

Marginal Rate of Technical Substitution Curves for Frequency Regulation Services

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Abstract—This paper presents a detailed methodology to develop Marginal Rate of Technical Substitution (MRTS) curves, which can be used to optimally determine the appropriate substitution of traditional regulation signals with fast regulation signals, considering different Energy Storage System (ESS) technologies and discharging times, scenarios, and seasons. The presented work is based on the Ontario Power System (OPS) managed by the Independent Electricity System Operator (IESO) of Ontario, Canada. Different comparisons of the MRTS curves are carried out and the criteria used to obtain 16 average optimized MRTS curves, four per season, are presented. Finally, the 16 MRTS curves obtained for the IESO and their parameters are presented, and the use of these curves is explained through an example.

Index Terms—Energy storage, fast regulation signal, frequency regulation, marginal rate of technical substitution, traditional regulation signal.

I. INTRODUCTION

In 2017, PJM Interconnection LLC (PJM) proposed the use of Marginal Rate of Technical Substitution (MRTS) curves to replace the Benefit Factor (BF) curve being used, which measured the relative value of fast Dynamic Regulation Signal (*RegD*) versus the slower Traditional Regulation Signal (*RegA*) resources, and was implemented in the regulation market [1]. Unlike the MRTS curves, the BF curve used by PJM is static across peak, no-peak, morning ramp and evening ramp scenarios and seasons. Most of the flaws in the Frequency Regulation (FR) market of PJM could be attributed to improper implementation of the BF curve between *RegA* and *RegD* resources, which caused both under- and over-payment for *RegD* resources, and over-procurement of *RegD* during some hours [2], [3]. The MRTS curves can be considered an improved version of the BF curve, but these have not been implemented, because of settlement related inconsistencies with Federal Energy Regulatory Commission (FERC) Order No.755 [4]. Since the Independent Electricity System Operator (IESO) of Ontario, Canada, is not under

FERC jurisdiction, it can take advantage of the proposed MRTS curves and learn from PJM experiences.

The IESO is at an early stage of implementation of fast FR using Energy Storage System (ESS) resources in the IESO-Administered Market (IAM) [5]. Therefore, it is important for the IESO to determine the benefits of including fast resources into the FR service provisions, and analyzing the trade-offs among resources with similar performance characteristics. In this context, the IESO can take advantage of heat maps as an analysis tool and combine them with the use of the MRTS curves, based on [6] and [7], to determine the appropriate value of FR capacity that the system needs, and the combination of fast and slow resources that can yield the same performance of the system at least cost, while avoiding over-procurement of FR resources. It is important to mention that the methodology proposed in this paper is generic, and thus can be used by other Independent System Operators (ISOs) with appropriate modifications and FR data.

The rest of this paper is organized as follows: Section II presents various definitions relevant to the presented work. The criteria and methodology used for generating MRTS curves for the IESO is presented in Section IV. Finally, Section V highlights the main contributions and conclusions of the presented work.

II. DEFINITIONS

The MRTS measures the reduction in one of the inputs vis-a-vis the increase in another input that is sufficient to maintain a constant level of output [8]. In the current paper, the MRTS curve represents the *RegA* capacity that must be substituted by one more unit of *RegD* while maintaining the same output level or performance metric selected, such as the Root Mean Squared Error (RMSE) of the Area Control Error (ACE). Thus, these MRTS curves help determining the *RegA* capacity in MW provided by conventional generators that can be replaced with *RegD* capacity from ESSs, so as to reduce the total FR capacity used, which should decrease the cost of procuring FR services for ISOs.

An isoquant is a curve in the input space that shows all possible combinations of inputs which produce the same level of output, and a family of isoquants is an isoquant map, as shown in Fig. 1, where Is_1 , Is_2 , and Is_3 are isoquants which

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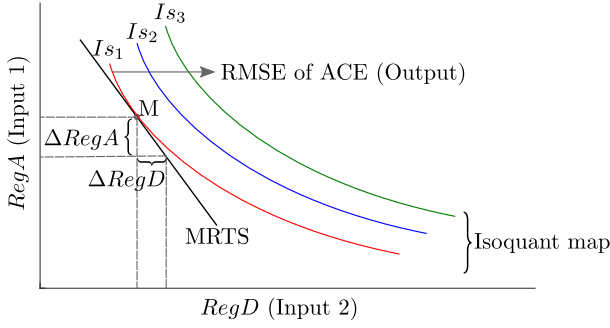


Fig. 1. Isoquant map.

together form an isoquant map. The higher the position of an isoquant, with respect to another, the higher its level of output [8], [9]; for example, in Fig. 1, Is_3 has a higher output value than Is_2 and Is_1 , while Is_2 has higher output than Is_1 .

