. 6.1 we consider detector A at a fixed proper distance from the black hole horizon and plot $\mathcal{C}(\rho_{AB})$ as a function of the proper distance between the two detectors. We observe that as the separation between the detectors grows, the entanglement between the detectors decreases. This behaviour is as expected since correlations in the vacuum state are small for spacetime points separated by a large distance, which can be seen from the BTZ Wightman function in (6.2). We also observe that the entanglement decreases more slowly for detectors with larger energy gap, but always vanishes for finite detector separation. This is a consequence of the fact that detectors with a larger gap are harder to excite by random fluctuations, so the correlations can dominate the transition probabilities of the detectors over larger separations; see (2.53).

In Fig. 6.2 we consider the two detectors, separated by a fixed proper distance, placed at various proper distances away from the horizon. For any given detector gap, we find that detectors closer to the horizon become less entangled than detectors further from the horizon. Moreover, there is a point where a “sudden death” of entanglement harvesting occurs: detectors closer to the horizon than a certain critical proper distance, $d_{\text{death}}(r_h, R_A)$, cannot become entangled. This is agreement with the results found by Hu and Yu [36]. Upon examination of the matrix elements of $\hat{\rho}_{AB}$, we find that there are two competing

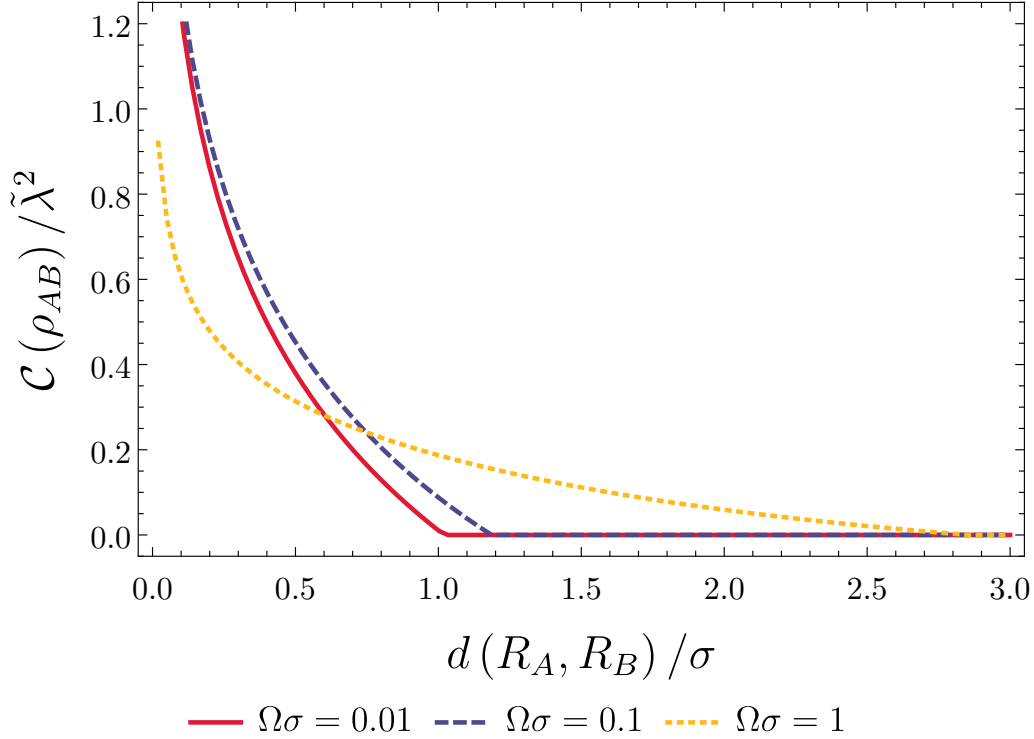


Figure 6.1: The concurrence $\mathcal{C}(\hat{\rho}_{AB})/\tilde{\lambda}^2$ between two detectors in the exterior region of the BTZ black hole is plotted as a function of the proper distance separating them; different energy gaps $\Omega\sigma$ of the detectors are shown. The proper distance between detector A and the horizon is $d(r_h, R_A) = \sigma$, and $\ell = 10\sigma$ and $M = 1$.

phenomena that lead to this result. First, detectors closer to the horizon experience a larger local Hawking temperature [33], and this manifests as an increase in the transition probability of the individual detectors. Second, there is a redshift effect for the correlation term X , which serves to decrease $|X|$ toward zero as the detectors approach the horizon; $|X|$ can be made arbitrarily small by placing the detectors at an appropriately close distance to the horizon. This can be seen in the prefactor K_X in (6.23), which will vanish as detector A approaches the horizon while keeping the proper separation between the two detectors fixed. Since entanglement requires that $|X| - \sqrt{P_A P_B} > 0$, both of these effects contribute to the sudden death of entanglement observed for detectors near the horizon.

The relative importance of the two effects depends on the detectors' energy gaps Ω , as is further highlighted in Fig. 6.3. In general, for a given proper separation between the

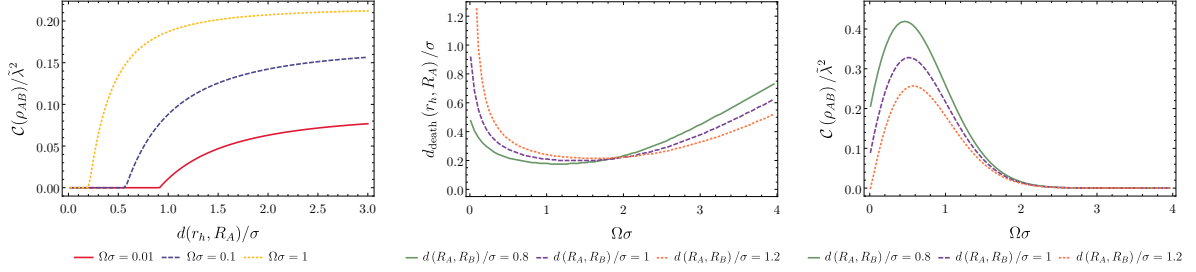


Figure 6.2: *Left:* The concurrence $\mathcal{C}(\hat{\rho}_{AB})$ between two detectors in the exterior region of the BTZ black hole is plotted as a function of the proper distance detector A is from the horizon; different energy gaps $\Omega\sigma$ of the detectors are shown. Here, the separation between the two detectors is fixed to be $d(R_A, R_B) = \sigma$. *Center:* A plot of the distance detector A is from the horizon $d(r_h, R_A)$ when the entanglement vanishes as a function of the detectors' energy gap $\Omega\sigma$ is shown for three choices of detector separation. *Right:* The concurrence in the large $d(r_h, R_A)$ limit is plotted as a function of the detectors' energy gap $\Omega\sigma$. In this plot, the concurrence has been evaluated at $d(r_h, R_A) = 100\sigma$. In all plots, we have taken $\ell = 10\sigma$ and $M = 1$.

detectors, in the limit when the detectors are far from the horizon, the values for $|X|$ and the transition probability decrease as the detectors' energy gaps are increased. Intuitively, this makes sense as detectors with a larger energy gap are less sensitive to fluctuations in the field. Furthermore, we see that as the detectors move toward the horizon, the transition probability displays a larger relative change with increasing gap. The decreasing sensitivity to field fluctuations means in turn that the detectors become more sensitive to Hawking radiation.

This leads to the following interpretation. For detectors with small energy gaps, it is the decrease in $|X|$, due to redshift effects, that is responsible for the sudden death of entanglement, since the transition probability remains effectively constant on the scales over which $|X|$ changes; see Fig. 6.3. For larger energy gaps, the effect of the local Hawking temperature on the transition probability is more pronounced, and the sudden death is a combined effect of the redshift decreasing $|X|$ and the local Hawking temperature increasing the transition probability of the detectors.

In fact, a close examination of the energy gap dependence shown in the middle plot of Fig 6.2 reveals that there is an optimal value of the gap that pushes the point of sudden death as close to the horizon as possible. This is a consequence of the competition between the increase in the sensitivity to the Hawking effect and the overall suppression of the background values of $|X|$ and the transition probabilities. The same effect also leads to an

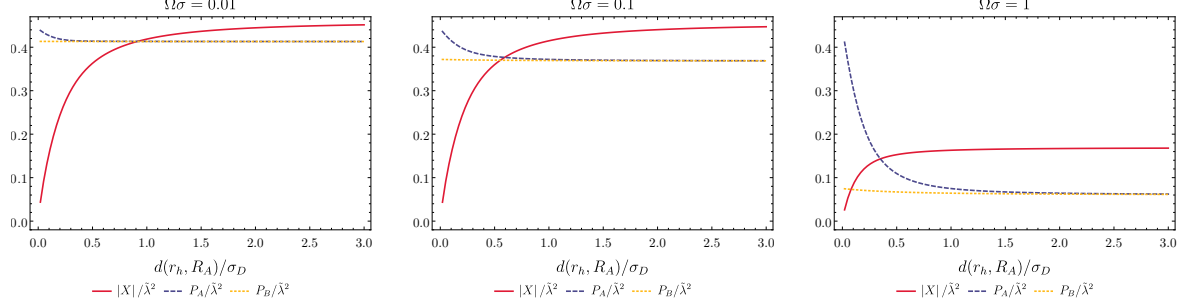


Figure 6.3: The transition probabilities P_A and P_B , and the coherence term $|X|$ are plotted as a function of the proper distance detector A is from the horizon for detectors with an energy gap of $\Omega\sigma$ of 0.01 (*Left*), 0.1 (*Center*), and 1 (*Right*). In all plots the detectors are separated by a proper distance of $d(R_A, R_B) = \sigma$, and $\ell = 10\sigma$ and $M = 1$.

optimal value of Ω for maximal entanglement extraction from the vacuum, shown in the right plot of Fig 6.2.

We have confirmed that the results in Fig. 6.2 are a property of the BTZ spacetime and not simply an AdS_3 effect. Furthermore, we find that all choices $M > 1$ do not affect any of the results quantitatively. Our calculations have assumed that the detectors are turned on for the same amount of time in their own rest frame. We have examined switching in other frames, e.g. in the black hole frame, and find this does not affect our basic results. Although the results have been presented for Dirichlet boundary conditions ($\zeta = 1$), they are qualitatively the same for the other boundary conditions.

Up until this point, we have only considered the case where $\Phi_A = \Phi_B$. Now we allow the angle between the detectors to vary. In this case, the proper separation, S , between the detectors is defined implicitly through the BTZ equivalent of the cosine law given by

$$\begin{aligned} \cosh(\sqrt{M}\Delta\phi) = & \left[\sqrt{R_B^2 - \ell^2 M} \sqrt{R_A^2 - \ell^2 M} (R_A^2 + R_B^2 - 2\ell^2 M (\cosh^2(S/\ell) + 1)) \right. \\ & \left. - \cosh(s/\ell) (2R_A^2 R_B^2 - 3(R_A^2 + R_B^2) + 4\ell^4 M^2) \right] \\ & \times \left[R_A R_B \left(R_A^2 + R_B^2 - 2\ell^2 M - 2\sqrt{R_A^2 - \ell^2 M} \sqrt{R_B^2 - \ell^2 M} \cosh(S/\ell) \right) \right]^{-1} \end{aligned} \quad (6.27)$$

where $\Delta\phi := \Phi_B - \Phi_A$.

In Fig. 6.4 we plot the concurrence as a function of the angle between the detectors, $\Delta\phi$, at various detector energy gaps. The shape of these plots depends heavily on the energy gap, with the maximum concurrence occurring when $R_B \approx R_A$ for small gaps, when R_B is a maximum for $\Omega\sigma$ near unity and for very large gaps, and when R_B is at a minimum for large gaps. These observations are summarized in Fig. 6.5. The discontinuities at $\Omega\sigma \approx 1.9$ and $\Omega\sigma \approx 5$, occur when the optimal configuration switches from detector B being as far from the horizon as possible to as close as possible and back.

These changes are the result of two competing effects. First, the transition probability of the detector increases when it is closer to the horizon due to the larger local Hawking temperature. The effect second results from the behaviour of the coherence, X , which is largely dominated by the behaviour of the prefactor, K_X . When $\Omega\sigma \leq 1$, K_X has a local maximum when $\gamma_A = \gamma_B$; however, when $\Omega\sigma > 1$, this becomes a local minimum, and two additional local maxima occur at $\gamma_B = \gamma_A (\Omega^2\sigma^2 \pm \sqrt{\Omega^4\sigma^4 - 1})$. At larger values of the energy gap, the smaller of the local maxima moves to smaller values of γ_B , and hence R_B . Additionally, the ratio between the local maximum and minimum values values of K_X increases exponentially with increasing $\Omega\sigma$, meaning at larger value of $\Omega\sigma$, the coherence term will be larger when the detector is closer to the horizon than R_A . However, the impact of the local noise of the detectors also becomes more significant at larger energy gaps, which forces the optimal value of R_B to be larger than R_A .

