

Aggregate Modeling of Thermostatically Controlled Loads for Microgrid Energy Management Systems

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Abstract—Second-to-second renewable power fluctuations can severely hinder the frequency regulation performance of modern isolated microgrids, as these typically have a low inertia and significant renewable energy integration. In this context, the present paper studies the coordinated control of Thermostatically Controlled Loads (TCLs) for managing short-term power imbalances, and their integration in microgrid operations through the use of aggregate TCL models. In particular, two computationally efficient and accurate aggregate TCL models are developed: a virtual battery model representing the aggregate flexibility of TCLs considering solar irradiance heat gains and wall/floor heat transfers, and a frequency transient model representing the aggregate dynamics of a TCL collection considering communication delays and the presence of model uncertainty and time-variability. The proposed aggregate TCL models are then used to design a practical Energy Management System (EMS) integrating TCL flexibility, and study the impact of TCL integration on microgrid operation and frequency control. Computational experiments using detailed frequency transient and thermal dynamic models are presented, demonstrating the accuracy of the proposed aggregate TCL models, as well as the economic and reliability benefits resulting from using these aggregate models to integrate TCLs in microgrid operations.

Index Terms—Load control, energy management system, virtual battery, short-term fluctuations, frequency dynamics, microgrid operation.

I. INTRODUCTION

MICROGRIDS are considered one of the key assets for developing a sustainable decentralized smart grid, as these hold the potential of enabling a reliable and efficient integration of Distributed Energy Resources (DERs). In this context, significant research efforts are currently being made in the design of Energy Management Systems (EMSs) for the economic and secure operation of autonomous microgrids,

which has resulted in modern EMS models that integrate advanced features such as phase imbalance, demand response, and thermal energy resources [1]–[5].

Contrary to traditional power systems, modern isolated microgrids are characterized by a low inertia with significant renewable energy integration, making their operation and control particularly challenging. As evidenced in [6], significant second-to-second renewable power fluctuations can be observed for geographically close plants, which when combined with a low system inertia, can result in a poor frequency regulation performance for the system [7]. Thus, there is a need for modern microgrid EMS models that take into account the impact of short-term power fluctuations on frequency dynamics.

Some prior work can be found in the context of microgrid EMS models embedding second-to-second renewable power fluctuations. Two-stage EMS models have been proposed in [8]–[10], for which dispatch decisions are obtained in an upper-stage problem, and short-term fluctuations are managed in a separate lower-stage problem. However, these works have the drawback that they are designed specifically for grid-connected microgrids, in which the support and inertia from the bulk utility grid is available, which is not applicable to isolated microgrids. To address this issue, EMS models capable of managing short-term fluctuations in isolated microgrid applications have recently been introduced, for which a strategic use of tailored reserves, renewable curtailment, and batteries is employed [11], as well as the use of explicit frequency-constraints for ensuring a proper frequency regulation performance [12]. However, these papers are limited to isolated microgrids with generators and batteries, neglecting other flexible technologies available for the management of power imbalances, such as modern load control techniques.

Within the wide spectrum of existing load control strategies, the aggregate control of Thermostatically Control Loads (TCLs), such as air conditioners and electric water heaters, are of particular interest for isolated microgrids, since TCLs comprise a significant share of the total residential demand [13], and can be used to manage second-to-second power imbalances while remaining non-disruptive to the end-user [14]. In this context, a series of different aggregate TCL control strategies can be found in the existing literature such as decentralized stochastic control [15], population-bin linear state-space models [16], and priority-stack control [17]. However, most of the aggregate TCL control models developed so far, including [15]–[17], assume a simplified first-order household thermal model in their design, which as discussed in

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[18], can significantly differ from the actual household thermal dynamics observed in practical settings. A few works on TCL control have used more accurate high-order household thermal models (e.g., [19]–[21]), which form the basis to the aggregate models proposed here.

Another relevant issue is the integration of aggregate TCL controllers within operational models such as EMSs, since building an optimization problem with constraints and variables for each TCL (up to 2000 TCLs for large distribution systems [22]) can result in a computationally demanding problem. To address this issue, the concept of *virtual battery* has been proposed in [17], [23]–[25], which aims at characterizing the set of reference signals that can be followed by the aggregate TCL controller in a compact and computationally efficient way through a state-of-charge equation with energy/power capacity limits. Virtual battery models have been successfully implemented in operational models for bulk power systems, resulting in single period and multi-period optimal power flow models incorporating reserves from TCLs [26], [27], as well as unit commitment models that incorporate the provision of both energy arbitrage and frequency response services from TCLs [28]. However, the virtual battery models considered in these works are based on simple first-order household thermal models, which can significantly differ from actual household thermal dynamics [18]. Furthermore, no frequency transient simulations are reported in the computational experiments of these works, which are particularly relevant for result validation in low-inertia systems with significant renewable energy integration, such as isolated microgrids [12]. It is also important to highlight that, in the context of isolated microgrids, previous works on frequency regulation provision from TCLs can be found (e.g., [20], [29]–[31]); however, these works neglect the capability of TCLs to provide energy shifting services to further improve microgrid operation, thus leaving a research gap.

Based on the aforementioned discussion, the present paper studies the integration of TCLs within microgrid operations through the use of computationally efficient aggregate models, focusing on the role that TCLs can play to manage second-to-second power fluctuations and their corresponding impact on frequency regulation. Furthermore, unlike previous works on aggregate TCL models, relevant practical challenges that arise during the implementation of a TCL control are considered in the design of the proposed aggregate models, in particular high-order household thermal dynamics, communication delays, time-variability, and uncertainty. Thus, the main contributions and objectives of this paper are as follows:

- A highly-accurate virtual battery model that captures detailed TCL thermal dynamics, such as solar irradiance heat gains and wall/floor heat transfers, is developed. The proposed virtual battery model is based on a novel decoupled household thermal model, which captures the main features of high-order household thermal models with a reduced complexity suitable for practical applications.
- A transient model for representing the frequency dynamics of an aggregate TCL control is designed, which incorporates practical challenges such as the time-varying and uncertain capacity limits of virtual battery models

[32], and the presence of communication delays in real-life aggregate TCL control implementations [29].

- Based on the proposed aggregate TCL models, an EMS model integrating TCL flexibility is developed for practical isolated microgrid applications, which is validated using detailed frequency transient and thermal dynamic models. The results show that the smart control of a collection of TCLs can translate in reduced operating costs while keeping an adequate frequency regulation performance in line with modern DER operating standards.

The rest of the paper is organized as follows: Section II discusses household thermal models, which include existing first- and third- order thermal models, as well as the novel decoupled thermal model. Section III presents the proposed aggregate TCL models, namely the virtual battery and frequency dynamic models. Section IV discusses the design of an EMS that considers TCL flexibility, which is based on the proposed aggregate TCL models. Section V presents the computational experiments and associated results, showcasing the benefits of integrating TCLs in microgrid operations through the use of aggregate TCL models. Finally, Section VI summarizes the main contributions and conclusions of the paper.

II. HOUSEHOLD THERMAL MODELS

There exists a wide range of thermal models for characterizing the heat dynamics of a household, ranging from simple first-order models to detailed higher-order models [18]. In this section, the thermal models commonly used in the aggregate TCL control literature are presented. Furthermore, a novel decoupled thermal model is introduced, which has improved computational performance and serves as a basis for developing a new virtual battery model in Section III.

A. First-Order Model

Papers discussing an aggregate TCL control typically assume a simple first-order model for representing inner household thermal dynamics (e.g., [16], [17], [22]–[25], [33]). In this model, only one state variable is considered, namely the household inner temperature θ^I , resulting in the following thermal dynamic equation:

$$C^I \frac{d\theta^I}{dt} = \frac{\theta^A - \theta^I}{R^A} + \eta^{AC} p^{AC} \quad (1)$$

where θ^A is the ambient temperature; C^I is the household's internal thermal capacitance; R^A is the thermal resistance between the household interior and ambient; η^{AC} is the TCL's coefficient of performance; and p^{AC} is the TCL's heating (+) or cooling (−) electric power, which is bounded by the TCL's rated power P^{AC} , and can be controlled in a discrete (ON/OFF) or continuous manner. Due to its simplicity, this model has been used for simulating large collections of TCLs (around 1000-2000 TCLs in [16], [17], [22], [23], [25]), as well as in the development of existing virtual battery models (e.g., [17], [23]–[25], [33]).

