Saturation-Dependent Thermal Conductivity of Southern Ontario Soils

by

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Author’s Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Soil thermal conductivity is an important parameter in geotechnical and environmental engineering applications, influencing the performance of underground energy storage, ground heat exchangers, and other subsurface thermal systems. Through geotechnical characterization and laboratory measurements, this study investigates the thermal conductivity of 20 soil samples collected from seven locations in Southern Ontario. The key soil properties, including texture, moisture content, and bulk density, were analyzed to understand their impact on thermal conductivity. Measured thermal conductivity values were compared with published regression-based and normalized models to assess their predictive accuracy across diverse soil types. A statistical evaluation incorporating root mean square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2) was performed to identify the best-performing models. The results indicate that *Lu et al.* (2014) and Yoon *et al.* (2018) describe the most reliable regression-based models, demonstrating strong correlations with measured data, minimum bias, and low error margins. Among normalized models, the Côté and Konrad (2005) model exhibited superior adaptability and lower prediction errors, while Johansen's (1975) model performed well but required calibration for extreme soil compositions. The results emphasize the significant influence of soil texture and moisture content on thermal conductivity, with silty and sandy soils exhibiting higher values due to their mineral composition and structural properties. The best-performing models effectively captured these variations, highlighting their applicability in geotechnical and environmental engineering.

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Dedication

This thesis is dedicated to my beloved wife, ***Dr. Fatematuj-Zannat,*** *whose unwavering love, patience, and support have been my most significant source of strength and inspiration. She has been my pillar, standing beside me through every challenge with unconditional love and encouragement. Her belief in me has been the driving force behind my perseverance, making this journey possible.*

I also extend my deepest gratitude to my parents, parents-in-law, and sister, whose constant support, sacrifices, and unwavering faith in me have shaped my aspirations and fueled my determination. Their love has been a guiding light, giving me the strength to pursue my dreams.

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List of Abbreviations

AD	Average Deviation
ASTM	American Society for Testing and Materials
HP	Heat Pulse
LSMs	Land-Surface Models
NSE	Nash-Sutcliffe Efficiency
RMSE	Root Mean Square Error
R^2	Coefficient of Determination
STC	Soil Thermal Conductivity
TLS	Transient Line Source
USDA	United States Department of Agriculture
XRD	X-Ray Diffraction
XRF	X-Ray Fluorescence

List of Symbols

ρ_b	Bulk Density
f_{clay}	Clay Content
χ and η	Constants
S_r	Degree of Saturation
λ_{dry}	Dry Thermal Conductivity
λ_{eff}	Effective Thermal Conductivity
d	Finite Dimensions of The Heat Source
ζ	Fitting Parameter
T_f	Freezing Point
q	Heat Flux
T_0	Initial Temperature
K_e	Kersten Number
$\widehat{y_i}$	Measured Value
δ and β	Parameters
n	Porosity
y_i	Predicted Value
f_{quartz}	Quartz Content
f_{sand}	Sand Content
λ_{sat}	Saturated Thermal Conductivity
k	Shape Factors
f_{silt}	Silt Content
λ_a	Thermal Conductivity of Air
λ_w	Thermal Conductivity of Water
α	Thermal Diffusivity
$t_{0.5}$	Time Required for The Rear Surface Temperature
∇T	Temperature Gradient
θ_v	Volumetric Water Content
C_v	Volumetric Heat Capacity
w	Water Content

Chapter 1
Introduction

1.1 Background and Motivation

Soil thermal properties are critically important in various disciplines, including engineering, environmental science, and agronomy, due to their significant influence on energy transfer and storage within the environment. In engineering, particularly in thermal construction projects, these properties are essential for designing and optimizing structures interacting with subsurface heat flows (Chen *et al*., 2025). The importance of understanding soil thermal conductivity extends to analyzing, designing, and simulating the long-term environmental impacts of buried engineering systems. These systems include high-power electric cables, ground-source heat pumps, district heating networks, gas and oil pipelines, nuclear waste disposal facilities, roads, airfields, and buildings (Tarnawski *et al.*, 2009). They are indispensable in developing geothermal energy systems, underground heat exchangers, and thermally stable foundations, where precise thermal dynamics determine the efficiency and durability of these systems (He *et al.*, 2018b; Kurelic *et al.*, 2014).

In thermal construction, these properties are pivotal for ensuring structural and operational efficiency, particularly in energy-related applications, such as geothermal heat exchangers (Rees *et al.*, 2000) and borehole thermal energy storage systems (Witte *et al.*, 2002). These systems depend on efficient heat transfer dynamics between the soil and the heat exchange system to maintain energy efficiency and operational stability. An efficient heat transfer system between the soil and the heat exchange system enhances energy efficiency in geothermal energy systems. Accurate thermal conductivity data is indispensable for designing and simulating soil-structure interactions under various thermal conditions.

For example, Oil and gas pipelines are vulnerable to waxing and hydrate formation when production fluids fall below specific temperature thresholds. To address this issue, insulation is essential to minimize rapid heat loss caused by the high thermal conductivity of bare pipes. Various insulation methods are available, including external coatings, pipe-in-pipe technology, electrical heating, and hot water annulus systems. However, these approaches are often costly and challenging to implement and maintain. An alternative, cost-effective solution is pipe burial, which utilizes the thermal properties of the surrounding soil. When combined with external coatings, pipe burial has been shown to provide thermal insulation comparable to pipe-in-pipe technology but at a significantly lower cost (Loch, 2000). Similarly, in constructing insulated foundations and energy piles, understanding soil thermal properties helps mitigate risks such as thermal degradation and uneven ground heat distribution (Campbell, 1985).

The performance of underground structures, including tunnels, pipelines, and buried utilities, is strongly influenced by the surrounding soil's ability to store and dissipate heat, further underscoring the importance of precise thermal characterization in their design (Ebel *et al.*, 2019).

Beyond engineering, soil thermal properties are essential in biological, chemical, and physical processes within the Earth's critical zone. They influence soil responses to environmental events such as forest fires, methane exchange, and root respiration, impacting global climate and hydrological cycles (Cheng *et al.*, 2020; Halloran *et al.*, 2016). The microclimate, influenced by soil thermal properties, also affects seed germination, seedling emergence, and subsequent stand establishment. These thermal properties are integral to agronomy (Abu-Hamdeh & Reeder, 2000; Ghuman & Lal, 1985).

Additionally, soil thermal properties mediate energy exchanges between the ground and atmosphere, influencing regional and global climate dynamics (Koster *et al.*, 2004; Harris *et al.*, 2009). So, soil thermal properties are fundamental to understanding and modeling energy transfer processes in engineering and environmental contexts. Their critical role in designing sustainable and efficient thermal systems, managing energy flows in agricultural and natural systems, and predicting ecological changes underscores the need for accurate measurement and modeling. This understanding enables the development of innovative solutions to challenges in energy systems, infrastructure development, and climate resilience.

1.2 Thermal Conductivity of Soils

Thermal conductivity plays a central role in governing heat transfer processes within soils, impacting the efficiency and stability of engineering systems. The three primary soil thermal properties, thermal conductivity (λ), volumetric heat capacity (C_v), and thermal diffusivity (α), are fundamental for understanding how soil interacts with and responds to thermal loads. Thermal conductivity, expressed in W/m·K quantifies the heat transfer rate through the soil under a unit temperature gradient and is fundamental for modeling soil heat flux (He *et al.*, 2018a; Verhoef *et al.*, 2012). Thermal conductivity plays a central role in governing heat transfer processes within soils, impacting the efficiency and stability of engineering systems. It is widely incorporated into land-surface models (LSMs) for simulating soil temperature and energy partitioning. It is especially crucial in energy-related applications, such as geothermal heat exchangers (Rees *et al.*, 2000) and borehole thermal energy storage systems (Witte *et al.*, 2002). Moreover, soil thermal conductivity is integral to managing freeze-thaw cycles in cold regions, as highlighted by Cote and Konrad (2005), who demonstrated its role in

infrastructure stability under such conditions. So, understanding the mechanisms of heat transfer in soils is fundamental to advancing geoenvironmental engineering practices.

1.2.1 Fundamental Principle

The transfer of heat through porous media, such as soils, and the associated changes in moisture content are vital considerations for many engineering structures. The thermal conductivity of soil is strongly influenced by moisture content, which is governed by two primary water retention mechanisms: adsorption and capillarity. The thermal conductivity remains relatively low when water is primarily adsorbed onto soil particle surfaces as thin films at low moisture levels. This is because the isolated water films and the surrounding air in soil pores contribute poorly to heat transfer. However, as soil moisture increases, the water films grow thicker, forming liquid bridges between particles due to capillary forces. At this stage, capillarity becomes dominant, enabling water to create a continuous network within the soil. This network allows liquid to flow when there are differences in concentration or pressure within the water.

Adsorption dominates at lower moisture levels, while capillarity is the primary mechanism at higher moisture levels when liquid bridges form, as shown in Figure 1. These liquid bridges significantly enhance heat transfer pathways by replacing air (a poor conductor) with water (a better conductor), leading to a sharp increase in thermal conductivity. When soil reaches the capillary saturation stage, water forms a continuous network within the pore spaces, enabling efficient heat transfer. At this point, the thermal conductivity increases linearly with moisture content, although the rate of increase diminishes compared to the earlier stages. This is because the primary contribution to thermal conductivity arises from replacing air with water rather than changes in the water distribution. However, beyond this point, additional moisture has minimal impact on thermal conductivity, as most air-filled pores have already been replaced.

The balance between adsorption and capillarity is influenced by soil properties such as granulometry (grain size) and chemical composition, with temperature having a minor role unless it exceeds 80°C. Heat transfer in soils primarily occurs through two mechanisms: conduction and convection. Conduction refers to transferring heat within a material due to internal molecular motion analogous to diffusion. At the same time, convection involves heat transfer caused by the movement of pore fluids (mass movement) (Chen *et al.*, 2025). The relative contribution of these mechanisms depends on the soil's grain size, with conduction dominating fine-grained soils and convection being more significant in coarse-grained soils (Savvidou, 1988).

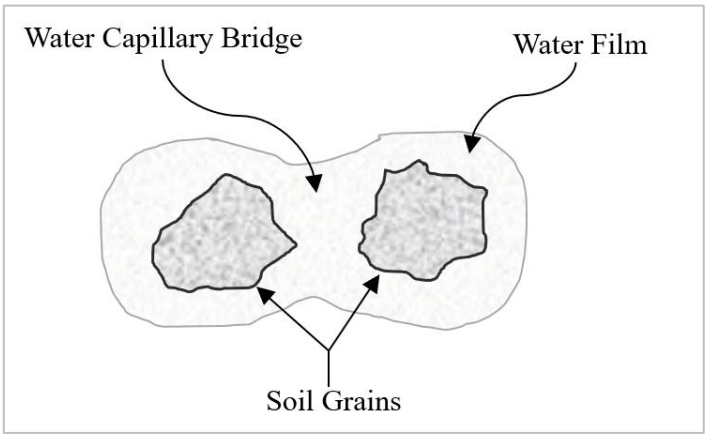


Figure 1: Capillary Bridge between Soil Particles.

Fine-grained soils (like clay or silt) primarily transfer heat through conduction because these soils have small particles packed tightly together, leaving little space for water to move. Coarse-grained soils (like sand or gravel) are more likely to transfer heat by convection because they have larger spaces (pores) between particles, allowing water or air to flow and carry heat. The general governing equation for heat transport through porous media is expressed as follows,

$$C_v * \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) \quad (1)$$

Where C_v is the volumetric heat capacity of the soil, T is temperature, t is time, and λ is the thermal conductivity, which depends mainly on soil texture, mineral composition, water content (w), and bulk density (ρ_b) (de Vries, 1963; Campbell, 1985). At similar w and (ρ_b) values, sandy soil usually has a higher λ value than clay soil. Soil with higher quartz content (f_q) usually has more thermal conductivity value because quartz has two times that of most other soil minerals (Campbell, 1985). A greater density (ρ_b) or a lower, more negligible porosity (n) leads to more excellent thermal conductivity, as more solid matter per unit volume results in better contact between soil particles.

1.2.2 Soil-Water-Heat Interaction Mechanisms

Soil is a heterogeneous medium where heat transfers through solid particles, water, and air. These phases interact differently under various environmental and loading conditions, resulting in complex thermal behaviors. Soil thermal conductivity as a function of soil water content (λ(θ)) has been extensively studied due to its importance in understanding soil heat transfer properties (Farouki, 1986). The λ(θ) curve is generally divided into distinct zones, each representing different water domains with unique thermal behavior. Tarnawski and Gori (2002) identified four primary zones within the λ(θ) curve: residual, transitory meniscus, micro/microporous capillary, and superfluous water domains.

In the residual water domain, water is tightly bound to the surfaces of soil particles by adhesion forces. This water forms a thin film that does not contribute significantly to thermal conductivity due to its minimal interaction with neighboring particles. As a result, the soil's thermal conductivity (λ) remains almost constant within this zone. The limited mobility and insignificant contribution of residual water to heat transfer explain the stability of λ in this phase.

As water content (w) increases, the system enters the transitory meniscus domain, where water forms bridges between adjacent soil particles. These water bridges enhance connectivity between particles, significantly improving heat conduction pathways. Consequently, λ increases sharply with increasing w, marking this domain as a critical transition phase where the thermal behavior of soil changes rapidly (Ewing and Horton, 2007). Water fills the soil pores in the capillary water domain, initially saturating larger macropores and extending to smaller capillaries. The thermal conductivity in this stage rises almost linearly with w but slower than the transitory meniscus domain. This reduced slope occurs because air replacement by water enhances thermal conductivity, but the contribution becomes progressively less pronounced as more pores are filled.

Finally, in the superfluous water domain, water occupies nearly all available pore space, including capillary and drainable pores. At this point, the system approaches saturation, and the thermal conductivity (λ) shows little to no change with further increases in water content. Stabilization occurs because the additional water no longer alters the dominant pathways for heat transfer, as most air voids have already been replaced. This progression from residual to superfluous water domains highlights the dynamic relationship between soil water content and thermal conductivity, illustrating the complex interplay between soil structure, moisture distribution, and heat transfer mechanisms.

In partially saturated soils, thin water films surrounding soil particles act as conductive bridges, enhancing heat transfer between grains. The extent and effectiveness of these films depend on soil suction and moisture distribution (Lu & Likos, 2004). Conduction through solid particles is the dominant mechanism in dry soils, relying on the thermal properties of the minerals and the degree of particle contact. In saturated or highly porous soils, convective heat transfer becomes significant. Moving water transports heat efficiently under hydraulic gradients, particularly in soils with high permeability. Conversely, the air phase is critical in dry or loosely packed soils, where air's low thermal conductivity reduces the overall heat transfer efficiency. Accurately characterizing these interactions requires advanced experimental techniques, such as transient heat flow tests and numerical modeling. These approaches provide insights into the complex interplay of soil, water, and heat, supporting precise predictions of thermal behavior in engineering applications (Mitchell & Soga, 2005).

1.3 Factors Affecting Thermal Conductivity

Soil thermal properties, including soil thermal conductivity (STC) and thermal diffusivity, are strongly influenced by various soil-specific factors. These factors include, but are not limited to, soil water content, bulk density, soil texture, organic matter content, mineral composition, and temperature.

1.3.1 Soil Composition and Texture

Soil composition and texture are foundational factors affecting soil thermal properties, such as soil thermal conductivity (STC) and thermal diffusivity. The size distribution of soil particles—categorized into sand, silt, and clay—plays a central role in determining heat transfer. Like sandy soils, coarse-textured soils typically exhibit lower thermal conductivity due to larger pore spaces that are less efficient at conducting heat. In contrast, fine-textured soils, such as clayey soils, have smaller pores and excellent water retention capacity, enhancing thermal conductivity under moist conditions (Abu-Hamdeh & Reeder, 2000).

The mineral composition of soil further influences thermal behavior, i.e., quartz-rich soils are known for their high thermal conductivity. In contrast, soils dominated by clay minerals or organic matter tend to have lower thermal conductivity. Directly measuring quartz content may require specialized equipment such as X-ray fluorescence (XRF) or X-ray diffraction (XRD), methods that are not always practical (Nikoosokhan *et al.,* 2015; Schönenberger *et al.,* 2012). As a result, various estimation schemes have been proposed to approximate quartz content without direct measurements. Studies have shown that the first scenario, quartz content set at half the sand content, generally produces better estimations than the other two, making it a preferred approach (Chen *et al.,* 2012; He *et al.,* 2021). However, this assumption should be applied cautiously for soils with high quartz sand content, as quartz's higher thermal conductivity can significantly affect the overall thermal properties of the soil. Previous research, including work by Lu *et al.* (2007) and Peters-Lidard *et al.* (1998), adopted similar approaches by assuming quartz content equal to the sand fraction or using 50% of the sand content as a practical approximation. These methods remain widely accepted due to their simplicity and effectiveness in the absence of direct quartz measurements (Tarnawski *et al.,* 2012; Luo *et al.,* 2009). This study assumes quartz content as half the sand content ($f_{quartz} = 0.5\, f_{sand}$) for estimating quartz content to predict the thermal conductivity (STC) models.

In addition, although beneficial for improving soil structure and fertility, organic matter generally decreases soil thermal conductivity due to its low density and poor heat conduction properties (Farouki, 1981). Bulk density, which measures the packing of soil particles, also impacts thermal conductivity.

Denser soils facilitate better particle-to-particle contact, enhancing heat transfer, whereas lower bulk-density soils, with more pore spaces, act as thermal insulators (Campbell *et al.*, 1994).

1.3.2 Saturation Levels and Moisture Content

Saturation levels and soil water content are among the most significant factors controlling the thermal properties of soil. Water has a higher specific heat capacity and thermal conductivity than air, making moisture content a critical determinant of soil heat transfer. In dry soils, air-filled pores dominate, resulting in low thermal conductivity. As water content increases, water displaces air, significantly enhancing thermal conductivity and volumetric heat capacity. Saturated soils exhibit the highest thermal conductivity due to the continuity of water pathways facilitating heat transfer (Campbell *et al.*, 1994). It is well known that λ_{eff} of unsaturated soils increases with water content because the thickening of the water films "bridges" soil particles and increases the heat flow path (or inter-particle contacts). These results in a sharp increase in thermal conduction (Singh and Devid, 2000). Bulk density interacts with moisture content to further influence thermal properties. Soils with higher bulk density generally retain less pore space, increasing contact between particles and enhancing heat conduction. Soil texture also determines water retention and saturation behavior; fine-textured soils, such as clays, retain more water and thus exhibit higher thermal conductivity under moist conditions than sandy soils, which drain quickly.

1.3.3 Temperature and Environmental Conditions

Temperature and environmental conditions significantly affect soil thermal properties as they alter the interactions between soil components, water, and air. For instance, the thermal conductivity of dry soils generally decreases with increasing temperature due to the expansion of air and reduced particle contact. Conversely, in moist soils, elevated temperatures can enhance thermal conductivity by reducing water viscosity and improving heat transfer pathways, especially near saturation (Abu-Hamdeh & Reeder, 2000).

Environmental factors such as seasonal temperature variations, freeze-thaw cycles, and vegetation coverage also play essential roles. Seasonal temperature changes can cause fluctuations in soil thermal properties due to water phase transitions and shifts in pore configurations. Freeze-thaw cycles, in particular, alter soil's structure and bulk density, impacting thermal conductivity and diffusivity. Soils with higher organic matter content tend to insulate against temperature extremes due to their low thermal conductivity. In contrast, mineral-rich soils, particularly those with high quartz content, exhibit stable and predictable thermal conductivity across a wide temperature range (Campbell *et al.*, 1994).

Among the various factors affecting soil thermal conductivity, pore water salinity is significant in saline or coastal environments. Saline conditions arise naturally or due to anthropogenic influences such as de-icing salts or industrial activities, which alter the pore fluid's physicochemical properties. Dissolved salts such as NaCl, KCl, or $CaCl_2$ increase the thermal conductivity of the pore water, particularly in saturated soils, thereby enhancing the overall thermal performance of the soil matrix (Farouki, 1986; Lu *et al.*, 2007). This enhancement is more pronounced in fine-grained soils like clays, where ionic interactions between saline water and adsorbed water layers promote more effective solid-fluid heat transfer. Moreover, salinity can influence soil structure through flocculation or aggregation, impacting porosity and heat conduction. While sandy soils show a relatively modest response, clayey soils are more sensitive due to their higher surface area and reactivity. Understanding these thermal behaviors is essential for geotechnical and geoenvironmental applications such as underground power cable design, geothermal system optimization, and evaluating ground stability in permafrost and coastal regions (Ochsner *et al.*, 2001). In this study, however, distilled water was used in laboratory testing to eliminate the effects of salinity, ensuring that other thermal conductivity influencing factors could be studied in isolation.

1.4 Thermal Conductivity Measurement Techniques

Thermal conductivity is an essential property that influences heat transfer in various materials. Accurate thermal conductivity measurement is vital for applications in fields like engineering, material science, and energy efficiency. Several techniques have been developed to measure this property, each with specific advantages and limitations. These methods differ in their principles, sensitivity, and suitability for different materials and conditions. By understanding the strengths and limitations of each technique, the most appropriate method can be selected based on the material and experimental needs. The following sections provide a detailed overview of these methods, outlining their principles, applications, and effectiveness in measuring thermal conductivity.

1.4.1 The steady-state method

The steady-state method measures thermal conductivity under steady thermal conditions, where the temperature distribution does not change with time. This approach involves applying a known heat flux through a material and measuring the resulting temperature gradient. Fourier's law of heat conduction forms the basis of this method.

$$q = -\lambda \frac{dT}{dx} \tag{2}$$

Here, q is the heat flux (W/m^2), λ is the thermal conductivity (W/m·K), and $\frac{dT}{dx}$ the temperature gradient (K/m). This method is reliable for materials with high thermal conductivity, such as metals. However, it requires precise control of boundary conditions and can be time-consuming due to the need to achieve steady-state conditions.

1.4.2 Transient hot-wire method

The transient hot-wire method measures thermal conductivity by analyzing the temperature response of a material subjected to a heat pulse from a line heat source (a thin wire). The wire acts as both the heater and the temperature sensor. The temperature rise is described by:

$$T - T_0 = \frac{q}{4\pi\lambda} ln(t) + d \quad (3)$$

Where T_0 is the initial temperature (°C), q is the heating input per unit length of the wire (W/m), λ is the thermal conductivity (W/m·K), and d is an empirical constant.

This method is ideal for measuring the thermal properties of fluids and gases due to its simplicity and speed. Transient-state thermal conductivity offers several advantages over steady-state methods. It allows for faster measurements since it doesn't require the system to reach thermal equilibrium. It is also more sensitive to a material's thermal properties, providing accurate results for materials with low or variable conductivity. Additionally, transient methods can be used with simpler geometries and are effective for insulating materials. They also enable measurements across various temperatures, making them well-suited for studying temperature-dependent materials. Transient methods are faster, more versatile, and better suited for specific materials. Transient methods have proved advantageous over steady-state methods because they are less likely to be affected by phase change and water redistribution, and the results are more accurate. In addition, they require shorter measurement times, less energy consumption, and lower equipment and maintenance costs. Therefore, transient methods are more amenable to measuring soil thermal properties in unfrozen soils at a full water range in laboratory and field environments (de Vries, 1952; Farouki, 1981; He *et al.*, 2018). The most often utilized transient method in the literature is the heat pulse method (He *et al.*, 2018b).

