

Competitive Project Portfolio Management

by

Mark Steven Zschocke

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Abstract

Although project portfolio management (PPM) has been an active research area over the past 50 years, budget allocation models that consider competition are sparse. Firms faced with the project portfolio management problem must not only consider their current projections for the returns from their projects' target markets, but must also anticipate that these returns can depend significantly on the investment decisions made by their competitors. In this thesis, we develop four Competitive PPM (CPPM) models wherein firms allocate resources between multiple projects and project returns are influenced by the actions taken by competitors.

In the first two CPPM problems, we assume all-or-nothing project investment decisions where firms fully commit to either a project targeting a mature or an emerging market and the investment amount is fixed (first model) or a decision variable (second model). In the final two CPPM problems, firms have a fixed budget which they allocate in a continuous manner between two markets (third model) or multiple markets (fourth model). The returns each firm obtains from investments into these markets are assumed to follow an s-shaped curve (first model), the Inada (1963) conditions (third model), or are determined based on linear demand functions (second and fourth model).

In the first model, two competing firms consider investing into two separate projects targeting a mature and an emerging market. We assume that firms have symmetric investment opportunities for each market and each firm simultaneously decides whether to invest in the mature or the emerging market. The returns from these markets are assumed to follow an s-shaped curve and depend on both firms' investment decision. We characterize the variety of interactions that may emerge in symmetric environments (e.g., Prisoner's Dilemma or Game of Chicken). For each game, we outline the CPPM strategy that can offer higher returns by

exploiting first-mover advantages, cooperation opportunities and aggressive choices. We also discuss the market conditions that lead to these games.

In the second model, a similar CPPM setting is considered where two symmetric firms face two target markets. However, we assume that demand for the emerging market is uncertain and may expand through firms' market entry (positive diffusion effects), while demand for the mature market is known with certainty and cannot expand. Firms decide when to invest, in which market to invest, and how much to invest into this market. Our analysis reveals that the existence of multiple investment opportunities may induce firms to delay their investment even in the absence of demand uncertainty, and that high diffusion effects coupled with low demand uncertainty can drive firms to invest early even if both firms could increase returns by delaying their investment. We then study the asymmetric case where firms differ with respect to their costs and diffusion effects and show some counter-intuitive results.

In the third CPPM problem, we consider continuous budget allocations and prove that while a monopoly firm bases its budget allocation decision solely on the marginal returns of the two markets, duopoly firms also account for their average returns from the two markets. This drives duopoly firms, in particular the firm with the smaller budget, to invest more heavily into the mature market. We show that as a firm's budget increases, the share of its budget that is invested into the mature market decreases while its competitor's investment into the mature market increases. This chapter also explores how changes to the market parameters and market uncertainty affect the resource allocation decision of firms under competition. Considering the special case of identical budgets, we prove that as the number of competing firms increases (with a fixed total budget), firms allocate an even greater share of their budget into the mature market.

The fourth model considers a general case where a number of budget-constrained firms engage in production decisions for multiple markets under competition. Each firm decides how much to produce for each market, subject to its budget constraint. We prove that firms produce greater quantities for markets with higher than average base demand and that these quantities are increasing in the number of competitors (assuming identical production capacities). With asymmetric production capacities, we numerically illustrate how firms with large production capacities may, instead, increase production into lower than average base demand markets. Furthermore, we characterize the increase in return firms can expect from budget increases and conjecture that if some markets are not served by all firms, the remaining firms reduce their production into those markets where some firms are not producing.

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TABLE OF CONTENTS

List of Tables	x
List of Figures.....	xi
1. Introduction.....	1
1.1. Motivating Example and Industry Cases	2
1.2. Structure of Thesis and Contributions	4
2. Literature Review.....	10
2.1. An Introduction to Project Portfolio Management	10
2.1.1. Areas of Application.....	11
2.1.2. Strengths and Weaknesses of PPM.....	12
2.2. Internally Focused Tools and Methods.....	13
2.2.1. Qualitative Tools and Methods.....	13
2.2.2. Quantitative Tools and Methods.....	15
2.3. Externally Focused Tools and Methods.....	17
2.3.1. Multidimensional Knapsack Problems	17
2.3.2. Real Options with Competition	19
2.3.3. Multiproduct and Multimarket Competition	19
2.3.4. Innovation Contests, All-pay auctions and R&D Races.....	22
2.4. Literature Review Conclusions.....	25
3. CPPM: Binary Investment Decisions	26
3.1. The Model.....	26
3.1.1. Market Parameters	27
3.1.2. CPPM Decisions	28
3.2. Potential Interactions between Firms.....	30
3.2.1. Market Parameters	36
3.3. Discussion and Results	38
3.3.1. CPPM Strategies	38
3.3.2. Market Sizes	40
3.4. Other Market Combinations	42
3.5. Non-symmetric Firms	43
3.6. Chapter 3 Conclusions	46
3.7. Proofs for Chapter 3.....	47

4. CPPM with Uncertain Returns: When, where and how much to invest	50
4.1. The Model	51
4.2. Model Analysis	55
4.3. Non-symmetric firms	65
4.3.1. Strategic Negligence	65
4.3.2. Multinational vs. Local Firm	67
4.4. Chapter 4 Conclusions	68
4.5. Proofs for Chapter 4.....	70
5. CPPM: Continuous Investment Decisions	87
5.1. The Monopoly Benchmark	88
5.2. Effect of Competition – The Duopoly Case	92
5.2.1. Budget Considerations.....	96
5.2.2. Effect of Market Uncertainty.....	99
5.2.3. Differences in Marginal Productivity	100
5.3. Oligopolies	102
5.4. Chapter 5 Conclusions	104
5.5. Proofs for Chapter 5.....	105
6. CPPM - Large Portfolios: Budget Constrained Multimarket Cournot Competition..	112
6.1. Unconstrained Multimarket Cournot Equilibrium Benchmark	113
6.2. Budget Constrained Multimarket Cournot Equilibrium	114
6.2.1. Effect of Competition on Firms with Symmetric Budgets	116
6.2.2. Effect of Competition on Firms with Asymmetric Budgets	119
6.2.3. Budget Adjustments.....	122
6.3. Market Abstaining	123
6.4. Budget constrained Multimarket Equilibrium - Varying Costs and Demand Slopes ...	127
6.4.1. Increasing Production Costs may Increase Profits	128
6.4.2. Beneficiaries from Changes in Demand slopes	130
6.5. Chapter 6 Conclusions	132
6.6. Proofs for Chapter 6.....	133
7. Conclusions and Future Directions	142
Appendix.....	145
Appendix A: Maple Code	145
CPPM: Binary Investment Decisions – Code for Figure 3.3	145

CPPM with Uncertain Returns – Code leading to Figure 4.3	145
CPPM: Continuous Investment Decisions - Code for Figure 5.3	147
CPPM: Multimarket Cournot Competition – Code for Figure 6.1 and Figure 6.2	148
Appendix B: List of Notations	150
All Chapters	150
CPPM: Binary Investment Decisions	150
CPPM with Uncertain Returns: When, where and how much to invest	150
CPPM: Continuous Investment Decisions	151
CPPM: Budget Constrained Multimarket Cournot Competition	151
References	152

LIST OF TABLES

Table 3.1: Returns of symmetric firms	29
Table 3.2: Example Game 1 in cardinal form.....	31
Table 3.3: Example Game 2 in cardinal form.....	31
Table 3.4: Example Game 2 in ordinal form	31
Table 3.5: Game of Convergence in ordinal form	33
Table 3.6: The Prisoner's Dilemma in ordinal form.....	33
Table 3.7: Game of Chicken in ordinal form.....	34
Table 3.8: The Stag Hunt in ordinal form.....	34
Table 3.9: Game of Spite in ordinal form	35
Table 3.10: Game of Union in ordinal form	35
Table 3.11: Game of Apology in ordinal form	42
Table 3.12: The Battle of Sexes in ordinal form.....	42
Table 3.13: Returns of non-symmetric firms.....	43
Table 4.1: Capacity game equilibria	56
Table 4.2: The Prisoner's Dilemma in ordinal form.....	85
Table 6.1: Equilibrium production summary.....	125
Table 6.2: Equilibrium production quantities and resulting profits.....	128
Table 6.3: New unit production cost parameters	129
Table 6.4: Equilibrium production quantities and profits with higher production costs	129
Table 6.5: Equilibrium production quantities and resulting profits.....	131
Table 6.6: Equilibrium production quantities and profits with differing demand slopes	132

LIST OF FIGURES

Figure 1.1: Overview of models	4
Figure 3.1: Market Characteristics.....	28
Figure 3.2: Four possible states of sole vs. joint market entry	32
Figure 3.3: All possible CPPM games.....	36
Figure 3.4: CPPM games	41
Figure 3.5: Return depending on budget size of Firm 1	46
Figure 4.1: Timeline of decisions	53
Figure 4.2: Firms' investment decisions in extensive form.....	54
Figure 4.3: Optimal investment strategies of market and timing game.....	59
Figure 4.4: Negligent vs. Attentive Firm.....	66
Figure 4.5: Multinational vs. Local Firm.....	67
Figure 5.1: Market returns of mature and emerging markets	90
Figure 5.2: Resource allocation of monopoly versus duopoly firms	96
Figure 5.3: Optimal resource allocation into mature market	98
Figure 5.4: Optimal resource allocation into mature market	100
Figure 5.5: Optimal resource allocation into mature market for different budgets and β	102
Figure 6.1: Total equilibrium production quantities as a function of N	118
Figure 6.2: Equilibrium profit as a function of N	119
Figure 6.3: Equilibrium production quantities as a function of N	120
Figure 6.4: Equilibrium profit as a function of N	122

1. INTRODUCTION

Project Management is about doing projects right;

Portfolio Management is about doing the right projects.

Cooper et al. (2000)

Allocating scarce resources over a range of project alternatives is an essential decision that all organizations make. Consequently, the decision that organizations face is not whether to engage in Project Portfolio Management (PPM) but how to engage in it. Approaches to PPM vary, ranging from ad-hoc resource allocation decisions by individual managers to formal processes that have been adopted throughout all levels of the firm. Although the success of PPM implementations has been mixed, any type of formal PPM process is better than ad-hoc decision-making (Cooper et al., 2004).

PPM was first examined by Lorie and Savage (1955) in the context of a capital budgeting problem in which firms choose from a selection of projects with different costs and returns, subject to a budget constraint. They acknowledged the complexity of the problem and used a trial-and-error method to obtain solutions to the problem. A decade later, Petersen (1967) used a knapsack formulation to solve the PPM problem and implemented an algorithm to find an optimal solution. However, the complexity of the model and the computational limitations of that time restricted him to smaller problems. Since these early days of PPM, many additional features have been incorporated into formal PPM models, including uncertainty with respect to project costs and returns, multiple resource constraints and the resulting threats of bottlenecks, correlations between projects in terms of both costs and returns, short-term versus long-term objectives and the degree to which projects fit the overarching strategy of the firm. We next

provide an illustrative example with the considerations and trade-offs associated with PPM problems.

1.1. MOTIVATING EXAMPLE AND INDUSTRY CASES

Let us assume that a firm (Firm A) is contemplating how to allocate its R&D budget for the upcoming year. Firm A's portfolio of projects includes, among others, an incremental project targeting a mature market and a radical project targeting an emerging market. Allocating resources across hundreds of possible projects is clearly difficult but even deciding between these two projects can be challenging. Suppose the radical project has a higher expected return than the incremental project but it also carries more risk. In addition, resources for the incremental project may be more readily available but the radical project, while more costly, may provide a better fit with the current strategy of the firm. The complexity of this decision has been addressed in many ways by academia and industry, yet a fundamental complexity is still missing from this description: the effect of competition. While Firm A is making its PPM decisions, its main competitor, Firm B, is contemplating its own portfolio of projects. Firm B may be deciding between two projects that target the same markets as Firm A's projects. In many instances, the portfolio decisions made by one firm will influence the outcome and profits of the other: both firms investing into a mature market can quickly saturate the market and lead to disappointing returns while joint entry into an emerging market may enlarge the market, resulting in increased profits for both firms.

There are many industries where two competing firms may be faced with these types of trade-offs in their portfolio decisions. Examples are soft drink firms (e.g., a new packaging project vs. a new drink creation project), airline carriers (e.g., a minor route addition vs. a new hub placement), and automotive firms (e.g., an interior design modification vs. a new motor

development). Although these examples and the description of possible outcomes are based on speculation, it is clear that firms that compete in the same markets will influence each other's returns with their portfolio decisions. Consequently, competition should influence firms' resource allocation decisions. We refer to PPM when faced with competition as Competitive Project Portfolio Management (CPPM). CPPM is the focus of this research. Throughout this thesis, we assume that each project in a firm's project portfolio represents an investment or production opportunity for a particular market. Consequently, we use the terms "investing into a project" or "investing into a market" interchangeably.

Firms that ignored or misjudged the effect of competition in their project selections have paid a steep price. For example, in the early 1980s, Frontier Airlines expanded beyond its Denver hub without anticipating the subsequent increase in competition in their core Denver market (Bulow et al., 1985). A decade later, DuPont focused too much of its estimated \$2 billion annual budget on projects aimed at improving existing lines of businesses, thereby making itself vulnerable to competitors that focused more on innovative projects (BusinessWeek, 2003).

Other firms made CPPM decisions by paying careful attention to their competitors' actions. For example, by the end of the 1990s PepsiCo specifically sought out markets in which the Coca Cola Company was not operating, thereby increasing their international revenue and returns dramatically (Yoffie, 2004). In the Niagara Wine Region of Canada, wineries have benefited from other wineries offering competing wine tasting services. Although this has increased competition in the region, the number of wineries in a region is a key driver of increasing tourism, thereby expanding the overall market (Getz and Brown, 2006). While

these examples are quite different, it is evident that management of project portfolios needs to account for the effects of competition.

1.2. STRUCTURE OF THESIS AND CONTRIBUTIONS

The structure of this thesis is as follows. Following this introduction, Chapter 2 reviews the literature on PPM. Chapters 3 – 6 study four different CPPM problems, each written in manuscript form and providing their own independent contributions. The differences between the models are presented in Figure 1.1.

Chapter	<i>3: Binary investments</i>	<i>4: When, where and how much to invest</i>	<i>5: Continuous resource allocation</i>	<i>6: Multimarket resource allocation</i>
# of Firms	2	2	$N \geq 2$	$N \geq 2$
# of Projects/Markets	2	2	2	$S \geq 2$
Investment Decision	Binary	Binary, continuous	Continuous	Continuous
Resource Constraint	Number of projects	Number of projects	Budget	Budget
Market Returns	S-shaped	Linear demand	Follow Inada (1963) conditions	Linear demand
Market Uncertainty	No	Yes	Yes	No
Diffusion Effects	Implied in return function	Explicitly modeled	No	No
Tools Used	Closed form solutions, Maple, Excel			

Figure 1.1: Overview of models

In Chapters 3 and 4, firms make all-or-nothing project investment decisions into either a mature or an emerging market and the investment amount is fixed (Chapter 3) or a decision variable (Chapter 4). In Chapters 5 and 6, firms allocate a fixed budget in a continuous manner

between two markets (Chapter 5) or multiple markets (Chapter 6). We assume that market returns follow an s-shaped curve (Chapter 3), the Inada (1963) conditions (Chapter 5), or are determined based on linear demand functions (Chapters 4 and 6). For each of these models, we further assume that firms have complete information about the characteristics of each other and the markets. While this is a strong assumption, it is reasonable because firms that compete in the same industry often have a similar understanding of the markets and can estimate each other's costs (see Wu and Parlar (2011) for treatment of incomplete cost information). The computational analysis for this thesis was completed in Maple and Excel (selected Maple code is included in Appendix A).

In Chapter 3, we study a duopoly in which symmetric firms consider project investment opportunities into two separate projects targeting a mature and an emerging market. Firms are constrained to invest in only one of the two markets and the markets provide returns according to an s-shaped function. Considering the case where firms have symmetric project investment opportunities, we find that various games may occur between the two firms (such as the Prisoner's Dilemma and the Game of Chicken) and that, given our assumptions, a pure strategy exists for all possible games. Each of these situations requires a different strategy. We further characterize the market parameters and investment opportunities that lead to particular strategic interactions between the two firms.

To gain insights in the more general case, we conduct a computational analysis where firms are constrained by an investment budget and have non-symmetric project investment opportunities. We show that in some instances, a firm may be better off when its competitor's budget increases.

In Chapter 4, we examine a setting where two firms are each considering two alternative project investments, one targeting a mature market and one an emerging market. However, in this model, demand for the emerging market is uncertain and subject to diffusion effects while the demand for the mature market is known with certainty and does not increase through either firm's entry. Our analysis reveals that the existence of multiple investment opportunities (i.e., multiple markets) in the investment portfolio can cause firms to delay their investment even if there is little to no demand uncertainty. This is in contrast to the existing literature which has considered a single market, where the strategic position gained from having the first mover advantage drives firms to invest early unless there are significant cost advantages to investing late or demand uncertainty is significant (Kulatilaka and Perottie, 1998; Swinney et al., 2011).

Our model also demonstrates that when an emerging market's high diffusion effect is coupled with low demand uncertainty, the competitive dynamics can drive firms to invest too early. Another challenging competitive dynamic can evolve if both firms prefer to invest as a monopoly in either market to Cournot investment in the respective other market. Such preferences lead to multiple equilibria and potentially poor outcomes if firms both invest into the same market. Our analysis of the market parameters reveals a key ratio (the difference between average demand and unit cost in the mature market divided by the difference between average demand and unit cost in the emerging market) that can help managers make timely decisions and focus their resources.

We proceed with numerical illustrations to highlight a number of important findings if firms are not symmetric. In the first scenario, two firms differ with respect to their costs in the mature market. Counterintuitively, a negligent firm that fails to keep its unit cost stable in the mature market may improve the dynamics of the game in a way that enables it to receive

higher returns than an attentive firm that maintains a low unit cost in both markets. In the second scenario, one of the firms has lower unit costs in both markets while the other firm generates stronger diffusion effects in the emerging market. We find that the latter firm can receive higher returns than the former, even if the former has a significant cost advantage in both markets.

In Chapter 5, firms face a continuous resource allocation challenge and must decide how to allocate their budget between two projects that target two separate markets. We first study the monopoly benchmark case and proceed with the portfolio decision faced by duopoly firms. This allows us to highlight the effect of competition on the PPM decision. We find that in contrast to a monopoly, which bases its decision on the marginal returns of the two markets, duopoly firms consider the marginal returns as well as the average returns from the two markets. This leads duopoly firms to invest more heavily into the mature market. The difference in investment strategy implies that duopoly firms receive lower total returns compared to a monopoly and that managers that fail to account for these competitive effects could harm their potential returns. This result reveals that under competition, firms have the incentive to allocate a greater share of their budget into the market that offers good returns for small investments. The firms get engaged in a seemingly unnecessary competition over their share of the mature market, resulting in overinvestment into this market and underinvestment into the emerging market, eventually leading to lower combined returns.

We further demonstrate the effect of firms' budgets on their investment decision when faced with competition. We first show that the share that a firm invests into the mature market is non-increasing in its own budget. This is quite intuitive, as a firm with a low budget will invest fully into the mature market, but, as its budget increases and the mature market saturates, it will shift investment into the emerging market. We then show the effect of the

competitor's budget on the investment decision. Firms are very sensitive to the size of their competitor's budget if it is small—a small increase in the competitor's budget can result in a large shift of the share of investment from the emerging into mature market. This is a strong defensive reaction aimed at protecting the average return from the mature market. However, if the competitor's budget is sufficiently high, changes to the competitor's budget affects firms' resource allocation decision only marginally.

We also demonstrate that firms may continue to invest significant resources into the emerging market even if returns from this market become highly uncertain. In addition, we show how the effect of changes to the rate at which markets become saturated depends on the firms' budget sizes. We then extend our findings to oligopolies and prove, for the special case where all firms have identical budgets, that the greater the number of firms competing, the more these firms invest into the project targeting the mature market.

In Chapter 6, we present a closed-form solution to the constrained resource allocation decision where firms are competing against each other in a number of product markets. Each firm has a fixed budget and unit production cost and decides how many units of each product to produce, where each product targets a different market. We assume no substitution effects between markets, but within each market the products of all firms are perfect substitutes. We further assume Cournot quantity competition between firms in each of these markets.

We first consider the symmetric case where all firms have identical production capacities (fixed budget / unit production cost) and prove that firms produce more (less) in markets with higher (lower) than average base demand, as expected. Surprisingly, however, production in high (low) demand markets increases (decreases) as the number of competitors increases (assuming fixed total budget). While competition always decreases firms' profits in unconstrained production settings, we prove that the competitive effect on firms' allocation

decisions reduces the total industry profits as the number of firms increases, even if total industry output is fixed.

With asymmetric production capacities, the effect of competition is more complex. We numerically illustrate how firms with large capacities may increase production into lower than average base demand markets. Furthermore, we characterize the budget's shadow price and prove that the shadow price is greater the smaller the current capacity of a firm.

In some circumstances, a particular market demand may be too low to attract investment by some firms, in particular by firms with low capacities. We conjecture that if some markets are not served by all firms, the remaining firms reduce their production into these markets (compared to a fully served market), taking advantage of the reduction in competition in those markets. We prove this result for an oligopoly setting with three firms and three markets.

By allowing demand slopes to differ by market and unit production costs to differ by firm and target market, we demonstrate how our key findings apply broadly and can illustrate certain counter-intuitive behaviour that may arise in more complex situations. For example, we show that an increase in firms' unit costs can lead to an increase in profits for all firms. In another example, we demonstrate how an increase in demand for a particular market can lead to lower profits for firms producing for that market and higher profits for those firms not producing for that market.

In Chapter 7, we conclude this thesis and discuss future directions for this work. A list of notation is available in Appendix B. PPM has been an active research field over the last 50 years and we believe that the CPPM research problem still offers many opportunities for new contributions that can help decision-makers faced with resource allocation problems.

2. LITERATURE REVIEW

2.1. AN INTRODUCTION TO PROJECT PORTFOLIO MANAGEMENT

There is vast literature dedicated to PPM and its tools and methods have been applied to a wide range of industries. In a review paper, Shane and Ulrich (2004) identified the area of PPM as the second largest stream of Management Science research within the field of technological innovation, product development, and entrepreneurship. Almost every organization must make resource allocation decisions between competing projects and with more organizations becoming project-oriented the need for PPM tools and methods continues to grow.

