

An Affine Arithmetic-Based Energy Management System for Cooperative Multi-Microgrid Networks

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Abstract—This paper presents an Energy Management System (EMS) for a Multi-Microgrid (MMG) system that considers power exchanges between a set of interconnected microgrids (MGs) in an Active Distribution Network (ADN), taking into account electricity demand and renewable energy generation uncertainties using an Affine Arithmetic (AA) approach. The deterministic EMS model is formulated as a cost minimization problem which includes detailed operational constraints of thermal generators and Energy Storage Systems (ESSs) within each MG, as well as power flow limits at the Point of Common Coupling (PCC), considering all power exchanges among the set of MGs and the ADN. The uncertainties are formulated in the AA domain to obtain an EMS model that is robust for a range of realizations of the uncertain parameters, with no need of statistical assumptions or repeated calculations, which can be solved with relatively low computational burden, as opposed to other approaches such as Monte Carlo Simulation (MCS). The proposed AA model is then tested and validated with data of a set of MGs in an ADN located in São Paulo, Brazil, through comparisons with the deterministic model, MCS, and a Two-Stage Stochastic Programming (TSSP) approach. Results show an execution time improvement in the AA model of approximately 70% when compared to a MCS approach, which is expected to be slower, while considering the same range of uncertainties. Furthermore, the operation cost of the overall system decreases, as expected, by approximately 63% when power exchanges are enabled, as opposed to the individual operation of each MG, demonstrating the economic benefit of MMG systems.

Index Terms—Active Distribution Networks, Affine Arithmetic, Energy Management System, multi-microgrids, uncertainty.

NOMENCLATURE

Sets

G	Sets of generators in each microgrid.
M	Set of microgrids.
S	Sets of ESS units in each microgrid.
T	Set of hours.

Indices

0	Center value of affine forms.
g	Index for thermal generators.
h	Partial deviation of affine forms.
m, n	Indices for microgrids.
s	Index for ESS units.
t	Index for operation hour.

Parameters

β	Power transfer cost coefficient [\$/kWh].
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Δt

$\hat{\cdot}$

$v_{m,g}^0$

w

η^c, η^d

ρ^{gs}, ρ^{gb}

$\underline{P}^c, \overline{P}^c$

$\underline{P}^d, \overline{P}^d$

$\underline{P}, \overline{P}$

$\underline{SoC}, \overline{SoC}$

$\varepsilon^D, \varepsilon^r$

b

c

C^{SD}, C^{SU}

D

DT, UT

DT^0, UT^0

L^D, L^U

p^D, p^r

P^{PCC}

P^r

RD, RU

Variables

SoC

P

P^c, P^d

P^e

Duration of periods in the optimization time span [h].

Affine form.

Initial status of thermal units.

Objective function weighting factor.

Charging and discharging efficiency of ESS units.

Grid selling and buying prices [\$/kWh].

Minimum and maximum charging power of ESS units [kW].

Minimum and maximum discharging power of ESS units [kW].

Minimum and maximum power output of thermal units [kW].

Minimum and maximum SoC of ESS units [kWh].

Noise symbols associated with demand and renewable generation uncertainties.

Coefficient of the linear term of generation cost functions [\$/kWh].

Coefficient of the constant term of generation cost functions [\$/h].

Shutdown and startup costs of thermal units [\$/h].

Power demand [kW].

Minimum down and up times of thermal units [h].

Number of periods that thermal units have been down and up prior to the first period of the optimization time span.

Number of periods that thermal units must be initially down and up due to their minimum down and up time constraints.

Number of noise symbols in the affine forms of demand and renewable generation.

Maximum power transfer capacity through the point of common coupling [kW].

Power generation from renewable energy sources [kW].

Ramp down and ramp up limits of thermal units [kW/h].

State of charge of ESS units [kWh].

Power output of thermal units [kW].

Charging and discharging power of ESS units [kW].

Power exchange between microgrids [kW].

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P^{gs}, P^{gb}	Power sold and bought by the main grid [kW].
SD, SU	Shutdown and startup decisions of thermal units.
v	Commitment status of thermal units (1 if the unit is up, 0 otherwise).
z^c, z^d	Charging and discharging status of ESS units.

I. INTRODUCTION

ENVIRONMENTAL concerns have motivated a gradual transformation of power systems, by replacing traditional fossil fuel-based energy sources, such as coal and natural gas, with renewable energy sources, such as wind and solar energy. Reducing the dependency on conventional power plants may be accomplished by integrating a large number of Distributed Energy Resources (DERs) that supply renewable energy in a reliable and localized manner [1]. However, the large-scale integration of DERs into existing power systems poses several technical challenges that must be addressed to fully exploit the potential of renewable energy.

The formation of microgrids (MGs) is an appealing approach to achieve high penetration of DERs without compromising the reliability of the system, while enhancing system resiliency. An MG is a cluster of DERs and loads operated in coordination to reliably supply electricity, while operating either in isolated or grid-connected modes, connected through the Point of Common Coupling (PCC) to the main power grid at the distribution level, enhancing the system resilience and reducing power transmission utilization and losses [2]. In an MG, the optimal operation is determined by the Energy Management System (EMS), which solves optimization problems such as the Unit Commitment (UC) and/or Economic Dispatch (EC) problems. These traditionally deterministic problems must be reformulated to account for uncertainties associated with load variability and renewable energy intermittence, which are some of the main technical challenges for the integration of DERs.

More recently, the coordinated operation of multiple MGs, connected to form a Multi-Microgrid (MMG) system, has attracted significant attention due to the potential improvement in system performance and mutual economic benefits for all participating MGs, when compared to their separate operation [3]. A comprehensive review on the potential of MMG systems to support the resiliency of the main power grid in the case of extreme events is presented in [4]. The resiliency is enhanced by connecting geographically-close MGs and coordinating their power transfers to ride through extreme events. Other survey papers provide an overview of the distributed control, communication, and topology of MMG systems, presenting models and control techniques such as graph theory, game theory, and consensus protocols [5]–[7].

The existing literature in MMG systems focuses on system planning, voltage and frequency control, communication in power sharing, and optimal energy management [8]. The present work corresponds to the last category, in which ongoing research on the modeling and coordination of the MMG EMS is taking place, considering demand and renewable generation variability, as well as different topological configurations.

The advantages of cooperative operation in MMG systems are explored in [9] and [10], where distributed algorithms based on Model Predictive Control (MPC) are proposed for coordinating the operation of MGs and a central aggregator. Simulation results show that the the daily operational costs of all MGs are reduced, and the computational burden is reduced when compared to a centralized approach. Similarly, an MPC-based model that determines the optimal power exchange scheduling among a set of MGs is proposed in [11], where the objective is to maximize the benefits from power transactions with the main grid and other MGs. A drawback of MPC-based approaches is their sensitivity to errors associated with demand and renewable generation forecasts, which depend on the prediction horizon, the prediction time step, and the quality of predictions. Thus, the correct characterization of prediction uncertainties is critical to attain satisfactory results with MPC techniques, which can be a significant challenge in the case of MMG systems, as MGs are subject to high variability of load and renewable generation.

A classical approach to handle uncertainties in MG scheduling problems is through stochastic formulations. In [12] and [13], uncertainties are represented by a set of scenarios, which are generated from probability distribution functions (pdfs) of the random variables, and two-stage cost minimization problems are formulated, with the first stage optimizing day-ahead energy transactions, while the second stage makes adjustments to the first stage decisions based on realizations of wind and Photovoltaic (PV) generation. In [14], a stochastic-predictive EMS for isolated microgrids is presented, which considers uncertainties through a two-stage decision process combined with a Receding Horizon Control (RHC) Optimal Power Flow (OPF); the first stage solves the UC problem, and the second stage solves the RHC OPF problem. Another stochastic approach that has been used to account for uncertainties in MMG EMS is Monte Carlo Simulation (MCS) [15], which is based on the generation of random samples from pdfs of the uncertain parameters. The optimization problem is solved repeatedly for each one of the random samples, and a statistical analysis is performed on the results. If a large number of solutions is calculated, the average value of the objective function and variables would eventually converge to their expected value. This approach yields accurate results but requires high computational effort [16]. Finally, Stochastic Programming (SP), which focuses on minimizing the expected value of objective function for various scenarios weighted by their corresponding probabilities and subject to the model constraints for each scenario considered, has also been applied to represent uncertainties in MMG EMS [15]. In the latter, a Two-Stage Stochastic Programming (TSSP) approach was also proposed, where the first stage includes integer decision variables only, while the second stage considers only continuous variables. The main challenge with SP-based techniques is to properly select the scenarios based on the probabilities. A drawback of all these stochastic approaches is the need to identify a pdf of the uncertain parameters, which may be a challenge considering that MMG systems are not yet commonplace in power grids.