1.4.3 Laser flash diffusivity method

The laser flash diffusivity method is widely used to measure the thermal diffusivity (α) of materials. A short laser pulse heats the front surface of a sample, and the resulting temperature rise on the rear surface is recorded as a function of time. The thermal diffusivity is calculated using:

$$q = \frac{d^2}{\pi^2 t_{0.5}} \tag{4}$$

The thermal conductivity can then be obtained by combining the diffusivity with specific heat capacity (c) and density (ρ) expressed as,

$$\lambda = \alpha.\rho.c \tag{5}$$

Here, d is the thickness of the sample (m), and $t_{0.5}$ is the time required for the rear surface temperature to reach half its maximum value (s).

1.4.4 Transient plane source method

The Transient Plane Source (TPS) method measures thermal properties using a flat sensor that acts as a heat source and a temperature sensor. The sensor is sandwiched between two sample halves, and a constant power is applied. The temperature rise of the sensor is monitored over time, and thermal conductivity (λ) and thermal diffusivity (α) are extracted from:

$$T(t) = \frac{q}{4\,\pi\lambda} ln(t\ +\ t_0) +\ d \tag{6}$$

This method is versatile, suitable for liquids, solids, and pastes, and allows simultaneous thermal conductivity and diffusivity measurement.

In this research work, a single proven heat transient source is used to determine the thermal conductivity (λ). It is based on the analytical solution of the heat conduction equation for a homogeneous, isotropic medium subjected to a line source of constant heat input (q). The process of heat transfer in a continuous medium is governed by Fourier's heat conduction law, which states that the heat flux (q) is proportional to the negative temperature gradient (∇T):

$$q = -\lambda.\nabla T \tag{7}$$

This forms the basis of the transient heat conduction equation, which describes the change in temperature over time in a homogeneous, isotropic medium. If in a medium of thermal diffusivity (α), the temperature is a function only of the time t and the distance r from the linear source, the relation between the variables is given by the Fourier equation in cylindrical coordinates. This equation assumes radial symmetry around the line heat source, making cylindrical coordinates (r, θ, z) expressed as

$$\frac{\partial T}{\partial t} = \alpha(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}) \tag{8}$$

Here, λ_ρ is the density (kg/m^3), and c is the specific heat capacity (J/kg·K). When a line heat source of constant heat input per unit length (q, in W/m) is embedded in an infinite, homogeneous medium, the

solution to the heat conduction equation gives the temperature rise as a function of radial distance (r) and time (t) (Carslaw & Jaeger, 1959) by,

$$T(r,t) - T_0 = \frac{q}{4\pi\lambda}\int_{t_0}^{t} \exp\left(-\frac{r^2}{4\alpha t}\right) * \frac{1}{t} dt \quad (9)$$

Here, t_0 is the correction factor for heat source dimensions and contact resistance, and r is t radial distance from the heat source (m). At distances close to the heat source ($r \to 0$), the exponential term diminishes, and the integral simplifies to a logarithmic relationship.

$$T - T_0 = \frac{q}{4\pi\lambda} ln(t + t_0) + d \quad (10)$$

Constant d accounts for the finite dimensions of the heat source and experimental inconsistencies (De Vries & Peck, 1952; Reece, 1996). For practical applications where $t_0 \ll t$, the logarithmic term simplifies to $\ln(t)$, yielding a linear relationship. This approximation forms the basis of the line heat source method, where the slope of the temperature (T) versus the natural logarithm of time (ln(t)) can be used to calculate thermal conductivity. Thermal conductivity is determined by analyzing temperature changes over two-time intervals,

$$T(t_2) - T(t_1) = \frac{q}{4\pi\lambda} ln(t_2) - ln(t_1) \quad (11)$$

$$\lambda = \frac{q}{4\pi} * \frac{ln(t_2) - ln(t_1)}{T(t_2) - T(t_1)} \quad (12)$$

The temperature distribution evolves differently during the cooling phase (after the heat input stops). Reece (1996) extended the line heat source method to include cooling behavior. The thermal conductivity during the cooling phase can be expressed as:

$$\lambda = \frac{q}{4\pi} * \frac{\ln\left(\frac{t_2 t_1}{t_1 - t_h}\right)}{T(t_2) - T(t_1)} \quad (13)$$

Here, t_h represents the duration of the heating before cooling begins. This approach captures heating and cooling dynamics, making the model more versatile. Several factors play an essential role in determining the accuracy of a heat conduction model. One of the key elements is thermal contact resistance, which occurs at the interface between the heat source and the medium. This resistance introduces an additional barrier to heat flow that must be carefully accounted for to ensure accurate predictions. Furthermore, the homogeneity and isotropy of the medium are essential considerations, as the assumption of uniform thermal properties may not hold in materials like soil, which often have

layered or heterogeneous structures. The variation in these properties can significantly affect the heat conduction process. In addition, the geometry and size of the sensor used to measure thermal conductivity play a vital role in influencing the heat flow pattern, as different sensor designs can lead to variations in how heat is distributed within the medium. Lastly, the finite size of the heat source necessitates the inclusion of empirical constants, such as d, in the analytical solution. These constants are vital for compensating for the inherent limitations of the model and ensuring that the results are as accurate as possible.

1.5 Empirical and Validated Models for Thermal Conductivity

Soil is a complex and heterogeneous material with varying proportions of solids, water, and air where heat transfer occurs. These phases interact differently under various environmental and loading conditions, resulting in complex thermal behavior. Thermal conductivity is a function of dry density, saturation, moisture content, mineralogy, temperature, particle size/arrangement, and the volumetric proportions of solid, water, and air phases. The interplay of these variables creates significant complexity, which is very challenging to measure accurately. Each factor uniquely contributes to heat transfer within the medium, requiring careful consideration in experimental and analytical approaches. In addition, this experimental process is often time-consuming, equipment-intensive, labor-demanding, and susceptible to errors. Consequently, there is a growing need for more straightforward yet reliable methods to estimate thermal conductivity based on easily accessible soil properties (Tarnawski *et al.*, 2009).

Over the years, research in soil thermal conductivity has provided valuable insights, particularly in linking moisture content, soil texture, and mineralogy with heat transfer properties. The significance of thermal conductivity in soil studies has been well-established in foundational research. Early contributions by Kersten (1949) highlighted the crucial role of soil thermal properties in heat transfer processes. Johansen's (1975) empirical models, which linked moisture content to thermal conductivity, remain central to the field, with subsequent studies, such as those by Farouki (1986) and Lu *et al.* (2014), furthering our understanding of the intricate relationships between soil characteristics and thermal behavior. Despite these advancements, challenges persist in obtaining accurate and reliable thermal conductivity measurements due to the heterogeneous nature of soils and the labor-intensive nature of current methods. This complexity underscores the need for more efficient and robust estimation techniques. Recent studies, such as those by Markert *et al.* (2017) and Lahouar and Jarray (2019), have integrated numerical modeling with empirical data to address these challenges. An extensive literature review investigated the available thermal conductivity models, explained in the

subsequent sections. These models developed to address the complex relationship between thermal conductivity and various soil properties were systematically categorized into three primary groups based on their underlying approaches and methodologies.

1.5.1 Theoretical and Semi-Theoretical Models

Soil thermal conductivity plays a vital role in controlling heat flow in soil and is essential for understanding various processes in agriculture, hydrology, and geotechnical engineering. Several semi-theoretical and empirical models have been proposed to estimate λ based on water content or the degree of saturation (S_r). Theoretical and semi-theoretical models are grounded in fundamental physical principles and mathematical formulations that describe heat transfer mechanisms in soils by incorporating soil composition, porosity, and mineralogical properties. Similarly, mixing models are based on combining the thermal properties of individual soil components, such as minerals, water, and air, using two main approaches. The first approach calculates the conductivity by averaging the thermal properties of the individual components of the soil, such as minerals, water, and air, based on their proportions. The second approach uses percolation theory to study how heat flows through the interconnected network of particles and pores in the soil, considering how these pathways affect the overall heat transfer, also known as Physically based models. Since physically based thermal conductivity predictions do not include empirical parameters, they are expected to apply to an extensive range of soils. Among all physically based models, De Vries (1963) introduced the foundational approach in soil thermal conductivity research, explained below.

1. de Vries (1963) model

De Vries (1963) treated soil as a heterogeneous mixture of ellipsoidal particles, with solid soil particles embedded within continuous air and/or water medium. This conceptualization accounts for the thermal interactions between different soil components. The thermal conductivity of the soil matrix is derived as a weighted average of the thermal conductivities of its elements, which include the solid phase (λ_s), water phase (λ_w), and air phase (λ_a) with their shape factors *(k)*, and corresponding volume fractions *(θ)*. The weighting factors are determined based on the volumetric fractions of each phase and their respective shape factors, which describe the ellipsoidal geometry of soil particles. This shape factor accounts for the influence of particle geometry on heat flow. The effective thermal conductivity (λ) of the soil where all soil grains are lumped into one solid phase can be expressed as,

$$\lambda = \frac{k_w\lambda_w\theta_w + k_s\lambda_s\theta_s + k_a\lambda_a\theta_a}{k_w\theta_w + k_s\theta_s + k_a\theta_a} \tag{14}$$

For dry conditions, $\theta_w = 0, \theta_a = n, k_w = 0, k_a = 1$

$$\lambda_{dry} = 1.25 * \frac{\lambda_a n + (1-n) * k_s \lambda_s}{n + (1-n) * k_s} \quad (15)$$

here,

$$k_s = \frac{2}{3} * \frac{\lambda_a}{\lambda_a + (\lambda_s - \lambda_a) * g_s} + \frac{1}{3} * \frac{\lambda_a}{\lambda_a + (\lambda_s - \lambda_a) * (1 - 2g_s)} \quad (16)$$

For saturated conditions, $\theta_w = n, \theta_a = 0, k_w = 1, k_a = 0$

$$\lambda_{sat} = \frac{\lambda_w + (1-n) * (k_s \lambda_s - \lambda_w)}{1 + (1-n) * (k_s - 1)} \quad (17)$$

$$k_s = \frac{2}{3} * \frac{\lambda_w}{\lambda_w + (\lambda_s - \lambda_w) * g_s} + \frac{1}{3} * \frac{\lambda_w}{\lambda_w + (\lambda_s - \lambda_w) * (1 - 2g_s)} \quad (18)$$

Although the De Vries model has been widely used and provides reasonable estimates of soil thermal conductivity, several limitations have been identified. One primary source of error arises from the assumption that soil particles are not in direct contact, which may not be accurate in compacted or cohesive soils. Farouki (1986) addressed this issue by suggesting a 25% increase in λ_{dry}. Additionally, the approximation method used by De Vries (1963) to estimate particle shape factors introduces further inaccuracies, as it oversimplifies the complex geometries of natural soil particles. Moreover, the model's performance decreases in extreme conditions, such as saturated soils or soils with very low moisture content, where additional factors (e.g., film water conductivity or heat flux through contact points) become significant. However, De Vries's (1963) model provides a robust framework widely adopted for estimating soil thermal conductivity due to its relatively simple formulation and reasonable accuracy under typical field conditions.

1.5.2 Linear and Non-Linear Regression Models

Regression-based linear and non-linear models rely on statistical relationships between thermal conductivity and measurable soil properties. These models are developed by fitting empirical data into mathematical equations, enabling them to capture complex, site-specific trends in thermal behavior. With the highest representation among the identified models, regression-based approaches demonstrate their versatility and effectiveness in predicting thermal conductivity under diverse conditions. The commonly used regression-based models mentioned in the scientific literature are discussed below.

1. Kersten (1949) Model

The Kersten (1949) model is one of the earliest empirical models developed for estimating the thermal conductivity of soils based on experimental results covering several samples of 19 different kinds of soil, ranging through crushed rock, pure sand (0%), nearly pure clay (98%) and peat. The experiments

were conducted on soil samples prepared under carefully controlled conditions of dry density and moisture content, where samples were placed within a cylindrical container designed to facilitate radial heat flow, ensuring precise and uniform thermal conductivity measurements. The model's foundation is rooted in experimental data, where thermal conductivity measurements were carried out for soils with varying moisture contents, densities, and compositions. Kersten's approach made it possible to relate thermal conductivity directly to measurable soil properties, making it highly suitable for engineering and field applications. A key feature of the Kersten model is its recognition of the dominant influence of soil moisture content on thermal conductivity. The model divides soil behavior into two regions: dry soils and moist to saturated soils. For dry soils, thermal conductivity (λ) is primarily influenced by the thermal properties of soil particles and the air-filled voids. Since air is a poor thermal conductor, the overall thermal conductivity in dry soils remains low. Conversely, as moisture content increases, water replaces air in the pores, significantly increasing thermal conductivity. Thermal conductivity reaches its maximum in saturated soils, governed by the combined effects of soil minerals and water.

The Kersten model provides empirical equations and fitting parameters tailored to specific soil types, including sands, silts, and clays. The line of division, in general, is based upon the silt and clay content: soils with 50 percent or more of silt and clay are in the fine textured group. The thermal conductivity also differs depending on whether the soil is frozen. In this model, In the work of Kersten 1949, the equations are given in imperial units. Farouki, 1981 has rewritten these with a conversion factor for metric units. The general mathematical expression for the thermal conductivity of moist soils in metric units for different soil for different conditions is given as:

For fine-grained soils, the thermal conductivity for w ≥ 1.7% is calculated as follows:

$$\lambda_{\text{eff}} = 0.144*(0.9 \log w - 0.2)*10^{0.624*\rho_d} \quad \text{[Unfrozen Soil]} \quad (19)$$

$$\lambda_{\text{eff}} = 0.144*(0.01*10^{1.373*\rho_d} + 0.085*10^{0.499*\rho_d}*w) \quad \text{[Frozen Soil]} \quad (20)$$

For granular soils of w ≥ 1%,

$$\lambda_{\text{eff}} = 0.144*(0.7 \log w + 0.4)*10^{0.624*\rho_d} \quad \text{[Unfrozen Soil]} \quad (21)$$

$$\lambda_{\text{eff}} = 0.144228*(0.076*10^{0.812*\rho_d} + 0.0032*10^{0.911*\rho_d}*w) \quad \text{[Frozen Soil]} \quad (22)$$

Here, w is the gravimetric water content (%), and ρ_d is dry bulk density (g/cm^3). Thermal conductivity is assumed to be 0.25 (W/m·K) when w <1.7% to avoid negative values of estimated λ_{eff} (Liu *et al.* 2021).

2. Donazzi *et al.* (1979)

Donazzi *et al.* (1979) proposed a model to estimate the thermal conductivity of various soils (λ_{eff}), including clays. The method utilizes an exponential relationship that considers the physical properties of the soil, such as the thermal conductivity of soil particles, porosity, and degree of saturation. The formula provided by Donazzi *et al.* (1979) is expressed as follows:

$$\lambda_{\mathrm{eff}} = 0.588 * \lambda_{\mathrm{s}}{}^{1-\mathrm{n}} * \exp\left[-3.08 * n * (1 - \mathrm{S_r})^2\right] \quad (23)$$

Here, λ_s represents the thermal conductivity of the solid soil particles. The parameter n corresponds to the soil porosity, representing the fraction of the volume occupied by voids. S_r indicates the degree of saturation, which is defined as the ratio of the volume of water to the volume of voids. The exponential term $exp[-3.08*n*(1-S_r)^2]$ serves as a correction factor, accounting for the complex interaction between porosity and saturation that influences the effective thermal conductivity. This model highlights the distinct contributions of the solid and liquid phases to the soil's thermal conductivity. This exponential factor further adjusts effective conductivity to reflect the nonlinear impact of unsaturated conditions, mainly when the degree of saturation is less than 100%.

3. Makowski and Mochlinski (1956) Model

Makowski and Mochlinski (1956) presented an empirical model for estimating the thermal conductivity of soils (λ_{eff}), extending the foundational concepts introduced by Kersten (1949). Their model emphasizes the effects of soil moisture content (w), bulk dry density (ρ_b), and clay content (f_{clay}) on thermal conductivity. Including clay content allows the model to adapt to varying soil types, particularly those with different proportions of fine-grained particles. Thermal conductivity is expressed as:

$$\lambda_{eff} = [a*log_{10}*w + b]*10^{0.624*\rho_d} \quad (24)$$

where *a* and *b* are related to the percentage of clay content (f_{clay}),

$$a = 0.1424 - 4.65 \times 10^{-4}*f_{clay}, \quad (25)$$

$$b = 0.0419 - 3.13 \times 10^{-4}*f_{clay}, \quad (26)$$

Thermal conductivity is assumed as 0.25 (W/m·K) when w <1.7% to avoid negative values of estimated λ_{eff} (Liu *et al.*, 2021). The model highlights the strong dependence of soil thermal conductivity on the logarithm of moisture content, reflecting the increased conductivity as water replaces air in soil pores. Additionally, the model incorporates dry density, acknowledging its role in reducing porosity and enhancing heat conduction pathways.

4. Kahr and Müller-Vonmoos (1982) model

Kahr and Müller-Vonmoos (1982) proposed an empirical model to estimate the thermal conductivity (λ_{eff}) of specific bentonite clays, namely Mx-80 Na-bentonite and Montigel Ca-bentonite. These clays are known for their unique properties, such as high swelling capacity and significant water retention, which influence their thermal behavior. The model considers both the bulk dry density (ρ_b) and the moisture content (w) of the soil as the primary influencing factors and is expressed as:

$$\lambda_{eff} = -0.56+0.6*\rho_b + 0.004*w * {\rho_b}^3 \quad (27)$$

Here, ρ_b is the bulk dry density of the soil (g/cm³), and w is the moisture content (%). The model demonstrates a linear relationship between thermal conductivity and dry density, with an additional cubic term accounting for the non-linear effects of high densities and water content. The negative constant (−0.56) reflects the low thermal conductivity of bentonite at low densities, where air-filled pores dominate heat transfer. Including moisture content highlights its significant role in enhancing thermal conductivity, as the presence of water reduces air-filled porosity and increases heat conduction through enhanced contact between soil particles. The cubic term accounts for the compounding effects of increasing density and moisture, leading to a rapid rise in thermal conductivity at higher values of these parameters. This model is particularly relevant for applications involving bentonite, such as engineered barriers in nuclear waste disposal and other geotechnical and environmental projects where heat transfer properties are critical. While the model provides a simple and practical means of estimating thermal conductivity, it is limited to bentonite clays. It does not account for factors like mineral composition, temperature variations, or saturation levels, which may also influence thermal conductivity.

5. Brakelmann (1984) model

Brakelmann's (1984) model, developed based on experimental data, provides a method for estimating the thermal conductivity of soils. This model accounts for the influence of porosity, saturation, and the solid components of the soil. His method has been used successfully for the planning of buried power cables. The thermal conductivity is expressed using the following equation:

$$\lambda_{eff} = 0.588^n * {\lambda_s}^{1-n} * exp\,[-3.08*n*(1-S_r)^2] \quad (28)$$

The exponential term in the equation accounts for the reduction in thermal conductivity due to unsaturation, which results in air-filled pore spaces. This relationship highlights the complex interaction between soil particles, water, and air in determining the overall thermal behavior of soils (Brakelmann, 1984). For the determination of the solid thermal conductivity (λ_s), the model incorporates the approach

described by Wessolek *et al.* (1994), which contained a weighted average of the thermal conductivities of the individual soil components, namely sand, silt, and clay, as follows:

$$\lambda_s = 0.0812 * f_{sand} + 0.054 * f_{silt} + 0.02 * f_{clay} \quad (29)$$

Here, f_{sand}, $f_{silt,}$ and f_{clay} represent the respective percentages of sand and clay in the soil composition.

6. Campbell (1985) model

The Campbell (1985) model provides an empirical approach for estimating soil's effective thermal conductivity (λ_{eff}). This model was derived from experimental data on five field soils from Washington state, USA, and widely used to estimate the effective thermal conductivity (λ_{eff}) of soils under varying moisture and composition conditions. This model is based on the premise that soil thermal conductivity depends significantly on its volumetric water content (θ_v) and the composition of its solid components, such as quartz, clay, and other minerals. The general form of the model is expressed as:

$$\lambda_{eff} = A + B*\theta_v - (A - D)\ exp\ [- (C * \theta v)^4] \quad (30)$$

A, B, C, and D are empirical constants, and θ_v represents the volumetric water content. Campbell (1985) showed that these parameters depend on the soil's composition, including minerals, organic matter, and clay content. Tarnawski and Leong (2012) further provided equations for calculating these parameters regarding soil properties, such as the volume fractions of quartz and other solid components.

$$A = \frac{0.57 + +1.73 * f_{quartz} + 0.93 * f_{others}}{1 - 0.74 * f_{quartz} - 0.49 * f_{others}} - 0.28 * f_s * (1 - f_s) \quad (31)$$

$$B = 1.06 * f_s,\ C = 1 + \frac{2.6}{\sqrt{f_{clay}}},\ D = 0.03 + 0.7 * f_s^2 \quad (32)$$

For soils with negligible quartz content and a typical particle density (ρ_s=2.65 g/cm^3), the parameters simplify to specific values: A=0.65−0.78ρ_b+0.60ρ_b^2, B = 1.06*ρ_b, and D=0.03+0.1ρ_b^2. This model has been widely used in numerical simulations for predicting soil thermal properties in environmental models, including soil–vegetation–atmosphere schemes (Haverd & Cuntz, 2010; Ross, 2014; Steele-Dunne *et al.*, 2010).

7. Chung and Horton (1987) model

Chung and Horton's (1987) model provides an empirical approach to estimating the effective thermal conductivity (λ_{eff}) of soils by considering only the volumetric water content (θ_v) using the following equation:

$$\lambda_{eff} = a + b * \theta_v + c * \sqrt{\theta_V} \quad (33)$$

Where a, b, and c are empirical parameters specific to soil characteristics. These parameters are defined for clay soils as a = −0.197, b = −0.962, and c = 2.521. Parameter a represents the baseline thermal conductivity of dry soil, b accounts for the linear effect of increasing water content, and c represents the nonlinear contribution due to water's higher thermal conductivity than air. Notably, the negative value of b indicates that air-filled pores can initially reduce thermal conductivity slightly at low moisture levels. The model also incorporates a safeguard for arid conditions (θ_v<0.04), where a constant thermal conductivity of 0.25 W /m.K is assumed to prevent negative values of λ_{eff}. This adjustment ensures realistic estimates under arid conditions. The Chung and Horton model has been incorporated into hydrological and heat transfer models, such as the Hydrological Soil Plant Atmosphere Continuum model (Yang, 2015) and the HYDRUS software (Simunek *et al.*, 1997), making it a valuable tool for simulating heat and moisture transfer processes in the soil-plant-atmosphere system. Overall, the model effectively captures the nonlinear relationship between soil thermal conductivity and moisture content, offering reliable predictions across various moisture conditions (Chung & Horton, 1987).

