

Design and Implementation of an Experimental Setup for Entanglement Harvesting

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Relativistic quantum information is an emerging field of study at the interface of relativity and quantum theories. One key result is the prediction of extracting entanglement from quantum fields known as “entanglement harvesting”. The Superconducting Quantum Devices (SQD) group has recently designed a qubit-coupler system, suitable for generating entanglement between two causally disconnected qubits, and developed an experimental protocol to measure entanglement harvesting.

In this thesis, developments to the experimental infrastructure aimed to facilitate the entanglement harvesting experiment will be presented. The proposed experiment is operated on a superconducting flux qubit device in a dilution refrigerator. We discuss the theoretical requirements for entanglement harvesting and outline the experimental proposal. We propose cryogenic electronics setup capable of providing all the of requirements for entanglement harvesting. We model the thermal state in the transmission line, and propose attenuation that will reach a low enough thermal noise to extract entanglement. We propose electronic cables capable of supplying biasing currents between 0-26.5 GHz, able to provide low dispersion of a 33 ps Gaussian pulse and low heating of static currents up to 5 mA, required for biasing an entanglement harvesting measurement. A new superconducting magnetic coil is designed and constructed, where we consider its impact on experimental operation. The thermal considerations of heating the dilution refrigerator and properly dissipating the heat are presented.

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Chapter 1

Introduction

The theories of quantum information and general relativity represent two major scientific revolutions of the last century. These paradigm shifts have led to some of the largest developments seen in the modern world. The understanding of quantum band structures enabled the development of the transistor and the entire emerging information age [7]. The development of relativity has been required for the deployment of satellites and Global Positioning System [8]. A significant subset of modern physics exploration requires these two theories. In just the last decade there have been four Nobel prizes related to quantum theory in 2014, 2016, 2022, and 2023 [9], and three Nobel prizes related to relativity in 2017, 2019, and 2020 [9]. However, despite the remarkable success of these two theories they are still largely incompatible [10].

Relativistic quantum information, which incorporates the concepts of quantum information and general relativity, seeks to bridge the gap between the two theories [10]. The blending of these methods can be seen to give rise to the effects of Hawking radiation, the Unruh effect, and predicts changes to the fidelity of quantum teleportation and entanglement protocols in relativistically accelerating systems [11]. Of particular interest to this thesis is the prediction that entanglement can occur between two causally disconnected systems [12]. Entanglement harvesting from a quantum field is also of key importance to measuring the topology of spacetime, quantum energy teleportation, and quantum meteorology [13, 14, 15, 16]. Despite the number of theoretical investigations into entanglement harvesting there has not been an experimental validation of this phenomenon.

Superconducting qubits are one of the most mature quantum computing architectures. A subset of superconducting qubits, flux qubits, are of particular interest for investigating relativistic quantum information and entanglement harvesting. Flux qubits, at their

most simple, are loops of superconducting wire, with states corresponding to clockwise and counter-clockwise currents. The magnetic fields of the persistent current states are the same as from a magnetic dipole. Hence, these devices can couple to electromagnetic waves similar to natural atoms, but have much greater engineering control over their properties. Recent work has shown flux qubits reaching the ultrastrong coupling regime, which is marked by the qubit's emission rate approaching its transition frequency [17]. This leads to novel physics beyond the rotating wave approximation [18, 19, 20]. Recently work in the superconducting quantum devices (SQD) group has led to the development of a superconducting flux qubit with a tunable coupler, which can drive the coupling strength between a qubit and transmission line from effectively decoupled up to the ultrastrong coupling regime [21]. This tunable coupler device is a key ingredient for experimentally testing entanglement harvesting. The feasibility of the entanglement harvesting experiment was explored in the thesis of Ren [22]. This thesis will discuss the design and implementation of the experimental infrastructure required to perform the entanglement harvesting experiment.

1.1 Outline of thesis

This thesis is organized as follows. Chapters 2 and 3 review the background information relevant for the thesis work. In sections 2.1 and 2.2 we begin with an introduction of the Josephson effect and superconducting quantum devices. We discuss the quantization of Josephson junction circuits and flux qubits in subsection 2.2.1. Next, we describe persistent current qubit operation and coupling noise channels in subsection 2.2.2, and DC-SQUIDs used for flux qubit readout in subsection 2.2.3. Chapter 3 introduces the theoretical background of entanglement harvesting, exploring the challenges of performing this measurement. The model for open transmission lines is derived, and a spin-boson Hamiltonian is presented to describe its coupling to a qubit. Finally, we discuss the modelling of the entanglement harvesting experiment discussed by Ren [22] and the implications surrounding its implementation.

Chapter 4 presents the results of the design work of the experimental setup for running an entanglement harvesting experiment. In section 4.1 the requirements for the experimental setup are outlined. Section 4.2 introduces the thermal modelling and assumptions used throughout the chapter. The microwave cables required for the experiment are presented in section 4.3. The temperature of the microwave cable between the two qubits is modelled in section 4.3.1. In subsection 4.3.2 we model the distortion of fast pulses introduced from the microwave cables. In subsection 4.4 the design of the direct current

(DC) wires is presented. We present a selection of wires to minimize heating, in subsection 4.4.1, and fixtures to thermalize the wires to the dilution refrigerator temperature stages, in subsection 4.4.2. A new superconducting coil is described in section 4.5, and the results of its testing is presented.

In chapter 5 the results are summarized, along with a discussion of the next steps with the implementation of the designs and outlook on its usage.

Chapter 2

Background on Superconducting Qubits

In this chapter, we outline the platform for experimental entanglement harvesting by introducing superconducting quantum devices. In section 2.1 the Josephson effect is introduced along with Josephson junctions. In section 2.2 the properties of superconducting qubits are derived, starting with the quantization of the anharmonic quantum oscillator in subsection 2.2.1. The persistent current qubit is introduced in subsection 2.2.2 and direct current superconducting quantum interference devices (DC-SQUID) are introduced in subsection 2.2.3. These devices are integral parts of the proposed entanglement harvesting experiment.

2.1 Superconductivity and the Josephson effect

The basis of the development of superconducting devices is superconductivity, the property of some metals to become perfect conductors at low temperatures. This effect arises from two electrons bound together, called Cooper pairs. Superconductivity was first theoretically described by Bardeen, Cooper, and Schrieffer [23], where if the electrons in a metal have an effective attractive interaction they form bosonics pairs. This theory importantly gives the macroscopic Cooper pair wavefunction as

$$\psi = |\psi|e^{i\phi} \tag{2.1}$$

where $|\psi|$ is the cooper pair density and ϕ is the phase of the wavefunction.

Superconducting quantum devices rely on the Josephson effect, when the electrons flow through thin potential barrier [24, 25]. Physically, a thin potential barrier can be constructed by interrupting a superconducting path with an insulator, non-superconducting metal, or a thin superconducting segment called a microbridge. These types of thin barriers are Josephson junctions, which are described by the Josephson effect. Consider two different superconducting islands, labelled with their superconducting phase as ϕ_1 and ϕ_2 , connected with a Josephson junction. The difference between the phases is defined as $\gamma = \phi_2 - \phi_1$. The dissipationless current through the junction is

$$I_s = I_c \sin \gamma, \quad (2.2)$$

where I_c is the critical current of the junction, the maximum current which can flow through the junction and have it remain superconducting. This is the first Josephson equation. The voltage across the junction is related to the change in the phase across the junction

$$V = \varphi_0 \frac{\partial \gamma}{\partial t}, \quad (2.3)$$

where φ is the reduced flux quantum. The flux quantum is defined as

$$\Phi_0 = \frac{h}{2|e|} \quad (2.4)$$

where h is Planck's constant and e is the charge of the electron. The reduced flux quantum is then defined as $\varphi_0 = \Phi_0/2\pi$.

Equations 2.2 and 2.3 show that the Josephson junction acts as a non-linear inductor. The partial derivative of current over time is

$$\frac{\partial I}{\partial t} = \frac{\partial I}{\partial \gamma} \frac{\partial \gamma}{\partial t} = I_C \cos \gamma \frac{V}{\varphi_0}. \quad (2.5)$$

This gives the voltage as,

$$V = L(\gamma) \frac{\partial I}{\partial t} \quad (2.6)$$

where $L(\gamma) = \varphi_0 / I_C \cos \gamma = L_J / \cos \gamma$. Here the Josephson inductance is defined as

$$L_J = \frac{\varphi_0}{I_C} \quad (2.7)$$

which is the inductance when γ is zero.

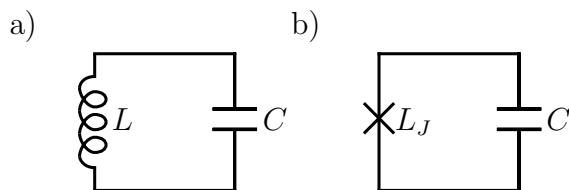


Figure 2.1: (a) The circuit diagram of LC circuit acting as a simple harmonic oscillator with inductance L , and capacitance C . (b) The effective circuit of a Josephson junction with a non-linear inductor L_J , which is a function of the phase across the junction and capacitor C . The cross in the figure represents a Josephson junction.

Josephson junctions also have a shunt capacitance across the junction, parallel to the non-linear inductance. This capacitance, C_J , is defined by the geometry of the junction. The Josephson charging energy associated with the capacitance is

$$E_C = \frac{(2e)^2}{C_J} \quad (2.8)$$

which results from the $2e$ charge of each Cooper pair.

2.2 The Superconducting flux qubit

This section will focus on qubits, quantum systems that are engineered to have two states which can be probed separately from other energy levels in the system. This section will introduce the quantization of superconducting circuits in subsection 2.2.1. The basic quantum circuit used in this thesis, the persistent current qubit, is derived in subsection 2.2.2. Finally the use of a DC-SQUID to readout a flux qubit will be discussed in subsection 2.2.3.

2.2.1 The anharmonic quantum oscillator

The quantization of superconducting qubits can be understood by first studying the quantum harmonic oscillator: a LC resonator.

The LC resonator is an inductor with inductance L and a capacitor with capacitance C , as shown in figure 2.1. When a current, I , is flowing through the inductor it will have a

magnetic flux of $\Phi = IL$. Using the magnetic flux as the canonical position for the system, it can be shown that the charge built up across the capacitor is the canonical momentum $p_\Phi = Q$. This gives potential energy term as

$$U_{LC} = \frac{\Phi^2}{2L} \quad (2.9)$$

and the kinetic energy term as

$$T_{LC} = \frac{p_\Phi^2}{2C}. \quad (2.10)$$

These energies can be used to solve Lagrange's equation to get the equation of motion. Using the Legendre transform to get the classical Hamiltonian, which is then quantized using the canonical quantization. This is a mapping from the phase-space of the dynamic variables to operators Hilbert space, denoted by the hat symbol ($\hat{}$). This process requires conserving the algebraic structure of the dynamic variables Φ and p_Φ . This consists of the Poisson bracket structure which is used to express the classical Hamiltonian evolution, $\{p_\Phi, \Phi\} = 1$. The related structure in the Hilbert space is the commutator, which under the canonical quantization mapping is

$$[\hat{p}_\Phi, \hat{\Phi}] = i\hbar \widehat{\{p_\Phi, \Phi\}} = i\hbar \mathbb{1}. \quad (2.11)$$

Hence, the quantized Hamiltonian is

$$\hat{H}_{LC} = \frac{\hat{p}_\Phi^2}{2C} + \frac{\hat{\Phi}^2}{2L}. \quad (2.12)$$

From this point on the dynamic variables will be assumed to be quantum and this notation will be neglected.

This Hamiltonian is the form of the commonly studied quantum harmonic oscillator [26]. By solving for the eigenstates and eigenenergies of the system, the Hamiltonian can be mapped to

$$E_n = (n + 1/2)\hbar\omega_{LC} \quad (2.13)$$

where n is a whole number representing the state and $\omega_{LC} = (LC)^{-1/2}$ is the resonant frequency of the system. These are the quantized levels which can be used for computation. However, as seen in equation 2.13, and illustrated in figure 2.2, the energy gap between each level is identical. This restricts the ability to probe individual energy transitions in the system.

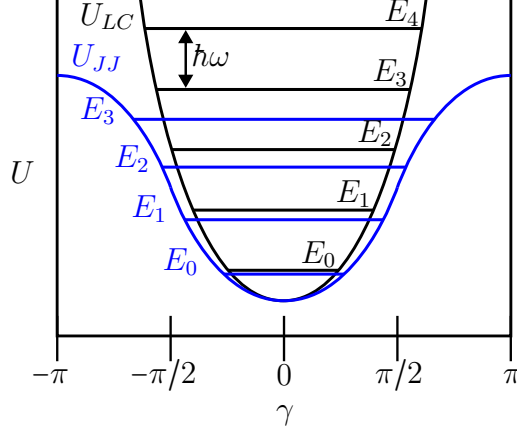


Figure 2.2: A comparison of the potential energies of the LC resonator as a black curve (U_{LC}) and the Josephson junction as a blue curve (U_{JJ}). The horizontal axis is the canonical position variable of the system, which is flux, ϕ , for the LC resonator and phase across the junction, γ , for the Josephson junction. The potential energies of the LC resonator are all equally spaced by $\hbar\omega$.

To be able to properly excite individual energy levels, a Josephson junction can be used as a non-linear inductor, as shown in equation 2.6. The potential energy using equations 2.2 and 2.6 is

$$U_{JJ} = \int_{-\infty}^t dt' I(t') V(t') = -\varphi_0 I_C \cos \gamma = -E_J \cos \gamma, \quad (2.14)$$

where the Josephson energy, $E_J = \varphi_0 I_C$, is introduced. The kinetic energy can be solved, as above in equation 2.10, using the capacitance across the junction, C_J . The kinetic energy of the Josephson junction is

$$T_{JJ} = \frac{1}{2C_J} Q^2 = \frac{C_J \varphi_0^2}{2} \dot{\gamma}^2 = \frac{E_C}{2} p_\gamma^2, \quad (2.15)$$

using the charging energy introduced in equation 2.8 and the conjugate momentum to γ . The resulting Hamiltonian after applying canonical quantization is

$$H_{JJ} = \frac{E_C}{2} p_\gamma^2 - E_J \cos \gamma. \quad (2.16)$$

The Josephson junction, has similar quantized levels to the LC circuit. The potential energy of a Josephson junction, in equation 2.16, has a cosign term instead of the squared

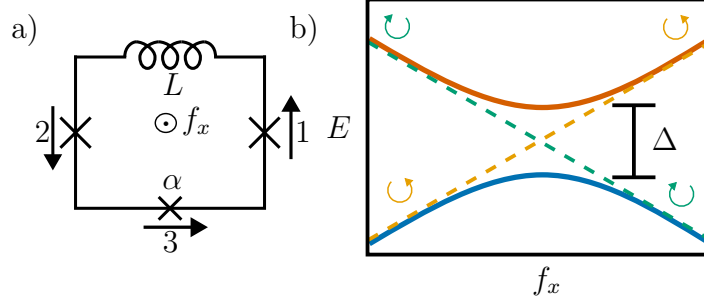


Figure 2.3: (a) Circuit diagram of a persistent current qubit where junction 1 and 2 are equal size and junction 3 is designed to have the Josephson energy α times the others. The loop has a geometric self inductance, L . (b) The avoided crossing in the energy levels near the symmetry point of the persistent current qubit controlled by the external flux. At the symmetry point the minimum gap has the frequency of Δ . The yellow dashed line corresponds to the counter clockwise current state, and similarly the teal dashed line corresponds to a clockwise current.

term in the LC circuit potential energy, in equation 2.12. This term causes different energy levels to occur. As shown in figure 2.2 the energy levels of the Josephson junction are all separated by different energies. This is called anharmonicity, and the larger the difference between the excitation energies the easier it is to probe individual states.

2.2.2 The persistent current qubit

The flux qubit is a superconducting quantum device used in the entanglement harvesting proposal and is the focus of this this thesis. The flux qubit is designed to have $E_J \gg E_C$ which leads to quantized energy levels in the flux basis and low charge noise sensitivity [27]. One key property of these type of devices is that they are sensitive to external flux biasing, which will be used to control the qubit system.