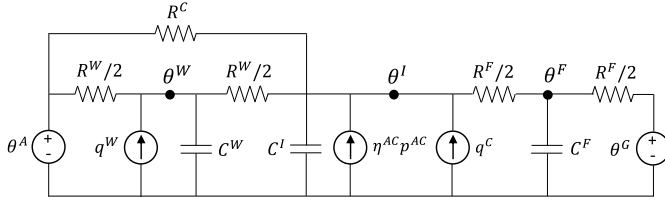


Fig. 1. Thermal circuit representation of detailed third-order model [21]

B. Third-Order Model

As discussed in [18], [21], [34], household thermal dynamics are not only affected by the ambient temperature, but also are affected by the solar radiation received by the household, the thermal capacitance of walls, and the heat transfer between the household's floor and the ground. To capture these additional aspects, a more detailed third-order model is needed, for which two additional state variables are added, namely, wall temperature θ^W and floor temperature θ^F . This results in the following thermal dynamic equations [21] (see Fig. 1):

$$C^I \frac{d\theta^I}{dt} = \eta^{AC} p^{AC} + q^C + \frac{\theta^A - \theta^I}{R^C} + \frac{\theta^W - \theta^I}{R^W/2} + \frac{\theta^F - \theta^I}{R^F/2} \quad (2a)$$

$$C^W \frac{d\theta^W}{dt} = q^W + \frac{\theta^A + \theta^I - 2\theta^W}{R^W/2} \quad (2b)$$

$$C^F \frac{d\theta^F}{dt} = \frac{\theta^G + \theta^I - 2\theta^F}{R^F/2} \quad (2c)$$

where q^C and q^W are the heating power from the solar radiation onto windows and walls, respectively; R^C is the thermal resistance of windows; R^W and C^W are the thermal resistance and capacitance of walls, respectively; R^F and C^F are the thermal resistance and capacitance of the floor, respectively; and θ^G is the ground temperature. Due to its improved modeling accuracy, this thermal model has been used to evaluate in detail the impacts of possible practical implementations of load management strategies (e.g., [21], [34]). However, due to its increased mathematical complexity, it has not been yet applied in studies with a large population of TCLs nor virtual battery models.

C. Decoupled Model

As previously discussed, one of the main drawbacks of the third-order model (2) is its increased mathematical complexity, which prevents its direct integration in virtual battery models with a large TCL population. Thus, to tackle this issue, a novel decoupled thermal model is proposed here, which captures the high-modeling accuracy of the original third-order model, while maintaining a low mathematical complexity suitable for virtual battery model design. The proposed decoupled thermal model is based on (2), for which the following properties commonly found in practical settings are exploited [20], [21], [34] (see Fig. 1):

1. Air conditioners automatically regulate the inner household temperature θ^I to match the thermostat's setpoint

$\theta^{I,SP}$. Furthermore, temperature deviations from the setpoint $|\theta^I - \theta^{I,SP}|$ are relatively small, as a narrow temperature deadband $\Delta\theta^{AC}$ is typically used in thermostats.

2. Modern air conditioner units have a significant heating/cooling rated power P^{AC} , resulting in inner household temperature dynamics operating in a relatively fast timescale in the order of seconds to minutes.
3. Ambient temperature θ^A and solar heat gains onto walls q^W operate in a relatively slow timescale in the order of minutes to hours. Furthermore, the ground temperature θ^G remains relatively constant during the day.
4. Household's walls and floor are compliant with modern thermal insulation standards for buildings, resulting in significant wall/floor thermal capacitances C^W and C^F .

Properties 1-4 result in inner household temperature deviations from the thermostat setpoint $|\theta^I - \theta^{I,SP}|$ having a negligible impact on the wall/floor thermal dynamics, as these small and fast deviations are filtered out by the wall/floor thermal capacitances C^W and C^F . Thus, thermal dynamic equations (2b) and (2c) can be approximated as follows:

$$C^W \frac{d\theta^W}{dt} \approx q^W + \frac{\theta^A + \theta^{I,SP} - 2\theta^W}{R^W/2} \quad (3a)$$

$$0 \approx \frac{\theta^G + \theta^{I,SP} - 2\theta^F}{R^F/2} \quad (3b)$$

where θ^I has been replaced by $\theta^{I,SP}$, eliminating the dependence on inner household temperature dynamics. Note that Property 3 results in wall thermal dynamics (3a) operating at a minutes/hours timescale, and floor temperature θ^F in (3b) remaining approximately constant during the day. Furthermore, observe that (3a) and (3b) can be solved analytically, yielding the following equations for wall and floor temperature evolution over time:

$$\theta^W(t) \approx \theta^{W,SS} + (\theta^{W,0} - \theta^{W,SS}) \exp(-\alpha^W t) \quad (4a)$$

$$\theta^F(t) \approx \theta^{F,SS} = (\theta^G + \theta^{I,SP})/2 \quad (4b)$$

where $\theta^{W,SS} = (\theta^A + \theta^{I,SP})/2 + q^W R^W/4$ is the steady-state wall temperature; $\theta^{W,0}$ is the initial wall temperature; $\theta^{F,SS}$ is the steady-state floor temperature; and $\alpha^W = 4/(R^W C^W)$. Hereinafter, the DT parameter will be used to refer to the time used for modeling wall/floor dynamics (e.g., $DT = 5\text{min}$).

On the other hand, Properties 1-4 result in inner household thermal dynamics (2a) operating in a much faster timescale than DT (e.g., 1s vs 5min), for which it can be assumed that wall and floor temperatures θ^W and θ^F remain relatively constant, resulting in the following decoupled representation:

$$C^I \frac{d\theta^I}{dt} \approx \eta^{AC} p^{AC} + q^{EQ} - \frac{\theta^I}{R^{EQ}} \quad (5a)$$

$$q^{EQ} = q^C + \frac{\theta^A}{R^C} + \frac{\theta^{W,DT}}{R^W/2} + \frac{\theta^{F,SS}}{R^F/2} \quad (5b)$$

$$\frac{1}{R^{EQ}} = \frac{1}{R^C} + \frac{1}{R^W/2} + \frac{1}{R^F/2} \quad (5c)$$

$$\theta^{W,DT} = \theta^{W,SS} + \frac{(\theta^{W,0} - \theta^{W,SS})}{\alpha^W DT} [1 - \exp(-\alpha^W DT)] \quad (5d)$$

where q^{EQ} and R^{EQ} are the equivalent heat gain and thermal resistance, respectively, and $\theta^{W,DT}$ is the average wall temperature during the slow DT timescale. Thus, the following equation can be used to describe the inner household temperature evolution over time:

$$\theta^I(t) \approx (\eta^{AC} p^{AC} + q^{EQ}) R^{EQ} + [\theta^{I,0} - (\eta^{AC} p^{AC} + q^{EQ}) R^{EQ}] \exp(-\alpha^I t) \quad (6)$$

where $\theta^{I,0}$ is the initial inner household temperature, and $\alpha^I = 1/(C^I R^{EQ})$.

Note that the proposed decoupled model (3)-(6) captures the dynamics of the original third-order model (2) with reduced complexity, as the differential equations are decoupled in this case. This not only translates into an improved computational performance, but also serves as a basis for deriving a practical virtual battery model that incorporates solar heat gains and wall/floor thermal dynamics, as discussed next.