8. Singh and Devid (2000) model

Singh and Devid's (2000) model provides an empirical framework for predicting the effective thermal conductivity of soil (λ_{eff}) based on soil bulk density (ρ_b) and gravimetric water content (w). The model divides the prediction into four cases depending on the soil's water content level. For soils with very low water content (w<2%), thermal conductivity is given as

$$\lambda_{eff} = a * 10^{0.6243 * \rho_b{}^{-3}} \quad (34)$$

Where a=0.219 for clay soils. This case reflects dry soil conditions where the solid particles of the soil dominate thermal conductivity. For intermediate water content (2% ≤ w ≤ 5%), the thermal conductivity is expressed as

$$\lambda_{eff} = b * 10^{0.6243 * \rho_b{}^{-3}} \quad (35)$$

Here, b=0.243 for 2% ≤w< 4% and b=0.276 for 4% ≤w≤ 5%. At this stage, some water in the soil enhances heat transfer due to water's higher thermal conductivity than air. For soils with high water content (w >5%), the model incorporates a logarithmic relationship to account for the dominant effect of water. The thermal conductivity is expressed as

$$\lambda_{eff} = (1.07\, log\, w + c) * 10^{0.6243 * \rho_b{}^{-3}} \quad (36)$$

Here, c=−0.73 for clay soils. This case represents a condition where water significantly contributes to heat transfer within the soil matrix. The model parameters depend on the soil texture, with the values explicitly provided for clay soils. Across all cases, bulk density (ρ_b) plays a crucial role in thermal conductivity, as denser soils have reduced air voids, allowing for more efficient heat conduction, which is captured by this factor $10^{0.6243*\rho_b{}^{-3}}$. This model effectively highlights the interplay between soil density, water content, and texture in determining thermal conductivity. It helps us understand soil thermal behavior in various conditions (Singh & Devid, 2000).

9. Caridad *et al.* (2014) Model

Caridad *et al.* (2014) introduced an empirical model for estimating the effective thermal conductivity (λ_{eff}) of seven clays in Spain. The model expresses the relationship between thermal conductivity and bulk density (ρ_b) of the clay samples through the linear equation:

$$\lambda_{eff} = 0.5873 * \rho_b - 0.0036 \tag{37}$$

This equation suggests that thermal conductivity increases linearly with increased bulk density. The constant term (−0.0036) accounts for baseline thermal conductivity behavior, while the coefficient 0.5873 represents the rate at which thermal conductivity changes per unit increase in bulk density. This model offers a simple yet effective approach to predicting the thermal performance of clay soils based on their density.

10. Lu *et al.* (2014) Model

Lu *et al.* (2014) model provides a predictive approach for estimating soil thermal conductivity (λ_{eff}) as a function of soil water content (w) based on experimental data collected from seventeen mineral soils with varying properties. These soils cover the full range of USDA soil texture classes, with bulk densities ranging from 1.06 to 1.6 g/cm³ and organic matter content from 0.09% to 3.2%. The model relies on a comprehensive equation incorporating soil porosity, bulk density, and texture to determine the thermal conductivity under different moisture conditions. The equation developed thermal conductivity using a nonlinear optimization procedure (Wolfram, 2003), written below.

$$\lambda = exp\,[\beta - w^{-\delta}] + \lambda_{dry} \tag{38}$$

Where λ_{dry} represents the thermal conductivity of dry soil. Thus, it does not influence the shape of the $\lambda(w)$ curve. The linear equation of Lu *et al.* (2007) can be applied to estimate λ_{dry} From the soil porosity, n. The parameters δ, $and\ \beta$, which influence the water content dependence of thermal

conductivity, are linked to the clay fraction (f_{clay}) and sand fraction (f_{sand}) respectively, which can be expressed.

$$\lambda_{dry} = 0.51 - 0.56 * n \tag{39}$$

$$\delta = 0.67 * f_{clay} + 0.24 \tag{40}$$

$$\beta = 1.97 * f_{sand} + 1.87 * \rho_b - 1.36 * \rho_b * f_{sand} - 0.95 \tag{41}$$

This experimental process accounted for soil structure, porosity, and texture variations, ensuring that the model accurately represents real-world conditions. The fixed parameter values in the model were calibrated to apply across a wide range of soil types, including sand, silt, and clay. Bertermann *et al.* (2018) later validated the model's applicability and used it to compare measured and calculated thermal conductivities within different soil grain size classes. The results demonstrated the model's effectiveness in predicting thermal conductivity based on soil properties and water content.

In their work, Donazzi *et al.* (1979) suggested a thermal conductivity value of 2.3 W/m·K for soil particles (λ_s) in clays, as referenced in subsequent studies such as Liu *et al.*. This value serves as a standard assumption for clays without specific experimental measures. Overall, this model is a valuable tool for predicting soil thermal behavior under varying environmental conditions, and it has applications in fields such as geotechnical engineering, energy storage systems, and soil science.

11. Yoon *et al.* (2018) Model

Yoon *et al.* (2018) proposed a regression model to estimate Korea's effective thermal conductivity (λ_{eff}). The model incorporates bulk density (ρ_b) and degree of saturation (S_r) as key thermal conductivity parameters. The model is expressed as the following equation.

$$\lambda_{eff} = 0.641 * \rho_b + 0.641 * S_r - 0.51 \tag{42}$$

This model indicates that higher bulk density and greater saturation levels increase thermal conductivity. The experiment's coefficient of regression suggests a reasonably strong fit with the observed data. Including bulk density and degree of saturation highlights the complex interplay of soil properties in determining thermal conductivity, making the model applicable to various clayed soil types under different conditions.

1.5.3 Normalized Models

Normalized models leverage dimensionless scaling techniques to correlate soil thermal conductivity with key influencing parameters, such as water content, density, and temperature. By normalizing

variables, these models simplify the complex interactions within the soil, improving their applicability across various soil types and conditions. The normalized approach offers an efficient and practical means of estimating thermal conductivity, making it a popular choice for field applications and engineering design. Some commonly used normalized models are discussed below.

1. Johansen (1975) model

The Johansen (1975) model, widely recognized for its simplicity and accuracy, provides an effective method for estimating the thermal conductivity of unsaturated soils. While Kersten (1949) did not test soils in the saturated state, he extrapolated his data to full saturation. Johansen (1975) analyzed Kersten's data and developed equations by introducing a normalized approach. This model simplifies the complex nature of soil heat transfer by relating effective thermal conductivity to the degree of soil saturation and its constituent materials expressed as,

$$\lambda_{eff} = K_e * \left(\lambda_{sat} - \lambda_{dry}\right) + S_r \tag{43}$$

Here, K_e represents the Kersten number, a dimensionless factor dependent on soil saturation (S_r), calculated as the ratio of volumetric water content (θ_v) to soil porosity (n). This K_e is dependent on the thermal condition of the soil. In saturated soils, the thermal conductivity of the frozen soil is significantly higher than that of the soil in its unfrozen state because the thermal conductivity of ice is approximately four times greater than water's (Farouki, 1981). Kersten's number is calculated based on clay percentage and state of saturation. If f_{clay} is above 5%, the soil is called fine-grained soil; otherwise, coarse soil. For unfrozen fine soils having $S_r > 0.1$,

$$K_e = log_{10}S_r + 1 \tag{44}$$

and unfrozen coarse soils having $S_r > 0.05$,

$$K_e = 0.7 * log_{10}S_r + 1 \tag{45}$$

In this study, $K_e = 0$ was assumed when $S_r < 0.05$ for coarse soil and $S_r < 0.1$ for fine soil to address numerical stability issues. This approach prevents computational errors associated with the logarithmic function when calculating K_e. Such a simplification ensures the model remains robust even under extremely low soil moisture conditions. The thermal conductivity of dry soil (λ_{dry}) is derived from the de Vriess (1963) by using λ_s =3.0 W/m·K, ρ_s= 2.70 gm/cm^3, k_s=0.053, and λ_a = 0.053 W/m·K

$$\lambda_{dry} = \frac{(0.135\rho_b + 0.0647)}{2.7 - 0.947\rho_b} \tag{46}$$

The thermal conductivity of saturated soil (λ_{sat}) is determined by the mass-weighted average of the thermal conductivities of quartz and other minerals present in the soil as the thermal conductivity of unfrozen soils at full saturation (λ_{sat}) is calculated using the following expression:

$$\lambda_{sat} = {\lambda_w}^{n} * {\lambda_s}^{1-n} \quad (47)$$

Here, λ_w=0.57 W/m·K is the thermal conductivity of water, and λ_s is the thermal conductivity of solid soil components. The thermal conductivity of the solid phase is approximated using a weighted average of the thermal conductivities of quartz (λ_q) and other soil minerals (λ_{other}), where q is the volume fraction of quartz, which is a critical factor because quartz has significantly higher thermal conductivity (7.7 W/m·K). In comparison, the other solids are set to 2 W/m·K. If the quartz content is less than 20% by volume, the thermal conductivity of the different soil minerals is adjusted to 3.0 W/m·K. Many studies have widely used and assessed this model (Chen, 2008). Although the application of the Johansen model has yielded various conclusions, all studies consistently indicate that this model is susceptible to quartz content due to the significantly higher thermal conductivity of quartz compared to other soil minerals. Typically, quartz content is determined using techniques such as X-ray fluorescence (XRF) and X-ray diffraction (XRD); however, these measurements are not routinely performed (Tarnawski *et al.*, 2012). As a result, researchers often estimate quartz content by assuming it is equivalent to the sand content (Lu *et al.*, 2007) or 50% of the sand content rather than conducting direct measurements (He et al., 2019; Luo *et al.*, 2009). According to Hu *et al.* (2017), the volume fraction of quartz in the soil can be estimated as a function of the sand fraction, represented by the following relationship:

$$f_q = 0.5 * f_{sand} \quad (48)$$

This approach reflects the significant influence of quartz content and soil composition on the thermal conductivity of the soil, which is flexible in handling both dry and saturated conditions and makes it applicable across a wide range of soil types and environmental settings. Furthermore, considering quartz content as a significant factor in thermal conductivity enhances its accuracy, given quartz's superior thermal conductivity relative to other soil minerals.

2. Farouki (1981) Model

This model is based on the modification of the normalized parameter of Kersten Number (K_e) of Johansen's (1975) model, which provides a comprehensive framework for estimating the effective thermal conductivity (λ_{eff}) of soils under varying conditions of saturation, temperature, and soil composition. The model incorporates the influence of water content through the degree of saturation (S_r) and distinguishes between saturated and dry soil states. For soils with a degree of saturation more

significant than 10^{-7}, the effective thermal conductivity is calculated as a weighted sum of the saturated thermal conductivity (λ_{sat}) and the dry thermal conductivity (λ_{dry}), modified by a scaling factor called the Kersten number (K_e). In contrast, when the degree of saturation is extremely low ($S_r \leq 10^{-7}$), the thermal conductivity of the soil is approximated as (λ_{dry}), representing an arid state expressed as,

$$\lambda_{eff} = \begin{cases} K_e*(\lambda_{sat}-\lambda_{dry}) + \lambda_{dry}, & S_r > 10^{-7} \\ \lambda_{dry}, & S_r \leq 10^{-7} \end{cases} \quad (49)$$

The Kersten number (K_e) is a key parameter in the model that accounts for the effect of saturation and temperature on the soil's thermal properties. When the soil temperature (T) is above the freezing point (T_f), K_e is calculated using a logarithmic relationship with the degree of saturation, defined as

$$K_e = log_{10}(S_r) + 1 \quad (50)$$

Below the freezing point ($T < T_f$), the Kersten number is directly proportional to the degree of saturation ($K_e=S_r$), reflecting the reduced mobility of water in frozen conditions. This model considers the thermal conductivity of the solid phase (λ_s), which depends on the proportions of sand and clay in the soil determined using a weighted average of the thermal conductivities of sand and clay.

$$\lambda_s = \frac{8.8*f_{sand}+2.92*f_{clay}}{f_{sand}+f_{clay}} \quad (51)$$

Here, f_{sand} and f_{clay} represent the respective fractions of sand and clay in the soil composition. This model is adaptable and widely applicable in geotechnical and thermal modeling studies. It integrates the effects of soil texture, water content, and temperature to estimate soil thermal conductivity under diverse environmental conditions.

3. Hu *et al.* (2001) Model

The model proposed by Hu *et al.* (2001) resembles the estimation of soil thermal conductivity by modifying the Johansen model. It introduces adjustments to key parameters, including the Kersten coefficient, and provides specific constants to enhance the accuracy of predictions. Hu *et al.* (2001) introduced a modified Kersten coefficient (K_e) that is expressed as a function of the degree of saturation (S_r) to account for the effect of soil moisture. This modification enhances the representation of water's influence on soil thermal conductivity.

$$K_e = 0.9878 + 0.1811\, ln(S_r) \quad (52)$$

Effective thermal conductivity (λ_{eff}) is subsequently calculated by combining saturated and dry thermal conductivity, weighted appropriately by the Kersten coefficient. This model incorporates slightly changed constants for the thermal conductivities of quartz, water, and air, ensuring its applicability across a diverse range of soil types and conditions. Here, λ_w=0.6 W/m·K is the thermal conductivity of water, λ_s=3.35 W/m·K is the thermal conductivity of solid components of the soil, and λ_{air}=0.0246 W/m·K is the thermal conductivity of air.

4. Côté and Konrad (2005) model

Côté and Konrad's (2005) model provides an approach for estimating the thermal conductivity of soils, both in saturated and dry states. This method relies on the earlier Johansen (1975) model, incorporating improvements in parameter estimation and empirical relationships. According to this model, the saturated thermal conductivity λ_{sat} is determined using Johansen's equations while assuming a thermal conductivity value of 0.6 (W/m·K) for water. Instead of directly calculating the thermal conductivity of solid particles, the model uses predefined values for different soil and rock materials, which are presented in Table 1.

Table 1: Average values for the thermal conductivity of solid particles (λ_s) (Côté and Konrad, 2005)

Material	λ_s [W/(m·K)]
Basalt	1.70
Granite	2.50
Limestone	2.50
Quartzite	5.00
Sandstone	3.00
Coal	0.26
Peat	0.25
Silt and Clay	2.90

A soil texture-dependent parameter κ is used to compute the practical degree of saturation Kefor unsaturated soils. The following equation gives the relationship:

$$K_e = \frac{\kappa * S_r}{1 + (\kappa - 1) * S_r} \quad (53)$$

Where S_r denotes the degree of saturation, and κ is determined based on the soil type, as shown in Table 2. The model also includes a formulation for estimating the thermal conductivity of dry soils (λ_{dry}) consisting of two parameters, χ, and η, whose values depend on the soil type. The equation for dry soil thermal conductivity is expressed as:

$$\lambda_{dry} = \chi * 10^{-\eta * n} \tag{54}$$

Here, χ and η are constants specific to different soil categories. Côté and Konrad's (2005) approach offers a robust and practical methodology for predicting the thermal conductivity of soils under varying moisture conditions, which is helpful in geotechnical engineering applications, where accurate thermal conductivity estimates are critical for the design of ground heat exchangers, buried pipelines, and other subsurface structures.

Table 2: ***Values for the parameter κ (Côté and Konrad, 2005)***

Material	κ
Gravels and coarse sands	4.60
Medium and fine sands	3.55
Silty and clayey soils	1.90
Organic fibrous soils (peat)	0.60

Table 3: Values for the parameters* χ and η *(Côté and Konrad, 2005)

Material	χ	η
Crushed rocks and gravel	1.70	1.80
Natural mineral soils	0.75	1.20
Organic fibrous soils (peat)	0.30	0.87

5. Yang *et al.* (2005)

Yang *et al.* (2005) model is fundamentally based on the Johansen (1975) model, which estimates the thermal conductivity (λ_{eff}) of soils using a weighted interpolation between the dry and saturated thermal conductivity values. While most of the framework of the Johansen model remains unchanged, Yang et al. introduced modifications to the formulation of the Kersten number (K_e) as given below.

$$K_e = exp[\, k_T(1 - \frac{1}{S_r})] \tag{55}$$

Here, k_T is an empirical coefficient set to 0.36 in the Yang *et al.* (2005) model. Yang *et al.* adopted a slightly modified equation for (λ_{dry}) initially proposed by Peters-Lidard *et al.* (1998).

$$\lambda_{dry} = \frac{(0.135\rho_b + 0.0647)}{2.7 - 0.947\rho_b} \tag{56}$$

This model offers an improved fit for the revised equation of K_e, incorporating an exponential relationship that better captures the thermal conductivity behavior across different saturation levels.

The enhancement yields greater accuracy, particularly for soils with intermediate degrees of saturation under experimental conditions with varying moisture levels. However, the model is not applicable in scenarios where S_r=0.

6. Markle *et al.* (2006)

Johansen's (1975) original formulation for K_e applies for saturation levels (S_r) above 0.1 for coarse soils but tends to underestimate thermal conductivity at lower saturation levels. To address this, Markle (2006) adopts an alternative formulation for K_e , based on Ewen and Thomas (1987), expressed as:

$$K_e = 1 - exp\,(-\zeta S_r) \tag{57}$$

Here, $\zeta = 8.9$ is a fitting parameter. This formulation provides a continuous relationship across the full range of saturation levels. However, discrepancies arise at full saturation when ζ > 4.5. To resolve this, the model modifies the equation for soil thermal conductivity for a full range of saturations, such as,

$$\lambda_{eff} = K_e * \left(\lambda_{sat} - \lambda_{dry}\right) + S_r * (\lambda_{sat} - \lambda_{Sr=1}) \tag{58}$$

Where $\lambda_{Sr=1}$ is evaluated using the unmodified Johansen equation.

1.6 Summary of Literature

This study builds on these efforts, aiming to refine and enhance our understanding and modeling of soil thermal conductivity, which is vital for a range of geotechnical and environmental engineering applications, including the design of underground energy systems, geothermal heat exchangers, and the analysis of heat flow in soil structures. Over the years, numerous empirical models have been proposed to capture the complex thermal behavior of different soil types. These models vary in their assumptions, required input parameters, and the degree to which they account for water content, temperature, and soil composition. As a result, each model carries distinct advantages and limitations that influence its suitability for specific conditions or project objectives.

Table 4 below compares widely recognized soil thermal conductivity models, offering insights into their fundamental assumptions, parameter requirements, and the contexts in which each model excels. This comprehensive overview not only highlights the strengths and drawbacks of each model but also underscores the importance of carefully selecting a model that will yield reliable and accurate predictions, ultimately enhancing the efficiency and safety of geotechnical and environmental projects.

Table 4: Comparative Overview of Different Soil Thermal Conductivity Models

Model Name	Advantages	Assumptions/Limitations
Kersten (1949)	• It approximates well, with most data points aligning well with measured values. • Only two parameters are required: Bulk density and Gravimetric water content. • Well-suited for soils categorized into two groups (frozen and unfrozen soils) based on sand content (high or low), without needing exact grain size distribution. • The model closely aligns with experimental data in many cases, making it a reliable first-order approximation for soil thermal conductivity.	• Thermal conductivity is assumed to be 0.25 W/m·K when the moisture content is less than 1.7% to avoid negative values. • Underestimates thermal conductivity in soils with a moisture content between 1.7% and 2.0% for denser soils. • It does not fully account for soil mineralogy, structure, and moisture distribution variations, limiting its accuracy in soils with complex compositions.
Donazzi *et al.*, 1979	• Captures nonlinear effects of saturation and porosity using key parameters like porosity and degree of saturation. • Provides strong correlation with measured values. • It applies to various soil types but assumes a consistent influence of porosity and saturation on thermal conductivity.	• The thermal conductivity of soil solid is assumed to be 2.3 W/m·K. • The model may not account for all factors influencing thermal conductivity, such as soil structure or mineral content.
Kahr and Müller-Vonmoos (1982)	• Effectively captures the influence of bulk dry density (ρ_b) and moisture content (w), especially in compacted and saturated conditions. • Highlights the low thermal conductivity of bentonite at low densities by reflecting the dominance of air-filled pores.	• Specific to bentonite clays, limiting its applicability to other soil types. • Does not fully capture the complex interactions of saturation or mineralogical variations in different soils.
Campbell (1985)	• Provides an empirical approach for estimating thermal conductivity (λ_{eff}) based on volumetric water content (θ_v) and soil composition (quartz and clay fractions). • Adaptable for various soil types and can be incorporated into environmental modeling simulations.	• Assumes a linear relationship between thermal conductivity and volumetric water content. • The exponential correction factor may not fully account for the complex interactions between moisture content and soil structure in all cases.
Chung and Horton (1987)	• Assuming a simplified approach, only volumetric water content is required. • Suitable for soils with varying moisture content and widely used in hydrological and heat transfer models like HYDRUS.	• Assumes a constant λ_{eff} of 0.25 W/m·K in dry conditions (θ_v<0.04), even though this may not be accurate for highly desiccated soils. • It tends to underestimate λeff at higher moisture content values, likely due to the model's simplicity.
Caridad *et al.* (2014)	• A simple linear model is based on bulk density, assuming a proportional relationship with thermal conductivity.	• Fails to capture variations in thermal conductivity due to moisture content or mineral composition. • It predicts TC through linear equations, although this dependence is nonlinear.
Lu *et al.* (2014)	• Based on water content (w), bulk density, porosity, and texture, incorporating key soil properties. • Strong alignment with measured data, with most data points falling within the ±10% range, indicating high predictive accuracy. • Effectively captures the nonlinear relationship between thermal conductivity and moisture content. • Calibration across different soil types improves generalization, making the model applicable to various scenarios.	• Minor deviations were observed, particularly in lower conductivity values, where the model slightly underestimates λ_{eff}. • It may require adjustments for highly organic soils or extreme moisture conditions.
Yoon *et al.* (2018)	• It uses bulk density and degree of saturation to estimate soil thermal conductivity, capturing their combined influence on heat transfer.	• Mainly used to determine the TC for various bentonite-based soil. It tends to overestimate λeff at higher conductivity values, particularly for specific soil types.

