

Validated Dynamic Stall Simulation of Pitching Low Reynolds Number Airfoils

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Deep dynamic stall is one of several complex behaviours that result in extreme variation of the aerodynamic loads on small wind turbine (SWT) blades during unsteady wind conditions. In this study, unsteady Reynolds-Averaged Navier-Stokes (URANS) simulations are performed for two low Reynolds number (Re) airfoils where sinusoidal pitching is applied to replicate the dynamic stall that occurs on rotating SWT blades. The SD 7037 airfoil is simulated at $Re = 4.1 \times 10^4$ and a pitching reduced frequency of $k = 0.08$, and the S833 airfoil is at $Re = 1.7 \times 10^5$ and $k = 0.06$. The simulated lift coefficient and dynamic stall timing agree with experimental data, which is attributed to the wall-normal resolution of the mesh and is an advancement from the early prediction of stall seen consistently in previous numerical studies. The accurate prediction of dynamic stall is found to be dependent on the correct simulation of the bursting of the laminar separation bubble (LSB), which initiates the complete separation of the boundary layer and the formation of a leading edge vortex. The $\gamma - Re_{\theta,t} k - \omega$ model combined with the use of a fine mesh at the airfoil leading edge results in an accurate simulation of the bursting LSB and the correct prediction of the deep dynamic stall.

Nomenclature

C_f	=	skin friction coefficient
C_L	=	lift coefficient
C_p	=	pressure coefficient
c	=	chord, m
f	=	pitch oscillation frequency, s^{-1}
k	=	reduced frequency
r	=	wind turbine local blade radius, m
Re	=	chord-based Reynolds number
T	=	cycle period, s

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Tu	=	turbulent intensity
U_∞	=	free-stream velocity, $m \cdot s^{-1}$
W	=	wind turbine relative air velocity, $m \cdot s^{-1}$
y^+	=	wall y-plus
α	=	angle of attack, deg
α_{amp}	=	pitching amplitude angle of attack, deg
α_{mean}	=	pitching mean angle of attack, deg
Δt	=	time step, s
ρ	=	density, $kg \cdot m^{-3}$
Ω	=	wind turbine blade rotation rate, s^{-1}
ω	=	pitching rate, s^{-1}
ω_z	=	z -vorticity, s^{-1}
Subscripts		
r	=	reattachment
s	=	separation
s, TE	=	trailing edge separation

I. Introduction

DYNAMIC stall produces extreme hysteresis in aerodynamic forces which poses a challenge for the design of blades for wind turbines, particularly small wind turbines (SWTs). SWTs typically operate in a chord-based Reynolds number (Re) range from 8×10^3 to 4×10^5 with blade lengths of 1.5 m to 8 m and low nacelle heights [1]. They are often located in residential or industrial areas where proximity to buildings introduces transient or complex inflow conditions such as gusts, wind shear and yaw misalignment.

Gradual increases in the incoming wind speed can result in static stall of localized sections of the wind turbine blade, with the loading following the behaviour predicted by the lift and drag curves used to design the blade. Dynamic stall of the blade can occur if the wind conditions change in a rapid transient manner which causes the angle of attack (AOA) to exceed the static stall AOA and the forces on the blade will deviate from the predictable static behaviour. This can lead to larger fluctuations in the forces on the blades, including the lift, drag and moment [2]. The dynamic loading can be detrimental to the performance of the blade as well as the lifespan since repeated fluctuations in loading increase the fatigue on the blade. This type of dynamic loading is also present on rotors of helicopters or drones, where the advancing and retreating sides of the rotor experience different relative wind speeds [3].

An additional concern for SWTs is that the low Re flow over the blades results in a transitional boundary layer

(BL), which is more complex than the turbulent BL that forms on large wind turbine blades. Low Re is defined as a chord-based Re of less than 5×10^5 and within this range, the airfoil BL is largely laminar or transitional. In general, the low Re BL can be considered to be less stable and therefore more sensitive to small changes in the inflow conditions. The combination of low Re flow and unsteady inflow conditions can cause significant variation in the loading of the blade, which in turn affects the power output and lifespan of the turbine.

Numerical simulation of dynamic stall using computational fluid dynamics (CFD) can be useful for analyzing the changes in the blade loading and understanding the key source behaviours behind the changes. At the present time, detailed simulations of rotating wind turbine blades are not practical and therefore analysis of dynamic stall is completed using simplified 2D geometries. Additionally, detailed transient experimental data of a full wind turbine blade is not available for validating full-blade simulations, whereas 2D dynamic stall data is available from wind tunnel testing. Once the simulations are validated, they can be used to test different blade designs or the impact of different parameters such as the rates and magnitudes of wind condition changes. This would be expensive and time consuming to perform experimentally and would require a combination of multiple methods to generate the same range of results as the simulations.

This work is a numerical study of the dynamic stall of SWT blade segments in unsteady wind conditions. The chosen CFD method is unsteady Reynolds-averaged Navier-Stokes (URANS) due to its ability to resolve transient behaviour while having a low computational cost in comparison to other methods such as large eddy simulation (LES). The simulations were performed for two different low Re airfoil profiles that are well-suited for use on SWTs, SD 7037 and S833, and results are validated against experimental data [4–6]. The analysis is centered around the prediction of the correct dynamic stall AOA, which requires a focus on the simulation of the development and eventual separation of the low Re BL. The accuracy of the URANS turbulence model is of high importance if it is to be used as a future design tool for SWT blades and unsteady load mitigation technologies such as a trailing edge flap [7].

A. Unsteady Blade Aerodynamics

Wind turbines of any size experience unsteady blade loading due to gusts as well as variation in wind speeds across the rotor plane such as wind shear and yaw misalignment. The relative air velocity, W , for a rotating SWT blade is a combination of wind speed, U_∞ , and the velocity caused by blade rotation, Ωr , as shown in Figure 1 [8]. These two components form a velocity triangle that dictates the effective AOA, α , of the blade, where an increase in wind speed and/or a decrease in rotational speed corresponds to an increase in AOA [9]. This concept is used to approximate changes in incoming wind speed through the change in effective AOA alone, greatly simplifying the analysis of complex wind patterns on a SWT blade.

This idealized method of analyzing the unsteady loading of a SWT blade uses a 2D blade with a pitching motion applied to correspond to the change in effective AOA that occurs during unsteady wind conditions. When representing

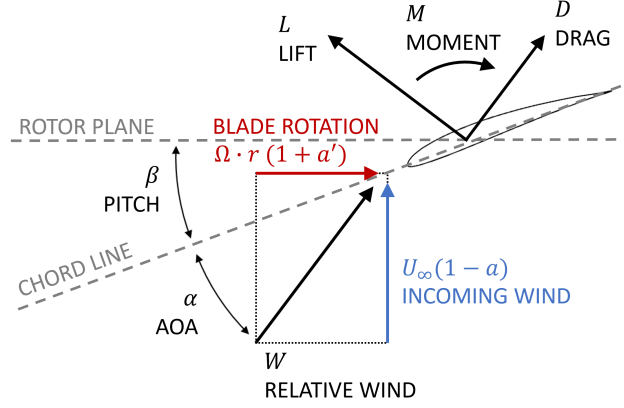


Fig. 1 Velocity triangle diagram for a wind turbine blade, including blade loading. Adapted from [8]

linear wind shear (horizontal or vertical) or yaw misalignment, the effective AOA oscillates in a sinusoidal pattern for each rotation of the blade [9]. For example, the change in effective AOA for a horizontal wind shear is shown in Figure 2, where the new idealized velocity triangles and effective AOAs are shown for the maximum wind speed (location II) and minimum wind speed (location IV) in the rotor plane. In this approach to representing AOA variation [9, 10], the pitching motion is governed by the following equation for effective AOA, α :

$$\alpha = \alpha_{mean} + \alpha_{amp} \sin(2\pi ft) \quad (1)$$

where α_{mean} is the average AOA, α_{amp} is the amplitude of the pitch oscillation and f is the frequency of oscillation. This oscillation in effective AOA for SWTs is often significant enough to initiate the dynamic stall process, which can cause extreme loading changes in comparison to static stall of SWT blades.

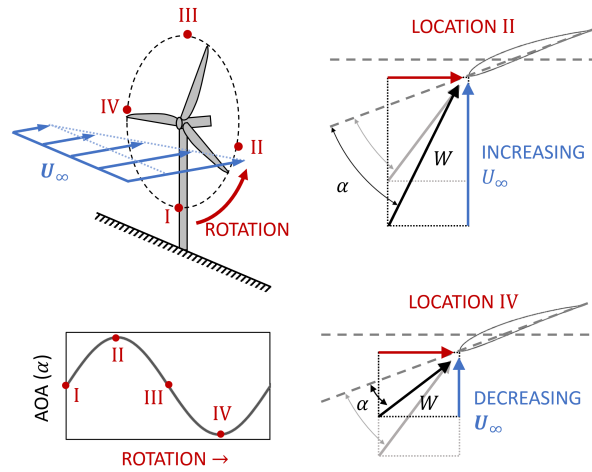


Fig. 2 Relationship between incoming wind shear and idealized change in effective AOA for a wind turbine blade

B. Dynamic Stall

Dynamic stall occurs during rapid changes in AOA, and creates hysteresis in the blade loading [2]. The changes in blade loading compared to the static behaviour and the degree of hysteresis that occurs are difficult to predict and thus hard to account for in the design of the SWT blade. An important parameter that impacts the degree of dynamic stall is the reduced frequency, k , of the pitching motion. Reduced frequency is defined as:

$$k = \frac{\pi f c}{U_{\infty}} \quad (2)$$

where f is the pitching frequency from Equation 1, c is the airfoil chord length and U_{∞} is the free stream velocity. A steady airfoil has a k of 0, and unsteady motion is defined by the range of $0.05 < k \leq 0.2$ [3]. Below 0.05 is considered quasi-steady behaviour, where behaviour can often be approximated using static analysis, and above 0.2 is highly unsteady, where acceleration effects dominate the flow. Dynamic stall is known to occur within the unsteady range, and the remainder of this paper will concern pitching frequencies within that range. The dynamic stall process is reported to be relatively insensitive to the airfoil Re, and is instead dependent on k and the range of AOAs of the pitching motion, as well as the airfoil geometry [2].