The MRTS of $RegA$ (Input 1) for $RegD$ (Input 2) in Fig. 1, at any point on the isoquant, is denoted by $MRTS_{RegD, RegA}$, and can be defined as the negative of the slope of the isoquant at that point. Also, defining the marginal product of $RegD$ as $MP_{RegD} = \frac{\partial Is_1}{\partial RegD}$ and the marginal product of $RegA$ as $MP_{RegA} = \frac{\partial Is_1}{\partial RegA}$, for any point on an isoquant Is_1 (e.g., M), maintains the output RMSE of ACE unchanged, i.e., $\Delta Is_1 = 0$. Hence, by linearizing Is_1 , it follows that [9]:

$$\begin{aligned} \Delta Is_1 &\approx MP_{RegD} \Delta RegD + MP_{RegA} \Delta RegA = 0 \\ \Rightarrow MP_{RegD} \Delta RegD &= -MP_{RegA} \Delta RegA \end{aligned} \quad (1)$$

Thus, the slope of the isoquant can be computed as follows:

$$\left. \frac{\Delta RegA}{\Delta RegD} \right|_{\Delta Is_1=0} = -\frac{MP_{RegD}}{MP_{RegA}} \quad (2)$$

And, since $MRTS_{RegD, RegA}$ is the negative of the slope of the isoquant, the following can be obtained:

$$MRTS_{RegD, RegA} = -\left. \frac{\Delta RegA}{\Delta RegD} \right|_{\Delta Is_1=0} = \frac{MP_{RegD}}{MP_{RegA}} \quad (3)$$

III. INTEGRATED MODEL

The Integrated Model used in this paper to obtain the heat maps and MRTS curves for the Ontario Power System (OPS) is illustrated in Fig. 2 [10], representing a large interconnected power system with ESS, particularly Flywheel Energy Storage System (FESS) and Battery Energy Storage System (BESS). It considers all relevant stages in the FR control process, including the State of Charge (SoC) management model of the ESSs and Communication Delays (CDs) in the signal transmission from/to the control center to/from the facilities contracted for FR. The CDs are represented as orange blocks in the figure.

The first block in the Integrated Model is the ACE calculation, whose input signals are the actual NI_a and scheduled NI_s net interchanges, the actual f_a and scheduled f_s frequencies, the inadvertent payback IP , and the interchange metering error IME . The output signal of this block is ACE, which feeds the Filter Φ that forces the ACE back to zero

through an optimal coordinated control between traditional and fast frequency response resources, thus optimizing the use of these resources while considering the SoC of fast resources and capacity limits of ESSs and Traditional Generators (TGs), while considering the conditions of the system. The first output signal of the Filter Φ is the traditional regulation signal limited $RegA_L$, which is an input to the aggregated model of the TGs contracted for regulation. The output of this block is the power of TG contracted for FR P_{TGr} , which together with the load of the system P_D , the generation total dispatch P_{GT} , and the power outputs of FESS P_{FESS} and BESS P_{BESS} are sent to the system block. The latter represents the primary frequency response of the elements in the system and the power in the tie-lines, which determine the signals NI_a and f_a , and are then send back to the ACE calculation block to close the FR control loop.

The second output of the Filter Φ is the dynamic regulation signal limited $RegD_L$. This signal is multiplied by a factor α_{BESS} for BESS or α_{FESS} for FESS, which indicate their respective capacity contributions to the total ESS FR capacity. The result of the product $RegD_L \alpha_{BESS}$ is sent to the Set-Point (SP) calculation block, together with the BESS FR capacity limit RC_{BESS} , the status availability of BESS AV_{BESS} , the BESS moving basepoint $BP_{m_{BESS}}$, and the maximum $\bar{M}_{BESS}$ and $\underline{M}_{BESS}$ minimum available capacity of BESS. The SP calculation block for the BESS guarantees that its FR capacity is comparable to the TGs FR capacity, and its output signal SP_{BESS} is essentially a scaled version of the $RegD_L$ signal, taking into account the SoC of the BESS. The SP_{BESS} signal is then sent to the BESS facility. The same applies for the FESS SP calculation block.

The last blocks in the Integrated Model correspond to the BESS and FESS models. These models include the SoC management model of the facilities, yielding for the BESS P_{BESS} , $SOCBESS$, $\bar{M}_{BESS}$, $\underline{M}_{BESS}$, AV_{BESS} , and $BP_{m_{BESS}}$, and similarly for the FESS. These last four signals are then fed back to the SP calculation block considering CDs.

IV. MRTS CURVES

A. Criteria

The IESO has been considering introduction of fast resources in its FR process since 2012. Therefore, they are interested in determining the benefit of adding fast resources to the FR process, and analyzing the trade-off resources that yield the same performance. The former can be achieved by using heat maps, while the latter can be addressed by using MRTS curves. For this purpose, the following criteria for generating heat maps and MRTS curves have been agreed upon with the IESO:

- The range of total FR capacity ($RegA + RegD$) considered is large, between 40 and 300 MW.
- Incremental steps of 5 MW of fast and slow FR capacity are considered when determining the $RegA$ - $RegD$ pairs of equivalent control.
- The performance metric selected for generation of heat maps is the ACE RMSE.

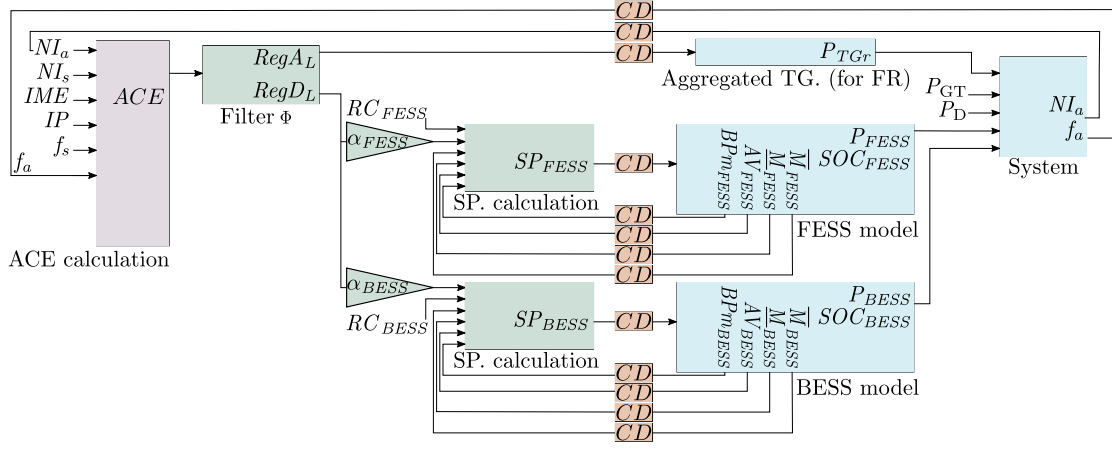


Fig. 2. Integrated Model [10].