	• Strong correlation with measured values, indicating good predictive accuracy. • It effectively accounts for variations in density and moisture, making it suitable for a range of bentonite-based soils under different saturation conditions.	• It relies on regression parameters derived from specific soil types, which may limit its applicability to soils with different compositions or extreme conditions.
Johansen (1975)	• Widely used for estimating soil thermal conductivity by incorporating soil saturation and quartz content. • Strong correlation with measured data, indicating high predictive accuracy. • Effectively handles dry and saturated conditions, making it applicable to various soil types. • Considers quartz content, which enhances accuracy due to the higher thermal conductivity of quartz compared to other soil minerals.	• The Kersten number is set to zero at lower saturation levels for numerical stability. • Sensitivity to the assumed quartz content, often estimated rather than directly measured, potentially introduces errors. • It does not explicitly account for organic matter content, which can affect heat transfer in certain soil types.
Farouki (1981)	• Modifies the Kersten number (Ke) normalized parameter to account for temperature effects on soil thermal properties using quartz using sand and clay percentages. • Incorporates soil composition, water content, and temperature for improved adaptability across diverse environmental conditions.	• The Kersten number is set to zero at lower saturation levels for numerical stability. • For very low saturation ($S_r \leq 10^{-7}$), it approximates soil's thermal conductivity as λ_{dry}, representing an arid state. • It tends to overestimate λ_{eff}, particularly for fine-textured soils with lower measured conductivity.
Hu *et al.* (2001)	• Refines the Kersten coefficient (K_e) to improve the model's representation of moisture effects on thermal conductivity (λ_{eff}). • Enhanced applicability to a broader range of soils, mainly due to its improved handling of moisture content. • Refined quartz, water, and air parameters improve the model's generalization.	• The thermal conductivity of soil is assumed to be 3.35 W/m·K. • Inconsistent predictions across different soil types and conditions suggest it may not fully account for soil texture and organic content variations. • It tends to overestimate λeff, particularly in high-moisture conditions.
Côté and Konrad (2005)	• Refines Johansen's approach by incorporating predefined values for solid particle thermal conductivity and a soil texture-dependent parameter (κ). • Provides accurate predictions across a range of soil types and moisture conditions. • Adaptable to various soil types, especially using predefined thermal conductivity values for solid particles. • Useful for practical applications in geotechnical fields, such as ground heat exchangers and buried pipelines.	• Some data scattering, particularly at lower thermal conductivity values, where slight underestimations occur. • Minor adjustments may be necessary for soils with highly organic or extreme conditions. • It may not accurately capture soil moisture variations if they are more complex than the assumed empirical relationships.
Yang *et al.* (2005)	• Introduces an exponential formulation for the Kersten number (K_e), improving the estimation of soil thermal conductivity, especially for intermediate moisture conditions. • It provides a smoother transition between dry and saturated states, expanding applicability to a broader range of soil moisture conditions.	• Not suitable for completely dry soil ($S_r = 0$), limiting its use in highly arid conditions. • There are slight underestimations at lower conductivity values and minor overestimations at higher values.
Markle *et al.* (2006)	• Refines Johansen's (1975) approach by introducing an alternative formulation for the Kersten number (K_e) conductivity for Sr<0.1 based on Ewen and Thomas (1987). • The model includes a refined equation for soil thermal conductivity at full saturation, helping to reduce discrepancies in high-moisture conditions.	• The fixed value of $\zeta = 8.9$ may not apply to all soil types, leading to noticeable deviations, including underestimations at moderate saturation levels and overestimations at mid-to-high conductivity values. • Considerable scattering of data points limits the model's precision, particularly under certain soil moisture conditions.

1.7 Comparisons Among Models

Accurate prediction of soil thermal conductivity relies on the availability and quality of various input parameters. Different empirical models require combinations of fundamental and derived parameters such as gravimetric water content, bulk density, temperature, and porosity. It is crucial to align model selection with the data that can be reliably obtained. Table 5 below provides a comprehensive overview of these parameter requirements for commonly used empirical soil thermal conductivity (STC) models. By highlighting the specific inputs needed by each model, this comparative summary serves as a valuable resource for researchers and practitioners seeking to select the most appropriate model based on data availability and the specific conditions of their study.

Table 5: Basic Input and Derived Parameters Required for Calculating Thermal Conductivity for Each Method

Type	Model Name	Gravimetric Water Content	Bulk Density	Grain Size Distribution	Temperature	Volumetric Water Content	Porosity	Saturation
Linear and non-linear regression STC Schemes	Kersten (1949)	Yes	Yes	No	No	No	No	No
	Donazzi *et al.*, 1979	No	No	No	No	No	Yes	Yes
	Kahr and Müller-Vonmoos (1982)	Yes	Yes	No	No	No	No	No
	Campbell (1985)	No	Yes	Yes	No	Yes	No	No
	Chung and Horton (1987)	No	No	No	No	Yes	No	No
	Caridad *et al.* (2014)	No	Yes	No	No	No	No	No
	Lu *et al.* (2014)	Yes	Yes	Yes	No	No	Yes	No
	Yoon et al. (2018)	No	Yes	No	No	No	No	Yes
Normalized STC Schemes	Johansen (1975)	No	Yes	Yes	No	No	Yes	Yes
	Farouki (1981)	No	Yes	Yes	Yes	No	Yes	Yes
	Hu *et al.* (2001)	No	Yes	No	No	No	Yes	Yes
	Côté and Konrad (2005)	No	No	No	No	No	Yes	Yes
	Yang *et al.* (2005)	No	Yes	No	No	No	Yes	Yes
	Markle *et al.* (2006)	Yes	Yes	No	No	No	No	No

1.8 Objectives of the Study

The primary aim of this thesis is to evaluate and compare the effectiveness of various published models for estimating the thermal conductivity of soils in Southern Ontario based on geotechnical characterization and laboratory measurements. This study comprehensively explains how soil properties, such as texture and moisture content, influence thermal conductivity by analyzing thermal conductivity data from different soil types and locations. Furthermore, it aims to recommend the most effective models for predicting soil conductivity across varied environmental conditions. The specific objectives of this thesis are:

- To perform geotechnical characterization and measure the thermal conductivity of 20 soil samples collected from seven locations, focusing on key properties affecting thermal conductivity.
- To compare measured thermal conductivity with regression-based and normalized models from the literature, assessing their predictive accuracy across the collected soil samples.
- To conduct statistical analysis to identify the best-performing models and evaluate their ability to fit the measured data accurately.
- To evaluate the impact of soil texture on thermal conductivity, highlighting model performance under varying environmental conditions.

Overall, this study identifies the most reliable models for predicting soil thermal conductivity, balancing simplicity and accuracy based on statistical analysis and comparison with measured data. The findings enhance predictive accuracy and offer practical applications in environmental and geotechnical fields.

1.9 Organization of the Thesis

This thesis is organized into four chapters, each addressing a distinct aspect of soil thermal conductivity. Each chapter builds upon the previous one, representing a comprehensive understanding of the topic. The overview of the thesis structure is summarized below.

Chapter 01: This chapter introduces the study, providing background on soil thermal conductivity, factors affecting it, measurement techniques, and an overview of empirical models, followed by the study's objectives and organizations.

Chapter 02: This chapter details the soil sample collection, experimental design, and calibration procedures, including data processing and error analysis for measuring thermal conductivity.

Chapter 03: This chapter presents the experimental results, compares the empirical models, and conducts statistical analysis to determine the most reliable models for predicting soil thermal conductivity.

Chapter 04: This chapter summarizes the study's findings, recommends the best-performing models, and offers suggestions for future research in soil thermal conductivity estimation.

Chapter 2
Materials and Methods

2.1 Introduction

This chapter details the collection of soil samples from various locations in Southern Ontario and their geotechnical characterization. It also outlines the experimental procedures, including sample preparation, thermal conductivity measurement, and calibration methods to ensure accuracy. Additionally, the chapter discusses the data processing and error analysis techniques used to model performance analysis.

2.2 Geographic and Climatic Context in Ontario

A varied geological landscape and a temperate climate with distinct seasonal variations characterize Southern Ontario. The region experiences warm, humid summers and cold, snowy winters, with climatic factors significantly altering soil moisture regimes, freeze-thaw cycles, and biological activity within the soil matrix (Environment Canada, 2019). These environmental conditions contribute to the dynamic behavior of soil thermal properties over time. To represent that variation, 20 soil samples were collected from different locations in Southern Ontario. Five soil samples from Chatham were collected, with textures ranging from fine to silty grains. In Harriston, two samples mainly consisted of loamy sand, a mix of sand, silt, and a small amount of clay. Norwich provided four samples, identified as silty clay or silty clay loam, indicating a higher clay content mixed with silt. Woodstock contributed two fine-grained soil samples known for their small particle sizes. Scarborough added four samples with textures from sandy clay loam to clay loam, showing a balance of sand, silt, and clay. From Kitchener, two samples were collected, ranging from loam to clay loam, indicating soils rich in organic material and balanced textures, as shown in the United States Department of Agriculture (USDA) soil classification triangle. Lastly, a single sample from London was classified as clay loam, with a higher clay content, making it denser and less permeable. The details of sand silt and clay proportions are shown in Appendix B.

The selected sampling locations lie within a geologically diverse area where variations in soil formation processes, such as glacial deposition and fluvial sedimentation, have led to the development of heterogeneous soil profiles. This heterogeneity is significant for thermal conductivity studies because differences in particle size distribution, organic content, and moisture retention capacities directly affect the heat transfer properties of the soil (Lachenbruch, 1965). Therefore, these locations' climatic and

geographic context provides a robust framework for examining the interplay between environmental conditions and soil behavior.

2.3 Collection of Soil Samples

This study collected soil samples from seven distinct locations in southern Ontario: Chatham, Harriston, Woodstock, Norwich, Scarborough, Kitchener, and London. A total of 20 soil samples from those locations in south Ontario were tested for thermal conductivity (λ) in controlled laboratory settings. These tests help us understand how well different soils conduct heat, which is essential for various environmental and engineering applications. At each location, soil samples were extracted using either manual drilling or auger boring techniques, depending on the site conditions, as illustrated in Figure 2. The sampling depth extended to around 5 meters below ground level, following the removal of the upper 0.5 meters to eliminate the influence of surface contamination and recent anthropogenic disturbances. Almost 20 soil samples were collected from those locations. This approach ensures that the collected samples accurately represent the in-situ soil properties and allow for a comprehensive evaluation of both near-surface and deeper stratigraphic layers (Das, 2010).



(a) (b)

Figure 2*: Soil Sample Collection a. Drilling and b. Sample Sealing*

The primary aim of this sampling strategy was to obtain a diverse and representative dataset of soil profiles, which could be used to assess geotechnical and thermal properties across a range of conditions. The study accounts for spatial variability in soil composition, moisture content, and density by targeting multiple depths and locations. These factors influence the soils' thermal conductivity and overall stability, affecting their suitability for various engineering applications.

2.4 Geotechnical Properties

Understanding the collected soil samples' geotechnical properties is essential for structural and environmental applications. Standard laboratory tests were performed to determine key parameters such as grain size distribution, moisture content, and bulk density. These properties are essential for assessing the mechanical stability of soils and interpreting their thermal performance. For example, moisture content and bulk density are directly related to thermal conductivity, as water has a higher thermal conductivity than air, and denser soils tend to have better heat conduction capabilities (Campbell, 1985). The grain size distribution indices also provide insights into the soil's ability to retain moisture and its susceptibility to compaction under load. This detailed geotechnical characterization allows a better understanding of how intrinsic soil properties influence thermal behavior under various environmental conditions (Das, 2010), as shown in Appendix A.

Table 6 also presents key properties for evaluating thermal conductivity, including essential soil properties, moisture content, bulk density, grain size distribution, and porosity. These parameters are critical for comprehensively understanding and predicting soil thermal behavior. Furthermore, the table effectively illustrates the influence of soil structure on thermal performance under varying environmental conditions, thereby facilitating the calibration of empirical models. This calibration enhances the precision of heat transfer analyses and the overall assessment of soil thermal behavior, particularly in geothermal applications.

Table 6: Selected Properties of Soil used to evaluate the Soil Thermal Conductivity

Serial No	Sample ID	Location	Porosity	Void Ratio	Sand (%)	Silt (%)	Clay (%)	Soil Classification	ρ_{dry} (gm/cc)
01	CH_01	Chatham	0.317	0.464	43.56	32.84	23.60	Loam	1.861
02	CH_02		0.288	0.405	12.25	42.37	45.38	Silty Clay	1.739
03	CH_03		0.332	0.498	19.58	40.84	39.58	Silty Clay Loam	1.782
04	CH_04		0.312	0.453	38.24	36.16	25.60	Loam	1.814
05	CH_05		0.316	0.462	31.89	38.53	29.58	Clay Loam	1.790
06	HAR_1	Harriston	0.316	0.461	53.24	25.71	21.05	Sandy Clay Loam	1.799
07	HAR_2		0.324	0.479	58.69	27.06	14.25	Sandy Loam	1.795
08	WOD_01	Woodstock	0.545	1.197	25.36	20.96	53.68	Clay	1.196
09	WOD_02		0.484	0.936	35.17	19.47	45.36	Clay	1.371

10	NOR_1	Norwich	0.335	0.504	39.57	29.18	31.25	Clay Loam	1.747
11	NOR_2		0.410	0.694	11.14	45.71	43.15	Silty Clay	1.567
12	NOR_3		0.373	0.594	16.47	48.34	35.19	Silty Clay Loam	1.646
13	NOR_4		0.364	0.572	30.26	34.31	35.43	Clay Loam	1.667
14	SCAR_01	Scarborough	0.378	0.607	28.36	39.73	31.91	Clay Loam	1.652
15	SCAR_02		0.354	0.548	36.54	30.72	32.74	Clay Loam	1.698
16	SCAR_03		0.308	0.446	46.51	27.62	25.87	Sandy Clay Loam	1.815
17	SCAR_04		0.335	0.504	44.17	32.36	23.47	Loam	1.735
18	KIC_01	Kitchener	0.317	0.464	37.59	33.76	28.65	Clay Loam	1.796
19	KIC_02		0.307	0.444	44.62	36.62	18.76	Loam	1.839
20	LON_01	London	0.385	0.626	38.33	31.78	29.89	Clay Loam	1.658

2.5 Classification and Grouping

Once transported to the laboratory, the collected field samples, often containing large aggregates, were initially broken into smaller fragments and cleaned of stones and rocks. These smaller aggregates were further processed using a soil crusher and gently disaggregated to preserve their natural structure as much as possible to represent subsurface conditions (ASTM International, 2013) accurately. These smaller aggregates were air-dried and sieved. Then, particle-size distribution analysis is conducted to quantify the proportions of sand, silt, and clay in each sample by ASTM D2487. This analysis typically involves two methods. First, the sand fraction is determined through sieving, where dried soil is passed through a series of sieves with progressively smaller apertures. Second, the silt and clay fractions are measured using a hydrometer or sedimentation test, which analyzes the rate of particle settlement in a liquid suspension, correlating this behavior to particle size. Once the particle-size data are obtained, the results are used for soil classification and grouping using the USDA soil classification scheme to evaluate soil thermal conductivity effectively. The USDA classification employs a soil triangle to graphically represent the relative proportions of sand, silt, and clay. The sand, silt, and clay proportions are plotted on the USDA soil texture triangle. Each triangle vertex represents 100% of one particle-size fraction, and the lines within the triangle delineate specific textural classes such as sandy loam, silty clay, and clay loam. Following this procedure, the USDA soil texture triangle was prepared to illustrate

the textural classes of the collected samples. Figure 3 shows that the red data points represent each sample's specific combinations of sand, silt, and clay percentages, indicating their classifications according to the USDA system. The plotted points reveal the variability in soil texture across the sampled locations.

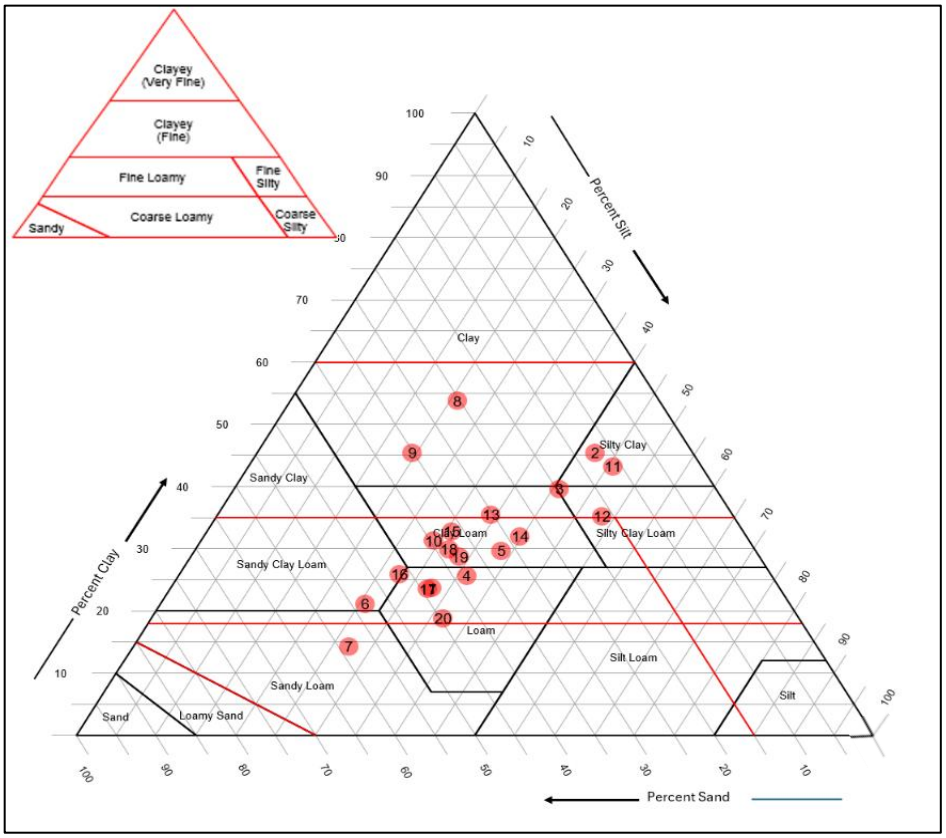


Figure 3: Textural Classification of soil according to the USDA system

This figure visually represents the soil classifications based on each sample's relative sand, silt, and clay proportions. The red circles within the triangle correspond to the specific soil samples collected and analyzed in this study. The diagram shows that most soil samples are clustered within the textural classes containing a higher proportion of fine sand. These classes include loam, sandy clay loam, and silty clay loam. The predominance of fine sand indicates that most collected soils exhibit textures conducive to moderate permeability and water retention, which are key factors in thermal conductivity studies. One distinct observation from the figure is that the soil sample collected from Harriston exhibits a comparatively higher sand content and plots closer to the sandy loam classification. This suggests that the Harriston soil has a coarser texture, which could result in relatively higher permeability and potentially lower water retention compared to the finer-textured soils observed in other locations. Such

variations are critical for understanding the influence of soil texture on thermal conductivity and other geotechnical behaviors.

Integrating the USDA soil classification system into this study offers a systematic framework for identifying site-specific engineering challenges and understanding soil performance under varying environmental conditions. This classification methodology aids in understanding both thermal conductivity and other geotechnical behavior by organizing soils into distinct groups based on their texture. This grouping of soils ensures that the relationship between soil texture and thermal conductivity is appropriately analyzed, as different soils exhibit varying thermal properties depending on their composition (Smith & Johnson, 2018). Soils rich in clay, for example, tend to have lower thermal conductivity than sandy soils due to differences in their respective particle sizes and moisture retention abilities. This study achieves analytical clarity by using the USDA soil classification system. It facilitates comparative research, enabling the identification of specific patterns in thermal conductivity across diverse soil types. Moreover, the grouping process helps recognize how these soil characteristics correlate with other physical and mechanical properties, including shear strength and compaction, offering insights into the broader implications for heat transfer efficiency in geothermal applications (Briaud & Smith, 2020). This approach enhances the accuracy of soil behavior assessments, ensuring practical and site-specific engineering designs.

2.6 Experimental Design and Procedure

2.6.1 Preparation of Soil Samples

The samples were first air-dried, then ground and sieved through a 2-mm mesh to ensure uniform particle size. Bulk density (ρ_b) was determined after oven-drying the samples at 105°C until a constant mass was achieved over 24 hours. For dry samples, the tapping method was employed, where the calculated amount of soil, based on its density and the volume of a cylindrical container (50.8 mm in diameter and 120 mm in height), was compacted by tapping the container's sides until the desired volume was reached. For samples with moderate moisture, the mixing method was used. Measured quantities of dry soil and distilled water were mixed thoroughly in a plastic bag and left to stand for 24 hours to ensure even moisture distribution, as shown in Figures 4 and 5. The mixture was then compacted in layers into the container using tapping and an acrylic rod to achieve the desired density. The degree of saturation was confirmed by weighing the sample from the sealed bags.



(a) Dying of Soil (b) 50% Saturated Soil (c) Hand mixing (100% Saturated)

Figure 4: Different Phases of Soil Sample Preparation

2.6.2 Saturation at Different Levels

The thermal conductivity of Each soil sample was measured at various degrees of saturation (S_r), representing different moisture levels from completely dry to fully saturated. The saturation values were 0, 0.1, 0.25, 0.5, 0.75, and 1.0 (Figure 5). This range allowed for a comprehensive assessment of how moisture content affects thermal conductivity. Each soil type was tested in triplicate under the same conditions to ensure data reliability.

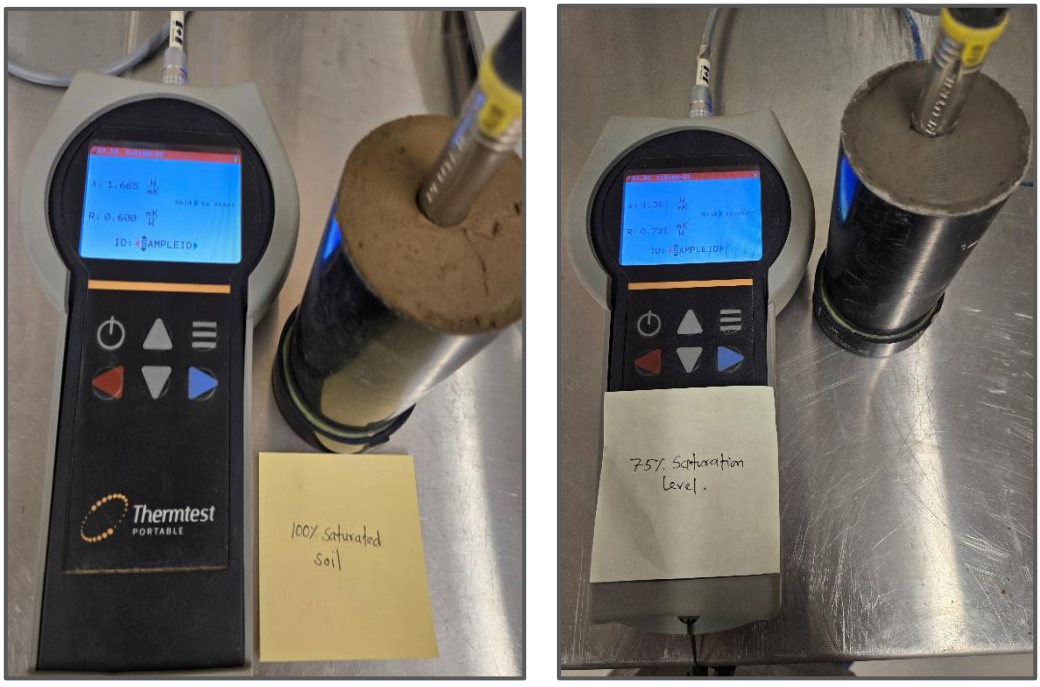


(a) 50% Saturated Soil (b) 75% Saturated Soil

Figure 5: Moisture Redistribution of Soil in Plastic Bags

2.6.3 Measurement of Thermal Conductivity

Thermal conductivity was measured using a Transient Line Source (TLS-100) thermo-time domain reflectometry probe, following the heat pulse (HP) method. The TLS-100 probe, compliant with ASTM D5334-22a, can measure thermal conductivity from 0.1 to 5.0 W/m·K. The probe consists of a stainless-steel tube with a length-to-diameter ratio of 100. The heater was activated for up to 120 seconds, and temperature changes were recorded using a T-type thermocouple. Then, the thermal conductivity was calculated from the recorded temperature data, and the heat generation rate of the heating wire was determined by the electric current and the wire's resistance. After heating the needle, the system automatically calculates the slope of the temperature versus the time curve and subsequently determines the thermal conductivity of the soil, as shown in Figure 6. The entire measurement process takes approximately 20 minutes to complete. Measurements were conducted on soil columns packed to achieve specific bulk densities and moisture contents. The columns were equilibrated in a temperature-controlled room at 25 ± 3°C before measurements. Each sample was measured three times, as shown in Appendix A, with a 5-to-10-minute interval between readings to allow the sample to return to isothermal conditions. The average of these readings was used for data analysis. After thermal conductivity measurements, the soil samples were oven-dried to determine the final bulk density and water content after oven-drying the soil samples at 105^0C to a constant mass for 24 hrs.