A particular flux qubit is called the persistent current qubit (PCQ), which is designed of a superconducting loop with three junctions in series [28, 29]. In the entanglement harvesting device two of these junctions are designed with the same parameters and the third with a junction α times the area. This is illustrated in figure 2.3 (a). The potential

of this system, with respect to the junction phases, is

$$U(\gamma_1, \gamma_2, \gamma_3) = -E_J (\cos \gamma_1 + \cos \gamma_2 + \alpha \cos \gamma_3) + \frac{\phi_0^2}{2L} (\gamma_1 + \gamma_2 + \gamma_3 + 2\pi f_x)^2. \quad (2.17)$$

In this equation the second term comes from the energy of the loop's self inductance. This term is expressed as the sum of the other phases in the superconducting loop, which is a result of the Cooper pairs wave function (Eq. 2.1) being single valued around a loop. The self inductance is thus dependant on the junction phases and the external flux threading the loop Φ_x ; written here as a normalized magnetic flux, $f_x = \Phi_x/\Phi_0$. Here the self inductance, L , of the loop is typically very small so the phase across this inductance will be taken to zero to minimize the energy. Hence the phase constraint is expressed as $\gamma_1 + \gamma_2 + \gamma_3 + 2\pi f_x = 0$. This constraint reduces the internal degrees of freedom to two. The resulting potential is thus

$$U(\gamma_1, \gamma_2) = -E_J (\cos \gamma_1 + \cos \gamma_2 + \alpha \cos (\gamma_1 + \gamma_2 + 2\pi f_x)) \quad (2.18)$$

The conjugate momentum associated with the two remaining phases are given as a vector $\mathbf{p} = (p_1, p_2)$ with an effective capacitance matrix of

$$\mathbf{C} = \begin{pmatrix} 1 + \alpha & \alpha \\ \alpha & 1 + \alpha \end{pmatrix} \quad (2.19)$$

giving the Hamiltonian of the PCQ as,

$$H = \frac{1}{2} \frac{E_C}{\hbar^2} \mathbf{p} \mathbf{C}^{-1} \mathbf{p}^T - E_J (\cos \gamma_1 + \cos \gamma_2 + \alpha \cos (\gamma_1 + \gamma_2 + 2\pi f_x)) \quad (2.20)$$

The potential energy is invariant under translations of 2π , and thus the entire energy landscape can be described by phases between $-\gamma$ to γ , as shown in figure 2.4. The two potential minimum are stable states associated with the classical clockwise and counter-clockwise current flows. These potential wells have the same energy which leads to an avoided crossing, as seen in figure 2.3 (b). This point is called the symmetry point and for this geometry occurs when $f_x = 0.5$. This is the biasing flux which gives the potential energy shown in figure 2.4 (a). The qubit near the symmetry point can then be described as a two level system with the Hamiltonian

$$H = -\frac{\varepsilon}{2} \sigma_z - \frac{\Delta}{2} \sigma_x \quad (2.21)$$

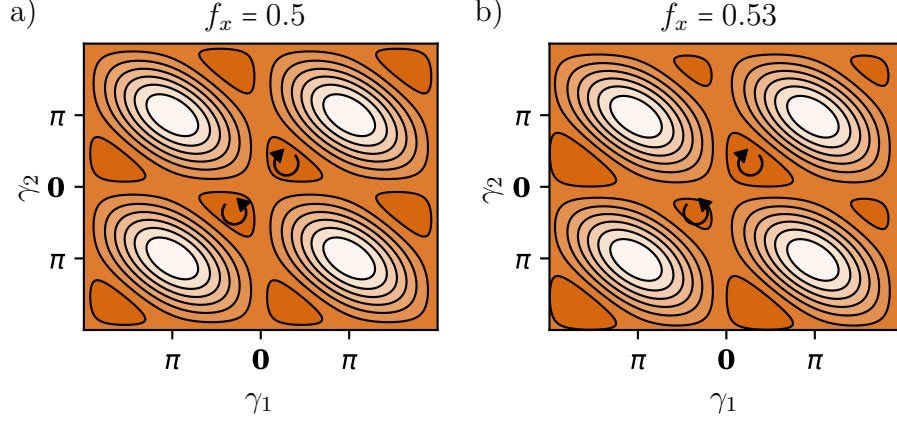


Figure 2.4: The potential energy of the persistent current qubit as a function of the independent phases γ_1 and γ_2 for a fixed value of f_x . (a) The potential energy at the symmetry point $f_x = 0.5$. (b) The potential energy away from the symmetry point $f_x = 0.53$.

where Δ is the minimum energy splitting and $\varepsilon = 2I_p\Phi_0(f_x - 0.5)$. Here we introduced the Pauli matrices

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2.22)$$

and

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.23)$$

which are operators on the clockwise ($|\odot\rangle$) and counter clockwise ($|\oslash\rangle$) current states. At the symmetry point, when $f_x = 0.5$, the Hamiltonian will only have a σ_x term. This results in the energy states being equal superposition of the two current states. The energy states are

$$|0\rangle = \frac{1}{\sqrt{2}} (|\odot\rangle - |\oslash\rangle), \quad (2.24)$$

$$|1\rangle = \frac{1}{\sqrt{2}} (|\odot\rangle + |\oslash\rangle), \quad (2.25)$$

where $|0\rangle$ is the ground state and $|1\rangle$ first excited state. The equal superposition of the two current states means that at the symmetry point these states have no magnetic moment.

When the system is biased away from the symmetry point the potential wells will separate in energies, as shown in figure 2.4 (b). When the two potential wells are no longer

equal in energy the two energy levels of the system can be associated with the classical clockwise and counter-clockwise currents as shown in figure 2.3 (b). This biasing regime can thus be used for readout of the PCQ qubit as the two energy levels will have different magnetic flux.

Typically operations are performed on the qubit in the symmetry point, and then the flux is adiabatically changed to a non-symmetric point for readout. For a process to be adiabatic it requires that the change is sufficiently slow so that the qubit is not kicked up or down to a different energy state. Therefore the occupation of the first and second excited states will not change and the anti-crossing energy states near the symmetry point will map directly to the persistent current states used in the readout.

Noise channels interacting with the PCQ are modelled by coupling to a general bath. The qubit Hamiltonian, equation 2.21, is diagonalized

$$H = -\frac{\hbar\omega_{01}}{2}\sigma_z + (\cos\theta\sigma_z + \sin\theta\sigma_x)\mathcal{F} + H_{bath} \quad (2.26)$$

where σ_z and σ_x are Pauli matrices with respect to the energy eigenstates of the PCQ, ω_{01} is the transition frequency between the lowest two energy states, $\tan\theta = \Delta/\epsilon$, and $\mathcal{F}$ is the bath operator defining the coupling to the PCQ. Typically the Born approximation is assumed, where the interaction Hamiltonian is small compared to the qubit and bath Hamiltonian. The Markov approximation is also applied, where the evolution of the interaction picture density matrix is slow compared to the bath operator. The power spectral density of the noise is

$$S(\omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle \mathcal{F}(\tau)\mathcal{F}(0) \rangle \quad (2.27)$$

which is the Fourier transform of the correlation function of the bath coupling function. Assuming the temperature is smaller than the energy gap of the system the excitation rate can be neglected. The relaxation rate is then

$$\Gamma_1 = \frac{\sin^2\theta}{\hbar^2} S(\omega_{10}) \quad (2.28)$$

and the dephasing rate is

$$\Gamma_\varphi = \frac{\cos^2\theta}{\hbar^2} S(0). \quad (2.29)$$

These inverse of these rates define the coherence time of a qubit, how long the qubit state lasts before losing its information to the quantum bath. The relaxation rate is how quickly the qubit decays back into its ground state by emitting energy into the bath. The dephasing rate is the rate the noise in the bath creates uncertainty in the phase of a superposition state.

2.2.3 Qubit state readout with DC-SQUIDS

As discussed in subsection 2.2.2, and illustrated in figure 2.3 (b), when biased away from the symmetry point the quantized states are associated with the clockwise and counter-clockwise currents. The difference in the sign of the magnetic flux generated by these states can be measured. This is typically done with a DC Superconducting Quantum Interference Device (DC-SQUID), a superconducting loop interrupted by two Josephson junctions.

There are two ways to operate the DC squid for readout. The first is in the dissipative regime. This works by supplying a biasing current I_b across the junction, which is split equally while there is no external flux. The magnetic flux from the qubit will modify the circulating current, I_{circ} , in the DC-SQUID, resulting in a current $I_b/2 \pm I_{circ}$ through the two arms of the DC-SQUID. When one of these currents becomes larger than I_C of the junctions they become dissipative and a voltage can be measured across them. This voltage corresponds to the sign and magnitude of the magnetic field from the qubit state [30]. However the dissipative regime the junction is resistive and will generate heat from the current flowing through it. This will generate quasiparticles that can disrupt the qubit and decrease coherence. Further, the device emits microwave signals which can further decrease the qubit coherence [22].

Alternatively, the DC-SQUID can be used like a flux dependant inductor, where the impedance of the DC-SQUID is dependant on the magnetic flux from the qubit state. When solving this circuit with the flux constraint $\gamma_1 + \gamma_2 + 2\pi f_x = 0$, and using the substitution $\gamma' = (\gamma_1 - \gamma_2)/2$, the kinetic energy is

$$U = -E_J [\cos \gamma_1 + \cos \gamma_2] = -2E_J \cos \pi f_x \cos \gamma'. \quad (2.30)$$

This flux dependence can be seen to control an effective Josephson energy and effective Josephson inductance

$$L_{J,eff} = 2L_J |\cos \pi f_x|. \quad (2.31)$$

A static biasing flux of $f_x = 0.5$ is used to maximize the change in inductance under the magnetic field from the qubit.

By placing this in a resonant circuit the change in the inductance will modify the resonant frequency. Thus, for the $|\odot\rangle$ and $|\ominus\rangle$ states there will be two distinct resonant frequencies of the resonant circuit, ω_0 and ω_1 . This is typically implemented by placing the DC-SQUID at the end of a quarter wavelength resonator where the resonator is capacitively coupled to a transmission line. As the frequency travelling through the transmission line approaches the resonant frequency it will be suppressed by a Lorentzian function. To do this as a single measurement the resonator can be probed at the frequency ω_0 which will

have be maximally suppressed for a qubit state 0 and not suppressed for a qubit state 1. These readout circuits are shown implemented into a experimental device in figure [4.1](#) in section [4.1](#).

Chapter 3

Background on entanglement harvesting

In this chapter, the theoretical basis of entanglement harvesting will be explored, a necessary step to understanding the technical requirements for successfully performing this experiment. In section 3.1 the concept of entanglement is introduced, along with entanglement harvesting and its restrictions. In section 3.2 the model for a 1-dimension waveguide will be derived, and in section 3.3 qubit-waveguide coupling will be explored. Section 3.4 will introduce previous work done to model and implement a measurement of entanglement harvesting between two persistent current qubits.

3.1 Entanglement harvesting

The study of quantum mechanics has led to many unintuitive predictions which are opposed to our classical understanding of the world. One such prediction is entanglement, which Einstein famously called “spukhafte fernwirkung” or “spooky action at a distance” [31]. This is the prediction that two systems have statistical correlation between their states even when they are removed from each other [32, 33]. As highlighted by experiments violating Bell’s inequalities the correlation does not arise from a hidden local variable, but from the two quantum systems being in a mixed state [34]. A mixed state is any state that cannot be separated into an independent combination of both of the two subsystems.

It was first described by Summers and Werner [32], in 1985, that the vacuum state of any free quantum field will have entanglement at two locations separated across space. The

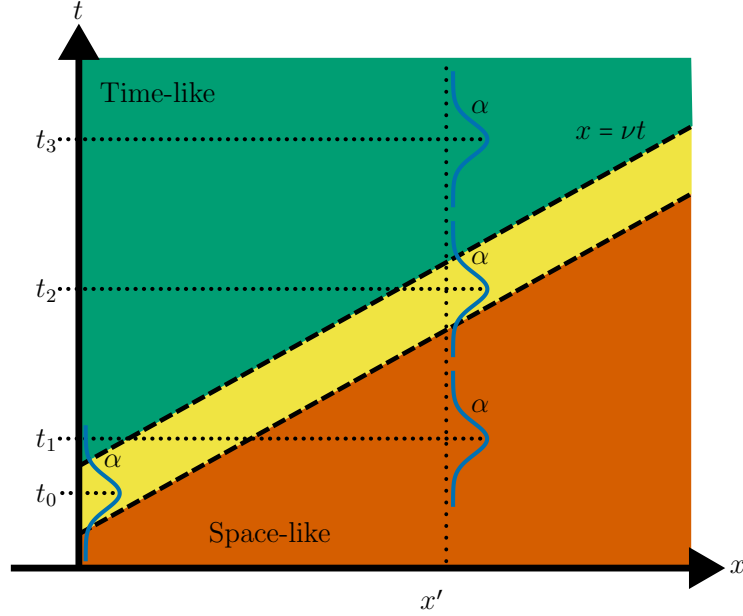


Figure 3.1: A space-time diagram for the separation of events. Diagonal dashed black lines represent the propagation of light at a speed of ν in this space. Events, α , occur over a time frame presented as a blue Gaussian curve. The reference event is at time t_0 and location $x = 0$. In the orange region the events are space-like separated, including the event illustrated at t_1 and cannot share information. Events in the yellow region can receive information directly, illustrated by the event at t_2 . The teal region, containing the event at t_3 , is where the events are time-like separated and can be impacted by the reference event, which happened in the past.

work by Valentini [12] and Reznik [35] formalized this prediction, describing the process wherein two quantum detectors can extract the entanglement present in the vacuum to become entangled themselves. The detectors used in these models are, very generally, two level systems which are coupled to the field for a small amount of time.

Additionally, it has predicted theoretically that entanglement harvesting from the vacuum is significantly smaller than the field-mediated communication between the two detectors. [36]. This result requires that the two probes used in entanglement harvesting must not be in direct communication. This introduces a problem for readout of the states, which would need to be completed before the causal contact occurs in the field. This cannot be done with the readout system discussed in subsection 2.2.3 which typically takes over 10

ns to perform readout [30]. A solution to this issue is presented by Sabin et al. [37] where they propose to decouple the detector from the field. Their proposal uses a flux qubit as a two level detector that can be coupled to a transmission line through a control of biasing flux. Their proposal requires coupling and decoupling from the field quickly enough to keep the two events space-like separated, as outlined in figure 3.1. This proposal is the basis for the experimental protocol for measuring entanglement harvesting. It has also been shown that the larger the coupling strength is the larger the entanglement that can be harvested [37].

These earlier analyses have been particularized to a massless scalar field, which corresponds to free travelling electromagnetic waves. This exploration has shown that it is also possible to harvest entanglement from a thermal state, but that there is a maximum temperature such that no entanglement can be harvested above it [38]. This is critical as it gives a range of temperatures where the thermal state is sufficiently close to the vacuum state of the quantum field. This is why the phrase “entanglement harvesting” is used instead of the phrase “vacuum harvesting”. This thesis will also use the term “quantum field” instead of “quantum vacuum” to emphasize this distinction.

These theoretical works outline the constraints for the experimental design. To measure entanglement harvesting we will need to implement a tunable coupler which can strongly couple a qubit to an electromagnetic field and is at a sufficiently low temperature. The coupling requirements are discussed further in section 3.3 and the temperature requirements are discussed in section 3.4.

3.2 Modelling a scalar massless field

The proposal of Sabin et al. (2010) [39], can be applied to harvesting entanglement out of the electromagnetic field in a transmission line. A transmission line is a waveguide designed to propagate electromagnetic waves. This corresponds to wave solutions which only one position and one time dimension, referred to as a 1+1 dimensional field [36].

There are two types of transmission lines which are used. Coplanar waveguides are patterned on a flat dielectric wafer from a central signal conductor with two grounding conductors on both sides [40]. These are used to construct quantum devices on superconducting chips. For travelling longer distances coaxial cables are commonly used. These have a cylindrical conductor for the signal surrounded by a layer of a dielectric material and another grounding conductor [40]. The impedance of both of these types of transmission line are defined by the dielectric chosen and the distance between the signal and ground

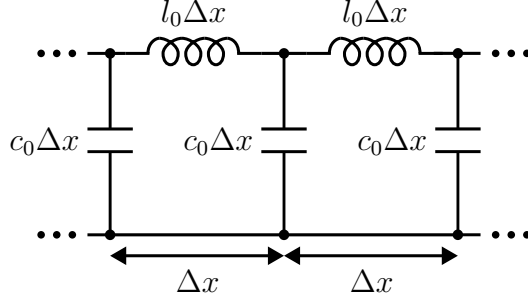


Figure 3.2: A circuit diagram of an infinitesimal segment of a transmission line. The transmission line has an inductance per length of l_0 and a capacitance per length of c_0 .

conductor. The standard impedance used is $50 \, \Omega$ as a compromise between maximizing the power transfer and minimizing the attenuation [40].

These two types of transmission lines can be modelled with a series of infinitesimally small inductors and capacitors. This model will be applied to the superconducting coplanar transmission lines around the device, so no resistance will be added to the model. This is illustrated in figure 3.2. Each Δx segment is characterized by an inductance l_0 and capacitance c_0 per unit length. Here the resistance of the Applying Kirchoff's laws to each section gives

$$V(x + \Delta x, t) = V(x, t) - l_0 \frac{\partial I(x, t)}{\partial t} \Delta x, \quad (3.1)$$

$$I(x + \Delta x, t) = I(x, t) - c_0 \frac{\partial V(x, t)}{\partial t} \Delta x. \quad (3.2)$$

In the continuum limit as $\delta x \rightarrow 0$,

$$\frac{\partial V(x, t)}{\partial x} = -l_0 \frac{\partial I(x, t)}{\partial t}, \quad (3.3)$$

$$\frac{\partial I(x, t)}{\partial x} = -c_0 \frac{\partial V(x, t)}{\partial t}, \quad (3.4)$$

gives the telegraphers equations.

These equations can be rewritten in terms of the node flux

$$\Phi(x, t) = \int_{-\infty}^t d\tau V(x, \tau), \quad (3.5)$$

which is written in terms of the voltage as

$$V(x, t) = \dot{\Phi}(x, t). \quad (3.6)$$

Then the current is given by equation 3.3 as

$$I(x, t) = -\frac{1}{l_0} \frac{\partial \Phi(x, t)}{\partial x}. \quad (3.7)$$

Now Eq. 3.4 can be rewritten as

$$0 = \ddot{\Phi} - \frac{1}{c_0 l_0} \frac{\partial^2 \Phi}{\partial x^2}, \quad (3.8)$$

which is a standard one dimensional wave equation. This can be solved by solutions of the form

$$\Phi(x, t) = \frac{1}{N} \sum_k \left(c_k e^{i(kx - \omega_k t)} + c_k^* e^{-i(kx - \omega_k t)} \right), \quad (3.9)$$

where $k \in 2\pi\mathbb{Z}/L$ represents the mode number for a transmission line with a length L . The value of c_k relates to the occupation of each mode solved by applying boundary conditions and ω_k is the mode frequency. The wave number and frequency are related with the speed of light in the transmission line

$$v = \omega_k / |k| = 1 / \sqrt{c_0 l_0}, \quad (3.10)$$

which is defined by the capacitance and inductance per unit length. Another important transmission line parameter is the impedance of the transmission line

$$Z_0 = \sqrt{l_0 / c_0}. \quad (3.11)$$

The values of the inductance and capacitance are chosen to set the impedance to $50 \, \Omega$, as discussed above.

The expression Eq. 3.8 is the equation of motion for the system, which can also be expressed as the Lagrangian

$$\mathcal{L} = \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \left[\frac{c_0}{2} (\dot{\Phi})^2 - \frac{1}{2l_0} \left(\frac{\partial \Phi}{\partial x} \right)^2 \right]. \quad (3.12)$$

With the conjugate momentum $p = c_0 \dot{\Phi}$, the canonical quantization can be applied to promote the dynamic variable into quantum variables with the commutation relation

$$[\Phi(x, t), p(x', t')] = i\hbar \delta(x - x') \delta(t - t') \quad (3.13)$$

using the same methods discussed in subsection 2.2.1.

