# On Optimal Online Policies in Energy Harvesting Communication Systems: Effects of Quality of Service and Battery Requirements

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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**Abstract**

We study the problem of finding optimal transmission policies in a point-to-point energy harvesting communication system with continuous energy arrivals in causal setting. In particular, we investigate bounds on the long-term achievable average throughput and corresponding power policies, where energy packets of random size arrive at the transmitter at random times, modelled as a compound Poisson dam.

In this work, we also account for battery life and quality of service of the users. We thus formulate non-linear constrained maximization problems. Specifically, we limit the instantaneous battery depletion rate (i.e., transmission power) as well as its variation to account for prolonging the battery life. Moreover, we limit the variation of instantaneous throughput to maintain it to a constant level to account for improving the quality of service.

Using the theory of calculus of variations as a powerful mathematical tool, we derive necessary conditions in the form of first order non-linear ODEs, for local and thus global optimality of solutions to the optimization problems. We also obtain numerical as well as analytical upper bounds for the problem of constrained proper functions of transmission power. Numerically solving the ODEs for the case of a Gaussian channel, we also compute achievable throughputs and locally optimal power policies as a function of battery capacity and remaining battery charge, respectively.

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**Dedication**

To my family

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# Chapter 1

# Introduction

## 1.1 Wireless Sensor Networks

A wireless sensor network is a group of small, lightweight wireless nodes, deployed in large numbers, interconnected by wireless networks. These nodes gather information about physical or environmental conditions, such as temperature, sound, pressure, humidity, etc. Recent advances in micro-electro mechanical system technology has made the building of these sensors possible. Each node consist of three main parts: i) a sensor part that senses the environment and gathers information, ii) a processing part that processes the sensed data, and iii) a communication part that enables the node to communicate with the neighbouring nodes. Due to the small size of each node, each individual node has a limited sensing region and processing power. However, since these nodes collaborate on collecting data, a large group of them can accurately monitor a wide region.
Wireless sensor networks can be used in different areas including health care [5], utilities, and remote monitoring. Wireless sensor networks open new possibilities in health care because they can collect data in a less invasive manner compared to other methods. They are also cost efficient which makes them a good choice for utilities, such as electricity grid, street lights, and municipal water. Remote monitoring applications include:

- environmental monitoring of air, water, and soil [6],
- structural monitoring for buildings and bridges [7],
- industrial machine monitoring [8],

- process monitoring,
- asset tracking.

### 1.1.1 Issues and Challenges in Designing a Sensor Network

The major issues and challenges in designing a sensor network are as follows:

- Each sensor can be located in a random spot and in general the whole sensor network doesn't need to conform to a regular topology. Once deployed, human intervention would be costly. Hence, the setup and maintenance of the network should be entirely autonomous.
- Available energy is one of the biggest concerns in operating sensor nodes. Due to remote location of sensors, they usually rely only on their battery for power, which in many cases cannot be replaced or recharged. Consequently, one should consider the available energy as an important factor while designing protocols. There are several works on this topic which let the designer to sacrifice the accuracy and transmission rate in favour of conserving energy.
- The detected events by the sensor nodes should be temporally ordered without ambiguity. Hence, all the sensors should be synchronized with each other, so that TDMA schedules can be imposed.
- The routing protocol used in a sensor network should be able to dynamically add or drop sensor nodes in their paths. This is an important factor since the failure of nodes is frequent and new nodes are added in replacement.
- Real-time communication over sensor networks must be supported through provision of guarantees on maximum delay, minimum bandwidth, or other QoS parameters.
- Since sensor networks have many military application, they often use secure protocols.

## 1.2 Energy Harvesting

Energy sources such as solar energy, wind energy and mechanical energy can often be found in abundance in the environment. However, unlike the energy from fossil fuels, the energy from these sources is often found in many small and unpredictable amounts. Due to this fact, until recently, these sources have not been considered as viable sources of energy to perform useful work.

Energy harvesting is the process by which minute amounts of energy are derived from external sources, and accumulated and stored for later use. As mentioned in section 1.1, wireless sensors use batteries as their source of energy and in most applications battery replacement is cumbersome or almost impossible. This issue can be solved by equipping the nodes with energy harvesting technology [9].

### 1.2.1 Common Sources of Energy Harvesting

Energy harvesting methods can be classified based on the source of energy that is scavenged. We list the most common renewable sources that are utilized.

1. Mechanical Vibration: Vibrations can be typically converted to electrical energy using three mechanisms: piezoelectric, electrostatic and electromagnetic.

    (a) Piezoelectric Materials: These materials have a unique property that lets them generate electrical energy when a mechanical load such as pressure or force is applied on them. Due to this property, piezoelectric materials have become a viable source of energy in energy harvesting. The earliest example of using these materials was extracting electrical energy from the impact of dropping a steel ball bearing onto a piezoelectric transducer [14]. Age, stress and temperature can affect the properties of piezoelectric materials. One of the advantages of using these materials is that they can generate electrical energy without relying on any other additional components. On the other hand, these materials are fragile and are prone to charge leakage [15].

    (b) Electromagnetic Energy Harvesting: This method utilizes the principle of electromagnetic induction to generate electrical energy. One can generate electromagnetic induction for energy harvesting with the help of permanent magnets, a coil and a resonating cantilever beam [16].

    (c) Electrostatic (Capacitive) Energy Harvesting: In this method electrical energy is generated by moving the plates of an initially charged varactor. Unlike piezoelectric materials, an initial source is needed to charge the capacitor. However, compared to the first two methods, electrostatic energy harvesters are cost efficient as they do not require a magnet or piezoelectric material that can be quite

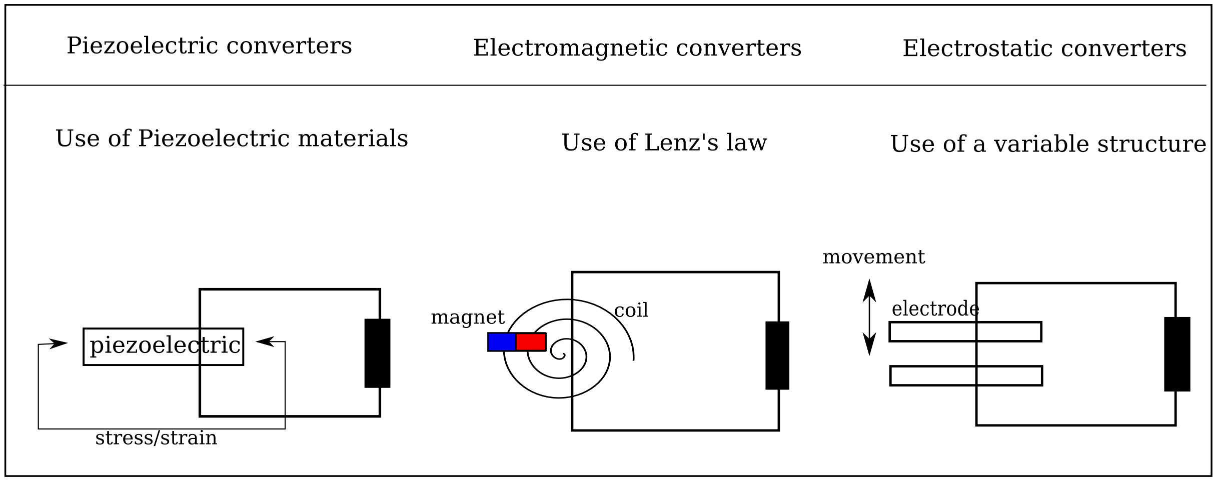


Figure 1.1: Mechanical-to-electrical converters [11]

expensive. Consequently, electrostatic energy harvesters can be an attractive solution to develop low-cost sensor networks.

In most cases, piezoelectric and electrostatic devices are used for small scale energy harvesters, while electromagnetic converters are utilized for larger devices. Figure 1 illustrates the three methods mentioned above.

2. Thermal Energy: Thermoelectric generators are able to generate electrical energy from temperature differences based on the principle of thermoelectricity. In [20] a method to generate electrical energy based on the temperature difference of soil and air is presented. Thermoelectric generators are considered to be reliable and usually require low maintenance. However, as indicated in Table 1.1, they have low energy conversion efficiency and due to this their usage is limited.

3. Solar: Light energy usually obtained from sunlight can be converted to electricity by photovoltaic cells. These cells exhibit a property called the photoelectric effect that allows them to absorb photons of light and generate electricity. This effect was first discovered by the French physicist, Edmund Bequerel, in 1839 and the first photovoltaic module was built by Bell Laboratories in 1954 [17] [18]. Compared to other methods of energy harvesting this method provides the highest power density as shown in Table 1.1. However, in this method the supply of energy is not guaranteed to be always available and it's much dependant on weather conditions and daylight.

Table 1.1: Energy Harvesting Sources [19]

| Harvesting Method | Power Density |
|---|---|
| Solar Cells | 15 mW/cm$^3$ |
| Piezoelectric | 330 $\mu$W/cm$^3$ |
| Vibration | 116 $\mu$W/cm$^3$ |
| Thermoelectric | 40 $\mu$W/cm$^3$ |

## 1.3 Effect of Discharging Pattern on Battery Life

The capacity of a battery or battery capacity is the amount of energy that can be stored in a battery, and is typically measured in either watt-hours (Wh) or kilowatt-hours (kWh). Battery capacity can also be measured in units of (Ah) and is defined as the number of hours for which a battery can sustain a current equal to the discharge rate at a fixed voltage called the nominal voltage of the battery [2]. However, the actual drain pattern can dramatically change the rated battery capacity that is reported on the battery data sheet. In the following we discuss battery capacity under different loading patterns.
In almost every battery the available capacity is highly dependant on the rate of discharge. Discharging the battery with a high discharge current can reduce the amount of energy that can be extracted from the battery or in other words it reduces the capacity of battery. Fig 1.3 approximately shows the output voltage of a typical battery as it is discharged with different values of continuous currents. A figure showing accurate values can be found in [10]. It can be seen that when discharge current is low and close to the rate stated in the battery data sheet the full capacity can be accessed before the voltage drops precipitously while for high discharge currents, the effective capacity in the battery is reduced [10].
There are many applications that require the battery to provide momentary loads at high power. GSM (Global System for Mobile Communications) for a mobile phone is such an example. However, compared to a continuous discharge rate, a pulsed discharge rate can dramatically decrease the number of cycles that a battery can be used before its capacity is diminished [3]. Fig 1.3 roughly shows the number of cycles a battery can last under different load conditions. The graph with accurate numbers can be found in [12].