III. AGGREGATED TCL CONTROL MODELS

In this section, models needed for the implementation of an aggregate TCL control are proposed. First, a novel virtual battery model for characterizing the aggregate flexibility of a TCL collection is presented. Then, practical challenges found in aggregate TCL control implementations, and how these can be modeled, are discussed. Finally, based on the previous modeling techniques, a new transient model for characterizing the frequency dynamics of an aggregate TCL control is developed.

A. Virtual Battery Model

A new virtual battery model is derived here using the decoupled thermal model presented in Section II-C. The proposed virtual battery model is based on [17], but adapted to incorporate the additional thermal dynamics of detailed high-order household thermal models.

As a first step, a continuous TCL power consumption p^{AC} in (5a) is assumed, which can be further split into steady-state baseline consumption $p^{AC,SS}$ and deviations from it Δp^{AC} , resulting in:

$$p^{AC} = p^{AC,SS} + \Delta p^{AC} \quad (7a)$$

$$0 \leq p^{AC} \leq P^{AC} \quad (7b)$$

where P^{AC} is the TCL rated electrical power. Note that throughout the paper, small case letters are used to identify variables in the model, whereas capital letters represent parameters. Thus, replacing (7a) in (5a), the following thermal-electric differential equation is obtained:

$$C^I \frac{d\theta^I}{dt} = \eta^{AC} (p^{AC,SS} + \Delta p^{AC}) + q^{EQ} - \frac{\theta^I}{R^{EQ}} \quad (8a)$$

$$p^{AC,SS} = \frac{1}{\eta^{AC}} \left(\frac{\theta^{I,SP}}{R^{EQ}} - q^{EQ} \right) \quad (8b)$$

where the TCL baseline consumption (8b) is obtained by assuming a steady-state condition with no TCL power deviations, i.e., $d\theta^I/dt = \Delta p^{AC} = 0$ and $\theta^I = \theta^{I,SP}$.

Defining the auxiliary variable $\Theta = C^I (\theta^I - \theta^{I,SP}) / \eta^{AC}$, and re-arranging (8), one has that:

$$\frac{d\Theta_k}{dt} = -\gamma_k^{AC} \Theta_k + \Delta p_k^{AC} \quad (9)$$

where the subsubscript k has been added for representing a particular TCL in the set of TCLs $\mathcal{K} = \{1, \dots, K\}$, and $\gamma_k^{AC} = (C_k^I R_k^{EQ})^{-1}$. Thus, (9) allows defining the following virtual battery model:

$$\frac{de^V}{dt} = -\gamma^V e^V - d^V \quad (10a)$$

$$-E^V \leq e^V \leq E^V, \quad -P^{V,C} \leq d^V \leq P^{V,D} \quad (10b)$$

where e^V , d^V , and γ^V are the virtual battery's stored energy, discharging power, and self-discharge rate, respectively; and E^V , $P^{V,C}$, and $P^{V,D}$ are the virtual battery's capacity limits for energy, charging power, and discharging power, respectively. The values of the above capacity limits can be determined based on analytical upper/lower bounds, which are presented next and are derived in the Appendix.

Upper bounds can be derived by overestimating the flexibility available from the TCL collection $\mathcal{K}$, yielding the following inequalities for the capacity limits of the virtual battery:

$$P^{V,D} \leq \sum_{k \in \mathcal{K}} p_k^{AC,SS} \quad (11a)$$

$$P^{V,C} \leq \sum_{k \in \mathcal{K}} (P_k^{AC} - p_k^{AC,SS}) \quad (11b)$$

$$E^V \leq \sum_{k \in \mathcal{K}} \left(1 + \frac{|\gamma_k^{AC} - \gamma^V|}{\gamma^V} \right) \frac{C_k^I \Delta \theta_k^{AC}}{\eta_k^{AC}} \quad (11c)$$

where $\Delta \theta_k^{AC}$ is the TCL's k temperature deadband. On the other hand, lower bounds can be determined by underestimating the flexibility of the TCL collection. Note, however, that in this case a range of possible lower bounds are available. In general, any $\underline{E}^V$, $\underline{P}^{V,C}$, and $\underline{P}^{V,D}$ satisfying the following expressions is a valid underestimator of the true virtual battery capacity limits:

$$\beta_k \underline{E}^V \leq \frac{C_k^I \Delta \theta_k^{AC}}{\eta_k^{AC}} \left(1 + \frac{|\gamma^V - \gamma_k^{AC}|}{\gamma_k^{AC}} \right)^{-1} \quad (12a)$$

$$\beta_k \underline{P}^{V,C} \leq P_k^{AC} - p_k^{AC,SS} \quad (12b)$$

$$\beta_k \underline{P}^{V,D} \leq p_k^{AC,SS} \quad (12c)$$

where $\beta_k \geq 0$ is a parameter indicating the share of the total regulation provided by each TCL, with $\sum_{k \in \mathcal{K}} \beta_k = 1$. For example, setting $\beta_k = p_k^{AC,SS} / \sum_{k \in \mathcal{K}} p_k^{AC,SS}$, which maximizes $\underline{P}^{V,D}$, results in:

$$\underline{P}^{V,D} = \sum_{k \in \mathcal{K}} p_k^{AC,SS} \quad (13a)$$

$$\underline{P}^{V,C} = \underline{P}^{V,D} \min_{k \in \mathcal{K}} \left\{ \frac{P_k^{AC} - p_k^{AC,SS}}{p_k^{AC,SS}} \right\} \quad (13b)$$

$$\underline{E}^V = \underline{P}^{V,D} \min_{k \in \mathcal{K}} \left\{ \frac{C_k^I \Delta \theta_k^{AC}}{\eta_k^{AC} p_k^{AC,SS}} \left(1 + \frac{|\gamma^V - \gamma_k^{AC}|}{\gamma_k^{AC}} \right)^{-1} \right\} \quad (13c)$$

which are valid underestimators of the exact virtual battery capacity limits. Note that (11) and (12), while similar to the equations in Theorems 5 and 6 of [17], differ in how γ_k^{AC} and $p_k^{AC,SS}$ are calculated, as these now incorporate the more detailed thermal dynamics of high-order household models.

It is important to highlight that a conservative estimate of the thermal parameters of a “typical” household are needed for computing the virtual battery capacity limits (13), which can either be obtained based on standard household dimensions and materials [21], or through statistical fitting methods [18]. However, if there are significant differences in the various building structures, different virtual battery models (one for each household archetype) may be needed to properly capture these disparities.

B. Practical Challenges

Another relevant issue to consider for the implementation of an aggregate TCL control are its inherent practical challenges, as these can hinder its flexibility and regulation capability. Thus, in what follows, two relevant practical challenges previously identified in real-life applications are discussed.

The first practical challenge relates to the time-variability and uncertainty present in virtual battery models, which stems from their dependence in external factors such as ambient temperature [32]. This applies also to the proposed virtual battery model, as its energy/power capacity limits are dependant on solar radiation heat gains q^C and q^W , and external temperatures θ^A and θ^G (see Sections II-C and III-A), all of which vary over the day and are subject to imperfect forecasts. Thus, to model these additional effects, subscripts for time-intervals $h \in \mathcal{H}$ and Monte Carlo (MC) scenarios $i \in \mathcal{I}$ are considered, yielding the following conservative virtual battery capacity limits:

$$P_h^{V,D,MC} = \min_{i \in \mathcal{I}} P_h^{V,D}(\xi_i) \quad (14a)$$

$$P_h^{V,C,MC} = \min_{i \in \mathcal{I}} P_h^{V,C}(\xi_i) \quad (14b)$$

$$E_h^{V,MC} = \min_{i \in \mathcal{I}} E_h^V(\xi_i) \quad (14c)$$

where ξ_i denotes the sample realization of scenario i , for which different values of $q^C(\xi_i)$, $q^W(\xi_i)$, $\theta^A(\xi_i)$, and $\theta^G(\xi_i)$ are generated and then used to compute the corresponding virtual battery capacity limits. Note that (14) uses a conservative minimum approach to ensure a reliable model. Furthermore, observe that capacity limits in (14) are assumed to remain constant for time-interval h , as external temperatures have relatively slow dynamics (in order of minutes to hours), and second-to-second fluctuations in solar radiation are filtered by the household's wall and internal thermal capacitances.