To address these challenges, alternative methods such as

Robust Optimization (RO) have been proposed, in which the goal is to optimize the worst-case scenario, thus ensuring that the system can operate adequately for any realization of the uncertain parameters. In [17] and [18], RO models for finding the optimal MG scheduling considering uncertainties associated with real-time market price signals, renewable energy sources, and forecasted demand are proposed. Immunity against the worst-case scenario is guaranteed if the uncertainties vary within specified bounds. In [19], an EMS for isolated MGs based on RO is presented, which combines the solution of an UC problem in the first stage, with a detailed OPF in the second stage. The main drawback of RO approaches is that the obtained solutions are highly conservative.

More recently, self-validated computing techniques have been proposed for dealing with uncertainties in power systems. The most simple self-validated computing model is Interval Arithmetic (IA) [20], in which an unknown quantity is represented by an interval. However, IA produces intervals that are much wider than the exact range of the computed function, making it unsuitable for certain applications [21]. Affine Arithmetic (AA) is a different self-validated model that has been proposed as an improvement to IA to overcome this overestimation effect. Thus, an AA-based OPF problem for MGs with uncertain load and generation is presented in [22], which yields intervals that can be used to estimate the necessary reserves that the MG should maintain to ensure that the demand is satisfied under uncertainties. In [23], the authors propose an AA-based EMS for isolated MGs, which can produce appropriate dispatch solutions by computing the noise symbols of the affine forms obtained from the AA UC solution, based on the actual realizations of demand, generation, and available reserves. The proposed AA-based method achieves cost-effective solutions, when compared to a deterministic model, an MPC approach, and stochastic optimization techniques. In [24], an RO and an AA-based optimization are compared to the traditional deterministic optimization, minimizing the maintenance and operation costs of an MG in a 24-hour time frame. Results show that the RO approach yields the most expensive solution, since it considers the worst-case scenario of the MG operation, while the resulting AA forms of the variables provide valuable information about the impact that each source of uncertainty has in the variable, which can be useful to perform sensitivity analyses. In [25], a scheduling model for an MMG system using AA to account for load and renewable generation uncertainty is presented, and simulation results show that the AA model provides less conservative solutions than its IA counterpart; however, the proposed model has several shortcomings, which are significant especially in the context of its practicality. For example, PCC constraints that limit the power transfer capacity of each MG are not considered, and the definition of power exchange variables only considers exchanges between two MGs, i.e., the formulation is not generalized for MMG systems. Furthermore, the model does not consider individual generation units within the MGs, and therefore there are no binary variables related to the generators' status. Only the total power outputs of the MGs are considered, with minimum and maximum limits enforced, meaning that MGs are always

generating, regardless of the exchange costs with the main grid and other MGs. There are no intertemporal constraints, such as ramping and minimum up/down time constraints and those associated with Energy Storage System (ESS) units, which are not considered; thus, ESS energy balance constraints, State of Charge (SoC), and charging/discharging power constraints are not present, with their associated binary variables.

One of the key differences between RO and AA is its formulation. RO is a mathematical framework used to handle uncertainty in optimization problems, focused on finding solutions that are resilient to variations in the input data or model parameters, for the worst-case scenario; this may lead to over-conservative solutions, which can be adjusted by means of a budget of uncertainty that gives control over the total deviation from forecasted values [24]. On the other hand, AA is a technique used to perform calculations with intervals or ranges of values, rather than precise numbers, which has been extended to solve optimization problems [26]. To overcome the over-conservativeness of traditional RO, adaptive RO techniques have been proposed, in which the optimization algorithm is capable of adjusting its decisions based on observed data or feedback [27]. There are limited number of papers in the literature comparing RO and AA approaches, such as [24] and the model of a Compressed Air Energy Storage (CAES) facility in [28]. However, the results in these papers show that the RO model, while presenting lower computational burden compared to AA, provide conservative schedules for an operator to choose depending on the budget of uncertainty, which is an issue in terms of having to select the most "appropriate" value of this parameter. On the other hand, the AA model produces an optimum schedule with the center value and the radius of the power dispatch variables, yielding intervals that properly reflect the forecast error ranges for all schedules, which define the actual dispatch at any point at which the error of the forecast is known, as demonstrated in the current paper, without the need for repeated simulations as in the case of RO with multiple budgets of uncertainty.

Based on the preceding discussion, the main objectives and contributions of this paper are as follows:

- Present a detailed MMG EMS model that considers thermal generation units, renewable generation, and ESS units within each MG, with realistic operational constraints, which is not the case for most existing papers on MMG operation.
- Include constraints that account for power flow limits at the PCC, which realistically consider all power exchanges among the set of MGs and the Active Distribution Network (ADN), which are ignored in the majority of MMG EMS models in the existing technical literature.
- Develop the proposed MMG EMS model in the AA domain, to account for uncertainties in demand and renewable generation, which requires using linearization techniques to overcome the nonlinear components associated with the AA formulation.
- Test and validate the AA model with electricity demand and renewable generation data from an existing ADN in São Paulo, Brazil, and describe the dispatch procedure to determine the optimal power generation, energy storage

and power exchanges from the AA solution, for specific realizations of the uncertain parameters.

It is important to mention that the proposed approach is compared here with respect to MCS and TSSP, as this allows validation of the results. It is not compared against RO models due to the fact that this does not allow to validate the proposed AA technique, and since it has been shown that AA is superior to RO techniques in previous papers [24], [28], given the aforementioned issues of conservative solutions and the choice of the budget of uncertainty.

The rest of the paper is organized as follows: Section II presents a review of concepts and theoretical background relevant to the new model and approach presented here. Section III discusses the deterministic and realistic formulation of the MMG EMS problem, and the corresponding formulation in the AA domain. The description of the test system and discussion of the results are presented in Section IV. Finally, Section V summarizes the main contributions and conclusions of the presented work.

II. BACKGROUND REVIEW

A. Multi-Microgrid Systems

MMG systems should be capable of operating in grid-connected or isolated modes, and, to leverage the benefits of MMGs discussed in Section I, all MGs should be able to exchange power with the main power grid and other MGs, while controlled/coordinated by the Distribution Network Operator (DNO). Two topologies of MMGs systems are usually reported in the literature: (1) a radial topology, in which each MG is connected directly to the main grid, with energy exchanges taking place through the distribution bus, and (2) a mesh topology, in which all MGs are interconnected to each other and also to the main grid directly, to exchange energy and information [8]. The radial topology, shown in Fig. 1, is the most realistic, considering the physical connection of MGs to a distribution system, and is thus selected here to guarantee the practicality of the proposed MMG model. The mesh topology is commonly assumed in the literature, as in [11] and [29], and while it is more general in theory, such configuration is nonexistent nor expected in practice because most if not all distribution systems are radially connected [30], due to their simple structure and economic operation, which facilitate the implementation of protection and voltage compensation schemes [31]. While it is true that meshed networks may accommodate the expected increase of distributed generation in the future [32], existing networks in which MMGs would be implemented are radial [33].

When minimizing operating costs in the MMG EMS problem, the main idea is the maximum utilization of available renewable generation, buying power at low prices and selling power at high prices, and taking advantage of the heterogeneity of load and generation profiles in each MG. These objectives are complementary and closely-related; hence, a common approach is to formulate a multi-objective problem that optimizes a combination of such objectives. The constraints considered largely depend on how the model is formulated. Thus, power and energy balance constraints are ubiquitous, but operating

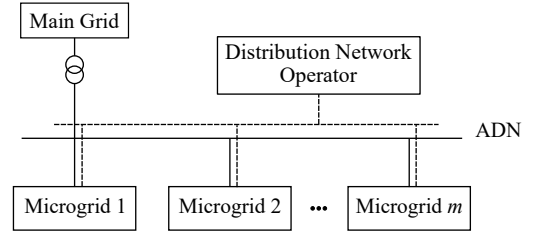


Fig. 1. MMG radial topology.

constraints of the MG components vary greatly, depending on the type of generation considered. In this context, it is crucial to properly take into account the limitation of power transfer capacity through the PCC when considering power exchanges among MGs and the ADN.