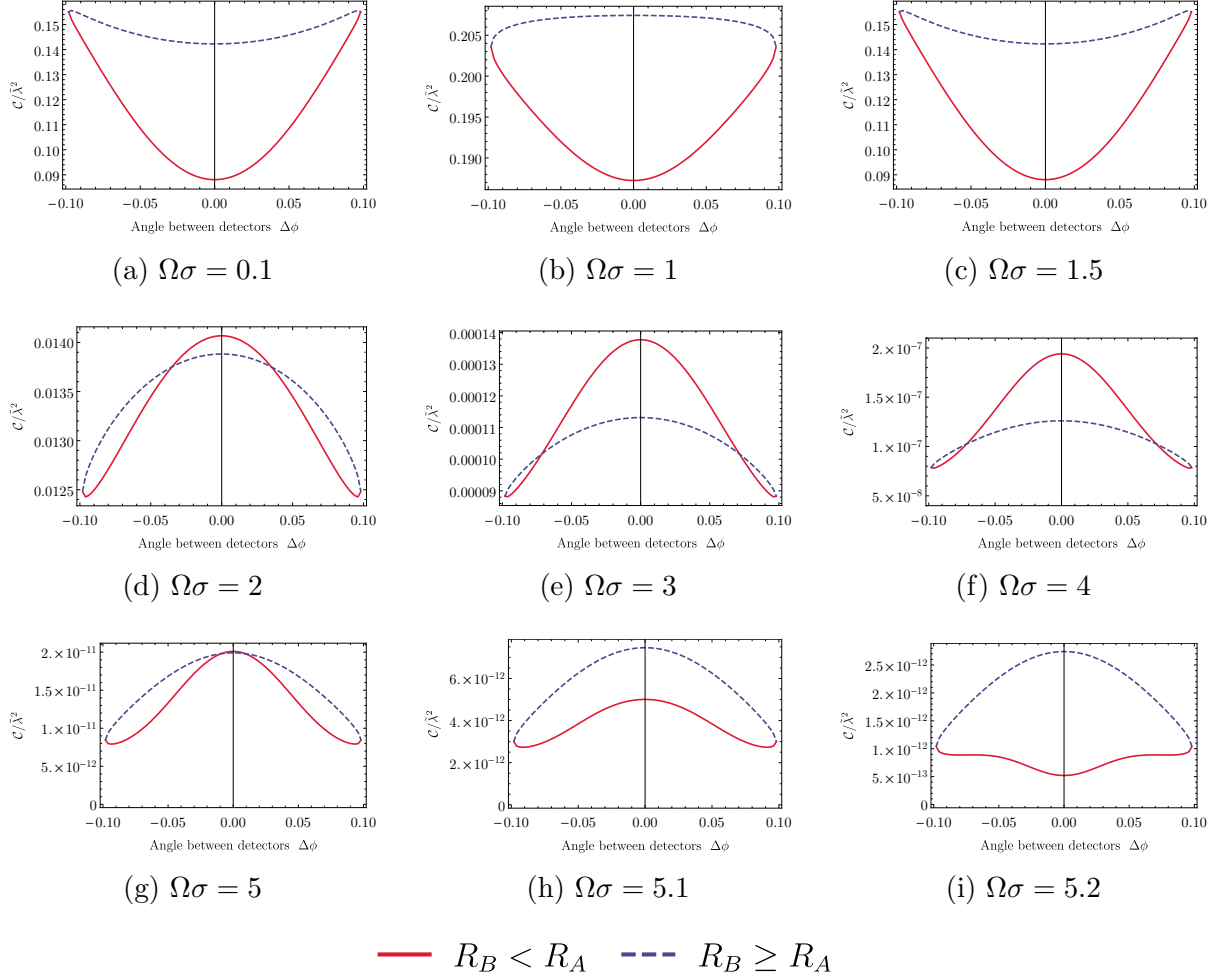


Figure 6.4: Here we plot the concurrence $\mathcal{C}(\hat{\rho}_{AB})/\tilde{\lambda}^2$ between two detectors as a function of the angle $\Delta\phi$ between them, as measured from the origin for various values of the energy gap $\Omega\sigma$. In all plots, detector A is fixed at $d(r_h, R_A) = 2\sigma$, the proper separation between the detectors is $S = \sigma$, and we have taken $\ell = 10\sigma$ and $M = 1$.

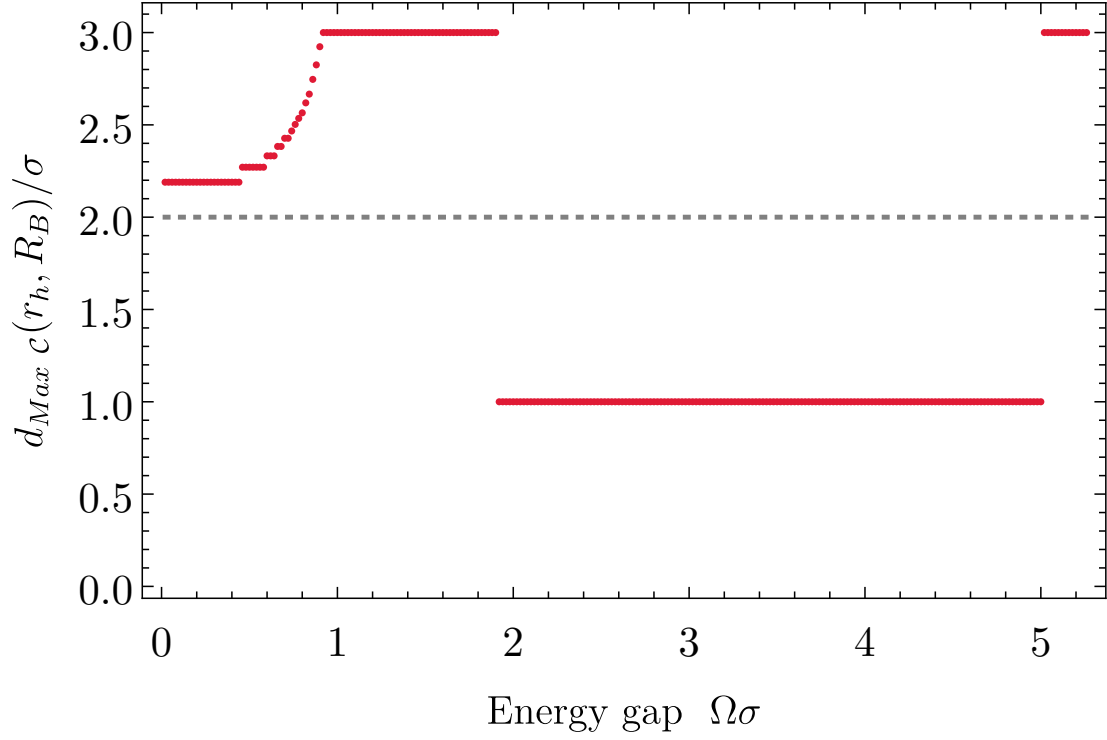


Figure 6.5: Here we plot (red dots) the proper distance of detector B from the horizon which maximizes concurrence $d_{Max} c(r_h, R_B)$, as a function of the energy gap of the two detectors $\Omega\sigma$. Detector A is fixed at $d(r_h, R_A) = 2\sigma$ which is marked in the grey dotted line, the proper separation between the detectors is $S = \sigma$, and $\ell = 10\sigma$ and $M = 1$.

6.3 Summary of results

We find that BTZ black holes inhibit entanglement harvesting. As two detectors with fixed proper separation move closer to the horizon, they encounter a point of “sudden death” where the harvested entanglement falls to zero. No entanglement can be harvested between this point and the horizon, though it is possible to minimize its proper distance from the horizon by choosing an optimal value of the detectors’ energy gap. We find this holds even for timelike separations of the detectors, where a field-mediated interaction might be expected to increase detector entanglement. For detectors with a small energy gap, this effect is primarily due to a decrease in the coherence term $|X|$ because of the influence horizon redshift.

It is possible to show explicitly that an analogous result holds for Rindler observers, and that a near-horizon limit of our results reduces to this case. This suggests an element of universality to our result, applicable to all black holes, with subleading contributions due to spacetime curvature modifying the dominant Rindlerian effect.

We also find a non-trivial dependence of the entanglement harvested by the detectors on the energy gap and the angle between them, even when they are arranged so that the proper separation between them is constant. This behaviour is due to competing effects between the local noise and non-local correlations of the detectors, which both depend on the energy gap and the distance from the horizon.

Chapter 7

Entanglement Harvesting with Coherently Delocalized Matter

The UDW detector model is often introduced, as it was in this thesis, as a simplification to the light-matter interaction, where no angular momentum is exchanged [49, 2, 62]. However, unlike an atom, the UDW detector has internal degrees of freedom that are quantum, but is described by a classical position, $\mathbf{x}_D$. That is, UDW detectors have classical center of mass degrees of freedom. One has to wonder, what is being lost, if anything, in the entanglement harvesting protocol when neglecting the quantum nature of the center of mass of approximate atoms.

A possible setup we have in mind here is the following: Consider two atoms, which are initially localized in a certain region of space via a center of mass position measurement, and are then left to evolve freely. The center of mass wave functions will spread and the detectors will dynamically delocalize in space. Now imagine that these two coherently delocalizing atoms interact with the electromagnetic vacuum. How much will the internal degrees of freedom of the atoms become entangled with each other? How will the result depend on the mass of the detectors, their initial localization, and their dynamical delocalization process? How will the results compare to the results for entanglement harvesting with classical center of mass degrees of freedom?

To study these questions, we will upgrade the standard UDW detector model using the generalized UDW detector described in [82]. From there, we will employ the usual entanglement harvesting protocol, coupling the internal degrees of freedom of a pair of generalized detectors to a quantum scalar field. We note that by simply upgrading the detectors' center of mass degree of freedom to a quantum variable and leaving the rest

of the interaction unchanged, we give up the connection to the light-matter interaction. In order for a generalized detector-field model to capture the light-matter interaction, it must include a Röntgen-like term that couples the detector's internal degrees of freedom, center of mass degrees of freedom, and the field [45]. However, we expect our toy model to provide some insight into the effects that quantum delocalization will have on the ability of a pair of detectors to harvest entanglement.

In this chapter, we use the convention that $\hbar = 1$, but retain a value c throughout our calculations in order to remind the reader that the virtual velocity of the detector is always non-relativistic. This is also set to 1 in the figures to better compare them with the previous literature.

7.1 Entanglement harvested from the vacuum by standard UDW detectors

Before taking into account the quantum delocalization of the centers of mass of the detectors, we will consider, for comparison, a pair of standard UDW detectors, labeled A and B whose center of mass degrees of freedom are described classically. We will take these two detectors to be static in $(3+1)$ -dimensional Minkowski space, separated by a distance $S := |\mathbf{x}_A - \mathbf{x}_B|$, and having identical energy gaps $\Omega_A = \Omega_B = \Omega$. We will also consider the case detectors are spatially smeared through identical spatial profile to better emphasize the difference between detector smearing and detector delocalization.

The detectors couple to the field through a switching function of compact support of the form

$$\chi(t) = \begin{cases} \sin(t/\sigma), & \text{for } 0 \leq t \leq \pi\sigma \\ 0 & \text{otherwise.} \end{cases} \quad (7.1)$$

This choice of switching function is almost identical to the cosine switching function used in chapter 4, and if pointlike detectors are used ($F_X(\mathbf{x}) \rightarrow \delta^{(3)}(\mathbf{x})$) will result in the same density matrix, up to a phase in the coherence term, in the case of a definite temporal order.

We choose a switching function with compact support in order to ensure that the interaction between the detectors and the field is switched on only during a compact time interval, $t \in [0, \pi\sigma]$, and we choose sine switching over cosine switching to ensure the time

interval begins at $t = 0$. The importance of both these choices will become apparent once we treat the detector's center of mass degrees of freedom as quantum.