Thus, the system Hamiltonian is

$$H = \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \left[\frac{1}{2c_0} p^2 - \frac{1}{2l_0} \left(\frac{\partial \Phi}{\partial x} \right)^2 \right], \quad (3.14)$$

which can be written it in terms of the photon creation and annihilation operators. The annihilation operator is defined as

$$b_k = \sqrt{\frac{2\omega_k}{\hbar}} c_k, \quad (3.15)$$

and the creation operator is its the conjugate transpose, $b^\dagger$. Thus the Hamiltonian can be written as

$$H = \sum_k \hbar \omega_k \left(b_k^\dagger b_k + \frac{1}{2} \right). \quad (3.16)$$

The commutation relation of the photon creation and annihilation operators is $[b_k, b_{k'}^\dagger] = \delta_{kk'}$. The Hamiltonian can also be extended to an infinite transmission line by taking $L \rightarrow \infty$. This has the effect of extending the integration domain and reducing the spacing between each k mode to an infinitesimal. This gives the system Hamiltonian as

$$\begin{aligned} H &= \int_{-\infty}^{\infty} dx \left[\frac{c_0}{2} (\ddot{\Phi})^2 - \frac{1}{2l_0} \left(\frac{\partial \Phi}{\partial x} \right)^2 \right] \\ &= \int_{-\infty}^{\infty} dk \hbar \omega_k \left(b_k^\dagger b_k + \frac{1}{2} \right). \end{aligned} \quad (3.17)$$

The process of taking the transmission line to being infinite also transform the sum seen in Eq. 3.9 into the integral

$$\Phi(x, t) = \int dk \sqrt{\frac{\hbar}{4\pi c_0 \omega_k}} \left(b_k e^{i(kx - \omega_k t)} + b_k^\dagger e^{-i(kx - \omega_k t)} \right), \quad (3.18)$$

where the constant term in this formulation comes from Eq. 3.15 and the normalization term.

This model contains all modes, which is not physical, as electromagnetic waves cannot propagate at arbitrarily high frequencies [41]. A cutoff is often included as an upper limit to the frequency integrals, but is often not justified from first principles [41]. This is applied here as

$$\Phi(x, t) = \int dk \sqrt{\frac{\hbar}{4\pi c_0 \omega_k}} C(k) \left(b_k e^{i(kx - \omega_k t)} + b_k^\dagger e^{-i(kx - \omega_k t)} \right), \quad (3.19)$$

where $C(k)$ is the unit-less cutoff function. This high frequency cutoff function is more general than a sharp cutoff.

3.3 Coupling a qubit to a transmission line

The spin-boson model is a widely applicable framework for studying the interaction of atoms to electromagnetic waves. This couples a spin system, which is modelled as a two level system, and a boson, which is modelled as a bath of quantum harmonic oscillators. The effective Hamiltonian describing the system and environment is

$$H_{SB} = H_{spin} + H_{int} + H_{boson} = -\frac{\hbar\omega}{2}\sigma_z + \sum_k (g_k^x \sigma_x + g_k^z \sigma_z)(b_k^\dagger + b_k) + \sum_k \hbar\omega_k b_k^\dagger b_k, \quad (3.20)$$

where the σ_z and σ_x are the Pauli matrices on the spin system and $b_k^\dagger$ and b_k are the creation and annihilation operators of the mode with wavenumber k in the bosonic bath. Here ω is the spin system energy gap. The terms g_k^x and g_k^y represent the transverse and longitudinal coupling strength between the spin and boson systems. It is expected that the longitudinal coupling will be negligible on the interaction with the transmission line [21]. The relaxation rate into the bosonics bath is

$$\Gamma_1 = \frac{2\pi}{\hbar^2} \sum_k (g_k^x)^2 \delta(\omega - \omega_k) = J(\omega), \quad (3.21)$$

where δ is the Dirac delta function. Assuming a ohmic bath the spectral density of the coupling terms can be defined as

$$J(\omega) = \pi\omega\alpha, \quad (3.22)$$

where the dimensionless coupling term α is introduced. In the Born-Markov approximation, and using Fermi's golden rule, the relationship between the coupling and qubit gap is

$$\alpha = \Gamma_1/\pi\Delta, \quad (3.23)$$

where Γ_1 is the relaxation rate of the spin system into the bosonic bath.

The value of α is of interest for characterizing the system. In elementary charged particles the coupling strength is set by the fine-structure constant $\alpha \sim 1/137$. In superconducting circuits coupled to a transmission line the coupling can be designed to be much greater or lower than the fine-structure constant.

In this thesis the spin-boson model will be applied to the interaction between a qubit and an open transmission line. The spin system will be a qubit and the bosonic bath will be a continuum of electromagnetic modes. Since the transmission line is open it does not have discrete modes and the summation over the wave numbers k in equation 3.20 will become a integration over all of space.

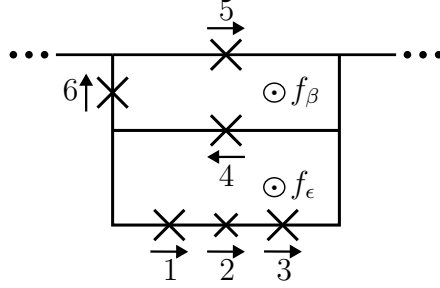


Figure 3.3: Circuit diagram of the tunable coupling device. The biasing fluxes are represented with f_β threading the coupling loop and f_ϵ threading the qubit loop. The crosses represent Josephson junctions. The triple dots represent the open transmission lines which extend out infinitely.

In the work by Janzen et al. [21] a tunable coupler is outlined. Janzen's design is shown in figure 3.3. The device is made up of two loops of superconducting aluminum and 6 Josephson junctions. The loop threaded by the normalized magnetic flux f_ϵ is a persistent current qubit. The waveguide transmission line is connected to the current using the loop threaded by the normalized magnetic flux f_β . The coupling loop connects the qubit to the open transmission line. The f_β applied changes the susceptibility of the coupling loop

$$1/L_C = \frac{1}{\Phi_0} \frac{\partial I_g}{\partial f_\beta}, \quad (3.24)$$

where I_g is the ground state current of the coupler loop. Through this coupling loop a current in the transmission line I_{TL} would induce a flux in the qubit loop, or vice-versa. The effective mutual inductance between the transmission line and the qubit is given by

$$M_{eff} = \frac{M_{qb}M_b}{L_C} = \frac{M_{qb}M_b}{\Phi_0} \frac{\partial I_g}{\partial f_\beta}, \quad (3.25)$$

where M_{qb} is the mutual inductance for the coupler-qubit interaction and M_b is the effective mutual inductance for the coupler-transmission line interactions. M_{qb} and M_b are dominated by the inductance of the shared junctions 4 and 5, respectively. For more details on the mapping of this circuit to a simpler coupling model see the thesis of Jiahao Shi [42]. Using equations 3.24 and 3.25 the coupling is optimized by increasing the dependence of I_g on the external flux while keeping the energy gap of this coupler sufficiently large so it remains in its ground state. The Josephson energy of junction 6 is set 0.52 times that of junctions 4 and 5 [21]. The resulting energy gap of the coupler is 23 GHz [42] which is much larger than the qubit energy of gap, which will be between 2-8 GHz [21].

Using the spatial transmission line Hamiltonian modelled with equation 3.17 in the spin boson model (equation 3.20) the full system Hamiltonian of the coupler-qubit system is

$$H = H_{qb} + H_{TL} + H_{int} = -\frac{\hbar\omega}{2}\sigma_z + H' + \int_{-\infty}^{\infty} \left[\frac{c_0}{2} (\dot{\Phi})^2 - \frac{1}{2l_0} \left(\frac{\partial\Phi}{\partial x} \right)^2 \right] + H_{int}, \quad (3.26)$$

where H' is the renormalization term [21],

$$H' = \frac{\Phi_0^2 \gamma_5^5}{8\pi^2 l_0 \delta x}, \quad (3.27)$$

where γ_5 is the phase across junction 5 shown in figure 3.3 and δx is a relevant length scale of the discrete LC model in section 3.2. The interaction term is given as

$$H_{int} = \frac{\Phi_0}{2\pi l_0} \gamma_5 \left. \frac{\partial\Phi}{\partial x} \right|_{x_{qb}}, \quad (3.28)$$

where x_{qb} is the location of the coupler loop on the transmission line. It will be assumed that $x_{qb} = 0$. This expression comes from the current in the transmission line in equation 3.7. Applying equation 3.18 to equation 3.28 gives

$$H_{int} = \frac{\Phi_0}{2\pi l_0} \gamma_5 \int dk \sqrt{\frac{\hbar}{4\pi c_0 \omega_k}} C(k) k \left(i b_k e^{i(kx - \omega_k t)} - i b_k^\dagger e^{-i(kx - \omega_k t)} \right), \quad (3.29)$$

which can be mapped to the spin-boson model. Restricting the Hamiltonian to the lowest two energy levels of the qubit system gives the coupling factor from equation 3.20 as

$$g_k = \frac{\Phi_0}{2\pi l_0} |\gamma_{5,10}| \sqrt{\frac{\hbar \omega_k}{2\pi c_0 v^2}}, \quad (3.30)$$

where the wave number is related to the frequency and wave velocity $k = \omega_k/v$ and $\gamma_{5,10}$ is the transverse coupling element of the phase operator. The decay rate from the qubit into the transmission line is

$$\Gamma_1 = \frac{\Phi_0^2}{4\pi^2 \hbar Z_0} |\gamma_{5,10}|^2 \omega, \quad (3.31)$$

where ω is the minimum energy gap of the qubit defined in equation 3.26. The dimensionless coupling term

$$\alpha = \frac{\Phi_0^2}{4\pi^3 \hbar Z_0} |\gamma_{5,10}|^2, \quad (3.32)$$

is only related to the phase operator through the shared coupler-transmission line junction.

Experimental validation of this design has been done in reference [21]. This device has been experimentally measured tuning the decay rate between 8.7 MHz and 1.85 GHz, representing a shift from being nearly decoupled to a strong coupling of through with just tuning in the β -flux in the coupling loop. The modeling done in by Janzen [43] suggests that the coupling possible with these devices may be even higher, which will be required to enhance entanglement harvesting measurements.

This also provides a constraint on the magnitude of other decoherence sources coupling to the qubit. To operate the tunable coupler it is required that any other noise channels only weakly couple to the qubit. This is required to ensure that the evolution of the Hamiltonian in equation 3.26 fully describes the system evolution, and can neglect any other noise channels. This is also required to ensure that the qubit coherence is sufficiently long for system readout. Thus, all components which couple to the qubit must have a relaxation or dephasing rate significantly lower than the minimum transmission line decay rate.

3.4 Realization of entanglement harvesting

In relativistic quantum information the Unruh-DeWitt (UDW) model is a toy model, used for light-matter interactions. In the language of this model a qubit is a monopole detector which interacts with light under the Hamiltonian

$$H_{int}(t) = \lambda(t)\sigma_x \int dx F(x - x_0) \frac{\partial \Phi(x, t)}{\partial x} \quad (3.33)$$

where $\lambda(t)$ is the coupling function and $F(x - x_0)$ is the spatial distribution of the detector. In particular this UDW model couples to the momentum term of the transmission line, which is inline with the qubit-transmission line coupling. Thus, this model can be mapped to equation 3.29. The spin-boson coupling is mapped to the coupling function, $g_k^x(t) = \lambda(t)$. The time dependence comes from the β -flux dependence of the ground state current as shown in equation 3.25. The frequency cutoff can be mapped to the Fourier transform of the spatial distribution of the detector. The frequency cutoff can be seen as an effective spatial smearing of the detector. This mapping allows the results discussed in section 3.1, many of which use the UDW detector model, to be applied to the qubit-coupler circuit.

In his thesis Ren [22] performed this mapping, fully modelling two tunable couplers on the same transmission line 12 cm apart. This work assumes a sharp cutoff frequency at 50 GHz. We will outline his treatment and results here, but refer to his thesis for full details

[22]. The system is initialized through relaxation into the qubits and the transmission line ground state while decoupled. A 10 μs wait allows for the qubits to decay to the ground state with high fidelity [22]. The transmission line will be in a thermal state is

$$\rho_{th} = e^{-\beta H} / \text{Tr}[e^{-\beta H}] \quad (3.34)$$

where $\beta = 1/k_B T_{th}$. The system's Hilbert state is a combination of the two qubits and the transmission line so the initial state

$$\rho_0 = \rho_{A,0} \otimes \rho_{B,0} \otimes \rho_{th,0} = \rho_{AB,0} \otimes \rho_{th,0}, \quad (3.35)$$

where the two qubit spaces are written together with $\rho_{AB} = \rho_A \otimes \rho_B$ and the transmission line starts in its thermal state ρ_{th} .

The system full system can be mapped as two UDW detectors with the Hamiltonian

$$H_I(t) = \sum_{v \in \{A,B\}} \lambda_v(t) \sigma_x^{(v)} \int dx F_v(x - x_v) \frac{\partial \Phi(x, t)}{\partial x} \quad (3.36)$$

with the coupling term λ_v , effective spatial smearing of the qubit F_v , and location x_v . These parameters are defined separately for the two qubits which will be labelled A and B . Using the interaction picture for the qubits states

$$\rho_I = e^{-iH_{qb}t/\hbar} \rho e^{iH_{qb}t/\hbar} \quad (3.37)$$

will evolve under the time evolution operator

$$U = \mathbb{1} + U^{(1)} + U^{(2)} + \dots \quad (3.38)$$

where $U^{(n)}$ is the n -th term in the Dyson expansion of the operator. The Dyson series operators are

$$U^{(1)} = -i \int_{-\infty}^{\infty} dt H_I(t) \quad (3.39)$$

and

$$U^{(2)} = - \int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' H_I(t) H_I(t'), \quad (3.40)$$

with all higher order terms neglected. The evolution of the two qubits is driven by the tuning of the coupling term in equation 3.36. It has been shown that a Gaussian switching function

$$\lambda_v(t) = \exp\left[-\frac{(t - t_v)^2}{2\tau^2}\right], \quad (3.41)$$

where t_v is the time of the coupling and τ is the width of the coupling time, harvests more entanglement than a sharp switching function [44]. The Gaussian pulse was chosen to have a time constant of $\tau = 33$ ps [22], such that the pulse can be generated with a fast arbitrary waveform generator. The length of the entire pulse, defined by the point where the Gaussian has a magnitude of 1.1% at its $3 - \sigma$ width, will be 200 ps. The Keysight M8195A AWG has control up to 25 GHz allowing for generation of 33 ps Gaussian pulses with little distortion.

The final qubit state of the experiment is

$$\rho_{AB} = \text{Tr}_\Phi [U \rho_0 U^\dagger] \quad (3.42)$$

where the transmission line states are traced out. The resulting state has the form

$$\rho_{AB} = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix} \quad (3.43)$$

where this matrix has the basis $\{|g_A g_B\rangle, |e_A g_B\rangle, |g_A e_B\rangle, |e_A e_B\rangle\}$. The biasing is then adjusted to push the qubits away from their symmetry points so that they can be measured by the adjacent DC-SQUID as outlined in subsection 2.2.3.

The final state in equation 3.43 can be analyzed for entanglement using the metric of concurrence. The concurrence is an entanglement monotone, which has the property that all separable states have a value of zero and is defined as

$$\eta(\rho) = 2 \max\{0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}\}. \quad (3.44)$$

This concurrence metric was used in Ren's thesis [22] to simulate the experimental procedure to optimize the design parameters.

The simulation done by Ren shows that for a device at 20 mK the concurrence of the final state is 0.01, which can be distinguished from a separable state. Further, the experimental simulations predict the decrease in concurrence with temperature. The temperature dependence of the concurrence is introduced with the dimensionless quantity $\hbar/k_B T_{th} T$. This is due to a deviation from the vacuum state, which is given by [22]

$$n(T_{th}) = \frac{1}{e^{\hbar\omega/k_B T_{th}} - 1} \quad (3.45)$$

where n is the occupation of thermal photons. When the number of thermal photons becomes significant the harvested entanglement is reduced. This corresponds to a decrease in the non-local interactions relative to the local interactions [38].

The number of thermal photons becomes significant at 20 mK, and the concurrence decreases. At 30 mK the concurrence goes to zero, and no entanglement is measurable due to this increase in thermal photons.. Hence, the device's environment must be below 20 mK to ensure ideal conditions for extracting entanglement from the quantum field. The temperature in the transmission line between the two qubits will be discussed further in subsection [4.3.1](#).

Additionally, it was shown in Ren's simulation that the readout fidelity of the experiment will increase the measurement repetitions required to read out an entangled state. A density of state simulated with 10^7 samples only has a non-zero concurrence 50% of the time when running the test at 20 mK. It is critical that the readout fidelity is not decreased. Within section [4.5](#) the biasing noise through the DC-SQUID, used in for qubit readout, will be discussed further.

Chapter 4

Cryogenic setup for entanglement harvesting experiment

In this chapter we will discuss the design and implementation of an experimental setup for entanglement harvesting. In section 4.1 the theoretical requirements of entanglement harvesting will be presented as they apply to experimental implementation, as discussed in chapter 3. The models for estimating heating and selecting the materials used in the design will be introduced in section 4.2. We will present the designs of the microwave cables required for the entanglement harvesting experiment in section 4.3. In subsection 4.3.1 we will discuss the thermal state in the transmission line between the two qubits, and how it depends on noise transferred from higher temperatures. In subsection 4.3.2 we will evaluate the distortion of the entanglement harvesting experimental biasing pulses due to attenuation, and choose cables to reduce it. In section 4.4 we describe the construction of wires for low frequency biasing currents. In subsection 4.4.1 we present designs of direct current (DC) wires which will minimize heat generated and in subsection 4.4.2 we will present designs which transport the heat generated to the refrigerator temperature stages.

4.1 Requirements for experimental entanglement harvesting

The focus of this chapter describing the requirements for the control of the qubit, coupler and readout to implement the entanglement harvesting protocol. As discussed in section

3.4, the implementation of the entanglement harvesting measurement protocol requires the control and readout of two qubits coupled to a transmission line.

The qubit-coupler device is shown in figure 4.1, denoted with black lines. As introduced in section 3.3 the coupling loop, biased by f_β , connects the qubit to the transmission line. This transmission line running between the two qubits will be connected to an input and output microwave cable, which enables scattering measurements of the qubit. As discussed in section 3.4, The input and output microwave cables must not increase the transmission line temperature above 20 mK for entanglement to be harvested from the quantum field.

When the qubit loop is biased away from its symmetry point it can be readout through the DC-SQUID, as described in subsection 2.2.3. The DC-SQUID, shown in purple in figure 4.1, is a part of quarter wavelength resonator. A pair of input and output microwave line are capacitively coupled to the DC-SQUID resonators near both qubits. This configuration allows for simultaneous readout with a single microwave line.