B. Affine Arithmetic

AA is a range analysis method suitable for handling multiple uncertainties, such as imprecise data, modeling errors, round-off errors, or truncation errors. It automatically keeps track of rounding or truncation errors for each computed quantity, as well as the correlations between those quantities [21]. In AA, a quantity x subject to uncertainty is represented by an affine form $\hat{x}$, which is a first-degree polynomial, as follows:

$$\hat{x} = x_0 + x_1\varepsilon_1 + \dots + x_p\varepsilon_p = x_0 + \sum_{h=1}^p x_h\varepsilon_h \quad (1)$$

The coefficient x_0 is the *central value* of the affine form, which represents the value that the unknown quantity is most likely to take. The coefficients x_h are the *partial deviations*, which define the magnitude of the corresponding uncertainty component. The variables ε_h are called *noise symbols*, and have unknown values in the interval $[-1,1]$. The number of noise symbols p depends on the affine form, which can use a different number of noise symbols, some of which may be shared with other affine variables. Each noise symbol represents an independent component of the total uncertainty of quantity x , and the corresponding x_h coefficients specify the magnitude of that component. A key feature of the AA model is that the same noise symbol may contribute to the uncertainty of two or more variables, reflecting dependencies among them.

According to the fundamental Invariance Theorem of AA, for every AA operation there is a single assignment of values from the $[-1,1]$ interval into each of the noise symbols that makes the value of every affine form equal to the true value of the corresponding variable [21]. Thus, the true value of quantity x is guaranteed to lie in the interval given by:

$$\hat{x} = [x_0 - \text{rad}(\hat{x}), x_0 + \text{rad}(\hat{x})] \quad (2)$$

$$\text{rad}(\hat{x}) = \sum_{h=1}^p |x_h| \quad (3)$$

Note that $\hat{x}$ yields the smallest interval that contains all possible values of x . This conversion discards any correlation between computed quantities present in their affine forms. The

quantity $\text{rad}(\hat{x})$ is called the *total deviation* or *radius* of the affine form $\hat{x}$, around its center value x_0 .

The first step to perform AA computations is to extend the elementary operations and functions to the affine domain, which can be combined to compute arbitrarily complex functions. For example, consider a generic function of two variables, $f(x, y)$. If the function is linear, the corresponding affine function can be obtained by expanding and rearranging the original noise symbols $\varepsilon_h \forall h \in (1, \dots, p)$ of the affine forms of the variables $\hat{x}$ and $\hat{y}$. However, if $f(x, y)$ is a nonlinear function, the corresponding affine extension cannot be expressed exactly as an affine combination of the original noise symbols; in this case, an affine function that approximates the original function must be identified, and an additional term to account for the approximation error is introduced.

C. Affine Arithmetic Optimization

Based on the previous description of affine extensions, a deterministic optimization problem can be reformulated to account for uncertainties, according to the theoretical framework introduced in [26]. The goal is to solve the following constrained optimization problem in the presence of uncertainties represented as affine forms:

$$\min_{\hat{\mathbf{z}}} \hat{f}(\hat{\mathbf{z}}) \quad (4a)$$

$$\text{s.t.} \quad \hat{g}_j(\hat{\mathbf{z}}) = 0 \quad \forall j \in (1, \dots, n) \quad (4b)$$

$$\hat{h}_k(\hat{\mathbf{z}}) < 0 \quad \forall k \in (1, \dots, m) \quad (4c)$$

where $\hat{\mathbf{z}}$ is the vector of unknown affine variables; $\hat{f}$ is the continuous and differentiable affine objective function; $\hat{g}_j(\hat{\mathbf{z}})$ and $\hat{h}_k(\hat{\mathbf{z}})$ are continuous and differentiable affine functions that represent the j -th equality and k -th inequality constraints, respectively.

To solve (4), the minimization and comparison operators must be extended into the affine domain, as proposed in [26]. Thus, similarity, inequality, and minimization operators are formally defined, which are used here to transform an AA optimization problem into a standard multi-objective nonlinear programming problem.

III. PROBLEM FORMULATION

The MMG system considered in this research is comprised of a set M of MGs, with each MG represented by the index m . All MGs are connected to the main distribution grid, and it is assumed that each MG can exchange power with the ADN and with all other MGs. The power exchanges between MGs take place through the main grid and their corresponding PCCs, as per the most realistic MMG radial topology. The optimization period is defined by the set of hours T , with each hour represented by the index t .

A. Deterministic Model

Each MG has a set of thermal generators G_m , such as diesel units, with each generator represented by index g . The

conventional quadratic cost function of thermal generators is replaced with the following linear cost function:

$$C_{m,g,t}(P_{m,g,t}) = b_{m,g}P_{m,g,t} + c_{m,g}v_{m,g,t} \quad \forall m \in M, g \in G_m, t \in T \quad (5)$$

where $b_{m,g}$ and $c_{m,g}$ are the variable and fixed costs, respectively, $P_{m,g,t}$ is the power output, and $v_{m,g,t}$ is a binary variable indicating the status of each generator. The quadratic cost coefficient of diesel units is usually very small, relative to the variable and fixed costs, and can be neglected without significantly affecting the results [34]; thus, this linear cost function is used to simplify the formulation of the AA optimization problem proposed here. Each MG also has a set of ESS units S_m , with each unit represented by the index s .

Based on the aforementioned assumptions, the objective function can be formulated as follows:

$$\begin{aligned} \min \quad & \sum_{t \in T} \sum_{m \in M} \left[\sum_{g \in G_m} (C_{m,g,t} \Delta t + C_{m,g}^{SD} SD_{m,g,t} \right. \\ & \left. + C_{m,g}^{SU} SU_{m,g,t}) + \sum_{n \in M \setminus m} \beta P_{n,m,t}^e \Delta t \right. \\ & \left. + (\rho_m^{gs} P_{m,t}^{gs} - \rho_m^{gb} P_{m,t}^{gb}) \Delta t \right] \end{aligned} \quad (6)$$

where $C_{m,g,t}$ is the cost function given by (5), Δt is the duration of each period in the optimization time span, assumed to be one hour here, based on existing MG UC models and the AA model complexity and interval characteristics, which allow to consider a larger Δt than other UC models in MGs; $C_{m,g}^{SD}$ and $C_{m,g}^{SU}$ are the shutdown and startup costs, respectively; and $SD_{m,g,t}$ and $SU_{m,g,t}$ are the shutdown and startup decisions, respectively. The term $\beta P_{n,m,t}^e$ represents the cost for the use of the distribution network in each power transfer, which should be paid to the distribution network operator by the receiving MG; β is a scalar chosen as small as possible to reduce the impact of this term in the objective function value, considering that the cost of power transfer is low; $P_{n,m,t}^e$ is the power exchange from MG n to MG m , where $n \in M \setminus m$, i.e., the index n denotes each MG in the set M different than MG m , to account for all possible combinations of power exchanges between MGs. The selling and buying prices of the main grid to and from each MG are denoted by ρ_m^{gs} and ρ_m^{gb} , respectively, and the power sold and bought by the main grid to and from each MG are represented by P_m^{gs} and P_m^{gb} , respectively. It is assumed here that there is no cost for the use of ESSs; however, such costs can be readily added to (6) to reflect ESS degradation [35], without loss of generality.

The supply-demand balance constraint ensures that the total generation in each MG is equal to its total demand at all hours, taking into account the power exchanges between MGs and the main grid, which can be formulated as follows:

$$\begin{aligned} & \sum_{g \in G_m} P_{m,g,t} + P_{m,t}^r + \sum_{s \in S_m} (P_{m,s,t}^d - P_{m,s,t}^c) \\ & + \sum_{n \in M \setminus m} P_{n,m,t}^e + P_{m,t}^{gs} = D_{m,t} + \sum_{n \in M \setminus m} P_{m,n}^e + P_{m,t}^{gb} \end{aligned} \quad \forall m \in M, t \in T \quad (7)$$

where $P_{m,t}^r$ is the renewable generation power and $D_{m,t}$ is the power demand in each MG, and $P_{m,s,t}^d$ and $P_{m,s,t}^c$ are the discharging and charging powers of ESS units, respectively.