(a). Silty Clay (b). Clay

Figure 6: Thermal Conductivity of Different Soil by TLS-100

2.6.4 Calibration Procedures

Calibration of the TLS-100 probe was performed to ensure accurate measurements. The calibration process involved using a standard reference material with a known thermal conductivity value. The TLS-100 sensor, consisting of a stainless-steel needle with an embedded heating element and temperature sensor, was inserted into the calibration material to ensure good thermal contact, as shown in Figure 7. The calibrated thermal conductivity of the reference material was 0.32 W/m·K. The measured thermal conductivity obtained from the probe was compared with this known value to determine the calibration coefficient, which was calculated as follows:

$$Calibration\ coefficient = \frac{Calibrated\ thermal\ conductivity}{Measured\ thermal\ conductivity}$$

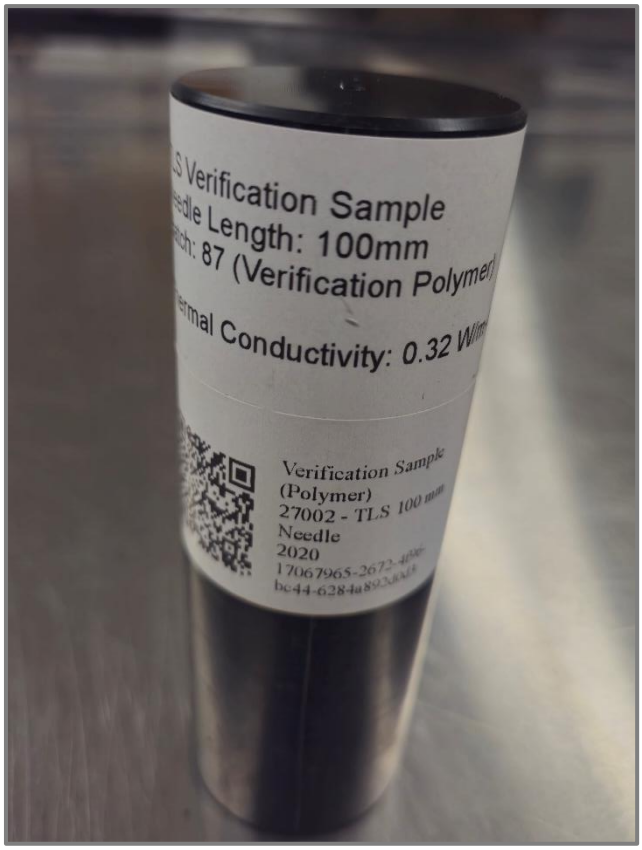


Figure 7: Calibration of TLS-100 by Standard Reference Material

This coefficient was used to adjust the probe readings for improved accuracy. Measurements were repeated to verify consistency, and calibration checks were regularly conducted throughout the study to maintain the reliability of the thermal conductivity data.

2.7 Post-Processing of Data

After completing the thermal conductivity tests, the saturation level of each soil sample at different moisture contents was calculated to ensure accurate moisture distribution. Then, the dry-out curve was created, showing the changes from completely dry to fully saturated conditions for all twenty soil samples. Besides the experimental measurements, the thermal conductivity of the soil samples was also calculated using well-known empirical formulas. These formulas included regression-based and normalized-based models, enabling a thorough comparison of different calculation methods. Detailed statistical analyses were conducted to confirm the experimental data's reliability and consistency. These analyses helped evaluate the precision of the data and identify any unusual results. Lastly, the experimental results were compared with theoretical models to verify the findings. This comparison helped confirm the accuracy of the experimental data and showed how closely the results matched theoretical predictions.

2.8 Error Analysis and Uncertainty Quantification Indices

A robust statistical analysis is performed to assess model performance and understand the associated uncertainties. Several key metrics are commonly employed to evaluate predictive accuracy and model efficiency, including Average Deviation (AD), Root Mean Square Error (RMSE), Nash-Sutcliffe Efficiency (NSE), or the coefficient of determination (R^2).

- **Average Deviations (AD) or Bias:** It measures the average absolute difference between observed and predicted values and provides a straightforward indication of the overall error without considering the direction of the error (over or under-prediction). AD reflects the typical prediction deviation, offering a precise, intuitive measure of model error. It is expressed as:

$$AD = \frac{\sum_{i=1}^{m} |y_i - \hat{y}_i|}{m}$$

Where y_i values are predicted by different thermal conductivity models, $\hat{y}_i$ are measured values, and m is the number of measured values. AD reflects the typical prediction deviation, offering a precise, intuitive measure of model error.

- **Root Mean Square Error (RMSE)**: RMSE is a widely used metric that measures the average magnitude of the error, giving more weight to significant errors. RMSE is more sensitive to larger errors because it squares the differences before averaging. This is useful in cases where considerable errors are smaller than unity. The following equation can express RMSE as:

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(y_i - \hat{y}_i)^2}$$

A lower RMSE indicates better predictive accuracy, with more minor deviations between predicted and observed values.

- **Coefficient of determination (R^2) or Nash-Sutcliffe Efficiency (NSE):** It was developed by Nash and Sutcliffe (1970) and assesses the performance of a model by using the variance of the prediction errors to the variance of the observed data. The NSE is a normalized statistic that assesses the predictive skill of a model and compares the model's predictions to the observed data relative to the mean of the observed data, which is given by:

$$NSE = 1 - \frac{\sum_{i=1}^{m}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{m}(y_i - \bar{y})^2}$$

where $\bar{y}$ is the mean measured value. NSE values range from -∞ to 1. A value of 1 means a perfect match between the model's predictions and the observed data. In contrast, a value close to or below 0 indicates poor model performance, meaning that the model's predictions are no better than those using the mean of the observed values as predictors. Overall, RMSE quantifies the deviation between predicted and observed values. AD measures the difference between the predicted and observed values. NSE assesses the relative magnitude of the residual variance between observed and predicted values (Nash and Sutcliffe, 1970). An NSE value closer to 1 indicates a minor discrepancy between predicted and observed values (Liu *et al.*, 2021).

Chapter 3
Experimental Results and Discussions

3.1 Introduction

This chapter provides a comprehensive overview of the data collected during the experimental phase of the study and compares it with the calculated thermal conductivity from different empirical models. It also represents the context for the subsequent analysis by describing the sampling locations, soil types, and experimental methods used to evaluate soil thermal conductivity under varying saturation conditions.

3.2 Dry-Out Curves for Different Soil Samples

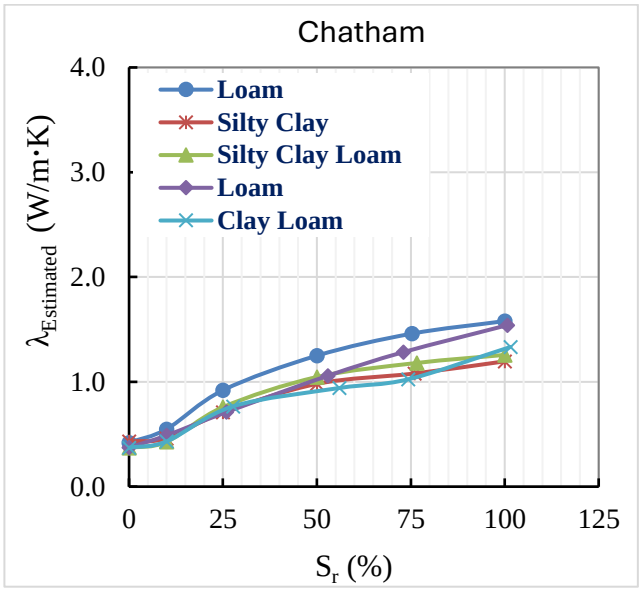
Dry-out curves are fundamental to understanding soil moisture content and thermal conductivity interactions. These curves depict how thermal conductivity varies as the degree of saturation increases from arid conditions (0%) to a fully saturated state (100%). The experimental data collected from the 20 soil samples across seven locations in Southern Ontario provide valuable insights into the thermal behavior of different soil types under varying moisture conditions. This section presents the dry-out curves for each soil sample and analyzes the observed trends in thermal conductivity across different saturation levels.

3.2.1 Trends Across Saturation Levels

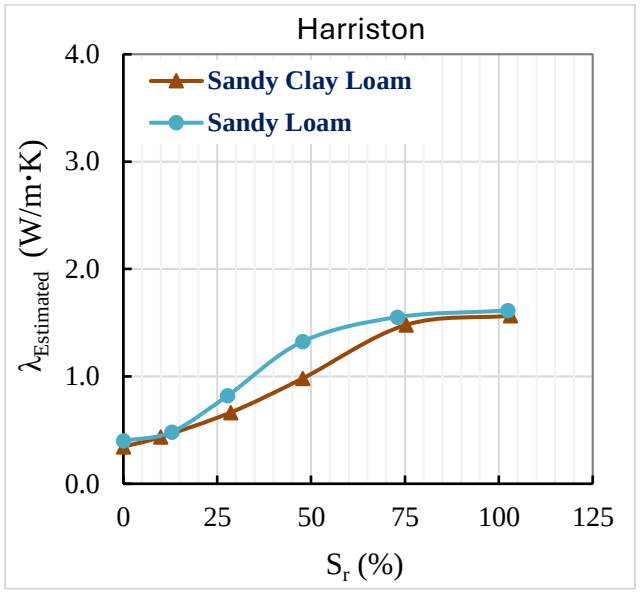
The dry-out curves for each soil sample were generated based on average thermal conductivity measurements conducted at specific degrees of saturation: 0%, 10%, 25%, 50%, 75%, and 100%, as shown in the figure below. These curves represent the relationship between average thermal conductivity and the calculated degree of saturation derived from the measured gravimetric moisture content. The desired degree of saturation was targeted for each measurement to ensure accuracy in the thermal conductivity assessment. The graphical representation of this data highlights the non-linear relationship between thermal conductivity and moisture content, influenced by soil composition and pore structure. The trends observed in these curves provide valuable insights into the heat transfer mechanisms within soils as they transition from dry to fully saturated conditions.

At very low saturation levels (0%–10%), the thermal conductivity of the soil samples reaches its minimum. In this dry state, soil pores are predominantly air-filled, with significantly lower thermal conductivity than water and soil minerals. Heat transfer in this phase occurs primarily through conduction within the solid soil particles, with minimal contribution from the air-filled voids.

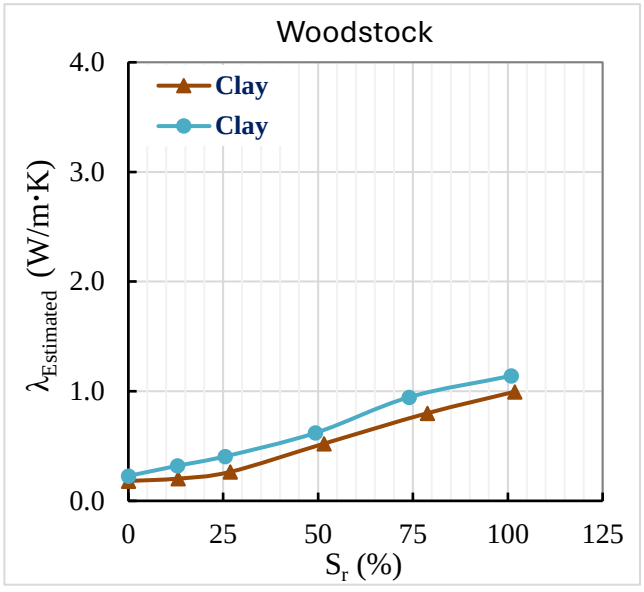
The thermal conductivity values in the dry state also vary depending on the soil's mineralogical composition. Soils with higher quartz content exhibit slightly higher thermal conductivity even under dry conditions due to quartz's inherently high thermal conductivity relative to other common soil minerals.



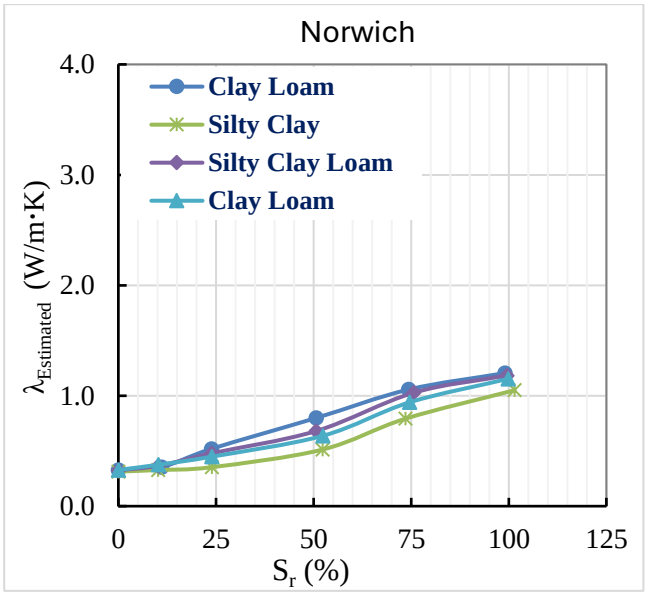
(a) ***Chatham***



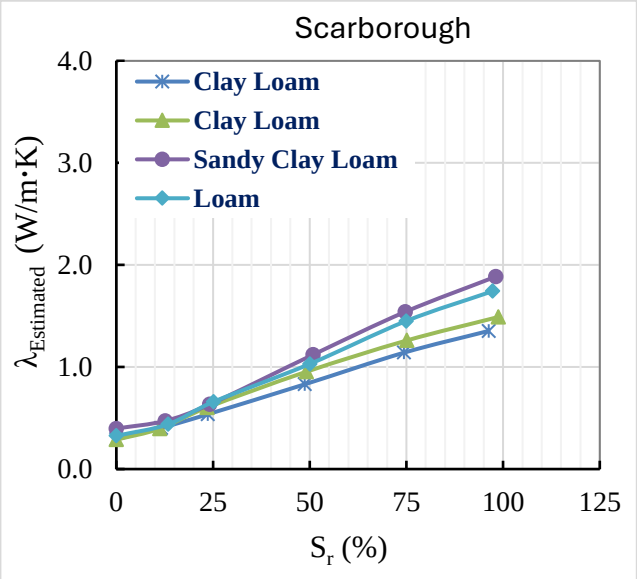
(b) ***Harriston***



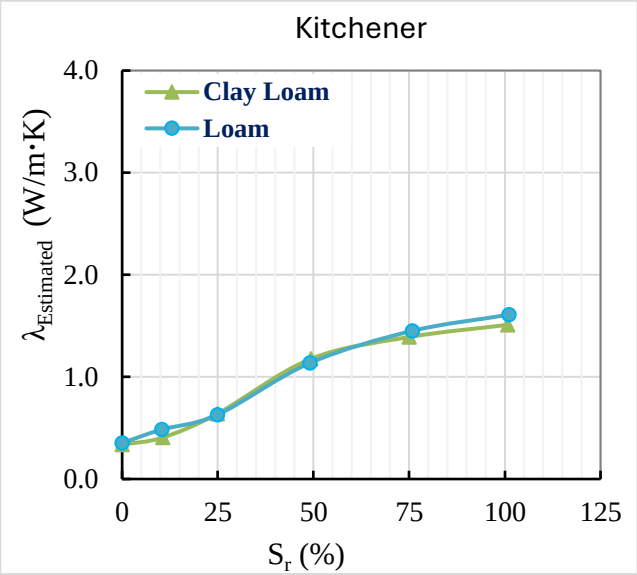
(c) ***Woodstock***



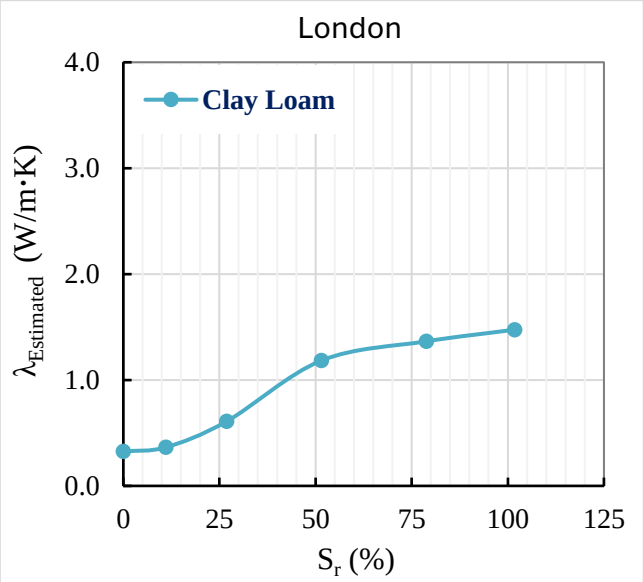
(d) ***Norwich***



(e) ***Scarborough***



(f) ***Kitchener***



(g) ***London***

Figure 8: ***Measured thermal conductivity of soil as a function of the degree of saturation for 20 soil samples collected from seven locations in Southern Ontario.***

Since direct measurement techniques such as X-ray fluorescence (XRF) and X-ray diffraction (XRD) are not performed, quartz content is estimated based on 50% of the sand fraction (He *et al.*, 2019; Luo *et al.*, 2009). Therefore, Harriston soil, which has the highest sand content, exhibited relatively higher thermal conductivity among all the tested soils, as shown in Figure 8b. As the degree of saturation

increases from 10% to 25%, a slight increase in thermal conductivity is observed. It can be attributed to the formation of thin water films around soil particles and within tiny pores, which begin to reconnect isolated thermal pathways. However, the thermal conductivity remains relatively low in this range because the water content is insufficient to establish continuous thermal bridges throughout the soil matrix. The increase in thermal conductivity during this phase is more pronounced in fine-grained soils, such as clays, which retain water more effectively within their micropores than coarser soils (Figure 8a to 8g).

A more significant rise in thermal conductivity occurs as the saturation level increases from 25% to 50%. This transitional phase is marked by the progressive filling of medium and large pores, allowing for the development of continuous water films that enhance heat transfer. The connectivity of water pathways within the soil matrix becomes more established, reducing thermal resistance and a noticeable increase in thermal conductivity. In sandy soils, this change is particularly sharp due to the rapid filling of large pore spaces. In contrast, in clayey soils, the increase is more gradual because of the finer pore structure and slower water redistribution. This trend is evident in Figure 8b, where the thermal conductivity of sand-rich soils rises more abruptly compared to clay-rich soils as moisture content increases. Between 50% and 75% saturation, the thermal conductivity continues to rise but at a decreasing rate. At this stage, most of the soil pores are filled with water, and the thermal pathways within the matrix are well connected. The marginal increases in thermal conductivity are due to the gradual filling of the remaining air-filled voids, which contributes less significantly to overall heat transfer than earlier phases. The influence of soil texture is still evident, with sandy soils approaching their peak thermal conductivity more quickly than finer-textured soils.

At full saturation (100%), the thermal conductivity of the soil samples reaches its maximum value. This is primarily because water, with its higher thermal conductivity than air, fills the soil pores, establishing continuous and efficient thermal pathways throughout the matrix. In this state, heat transfer occurs predominantly through the combined conduction of water and soil minerals, minimizing thermal resistance. The magnitude of thermal conductivity at full saturation varies based on the soil's mineral composition and texture. Due to quartz's superior thermal conductivity, soils with higher quartz content exhibit higher thermal conductivity than soils with lower quartz content. Additionally, finer-grained soils may retain slightly more water in micropores, influencing the overall conductivity.

3.2.2 Variation with Soil Compositions

The analysis of the thermal conductivity trends observed in the laboratory tests indicates that soil texture and mineralogical composition significantly influence the variation of thermal conductivity with

saturation, shaping the dry-out curves and determining their overall slope. Soil texture, defined by the relative proportions of sand, silt, and clay, directly influences the thermal conductivity response to moisture variation. The plotted data further highlight distinct trends based on soil type, emphasizing the impact of pore structure and water retention capacity on heat transfer.

Clayey soils exhibit more gradual increases in thermal conductivity due to their fine pore structures and higher water retention capabilities. Micropores allow these soils to retain moisture longer, moderating the rate of thermal conductivity change across varying saturation levels. In contrast, sandy soils, characterized by larger pores and lower water retention capacity, exhibit a sharp increase in thermal conductivity as saturation rises. This behavior is attributed to the rapid formation of thermal pathways as the coarse pores quickly fill with water, enhancing heat transfer. This trend was observed in Harriston and one Scarborough soil, which contains relatively higher sand content, as depicted in Figures 8b and 8e. Loamy soils, which consist of a balanced mixture of sand, silt, and clay, demonstrate intermediate behavior with moderate slopes in their dry-out curves. Their mixed texture facilitates water retention and drainage, resulting in thermal conductivity trends that reflect characteristics of both sandy and clayey soils.

Beyond texture, mineralogical composition is crucial in influencing thermal conductivity, particularly the presence of thermally conductive minerals such as quartz. Quartz, with its significantly higher thermal conductivity compared to other soil minerals, helps maintain relatively higher conductivity values even at lower moisture levels. This study estimated quartz content indirectly based on the sand fraction, following established methodologies in soil thermal studies. The effect of quartz content on thermal conductivity is especially evident in sandy soils, where higher sand fractions correlate with increased conductivity, as shown in Figure 8b. These variations in thermal conductivity, influenced by soil composition, highlight the complex interactions between mineral content, pore structure, and moisture dynamics. Understanding these relationships is essential for applications such as geothermal energy systems, where soil thermal properties directly affect heat exchange efficiency. The subsequent sections will compare the estimated thermal conductivity with empirical models, offering further insights into the findings from the seven sampling locations in Southern Ontario.