The dynamic stall process can be divided into two regions: the pitch up motion where AOA is increasing, and the pitch down motion where AOA is decreasing. In the pitch up motion, the C_L increases linearly as it would for static changes in AOA, however the C_L magnitude is often higher than the static lift curve for a given AOA. After the static stall AOA, the dynamic C_L continues to increase due to the formation of a leading edge vortex (LEV), which is also referred to as a dynamic stall vortex (DSV) [11]. This LEV causes the C_L to increase well above the maximum static C_L and delays the stall to a higher AOA. Dynamic stall will occur due to the shedding of the LEV and is categorized as light stall or deep stall depending on the severity of the stall behaviour [12]. Light dynamic stall occurs when the AOA is close to the static stall AOA, resulting in minimal C_L hysteresis and an airfoil wake which is on the order of the airfoil thickness [2]. Deep dynamic stall can occur when the AOA far exceeds the static stall AOA and the severity of the stall results in the width of the airfoil wake being on the order of the airfoil chord length [2]. Once deep stall occurs, there is a prolonged recovery period with low C_L , and the airfoil does not regain its lift properties until the pitch down motion has reached a low AOA in the pitching cycle, ending the hysteresis.

Dynamic stall from sinusoidal pitching can create a repeating pattern of extreme loading in comparison to the expected static behaviour. Understanding how to mitigate or interrupt the loading changes in the dynamic stall process can lead to improvements in blade design or control strategies to improve aerodynamic performance and reduce the fatigue on SWT blades.

C. Simulation of Low Re Boundary Layers

For low Re applications such as SWTs, the CFD method must be able to correctly predict the natural unforced transition of the BL, which is a greater challenge in comparison to high Re simulations where the BL transitions to turbulence over a short distance close to the airfoil leading edge (LE). A particularly challenging behaviour to capture is transitional separated flow, also known as a laminar separation bubble (LSB), shown in Figure 3 [13]. In a LSB, the laminar BL separates due to the adverse pressure gradient, at which point the transition to turbulence is triggered through a Kelvin-Helmholtz (K-H) instability in the separated BL [14, 15]. After the growth of the K-H instability, the BL reattaches as a turbulent BL. As the airfoil approaches stall, the adverse pressure gradient moves upstream towards the LE which causes the LSB to also move towards the LE.

At a certain critical AOA, the separated laminar BL will not have sufficient development of the K-H instability to result in BL reattachment, leaving a fully separated BL. This phenomenon is known as the bursting or breakdown of the LSB [14], since the disappearance of the reattachment point means there is no longer a defined end point of the separation bubble. When the bursting of the LSB occurs independently from turbulent BL separation at the trailing edge (TE), it results in an abrupt loss of lift and is classified as LE stall [16]. Depending on the airfoil geometry and Re, TE stall can occur on airfoils with a LSB, where the turbulent BL separation progresses upstream resulting in a loss of lift without bursting the LSB [14]. These definitions of LSB related stall behaviour were determined from analysis of several data sets at a high Re of 5.8×10^6 [14, 16] where the streamwise length of the LSB is on the order of $0.01c$ and is located at the LE [14]. Since the length of the LSB is known to increase with decreasing Re, it can be expected that the reattachment location of a long, low Re, LSB may interact with the turbulent BL TE separation location and result in a combination of LE stall and TE stall behaviours. For the purposes of this paper, the definition of LSB bursting includes the abrupt BL separation that occurs when the TE separation approaches the reattachment location of the LSB. This complex method of natural BL transition and LSB bursting is difficult to model with traditional URANS models, leading some researchers to opt for the more computationally expensive LES to capture the dynamic stall behaviour for low Re airfoils [17–20].

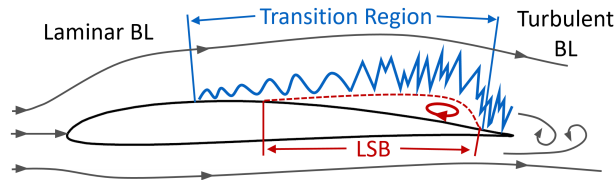


Fig. 3 Boundary layer transition (blue) over a cambered airfoil, including LSB details (red). Reprinted with permission from [13]

D. Simulation of Dynamic Stall

In numerical simulations, dynamic stall is induced through the motion of the entire domain using Equation 1 to replicate the expected change in effective AOA for the given application. This technique has been previously used in LES simulations of dynamic stall [17–22] which captures high levels of detail in the formation and shedding of the LEV. The LES simulations by Kim and Xie, Boye and Xie, Geng *et al.*, and Huang *et al.* [17, 19–21] examined the sinusoidal pitching of a NACA 0012 airfoil at a Re of 1.35×10^5 and $k = 0.1$ to gain insight into the unsteady loading of wind turbine blades. The predicted C_L through the pitching cycles were in general agreement with the experimental data in Lee and Gerontakos [23], however all the LES simulations predict early stall, likely due to an early shedding of the LEV. Kim and Xie [17], who also simulated $k = 0.025$ and 0.05 , attributed the differences in stall AOA to experimental error due to the effect of high frequency pitching on the surface pressure measurement accuracy. When looking at the latter part of the pitching cycle, Boye and Xie [20] identified additional LEVs shedding after the initial LEV that occurs at stall, which contributes to the unsteady loading of the wind turbine blade.

It is common for these high fidelity simulations to examine only the pitch up (or ramp-up) motion to isolate the shedding of the LEV from the pitch down motion. This is the case for the LES studies by Visbal and Garmann [18] and Benton and Visbal [22] and the delayed detached eddy simulation (DDES) by Batther and Lee [24]. These works simulate a NACA 0012 airfoil at a Re of 2×10^5 undergoing rapid pitch up motion, with Visbal and Garmann concluding that the LSB formation and subsequent bursting are critical for the formation of the LEV in their case. Contrary to this, the Lee and Gerontakos [23] experimental analysis of the NACA 0012 airfoil at a Re of 1.35×10^5 and $k = 0.1$ concluded that the LSB bursting was not involved in the initiation of the LEV, while noting that LSB bursting is commonly believed to play a role. Instead, they found the upstream progression of the flow reversal under the separated turbulent BL resulted in the breakdown of the BL (i.e. TE stall) that then led to the formation of the LEV. Visbal and Garmann had no TE BL separation occur, indicating LE stall behaviour which may be due to the constant rate of pitch-up motion versus the decelerating pitch rate occurring during LEV formation in the Lee and Gerontakos experiments.

Due to the potential importance of the LSB in the formation of the LEV, Visbal and Garmann call into question the ability of URANS turbulence models to accurately predict the onset of dynamic stall. Batther and Lee showed that DDES, which uses URANS at the airfoil surface to reduce computational cost, is able to capture the complex separated shear layer BL transition that is required to accurately simulate the LSB behaviour. However, they cautioned that they did not assess the sensitivity of the results to different URANS transition models and were validating the results against LES rather than experimental data.

Similar to the LES studies mentioned above, there have been many URANS simulations completed using the parameters of the NACA 0012 case reported in Lee and Gerontakos [23] for $k = 0.1$ [21, 25–28] and $k = 0.05$ [29]. Overall, these URANS studies suffered from the same early prediction of stall that was present in the LES simulations, with the exception of Xu *et al.* [29] which predicted the correct stall AOA but also predicted a second LEV that was not

present in the validating experimental data. All URANS simulations cited above used a variation of the shear stress transport (SST) $k - \omega$ turbulence model. Honarmand *et al.* [26] used the base SST model, while Gharali and Johnson [25] applied the low Re correction to better account for the BL transition and Wu *et al.* [27] used the γ SST model which adds an additional intermittency, γ , equation to aid the onset of transition. The remaining simulations [21, 28, 29] used the $\gamma - Re_{\theta_t}$ SST model [30], which adds two additional equations to improve the triggering of BL transition, including cases with transitional separated flow. A comparative study of RANS models by Liu *et al.* [31] found that the $\gamma - Re_{\theta_t}$ SST $k - \omega$ model was most accurate compared to the γ SST and low Re correction options for predicting a LSB on static low Re airfoils. As Xu *et al.* [29] predicted the correct stall AOA with the $\gamma - Re_{\theta_t}$ model and others using the same model did not, it is possible that other factors such as higher mesh density at the LE are required to resolve the LSB at the LE to accurately capture the LEV formation and dynamic stall AOA.

Additional URANS studies have been completed for different dynamic pitching cases, including the Gharali and Johnson [4] simulation of the SD 7037 airfoil at $Re = 4 \times 10^4$ and $k = 0.08$ using $\gamma - Re_{\theta_t}$ SST. The simulations accurately predicted the stall AOA as well as the timing of the formation of a second LEV. Tseng and Cheng [32] simulated the NACA 0012 airfoil at $Re = 2.5 \times 10^6$ and $k = 0.15$ using the SST model with the low Re correction. The prediction of the stall location is in agreement with the experimental results, however there is larger disagreement in the downstroke with an over-prediction of the C_L during the formation of a second LEV. A NACA 0015 study at $Re = 2 \times 10^5$ and $k = 0.1$ was completed by Tseng *et al.* [33] using the $\gamma - Re_{\theta_t}$ SST model. The study had a focus on the BL transition and LSB that is present at the LE ahead of the LEV formation, and found that the base formation of the SST $k - \omega$ model prematurely predicted suction collapse and stall in comparison with the $\gamma - Re_{\theta_t}$ SST model. Analysis of the dynamic stall found that the LSB reattachment location progressed downstream simultaneously with the upstream progression of the turbulent BL TE separation until eventual bursting of the LSB and suction collapse. Tseng *et al.* also had a similar conclusion to Visbal and Garmann [18] that the formation and bursting of the LSB is critical to predicting the correct stall AOA.

II. Numerical Method

The simulations reported here were performed using the commercial software ANSYS Fluent, release 19.2, and the chosen URANS turbulence model was the $\gamma - Re_{\theta_t}$ SST $k - \omega$ model, known as the transition SST model in Fluent. The transition SST model [34, 35] is a four-equation model that expands on the standard SST model [36] and is designed to account for the transitional BLs that develop at low Re and can predict localized regions of separation, including a LSB. The accuracy of the prediction of the LSB was assessed through test cases of the SD 7037 airfoil at $Re = 4.1 \times 10^4$ using the following RANS models available in Fluent: Spalart Allmaras (S-A), strain-based S-A, SST, SST with a low Re correction and transition SST. The S-A and base SST models were unable to predict laminar BL separation and the low Re correction was able to predict laminar separation, but not reattachment. The transition SST model was the only

model tested that was capable of predicting the LSB for this case, which reflects the findings of Liu *et al.* [31] and Tseng *et al.* [33].

The simulations were run in the incompressible form, and all residuals were set at 1×10^{-5} . The semi-implicit method for pressure-linked equations - consistent (SIMPLEC) pressure-velocity coupling scheme was used, with second-order bounded central differencing scheme (CDS) as the discretization scheme and bounded second order implicit as the time integration method. To decrease computing time, simulations were run using a high performance computer cluster in the Digital Research Alliance of Canada network [37].