After the interaction, that is after time $\pi\sigma$, the detector is “off” again and we calculate the final state of the joint state of the two detectors. Since we are interested in calculating the entanglement negativity (Eq. (2.55)) that has been accumulated by the interaction, we specifically calculate transition probabilities and the coherence term to lowest order in the coupling strength as follows,

$$\begin{aligned} P^c &= \frac{\lambda^2}{(2\pi)^3} \int \frac{d^n \mathbf{k}}{2c|\mathbf{k}|} \left| \int_0^{\pi\sigma} dt \tilde{F}(\mathbf{k}) \sin\left(\frac{t}{\sigma}\right) e^{it(c|\mathbf{k}|+\Omega)} e^{i\mathbf{k}\cdot\mathbf{x}_D} \right|^2 \\ &= \frac{\lambda^2 \sigma^2}{(2\pi)^3} \int \frac{d^3 \mathbf{k}}{2c|\mathbf{k}|} |\tilde{F}(\mathbf{k})|^2 A(\mathbf{k}), \end{aligned} \quad (7.2)$$

$$\begin{aligned} X^c &= -\frac{\lambda^2}{(2\pi)^3} \int \frac{d^n \mathbf{k}}{2c|\mathbf{k}|} \int_0^{\pi\sigma} dt \int_0^t dt' |\tilde{F}(\mathbf{k})|^2 \sin\left(\frac{t}{\sigma}\right) \sin\left(\frac{t'}{\sigma}\right) e^{i\Omega(t+t')} e^{-ic|\mathbf{k}|(t-t')} \\ &\quad \times (e^{i\mathbf{k}\cdot(\mathbf{x}_A-\mathbf{x}_B)} + e^{-i\mathbf{k}\cdot(\mathbf{x}_A-\mathbf{x}_B)}) \\ &= -\lambda^2 \sigma^2 e^{i\pi\Omega\sigma} \int \frac{d^3 \mathbf{k}}{c|\mathbf{k}|} \frac{e^{2i\mathbf{k}\cdot\mathbf{x}_0} \tilde{F}(\mathbf{k})^2 B(\mathbf{k})}{1 - \sigma^2(\Omega + c|\mathbf{k}|)^2}, \end{aligned} \quad (7.3)$$

where we use the superscript c to denote the classical nature of the detectors' center of mass, $\tilde{F}(\mathbf{k})$ denotes the Fourier transformation of the spatial smearing profile $F(\mathbf{x})$ and where we defined the following functions:

$$A(\mathbf{k}) := \frac{1 + \cos(\pi\sigma(\Omega + c|\mathbf{k}|))}{(\sigma^2(\Omega + c|\mathbf{k}|)^2 - 1)^2}, \quad (7.4)$$

$$B(\mathbf{k}) := \frac{i(2\Omega + c|\mathbf{k}|) \sin(\pi\Omega\sigma)}{2\Omega(1 - \Omega^2\sigma^2)} + \frac{e^{-i\pi\Omega\sigma} + e^{-i\pi c|\mathbf{k}|\sigma}}{1 - \sigma^2(\Omega - c|\mathbf{k}|)^2}. \quad (7.5)$$

We now specify a spatial smearing function according to which the detectors couple to the field to be a Gaussian smearing profile of standard deviation $L/2$:

$$F(\mathbf{x}) = \left(\frac{2}{\pi L^2} \right)^{3/2} e^{-2x^2/L^2}. \quad (7.6)$$

From now on, we refer to L as the “width” of the smearing profiles. Since the smearing profiles are normalized, we may interpret them as classical probability distributions, according to which the detectors couple to the quantum field.

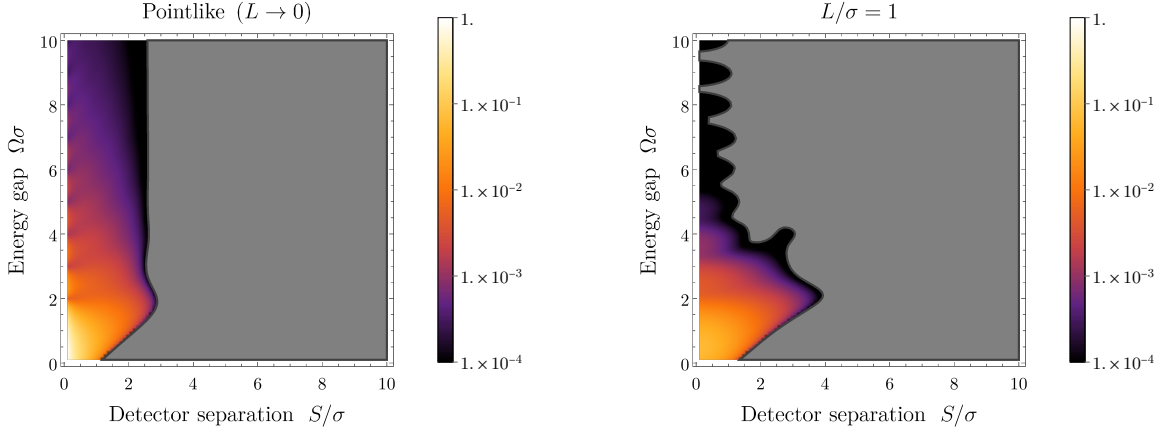


Figure 7.1: The negativity $\mathcal{N}(\hat{\rho}_{AB})/\lambda^2$ after the end of the interaction, as a function of the energy gap Ω and the separation S of the detectors, plotted (*left*) for pointlike UDW detectors and (*right*) for spatially smeared UDW detectors with $L = \sigma$. The regions of zero negativity are marked in grey.

With the spatial profile selected, we calculate the transition probabilities and the coherence term which both depend on the width L of the smearing profiles, the energy gap Ω of the detectors and the total interaction time $\pi\sigma$, and the coherence term also depends on the separation S of the two detectors:

$$P_L^c = \frac{\lambda^2 \sigma^2}{2\pi^2 c} \int_0^\infty dk k e^{-\frac{L^2 k^2}{4}} A(k), \quad (7.7)$$

$$X_L^c = -\frac{\lambda^2 \sigma^2 e^{i\pi\Omega\sigma}}{2\pi^2 c S} \int_0^\infty dk \frac{\sin(kS) e^{-\frac{L^2 k^2}{4}} B(k)}{1 - \sigma^2(\Omega + ck)^2}. \quad (7.8)$$

We use the subscript L to indicate that we are using Gaussian smearing profiles. In the limit of very narrow profiles ($L \rightarrow 0$), we recover the transition probabilities and the coherence term for pointlike UDW detectors 4:

$$P_0^c = \frac{\lambda^2 \sigma^2}{2\pi^2 c} \int_0^\infty dk k A(k), \quad (7.9)$$

$$X_0^c = -\frac{\lambda^2 \sigma^2 e^{i\pi\Omega\sigma}}{2\pi^2 c S} \int_0^\infty dk \frac{\sin(kS) B(k)}{1 - \sigma^2(\Omega + ck)^2} \quad (7.10)$$

The negativity after the end of the interaction, for both pointlike, which will serve as

a baseline comparison, and Gaussian smeared UDW detectors, is plotted in Fig. 7.1, as a function of the energy gap Ω of the detectors and their separation S . We note that when sine switching (Eq. (7.1)) is chosen, pointlike UDW detectors cannot harvest entanglement when they are spacelike separated (see Fig. 4.3 (lower left)); however spacelike entanglement harvesting is possible for a different choice of setup, for example choosing a Gaussian switching function [61, 62].

The Gaussian smearing profile given in equation (7.6) provides a simple modification to the UDW detectors that is distinct from introducing delocalization of their center of mass. We find that the smeared detectors can harvest entanglement for larger values of separation of the center of their distributions than the pointlike detectors. Physically, this is expected, since the smeared detectors have a spatial extent. Additionally, we find that the negativity decreases with increasing widths, L , of the smearing profiles. Intuitively, we can understand this behaviour as follows: spatially smeared UDW detectors average the quantum field fluctuations over extended spatial regions, and the larger these spatial regions are, the smaller the averaged value of fluctuations, and the less entanglement the detectors can harvest from the quantum field. In the negativity plots in Fig. 7.1 we further observe a resonance-like behaviour, for energy gaps that are multiples of the switching scale σ , which are manifest as the ripples in the negativity plots. The ripples are more pronounced in the case of the smeared detectors due to the overall reduced negativity.

7.2 Entanglement harvested by coherently delocalized detectors from the vacuum

The results for entanglement harvesting in the previous section relied on the assumption that the center of mass degrees of freedom of the matter systems under investigation are classical. In this section, we want to study how the process of entanglement harvesting is affected when the center of mass motion of the two detectors is quantum uncertain.

We now consider the center of mass degrees of freedom of the two detectors to be quantized which will allow the detectors to coherently delocalize, . We let $\hat{\mathbf{x}}_d$ and $\hat{\mathbf{p}}_D$ denote the center of mass position and momentum operators of the two detectors and take assume that the two detectors have equal mass, M . The detectors' center of mass each behave as free particles, and evolve with the free Hamilton

$$\hat{H}_0 = \frac{\hat{\mathbf{p}}_D^2}{2M}. \quad (7.11)$$

The detectors couple to the scalar field in the same manor as UDW detectors, via the monopole moment coupling. However, the field operators now take the center of mass position operators of the two detectors as their argument:

$$\hat{H}_{\text{int}}(t) = \lambda \chi(t) \sum_{D=A,B} \hat{\mu}_D(t) \hat{\phi}(\hat{\mathbf{x}}_D, t) \quad (7.12)$$

We employ the spectral theorem of functional calculus [90, 58, 82] to make sense of the operator valued operator $\hat{\phi}(\hat{\mathbf{x}}_D, t)$ as follows,

$$\hat{\phi}(\hat{\mathbf{x}}_D, t) := \int d^3 \mathbf{x}_D \hat{\mathcal{P}}(\mathbf{x}_D, t) \hat{\phi}(\mathbf{x}_D, t), \quad (7.13)$$

where the operators $\hat{\mathcal{P}}(\mathbf{x}_D, t)$ are given by

$$\hat{\mathcal{P}}(\mathbf{x}_D, t) := |\mathbf{x}_D(t)\rangle \langle \mathbf{x}_D(t)| = \int \frac{d^3 \mathbf{p} d^3 \mathbf{q}}{(2\pi)^3} e^{-i(\mathbf{p}-\mathbf{q}) \cdot \mathbf{x}_D + it \frac{\mathbf{p}^2 - \mathbf{q}^2}{2M}} |\mathbf{p}\rangle \langle \mathbf{q}|. \quad (7.14)$$

We again assume that initially the two detectors are in their ground states and the field is in its vacuum state. We further let $|\varphi_D\rangle$ denote the initial center of mass states of the two detectors. The initial state of the system then reads

$$\hat{\rho}_0 = |\varphi_A\rangle_{\text{COM}_A} \langle \varphi_A| \otimes |g_A\rangle_A \langle g_A| \otimes |\varphi_B\rangle_{\text{COM}_B} \langle \varphi_B| \otimes |g_B\rangle_B \langle g_B| \otimes |0\rangle_\phi \langle 0|. \quad (7.15)$$

We can express the initial center of mass states both in terms of the initial center of mass wave functions in the position and in the momentum representation:

$$|\varphi_D\rangle = \int d^3 \mathbf{x} \varphi_D(\mathbf{x}) |\mathbf{x}\rangle = \int d^3 \mathbf{p} \tilde{\varphi}_D(\mathbf{p}) |\mathbf{p}\rangle. \quad (7.16)$$

Again, we are interested in the final state of the internal degrees of freedom of the two detectors, and in particular, if they become entangled as a result of interacting with the quantum field. Therefore, after the interaction, we trace out the field and the center of mass degrees of freedom to once again obtain, to lowest order in the coupling strength,

$$\hat{\rho}_{AB} = \text{Tr}_{\phi, \text{COM}_A, \text{COM}_B} [\hat{U} \hat{\rho}_0 \hat{U}^\dagger] = \begin{pmatrix} 1 - P_A - P_B & 0 & 0 & X^* \\ 0 & P_B & C^* & 0 \\ 0 & C & P_A & 0 \\ X & 0 & 0 & 0 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (7.17)$$

We find the transition probabilities P_D of the two detectors to be

$$P_D = \frac{\lambda^2}{(2\pi)^3} \int_0^{\pi\sigma} dt \int_0^{\pi\sigma} dt' \chi(t)\chi(t') \int d^3p \int \frac{d^3\mathbf{k}}{2c|\mathbf{k}|} |\tilde{\varphi}_D(\mathbf{p})|^2 e^{i(t-t')\left(\Omega+c|\mathbf{k}|+\frac{\mathbf{k}^2-2\mathbf{p}\cdot\mathbf{k}}{2M}\right)} \quad (7.18)$$

and we note that they depend on their center of mass states only through their respective momentum probability distributions $|\tilde{\varphi}_D(\mathbf{p})|^2$. Unlike the transition probabilities, we find that the coherence term X also depends on the phases of the initial center of mass momentum wave functions:

$$X = -\frac{\lambda^2}{(2\pi)^3} \int_0^{\pi\sigma} dt \int_0^{t_1} dt' \chi(t)\chi(t') \int d^3\mathbf{p}_1 d^3\mathbf{p}_2 \int \frac{d^3\mathbf{k}}{2c|\mathbf{k}|} \tilde{\varphi}_A(\mathbf{p}_1 + \mathbf{k}) \tilde{\varphi}_B(\mathbf{p}_2 - \mathbf{k}) \\ \times \tilde{\varphi}_A^*(\mathbf{p}_1) \tilde{\varphi}_B^*(\mathbf{p}_2) e^{i(t+t')\left(\Omega-\frac{\mathbf{k}\cdot(\mathbf{p}_1+\mathbf{p}_2)}{2M}-\frac{\mathbf{k}^2}{2M}\right)} \left[e^{-i(t-t')\left(c|\mathbf{k}|-\frac{\mathbf{k}\cdot(\mathbf{p}_1+\mathbf{p}_2)}{2M}\right)} + e^{-i(t-t')\left(c|\mathbf{k}|+\frac{\mathbf{k}\cdot(\mathbf{p}_1+\mathbf{p}_2)}{2M}\right)} \right] \quad (7.19)$$

The phases of the momentum wave functions carry the position information of the two wave functions, and as expected, the coherence term thus depends on the spatial locations of the two center of mass wave packets. While the transition probabilities of the detectors respectively only depend on the properties of one detector, the coherence term thus depends on the properties of both detectors.