PPM is a challenging problem for organizations for a range of reasons: project returns are uncertain and can be hard to estimate (Cooper et al., 2001), allocating scarce resources efficiently without creating bottlenecks is crucial to success (Adler et al., 1995), correlations between projects can affect their cost and return estimates (Loch and Kavadias, 2002), the sequence of projects may be important (Kavadias and Loch, 2003), and there are a many behavioural issues that affect project portfolio decisions since the outcomes can heavily influence the careers of decision-makers (Sanwal, 2007). On top of these challenges, the actions of competitors can heavily influence the outcome of PPM decisions (Zhu and Weyant, 2003).

In this literature review, we will describe where PPM has been applied and illustrate its strengths and weaknesses. Subsection 2.2 outlines the key internally focused PPM tools and Subsection 2.3 outlines the PPM tools and methods that are externally focused and consider the effect of competition.

2.1.1. AREAS OF APPLICATION

Although there is a need for PPM in nearly every organization, some industries are particularly suitable candidates for a PPM implementation. Such industries can be characterized as being heavily project-based, where these projects have benefits that are hard to measure, pose significant risks and the number of potential projects generally exceeds the available budget. Unsurprisingly, PPM tools and methods were first developed and employed by such industries.

Perhaps the first area that witnessed a broad application of PPM was the information technology sector. ITPM (information technology portfolio management) was first introduced by McFarlan (1981) but has since garnered significant attention from academia (see for example Bardhan et al., 2004). The IT sector was a suitable candidate because the benefits of IT projects are often hard to quantify yet the costs of IT projects are large, forcing organizations to make tough choices on which IT projects to implement and which ones to defer or decline.

Another massive application area is the pharmaceutical industry (see for example Blau et al., 2004). Pharmaceutical companies are constantly weighing off competing projects progressing down their R&D pipeline. Similar to the IT sector, these projects are very expensive, the eventual return is highly uncertain, and due to resource constraints, not every idea or prototype can be further developed.

PPM decisions are not only crucial at the strategic level but are equally important at the operational level. For example, Loch et al. (2001) applied their PPM methodology to the transmission predevelopment group at BMW in Munich, Germany. In their case study, the transmission group had to select among new technologies and improvements to current technologies to develop the new year-2000 powertrains. Each technology proposal represented

a project with its own distinct potential benefits and challenges. Choosing the right combination of these projects was crucial for developing an efficient, high-quality transmission system.

2.1.2. STRENGTHS AND WEAKNESSES OF PPM

In a broad survey of business units, Cooper et al. (2004) discovered that top performing business units (in terms of NPD success and profitability) were eight times more likely to have implemented a formal and systematic portfolio management process compared to poorly performing business units. It is generally recognized that PPM leads to better resource allocation, closer alignment of projects and overarching business goals, better communication between groups within organizations and generally higher profitability (Sanwal, 2007; Cooper et al., 2001).

In spite of these benefits, PPM tools and methods have not been consistently implemented in industry. Although successful business units are much more likely to have a formal PPM process in place than their less successful counterparts, the actual percentage of successful business units with a formal PPM process is only 31% (Cooper et al., 2004). This lack of PPM implementation has been widely recognized (e.g., Schmidt and Freeland, 1992; Loch et al., 2001). The need for attention to project portfolio and resource allocation decisions has been highlighted by Krishnan and Loch (2005) in a retrospective look at Production and Operations Management articles on new product development.

Since the first paper on PPM by Lorie and Savage (1955), many new tools and methods have been developed and proposed both by academia and by industry, each with their own set of strengths and weaknesses. Some rely on highly uncertain financial data, others reduce real world complexities aggressively, and others are easily manipulated by decision-makers. The

inherent problem with PPM tools and methods is very clearly illustrated by a study conducted by Wind et al. (1983). They demonstrated that project recommendations can vary significantly depending on the PPM tool used to make a project selection. In their study of 15 business units of a large Fortune 500 multinational industrial firm, only 1 out of 15 projects was consistently identified as a clearly worthwhile project to pursue.

2.2. INTERNALLY FOCUSED TOOLS AND METHODS

As the research area matured, the methodologies used to analyze PPM branched into two distinct paths: qualitative and quantitative approaches. The former path has experienced the development of qualitative tools and methods which combine financial data with other aspects, such as strategic fit of projects. These developments aim for a more complete characterization of individual projects. The quantitative research stream addresses shortcomings of early PPM models by capturing further mathematical complexities. However, both branches remained focused on firm-internal aspects of the decision and did not consider competition. Subsection 2.2.1 describes the key qualitative tools and methods that have been developed and Subsection 2.2.2 focuses on quantitative PPM tools and methods.

2.2.1. QUALITATIVE TOOLS AND METHODS

A great number of qualitative tools and methods for PPM have been developed. Due to space constraints, we will only highlight some of the key tools that are most commonly applied and discuss the strengths and weaknesses identified by Cooper et al. (2001), who provide an excellent review of a large number of qualitative PPM tools and methods.

One very common tool is the balanced scorecard (Kaplan and Norton, 2001), where managers score projects on a number of individual dimensions such as project return, risk,

strategic fit, resource requirements, and technological complexity. By summing the rankings, a total score is calculated for each project which then allows for easy comparison between projects. Due to the simplicity of the method, it is popular in industry; however, the final score is susceptible to bias by the managers that rank the projects. Furthermore, this method ignores balance within the portfolio (e.g., between innovative products vs. improvements products).

One PPM tool that is specifically targeted at establishing a well balanced project portfolio is the two-dimensional matrix, originally developed by the Boston Consulting Group (BCG) in the early 1970s (Ghemawat, 2002). First called the “Growth-Share Matrix” by BCG, this method was later adapted to PPM and individual projects were mapped onto a matrix using a wide range of dimensions such as risk vs. reward (Blau et al., 2004), project structure vs. technology level (McFarlan, 1981), and technological attractiveness vs. technological competitiveness (Jolly, 2003). These matrices are popular with managers because they bring clarity to a complex problem and are focused on balancing the portfolio. However, managers can easily be overloaded with too many graphs and this effect is exacerbated by the fact that these maps do not lead to a clear recommendation on which projects to pursue or terminate.

For organizations that want to enforce a balance of projects that match their broader organizational goals, the strategic bucket method of PPM was developed (Cooper et al., 2001). This method is comprised of three steps: first, strategic buckets are defined such as “radical products”, “incremental product improvements”, and “maintenance products”. Second, resources are allocated between these buckets. And lastly, all projects in the firm’s portfolio are allocated to the appropriate bucket and the project portfolio of each bucket is optimized by choosing the most suitable project candidates until the resources of the strategic bucket have been exhausted. In spite of its popularity in industry, the strategic bucket method forces many

tough decisions on managers who must select appropriate buckets, distribute available resources over these buckets, and finally optimize projects portfolios within those buckets.

2.2.2. QUANTITATIVE TOOLS AND METHODS

Another set of researchers tried to address shortcomings of early PPM models by adding further mathematical complexities. Initially, economic and financial models of PPM used net present value (NPV) (Chun, 1994; Sharpe and Keelin, 1998). Although these models aim to maximize the future return of projects, they do not work well in all situations, particularly when financial data is not available or uncertain (Dickinson et al., 2001).

In response to criticism of the initial financial models, PPM tools and methods were expanded to more accurately reflect real world complexities. One of the first extensions was the consideration of risk of the individual projects as well as the risk preference of the decision-maker (Brandon, 2006; Graves and Ringuest, 1991). A further complexity of the PPM problem is that the cost of a particular project can depend on decisions made about other projects. For example, if two projects require the same resource, procurement may get volume discounts when purchasing this resource. Loch and Kavadias (2002) have created a model that takes such correlations into account. Chun (1994) made the point that the timing of tasks within projects is crucial and that these need to be considered when making PPM decision. For example, if two projects have tasks that can be streamlined effectively, this should be recognized as an advantage when choosing which projects to pursue.

Another major branch of PPM models focuses on the timing of decisions when making project selection choices. Researchers have drawn analogies between financial options (such as stock options) and the project selection process, coining the term Real Options (Bardhan et al., 2004; Smit and Trigeorgis, 2004; Dixit and Pindyck, 1994). In a real options framework, a

small investment can be made towards a particular project to “keep the project alive”. Should market conditions evolve favourably, the project can then be pursued fully and large profits can be made. If market conditions are poor, the project can be terminated and the small initial investment is lost. Another PPM approach focuses on the timing of decision by using decision trees (Brandon, 2006; Zhu and Weyant, 2003).

These additional complexities led to the need for more sophisticated mathematical programming and optimization techniques for PPM, including mixed integer programs (Beaujon et al., 2001), non-linear integer programs (Dickinson et al., 2001) and dynamic programming techniques (Loch and Kavadias, 2002). These mathematical programming techniques have been applied in a number of organizations (Schmidt and Freeland, 1992; Loch et al., 2001) but are not widely used (Loch et al., 2001). Just as the simple NPV models, sophisticated mathematical models tend to rely heavily on financial data (Cooper et al., 2001). The reliance on estimates is particularly concerning as these models lack robustness: small changes in the inputs (e.g., the expected return of a project) can have large effects on the solution provided by the mathematical optimization technique. The lack of transparency and the complexity of these models make decision-makers hesitant to fully trust the model recommendations. Although many of these complex economic and financial models have gone beyond merely considering expected return of projects, one of the critical challenges of the PPM problem is that decision-makers are often pursuing more than one objective.

To address this need, multi-criteria decision making tools and methods were developed that consider multiple objectives simultaneously, such as revenue maximization, risk minimization, matching existing resources and competencies with project selection, and choosing projects that align with the stated broad strategy of an organization.

One method of comparing projects based on multiple criteria is to represent each aspect of the decision by a utility function. However, creating utility functions is difficult and has some of the same drawbacks as the balanced scorecard in terms of its vulnerability to biased decision-makers. Another way to reduce the complexity of this multi-criteria problem is the use of the Analytical Hierarchy Process (AHP) that decomposes the PPM decision into smaller sub-problems. This approach allows decision-makers to more easily compare projects and has thus been widely applied to PPM (Saaty, 1994). However, the implementation of AHP is time intensive and can lead to bad decisions (Cheng et al., 2002).

2.3. EXTERNALLY FOCUSED TOOLS AND METHODS

In the research noted so far, competitive forces are only captured implicitly or in passing. Some qualitative tools can capture the impact of competition: for example, the balanced scorecard could easily include a category in which managers are asked to rate the expected aggression level of competitors' actions. In the previously described quantitative tools, competition has only been considered implicitly: for example, risk can be thought to include the risk of some form of negative competitive response. However, none of these tools consider strategic interactions between competitors explicitly. To date, the field of externally focused, quantitative tools and methods for project portfolio management remains understudied – notable exceptions are described below.

2.3.1. MULTIDIMENSIONAL KNAPSACK PROBLEMS

Multidimensional knapsack problems (MKP), a more general form of the standard 0-1 knapsack problem, have many applications within PPM. For example, the standard PPM knapsack problem of maximizing revenue subject to a budget constraint could be expanded to

a multidimensional PPM knapsack problem in which revenue is maximized subject to a financial budget and a manpower resource constraint. However, few MKP have been applied to strategic decision with multiple decision makers. See Freville (2004) for an excellent review of MKP.

Meier et al. (2001) apply a knapsack formulation to a situation with multiple possible states (or scenarios). Using their model, external forces could be considered by treating these states as different levels of competitive response. However, this does not allow for strategic interactions in which multiple decision-makers are influencing each others choices. A recent paper by Gibson et al. (2009) considers competitive actions more explicitly. In their model, multiple decision-makers make sequential decisions on how to allocate their resources over indivisible objects. This multidimensional knapsack problem is very hard to solve to optimality and the authors are forced to simulate the actions of competitors and then apply an efficient search heuristic to find good solutions. Although this problem is close to the PPM problem, it differs in that as soon as one decision-maker chooses a particular object (or project), this object is no longer available to the competition. One could argue that the first-mover advantage in certain product categories would have a similar effect by basically removing the incentive of the other firm to pursue the same project; however, their framework is generally more suitable to their own example, namely a sports draft: teams have a certain budget to spend on new players of various costs and as soon as one team chooses a player, that player is off the market.

Apart from this pioneering paper, little has been done on the multidimensional knapsack problem with multiple decision-makers. Even the very comprehensive book on knapsack problems by Kellerer et al. (2004) does not make reference to any such model. This is perhaps

an expression of how difficult even the single decision-maker multidimensional knapsack problem is.

2.3.2. REAL OPTIONS WITH COMPETITION

In the field of real options, some attempts have been made to include competitive forces. Kulatilaka and Perotti (1998) analyzed a duopoly setting where firms trade-off the benefit of delaying investment until uncertainty is resolved with the advantage of investing early and becoming the leader in the market. They showed that, contrary to earlier work on real options, an increase in market uncertainty does not necessarily increase the benefit of delaying investment. Their work has been extended in multiple ways, such as in the asymmetric information case where firms' knowledge of the market uncertainties varies (Zhu and Weyant, 2003), and the analysis of the particular dynamics when an established firm competes with a start-up firm, where the former firm maximizes returns and the latter firm maximizes its probability of survival (Swinney et al., 2011).

Although this problem framework is directly embedded in resource allocations towards new products, the decision evolves around a single market. However, in a PPM problem, firms are considering how to allocate resources between multiple projects that may compete in different markets.

2.3.3. MULTIPRODUCT AND MULTIMARKET COMPETITION

Tangential to the operations research literature on PPM, multiple papers in the economics literature have addressed the resource allocation decision where firms decide how much to invest (or produce) in multiple markets. Analogous to the PPM problem, each target market can be viewed as a separate project. Although most models of competition in the economic literature have focused on single-market settings, the lack of multiproduct or multimarket

models was noted early by Brander and Eaton (1984). They focused on the demand effects, noting that “interactions between demand for different products, and the associated strategic effects, are important determinants of the products a single firm will produce.” They found that firms may produce very similar products as a strategic preemptive move to keep competition out of their market. However, they also showed that the threat of increased competition can drive firms to invest in distant product substitutes to increase competition for potential entrants who may be contemplating entering markets with new products. Bulow, Geanakoplos and Klemperer (1985) further revealed the complex effects of competition in a multiproduct setting by demonstrating how a firm’s opportunity in one market may influence its competitor’s action in unrelated markets in surprising ways. For example, a firm whose competitor experiences an increase in demand in one of its markets may experience higher profits even if it is not itself competing in the market with increased demand; meanwhile, the competitor who is experiencing an increase in demand in one of its market may actually realize lower profits than before. Bulow et al. (1985) note that key factors in these dynamics are whether products are “strategic substitutes” or “strategic complements”. The focus on product differentiation was continued by Dobson and Waterson (1996), who refined this approach by distinguishing between intraproduct and interproduct rivalry. The former captures the degree to which retailers’ products are viewed as independent or close substitutes, while the latter categorizes how similar the products that are sold in the same store are perceived, ranging from substitutes, to demand-unrelated, to complements. Their key contribution was to demonstrate that the extent of competition directly influences the degree of product diversification. This is complementary to the traditional view that the level of diversification that firms exhibit is mainly driven by risk sharing and asset utilization.

Shaked and Sutton (1990) argued that substitutability has two distinct impacts. First, an “expansion effect” is brought about by the demand for a new product minus the loss of sales from existing products, and a “competition effect” is caused by new entrants. Similar to other work (Brander and Eaton, 1984; Dobson and Waterson, 1996), they developed a two stage model where firms first decide which products to produce and subsequently set the quantity and price of the selected products. They asserted that by modeling the expansion and competition effect explicitly, more intuitive results can be attained. Bernheim and Whinston (1990) pointed out that earlier research on industrial behaviour had focused on internal features of a particular market such as demand condition, concentration, and barriers-to-entry, without considering important external factors to that market. They demonstrated that multimarket contact between firms increases collusive behaviour of firms. Another contribution was brought forward by Zhang and Zhang (1996), who established stability conditions for the Cournot-Nash equilibria in the multiproduct case, highlighting that without these conditions the Nash equilibria may not predict firms’ actions accurately.

A few decades after the onset of research on multiproduct firms, Johnson and Myatt (2006) found that only limited progress had been made in understanding competition among multiproduct firms. Following their previous work (Johnson and Myatt, 2003), they modeled an “upgrade approach” where product lines are differentiated by quality levels, rather than actual products. Through this formulation they were able to demonstrate that many results from the single product setting hold in the multiproduct setting as well. However, they acknowledged that their findings were based on a single product with differing quality levels and that the effect of competition on the production of different products is far more complex.

Bulow et al. (1985) noted that their work on multiproduct oligopolies can be applied to areas such as royalties and license fees, international trade, natural resource markets, and – “most obvious[ly]” – to product portfolio selection. In a product portfolio selection context (which is akin to the PPM problem), firms decide which product to produce and in which quantity to produce those products, while considering competition. A key characteristic of this problem—which is missing in the previously mentioned literature—is that firms are subject to production constraints and need to allocate their limited resources in the most efficient way. To the best of our knowledge, the only paper that addresses our proposed problem is by Laye and Laye (2008), who noted that “despite the simplicity of this setting, this problem is not solved analytically in the literature” and further stated that solutions to the problem are “not a priori obvious”. Laye and Laye (2008) proved the existence of a unique capacity constrained Cournot-Nash equilibrium by transforming the model into a transportation problem formulation; however, they did not explore the properties of the solution.

2.3.4. INNOVATION CONTESTS, ALL-PAY AUCTIONS AND R&D RACES

Another approach to solving the resource allocation problem while considering competition comes from the field of innovation contests, all-pay auctions and R&D races. In innovation contests, firms engage in R&D ‘experiments’ and the firm with the best resulting product wins a prize (Boudreau et al., 2011). Similar to the PPM problem, firms must decide how many resources to invest in these projects. However, innovation contests are centered on a single project and thus do not include the type of trade-off decisions required between projects in the PPM problem.

In a more abstract sense, innovation contests can be seen as all-pay auctions. In an all-pay auction, all players place a certain bid, the highest bidder wins but all bidders must pay their

bid (Baye et al., 1996). Analogously, in innovation contests all players expend resources developing their prototype (submitting their bid), the best innovation wins the prize (highest bid wins), yet the expended resources of all players are non-refundable (all players pay their bid). The all-pay auction literature typically focuses on finding equilibrium states and measuring expected revenue. Baye et al. (1996) demonstrate that with more than two players, a continuum of asymmetric equilibria exists and that the revenue equivalence theorem does not hold in this case. Boudreau (2011) shows that restricting bids to discrete amounts can be advantageous to all involved parties.

R&D races are similar to innovation contests and all-pay auctions except that players are racing to secure a patent (Harris and Vickers, 1985; Grossman and Shapiro, 1987). Examples of R&D races can be found in the pharmaceutical industry where multiple firms are racing to develop a new drug. The first company to successfully claim a patent secures the complete market while the other firms are forced to abandon their efforts, losing their investments to date. Grossman and Shapiro develop a dynamic model that shows how incumbents try to prevent challengers from securing patents and model this defence as a series of small bids (or investments in R&D) that depend on competitive action.

Although innovation contests, all-pay auctions and R&D races consider resource allocation in the context of competition on R&D type projects, each firm only has one project and these projects are competing in the same market. Furthermore, the winner takes all: the non-winning firms lose their investment or bid and receive nothing in return.

The exception in this research stream is the paper by Ali et al. (1993) who more directly work in a PPM framework. In their model, two firms compete with each other and each firm has a portfolio of projects to choose from. This portfolio consists of two projects: one of Type

A (pioneering product) and one of Type B (modification product). Firms must decide which project to invest in and their payoff will depend on each other's decision as well as the timing of their decision. While they are alone in the market, they derive monopolistic profits; when their competitor joins the market, they derive duopolistic profits. These duopolistic profits will depend on whether they invested in the same type of project or not. Furthermore, there is technical uncertainty which affects the duration of time for a successful project completion. On average, pioneering projects are expected to take longer than modification products but in any particular instance, the reverse could occur. Ali et al. were unable to express the Nash equilibrium in closed form and thus used a numeric analysis to derive insights into the problem. Their analysis is very comprehensive and examines a wide range of possible scenarios to establish the effect on equilibria and to provide optimal strategies under a wide range of scenarios. However, their work is also focused on a single market setting.

Gerchak and Parlar (1999) also focused on R&D races and developed a model that considers more than two projects. In their case, firms allocate their budget over the range of available projects in a continuous manner. Given the setting of an R&D race, they assume a "winner takes all" framework where investments into a project increase the likelihood of securing a market, thereby excluding the competition from that market. Finally, Selove (2010) proposed a dynamic investment model in which duopoly firms compete in two market segments and decide in which segment to invest. Selove assumed that market returns are increasing and that due to small random fluctuations, each firm will initially achieve a higher return in different markets. His framework offers an explanation for why firms focus on different markets and continue to invest in markets where they have already established their presence. As described by Selove, there are markets that provide increasing returns due to, for

example, reputation effects or learning curves; however, many markets provide decreasing returns.

2.4. LITERATURE REVIEW CONCLUSIONS

As demonstrated by the broadness of this literature review, the project portfolio management problem has attracted interest from many different researchers. In spite of the large amount of literature on this topic, a significant gap exists: most of the quantitative models of PPM do not consider competitive forces. Furthermore, existing work on PPM models with competition has focused on special cases, such as single market settings and winner-takes-all models. PPM models where firms compete in multiple markets and those market returns depend on competitive actions are scarce.

The need for more work on CPPM models has been addressed by multiple authors. As early as the 1990s, Weinberg (1990) acknowledged the need for resource allocation models that include competitive response. Bower and Gilbert (2005) agreed that “to date, the potential for linkage [between external players and success] to work on resource allocation process has not been exploited” (p. 19). Zhu and Weyant (2003) emphasized that competitive forces are critical to the PPM model. And in their recent review and agenda for *Marketing Science* on the topic of research and innovation, Hauser et al. (2006) declared that a major research challenge is “merging game-theoretic ideas with the real challenges in selecting a line of complex products” (p. 698).

In this thesis, we demonstrate how dramatically competition can impact the PPM decisions of firms, often in surprising ways. In the next chapter, we begin with a model where two firms decide which of two projects to fund and demonstrate that—even in this simple setting—competition can add significant complexity to the PPM decision.

3. CPPM: BINARY INVESTMENT DECISIONS

In this chapter, we study a duopoly in which firms consider project investment opportunities into two projects targeting, separately, a mature and an emerging market. Projects are assumed to have a fixed investment requirement and firms are resource constrained, restricting them to invest in only one of the two markets. Consequently, the key decision faced by the firms is whether to invest into the mature or into the emerging market.