The generation limits of thermal units are represented by:

$$v_{m,g,t} \underline{P}_{m,g} \leq P_{m,g,t} \leq \bar{P}_{m,g} v_{m,g,t} \quad \forall m \in M, g \in G_m, t \in T \quad (8)$$

where $\underline{P}_{m,g}$ and $\bar{P}_{m,g}$ are the minimum and maximum power outputs, respectively. Ramping limits of thermal units are necessary in the proposed model, given the one-hour interval used, and are represented by:

$$P_{m,g,t} - P_{m,g,t+1} \leq RD_{m,g} \Delta t + SD_{m,g,t+1} \underline{P}_{m,g} \quad \forall m \in M, g \in G_m, t, t+1 \in T \quad (9)$$

$$P_{m,g,t+1} - P_{m,g,t} \leq RU_{m,g} \Delta t + SU_{m,g,t+1} \underline{P}_{m,g} \quad \forall m \in M, g \in G_m, t, t+1 \in T \quad (10)$$

where $RD_{m,g}$ and $RU_{m,g}$ are the ramp down and ramp up limits, respectively; and the coordination of shutdown, startup, and status variables is enforced by:

$$SU_{m,g,t} + SD_{m,g,t} = v_{m,g,t} - v_{m,g,t-1} \quad \forall m \in M, g \in G_m, t, t-1 \in T \quad (11)$$

$$SU_{m,g,t} + SD_{m,g,t} \leq 1 \quad \forall m \in M, g \in G_m, t \in T \quad (12)$$

The minimum downtime and uptime requirements of thermal units, which are also needed due to the one-hour interval used, are imposed by the following constraints [36]:

$$\sum_{t=1}^{L_{m,g}^D} v_{m,g,t} = 0 \quad \forall m \in M, g \in G_m \quad (13)$$

$$\sum_{t'=t}^{t+DT_{m,g}-1} (1 - v_{m,g,t'}) \geq DT_{m,g} SD_{m,g,t} \quad \forall m \in M, g \in G_m, t = (L_{m,g}^D + 1), \dots, (T - DT_{m,g} + 1) \quad (14)$$

$$\sum_{t'=t}^T (1 - v_{m,g,t'} - SD_{m,g,t}) \geq 0 \quad \forall m \in M, g \in G_m, t = (T - DT_{m,g} + 2), \dots, T \quad (15)$$

$$\sum_{t=1}^{L_{m,g}^U} (1 - v_{m,g,t}) = 0 \quad \forall m \in M, g \in G_m \quad (16)$$

$$\sum_{t'=t}^{t+UT_{m,g}-1} v_{m,g,t'} \geq UT_{m,g} SU_{m,g,t} \quad \forall m \in M, g \in G_m, t = (L_{m,g}^U + 1), \dots, (T - UT_{m,g} + 1) \quad (17)$$

$$\sum_{t'=t}^T (v_{m,g,t'} - SU_{m,g,t}) \geq 0 \quad \forall m \in M, g \in G_m, t = (T - UT_{m,g} + 2), \dots, T \quad (18)$$

where $L_{m,g}^D = \min\{T, (DT_{m,g} - DT_{m,g}^0)(1 - v_{m,g}^0)\}$ is the number of hours that the generator must be down after considering its downtime before solving the problem, $DT_{m,g}$ is its minimum downtime, $DT_{m,g}^0$ is the number of hours it has been down prior to the first hour, and $v_{m,g}^0$ is its initial commitment status. Constraint (13) ensures that the unit satisfies its minimum downtime at the beginning of the optimization period, and are only included when $L_{m,g}^D \geq 1$; constraint (14) enforces the downtime requirement for all subsequent periods of size $DT_{m,g}$, and are only included if $(L_{m,g}^D + 1) < (T - DT_{m,g} + 1)$ and $DT_{m,g} > 1$; and constraint (15) enforces the minimum downtime for the last $(DT_{m,g} - 1)$ hours. Similarly, $L_{m,g}^U = \min\{T, (UT_{m,g} - UT_{m,g}^0)v_g^0\}$ is the number of hours that the generator must be up after considering its uptime before solving the problem, $UT_{m,g}$ is its minimum uptime, and $UT_{m,g}^0$ is the number of hours it has been up prior to the first hour. Constraint (16) ensures that the unit satisfies its minimum uptime at the beginning of the optimization period, and are only included when $L_{m,g}^U \geq 1$; constraint (17) enforces the uptime requirement for all subsequent periods of size $UT_{m,g}$, and are only included if $(L_{m,g}^U + 1) < (T - UT_{m,g} + 1)$ and $UT_{m,g} > 1$; and constraint (18) enforces the minimum uptime for the last $(UT_{m,g} - 1)$ hours. Note that depending on the size of the thermal units and the time frame of their response, as well as the EMS time intervals, the minimum up and down time constraints may not be necessary in the MMG EMS formulation. Hence, the inclusion of these constraints would depend on the characteristics of the installed units and the EMS implementation, and are thus considered here for modeling completeness.

The following expressions impose the energy balance, status coordination, and operating limits of the ESS units:

$$SoC_{m,s,t+1} - SoC_{m,s,t} = [P_{m,s,t}^c \eta_{m,s}^c - P_{m,s,t}^d / \eta_{m,s}^d] \Delta t \quad \forall m \in M, s \in S_m, t, t+1 \in T \quad (19)$$

$$\underline{SoC}_{m,s} \leq SoC_{m,s,t} \leq \overline{SoC}_{m,s} \quad \forall m \in M, s \in S_m, t \in T \quad (20)$$

$$\underline{P}_{m,s}^c z_{m,s,t}^c \leq P_{m,s,t}^c \leq \overline{P}_{m,s}^c z_{m,s,t}^c \quad \forall m \in M, s \in S_m, t \in T \quad (21)$$

$$\underline{P}_{m,s}^d z_{m,s,t}^d \leq P_{m,s,t}^d \leq \overline{P}_{m,s}^d z_{m,s,t}^d \quad \forall m \in M, s \in S_m, t \in T \quad (22)$$

$$z_{m,s,t}^c + z_{m,s,t}^d \leq 1 \quad \forall m \in M, s \in S_m, t \in T \quad (23)$$

where $SoC_{m,s,t}$ is the SoC, with $\underline{SoC}_{m,s}$ and $\overline{SoC}_{m,s}$ representing its minimum and maximum limits, respectively; $\eta_{m,s}^c$ and $\eta_{m,s}^d$ are the charging and discharging efficiencies, respectively; $\underline{P}_{m,s}^d$ and $\overline{P}_{m,s}^d$ are the minimum and maximum discharging power limits, respectively; $\underline{P}_{m,s}^c$ and $\overline{P}_{m,s}^c$ are the minimum and maximum charging power limits, respectively; and $z_{m,s,t}^c$ and $z_{m,s,t}^d$ are binary variables representing the charging and discharging status. Constraint (19) guarantees the energy balance for ESS units at all periods; constraint (20) enforces the minimum and maximum SoC limits; constraints (21) and (22) set the minimum and maximum boundaries for

charging and discharging power, respectively; and constraint (23) ensures that the ESS units are never charging and discharging simultaneously.

The limits of power transfer capacity through the PCC of each MG are enforced by the following constraints:

$$\sum_{n \in M \setminus m} P_{m,n,t}^e + P_{m,t}^{gb} \leq P_m^{PCC} \quad \forall m \in M, t \in T \quad (24)$$

$$\sum_{n \in M \setminus m} P_{n,m,t}^e + P_{m,t}^{gs} \leq P_m^{PCC} \quad \forall m \in M, t \in T \quad (25)$$

$$P_{m,n,t}^e P_{n,m,t}^e = 0 \quad \forall m \in M, n \in M \setminus m, t \in T \quad (26)$$

where P_m^{PCC} is the maximum power transfer capacity through the PCC of MG m . Constraint (24) ensures that the power outflow (from MG m to the other n MGs and the main grid) is within the required limits, and (25) ensures that the power inflow (from all n MGs and the main grid to MG m) is within the required limits. Simultaneous power exchanges in opposite directions between MGs are prevented by (26), which is a nonlinear expression that can be readily linearized using McCormick envelopes [37]. Finally, the following constraints define the model's binary and nonnegative variables:

$$v_{m,g,t}, SD_{m,g,t}, SU_{m,g,t}, z_{m,s,t}^c, z_{m,s,t}^d \in \{0, 1\} \quad \forall m \in M, g \in G_m, s \in S_m, t \in T \quad (27)$$

$$P_{m,n,t}^e, P_{m,t}^{gs}, P_{m,t}^{gb} \geq 0 \quad \forall m \in M, n \in M \setminus m, t \in T \quad (28)$$

B. Affine Arithmetic Model

In order to introduce uncertainties related to electricity demand and renewable generation in the model proposed in Section III-A, the AA formulation is developed by first expressing the parameters in their affine form, as follows:

$$\widehat{D}_{m,t} = D_{0,m,t} + D_{1,m,t} \varepsilon_{m,t}^D \quad \forall m \in M, t \in T \quad (29)$$

$$\widehat{P}_{m,t}^r = P_{0,m,t}^r + P_{1,m,t}^r \varepsilon_{m,t}^r \quad \forall m \in M, t \in T \quad (30)$$

where $D_{0,m,t}$ and $P_{0,m,t}^r$ are the central values of the affine forms of demand and renewable generation, respectively, equal to the deterministic parameters; $D_{1,m,t}$ and $P_{1,m,t}^r$ are their partial deviations; and $\varepsilon_{m,t}^D$ and $\varepsilon_{m,t}^r$ are the corresponding noise symbols. It is realistically assumed that the sources of uncertainty are independent, and thus a single and distinct noise symbol for each parameter is used. It is also assumed for simplicity that there is only one renewable generation lumped source per MG, which is a reasonable and practical assumption given the relatively small geographical area covered by each MG, and the fact that renewable generation in MGs in urban ADNs primarily consists of solar PV generators. However, the model can be readily modified to consider multiple and independent renewable generation sources, which would make the problem unrealistically more complex.