## 1.4 Literature Review

In this section, we summarize various studies regrading energy management in energy harvesting. In general the efforts in the this area can be categorized into two scenarios:

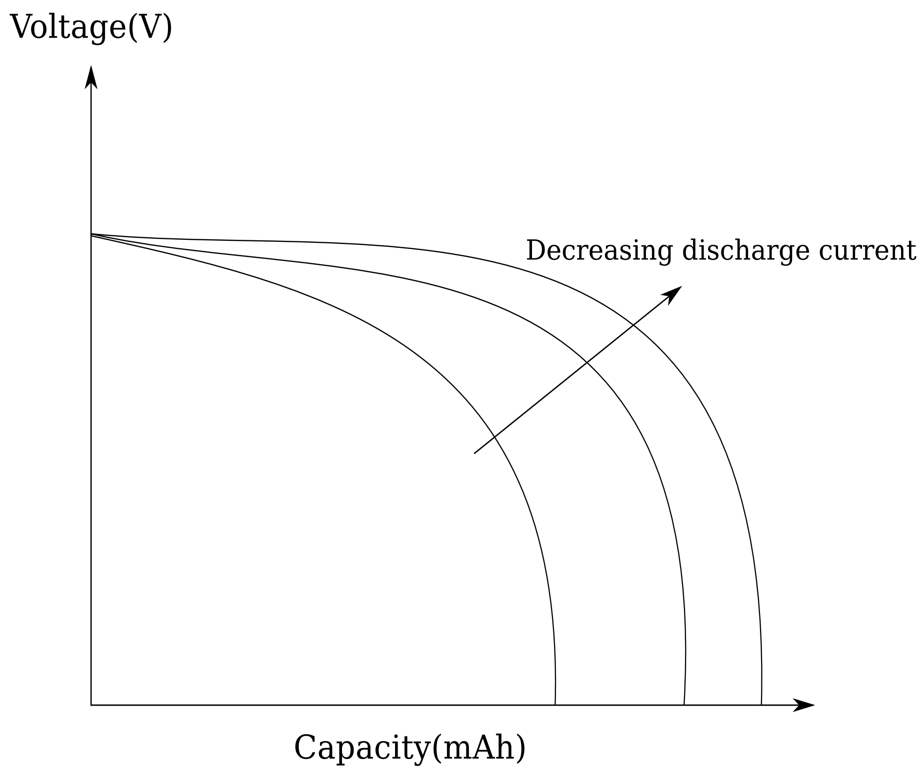


Figure 1.2: Continuous Discharge Patterns.

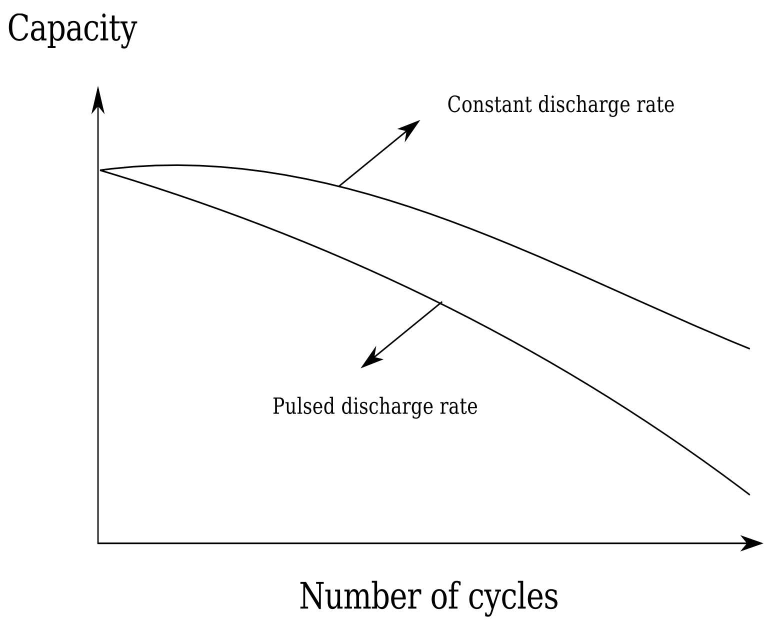


Figure 1.3: Cycle life of battery under different load conditions.

- Offline scenario: Investigations that lie into this scenario assume that all the information regarding the energy harvesting profiles of the nodes are known deterministically before the transmission starts. In particular, all the results are deduced based on the prior knowledge of the exact arrival times and the amount of harvested energy. Although these results may not have practical application due to the unrealistic assumptions, One can use them as an upper bound for the achievable performance in more realistic scenarios.

- Online scenario: In this scenario, the device only has causal knowledge of the energy harvesting profiles of the nodes. In other words, the device only knows the history and current state of the device. However, in some works it is assumed that there is stochastic information about the harvesting process.

### 1.4.1 Offline Energy Management

The general communication model considered in these works is a fading channel with additive Gaussian noise where transmission is continuous and the rate can be controlled via power. Specifically, the transmitter selects a power control policy $P(t)$, achieving

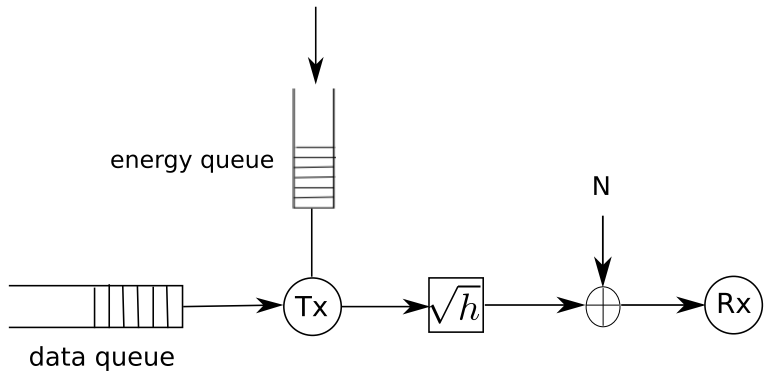


Figure 1.4: Energy harvesting system model

transmission rate $r(p(t))$ where $r(.)$ is referred to as power rate function. The transmitter stores the arriving energy in an energy queue and it also has another queue for storing the arriving data. Fig. 1.4 depicts the model.

In [22] an optimal power allocation is found that maximizes the total data transferred under a deadline $T$. The optimization problem defined in this work is subject to two constraints, the energy causality constraint and the finite-storage constraint. That is to say that the battery is not able to store more energy than its capacity and energy can not be utilized before it is stored in the battery. These two constraints are illustrated in Fig. 1.5. The upper staircase represents the causality constraint and the lower stair case represents the finite-storage constraint. Any feasible power allocation should lie in the tunnel between the two staircases. It is shown that due to the concavity of the power rate function, the optimal policy is the tightest string that lies in the feasible tunnel.

The same model is also considered in [23]; however, in [23] a power allocation that minimizes the transmission time $T$ which is the time required to send a certain amount of data is found. In [23], two scenarios are considered, in the first scenario all the data is available at time $t = 0$ and in the second scenario, packets of data arrive during the transmission. It is shown that the optimal transmission power does not change between data packets or energy arrivals. Based on this property different algorithms are proposed to obtain the optimal transmission power.

In reference [24], authors solve the same maximization problem as [22] using Lagrange multipliers and present another approach for finding the optimal transmission power, called directional water-filling algorithm. In this approach, due to the concavity of the rate function, the goal is to allocate the water(energy) equally in a rectangle, which corresponds to the concept of time. At the points of energy arrivals there are walls with right permeable taps, and these taps only permit the water to flow from right to left. In other words the

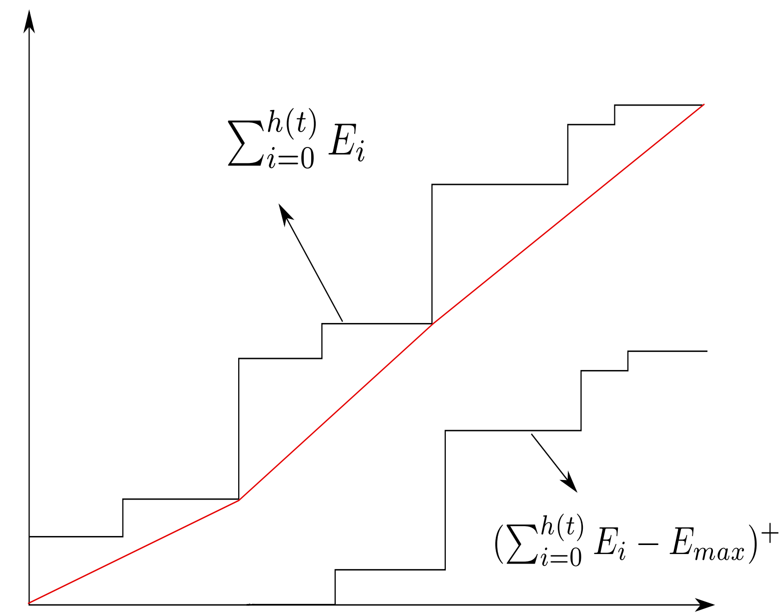


Figure 1.5: Feasible tunnel [10]

energy can be used and stored however, the energy arriving in the future can not be used and this implements the causality constraint. Moreover, in [24] it is assumed that these taps can not transfer more than $E_{max}$ amount of water, and this handles the finite-storage constraint. Fig. 1.6 depicts different possible scenarios in this algorithm. Then in [24], the authors use this approach to find the optimal policy for the fading channel, assuming the channel state changes M times through the transmission time.

The studies mentioned assume that the transmitter gathers the harvested energy at discrete times. A more recent work, [25], considers the same framework with continuous energy arrivals. Using convex analysis, it first shows that the optimal policy for an infinite capacity battery is the boundary of the convex hull of the region above $h(t)$ where $h(t)$ represent the cumulative harvested energy up to time $t$. Following a similar approach, an optimal policy is presented for the finite capacity battery. In both cases, the optimal policy has similar structure to the optimal policy obtained in [22]. Fig.1.7 shows an example of an optimal policy obtained for continuous energy arrivals.