TABLE I
SCENARIOS CONSIDERED FOR THE GENERATION OF THE IESO'S MRTS CURVES.

Scenarios	Summer	Fall	Winter	Spring
Peak hours	6 am to 10 pm			
Non-peak hours	10 pm to 6 am			
Morning ramp	5 am to 8 am			
Evening ramp	6 pm to 9 pm	8 pm to 11 pm		

TABLE II
SELECTED REPRESENTATIVE DAYS FOR GENERATING THE MRTS CURVES

Day	Month	Season	Year
3 (Wednesday)	4 (April)	Spring	2019
15 (Thursday)	8 (August)	Summer	2019
24 (Tuesday)	9 (September)	Fall	2019
4 (Tuesday)	2 (February)	Winter	2020

- The ACE RMSE obtained for a 100 MW $RegA$ and 0 MW $RegD$ is used to generate the isoquant, which is consistent with the current FR service scheduled by the IESO [11].
- The following three features are considered in the MRTS curve generation for different ESS technologies and discharging times, scenarios, and seasons:
 - FESSs and BESSs are considered for the studies, one at a time, since their SoC management is different. An FESS with discharging time of 15 min and a BESSs with discharging times of 15 min, 1 h, 2 h and 4 h have been selected for this study.
 - The scenarios considered for each representative day are peak hours, non-peak hours, morning ramp hours, and evening ramp hours, as per Table I.
 - One representative typical day is selected per season, which is a non-holiday weekday.

Thus, 16 MRTS curves can be generated per ESS technology and discharging time, yielding a total of 80 MRTS curves.

- A multi-segment approach is used when generating the MRTS curves.

Although this paper does not focus on electricity markets, it is worth mentioning that the IESO can use the MRTS curves together with pricing data to determine the optimal combination of $RegD$ and $RegA$ resources to meet the system FR requirement, while maintaining the same level of FR performance at minimum cost. Thus, when solving for the opti-

imum combination of resources, $RegD$ would substitute $RegA$ when $RegD$ is cheaper. The $RegD$ resources determined using MRTS curves can be assigned a base payment and a mileage payment component, to provide a fair compensation for the service actually provided by fast resources. This analysis is left as future research stemming from the work presented in this paper. The value of $RegD$ capacities should also be kept in mind when planning for potential increments in FR capacity.

The data used in this paper was provided by the IESO, and corresponds to March 1, 2019 to February 28, 2020, re-sampled to 1 s resolution. The following is the methodology used to select the representative days per season that are used to generate the MRTS curves:

- 1) Consider the load curve data for all the days within each season.
- 2) Find the mean of the load curves for the season.
- 3) Calculate the Mean Absolute Error (MAE) between the load curve of each day and the mean load curve for a season.
- 4) Select the day with the lowest MAE with respect to the mean load curve.
- 5) Verify that the selected days are weekdays and not holidays; if a holiday or weekend, select a day with the next lowest MAE.

Table II presents the selected representative days for all seasons after applying this methodology to each season.

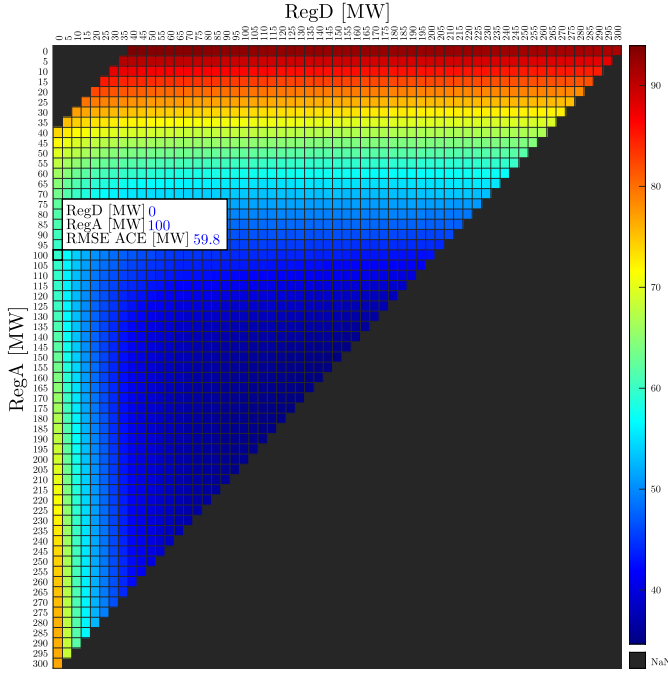


Fig. 3. Heat map for fall season, peak hours scenario, and 4 h BESS.