Once the uncertain parameters have been expressed in their affine form, all the continuous variables in the problem are redefined to include terms corresponding to each one of the noise symbols appearing in the parameters, as follows:

$$\widehat{P}_{m,g,t} = P_{0,m,g,t} + P_{1,m,g,t} \varepsilon_{m,t}^D + P_{2,m,g,t} \varepsilon_{m,t}^r \quad \forall m \in M, g \in G_m, t \in T \quad (31)$$

$$\widehat{P}_{m,t}^{gs} = P_{0,m,t}^{gs} + P_{1,m,t}^{gs} \varepsilon_{m,t}^D + P_{2,m,t}^{gs} \varepsilon_{m,t}^r \quad \forall m \in M, t \in T \quad (32)$$

$$\widehat{P}_{m,t}^{gb} = P_{0,m,t}^{gb} + P_{1,m,t}^{gb} \varepsilon_{m,t}^D + P_{2,m,t}^{gb} \varepsilon_{m,t}^r \quad \forall m \in M, t \in T \quad (33)$$

$$\widehat{P}_{m,n,t}^e = P_{0,m,n,t}^e + P_{1,m,n,t}^e \varepsilon_{m,t}^D + P_{2,m,n,t}^e \varepsilon_{m,t}^r \quad \forall m \in M, n \in M \setminus m, t \in T \quad (34)$$

And similarly for $\widehat{SoC}_{m,s,t}$, $\widehat{P}_{m,s,t}$, and $\widehat{P}_{m,s,t}^d$. In all variables, the first term is the central value, the second term is the partial deviation due to demand uncertainty, and the third term is the partial deviation due to renewable generation uncertainties.

According to the definition of the AA minimization operator in [26], the AA MMG EMS problem can be formulated as a multi-objective programming problem that minimizes the affine central value and the affine radius. To solve this multi-objective optimization problem, a possible approach is to decompose the problem in two subsequent stages: The first stage finds the optimal central values, assuming a “nominal state” of the system in which the uncertainties are not considered; and the second stage finds the optimal values of the partial deviations, corresponding to a “perturbed state” of the system in which uncertainties are taken into account [26]. However, the binary variables and intertemporal constraints present in the MMG EMS problem do not allow such decomposition because the solution of one stage may be infeasible for the other; hence, the objective function is instead formulated as a weighted function of both the center values and the affine radius [23], as follows:

$$\begin{aligned} \min \quad & \sum_{t \in T} \sum_{m \in M} \left\{ w \left[\sum_{g \in G_m} [(b_{m,g} P_{0,m,g,t} + c_{m,g} v_{m,g,t}) \Delta t \right. \right. \\ & \quad \left. \left. + C_{m,g}^{SD} SD_{m,g,t} + C_{m,g}^{SU} SU_{m,g,t}] \right. \right. \\ & \quad \left. \left. + \left(\sum_{n \in M \setminus m} \beta P_{0,n,m,t}^e + \rho_m^{gs} P_{0,m,t}^{gs} - \rho_m^{gb} P_{0,m,t}^{gb} \right) \Delta t \right] \right. \\ & \quad \left. + (1-w) \left[\sum_{g \in G_m} b_{m,g} (|P_{1,m,g,t}| + |P_{2,m,g,t}|) \Delta t \right. \right. \\ & \quad \left. \left. + \left(\sum_{n \in M \setminus m} \beta (|P_{1,n,m,t}^e| + |P_{2,n,m,t}^e|) \right. \right. \right. \\ & \quad \left. \left. \left. + \rho_m^{gs} (|P_{1,m,t}^{gs}| + |P_{2,m,t}^{gs}|) \right. \right. \right. \\ & \quad \left. \left. \left. - \rho_m^{gb} (|P_{1,m,t}^{gb}| + |P_{2,m,t}^{gb}|) \right) \Delta t \right] \right\} \end{aligned} \quad (35)$$

where w is a weighting factor that can be chosen according to the desired conservativeness of the solution with respect to uncertainties. For example, a weighting factor closer to 1 leads to cost-effective solutions that are sensitive to uncertainties, i.e., the AA objective function approximates the deterministic model more closely, while a value close to 0 leads to solutions that are more expensive but more protected from uncertainties, i.e., larger solution intervals. By trial and error, based on the MCS results, the weighting factor for the test system was set

to $w = 0.9$, which yielded similar results as the MCS approach so that proper comparisons could be carried out.

The supply-demand balance constraint (7) is formulated in the AA domain by expanding the affine forms and collecting terms corresponding to the central values and partial deviations of variables and parameters, according to the AA numerical computation model described in Section II-C. Thus, applying the AA similarity operator produces the following set of constraints:

$$\begin{aligned} & \sum_{g \in G_m} P_{0,m,g,t} + P_{0,m,t}^r + \sum_{s \in S_m} (P_{0,m,s,t}^d - P_{0,m,s,t}^c) \\ & + \sum_{n \in M \setminus m} P_{0,n,m,t}^e + P_{0,m,t}^{gs} = D_{0,m,t} + \sum_{n \in M \setminus m} P_{0,m,n,t}^e \\ & + P_{0,m,t}^{gb} \quad \forall m \in M, t \in T \end{aligned} \quad (36)$$

$$\begin{aligned} & \sum_{g \in G_m} P_{1,m,g,t} + \sum_{s \in S_m} (P_{1,m,s,t}^d - P_{1,m,s,t}^c) + \sum_{n \in M \setminus m} P_{1,n,m,t}^e \\ & + P_{1,m,t}^{gs} = D_{1,m,t} + \sum_{n \in M \setminus m} P_{1,m,n,t}^e + P_{1,m,t}^{gb} \\ & \quad \forall m \in M, t \in T \end{aligned} \quad (37)$$

$$\begin{aligned} & \sum_{g \in G_m} P_{2,m,g,t} + P_{1,m,t}^r + \sum_{s \in S_m} (P_{2,m,s,t}^d - P_{2,m,s,t}^c) \\ & + \sum_{n \in M \setminus m} P_{2,n,m,t}^e + P_{2,m,t}^{gs} = \sum_{n \in M \setminus m} P_{2,m,n,t}^e + P_{2,m,t}^{gb} \\ & \quad \forall m \in M, t \in T \end{aligned} \quad (38)$$

Note that the single power balance constraint in the deterministic problem has been replaced here by three constraints: One for the central values, one for the partial deviations associated with the demand uncertainty, and one for the partial deviations associated with the renewable generation uncertainty. These three constraints are required because the power balance constraint is an equality constraint in the AA domain and thus, as per the AA similarity operator definition mentioned in Section II-C, two affine forms are equal to each other if their central values and partial deviations are the same. Therefore, one constraint corresponds to the equality of central values, while the other two constraints correspond to the equality of the partial deviations. In general, the number of sources of uncertainty will determine the number of additional equality constraints required in the AA formulation.

The generation limit constraints (8) are reformulated based on the minimum and maximum values that the affine forms $\hat{P}_{m,g,t}$ can reach, according to the AA inequality operator, as follows:

$$P_{0,m,g,t} - |P_{1,m,g,t}| - |P_{2,m,g,t}| \geq \underline{P}_{m,g} v_{m,g,t} \quad \forall m \in M, g \in G_m, t \in T \quad (39)$$

$$P_{0,m,g,t} + |P_{1,m,g,t}| + |P_{2,m,g,t}| \leq \bar{P}_{m,g} v_{m,g,t} \quad \forall m \in M, g \in G_m, t \in T \quad (40)$$

Similarly, the ramping constraints (9) and (10) in AA form can be reformulated as:

$$\begin{aligned} & P_{0,m,g,t} + |P_{1,m,g,t}| + |P_{2,m,g,t}| \leq P_{0,m,g,t+1} \\ & + RD_{m,g} \Delta t + SD_{m,g,t+1} \underline{P}_{m,g} \\ & - |P_{1,m,g,t+1}| - |P_{2,m,g,t+1}| \\ & \quad \forall m \in M, g \in G_m, t, t+1 \in T \end{aligned} \quad (41)$$

$$\begin{aligned} & P_{0,m,g,t+1} + |P_{1,m,g,t+1}| + |P_{2,m,g,t+1}| \leq P_{0,m,g,t} \\ & + RU_{m,g} \Delta t + SU_{m,g,t+1} \underline{P}_{m,g} \\ & - |P_{1,m,g,t}| - |P_{2,m,g,t}| \\ & \quad \forall m \in M, g \in G_m, t, t+1 \in T \end{aligned} \quad (42)$$

Constraints (11)–(18), which correspond to the status coordination and minimum uptime/downtime of thermal units, only involve binary variables and are thus unaffected by the formulation of continuous variables as affine forms. Hence, these constraints do not need modifications to be included in the AA model.