A second practical challenge relates to communication delays, as these directly impact the response time of aggregate TCL control implementations. As reported in [29], communication delays can reach values of up to 1s in real-life applications, making their incorporation in frequency dynamic models necessary, since the timescale of these dynamics are in the order of milliseconds to seconds in isolated microgrids [12]. To model this effect, an additional communication delay

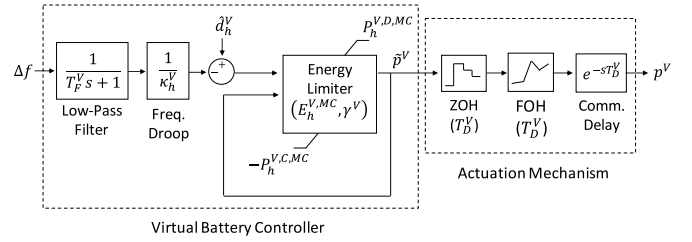


Fig. 2. Block diagram of aggregate TCL control model.

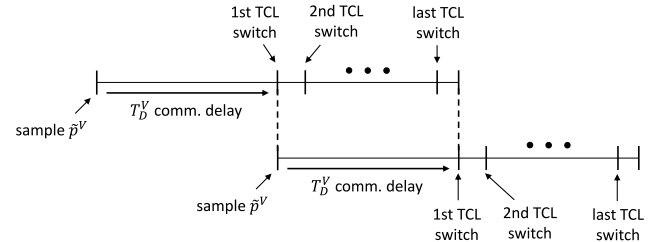


Fig. 3. Coordinated TCL actuation mechanism with communication delays.

block is considered in the proposed frequency dynamic model presented next.

C. Frequency Dynamic Model

Based on the virtual battery model and practical challenges discussed in Sections III-A and III-B, a novel transient model for characterizing the frequency dynamics of an aggregate TCL control is developed here. The proposed transient model is presented in Fig. 2, which considers frequency deviation measurements Δf and virtual battery discharge power output p^V , as well as a Virtual Battery Controller (VBC) and an Actuation Mechanism, which are described next.

The VBC block is composed of: (i) an Energy Limiter block, in which virtual battery state-of-charge (10a) and capacity limits (14) are integrated; (ii) an external discharging power set-point d_h^V ; (iii) a Frequency-Droop block with droop constant κ_h^V ; and (iv) a Low-Pass Filter with time constant T_F^V . The Actuation Mechanism represents the change in dynamics resulting from implementing the virtual battery regulation instruction $\tilde{p}^V$ across the different TCLs, and is composed of: (i) a Zero-Order Hold (ZOH) block with sampling period T_D^V , which is used to represent the inherent discrete-time nature of the control, as residential TCLs typically operate using a ON/OFF switching logic and have short cycling constraints [15]–[17]; (ii) a First-Order Hold (FOH) block, which is used to represent that ON/OFF switching signals are intentionally spread across a T_D^V time interval to prevent large step-wise power changes in the system; and (iii) a Communication Delay block with time delay T_D^V (see Section III-B). Note that the parameter T_D^V is intentionally chosen to be consistent in the previous three blocks, so that a coordinated system response is ensured, as illustrated in Fig. 3.

IV. ENERGY MANAGEMENT SYSTEM

In this section, a novel microgrid EMS embedding the flexibility of an aggregate TCL control is designed. First,

the optimization model considered for the EMS is presented, which builds on the models previously developed in Section III. Then, the general architecture needed for implementing the aggregate TCL control and EMS is discussed.

A. Optimization Model

As previously discussed, modern EMS models for isolated microgrids require the incorporation of frequency dynamics in their decision-making process, as these systems are particularly prone to poor frequency regulation performance. Thus, the proposed EMS model builds on the frequency-constrained EMS model described in [12], which has been adapted to integrate the flexibility of an aggregate TCL control. The specific changes made to the the original frequency-constrained EMS model are described next.

First, the constraints associated to the virtual battery model (10) are defined $\forall h \in \mathcal{H}$ as follows:

$$\hat{e}_h^V = (1 - \gamma^V \Delta T_h) \hat{e}_{h-1}^V - \Delta T_h \hat{d}_h^V + \Delta \hat{e}_h^V \quad (15a)$$

$$-\mu E_h^{V,MC} \leq \hat{e}_h^V \leq \mu E_h^{V,MC} \quad (15b)$$

$$\hat{d}_h^V + \Delta p_h^{V,\chi,\uparrow} + \Delta p_h^{V,r,\uparrow} \leq \mu P_h^{V,D,MC} \quad (15c)$$

$$\hat{d}_h^V - \Delta p_h^{V,\chi,\downarrow} - \Delta p_h^{V,r,\downarrow} \geq -\mu P_h^{V,C,MC} \quad (15d)$$

where ΔT_h is the time step of dispatch time-interval h (typically in the range of minutes in practice); $\hat{e}_h^V$ is the virtual battery's stored energy at the end of time-interval h ; $\hat{d}_h^V$ is the discharge power set-point for the VBC during time-interval h (see Fig. 2); $\Delta p_h^{V,\chi,\uparrow/\downarrow}$ and $\Delta p_h^{V,r,\uparrow/\downarrow}$ are the upward/downward forecast-error and regulation reserves provided by the virtual battery; and $\mu \in [0, 1]$ is a safety derating factor to take into account potential model mismatches for the virtual battery model (e.g., thermal parameter estimation errors or room occupancy heat gains). Note that (15) also includes the decision variable $\Delta \hat{e}_h^V$, which represents the expected increase in the virtual battery's stored energy due to reserve provision, defined as follows:

$$\Delta \hat{e}_h^V = \frac{\varphi_h^X}{2} \Delta T_h \left(\Delta p_h^{V,\chi,\downarrow} - \Delta p_h^{V,\chi,\uparrow} \right) + \frac{\varphi_h^r}{2} \Delta T_h \left(\Delta p_h^{V,r,\downarrow} - \Delta p_h^{V,r,\uparrow} \right) \quad (16)$$

where $\varphi_h^X \in [0, 1]$ and $\varphi_h^r \in [0, 1]$ are the Expected Reserve Utilization (ERU) parameters for forecast error and regulation reserves, respectively, as per [11], [12].

To ensure enough energy is saved in the virtual battery for reserve provision, the following additional conservative constraints are also considered $\forall h \in \mathcal{H}$:

$$(1 - \gamma^V \Delta T_h) \hat{e}_{h-1}^V - \Delta T_h \hat{d}_h^V + \Delta T_h \left(\Delta p_h^{V,\chi,\downarrow} + \frac{1}{2} \Delta p_h^{V,r,\downarrow} \right) \leq \mu E_h^{V,MC} \quad (17a)$$

$$(1 - \gamma^V \Delta T_h) \hat{e}_{h-1}^V - \Delta T_h \hat{d}_h^V - \Delta T_h \left(\Delta p_h^{V,\chi,\uparrow} + \frac{1}{2} \Delta p_h^{V,r,\uparrow} \right) \geq -\mu E_h^{V,MC} \quad (17b)$$

Furthermore, an energy neutrality constraint for the virtual battery is included, as follows:

$$\hat{e}_H^V = E_0^V \quad (18)$$

where E_0^V is the initial stored energy in the virtual battery, and H is the last time index of the set of time intervals $\mathcal{H}$.