B. Methodology

The method proposed here to develop the MRTS curves for the OPS is based on the concepts from [6] and [7]. Thus, the first step is to run the Integrated Model presented in [10] considering existing CDs for one ESS technology at a time, varying the FR capacity of fast and slow regulation resources for each scenario and season, for a total of 80 cases. The capacity of *RegA* and *RegD* resources is increased in steps of 5 MW, for a total FR capacity varying in the chosen range of 40 MW to 300 MW. The initial state of the system corresponds to a point where the FR resources are sufficient for the interval of analysis, thus allowing *RegD* and *RegA* trading. The initial ACE for all the simulations within the same case is selected as the initial ACE for the corresponding representative day at 100 MW FR requirement, i.e., *RegA* = 100 MW and *RegD* = 0 MW. The RMSE of the ACE for the interval of analysis is the performance metric, which is calculated, saved and used to generate heat maps considering one day of data with 1 s resolution.

1) *Heat Maps*: Heat maps allow visualizing the value of the performance metric in color, on a two-dimensional plot of *RegA* versus *RegD*. These heat maps contain information on *RegA* and *RegD* capacity, and the value of the RMSE of the ACE, as illustrated in Fig. 3 for one particular case; the rest of the heat maps for all cases analyzed in this work are presented in [12]. These heat maps allow evaluating the impact of different combinations of *RegA* and *RegD* capacities on the RMSE of the ACE, with the darkest blue indicating a mix of fast and slow resources that yields the lowest RMSE of the ACE, while the darkest red indicates the opposite. Note that the ideal combination of *RegA* and *RegD* capacities, which yields

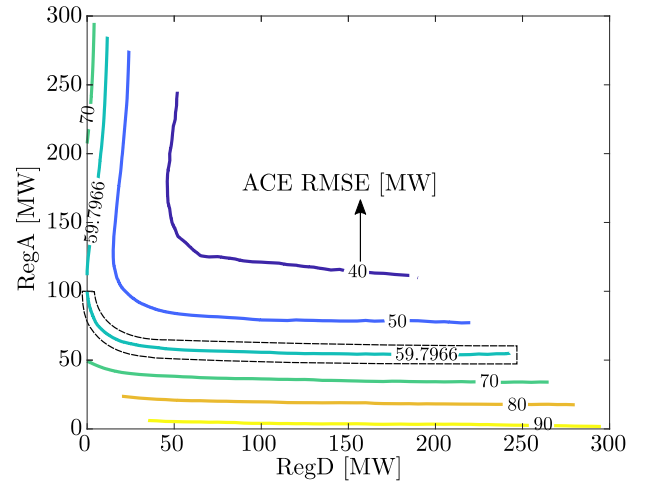


Fig. 4. Example of an isoquant map for different values of ACE RMSE.

the lowest RMSE of the ACE, may not be feasible because of high cost of procurement. A comprehensive analysis of all heat maps for different ESS technologies and discharging times, scenarios and seasons, allows selecting the optimal combination of FR resources that results in the most benefit to the system, taking into account the procurement cost and capacity limits of *RegA* and *RegD* resources.

Generating each heat map, e.g., Fig. 3, requires 1802 simulations of the Integrated Model considering the scenarios described in Section IV-A, which translate in computational times of approximately 3 to 4 h for morning and evening ramp scenarios, and up to about 10 h for peak hour scenarios. A computer with a processor Intel(R) Core(TM) i9-9900K CPU @3.60 Hz, 32 GB RAM, and 64-bit operating system was used to generate the heat maps.

In the heat maps, as the one presented in Fig. 3, it can be observed that for a given FR capacity, when *RegD* capacity is increased, it results in an improvement in the performance metric; however, beyond a certain value, the performance deteriorates, which translates into reduced returns from using more *RegD* resources. The performance deterioration beyond a certain value of *RegD* is explained by the fact that these resources are typically energy limited (such as FESS and BESS), and thus will reach their SoC limits, resulting in increased ACE; hence, energy limited *RegD* resources need to be complemented with *RegA* resources [6]. Heat maps can be a powerful analysis tool for ISOs looking to expand their FR capacity. Once the selection of performance metric and combination of FR resources is determined, the MRTS curves can be created, as discussed next.

2) *MRTS Curves*: After generating the heat maps for all cases, the incremental step in the meshgrid data corresponding to *RegA* and *RegD* capacity is decreased from 5 MW to 0.02 MW, using interpolation of two-dimensional data in meshgrid format to obtain interpolated values of ACE RMSE; the purpose of this exercise is to increase the number of data points to generate a more accurate isoquant map. The isoquant

map can then be generated from the interpolated data, as illustrated in Fig. 4, which includes the isoquant corresponding to the value of ACE RMSE of 59.7966 MW (obtained for $RegA = 100$ MW and $RegD = 0$ MW highlighted in Fig. 3). The isoquant determines the $RegA$ - $RegD$ pairs of equivalent control, i.e., equivalent ACE RMSE. When an isoquant is split in two parts, only the part that allows analyzing the impact of reducing $RegA$ resources and replacing them by $RegD$ resources on ACE RMSE is considered; this part of the curve is enclosed in Fig. 4 for the ACE RMSE of 59.7966 MW.

A multi-segment approach is used to generate the MRTS curves; the single-segment approach was not considered to avoid the same issues that PJM had in its implementation, wherein the BF curve resulted in over-procurement of fast FR resources [2], [3]. Thus, 6 $RegD$ MW-intervals are considered with cuts at 5, 10, 25, 50 and 75 MW on the $RegD$ axis, producing the following intervals: $[0\ 5]$, $[5\ 10]$, $[10\ 25]$, $[25\ 50]$, $[50\ 75]$, and $[75\ Hv1]$, where $Hv1$ is the highest value of $RegD$ for the isoquant, as illustrated in Fig. 5, where each segment of the isoquant corresponding to a different $RegD$ MW-interval is denoted with a different color. For some isoquants, the value of $Hv1$ may be less than 75 MW, which will reduce the total number of $RegD$ MW-intervals.