Constraints (19), which are intertemporal equality constraints, are redefined in the AA domain by equalizing the central values and affine radius, as follows:

$$\begin{aligned} & SoC_{0,m,s,t+1} = SoC_{0,m,s,t} + [P_{0,m,s,t}^c \eta_{m,s}^c \\ & - P_{0,m,s,t}^d / \eta_{m,s}^d] \Delta t \\ & \quad \forall m \in M, s \in S_m, t, t+1 \in T \end{aligned} \quad (43)$$

$$\begin{aligned} & |SoC_{1,m,s,t+1}| + |SoC_{2,m,s,t+1}| = |SoC_{1,m,s,t}| \\ & + |SoC_{2,m,s,t}| + [(|P_{1,m,s,t}^c| + |P_{2,m,s,t}^c|) \eta_{m,s}^c \\ & - (|P_{1,m,s,t}^d| + |P_{2,m,s,t}^d|) / \eta_{m,s}^d] \Delta t \\ & \quad \forall m \in M, s \in S_m, t, t+1 \in T \end{aligned} \quad (44)$$

The operational limits of ESS units, defined by (20)–(22), are formulated in the AA domain according to the minimum and maximum values that the affine forms can take, as follows:

$$SoC_{0,m,s,t} - |SoC_{1,m,s,t}| - |SoC_{2,m,s,t}| \geq \underline{SoC}_{m,s} \quad \forall m \in M, s \in S_m, t \in T \quad (45)$$

$$SoC_{0,m,s,t} + |SoC_{1,m,s,t}| + |SoC_{2,m,s,t}| \leq \bar{SoC}_{m,s} \quad \forall m \in M, s \in S_m, t \in T \quad (46)$$

$$P_{0,m,s,t}^c - |P_{1,m,s,t}^c| - |P_{2,m,s,t}^c| \geq \underline{P}_{m,s} z_{m,s,t}^c \quad \forall m \in M, s \in S_m, t \in T \quad (47)$$

$$P_{0,m,s,t}^c + |P_{1,m,s,t}^c| + |P_{2,m,s,t}^c| \leq \bar{P}_{m,s} z_{m,s,t}^c \quad \forall m \in M, s \in S_m, t \in T \quad (48)$$

$$P_{m,s,t}^d - |P_{1,m,s,t}^d| - |P_{2,m,s,t}^d| \geq \underline{P}_{m,s} z_{m,s,t}^d \quad \forall m \in M, s \in S_m, t \in T \quad (49)$$

$$P_{m,s,t}^d + |P_{1,m,s,t}^d| + |P_{2,m,s,t}^d| \leq \bar{P}_{m,s} z_{m,s,t}^d \quad \forall m \in M, s \in S_m, t \in T \quad (50)$$

Constraint (23) is included without modifications in the AA model since it only involves binary variables.

As before, based on the minimum and maximum values that the corresponding affine forms can take, (24) and (25) can be reformulated as follows:

$$\begin{aligned} \sum_{n \in M \setminus m} (P_{0,m,n,t}^e + |P_{1,m,n,t}^e| + |P_{2,m,n,t}^e|) \\ + P_{0,m,t}^{gb} + |P_{1,m,t}^{gb}| + |P_{2,m,t}^{gb}| \leq P_m^{PCC} \end{aligned} \quad (51)$$

$$\forall m \in M, t \in T$$

$$\begin{aligned} \sum_{n \in M \setminus m} (P_{0,n,m,t}^e + |P_{1,n,m,t}^e| + |P_{2,n,m,t}^e|) \\ + P_{0,m,t}^{gs} + |P_{1,m,t}^{gs}| + |P_{2,m,t}^{gs}| \leq P_m^{PCC} \end{aligned} \quad (52)$$

$$\forall m \in M, t \in T$$

To include (26) in the AA model, which is a nonlinear constraint, it is assumed that the product of two noise symbols in the expansion of the affine operation is equal to 1, which is a conservative approximation of the non-affine operation that provides more protection against internal sources of error and allows a more straightforward implementation. This results in the following expression:

$$\begin{aligned} P_{0,m,n,t}^e \sum_{h=0}^2 P_{h,n,m,t}^e + P_{1,m,n,t}^e \sum_{h=0}^2 P_{h,n,m,t}^e \\ + P_{2,m,n,t}^e \sum_{h=0}^2 P_{h,n,m,t}^e = 0 \quad \forall m \in M, n \in M \setminus m, t \in T \end{aligned} \quad (53)$$

Finally, the nonnegativity constraints (28) are reformulated to ensure that the minimum value that the affine forms can take is greater than zero, as follows:

$$\begin{aligned} P_{0,m,n,t}^e - |P_{1,m,n,t}^e| - |P_{2,m,n,t}^e| \geq 0 \\ \forall m \in M, n \in M \setminus m, t \in T \end{aligned} \quad (54)$$

$$\begin{aligned} P_{0,m,t}^{gb} - |P_{1,m,t}^{gb}| - |P_{2,m,t}^{gb}| \geq 0 \\ \forall m \in M, t \in T \end{aligned} \quad (55)$$

$$\begin{aligned} P_{0,m,t}^{gs} - |P_{1,m,t}^{gs}| - |P_{2,m,t}^{gs}| \geq 0 \\ \forall m \in M, t \in T \end{aligned} \quad (56)$$

The absolute value operators introduced by the AA formulation complicate the problem by making it nonlinear. To avoid this, a linearization procedure is applied to remove these values in the objective function and constraints, based on [38]. This process involves the inclusion of auxiliary variables and constraints for each absolute value. The description of the procedure and the complete formulation after linearization are omitted here for brevity; however, it is worth noting that the number of constraints and complexity of the problem significantly increase in proportion to the number of MGs and sources of uncertainty considered.

IV. RESULTS AND DISCUSSION

To test the proposed models, a system comprised of three connected MGs was analyzed. Each MG has different demand and renewable generation profiles, considering a 24-hour optimization period. Furthermore, each MG contains a set of

three thermal units with distinct generation cost functions and operating limits, and a set of two ESS units. For this system, if two sources of uncertainty are considered per MG, the number of constraints is 40,464; if an additional source of uncertainty is considered, the number of constraints significantly increases to 708,016. The proposed models were implemented using the open-source modeling package Pyomo (version 6.4.2), and solved using the Gurobi solver (version 10.0.1) on a server with dual Intel Xeon Silver 4214 processors with a base speed of 2.20 GHz.

A. Test System

The electricity demand and renewable generation profiles of each MG were obtained from measured data in an ADN located in the state of São Paulo, Brazil, where the medium voltage grid was clustered to identify potential independent MGs [15]. The available data were demand and irradiance measurements, which were used to obtain the required profiles according to [39]. A nominal power output of 24 kW for a PV array in each MG was considered, based on the characteristics of the test system. Fig. 2 shows the demand and PV generation profiles for all MGs. Observe that there is no significant correlation between the uncertain parameters because the maximum or minimum demand values do not occur at the same hours as the maximum or minimum PV generation values. This observation is confirmed by the Pearson correlation coefficients, which are -0.25, 0.56, and -0.17 for MG1, MG2, and MG3, respectively, which measure the correlation between two sets of data [40], and are computed for each MG as the covariance of demand and PV generation, divided by the product of their standard deviations. These correlation coefficients can take values within [-1, 1], with values close to zero indicating no correlation between the two sets of data. Hence, the computed values indicate a weak correlation between the uncertain parameters, which supports the choice of single and distinct noise symbols for each source of uncertainty in the AA formulation. If the correlation coefficients were higher in magnitude, thus indicating a strong correlation, this could be readily introduced in the AA formulation through the definition of shared noise symbols in the affine forms of the uncertain parameters, which would increase the complexity of the problem. All the other parameters in the deterministic model were adapted from various references and adjusted to be adequate for the available demand and renewable generation profiles. The PCC limits were set to 100 kW, 200 kW, and 100 kW for MG1, MG2, and MG3, respectively, to ensure sufficient capacity for power exchanges in this case.