We can now specify the initial center of mass wave functions for the two detectors. Let us consider a pair of detectors whose center of mass position wave functions are Gaussian wave packets of initial width L , respectively centered around $\mathbf{x}_D$, at a separation $S := |\mathbf{x}_A - \mathbf{x}_B|$:

$$\varphi_D(\mathbf{x}) = \left(\frac{2}{\pi L^2}\right)^{3/4} e^{-\frac{|\mathbf{x}+\mathbf{x}_D|^2}{L^2}}, \quad (7.20)$$

$$\tilde{\varphi}_D(\mathbf{p}) = \left(\frac{L^2}{2\pi}\right)^{3/4} e^{-\frac{\mathbf{p}^2 L^2}{4} + i\mathbf{p}\cdot\mathbf{x}_D} \quad (7.21)$$

Since the momentum probability distributions are the same for both detectors,

$$|\tilde{\varphi}_D(\mathbf{p})|^2 = L^3/(2\pi)^{3/2} e^{-\mathbf{p}^2 L^2/2}. \quad (7.22)$$

We thus find that the excitation probabilities of the two detectors are equal, $P_A = P_B = P$.

We can now see why it is important to employ a switching function of compact support.

Under the free quantum mechanical time evolution, the wave packets in equation (7.21) start out completely delocalized in space for $t \rightarrow -\infty$, then flow together to Gaussians of width L at time $t = 0$, and finally spread again into completely delocalized states for $t \rightarrow \infty$. If we employed a switching function of non-compact support, such as a Gaussian switching function, we would need to consider the completely delocalized center of mass wave packets at time $t \rightarrow -\infty$ as the initial center of mass states. However, our goal is to study how the entanglement harvesting protocol is modified by the spreading of the center of mass, we need to consider a localized wave packets at the start of interaction. We therefore use the switching function (Eq. (7.1)) to ensure that the interaction is switched on at precisely $t = 0$ when the wave packets of the detector are the most localized.

With these considerations, we obtain

$$P = \frac{\lambda^2 \sigma^2}{\pi c} \int_0^\infty dp p^2 |\tilde{\varphi}_D(p)|^2 \mathcal{U}(p) \quad (7.23)$$

for the transition probabilities, where we define the template function

$$\mathcal{U}(p) := \int_{-1}^1 dz \int_0^\infty dk k \left[1 + \cos \left(\pi \sigma \left(\mathcal{F} - \frac{k p z}{M} \right) \right) \right] \times \left[\sigma^2 \left(\mathcal{F} - \frac{k p z}{M} \right)^2 - 1 \right]^{-2} \quad (7.24)$$

p is the magnitude of the detector's recoil momentum and we define $\mathcal{F} := \Omega + ck + k^2/(2M)$ for convenience of notation. We refer to $\mathcal{U}(p)$ as a “template function” following the work of [82] due to the fact that it is independent of the center of mass states of the detectors. For the coherence term we obtain,

$$X = \frac{\lambda^2 \sigma^2 L^2}{c(2\pi)^3} \int_{-\infty}^\infty dp_1 \int_{-\infty}^\infty dp_2 \int_0^\infty dk \frac{\sin(kS)}{S} e^{-L^2(p_1^2 + p_2^2)/2} e^{-L^2 k^2/4} \mathcal{V}(k, p_1, p_2), \quad (7.25)$$

where again we defined a template function,

$$\mathcal{V}(k, p_1, p_2) := \sum_{j=0}^1 \frac{e^{i\pi\alpha\sigma}}{1 - \sigma^2(\alpha + \beta_j)^2} \left(\frac{i(2\alpha + \beta_j) \sin(\pi\alpha\sigma)}{2\alpha(1 - \alpha^2\sigma^2)} + \frac{e^{-i\pi\alpha\sigma} + e^{-i\pi\beta_j\sigma}}{1 - \sigma^2(\alpha - \beta_j)^2} \right) \quad (7.26)$$

and we defined $\alpha := \Omega - k(p_1 - p_2)/(2M)$ as well as $\beta_j := ck + (-1)^j k(p_1 + p_2)/(2M)$ for convenience of notation.

Since we work within a framework in which the center of mass dynamics are described by the Schrödinger equation, we need to ensure that the virtual center of mass velocities

are well within the non-relativistic regime. That is, we need to ensure that the momentum probability distributions $|\tilde{\varphi}_D(\mathbf{p})|^2$ have contributions only for momenta corresponding to virtual velocities much smaller than the speed of light. Let us here restrict the virtual velocities to velocities no larger than one percent of the vacuum speed of light, $v := p/M \leq 0.01c$. The Gaussian momentum probability distributions of the detectors have a standard deviation of $1/L$. We can thus assume to a very good approximation (within 3.5 standard deviations away from the mean) that the center of mass momenta $\mathbf{p}$ in the probability distributions satisfy $pL \lesssim 3.5$. The non-relativistic regime therefore corresponds to parameters L and M satisfying

$$LMc \gtrsim 3.5 \times 10^2. \quad (7.27)$$

We note that the center of mass wave function of a coherently delocalized detector spreads faster for smaller L and M , that is, for initially more sharply localized detectors and for smaller detector masses. As a consequence, for a given detector mass, there is a minimal initial localization width we can consider while staying within the non-relativistic regime for the virtual center of mass velocities.

Provided that we chose appropriate parameters M and L , we can consider the template functions $\mathcal{U}$ and $\mathcal{V}$ for non-relativistic virtual center of mass velocities. We Taylor expand $\mathcal{U}$ around $p/(Mc) = 0$, and we Taylor expand $\mathcal{V}$ around both $p_1/(Mc) = 0$ and $p_2/(Mc) = 0$. To second order in $(LMc)^{-1}$, we obtain the following simplified expressions for the transition probabilities and the coherence term:

$$P = \frac{\lambda^2 \sigma^2}{4\pi^2 c} \left[\mathcal{U}(0) + \frac{3}{2L^2} \frac{\partial^2 \mathcal{U}}{\partial p^2}(0) + \mathcal{O}((LMc)^{-4}) \right], \quad (7.28)$$

$$X = \frac{\lambda^2 \sigma^2}{4\pi^2 c} \int_0^\infty dk \frac{\sin(kS)}{S} e^{-L^2 k^2/4} \left[\mathcal{V}(k, 0, 0) + \frac{C(k)}{(LMc)^2} + \mathcal{O}((LMc)^{-4}) \right], \quad (7.29)$$

where we define

$$C(k) := \frac{M^2 c^2}{2} \left[\frac{\partial^2 \mathcal{V}}{\partial p_1^2}(k, 0, 0) + \frac{\partial^2 \mathcal{V}}{\partial p_2^2}(k, 0, 0) \right] \quad (7.30)$$

for notational convenience.

As shown in Fig. 7.2, we find that the transition probabilities decrease, both for increasing energy gaps Ω and decreasing detector masses M . Intuitively, this behavior can be explained as follows: switching the interaction on and off breaks time translation invariance and therefore provides energy for the excitation and the recoil of the detectors and the

excitation of the field. The kinetic energy of the recoil becomes larger for smaller detector masses. Since the excitation of the detector is always accompanied by the emission of a field quantum and the recoil of the detector, the excitation process becomes energetically more expensive for larger energy gaps and smaller detector masses. In the limit of infinitely small energy gaps, the detectors essentially turn into simple charges and all the switching energy can go into the recoil of the detectors and the excitation of the field. Similarly, in the limit of infinitely large detector masses, the kinetic recoil energy tends to zero and all the switching energy can go into the excitation of the field and the internal degrees of freedom. When considering the transition probability of a very massive detector, it is justifiable to neglect the recoil of the detector and to model the center of mass degrees of freedom classically. This can be seen in Fig. 7.2, in the limit of large detector masses the transition probability approaches the excitation probability of a pointlike UDW detector.

In Fig. 7.3, we further find that the coherence term is suppressed both in the separation of the two detectors and in the initial center of mass delocalization widths. Intuitively, this is because the amplitude of the quantum field fluctuations (which correlate the quantum field amplitudes at different points in space and time) decrease with the fluctuation size; the more delocalized the detectors are, the larger are the spatial regions in which the two detectors probe the quantum field fluctuations, and the smaller is thus the coherence term. In fact, as we can see from Eq. (7.29), the coherence term for coherently delocalized detectors is Gaussian suppressed in the initial delocalization width. This is in contrast to the transition probabilities, whose leading order term does not depend on the initial delocalization width at all, as we can see from Eq. (7.28). Therefore, we can conclude that the ability of quantum delocalized detectors to harvest entanglement from the vacuum is Gaussian suppressed in the initial center of mass delocalization.

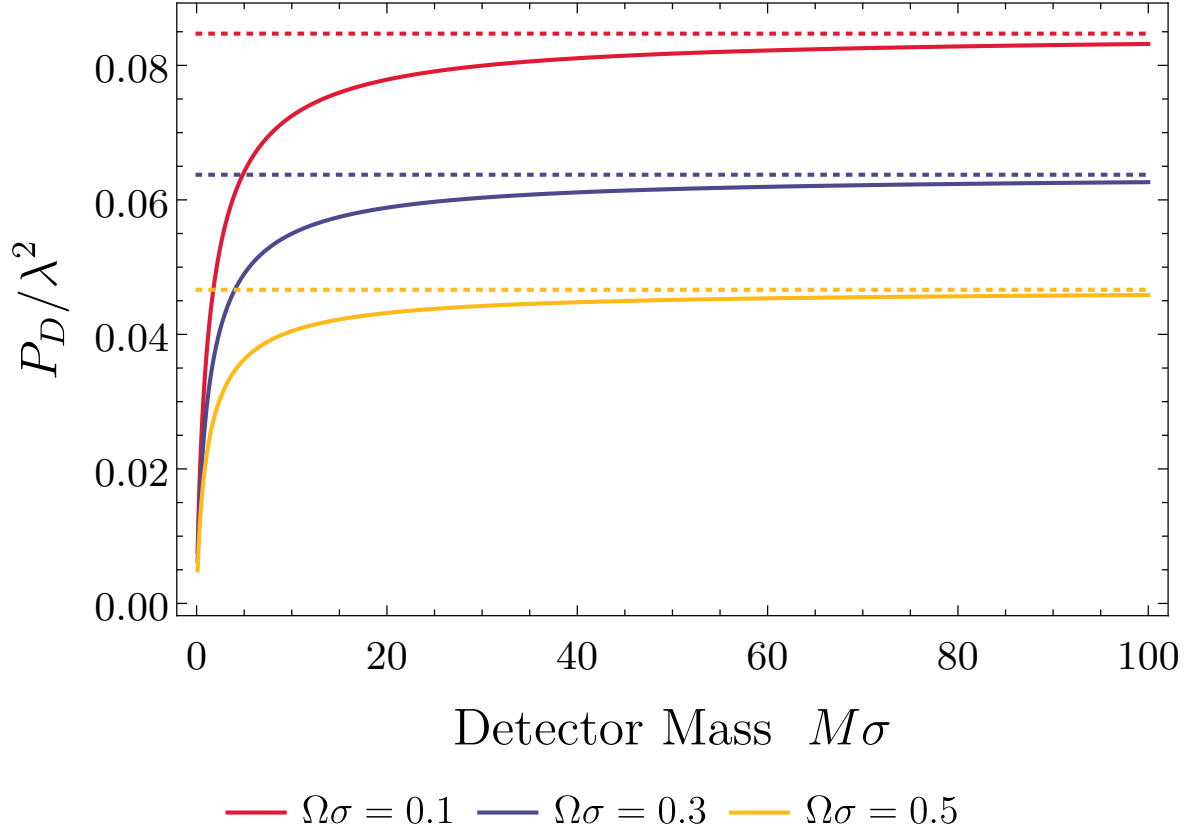


Figure 7.2: The transition probability, P_D , for a detector as a function of its mass M , after the end of the interaction, for different energy gaps and with $L = 1000\sigma$. The dotted lines represent the excitation probabilities of pointlike UDW detectors with the same energy gaps as the respective massive detectors.