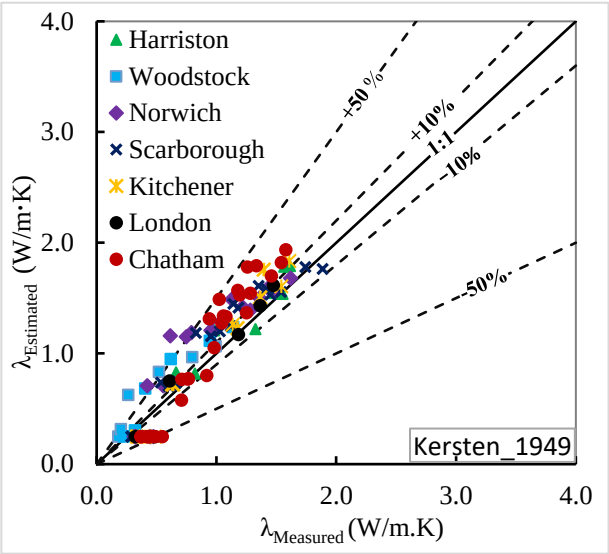
3.3 Comparison with Empirical Models

The measured thermal conductivity results were compared using fourteen empirical models encompassing normalized and regression-based models. These models were selected based on their prominence in existing literature and demonstrated applicability to diverse soil types and environmental conditions. The primary objective of this validation process was to evaluate the accuracy, reliability,

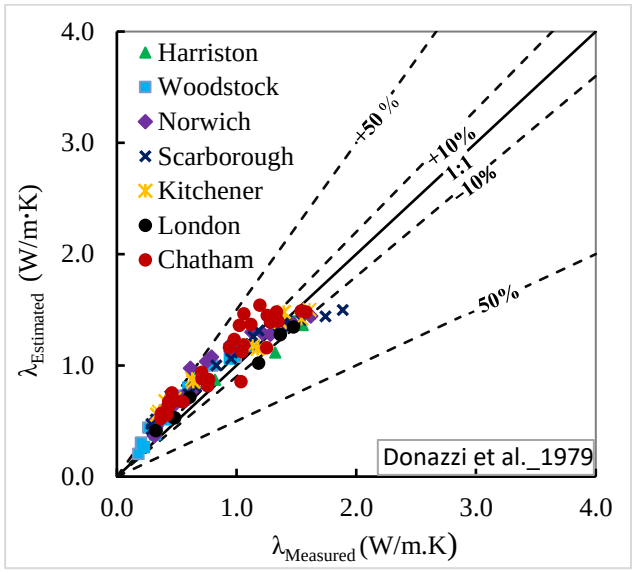
and robustness of each model in predicting soil thermal conductivity across varying degrees of saturation. The measured data served as the benchmark for assessing model performance, providing a comprehensive basis for comparison. A solid line (1:1 line) was created to facilitate this comparison, representing a perfect agreement between the measured and estimated values. Additionally, dashed lines indicating deviations of +10%, -10%, +50%, and -50% were used to define acceptable or extreme variations. This not only highlights the predictive capabilities of the models but also identifies their limitations in different soil compositions and moisture conditions, thereby offering critical insights into their practical utility in soil thermal studies.

3.3.1 Linear or nonlinear regression-based models

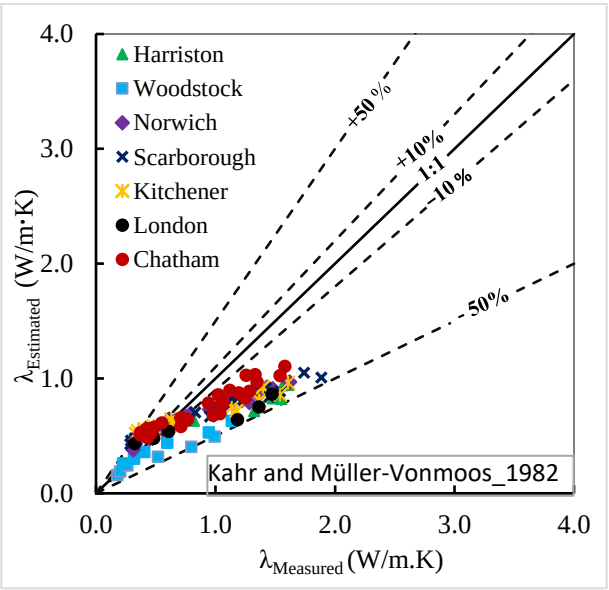
The Kersten (1949) model provides one of the earliest empirical estimations of thermal conductivity, incorporating the effects of moisture content and dry bulk density. The model demonstrates a generally good approximation, with most data points aligning along the 1:1 line. However, noticeable deviations occur, particularly at the lower and higher saturation levels (Figure 9a). This discrepancy is likely due to the model's empirical nature and assumption of a minimum conductivity threshold (0.25 W/m·K) under dry conditions. Despite its limitations in accounting for mineralogical and structural variations, the model remains widely used due to its simplicity and empirical validation in engineering applications. Donazzi *et al.* (1979) introduced an exponential model incorporating key physical properties such as soil particle conductivity, porosity, and degree of saturation to address these limitations. This model improves predictive accuracy by capturing the nonlinear effects of porosity and saturation through an exponential correction factor. However, deviations persist, with overestimations at lower saturation values, as evidenced by data scattering within the bounds (Figure 9b). A notable advantage of this model is its ability to capture the nonlinear effects of porosity and saturation through an exponential correction factor, enhancing its applicability in various soil conditions. However, the observed deviations suggest that the model may be less reliable for extreme soil compositions or conditions where additional influencing factors, such as mineral composition and soil structure, play a significant role.



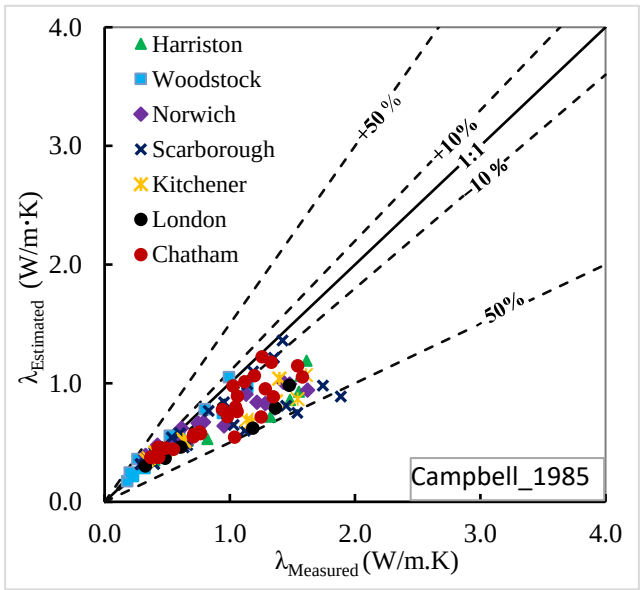
(a) *Kersten (1949)*



(b) *Donazzi et al. (1979)*



(c) *Kahr & Müller-Vonmoos (1982)*



(d) *Campbell (1985).*

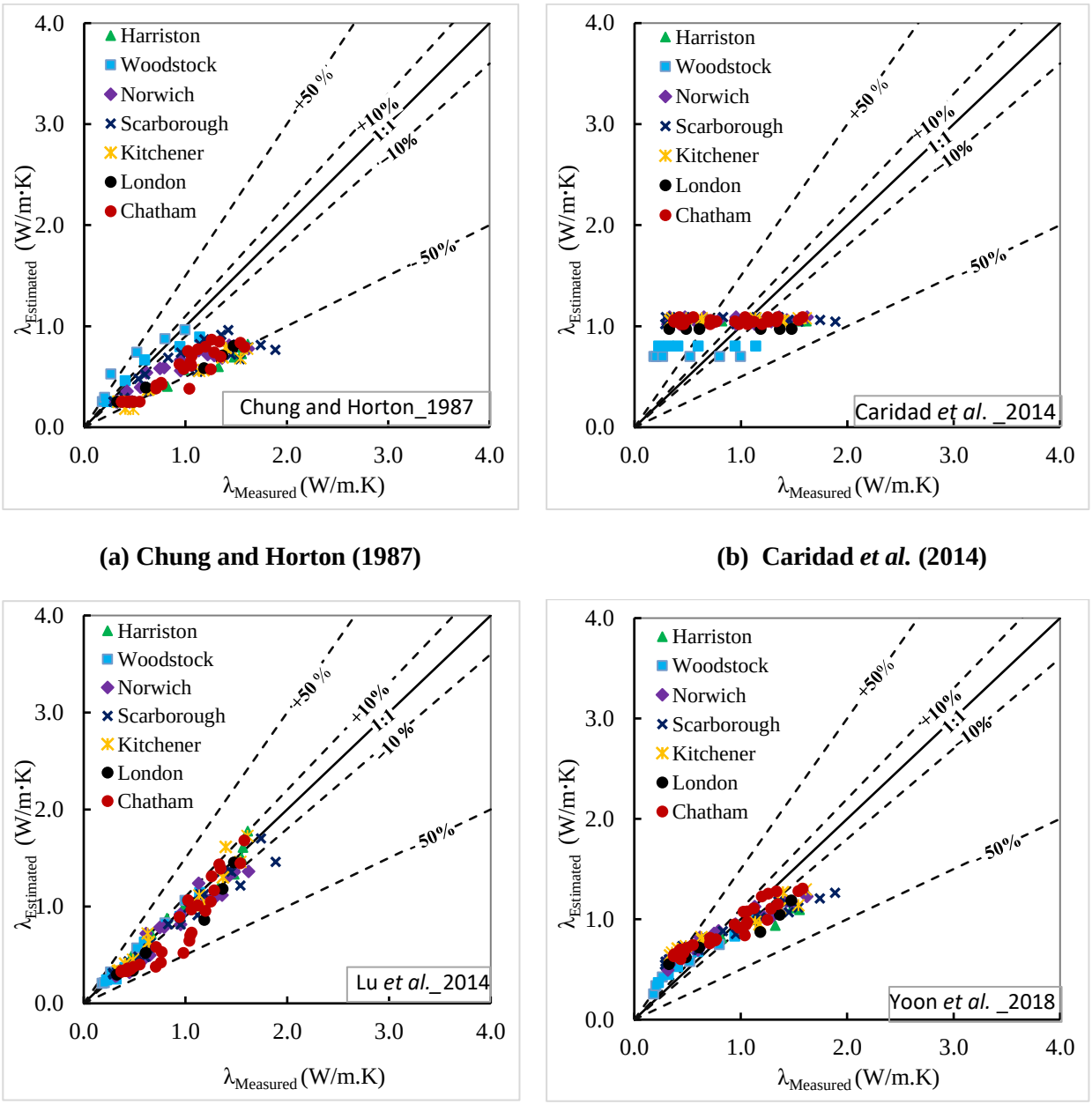
Figure 9: ***Comparison of measured soil thermal conductivity with linear and non-linear regression-based model predictions.***

Focusing specifically on bentonite clays, Kahr and Müller-Vonmoos's (1982) model, based on bulk dry density and moisture content, captures the increase in thermal conductivity with higher density and moisture, particularly in compacted and saturated conditions. However, deviations indicate a tendency to underestimate thermal conductivity in some instances, suggesting its applicability is limited to bentonite-based materials, as shown in Figure 9(c). Moreover, the model does not explicitly consider

saturation effects or mineralogical variations, which may contribute to discrepancies in its predictions for different soil types. Expanding on the role of soil composition, Campbell (1985) introduced an empirical model incorporating volumetric water content, quartz, and clay fractions. While it generally aligns well with measured values, noticeable overestimations occur at higher saturation levels, as Figure 9(d) depicts. The model's adaptability across different soil types and integration into numerical simulations make it a valuable tool for environmental modeling. However, the model relies on empirical constants, which may not generalize well to all soil compositions, particularly in highly organic or extreme mineralogical conditions. Furthermore, its exponential correction factor may not fully capture complex interactions between moisture and soil structure, leading to predictive inaccuracies.

The Chung & Horton (1987) model estimates thermal conductivity primarily based on volumetric water content (θ_v), integrating linear and nonlinear moisture effects. While it effectively captures the nonlinear increase in thermal conductivity with moisture, deviations are evident, mainly underestimating at high moisture levels due to the reliance on a limited set of empirical parameters (Figure 10a). The model assumes a constant λ_{eff} in dry conditions ($\theta_v < 0.04$), which may not fully account for the behavior of desiccated soils. Although widely implemented in hydrological and heat transfer models like HYDRUS, its lack of explicit consideration for soil composition variations limits its applicability across diverse soil types. In contrast, Caridad *et al.* (2014) proposed a linear model that estimates thermal conductivity based on bulk density, assuming a direct proportional relationship, as shown in Figure 10b(b). However, a significant drawback is its inability to account for moisture variations, resulting in a nearly constant thermal conductivity across different soil conditions. This oversimplification leads to substantial deviations, particularly at lower and higher saturation levels. The clustering of data points suggests that this model is most suitable for specific clay types with uniform thermal behavior but lacks the adaptability needed for a broader range of soil conditions.

Lu *et al.* (2014) introduced a more refined approach, incorporating water content, bulk density, porosity, and soil texture into the estimation process. This model demonstrates strong agreement with measured values, with most data points falling within the ±10% range, indicating high predictive accuracy, as shown in Figure 10(c). The model improves upon previous empirical methods by capturing the nonlinear relationship between thermal conductivity and moisture. However, slight underestimations occur at lower moisture levels. While universally applicable across different environmental conditions, the model may require recalibration for highly organic soils or extreme moisture conditions. Expanding on these advancements, Yoon *et al.* (2018) developed a regression-based model that integrates bulk density and degree of saturation.



(a) Chung and Horton (1987)

(b) Caridad *et al.* (2014)

(c) Lu *et al.* (2014)

(d) Yoon *et al.* (2018)

Figure 10: Comparison of measured soil thermal conductivity with linear and non-linear regression-based models.

This model effectively captures the combined influence of density and moisture, making it particularly suitable for clayed soils under varying saturation levels. The strong correlation between estimated and measured values confirms its reliability (Figure 10d). However, slight overestimations occur at a higher level of saturation. Although the model balances simplicity and accuracy, its dependence on regression

parameters derived from specific soil types may limit its generalizability, necessitating recalibration for applications in different geological settings.

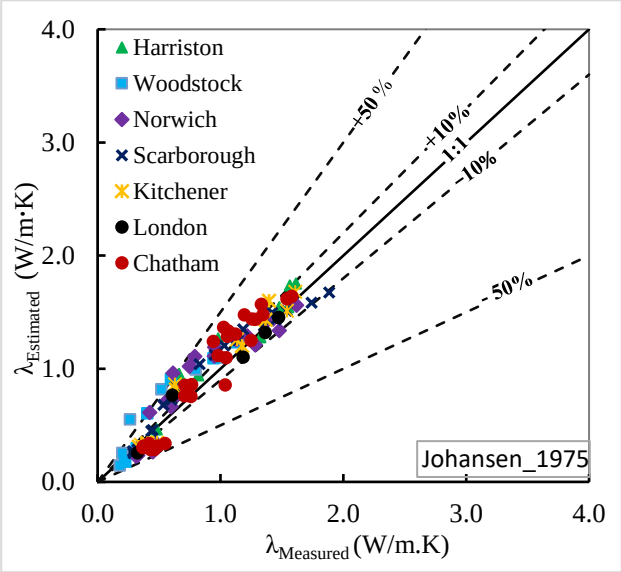
3.3.2 Normalized Model

The Johansen (1975) model is a widely recognized method for estimating soil thermal conductivity, incorporating the effects of soil saturation and quartz content. By analyzing Kersten's data, Johansen introduced a normalized approach to improve predictive accuracy. The comparison of estimated and measured values shows a strong correlation along the 1:1 line, demonstrating reliable performance, as shown in Figure 11a. However, slight data scattering is observed, particularly at lower levels of saturation where underestimation occurs. One of the model's key strengths is its ability to effectively estimate λ_{eff} for dry and saturated soils, making it applicable across various soil types. Despite these advantages, its reliance on estimated quartz content rather than direct measurements introduces potential errors. Overall, the model adopts a simplified approach for dry soil conductivity and remains a robust and widely validated reference for geotechnical and environmental applications.

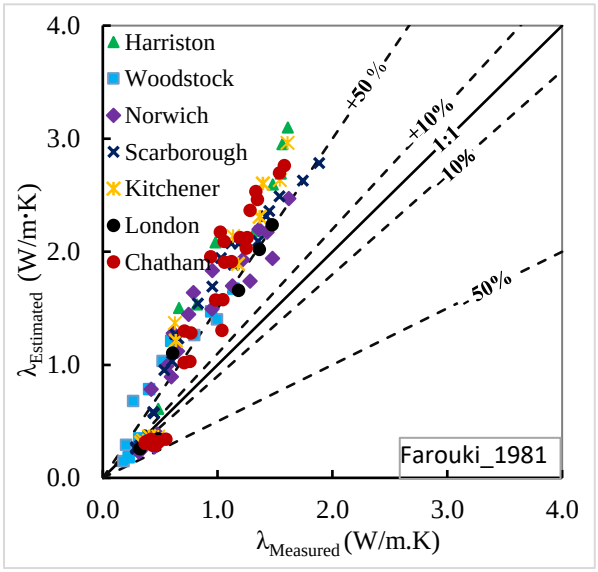
Building on Johansen's framework, Farouki (1981) and Hu *et al.* (2001) introduced modifications to enhance soil thermal conductivity estimation. However, both models overestimate λ_{eff}, particularly at higher moisture levels. Farouki (1981) refined the Kersten number (K_e) by incorporating soil composition, water content, and temperature effects, increasing adaptability to diverse environmental conditions. However, its reliance on a logarithmic function for K_e results in inaccuracies and overestimation in extremely dry soils, especially fine-textured soils. Similarly, Hu *et al.* (2001) modified K_e to represent moisture effects better, improving general applicability across various soil types. Despite this refinement, the model exhibits significant deviations from measured values, with most data points falling outside the +50% range and a clear trend of overestimation in moderate-moisture conditions. While both models contribute to improved predictive capabilities, their overestimation tendencies indicate further refinement to enhance accuracy across different soil compositions and moisture levels, as illustrated in Figures 10(b) and 10(c).

The Côté and Konrad (2005) model improves Johansen's framework by incorporating predefined values for the thermal conductivity of solid particles and introducing a soil texture-dependent parameter (κ) for unsaturated conditions. These refinements enhance adaptability across different soil types, improving accuracy in predicting λ_{eff} under varying moisture conditions. The comparison between estimated and measured values shows a relatively good correlation along line 1:1, with most data points clustering within the range, as illustrated in Figure 11(d). However, deviations are observed, particularly at lower moisture values, where slight underestimations occur. This model's reliance on

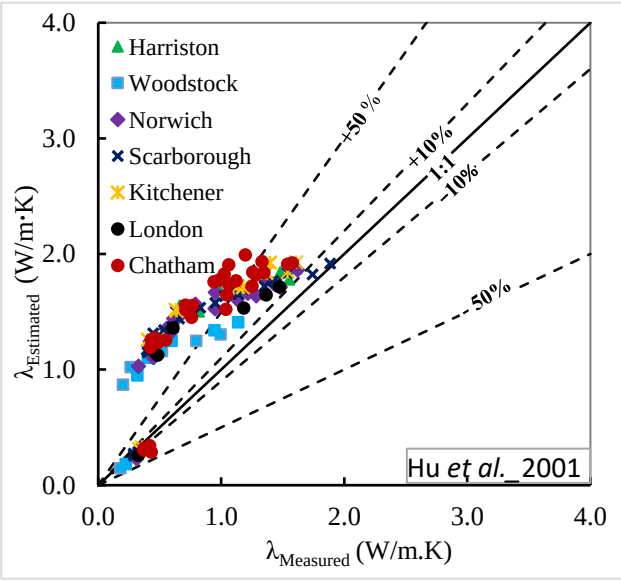
predefined thermal conductivity values may introduce errors if field conditions differ from assumed parameters. While this approach is valuable for geotechnical applications such as ground heat exchangers and buried pipelines, minor adjustments may be necessary for highly organic or extreme soil conditions. Similarly, Yang *et al.* (2005) modified Johansen's model by introducing an exponential formulation for the Kersten number (K_e), enhancing the representation of soil saturation effects.



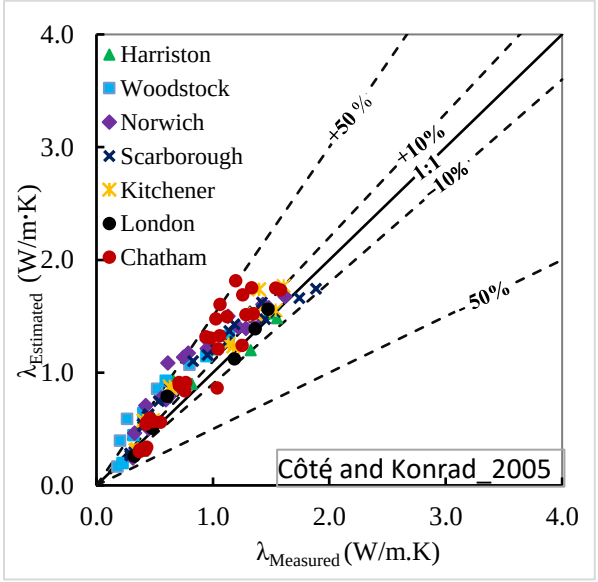
(a) Johansen (1975)



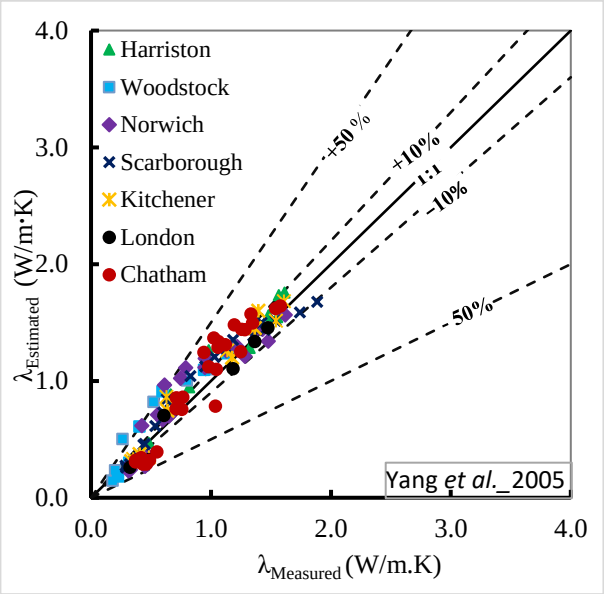
(b) Farouki (1981)



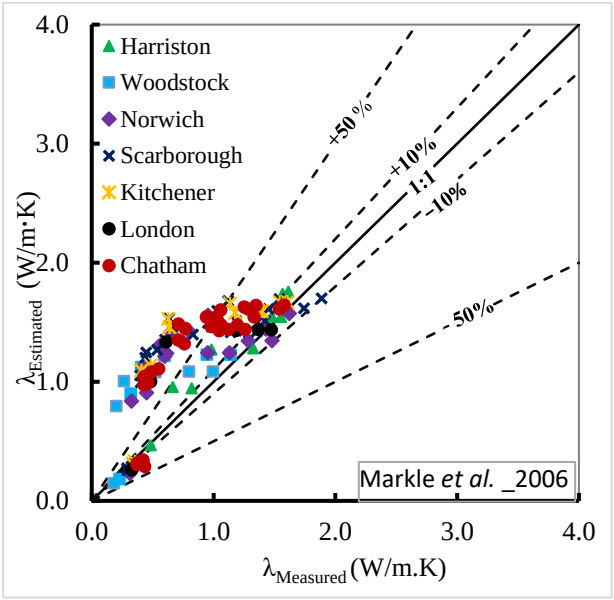
(c) Hu *et al.* (2001)



(d) Côté and Konrad (2005)



(e) Yang *et al.* (2005)



(f) Markle *et al.* (2006)

Figure 11: Comparison of measured soil thermal conductivity with normalized-based models.