This chapter is organized as follows. In Section 3.1 we develop a CPPM model for the symmetric case where firms have equally sized project opportunities. Using this model, we provide the optimal CPPM decisions under a range of market dynamics in Section 3.2, discuss our findings in Section 3.3 and describe other possible market states in Section 3.4. In Section 3.5 we provide some numerical results from a CPPM model where firms have non-symmetric project opportunities and in Section 3.6 we draw conclusions from our work on the binary investment case.

3.1. THE MODEL

We study a duopoly in which the firms consider investments in projects targeting two markets. Each firm n (where $n = 1, 2$) has a portfolio with two project alternatives, one targeting each market. Firm n , $n \in \{1, 2\}$, has project investment opportunity of $P_{n,s}$ to pursue a project targeting Market s , where $s \in \{M, E\}$. We assume that each project has a well developed business case with a fixed required investment and thus take $P_{n,s}$ as given. Firms are faced with the strategic decision of selecting only one of the two projects. This type of restriction is often the consequence of budget limits and other resource constraints. In our game-theoretical

model, each firm seeks to invest in the project that maximizes its return, given the portfolio of its competitor.

3.1.1. MARKET PARAMETERS

Each market s offers total returns according to a function $f_s(z_s)$, where $z_s = \sum_{n=1}^2 P_{n,s}$ is the total investment by both firms into market s . We assume that f is s-shaped: investing more heavily into a market is initially associated with economies of scale – in this range, returns convexly increase in z ; after a tipping point is reached, larger projects are associated with decreasing returns to scale as the market becomes saturated – returns concavely increase in z within this range. The s-shape of the diffusion of new products has been widely shown empirically (Mahajan and Wind, 1986) and has been previously applied to the PPM problem (Savin and Terwiesch, 2005). Typically, the s-shape relates to cumulative sales or demand for a product over time but it has also been used to model the return on product investment within a project portfolio management setting (Agrah and Geunes, 2009).

Using an s-shaped return function in a PPM setting implies the following two assumptions: first, projects never incur a negative return. Indeed, firms typically only maintain projects in their portfolio that have an expected positive return; second, returns experience diminishing returns for large investments but not declining returns due to over-investment. Overinvestment into a market can lead to falling returns and eventually even negative returns; however, we assume that only profitable projects are considered in a firm's project portfolio.

More specifically, we use the s-shaped return function for market s of the following form, $f_s(z_s) = a_s e^{-\frac{b_s}{z_s}}$, where a_s is the maximum return potential of market s and b_s determines the inflection point (which is $b_s/2$). We assume that one of the markets is mature and the other emerging, where the emerging market has both a higher overall return and a later inflection

point than the mature market. As depicted in Figure 3.1, the mature Market M promises quick returns for small projects (such as adding a feature to an existing product) but quickly becomes saturated. Conversely, the emerging Market E can provide substantial returns for large projects but a significant project investment is required to develop the market. Firms face interesting tradeoffs between harvesting the mature market (but risking overinvestment) or building the emerging market (but risking exploitation).

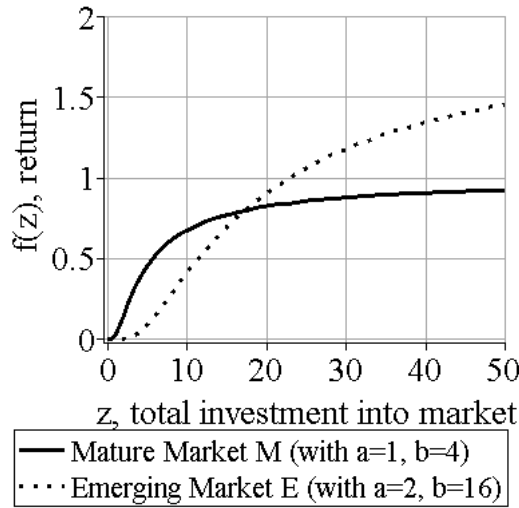


Figure 3.1: Market Characteristics

3.1.2. CPPM DECISIONS

Given the project opportunities $P_{n,s}$ we investigate the binary decision for each firm n :

$$v_{n,s} = \begin{cases} 1 & \text{if firm } n \text{ invests in the project targeting market } s, \\ 0 & \text{if firm } n \text{ does not invest in the project targeting market } s, \end{cases}$$

with $\sum_s v_{n,s} = 1$ for all n (firms must invest into one of the projects).

The returns from each market are shared proportionally to the investments made by the two firms, leading to the following return maximizations for each firm n :

$$\max_{v_{n,M}, v_{n,E}} \pi_n = \frac{v_{n,M} P_{n,M}}{\sum_{k=n,-n} v_{k,M} P_{k,M}} a_M e^{\frac{-b_M}{\sum_{k=n,-n} v_{k,M} P_{k,M}}} + \frac{v_{n,E} P_{n,E}}{\sum_{k=n,-n} v_{k,E} P_{k,E}} a_E e^{\frac{-b_E}{\sum_{k=n,-n} v_{k,E} P_{k,E}}}.$$

This formulation of our CPPM model captures significant real world complexity in a compact form. In particular, if firms invest in projects that target the same market within the convex part of the market return curve, they experience higher returns than if they entered that market alone. This is a reflection of the fact that competition can increase category credibility, thereby aiding diffusion (Carpenter and Nakamoto, 1989). However, if firms invest in projects that target the same market within the concave part of the market return curve, they experience lower returns compared to sole entry into the market. Such decreasing returns from competitive market entry are common in mature markets.

For tractability, we assume that firms have identical investment opportunities for each market, i.e., $P_{1,M} = P_{2,M} = P_M$ and $P_{1,E} = P_{2,E} = P_E$. While this is a special case, duopolies often tend to match one another's moves (Lieberman and Asaba, 2006) and may thus have similar projects in their portfolio. In Section 3.5 we study the case where firms have non-symmetric project investment opportunities. Here, we set $\alpha \equiv a_E/a_M$ and $\beta \equiv b_E/b_M$ with $\alpha, \beta > 1$. In other words, α and β capture the relative differences in the return potentials and inflection points, respectively, of the market return functions of the mature and emerging markets. The firms' returns are shown in Table 3.1 and for simplicity we define firm i 's strategy $v_{i,M} = 1$ as M and $v_{i,E} = 1$ as E , $i = 1, 2$.

Returns of symmetric firms		Firm 2	
		M	E
Firm 1	M	$\frac{a_M}{2} e^{\frac{-b_M}{2P_M}}, \frac{a_M}{2} e^{\frac{-b_M}{2P_M}}$	$a_M e^{\frac{-b_M}{P_M}}, \alpha a_M e^{\frac{-\beta b_M}{P_E}}$
	E	$\alpha a_M e^{\frac{-\beta b_M}{P_E}}, a_M e^{\frac{-b_M}{P_M}}$	$\frac{\alpha a_M}{2} e^{\frac{-\beta b_M}{2P_E}}, \frac{\alpha a_M}{2} e^{\frac{-\beta b_M}{2P_E}}$

Table 3.1: Returns of symmetric firms

3.2. POTENTIAL INTERACTIONS BETWEEN FIRMS

Using the returns from Table 3.1, we can derive the possible Nash equilibria of the project selection decision. We take a normative/prescriptive approach to analyzing the potential interactions between firms. Under the normative framework, we assume firms are fully informed of the potential returns and are rational decision-makers. Under the prescriptive framework, we recognize that firms compete with each other over extended periods: although firms are unlikely to face the identical project selection at a future point of time, past portfolio decisions will influence their competitor's perceptions of their likely future strategies. Furthermore, we assume that firms' utilities may be influenced by matters above and beyond their project returns. For example, a firm may prefer slightly lower project returns if this results in drastically lower returns for its key competitor.

In a PPM setting, mixed strategies in which players make their project selection randomly, based on a fixed probability, are not realistic. Mixed strategies would imply that managers set probabilities with which they want to pursue certain projects and then role the dice to make their actual choice. Consequently, we focus on pure strategies in this chapter. Though finding Nash equilibria (NE) can be challenging in itself (Savani and Stengel, 2006), we further argue that the analysis should go beyond calculating the pure-strategy NE and that the structure of the individual outcomes must be characterized in more detail; otherwise, significant complexity of the nature of the interaction is lost. Consider, for example, the two instances presented in Table 3.2 and Table 3.3. In both games, the NE is $[E, E]$ and the returns are $[3, 3]$. Structurally, however, the games are quite distinct. In Example Game 1 (Table 3.2), the NE represents the best outcome possible for both players. By contrast, Example Game 2 (Table 3.3) is a form of the famous Prisoner's Dilemma (Rapoport, 1980). Although neither

player can improve from [3, 3] unilaterally, this outcome is not Pareto-efficient and both players can receive a higher payoff [6, 6] in outcome [M, M].

Game 1		Firm 2	
		M	E
Firm 1	M	0, 0	1, 2
	E	2, 1	3, 3

Table 3.2: Example Game 1 in cardinal form

Game 2		Firm 2	
		M	E
Firm 1	M	6, 6	0, 10
	E	10, 0	3, 3

Table 3.3: Example Game 2 in cardinal form

To facilitate the analysis, we represent the potential interactions between firms in ordinal form. Any cardinal 2x2 game can be represented as an ordinal 2x2 game by ranking the outcomes from the highest payoff, 4, to the lowest payoff, 1 (Fraser, 1994). For example, Table 3.4 shows Game 2 (from Table 3.3) in ordinal form and all the dynamics of Game 2 in cardinal form still hold (e.g., NE is still [E, E] and that outcome is Pareto-dominated by [M, M]).

Game 2		Firm 2	
		M	E
Firm 1	M	3, 3	1, 4
	E	4, 1	2, 2

Table 3.4: Example Game 2 in ordinal form

For all possible CPPM interactions, we have the following results:

Theorem 3.1: *In a duopoly market where firms have two identical project opportunities at least one pure-strategy equilibrium exists.*

All proofs appear in Section 3.7.

In this symmetric game, only six payoff comparisons are required to characterize any possible ordinal game. We first derive the condition under which both firms prefer lone investment into mature Market M, $\pi_1[M, E] > \pi_1[M, M]$ and $\pi_2[E, M] > \pi_2[M, M]$. Using

Table 3.1, we have $a_M e^{\frac{-b_M}{P_M}} > \frac{a_M}{2} e^{\frac{-b_M}{2P_M}}$ and thus $P_M > \overline{P_M} \equiv \frac{b_M}{2 \ln 2}$. Note that $\overline{P_M}$ is greater

than the inflection point, $\frac{b_M}{2}$. Thus, as b_M increases, a greater project investment opportunity into the mature market must be taken to justify lone investment into the mature market. Particularly, the project opportunity must be greater than the inflection point of the market return function. Similarly, we define $\overline{P_E} \equiv \frac{\beta b_M}{2 \ln 2}$ as the maximum project investment size into the emerging market that justifies joint investment into the emerging market by both firms. Based on $\overline{P_M}$ and $\overline{P_E}$, the investment space is segmented into four quadrants, as depicted in Figure 3.2. We first focus our analysis on State 1, where firms prefer to harvest the mature Market M alone but would like their competitor to help diffusion in the emerging Market E . Others have recognized the importance of this state; e.g., Shankar et al. (1998) analyzed the game between a pioneer and a late mover where the competitor could help or hinder diffusion of the product. The other states are briefly discussed in Section 3.4.

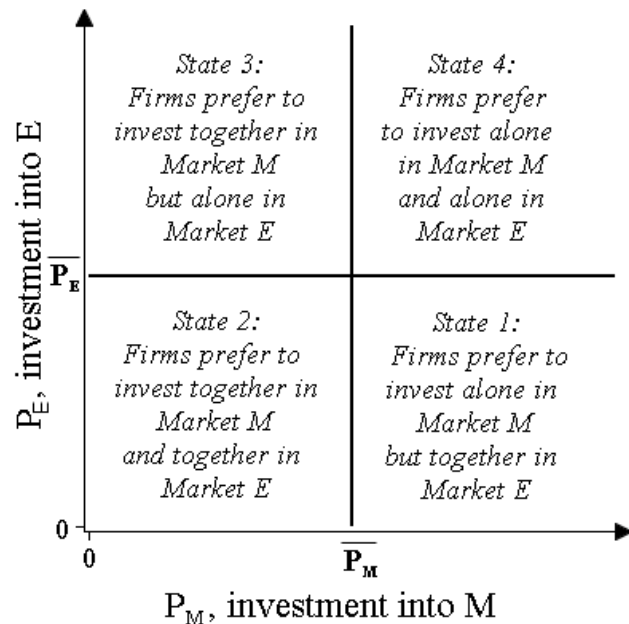


Figure 3.2: Four possible states of sole vs. joint market entry

Let us start the analysis of State 1 with the case where firms consider an investment of size $P_M > \overline{P_M}$ into the mature market and a minimal investment $P_E \rightarrow 0$ into the emerging market. This instance is represented by the ordinal game of Convergence (Hamburger, 1979). Given the assumptions of State 1, $\pi_1[E, E] > \pi_1[E, M]$ and $\pi_1[M, E] > \pi_1[M, M]$, and since $P_E \rightarrow 0$, we have $\pi_1[M, M] > \pi_1[E, E]$. Due to symmetry, this fully characterizes the game (Table 3.5). The game of Convergence has a NE at $[M, M]$ which is also the Pareto-optimal outcome of this game. Pursuing Project M leads to either the highest or second highest return (depending on the competitor's decision), and pursuing Project E leads to either the lowest or second lowest return.

Convergence		Firm 2	
		M	E
Firm 1	M	3, 3	4, 1
	E	1, 4	2, 2

Table 3.5: Game of Convergence in ordinal form

The game shifts to the Prisoner's Dilemma (Table 3.6) once $\pi_1[E, E] > \pi_1[M, M]$, implying:

$$P_E > \frac{\beta}{\frac{2 \ln \alpha}{b_M} + \frac{1}{P_M}} \quad (3.1)$$

Prisoner's Dilemma		Firm 2	
		M	E
Firm 1	M	2, 2	4, 1
	E	1, 4	3, 3

Table 3.6: The Prisoner's Dilemma in ordinal form

The Prisoner's Dilemma has the same NE as the game of Convergence $[M, M]$, yet the outcome is much less predictable. For example, the outcome $[E, E]$ Pareto-dominates $[M, M]$ as both players receive a higher return in $[E, E]$. However, $[E, E]$ is an unstable outcome in the sense that both firms can achieve higher returns by deviating to Project M . Recommendations

have been made how the Pareto-optimal outcome in the Prisoner's Dilemma can be reached in repeated games (Rapoport, 1980).

If firms have an even more significant project for the emerging market, two possible games could emerge: the Game of Chicken or the Stag Hunt. The former (Table 3.7) occurs if $\pi_1[E, M] > \pi_1[M, M]$, which implies:

$$P_E > \frac{\beta}{\frac{\ln(2\alpha)}{b_M} + \frac{1}{2P_M}}, \quad (3.2)$$

while the latter (Table 3.8) occurs if $\pi_1[E, E] > \pi_1[M, E]$, implying:

$$P_E > \frac{\beta}{2 \left(\frac{\ln(0.5\alpha)}{b_M} + \frac{1}{P_M} \right)}. \quad (3.3)$$

Game of Chicken		Firm 2	
		M	E
Firm 1	M	1, 1	4, 2
	E	2, 4	3, 3

Table 3.7: Game of Chicken in ordinal form

Stag Hunt		Firm 2	
		M	E
Firm 1	M	2, 2	3, 1
	E	1, 3	4, 4

Table 3.8: The Stag Hunt in ordinal form

Both games have two NE: however, in the Stag Hunt (Oye, 1985) the NE in $[E, E]$ dominates the NE in $[M, M]$ whereas the Game of Chicken (Rapoport and Chammah, 1966) does not have a dominant NE. Both games are hard to predict and the outcome can be influenced by other strategic considerations. Although the Stag Hunt has a dominant NE $[E, E]$, a firm can strategically reduce the return of the other firm by choosing Project M . Furthermore, fearing a competitor defection to M , a firm may initiate defection to M to prevent receiving its lowest possible return. In the Game of Chicken, the two NE are $[M, E]$ and $[E, M]$. One could argue that the first-mover decides the outcome of this game: for example, if Firm 1 chooses Project M , then Firm 2's best action is to choose Project E . Therefore, Firm 1 can secure the highest possible return for itself, $\pi_1[M, E]$ by irrevocably committing first to

Project M . However, this outcome is particularly frustrating for Firm 2. Not only does Firm 2 receive the second worst possible return, its rival is receiving the highest possible return. Consequently, Firm 2 may be driven to choose Project M as well, leading to the worst possible outcome for both firms $[M, M]$. Experiments have confirmed this danger (Rapoport and Chammah, 1966).

With an even larger project opportunity into the emerging market, the game evolves into the game of Spite (Hamburger, 1979). The game of Spite (Table 3.9) occurs once both (3.2) and (3.3) hold. Spite has a single NE in $[E, E]$ and this outcome provides the highest return for both firms. Firms can deviate to M to punish the other firm but this strategy is much less effective than in the Stag Hunt because the other firm still receives the third highest return. Furthermore, if both try to punish each other, the game results in the worst possible outcome for both firms, $[M, M]$.

Spite		Firm 2	
		M	E
Firm 1	M	1, 1	3, 2
	E	2, 3	4, 4

Table 3.9: Game of Spite in ordinal form

With an even greater project investment for the emerging market, the game becomes a previously unnamed game to which we refer as the game of Union (Table 3.10); the game of Union occurs if $\pi_1[E, M] > \pi_1[M, E]$, which implies:

$$P_E > \frac{\beta}{\frac{\ln \alpha}{b_M} + \frac{1}{P_M}}. \quad (3.4)$$

Union		Firm 2	
		M	E
Firm 1	M	1, 1	2, 3
	E	3, 2	4, 4

Table 3.10: Game of Union in ordinal form

Similar to the game of Spite, the game of Union has a unique NE in $[E, E]$ which is also the Pareto-optimal outcome. A firm that deviates to M reduces its own returns greater than that of its competitor (e.g., $\pi_1[M, E] < \pi_2[M, E]$) and choosing Project E leads to the highest or second highest return irrespective of the competitor's action. Figure 3.3 shows all possible games depending on the project investment opportunities P_M (project targeting mature market) and P_E (project targeting emerging market), as they occur in State 1.

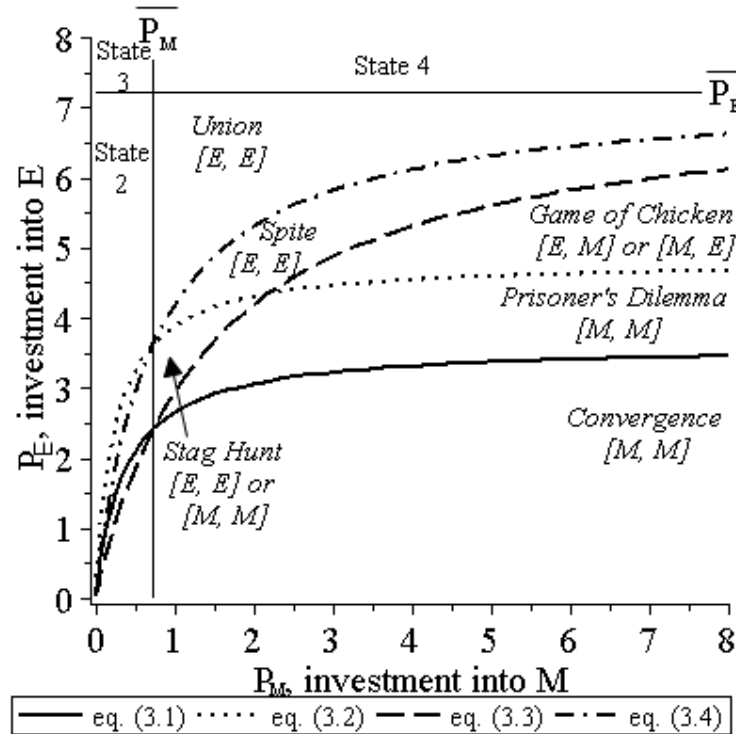


Figure 3.3: All possible CPPM games in State 1 and their Nash Equilibria in brackets depending on project opportunities P_M and P_E with $b_M = 1$, $\alpha = 4$ and $\beta = 10$

3.2.1. MARKET PARAMETERS

Equations (3.1), (3.2), (3.3) and (3.4) depend on the following parameters: b_M , β , and α . Thus, we have the following results.

Proposition 3.1: *In the symmetric CPPM with $\alpha \equiv \frac{a_E}{a_M}$, a_M and a_E have no impact on the ordinal games and merely change the absolute values of the returns.*

Consequently, CPPM strategies are not dependent on the absolute value of the potential market returns a_s ($s = M, E$). However, a_s does influence the absolute returns and may thus reveal the value of taking strategic action.

Proposition 3.2: (i) Increasing both b_M and P_M by a factor θ leads to the same ordinal game for any P_M and P_E . (ii) Increasing both β and P_E by a factor θ leads to the same ordinal game for any P_M and P_E .

The parameters b_M and b_E alone dictate the required project investment to acquire a certain percentage of the maximum market return potential. In particular, by solving the market return function $f_s(z_s) = a_s e^{-\frac{b_s}{z_s}} = ca_s$ for z_s , we get $z_s = \frac{b_s}{-\ln c}$ which is the required investment to acquire $c\%$ of the maximum market return a_s . Proposition 3.2 implies that if we keep the ratios $\frac{b_M}{P_M}$ and $\frac{b_E}{P_E}$ constant (i.e., keep c constant), changes to the slopes of the return curve have no effect on the CPPM strategy. In effect, changes to these ratios re-scale the x- axis and y-axis of Figure 3.3, respectively.

Proposition 3.3: Given any market parameter set, all previously identified ordinal games exist for some range of P_M and P_E , except for the Game of Chicken which can occur only if $\alpha < 8$.

Proposition 3.3 implies that if $\alpha > 8$, then all games have a dominant Nash equilibrium. This is partly driven by the fact that a large α implies that one market has a far higher market return potential than the other.

3.3. DISCUSSION AND RESULTS

3.3.1. CPPM STRATEGIES

As demonstrated in Section 3.2, market dynamics can critically impact optimal project selection strategy. Not only do the possible interactions with competitors affect which project should be chosen but they also dictate the optimal timing of the decision, the long term considerations and the communication strategy. Some studies have found that the main value of PPM models is the helpful insight they can provide, not necessarily the final selection recommendation for a specific case (Loch et. al., 2001; Beaujon et al., 2001). The important aspect of PPM models has also been described as collecting data and helping decision makers think through the decision, not mathematical optimization in isolation (Coldrick et al., 2005). In the same spirit, this section focuses on the general insight that can be derived from our CPPM model and how this should guide firms' CPPM strategies. Specifically, we look more carefully into the various games that may emerge and the insights they provide.