The works we summarized above study the case of a single-user channel. In [26], the energy harvesting problem with a broadcast channel is considered. It is shown that the

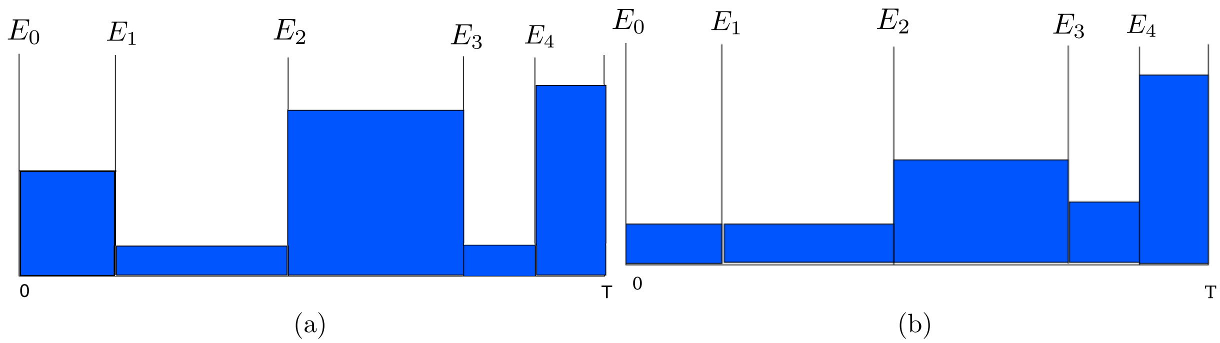


Figure 1.6: Example of directional water-filling algorithm. (a) Initial levels. (b)Final levels after running the algorithm

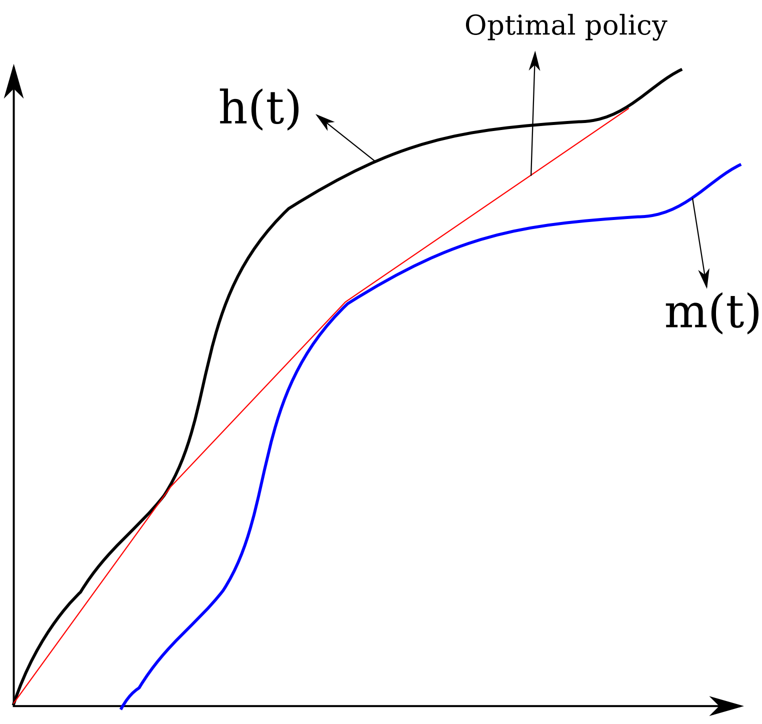


Figure 1.7: Optimal transmission policy

optimal total transmit power is the same as the optimal transmit power with a single-user channel. The authors prove that the optimal solution split the power based on a cut-off level. For example, for the case of two users, if the optimal total transmit power is lower than the cut-off level all the power will go to the stronger user and all the power above the the cut-off level goes to the weaker user. The case of the multiple access channel is studied in [27]. There, a proposed generalized iterative backward water-filling algorithm simplify the problem of minimizing the transmission completion time into a convex optimization problem and obtain the optimum energy management. In [28], the authors consider the case of a two-user interference channel. An iterative algorithm to maximize the short term sum-throughput is used. In [29], the authors consider the classic three-node relay channel where both the relay and source harvest energy and can adapt their transmission power instantaneously. An optimal transmission scheme that maximizes the short term throughput is presented for the case of full-duplex relay. The more general case is studied in [39]. Authors study the case where the source has the option to share its harvested energy with the relay. It is assumed that energy transfer efficiency is $\alpha$ i.e., when the source node sends $\sigma$ amount of energy the relay receives $\alpha \times \sigma$, where $\alpha \leq 1$. The problem is formulated in a similar fashion to [39], modifying the causality constraint to account for the shared energy between the source and the relay. Then some necessary conditions for the optimal solution are found. Moreover, two specific scenarios are studied in detail. In the first scenario, relay energy arrivals are more frequent at the beginning and decrease towards the end of the communication and an optimal solution for this case is presented. In the second scenario, it is assumed that the source only harvests a packet of energy at the beginning and through the rest of the communication, the relay harvests all the energy. In this case, it is argued that in the optimal scenario, the source should share $\sigma^*$ amount of its energy with the relay at the beginning of the transmission, where $\sigma^*$ can be calculated by solving a fixed point equation.

In [30] it is assumed that the transmitter spends a processing energy cost when it is transmitting. This adds an additional processing cost condition to the causality and finite-storage conditions. The authors find the optimal policy which maximizes the throughput. A directional glue pouring algorithm is introduced to solve the maximization problem which is a modification to the glue pouring algorithm in [31], where the threshold power level $p^*$ used in the algorithm is calculated by solving the equation:

$$\frac{\log(1+p^*)}{p^*+\epsilon} = \frac{1}{1+p^*}, \tag{1.1}$$

and $\epsilon$ is the processing energy cost. Moreover, glue is permitted to only flow to the right in order to conserve the causality condition.

In [32], authors take into account various constraint on the battery and practical imperfections that it might have. In particular, for constant leakage rate it is shown that by modifying the feasible tunnel introduced in [22], the optimal policy can be achieved with a similar procedure.

### 1.4.2 Online Energy Management

In this section, we summarize works that study the problem of energy management in the online scenario, i.e., unlike the offline case the system has only causal knowledge of harvesting profile of the nodes and should adopt the power policy based on the system state.

In [33], authors consider a slotted-time system where in each slot the transmitter has a data packet to send, it can either send the packet in that time slot or it should discard it. Moreover each data packet has a random importance value $V_k$ and battery status in slot $k$ is determined by $B_{k+1} = \min\{[B_k - Q_k]^+ + E - k, E_{max}\}$, where $E_{max}$ is capacity of battery, $Q_k$ is the action process which is one if the data packet is transmitted at slot $k$ and zero otherwise, and $E_k$ is the amount of harvested energy during slot $k$. The goal is to find an action process which maximizes the long-term average reward, defined as

$$G(\mu) = \lim_{K\to\infty} \inf \frac{1}{K}\mathbb{E}\Bigg[\sum_{k=0}^{K-1} Q_k V_k\Bigg], \tag{1.2}$$

where the action process is drawn according to $\mu$. The optimal strategy can be numerically calculated using techniques, such as linear programming. However, a three-level suboptimal policy is presented which is conservative when the battery is low and aggressive when the battery is in high energy states. In [34], the authors study the special case of the mentioned problem when the system consists of two energy harvesting transmitters. It is shown that the optimal policy dictates the transmission of data when the importance of the packet is above a given threshold.

In [35], the authors study the problem of throughput maximization in a two-hop amplify-and-forward relay network where the channel is assumed to be a fading channel represented by a first-order Markov chain. The finite state space $\mathcal{S}$ is defined as

$$\mathcal{S} = \mathcal{B}_s \times \mathcal{B}_r \times \mathcal{G}_{sr} \times \mathcal{G}_{rd}$$

where $\mathcal{B}_s \triangleq \{0, \mathcal{B}_s^{max}/n, \ldots, \mathcal{B}_s^{max}\}$ and $\mathcal{B}_r \triangleq \{0, \mathcal{B}_r^{max}/n, \ldots, \mathcal{B}_r^{max}\}$ are the sets of battery levels at the source and relay respectively and $\mathcal{G}_{sr} \triangleq \{g_{sr}^1, g_{sr}^2, \ldots, g_{sr}^m\}$, $\mathcal{G}_{rd} \triangleq \{g_{rd}^1, g_{rd}^2, \ldots, g_{rd}^m\}$

are the sets of possible states of S-R and R-D links represented by the first-order Markov chain. Using the defined state space the problem is converted into a discrete dynamic programming and an optimal power policy is obtained.
In [42], authors study the case of lossy joint source-channel coding in a point-to-point channel. It is assumed that the arrival times of energies follows a Poisson process, with the battery charge modelled as a compound Poisson dam. Using this model and calculus of variation technique two non-linear ODEs are derived that characterize the optimal power policy and mismatch factor minimizing the distortion at the receiver. Multiple access communication system with energy harvesting nodes is considered in [46]. The authors consider a similar model for the energy arrivals, with a similar approach to [42], they show that the optimal policy which maximize the throughput can be derived by solving a system of simultaneous partial-integro-differential equations. They present an iterative algorithm to solve the equations.
The works we mentioned above all assume that energy harvesting devices have perfect knowledge of the state-of-charge in the battery. However, in practical world this is not always the case, as there might be up to 30% error estimating the state of charge in some batteries [36]. In [37], the authors investigates the performance of different transmission policies with imperfect knowledge of the state of charge. It has been shown that the loss is negligible compared to the case with perfect knowledge about the state-of-charge. In [38], the authors consider the case where multiple sensors randomly access the channel to send their data packets with random importance to the Fusion Center (FC). The authors study the problem of designing optimal random access policies that maximize the network utility, assuming simultaneous transmission from multiple nodes causes collision and packet loss. The problem is formulated as a game and using techniques in game theory the local optimum is found and an algorithm to compute it is presented.
In [40], the authors consider the realistic case where the battery suffers from battery degradation that can decrease the lifetime of the harvesting device. Using Markov chains, a general framework to represent the degradation status of the battery is introduced. Based on this framework an optimization problem to maximize the battery lifetime subject to a minimum quality of service is formulated. Then the authors show that the optimization problem can be solved using sequential linear programming optimization algorithm.

## 1.5 Contributions

In this thesis, we derive transmission power policies that maximize the long-term average transmission rate while forcing constraints to improve battery life as well as user quality of service. We consider a simple point-to-point communication system, where the transmitter uses an energy harvesting module to scavenge and save energy for communication tasks. The major contributions of this thesis are as follows:

- To avoid policies that can result in shortening battery life, we consider a new constraint (compared to previous work) that limits the variations of the battery depletion rate (i.e., transmission power). This in turn decreases the internally dissipated power in the battery and thus improves battery efficiency and life time.
- In a separate problem, we limit the maximum instantaneous discharge rate. This is justified by the fact that most batteries have a maximum safe discharge rate.
- We also consider a problem where variations in the instantaneous throughput are constrained. This new constraint accounts for the quality of service provided to the users.
- We apply a calculus of variations technique combined with the Lagrange multiplier method to derive necessary conditions for achievable locally optimal transmission policies that maximize long-term average throughput in each optimization problem.
- Using positive recurrence arguments for the storage process, we prove a lemma which helps derive an analytical upper bound for the problem of peak power constraint. We also show that under certain conditions this bound meets another bound derived in a previous work [46].
- We show for the special case of the Shannon rate function that the variance of the instantaneous rate is bounded above by the variance of the power. This, in particular, shows that for the special case of a Gaussian channel that a relatively small variance in the transmission power also achieves a good quality of service.
- For the case of a Gaussian channel, we numerically show that by capping the peak power, the gap between average achievable throughput and the upper bound is vanishing. Numerical simulations also reveal that the gap between average throughput with limited power variance and that of limited peak power is almost negligible.

## 1.6 Thesis Organisation

The remainder of this thesis is organized as follows. In chapters 2, we state our communication model as well as the storage model. Specifically, we consider a point-to-point communication system with non-causal knowledge of the energy arrivals and by modelling the arrivals as a Posisson process we model our battery charge processes. Moreover, we formulate our problem as a maximization problem. In chapter 3, we study the problem with three different constraint. First we consider the problem of throughput maximization while we constrain the variance of power policy. Second, we solve the problem while we limit the peak power. Finally, we derive a necessary condition for the optimal policy that maximizes the throughput while the variance of transmission rate is limited. Moreover, we provide an algorithm which can be used to find the achievable solution. We provide our simulations and numerical results in chapter 4. Finally we conclude our work in 5.