B. Comparative Analysis

To validate the proposed AA model, comparisons of the deterministic solution, MCS, and the AA solution intervals are presented next. Furthermore, an SP approach was implemented, to compare the objective function values of all models. To carry out the MCS, upper and lower bounds for the uncertain parameters are first defined, considering $\pm 10\%$ for demand, and $\pm 25\%$ for PV generation, at each MG and hour, which are typical variation ranges for these parameters.

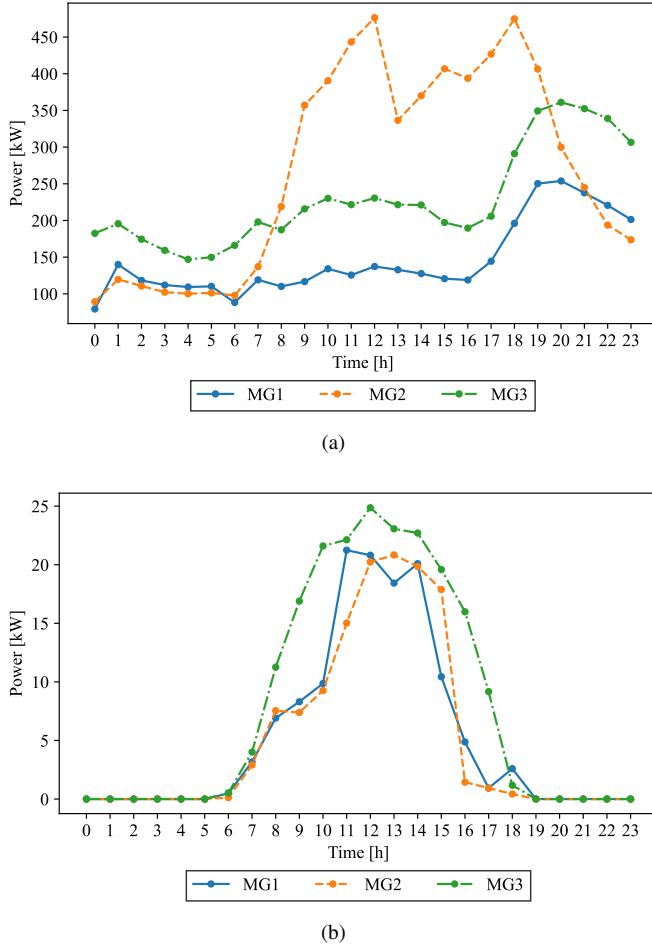


Fig. 2. (a) Demand and (b) PV generation profiles for all MGs.

Then, a set of random profiles within the defined bounds are created, considering a uniform distribution. A total of 1,000 random profiles were generated, which were found to be enough to observe convergence of the objective function and all variables through trial and error. To compare the output of MCS, intervals for each variable are obtained from their corresponding histograms by discarding the least probable solutions, which results in narrower and more significant intervals than taking the minimum and maximum values, with no assumption of a normal distribution, as some variables do not follow such distribution.

The TSSP model was formulated based on [15], [41], where the first stage includes the binary decision variables for shutdown, startup, commitment status of thermal units, and charging/discharging state of ESS units; and in the second stage, the decision variables are the power dispatches of thermal units, the power bought/sold by the main grid from and to each MG, the power exchanges between MGs, and the SoC and charging/discharging powers of ESS units. Constraints related only to the binary variables, such as shutdown/startup and minimum up/down time logic, correspond to the first stage, while constraints related to the dispatch variables subject to uncertainty, such as power balance constraints, generation limits, ramping constraints, PCC constraints, and charging/discharging limits, correspond to the second stage,

TABLE I
SCENARIOS AND PROBABILITIES FOR TSSP MODEL

	Scenarios			
	S1	S2	S3	S4
Demand	1.0622	1.051	1.042	1.0362
PV	6.5226	6.5152	6.5086	6.5122
Probability	0.025	0.083	0.142	0.042

	Scenarios				
	S5	S6	S7	S8	S9
Demand	1.0302	1.0342	1.0242	1.0192	1.0092
PV	6.5094	6.5094	6.5094	6.5104	6.5104
Probability	0.128	0.123	0.143	0.144	0.17

and are thus defined for every scenario. The scenarios and their associated probabilities were obtained based on [15], where a moment matching approach was applied to generate scenarios with relevant states for demand and renewable generation, for the same MMG system in São Paulo, Brazil, considered in this work; the scenarios were built to ensure that the probability of each scenario is greater than 2%, thus discarding scenarios with extremely low probability of occurrence, and 9 scenarios were chosen that guarantee neither under nor overspecification of the statistical measures [15]. Table I shows the factors for demand and PV generation corresponding to each scenario used in the TSSP model, and the associated probabilities. The base profiles shown in Fig. 2 are multiplied by these factors and assigned the given probability value.

For the AA model, the partial deviations of all uncertain parameters must be specified. In this case, to match the input of the MCS, the same upper and lower bounds are considered, i.e., the values of the partial deviations of the affine forms in (29) and (30) produce an identical interval to the one defined by the upper and lower bounds of MCS. As previously mentioned, a weighting factor of $w = 0.9$ in (35) was chosen because it yields an objective function value close to the one obtained in the MCS solution, which also considers parameter uncertainties.

A comparison of the deterministic, MCS, TSSP, and AA solutions is shown in Fig. 3 for the power output of thermal unit G1 in MG1. The TSSP “interval” corresponds to the dispatch solutions associated with Scenario 1 and Scenario 9, which have the highest and lowest demand factors, respectively. Observe that the AA intervals are not always enveloping the MCS intervals, nor the TSSP and deterministic solutions. This may seem counterintuitive, since the AA solution is robust for the full range of uncertainties considered, and is thus expected to be the most conservative. However, in the case of MMGs, the power generation and exchanges of all MGs are coupled through the power balance constraints, and when the AA interval does not envelop the deterministic solution, that mismatch will be compensated by the interval of another variable that results in a lower cost while satisfying the constraints of the problem. Furthermore, due to the formulation of the AA problem, which minimizes the magnitudes of the partial deviations, the resulting intervals will be as narrow as possible, while still ensuring the feasibility of the problem for any realization of the uncertain parameters. On the other hand, the MCS intervals are wider because they cover a range of solutions for each individual variable, while the AA solution

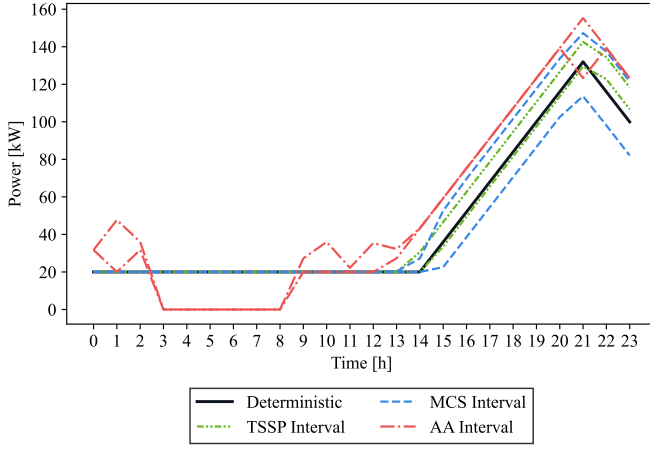


Fig. 3. Solution comparison for thermal unit G1 in MG1.

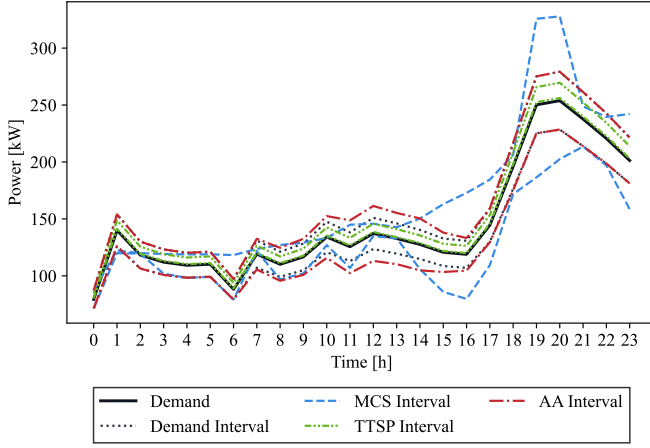


Fig. 4. Total generation and demand in MG1.

gives the narrowest interval for all variables. A solution within the AA interval may not be within the solution boundaries of MCS, due to the flexibility of the AA model to find the optimal solution contributing to minimizing the cost function. Based on this analysis, the resulting patterns of all variables are consistent with the AA formulation, which validates the proposed AA model.