Based on the frequency dynamic controller presented in Section III-C, constraints for characterizing the maximum deliverable short-term regulation of the virtual battery can be also defined $\forall h \in \mathcal{H}$ as follows (see Fig. 2):

$$\Delta p_h^{V,r,\uparrow/\downarrow} \leq \frac{\overline{\Delta f}}{f_0} \frac{1}{\kappa_h^V} \quad (19a)$$

$$\kappa_h^V = \bar{\kappa}^V / \min \left\{ P_h^{V,D,MC}, P_h^{V,C,MC} \right\} \quad (19b)$$

where f_0 is the microgrid's nominal frequency, and $\overline{\Delta f}$ is the maximum frequency deviation limit imposed by DER operating standards. Note that in (19), the frequency-droop constant κ_h^V is adjusted based on the virtual battery's power capacity limits for dispatch time-interval h , where $\bar{\kappa}^V$ is a fixed droop value in line with modern operating DER standards [35].

Using the virtual battery constraints (15)-(19), system power and reserve balance constraints can be defined. For power balance, the following constraint is considered $\forall h \in \mathcal{H}$:

$$\sum_{b \in \mathcal{B}} \sum_{l \in \mathcal{L}} (d_{b,l,h}^B - c_{b,l,h}^B) + \sum_{g \in \mathcal{G}} \hat{p}_{g,h}^G + \hat{d}_h^V = \hat{D}_h - \hat{p}_h^W - \hat{p}_h^S - l_{s,h} \quad (20)$$

where $d_{b,l,h}^B$ and $c_{b,l,h}^B$ are the discharging and charging power for battery's b partition l ; $\hat{p}_{g,h}^G$ the average power set-point for generator g ; $\hat{p}_h^W$ and $\hat{p}_h^S$ the forecast average wind and solar powers, respectively; and $\hat{D}_h$ and $l_{s,h}$ are the forecast average demand and load shedding set-points, respectively.

The forecast-error reserve balance can be represented as follows:

$$\zeta_h^X = \sum_{g \in \mathcal{G}} \Delta p_{g,h}^{G,\chi,\uparrow/\downarrow} + \sum_{b \in \mathcal{B}} \sum_{l \in \mathcal{L}} \left(\Delta p_{b,l,h}^{B,D,\chi,\uparrow/\downarrow} + \Delta p_{b,l,h}^{B,C,\chi,\uparrow/\downarrow} \right) + \Delta p_h^{V,\chi,\uparrow/\downarrow} \quad \forall h \in \mathcal{H} \quad (21)$$

where ζ_h^X is the total forecast-error reserve requirement; $\Delta p_{g,h}^{G,\chi,\uparrow/\downarrow}$ are the upward/downward forecast-error reserves provided by generator g ; $\Delta p_{b,l,h}^{B,D,\chi,\uparrow/\downarrow}$ are the upward/downward forecast-error reserves provided by increasing/decreasing battery's b discharging; and $\Delta p_{b,l,h}^{B,C,\chi,\uparrow/\downarrow}$ are the upward/downward forecast-error reserves provided by decreasing/increasing battery's b charging.

Finally, the regulation reserve balance is enforced using the following two separate constraints:

$$\zeta_h^r = \sum_{g \in \mathcal{G}} \Delta p_{g,h}^{G,r,\uparrow/\downarrow} + \sum_{b \in \mathcal{B}} \sum_{l \in \mathcal{L}} \left(\Delta p_{b,l,h}^{B,D,r,\uparrow/\downarrow} + \Delta p_{b,l,h}^{B,C,r,\uparrow/\downarrow} \right) + \Delta p_h^{V,r,\uparrow/\downarrow} + \Delta p_h^{PFC,r,\uparrow/\downarrow} + \Delta p_h^{VFC,r,\uparrow/\downarrow} \quad \forall h \in \mathcal{H}^r \quad (22a)$$

$$\zeta_h^r = \sum_{g \in \mathcal{G}} \Delta p_{g,h}^{G,r,\uparrow/\downarrow} + \sum_{b \in \mathcal{B}} \sum_{l \in \mathcal{L}} \left(\Delta p_{b,l,h}^{B,D,r,\uparrow/\downarrow} + \Delta p_{b,l,h}^{B,C,r,\uparrow/\downarrow} \right) + \Delta p_h^{V,r,\uparrow/\downarrow} \quad \forall h \in \mathcal{H} \setminus \mathcal{H}^r \quad (22b)$$

where ζ_h^r is the total regulation reserve requirement; $\Delta p_{g,h}^{G,r,\uparrow/\downarrow}$ are the upward/downward regulation reserves provided by generator g ; $\Delta p_{b,l,h}^{B,D,r,\uparrow/\downarrow}$ are the upward/downward

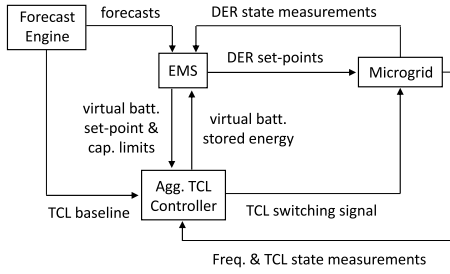


Fig. 4. EMS and aggregate TCL control architecture.

regulation reserves provided by increasing/decreasing battery's b discharging; and $p_{b,l,h}^{B,C,r,\uparrow/\downarrow}$ are the upward/downward regulation reserves provided by decreasing/increasing battery's b charging. Note that (22) is differentiated for the first $\mathcal{H}^r \subseteq \mathcal{H}$ dispatch time-intervals, for which additional short-term regulation capabilities $p_h^{PFC,r,\uparrow/\downarrow}$ and $\Delta p_h^{VFC,r,\uparrow/\downarrow}$ are available.

Based on the above constraints, an EMS model incorporating the flexibility of TCLs can be formulated. The model is based on [12], for which constraints (15)-(22) have been added yielding the following Mixed Integer Linear Programming (MILP) problem that can be solved using off-the-shelf solvers:

$$\begin{aligned}
 & \min_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \quad \mathcal{F}(\mathbf{x}, \mathbf{y}) \\
 & \text{s.t.} \quad \mathbf{x} \in \{0, 1\}^n, \mathbf{y} \geq 0 \\
 & \quad \text{Virtual battery and balancing: (15)-(22)} \\
 & \quad \text{Others: eqs. (6)-(9), (15)-(24), (30)-(31),} \\
 & \quad \text{and (33)-(35) from ref. [12]}
 \end{aligned}$$

where the objective function $\mathcal{F}(\cdot)$ includes thermal generation, battery degradation, and load shedding costs; $\mathbf{x}$ are binary decision variables related to generator dispatch; $\mathbf{y}$ are non-negative continuous decision variables, which include Δp^V ; and $\mathbf{z}$ are free continuous decision variables, which include $\hat{e}^V$ and $\hat{d}^V$. Due to space limitations, the reader is referred to [12] for details on the others variables and constraints of the EMS model.

B. Implementation

The general architecture required to implement the proposed EMS with aggregate TCL control is illustrated in Fig. 4. The EMS includes the optimization model described in Section IV-A, for which dispatch set-points for the DER units and virtual battery are regularly re-calculated and broadcasted based on a Model Predictive Control (MPC) approach [1]–[3]. Note that, as in [17], the proposed architecture follows a centralized approach for TCL control, in which direct communication links between the Aggregate TCL Controller and the different TCLs are needed to ensure a reliable co-ordinated control. Furthermore, observe that local frequency measurements are required for the Aggregate TCL Controller, such that frequency regulation services can be provided by it.

V. COMPUTATIONAL EXPERIMENTS

In this section, simulations to evaluate the benefits of the proposed virtual battery and EMS models are presented and

discussed. First, the general settings considered in the experiments are described. Then, the proposed decoupled household thermal model and virtual battery model are benchmarked against the current state-of-art models. Finally, the implementation of an aggregate TCL control for improved microgrid operations is simulated, and its benefits quantified.

A. General Settings

The experiments are performed assuming the households used for the priority-stack TCL control presented in [17] are located in Kasabonika Lake First Nation (KLFN) (53°35'N 88°39'W) during the fall season [36], given that detailed renewable generation and load data is available for this community, which represents a “typical” isolated microgrid in Canada. The test microgrid corresponds to a CIGRE medium voltage benchmark system, with a 6153 kW peak load, 3 diesel generators (4700 kW combined capacity), 2 combined heat-and-power units (810 kW combined capacity), and a single 1324kW/1324kWh lithium-ion battery [12].