The dynamic change in AOA is represented through the sinusoidal rotation of the computational domain with respect to the global coordinate system using Equation 1. This motion is applied using the Mesh Motion feature in Fluent, where the rotational speed in rad/s is provided for each time step. The axis of rotation is located at the 1/4 chord location of the airfoil, which is the rotation axis location for both experimental data sets. The simulations were run for at least 5 complete pitching cycles, where the first 1-2 cycles are an initialization and the final 3 cycles, which had no variation, are used for the analysis.

A. Case Geometries

The first airfoil profile selected for this study is SD 7037, which is designed to operate at low Re and has loading behaviour well suited for SWTs [38]. To replicate the experimental conditions, the airfoil has a chord length, c , of 0.0258 m and a Re of 4.1×10^4 , and the pitching cycle has a mean AOA of 11° and amplitude of 11° . The frequency of the pitching cycle results in a reduced frequency of $k = 0.08$, which is within the unsteady range. The validating experimental data was obtained through particle image velocimetry (PIV) using a closed loop wind tunnel with a dynamic pitching constant-chord blade [4, 6].

The second airfoil profile is S833 which is part of the NREL family of airfoils designed for a SWT with a rotor diameter of 1 to 3 m [39]. The simulations are at a chord of 0.178 m and a Re of 1.7×10^5 , which is approximately four times larger than the SD 7037 simulations but still falls within the low Re range of $Re < 5 \times 10^5$. The validating experimental data was reported by Samara and Johnson [5] and used a pitching cycle with a mean AOA of 10° and an amplitude of 10° . The reduced frequency for this case is $k = 0.06$ which falls within the unsteady range.

The simulation time step size for each airfoil was determined through numerous test cases, and was selected based on a balance of number of time steps required per pitching cycle, and the number of solution iterations required for each time step. The SD 7037 simulations use a time step of 5×10^{-5} s and the S833 simulation uses a larger time step of 1×10^{-4} s due to the longer duration of the cycle. The complete details of each airfoil and its corresponding pitching motion are summarized in Table 1.

Table 1 Geometries and numerical details

Airfoil Profile		SD 7037	S833
Reynolds Number	Re	4.1×10^4	1.7×10^5
Chord	c	0.0258 m	0.178 m
Inflow Velocity	U_∞	23.8 m/s	15 m/s
Turbulent Intensity	Tu	1%	0.1%
Reduced Frequency	k	0.08	0.06
Cycle Period	T	0.043 s	0.62 s
Mean Angle of Attack	α_{mean}	11°	10°
Pitching Amplitude	α_{amp}	11°	10°
Time Step	Δt	5×10^{-5} s	1×10^{-4} s

B. Meshing and Boundary Conditions

The simulation domain for each airfoil is a 2D C-mesh with 20 chord offsets as shown in Figure 4 (a), with close up views of the mesh near the airfoils in Figure 4 (b) and (c). The vertical outflow surface was defined as a pressure outlet and the surface of the airfoil was set as a no-slip wall. The remaining outer boundaries of the C-mesh were set as velocity inlets, with the velocity magnitude and direction remaining a constant horizontal velocity in the global reference frame. The pitching motion was applied purely through the rotation of the domain, meaning no change in inlet velocity direction is required. This is identical to the method of sinusoidal pitching used in the validating experimental data, where the airfoil is pitched relative to the constant horizontal velocity of the wind tunnel [4, 5].

The inflow turbulence conditions were set to account for the dissipation of inflow turbulence between the inlet and the airfoil, which requires specifying an elevated turbulent intensity at the domain inlets. For the SD 7037 case, this required the use of a turbulent intensity of $Tu = 12\%$ at the inlet to achieve a value of 1% at a location $0.9c$ upstream of the LE to agree with the experimental data [40]. For the S833 case, a value of $Tu = 0.12\%$ was used at the inlet which dissipated to 0.1% at a location of $1.7c$ upstream of the LE [5].

The mesh consists of 6.8×10^4 cells and the airfoil has 300 nodes along the chord length with a wall y^+ of less than 0.5 for the AOAs with an attached BL. Assessment of the URANS simulation of the LSB determined that the LSB prediction benefits from a finer mesh than is typically recommended for URANS analysis. For this reason, the near-wall mesh sizing is identical to that used in a previous 3D LES study of the LSB over the SD 7037 airfoil [13]. Due to the mesh-dependent subgrid filtering used during LES simulations, the near-wall mesh spacing had been optimized for up to $y^+ = 20$ rather than only ensuring that the first node height was at $y^+ < 1$. This required the most grid refinement in the wall-normal direction near the airfoil LE, where the thin BL has significant velocity and pressure gradients.

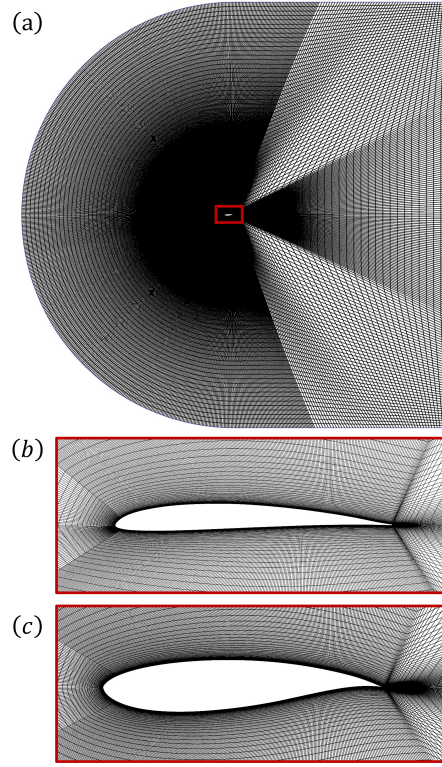


Fig. 4 (a) C-Mesh distribution for both airfoils (b) mesh quality around SD 7037 airfoil and (c) mesh quality around S833 airfoil

1. Grid Coarsening Study

A grid coarsening study was completed to determine the sensitivity of the dynamic stall process to the mesh density by increasing the wall y^+ and decreasing the wall-normal and chordwise nodes in the airfoil BL. The key parameter monitored through the study was the magnitude and AOA of the peak C_L in the dynamic stall cycle, which is used as an indicator of the correct timing of the simulation of the LEV formation and the subsequent drop in lift that occurs at stall. The first mesh version altered the first layer height (FLH) at the airfoil surface, where the wall-normal height of the first node was doubled to increase the wall y^+ from $y^+ < 0.5$ to $y^+ < 1.2$. The second mesh version reduced the number of wall-normal nodes (WNN) within a wall-normal distance of $0.4c$ from the airfoil surface from 50 to 25 and has the same $y^+ < 1.2$ as the FLH mesh. The third and final mesh used the wall-normal spacing of the WNN mesh and coarsened the number of chordwise nodes (CWN) from the original value of 300 nodes along the chord to 150 nodes. The mesh details are summarized in Table 2 and all mesh versions had a wall-normal expansion ratio set at 1.1.

The C_L values through the pitching cycle for each coarsened mesh version are shown in Figure 5 and are compared against the C_L cycles from the original mesh and the experimental data [4, 5]. The FLH case resulted in no change in the magnitude or timing of the C_L cycle for both airfoils which indicates that the low y^+ of the original LES-quality mesh is not necessary for the correct peak C_L AOA prediction. The alteration that resulted in the prediction of early

Table 2 Grid coarsening study details

Mesh Version	y^+	BL Nodes	Chordwise Nodes
Original	<0.5	50	300
FLH	<1.2	50	300
WNN	<1.2	25	300
CWN	<1.2	25	150

peak C_L and stall for both airfoils was the reduction of the number of wall-normal nodes in the WNN case. This resulted in peak C_L AOA that occurred earlier in the pitching cycle by 0.9° for the SD 7037 airfoil (Fig. 5b) and 0.8° for the S833 airfoil (Fig. 5d). The alteration of the chordwise nodes in the CWN mesh did not alter the peak C_L AOA in any significant manner beyond the change already seen from the WNN results.

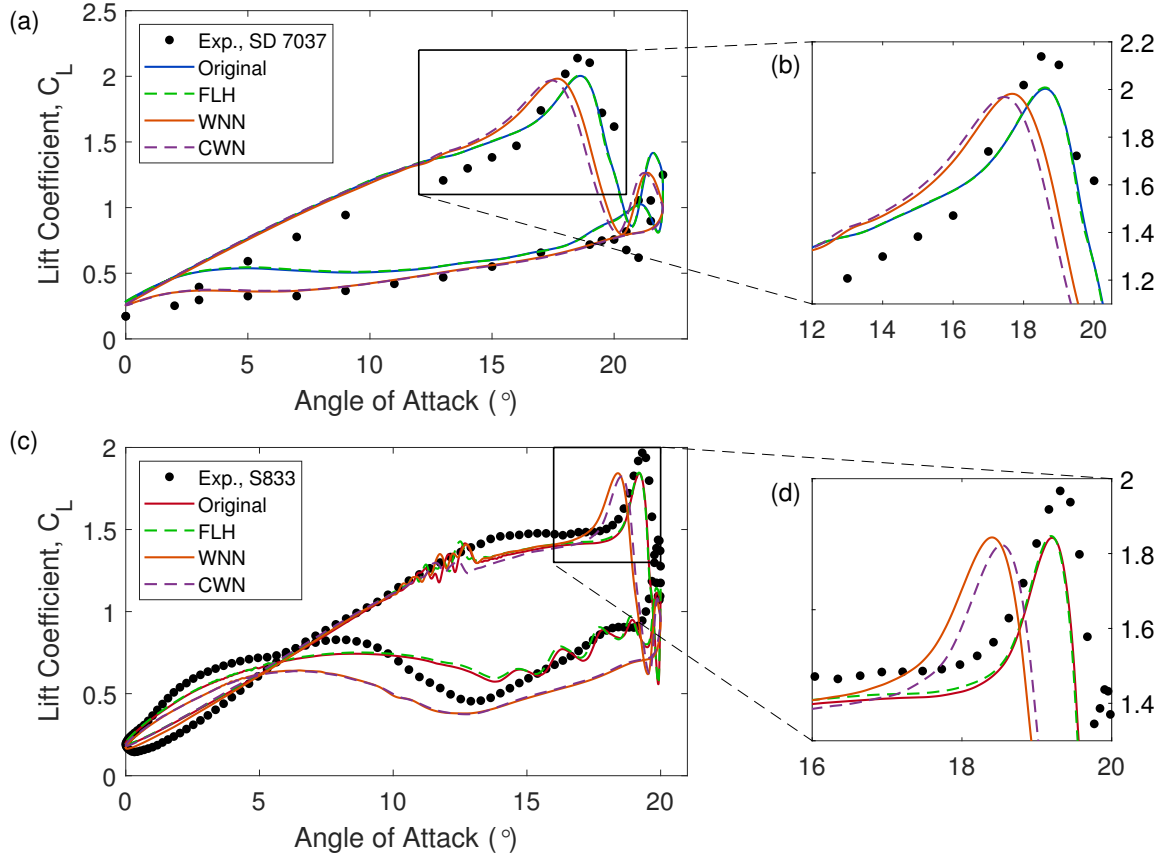


Fig. 5 Grid coarsening study results for the dynamic C_L cycle compared against experimental data for (a) SD 7037 [4] and (c) S833 [5]. Detailed views of the C_L peak are shown for (b) SD 7037 and (d) S833.