Each segment of the isoquant may be approximated by a quadratic function, as shown in Fig. 5. In order to determine the coefficients of these functions, a constrained linear least-squares problem can be formulated for each segment u as follows:

$$\min_{p_1} \frac{1}{2} \|C_1 p_1 - d_1\|_2^2 \quad (4a)$$

$$\text{s.t.} \quad Aeq_1 p_1 = Beq_1 \quad (4b)$$

$$\min_{p_u} \frac{1}{2} \|C_u p_u - d_u\|_2^2 \quad \forall (u > 1) \wedge (u < T) \quad (5a)$$

$$\text{s.t.} \quad Aeq_u p_u = Beq_u \quad \forall (u > 1) \wedge (u < T), \quad (5b)$$

$$A_u p_u \leq B_u \quad \forall (u > 1) \wedge (u < T) \quad (5c)$$

$$\min_{p_T} \frac{1}{2} \|C_T p_T - d_T\|_2^2 \quad (6a)$$

$$\text{s.t.} \quad Aeq_T p_T = Beq_T, \quad (6b)$$

$$A_T p_T \leq B_T \quad (6c)$$

where

$$C_u = \begin{bmatrix} x_{2,u}^2 & x_{2,u} & 1 \\ \vdots & \vdots & \vdots \\ x_{r,u}^2 & x_{r,u} & 1 \\ \vdots & \vdots & \vdots \\ x_{N_u-1,u}^2 & x_{N_u-1,u} & 1 \end{bmatrix} \quad \forall u \quad (7)$$

$$p_u = \begin{bmatrix} a_u \\ b_u \\ c_u \end{bmatrix} \quad \forall u \quad (8)$$

$$d_u = \begin{bmatrix} y_{2,u} \\ \vdots \\ y_{r,u} \\ \vdots \\ y_{N_u-1,u} \end{bmatrix} \quad \forall u \quad (9)$$

$$Aeq_1 = \begin{bmatrix} x_{N_1,1}^2 & x_{N_1,1} & 1 \\ x_{1,2}^2 & x_{1,2} & 1 \\ x_{1,1}^2 & x_{1,1} & 1 \end{bmatrix} \quad (10)$$

$$Aeq_u = \begin{bmatrix} x_{N_u,u}^2 & x_{N_u,u} & 1 \\ x_{1,u+1}^2 & x_{1,u+1} & 1 \\ 2x_{1,u} & 1 & 0 \end{bmatrix} \quad \forall (u > 1) \wedge (u < T) \quad (11)$$

$$Aeq_T = \begin{bmatrix} x_{N_T,T}^2 & x_{N_T,T} & 1 \\ 2x_{1,T} & 1 & 0 \end{bmatrix} \quad (12)$$

$$Beq_1 = \begin{bmatrix} y_{N_1,1} \\ y_{1,2} \\ y_{1,1} \end{bmatrix} \quad (13)$$

$$Beq_u = \begin{bmatrix} y_{N_u,u} \\ y_{1,u+1} \\ 2a_{u-1}x_{1,u} + b_{u-1} \end{bmatrix} \quad \forall (u > 1) \wedge (u < T) \quad (14)$$

$$Beq_T = \begin{bmatrix} y_{N_T,T} \\ 2a_{T-1}x_{1,T} + b_{T-1} \end{bmatrix} \quad (15)$$

$$A_u = \begin{bmatrix} 2x_{1,u} & 1 & 0 \\ 2x_{N_u,u} & 1 & 0 \\ 2x_{N_u,u} - 2x_{1,u} & 0 & 0 \end{bmatrix} \quad \forall u > 1 \quad (16)$$

$$B_u = \begin{bmatrix} 2a_{u-1}x_{1,u} + b_{u-1} \\ 2a_{u-1}x_{1,u} + b_{u-1} \\ 0 \end{bmatrix} \quad \forall u > 1 \quad (17)$$

In this formulation, each isoquant data point is defined in the $RegD$ - $RegA$ space, given by $(x_{r,u}, y_{r,u})$, where u is the segment in a $RegD$ MW-interval in the isoquant, r is the isoquant data point in a segment u , and $x_{r,u}$ and $y_{r,u}$ are the values of $RegD$ and $RegA$, respectively, corresponding to the r isoquant point in segment u . The parameters T and N_u are the total number of segments in the isoquant of analysis and the total number of isoquant data points in the segment u , respectively. The vector p_u , as per (8), contains the coefficients a_u , b_u , and c_u of the quadratic function approximating the segment u at the isoquant.

The first equality constraints in (4b), (5b), and (6b) fix the last isoquant point in segment u ($r = N_u$). The second equality constraints in (4b) and (5b) ensure that the segments approximating the isoquant are connected, by considering the first point ($r = 1$) of the segment $u + 1$. The third equality constraint in (4b) fixes the first isoquant point for the segment $u = 1$. The last equality constraints in (5b) and (6b) ensure that the derivative of the quadratic function approximating the segment u evaluated at its first point is equal to the derivative of the quadratic function approximating the segment $u - 1$.