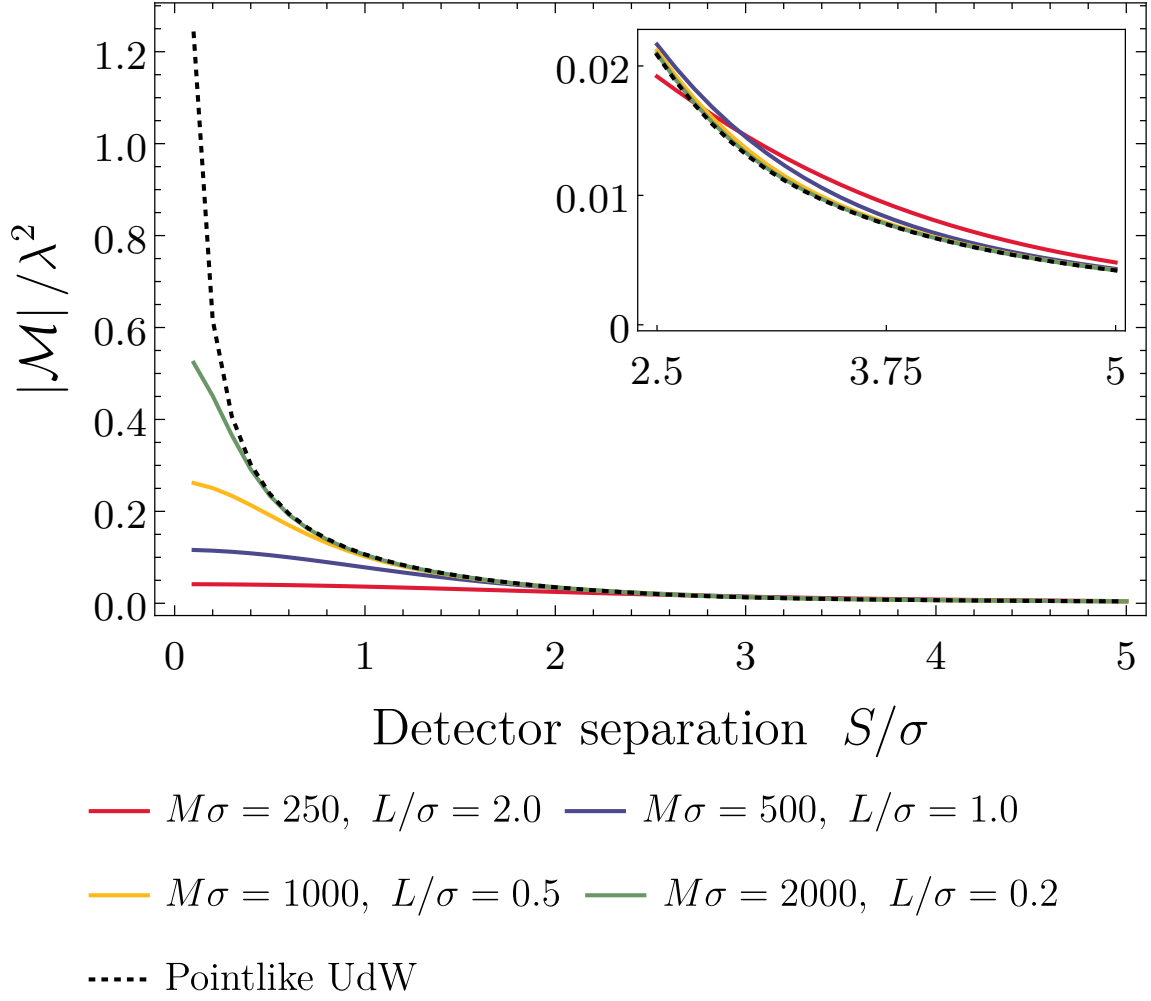


Figure 7.3: The coherence term, X , for pointlike UDW detectors as well as for massive detectors, after the end of the interaction, as a function of the detector's separation S (with $\Omega\sigma = 0.1$). For the massive detectors, we chose different values for M and L such as to keep their product constant ($ML = 500$), which fixes the virtual velocities at which the detectors dynamically delocalize.

7.3 Recovering entanglement harvesting for UDW detectors

Now we will consider a few different limits that one could take in order to recover the entanglement harvesting results for UDW detectors, that is, detectors with a classical center of mass. In the previous section we saw as the detector mass increased, the transition probability of the detectors approached the UDW transition probability, so we will start by considering the $M \rightarrow \infty$ limit while keeping the initial delocalization width, L , fixed.

One might expect to recover the classical behaviour of UDW detectors in this limit, since the dynamical quantum center of mass delocalization process becomes very slow: the virtual center of mass velocities satisfy $v \lesssim 3.5/(LM)$ and thus tend to zero in this limit. Even though the detectors each have a finite initial delocalization width, their center of mass wave packets do not coherently spread any further.

We do indeed find that the transition probabilities and the coherence term respectively reduce to the transition probabilities and the coherence term for UDW detectors. However, there is a twist: The excitation probabilities reduce to the excitation probabilities for pointlike UDW detectors,

$$\lim_{M \rightarrow \infty} P = P_0^c, \quad (7.31)$$

while the coherence term reduces to the coherence term for Gaussian smeared UDW detectors,

$$\lim_{M \rightarrow \infty} X = X_L^c. \quad (7.32)$$

Thus, in the limit of very large detector masses and for finite initial delocalization widths L , the negativity neither reduces to the negativity for a pair of pointlike UDW detectors, nor to the negativity for a pair of Gaussian smeared UDW detectors.

We can understand this behavior as follows. In the infinite mass limit, the kinetic energy of the recoil of the detectors tends to zero. The center of mass degrees of freedom no longer play a role in the energy balance of the excitation process of the detectors, and the recoil of the detector becomes negligible. In this limit, we can interpret the center of mass probability distributions as classical probability distributions for the position of the two pointlike UDW detectors. Since the transition probability of a pointlike UDW detector is independent of the position of the detector, we recover the results for pointlike UDW detectors for the transition probabilities. However, the nonlocal correlations shared by two pointlike UDW detectors depend on the detector separation, and therefore they will also depend on the classical position probability distributions for the two detectors. Consequently, in the large mass limit the coherence term does not reduce to the coherence

term for pointlike UDW detectors, but rather to the coherence term for spatially smeared UDW detectors.

Recall that Gaussians of width L approach delta distributions in the limit $L \rightarrow 0$. Clearly, we should be able to recover the entanglement harvesting results for a pair of pointlike UDW detectors, in the limit of very large detector masses and center of mass distributions that are very sharply peaked (and thus essentially completely localized) at all times. However, we need to approach this limit in a way that ensures that the virtual center of mass velocities stay within the non-relativistic regime identified in Eq. (7.27). In other words, we need $M \rightarrow \infty$ faster than $L \rightarrow 0$. To achieve this, we define

$$m := M\gamma, \quad l := L/\gamma, \quad (7.33)$$

with γ a unitless regularization factor and with m and l constants satisfying $lmc \gtrsim 3.5 \times 10^2$. Letting $\gamma \rightarrow 0$ then lets the initial center of mass localization become very sharp ($L \rightarrow 0$) and the detector masses become very large ($M \rightarrow \infty$), while keeping the virtual center of mass velocities fixed and therefore non-relativistic. In the limit $\gamma \rightarrow 0$, the transition probabilities and the coherence term reduce to

$$\begin{aligned} P \rightarrow P_0^c + \frac{\lambda^2 \sigma^2}{4\pi^2 c (lmc)^2} \int_0^\infty dk \frac{\sigma^2 c^2 k^3}{((\Omega + ck)^2 \sigma^2 - 1)^4} & \left(20(\Omega + ck)^2 \sigma^2 + 4 \right. \\ & + \cos(\pi\sigma(\Omega + ck)) [20(\Omega + ck)^2 \sigma^2 + 4 - \pi^2((\Omega + ck)^2 \sigma^2 - 1)^2] \\ & \left. + 8(\Omega + ck)\pi\sigma[(\Omega + ck)^2 \sigma^2 - 1] \sin(\pi\sigma(\Omega + ck)) \right) + \mathcal{O}((lmc)^{-4}), \end{aligned} \quad (7.34)$$

$$X \rightarrow X_0^c + \frac{\lambda^2 \sigma^2}{4\pi^2 c (lmc)^2} \int_0^\infty dk \frac{\sin(kS)}{S} C(k) + \mathcal{O}((lmc)^{-4}). \quad (7.35)$$

By letting the virtual center of mass velocities go to zero, we can then describe two detectors whose center of mass degrees of freedom are localized very sharply at all times. We find that by taking the limit $\gamma \rightarrow 0$ first and then taking the limit $1/(lmc) \rightarrow 0$, we recover the transition probabilities and the coherence term for two pointlike UDW detectors, $P \rightarrow P_0^c$ and $X \rightarrow X_0^c$. Therefore, we have identified the limit in which entanglement harvesting for a pair of coherently delocalized detectors reduces to entanglement harvesting for a pair of pointlike UDW detectors. On the other hand, we find that there is no limit in which the results reduce to entanglement harvesting for a pair of spatially smeared UDW detectors. This confirms what we mentioned before, namely that spatial smearing profiles are appropriate to model the finite spatial extent of atoms due to their electronic orbitals [62], but not to model the coherent center of mass delocalization of an atom [82].

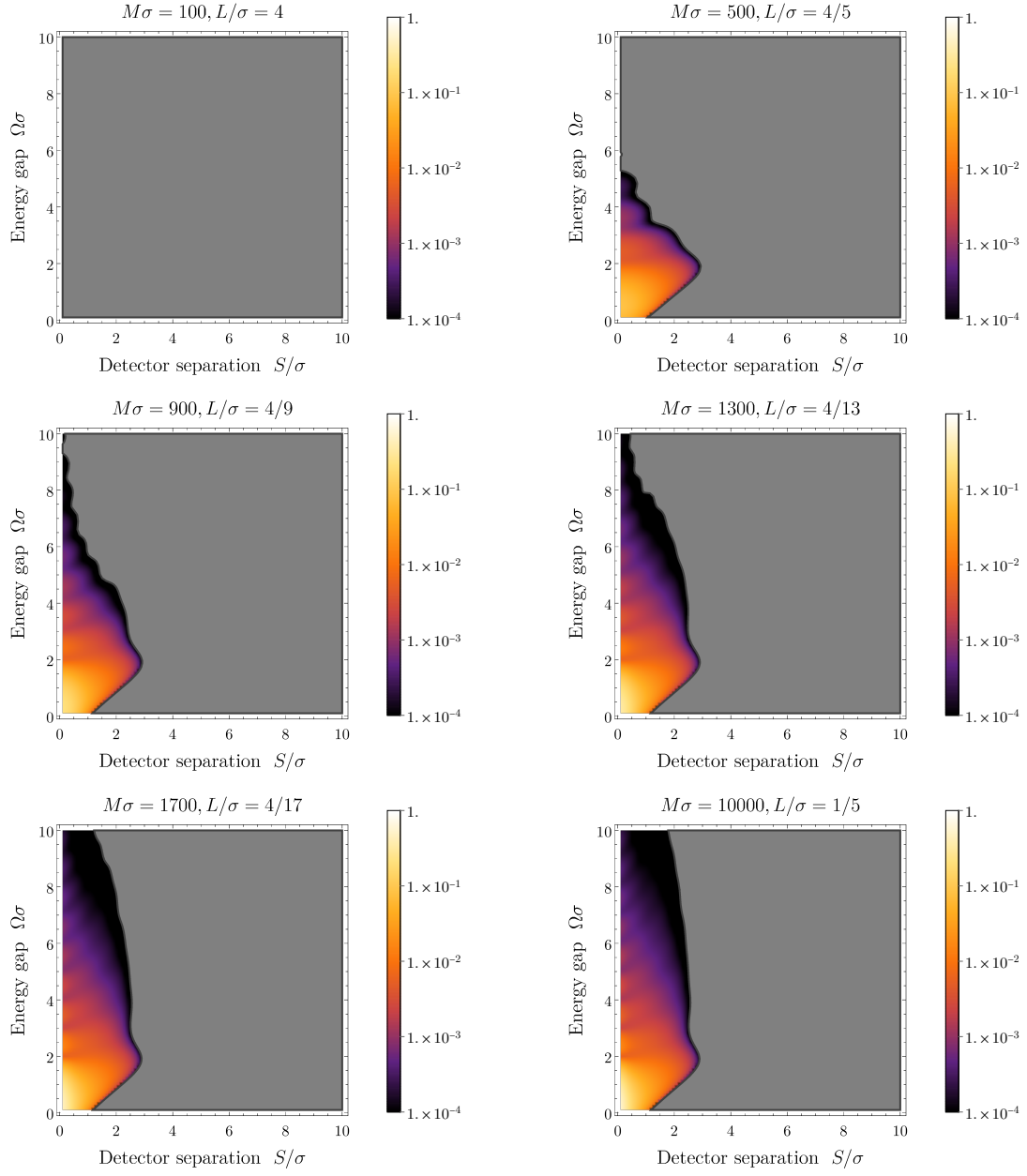


Figure 7.4: The negativity $\mathcal{N}(\hat{\rho}_{AB})/\lambda^2$ for two coherently delocalizing detectors, after the end of the interaction, plotted as a function of the energy gap Ω and the separation S of the two detectors. Regions of zero negativity are marked in grey. We chose the detector masses M and the initial center of mass localization widths L so that the unitless regularization factor γ decreases from left to right and from top to bottom.

In Fig. 7.4, we plotted the entanglement negativity for two coherently delocalizing detectors, as a function of the energy gap and the separation of the detectors. We can clearly see how the negativity reduces to the negativity for a pair of pointlike UDW detectors, when first letting $\gamma \rightarrow 0$ and then also letting $1/(lmc) \rightarrow 0$. We also observe that entanglement harvesting is indeed highly suppressed in the initial center of mass delocalization width.

Overall, we find that entanglement harvesting is suppressed for coherently delocalized detectors (and thus for actual physical matter systems such as atoms, ions or molecules), compared to entanglement harvesting for UDW detectors, whose center of mass degrees of freedom are assumed to be classical. An intuitive explanation for this suppression might be the following. We here focused on the entanglement harvested by the internal degrees of freedom of the two detectors. However, further (bipartite as well as multipartite) entanglement could potentially build up between the respective internal and center of mass degrees of freedom of the two detectors. This entanglement, which remains to be calculated, might build up at the expense of entanglement between the internal degrees of freedom of the two coherently delocalized detectors.