The number of nodes in the BL were compared in the attached laminar BL upstream of LSB formation for a range of dynamic AOA before the peak C_L . For the SD 7037 airfoil this was assessed at $x/c = 0.02$ from 0° to 9° AOA and the S833 comparison is completed for 0° to 13° AOA at the LE ($x/c = 0.002$) due to the LSB location at high AOA. The

number of wall-normal nodes in the attached BL was 10 to 11 for the original mesh, 7 to 9 for the FLH mesh and 6 to 7 for the WNN and CWN meshes. Therefore in both cases a minimum of 8 nodes within the laminar BL at the airfoil LE is required to prevent the early stall behaviour seen in the WNN and CWN mesh versions. The impact of the wall-normal resolution of the BL is evident in the dimensionless BL thickness, δ/c , where the original mesh has a value of 0.004 for the SD 7037 airfoil at $x/c = 0.02$ and 0.0012 for the S833 airfoil at $x/c = 0.002$. The BL thickness at those locations remained constant for the FLH SD 7037 mesh but increased by a factor of 1.25 for the FLH S833 mesh. The larger change in wall-normal spacing in the WNN and CWN mesh versions increased the BL thickness by a factor of 1.5 for the SD 7037 simulation and 1.7 for the S833 simulation. These changes in the simulated BL thickness at the airfoil LE will alter the downstream development of the BL transition and the LSB. The importance of the correct LSB simulation at high AOA and its relevance to the accurate prediction of the LEV will be discussed at length in the results section.

The WNN and CWN results in Figure 5 show a similar early C_L peak behaviour to previous URANS studies [21, 25–28] where the prediction of the peak C_L AOA occurs approximately 2° early in comparison with experimental data. These prior studies were completed at Re values that fall between the 4.1×10^4 of the SD 7037 simulation and the 1.7×10^5 of the S833 simulation, and therefore it is expected that the requirement of 8 nodes within the laminar BL at the airfoil LE would be applicable to those prior studies. Through this study, it can likely be concluded that the improvement in the stall AOA prediction gained through the usage of the LES-quality mesh is due to the increased wall-normal mesh density in the BL. The original mesh and the FLH mesh both provide accurate predictions of peak C_L AOA and feature the same total number of elements and associated computational expense. However, since the original mesh has 10 to 11 nodes in the laminar BL compared to the 7 to 9 in the FLH mesh, the remainder of the simulations in this paper are completed using the original LES-quality mesh to better meet the new recommendation of 8 nodes in the laminar BL.

III. Results

A. Static Validation

The simulation method was first validated for a range of static AOA against experimental data to determine the ability of the model to predict the low Re BL. The overall aerodynamic behaviour was assessed through the comparison of the simulated lift coefficient, C_L , and pressure coefficient, C_p , with experimental data. The SD 7037 static results are presented in Figure 6 and the validating experimental data was collected using PIV at the Re of 4.1×10^4 by Ghorbanishohrat [6] and Gharali and Johnson [4]. The average difference between the simulated and experimental [4] C_L results presented in Figure 6(a) is 0.16 ± 0.13 , with better agreement at the high AOA where the deep dynamic stall behaviour will occur in the dynamic case. Above 10° AOA, there is a discrepancy in the severity of stall with Gharali and Johnson measuring a greater loss in C_L than is predicted by the URANS model. For the Ghorbanishohrat data, the

slope in the linear region of the C_L curve is in agreement with simulated results.

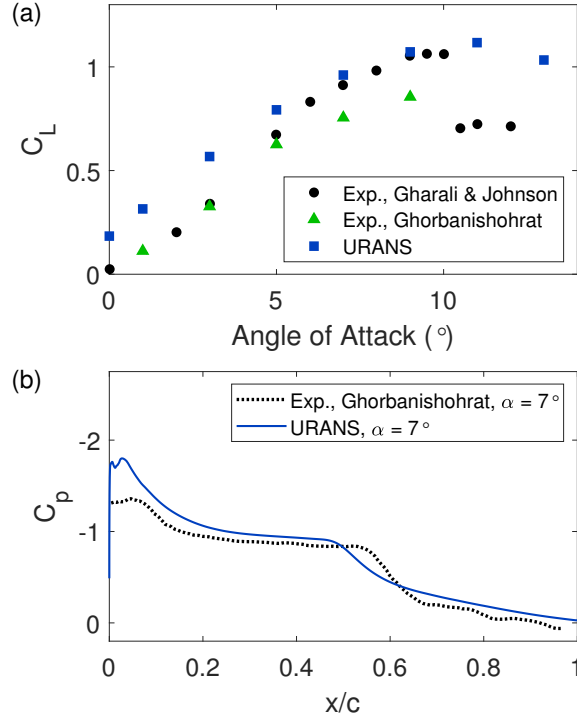


Fig. 6 Static results for SD 7037 at $Re = 4.1 \times 10^4$ for (a) C_L plotted against experimental data by Gharali and Johnson [4] and Ghorbanishohrat [6] and (b) suction side pressure coefficient at AOA = 7° plotted against experimental data [6]

The pressure coefficient, C_p , is used to validate the accuracy of the BL simulation along the airfoil surface, including the prediction of a LSB. This is a particularly important parameter when it comes to investigating the behaviour leading to the deep dynamic stall as the LSB bursting has been identified as a precursor to the LEV that causes stall at low Re [18, 33]. A C_p versus x/c plot with a fully formed LSB will feature an indicative transition ramp [41], which begins with the levelling off of the C_p curve at the location of laminar separation followed by a sharp pressure recovery at reattachment. The C_p plot for the suction side (top) of the SD 7037 airfoil at 7° AOA in Figure 6(b) shows the successful simulation of the LSB identified experimentally by Ghorbanishohrat [6]. The selected URANS model is able to correctly predict the location of laminar BL separation, as seen by the agreement of the C_p gradient in the early stages of the transition ramp. In the selected AOA, the location of reattachment is at approximately $x/c = 0.61$ which is earlier than the experimental location at approximately $x/c = 0.67$. This difference in reattachment location is likely to occur since the separated BL is more sensitive to simulation parameters such as the inflow turbulence [42], where a higher inflow turbulence will lead to earlier BL reattachment [43–46]. These errors will be accentuated in cases with an elongated LSB, which is the case for the SD 7037 airfoil at this Re . It should also be noted that the experimental suction peak measurements for the SD 7037 case are expected to be a lower magnitude than the simulated values due to

the experimental method of C_p calculation [6], which has reduced accuracy near the LE due to low PIV resolution in comparison to the local BL thickness. Even though there is a difference in magnitude for the first 10% of the airfoil, the average difference in C_p is 0.12 with a standard deviation of 0.08 and the maximum difference occurring at the LE with a value of 0.47.

The S833 C_L and C_p results are presented in Figure 7 and are validated against experimental data published by Samara and Johnson [5]. These experiments used differential pressure taps, meaning the C_p at each x/c location is a difference between a tap on the pressure side (bottom) and the suction side (top) of the airfoil. A comparison with the simulated results at a static AOA of 7° is shown in Figure 7(b) where the average difference in ΔC_p with the simulated values is 0.1 ± 0.06 . During the validation process, it was found that the excellent agreement in ΔC_p did not lead to the same agreement with the experimental C_L results [5] shown as open circles in Figure 7(a). Since the validation of the dynamic cycle is heavily dependent on the C_L values, the decision was made to recalculate the C_L using the available ΔC_p data. This is presented as filled circles in Figure 7(a), and results in an average C_L difference of 0.05 ± 0.03 compared to the simulated values, which is more consistent with the difference in the ΔC_p data. The S833 experimental C_L values used for the remainder of this paper, including the dynamic case, are the recalculated values using the ΔC_p data [5].

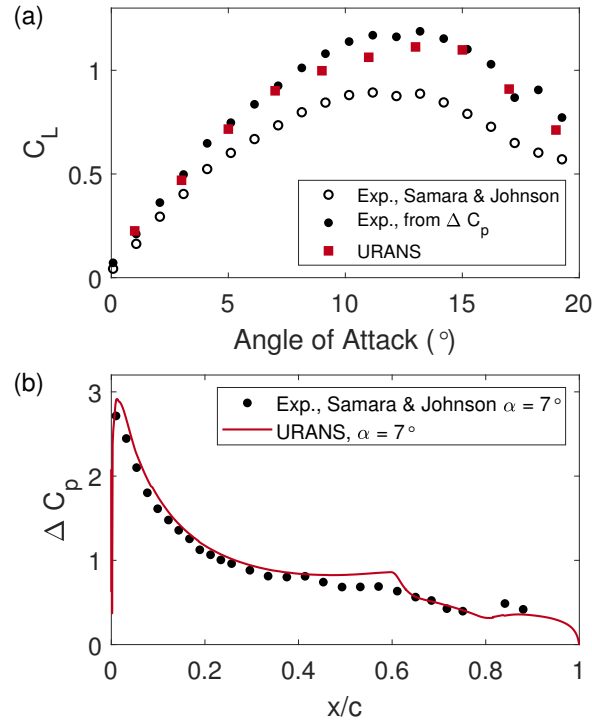


Fig. 7 Static results for S833 at $Re = 1.7 \times 10^5$ for (a) C_L plotted against experimental data by Samara and Johnson [5] and C_L calculated from published ΔC_p data [5] and (b) ΔC_p at AOA= 7° plotted against experimental data [5]

The use of ΔC_p leads to difficulty in separating the development of the pressure side and suction side BLs. Examination of the simulated results identified a LSB on both the suction and pressure sides of the airfoil, with the location of the LSB on the suction side at x/c from 0.29 to 0.63 and at x/c from 0.58 to 0.88 for the pressure side. The location of the LSB on the suction side progresses upstream with increasing AOA whereas the pressure side LSB moves downstream with increasing AOA and is no longer present above 10° AOA. Therefore the pressure side LSB is not of concern during the high AOAs associated with dynamic stall for the S833 case, and focus will be placed on the suction side LSB for the remainder of this paper.