# Chapter 2

# Preliminaries

## 2.1 Communication Model

We consider a point-to-point communication system with a transmitter-receiver pair, where the transmitter uses an energy harvesting module, i.e, a storage unit of finite capacity which is capable of harvesting ambient energy from its environment, and the receiver has enough energy to decode its received signal at any rate. Furthermore, we assume that the instantaneous rate of the transmitter is only a function of instantaneous transmission power, i.e., $r(p)$, where the function $r(.)$ satisfies the following conditions:

[R1] $r(0) = 0$,

[R2] $\forall\, p > 0, \;\; r(p) > 0$,

[R3] $r(p)$ is three times continuously differentiable,

[R4] $r(p)$ is non-decreasing,

[R5] $r(p)$ is concave.

In particular, the Shannon rate function, $r(p) = \log_2(1 + p/N_0)$, which will be used later in the thesis, satisfies the above conditions. We also assume that the communication is carried over a sufficiently large block length so that $r(p)$ has operational significance. Moreover, the transmission power (average power constraint) is fixed over each block, but it can change from block to block. We define the long term average transmission rate as

$$R := \lim_{T\to\infty} \frac{1}{T} \int_0^T r(p(s))ds. \tag{2.1}$$

## 2.2 Storage Model

We assume the transmitter has a battery of finite size $L$ to store the harvested energy, and thus the consumed power of the transmitter will become a function of available energy in the battery denoted by $Z(t)$. Consequently, we rewrite (2.1) as

$$R := \lim_{T\to\infty} \frac{1}{T} \int_0^T r(p(Z(s)))ds. \tag{2.2}$$

We further assume that packets of energy $\{U_i\}_{i=0}^{\infty}$ arrive at discrete and i.i.d. random time instants $\{T_i\}_{i=0}^{\infty}$, where $T_0 < T_1 < \cdots$, which are a homogeneous Poisson point process with intensity $\delta$. Energy packets are also i.i.d. with cumulative distribution function $B(x) = \Pr(U \leq x)$. One can thus express the total harvested energy up to the time $t$ as follows

$$A(t) = \sum_{i=0}^{N(t)} U_i, \tag{2.3}$$

where $N(t)$ is an integer such that $T_{N(t)-1} \leq t < T_{N(t)}$.

**Remark 1** *Note that $\{A(t) : t \geq 0\}$ is a compound Poisson process with rate $\delta$, and thus $\mathbb{E}(A(t)) = \delta t \mathbb{E}(U)$ (i.e., expected energy arrival up until time $t$).*

While the assumption of exponential distribution makes the problem analytically more tractable, it is also justifiable in the sense that larger energy packets are less likely to occur and vice versa. Thus, the stored energy in the battery is described by

$$X(t) = X(0) + A(t) - \int_{0^+}^{t} p(Z(s))ds - R(t), \tag{2.4}$$

where $R(t)$ is a non-negative, non-decreasing and continuous-time process known as a reflection process that ensures $X(t)$ always remains inside the battery capacity, i.e., $X(t) \in [0, L]$[43]. These assumptions let us apply the following theorem.

**Theorem 1** *For a battery with finite capacity $L$ and transmission power $p(x)$ such that $\sup_{0<x\leq L} p(x) < \infty$, $\forall L > 0$ and $p(0) = 0$, the storage process $Z(t)$ is positive recurrent. Moreover, there exists a unique stationary measure $\pi(x)$ such that*

$$\pi(x) = \pi_0 + \int_{0^+}^{x} f(u)du, \tag{2.5}$$

*where $\pi(x)$ is continuous on $(0, L]$ and has an atom at $x = 0$ of size $\pi_0$. In addition,*

$$f(x) = \frac{\delta}{p(x)}\left\{\pi_0(1 - B(x)) + \int_{0^+}^{x}(1 - B(x - y))f(y)dy\right\}. \tag{2.6}$$

From now on, we assume energy arrivals are exponentially distributed with parameter $\lambda$, i.e., $B(x) = 1 - e^{-\lambda x}$. Therefore, (2.6) reduces to

$$f(x) = \frac{\delta e^{-\lambda x}}{p(x)}\big(\pi_0 + \int_{0^+}^{x} e^{\lambda y} f(y)dy\big). \tag{2.7}$$

Thus, one can see that long term average rate in (2.1) almost surely converges to

$$R := r(0)\pi_0 + \int_{0^+}^{L} r(p(u))f(u)du. \tag{2.8}$$

## 2.3 Problem Formulation

Our objective in this thesis is to derive achievable constrained policies with good performance, where the measure of performance is taken to be the long term average rate. We formulate our problem as

$$\sup_{p(x),\pi_0} \int_{0}^{L} r(p(x))f(x)dx \tag{2.9}$$

$$\text{s.t. } f(x) = \frac{\delta e^{-\lambda x}}{p(x)}\Big(\pi_0 + \int_{0^+}^{x} e^{\lambda u} f(u)du\Big) \tag{2.10}$$

$$\pi_0 + \int_{0^+}^{L} f(u)du = 1 \tag{2.11}$$

$$h(p, f) \leq h_0 \tag{2.12}$$

$$f(x) \geq 0, \pi_0 \geq 0. \tag{2.13}$$

By choosing a suitable functional $h$ we ensure that the resulting policy satisfies our desirable criteria which will be defined in the following sections. In what follows, we first pick $h$ to be the variance function and then the identity function to account for the battery discharge rate.

# Chapter 3

# Achievable Policy with constrained variance

From the viewpoint of battery life time, discharging the battery at a constant rate is better than a pulse or a momentary high load. Since internal power dissipation is less in constant discharging compared to pulsed discharging, therefore this issue has not been taken into consideration in previous works. Moreover, when the Shannon rate function is used, we prove that the variance of the transmission rate is bounded by the variance of the transmission power. With this intuition, by limiting the variance of transmission power, we can limit the variance of the transmission rate and obtain good performance as well. In this section, we derive a necessary condition for the optimal policy when the variance is bounded by same constant $v_0$. Specifically, we formulate the problem as

$$\sup_{p(x),\pi_0} \int_0^L r(p(x))f(x)dx \tag{3.1}$$

$$\text{s.t. } f(x) = \frac{\delta e^{-\lambda x}}{p(x)}\Big(\pi_0 + \int_{0^+}^x e^{\lambda u} f(u)du\Big) \tag{3.2}$$

$$\pi_0 + \int_{0^+}^L f(u)du = 1 \tag{3.3}$$

$$Var(p(X)) \leq h_0 \tag{3.4}$$

$$f(x) \geq 0, \pi_0 \geq 0. \tag{3.5}$$

Consider the change of variable

$$f(x) := g(x)e^{-\lambda x}, \tag{3.6}$$

then one can verify from (2.7) that

$$p(x) = \frac{\delta G(x)}{g(x)}, \tag{3.7}$$

where we defined $G(x) = \pi_0 + \int_{0+}^{x} g(u)du$. Thus problem(3.1)-(3.5) reduces to

$$\sup_{p(x),\pi_0} \int_{0+}^{L} r\left(\frac{\delta G(x)}{g(x)}\right) g(x)e^{-\lambda x}dx \tag{3.8}$$

$$\text{s.t. } G(x) = \pi_0 + \int_{0+}^{x} g(u)du \tag{3.9}$$

$$\pi_0 + \int_{0+}^{L} g(u)e^{-\lambda u}du = 1 \tag{3.10}$$

$$Var\left(\frac{\delta G(X)}{g(X)}\right) \leq v_0 \tag{3.11}$$

$$g(x) \geq 0, \pi_0 \geq 0, \tag{3.12}$$

The objective is now to find a necessary condition for optimality of $G(x)$. In order to deal with the variance condition, we use the Lagrange multiplier method. Specifically, for some non-positive $\eta$ solving (3.8) is equivalent to solving the following.

$$\sup_{p(x),\pi_0} \int_{0+}^{L} r\left(\frac{\delta G(x)}{g(x)}\right) g(x)e^{-\lambda x}dx + \eta\left(Var\left(\frac{\delta G(X)}{g(X)}\right) - v_0\right) \tag{3.13}$$

$$\text{s.t. } G(x) = \pi_0 + \int_{0+}^{x} g(u)du \tag{3.14}$$

$$\pi_0 + \int_{0+}^{L} g(u)e^{-\lambda u}du = 1 \tag{3.15}$$

$$g(x) \geq 0, \pi_0 \geq 0. \tag{3.16}$$

Assume that $G(x)$ is an extremum for the problem. Now consider the perturbation of the form

$$\widehat{G(x)} = G(x) + \epsilon\mu(x), \tag{3.17}$$

where $\epsilon$ is a small positive constant and $\mu(x)$ is an arbitrary continuous function over $(0, L]$ such that

$$\int_{0^+}^{L} \mu'(u) e^{-\lambda u} du = 0, \tag{3.18}$$

$$\mu(0^+) = \mu(L) = 0. \tag{3.19}$$

Note that (3.18) accounts for feasibility of $\widehat{G(x)}$ with respect to (3.15), where $G'(x) = g(x)$ Consequently, our perturbed objective function should have an extrema at $\epsilon = 0$, i.e.,

$$\frac{\partial}{\partial \epsilon}\Big( \int_{0^+}^{L} r\left(\frac{\delta \widehat{G(x)}}{\widehat{g(x)}}\right) \widehat{g(x)} e^{-\lambda x} dx + \eta\left(Var\left(\frac{\delta \widehat{G(X)}}{\widehat{g(X)}}\right) - v_0\right)\Big)\Big|_{\epsilon=0} = 0, \tag{3.20}$$

where

$$\widehat{g(x)} = \widehat{G(x)}'. \tag{3.21}$$

Equation (3.20) can be simplified as

$$\begin{aligned}\int_{0^+}^{L} \Big( r\left(\frac{\delta G(x)}{g(x)}\right) e^{-\lambda x} \mu'(x) + r'\left(\frac{\delta G(x)}{g(x)}\right) e^{-\lambda x}\left(\delta \mu(x) - \frac{\delta G(x)\mu'(x)}{g(x)}\right)\Big) dx \\ + \eta\Big( \int_{0^+}^{L} \left(\frac{2\delta^2 G(x) e^{-\lambda x}\mu(x)}{g(x)} - \frac{\delta^2 G(x)^2 e^{-\lambda x}\mu'(x)}{g(x)^2}\right) dx \\ - 2\int_{0^+}^{L} \delta G(x) e^{-\lambda x} dx \int_{0^+}^{L} \delta \mu(x) e^{-\lambda x} dx \Big) = 0.\end{aligned} \tag{3.22}$$