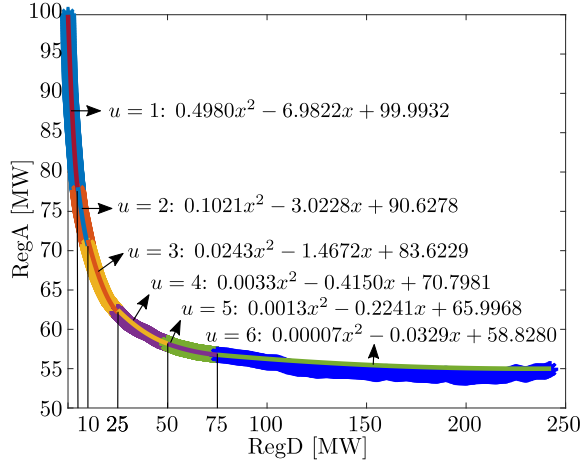


Fig. 5. Example of an isoquant divided in six segments and their corresponding quadratic approximations.

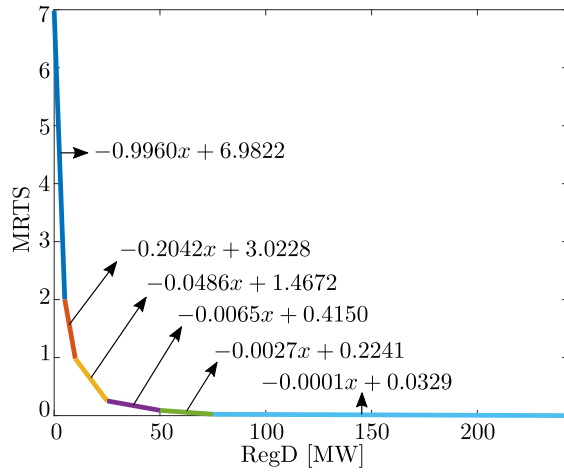


Fig. 6. Example of MRTS curve formed by the negative of the derivatives of the approximate quadratic functions.

evaluated at its last point, which is equal to the first point of the segment u . This is to ensure that the segments on the MRTS curve are connected to each other.

The first two inequality constraints in (5c) and (6c) ensure that the derivative of the quadratic function approximating the segment u , evaluated at its first ($r = 1$) and last ($r = N_u$) points, are lower or equal to the derivative of the quadratic function approximating the segment $u - 1$ evaluated at its last point, which is equal to the first point of the segment u . The third inequality constraints in (5c) and (6c) ensure that the derivative of the quadratic function approximating the segment u , evaluated at its last point, minus the same derivative evaluated at its first point, is lower or equal to zero, which ensures that the segments of the MRTS have a negative slope, as desired.

Each MRTS curve is formed by the negative of the derivatives of the quadratic functions approximating the segments in the isoquant of interest (see Fig. 6). The parameters for all the linear functions corresponding to the MRTS curves for the cases studied in this paper are presented in [12].

The formulation in (4) to (6) is applied to each segment u in the isoquant for all the 80 cases considered in this work, for different ESS technologies and discharging times, scenarios, and seasons. However, having 80 MRTS curves is not practical; thus, several comparisons were then carried out to determine similarities among the MRTS curves that allow establishing a criterion to reduce the number of curves. The comparisons, which are presented in [12], correspond to peak vs non-peak hour scenarios per season for different ESS technologies and discharging times; morning vs evening ramp scenarios per season for different ESS technologies and discharging times; and different ESS technologies and discharging times per season for all scenarios. When the curves are compared considering the latter criteria, the highest similarities are observed; hence, average MRTS curves can be obtained in this case for each scenario and season, resulting in a total of 16 average MRTS curves. To obtain these curves, the average of each segment of all the MRTS curves that belong to the same scenario and season, for different ESS technologies and discharging times, is calculated (see black line in Fig. 7). However, the average segments are not necessarily connected to each other, and the maximum value of $RegD$ capacity considered may not be the same for all the curves, resulting in a discontinuity at the last segment, as shown in Fig. 7. To correct this problem, the average for each segment is used in the formulation of the following constrained linear least-squares problem:

$$\min_{\hat{p}_1} \frac{1}{2} \|\hat{C}_1 \hat{p}_1 - \hat{d}_1\|_2^2 \quad (18a)$$

$$\text{s.t.} \quad \hat{A}eq_1 \hat{p}_1 = \hat{B}eq_1 \quad (18b)$$

$$\min_{\hat{p}_v} \frac{1}{2} \|\hat{C}_v \hat{p}_v - \hat{d}_v\|_2^2 \quad \forall (v > 1) \wedge (v < L) \quad (19a)$$

$$\text{s.t.} \quad \hat{A}eq_v \hat{p}_v = \hat{B}eq_v \quad \forall (v > 1) \wedge (v < L) \quad (19b)$$

where

$$\hat{C}_v = \begin{bmatrix} x_{2,v} & 1 \\ \vdots & \vdots \\ x_{q,v} & 1 \\ \vdots & \vdots \\ x_{O_v-1,v} & 1 \end{bmatrix} \quad \forall v \quad (20)$$

$$\hat{p}_v = \begin{bmatrix} \hat{a}_v \\ \hat{b}_v \end{bmatrix} \quad \forall v \quad (21)$$

$$\hat{d}_v = \begin{bmatrix} y_{2,v} \\ \vdots \\ y_{q,v} \\ \vdots \\ y_{O_v-1,v} \end{bmatrix} \quad \forall v \quad (22)$$

$$\hat{A}eq_v = \begin{bmatrix} x_{O_v,v} & 1 \\ x_{1,v} & 1 \end{bmatrix} \quad \forall v \quad (23)$$