There are two fast biasing lines, shown as teal in figure 4.1, which individually bias the qubit and coupler loops using on-chip inductances. These biasing lines combine a high frequency signal from a microwave line and a DC signal through a bias tee. The microwave cables must be able to propagate a 33 ps Gaussian pulse with minimal distortion, which is required for the entanglement harvesting protocol as discussed in 3.4. The mutual coupling between the on-chip biasing lines and superconducting loops is designed to be around 1×10^{-12} Wb/A Therefore, the DC wires must be able to conduct up to ± 5 mA to allow for sweeping 5 flux quantum. Being able to sweep through multiple flux quantum is ideal for calibration of the flux biasing.

The superconducting magnetic coil is presented in figure 4.1, in orange. This coil has the advantage of supplying low noise magnetic flux which is constant across the entire device. This will be used to supply constant offsets to the experiment. As discussed in section 3.3 noise in the magnetic flux will introduce decoherence in the qubit. The noise generated by currents in the coil, and the fast biasing lines, will need to be evaluated to ensure there is no effect on the qubit operation. Additionally, as discussed in subsection 2.2.3, flux noise modified the effective Josephson inductance of the DC-SQUID. The coil will be designed to not introduce flux bias noise that can impact the readout.

4.2 Thermal modelling of heating

To run a dilution refrigerator at its lowest possible temperatures there are constraints on heat being generated, which must be considered. This section will introduce the models

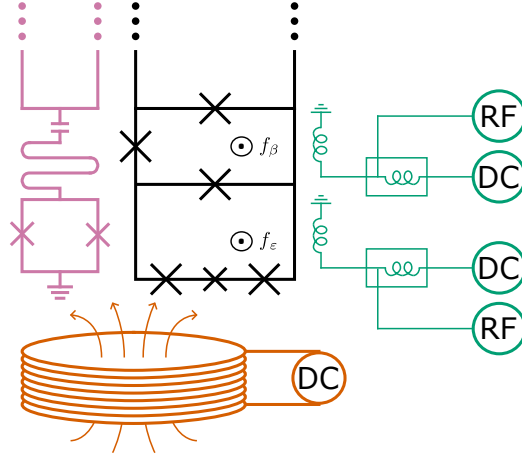


Figure 4.1: Circuit diagram of the tunable coupling device with biasing and readout. The readout circuit is represented in purple. The readout circuit includes a DC-SQUID loop at the end of a quarter wavelength resonator which is capacitively coupled to a microwave line. The triple dots in the qubit and readout circuits represent infinite microwave lines. The orange circuit represents the superconducting coil, which provides magnetic bias to the qubit loop, coupler loop, and DC-SQUID loop. The magnetic field is generated with a current from a DC source. The on-chip biasing lines are represented in teal. The two biasing lines couple are designed to primarily bias one of the coupler and qubit loops. These biasing lines use a bias tee to combine signals from high frequency (RF) and low frequency (DC) sources.

Temperature Stage	Typical Temperature (K)	Cooling Power (mW)
50K	50.46	1000
3K	3.41	100
Still	0.434	1
50mK	0.092	0.1
Mixing chamber	0.00086	0.05

Table 4.1: Overview of temperatures and excess cooling power estimates in the Leiden Cryogenic CF-CS81 dilution refrigerator. ‘Typical Temperature’ is the temperature of stages in test condensations of the systems prior to installing anything in the system. ‘Cooling power’ is an estimate of the excess cooling power of each stage from tests done on the Leiden Cryogenic CF650 dilution refrigerator.

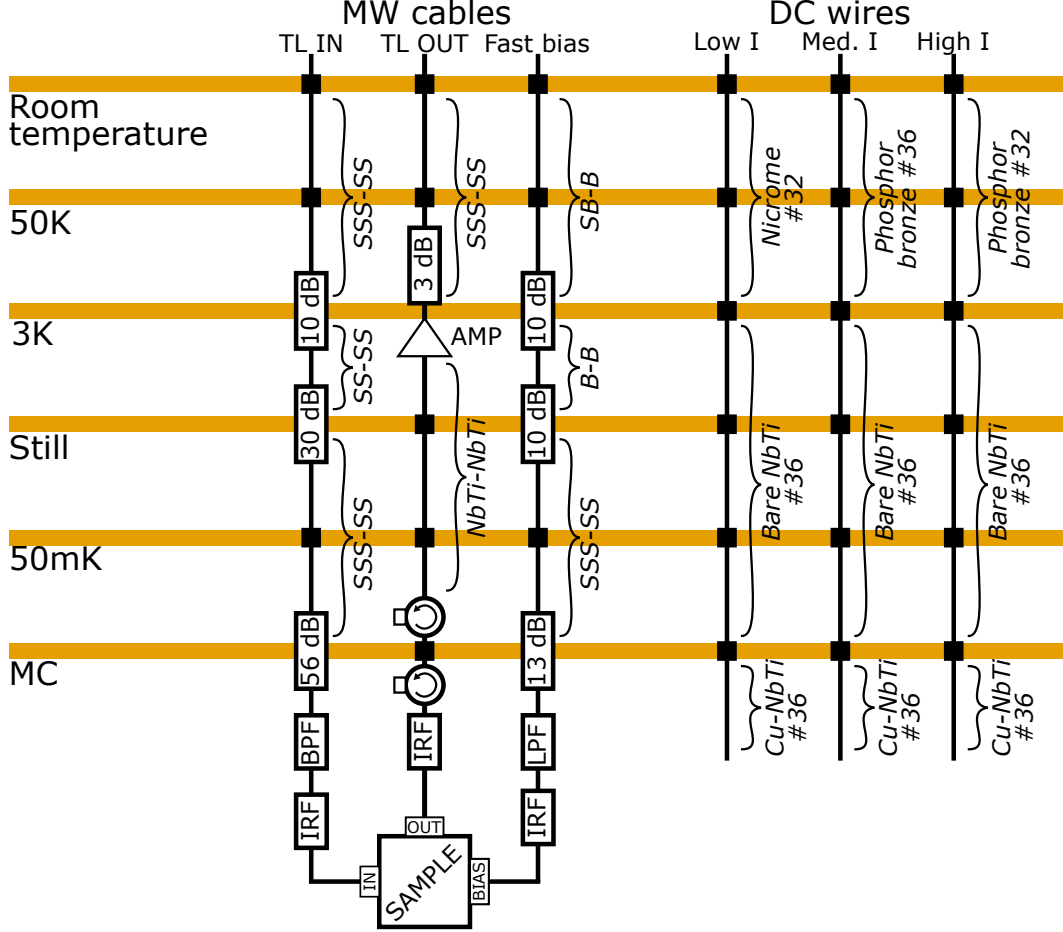


Figure 4.2: A Leiden Cryogenics CF-CS81 dilution refrigerator schematic of temperature stages with example wiring installed. The stages are labelled with their names in table 4.1. The microwave cables (MW) installed include a transmission line (TL) going in and out, along with a fast bias line. These contain cryogenic attenuators denoted with their attenuation in dB. All lines also contain infrared filters (IRF). The input line contains a MicroTronics 1.8-13 GHz bandpass filter (BPF). The output line contains two Quinstar isolators at the mixing chamber and an LNF amplifier at the 3K stage. The fast bias line has a MicroTronics 23 GHz lowpass filter (LPF). See subsections 4.3.1 and 4.3.2 for details about the microwave cable materials. The DC wires have their wire materials listed with the diameters as American Wire Gauge thicknesses. See subsection 4.4.1 for details about wire materials.

used for Joule heating which is generated from currents, and thermal leakage which occurs between different temperatures.

A dilution refrigerator has six temperature stages, as shown in figure 4.2, which get progressively colder. The temperature of each of these stages prior to installing any experimental cables in table 4.1. The estimated cooling powers of each temperature stage is presented in table 4.1. For the operation of the dilution refrigerator at its base temperature, it is critical that these powers are not exceeded. At the mixing chamber the cooling power is proportional to the square of the temperature. Hence, the cooling power available at the mixing chamber is dependant on the desired temperature of the mixing chamber. All components which introduce heat will need to be carefully considered as significant heating of the dilution refrigerator will impact the base temperature that the experiment is operated at. Images of dilution refrigerators can be found on the manufacturer's website [45]. Additionally, more details about dilution refrigeration properties and operation can be found in the reference text by Pobell [1].

Thermal leakage in a bulk material is defined between a higher temperature T_1 and a lower temperature T_2 as

$$\dot{Q}_{leak} = \sum_i \frac{A_i}{L} \int_{T_1}^{T_2} \kappa_i(T) dT = \sum_i \int_{T_1}^{T_2} \frac{1}{R_{th,i}(T)} dT, \quad (4.1)$$

where κ_i is the thermal conductivity of the material, A_i is the cross sectional area and L is the length. This can also be written with the thermal resistance, $R_{th,i} = L/A_i\kappa_i$. This heating is a sum of the all the materials connecting the two temperatures, denoted with the index i . This equation assumes that the materials running between temperature stages are isolated from each other. If they are thermally connected the material with the lowest thermal resistance assumed to dominate. We also assume that no energy is generated along the wire, and that the only transfer of heat is from the thermal transport. The thermal conductivity is a temperature dependant property of a material which describes its ability to transport energy. The thermal conductivities of a number of common materials for cryogenic use are presented in figure 4.3, and can be understood by studying microscopic scattering dynamics. Energy can be carried by lattice vibrations, called phonons, or electrons [1].

At high temperatures in conductors and at all temperatures in insulators, the phonon scattering dominates. The formula for phonon scattering is

$$\kappa_{ph}(T) = \frac{1}{3} \frac{C_{ph} v_s}{V} \lambda_{ph} \propto T^3 \lambda_{ph}(T) \quad (4.2)$$

where κ_{ph} is the thermal conductivity, C_{ph} is the phonon's heat capacity, v_s is the sound velocity of phonons, V is the volume and λ_{ph} is the mean free path of the phonons [1]. The

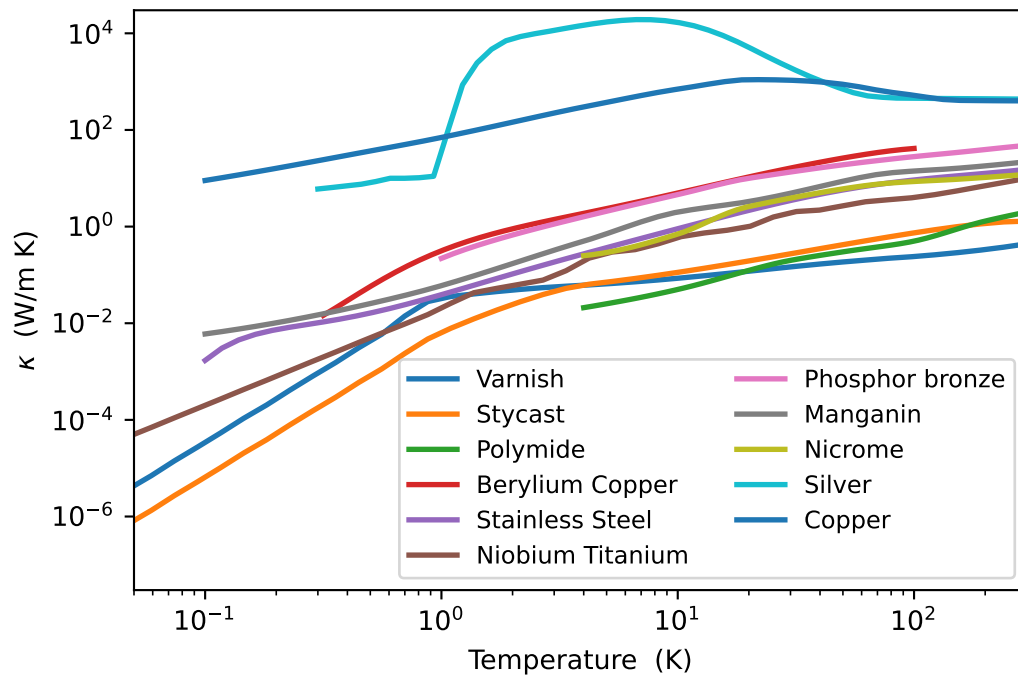


Figure 4.3: Values of thermal conductivity of a number of common materials used in cryogenic settings. These curves are cubic interpolations of data from sources [1, 2, 3]. The curves from Stainless-Steel and Beryllium copper are plots of curves given in sources [4, 5]

cubic dependence on temperature is from the specific heat of phonons. The decrease in the phonon occupation with lower temperature results in an increasing mean free path. The increasing mean free path results in an increasing thermal conductivity in some materials, which have local maximum in thermal conductivity due to this effect [1]. As the temperature decreases the mean free path reaches a maximum where the scattering of phonons off of lattice defects becomes dominant.

In conductive materials electron-phonon scattering is the dominant energy transfer mechanism. The formula for electron-phonon scattering is

$$\kappa_e = \frac{1}{3} \frac{C_e \nu_e}{V} \lambda_e \propto T \lambda_e(T) \quad (4.3)$$

where κ_e is the thermal conductivity, C_e is the electron's heat capacity, ν_e is the velocity of the electrons, V is the volume and λ_e is the mean free path for the electrons. Electron-phonon scattering is dominant in metals at high temperatures. At low temperatures the occupation of the phonon decreases. The increasing mean free path results in a local maximum in the thermal conductivity [1]. A local maximum can be seen in figure 4.3 in the thermal conductivity of silver between 2 K and 10 K, and copper between 20 K and 50 K. This transition occurs above 10 K for all metals [1]. At lower temperature the thermal conductivity is dominated by the scattering of electrons and lattice defects, where λ_e is constant.

Further, there is a special case in the electronic heat transfer. In superconductors the Cooper pairs all sit at the same low energy state, which means they cannot conduct heat. The heat capacity is then proportional to the number of unbound electrons. Hence, the heat capacity in superconducting metals is

$$\kappa_{e,s} = \kappa_e e^{-\Delta_S/k_B T} \propto T \lambda_e(T) e^{-\Delta_S/k_B T}, \quad (4.4)$$

where Δ_S is the energy of the superconducting gap and k_B is Boltzmann's constant [1].

When two surfaces at different temperatures are held together there is thermal leakage between them. The thermal conductivity is related to the material, surface roughness, clamping force, and oxide formation [46, 47, 48]. It has been found that to improve thermal contact, copper should be gold plating to minimize surface oxide formation [47]. The surfaces should be compressed to reach a pressure of 8 MPa, which has been found to maximize the thermal conductivity [48].

The thermal resistance between two contacting surfaces is given as

$$R_{th} = \frac{\rho_N}{A}, \quad (4.5)$$

where ρ_N is the thermal resistivity between the two surfaces. ρ_N is based on parameters such as materials, surface roughness, and clamping force, but there is no general model. Therefore, values will be taken from experiments which have a similar clamping force, around 8 MPa, and gold-plated copper surfaces used in our setup. Measurements by Siddappa et al. [46] are applicable for temperatures above 50K and measurements by Dhuley et al. [47] for values around 4.2 K. The resulting thermal resistance between two surfaces can be substituted into equation 4.1 to obtain the thermal leakage.

Joule heating results from currents flowing through resistive metal, and is given as

$$\dot{Q}_{Joule} = RI^2 = \frac{l}{A\sigma_e}I^2, \quad (4.6)$$

where I is the current and R is the resistance. The resistance is written as a function of the metal's electrical conductivity σ_e , length l . For wires running between two temperatures, an upper bound on the Joule heating can be taken by using the maximum resistance in the temperature range.

The microscopic model of the electrical conductivity is based on electron scattering as

$$\sigma(T) = \frac{ne^2\nu_e}{m_e}\lambda_e(T), \quad (4.7)$$

where n the electron density per unit volume and m_e is the electron mass [1]. Here the only temperature dependence is due to the electron mean free path, thus the electrical conductivity has a minimum value when the electron-defect scattering dominates.

For microwave cables a simple resistor does not model the Joule heating from a propagating signal. The transmission line model presented in section 3.2 can be updated to include resistances. An infinitesimal resistance in series is added related to the cable's electrical conductivity in series with the inductor. A resistance is added in parallel to the capacitance related to the dielectric loss between the signal and ground conductors [40]. The current flowing through these resistors will generate heat.

A microwave of power, P_0 , will be attenuated through a lossy microwave cable according to

$$P(x) = P_0e^{-2\alpha x}, \quad (4.8)$$

according to the cable's attenuation constant which is approximated as

$$\alpha = \frac{1}{2} \left(\frac{R}{Z_0} + GZ_0 \right), \quad (4.9)$$

where R represents the resistance along the conductors and G represented the shunt conductance between the signal and ground conductors [40].

Typically, the power lost through a microwave component is quantified as a single number such that

$$e^{-2\alpha l} = 10^{-A/10}, \quad (4.10)$$

where A is the attenuation in decibels and l is the length of the component. Hence, the heating from attenuation for microwave cables is

$$\dot{Q}_{Joule} = P(l) - P_0 = (1 - 10^{-A/10}) P_0. \quad (4.11)$$

The total heating of due to a DC wire or microwave cable carrying current is then

$$\dot{Q} = \dot{Q}_{leak} + \dot{Q}_{Joule}. \quad (4.12)$$

This assumes that the Joule heating is not large enough to cause an impact on the transfer of heat, and that all of the energy generated from Joule heating is be deposited at the lower temperature stage. This is justifiable when the Joule heating is less than the thermal leakage. In this case the energy deposited from resistance will be transferred away to the lower temperature as it is generated.

4.3 Microwave cables

In this section we present the design of the microwave control for flux qubit experiments. The primary focus is on the transmission lines used to propagate microwaves from the room temperature equipment to the experiment for qubit control and readout.

4.3.1 Noise temperature in the transmission line

The temperature of the transmission line which connects the two qubits is critical for implementation of entanglement harvesting. As discussed in section 3.4 it is ideal to keep the temperature of the transmission line lower than 20 mK. Here we model the temperature of the transmission line based on the heat transferred from higher energy stages.

As presented in equation 3.35 the transmission line is assumed to be in a thermal state at the beginning of the experiment to measure entanglement harvesting. A thermal state

is defined by the number of thermal photons, which are given by Bose-Einstein statistics, as [49]

$$n_{th}(T, \omega) = 1 / (e^{\hbar\omega/k_B T} - 1), \quad (4.13)$$

where ω is the frequency of the phonon and T is the microwave line temperature. Bose-Einstein statistics gives a relationship between temperature and number of thermal photons.

From this equivalence the transmission line will have a higher effective temperature compared to its environment due to photons travelling from higher temperatures. At a temperature stage, i , the thermal photons from the hotter $i-1$ stage will either continue to propagate, or dissipate and thermalize to the temperature of the attenuator. The resulting thermal photon occupation number is thus [50]

$$n_i(\omega) = (1 - 10^{-A_i/10})n_{th}(T, \omega) + 10^{-A_i/10}n_{i-1}(\omega) \quad (4.14)$$

where A_i is the attenuation of an attenuator placed at the i -th temperature stage.