Given the excellent agreement of the static results across a range of AOAs, particularly at the location of stall, there is confidence that the transition SST model has sufficient accuracy in the prediction of the low Re BL and associated loading to proceed to the more complex dynamic simulation cases.

B. Deep Dynamic Stall

The dynamic stall simulation results are presented as C_L values throughout the sinusoidal pitching cycle. The classic presentation of these results is to plot the C_L versus AOA, as this draws a direct comparison to the static C_L behaviour as a function of AOA in Figures 6(a) and 7(a). However, when examining deep dynamic stall, the stall behaviour is often compressed into a narrow range of AOA near the peak AOA of the pitching motion. This can obscure the stall behaviour and complicate the validation of the results, especially if the C_L during the pitch up and pitch down motions cross each other forming loops in the C_L versus AOA plot. Therefore, when examining deep dynamic stall, it is practical to also plot the C_L values with respect to time.

The results of the SD 7037 sinusoidal pitching case are presented in Figure 8 and are validated against the experimental results [4] as well as previous URANS results by Gharali and Johnson [4]. The full details of the sinusoidal pitching can be found in Table 1. Figure 8(a) shows the C_L as a function of AOA and Figure 8(b) shows C_L as a function of non-dimensional time, t/T , where T is the period of the pitching cycle. The location of maximum pitching rate, ω_{max} , is labelled in both cycles, and Figure 8(b) has a vertical dashed line to indicate the maximum AOA of the cycle which corresponds to the initiation of the pitch down motion. In comparison to the static simulations, the dynamic C_L begins to exceed the static C_L at approximately 9° AOA. The average difference in C_L between the simulated results and the experimental data is 0.15 ± 0.09 and is similar to the C_L difference in the static results, showing that the dynamic motion is not adding additional error to the simulation. Overall the C_L trend throughout the pitching cycle is well replicated in the simulation, particularly the AOA of the lift peak that occurs before stall.

The lift peak at 18.6° AOA and subsequent drop in lift are associated with the formation and shedding of a large LEV, respectively [2]. After the bursting of the suction side LSB (labelled Burst in Figure 8), the BL is completely separated, leading to the roll up of the BL to form the LEV. The LEV resulted in a 79% increase in maximum C_L above the static value of 1.12 and a delay in the AOA of the maximum C_L by approximately 7° in comparison with the

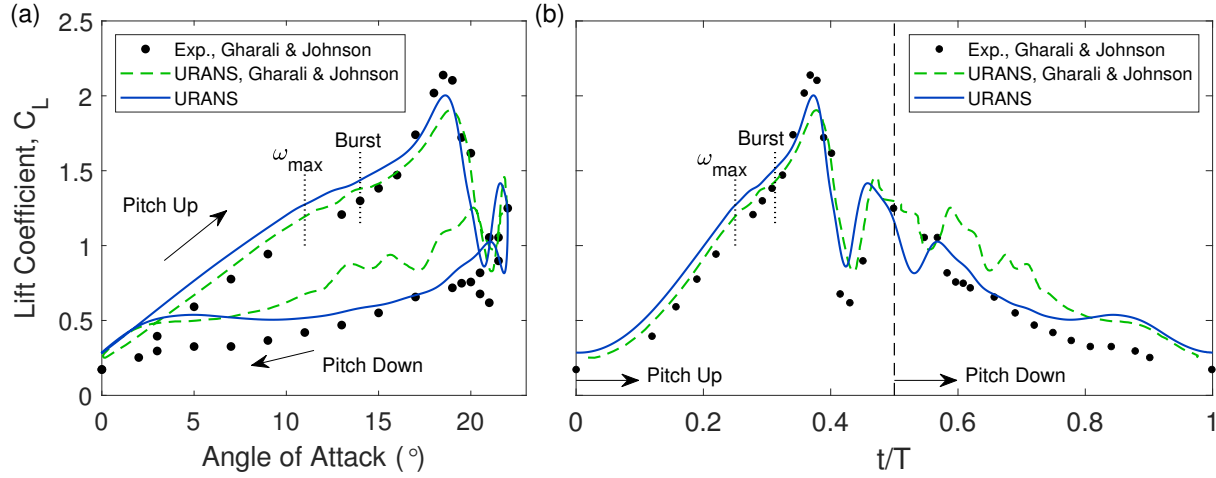


Fig. 8 URANS predicted dynamic lift coefficient cycle for the SD 7037 airfoil at $Re = 4.1 \times 10^4$ and $k = 0.08$ compared with experimental [4] and URANS data [4] versus (a) AOA and (b) non-dimensional time t/T

static results. After the LEV is shed, the C_L rapidly drops by 1.14 in comparison to the drop of 1.52 experimentally, confirming the presence of deep stall. Before reaching the maximum AOA in the pitching cycle, the C_L increases again due to the formation of a secondary LEV.

The accuracy of the prior URANS results by Gharali and Johnson [4] is high in comparison to URANS simulations by other researchers, and yet the current URANS results show some further improvements. Notably, the prediction of maximum C_L and its AOA is closer to the experimental results and the C_L magnitude through the pitch down motion is in better agreement. The simulation by Gharali and Johnson was completed using ANSYS Fluent, release 13, using the same numerical method as the current simulations and it is expected that improvements have been made in the software between release 13 and the release 19.2 used currently. A more likely factor for the improvement in the results is the mesh parameters both near to the airfoil and between the inlet and the airfoil. The mesh distribution used for the current simulations was from previous 3D LES simulations [13], which had been refined based on different criteria than a traditional URANS simulation. LES can be overly dissipative if the mesh sizing is large, therefore the mesh refinement extends further into the BL and the expansion of cell size towards the inlet is limited [47]. For the URANS simulations, this refined mesh likely resulted in improvements in the accuracy of the LSB simulation and the inflow turbulence propagation from the inlet to the airfoil. In the current simulations, higher inflow turbulence at the airfoil would result in the damping of the bluff body vortex shedding that happens after deep stall, which could explain the improved accuracy of the C_L magnitude in the pitch down motion for the current simulation.

The C_L results for the S833 dynamic pitching case are shown in comparison to the experimental results [5] in Figure 9(a) for C_L versus AOA and Figure 9(b) for C_L versus t/T . As with the SD 7037 case, the location of ω_{max} and LSB bursting are labelled in both plots along with the vertical dashed line for location of maximum AOA in Figure 9(b).

The experimental C_L values were calculated from the experimental ΔC_p [5] for the reasons explained in detail in the static validation section of this paper. The simulated C_L values align well with the experimental results and the average difference in C_L is 0.24 ± 0.29 , though the difference is increased when compared to the static simulation results. This pitching cycle causes deep dynamic stall to occur, with a LEV forming between the indicated burst location and the peak C_L followed by a rapid drop in C_L as the LEV is shed. For this cycle, the C_L begins to exceed the static lift at approximately 10° AOA, the maximum C_L is 84% higher than the static value of 1.0 and the delay in the peak C_L location is 6.2° .

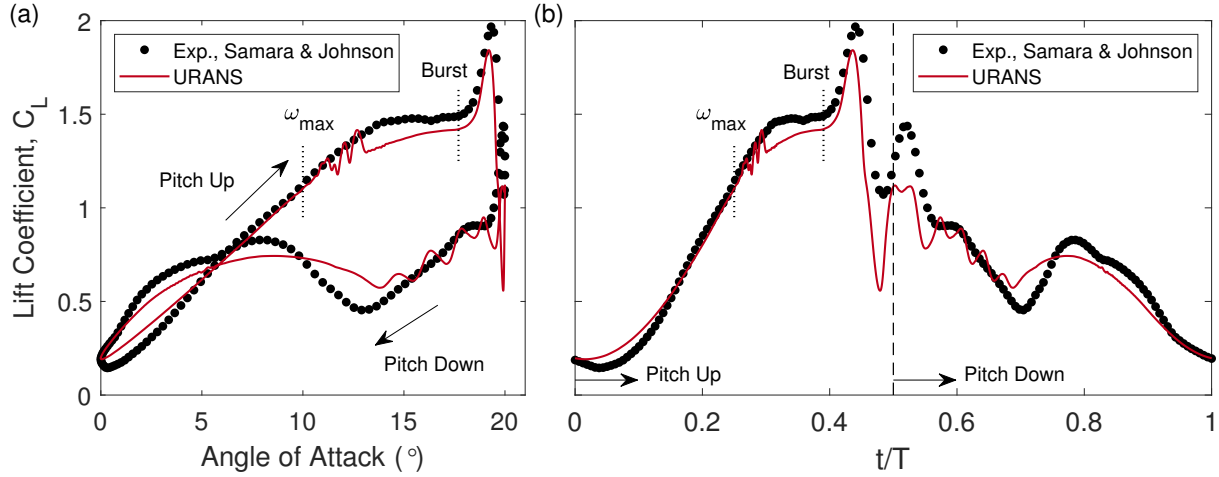


Fig. 9 URANS predicted dynamic lift coefficient cycle for the S833 airfoil at $Re = 1.7 \times 10^5$ and $k = 0.06$ compared with experimental data [5] versus (a) AOA and (b) non-dimensional time t/T

The fluctuations in C_L during the pitch up motion from approximately 10° to 14° AOA are likely due to instability in the reattachment point of the suction side LSB, identified through the simulated C_p behaviour. A sample of the C_p behaviour is shown in Figure 10(a) comparing the ΔC_p to the experimental results [5]. The instability begins at the location of LSB reattachment ($x/c = 0.50$) but the overall trend is in agreement with the experimental pressure data with the average difference in ΔC_p being 0.16 ± 0.13 . This BL instability was also observed by Visbal and Garmann [18] in their LES study, however in that case the fluctuations in C_p do not result in fluctuations in C_L during the pitch up motion. This is likely due to the LES method introducing additional randomization and spanwise variation in the turbulent structures in comparison to the 2D Reynolds-averaged approach. For the URANS results, the fluctuations in C_L were identical in each simulated pitching cycle and therefore did not disappear when averaging the cycles. Tests of higher inflow turbulence as well as chordwise grid refinements were completed to determine if these fluctuations were a result of numerical errors. ANSYS Fluent specifically recommends a turbulent intensity greater than 0.1% around the airfoil, and while tests at $Tu = 0.7\%$ caused the damping of the C_p fluctuations, they were still present in the BL. It should be noted that these fluctuations begin immediately following the location of ω_{max} for the pitch up motion,

giving further confidence in the assertion that the fluctuations are due to the flow physics and not due to numerical error. The impact of the location of maximum pitch up motion will be discussed in the next section.