The Prisoner's Dilemma: The NE predicts that firms both invest in the project that targets the mature market. However, both firms could increase their returns by investing in the emerging market together. The crux of the Prisoner's Dilemma is that even if the firms recognize this superior outcome, they both have an incentive to "cheat" and harvest the mature market. The key to a better outcome is to convince the competitor to cooperate and enter the emerging market together. In a one-time emerging market entry decision, this could be done by contractually committing to investing in the emerging market (where allowed). If firms are in this situation repeatedly, the optimal strategy is the "tit-for-tat" strategy (Rapoport, 1980) in which firms start by cooperating, retaliate against defection and return to cooperation once the

other firm cooperates. In effect, an active communication and cooperation strategy is required to reach a better outcome in the Prisoner's Dilemma.

The Stag Hunt: Firms are not just trying to maximize their own short-term returns but have a long-term perspective and may purposely try to slow the progress of their competitors. Although the NE (both firms investing in the emerging market) leads to the highest return for both firms – a seemingly stable outcome – a firm that instead invests in the mature market still receives its second highest return yet leaves the other firm in the worst possible outcome (investing in the emerging market alone). This might be a tempting strategy for a firm that takes a long-term perspective and determines that the lower immediate return is outweighed by the effect of impeding the progress of the competitor. A firm may also fear that the other firm will take this action and thus defect to the mature market as a defensive strategy. Consequently, a firm seeking or fearing aggressive action in the marketplace should consider defecting to the mature market.

Game of Chicken: A timely portfolio decision can significantly impact the outcome of the game. As soon as the first mover has committed to investing in the mature market, the competitor's best option is to invest in the emerging market. This outcome gives the first mover the best possible return and the follower the second worst return. However, this strategy is not without risk for the leader. The follower can punish the leader by entering the mature market as well, thereby reducing the leader's return from highest to lowest, costing the follower only the difference between the lowest and second lowest return. Consequently, the Game of Chicken is a dangerous CPPM situation as there is a real chance that it may end with both firms over-investing in the mature market and receiving their lowest possible return.

Convergence, Spite, Union: the games of Convergence, Spite and Union all have NE outcomes that are likely to play out. These games do not provide additional strategic opportunities or dangers that could be exploited or mitigated against.

3.3.2. MARKET SIZES

By estimating the market return functions and the project opportunities of the competitor, firms can determine the resulting CPPM game and adjust their strategy accordingly. This subsection attempts to provide general guidance as to when certain games are more likely to occur. As established by Propositions 3.1 and 3.2, the key determinant of the type of CPPM game is the relative difference in market size, α . As Figure 3.4 shows, the parameter sets that lead to certain CPPM games depends greatly on α ; however, the areas in which certain CPPM games occur are unbounded within the parameter space. To draw some conclusions, we first find the value of P_M for which the range of P_E values that lead to a particular CPPM game is maximized. Subsequently, we determine the value of α that maximizes that range.

If the factor difference in the maximum return of the two markets, α , approaches 1, the CPPM decision will likely require no strategic action: if the markets offer equal returns ($\alpha = 1$), equations (3.1), (3.2), (3.3) and (3.4) pass through the point $[\overline{P}_M, \overline{P}_E]$. Consequently, for all parameters, the CPPM game will be Convergence which requires no strategic considerations (see top left of Figure 3.4). On the other hand, if α is very large, the CPPM decision will likely require no strategic action: as shown in Proposition 3.3, for $\alpha > 8$, the Game of Chicken does not exist for any parameter combination. Furthermore, the greater α , the smaller the Prisoner's

Dilemma region defined by $\frac{\beta}{\frac{2 \ln \alpha}{b_M} + \frac{1}{P_M}} < P_E < \min \left[\frac{\beta}{\frac{\ln(2\alpha)}{b_M} + \frac{1}{2P_M}}, \frac{\beta}{2 \left(\frac{\ln(0.5\alpha)}{b_M} + \frac{1}{P_M} \right)} \right]$ and

although the Stag Hunt region $\frac{\beta}{\frac{\ln(2\alpha)}{b_M} + \frac{1}{2P_M}} < P_E < \frac{\beta}{2\left(\frac{\ln(0.5\alpha)}{b_M} + \frac{1}{P_M}\right)}$ initially stretches for

larger α (as shown in Figure 3.4), for very large α , this area goes toward zero as well.

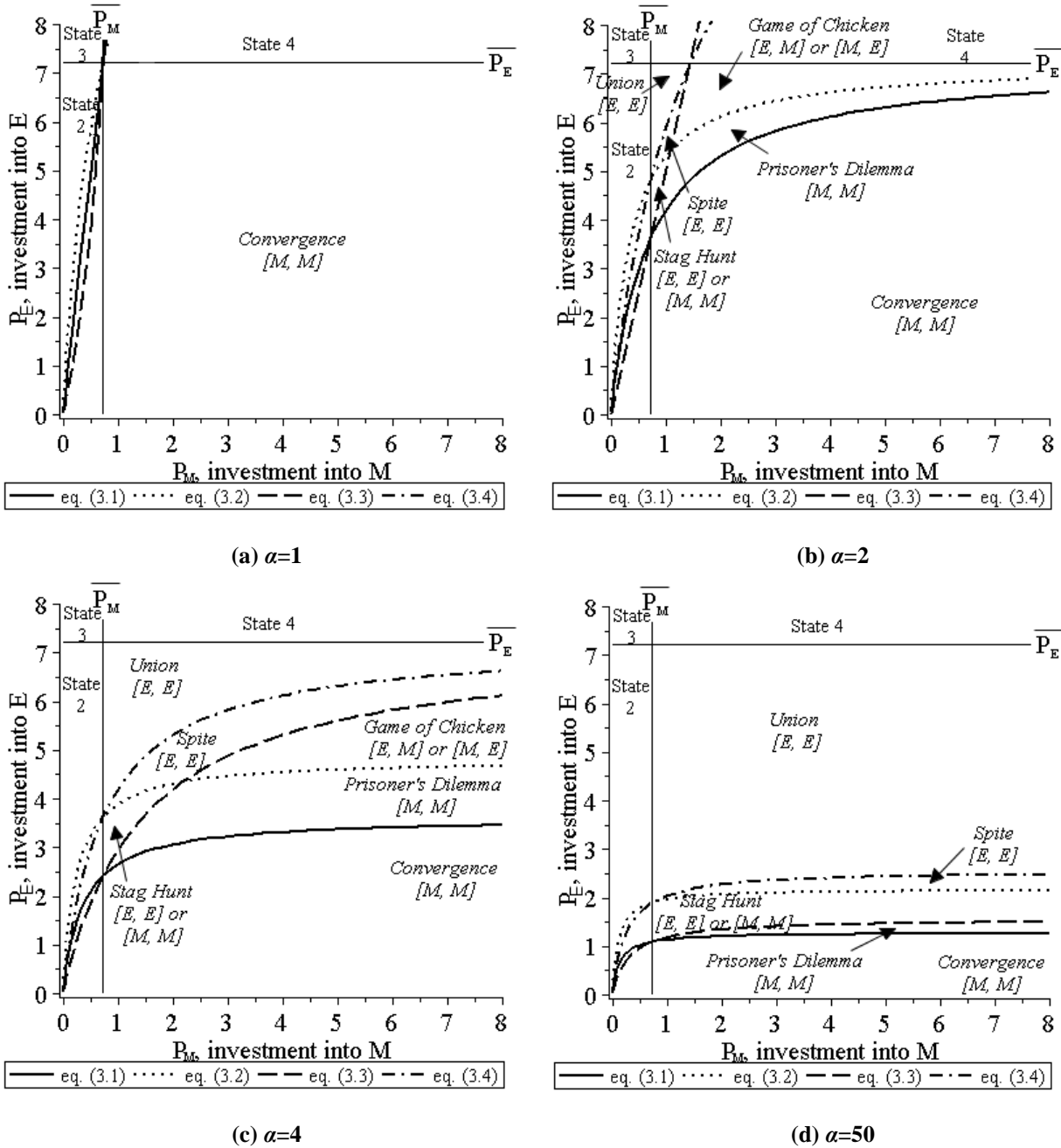


Figure 3.4: CPPM games with $b_M=1, \beta=10$

Consequently, for most parameters and project opportunities, the firms will be engaged in a CPPM game with a clear NE that does not require strategic considerations. The CPPM games that require strategic choices are most likely to occur if the difference between the maximum return potential of the emerging and mature market is neither very small nor very large.

3.4. OTHER MARKET COMBINATIONS

In Sections 3.2 and 3.3, we focused our analysis on the case in which firms prefer either building the emerging market together or harvesting the mature market without competition. Figure 3.2 shows the three other states. Changes in b_M , b_E or β effect the games in accordance to Propositions 3.1 and 3.2 in these states as well. Therefore, this section focuses solely on the impact of the relative size of the markets, α , on the existence of games. In State 2, firms would rather invest together than alone in both markets. Consequently, all possible games in this state have NE outcomes in which both firms invest into the same market and there is always one dominant NE. In State 3, both firms would rather be together in the mature market but alone in the emerging market. In this state, the firms are playing the game of Convergence for any possible combination of parameters. As described earlier, the outcome of this game is very predictable and cannot be influenced. Finally, in State 4 firms want to be alone in both markets. For $\alpha > 4$, all parameters lead to the game of Hero (Rapoport, 1967), with the highly predictable outcome of both firms investing in the emerging market. However, for $\alpha < 4$, the game of Apology (Table 3.11) and the Battle of the Sexes (Table 3.12) exist (Rapoport, 1967).

Apology		Firm 2	
		M	E
Firm 1	M	1, 1	3, 4
	E	4, 3	2, 2

Table 3.11: Game of Apology in ordinal form

Battle of the Sexes		Firm 2	
		M	E
Firm 1	M	1, 1	4, 3
	E	3, 4	2, 2

Table 3.12: The Battle of Sexes in ordinal form

These games are coordination games in which two NE exists, $[M, E]$ and $[E, M]$. Either outcome results in firms getting one of their two highest possible returns; conversely, if they enter either market together they receive one of their two lowest possible returns. Therefore, coordination and communication is critical in these games. Furthermore, a first-mover advantage exists and the first firm to make a commitment can secure its highest possible return.

3.5. NON-SYMMETRIC FIRMS

So far we have assumed that both firms have identical project investment opportunities into the mature and emerging market. While this scenario is an important special case of the more general CPPM problem, firms' project investment opportunities may differ. In this section we relax the symmetry assumption to gain some insights into the more general setting. In many instances, firms have fixed budgets which they can allocate over a range of projects. We assume that each of the two firms has a budget constraint $B_n, n \in \{1,2\}$, which it can invest, completely, in either the mature or the emerging market. As before, if the firms invest in the same market, their share of the market return is proportional to their share of investment. The resulting returns are shown in Table 3.13.

Returns of non-symmetric firms		Firm 2	
		M	E
Firm 1	M	$\frac{a_M B_1}{B_1 + B_2} e^{\frac{-b_M}{B_1+B_2}}, \frac{a_M B_2}{B_1 + B_2} e^{\frac{-b_M}{B_1+B_2}}$	$a_M e^{\frac{-b_M}{B_1}}, \alpha a_M e^{\frac{-\beta b_M}{B_2}}$
	E	$\alpha a_M e^{\frac{-\beta b_M}{B_1}}, a_M e^{\frac{-b_M}{B_2}}$	$\frac{\alpha a_M B_1}{B_1 + B_2} e^{\frac{-\beta b_M}{B_1+B_2}}, \frac{\alpha a_M B_2}{B_1 + B_2} e^{\frac{-\beta b_M}{B_1+B_2}}$

Table 3.13: Returns of non-symmetric firms

Although the number of parameters remains equal to that in the symmetric setting, the complexity of the problem increases dramatically. The number of possible ordinal games

increases from 6 in the symmetric case to 26 (!). Furthermore, Theorem 3.1 does not hold anymore and for two of the new ordinal games no pure NE exists. Similarly, Proposition 3.2 does not hold in this setting. Lastly, we cannot longer solve the comparisons between returns analytically. Consequently, we resort to computational analysis to derive insights into the CPPM problem with non-symmetric firms.

The key question we address here is how the difference in budget size between two firms affects their CPPM decision and profitability. Given the total budget available to both firms to invest in either market, we set Firm 1's proportional share to r , so $r = \frac{B_1}{B_1 + B_2}$. Due to space considerations, we highlight the key findings of our analysis using Figure 3.5, wherein Firm 1's returns are plotted for varying budget constraints. Note, in this figure, for a given value of r , as Firm 1's budget increases, Firm 2's budget increases proportionally.

Quite naturally, we find that small budgets drive both firms to joint investment in the mature market (which provides higher initial returns). Since market returns are shared proportional to investment, when both firms jointly invest in the mature market, which occurs in Figure 3.5 for B_1 smaller than about 1.1, an increase in r is associated with an increase in Firm 1' returns.

With a sufficient increase in budget, the high-budget firm shifts its investment from the joint investment into the mature market to a lone investment into the emerging market, leaving the small-budget firm alone in the mature market. At that point, the transition in return of the high-budget firm is smooth, while the small-budget firm experiences a significant increase in its return, as it solely harvests the mature market. Consider for example the case of $r = 0.2$ (resp., $r = 0.8$) in Figure 3.5. Once B_1 exceeds about 1.1 (resp., 4.4), Firm 2 (resp., Firm 1) enters the emerging market and leaves Firm 1 (resp., Firm 2) alone in the mature market. At

this point, the profit of Firm 1 spikes up (resp., transitions upwards). So counter-intuitively, for intermediate range of total budget, firms could be better off competing against a competitor with a larger budget than against a competitor with a smaller budget—evidently, in Figure 3.5, when Firm 1's budget is between about 1.1 and 3.5, the return when $r = 0.2$ is larger than when $r = 0.5$ or 0.8.

On the other hand, when total budgets are relatively large, by contrast with the above result, we find that firms are better off competing against a competitor with a smaller budget—which occurs in Figure 3.5 for B_1 greater than about 5.8, where an increase in r is associated with an increase in Firm 1's returns.

In the symmetric case, where $r = 0.5$, the return for budgets between about 1.1 and 3.5 is lower than for $r = 0.2$ or 0.8 as both firms continue to invest in the mature market although they could both improve their returns by joint investment into the emerging market (the Prisoner's Dilemma). For budgets between about 3.5 and 4.5, there are two NE, both of which are non-symmetric such that one firm invests in the mature market and the other in the emerging market. The firm that secures the mature market experiences a spike in its returns (denoted leader in the figure). For budgets larger than about 4.5 both firms invest into the emerging market.

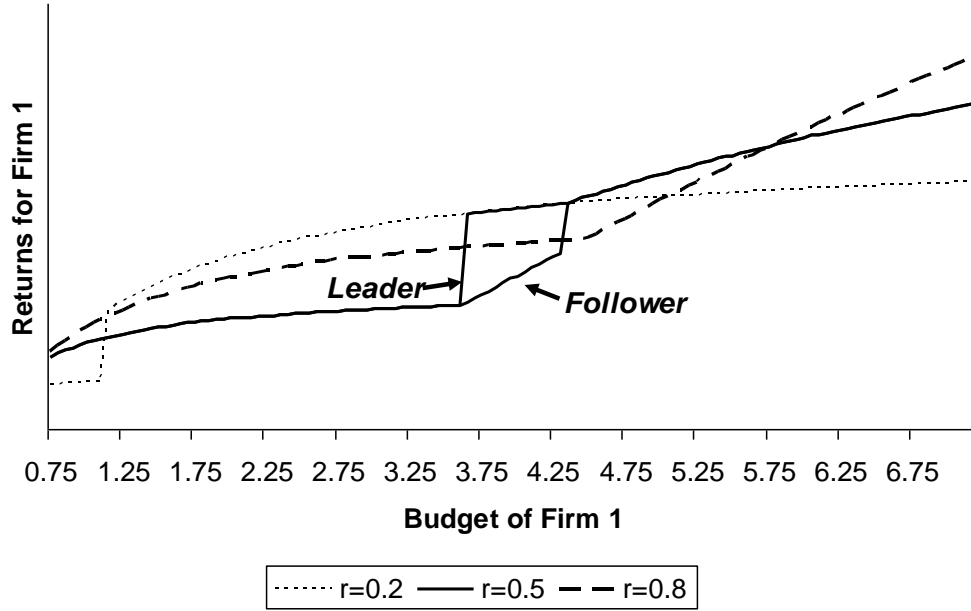


Figure 3.5: Return depending on budget size of Firm 1 with $\alpha = 4$, $\beta = 8$, $B_2 = (B_1 - rB_1)/r$

The other major assumption in this chapter is the focus on duopoly dynamics. In many instances, there can be more than two firms competing with each other in multiple markets which increases the complexity of the decision substantially. In binary investment settings, the resulting three-dimensional games are particularly hard to solve. We analyze oligopoly settings in Chapters 5 and 6.

3.6. CHAPTER 3 CONCLUSIONS

Competitive forces greatly influence the optimal strategy of the general PPM problem. We have demonstrated that even under simplifying assumptions the analysis of the CPPM problem is fairly complex. Furthermore, we have shown that in a duopoly setting where firms have symmetric project investment opportunities there are a several possible interaction types that can occur. Each of these situations requires different strategies and careful decision-making. Relaxing the symmetry assumption, and allowing for different budgets which are invested

solely into one of the projects, we numerically illustrated that a firm might be better off when the competitor's budget increases.

Apart from the market potential and rate of return, markets may also differ with respect to the certainty with which these returns can be expected. For example, emerging markets can be subject to significant volatility and uncertainty. Returns from project targeting such markets may be harder to estimate accurately than returns from projects targeting mature markets in which substantial market research has been completed. The next chapter explores how such uncertainty of project returns affects the CPPM decision of firms.

3.7. PROOFS FOR CHAPTER 3

Proof of Theorem 3.1: By contradiction. If there was no pure strategy, at least one of the firms must have an incentive to switch actions from any possible outcome of the game. In other words, starting at any of the four outcomes in Table 3.1, firms would want to switch their project choice in a full clockwise or counter-clockwise rotation through all the outcomes. A counter-clockwise rotation would require $\pi_1[M, M] < \pi_1[E, M]$, $\pi_2[E, M] < \pi_2[E, E]$, $\pi_1[E, E] < \pi_1[M, E]$, and $\pi_2[M, E] < \pi_2[M, M]$. Due to symmetric project choices, $\pi_1[E, M] = \pi_2[M, E]$ and $\pi_1[M, M] = \pi_2[M, M]$. Therefore, $\pi_1[M, M] < \pi_1[E, M]$ and $\pi_2[M, E] < \pi_2[M, M]$ cannot hold simultaneously. Similarly, the clockwise rotation $\pi_1[M, M] > \pi_1[E, M]$, $\pi_2[M, E] > \pi_2[M, M]$, $\pi_1[E, E] > \pi_1[M, E]$, and $\pi_2[E, M] > \pi_2[E, E]$ cannot hold. \square

Proof of Proposition 3.1: Equations (3.1), (3.2), (3.3) and (3.4) are all independent of a_M and a_E and solely dependent on the project opportunities P_M and P_E and the parameters b_M, β , and α . \square

Proof of Proposition 3.2: (i) Take for example equation (3.1): increasing P_M, P_E and b_M by θ

leads to $P_E > \frac{\theta\beta}{\frac{2\ln\alpha}{\theta b_M} + \frac{1}{\theta P_M}}$ and by dividing by θ leads to the original equation

$P_E > \frac{\beta}{\frac{2\ln\alpha}{b_M} + \frac{1}{P_M}}$. This holds for equations (3.2), (3.3) and (3.4) as well.

(ii) Take for example equation (3.1): increasing P_E and β by θ leads to $\theta P_E > \frac{\theta\beta}{\frac{2\ln\alpha}{b_M} + \frac{1}{P_M}}$

and by dividing by θ leads to the original equation $P_E > \frac{\beta}{\frac{2\ln\alpha}{b_M} + \frac{1}{P_M}}$. This holds for equations

(3.2), (3.3) and (3.4) as well. □

Proof of Proposition 3.3: We need to compare all equations (3.1), (3.2), (3.3), and (3.4)

(shown in Figure 3.3):

(3.1) is less than (3.2) if $\frac{\beta}{\frac{2\ln\alpha}{b_M} + \frac{1}{P_M}} < \frac{\beta}{\frac{\ln(2\alpha)}{b_M} + \frac{1}{2P_M}}$, which simplifies to $-\ln\frac{\alpha}{2} < \frac{b_M}{2P_M}$.

For $\alpha > 2$, we get $P_M > \frac{b_M}{2\ln\frac{2}{\alpha}}$ which holds for all positive P_M . For $\alpha < 2$, we get $P_M < \frac{b_M}{2\ln\frac{2}{\alpha}}$

and inserting $P_M = \frac{b_M}{2\ln\frac{2}{\alpha}}$ into (3.2), we get $P_E < \frac{\beta b_M}{2\ln 2}$, which defines the feasible region

$P_E < \overline{P_E}$. Therefore, in the feasible region, (3.1) is always less than (3.2).

(3.1) is less than (3.3) if $\frac{\beta}{\frac{2\ln\alpha}{b_M} + \frac{1}{P_M}} < \frac{\beta}{2\left(\frac{\ln(0.5\alpha)}{b_M} + \frac{1}{P_M}\right)}$, which simplifies to

$P_M > \frac{b_M}{2\ln 2}$. This is always true in the feasible region $P_M > \overline{P_M}$.

(3.1) is less than (3.4) if $\frac{\beta}{\frac{2\ln\alpha}{b_M} + \frac{1}{P_M}} < \frac{\beta}{\frac{\ln\alpha}{b_M} + \frac{1}{P_M}}$ which is true for all possible

parameters.

(3.2) is less than (3.3) if $\frac{\beta}{\frac{\ln(2\alpha)}{b_M} + \frac{1}{2P_M}} < \frac{\beta}{2\left(\frac{\ln(0.5\alpha)}{b_M} + \frac{1}{P_M}\right)}$, which simplifies to

$$\frac{3b_M}{2P_M} < \ln \frac{8}{\alpha}. \text{ For } \alpha < 8, \text{ this holds for } P_M > \frac{b_M}{2 \ln \frac{2}{\sqrt[3]{\alpha}}}$$

region $P_M > \overline{P_M}$. For $\alpha > 8$, P_M would have to be negative for the inequality to hold, therefore the Game of Chicken only exists for $\alpha < 8$.