The microwave cables labelled transmission line in and transmission line out, as shown in figure 4.2, will carry thermal photons from higher temperatures to the sample. We consider the occupation number of 5 GHz thermal photons at room temperature and find it must be attenuated by at least 83 dB to have an effective temperature under 20 mK at the experimental device. This attenuation assumes that all of the attenuation is at the mixing chamber stage. For the transmission line in, this will come from placing cryogenic attenuators along the microwave cable between the room temperature stage and the mixing chamber stage. In the case of the transmission out line the two isolators each have a isolation greater than 58 dB. Thus, the thermal photons travelling down the transmission line out can be neglected.

Microwave signals will propagate through the transmission line in and cause Joule heating, as given by equation 4.11. The attenuation should be distributed across a number of temperature plates to reduce the amount of energy deposited at the mixing chamber temperature stage. In figure 4.4 we model the effective temperature of the transmission line is presented with different attenuations at the still and mixing chamber temperature stages. Here a constant 10 dB attenuation is also placed at the 3K stage. The black dashed line is plotted for the configurations that give an effective temperature of 20 mK. We will thus place 10 dB attenuation at the 3K, 30 dB at the still plate and 56 dB at the mixing chamber plate to attain a 19.2 mK transmission line between the two qubits.

To get an estimate of the Joule heating due to these attenuation assume a -4 dBm pulse is applied at the top of the transmission input line. This will have a power of -100 dBm

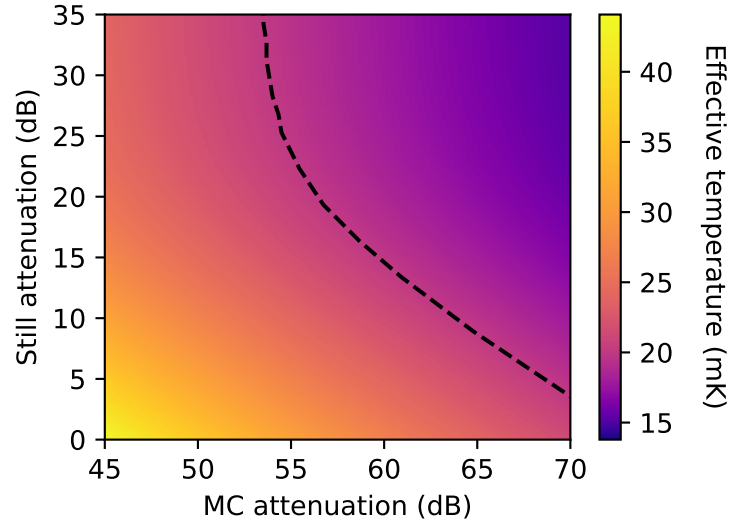


Figure 4.4: Effective temperature in mK in the transmission line of 5 GHz photons with 10 dB of attenuation at the 3K temperature stage, varying the attenuation thermalized at the mixing chamber and still stages. The dashed black line denotes a temperature of 20 mK in the transmission line.

Temperature stage	Attenuation (dB)	Joule heating (μW)
3K	10	360
Still	30	40
Mixing chamber	56	0.04

Table 4.2: Attenuation in dB in the input transmission line, and the temperature stage they are thermalized to. The Joule heating is generated from a -4 dBm pulse.

Temperature stages	Cable material	Thermal Leakage
RT - 50K	Silver-plated stainless steel	22.4 mW
50K - 3K	Silver-plated stainless steel	7.4 mW
3K - Still in-line	Stainless steel	6.7 μW
Still - 50mK in-line	Silver-plated stainless steel	222 nW
50mK - MC in-line	Silver-plated stainless steel	2.6 nW
3K - MC out-line	Niobium Titanium	0.91 μW

Table 4.3: Microwave cables between the temperature stages for the input and output lines with thermal leakage at the lower temperature stage. The cable materials listed are for the coaxial cable's inner conductor. The outer conductor is the same material, without the silver plating if applicable.

at the sample, which is significantly above the typical power of -120 dBm used for qubit transmission measurements. The resulting heating is calculated with equation 4.6 for each attenuator. The heating is shown in table 4.2, and is significantly below the cooling power of the temperature stages presented in table 4.1.

The Joule heating due to the microwave cables is based on their attenuation at the pulse frequency applied. The cable attenuations are presented at room temperature in figure 4.6 (b). The measured values for the cables are close to the expected attenuation based on cable length. The niobium titanium cable is the furthest from the expected value, and has 0.4 dB less attenuation than expected. The other cables are within ± 0.2 dB for values under 10 GHz. Above 10 GHz the measured attenuation values increase compared to the expected values, with between 0.3 dB to 0.8 dB more attenuation. The measured attenuation of these cables are much smaller than the attenuation from the proposed attenuators around 5 GHz. Additionally, the cable's attenuation will decrease from the room temperature values at cryogenic temperatures. Hence, the Joule heating from the microwave cables can be neglected. The cables can then be optimized to reduce the leakage with equation 4.1. The resulting cables are presented in table 4.3.

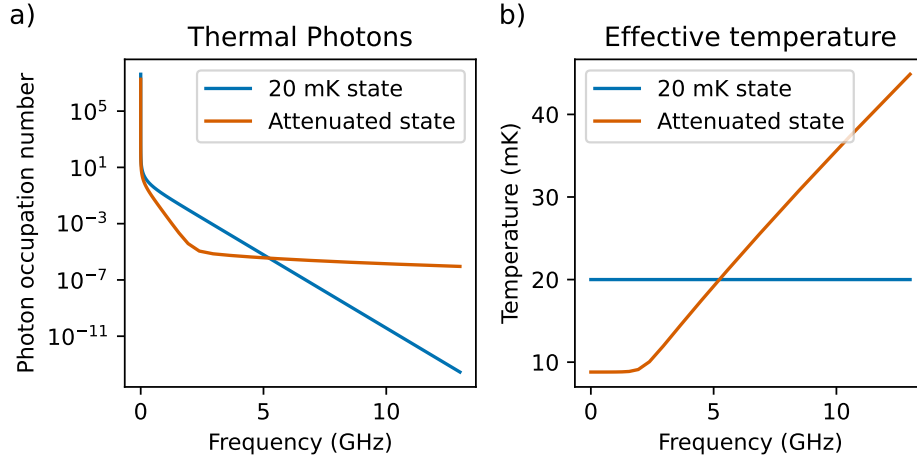


Figure 4.5: Thermal photons through the transmission line in, with the attenuation listed in table 4.2. (a) The thermal photon occupation number and (b) the effective temperature in the line is compared to the 20 mK thermal state.

The effective temperature of the transmission line is dependant on the frequency of the photon. In figure 4.5 (a) the thermal photon occupation number is presented over the frequency bandwidth of the transmission line. This presents the number of thermal photons in a transmission line with the attenuation in table 4.2, compared to a 20 mK thermal state. The number of thermal photons attenuated by decreases with temperature, but not as quickly as the 20 mK thermal state. In figure 4.5 (b) it can be seen that the attenuated signal only has an effective thermal temperature of 20 mK around 5 GHz. The effective temperature increases for higher frequencies.

We recommended that another test of the tunable coupler, following the results published by Janzen et al. [21], is done to study the effective temperature in the transmission line prior to performing the entanglement harvesting protocol. This previous result only reached a qubit temperature of 50 mK. The test performed is a measurement of the scattering off of the qubit through the transmission line coupled to the qubit. A transmission sweep can be fit to equation (12) in reference [21] to extract the effective temperature of the qubit. Here the qubit temperature is measured, which is assumed to be the same as the transmission line temperature. In general the qubit temperature is a combination of the temperatures of all the noise channels it is interacting with. However in this case the qubit-transmission line coupling is the strongest [21], satisfying the assumption that the qubit is in thermal equilibrium with the transmission line.

The gap frequency of the qubit in its symmetry point changes as the coupling is biased. As the transmission measurements extract the transmission line temperature at the minimum gap of the qubit, this sweep will generate the transmission line temperature as a function of frequency. The effective temperature can be validated against the model presented in figure 4.5 (b). If it is found that the transmission line has a temperature significantly above 20 mK then the transmission line should not be connected to any higher temperature stages. For the entanglement harvesting protocol it is not required to probe the qubits through this transmission line. Hence, the input and output of the experimental device can be terminated. This preserves the infinite transmission line modelled in section 3.2, and will keep the experiment thermalized at the temperature of the mixing chamber.

4.3.2 Pulse distortion in fast biasing cables

In this subsection we will evaluate the materials used to construct the microwave cables used to transmit the fast flux biasing pulses required for the entanglement harvesting pulses. Frequency dependant attenuation can cause distortion of the pulses. The attenuation as a function of frequency can be inverse Fourier transformed and convoluted with the pulse to return the distorted pulse. We present microwave cables designed to minimize this detrimental effect.

The frequency dependant attenuation of a number of microwave cable is presented in figure 4.6 (a). These attenuation values presented are for a 54 cm segment of cable at 300 K. This is the length of a microwave cable between the room temperature and 3K stages. This is an overestimate of attenuation, as the cable's resistance will decrease with the temperature.

In the entanglement harvesting protocol, Gaussian pulses with a time constant of 33 ps are required for the fast switching of the qubit transmission line coupling strength. These current pulses drive the flux bias in the tunable coupler devices, as shown in figure 3.3, tuning the qubit-transmission line interaction from decoupled to strong coupling. These pulses have significant distortion based on the attenuation in the cables. In figure 4.7 (a) we investigate the Gaussian signal's distortion while passing through the microwave cables presented in figure 4.6 (a). As expected the cable materials with more attenuation at high frequencies will cause more spreading of the Gaussian pulse.

We quantify the spreading of the pulses using the 1σ and 3σ widths of the attenuated pulses. The Gaussian pulses, introduced in equation 3.41, are distorted into a non-Gaussian pulse. These widths are defined for non-Gaussian pulses using the amplitude of a Gaussian pulse associated with these widths. In table 4.4 it can be seen that the cable materials do not

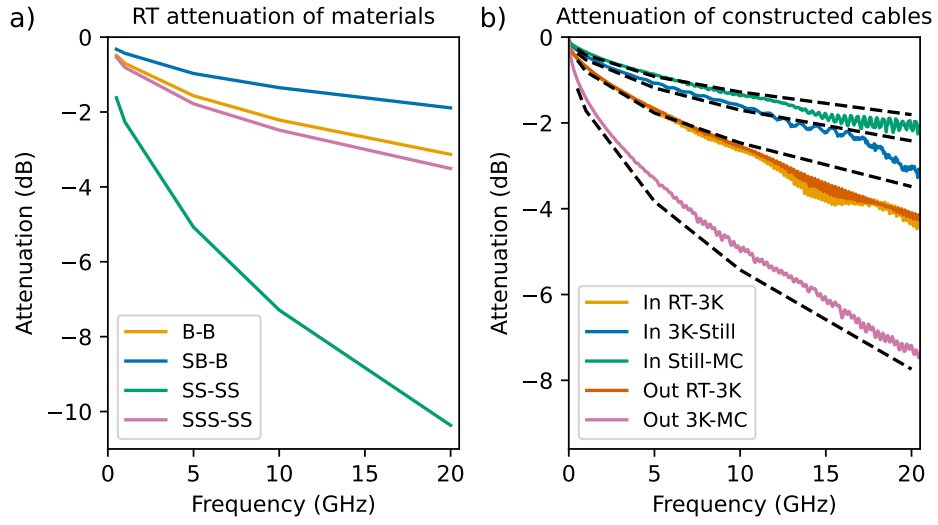


Figure 4.6: (a) Attenuation of 0.54 m segments of beryllium copper (B-B), silver-plated beryllium copper (SB-B), stainless steel (SS-SS), silver-plated stainless steel (SSS-SS) cables as a function of frequency, based on manufacturer specifications. This is the length of a cable between the room temperature and 3K stages. (b) Measurements of constructed microwave cables labelled by the temperature stages they run between. The black dashed lines are the expected attenuation of the cables based on their length. These measurements were done at room temperature.

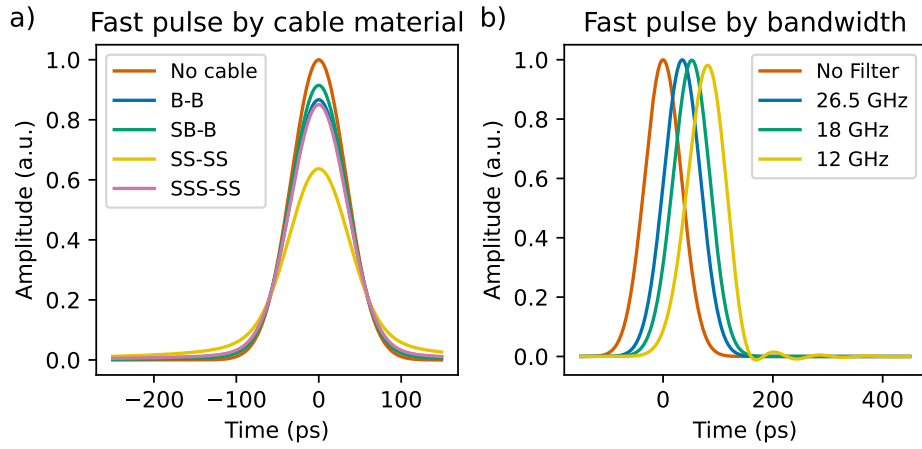


Figure 4.7: Distortion of high frequency Gaussian pulses, of arbitrary amplitude, through different microwave cables and components. (a) Distortion of a Gaussian pulse, with time constant 33 ps, through 0.54 m cables of different materials at room temperature. This includes beryllium copper (B-B), silver-plated beryllium copper (SB-B), stainless steel (SS-SS) and silver-plated stainless steel (SSS-SS). (b) The distortion of a 33 ps Gaussian pulse through a variety of connector bandwidths, listed by the high cutoff frequency. The connectors all have pass bands to 0 GHz.

Cable material	1σ width (ps)	3σ width (ps)
No cable	67	200
Beryllium copper	69	261
Silver-plated beryllium copper	67	227
Stainless steel	73	577
Silver-plated stainless steel	69	281

Table 4.4: Distortion of a Gaussian pulse through different cable materials. The 1σ and 3σ points are defined as the widths where the pulse reaches 60.7% and 1.1% of its maximum amplitude, respectively.

Temperature stages	Cable material	Thermal leakage
RT - 50K	Silver-plated Beryllium Copper	55.7 mW
50K - 3K	Silver-plated Beryllium Copper	14.1 mW
3K - Still	Beryllium Copper	33.9 μ W
Still - 50mK	Silver-plated stainless steel	222 nW
50mK - MC	Silver-plated stainless steel	2.6 nW

Table 4.5: Microwave cables between the temperature stages for the fast biasing lines with thermal leakage at the lower temperature stage. The cable material listed is the coax cable's inner conductor. The outer conductor is the same material, without silver plating if applicable.

significantly change the 1σ width but can significantly increase the 3σ width. These pulses are changing the biasing of the coupler loop from decoupled from the transmission line to strongly coupled, and back to the decoupling point. The 3σ width is thus the total time that the coupler loop is biased away from its decoupling point. Keeping this value under 1 ns is required to ensure the qubits do not directly interact during the entanglement harvesting measurement, as illustrated in figure 3.1.

The pulse distortion can be computed for all the proposed materials listed in figure 4.6, between all temperature stages. Table 4.5 shows the selected cables for each segment in the dilution refrigerator. This combination will result in a 33 ps Gaussian pulse spreading to have a 1σ width of 69 ps and a 3σ width of 284 ps. This is sufficient to maintain the space-like separation of the two qubits interacting with the transmission line. The beryllium copper and silver-plated stainless steel were chosen to reduce thermal leakage, while not introducing significant distortion.

Now that the attenuation of the cables have been selected, we consider the connectors

used. Microwave components can be made with a number of connector types which all have different bandwidths. SMA connectors commonly have a 0-18 GHz bandwidth, but are also sold as 0-12 GHz and 0-26.5 GHz variants [51]. The distortion of a 33 ps Gaussian pulse are shown in figure 4.7 (b). Here it can be seen that there is significant “ringing” at the end of pulse for the 0-12 GHz bandwidth connector. For the entanglement harvesting experiment connectors with a bandwidth of 0-18 GHz are sufficient for the flux control cables. The SMA variants specified up to 26.5 GHz can be considered for further improvements, as they will allow transmission of arbitrary fast biasing signals from the AWG which is limited to 25 GHz. The proposed design is implemented with SMA connectors which have a 0-26.5 GHz passband, which is sufficient for the entanglement harvesting experiment, and introduces the possibility of transmitting faster pulses in future experiments.

In summary, we have presented a set of microwave cables and connectors capable of providing the biasing pulses required to operate the entanglement harvesting measurement. These can provide the 33 ps Gaussian flux pulses required for biasing the tunable coupler in the experiment. Additionally, the thermal leakage was considered to ensure proper operation of the dilution refrigerator.

4.4 DC biasing circuit

In this section we present a design of the direct current (DC) wires required to provide the static magnetic biasing required for the entanglement harvesting experiment. This section presents the construction of the DC wires and mounting systems, evaluating the heating and noise as it affects the entanglement harvesting experiment.

4.4.1 DC wires between temperature stages

Biasing currents of up to 5 mA are required for running the entanglement harvesting experiment. For these currents the Joule heating in the DC wires must be carefully considered running from room temperature electronics to the experiments at cryogenic temperatures can introduce heat into the dilution refrigerator and noise into the experiment. It is critical that the heating is kept significantly below the cooling power of the fridge temperature stages. We consider all heating from the DC wiring to minimize heating of the cold stages use this to select the ideal materials to construct the DC wiring.