Integrating the l.h.s of (3.18) by parts and using (3.19), we obtain

$$\int_{0^+}^{L} \mu'(x) e^{-\lambda x} dx = e^{-\lambda x}\mu(x)|_{0^+}^{L} + \lambda \int_{0^+}^{L} \mu(x) e^{-\lambda x} dx = 0, \tag{3.23}$$

or equivalently,

$$\int_{0}^{L} \mu(x) e^{-\lambda x} dx = 0. \tag{3.24}$$

Replacing (3.24) into (3.22), we obtain

$$\int_{0^+}^{L}\left(r\left(\frac{\delta G(x)}{g(x)}\right)e^{-\lambda x}\mu'(x)+r'\left(\frac{\delta G(x)}{g(x)}\right)e^{-\lambda x}\big(\delta\mu(x)-\frac{\delta G(x)\mu'(x)}{g(x)})\Big)dx$$
$$+\eta\Bigg(\int_{0^+}^{L}(\frac{2\delta^2 G(x)e^{-\lambda x}\mu(x)}{g(x)}-\frac{\delta^2 G(x)^2 e^{-\lambda x}\mu'(x)}{g(x)^2})dx\Bigg)=0. \tag{3.25}$$

Integrating by parts, we have

$$\Big(e^{-\lambda x}r(p(x))-e^{-\lambda x}r'(p(x))p(x)-\mu p^2(x)\lambda e^{-\lambda x}\Big)\mu(x)\Big|_{0^+}^{L}-\int_{0^+}^{L}e^{-\lambda x}\Big(\lambda r(p(x))+\delta r'(p(x))$$
$$-\lambda r'(p(x))p(x)+p(x)p'(x)r''(p(x))+2\delta\eta p(x)-2\eta p(x)p'(x)-\lambda\eta p^2(x)\Big)\mu(x)dx=0 \tag{3.26}$$

Using (3.19), (3.24) and based on the fundamental lemma of calculus of variations we have

$$\lambda r(p(x))+\delta r'(p(x))-\lambda r'(p(x))p(x)+p(x)p'(x)r''(p(x))$$
$$+2\delta\eta p(x)+2\eta p(x)p'(x)-\lambda\eta p^2(x)=K, \tag{3.27}$$

where $K$ is a free parameter. For specific values of $K$ and $\eta$ one can compute from (3.27) an optimum for the problem (3.8)-(3.12). In other words, if we think of $p(x)$ as a function of $K$ and $\eta$ our objective becomes

$$\max_{K,\eta}\int_{0^+}^{L}r(p(x,K,\eta))f(x,K,\eta)dx \tag{3.28}$$
$$Var(p(X,K,\eta))<v_0, \tag{3.29}$$

where we have reduced our problem from an optimization over a function space into an optimization over two parameters in $\mathbb{R}$.

## 3.1 Upper bound

In the previous section, we proposed a method to derive a policy with good performance, which requires us to search over two parameters. We now derive an upper bound for this problem. Using Jensen's inequality, and due to [R5], we have

$$\mathbb{E}_\pi(r(p(X))) \leq r(\mathbb{E}_\pi(p(X)), \tag{3.30}$$

where $\mathbb{E}_\pi(.)$ is the expectation with respect to the stationary distribution $\pi$. Moreover, due to [R4] we have

$$\sup_{p(x),\pi_0} \mathbb{E}_\pi(r(p(X))) \leq r(\sup_{p(x),\pi_0} \mathbb{E}_\pi(p(X)). \tag{3.31}$$

In order to calculate the term $\sup_{p(x),\pi_0} \mathbb{E}_\pi(p(X))$, we write it as a Lagrange multiplier problem

$$\sup_{p(x),\pi_0} \int_{0^+}^{L} \delta G(x) e^{-\lambda x} dx + \Lambda \left( Var(\frac{\delta G(X)}{g(X)}) - v_0 \right) \tag{3.32}$$

$$\text{s.t. } G(x) = \pi_0 + \int_{0^+}^{x} g(u) du \tag{3.33}$$

$$\pi_0 + \int_{0^+}^{L} g(x) e^{-\lambda x} dx \tag{3.34}$$

$$g(x) \geq 0, \pi_0 \geq 0, \tag{3.35}$$

where $\Lambda$ is the Lagrange multiplier. Now our goal is to find the critical values of the problem (3.32)-(3.35). Similar to the previous section, let $G(x)$ be extremum for the problem and define the following perturbation

$$\widehat{G(x)} = G(x) + \epsilon \mu(x), \tag{3.36}$$

where $\epsilon$ is a small positive constant and $\mu(x)$ is an arbitrary continuous function over $(0, L]$ such that

$$\int_{0^+}^{L} \mu'(u) e^{-\lambda u} du = 0, \tag{3.37}$$

$$\mu(0^+) = \mu(L) = 0. \tag{3.38}$$

Note that (3.37) accounts for feasibility of $\widehat{G(x)}$ with respect to (3.34). Since $G(x)$ is an extremum for the objective function, we have

$$\frac{\partial}{\partial \epsilon} \left( \int_{0^+}^{L} \delta \widehat{G(x)} e^{-\lambda x} dx + \Lambda \left( Var \left( \frac{\delta \widehat{G(X)}}{\widehat{g(X)}} \right) - v_0 \right) \right) \Big|_{\epsilon=0} = 0, \tag{3.39}$$

gives us a necessary condition for a local and thus global optimal solution, where $\widehat{G(x)} = G(x) + \epsilon\mu(x)$. Expanding (3.39) yields

$$\int_{0^+}^{L} \delta e^{-\lambda x}\mu(x)dx + \eta\bigg(\int_{0^+}^{L}\big(\frac{2\delta^2 G(x)e^{-\lambda x}\mu(x)}{g(x)} - \frac{\delta^2 G(x)^2 e^{-\lambda x}\mu'(x)}{g(x)^2}\big)dx \\ - 2\int_{0^+}^{L}\delta G(x)e^{-\lambda x}dx\int_{0^+}^{L}\delta\mu(x)e^{-\lambda x}dx\bigg) = 0. \tag{3.40}$$

Using (3.24) and integrating by parts we obtain

$$\Lambda p^2(x)e^{-\lambda x}\mu(x)\Big|_{0^+}^{L} - \int_{0^+}^{L}\Big(2\delta\Lambda p(x) - 2\Lambda p(x)p'(x) - \lambda\Lambda p^2(x)\Big)\mu(x)dx = 0$$

Performing some simplifications we get

$$2\delta\Lambda p(x) - 2\Lambda p(x)p'(x) - \lambda\Lambda p^2(x) = K_2, \tag{3.41}$$

where $K_2$ is a free parameter similar to $K$ in (3.27). One can rewrite (3.41) as

$$p'(x) = \alpha\frac{1}{p(x)} + \lambda p(x) - \delta, \tag{3.42}$$

where $\alpha = \frac{-K_2}{2\Lambda}$. Thus, similar to (3.28), solving (3.32)-(3.35) is now equivalent to solving the following problem

$$\max_{\alpha} \int_{0^+}^{L} p(x,\alpha)f(x,\alpha)dx \tag{3.43}$$

$$\text{s.t. } Var(p(X,\alpha)) \leq v_0. \tag{3.44}$$

Different values of $\alpha$ corresponds to different values of the Lagrange multiplier $\Lambda$. In the following lemma we show that both the objective function and $Var(p(x))$ are either increasing or decreasing with respect to the $\alpha$, hence finding an $\alpha_0$ that satisfies the condition (3.44) with equality solves the maximization problem above.

**Lemma 1** *Changing $\alpha$ will either increase or decrease both $\mathbb{E}_\pi(p(X,\alpha))$ and $Var(p(X,\alpha))$ at the same time.*

*Proof*: We should note that due to the inequality sign of the constraint only the negative values of $\Lambda$ would be acceptable. Assume to the contrary that for two random values $\alpha_1$ and $\alpha_2$ we have $\mathbb{E}_\pi(p(X,\alpha_1)) > \mathbb{E}_\pi(p(X,\alpha_2))$ and $Var(p(X,\alpha_1)) < Var(p(X,\alpha_2))$. Let's assume $\alpha_2$ corresponds to $\Lambda_2$, now since $\Lambda_2$ is negative we'll have

$$\mathbb{E}_\pi(p(X,\alpha_2)) + \Lambda_2 Var(p(X,\alpha_2)) < \mathbb{E}_\pi(p(X,\alpha_1)) + \Lambda_2 Var(p(X,\alpha_1)),$$

which is a contradiction since we assumed $\alpha_2$ maximizes $\mathbb{E}_\pi(p(X,\alpha_2)) + \Lambda_2 Var(p(X,\alpha_2))$. The other possibility is $\mathbb{E}_\pi(p(X,\Lambda_1)) < \mathbb{E}_\pi(p(X,\Lambda_2))$ and $Var(p(X,\Lambda_1)) > Var(p(X,\Lambda_2))$ which by a similar argument leads to a contradiction, which proves the lemma.

## 3.2 Achievable Policy with peak power constraint

Another way of decreasing the variation of the power is to limit the peak power. Moreover, most batteries have a maximum safe discharge rate. Here, we maximize the long term average rate while we constrain the power policy to be bounded by a finite number, say $M$. We formulate the problem as follows

$$\sup_{p(x),\pi_0} \int_{0^+}^{L} r(p(x))f(x)dx \tag{3.45}$$

$$f(x) = \frac{\delta e^{-\lambda x}}{p(x)}\Big(\pi_0 + \int_{0^+}^{x} e^{-\lambda u} f(u)du\Big) \tag{3.46}$$

$$\pi_0 + \int_{0^+}^{L} f(x)dx = 1 \tag{3.47}$$

$$p(x) \leq M \tag{3.48}$$

$$f(x) \geq 0, \pi_0 \geq 0. \tag{3.49}$$

In order to deal with the inequality constraint (3.48), we use the Kuhn-Karush-Tucker condition and reformulate the problem as below

$$\sup_{p(x),\pi_0} \int_{0^+}^{L} r(p(x))f(x)dx + \phi(x)(p(x) - M) \tag{3.50}$$

$$\text{s.t. } f(x) = \frac{\delta e^{-\lambda x}}{p(x)}\Big(\pi_0 + \int_{0^+}^{x} e^{-\lambda u} f(u)du\Big) \tag{3.51}$$

$$\pi_0 + \int_{0^+}^{L} f(x)dx = 1 \tag{3.52}$$

$$f(x) \geq 0, \pi_0 \geq 0, \tag{3.53}$$

where $\forall x \in (0, L]$, $\phi(x) \geq 0$. Thus, by the complementary slackness we must have $\phi^*(x)\big(p^*(x) - M\big) = 0$, where $*$ denotes the optimal function. If $\phi^*(x) = 0$ then the problem simplifies to the problem defined in [41]. As a consequence of this, the solution will be a piecewise function where it either satisfies

$$\lambda r(p(x)) + \delta r'(p(x)) - \lambda r'(p(x))p(x) + p(x)p'(x)r''(p(x)) = K, \tag{3.54}$$

which is obtained in [41] or we have $p^*(x) = M$. Since the optimal policy obtained in [41] is increasing and the initial value of the optimal solution, $p^\dagger(0^+)$, is less than $M$, our solution will be in the form below

$$p^*(x) = \begin{cases} p^\dagger(x) & \text{if } p^\dagger(x) < M, \\ M & \text{if } p^\dagger(x) > M. \end{cases} \tag{3.55}$$

Where, $p^\dagger(z)$ is calculated in [41].