This adjustment provides a smoother transition between dry and saturated states, improving predictive accuracy, particularly in intermediate moisture conditions. The estimated and measured values comparison shows a relatively strong correlation along line 1:1, with slight underestimations at lower saturation levels, as shown in Figure 11(e). However, the revised λ_{dry} equation improves accuracy, and some deviations indicate that further adjustments may be required for soils with unique thermal properties.

The Markle *et al.* (2006) model refines Johansen's method by introducing an alternative formulation for K_e, improving the representation of λ_{eff} across varying saturation levels. While the model correlates well with measured values, noticeable deviations occur, especially at moderate saturation levels (Figure 11f). Its key advantage is improved performance in low-saturation conditions, addressing a limitation in Johansen's model. Additionally, the revised equation for thermal conductivity at full saturation reduces discrepancies at higher moisture levels. The empirical fitting parameter (ζ = 8.9) may also require calibration for different soil types, limiting its universal applicability.

3.4 Comparison of Models

Various models have been developed to estimate soil thermal conductivity (λ_{eff}), each offering distinct advantages and limitations depending on environmental conditions, soil types, and moisture content. Regression-based models, such as Kersten (1949), provide a solid foundation but tend to underestimate

λ_{eff} in dry soils and overestimate it in saturated conditions. Donazzi *et al.* (1979) enhance this approach by incorporating factors like soil particle conductivity and saturation, though it still tends to overestimate λ_{eff} at high moisture levels. Kahr & Müller-Vonmoos (1982) perform well for clays, but their applicability is more limited to such specific soils and does not fully account for mineralogical variations. Campbell (1985) is adaptable to various soil types but may require calibration because it relies on empirical constants. Chung & Horton (1987) successfully incorporate linear and nonlinear moisture effects but tend to underestimate λ_{eff} at high moisture levels. Caridad *et al.* (2014) provide a more straightforward, bulk-density-based approach, but it may not fully capture moisture variations, leading to less accurate predictions in soils with significant moisture fluctuations. Lu *et al.* (2014) and Yoon *et al.* (2018) improve accuracy by considering additional factors like soil texture and bulk density. However, soils with high organic content or extreme moisture conditions may need refinement.

Normalized models, such as Johansen (1975), perform well across various soils. However, they may underestimate λ_{eff} in dry conditions and overestimate it in medium to high-saturated soils, mainly due to the assumption of estimated quartz content. Farouki (1981) builds on this by refining the Kersten number (Ke) but still tends to overestimate λ_{eff}, particularly in high-moisture environments. Hu *et al.* (2001) further refine this number and show similar behavior to Farouki (1981). Côté and Konrad (2005) introduce a soil texture-dependent parameter, but some deviations persist in complex moisture variations or extreme mineral compositions. Yang *et al.* (2005) improve the handling of moisture variations, particularly for partially saturated soils, but still require little calibration in dry conditions. Markle *et al.* (2006) refine Johansen's approach to provide better predictions in low-saturation conditions, but it still tends to overestimate λ_{eff} under higher moisture levels.

Both regression-based and normalized models present valuable frameworks for estimating soil thermal conductivity, each with unique strengths depending on the soil type and moisture conditions. Both regression-based and normalized models offer valuable approaches, but most require refinement to improve their accuracy in diverse situations. Statistical analysis will help quantify these discrepancies and guide the selection of the most reliable models, as discussed in the following sections. Overall, the models show differing levels of adaptability depending on the moisture conditions, soil texture, and mineral composition. The statistical analysis of model performance will provide a quantitative evaluation of these variations, helping to identify the most reliable models and determine necessary refinements for improved accuracy in specific applications.

3.5 Statistical Analysis of Model Performance

3.5.1 Methodology for Model Performance

The comparison of regression-based and normalized models across locations and soil types highlights variations in model performance, emphasizing the need for statistical analysis to evaluate accuracy. The statistical analysis of model performance was conducted using rigorous quantitative methods to assess each empirical model's predictive accuracy and reliability. Key statistical metrics, including the root mean square error (RMSE), mean absolute error (MAE)/Bias, and the coefficient of determination/efficiency (R^2), were employed to quantify the deviations between the measured thermal conductivity values and the corresponding model predictions. These metrics provided a robust framework for assessing model performance, enabling a comprehensive evaluation of the precision and bias inherent in the models' predictions. These statistical tools facilitated an objective comparison, ensuring the evaluation process was thorough and scientifically rigorous.

Beyond these quantitative metrics, the models were also evaluated based on their ability to replicate the observed trends in thermal conductivity as a function of saturation. This dual approach provided a holistic model performance evaluation by combining quantitative accuracy with qualitative trend analysis. The insights gained from this comprehensive statistical analysis were critical in identifying the most reliable and robust models for predicting soil thermal conductivity under the diverse conditions observed in Southern Ontario.

3.5.2 Selection of Models

The selection of models for detailed statistical comparison was guided by their performance during initial validation tests and their established credibility within the scientific literature on soil thermal conductivity. Models that demonstrated consistent accuracy across multiple soil types and a wide range of saturation levels were prioritized for analysis. Additionally, models representing diverse methodological approaches, including those based on normalized parameters and regression-based formulations, were included to assess predictive strategies comprehensively. This selection process ensured a balanced representation of models, capturing a broad spectrum of theoretical foundations and empirical methodologies, as discussed below.

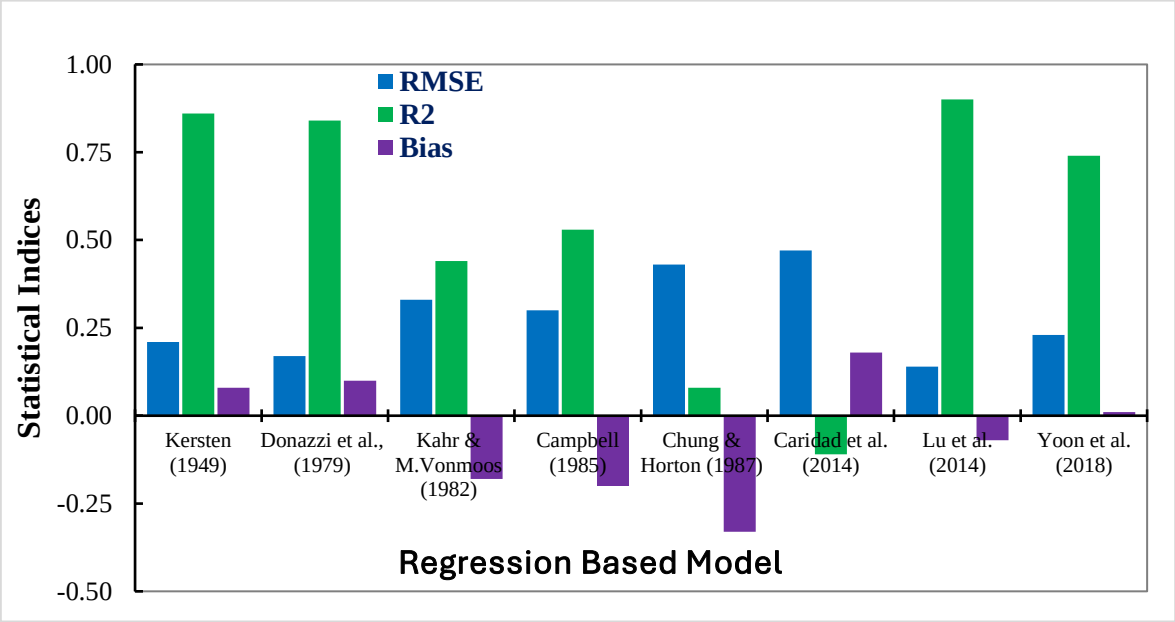


Figure 12: Performance analysis of different models based on statistical indices for regression-based models.

In the Regression-Based Models shown in Figure 12, Kersten (1949) shows the highest R^2, indicating a good fit to the measured data. However, it tends to underestimate λ_{eff} at low saturation values and overestimate at higher values, reflected by its higher RMSE. This model's simplicity makes it suitable for general soil types but requires calibration for extreme moisture conditions. Kahr & Müller-Vonmoos (1982) perform similarly, but its bias suggests that it may also underestimate conductivity under certain conditions. Campbell (1985) and Chung & Horton (1987), while offering lower R^2 values, indicate that these models struggle with moisture variations and display significant underestimations, as indicated by their bias. Lu *et al.* (2014) and Donazzi *et al.* (1979) show improved performance with higher R^2 and lower RMSE values, although their bias values suggest that some deviations exist, particularly under high moisture conditions. Caridad *et al.* (2014) present a simpler approach but show the least effectiveness in predicting soil conductivity, indicated by its low R^2 and high RMSE.

In the Normalized Models, as illustrated in Figure 13, Johansen (1975) stands out due to its high R^2 and relatively low RMSE, indicating good overall performance in diverse soil types. However, its bias value suggests that it overestimates conductivity in highly saturated soils, an expected limitation due to the model's reliance on quartz content. Farouki (1981) shows a high R^2 but tends to underestimate λ_{eff} at high moisture levels, as indicated by the bias and higher RMSE. Hu et al. (2001) exhibit the highest RMSE and a significant bias, showing substantial overestimation of thermal conductivity at high moisture conditions. This suggests that the model's modifications to the Kersten number are not fully

capturing variations in moisture and soil composition. Côté and Konrad (2005) and Yang *et al.* (2005) demonstrate better performance, having a good balance of R^2 and RMSE, indicating it is reliable across a broad range of soil types. Markle *et al.* (2006) perform better for partially saturated soil, although it requires significant calibration, especially in medium moisture levels, as shown by its higher RMSE.

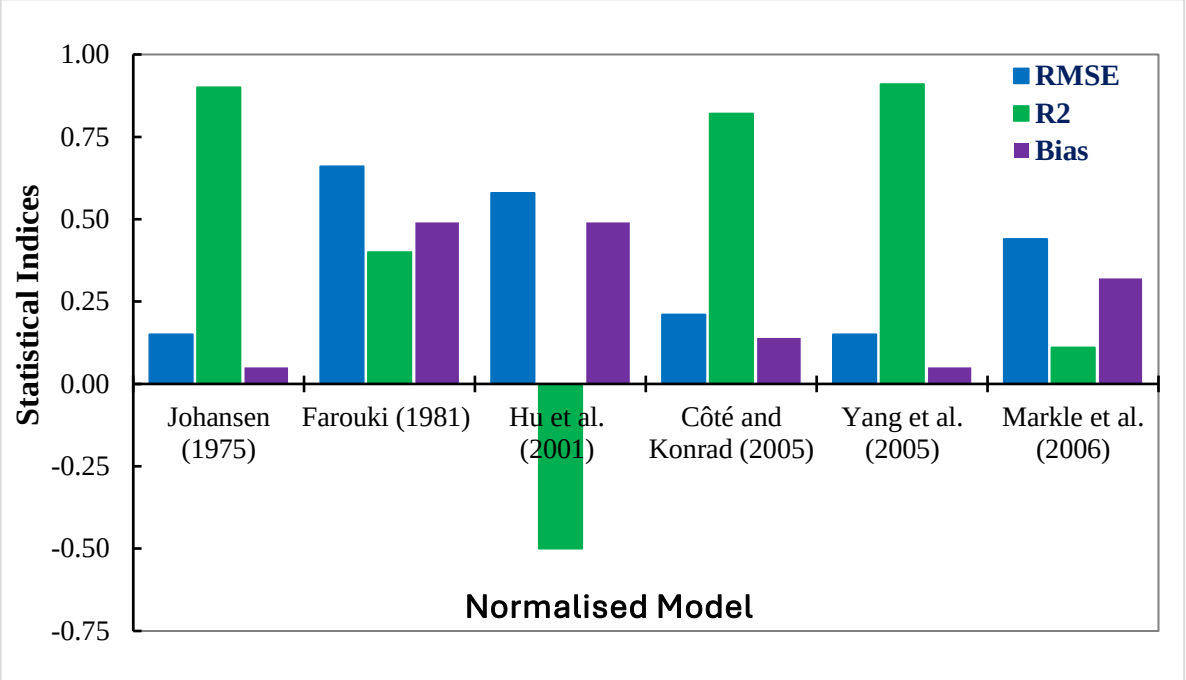
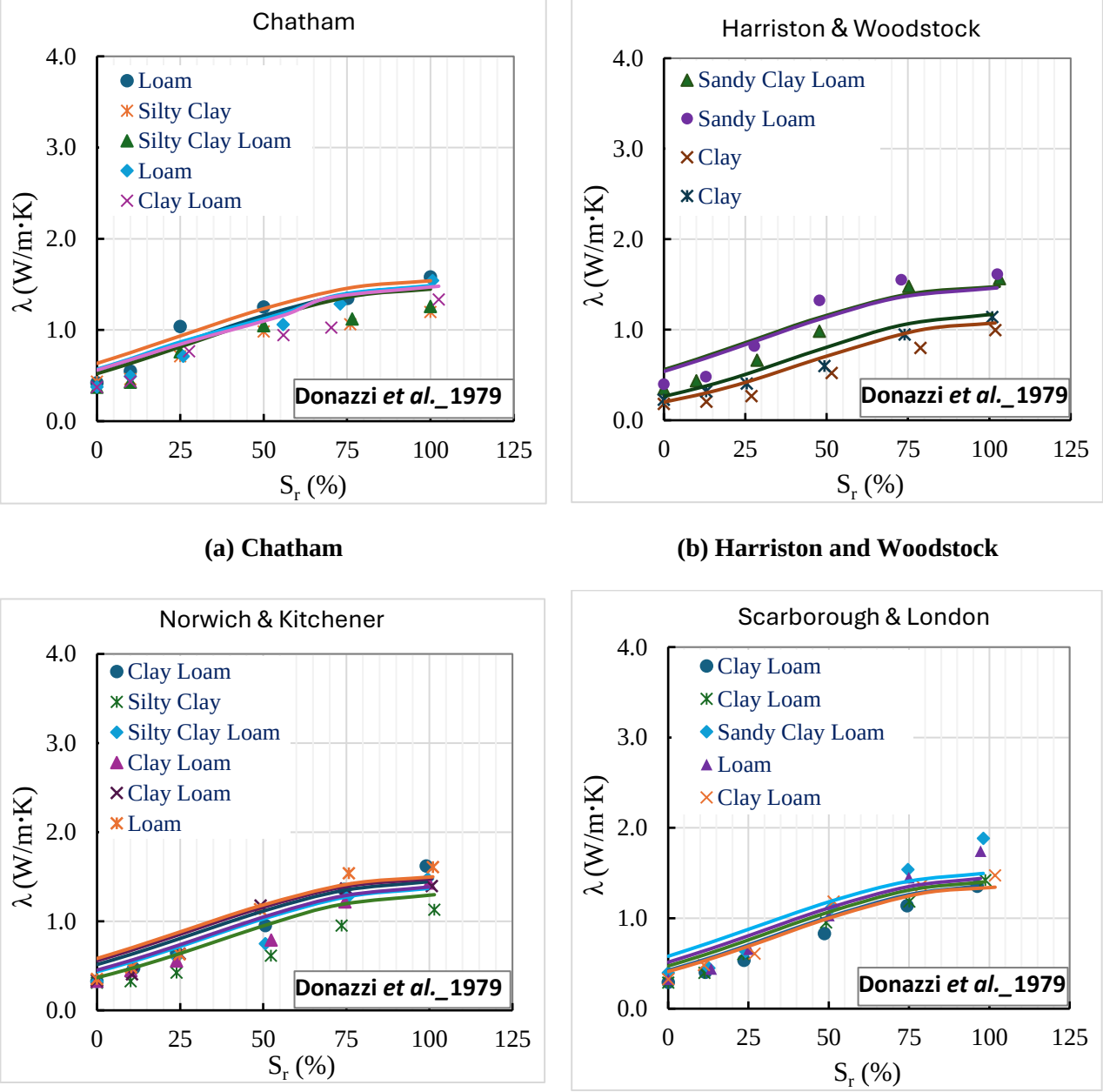


Figure 13: Performance analysis of different models based on statistical indices for normalized-based models.

3.5.3 Selected Regression-Based Models

The comparison between measured and calculated thermal conductivity for the two best-performing models, Donazzi *et al.* (1979) and Lu *et al.* (2014), represents their ability to predict soil thermal conductivity (λ_{eff}) under varying soil conditions, as illustrated in the following two figures. In these comparisons, the solid lines represent the variation of calculated thermal conductivity. In contrast, the markers without any lines represent the measured thermal conductivity for each soil type, with the same color used for both calculated and measured values. This visual representation highlights the models' performance in capturing the thermal conductivity behavior across different soil types and moisture conditions. The Donazzi *et al.* (1979) model exhibits a strong correlation between the predicted and measured thermal conductivity (λ_{eff}), effectively accounting for key factors such as soil texture and degree of saturation. This model demonstrates its ability to handle a diverse range of soil types across

multiple locations, with the regression lines aligning well with the measured data, particularly for soil types such as clay loam and loam.



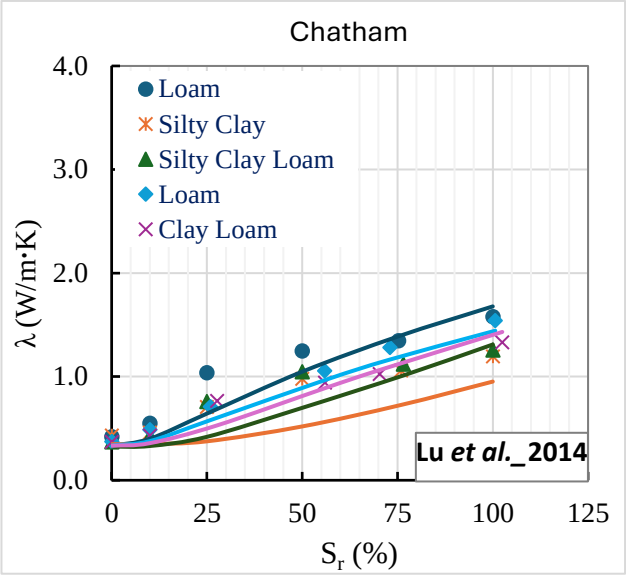
(a) Chatham **(b) Harriston and Woodstock**

(c) Norwich and Kitchener **(d) Scarborough and London**

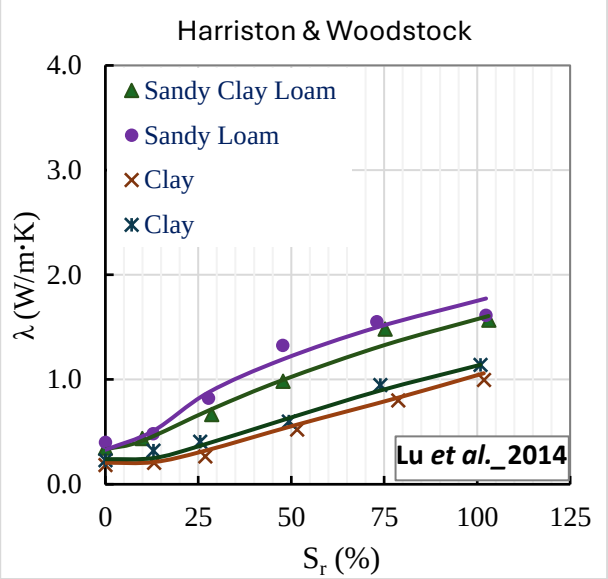
Figure 14: A comparison of measured thermal conductivity-degree of saturation relationship by Donazzi et al. (1979)

However, some deviations are evident under high-moisture conditions, notably in locations such as Chatham, Harriston, and Kitchener, where soils like silty clay and sandy loam show slight discrepancies

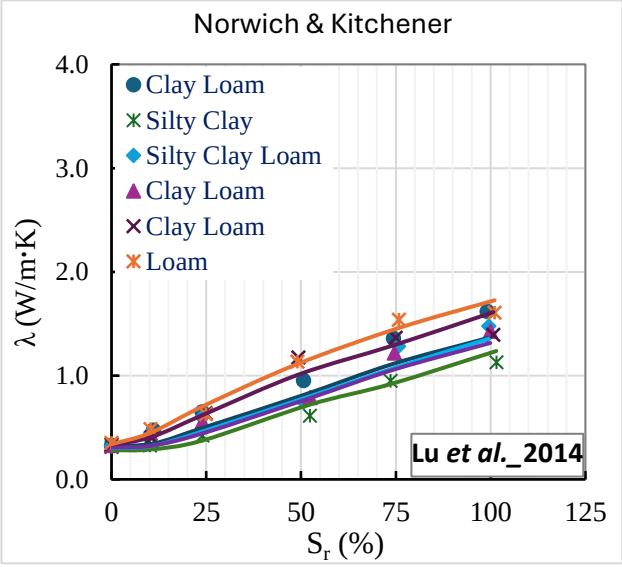
in predicted λ_{eff}. Despite these minor deviations, the model demonstrates robustness and reliability, making it suitable for various environmental conditions.