The wire material must be chosen to minimize the heating in equation 4.12. Common materials for cryogenic wiring are nicrome, manganin, and phosphor bronze, listed here

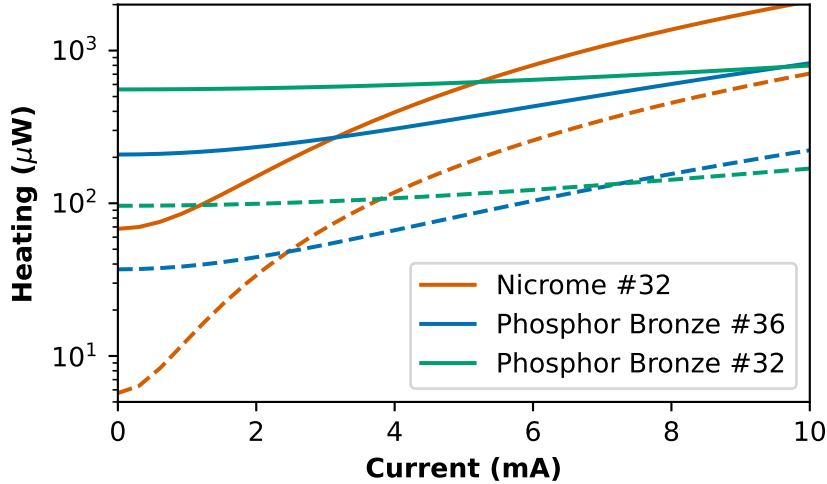


Figure 4.8: Heating of different conductor materials and thicknesses based on current. Here the solid lines represent a wire between the room temperature and 50K stages and the dashed line represents wires between the 50K and 3K temperature stages.

from highest to lowest thermal resistance. These wires have typical diameters between 0.127 mm and 0.254 mm. These wire diameters are also listed in this thesis using their American Wire Gauge (AWG), which are typically between #36 or #30.

We calculate the thermal leakage for all these wire materials and wire diameters. These estimates use the assumptions introduced in section 4.2, where the thermal leakage is independent of the Joule heating and the Joule heating is all deposited at the lower temperature stage. Additionally, we also take the resistance of the entire wire to be equal to its value at the high temperature state. These assumptions will generate an upper bound on the thermal leakage and Joule heating.

The thermal leakage is constant for each wire and the Joule heating is dependant on the current running through the wires, as shown in equation 4.6. The total heating from a DC wire is plotted in figure 4.8. Here we present the three wire material-diameter combinations with the lowest heating. The solid lines represent the heating dissipated at the 50K stage, due to the wire running from room temperature. The dashed lines represent the heating of the 3K stage, from the wires running from the 50K stage. The decreased thermal conductivity of nicrome leads to it being advantageous with currents under 2.5 mA. For wires typically supplying currents between 2.5 mA and 7.5 mA #36 gauge phosphor bronze wire is best. Currents higher than 7.5 mA are not often used for biasing experiments, but

Temperature stages	Current range (mA)	Wire material and gauge	Thermal leakage	Joule heating	Total heating
RT - 50K	0-2.5	Nicrome #32	68 μ W	0-127 μ W	68-195 μ W
RT - 50K	2.5-7.5	Phosphor Bronze #36	208 μ W	38-348 μ W	246-556 μ W
RT - 50K	>7.5	Phosphor Bronze #32	557 μ W	>135 μ W	>692 μ W
50K - 3K	0-2.5	Nicrome #32	6 μ W	0-44 μ W	6-50 μ W
50K - 3K	2.5-7.5	Phosphor Bronze #36	37 μ W	12-104 μ W	49-141 μ W
50K - 3K	>7.5	Phosphor Bronze #32	96 μ W	>41 μ W	>137 μ W
3K - Still	any	Bare NbTi #36	12 nW	-	12 nW
Still - 50mK	any	Bare NbTi #36	190 pW	-	190 pW
50mK - MC	any	Bare NbTi #36	49 pW	-	49 pW
Below MC	any	Copper stabilized NbTi #32	-	-	-

Table 4.6: List of wire materials and gauge used between temperature stages in the fridge; room temperature (RT), 50K, 3K, Still, 50mK and mixing chamber (MC). The wire gauges are listed in American Wire Gauge (AWG). The wiring is separated into different current ranges when relevant to minimize the heating. The thermal leakage is the heating from thermal conductivity through the cables and the Joule heating is the heating from the cable resistance. There is no leakage below the mixing chamber as all wiring will be thermalized to the same temperature.

can occur when needing to bias microwave amplifiers, and it is best to use #32 gauge phosphor bronze wires. These results are summarized in table 4.6 and included in figure 4.2.

For wiring under 3K, superconducting wiring can be used. The resistance going to zero results in no Joule heating, as discussed in equation 4.4, and the thermal leakage is exponentially suppressed. Superconducting niobium titanium (NbTi) wire bare NbTi wire is used between the temperature stages. For wiring under the mixing chamber, which is not running between different temperature stages, a copper stabilized NbTi wire can be used. This wire has a NbTi core which is surrounded by a thin layer of copper. This copper layer makes the wire easier to solder and less brittle. The critical current of these wires is typically in the ampere range, which is not a limitation for experimental currents which are only in the milliamperage range. Alternatively, copper wire can be used to make connections under the mixing chamber and will not have significant Joule heating for currents under 1 mA.

4.4.2 Connection and thermalization

As discussed in subsection 4.4.1, DC wires between two temperatures generate heat through thermal leakage and Joule heating. This heating must be dissipated to the temperature stages efficiently to prevent temperature increases from occurring in the dilution refrigerator. If the wire is not properly thermally connected to the lower temperature stage then the heat will continued to be transported down to lower temperature stages or the temperature of the wire will increase, which will also increase thermal leakage. If the wires are hotter than the surrounding dilution refrigerator it will increase the thermal noise in the wire. We will consider the thermal noise as it impacts the coherence of the qubit used in the entanglement harvesting experiment.

Resistive circuit elements generate Johnson-Nyquist thermal noise [52], according to the equation

$$S_V(\omega) = \hbar\omega \text{Re}[Z] \left(\coth\left(\frac{\hbar\omega}{2k_B T}\right) - 1 \right), \quad (4.15)$$

where Z is the impedance, k_B is Boltzmann's constant and T is the temperature of the component. The noise increases with temperature and impedance of electrical resistances, which highlights the importance of creating low resistance connections in the DC wiring and minimizing their temperatures.

We account for the thermal noise of all resistances along the wiring, including 5230 Ω through a low-pass filter at room temperature, the resistances of the DC wires, and 10

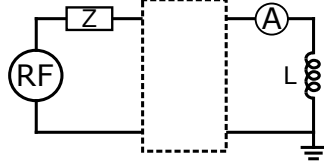


Figure 4.9: Circuit schematic of a PSPICE simulation to extract the propagation of the Johnson-Nyquist noise. The impedance, Z , is modelled with a RF voltage source in series. The dashed box represents any combination of passive electrical components connecting the impedance to the inductance L . This inductance is coupling to the desired superconducting loop. The current through the inductor is measured with an ampmeter, denoted here as A .

$\text{m}\Omega$ as an upper bound for the connectors between different DC wires at the 3K and MC temperature stages. These connectors will be discussed further in subsection 4.4.2. All of the wires will be assumed to be held at the temperature of the temperature stages they are connected to.

We model all resistive components in a PSPICE simulation to understand how the Johnson-Nyquist noise propagates through the DC wires and components. The propagation of the noise to the qubit is shown in figure 4.9. This circuit shows the Johnson-Nyquist noise generated from an impedance modelled as an impedance in series with a voltage source. This voltage is then propagated through the wiring and other passive components, denoted here with a dashed box. Finally the current across the coupling inductor can be readout. The simulation thus generates current across the coupling inductance as a transfer function, $I_L(\omega)$, through the elements of the current biasing setup. The resulting flux noise at the qubit is then

$$S_f(\omega) = \left(\frac{M}{\Phi_0}\right)^2 \left(\frac{I_L(\omega)}{1 \text{ V}}\right)^2 S_V(\omega) \quad (4.16)$$

where M is the mutual inductance between the inductor and the qubit. This can be transformed into the noise in the minimum energy gap, ε , introduced in equation 2.21. The noise in the qubit gap is then

$$S(\omega) = (I_P \Phi_0)^2 S_f(\omega), \quad (4.17)$$

where I_P is the persistent current of the qubit. Using equations 2.28 and 2.29 the decoherence can be estimated.

For the on-chip biasing lines, which have a coupling to the qubit loops on the order of 1×10^{-12} Wb/A, the relaxation rate is 0.1 Hz and the dephasing rate is 18 kHz. These calculations assume the qubit at the symmetry point ($\Delta \gg \varepsilon$) when calculating the relaxation rate and away from the symmetry point ($\Delta \ll \varepsilon$) for the relaxation rate, which gives an upper bound on the decoherence rates. In this simulation the biasing noise is dominated by the room temperature resistance. The source noise is also included, but found to not be significant. These noises can be compared to the relaxation rate from the qubit into the transmission line, which was measured to be above 10 MHz [21]. Thus, these noises have a negligible affect on the qubit.

To reduce the thermal noise from wire and connectors, proper thermal connections are required to keep the temperatures these components as close as possible to the dilution refrigerator temperature stages. We will design these thermal anchoring components to maximize heat transport.

This conduction of heat is defined by thermal conduction of the materials and the contact between different surfaces, as discussed in section 4.2. For a wire or connector heated by q , its temperature will increase the thermal leakage in equation 4.1 until it is sufficiently large, such that $q = \dot{Q}_{leak}$. This assumes that the thermal resistance through the connection will be significantly smaller than thermal resistance continuing down the wire to lower temperature stages. We find that this is true for our designed thermal anchors.

We find the thermal resistance through a bulk material using equation 4.1, which can be solved analytically for the high temperature, T_H , using the functional form of κ . To approximate the high temperature the minimum κ value around T_L can be taken to provide an upper bound on T_H . Additionally, κ can be approximated linearly or quadratically around T_L to get reasonable estimates of the high temperature.

Therefore the temperature of the heated component is

$$T_H = T_C + \sum_i R_i q, \quad (4.18)$$

where R_i is summed over all the thermal resistances in series between the cold temperature stage and the high temperature stage. In most cases the thermal resistance of one material or connection will dominate, and equation 4.18 can be solved with just that resistance.

We present a thermal bobbin, shown in figure 4.10 (a), which is used to thermalize wires to the temperature stages. The cross section of a thermal bobbin thermalization is shown in figure 4.10 (b). Shown in figure 4.10 (a), these OFHC copper cylinders are designed to secure the wire wrapped around it with a cryogenic epoxy [2]. A number of these bobbins can be secured to a single mount, which is then screwed into the dilution refrigerator. The

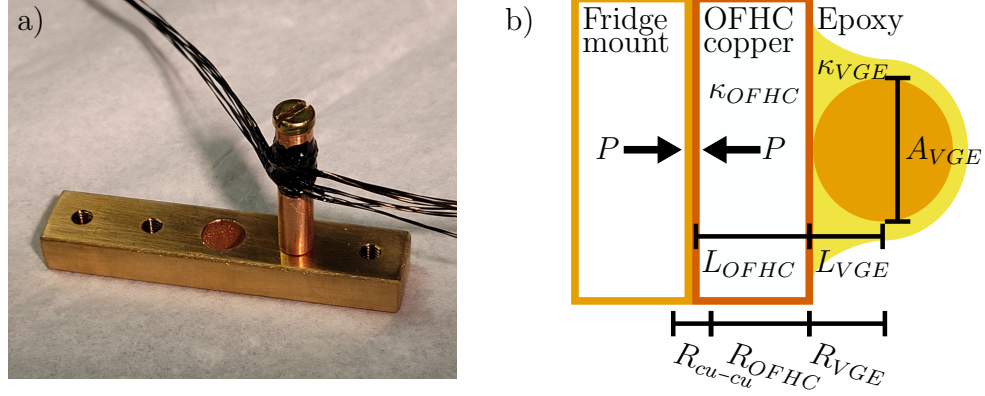


Figure 4.10: Design of the thermal anchoring bobbin. (a) A constructed thermal anchoring bobbin thermalizing 6 twisted pairs. The thermal anchor is mounted onto a gold plated OFHC copper block designed to organize up to 4 of these thermal anchoring bobbins. (b) A schematic of the thermal resistances in the thermal bobbin with the cross section of the wire and the epoxy securing it, the path through the copper bobbin, and contact resistance to the dilution refrigerator.

mount shown in figure 4.10 (a) has 4 threaded holes for thermal bobbins, one of which is shown in use.

We calculate the temperature of the wire using the thermal resistance through the epoxy and the copper, along with the contact resistance between the thermal bobbin and the mount. An example calculation is shown in Appendix B.1 for a thermal bobbin at the 50K temperature stage. A schematic of the thermal resistances is shown figure 4.10 (b). In this case the thermal resistance through the VGE-7031 varnish is the largest, resulting in the temperature of the wire being held at

$$T = 50 \text{ K} + 152 \text{ K/W} \dot{Q}. \quad (4.19)$$

We also present two thermal anchors which are designed around commercial DC connectors. The Fischer Elektronik MK201 and MK204 branded DC connectors are the built-in DC connections in the CF-CS81 dilution refrigerator. These are limited in their connection density, and limiting for making the number of connections required for more complicated experiments. The CRM-Direct circular connectors solve this problem as they are designed with 4 pins to 30 pins, which improves the density of electrical connections.

Currents through these connectors cause heating due to the contact resistance between the connector pins. The contact resistance of are difficult to estimate since the geometries are more complicated compared to the gold-plated copper surface mounting discussed above. In the case of DC connectors, the friction forces holding the surfaces together are typically much less than the clamping force for mounting. Estimates from Elektronik place these connectors at less than $10\text{ m}\Omega$ of contact resistance which will serve as an upper bound for designing mounting systems for these devices.

We present a mounting system for CMR connectors, using clamps to mount the connectors to a plate, as shown in figure 4.11. This connector has the advantage of being able to connect several conductors at once in a compact design. The CS81 dilution refrigerator wiring setup includes 12-pin connectors. We perform an example calculation for this connector at the 3K stage in Appendix B.2, where thermal resistance through the PEEK connector is 30.4 K/mW . When the heating through the 12 wires approaches 1 mW the temperature change can become significant. In this case it is recommended to use a thermal bobbin as described above to dissipate the heat from the above plates.

This type of connector mounting system can also be used at lower temperatures, however the high thermal resistance of PEEK can cause issues with heat dissipation. If there is low current flowing through the wires then these connectors can be used for low temperature. For example, at the mixing chamber a connector will have a thermal resistance of $0.48\text{ mK}/\mu\text{W}$. This heating can be managed with a thermal bobbin, as discussed above, but will become too much for currents above $10\text{ }\mu\text{A}$. This sets an upper limit on the current the can be used for bias flux qubits with this connector mount. This connector type cannot be used at the mixing chamber, due to the currents required for the entanglement harvesting experiment.

We present a mounting and breakout system for the Fischer Elektronik connectors is designed to be modular, allowing for each twisted pair to be added and removed individually, and to support higher currents up to the mA range. Figure 4.12 shows the configuration of the connectors in the breakout system. Each horizontal row is designed for a pair of conductors. The connectors are wired together through traces that run along the back side of the PCB. These traces are attached to a OFHC copper plate with a thin layer of Stycast to thermally connect them to the temperature stage of the fridge. We use thin 0.125 mm strips of Teflon between the PCB traces and the copper surface to create a uniform thickness of Stycast and to ensure there is no electrical contact. This breakout board has the advantage of being modular, allowing connections to be moved or modified without needing to change the entire wire bundle.

This style of breakout board has the disadvantage of being very large. The size is re-

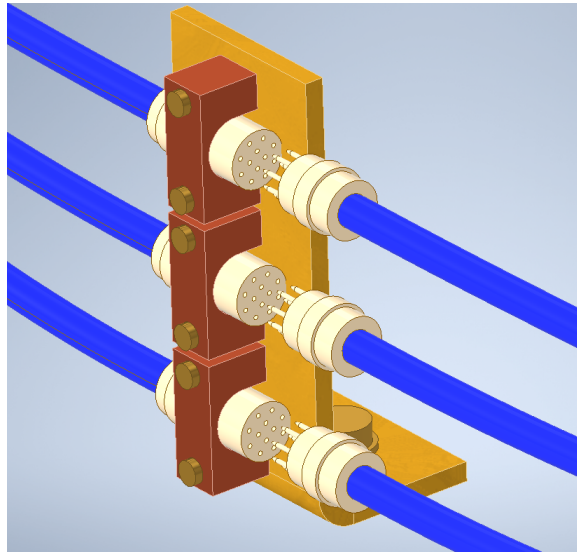


Figure 4.11: Inventor diagram of the breakout system used to hold the CMR-Direct circular cryogenic connectors. The white represents the PEEK of the connectors. The orange is the OFHC copper clamping components. The yellow is gold plated OFHC copper. The blue represents the bundle of wires soldered to the connectors. The clamps are secured to the mounting board with brass M2 screws and the mounting board has a brass M5 screw to attach to the dilution refrigerator stage.

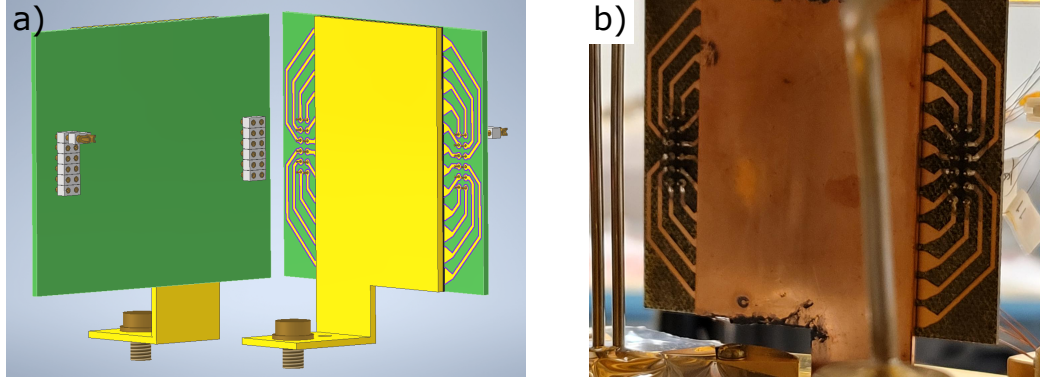


Figure 4.12: (a) Inventor diagram of the breakout system used with the Fischer Elektronik MK204 and MK201 connectors from the front and back views. This is designed for 6 twisted pairs and includes 1 example male connector. The green represents an FR-4 PCB, the yellow represents gold-plated copper, including the PCB traces, and the OFHC copper mounting bracket. The PCB traces are epoxied to the mounting bracket with Stycast, represented here with a thin black layer. Here the mounting bracket is connected to the dilution refrigerator stage with a brass M5 screw. (b) Version of this breakout system implemented in the Leiden Cryogenic CF650 dilution refrigerator.

quired to ensure there is enough thermal conduction at the mixing chamber. This breakout board design was implemented for supplying biasing currents in a recent experiment and mounted on the mixing chamber and worked as designed. We have tested all 12 pins and this type of connection does not lead to excess heating of the mixing chamber at currents up to the 5 mA. This amount of heating will lead to heating of the connector to 26.7 mK, which can be managed with a thermal anchoring bobbin after the connector to ensure there is no additional heat transported towards the experiment. See Appendix B.3 for details on this calculation. This connector will satisfy the current requirements for running the entanglement harvesting protocol.