(3.2) is less than (3.4) if $\frac{\beta}{\frac{\ln(2\alpha)}{b_M} + \frac{1}{2P_M}} < \frac{\beta}{\frac{\ln\alpha}{b_M} + \frac{1}{P_M}}$, which simplifies to $P_M > \frac{b_M}{2\ln 2}$. This

always holds in the feasible region $P_M > \overline{P_M}$.

(3.3) is less than (3.4) if $\frac{\beta}{2\left(\frac{\ln(0.5\alpha)}{b_M} + \frac{1}{P_M}\right)} < \frac{\beta}{\frac{\ln\alpha}{b_M} + \frac{1}{P_M}}$, which simplifies to $\ln \frac{4}{\alpha} < \frac{b_M}{P_M}$.

For $\alpha > 4$, we get $P_M > \frac{b_M}{\ln \frac{4}{\alpha}}$ which holds for all positive P_M . For $\alpha < 4$, we get $P_M < \frac{b_M}{\ln \frac{4}{\alpha}}$

and inserting $P_M = \frac{b_M}{\ln \frac{4}{\alpha}}$ into (3.4), we get $P_E < \frac{\beta b_M}{2\ln 2}$, which defines the feasible

region $P_E < \overline{P_E}$. Therefore, in the feasible region, (3.3) is always less than (3.4). \square

4. CPPM WITH UNCERTAIN RETURNS: WHEN, WHERE AND HOW MUCH TO INVEST

An additional challenge to the CPPM problem is the potential for uncertainty of project returns. Some markets are inherently more uncertain than others and the returns from projects targeting such markets can be hard to estimate. Depending on the size of the project portfolio and the objective of the decision maker, the uncertainty of project returns makes the PPM problem an extremely challenging problem (Carraway et al., 1993). One simple, yet effective, way of dealing with the complexity is to present decision makers with a number of Pareto-efficient outcomes, comparing expected net present value and the standard deviation of returns of different project bundles (Walls, 2004). However, the degree of uncertainty of project returns may change as new information is gained over time. Consequently, the decision of when to invest can become an important decision variable (Dixit and Pindyck, 1994).

Furthermore, the amount of resources to be invested into a project may not be fixed. Although firms may have a well developed business case for each project, project budgets can be flexible and may be adjusted in order to best suit the situation. Combined with the market choice and timing decision, this adds a third decision variable to the CPPM problem.

Competition affects all aspects of the investment decision: firms that invest early may capture a first mover advantage but are subject to demand uncertainty; firms may decide to compete directly against their competitor in a particular market or try to avoid competition by deviating to other markets; finally, the size of the optimal investment can vary significantly depending on, for example, whether a firm can gain a monopoly position or not. The

complexity of these tradeoffs, in combination with the potentially dramatic impact of such a key decision on a firm's long term returns, makes this a hard and important problem.

We continue with the setting where two firms are each considering two alternative investments, one targeting a mature market and one an emerging market. However, in this model, demand for the emerging market is uncertain and subject to diffusion effects while the demand for the mature market is known with certainty and does not increase through either firm's entry. Furthermore, firms do not only decide on the timing and target market of their investment, but also set the optimal investment amount.

This chapter is organized as follows. The next section sets up our model and outlines its assumptions. Section 4.2 analyzes the equilibria investment strategies and presents the key findings. Section 4.3 explores the case of non-symmetric firms and Section 4.4 contains our conclusions.

4.1. THE MODEL

Assume there are two firms, Firm 1 and Firm 2, each seeking to make a capacity investment. In their decision making processes, two binary choices need to be made: when to invest and in which market to invest. Let t_n denote the investment timing of firm $n \in \{1,2\}$, with $t_n \in \{I, D\}$, where I and D denote early and delayed investment decisions, respectively. Further, let s_n denote the market choice of firm n , with $s_n \in \{M, E\}$, where M and E denote investment into the mature and emerging market, respectively. Based on timing and market decisions, sixteen possible decision outcomes are possible. Let $[t_1 t_2 : s_1 s_2]$ denote the timing and market decisions of the two firms. Following the binary decisions of when and where to invest, each Firm n sets the size of its capacity expansion for its chosen market, k_n . Following Swinney et al. (2011),

we assume that firms produce at their full capacity and release all units to the market. In addition, we assume that the unit cost is the same for both firms, independent of the time of investment, but differs for the two markets, denoted c_M for the mature and c_E for the emerging market (we briefly study the case of non-symmetric firms in Section 4.3).

The realized price in market s , p_s , is determined by the linear demand function $p_s(k_1, k_2) = A_s - k_1 - k_2$, where A_s , the demand intercept, is a continuous random variable with distribution F , with mean $(\mu_s + x_s \cdot v_s)$ and variance σ_s^2 , where x_s is the number of firms investing in market s (early and delayed) and each firm's market investment creates an additional demand v_s through diffusion effects. Since the mature market M is a well-established, highly predictable, market, we assume that $v_M = 0$ and that $\sigma_M^2 = 0$. Hence, the demand intercept A_M is known with certainty to be μ_M . In contrast to the mature market, diffusion effects occur in the emerging market and the more firms enter the market, the stronger these effects can be (Krishnan et al., 2000; Carpenter and Nakamoto, 1989; Shankar et al., 1999). Furthermore, demand in emerging markets is difficult to predict and thus uncertain. To simplify the notation, we drop the subscript E from v_E and σ_E^2 and write the mean and variance of the emerging market as v and σ^2 , respectively. Similar to previous work, we shall require that $A_E - k_1 - k_2 > 0$ to guarantee a non-negative price (Gerchak, 2010). Both firms are rational decision makers that aim to maximize their expected returns:

$$\pi_n^{t_1 t_2; s_1 s_2} = E \left[\sum_{s=M,E} (p_s(k_1, k_2) - c_s) \cdot k_n \right], \forall n.$$

The sequence of decisions made by the firms is composed of three stages. In the first stage, the *timing game*, both firms decide simultaneously whether to invest early or delay investment. Firms that decide to invest early into the emerging market will set their production quantity, i.e. the production capacity k_n , before the demand uncertainty has been resolved. By delaying

their investment decision, they can make their capacity decision after the actual demand has been observed. Since the demand of the mature market is known with certainty, delaying investment into that market does not provide additional information. In the second stage, the *market game*, firms decide which of the two projects in their project portfolio to pursue, i.e. in which markets to invest in capacity. The firms that invest early make their decision before firms that delay investment. In the third and final stage, the *capacity game*, firms set the size of the capacity investment. The form of the capacity game depends on the strategic decisions made by the two firms in the earlier timing and market games. If both firms decide to invest early or both decide to delay investment, and subsequently decide to invest in the same market, then they obtain Cournot returns; if one firm invests early and one delays investment, and subsequently both firms invest in the same market, then these firms are engaged in a Stackelberg game; if the firms invest in different markets, both receive monopoly returns from their respective markets. The decisions made in these three stages are accompanied by significant resource commitments and are thus credible and irreversible. Figure 4.1 shows the timeline of the sequence of decisions. Note that there are alternative sequences of events that could be modeled, which would impact the nature of the strategic decisions. For this work we modeled our assumptions, nomenclature and event sequence regarding the timing and capacity games following related work in Kulatilaka and Perottie (1998) and Swinney et al. (2011).

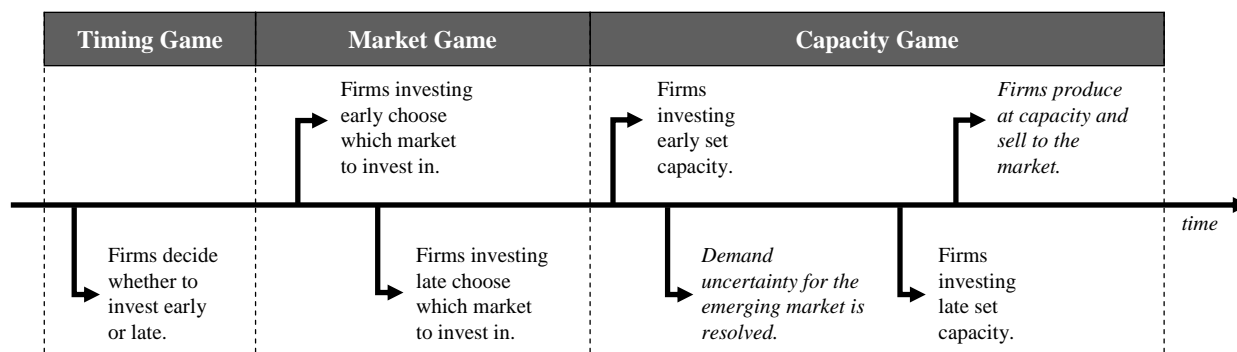


Figure 4.1: Timeline of decisions (non-decision events are in italic)

Figure 4.2 shows the firms' decisions (corresponding to Figure 4.1) in extensive form.

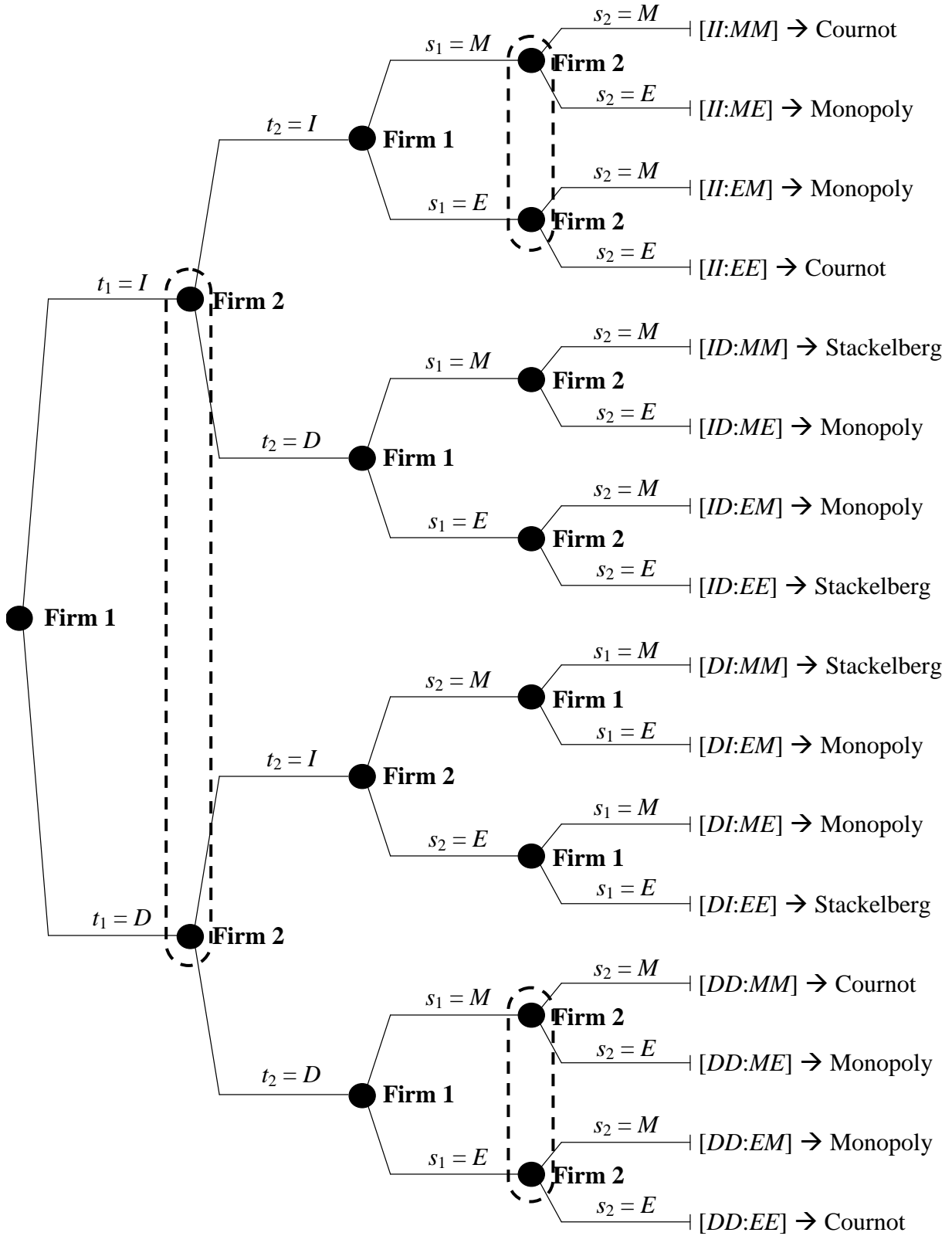


Figure 4.2: Firms' investment decisions in extensive form

4.2. MODEL ANALYSIS

To find the sub-game perfect Nash equilibria (SPNE) strategies, we solve the model backwards. We first find the equilibria capacity expansions, k_n^* , for each of the sixteen possible outcomes of the timing and market games. The equilibria capacity expansion size for each firm depends on its own timing and market decisions as well as the decisions of its counterpart. In addition to these competitive aspects, the uncertainty of demand in the emerging market influences the size of the capacity expansion. The following lemma lists all possible outcomes and corresponding capacity decisions and returns. To simplify the expressions, we denote $\Delta_s \equiv \mu_s - c_s, \forall s$.

Lemma 4.1: *Consider the capacity games. The equilibrium capacities and returns contingent on timing and market outcomes are characterized in Table 4.1.*

Outcome $[t_1 t_2 : s_1 s_2]$	Type of interaction	Optimal capacity expansion decision	Expected Return
$[II:EE]$	<i>Cournot</i>	$k_n^* = \frac{\Delta_E + 2\nu}{3}, \forall n$	$\pi_n^{II:EE*} = \frac{(\Delta_E + 2\nu)^2}{9}, \forall n$
$[DD:EE]$	<i>Cournot</i>	$k_n^* = \frac{A_E - c_E}{3}, \forall n$	$\pi_n^{DD:EE*} = \frac{\sigma^2 + (\Delta_E + 2\nu)^2}{9}, \forall n$
$[II:MM]$ or $[DD:MM]$	<i>Cournot</i>	$k_n^* = \frac{\Delta_M}{3}, \forall n$	$\pi_n^{II:MM*} = \pi_n^{DD:MM*} = \frac{\Delta_M^2}{9}, \forall n$
$[II:EM]^\#$ or $[ID:EM]^\#$	<i>Monopoly</i>	$k_1^* = \frac{\Delta_E + \nu}{2}$ $k_2^* = \frac{\Delta_M}{2}$	$\pi_1^{II:EM*} = \pi_1^{ID:EM*} = \frac{(\Delta_E + \nu)^2}{4}$ $\pi_2^{II:EM*} = \pi_2^{ID:EM*} = \frac{\Delta_M^2}{4}$
$[DI:EM]^\#$ or $[DD:EM]^\#$	<i>Monopoly</i>	$k_1^* = \frac{A_E - c_E}{2}$ $k_2^* = \frac{\Delta_M}{2}$	$\pi_1^{DI:EM*} = \pi_1^{DD:EM*} = \frac{\sigma^2 + (\Delta_E + \nu)^2}{4}$ $\pi_2^{DI:EM*} = \pi_2^{DD:EM*} = \frac{\Delta_M^2}{4}$

$[ID:EE]^\#$	<i>Stackelberg</i>	$k_1^* = \frac{\Delta_E + 2\nu}{2}$ $k_2^* = \frac{A_E - c_E}{4}$	$\pi_1^{ID:EE*} = \frac{(\Delta_E + 2\nu)^2}{8}$ $\pi_2^{ID:EE*} = \frac{4\sigma^2 + (\Delta_E + 2\nu)^2}{16}$
$[ID:MM]^\#$	<i>Stackelberg</i>	$k_1^* = \frac{\Delta_M}{2}$ $k_2^* = \frac{\Delta_M}{4}$	$\pi_1^{ID:MM*} = \frac{\Delta_M^2}{8}$ $\pi_2^{ID:MM*} = \frac{\Delta_M^2}{16}$

[#] Given that firms have symmetric costs, these equilibria also hold if the strategies of Firm 1 and Firm 2 are reversed. That is, each such outcome represents two outcomes.

Table 4.1: Capacity game equilibria

All proofs appear in Section 4.5.

Using the equilibria strategies of the capacity games from Lemma 4.1, we next find the equilibria strategies of the market game contingent on the outcome of the timing game. For example, if the outcome of the timing game was that both firms invest early, then there are four possible market game outcomes: both firms invest into the mature market, both firms invest into the emerging market, or Firm 1 invests into the mature and Firm 2 invests into the emerging market (and vice versa). If both firms invest at the same time, then the resulting market game is a simultaneous move game; if one firm invests early and one late, we analyze the corresponding sequential game.

Using the expected returns of the established SPNE of the market games for all four possible timing game outcomes, we can analyze the equilibria of the timing game itself. The timing decision is always made simultaneously by the two firms. The following two propositions characterize all possible equilibria strategies, which could either be unique (Proposition 4.1) or give rise to multiple equilibria strategies (Proposition 4.2). We first characterize the parameter settings that yield a unique SPNE equilibrium.

Proposition 4.1: *There are three distinct regions in the parameter space that have a unique SPNE. Specifically, the unique SPNE are:*

1. [II:MM] if $\sigma^2 < \frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2$;
2. [II:EE] if $\Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} < \sigma^2 < \frac{(\Delta_E + 2\nu)^2}{8}$ or $\left(\sigma^2 < \min \left[\frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \right] \text{ and } \nu > \frac{3\Delta_M}{4} - \frac{\Delta_E}{2} \right)$;
3. [DD:EE] if $\max \left[\frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2, \frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2 \right] < \sigma^2 < \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ or $\sigma^2 > \max \left[\frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}, \frac{(\Delta_E + 2\nu)^2}{8} \right]$.

All of the outcomes listed in Proposition 4.1 are symmetric with respect to both the timing and market decisions. The symmetric outcome [DD:MM] is not an equilibrium strategy since the demand in the mature market is known with certainty and firms are thus not exposed to demand variance in this market; therefore, they have no incentive to delay investment. Moreover, investing early gains them a leadership position in the market. Thus, [DD:MM] is not an equilibrium strategy because firms have an incentive to change their strategy and invest early. We now characterize the remaining regions where multiple equilibria exist.

Proposition 4.2: *There are three regions in the parameter space that have multiple SPNE. Specifically, the SPNE are:*

1. [ID:ME] or [DI:EM] if

$$\frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2 < \sigma^2 < \min \left[\Delta_M^2 - (\Delta_E + \nu)^2, \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2 \right];$$

2. $[DD:ME]$ or $[DD:EM]$ if $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ and $\left[\left(\sigma^2 > \Delta_M^2 - (\Delta_E + \nu)^2 \right. \right.$
 $\left. \left. \text{and } \nu < \Delta_M - \Delta_E \right) \text{ or } \left(\sigma^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \text{ and } \nu > \Delta_M - \Delta_E \right) \right]$;
3. $[II:EM]$ or $[II:ME]$ if $\sigma^2 < \min \left[\frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \right]$ and
 $\Delta_M - \Delta_E < \nu < \frac{3\Delta_M}{4} - \frac{\Delta_E}{2}$.

As is evident from Proposition 4.2, the outcome pairs $[ID:MM]$ or $[DI:MM]$, $[ID:EM]$ or $[DI:ME]$ and $[ID:EE]$ or $[DI:EE]$ are not equilibria strategies. In the case of the first outcome pair, $[ID:MM]$ or $[DI:MM]$, the firm that delays investment into the mature market becomes the Stackelberg follower. Since delaying investment does not provide additional information about the demand for the mature market, firms can improve their position by investing early into the mature market, in which case they are engaged in a Cournot competition. Similarly, for the pair of outcomes $[ID:EM]$ and $[DI:ME]$, the firm that delays its investment can secure at least the same return by investing early into the mature market, i.e., $[II:EM]$ or $[II:ME]$. Finally, the pair of outcomes $[ID:EE]$ and $[DI:EE]$ imply that one firm invests early in spite of the demand uncertainty in the emerging market. In this case, the other firm is driven to follow suit to avoid being the Stackelberg follower, leading to the outcome $[II:EE]$. The details of these statements are in the proofs of Propositions 4.1 and 4.2.

The equilibria regions from Propositions 4.1 and 4.2 are illustrated in Figure 4.3 for a number of possible parameter instances. The x -axis depicts ν , the diffusion rate of the emerging market, and the y -axis depicts σ^2 , the variance of demand in the emerging market. These parameters define the key differences between the mature and the emerging market: the benefit of competitive entry into the emerging market is increasing in ν , and the benefit of delaying investment into the emerging market is increasing in σ^2 . In Figure 4.3, the unit cost

be the same in both markets, $c_M = c_E = 0.5$, and the demand in the emerging market, μ_E , is 1. The four sub-figures demonstrate the impact as μ_M is varied from 0.8 to 1.6. Figure 4.3 also reveals how ν and σ^2 affect the strategy of the two firms in terms of timing and market choices. The thick solid lines depict the separation between unique and multiple equilibria regions, as defined by Propositions 4.1 and 4.2, respectively.