We now find an upper for this problem using the following lemma.

**Lemma 2** *Let's assume $X_1(t)$ and $X_2(t)$ result from the same energy arrivals $A(t)$ by applying two different policies $p_1(x)$ and $p_2(x)$ respectively. Now if $\forall x \in (0, L]$, $p_1(x) \geq p_2(x)$, where both are continuous on this interval and $X_2(0) \geq X_1(0)$ then $E_{\pi_1}(p_1(X)) \geq E_{\pi_2}(p_2(X))$.*

*Proof*: See Appendix A.

Consider the following choice of $p(x)$

$$\widetilde{p(x)} = \begin{cases} M & \text{if } x > 0, \\ 0 & \text{if } x = 0. \end{cases} \tag{3.56}$$

Based on (3.55) and Lemma 2 we thus have

$$\sup_{p(x) \leq M, \pi_0} \mathbb{E}_\pi(r(p(X))) < r(\mathbb{E}_\pi(\widetilde{p(X)}), \tag{3.57}$$

where the left hand side is the optimization problem (3.45)-(3.49). Using the derivations (4.2) and (4.1), to be shown later in section VII, we calculate $\mathbb{E}_\pi(\widetilde{p(X)})$ as follows

$$\mathbb{E}_\pi(\widetilde{p(X)}) = \int_{0^+}^{L} \widetilde{p(x)} f(x)dx = \int_{0^+}^{L} \Big(\pi_0 \delta e^{\lambda x} \exp(\int_{0^+}^{L} \frac{\delta}{p(u)} du)\Big) dx, \tag{3.58}$$

substituting (3.56) into (3.58) we obtain

$$\mathbb{E}_\pi(\widetilde{p(X)}) = \pi_0\delta\int_{0^+}^{L} e^{-\lambda x}\exp(\int_{0^+}^{x}\frac{\delta}{M}du)dx = \pi_0\delta\int_{0^+}^{L} e^{-(\lambda-\frac{\delta}{M})x}dx \tag{3.59}$$

$$= \frac{\pi_0\delta M}{M\lambda-\delta}\left(1-e^{-(\lambda-\frac{\delta}{M}L)}\right) \tag{3.60}$$

Similarly $\pi_0$ is calculated as

$$\pi_0 = \left[1+\int_{0^+}^{L}\frac{\delta e^{-\lambda x}}{M}\exp(\int_{0^+}^{x}\frac{\delta}{M}du)dx\right]^{-1} = \left[1+\frac{\delta}{M}\int_{0^+} Le^{-(\lambda-\frac{\delta}{M})x}dx\right]^{-1} \tag{3.61}$$

$$= \big(M\lambda-\delta\big)\big(M\lambda-\delta e^{(\lambda-\frac{\delta}{M})L}\big). \tag{3.62}$$

Hence (3.60) simplifies to

$$\mathbb{E}_\pi(\widetilde{p(X)}) = \frac{\delta M(e^{\lambda L}-e^{\frac{\delta L}{M}})}{M\lambda e^{\lambda L}-\delta e^{\frac{\delta L}{M}}} \tag{3.63}$$

**Remark 2** *Note that by calculating the following limit we are also able to get an upper bound for the unconstrained problem i.e., problem (2.9) when constraint (2.12) is ignored.*

$$\lim_{M\to\infty}\frac{\delta M(e^{\lambda L}-e^{\frac{\delta L}{M}})}{M\lambda e^{\lambda L}-\delta e^{\frac{\delta L}{M}}} = \frac{\delta}{\lambda}(1-e^{-\lambda L}). \tag{3.64}$$

*Consequently, we have*

$$\sup_{p(x),\pi_0}\mathbb{E}_\pi(r(p(X))) \le r\left(\frac{\delta}{\lambda}\left(1-e^{-\lambda L}\right)\right). \tag{3.65}$$

*This upper bound was also calculated in [46].*

## 3.3 Achievable Policy with constraint on Rate Variance

In the previous sections, our objective was to prolong the battery life by limiting the variations in the transmission power. However there are situations where we wish to maintain a near constant rate of transmission. Here, we aim to achieve this by finding efficient policies with limited transmission rate variance. Specifically, we formulate the problem as follows

$$\sup_{p(x),\pi_0} \int_{0^+}^{L} r\left(\frac{\delta G(x)}{g(x)}\right) g(x)e^{-\lambda x}dx \tag{3.66}$$

$$\text{s.t. } G(x) = \pi_0 + \int_{0^+}^{z} g(u)du \tag{3.67}$$

$$\pi_0 + \int_{0^+}^{L} g(x)e^{-\lambda x}dx = 1 \tag{3.68}$$

$$Var\left(r\left(\frac{\delta G(X)}{g(X)}\right)\right) \le v_0 \tag{3.69}$$

$$g(x) \ge 0, \pi_0 \ge 0. \tag{3.70}$$

A similar approach to that of (3.20) can be adopted to solve the above problem. Consider the perturbation of the form $G(x) = \widehat{G(x)} + \epsilon H(x)$, and define $H'(x) = h(x)$. Hence, by perturbing the objective function we get

$$\begin{aligned}
&\int_{0^+}^{L}\left(e^{-\lambda x}h(x)r\left(\frac{\delta\, G(x)}{g(x)}\right) + e^{-\lambda x}g(x)r'\left(\frac{\delta\, G(x)}{g(x)}\right) \times \left(\frac{\delta\, H(x)}{g(x)} - \frac{\delta\, G(x)h(x)}{(g(x))^2}\right)\right)dx + \\
&\theta\left(\int_{0^+}^{L}\left(2\, e^{-\lambda x}g(x)r\left(\frac{\delta\, G(x)}{g(x)}\right)r'\left(\frac{\delta\, G(x)}{g(x)}\right)\left(\frac{\delta\, H(x)}{g(x)} - \frac{\delta\, G(x)h(x)}{(g(x))^2}\right)\right.\right. \\
&+\left.\left(r\left(\frac{\delta\, G(x)}{g(x)}\right)\right)^2 h(x)e^{-\lambda x}\right)dx - 2\int_{0}^{L} e^{-\lambda x}g(x)r\left(\frac{\delta\, G(x)}{g(x)}\right)dx \\
&\times \left.\int_{0^+}^{L}\left(e^{-\lambda x}h(x)r\left(\frac{\delta\, G(x)}{g(x)}\right) + e^{-\lambda x}g(x)r'\left(\frac{\delta\, G(x)}{g(x)}\right)\left(\frac{\delta\, H(x)}{g(x)} - \frac{\delta\, G(x)h(x)}{(g(x))^2}\right)dx\right)\right) = 0,
\end{aligned} \tag{3.71}$$

where $\theta$ is our Lagrange multiplier. Now let

$$\xi := \int_{0^+}^{L} e^{-\lambda x}g(x)r\left(\frac{\delta\, G(x)}{g(x)}\right)dx. \tag{3.72}$$

Following the approach used in the previous sections, based on the fundamental lemma of the calculus of variations, we obtain the following equation.

$$\begin{aligned}
&\lambda\, r(p(x)) - \lambda\, r'(p(x))p(x) + r''(p(x))p'(x)p(x) - \theta \Big( 2\,\lambda\, r(p(x))r'(p(x))p(x) \\
&- 2\,(r'(p(x)))^2 p'(x)p(x) - 2\, r(p(x))(r''(p(x))p'(x)p(x) - (r(p(x)))^2 \lambda \\
&- 2\,\xi\, \big( -\lambda\, r(p(x)) + \lambda\, r'(p(x))p(x) - r''(p(x))p'(x)p(x) \big) \Big) + r'(p(x))\delta \\
&+ \theta \left( 2\, r(p(x))r'(p(x))\delta - 2\,\xi\, r'(p(x))\delta \right) = K_3, \qquad (3.73)
\end{aligned}$$

where $K_3$ is a free parameter. Since $\xi = \mathbb{E}_\pi(r(p(X)))$, solving the above equation is not as straight forward as solving (3.27).
In section 4.3, we present a recursive algorithm to solve this equation.
Solving the equation (3.73) will be a demanding task. However, based on the following theorem, for some rate functions, by solving (3.8)-(3.12) we also obtain policies with good performance while the variance of the rate is limited as well.

**Theorem 2** *Let $X$ be a random variable. If $f$ and $g$ are non-negative, differentiable, convex functions such that $f' > 0$, $g' > 0$, $f < g$ and $f' < g'$ over the convex hull of the range of $X$, then*

$$Var(f(X)) \leq Var(g(X)).$$

*Proof*: See Appendix 2.

In particular, if we let $f(x) = \ln(1+x)$ and $g(x) = x$ we get that

$$Var(\ln(1+X)) \leq Var(X), \qquad (3.74)$$

equivalently,

$$(\ln 2)^2 Var\left(\log_2\left(1 + \frac{P}{N}\right)\right) \leq Var\left(\frac{P}{N}\right)$$

which results in

$$Var\left(\log_2\left(1 + \frac{P}{N}\right)\right) \leq \frac{1}{(\ln 2)^2 N^2} Var\left(P\right).$$

# Chapter 4

# Numerical Results

In this section, we aim to solve (3.27), (3.42) and (3.73) in order to obtain efficient power policies. It appears there is no closed form solution for these due to the non-linear nature of the equations. Thus, we rely on numerical solutions. Using (3.7), we have

$$\int_{0^+}^{x} \frac{1}{p(u)} du = \int_{0^+}^{x} \frac{g(u)}{\delta G(u)} du, \tag{4.1}$$

which simplifies to

$$G(x) = \pi_0 \exp\left(\int_{0^+}^{x} \frac{\delta}{p(u)} du\right). \tag{4.2}$$

Differentiating (4.2) with respect to $x$, results in

$$g(x) = \pi_0 \frac{\delta}{p(x)} \exp\left(\int_{0^+}^{x} \frac{\delta}{p(u)} du\right). \tag{4.3}$$

And $\pi_0$ can be computed from

$$\pi_0 = \left[1 + \int_{0^+}^{L} \frac{\delta e^{-\lambda x}}{p(x)} \exp\left(\int_{0^+}^{x} \frac{\delta}{p(u)} du\right) dx\right]^{-1}. \tag{4.4}$$

In the following, we consider the special case of the Shannon rate function $r(p) = \log(1 + p/N)$ where $N = 1$. Moreover, we assume $\delta = \lambda = 1$. The initial value for the power policy is chosen to be $p(0^+) = 0.001$. Picking a small initial value for the power policy can be justified by the intuition that a small amount of energy in the battery results in a small depletion rate $p(x)$, otherwise the battery will deplete rapidly which leads to a non-efficient policy. Numerical simulations also show that for sufficiently small values of $p(0^+)$ the best throughput is achieved [46].