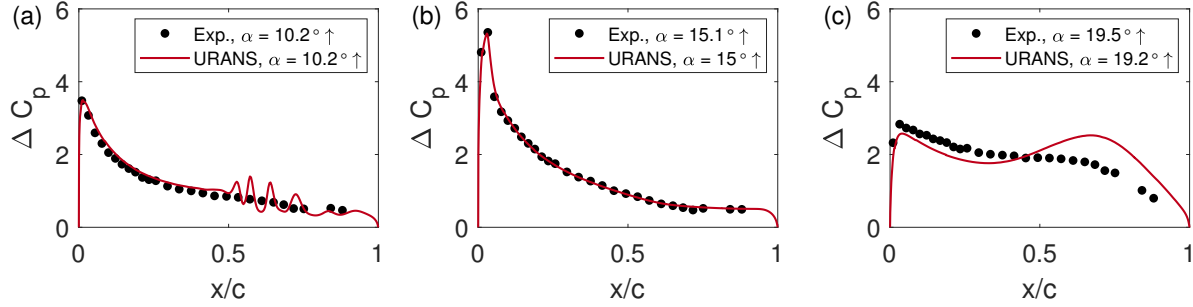


Fig. 10 URANS predicted dynamic ΔC_p for the S833 airfoil at $Re = 1.7 \times 10^5$ and $k = 0.06$ compared with experimental data [5] for AOAs: (a) $10^\circ \uparrow$ (b) $15^\circ \uparrow$ (c) $19^\circ \uparrow$

The pressure behaviour was also validated for an AOA of 15° in the region of levelled off C_L before the LSB burst occurs (Figure 10(b)), and at the peak C_L (Figure 10(c)). At 15° the difference in ΔC_p is 0.05 ± 0.07 and at the peak C_L it is 0.39 ± 0.23 . At the peak C_L location, the simulated LEV is located from approximately $x/c = 0.4$ to the TE, which can be seen due to the increase in suction pressure in that region. A similar increase in pressure can be seen in the experimental data, however it is not as strong as the simulated result. The AOAs associated with the downstream propagation of the LEV in the experimental data were reported to have a high standard deviation of ΔC_p [5]. Any differences in the pitching cycle due to experimental factors, such as cycle repeatability, could have resulted in small differences in the timing of the LEV shedding. Since the reported ΔC_p is averaged across all experimental cycles [5], the small variation in the location of the LEV would result in the lower ΔC_p magnitudes due to the wash out of the suction pressure during cycle averaging.

Previous researchers completing URANS [21, 25–28] and LES [17, 19–21] simulations of similar sinusoidal pitching low Re airfoils have had poor agreement with the location of peak C_L , and consistently predict that the lift peak occurs before reference experimental data. In comparison, both presented cases are within 0.1° of the experimental peak lift AOAs. This indicates the timing of key behaviours in the dynamic stall cycle are simulated correctly using this URANS method and therefore a computationally expensive LES simulation is not required. These behaviours include the formation and bursting of the LSB and the formation and shedding of the LEV. The details of these behaviours are analysed in greater detail in the remaining results sections now that the simulation of the overall dynamic stall behaviour has been validated.

C. SD 7037 Dynamic LSB

The LSB formed on the suction side of the SD 7037 airfoil is inherent to the low Re BL and airfoil curvature and is therefore present in both the static and dynamic BLs. Due to the rapid pitching motion altering the velocity and pressure

gradients in the BL, the development and movement of the LSB is delayed in comparison to the static behaviour. To quantify this delay, the movement of the LSB with AOA is tracked in the static and dynamic cases by plotting the locations of laminar BL separation, x_s , and reattachment, x_r , as well as the location of TE separation of the turbulent BL, $x_{s,TE}$. These values are determined using the skin friction coefficient, C_f , in the chordwise direction since this provides the exact locations of BL separation and reattachment. The regions of the airfoil with an attached BL have a positive C_f , which means that regions of separation can be identified where $C_f \leq 0$. This method of BL development tracking has been previously used in the static experiments by Lee and Gerontakos [23] and the dynamic URANS simulations by Tseng *et al.* [33]. It is also possible to infer this behaviour from contour plots of C_p or C_f , such as in the LES work by Visbal and Garmann [18] and the experimental work by Samara and Johnson [5].

The results for the SD 7037 case are presented in Figure 11, with labels for the ω_{max} and LSB bursting locations included to align with the C_L cycles in Figure 8. The analysis of the dynamic LSB is split into the AOAs before and after ω_{max} as there is distinct behaviour that occurs depending on the AOA of the pitching up motion and whether the pitching motion is accelerating or decelerating (i.e. before or after ω_{max} , respectively). To draw conclusions about the impact of the sinusoidal pitching motion on the suction side BL, the results are compared against the results from the static simulations presented earlier in this paper.

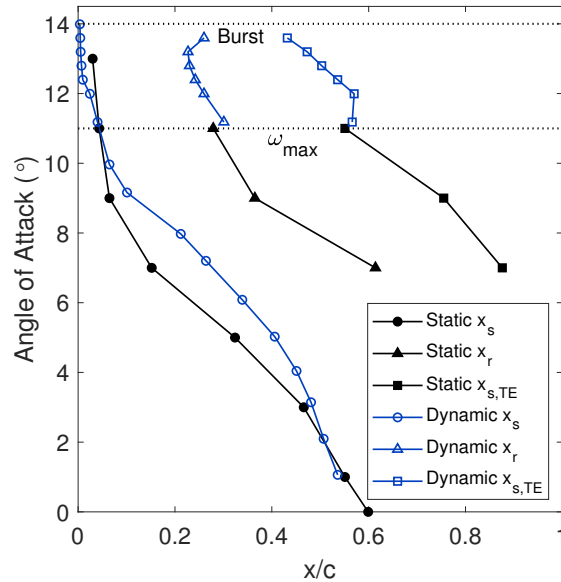


Fig. 11 Comparison of BL development in the static and dynamic pitch up motion for the SD 7037 airfoil at $Re = 4.1 \times 10^4$. x/c locations of laminar separation (x_s), reattachment (x_r) and trailing edge separation ($x_{s,TE}$) with connecting lines for visualization only

The static SD 7037 results at low AOAs (shown in Figure 11) have a BL that undergoes laminar separation but does not reattach to form a LSB. The location of laminar separation moves upstream with increasing AOA, as expected, and

reaches the LE as the AOA approaches the peak C_L AOA of 11° . The static LSB is first captured at the AOA of 7° with a length of $0.46c$ and the BL reattaches for a short distance before turbulent BL separation occurs at $x/c = 0.88$. The LSB x_r and TE separation move upstream at a similar pace to x_s , again until the peak C_L AOA of 11° is reached. By the next simulated AOA of 13° the LSB has burst due to the TE separation location meeting the LSB reattachment location, which results in a fully separated BL.

The dynamic SD 7037 results for the AOAs before the location of ω_{max} exhibit a minor delay in x_s in comparison to the static results, reaching a maximum of $0.11c$ at an AOA of 7° . A more significant difference between the static and dynamic results is that the dynamic case does not form a LSB before reaching the location of ω_{max} . This indicates that the acceleration of the rate of the pitch up motion is having a significant impact on the development of the BL transition required to form the LSB. Immediately after reaching ω_{max} , the dynamic case forms a LSB and develops a region of turbulent BL separation at the TE. The LSB and TE separation locations are equal to the locations for the static case at the same AOA. This is assumed to be caused by the deceleration of the pitching motion that occurs after ω_{max} , which lessens the dynamic effect on the BL.

After reaching ω_{max} , the LSB length reduces from $0.26c$ to $0.22c$ and moves $0.04c$ upstream. At the same time, the TE separation location progresses upstream by $0.13c$ which is approximately 3 times the rate of the LSB motion. At 13.6° AOA, there is a slight expansion of the LSB towards the TE separation, with the x_r changing from 0.23 to 0.26 , indicating the initiation of the type of LSB bursting seen in LE stall [14]. By 14° AOA, the LSB of the dynamic SD 7037 case has burst from the combination of the expanding LSB and the upstream progression of the TE separation. Visbal and Garmann [18] and Tseng *et al.* [33] both capture similar amounts of LSB expansion occurring before LSB burst in their dynamic simulations. Additionally, Tseng *et al.* [33] identified two regions during the pitch up motion with distinct BL behaviour, using the first appearance of TE separation as the division rather than the location of ω_{max} used in the current work. For their cycle, the TE separation begins 1.8° after the location of ω_{max} , and the location of ω_{max} marks initiation of a rapid reduction in the length of the long LSB that formed during the low AOAs of the cycle. The NACA 0015 case at $Re = 2 \times 10^5$ and $k = 0.1$ by Tseng *et al.* and the current SD 7037 case at $Re = 4.1 \times 10^4$ and $k = 0.08$ have a similar abrupt change in the LSB location after reaching the ω_{max} of the pitch up motion, showing that this LSB behaviour is likely inherent to the BL behaviour during sinusoidal pitching at low Re.

D. S833 Dynamic LSB

The same analysis of the motion of the LSB can be completed for the S833 case to determine the role of the LSB in dynamic stall at low Re. The static S833 results shown in Figure 12 exhibit similar behaviour to the SD 7037 case, though with minor differences which can likely be attributed to the difference in Re. The S833 case has approximately 4 times the Re of the SD 7037 case, and a higher Re is known to result in shorter LSB lengths as well as reduced LSB thickness [14]. The maximum static LSB length for the S833 case is $0.33c$ in comparison with the $0.46c$ of the

SD 7037 case. The LSB also progresses towards the leading edge at a much higher rate with respect to AOA when compared to the SD 7037 case. This results in the S833 static LSB remaining stationary with a reattachment location of approximately $x/c = 0.05$ to 0.1 for AOAs ranging from 11° to 15° . The static LSB has burst by 17° AOA, which is several degrees after the peak C_L AOA of 13° . In comparison to the SD 7037 static case, the S833 case has a longer delay in the bursting of the LSB after peak C_L . This is a result of the short LSB at the LE that forms at the higher Re, since that requires the TE separation point to progress further upstream to cause the LSB to burst.