7.4 Entanglement harvested by coherently delocalized detectors from the ground state of a medium

Experimentally verifying entanglement harvesting from the vacuum is a difficult task [54, 55, 71, 50]. It might be more feasible to experimentally observe entanglement harvesting from the ground state of a medium, e.g., by sending atoms through a thin foil or a Bose-Einstein condensate. Here, we want to shed light on whether or not the internal degrees of freedom of quantum delocalized atoms might become entangled through their interactions with the ground state of a medium.

In the previous sections, we modelled the electromagnetic field via a simple scalar quantum field with dispersion relation $\omega = c |\mathbf{k}|$, where c stands for the vacuum propagation speed of light. We will here model a medium via a scalar quantum field with dispersion relation $\omega = c_s |\mathbf{k}|$, where $c_s < c$ is the wave propagation speed in the medium. The propagation of waves in the scalar field could then for instance model the propagation of light in a medium, or the propagation of sound in a phononic field, both of which are known to propagate slower than light in the vacuum. Repeating the calculations we performed in

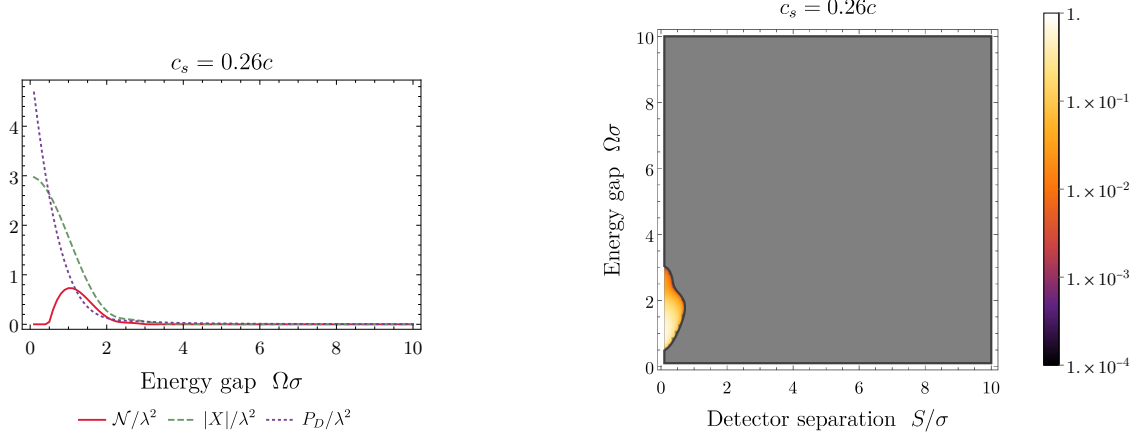


Figure 7.5: We consider two detectors (with detector masses $M\sigma = 900$ and initial localization widths $L\sigma = 4/9$) in a medium with wave propagation speed $c_s = 0.26c$. We plot (*left*) the transition probability P , the coherence term X and the negativity $\mathcal{N}(\hat{\rho}_{AB})/\lambda^2$, as function of the energy gap Ω and for a detector separation $S = 10\sigma$, and (*right*) the negativity $\mathcal{N}(\hat{\rho}_{AB})/\lambda^2$, after the end of the interaction, as a function of the energy gap and the detector separation. The region of zero negativity is marked in grey.

the previous section, we obtain the transition probabilities

$$P = \frac{\lambda^2 \sigma^2}{4\pi^2 c_s} \left[\mathcal{U}(0) + \frac{3}{2L^2} \frac{\partial^2 \mathcal{U}}{\partial p^2}(0) + \mathcal{O}((LMc)^{-4}) \right] \quad (7.36)$$

and the coherence term

$$X = \frac{\lambda^2 \sigma^2}{4\pi^2 c_s} \int_0^\infty dk \frac{\sin(kS)}{S} e^{-L^2 k^2/4} \left\{ \mathcal{V}(k, 0, 0) + \frac{1}{2L^2} \left[\frac{\partial^2 \mathcal{V}}{\partial p_1^2}(k, 0, 0) + \frac{\partial^2 \mathcal{V}}{\partial p_2^2}(k, 0, 0) \right] + \mathcal{O}((LMc)^{-4}) \right\}, \quad (7.37)$$

where $\mathcal{U}$ and $\mathcal{V}$ are defined as in Eq. (7.24) and Eq. (7.26), with the exception that c is being replaced by c_s in the definitions of $\mathcal{U}$ and β_j .

In Fig. 7.5 we plot the transition probability, the coherence term and the negativity for a pair of coherently delocalized detectors in a medium with wave propagation speed $c_s = 0.26c$. We find that the negativity is significantly suppressed, compared to the negativity in Fig. 7.4, in which the detectors were in the vacuum. We explain this behaviour as

follows: transforming into the quantum uncertain rest frame of the delocalizing detectors, the phononic ground state transforms non-trivially into an excited field state, which might be more entangled than the phononic ground state. The non-local correlations thus potentially increase, but at the same time, the “noisy” transition probabilities also increase. For a choice of $c_s = 0.26c$ these two competing effects result in a significant reduction of the negativity.

Both light and sound can be slowed down significantly in media (e.g. light in crystals or sound in Bose-Einstein condensates), to the extreme of being stopped completely [44, 4]. The detectors in such media could coherently delocalize with virtual velocities that are comparable to, or even larger than, the propagation speed in the medium, $v \gtrsim c_s$, while remaining well within the non-relativistic regime, $v \leq 0.01c$. Gaussian center of mass wave packets with support for supersonic virtual center of mass velocities are ones for which $LMc_s \gtrsim 3.5$, while the non-relativistic regime is characterized by $LMc \gtrsim 3.5 \times 10^2$.

In Fig. 7.6 we plotted the transition probability, the coherence term and the negativity for two detectors in a medium whose wave propagation speed is 1% of the vacuum speed of light. We chose the parameters so that the maximal virtual center of mass velocities in the Gaussian wave packet are close to the speed of sound in the medium ($LMc_s = 4$), while staying well within the non-relativistic regime ($LMc = 400$). We find that while the coherence term is significantly enhanced, the transition probabilities absolutely dominate, so overall the negativity vanishes. We thus find that if the wave propagation speed in the medium is too small, the internal degrees of freedom of a pair of coherently delocalizing detectors cannot become entangled with each other.

Based on these results, we conjecture that it is generally harder for detectors to harvest entanglement from a medium than from the vacuum. Entanglement harvesting experiments in media might however still be worth considering, given that they may be more easily conducted than the harvesting of entanglement from the vacuum.

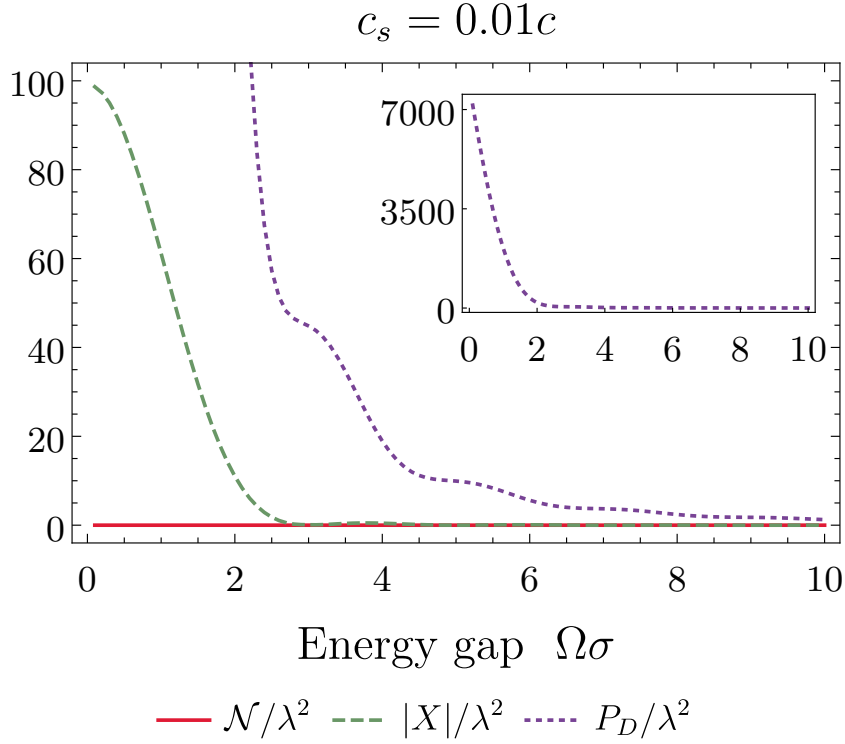


Figure 7.6: We consider two detectors (with detector masses $M\sigma = 900$, initial localization widths $L\sigma = 4/9$ and detector separation $S = \sigma/10$) in a medium with wave propagation speed $c_s = 0.01c$. We plot the transition probability P and the coherence term X , and the negativity, $\mathcal{N}(\hat{\rho}_{AB})/\lambda^2$, as function of the energy gap. We find that the negativity vanishes for this choice of parameters.

7.5 Summary of results

We studied the ability of two quantum delocalized detectors to become entangled with one another, via their respective interaction with a scalar quantum field. We found that the process of entanglement harvesting is affected by the coherent delocalization of matter and, in particular, that delocalized detectors harvest less entanglement than detectors whose center of mass degrees of freedom are assumed to behave classically.

We identified the limit in which the results for entanglement harvesting for coherently delocalized detectors reduce to the results for detectors with classical external degrees of freedom: For two detectors of very large mass, whose center of mass wave functions are initially very sharply peaked and which dynamically delocalize very slowly, we recover the negativity for two pointlike UDW detectors. This limit corresponds to detectors whose centers of mass are essentially completely localized at all times.

Finally, we discussed entanglement harvesting in media, where we found that entanglement harvesting for coherently delocalized detectors decreases with decreasing wave propagation speeds.

Chapter 8

Conclusions and Outlook

The goal of this thesis was to use a pair of Unruh-DeWitt detectors and the entanglement harvesting protocol to learn about properties of the quantum scalar field that they couple to and the underlying spacetime. We have also been interested in tuning the detectors so that they are best able to detect these properties.

In chapter 3, we coupled the detectors to a bandlimited scalar field in $(3+1)$ Minkowski space. We found that due to the finite size of the field degrees of freedom, the detector were entangled after the interaction when they were separated by distances that would give leave them in a separable state in the non-bandlimited case. This property can be used to put an upper bound on the value of the bandlimit.

From here it would be interesting to apply a covariant bandlimit, such as the one described by [37, 15] to the UDW model, at least for a single detector. This work may also have applications in superconducting circuits, where it was shown using a UDW detector coupled to the conjugate momentum of a scalar field that the transition probability of a superconducting qubit can provide information about the UV behaviour (cutoff) of the open transmission line [52], so the extension to a pair of qubits is very natural.

In chapter 4, we coupled the detectors to a scalar field in $(3+1)$ Minkowski space through a quantum controlled switching function. This allows the detectors to switch in a superposition times, for example, “ A before B ” and “ B before A ”. When the switch in a quantum controlled superposition, we find that they are able to harvest more entanglement then the when they are in a classical mixture, and can even appear to violate a no-go theorem and harvest entanglement with quantum controlled superposition of a Dirac delta switching.

A natural extension of this work, is to apply it to a situation where the detectors are redshifted relative to each other, such as if they moving with different velocities or in a curved spacetime.

In chapter 5, we studied the entanglement harvesting protocol in AdS_3 . We found that the concurrence between the detectors is maximum for an optimal AdS length which depends on the boundary conditions of the field. We also find “islands of separability”, a small location in the parameter space where the detectors cannot become entangled, which occurs in the an Ads length $\ell \approx 5/2\sigma$. The physics behind these still needs to be fully understood.

One extension of this work would be to consider Dirac δ -switching functions and solving the time evolution operator non-perturbatively. Although entanglement harvesting would no be possible in this regime, the results for the transition probabilities of the detectors would provide a good cross check to the work presented here.

In chapter 6, we carried out the first study of the entanglement harvesting protocol in a black hole spacetime. We found that BTZ black holes inhibit entanglement. When the detectors are moved close to the horizon while keeping the their proper distance constant, the amount of entanglement harvested at the end of the interaction rapidly falls to zero, due to the local Hawking temperature increasing the transition probability of the detectors and the redshift reducing non-local correlations.

There are still many extensions to this area of study, for example, what happens to the entanglement between the detectors if we initialize them in a maximally mixed state rather than a separable state. Alternatively, we can consider what happens if one or both of the detectors are fall into the black hole.

Finally, in chapter 7, we upgraded our detector model to included a quantum center of mass degree of freedom. This upgrade, in general, makes it much more difficult for the internal degrees of freedom of a pair of detectors to become entangled through local interactions with the field. We conjecture this is because the center of mass degrees of freedom preferentially become entangled. So, the obvious next step would be to quantify that entanglement. This will likely be a technically difficult to do since the momentum operators have an unaccountably infinite Hilbert space. However, this could be worked around, by post-selecting on a specific recoil momentum or discretizing the momentum space. This upgrade could also be applied to a more realistic entanglement harvesting model such as the hydrogen-like atom and electromagnetic field described [62, 45].