A collection of $K = 1800$ air conditioners operating in heating mode is considered for the aggregate TCL control, which results from considering KLFN's population of 914 people, 3 inhabitants per household, and a scaling factor of $5.933 = 6153\text{kW}/1037\text{kW}$ based on the peak loads of the CIGRE-based microgrid used here and the KLFN microgrid from which renewable generation and load data is extracted. Air conditioner parameters η^{AC} , P^{AC} , $\Delta\theta^{AC}$ are extracted from [17], and a discrete ON/OFF deadband temperature control with a thermostat setpoint $\theta^{I,SP} = 21^\circ\text{C}$ is assumed. Thermal household parameters R^W , R^C , R^F , C^W , C^I , C^F and time-variant variables θ^A , q^W , q^C are calculated using the procedure described in [21], assuming a typical North American house with a 12m x 9m surface area and 5.4m height (2 floors). To represent heterogeneity in the TCL population, random samples of house length, width and height, as well as η^{AC} , P^{AC} , $\Delta\theta^{AC}$, and $\theta^{I,SP}$ parameters, are generated from independent uniform distributions with a maximum deviation of $\pm 5\%$ around their nominal values. A $DT = 5\text{min}$ timescale is used for the decoupled thermal model (3)-(6), resulting in variables $\theta^{W,0}$, $\theta^{W,SS}$, $\theta^{W,DT}$, and q^{EQ} being updated every 5min. A constant ground temperature of $\theta^G = 2.2^\circ\text{C}$ is considered based on KLFN's location [37].

Thermal dynamic simulations are run using the Julia programming language [38] and the DifferentialEquations package [39]. The EMS optimization model is solved in Julia using the JuMP package [40] and the Gurobi solver [41]. Frequency transient simulations are run using MATLAB-Simulink [42]. All simulations are performed on a PC with an Intel Core i7 3.20-GHz processor and 16 GB of RAM under a 64-bit Windows 10 operating system.

B. Comparison of Thermal Models

The accuracy of the first-order and decoupled thermal models presented in Section II is evaluated here. For this, one full-day simulation of the thermal dynamics of a typical household with nominal parameters is performed, and the

TABLE I
COMPUTATION TIMES FOR THERMAL MODELS

Thermal model	Mean time [ms]	Max. time [ms]
Third-order	568	619
Decoupled	97	114

inner, wall and floor temperatures resulting from the different models are compared.

Fig. 5 illustrates the inner, wall, and floor temperatures resulting from the full-day simulation of the first-order, decoupled, and third-order thermal models for two periods of the day, i.e., midday and midnight. Note that the first-order model significantly differs from the highly accurate third-order model after 00:30, suggesting that aggregate TCL control techniques based on a first-order thermal model would show a degraded performance in practical applications. In contrast, observe the close resemblance between the proposed decoupled model and third-order model obtained throughout the day, indicating that the proposed decoupled model is adequate for developing more precise aggregate TCL control techniques, including the proposed virtual battery model presented in Section III-A.

To further illustrate the advantages of the proposed decoupled thermal model, the computation times required to perform the aforementioned full-day simulation were recorded and compared to the ones required for the more detailed and complex third-order model. The resulting mean and maximum computation times are presented in Table I, which are obtained from repeating the full-day simulation 100 times. Observe how the proposed decoupled model yields a significant speed-up in computation times (≈ 5.5 times faster), as temperatures can readily be evaluated through analytical expressions (4) and (6), avoiding the need for time-consuming numerical solutions of the differential equations of the third-order model. It is also important to highlight that the steps used to derive the virtual battery model in Section III-A are based on the simplified mathematical structure of the decoupled thermal model, and thus do not apply for the more complex third-order thermal model.

C. Comparison of Virtual Battery Models

In this section, the virtual battery model presented in Section III-A is compared against the current state-of-art model presented in [17] in terms of modeling accuracy. For the comparison, the implementation of both virtual battery models within an aggregate TCL controller is simulated, and their corresponding estimations of TCL load baseline and battery capacity limits are evaluated. For the current state-of-art virtual battery model, the TCL load baseline is calculated from the steady-state condition of (1) [17]:

$$p_0^{AC,SS} = (\theta^I - \theta^A) / \eta^{AC} R^A, \quad (23)$$

and battery capacity limits are computed based on the “Maximize n_- (Sufficient)” battery model presented in [17], with $R^A = (1/R^C + 1/R^W)^{-1}$ and $\gamma^V = (1/K) \sum_{k \in \mathcal{K}} \gamma_k^{AC}$. In contrast, for the proposed virtual battery model, the TCL load baseline is calculated using (8b), and battery capacity limits are computed based on (13), for which $\gamma^V =$

$(1/K) \sum_{k \in \mathcal{K}} \gamma_k^{AC}$ is also assumed. Thermal simulations are performed using the decoupled household thermal model (see Sections II-C and V-B).

The simulation results for the virtual battery model in [17] are illustrated in Fig. 6a, for which a 2 MW discharging power is requested from the aggregate TCL controller. Note that due to an inaccurate TCL load baseline estimation, the controller incorrectly delivers an additional 457 kW discharging power (22.8% error), resulting in a potentially significant generation-load imbalance for microgrids, whose size are typically in the range of hundreds/thousands of kW. Furthermore, observe that while the virtual battery operates within its energy capacity limits until $T_{lim} = 12.5$ min, the controller loses its tracking capability significantly earlier at $T_{loss} = 9.4$ min. This indicates an inaccurate virtual battery modeling, as the controller should lose its tracking capability only after surpassing the virtual battery’s capacity limits.

The results for the proposed virtual battery model are presented in Fig. 6b, for which the same 2 MW discharging signal is requested. Note that in this case, the controller correctly delivers the 2 MW discharging power, as an accurate baseline estimation is now used. Furthermore, observe that the controller’s tracking capability is lost at $T_{loss} = 13.9$ min, which is only 1.1min after the virtual battery reaches its energy capacity limit ($T_{lim} = 12.8$ min). This is consistent with the conservativeness of the lower bounds (13) used for computing such limits, indicating an accurate virtual battery modeling.

D. Integration in Microgrid Operations

The benefits of harvesting TCL flexibility for an enhanced microgrid operation are evaluated in this section. For this purpose, the implementation of two different EMS models is simulated: (i) the current state-of-art EMS model presented in [12], which incorporates frequency-dynamics but neglects TCL flexibility; and (ii) the novel EMS model presented in Section IV-A, which integrates frequency-dynamics and the flexibility stemming from a coordinated TCL control. Simulations are performed for one full day considering the same settings of the Base case in the computational experiments of [12], which include employing a Model Predictive Control (MPC) scheme with a 5min update rate for the EMS; as well as the use of real measurements from the KLFN microgrid [36], detailed frequency transient models, and frequency and Rate-of-Change-of-Frequency (RoCoF) limits in line with modern DER operating standards [35].