In summary, we have presented the thermal noise that can introduce decoherence in the qubit resulting when supply DC biasing currents and magnetic flux to the experimental device. We then present and evaluate three designs for the thermal anchoring, designed to allow the high currents required for implementing the entanglement harvesting protocol while keeping the components well thermalized to the temperature stages.

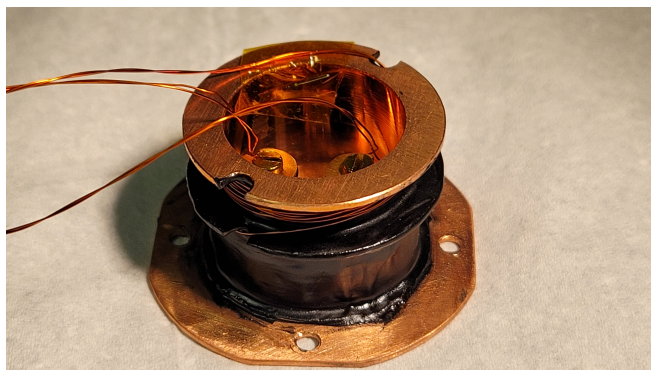


Figure 4.13: Image of constructed main-coil with two mini-coils mounted inside.

4.5 Biasing coils

Superconducting coils are used to supply a uniform and low noise magnetic field across the entire experimental device. These coils provide a constant magnetic field uniform across the entire device, biasing the qubit, coupler, and DC-SQUID loops. The coils are designed to provide static, low noise magnetic fields. These contrast the on-chip biasing inductances discussed above, which only provide local biasing to the device. We present designs for a coil using two different wire thicknesses and evaluate the impact on qubit coherence and DC-SQUID operation in the entanglement harvesting experiment.

Superconducting wires are used in the coil construction to remove the resistance that would generate heat. These coils are made from copper stabilized mono-filament superconducting wires; the copper stabilizing improves the tensile strength of the NbTi wire. The joint of the superconducting wire can be made by soldering the copper stabilizing layer. There are methods to make fully superconducting junctions [53, 54, 55, 56, 57] but these are not required in this context.

The requirements for experimental operation are being able to sweep the magnetic bias through multiple flux quanta, and having minimal noise. Previous coil designs used $50\ \mu\text{m}$ diameter wire. However they are difficult to produce, due to the 2500 wire turns required and the 0.1 N tension limit of the wires [58]. Due to these challenges a similar coil will be designed to use a larger $254\ \mu\text{m}$ diameter wire.

The coil geometry is shown in figure 4.13. Using the coil designed with the thin wire requires 2500 coil turns and over 200 meters of wire. This design has a mutual inductance of $700 \times 10^{-12}\ \text{Wb/A}$ with the DC-SQUID loop. Thus, a flux quantum of biasing requires $3\ \mu\text{A}$ of current. For a coil made with the same geometry but a thicker wire parameters will

	Thin wire	Thick wire
Wire diameter	50 μm	254 μm
SQUID Biasing Current	3 $\mu\text{A}/\Phi_0$	77 $\mu\text{A}/\Phi_0$
Coil turns	2500	100
Inductance	0.250 H	0.39 mH
Relaxation rate	4 μHz	2 mHz
Dephasing rate	3 Hz	420 Hz

Table 4.7: Estimates for coil parameters of two coils designed with the same geometry and different wire diameters.

need to be scaled to generate the same flux biasing. The number of coil turns in the same size bobbin will scale with the inverse of the diameter squared and the current needed will scale with the diameter squared [6]. A coil designed with a wire thickness of a 254 μm diameter wire will then have 100 coil turns and require a current of 77 μA to bias 1 flux quantum.

The inductance of the two different types of coils can be calculated as

$$L = \frac{\mu_0 N^2 A}{l} k_L, \quad (4.20)$$

where A is the area of the coil, l is the length of the coil, and k_L is Nagaoka's Coefficient, which is approximated as 0.633 for the coil geometry [59]. This gives an inductance of 0.25 H for the thin wire and 0.39 mH for the thick wire. These inductances are relevant for the impedance of the coil, which contributes to the thermal noise in the coil and shielding of the source noise.

We calculate the noise spectral density from biasing a device with these coil designs using the noise of a custom-designed current buffer, and the thermal noise in the inductor. We measure the voltage noise from the current source. See the measurement configuration for this measurement in figure 4.14 (b). The voltage bias is converted to a current, which flows through a 12 Ω resistor, used to approximate the series resistance in the dilution refrigerator. The voltage across the resistance is amplified using a Stanford Research SR560 amplifier and measured with a Stanford Research SR780 spectrum analyzer. The voltage noise of the source can be seen in figure 4.14 (a). The voltage noise spectra are taken for two different configurations associated with the current level of the thin wire with 3 μA and with the thick wire with 77 μA . For the thin wire configuration a gain of 10^4 is used and with the thick wire configuration a gain of 10^3 is used. The voltage noise presented here has divided out the gain. We attribute the significant different in 60

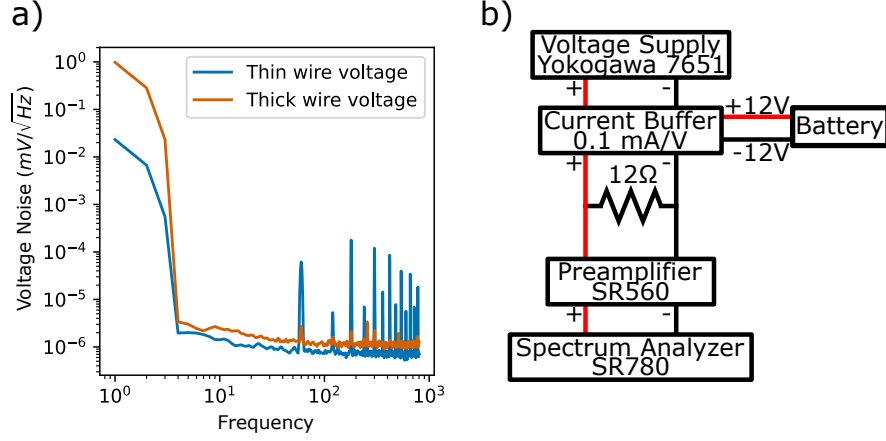


Figure 4.14: (a) The measurements of source noise with the voltage applied associated with the DC-SQUID flux quantum of the coils designed with a thick and thin wire diameter. (b) Measurement schematic. The voltage is applied from a Yokogawa 7651. The current buffer and battery are custom designed to supply 0.1 mA/V. The low noise preamplifier is a Stanford Research SR560 and the spectrum analyzer is a Stanford Research SR780.

Hz noise measured here to the SR560, which switches to a high gain mode between the two measurements. We also expect that the 60 Hz noise is dominated by the SR650 and SR780, which will not be present when providing experimental biasing. The source noise is the propagated to flux noise using equation 4.16. Here the larger inductance of the coil designed with the thick wire will have a larger impedance than the coil designed with the thin wire. This shields the coil from the voltage noise measured from the source.

The thermal noise from the coil impedance is calculated using the parallel resistance of the source and the inductor as

$$Z = \left(\frac{1}{R} + \frac{1}{i\omega L} \right)^{-1} = \frac{iR\omega L}{R + i\omega L}. \quad (4.21)$$

Here the current buffer impedance of 10 k Ω is used. The thermal noise for the coil is then calculated using equation 4.15.

We sum the thermal and source noise and then convert to noise in the qubit and DC-SQUID using equation 4.17. The noise calculations done here are only treating the effect of noise on a single measurement. For the entanglement harvesting experimental protocol longer time scale noise has not been considered.

For treating the qubit noise the mutual inductance, M , will be 150×10^{-12} Wb/A with

the coil designed with the thin wire and 5.7×10^{-12} Wb/A for the coil with the thick wire. This is an order of magnitude smaller than the DC-SQUID mutual inductance, due to the smaller size of the qubit. The decoherence rates from these noise channels can be solved with equations 2.28 and 2.29. We get the upper bounds on the decoherence rates by assuming the decoherence rate when the qubit is in the symmetry point ($\Delta \gg \varepsilon$) and the dephasing rate when the qubit is away from the symmetry point ($\Delta \ll \varepsilon$). In this case instead of zero frequency the lowest frequency relevant for the operation will be the longest time scale. The readout time is typically between 10 to 200 ns [30], which sets a minimum frequency of $2\pi/200 \text{ ns} = 31 \text{ MHz}$. The coil constructed with the thin wire has the relaxation rate of $4 \mu\text{Hz}$ and dephasing rate 3 Hz. For the thick wire, the relaxation rate is 2 mHz and dephasing rate of 420 Hz. The noise is dominated by the thermal noise in the coil's inductance. This shows that the coil produced with thin wire produces less flux noise in the qubit, however both coils have a negligible effect on the qubit coherence.

The coil noise will also influence the readout of the qubit state as flux noise will change the readout biasing. The uncertainty in the flux using equation 4.16 is calculated as

$$\langle f^2 \rangle = \int_{\omega_l}^{\omega_h} d\omega S_f(\omega), \quad (4.22)$$

where $\langle f^2 \rangle$ is the mean squared deviation of the flux, ω_l is the low frequency range set by the maximum readout time, and ω_h is a high frequency cutoff. As above, the minimum frequency will be set by the readout time at 31 MHz. The maximum frequency used in literature for high frequency cutoff is 50 GHz [41]. This results in 13 $\text{n}\Phi_0$ of noise for the coil designed with the thin wire, and 280 $\text{n}\Phi_0$ for the thick wire coil. Using the simulations done by Shi [42], of the readout of a tunable coupling device at 4 GHz, the effect on readout can be estimated. At this biasing the two qubit states have a separation of 470 kHz. From the same simulation the noise from the coil will result in a shift in the qubit resonant frequency [42]. The coil using thin wire will result in 90 Hz resonance shift and the coil using thick wire will result in 2.1 kHz resonance shift. The coil designed with the thin wire will lead to lower noise, but the coil designed with the thicker wire will still lead to negligible shifting of the resonance point of the DC-SQUID for readout.

With the results from the noise modeling it was decided that a new biasing coil could be constructed using the thicker 254 μm diameter wire. The constructed coil is shown in figure 4.13. This was made of an OFHC bobbin, which also was bored out to allow for two smaller bobbins to fit inside. These smaller bobbins are only 6 mm in diameter and can supply local biasing to the chip. The larger coil is also designed to have a secondary bobbin above, designed for winding and securing extra coil length. This tension was maintained under 1 N during the coiling process. 120 turns of wire were added to this coil, up from the

100 originally designed for. This means the expected biasing current through the SQUID is expected to be $66 \mu\text{A}/\Phi_0$. This coil was used to bias a $625 \mu\text{m}^2$ SQUID loop and swept through several flux quantum of bias in an experiment. The coils have a SQUID biasing current of $93 \mu\text{A}/\Phi_0$. The measured mutual inductance is only 71% of the estimated value. As of the writing of this thesis no tests of biasing qubit with this coil have been performed to validate the noise estimates. See Appendix [A.3](#) for more information on the coil construction. This coil as constructed will provide the global flux biasing required for the static biasing of the tunable coupler loops and the DC-SQUID as specified for the entanglement harvesting protocol.

Chapter 5

Conclusion

5.1 Summary

In this thesis we have proposed a full cryogenic experimental wiring setup, for the validation of the entanglement harvesting phenomenon. We have also derived a novel Hamiltonian for the operation of the tunable coupler with time-dependant flux biasing.

In chapters 2 and 3 the theoretical building blocks for the entanglement harvesting experiment were introduced. This included a discussion on the quantization of Josephson junctions, the operation of the persistent current qubit and the readout of qubit states with DC-SQUIDS. The basics of the entanglement harvesting theory were discussed, along with challenges of perform this measurement. The model of a qubit-transmission line coupling system was discussed, outlining the results from the Superconducting Devices Group (SQD) group in Janzen et al. [21]. Finally, the work of Ren was discussed implementing a protocol to measure entanglement harvesting with the tunable coupler system [22].

Chapter 4 presents the designs of cryogenic electronics improvements required for implementing the entanglement harvesting protocol. The models of thermal dissipation are presented. Choices for the microwave cables and attenuation for the transmission lines and the fast biasing lines are presented. The transmission line designs reduce the thermal state sufficiently to extract entanglement. The fast biasing lines allow for the fast pulses to operate the entanglement harvesting experiment, with discussions on extending the cables to support event faster pulses. Choices of DC wires to minimize heating of the biasing wires are presented, along with mechanical designs of the thermalization and connections for the DC wires. This selection of wires and connectors are able to support currents up to 5 mA,

with connectors to break the wires at the 3K and MC temperature stages. The design of a superconducting coil is presented and its noise is modelled to ensure it will satisfy the requirements of the entanglement harvesting experiment. All the designs are evaluated for impact on the experiment, and to minimize the heat generated. All the hardware presented is designed to the requirements of the entanglement harvesting experiment but we expect that these developments will become applicable for many future experiments studying the spin-boson model and relativistic quantum information.

5.2 Outlook

In this thesis developments to the hardware are presented as corequisites to the work of Ren [22] for implementing the entanglement harvesting protocol. Initially the intent was to install all these components into the CF-CS81 dilution refrigerator, due to its larger experimental mounting space and cooling power. The designed components could not be installed and tested due to cooling issues which arose. Condensation failed during the installation and testing of 36 DC wires and two transmission in and out lines. We determined that this was due to a liquid helium leak in the dilution circuit, which required added wires to be removed and the dilution refrigerator to be sent for repairs. Instead designs for the DC connection shown in figure 4.12 and superconducting coils shown in figure 4.13 were installed in the CF650 dilution for testing, results of which were presented in this thesis. It is recommended that the remaining required components are installed in the CF650 to allow it to preform the entanglement harvesting experiment. After the hardware requirements are implemented into the dilution refrigerator, the device designed to measure entanglement harvesting by Janzen and Ren can be fabricated and measured.

In the longer term we anticipate that the improvements to the experimental hardware will lead to improved capabilities of qubit control. Particularly, with respect to the increased capability for strong and fast biasing of flux qubits [60, 61, 62]. This is very interesting when combined with the tunable coupler able to reach ultra-strong coupling, which is able to probe this new regime of light-matter interactions where many standard models break down [63]. Further, the methods discussed for prescribing the time-varying flux will be invaluable for studying fast biasing protocols.

The results of the entanglement harvesting experiment itself will open up potential protocols to study other proposals from relativistic quantum information. The dynamics of entanglement harvesting is very sensitive to the state of the transmission line, which could allow for very sensitive measurements of vibrations of the system. Additionally, there have been proposals for quantum energy teleportation, extracting energy from the quantum

vacuum [15]. This would follow from the recent experimental validation of quantum energy teleportation [64, 65].

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APPENDICES

Appendix A

Component design specifications

In this appendix we extend on the details surrounding the designed components in section 4. This includes the general dimensions of the designed components and reasoning for designed components. For commercially available components the product numbers and general specifications will be described.

A.1 Microwave cables and connectors

In subsections 4.3.1 and 4.3.2 designs for microwave cables and connectors were given. In these designs it was assumed that the cables are coaxial. These have an inner conductor diameter of 0.51 mm, surrounded by a PTFE insulator with a 1.67 mm diameter, surrounded by a conductor of 2.19 mm diameter [3]. The inner conductor can be silver-plated, which reduces attenuation. The plating is assumed to be 1 μm thick [66]. This thickness was quotes as being used with a thinner 0.11 mm inner conductor, and is assumed to be the same for the thickness of cables used here.

For all cryogenic uses there are 5 material configurations considered. Stainless steel conductors, beryllium copper conductors and niobium titanium conductors, with additional options to silver-plate the inner conductor of the stainless steel and beryllium copper cables. Cryogenic microwave cables have previously been procured from Coax Co., LTD. The product numbers are listed in table A.1. All cable options are compared with the thermal leakage using equation 4.1, assuming that the materials used to make the coaxial cable are not in thermal contact. This further assumes that the materials all thermalized at each temperature stage of the dilution refrigerator. This The materials area all compared to

Cable short name	Inner conductor materials	Coax Co. Product number
B-B	Beryllium copper	SC-219/50-B-B
SB-B	Silver-plated beryllium copper	SC-219/50-SB-B
SS-SS	Stainless steel	SC-219/50-SS-SS
SSS-SS	Silver-plated stainless steel	SC-219/50-SSS-SS
NbTi-NbTi	Niobium titanium	SC-219/50-NbTi-NbTi

Table A.1: Types of materials used for coaxial cables including their product numbers from Coax Co., TLD [3]. The outer conductor is the same metal as the inner conductor, without any plating.

Cable short name	300 K attenuation	300 K heating	4K attenuation	4 K heating
B-B	2.9 dB/m	124 μ W	2.2 dB/m	2.3 μ W
SB-B	1.8 dB/m	82 μ W	0.6 dB/m	0.7 μ W
SS-SS	9.4 dB/m	280 μ W	5.9 dB/m	6.0 μ W
SSS-SS	3.3 dB/m	138 μ W	1.5 dB/m	1.6 μ W
NbTi	9.6 dB/m	183 μ W	< 0.3 dB/m	< 0.3 μ W

Table A.2: Joule heating from coaxial cable attenuation. The attenuation values are at a frequency of 5 GHz, from [3]. Using equation 4.11 to calculate the heat dissipated through the cables. At 300 K the cable a test pulse of -4 dB is calculated through cable of length is 56 cm. At 4 K a -14 dB test pulse goes through a 12 cm cable.

ensure that the thermal leakage is sufficiently small relative to the cooling powers presented in table 4.1. This requires using values of thermal conductivity from literature. The values used in this calculation are presented in figure 4.3.