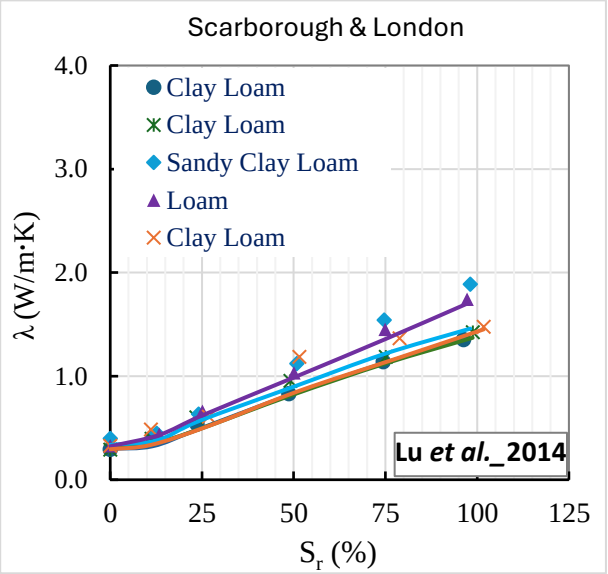
(a) Chatham



(b) Harriston and Woodstock



(c) Norwich and Kitchener



(d) Scarborough and London

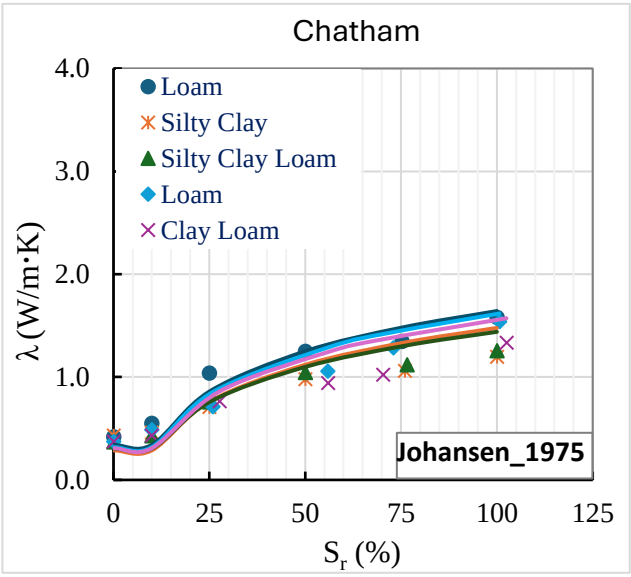
Figure 15: A comparison of measured thermal conductivity-degree of saturation relationship by Lu et al. (2014)

The comparison of measured and calculated thermal conductivity for the Lu *et al.* (2014) model demonstrates a strong fit between the predicted and actual values. This model accounts for multiple factors, including soil texture and the degree of saturation, enhancing its ability to handle various soil

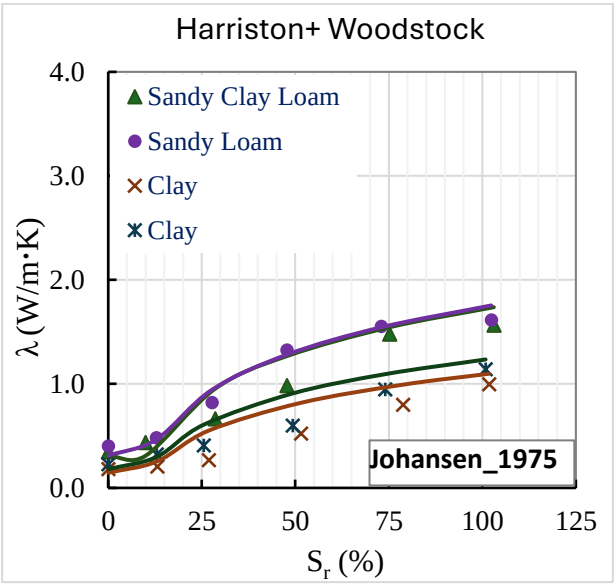
types from numerous locations. Across all locations with varying moisture content, Lu *et al.* (2014) consistently show good agreement with the measured data. Notably, the regression lines closely follow the measured data points for soil types like loam and clay loam, indicating high accuracy. However, slight deviations are evident in certain high-moisture conditions. For example, in soils like silty clay and sandy loam in locations such as Chatham and Harriston, the model slightly underestimates λ_{eff}, as shown in Figures 15(a) and 15(b). Despite these minor deviations, the model performs well across diverse conditions, further supporting its robustness.

3.5.4 Normalized Based Models

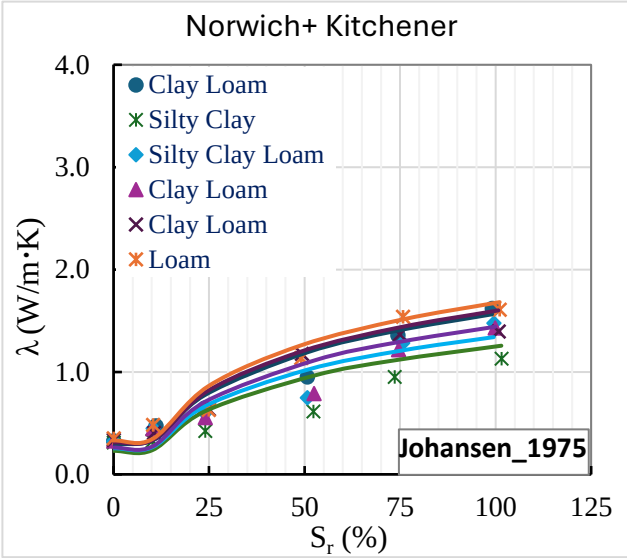
The Johansen (1975) and Yang *et al.* (2005) models are the best-performing normalized models for estimating soil thermal conductivity (λ_{eff}). They show good accuracy across different soil types and conditions. The following figures illustrate the models' predictive capabilities. The solid lines represent the predicted thermal conductivity values, while the markers without lines represent the measured values. The calculated and measured data points for each soil type are shown with matching colors, clearly showing how well the models align with the actual thermal conductivity measurements across different soil types and moisture levels. This helps to compare how well the models predict the thermal conductivity of soil in various conditions.



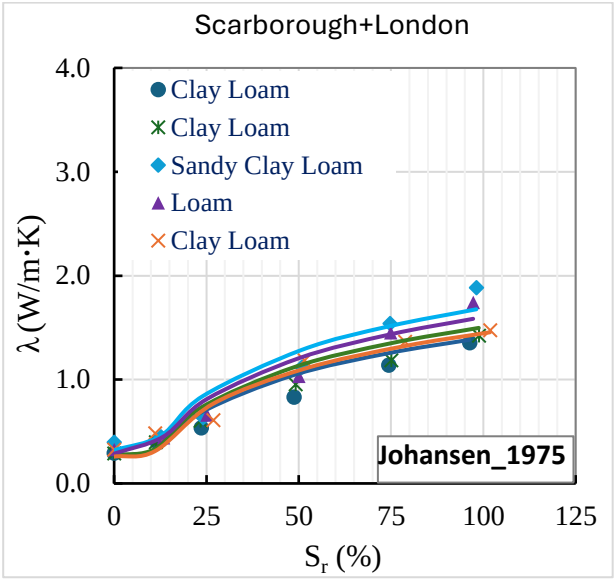
(a) Chatham



(b) Harriston and Woodstock



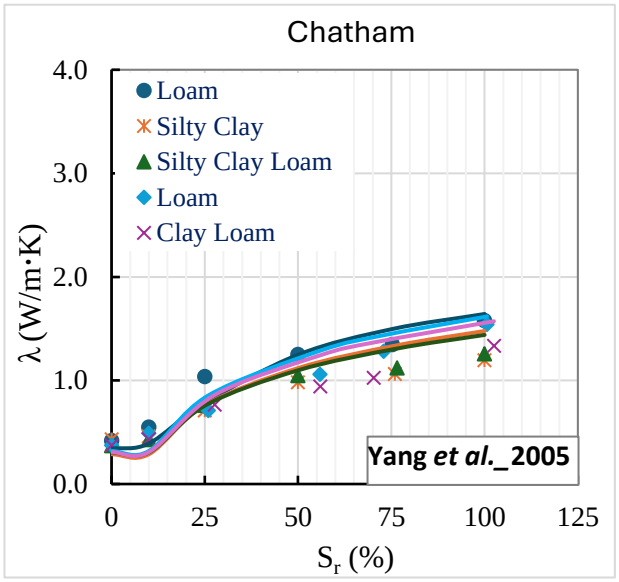
(c) Norwich and Kitchener



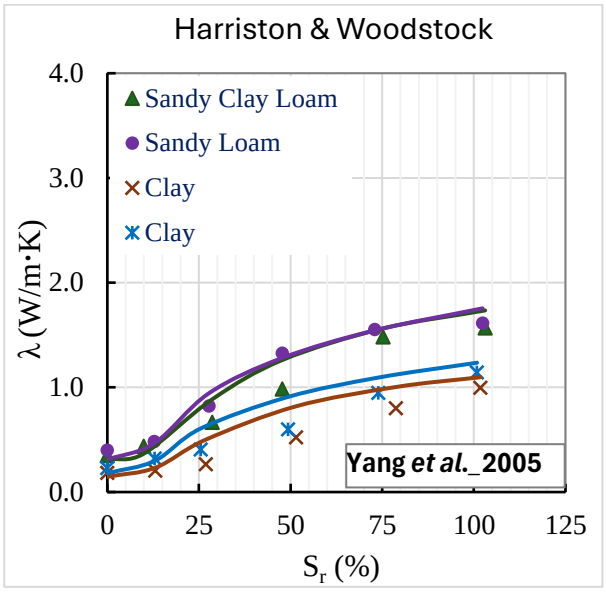
(d) Scarborough and London

Figure 16: A comparison of measured thermal conductivity-degree of saturation relationship by Côté and Konrad (2005)

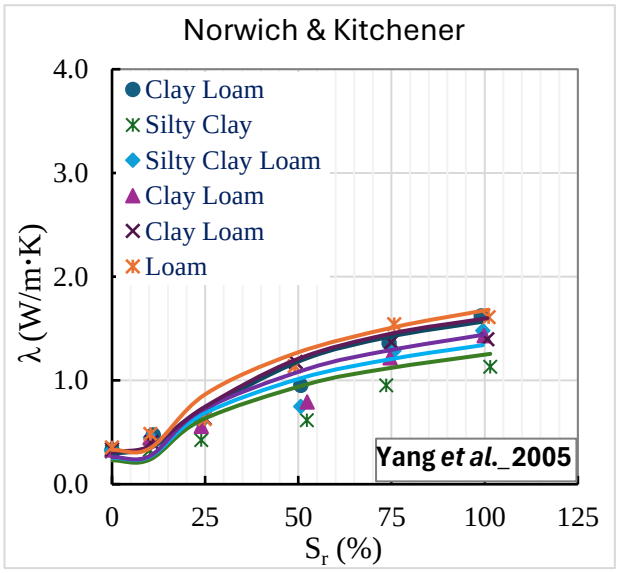
The Johansen (1975) model effectively predicts soil thermal conductivity across various soil types and conditions. The soil texture and saturation align well with the measured data, especially for clay loam and loam soils. However, slight underestimations occur in high-moisture conditions, particularly for sandy loam and silty clay soils, as depicted in Figures 16(b) and 16(c). Despite these minor discrepancies, the model remains reliable and suitable for different soil environments. The Yang *et al.* (2005) model accurately estimates soil thermal conductivity (λ_{eff}) across various soil types and moisture conditions by considering key factors such as soil texture and saturation. Figure 17 shows that the regression lines align closely with the measured data points, indicating high accuracy. While minor deviations are observed in high-moisture conditions, such as silty clay and clay in locations like Chatham and Harriston (Figure 17a and 17b), these differences are minimal and do not detract from the model's overall strong performance. Overall, Yang *et al.* (2005) provide a reliable and robust approach for predicting soil thermal conductivity across diverse environmental conditions.



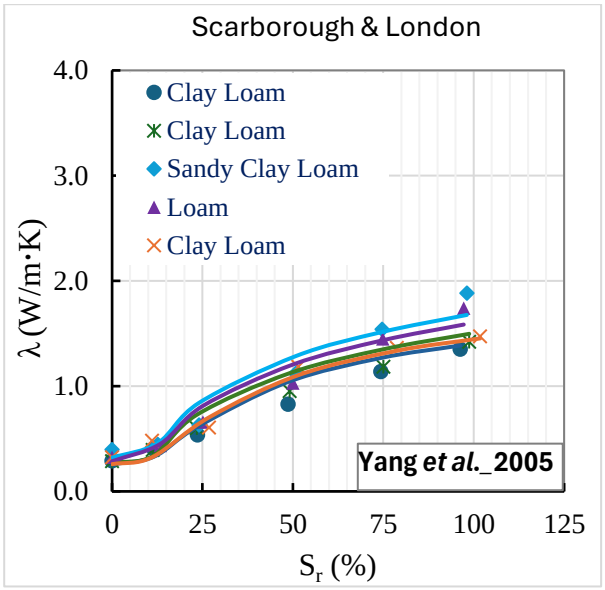
(a) Chatham



(b) Harriston and Woodstock



(c) Norwich and Kitchener



(d) Scarborough and London

Figure 17: A comparison of measured thermal conductivity-degree of saturation relationship by Yang et al. (2005)

Chapter 4
Conclusions and Recommendations

4.1 General

This study examined the thermal conductivity of soils collected from various locations in Southern Ontario, integrating geotechnical characterization and laboratory-based thermal conductivity measurements. The experimental results are systematically compared with established regression-based and normalized thermal conductivity models to assess their predictive accuracy and applicability. A rigorous statistical analysis is conducted to identify the models best aligned with the measured data. Based on the study's results, the following conclusions are discussed below.

4.2 Conclusions

This study systematically compared measured thermal conductivity values with predictions from both regression-based and normalized models, evaluating their accuracy across various soil samples. The results demonstrate that while many models align well with measured data, discrepancies exist due to differences in soil composition, moisture content, and model assumptions. These differences highlight the importance of selecting models that best account for specific soil properties and environmental conditions.

The statistical evaluation of regression-based and normalized thermal conductivity models identified the most reliable models for predicting soil thermal conductivity under varying conditions. Donazzi *et al.* (1979) and Lu *et al.* (2014) emerged as the best-performing regression-based models, exhibiting low RMSE, high R^2 values, and minimal bias, making them suitable for various soil textures and moisture conditions. Similarly, Johansen (1975) and Yang et al. (2005) effectively captured variations across diverse soil types among the normalized models. Statistical validation using RMSE, MAE, and R^2 reinforced the reliability of these models, emphasizing the importance of calibration and refinement.

The analysis further confirmed that Côté and Konrad (2005) performed well but required additional calibration for extreme soil compositions. These findings provide valuable insights into model selection and optimization, highlight the significance of statistical validation in model selection, and reinforce the need for calibration and refinement to enhance predictive accuracy. The identified models can be practical tools for improving soil thermal conductivity estimation in geoenvironmental applications.

Lastly, evaluating soil texture's impact on thermal conductivity revealed notable variations in model performance across different environmental conditions. Experimental results indicated that silty and sandy loam soils exhibited higher thermal conductivity due to their increased proportion of solid

minerals. However, despite their high porosity and lower water retention capacity, sandy soils showed a slight reduction in thermal conductivity. These variations directly influenced the accuracy of predictive models, particularly under fluctuating moisture conditions. These models consistently aligned with measured data, accurately reflecting the higher thermal conductivity observed in silty soils and the comparatively lower values recorded for sandy soils.

These findings emphasize the necessity of selecting models that critically influence soil texture, moisture content, and bulk density on thermal conductivity. The identified best-performing models provide valuable predictive tools for soil thermal conductivity estimation, offering practical applications in geotechnical engineering, environmental studies, and energy-related infrastructure.

4.3 Future Recommendations

Based on this study's findings, the following recommendations are proposed to enhance further the accuracy and applicability of soil thermal conductivity models: First, the best-performing models identified in this study should undergo further refinement and calibration using additional experimental data. Expanding the dataset to include a broader range of soil types, mainly highly organic and coarse-textured soils, will improve predictive accuracy and ensure broader applicability across diverse environmental conditions and geographic regions.

Additionally, further research should explore the impact of hysteresis on heat transport in soils under controlled laboratory conditions. Understanding hysteresis effects is crucial for improving thermal performance predictions, particularly for underground energy storage systems and ground heat exchanger applications. Moreover, future studies should examine the influence of external factors, including temperature fluctuations and soil compaction, on thermal conductivity. Incorporating these variables will help develop more comprehensive and adaptable models. Addressing these aspects will refine predictive methodologies, leading to more accurate engineering designs and sustainable environmental applications.

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Appendix A

Table: Geotechnical Properties and Measured Thermal Conductivity of Soil Samples at Varying Saturation Levels

Serial No	Sample ID	Location	Porosity	Void Ratio	ρ_{dry} (gm/cc)	Average λ (W/m·K)						Calculated S_r (%)						Calculated w (%)					
						0	10	25	50	75	100	0	10	25	50	75	100	0	10	25	50	75	100
01	CH_01	Chatham	0.317	0.464	1.861	0.42	0.55	0.92	1.25	1.46	1.58	0.00	10.00	25.00	50.00	75.25	100.00	0.00	1.78	4.45	8.89	13.34	17.79
02	CH_02		0.288	0.405	1.739	0.43	0.46	0.71	0.98	1.08	1.20	0.00	10.00	25.00	50.00	75.97	100.00	0.00	1.55	3.87	7.75	11.62	15.49
03	CH_03		0.332	0.498	1.782	0.37	0.43	0.76	1.05	1.18	1.26	0.00	10.00	25.00	50.00	76.53	100.00	0.00	1.91	4.78	9.55	14.33	19.11
04	CH_04		0.312	0.453	1.814	0.37	0.49	0.71	1.06	1.28	1.54	0.00	10.00	25.92	55.87	72.98	100.66	0.00	1.72	4.55	9.60	12.56	17.30
05	CH_05		0.316	0.462	1.790	0.37	0.43	0.77	0.94	1.02	1.33	0.00	10.00	27.65	55.94	70.30	102.50	0.00	1.77	4.75	9.87	12.54	18.88
06	HAR_1	Harriston	0.316	0.461	1.799	0.346	0.435	0.664	0.982	1.480	1.565	0.00	9.93	28.60	47.77	75.24	103.08	0.00	1.75	5.06	8.38	13.21	18.09
07	HAR_2		0.324	0.479	1.795	0.400	0.482	0.820	1.324	1.552	1.613	0.00	12.86	27.75	47.73	72.95	102.38	0.00	1.81	5.01	8.62	13.17	18.48
08	WOD_01	Woodstock	0.545	1.197	1.196	0.181	0.203	0.265	0.522	0.799	0.995	0.00	13.07	26.87	51.53	78.79	101.79	0.00	4.56	12.20	23.47	35.91	49.19
09	WOD_02		0.484	0.936	1.371	0.227	0.320	0.406	0.620	0.946	1.140	0.00	12.95	25.48	49.31	73.98	100.87	0.00	3.53	8.99	17.39	26.20	35.58
10	NOR_1	Norwich	0.335	0.504	1.747	0.328	0.474	0.650	0.953	1.360	1.620	0.00	11.06	23.91	50.66	74.36	99.06	0.00	1.92	4.80	9.59	14.39	18.81
11	NOR_2		0.410	0.694	1.567	0.313	0.327	0.423	0.615	0.951	1.129	0.00	10.16	23.93	52.30	73.58	101.50	0.00	2.62	6.26	14.44	18.93	26.81
12	NOR_3		0.373	0.594	1.646	0.325	0.450	0.598	0.748	1.284	1.479	0.00	10.28	23.91	50.66	75.68	99.51	0.00	2.26	5.66	11.32	16.98	22.53
13	NOR_4		0.364	0.572	1.667	0.328	0.447	0.556	0.790	1.218	1.430	0.00	10.25	23.93	52.41	74.59	99.89	0.00	2.18	5.28	11.44	16.54	21.81
14	SCAR_01	Scarborough	0.378	0.607	1.652	0.292	0.403	0.536	0.830	1.140	1.353	0.00	11.57	23.61	48.67	74.37	96.24	0.00	2.08	5.21	10.42	15.64	20.07
15	SCAR_02		0.354	0.548	1.698	0.288	0.396	0.602	0.953	1.260	1.490	0.00	11.24	23.43	49.12	75.08	98.76	0.00	2.57	5.36	11.24	17.18	22.59
16	SCAR_03		0.308	0.446	1.815	0.397	0.472	0.632	1.120	1.540	1.886	0.00	12.64	24.11	50.85	74.67	98.11	0.00	1.70	4.25	8.49	12.74	16.66
17	SCAR_04		0.335	0.504	1.735	0.325	0.438	0.659	1.030	1.450	1.74	0.00	13.34	25.13	50.01	74.90	97.24	0.00	2.50	4.85	9.66	14.47	22.72
18	KIC_01	Kitchener	0.317	0.464	1.796	0.336	0.406	0.64	1.176	1.39	1.51	0.00	10.58	24.85	49.27	74.930	100.730	0.00	1.87	4.38	8.69	12.58	17.62
19	KIC_02		0.307	0.444	1.839	0.353	0.484	0.629	1.136	1.45	1.61	0.00	10.31	24.90	49.06	75.76	101.06	0.00	1.72	4.16	8.20	12.67	16.89
20	LON_01	London	0.385	0.626	1.658	0.325	0.365	0.609	1.184	1.365	1.475	0.00	11.07	26.87	51.53	78.79	101.79	0.00	2.36	5.72	11.32	17.38	23.48

Appendix B

Table: Soil Classification of Collected Samples as Per United States Department of Agriculture (USDA)

Serial No	Sample ID	Site No	Location	Sand (%)	Silt (%)	Clay (%)	Soil Classification
01	CH_01	01	Chatham	43.56	32.84	23.60	Loam
02	CH_02			12.25	42.37	45.38	Silty Clay
03	CH_03			19.58	40.84	39.58	Silty Clay Loam
04	CH_04			38.24	36.16	25.60	Loam
05	CH_05			31.89	38.53	29.58	Clay Loam
06	HAR_1	02	Harriston	53.24	25.71	21.05	Sandy Clay Loam
07	HAR_2			58.69	27.06	14.25	Sandy Loam
08	WOD_01	03	Woodstock	25.36	20.96	53.68	Clay
09	WOD_02			35.17	19.47	45.36	Clay
10	NOR_1	04	Norwich	39.57	29.18	31.25	Clay Loam
11	NOR_2			11.14	45.71	43.15	Silty Clay
12	NOR_3			16.47	48.34	35.19	Silty Clay Loam
13	NOR_4			30.26	34.31	35.43	Clay Loam
14	SCAR_01	05	Scarborough	28.36	39.73	31.91	Clay Loam
15	SCAR_02			36.54	30.72	32.74	Clay Loam
16	SCAR_03			46.51	27.62	25.87	Sandy Clay Loam
17	SCAR_04			44.17	32.36	23.47	Loam
18	KIC_01	06	Kitchener	37.59	33.76	28.65	Clay Loam
19	KIC_02			44.62	36.62	18.76	Loam
20	LON_01	07	London	38.33	31.78	29.89	Clay Loam

Appendix C

Table: Statistical Performance Evaluation of Different Models to Predict the Thermal Conductivity of Soil at Varying Moisture Content

Serial No	Model Type	Model Name	Statistical indices		
			Bias	R^2	RMSE
01	Regression-Based	Kersten (1949)	0.08	0.86	0.21
02		Donazzi *et al.* (1979)	0.10	0.84	0.17
03		Kahr & M.Vonmoos (1982)	-0.18	0.44	0.33
04		Campbell (1985)	-0.20	0.53	0.30
05		Chung & Horton (1987)	-0.33	0.08	0.43
06		Caridad *et al.* (2014)	0.18	-0.11	0.47
07		Lu *et al.* (2014)	-0.07	0.90	0.14
08		Yoon *et al.* (2018)	0.01	0.74	0.23
09	Normalized Based	Johansen (1975)	0.05	0.90	0.15
10		Farouki (1981)	0.49	0.40	0.66
11		Hu *et al.* (2001)	0.49	-0.50	0.58
12		Côté and Konrad (2005)	0.14	0.82	0.21
13		Yang *et al.* (2005)	0.05	0.91	0.15
14		Markle *et al.* (2006)	0.32	0.11	0.44