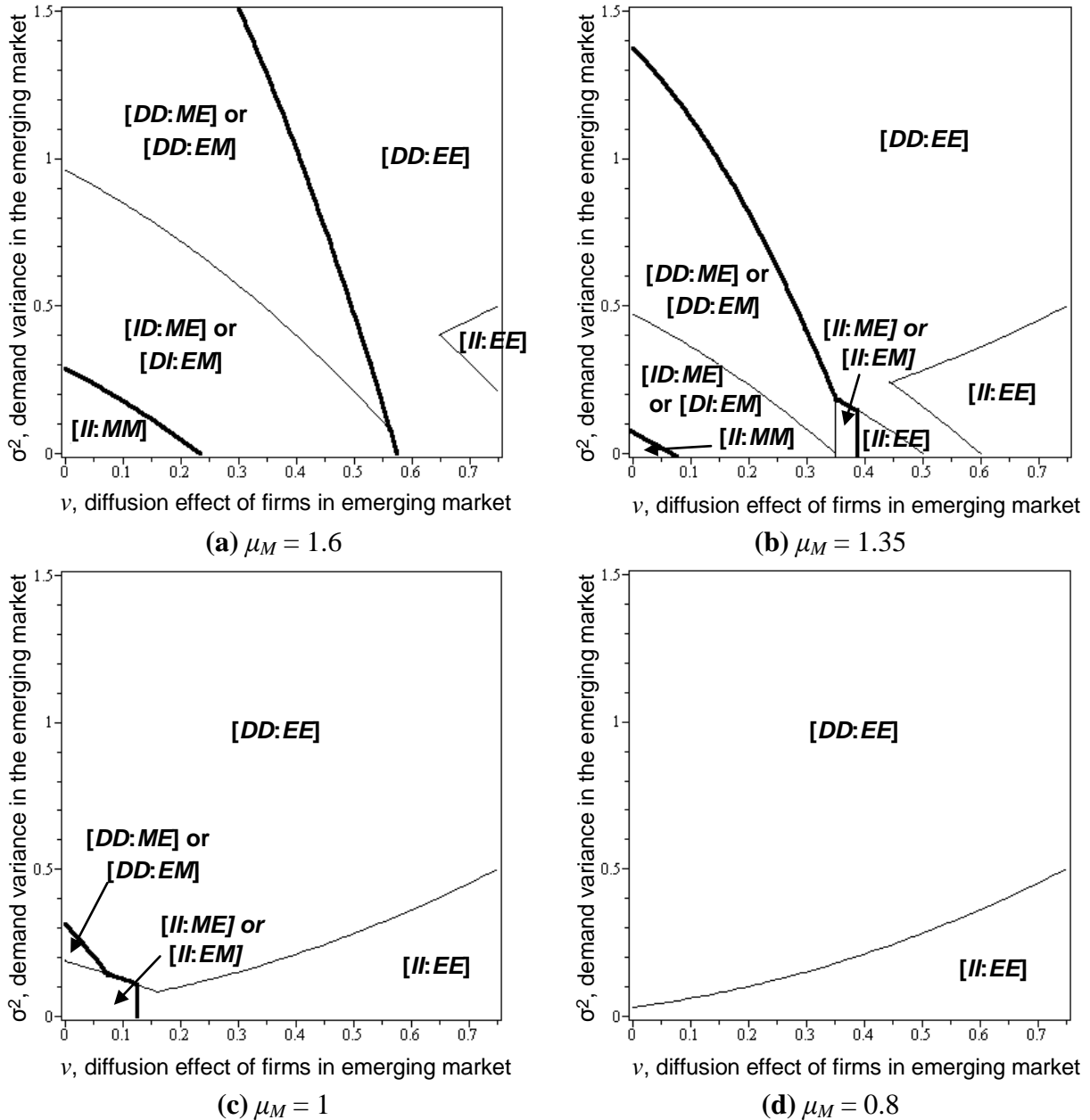


Figure 4.3: Optimal investment strategies of market and timing game for $\mu_E = 1$ and $c_M = c_E = 0.5$.

Note: thick solid lines depict the separation between unique and multiple equilibria regions.

According to Figure 4.3, some of the regions defined in Proposition 4.1 and 4.2 are eliminated for certain parameter values. For example, as the expected demand intercept of the mature market, μ_M , decreases in relation to that of the emerging market, μ_E , the region where at least one of the firms invests into the mature market shrinks (Figures 4.3a – 4.3c) and is eventually eliminated (Figure 4.3d).

Figure 4.3b depicts an instance where all seven regions defined in Propositions 4.1 and 4.2 are possible. As the following discussion demonstrates, the shape of these equilibria regions can be counter-intuitive. An increase in demand uncertainty is typically associated with an increase in the value of delaying investment (Dixit and Pindyck, 1994). Consequently, if firms invest early into the emerging market, then we expect that a sufficient increase in the demand uncertainty, σ^2 , will induce firms to switch to delaying their investment into this market. Since the returns from delayed investment into the emerging market are increasing in σ^2 (as shown in Lemma 4.1), we also expect an increase in σ^2 to drive firms to shift from the mature market into the emerging market. As Figure 4.3 illustrates, this is generally true in our model; but, surprisingly, we find that this does not hold true universally. For example, one can observe from Figure 4.3a (4.3b, respectively) that if $\nu = 0.75$ (0.55, respectively) and σ^2 increases from 0.1 to 0.3, then the equilibrium strategy changes from $[DD:EE]$ to $[II:EE]$. Despite an increase in demand uncertainty, firms switch from delayed investment to investing *early*! If $\sigma^2 = 0.1$, then a firm that invests early into the emerging market causes the other firm to invest into the mature market instead, thereby not only adding exposure to demand uncertainty but also foregoing the diffusion effects of the other firm. Therefore, both firms prefer to delay investment into the emerging market. By contrast, if $\sigma^2 = 0.3$, then demand uncertainty is sufficiently high so that a firm that invests early into the emerging market can rely on the other

firm pursuing delayed investment into the emerging market (thus adding its demand diffusion). Consequently, in this instance, firms switch from delayed to early investment as demand uncertainty increases.

As mentioned above, an increase in the diffusion effect, ν , enhances the attractiveness of the emerging market and, as a result, we expect firms to shift their investment from the mature market to the emerging market. Indeed, we observe this pattern in Figure 4.3. However, an increase in ν has a non-trivial effect on the timing of the investment decision. Generally, if σ^2 is large, we anticipate firms to delay their investment decision and if σ^2 is small, firms will invest early to secure the Stackelberg leader position. However, Figures 4.3a and 4.3b reveal that when σ^2 is small, firms modify their investment timing decisions as ν increases. Consider in Figure 4.3b the case where $\sigma^2 = 0.01$ and either $\nu < 0.06$ or $\nu > 0.59$: firms invest early, as expected. However, for intermediate values of ν , the optimal strategy of the timing game transitions to one firm investing early and one late ($0.06 < \nu < 0.34$), to both delaying investment ($0.34 < \nu < 0.35$), to both investing early ($0.35 < \nu < 0.49$), to both investing late ($0.49 < \nu < 0.59$). The existence of multiple viable projects (i.e., markets) in the portfolio is key to this complexity: if one market is far superior to the other, the pattern of optimal investment strategies becomes more predictable. Namely, as μ_M decreases, the timing complexity resolves, and, as exhibited in Figures 4.3c and 4.3d, we only have transitions from both delaying investment to both investing early. In summary, the decision of investing early and gaining a leadership position versus delaying investment to eliminate demand uncertainty is clearly dependent on the market parameters and can vary in unpredictable ways. While some of the tradeoffs are understood (Craig, 1995), the debate on optimal strategy continues in

the literature. Figure 4.3 supports the view that there is no universal optimal strategy and that the specific market situation should guide optimal timing decisions.

The existence of multiple investment opportunities in the portfolio also critically impacts the degree of uncertainty required for firms to delay investment into the emerging market. For example, for $0.5 < \nu < 0.6$ in Figure 4.3b, firms choose to delay investment even if $\sigma^2 = 0$. This result is driven by the fact that a firm that invests early into the emerging market—thereby claiming leadership—cannot rely on the other firm investing into the same market and adding its demand diffusion. Instead, the other firm has the option of achieving monopolistic returns in the mature market. If the diffusion effect of competitive entry into the emerging market is sufficiently high, the prospect of reduced demand (by the other firm deviating to the mature market) may be sufficient to prevent firms from pursuing the leader position in the emerging market through early investment. This leads to the outcome in which both firms delay investment into the emerging market in order to eliminate demand uncertainty and to take advantage of the diffusion effects. However, in the presence of high diffusion effects, firms may invest early into the emerging market even under relatively high demand uncertainty (for example at $\sigma^2 = 0.2$ and $\nu = 0.7$ in Figure 4.3b). We have the following theorem.

Theorem 4.1: *If $\Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} < \sigma^2 < \frac{(\Delta_E + 2\nu)^2}{8}$, then firms are trapped in a Prisoner's Dilemma situation where both firms invest early into the emerging market instead of the Pareto-optimal outcome of both pursuing delayed investment into the emerging market.*

In the parameter region defined by Theorem 4.1 (which is the region labeled [II:EE] at the bottom right of Figures 4.3a and 4.3b and parts of that region in Figures 4.3c and 4.3d), the diffusion effect is large enough for both firms to prefer to be a follower in the emerging market

over being a monopolist in the mature market. At the same time, the demand uncertainty is not high enough to warrant delayed investment into the emerging market. Even though both firms would receive higher returns by delayed investment, the additional return from being the Stackelberg leader (or the fear of only receiving follower returns) drives both firms to instead invest early into the emerging market. This dynamic is well documented in the literature: Lilien and Yoon (1990) empirically show that if markets are in the introductory or growth phase—i.e., diffusion effects are high—early entrants perform better than later entrants. Smit and Trigeorgis (2004) demonstrate how uncertainty in the market, and the advantage a leader might acquire, leads to outcomes where firms sacrifice substantial returns by investing prematurely. Emerging markets are particularly vulnerable to this dynamic as the early investor can build a dominant position through, for example, patents or distribution channels. The risk of being left behind, and potentially even blocked out of a market, drives firms to invest early even if demand uncertainty is quite substantial. An example of this market dynamic was the race to develop superior memory chips between a collaboration of Hitachi, Mitsubishi Electric and Texas Instruments and a collaboration of NEC, Lucent Technologies and Samsung. Both partnerships could have received higher returns if they had tempered the pace of investment; however, the stakes were too high and both partnerships invested significant resources at a very early stage of the technology (Smit and Trigeorgis, 2004).

The following theorem defines a region which is characterized by asymmetric market choices.

Theorem 4.2: *If $\frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2 < \sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$, the SPNE are such that the two firms invest in different markets.*

In the parameter region defined by Theorem 4.2, each firm prefers a monopoly position in the emerging market over Cournot investment into the mature market, and also prefers to be the monopolist in the mature market over delayed Cournot investment into the emerging market. The dynamics in this region are classified as coordination games (Cooper, 1999). In equilibrium, each firm pursues a different market but it is not clear which of the two equilibria strategies is pursued. An example of firms that have successfully pursued asymmetric investment strategies are the Coca Cola Company and PepsiCo. As described in the introduction, when in the late 1990s PepsiCo began pursuing a strategy of entering different markets than the Coca Cola Company, its international revenue and returns rose dramatically (Yoffie, 2004).

These asymmetric equilibria also result in asymmetric returns to the firms. Next, we discuss bounds on the difference of returns between the two firms. In particular, we can prove that one of the firms may receive up to 2.25 times greater returns than its competitor. Consequently, it may be justified to employ significant resources to secure the more desirable investment strategy equilibrium. In such a market dynamic, reliable competitive intelligence and an effective communication strategy are key in achieving a good outcome.

The key measure in defining the equilibria regions of Propositions 4.1 and 4.2 is the difference between the average demand and unit cost in the mature market, Δ_M , and the difference between the average demand and unit cost in the emerging market, Δ_E . The following theorem states the role of the ratio of these two measures.

Theorem 4.3: (i) If $\frac{\Delta_M}{\Delta_E} < \frac{3}{2}$, then the firms never invest into the mature market together; (ii) if $\frac{\Delta_M}{\Delta_E} < \frac{2}{3}$, then the firms always invest into the emerging market together.

Although a detailed analysis is required to reveal the optimal investment strategy for any particular situation, the ratio Δ_M/Δ_E can quickly reduce the complexity of the problem. The ratio provides quick direction without requiring decision-makers to first determine the degree of demand uncertainty and diffusion effects present in the emerging market. In particular, if this ratio is less than $3/2$, both firms investing into the mature market is never an optimal strategy and the mature market should only be considered if a firm thinks that it can secure the monopoly position in that market. Furthermore, if this ratio is less than $2/3$, firms know that even sole investment into the mature market is never an optimal strategy. Consequently, firms can shift their attention and resources fully onto the emerging market and the timing of their investment decision.

4.3. NON-SYMMETRIC FIRMS

In this section, we turn to a more general case where firms are not symmetric in terms of cost and diffusion effects. Let $c_{s,n}$ denote the unit cost of firm n for the product targeting market s , and let v_n denote firm n 's diffusion effect in the emerging market. With these assumptions, the number of model parameters increases from five to nine and the model becomes too complex for a closed form solution. We present two numerical instances that represent a broad range of scenarios to highlight some of the dynamics that can occur in the non-symmetric setting. In the first instance, we consider competition between a negligent and an attentive firm (as defined below), and in the second example, a multinational firm and a local firm compete.

4.3.1. STRATEGIC NEGLIGENCE

In this example, we analyze a negligent firm that allows its costs in the mature market to rise in comparison to its attentive competitor. Unit costs may rise due to internal factors, such as

insufficient investment in continuous employee training and maintenance of equipment, or due to external factors, such as an increase in the price of raw resources required in production. For this example, we let $c_{M,1} = 1.2 \cdot c_{M,2}$, $c_{E,1} = c_{E,2}$ and $v_1 = v_2 = v$. Figure 4.4 depicts the ranking of returns for the two firms depending on the uncertainty and diffusion effects of the emerging market.

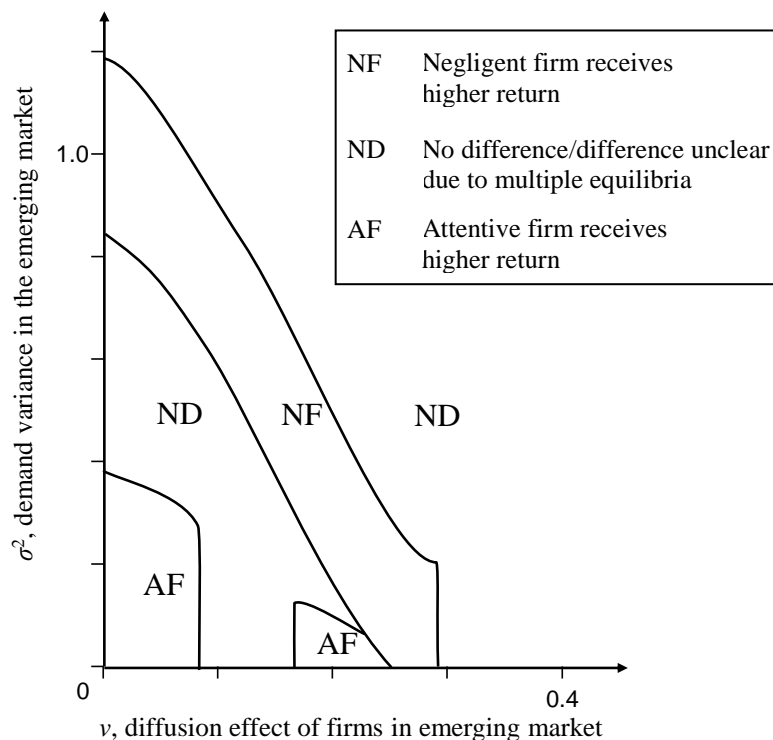


Figure 4.4: Negligent vs. Attentive Firm, with $\mu_M=1.35$, $\mu_E=1$, $c_{M,2}=c_{E,2}=c_{E,1}=0.5$, $c_{M,1}=0.6$

In the region denoted “NF” in Figure 4.4, higher unit costs in the mature market lead to an *advantage* for the negligent firm since it can commit to investment into the emerging market irrespective of the decision of the attentive firm. In this region, the attentive firm yields and invests into the less attractive mature market. This dynamic only changes if the emerging market is adequately attractive (high σ^2 and v), such that even the attentive firm prefers Cournot investment into the emerging market over sole investment into the mature market, or if the emerging market is sufficiently unattractive such that even the negligent firm invests into

the mature market. Consequently, increasing unit costs in a mature market (either through negligence or intentionally) can be a profitable strategy for certain market parameters.

4.3.2. MULTINATIONAL VS. LOCAL FIRM

Multinationals typically have significant cost advantages due to economies of scale and their experiences in other markets; meanwhile, local firms can be very successful in increasing the demand in their home markets due to their local knowledge and relationships with other key stakeholders such as suppliers (Craig, 1995). While demand in the mature market cannot be increased by either firm, we assume that the local firm has ability to increase demand in the emerging market to a much greater degree than the multinational. In line with the existing literature, we assume that $c_{s,1} = 0.8 \cdot c_{s,2} \forall s$ and $v_2 = 3v_1$, where Firm 1 is the multinational and Firm 2 is the local firm. Figure 4.5 depicts the return ranking depending on the uncertainty and diffusion effects in the emerging market.

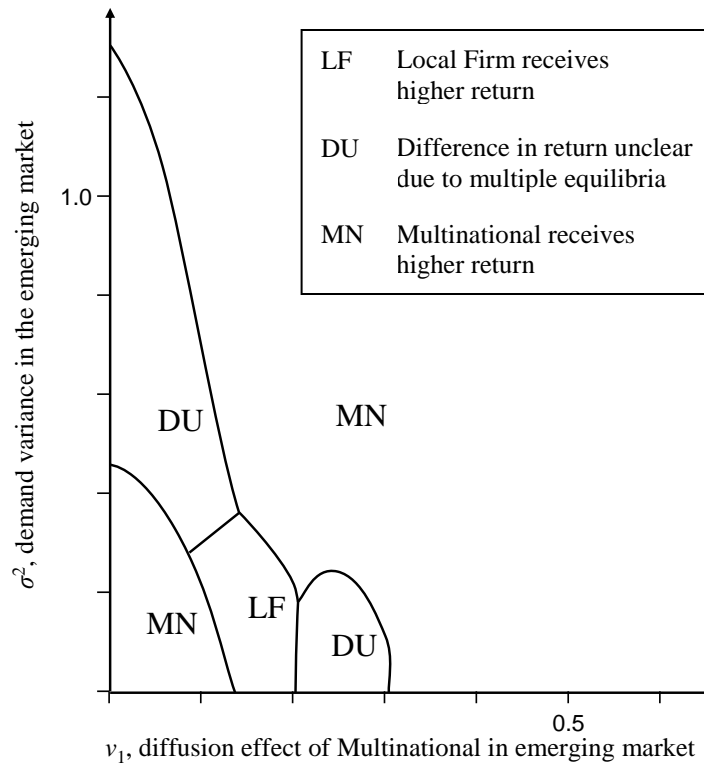


Figure 4.5: Multinational vs. Local Firm, with $\mu_M=1.35$, $\mu_E=1$, $c_{M,2}=c_{E,2}=0.5$, $c_{M,1}=c_{E,1}=0.4$, $v_2=3v_1$

As Figure 4.5 reveals, the multinational firm generally receives higher returns if both σ^2 and v_n are either high or low. In these regions, the investment decisions of the two firms are symmetric, and the multinational receives higher returns due to its cost advantage. However, we do find a region where the local firm has greater returns. This occurs when demand uncertainty is sufficiently low and diffusion effects are at some intermediate level. In the region labeled “LF”, the local firm commits to investing into the emerging market and pushes the multinational into the mature market. This occurs because the local firm can build the emerging market to be more profitable than the multinational can, due to its superior diffusion effect. However, if the emerging market becomes too attractive (very high uncertainty and/or high diffusion effect), then the multinational joins into the emerging market, using its lower costs to receive higher returns.

4.4. CHAPTER 4 CONCLUSIONS

In this chapter, we developed a model that combines the problem of deciding when, where and how much to invest while considering competition. With this comprehensive model, we are able to extend and modify previous findings. In particular, while the real options literature suggests that firms only delay investment if there is significant uncertainty in the market demand (see, for example, Kulatilaka and Perotti, 1998; Swinney et al., 2011), our analysis shows that firms may decide to delay investment even if there is little uncertainty in the market. In a single market setting, firms are tempted to invest early to gain a leader position and avoid being the follower; however, the existence of two investment alternatives in the portfolio provides firms with the flexibility to wait until demand uncertainty has been resolved, even if this uncertainty is not high.

Our analysis also provides support for the view that by focusing on internal considerations—and mostly ignoring competition—the PPM literature is missing an important aspect of the investment decision (Bower and Gilbert, 2005; Hauser et al., 2006). In particular, if diffusion effects are high, firms may invest early into the emerging market even if demand uncertainty is relatively high. Although delaying investment would lead to higher returns for both firms, both firms want to achieve even higher returns by becoming the leader in the market. Asymmetric investment is important in instances where there are multiple equilibria strategies and both firms seek to invest as a monopoly in either market. If the coordination of investments fails, a poor outcome for both firms can ensue. This challenge is exacerbated by the fact that one market is typically more attractive than the other and thus both firms prefer to be the monopolist in the more attractive market.

We also derived a straightforward ratio which can help decision makers focus their resources early into one market. In particular, if the ratio Δ_M/Δ_E is less than $2/3$, then firms never invest into the mature market and can focus on the emerging market. Since significant resources can be spent contemplating decisions and acquiring the necessary information to make an informed decision, a method of accelerating the decision process can lead to significant cost savings.

Although this model constrains the number of projects (i.e., markets) that firms can invest in, firms are not constrained with respect to the amount of resources they can invest into their chosen market. However, in many circumstances, firms have a limited R&D budget with which to invest in new projects. This shifts the problem from a binary investment decision of which project to pursue to a continuous investment decision of how to allocate a limited budget over multiple projects. Furthermore, we have again focused on duopoly settings in this

chapter. Expanding this model to oligopoly settings may reveal additional challenges as coordination and cooperation between firms become harder to achieve. In the next chapter, we address the continuous resource allocation problem and analyze duopoly as well as oligopoly settings.

4.5. PROOFS FOR CHAPTER 4

Proof of Lemma 4.1: We prove the optimal capacity decision for each outcome separately.

[II:EE]: The objective of firm n , $\max_{k_{E,n} \geq 0} E[(A_E - k_{E,1} - k_{E,2} - c_E)k_{E,n}]$, is concave in $k_{E,n}, \forall n$. Hence, the optimal Cournot capacity investments are $k_{E,1}^* = k_{E,2}^* = \frac{\Delta_E + 2\nu}{3}$, and $\pi_n^{II:EE^*} = \frac{(\Delta_E + 2\nu)^2}{9}, \forall n$.

[DD:EE]: The objective of firm n , $E\left[\max_{k_{E,n} \geq 0} ((A_E - k_{E,1} - k_{E,2} - c_E)k_{E,n})\right]$, is concave in $k_{E,n}, \forall n$. The optimal capacity investments are $k_{E,1}^* = k_{E,2}^* = \frac{A_E - c_E}{3}$, which are the Cournot outputs and depend on the realized demand in the emerging market. Therefore,

$$\pi_n^{DD:EE^*} = E\left[\frac{(A_E - c_E)^2}{9}\right] = \frac{(E[A_E - c_E])^2 + \text{Var}(A_E - c_E)}{9} = \frac{(\Delta_E + 2\nu)^2 + \sigma^2}{9}, \forall n.$$

[II:MM] or [DD:MM]: Similar to the proofs for [II:EE] and [DD:EE], except that $\sigma^2 = 0$ and $\nu = 0$. Therefore, $k_{M,1}^* = k_{M,2}^* = \frac{\Delta_M}{3}$ and $\pi_n^{II:MM^*} = \pi_n^{DD:MM^*} = \frac{\Delta_M^2}{9}$.