In conclusion, the answer to, “What can detectors detect?” is, “They can detect many interesting things and we haven’t yet pushed them to their full potential”.

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APPENDICES

Appendix A

Additional Calculations for Chapter 4

A.1 Determining $M_{DE,ij}(k)$

Due to the time ordering, the expression for $M_{DE,ij}(k)$ depends on the time delay between detector D and detector E (for $D = A, B$ and $E = A, B$ and $D \neq E$). The four different cases are illustrated in figure A.1

For cosine switching (4.22), case #1 occurs when $(T_{D,i} - T_{E,j}) < -\pi\eta/2$. Here

$$M_{DE,ij} = 0 \quad (\text{A.1})$$

(for those particular values of D , E , i , and j).

Case #2 occurs when $(T_{D,i} - T_{E,j}) > \pi\eta/2$, and

$$M_{DE,ij}(k) = -\frac{\lambda^2\eta^2}{4\pi^2} \frac{e^{i(\Omega_D T_{D,i} + \Omega_E T_{E,j})} e^{-ik(T_{D,i} - T_{E,j})}}{(1 - (\eta/2)^2(k - \Omega_D)^2)(1 - (\eta/2)^2(k + \Omega_E)^2)} \frac{\sin(kS)}{S} \\ \times \cos\left(\frac{\pi\eta}{4}(k - \Omega_D)\right) \cos\left(\frac{\pi\eta}{4}(k + \Omega_E)\right). \quad (\text{A.2})$$

Case #3a occurs when $-\pi\eta/2 < (T_{D,i} - T_{E,j}) < 0$, and

$$M_{DE,ij} = -\frac{\lambda^2\eta}{8\pi^2} \frac{\sin(kS)}{S} \left(I_1^{(3a)} + I_2^{(3a)} + I_3^{(3a)} \right) \quad (\text{A.3})$$

where

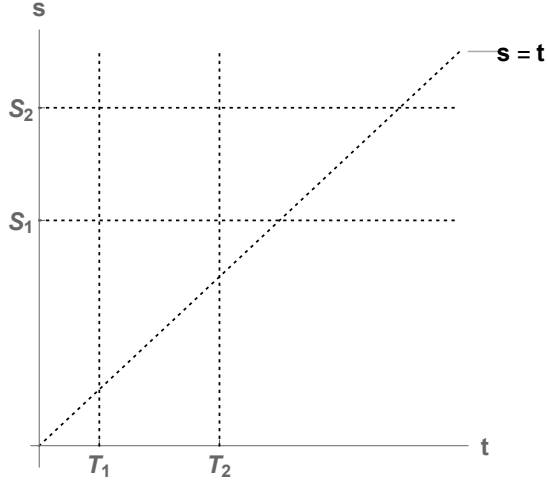
$$\begin{aligned}
I_1^{(3a)} &= \frac{\eta e^{i\pi\eta(\Omega_D - \Omega_E)/4} e^{i(\Omega_D T_{D,i} + \Omega_E T_{E,j})} e^{-ik(T_{D,i} - T_{E,j} + \pi\eta/2)}}{2 (1 - (\eta/2)^2(k - \Omega_D)^2) (1 - (\eta/2)^2(k + \Omega_E)^2)} \\
I_2^{(3a)} &= \frac{e^{i(T_{D,i} + \pi\eta/4)(\Omega_D + \Omega_E)}}{(\Omega_D + \Omega_E) (1 - (\eta/2)^2(k + \Omega_E)^2) (4 - (\eta/2)^2(\Omega_D + \Omega_E)^2)} \\
&\quad \times \left[\frac{\eta}{2} (2k + \Omega_D + 3\Omega_E) \cos\left(\frac{2(T_{D,i} - T_{E,j})}{\eta}\right) \right. \\
&\quad \left. - i(2 + (\eta/2)^2(k + \Omega_E)(\Omega_D + \Omega_E)) \sin\left(\frac{2(T_{D,i} - T_{E,j})}{\eta}\right) \right] \\
I_3^{(3a)} &= -\frac{e^{i(T_{E,j} - \pi\eta/4)(\Omega_D + \Omega_E)}}{(\Omega_D + \Omega_E) (1 - (\eta/2)^2(k - \Omega_D)^2) (4 - (\eta/2)^2(\Omega_D + \Omega_E)^2)} \\
&\quad \times \left[\frac{\eta}{2} (2k - 3\Omega_D - \Omega_E) \cos\left(\frac{2(T_{D,i} - T_{E,j})}{\eta}\right) \right. \\
&\quad \left. - i(2 - (\eta/2)^2(k - \Omega_D)(\Omega_D + \Omega_E)) \sin\left(\frac{2(T_{D,i} - T_{E,j})}{\eta}\right) \right]. \tag{A.4}
\end{aligned}$$

Case #3b occurs when $0 < (T_{D,i} - T_{E,j}) < \pi\eta/2$, and

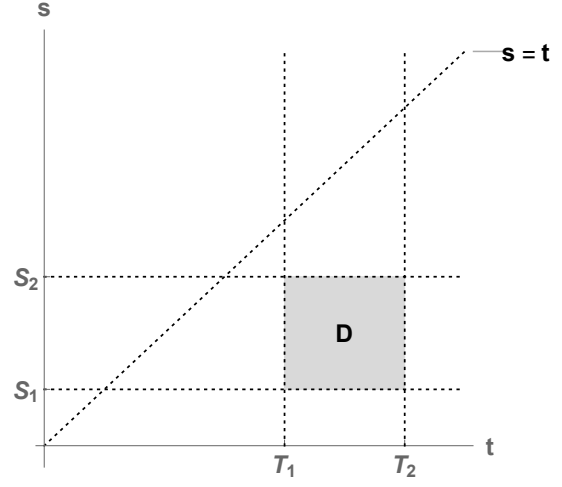
$$M_{DE,ij}(k) = -\frac{\lambda^2 \eta \sin(kS)}{8\pi^2 S} (I_1^{(3b)} + I_2^{(3b)} + I_3^{(3b)}) \tag{A.5}$$

where

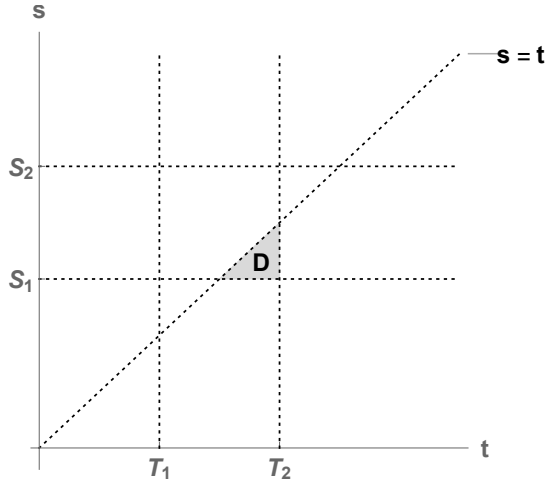
$$\begin{aligned}
I_1^{(3b)} &= \frac{\eta}{2} \frac{e^{-i\pi\eta(\Omega_D - \Omega_E)/4} e^{i(\Omega_D T_{D,i} + \Omega_E T_{E,j})} e^{-ik(T_{D,i} - T_{E,j} - \pi\eta/2)}}{(1 - (\eta/2)^2(k - \Omega_D)^2)(1 - (\eta/2)^2(k + \Omega_E)^2)} \\
I_2^{(3b)} &= \frac{e^{i(T_{D,i} - \pi\eta/4)(\Omega_D + \Omega_E)}}{(\Omega_D + \Omega_E)(1 - (\eta/2)^2(k + \Omega_E)^2)(4 - (\eta/2)^2(\Omega_D + \Omega_E)^2)} \\
&\quad \times \left[\frac{\eta}{2} (2k + \Omega_D + 3\Omega_E) \cos\left(\frac{2(T_{D,i} - T_{E,j})}{\eta}\right) \right. \\
&\quad \left. - i(2 + (\eta/2)^2(k + \Omega_E)(\Omega_D + \Omega_E)) \sin\left(\frac{2(T_{D,i} - T_{E,j})}{\eta}\right) \right] \\
I_3^{(3b)} &= -\frac{e^{i(T_{E,j} + \pi\eta/4)(\Omega_D + \Omega_E)}}{(\Omega_D + \Omega_E)(1 - (\eta/2)^2(k - \Omega_D)^2)(4 - (\eta/2)^2(\Omega_D + \Omega_E)^2)} \\
&\quad \times \left[\frac{\eta}{2} (2k - 3\Omega_D + \Omega_E) \cos\left(\frac{2(T_{D,i} - T_{E,j})}{\eta}\right) \right. \\
&\quad \left. - i(2 - (\eta/2)^2(k - \Omega_D)(\Omega_D + \Omega_E)) \sin\left(\frac{2(T_{D,i} - T_{E,j})}{\eta}\right) \right] \tag{A.6}
\end{aligned}$$



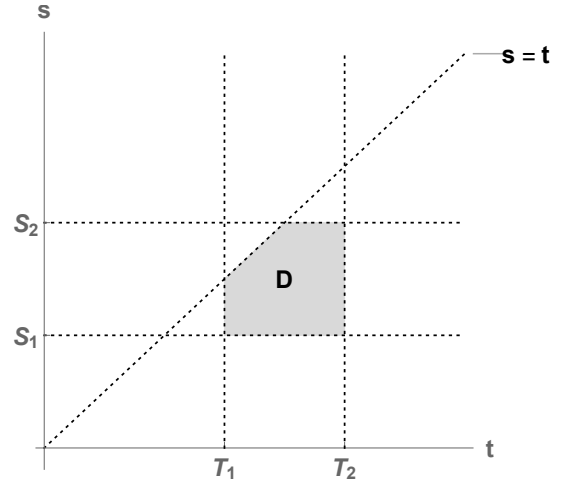
(a) Case 1: $S_1 > T_2$



(b) Case 2: $T_1 > S_2$



(c) Case 3a: $T_2 > S_1 > T_1$



(d) Case 3b: $S_2 > T_1 > S_1$

Figure A.1: The 4 different types of time domains D for the integration of $M_{DE,ij}(\mathbf{k})$

A.2 Showing that $P_D^{(\text{Tr})} \geq P_D(+)$

The matrix element $P_{D,ij}$ (Eq. (4.6)) can be rewritten as the L^2 inner product

$$\begin{aligned}
P_{D,ij} &= \lambda_D^2 \int \frac{d^n \mathbf{k}}{2|\mathbf{k}|} \left[\left(\int_{-\infty}^{\infty} dt \frac{\chi_{D,i}(t)}{\sqrt{(2\pi)^n}} e^{it(\Omega_D + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_D(t)} \right) \left(\int_{-\infty}^{\infty} dt' \frac{\chi_{D,j}(t')}{\sqrt{(2\pi)^n}} e^{it'(\Omega_D + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_D(t')} \right)^* \right] \\
&= \int d^n \mathbf{k} \left[\left(\frac{\lambda}{\sqrt{2|\mathbf{k}|}} \int_{-\infty}^{\infty} dt \frac{\chi_{D,i}(t)}{\sqrt{(2\pi)^n}} e^{it(\Omega_D + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_D(t)} \right) \right. \\
&\quad \times \left. \left(\frac{\lambda}{\sqrt{2|\mathbf{k}|}} \int_{-\infty}^{\infty} dt' \frac{\chi_{D,j}(t')}{\sqrt{(2\pi)^n}} e^{it'(\Omega_D + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_D(t')} \right)^* \right] \\
&= \int d^n \mathbf{k} L_{D,i}(\mathbf{k}) L_{D,j}^*(\mathbf{k}) \\
&= \langle L_{D,i} | L_{D,j} \rangle
\end{aligned} \tag{A.7}$$

where

$$L_{D,i}(\mathbf{k}) := \frac{\lambda}{\sqrt{2|\mathbf{k}|}} \int_{-\infty}^{\infty} dt \frac{\chi_{D,i}(t)}{\sqrt{(2\pi)^n}} e^{it(\Omega_D + |\mathbf{k}|)} e^{-i\mathbf{k} \cdot \mathbf{x}_D(t)}. \tag{A.8}$$

Now, we want to show

$$\begin{aligned}
P_D^{(\text{Tr})} \geq P_D^{(+)} &\iff \frac{1}{2}(P_{D,00} + P_{D,11}) \geq \frac{1}{4}(P_{D,00} + P_{D,01} + P_{D,10} + P_{D,11}) \\
&\iff \frac{1}{4}(P_{D,00} - P_{D,01} - P_{D,10} + P_{D,11}) \geq 0 \\
&\iff \langle L_{D,0} | L_{D,0} \rangle - \langle L_{D,0} | L_{D,1} \rangle - \langle L_{D,1} | L_{D,0} \rangle + \langle L_{D,1} | L_{D,1} \rangle \geq 0 \\
&\iff \langle (L_{D,0} - L_{D,1}) | (L_{D,0} - L_{D,1}) \rangle \geq 0
\end{aligned} \tag{A.9}$$

which is always true since

$$\langle (L_{D,0} - L_{D,1}) | (L_{D,0} - L_{D,1}) \rangle = \|L_{D,0} - L_{D,1}\|^2 \geq 0. \tag{A.10}$$

Appendix B

Derivation of (5.15) and (5.17)

In this appendix we derive the numerical form of the transition probability P_D and matrix element X defined in Eqs. (2.24) and (2.26) for detectors on static trajectories in AdS_3 .