The resulting thermal leakage, can be compared with the amount of Joule heating from attenuation through the cables. The values of attenuation per meter are shown in table A.2 at 5 GHz, which is the typical gap frequency of the qubit, and is used for qubit scattering test. An upper bound on the Joule heating is presented in table A.2. The cable lengths are based on the distance between the room temperature stage and the 50K stage, which is 56 cm, and the distance between adjacent low temperature stages, which are 12 cm. These estimates presented are using very high power test pulses, where typically pulses will have less power than this. When using pulses attenuated by attenuators presented in subsections 4.3.1 and 4.3.2 it is found that the Joule heating more than an order of magnitude lower than the leakage. As such the Joule heating will not be considered for minimizing heating.

It should be noted that in some context using a cable with more leakage is preferred. This is discussed in the context of pulse distortion in subsection 4.3.2. Additionally, at temperatures under 0.1 K stainless steel coaxial cables, with and without silver plating, and Niobium titanium coaxial cables have very similar leakage. In these cases it becomes advantageous to use a lower attenuation cable. The lack of attenuation in a niobium titanium cable is useful for output lines to remove the attenuation of low power signals, improving the noise floor. However, there are additional difficulties with constructing NbTi cables. Niobium titanium has low solderability [67] and cables must be constructed with special SMA crimp connections. This is compared to stainless steel which can be soldered using a special solder flux, which has been shown to be reliable after extensive use in previous constructed cables. As such silver-plated stainless steel cables will be for low temperature coaxial cables. Niobium titanium cables will only be used for the output lines. These results are summarized in figure 4.2.

As discussed in subsection 4.3.2, the bandwidth of connectors must be considered for propagating signals distortion and attenuation. Other microwave components placed in the fast biasing lines must have a high bandwidth to ensure that the signal is not distorted, as shown in figure 4.7 (b).

The cables use connectors Rosenberger 32K101-271L5 for female and Rosenberger 32S102-271L5 for male SMA connectorization with soldering. Additionally, the TE Connectivity 2001-5443-02 is used for making SMA connections with niobium titanium, which requires crimp connections. Cables constructed with all of these connections are presented in figure 4.6 (b). From this figure "In RT-3K" and "Out RT-3K" are constructed with two Rosenberger male connectors. "In 3K-Still" and "In Still-MC" use 1 of each of the Rosenberger connectors. "Out 3K-MC" uses two TE connectivity crimp connectors. All of these cables show a no cutoff, or significant deviation from the ideal cable, shown as dashed lines in figure 4.6. Therefore these connectors will also be used for the fast bias lines.

In the input and output lines attenuators from XMA Corporation 2082-6418-dB-CRYO are used. These attenuators have a bandwidth of 0-18 GHz, compared to the XMA Corporation 2682-6460-dB-CRYO which have a 0-26.5 GHz band width. These higher bandwidth attenuators will be used in the fast biasing lines. Finally, the low pass filter (LPF) in the fast flux lines, as shown in figure 4.2, will use a custom requested Microtronics 0-23 GHz filter.

A.2 DC wire and connectors

In subsection 4.4.1 the wires used for biasing are discussed in the context of heating. These wires are commercially available from suppliers Lakeshore Cryogenics and Supercon Inc. In figure 4.3 the thermal conductivity is given for these materials. The leakage between temperature stages is given by equation 4.1. The lengths of the DC wires will be slightly above the separation between the dilution refrigerator stages. Between the room temperature connector and the 50K stage is 56 cm, and the DC wire will be assumed to be 60 cm. Between the rest of the stages there is a gap of 12 cm and DC wire is assumed to be 21 cm. The area of the wire material comes from the diameters given in table A.3.

Additionally, the DC wires have Joule heating when supplying current. This is given by equation 4.6 using the resistance values presented in table A.3. It is assumed that the resistance is equal to the higher temperature stage, to get an upper bound on the Joule heating. For wires between room temperature and 50K the resistance of 305 K is used, between 50K and 3K the resistance at 77 K is used, and the 4.2 K resistance is used for all lower wires.

These two heating sources can be added together. Assuming that all heat is deposited at the lower temperature stage, which will always be the limiting cooling power, the ideal wire can be selected. Keeping the lengths fixed the current can be swept for all of the cable options. Due to the wide spread of electrical resistance/length, and conversely thermal conductivity $\times$ length a wire material and diameter can be found to achieve low heating. The results of this calculation are presented in figure 4.8.

For wires lower than the 3K stage we use superconducting wiring as the thermal leakage is thermally suppressed without adding any resistance. These are purchased from Supercon Inc. either with a copper stabilizing layer, as PN T48B-M-0.127, or without, as PN SC-NbTi-0.127mm-HML. These wires have critical currents above 1 ampere, which is significantly above the required currents for flux qubit experiments. The bare wire, without any copper stabilizer is used between temperature stages of the dilution refrigerator, as copper would create significant thermal leakage. The copper stabilized NbTi wire is used in wiring under the mixing chamber, where there is no concern of leakage.

The construction of NbTi wires can be challenging due to strong oxides which form on the surface [67]. We use Cohler Superbrite Rhodium Plating Solution to deposit a thin layer of Rhodium onto the NbTi wire, which allows for the wires to be soldered together or to a connector. This method is more time consuming compared to the copper stabilized NbTi, which can be soldered directly.

In subsection 4.4.2 the designs for thermalization of heat from DC wires are presented.

Cable Material	AWG	Diameter (mm)	R @ 305 K (Ω/m)	R @ 77 K (Ω/m)	R @ 4.2 K (Ω/m)
Phosphor bronze	32	0.203	4.02	3.45	3.34
Phosphor bronze	36	0.127	10.3	8.83	8.56
Nichrome	32	0.203	34.0	33.4	33.2
Copper	30	0.254	0.32	0.04	0.003
Cooper	34	0.160	0.81	0.101	0.0076
Manganin	30	0.254	9.69	9.13	8.64
Manganin	32	0.203	15.1	14.3	13.5
Manganin	36	0.127	38.8	36.5	34.6

Table A.3: Cryogenic DC wires sold by Lakeshore Cryogenics. The materials and American Wire Gauge (AWG), which specifies the wire diameter, give the cable properties. The specified resistance per unit length of wire are given at a number of different temperatures.

The thermalization of these designs are presented in Appendix B.

A.3 Superconducting coil construction

The superconducting coils discussed in section 4.5 will be discussed further here. They are constructed with superconducting wire purchased from Supercon Inc. The wires are mono-filament, with a copper stabilizing over the niobium titanium core in a ratio of 1.5:1 Cu:NbTi. The wires are either T48B-M-0.050 or T48B-M-0.254, which have a diameter of 50 μm or 254 μm .

The coil geometry will be the same as previous coil's used to operate global flux qubit biasing. This coil has an inner diameter of 25 mm and a coil length of 10 mm. This size is chosen to sure that the magnetic field perpendicular to our experimental sample is uniform. In figure A.1 the field on the coil's axis is perpendicular from the experimental chip, which is placed 10 mm below the bottom of the coil. The black dashed lines in figure A.1 represent the shape of a 7 mm experimental chip. Here the magnetic field only changes by 2% from the edge of the chip to the center. Typically the superconducting loops are significantly smaller than the chip width. Here this magnetic field is sufficiently uniform across the experimental sample.

The coil's construction requires milling the bobbin out of oxygen free high conductivity (OFHC) copper, which will hold the material on a lathe, and finishing with a sanding up to 1000 grit, to ensure that no sharp edges exist. Coil construction can fail if there is a

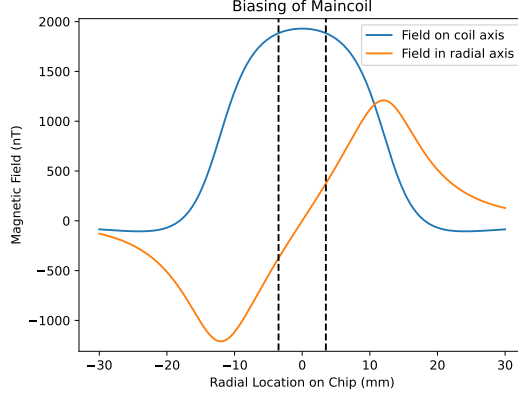


Figure A.1: The biasing of a the coil constructed with $254\ \mu\text{m}$ diameter wire. This assumes that the coil has a current density uniformly distributed along a continuous solenoid. This is computed using the treatment in reference [6]. The dashed lines represent the extents of a $7\ \text{mm} \times 7\ \text{mm}$ chip, which is the typical size of experimental devices.

sharp edge which cuts through the wire’s dielectric and causes a short to ground. Then a thin layer of stycast is added to the bobbin, where the wire will be mounted. This additionally protects the wire. This layer is designed to be as thin as possible. Here we aim for less than $50\ \mu\text{m}$ to minimize thermal resistance between the wire and the coil bobbin. Next, the wire is wound using a coil winder. The wire is directed by a rotating wheel to the coil bobbin. This rotating wheel provides $2\ \text{N}$ of tension. This is lower than the $3.9\ \text{N}$ maximum winding tension of the $254\ \mu\text{m}$ diameter wire and higher than the $0.9\ \text{N}$ maximum tension of the $50\ \mu\text{m}$ diameter wire [58]. Further, we want to apply between 50-80% of the maximum tension for our wire, which means the tension of the winding setup will need to be adjusted and reduced to construct a wire with the thinner diameter.

To generate the same coil geometry the number of turns will reduce by the diameter squared. Due to the packing efficiency of circles this geometry will not be identical, and using a thicker wire will result in a less dense winding. This assumption likely leads to the resulting discrepancy between the designed vs measured biasing. Conversely, the current through the coil will need to scale with the diameter squared. Hence, we expect the resulting mutual inductance between the coil and a superconducting loop in the sample to scale with the diameter squared. The results of this scaling is shown in table 4.7.

The inductance of the coil is relevant for computing the noise from the coil, which is discussed in subsection 4.5. The inductance of the coil is given by equation 4.20, which uses the Lundin Handbook approximation of Nagaoka’s coefficient. This coefficient is ap-

proximated as

$$k_L = \frac{2}{\pi} \frac{l}{D} \left(\frac{\left[\ln \left(4 \frac{D}{l} \right) - \frac{1}{2} \right] \left[1 + 0.383901 \left(\frac{l}{D} \right)^2 + 0.017108 \left(\frac{l}{D} \right)^4 \right]}{1 + 0.258952 \left(\frac{l}{D} \right)^2} + 0.093842 \left(\frac{l}{D} \right)^2 + 0.002029 \left(\frac{l}{D} \right)^4 - 0.000801 \left(\frac{l}{D} \right)^6 \right), \quad (\text{A.1})$$

which is valid for short coils, which have a diameter D greater than their length l . Since the coils are assumed to have the same geometry regardless of wire the Nagaoka's coefficient is 0.472 for both. Hence, the inductance is only based on the number of turns,

$$L = \frac{\mu_0 N^2 A}{l} k_L, \quad (\text{A.2})$$

where μ_0 is the vacuum permeability, N is the number of wire turns in the coil, A is the area of the coil, and l is the length of the coil. The inductance of a coil constructed with a 254 μm diameter wire is 0.39 mH and a coil with a 50 μm diameter wire is 0.25 H.

The main text presents the conclusion that a coil designed with a thicker 254 μm diameter wire will have sufficiently low noise for the operation of the entanglement harvesting experiment and operation of other qubit control experiments. Based on this conclusion a coil was constructed with the 254 μm diameter wire with 120 turn. The measured coil mutual coupling is only $66 \mu\text{A}/\Phi_0$ between the coil and a $625 \mu\text{m}^2$ SQUID loop in a device. The estimated mutual inductance is $77 \mu\text{A}/\Phi_0$ ($120/100$) = $93 \mu\text{A}/\Phi_0$. This is an decrease of $(66 \mu\text{A}/\Phi_0) / (93 \mu\text{A}/\Phi_0) = 71\%$ from the estimated mutual inductance. It is expected that the magnetic biasing is lower than expected due to worse packing of the turns of the coil. With the scaling of the turns and current the total current flowing around a cross section of the coil is equal, however due to inefficient packing of the wires. It is expected that the coil winding has approximately square packing, which will lead to empty spaces equal to $1 - \pi/4 = 21\%$. This is expected to be the main contributor to the decrease in the coil biasing current. Further tests with qubits are needed to fully validate the coil operation.

Appendix B

Thermalization calculations

In this appendix the calculations of thermal resistances are presented. These are related to the thermal anchoring designs presented in subsection 4.4.2.

B.1 Thermal bobbin thermal mounting

The thermal bobbins shown in figure 4.10 have a number of thermal resistances. Here a thermal bobbin will be modelled at 50 K with a 0.127 mm diameter wire wrapped around it 1 time and held by VGE-7031 varnish. The thermal bobbin is typically 2-3 cm long, has an outer diameter of 4.76 mm and an inner diameter of 3.15 mm.

At 50 K the thermal conductivity of VGE-7031 varnish is 0.22 W/m·K [2], the thermal area is the diameter of the wire by the diameter, and the thermal length is half the diameter of the wire at 0.67 mm. The thermal resistance through the VGE-7031 varnish, using equation 4.1, is

$$R_{VGE} = \frac{63.5 \mu\text{m}}{\pi(4.76 \text{ mm})(127 \mu\text{m})(0.22 \text{ W/m} \cdot \text{K})} = 152 \text{ K/W.} \quad (\text{B.1})$$

The thermal conductivity of OFHC copper is 1000 W/m·K [1], and the thermal bobbin the thermal resistance through the copper tube is

$$R_{OFHC} = \frac{3 \text{ cm}}{\pi[(4.76 \text{ mm})^2 - (3.15\text{mm})^2](1000 \text{ W/m} \cdot \text{K})} = 7.5 \text{ K/W.} \quad (\text{B.2})$$

The contact thermal resistance comes from the torque applied by the M3 brass screw threading into copper. These materials have a coefficient of friction around is around 0.5 [68]. With a torque or 1 N·m and thread radius of 1.6 mm the pressure is than 8 MPa. The thermal conductivity between the two copper surfaces is given by equation 4.5 as [46]

$$R_{Cu-Cu} = (10\text{mm}^2\text{K/W})/\pi [(4.76 \text{ mm})^2 - (3.15\text{mm})^2] = 0.25 \text{ K/W}. \quad (\text{B.3})$$

Here the maximum thermal resistance is through the epoxy, which can be taken as the thermal resistance of the entire thermal connection. It should be noted that the wire diameter divides out of the equation of R_{VGE} , meaning the thermal anchor is only dependant on its temperature and the number of wire windings.

If a single phosphor bronze #32 wire with 10 mA is applied the temperature of the wire would be

$$T_H = 50.46 \text{ K} + (0.8 \text{ mW})(152 \text{ K/W}) = 50.58 \text{ K}, \quad (\text{B.4})$$

which is a small temperature gradient, only a 0.2% increase from the dilution refrigerator stage temperature.

B.2 CMR connector thermal mounting

The connector mount using CMR-direct branded circular connectors is presented in figure 4.11. The geometry of these connectors makes it difficult to define the area and length of the thermal path. The conductor inside the PEEK has a surface area of $\pi \times 1 \text{ mm} \times 10 \text{ mm}$, and half this value will be used as the thermal cross-sectional area. The most central conductors are 1 mm from the outside edge, which will be the length of the thermal path, L in equation 4.1. The resistance through the connectors is lower than 10 m Ω and generates 10 nW. This heat is smaller than the worst case 137 μW from leakage and Joule heating through the wires from higher temperatures.

The thermal resistance is dominated the PEEK dielectric, which has a thermal conductivity of

$$\kappa(T)_{PEEK} = 0.18 \text{ mW/m} \cdot \text{K} \frac{T^2}{(1 \text{ K})^2}, \quad (\text{B.5})$$

from measurements made at 1 K [69]. This gives the thermal resistance at 3.4 K as

$$R = \frac{1 \text{ mm}}{(\pi \times 1 \text{ mm} \times 10 \text{ mm}/2) (0.0021 \text{ W/m} \cdot \text{K})} = 30.4 \text{ K/mW}. \quad (\text{B.6})$$

The worst case heating from the wires of $137 \mu\text{W}$ heats the conductor up to 7.1 K. Due to this excess heating, it is recommended to use a thermal bobbin as described above to dissipate the heat from the above plates. When this connector is just dissipating the 10 nW it only had negligible heating above the 3.4 K temperature stage.

B.3 Fischer Elektronik connector thermal mounting

The connector mount which uses the Fischer Elektronik MK204/201 connectors is shown in figure 4.12. An example of this connector is as follows. For a current flowing through a twisted pair: there will be 4 MK204/MK201 mated connections which the current flows through, two for each of the signal and return lines, which gives $40 \text{ m}\Omega$ of resistance. Designing for up to 1 mA current will generate 40 nW of power, which is larger than the 49 pW generated through dissipation down the wires, as shown in table 4.6. The thermal conductivity of Stycast dominates the thermal resistance at these low temperatures. It can be modelled with a T^2 dependence [1] as

$$\kappa(T)_{\text{Stycast}} = 6.5 \text{ mW/m} \cdot \text{K} \frac{T^2}{(1 \text{ K})^2}, \quad (\text{B.7})$$

based on the measured value at 1 K [2]. Each trace is 4 mm wide and 50 mm long. This gives a thermal resistance of

$$R = \frac{0.125 \text{ mm}}{(4 \text{ mm} \times 50 \text{ mm}) (4.8 \times 10^{-7} \text{ W/m} \cdot \text{K})} = 1.3 \text{ mK/nW}. \quad (\text{B.8})$$

The thermal resistance reported above is an over estimate, as it assumes that the Stycast is fixed at the low temperature. The change in thermal conductivity with temperature is accounted for by solving equation 4.1 with the functional form of the thermal conductivity, and results in a temperature of the connector at 26.7 mK. This is an effective thermal resistance of 0.67 mK/nW. This temperature pocket will result in thermal dissipation of 0.36 nW·m flowing down a NbTi copper stabilized AWG #32 wire. If this is wired to a device the wire should be thermalized with a thermal anchoring bobbin as shown in figure 4.10, to reduce heat flowing to the sample. At this temperature the thermal bobbin using VGE has a thermal resistance is 15.4 mK/nW which is divided by the number of windings of the wire. Two or three winding will be sufficient to thermalize the heat from the connector breakout board and ensure no heat flows to the sample.

It is important to ensure that this thermal contact is better than the thermal contact along the wire, to prevent heat from dissipating towards the experimental sample. The

thermal resistance of a 0.127 mm diameter copper stabilized niobium titanium wire travelling 10 cm is 13 mK/nW. This assumes a thermal conductivity with $RRR = 100$ [1] held at 8.6 mk.