[II:EM] or [ID:EM] and [DI:EM] or [DD:EM]: Similar to previous proofs of the other outcomes, only that firms are monopolies in their respective markets. Therefore, the optimal capacity investment for the firm investing into the mature market is $\frac{\Delta_M}{2}$, for the firm investing early into the emerging market is $\frac{\Delta_E + \nu}{2}$, and for the firm pursuing delayed

investment into the emerging market is $\frac{A_E - c_E}{2}$. Hence, we have

$$\pi_2^{II:EM^*} = \pi_2^{ID:EM^*} = \pi_2^{DI:EM^*} = \pi_2^{DD:EM^*} = \frac{\Delta_M^2}{4}, \quad \pi_1^{II:EM^*} = \pi_1^{ID:EM^*} = \frac{(\Delta_E + v)^2}{4}, \text{ and}$$

$$\pi_1^{DI:EM^*} = \pi_1^{DD:EM^*} = \frac{\sigma^2 + (\Delta_E + v)^2}{4}.$$

[*ID:EE*] and [*ID:MM*]: Similar to the previous proofs, but the firm that invests early receives Stackelberg-leader returns and the firm that delays investment receives Stackelberg-follower returns.

Therefore, the optimal capacity investments for the leader are $\frac{\Delta_M}{2}$ and $\frac{\Delta_E + 2v}{2}$, for the mature and emerging market respectively. Similarly, the optimal capacity investments for the follower are $\frac{\Delta_M}{4}$ and $\frac{A_E - c_E}{4}$. Therefore, we have $\pi_1^{ID:EE^*} = \frac{(\Delta_E + 2v)^2}{8}$, $\pi_1^{ID:MM^*} = \frac{\Delta_M^2}{8}$,

$$\pi_2^{ID:EE^*} = \frac{4\sigma^2 + (\Delta_E + 2v)^2}{16}, \text{ and } \pi_2^{ID:MM^*} = \frac{\Delta_M^2}{16}. \quad \square$$

Preliminaries for the Proofs of Propositions 4.1 and 4.2: We establish the following

Lemmas which are required for the proofs of Propositions 4.1 and 4.2. For the remainder of the proofs, we state the condition in terms of the model variables (Δ_s, σ, v) , followed by the equivalence expressed in terms of the return statements, $\pi_n^{t_1 t_2 : s_1 s_2}$, in square brackets using

Lemma 4.1.

Lemma 4.A1: $\sigma^2 < \frac{4}{9}\Delta_M^2 - (\Delta_E + v)^2$ [$\Leftrightarrow \pi_1^{DD:MM} > \pi_1^{DD:EM}$] implies

$$\sigma^2 < \frac{9}{4}\Delta_M^2 - (\Delta_E + 2v)^2 \text{ [$\Leftrightarrow \pi_1^{DD:ME} > \pi_1^{DD:EE}$].}$$

Proof: The Lemma always hold if

$$\frac{9}{4}\Delta_M^2 - (\Delta_E + 2v)^2 > \frac{4}{9}\Delta_M^2 - (\Delta_E + v)^2. \quad (4.P1)$$

We can reduce the LHS of (4.P1) by increasing v , but given $\sigma^2 < \frac{4}{9}\Delta_M^2 - (\Delta_E + v)^2$ and the condition $\sigma^2 \geq 0$, we have $v \leq \frac{2}{3}\Delta_M - \Delta_E$. For the largest feasible v , $v = \frac{2}{3}\Delta_M - \Delta_E$, (4.P1) turns into $\left(\frac{3}{2}\Delta_M\right)^2 > \left(\frac{4}{3}\Delta_M - \Delta_E\right)^2$, which holds for all $\Delta_E > 0$. \square

The following Lemmas 4.A2 and 4.A3 compare returns in potential timing game outcomes for firms investing into the mature and the emerging market, respectively.

Lemma 4.A2: Returns from the mature market have the following properties for all

- $n \in \{1,2\}, t \in \{I, D\}, \sigma^2 > 0$ and $v > 0$: (i) $\pi_n^{II:MM} = \pi_n^{DD:MM}$; (ii) $\pi_1^{It:ME} = \pi_1^{Dt:ME}$;
(iii) $\pi_2^{It:EM} = \pi_2^{tD:EM}$; (iv) $\pi_1^{tt:ME} > \pi_1^{ID:MM} > \pi_1^{DD:MM} > \pi_1^{DI:MM}$; and
(v) $\pi_2^{tt:EM} > \pi_2^{DI:MM} > \pi_2^{DD:MM} > \pi_2^{ID:MM}$.

Proof: Follows immediately from Lemma 4.1. \square

Lemma 4.A3: Returns from the emerging market have the following properties for all

- $n \in \{1,2\}, t \in \{I, D\}$: (i) $\pi_n^{DD:EE} > \pi_n^{II:EE}$; (ii) $\pi_1^{Dt:EM} > \pi_1^{It:EM}$; (iii) $\pi_2^{tD:ME} > \pi_2^{tI:ME}$;
(iv) $\pi_1^{ID:EE} > \pi_1^{II:EE}$; (v) $\pi_2^{DI:EE} > \pi_2^{II:EE}$; (vi) $\pi_1^{It:EM} = \pi_1^{Dt:EM}$; (vii) $\pi_2^{tI:ME} = \pi_2^{tD:ME}$;
(viii) $\pi_1^{Dt:EM} = \pi_1^{It:EM}$; and (ix) $\pi_2^{tD:ME} = \pi_2^{tI:ME}$.

Proof: Follows immediately from Lemma 4.1. \square

Lemma 4.A4: Given symmetric firms, the conditions $\pi_1^{tt:EE} > \pi_1^{tt:ME}$ and $\pi_1^{tt:EE} > \pi_1^{tt:MM}$ imply a dominant Nash equilibrium at $[tt:EE]$, $\forall t$.

Proof: Since $\pi_1^{tt:EE} > \pi_1^{tt:ME}$ and, due to symmetry, $\pi_2^{tt:EE} > \pi_2^{tt:EM}$, $[tt:EE]$ is as Nash equilibrium while $[tt:ME]$ and $[tt:EM]$ is not a Nash equilibrium, $\forall t$. Due to $\pi_1^{tt:EE} > \pi_1^{tt:MM}$, $[tt:MM]$ is dominated by $[tt:EE]$, $\forall t$. \square

Lemma 4.A5: Given symmetric firms, the conditions $\pi_1^{DD:ss} > \pi_1^{ID:ss}$ and $\pi_1^{DD:ss} > \pi_1^{II:ss}$ imply a dominant Nash equilibrium at $[DD:ss]$ $\forall s$.

Proof: Since $\pi_1^{DD:ss} > \pi_1^{ID:ss}$ and due to symmetry, $[DD:ss]$ is a Nash equilibrium while $[ID:ss]$ and $[DI:ss]$ is not a Nash equilibria, $\forall s$. Due to $\pi_1^{DD:ss} > \pi_1^{II:ss}$, $[II:ss]$ is dominated by $[DD:ss]$, $\forall s$. □

Lemma 4.A6: If $\sigma^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ [$\Leftrightarrow \pi_2^{ID:EE} > \pi_2^{ID:EM}$] and $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_2^{DD:EM} > \pi_2^{DD:EE}$], then $\nu < \frac{\sqrt{2}\Delta_M - \Delta_E}{2}$ [$\Leftrightarrow \pi_1^{ID:ME} > \pi_1^{ID:EE}$].

Proof: The RHSs of the conditions $\sigma^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ and $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$

intersect at $\nu = \frac{\sqrt{\frac{5}{3}}\Delta_M - \Delta_E}{2}$. Since the former inequality decreases faster than the latter in ν ,

$\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ and $\sigma^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ can only hold prior to this intersection

point, $\nu < \frac{\sqrt{\frac{5}{3}}\Delta_M - \Delta_E}{2}$. Since $\frac{\sqrt{2}\Delta_M - \Delta_E}{2} > \frac{\sqrt{\frac{5}{3}}\Delta_M - \Delta_E}{2}$, the Lemma holds. □

Proof of Proposition 4.1: For each of the regions, we solve the game backward, recalling the optimal capacity decisions from Lemma 4.1. First, we prove the equilibria strategies of the market game for each of the four possible outcomes of the timing game $[t_1t_2]$ and then we prove the equilibrium strategy for the timing game itself. We assume that firms do not pursue weakly dominated strategies and that if there are two Nash equilibria where one provides higher returns for both firms, then firms pursue the Pareto efficient Nash equilibrium. Since the firms are symmetric with respect to their production costs, we immediately extend any conclusion that is made from Firm 1's perspective to Firm 2, and equilibria strategies for the timing game outcome $[DI]$ are equivalent to the strategies of $[ID]$.

Region 1: We prove that if $\sigma^2 < \frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2$, the equilibrium strategy is $[II:MM]$.

Timing Game outcome	Proof of respective market game equilibria strategy for given timing game outcome	Market Game strategy
[DD]	<p>The condition $\sigma^2 < \frac{4}{9}\Delta_M^2 - (\Delta_E + \nu)^2$ [$\Leftrightarrow \pi_1^{DD:MM} > \pi_1^{DD:EM}$] implies</p> <p>$\sigma^2 < \frac{9}{4}\Delta_M^2 - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_1^{DD:ME} > \pi_1^{DD:EE}$] by Lemma 4.A1.</p> <p>Consequently, if both firms delay investment, Firm 1 prefers investing in the mature market regardless of Firm 2's market decision.</p>	[MM]
[II]	<p>The condition $\sigma^2 < \frac{4}{9}\Delta_M^2 - (\Delta_E + \nu)^2$ [$\Leftrightarrow \pi_1^{DD:MM} > \pi_1^{DD:EM}$] implies $\pi_1^{II:MM} = \pi_1^{DD:MM} > \pi_1^{DD:EM} > \pi_1^{II:EM}$ by Lemmas 4.A2.i and 4.A3.ii.</p> <p>The additionally established condition $\sigma^2 < \frac{9}{4}\Delta_M^2 - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_1^{DD:ME} > \pi_1^{DD:EE}$] implies $\pi_1^{II:ME} = \pi_1^{DD:ME} > \pi_1^{DD:EE} > \pi_1^{II:EE}$ by Lemmas 4.A2.ii and 4.A3.i. Consequently, if both firms invest early, Firm 1 prefers to invest in the mature market regardless of Firm 2's market decision.</p>	[MM]

Although $\pi_1^{DD:MM} = \pi_1^{II:MM}$, delaying investment is a dominated strategy because if Firm 1 delays but Firm 2 invests early, Firm 1 receives lower returns than in the $[II:MM]$ outcome regardless of its market decision (the above conditions and Lemma 4.A2 imply

$\pi_1^{DI:MM} < \pi_1^{II:MM}$ and $\pi_1^{DI:EM} = \pi_1^{DD:EM} < \pi_1^{DD:MM} = \pi_1^{II:MM}$). Consequently, the unique strategy

investment equilibrium when $\sigma^2 < \frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2$ is $[II:MM]$.

Region 2: We first prove that if $\sigma_E^2 < \min \left[\frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \right]$ and

$\nu > \frac{3\Delta_M}{4} - \frac{\Delta_E}{2}$, the equilibrium strategy is [II:EE].

Timing Game outcome	Proof of respective market game equilibria strategy for given timing game outcome	Market Game strategy
[DD]	<p>The condition $\nu > \frac{3\Delta_M}{4} - \frac{\Delta_E}{2}$ [$\Leftrightarrow \pi_1^{II:EE} > \pi_1^{ID:ME}$] implies $\pi_1^{DD:EE} > \pi_1^{II:EE} > \pi_1^{ID:ME} = \pi_1^{DD:ME}$ by Lemmas 4.A3.i and 4.A2.ii. Similarly, the condition $\sigma_E^2 < \frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_1^{ID:EM} > \pi_1^{DD:EE}$] and the above condition $\pi_1^{II:EE} > \pi_1^{ID:ME}$ imply $\pi_1^{DD:EM} > \pi_1^{ID:EM} > \pi_1^{DD:EE} > \pi_1^{II:EE} > \pi_1^{ID:ME} > \pi_1^{DD:MM}$ by Lemmas 4.A3.ii, 4.A3.i, and 4.A2.iv.</p> <p>Consequently, if both firms delay investment, Firm 1 prefers to invest in the emerging market regardless of Firm 2's market decision.</p>	[EE]
[II]	<p>The above conditions $\pi_1^{II:EE} > \pi_1^{ID:ME}$ and $\pi_1^{ID:EM} > \pi_1^{DD:EE}$ imply $\pi_1^{II:EE} > \pi_1^{ID:ME} = \pi_1^{II:ME}$ and $\pi_1^{ID:EM} > \pi_1^{DD:EE} > \pi_1^{II:EE} > \pi_1^{ID:ME} > \pi_1^{II:MM}$ by Lemmas 4.A2.ii, 4.A3.i, and 4.A2.iv. Therefore, if both firms invest early, Firm 1 prefers to invest in the emerging market regardless of Firm 2's market decision.</p>	[EE]
[ID]	<p>The above conditions $\pi_1^{II:EE} > \pi_1^{ID:ME}$ and $\pi_1^{ID:EM} > \pi_1^{DD:EE}$ imply $\pi_1^{ID:EM} > \pi_1^{DD:EE} > \pi_1^{II:EE} > \pi_1^{ID:ME} > \pi_1^{ID:MM}$ by Lemmas 4.A3.i and 4.2.iv, which means $\pi_1^{ID:EM} > \pi_1^{ID:ME}$ and $\pi_1^{ID:EM} > \pi_1^{ID:MM}$. If the condition $\sigma_E^2 < \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ [$\Leftrightarrow \pi_2^{ID:EE} < \pi_2^{ID:EM}$] holds, then</p>	[EM]

	<p>Firm 1 knows that Firm 2 invests into the mature market if Firm 1 invests into the emerging market early. Since we have established that Firm 1 prefers this outcome to either possible outcome if Firm 1 invests into the mature market instead ($\pi_1^{ID:EM} > \pi_1^{ID:ME}$ and $\pi_1^{ID:EM} > \pi_1^{ID:MM}$) the SPNE of this timing game outcome is $[EM]$.</p>	
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Since we have $\pi_1^{ID:EM} > \pi_1^{DD:EE}$, and $\pi_1^{II:EE} > \pi_1^{ID:ME}$ implies $\pi_1^{II:EE} > \pi_1^{ID:ME} = \pi_1^{DI:ME}$ (by Lemma 4.A2.ii), Firm 1 prefers to invest early regardless of Firm 2's timing choice, and thus the equilibrium strategy when $\sigma_E^2 < \min \left[\frac{9(\Delta_E + 2\nu)^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \right]$ and $\nu > \frac{3\Delta_M}{4} - \frac{\Delta_E}{2}$ is $[II:EE]$.

Next, we prove that if $\Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} < \sigma^2 < \frac{(\Delta_E + 2\nu)^2}{8}$, the equilibrium strategy is $[II:EE]$ as well.

Timing Game outcome	Proof of respective market game equilibria strategy for given timing game outcome	Market Game strategy
$[DD]$	<p>One can immediately observe that $\sigma_E^2 < \frac{(\Delta_E + 2\nu)^2}{8}$</p> <p>$[\Leftrightarrow \pi_1^{ID:EE} > \pi_1^{DD:EE}]$, implies $\sigma_E^2 < \frac{7(\Delta_E + 2\nu)^2}{36}$ $[\Leftrightarrow \pi_1^{II:EE} > \pi_1^{DI:EE}]$.</p> <p>Coupled with the condition $\sigma_E^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$</p> <p>$[\Leftrightarrow \pi_1^{DI:EE} > \pi_1^{DI:ME}]$ and Lemmas 4.A3.i and 4.A2.ii this implies $\pi_1^{DD:EE} > \pi_1^{II:EE} > \pi_1^{DI:EE} > \pi_1^{DI:ME} = \pi_1^{DD:ME}$. Since, by Lemma 4.A2.iv, we also have $\pi_1^{DD:ME} > \pi_1^{DD:MM}$, Lemma 4.A4 implies that both firms invest into the emerging market.</p>	$[EE]$

[II]	<p>Since the above conditions $\pi_1^{II:EE} > \pi_1^{DI:EE}$ and $\pi_1^{DI:EE} > \pi_1^{DI:ME}$ imply $\pi_1^{II:EE} > \pi_1^{DI:EE} > \pi_1^{DI:ME} = \pi_1^{II:ME}$ and $\pi_1^{II:ME} > \pi_1^{II:MM}$ (by Lemmas 4.A2.ii and A2.iv), Lemma 4.A4 implies that both firms invest into the emerging market.</p>	[EE]
[ID]	<p>Since the above conditions $\pi_1^{ID:EE} > \pi_1^{DD:EE}$, $\pi_1^{II:EE} > \pi_1^{DI:EE}$ and $\pi_1^{DI:EE} > \pi_1^{DI:ME}$ imply $\pi_1^{ID:EE} > \pi_1^{DD:EE} > \pi_1^{II:EE} > \pi_1^{DI:EE} > \pi_1^{DI:ME} = \pi_1^{ID:ME}$ (by Lemmas 4.A3.i and 4.A2.ii), and we also have $\pi_1^{ID:ME} > \pi_1^{ID:MM}$ (by Lemma 4.A2.iv) and the condition $\sigma_E^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ [$\Leftrightarrow \pi_2^{ID:EE} > \pi_2^{ID:EM_1}$], [ID:EE] is the unique SPNE.</p>	[EE]

Since we have established $\pi_1^{ID:EE} > \pi_1^{DD:EE}$ and $\pi_1^{II:EE} > \pi_1^{DI:EE}$, Firm 1 prefers to invest early regardless of Firm 2's timing decision, and thus the equilibrium strategy when

$$\Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} < \sigma^2 < \frac{(\Delta_E + 2\nu)^2}{8} \text{ is [II:EE].}$$

Region 3: We prove that if

$$\max \left[\frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2, \frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2 \right] < \sigma^2 < \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \text{ or}$$

$$\sigma^2 > \max \left[\frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}, \frac{(\Delta_E + 2\nu)^2}{8} \right], \text{ the equilibrium strategy is [DD:EE].}$$

Timing Game outcome	Proof of respective market game equilibria strategy for given timing game outcome	Market Game strategy
[DD]	Given the condition $\sigma^2 > \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_1^{DD:EE} > \pi_1^{DD:ME}$]	[EE]

	<p>and since $\pi_1^{DD:ME} > \pi_1^{DD:MM}$ (by Lemma 4.A2.iv), Lemma 4.A4 implies that Firm 1 always invests into the emerging market.</p>	
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The earlier condition $\pi_1^{DD:EE} > \pi_1^{DD:ME}$ implies $\pi_1^{DD:EE} > \pi_1^{DD:ME} = \pi_1^{II:ME}$ (by Lemma 4.A2.ii) and $\pi_1^{DD:EE} > \pi_1^{DD:ME} > \pi_1^{II:MM}$ (by Lemma 4.A2.iv); furthermore, the condition $\sigma^2 > \frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_1^{DD:EE} > \pi_1^{ID:EM}$] implies $\pi_1^{DD:EE} > \pi_1^{ID:EM} = \pi_1^{II:EM}$ (and Lemma 4.A3.vi) and we have $\pi_1^{DD:EE} > \pi_1^{II:EE}$ (by Lemma 4.A3.i). Consequently, the outcome $[DD:EE]$ yields returns greater than any possible market game outcome of the timing game $[II]$.

The earlier condition $\pi_1^{DD:EE} > \pi_1^{DD:ME}$ implies $\pi_1^{DD:EE} > \pi_1^{DD:ME} = \pi_1^{ID:ME}$ (by Lemma 4.A2.ii) and $\pi_1^{DD:EE} > \pi_1^{DD:ME} > \pi_1^{ID:MM}$ (by Lemma 4.A2.iv) and we also have $\sigma^2 > \frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_1^{DD:EE} > \pi_1^{ID:EM}$]; in addition, $\sigma^2 < \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ [$\Leftrightarrow \pi_2^{ID:EE} < \pi_2^{ID:EM}$] implies that $[ID:EE]$ is not attainable because Firm 2 will switch to $[ID:EM]$. Therefore, Firm 1 prefers $[DD:EE]$ over any possible market game outcome of the timing game $[ID]$. From Lemma 4.A5, the outcome when

$$\max \left[\frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2, \frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2 \right]^2 < \sigma^2 < \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \text{ or}$$

$$\sigma^2 > \max \left[\frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}, \frac{(\Delta_E + 2\nu)^2}{8} \right] \text{ is } [DD:EE]. \quad \square$$

Proof of Proposition 4.2:

Region 1: We prove that if

$$\frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2 < \sigma^2 < \min \left[\Delta_M^2 - (\Delta_E + \nu)^2, \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2 \right],$$

the equilibria strategies are $[ID:ME]$ and $[DI:EM]$.

Timing Game outcome	Proof of respective market game equilibria strategy for given timing game outcome	Potential Market Game strategies
[DD]	If $\sigma^2 > \frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2$ [$\Leftrightarrow \pi_1^{DD:EM} > \pi_1^{DD:MM}$] and $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_2^{DD:EM} > \pi_2^{DD:EE}$], then $[DD:EM]$ is a Nash equilibrium and, due to symmetry, so is $[DD:ME]$.	[EM] or [ME]
[II]	The condition $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_1^{DD:ME} > \pi_1^{DD:EE}$] implies $\pi_1^{II:ME} = \pi_1^{DD:ME} > \pi_1^{DD:EE} > \pi_1^{II:EE}$ by Lemmas 4.A2.ii and 4.A3.i. Therefore, $[II:EE]$ is not a Nash equilibrium. This leaves $[II:ME]$, $[II:EM]$ and $[II:MM]$ as potential outcomes.	[ME] or [EM] or [MM]
[ID]	One can immediately observe that the condition $\sigma^2 < \Delta_M^2 - (\Delta_E + \nu)^2$ [$\Leftrightarrow \pi_1^{DD:ME} > \pi_1^{DD:EM}$] implies $\sigma^2 < \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ [$\Leftrightarrow \pi_2^{ID:EM} > \pi_2^{ID:EE}$]. These conditions further imply $\pi_1^{ID:ME} = \pi_1^{DD:ME} > \pi_1^{DD:EM} > \pi_1^{ID:EM}$ (by Lemmas 4.A2.ii and 4.A3.ii) and $\pi_2^{ID:ME} = \pi_2^{DD:ME} > \pi_2^{DD:MM} > \pi_2^{ID:MM}$ (by Lemmas 4.A2.ii and 4.A2.iv); therefore, $[ID:ME]$ is a unique SPNE.	[ME]

Given the number of potential market games defined for each possible timing game, there are six potential outcome combinations that we need to analyze (two outcomes from $[DD]$ · three outcomes from $[II]$ · a single outcome from $[ID]$).