Fig. 4 shows a comparison of the total demand, with its corresponding uncertainty range, and the total generation intervals obtained from MCS, the TSSP, and AA models, for MG1. Note that, as in Fig. 3, the TSSP interval corresponds to the scenarios with highest and lowest demand factors, which represent a narrower uncertainty range than the MCS and AA approaches, as reflected in the illustrated results. Note that by considering the AA intervals for the aggregated generation in the MG, the total demand and its uncertainty interval are satisfied, thus demonstrating that the AA solution is in fact robust for any realization of the uncertain parameters when the whole system operation is considered. On the other hand, the MCS intervals are still wider at certain hours, which can be the case depending on the distribution of the solutions. The operating costs and computational performance

TABLE II
OPERATING AND COMPUTATIONAL COST COMPARISON

	Deterministic	MCS	TSSP	AA
Operating Cost [\$]	4,665	4,691	4,712	4,598 [4,774, 4,423]
Computational Cost [mm:ss]	00:09	19:23	00:14	03:07

are illustrated in Table II, where it is shown that the AA model provides a solution approximately 70% faster than MCS, while considering the same range of uncertainties, which is a major advantage of the proposed AA model. The objective function values of the deterministic, MCS, TSSP, and AA models are similar, with the objective function of the AA model being slightly lower due to the weighting factor of the central and affine radius terms, which in principle correspond to intervals. The cost of the deterministic model is obtained by evaluating the objective function with the optimal values found from the problem's solution; the operating cost of the MCS is computed as the average of the objective function values for the 1,000 simulations performed; the operating cost of the TSSP model is its objective function value evaluated at the optimal solution, which minimizes the shutdown and startup costs in the first stage, and the product of the dispatch costs and their probabilities for each scenario in the second stage; the AA operating cost is obtained from evaluating the AA objective function (35) with the corresponding optimal values of central values and partial deviations of the affine forms of the solution variables, yielding an interval computed by adding and subtracting the affine radius from the central value.

Note that the TSSP model yields a similar objective function value than the AA model; however, it requires a careful scenario generation and selection process based on the statistical properties of the uncertain parameters, which may miss possible relevant scenarios. On the contrary, the AA-based approach does not present these drawbacks, as it directly considers the complete range of uncertainties through the definition of affine forms, which is a significant advantage of the proposed model.

C. Dispatch from the AA Solution

To demonstrate the utility of the proposed AA model, a dispatch procedure can be implemented to convert the resulting affine forms to particular solutions for specific realizations of the uncertain parameters. This involves assigning values in the $[-1, 1]$ interval to each noise symbol and substituting in the affine forms for finding a specific dispatch solution. To exemplify this, the noise symbols assignment can be made as follows:

$$\varepsilon_{m,t}^D = \frac{D_{m,t} - D_{0,m,t}}{D_{1,m,t}} \quad \forall m \in M, t \in t \quad (57)$$

where $D_{m,t}$ is the realization of the uncertain demand, $D_{0,m,t}$ is the central value, and $D_{1,m,t}$ is the corresponding partial deviation. Equation (57) also applies for the other uncertain parameters. Thus, assuming that a profile deviates from the deterministic hourly profiles, (57) can be used to compute all noise symbols associated with the two sources of uncertainty.

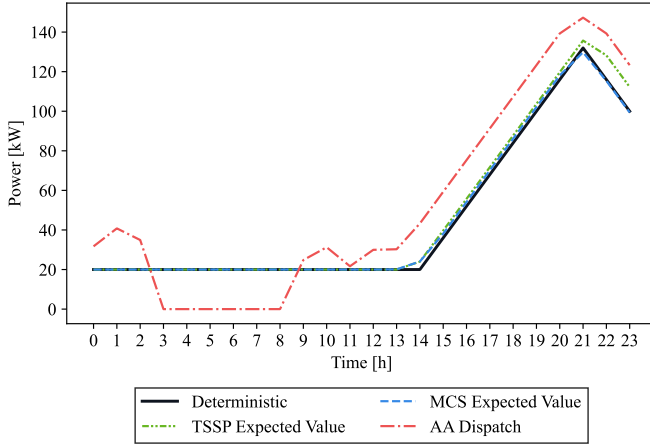


Fig. 5. Dispatches of thermal unit G1 in MG1.

For example, the affine form of the power output of generator G1 in MG1 at hour 10 is $\hat{P}_{1,1,10} = 28.01 + 5.53\varepsilon_{1,10}^D - 2.47\varepsilon_{1,10}^r$. Note that this particular variable is more sensitive to demand uncertainty than renewable generation uncertainty, since the partial deviation corresponding to the demand has the largest absolute value. Similar affine forms of all the continuous variables in the problem are available from the optimal solution of the AA problem. Hence, substituting the values of noise symbols found from (57) for each uncertain parameter, in all affine variables, yields a particular dispatch solution. To demonstrate this, a realization of the uncertain parameters is chosen arbitrarily, assuming a deviation of +5% in demand and -5% in PV generation, for all MGs. Fig. 5 shows the resulting dispatch after applying the described procedure, as well as the corresponding deterministic, MCS, and TSSP solutions. The TSSP solution is the expected value of the dispatches associated with all the scenarios considered, which closely follows the deterministic dispatch, similarly to the MCS solution. As can be seen, the affine forms obtained from the solution of the AA model can be used to find the optimal dispatch for specific realizations of the parameters within the range of uncertainty considered, with no need for solving the problem again for different realizations, and with lower computational burden with respect to the MCS. Furthermore, the resulting affine forms provide valuable information regarding the sensitivity of variables to each source of uncertainty.

D. Individual and Cooperative Operation

Finally, the benefits of cooperative operation are demonstrated by comparing the deterministic cost of MGs operating on their own, with no power exchanges, against the cost when power exchanges are enabled. For this test, the PCC limits of all MGs were set to zero to restrict the power exchanges and ensure that all loads are satisfied only with local generation in each MG. The operating costs for all MGs are depicted in Table III, where it can be observed that, as expected, the operating cost of all MGs is reduced as a consequence of the power exchanges, with the most drastic

TABLE III
OPERATING COSTS OF INDIVIDUAL VS. COOPERATIVE MGs

	Individual	Cooperative	Difference
MG1 [\$]	1,124.96	708.10	416.86
MG2 [\$]	6,801.99	1,897.07	4,904.92
MG3 [\$]	4,362.81	2,059.93	2,302.88
Total Cost [\$]	12,289.76	4,665.01	7,624.75

improvement occurring in MG2, since this MG has the most expensive generation units, and thus, through exchanges, it is able to supply its demand with imports from the main grid and other MGs, avoiding its higher local generation costs. These costs are computed by evaluating the objective function (6) at the optimal solution when power exchanges are enabled and disabled, for the overall system and for each MG, which corresponds to the optimal solution values for each MG m , i.e., the part of the overall objective function associated with the operating cost of each MG.

Depending on the parameters of each MG, it may be possible that the operating cost of some MGs increases when power exchange are considered, because they need to increase their local generation to supply additional power needed by other MGs. However, the total cost of the system is expected to decrease or stay the same. This is a consequence of the centralized formulation of the EMS problem, which optimizes the overall objective function considering all MGs as a single system. The degree of improvement will depend on the local generation costs of each MG, their demand and renewable generation profiles, the main grid buying and selling prices, and the PCC limits. The possibility of power exchanges allows MGs to benefit from higher or lower costs to meet their demand.

V. CONCLUSION

This paper presented an EMS model that considers power exchanges between a set of MGs in an ADN, taking into account uncertainties associated with electricity demand and renewable energy generation with an AA-based approach. The main contributions of this work are the formulation of the EMS that realistically considers the power limit constraints at the PCC of each MG, under the most practical MMG system topology, and the subsequent formulation of the model in the AA domain. The results show that the AA EMS is robust under a range of possible realizations of the uncertain parameters, and can be solved with relatively low computational burden and short execution time. Furthermore, an advantage of this approach is that the same affine forms can be used to find a dispatch for different realizations of the uncertain parameters, with no need to repeatedly solve the optimization problem. Finally, it is shown that the operating cost of the MMG system improves when power exchanges between MGs are enabled, with respect to their individual operation, as expected, which confirms the potential benefits for ADNs when clustering parts of the network to form an MMG system.

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