## 4.1 Constrained Variance for Power

In this section, we numerically solve equations (3.27) and (3.42) in order to find an achievable optimal power policy and an upper when the variance of power is constrained. We first calculate the upper bound. Based on Lemma 1, our search over $\Lambda$ and $K_2$ is limited to finding a certain variance for which the throughput is maximum. Next, we perform an extensive search over $\eta$ and $K$ in (3.27) to find the optimal values. Fig. 4.1 shows the throughput achieved by the optimal policy as well as the upper bound as a function of $v_0$. It can be seen that for small variances we almost have a tight upper bound. Moreover, as the plot shows we can obtain policies with small variances that provide good performance. Fig. 4.2 illustrates policies that result, for different variances.

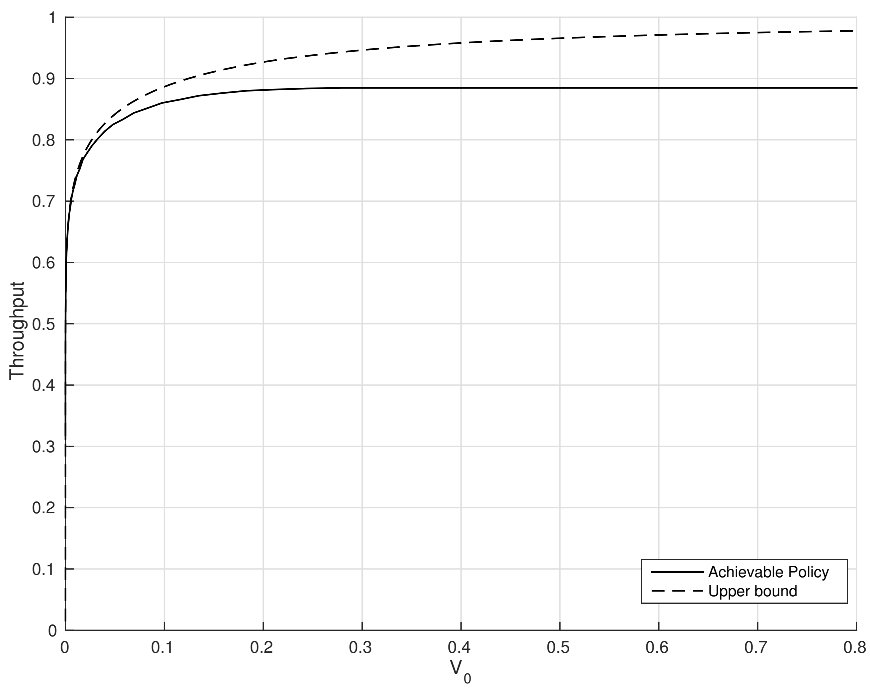


Figure 4.1: The optimal throughput as well as the upper bound as a function of $v_0$.

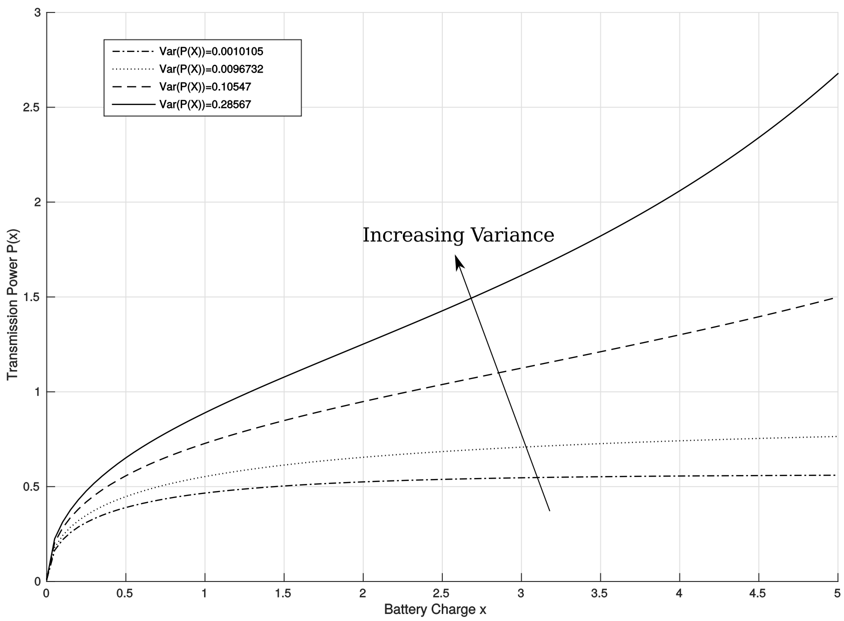


Figure 4.2: The optimal policy for different values of variance.

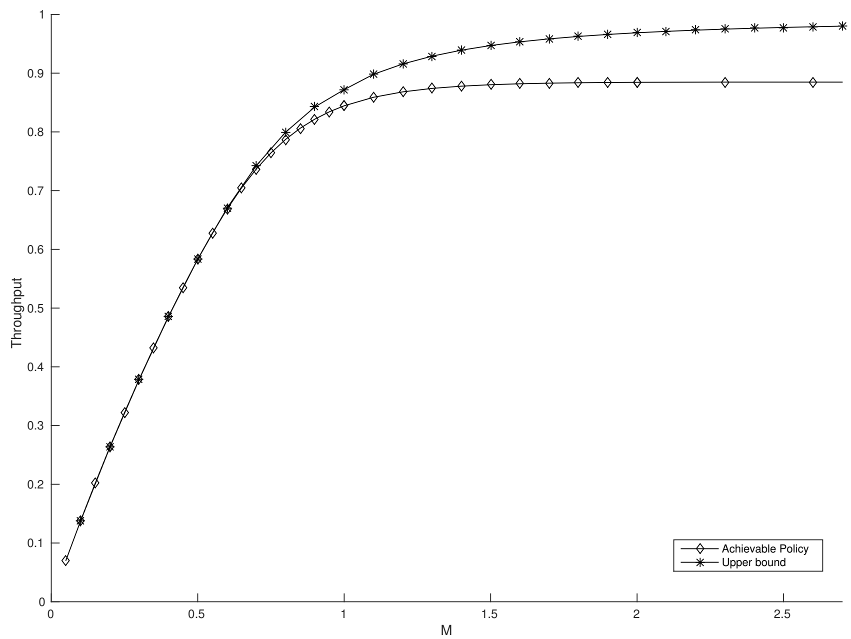


Figure 4.3: The optimal throughput for different values of $M$.

## 4.2 Peak Power Constraint

We now consider the problem of finding an optimal transmission policy which maximize the throughput while the power is bounded by a constant $M$. From the results shown in section 3.2, in order to obtain the optimal policy for this problem we need to bound the policy obtained in [41] by a constant $M$. Fig. 4.3 shows the optimal throughput as well as the upper bound obtained from equation (3.63) as functions of $M$.

Fig. 4.4 shows that by limiting the peak power, we also reduce the variance of the power, moreover, we are able to achieve values that are almost identical to our results with constrained variance in the previous subsection. Consequently, by only limiting the peak power we obtain a near optimal lower bound. Based on this observation we can conclude that most of the variation in transmission power is due to large transmission power when battery has close to full charge.

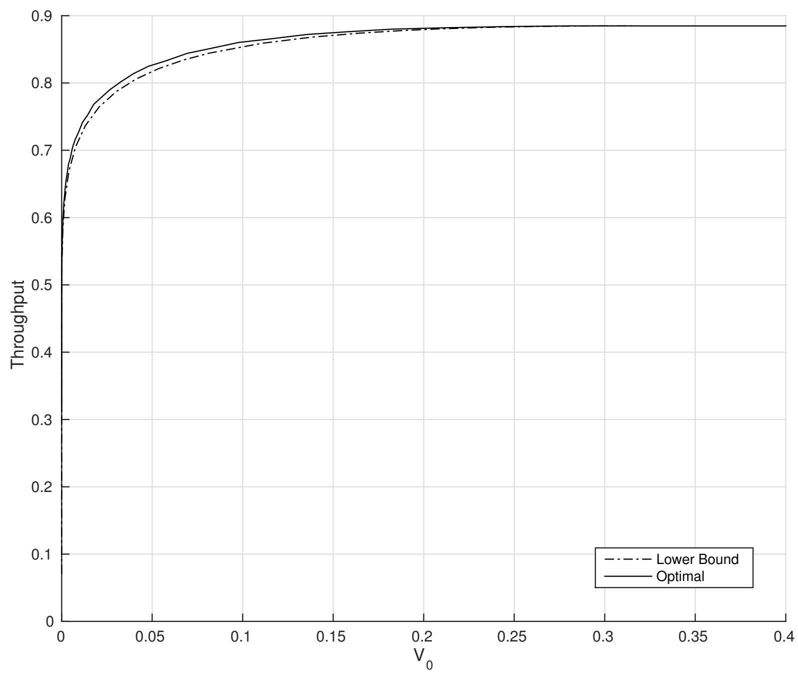


Figure 4.4: The throughput for the optimal case and limited peak power when the variance of power varies in the range $0 < v_0 < 1$.

## 4.3 Constrained Variance on Throughput

As we showed in Section 3.3, to obtain an optimal policy when the variance of throughput is constrained, we need to solve equation (3.73). Algorithm 1 below provides an algorithm to do so.

**Algorithm 1** Recursive algorithm to numerically solve (3.73).
initialize $p(x)$ with some arbitrary function ;
compute (4.2) and (4.4) ;
**while** *termination criterion is not satisfied* **do**
    calculate (3.72);
    update $p(x)$ by solving (3.73) for optimized values of $K_3$ and $\theta$ such that the constraint is satisfied.;
    update (4.2) and (4.4) ;
**end**

Fig. 4.5 depicts the optimal throughput achieved, when the variance of the transmission rate is bounded. As already mentioned, we expect the policies achieved in section 4.1 to show good performance in this scenario as well. Fig. 4.5 compares the performance of the three different policies when the variance of the transmission rate is bounded by $v_0$.

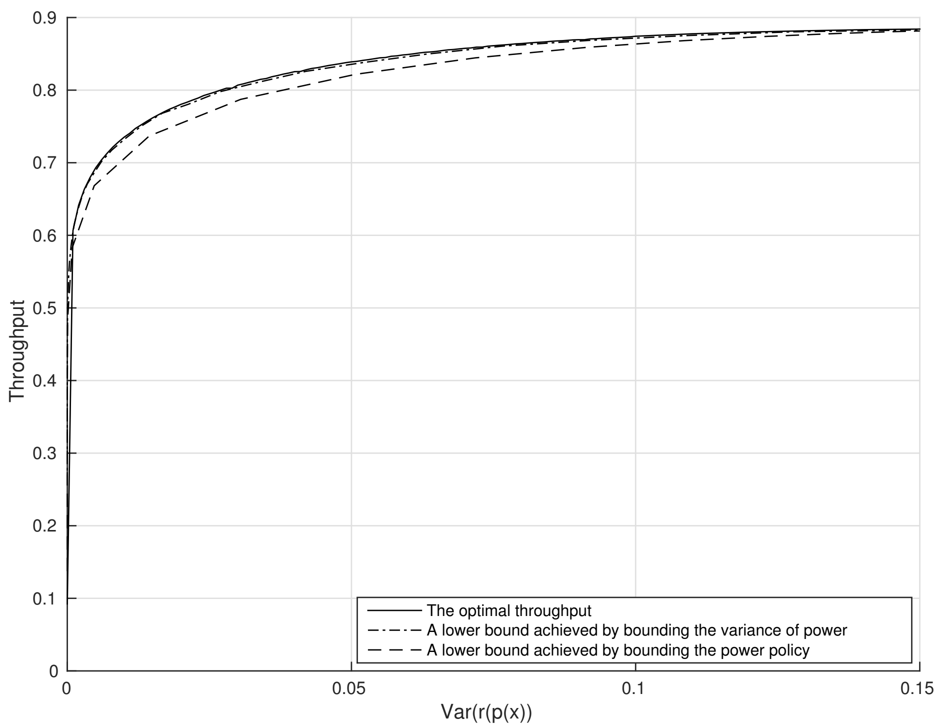


Figure 4.5: The throughput achieved by policies that are derived in different methods.