For the EMS model integrating TCL control, the frequency transient model presented in Section III-C is considered, with $\kappa_h^V = 5\%$ [35], and two different communication delays $T_D^V = 1$ [29] and $T_D^V = 0.1$ [43], for which the low-pass filter is tuned to ensure a stable system response, yielding $T_F^V = 5$ and $T_F^V = 1$, respectively. Virtual battery capacity limits (14) are determined using 100 Monte Carlo samples drawn from two independent zero-mean normal distributions for solar irradiance and ambient temperature forecast errors, whose standard deviations are extracted from [11] and [44], respectively; with 100 being the number of samples at which the process converged. Furthermore, a safety derating factor

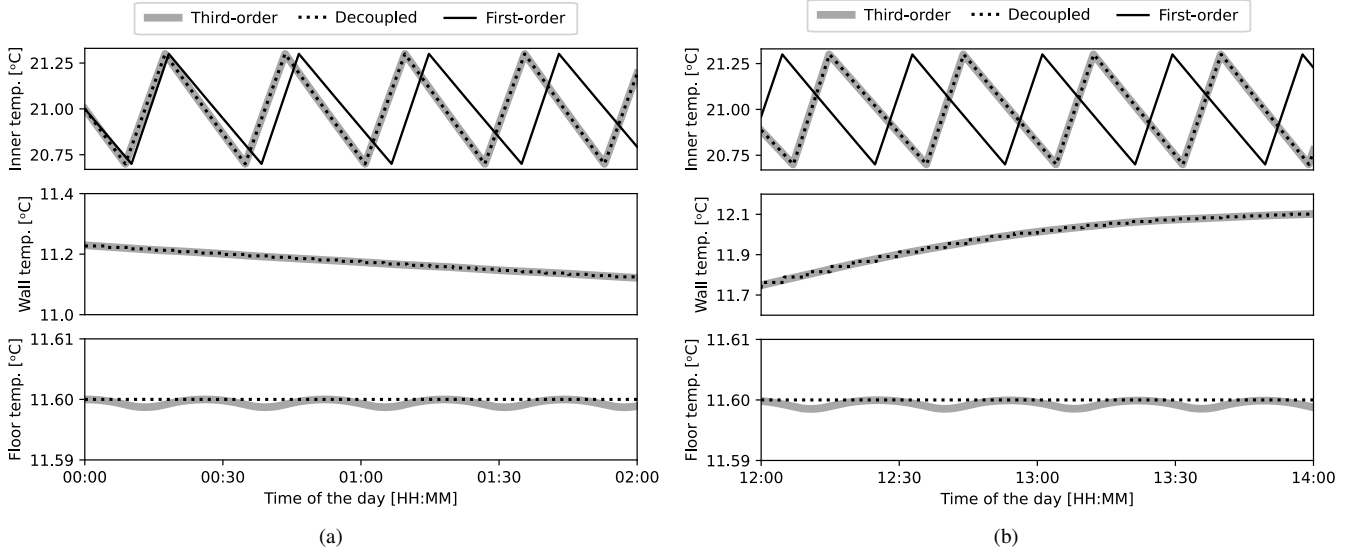


Fig. 5. Household's inner, wall, and floor temperatures for the first-order, third-order, and decoupled thermal models during (a) midnight and (b) midday.

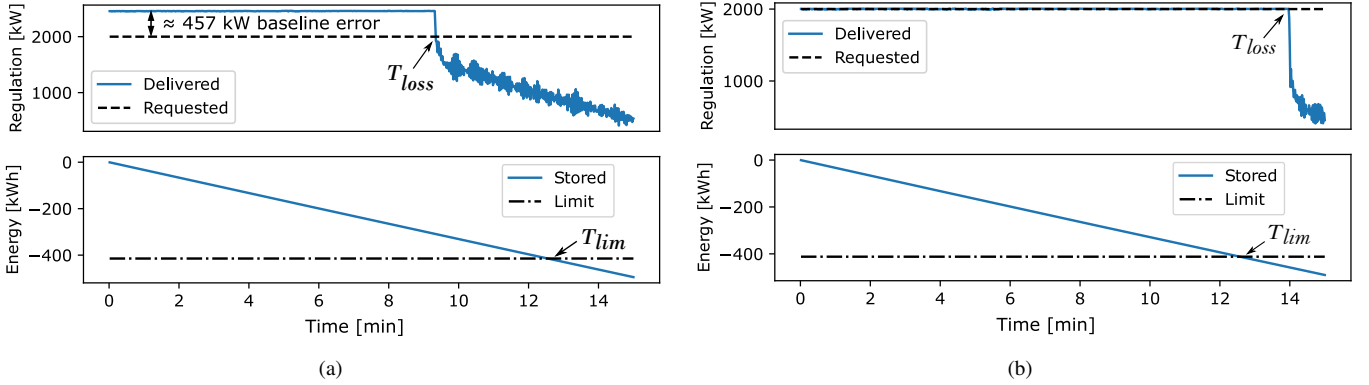


Fig. 6. Regulation and stored energy for (a) the virtual battery model in [17] and (b) the proposed virtual battery model.

of $\mu = 0.95$ is considered for the EMS's virtual battery constraints (15).

A summary of the simulation results is presented in Table II, for which the current state-of-art EMS model without TCL flexibility (first row) is compared against the proposed EMS model embedding TCL flexibility with a communication delay of $T_D^V = 1s$ (second row), and $T_D^V = 0.1s$ (third row). The table includes the following metrics:

- Total costs resulting from daily microgrid operation, which include the costs stemming from thermal generation and battery degradation (Oper. costs), as well as the cost stemming from renewable curtailment and load shedding (RC & LS costs).
- Total accumulated renewable curtailment and load shedding for the simulated day.
- Total non-compliance time, which measures the amount of time for which either frequency or RoCoF limits ($\pm 1.2\text{Hz}$ and $\pm 0.5\text{Hz}$, respectively) are violated.
- Mean and maximum computation times required to solve an MPC iteration of the EMS optimization problem.

From the results, it can be observed that the proposed EMS model integrating TCL flexibility enables daily savings of 908

to 1039 USD (5.3%-6.1%) for a similar frequency regulation security level (non-compliance $\leq 86s$) at sufficiently low computation times for practical applications, since these are much less than the 5min MPC update rate.

The dispatch profiles resulting from the current state-of-art EMS model without TCL flexibility, and the proposed EMS model integrating TCL flexibility ($T_D^V = 1s$), are presented in Figs. 7a and 7b, respectively. Observe how the virtual battery tends to replace the “conventional” battery, as the former can provide energy shifting services without incurring into battery degradation costs. Furthermore, observe how the incorporation of the virtual battery allows injecting more renewable generation into the system, which is reflected in the reduced renewable curtailment and total costs presented in Table II.

E. Validation of TCL Tracking Capability

In this section, the simulation results of Section V-D are used to evaluate the tracking capability of the aggregate TCL controller in practical microgrid operations. For this test, the TCL regulation signal p^V resulting from the frequency transient simulations is used as an input for the aggregate

TABLE II
EMS MODEL COMPARISON

Virtual Batt.	Comm. Delay [s]	Total costs [kUSD]	Oper. costs [kUSD]	RC & LS costs [USD]	Renew. curt. [kWh]	Load shed. [kWh]	Total non-compl. [s]	Comp. time [s]	
No	-	17.02	16.92	106	1321	1.15	86	Mean	Max.
Yes	1	16.12	16.05	62	573	1.83	79	1.3	9.4
Yes	0.1	15.99	15.95	36	336	1.06	72	1.3	9.4