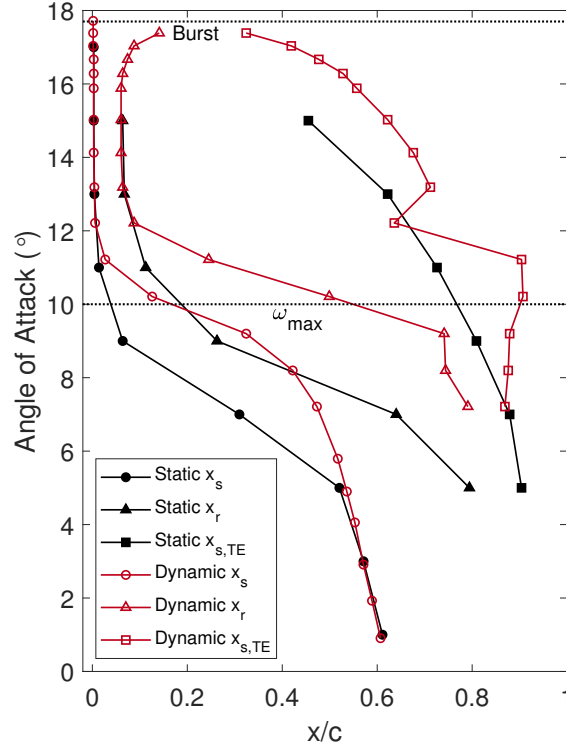


Fig. 12 Comparison of BL development in the static and dynamic pitch up motion for the S833 airfoil at $Re = 1.7 \times 10^5$. x/c locations of laminar separation (x_s), reattachment (x_r) and trailing edge separation ($x_{s,TE}$) with connecting lines for visualization only

The dynamic S833 results before the location of ω_{max} show a delay in x_s in comparison to the static results. The maximum simulated delay is $0.26c$, which is over 2 times larger than the SD 7037 case. The reduced frequency, k , for the S833 case is 0.06 whereas the SD 7037 case is 0.08, so it would be expected that the higher reduced frequency of the pitching motion would result in a greater delay in the x_s for the SD 7037 case. However, it seems that the higher Re of the S833 case had a larger impact on the dynamic delay of x_s caused by the faster upstream progression of the LSB location seen in the static AOAs. Due to the shorter LSB present at higher Re, the LSB in the dynamic S833 case forms before the location of ω_{max} at an AOA of 7.2° . The x_r remains relatively constant at a location of $x/c = 0.75$ until reaching ω_{max} and also has TE separation that remains constant at around $x/c = 0.88$. The formation of the LSB

before ω_{max} was also seen in the URANS simulation at a similar Re of 2×10^5 by Tseng *et al.* [33].

On reaching ω_{max} , there is a rapid change in the BL behaviour associated with the deceleration of the pitching motion that was seen in the SD 7037 case and the work by Tseng *et al.* [33]. Both the x_s and x_r progress upstream to match with the LSB location for the S833 static simulations. This rapid change in x_r corresponds to the previously discussed fluctuations in C_L seen in Figure 9 during the pitch up motion from 10° to 14° AOA, indicating that the oscillations in C_p in Figure 10(a) are a result of the flow physics associated with this rapid motion of the x_r location. After 13° , the dynamic LSB remains stationary at a length of $0.06c$ until 16.3° where the LSB slowly expands to $0.14c$ before bursting. The AOAs of constant LSB length correspond to a region of approximately constant C_L in Figure 9 (a). Due to the strong LE suction peak (Figure 10 (b)) associated with the LE LSB, the upstream motion of the TE separation has little impact on the C_L between the locations of ω_{max} and LSB burst. During this time, the TE separation location progresses upstream from $x/c = 0.71$ to 0.32 before the LSB bursts at 17.7° .

E. Dynamic Bursting of the LSB

For the SD 7037 and S833 cases, the bursting of the LSB results in the complete separation of the suction side BL when the BL separation location changes from the TE separation location, $x_{s,TE}$, to the laminar separation location, x_s . This change in BL separation location is $0.43c$ for the SD 7037 case and $0.32c$ for the S833 case, as shown in Figures 11 and 12. The complete separation of the suction side BL is the driving force behind the LEV formation [11] and therefore having the URANS simulation predict the correct LSB bursting location is crucial in predicting the correct dynamic stall AOA. From previous test cases, it was determined that simulating the correct LSB size and location using URANS requires further grid refinement than is traditionally recommended. This is particularly important in the thin BL that forms close to the LE where higher Re airfoils, such as the S833 case, will have a short and thin LSB. The error introduced in the simulation of the LSB at the LE will lead to an incorrect prediction of the LSB bursting AOA, which is likely the reason that previous works [17, 19–21, 25–28] have consistently predicted early dynamic stall in comparison to experimental data. Therefore specific attention must be given to the LSB simulation for these dynamic stall cases in order to predict the correct dynamic stall AOA, regardless of whether the numerical method is LES or URANS.

As was previously stated, the mesh used for the current simulations uses the mesh distribution from the 3D LES simulations of the SD 7037 airfoil at $Re = 4.1 \times 10^4$ [13], which includes a refined grid for $y^+ < 20$ at the airfoil LE. The results of the grid coarsening study, shown in Figure 5, indicated that the simulation is sensitive to the wall-normal grid resolution, and the changes in the C_L cycle were only seen after the location of ω_{max} for each case. The early formation of the C_L peak in the WNN and CWN simulations indicates that the coarse mesh in the wall-normal direction is numerically causing the early bursting of the LSB and is leading to premature LEV formation in comparison to the experimental data. The use of the LES-quality BL refined mesh is therefore critical to the URANS simulation of the correct dynamic stall location, and is considered the main factor in the successful simulation of the dynamic stall

location for both cases.

F. LEV Formation

The LEV formation occurs between the location of LSB bursting and the peak C_L for each cycle (shown in Figures 8 and 9), and it is the growth and shedding of this LEV that is the source of the extreme load fluctuations that occur during deep dynamic stall. The formation of the LEV is shown in Figure 13 and Figure 14 through a combination of chordwise C_f and the non-dimensional z -vorticity, $\omega_z c/U_\infty$, where the z -direction is aligned with the spanwise direction. The chordwise C_f is used to identify the location of the LSB before bursting and identify the regions of strong reverse flow that occur between the LEV and the airfoil surface. The non-dimensional z -vorticity is used to visualize the formation and growth of the LEV. The locations chosen for analysis were selected from each case to highlight the same key moments in the LEV formation and therefore comparisons can be drawn between the two cases.

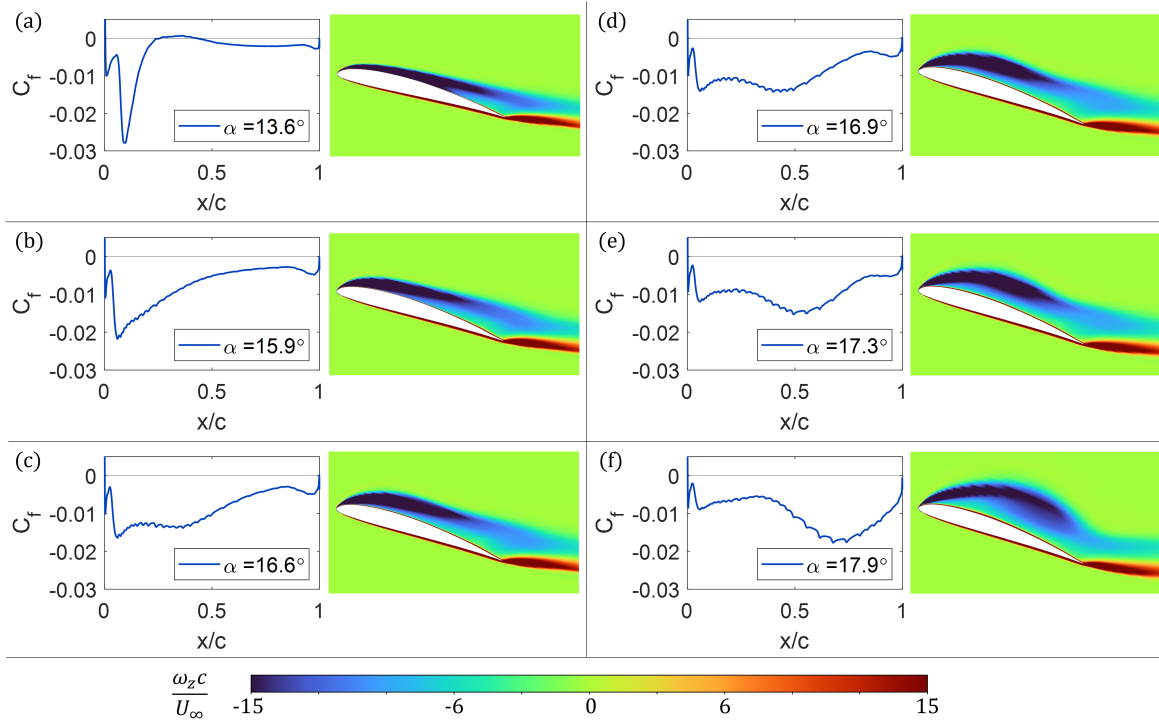


Fig. 13 URANS dynamic C_f and non-dimensional z -vorticity contours of the LEV formation of the SD 7037 airfoil at $Re = 4.1 \times 10^4$ and $k = 0.08$ for AOAs: (a) 13.6° (b) 15.9° (c) 16.6° (d) 16.9° (e) 17.3° and (f) 17.9°

Location (a) shows the behaviour immediately before the bursting of the LSB, and location (b) shows the fully separated BL after the burst occurs. At location (a) the LSB reattachment location, x_r , is moving downstream as shown in Figures 11 and 12. From the negative C_f at location (b) for both cases, it is clear that the remnants of the LSB continue to expand in the downstream direction once the BL is fully separated. This effect is more pronounced for the S833 case, where the reverse flow for $x/c > 0.5$ remains relatively unchanged between location (a) and (b), but the