Beginning with the definition of the transition probability P_D in Eq. (2.24), we can express P_D in terms of the integration variables $u := \tau_D$ and $s := \tau_D - \tau'_D$ and evaluate the integral over u

$$\begin{aligned}
P_D &:= \lambda^2 \int d\tau_D d\tau'_D \chi_D(\tau_D) \chi_D(\tau'_D) e^{-i\Omega(\tau_D - \tau'_D)} W(x_D, x'_D) \\
&= 2\lambda^2 \int_{-\infty}^{\infty} du \chi_D(u) \text{Re} \left[\int_0^{\infty} ds \chi_D(u - s) e^{-i\Omega s} W(x_D, x'_D) \right] \\
&= 2\lambda^2 \sqrt{\pi} \sigma \gamma_D \text{Re} \int_0^{\infty} d(\Delta t) e^{-\gamma_D^2 \Delta t^2 / (4\sigma^2)} e^{-i\Omega \gamma_D \Delta t} W(x_D, x'_D), \tag{B.1}
\end{aligned}$$

where in the last equality we have expressed the remaining integral in terms of the integration variable $\Delta t := t - t' = s/\gamma_D$. Upon substituting the AdS_3 Wightman function given in Eq. (5.10) into Eq. (B.1), it is seen that the transition probability can be expressed as a difference of two terms

$$P_D = P_D^- - \zeta P_D^+ \tag{B.2}$$

where

$$P_D^- := \lambda^2 \frac{\gamma_D \sigma}{2\sqrt{2\pi}\ell} \text{Re} \int_0^{\infty} d(\Delta t) e^{-\gamma_D^2 \Delta t^2 / (4\sigma^2)} \frac{e^{-i\Omega \gamma_D \Delta t}}{\sqrt{\sigma(x_D, x'_D)}}, \tag{B.3}$$

and

$$P_D^+ := \lambda^2 \frac{\gamma_D \sigma}{2\sqrt{2\pi}\ell} \operatorname{Re} \int_0^\infty d(\Delta t) e^{-\gamma_D^2 \Delta t^2 / (4\sigma^2)} \frac{e^{-i\Omega\gamma_D \Delta t}}{\sqrt{\sigma(x_D, x'_D) + 2}}. \quad (\text{B.4})$$

Using Eqs. (5.11) and (5.6), and the detector's trajectory given in Eq. (5.12), we may express the denominators appearing in the integrands defining $P_D^\pm$ as

$$\sqrt{\sigma(x_D, x'_D)} = \gamma_D [\alpha_D^- + \cos(\Delta t/\ell - i\epsilon)]^{1/2}, \quad (\text{B.5})$$

$$\sqrt{\sigma(x_D, x'_D) + 2} = \gamma_D [\alpha_D^+ + \cos(\Delta t/\ell - i\epsilon)]^{1/2}, \quad (\text{B.6})$$

where we have made explicit the $i\epsilon$ dependence indicating the appropriate branch cut [42] and defined $\alpha_D^\pm := [-(R_D/\ell)^2 \pm 1]/\gamma_D^2$; note that from the definition of γ_D below Eq. (5.12) it is seen that $\alpha_D^- = -1$.

Let us first express P_D^- in terms of the dimensionless integration variable $y := \Delta t/\ell$

$$P_D^- = \frac{\lambda^2 \sigma}{2\sqrt{2\pi}} \operatorname{Re} \int_0^\infty dy \frac{e^{-a_D y^2} e^{-i\beta_D y}}{\sqrt{-1 + \cos(y - i\epsilon)}} \quad (\text{B.7})$$

where $\beta_D := \gamma_D \ell \Omega$ and $a_D := \ell^2 \gamma_D^2 / 4\sigma^2$.

Before we continue, we note that if we only consider the principal value of the square root, then integrals of the form

$$\int_{-1}^1 dy \frac{f(y)}{\sqrt{\sin^2(y)}} = \int_{-1}^1 dy \frac{f(y)}{|\sin(y)|} = \int_{-1}^0 dy \frac{f(y)}{-\sin(y)} + \int_0^1 dy \frac{f(y)}{\sin(y)} \quad (\text{B.8})$$

are infinite, and the Cauchy principle value of integration cannot be applied to correct this. However, P_D is calculated using the Wightman function, which is a tempered distribution (i.e. P_D^- must be finite). In order to correct this, we require that

$$\int_{-1}^1 dy \frac{f(y)}{\sqrt{\sin^2(y)}} := \int_{-1}^1 dy \frac{f(y)}{\sin(y)} \quad (\text{B.9})$$

Now under this condition, the denominator may be simplified

$$\sqrt{-1 + \cos(y - i\epsilon)} = \sqrt{-2 \sin^2\left(\frac{y}{2} - i\epsilon\right)} = i\sqrt{2} \sin\left(\frac{y}{2} - i\epsilon\right), \quad (\text{B.10})$$

Direct application of Sokhotsky's formula

$$\lim_{\epsilon \rightarrow 0^+} \int_a^b dx \frac{f(x)}{x \pm i\epsilon} = \mp i\pi \int_a^b dx f(x) \delta(x) + \text{PV} \int_a^b dx \frac{f(x)}{x}. \quad (\text{B.11})$$

yields the identity

$$\frac{1}{\sin(x - i\epsilon)} = \text{PV} \frac{1}{\sin x} + i\pi \sum_{n \in \mathbb{Z}} (-1)^n \delta(x - n\pi), \quad (\text{B.12})$$

which when combined with Eq. (B.7) allows for the simplification of P_D^- to

$$P_D^- = \frac{\lambda^2 \sigma}{4\sqrt{\pi}} \left[-\text{PV} \int_0^\infty dy \frac{e^{-a_D y^2} \sin(\beta_D y)}{\sin(y/2)} + \pi \sum_{n \in \mathbb{Z}} (-1)^n \cos(2n\pi\beta_D) e^{-4n^2\pi^2 a_D} \right]. \quad (\text{B.13})$$

Turning our attention to P_D^+ , we note that it may also be rewritten in the form

$$P_D^+ := \frac{\lambda^2 \sigma}{2\sqrt{2\pi}} \text{Re} \int_0^\infty dy \frac{e^{-y^2 a_D} e^{-i\beta_D y}}{\sqrt{\alpha_D^+ + \cos(y - i\epsilon)}}. \quad (\text{B.14})$$

Note that $|\alpha_D^+| \neq 1$ for finite ℓ , in which case the singularities appearing in the above integrand are integrable and may take $\epsilon \rightarrow 0$. Again, by requiring that the Wightman function is a tempered distribution, we use Eq. (B.9), which implies

$$\sqrt{\cos y + \alpha_D^+} = \begin{cases} |\sqrt{\cos y + \alpha_D^+}|, & y \in (0, \pi - \Theta_D^+) \\ i|\sqrt{-\cos y - \alpha_D^+}|, & y \in (\pi - \Theta_D^+, \pi + \Theta_D^+) \\ -|\sqrt{\cos y + \alpha_D^+}|, & y \in (\pi + \Theta_D^+, 3\pi - \Theta_D^+) \\ -i|\sqrt{-\cos y - \alpha_D^+}|, & y \in (3\pi - \Theta_D^+, 3\pi + \Theta_D^+) \\ \vdots & \vdots \end{cases} \quad (\text{B.15})$$

where $\Theta_D^+ := \arccos \alpha_D^+$. Using Eq. (B.15), P_D^+ can be written in the form

$$P_D^+ = \frac{\lambda^2 \sigma}{2\sqrt{2\pi}} \text{Re} \left[\int_0^{\pi + \Theta_D^+} dy \frac{e^{-a_D y^2} e^{-i\beta_D y}}{\sqrt{\cos y + \alpha_D^+}} + \sum_{n \in \mathbb{Z}^+} (-1)^n \int_{\Theta_D^+ + (2n-1)\pi}^{\Theta_D^+ + (2n+1)\pi} dy \frac{e^{-a_D y^2} e^{-i\beta_D y}}{\sqrt{\cos y + \alpha_D^+}} \right]. \quad (\text{B.16})$$

Combining Eqs. (B.13) and (B.16) yields the transition probability stated in Eq. (5.15).

We now evaluate the matrix element X defined in Eq. (2.26). Taking the switching function to be the Gaussian functions given in Eq. (5.14), X may be simplified to

$$\begin{aligned}
X &= -\lambda^2 \gamma_A \gamma_B \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' \left[e^{-(t-t_0/2)^2 \gamma_B^2 / 2\sigma^2} e^{-(t'+t_0/2)^2 \gamma_A^2 / 2\sigma^2} e^{-i\Omega(\gamma_B t + \gamma_A t')} W(x_A(t'), x_B(t)) \right. \\
&\quad \left. + e^{-(t+t_0/2)^2 \gamma_A^2 / 2\sigma^2} e^{-(t'-t_0/2)^2 \gamma_B^2 / 2\sigma^2} e^{-i\Omega(\gamma_A t + \gamma_B t')} W(x_B(t'), x_A(t)) \right] \\
&= -\lambda^2 2\sqrt{2\pi} \sigma \frac{\gamma_A \gamma_B}{\sqrt{\gamma_A^2 + \gamma_B^2}} e^{-\frac{\sigma^2 \Omega^2}{2} \frac{(\gamma_A + \gamma_B)^2}{\gamma_A^2 + \gamma_B^2} - \frac{t_0^2}{2\sigma^2} \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} + i \frac{\Omega t_0}{2} \frac{(\gamma_A + \gamma_B)^2 (\gamma_A - \gamma_B)}{\gamma_A^2 + \gamma_B^2}} \\
&\quad \times \int_0^{\infty} ds \cosh \left[\left(i \frac{\gamma_A \gamma_B (\gamma_A - \gamma_B)}{\gamma_A^2 + \gamma_B^2} \Omega - \frac{t_0}{\sigma^2} \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} \right) s \right] e^{-\frac{s^2}{2\sigma^2} \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2}} W(x_A(t'), x_B(t)), \tag{B.17}
\end{aligned}$$

where in arriving at the last equality we have introduced the integration variables $u := t$ and $s := t - t'$ and carried out the integration over u .

Upon substituting the spacetime trajectories of the static detectors given in Eq. (5.12) into the Wightman function in Eq. (5.10), the denominators become

$$\begin{aligned}
\sqrt{\sigma(x_A, x'_B)} &= \sqrt{\gamma_A \gamma_B} \left[\alpha_X^- + \cos(s/\ell - i\epsilon) \right]^{1/2}, \\
\sqrt{\sigma(x_A, x'_B) + 2} &= \sqrt{\gamma_A \gamma_B} \left[\alpha_X^+ + \cos(s/\ell - i\epsilon) \right]^{1/2}, \tag{B.18}
\end{aligned}$$

where $\alpha_X^{\pm} := [-R_A R_B / \ell^2 \pm 1] / \gamma_A \gamma_B$. Using Eqs. (B.18) and (5.10), X simplifies to

$$X = -\frac{\lambda^2 \sigma}{2\sqrt{\pi}} K_X \int_0^{\infty} dy e^{-a_X y^2} \cosh((\Delta_T + i\beta_X)y) \left[\frac{1}{\sqrt{\alpha_X^- + \cos(y - i\epsilon)}} - \zeta \frac{1}{\sqrt{\alpha_X^+ + \cos(y - i\epsilon)}} \right], \tag{B.19}$$

where we have introduced the integration variable $y := s/\ell$ and defined

$$\begin{aligned}
K_X &:= \sqrt{\frac{\gamma_A \gamma_B}{\gamma_A^2 + \gamma_B^2}} \exp \left(-\frac{\Omega^2 \sigma^2}{2} \frac{(\gamma_A + \gamma_B)^2}{\gamma_A^2 + \gamma_B^2} - \frac{t_0^2}{2\sigma^2} \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} + i \frac{\Omega t_0}{2} \frac{(\gamma_A + \gamma_B)^2 (\gamma_A - \gamma_B)}{\gamma_A^2 + \gamma_B^2} \right) \\
a_X &:= \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} \frac{\ell^2}{2\sigma^2} \\
\Delta_T &:= -\frac{t_0 \ell}{\sigma^2} \frac{\gamma_A^2 \gamma_B^2}{\gamma_A^2 + \gamma_B^2} \\
\beta_X &:= \frac{\gamma_A \gamma_B (\gamma_A - \gamma_B)}{\gamma_A^2 + \gamma_B^2} \ell \Omega.
\end{aligned} \tag{B.20}$$

Through the methods similar to those used to obtain P_D^+ , the matrix element X can be brought into the form given in Eq. (5.17).