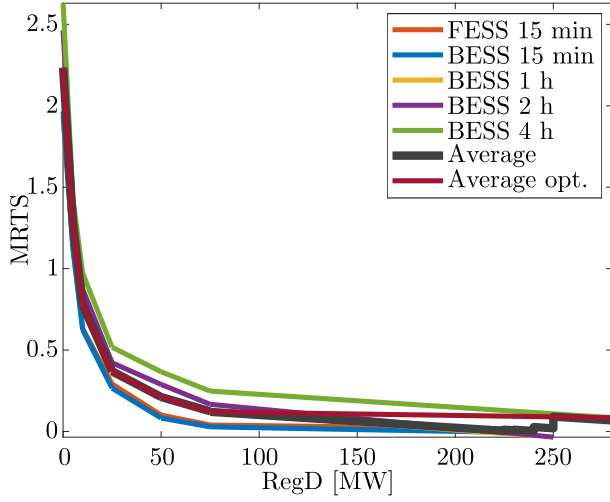


Fig. 7. MRTS curves for different ESS technologies and discharging times, for winter season and peak hours scenario, plus their average MRTS curve and average optimized MRTS curve.

$$\hat{Beq}_1 = \begin{bmatrix} y_{O_1,1} \\ y_{1,1} \end{bmatrix} \quad (24)$$

$$\hat{Beq}_v = \begin{bmatrix} y_{O_v,v} \\ \hat{a}_{v-1}x_{1,v} + \hat{b}_{v-1} \end{bmatrix} \quad \forall (v > 1) \wedge (v < L) \quad (25)$$

v is a segment of the average MRTS curve, q corresponds to the points in each average MRTS curve for segment v , and $x_{q,v}$ and $y_{q,v}$ are the values of $RegD$ and MRTS, respectively, for a q point in segment v . The parameters L and O_v denote the total number of segments v in the average MRTS curve and the total number of average MRTS curve data points in the segment v , respectively. The vector $\hat{p}_v$, as per (21), contains the coefficients $\hat{a}_v$, and $\hat{b}_v$ of the linear function approximating the segment v in the average MRTS curve.

The first equality constraints in (18b) and (19b) fix the last average MRTS curve point for the segment v ($q = m_v$). The second equality constraint in (18b) ensures that the first average MRTS curve data point of segment v ($q = 1$) is a fixed value. The second equality constraint in (19b) ensures that all the segments within the average MRTS curves are connected, by making the linear function approximating the segment v evaluated at its first point equal to the value of the linear function approximating the segment $v - 1$ evaluated at its last point, which is equal to the first point of segment v .

The parameters of the linear equation for each segment of the 16 average optimized MRTS curves are presented in Table III. In this table, the 6 segments of the average optimized MRTS curve correspond to the intervals $[0 \ 5]$, $[5 \ 10]$, $[10 \ 25]$, $[25 \ 50]$, $[50 \ 75]$, and $[75 \ Hv2]$, where $Hv2$ is the highest value of $RegD$ for the average optimized MRTS curve. Fig. 8 presents all the average optimized MRTS curves.

The MRTS curves in Fig. 8 help determine the $RegA$ MW capacity that can be replaced by $RegD$ MW capacity to produce the same performance metric, while decreasing the total FR capacity used. The average optimized MRTS curves have different starting points on the y-axis, which means

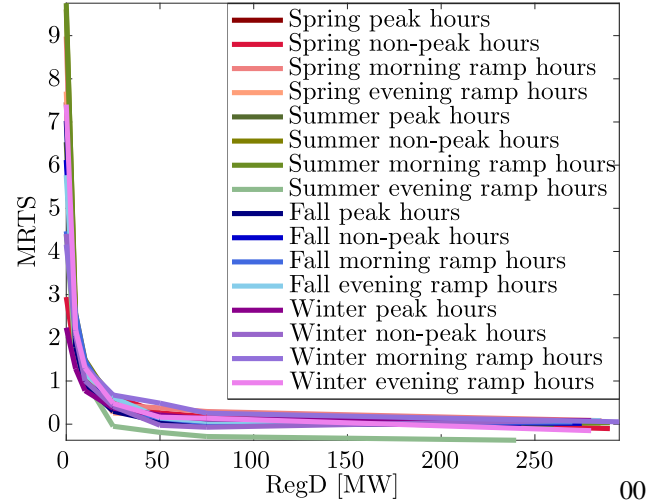


Fig. 8. Average optimized MRTS curves for all seasons and scenarios. MW to 84.05 MW; note that the 33.6 MW of $RegA$ capacity is the area under the curve, calculated using data from Table III.

Fig. 8. Average optimized MRTS curves for all seasons and scenarios.