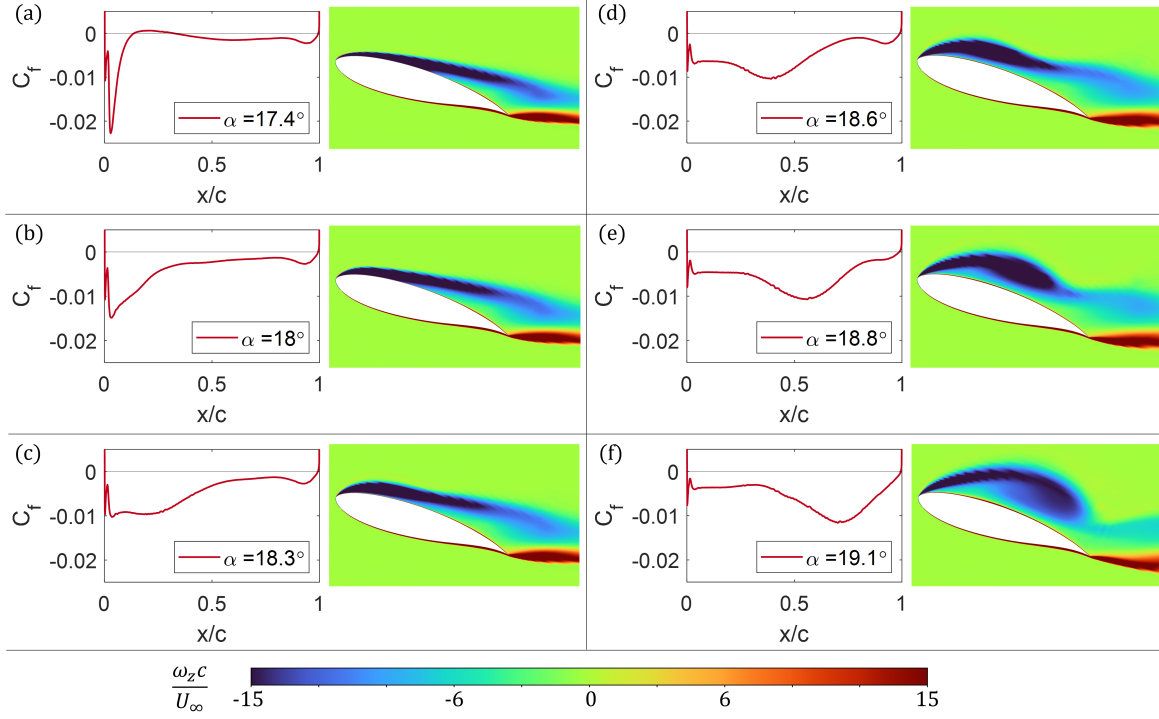


Fig. 14 URANS dynamic C_f and non-dimensional z -vorticity contours of the LEV formation of the S833 airfoil at $Re = 1.7 \times 10^5$ and $k = 0.06$ for AOAs: (a) 17.4° (b) 18.0° (c) 18.3° (d) 18.6° (e) 18.8° and (f) 19.1°

region of negative C_f associated with the LSB has expanded from $x/c = 0.14$ to approximately $x/c = 0.25$.

The separation of the BL results in an unstable shear layer that begins to roll up to form the LEV in location (c), which is visualized for both cases in the thickening of the LE region of negative z -vorticity as well as the bump of increasing reverse flow seen in the range of $x/c = 0.2$ to 0.5 in the C_f plot. Since the LEV first appears from the region of negative C_f associated with the downstream expansion of the bursting LSB, this further highlights the need to take great care in simulating the LSB in these dynamic stall cases. Not only does the LSB bursting generate the separated BL that rolls up into the LEV, but it also determines the chordwise location where the LEV first appears.

Locations (d) to (f) show the chordwise progression of the LEV as both cases approach the peak C_L in the pitching cycle. For the S833 case, the negative C_f peak at the wall in Figure 14(f) at 19.1° AOA corresponds to the suction peak shown in the ΔC_p in Figure 10(c) at 19.2° AOA. The LEV motion can be tracked through either a peak in negative chordwise C_f or a suction peak in the C_p , and is often shown through contour plots [5, 18] to condense the full pitching cycle into a single image.

The C_f trends and the z -vorticity magnitudes are very similar for the two cases except that the Z -vorticity visualization of the SD 7037 case shows less growth of the LEV in the wall-normal direction. This is best visualized in the z -vorticity contours of Figure 13 and Figure 14 at locations (e) and (f). There are several factors that could have an impact on the growth of the LEV that results in differences in the size of the LEV in the wall-normal direction between the two cases.

One difference between the cases is that the peak C_L occurs closer to the ω_{max} for the SD 7037 cycle than it does for the S833 cycle, with the SD 7037 peak C_L occurring when $\omega = 0.7\omega_{max}$ and the S833 peak C_L when $\omega = 0.4\omega_{max}$. The higher percentage of ω_{max} at peak C_L for the SD 7037 case will result in a larger dynamic effect on the LEV formation, and is a factor in the reduction of the growth of the LEV in the wall-normal direction. Also, the reduced frequency for the SD 7037 case is higher than the S833 case, and therefore the expectation is that even if the LEV formation occurred at the same location within the pitching cycle, that the SD 7037 case would have a thinner LEV due to increased dynamic effects. Additionally, the lower Re results in a lower momentum BL in comparison to the S833 case which has a short LSB and a longer chordwise distance for the turbulent BL to develop. Since the separated shear layer roll-up is the driving force in the LEV growth [11], the higher momentum in the S833 BL leads to a higher energy and therefore a LEV with greater wall-normal growth.

The SD 7037 and S833 cases have differing parameters (Re, k) and slight differences in the growth of the LEV, however the overall behaviour of the LEV is similar. The cycle location and the growth of the LEV are clearly simulated accurately using the URANS method, since the C_L results in Figures 8 and 9 agree with the experimental location and magnitude of the dynamic C_L peak that forms after the bursting of the LSB.

IV. Conclusion

Two low Re airfoils with an applied sinusoidal pitching motion were simulated using URANS to analyze the deep dynamic stall process that can occur on SWT blades. The chosen SD 7307 and S833 airfoils are designed for use in the Re range in which SWTs operate and the applied sinusoidal pitching cycle parameters were selected to replicate the change in effective AOA of a SWT in unsteady wind conditions. Previous URANS and LES simulations of similar low Re pitching airfoils have struggled to predict the correct dynamic stall AOA, which limits the usage of the simulations as a design tool for mitigation of the unsteady aerodynamic forces that occur during deep dynamic stall. There is also contradicting findings on the importance of the LSB to the initiation of the LEV and the subsequent dynamic stall, therefore extra attention was given to the LSB simulation especially since it is a complex behaviour to predict using URANS methods.

The transition SST $k - \omega$ model, also known as $\gamma - Re_{\theta t}$, was selected after verification of the LSB simulation and lift coefficient for a range of static AOAs. The dynamic pitching simulation for the SD 7037 airfoil was run at a Re of 4.1×10^4 with a k of 0.08 and an AOA range from 0° to 22° . The S833 pitching simulation was run at a higher Re of 1.7×10^5 with a k of 0.06 and an AOA range from 0° to 20° . The simulated C_L through both the pitching cycles agreed with the experimental values and the location of the peak C_L AOA was within 0.1° of the experimental location. The agreement of the stall location is often obscured by the method of plotting the C_L versus AOA, and the results in this paper were specifically plotted versus cycle time to highlight the excellent agreement throughout the whole dynamic stall process. This is a significant improvement over previously published LES and URANS simulations,

which consistently predicted early dynamic stall in comparison with experimental data. A grid coarsening study was completed that showed a similar erroneous prediction of early dynamic stall occurred when the mesh in the BL was coarsened in the wall-normal direction, and that a minimum of 8 nodes in the laminar BL at the airfoil LE are required to predict the correct peak C_L AOA.

Analysis was completed of the BL and LSB leading up to the complete separation of the BL that results in the formation of the large LEV that causes the peak C_L and dynamic stall. The location of the LSB during the pitch up motion was compared with the static LSB location through the tracking of its separation and reattachment locations as well as the TE turbulent BL separation location. It was found that for the SD 7037 and S833 airfoils in the static and dynamic cases, the complete BL separation occurs when the TE separation point meets the LSB reattachment point, causing the bursting of the LSB. In the dynamic cases, the LSB had a slight expansion in length before bursting due to the LSB reattachment location moving downstream in a similar manner to the bursting that occurs in LE stall. These findings show that any errors in the simulation of the LSB length and its expansion before bursting will result in incorrect timing of the LSB bursting and complete BL separation. Incorrect prediction of the BL separation in the pitching cycle will result in errors in the prediction of the LEV and the AOA of dynamic stall.

While examining the dynamic LSB motion, it was also found that during the pitch up motion there is a distinction in the behaviour of the LSB and BL depending on if the pitch rate is accelerating or decelerating. In the accelerating region that occurs before reaching the maximum pitch rate, ω_{max} , the LSB and BL development was delayed in comparison to the equivalent static AOA. In the SD 7037 case, this prevented the formation of the LSB and in the S833 case the LSB formation was delayed, with the reattachment location remaining relatively constant with increasing AOA. After reaching ω_{max} , the deceleration of the pitching rate caused rapid motion of the LSB to the LE of the airfoil, and also resulted in the location of the TE separation quickly approaching the reattachment location of the LSB. This abrupt shift in behaviour is seen in both the SD 7037 and S833 cases, regardless of the differences in airfoil shape, Re , or k , indicating that the acceleration and deceleration of the pitching motion is the driving force behind this phenomenon.

Analysis was also completed on the formation of the LEV after the LSB bursts and the BL separates for both simulated cases. It was found through the examination of the simulated C_f in the chordwise direction that the region of strong reverse flow associated with the LSB was still present after the LSB had burst. The remnants of the LSB continued to expand downstream in the same manner as the motion of the LSB reattachment location preceding the bursting of the LSB. It is from the downstream end of the remnants of the bursting LSB that the first evidence of LEV is detected. This yet again highlights the importance of the correct LSB simulation during dynamic stall, since the length of the LSB at the time of bursting must be correct in order to result in the correct chordwise location of LEV formation.

Once the LEV formed, the S833 case predicted greater growth of the LEV in the wall-normal direction in comparison with the SD 7037 case. This was attributed to the lessened dynamic effect due to the lower k and because the shedding of the LEV occurred later in the pitching cycle, meaning the pitch rate was at a lower magnitude relative to ω_{max} .

The presented analysis thoroughly validated the ability of the $\gamma - Re_{\theta l} k - \omega$ URANS model to simulate deep dynamic stall of low Re airfoils. The success is attributed to the use of a LES-quality mesh, as this was crucial to the simulation of the LSB formation, motion and bursting. A new requirement of a minimum wall-normal mesh resolution of 8 nodes in the laminar BL at the airfoil LE was established for the prediction of the correct dynamic stall AOA, since the bursting of the small LSB in the thin LE BL initiates the LEV that causes the peak C_L and deep dynamic stall. The accurate simulation of the LSB eliminated the issue of early prediction of dynamic stall and further strengthens the viability of this low-cost simulation as a design tool for methods to mitigate the variation in the aerodynamic loading of SWT blades under challenging unsteady wind conditions.

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