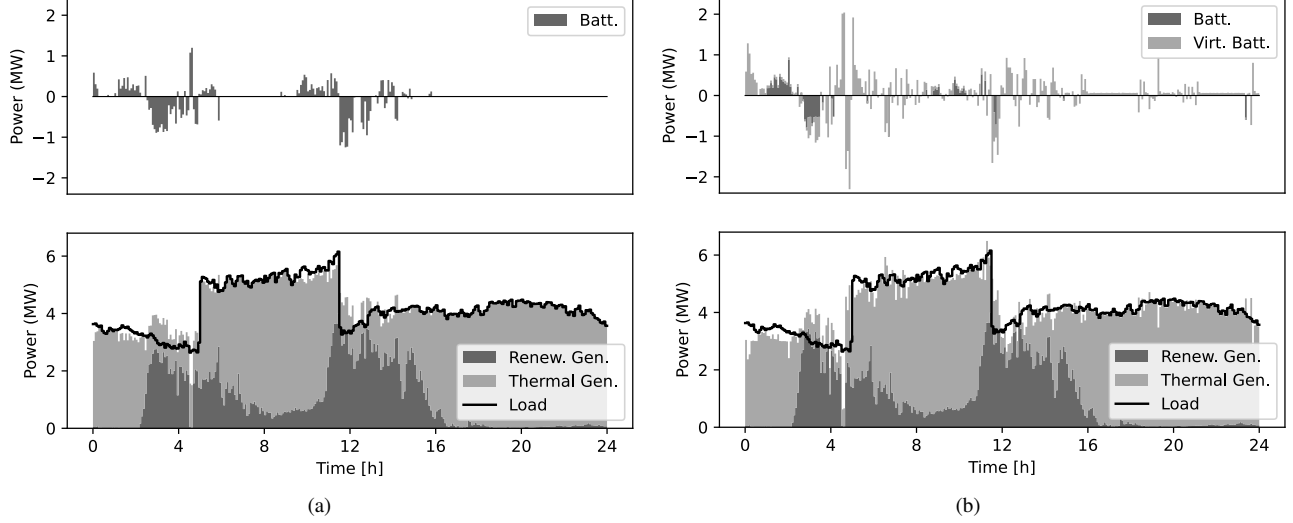


Fig. 7. Dispatch profiles for (a) current state-of-art EMS without TCL flexibility, and (b) proposed EMS model with TCL flexibility and $T_D^V = 1s$.

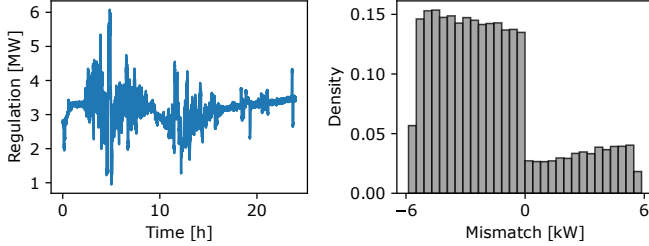


Fig. 8. Estimated regulation provided by the TCL aggregation and mismatch between estimated and actually delivered TCL regulation

TCL controller, and the response of each of the 1800 TCLs composing the TCL aggregation is simulated in detail considering high-order household thermal dynamics and a deadband ON/OFF control. With these results, the differences between the estimated TCL regulation considered in the frequency transient simulations, and actual regulation provided by the TCL population can be calculated.

Fig. 8 illustrates the resulting mismatches between estimated and actual TCL response, considering a communication delay of $T_D^V = 1s$ (second row in Table II). Observe that the TCLs are capable of accurately tracking the requested regulation signal, with a maximum error of approximately the rated power of one TCL (≈ 6 kW). This demonstrates the validity of the proposed virtual battery model and frequency transient model for representing the flexibility and frequency dynamics of an aggregate TCL control.

VI. CONCLUSION

The present paper discussed the integration of TCLs within EMS models for isolated microgrids, and its associated benefits and impacts on operations and frequency control. For this purpose, two computationally efficient aggregate TCL models were developed, which describe the flexibility and frequency dynamics of aggregated TCLs, while taking into account relevant practical challenges such as high-order household thermal dynamics, communication delays, and the presence of model uncertainty and time-variability. These aggregate models were then used to design a practical microgrid EMS incorporating TCLs, and analyze its corresponding impact on microgrid operation and frequency regulation. Computational experiments on a realistic isolated microgrid using detailed thermal and frequency transient models demonstrated the high modeling accuracy of the proposed aggregate TCL models, and showed that an EMS embedding TCL flexibility can enable significant daily savings in the order of 5%-6%, while meeting frequency regulation security levels in line with modern DER operating standards.

APPENDIX

The steps used to determine the energy and power capacity limits of the virtual battery model are presented here, which are based on the derivation originally made in [17] for simple first-order household thermal models. For the derivation, (9) and (10a) are mapped to the Laplace domain, as follows:

$$s \Theta_k(s) = -\gamma_k^{AC} \Theta_k(s) + \Delta p_k^{AC}(s) \quad (24a)$$

$$s e^V(s) = -\gamma^V e^V(s) - d^V(s) \quad (24b)$$

where $\Theta_k(0) = e^V(0) = 0$ assuming $\theta^I(0) = \theta^{I,SP}$.

A. Upper Bounds

Define the virtual battery power discharge as the sum of the TCLs baseline deviation, as follows:

$$d^V = - \sum_{k \in \mathcal{K}} \Delta p_k^{AC} \quad (25)$$

Combining (25) and (24), the following expression for the virtual battery's stored energy can be obtained:

$$e^V(s) = \sum_{k \in \mathcal{K}} \left(1 + \frac{\gamma_k^{AC} - \gamma^V}{s + \gamma^V} \right) \Theta_k(s) \quad (26)$$

for which the property $\Pi(s) = X(s)Y(s) \Rightarrow \|\pi(t)\|_\infty \leq \|x(t)\|_1 \|y(t)\|_\infty$ can be exploited, where $\pi(t)$, $x(t)$, and $y(t)$ are the inverse Laplace transforms of $\Pi(s)$, $X(s)$ and $Y(s)$, respectively. Consequently, the following inequality for the virtual battery's energy capacity limit can be obtained:

$$\|e^V(t)\|_\infty \leq \sum_{k \in \mathcal{K}} \left(1 + \frac{|\gamma_k^{AC} - \gamma^V|}{\gamma^V} \right) \frac{C_k^I \Delta \theta_k^{AC}}{\eta_k^{AC}} \quad (27)$$

since $\|\Theta_k(t)\|_\infty = C_k^I \Delta \theta_k^{AC} / \eta_k^{AC}$, as residential TCLs typically have a deadband control with a maximum temperature range of $\Delta \theta_k^{AC}$.

The virtual battery's power capacity limits can then be obtained from combining (7) and (25), resulting in:

$$\sum_{k \in \mathcal{K}} (p_k^{AC,SS} - P_k^{AC}) \leq d^V \leq \sum_{k \in \mathcal{K}} p_k^{AC,SS} \quad (28)$$

B. Lower Bounds

The baseline deviation for each TCL can be defined as a share of the total virtual battery discharge as follows:

$$\Delta p_k^{AC} = -\beta_k d^V \quad (29)$$

where $\beta_k \geq 0$ and $\sum_{k \in \mathcal{K}} \beta_k = 1$. Combining (24) and (29) yields:

$$\Theta_k(s) = \sum_{k \in \mathcal{K}} \beta_k \left(1 + \frac{\gamma^V - \gamma_k^{AC}}{s + \gamma_k^{AC}} \right) e^V(s) \quad (30)$$

for which the property $\Pi(s) = X(s)Y(s) \Rightarrow \|\pi(t)\|_\infty \leq \|x(t)\|_1 \|y(t)\|_\infty$ can again be exploited, resulting in:

$$\|\Theta_k(t)\|_\infty \leq \beta_k \left(1 + \frac{|\gamma^V - \gamma_k^{AC}|}{\gamma_k^{AC}} \right) E^V \quad (31)$$

where the virtual battery's energy capacity limit $E^V = \|e^V(t)\|_\infty$ can be chosen such that:

$$\beta_k E^V \leq \frac{C_k^I \Delta \theta_k^{AC}}{\eta_k^{AC}} \left(1 + \frac{|\gamma^V - \gamma_k^{AC}|}{\gamma_k^{AC}} \right)^{-1} \quad (32)$$

in order to guarantee that that TCLs remain within their temperature deadband, i.e., $\|\Theta_k(t)\|_\infty \leq C_k^I \Delta \theta_k^{AC} / \eta_k^{AC}$.

The virtual battery's power capacity limits can then be obtained by combining (7) and (29), resulting in:

$$\beta_k P^{V,C} \leq P_k^{AC} - p_k^{AC,SS} \quad (33a)$$

$$\beta_k P^{V,D} \leq p_k^{AC,SS} \quad (33b)$$

which ensures electric power consumption of each TCL will remain within its limits, i.e., $0 \leq p_k^{AC} \leq P_k^{AC}$.

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