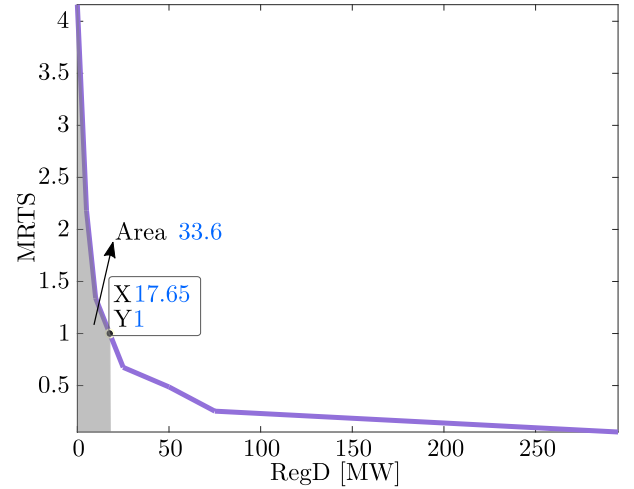


Fig. 9. MRTS average optimized curve interpretation for winter morning ramp hours.

that for different seasons and scenarios, different amounts of $RegA$ capacity can be replaced by one MW of $RegD$. Considering that FR provided by fast resources is likely to be more expensive, it is reasonable to consider $MRTS = 1$, as the minimum value at which it would make sense to replace $RegA$ resources. Indeed, $MRTS = 1$ indicates that substituting $RegA$ by $RegD$ would yield the same system performance. For example, in Fig. 9, it can be observed that 17.65 MW of $RegD$ capacity can replace 33.6 MW of $RegA$ capacity, while still maintaining the same ACE RMSE, thereby reducing the required FR capacity from 1

The criteria and methodology presented here can be modified for its application in other systems. Depending on the FR capacity considered for analysis, the range of total FR capacity can be reduced or increased, which may call for

TABLE III
PARAMETERS OF AVERAGE OPTIMIZED MRTS CURVES.

Scenarios	Segments v	Spring		Summer		Fall		Winter	
		$2\hat{a}$	$\hat{b}$	$2\hat{a}$	$\hat{b}$	$2\hat{a}$	$\hat{b}$	$2\hat{a}$	$\hat{b}$
Peak Hours	1	-0.9948	7.4573	-0.9621	6.5317	-1.0477	7.025	-0.1901	2.2347
	2	-0.2326	3.6458	-0.1411	2.4266	-0.175	2.6615	-0.1015	1.7921
	3	-0.0563	1.883	-0.0422	1.4375	-0.0409	1.3209	-0.0271	1.0478
	4	-0.0119	0.7736	-0.0108	0.653	-0.0095	0.5358	-0.0063	0.5286
	5	-0.0023	0.2955	-0.0017	0.1959	-0.0023	0.1764	-0.0036	0.3933
	6	-0.0006	0.1624	-0.0007	0.1251	0	0.0046	-0.0002	0.1334
Non-peak Hours	1	-0.2796	2.9446	-1.4117	9.6077	-0.7961	6.1186	-0.4735	4.4091
	2	-0.1078	2.0858	-0.2105	3.6014	-0.1764	3.0202	-0.2075	3.0788
	3	-0.0235	1.2426	-0.0668	2.1649	-0.0517	1.7735	-0.0395	1.3985
	4	-0.0158	1.0495	-0.0153	0.8766	-0.0131	0.8082	-0.0175	0.8505
	5	-0.0036	0.4428	-0.0024	0.2331	-0.0029	0.2996	-0.0018	0.0623
	6	-0.0013	0.2667	-0.0001	0.0613	-0.0003	0.1039	0.0008	-0.1313
Morning Ramp Hours	1	-1.383	8.9788	-1.4773	9.7516	-0.3826	4.4678	-0.3955	4.1596
	2	-0.2133	3.1305	-0.1939	3.3346	-0.2162	3.636	-0.1687	3.0258
	3	-0.0367	1.3643	-0.0513	1.9079	-0.072	2.1935	-0.0442	1.7811
	4	-0.0028	0.5182	-0.0213	1.1577	-0.0102	0.6504	-0.0075	0.8631
	5	-0.0033	0.543	-0.0007	0.1306	-0.0043	0.3525	-0.0093	0.9522
	6	-0.001	0.3684	-0.0002	0.0937	0.0001	0.0244	-0.0009	0.3225
Evening Ramp Hours	1	-1.1646	7.7037	-0.5701	4.3952	-0.7214	5.7682	-1.0458	7.4031
	2	-0.1251	2.5063	-0.1038	2.0638	-0.199	3.1561	-0.1746	3.0471
	3	-0.0665	1.9203	-0.0716	1.7421	-0.0364	1.5304	-0.0546	1.8473
	4	-0.002	0.3076	-0.0057	0.0932	-0.0189	1.091	-0.0128	0.8026
	5	-0.0081	0.6109	-0.0038	0.0001	-0.0034	0.3162	-0.001	0.2112
	6	0.0002	-0.0106	-0.0005	0.2471	0.0001	0.0566	-0.0014	0.2418

more or less detail in the analysis; hence, the incremental step should be reduced or increased, accordingly. The RMSE is used in this study as the performance metric, but it can be replaced by other metrics that provide an adequate measure of the ACE performance. The value of FR capacity used to generate the isoquant will likely vary for different systems based on their current FR capacity. In addition, other types of ESS technologies and scenarios can be considered, and the intervals of analysis in the multi-segment approach may vary depending on the values of the isoquant for other systems.

V. CONCLUSIONS

This paper presented a detailed methodology to obtain heat maps and MRTS curves for a bulk power system. While the proposed methodology was demonstrated for the OPS, it is generic and could be applied to other ISOs with appropriate modifications. The process and criteria used to obtain 16 average optimized MRTS curves from 80 cases, as per IESO requirements, was also detailed. The proposed heat maps and MRTS curves are analysis tools to allow ISOs to select the desired performance metric and the combination of slow *RegA* and fast *RegD* FR resources to achieve it, while still reducing the total required FR capacity.

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