# Chapter 5

# Conclusion

In this thesis, we have studied the problem of designing optimal transmission policies, in a point-to-point energy harvesting communication system, while considering more practical scenarios to account for quality of service and battery requirements compared to previous works.
We have found the structure of achievable transmission power policies, which maximize the long-term average throughput, while forcing constraints on appropriate functional of the power policy to cope with battery requirements and also to improve the quality of service. More specifically, we have used a calculus of variations technique to derive power policies that achieve good performance and are adapted to the remaining charge in the battery.
In order to improve the battery efficiency and lifetime, we have investigated limiting the transmission power as well as its variation. Moreover, we have derived interesting analytical and numerical upper bounds for these cases. In particular, we have numerically shown that under certain conditions the analytical upper bound achieves another bound previously derived in a different work for the unconstrained problem. We have also accounted for the quality of service, in a separate optimization problem, by constraining variations in instantaneous throughput.
We have considered the case of a Gaussian channel for which we numerically showed that for different values of the variance of the transmission power, as the remaining battery charge increases the transmission power increases as well. Numerical results have also shown that values of achievable throughput under the constraint of limited variance of the transmission power are almost identical to those of limited peak power. We thus conclude that most of the variation in transmission power is due to large large transmission power at high battery charge. Future extensions include considering more general energy arrival models as well as deriving better bounds for the problem of limiting the variance of the

throughput.

Another extension is to consider the outage probability as a quality of service and find achievable solutions which maximize the throughput while minimize the outage probability.

# APPENDICES

## 5.1 Appendix 1

*Proof*: We first prove that $\forall t \in [0, \infty),\ X_2(t) \geq X_1(t)$. Define $f(t) = X_2(t) - X_1(t),\ for t \in (0, T_0)$ where $T_0$ is the first energy arrival time. $f(t)$ is a continuous function on $(0, T_0)$. We thus need to show that $f(t) \geq 0$. Assume to the contrary that there exists a point, say $t_0 \in (0, T_0)$, such that $f(t_0) < 0$. Since $f(t)$ is continuous there exists a neighbourhood around $t_0$, say $(p, q)$, over which $f(t)$ is negative and $(p, q) \subset (0, t_0)$. Let's say $(\alpha, \beta)$ is the biggest interval containing $t_0$ such that $\forall t \in (\alpha, \beta),\ f(t) < 0$, and since $f(0) \geq 0$ we must have $\alpha \geq 0$. Fig. 5.1 illustrates this case. One can easily see that this requires us to have $f(\alpha) = 0$. In addition, we have that $X_1(\alpha) = X_2(\alpha) > 0$. This is due to the fact that for $t \in [0, T_0)$, $X(t)$ is decreasing and once it hits zero it remains there. Thus $p(X_1(t))$ and $p(X_2(t))$ are continuous at $t = \alpha$. By definition we know that $X_k(t) = X_{k,0} - \int_{0^+}^{t} p(X_k(u))du,\ for\ t \in (0, T_0),\ k = 1, 2$. Thus $X_k'(\alpha) = -p(X_k(\alpha)),\ k = 1, 2$. Also, by our assumption we have that $X_1'(\alpha) = -p_1(X_1(\alpha)) \leq -p_2(X_2(\alpha)) = X_2'(\alpha)$. On the other hand,

$$
\begin{aligned}
X_1'(\alpha) &= \lim_{\epsilon \to 0^+} \frac{X_1(\alpha + \epsilon) - X_1(\alpha)}{\epsilon} \\
&\geq \lim_{\epsilon \to 0^+} \frac{X_2(\alpha + \epsilon) - X_2(\alpha)}{\epsilon} = X_2'(\alpha),
\end{aligned}
$$

which is a contradiction. Therefore $for\ t \in (0, T_0)$, $X_2(t) \geq X_1(t)$. By induction, this can also be proved for every interval $(T_i, T_{i+1})$, since $X_2(T_i) \geq X_1(T_i)$. In addition, one can

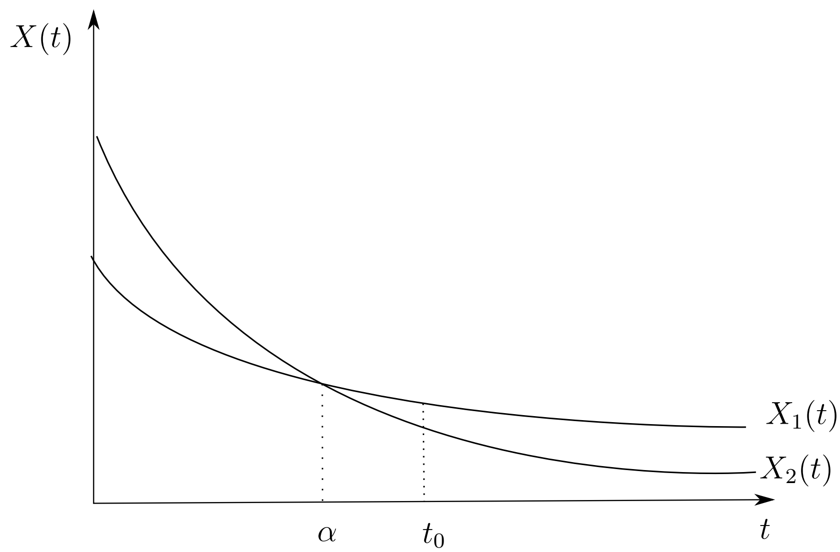


Figure 5.1: Available Energy in Battery at time $t$

write $\mathbb{E}_\pi(p(X_k)),\ k = 1, 2$ as

$$\begin{aligned}
\mathbb{E}_\pi(p(X_k)) &= \lim_{T\to\infty} \frac{1}{T} \int_0^T p(X_k(u))du \\
&= \lim_{T\to\infty} \frac{1}{T} \left(X_k(0) + A(T) - X_k(T) - R_k(T)\right) \\
&= \lim_{T\to\infty} \frac{1}{T} \left(A(T) - X_k(T) - R_k(T)\right),
\end{aligned}$$

where $R(t)$ is the reflection process defined in section 2.2. Since, we have $X_2(t) \geq X_1(t), \forall t \geq 0$, the amount of overflow of the first policy is always less than or equal to that of the second policy (i.e., $R_1(t) \leq R_2(t), \forall t$). We thus conclude that $\mathbb{E}_{\pi_1}(p_1(X)) \geq \mathbb{E}_{\pi_2}(p_2(X))$ and the proof is complete.

## 5.2 Appendix 2

**Lemma 3** *For any random variable $X$, if $f(X,.)$ is a differentiable, convex decreasing function on $[a,b]$ and its derivative at $a$ exist and is bounded, then*

$$\frac{\partial}{\partial \epsilon}\mathbb{E}[f(X,\epsilon)] = \mathbb{E}[\frac{\partial}{\partial \epsilon}f(X,\epsilon)] \tag{5.1}$$

*Proof*: We bring the proof from [50]
Let $g(x,\epsilon) = \frac{\partial}{\partial \epsilon}f(x,\epsilon)$ For $\epsilon \in [a,b)$, let

$$m_n(x,\epsilon) = (n+N_\epsilon)\Big[f(x,\epsilon+\frac{1}{n+N_\epsilon}) - f(x,\epsilon)\Big] \tag{5.2}$$

where $N_\epsilon = \frac{2}{b-\epsilon}$, and for $\epsilon = b$, let

$$m_n(x,\epsilon) = (n+N_\epsilon)\Big[f(x,\epsilon) - f(x,\epsilon-\frac{1}{n+N_\epsilon})\Big] \tag{5.3}$$

where $N_\epsilon = \frac{2}{b-a}$. Clearly the sequence $m_{n\{n\geq 1\}}$ converges point-wise to $g$. Since with probability 1, $f(X,.)$ is convex and decreasing, and (by the hypothesis of boundedness) $|m_n(X,\epsilon)| \leq |g(X,a)| \leq M$ for all $\epsilon \in [a,b]$. By Lebesgue's Dominated Convergence Theorem

$$\mathbb{E}[\frac{\partial}{\partial \epsilon}f(X,\epsilon)] = \mathbb{E}[g(X,\epsilon)] = \lim_{n\to\infty}\mathbb{E}[m_n(X,\epsilon)] = \frac{\partial}{\partial \epsilon}\mathbb{E}[f(X,\epsilon)] \tag{5.4}$$

and the proof is complete.

*Proof of lemma 2*: Define

$$h(x,\epsilon) = \epsilon f(x) + (1-\epsilon)g(x),$$

then $h(x,\epsilon)$ is an increasing function with respect to $\epsilon$. On the other hand, we have that

$$\frac{\partial}{\partial x}\frac{\partial}{\partial \epsilon}h(x,\epsilon) = f'(x) - g'(x) < 0.$$

Consequently, $for \epsilon \in [0,1]$, $\frac{\partial}{\partial \epsilon}h(x,\epsilon)$ is a decreasing function with respect to $x$. Now

$$\frac{\partial}{\partial \epsilon}Var(h(X,\epsilon)) = \frac{\partial}{\partial \epsilon}\{\mathbb{E}(h(X,\epsilon)^2) - (\mathbb{E}(h(X,\epsilon))^2\},$$

using Lemma 3, we have

$$\frac{\partial}{\partial\epsilon}Var(h(X,\epsilon)) = \mathbb{E}\left(2h(X,\epsilon)\frac{\partial}{\partial\epsilon}h(X,\epsilon)\right) - 2\mathbb{E}(h(X,\epsilon)]\mathbb{E}\left(\frac{\partial}{\partial\epsilon}h(X,\epsilon)\right) < 0.$$

Where the last inequality is due to a theorem which states, $\mathbb{E}(u(X)w(X)) \leq \mathbb{E}(u(X))\mathbb{E}(w(X))$ when $u(x)$ is an increasing function and $w(x)$ is decreasing.
From the above statement

$$Var(h(X,0)) \geq Var(h(X,1))$$

or equivalently

$$Var(g(X)) \geq Var((f(X)),$$

which completes our proof.

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