If the outcomes are $[DD:ME]$ and $[II:ME]$, then the above condition $\pi_1^{DD:ME} > \pi_1^{DD:EM}$ implies $\pi_1^{II:ME} = \pi_1^{DD:ME} > \pi_1^{DD:EM} = \pi_1^{DI:EM}$ (by Lemmas 4.A2.ii and 4.A3.viii) and we also have $\pi_1^{ID:ME} = \pi_1^{DD:ME}$ (by Lemma 4.A2.ii); therefore, delaying is a weakly dominated strategy for Firm 1. Since we have established that Firm 1 will invest early and since we have $\pi_2^{ID:ME} > \pi_2^{II:ME}$ (by Lemma 4.A3.iii), the equilibrium outcome is $[ID:ME]$. By the same logic, if the outcomes are $[DD:EM]$ and $[II:EM]$, the equilibrium outcome is $[DI:EM]$.

If the outcomes are $[DD:EM]$ and $[II:ME]$, then the above condition $\pi_1^{DD:ME} > \pi_1^{DD:EM}$ implies $\pi_1^{II:ME} = \pi_1^{DD:ME} > \pi_1^{DD:EM} = \pi_1^{DI:EM}$ (by Lemmas 4.A2.ii and 4.A3.viii) and $\pi_1^{ID:ME} = \pi_1^{DD:ME} > \pi_1^{DD:EM}$ (by Lemma 4.A2.ii); therefore, delaying is a strictly dominated strategy for Firm 1 and since we have $\pi_2^{ID:ME} > \pi_2^{II:ME}$ (by Lemma 4.A3.iii), the equilibrium outcome is $[ID:ME]$. By the same logic, if the outcomes are $[DD:ME]$ and $[II:EM]$, the equilibrium outcome is $[DI:EM]$.

If the outcomes are $[DD:ME]$ and $[II:MM]$, then the above condition $\pi_1^{DD:EM} > \pi_1^{DD:MM}$ implies $\pi_1^{II:MM} = \pi_1^{DD:MM} < \pi_1^{DD:EM} = \pi_1^{DI:EM}$ (by Lemmas 4.A2.i and 4.A3.viii) and we also have $\pi_1^{ID:ME} = \pi_1^{DD:ME}$ (by Lemma 4.A2.ii); therefore, investing early is a weakly dominated strategy for Firm 1 and since the earlier condition $\pi_1^{DD:ME} > \pi_1^{DD:EM}$ implies $\pi_2^{DI:EM} = \pi_2^{DD:EM} > \pi_2^{DD:ME}$ (by Lemma 4.A2.ii), the equilibrium outcome is $[DI:EM]$. By the same logic, if the outcomes are $[DD:EM]$ and $[II:MM]$, the equilibrium outcome is $[ID:ME]$.

Consequently, the equilibria strategies when $\frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2 < \sigma^2 < \min \left[\Delta_M^2 - (\Delta_E + \nu)^2, \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2 \right]$ are $[ID:ME]$ and $[DI:EM]$.

Region 2: We prove that if $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ and $\left[\left(\sigma^2 > \Delta_M^2 - (\Delta_E + \nu)^2 \right. \right.$
and $\left. \left. \nu < \Delta_M - \Delta_E \right) \right]$ or $\left(\sigma^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \right.$ and $\left. \left. \nu > \Delta_M - \Delta_E \right) \right]$, the equilibria strategies are $[DD:ME]$ and $[DD:EM]$.

Timing Game outcome	Proof of respective market game equilibria strategy for given timing game outcome	Potential Market Game strategies
[DD]	The condition $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_1^{DD:ME} > \pi_1^{DD:EE}$] implies that $[DD:EE]$ is not a Nash equilibrium. Similarly, the condition $\sigma^2 > \Delta_M^2 - (\Delta_E + \nu)^2$ [$\Leftrightarrow \pi_1^{DD:EM} > \pi_1^{DD:ME}$] implies $\pi_1^{DD:EM} > \pi_1^{DD:ME} > \pi_1^{DD:MM}$ (by Lemma 4.A2.iv); therefore, $[DD:MM]$ is not a Nash equilibrium.	[EM] or [ME]
[II]	The above condition $\pi_1^{DD:ME} > \pi_1^{DD:EE}$ implies $\pi_1^{II:ME} = \pi_1^{DD:ME} > \pi_1^{DD:EE} > \pi_1^{II:EE}$ (by Lemmas 4.A2.ii and 4.A3.i), and thus $[II:EE]$ is not a Nash equilibrium.	[EM] or [ME] or [MM]
[ID]	We begin by analyzing the condition set $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ and $\sigma^2 > \Delta_M^2 - (\Delta_E + \nu)^2$ and $\nu < \Delta_M - \Delta_E$:	[ME]

<p>The condition $\sigma^2 > \Delta_M^2 - (\Delta_E + \nu)^2$ [$\Leftrightarrow \pi_2^{DD:ME} > \pi_2^{DD:EM}$] implies $\pi_2^{ID:ME} = \pi_2^{DD:ME} > \pi_2^{DD:EM} > \pi_2^{ID:MM}$ by Lemmas 4.A3.ix and 4.A2.v.</p> <p>If we had $\sigma^2 < \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ [$\Leftrightarrow \pi_2^{ID:EM} > \pi_2^{ID:EE}$], then the above condition $\pi_2^{ID:ME} > \pi_2^{ID:MM}$ and $\nu < (\Delta_M) - (\Delta_E)$ [$\Leftrightarrow \pi_1^{ID:ME} > \pi_1^{ID:EM}$] would imply a SPNE at $[ID:ME]$. However, if $\pi_2^{ID:EM} > \pi_2^{ID:EE}$ does not hold, we still have $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_2^{DD:EM} > \pi_2^{DD:EE}$] and according to Lemma 4.A6 we thus have $\pi_1^{ID:ME} > \pi_1^{ID:EE}$. Combined with $\pi_1^{ID:ME} > \pi_1^{ID:EM}$ and $\pi_2^{ID:ME} > \pi_2^{ID:MM}$, this also implies a SPNE at $[ID:ME]$.</p> <p>Next, we analyze the condition set $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ and $\sigma^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ and $\nu > \Delta_M - \Delta_E$:</p> <p>According to Lemma 4.A6, $\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ [$\Leftrightarrow \pi_2^{DD:EM} > \pi_2^{DD:EE}$] and $\sigma^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ [$\Leftrightarrow \pi_2^{ID:EE} > \pi_2^{ID:EM}$] can only both hold if $\nu < \frac{\sqrt{2}\Delta_M - \Delta_E}{2}$ [$\Leftrightarrow \pi_1^{ID:ME} > \pi_1^{ID:EE}$]. Additionally, the condition $\nu > \Delta_M - \Delta_E$ [$\Leftrightarrow \pi_2^{DI:ME} > \pi_2^{DI:EM}$] implies $\pi_2^{ID:ME} > \pi_2^{DI:ME} > \pi_2^{DI:EM} > \pi_2^{ID:MM}$ by Lemmas 4.A3.iii and 4.A2.v.</p> <p>Therefore $[ID:ME]$ is the SPNE.</p>	
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The above conditions $\pi_1^{DD:EM} > \pi_1^{DD:ME}$ and $\pi_1^{ID:EM} > \pi_1^{ID:ME}$ imply $\pi_1^{DD:EM} > \pi_1^{DD:ME} = \pi_1^{ID:ME}$ and $\pi_1^{DD:EM} > \pi_1^{ID:EM} > \pi_1^{ID:ME}$ by Lemmas 4.A2.ii and 4.A3.ii, respectively. Since we

also have $\pi_1^{DD:ME} = \pi_1^{ID:ME}$ (by Lemma 4.A2.ii), $[DD:ME]$ and $[DD:EM]$ are equilibria strategies. Since the conditions $\pi_1^{DD:EM} > \pi_1^{DD:ME}$ and $\pi_1^{ID:EM} > \pi_1^{ID:ME}$ also imply $\pi_1^{DI:EM} = \pi_1^{DD:EM} > \pi_1^{ID:EM} = \pi_1^{II:EM} = \pi_1^{ID:EM} > \pi_1^{ID:ME} = \pi_1^{DD:ME} = \pi_1^{II:ME} > \pi_1^{II:MM}$ (by Lemmas 4.A3.viii, 4.A3.ii, 4.A2.ii, and 4.A2.iv), we also have $\pi_1^{DI:EM} > \pi_1^{II:ME}$, $\pi_1^{DI:EM} > \pi_1^{II:MM}$ and $\pi_1^{DI:EM} > \pi_1^{II:EM}$; therefore, $[DD:ME]$ and $[DD:EM]$ are the only equilibria strategies when

$$\sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2 \text{ and } \left[\left(\sigma^2 > \Delta_M^2 - (\Delta_E + \nu)^2 \text{ and } \nu < \Delta_M - \Delta_E \right) \text{ or } \left(\sigma^2 > \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \text{ and } \nu > \Delta_M - \Delta_E \right) \right].$$

Region 3: We prove that if $\sigma^2 < \min \left[\frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \right]$ and

$$\Delta_M - \Delta_E < \nu < \frac{3\Delta_M}{4} - \frac{\Delta_E}{2}, \text{ the equilibria strategies are } [II:EM] \text{ and } [II:ME].$$

Timing Game outcome	Proof of respective market game equilibria strategy for given timing game outcome	Potential Market Game strategies
[DD]	The condition $\nu > \Delta_M - \Delta_E$ [$\Leftrightarrow \pi_1^{ID:EM} > \pi_1^{ID:ME}$] implies $\pi_1^{DD:EM} > \pi_1^{ID:EM} > \pi_1^{ID:ME} > \pi_1^{DD:MM}$ by Lemmas 4.A3.ii and 4.A2.iv; therefore, $[DD:MM]$ is not a Nash equilibrium.	[EM] or [ME] or [EE]
[II]	The above condition $\pi_1^{ID:EM} > \pi_1^{ID:ME}$ also implies $\pi_1^{II:EM} = \pi_1^{ID:EM} > \pi_1^{ID:ME} > \pi_1^{II:MM}$ by Lemmas 4.A3.vi and 4.A2.iv, and the condition $\nu < \frac{3\Delta_M}{4} - \frac{\Delta_E}{2}$ [$\Leftrightarrow \pi_1^{ID:ME} > \pi_1^{II:EE}$] implies $\pi_1^{II:ME} = \pi_1^{ID:ME} > \pi_1^{II:EE}$ by Lemma 4.A2.ii; therefore, $[II:EM]$ is a Nash equilibrium. Due to symmetry, $[II:ME]$ is also a Nash equilibrium.	[EM] or [ME]

[ID]	<p>Since the above conditions $\pi_1^{ID:EM} > \pi_1^{ID:ME}$ implies $\pi_2^{ID:ME} >$ $\pi_2^{DI:ME} > \pi_2^{DI:EM} > \pi_2^{ID:MM}$ (by Lemmas 4.A3.iii and 4.A2.v) and we also have $\sigma^2 < \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4}$ [$\Leftrightarrow \pi_2^{ID:EM} > \pi_2^{ID:EE}$], the SPNE is [<i>ID:EM</i>].</p>	[EM]
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Regardless of the market game outcome under [*DD*], one of the firms has an incentive to invest early since the above condition $\pi_1^{ID:EM} > \pi_1^{ID:ME}$ implies $\pi_1^{ID:EM} > \pi_1^{ID:ME} = \pi_1^{DD:ME}$ (by Lemma 4.A2.ii) and $\pi_2^{DI:ME} > \pi_2^{DI:EM} = \pi_2^{DD:EM}$ (by Lemma 4.A2.iii), and we also have

$$\sigma^2 < \frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2 \quad [\Leftrightarrow \pi_1^{ID:EM} > \pi_1^{DD:EE}].$$

From the outcome [*ID:EM*], Firm 2 has

an incentive to invest early since the condition $\pi_1^{ID:EM} > \pi_1^{ID:ME}$ implies $\pi_2^{II:ME} = \pi_2^{DI:ME} >$
 $\pi_2^{DI:EM} = \pi_2^{ID:EM}$ (by Lemmas 4.A3.vii and 4.A2.iii), and we also have $\pi_2^{II:EM} = \pi_2^{ID:EM}$ (by

Lemma 4.A2.iii). Only from the outcomes [*II:EM*] and [*II:ME*] neither of the firms has an incentive to switch strategy. Therefore, [*II:EM*] and [*II:ME*] are the equilibria strategies when

$$\sigma^2 < \min \left[\frac{9(\Delta_E + \nu)^2}{4} - (\Delta_E + 2\nu)^2, \Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} \right] \text{ and } \Delta_M - \Delta_E < \nu < \frac{3\Delta_M}{4} - \frac{\Delta_E}{2}. \quad \square$$

Proof of Theorem 4.1: When $\Delta_M^2 - \frac{(\Delta_E + 2\nu)^2}{4} < \sigma^2 < \frac{(\Delta_E + 2\nu)^2}{8}$, then for any timing decision by the two firms, both invest into the emerging market (by the proof of Region 2 of Proposition 4.1). Consequently, we focus only on the timing decision and prove that it has the same ordinal form as in the Prisoner's Dilemma (see Table 4.2).

Prisoner's Dilemma		Firm 2	
		I	D
Firm 1	I	2, 2 $(\pi_1^{II:EE}, \pi_2^{II:EE})$	4, 1 $(\pi_1^{ID:EE}, \pi_2^{ID:EE})$
	D	1, 4 $(\pi_1^{DI:EE}, \pi_2^{DI:EE})$	3, 3 $(\pi_1^{DD:EE}, \pi_2^{DD:EE})$

Table 4.2: The Prisoner's Dilemma in ordinal form

One can immediately observe that the condition $\sigma^2 < \frac{(\Delta_E + 2\nu)^2}{8}$ [$\Leftrightarrow \pi_1^{ID:EE} > \pi_1^{DD:EE}$]

implies $\sigma^2 < \frac{7(\Delta_E + 2\nu)^2}{36}$ [$\Leftrightarrow \pi_1^{II:EE} > \pi_1^{DI:EE}$]. By Lemma 4.A3.i, we also have

$\pi_1^{DD:EE} > \pi_1^{II:EE}$. Given symmetric firms, this fully characterizes the Prisoner's Dilemma,

where the equilibrium strategy is $[II:EE]$, but this outcome is Pareto-inferior to $[DD:EE]$. \square

Proof of Theorem 4.2: The condition $\frac{4\Delta_M^2}{9} - (\Delta_E + \nu)^2 < \sigma^2 < \frac{9\Delta_M^2}{4} - (\Delta_E + 2\nu)^2$ defines a

region that is a subset of Regions 1-3 of Proposition 4.2. For $\nu < \Delta_M - \Delta_E$, this region is

comprised of Regions 1 and 2 of Proposition 4.2. For $\nu > \Delta_M - \Delta_E$, this region is a subset of

Regions 2 and 3 of Proposition 4.2. Therefore, there are always two SPNE in this region. \square

Proof of Theorem 4.3: (i) Firms will only both invest into the mature market if

$\pi_n^{DI:EM} < \pi_n^{II:MM} \quad \forall n \quad \left(\frac{\sigma^2 + (\Delta_E + \nu)^2}{4} < \frac{\Delta_M^2}{9} \right)$. Intuitively, the Cournot returns from the

mature market must exceed the highest possible return from monopoly investment into the

emerging market, which is achieved by delayed investment into the emerging market. The

LHS of the expression in brackets can be minimized by setting σ^2 and ν to zero. The equation

then rearranges to $\frac{\Delta_M}{\Delta_E} > \frac{3}{2}$. This implies that even if the uncertainty and diffusion effects of

the emerging market are zero, the ratio $\frac{\Delta_M}{\Delta_E}$ must be greater than 3/2 for both firms to invest in

the mature market. Conversely, if this ratio is less than $3/2$, firms will not both invest in the mature market for any market parameters. (ii) All regions where the outcome of the market game is not $[EE]$ (defined by Proposition 4.2 and Region 1 in Proposition 4.1) are constrained by $\pi_1^{II:EE} < \pi_1^{II:ME} \left(\frac{\Delta_E + 2v}{3} < \frac{\Delta_M}{2} \right)$. Intuitively, the monopoly return is the best possible return from the mature market and by investing early into the emerging market a firm can guarantee at least Cournot returns. Without the condition $\pi_1^{II:EE} < \pi_1^{II:ME}$, firms have no incentive to invest in anything but the emerging market. However, if $\frac{\Delta_M}{\Delta_E} < \frac{2}{3}$, then $\frac{\Delta_E + 2v}{3} < \frac{\Delta_M}{2}$ cannot hold for any feasible value of $v \geq 0$. Consequently, if $\frac{\Delta_M}{\Delta_E} < \frac{2}{3}$, then both firms invest into the emerging market. □

5. CPPM: CONTINUOUS INVESTMENT DECISIONS

In some instances, firms have a well developed business case for each project opportunity and require a fixed amount of resources to execute a particular project (see Chapter 3). In other instances, the optimal investment into a particular project, or market, is based on the available market parameters (see Chapter 4). In both cases, we have assumed that firms face binary investment decision of which projects to fund and are constrained by the number of projects that they can invest in. Namely, firms choose between investing fully into either the emerging market or the mature market.

However, in many situations, firms' project investments can be scaled according to the amount of resources a firm is willing or able to invest (Loch and Kavadias, 2010). Under this framework, firms do not just decide whether to fund a project, but *how much* to invest into each project, subject to their budget constraint. The model in this chapter still assumes two firms with projects that target a mature and an emerging market, but firms decide how to distribute their budget over the two markets in a continuous manner.

This chapter is organized as follows. In the next section, we establish the benchmark case by discussing the optimal budget allocation of a monopoly. In Section 5.2 we develop the competitive PPM model that considers the budget allocation problem faced by duopoly firms. We are then able to derive insights regarding the effect of competition. Section 5.3 analyzes the oligopoly setting and Section 5.4 contains conclusions for this chapter.

5.1. THE MONOPOLY BENCHMARK

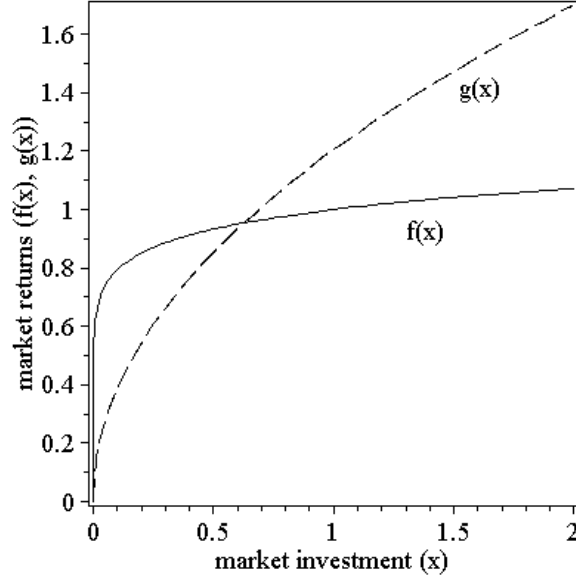
Consider a monopoly faced with two project investment opportunities: one targeting a mature market and another targeting an emerging market. Endowed with an investment budget of B_M , all of which is to be invested, the monopoly needs to decide what share $r_M \in [0,1]$ of its budget to allocate to the project targeting the mature market, where the remaining $1 - r_M$ of the budget is invested in the project targeting the emerging market. Assuming that project investments can be scaled according to the available budget, our focus is on how a monopoly allocates a given budget between the two projects.

The mature and emerging markets offer returns for an investment amount x according to functions $f(x) = ax^\alpha$ and $g(x) = bx^\beta$, respectively, where $\alpha, \beta \in (0,1)$. Thus, the market return functions follow the Inada conditions (Inada, 1963): (i) zero investment into a market results in zero returns from that market, $f(0), g(0) = 0$; (ii) increasing investment into a market always results in higher total returns, $f', g' > 0$; (iii) the marginal returns are decreasing, $f'', g'' < 0$; and (iv) the functions are continuously differentiable and the limit of the derivative towards zero is positive infinity and the limit towards positive infinity is zero.

The parameters a and b define the market potential of the mature and emerging markets, respectively: the greater the market potential, the greater the returns from that market at any investment level. We characterize the emerging market as the market with the greater market potential, $b > a$, however, the emerging market is also more risky than the well-understood mature market. To capture this uncertainty, we let p , $p \in (0,1)$, denote the probability that the emerging market has the expected return function $g(\cdot)$ and assume that with probability $1 - p$ the emerging market provides no returns.

The parameters α and β , represent the degree of homogeneity of the mature and emerging markets' return functions, respectively. In the context of this resource allocation problem, we use these parameters to define the marginal productivity of R&D investment of the respective markets. A low marginal productivity (i.e., a low value of α or β) implies that the market provides significant returns for small investments but then experiences diminishing returns quickly. By contrast, markets with high marginal productivity may initially provide lower returns but continue to provide significant returns for large investments. We assume that the emerging market has a higher marginal productivity than the mature market, $\alpha < \beta$, since the mature market provides less opportunities for significant product or service improvements or new product developments.

As depicted in Figure 5.1 (for $\alpha = 0.1$, $\beta = 0.5$, $a = 1$, $b = 2$, $p = 0.6$), these assumptions generally imply that the mature market provides larger returns than the emerging market for small investments. For example, a small product change to an existing product may provide some quick returns in a mature market in which the firm has already established a customer base, whereas this product change would not see large returns in an emerging market without significant investment into marketing of the new product. By contrast, significant investment into a radically new product may lead to disappointing returns in a mature market, whereas such a new product may help develop the emerging market and lead to very high returns. Although a large investment into the emerging market can lead to the highest possible return for a firm, investment into an emerging market is not without risk. Note that we are not attempting to fully capture the characteristics of a mature versus an emerging market but are using this terminology to facilitate the discussion.



**Figure 5.1: Market returns of mature and emerging markets;
 $\alpha = 0.1, \beta = 0.5, a = 1, b = 2, p = 0.6$**

The risk neutral monopoly aims to maximize its total expected return $E[\pi_M] = a(r_M B_M)^\alpha + pb((1-r_M)B_M)^\beta$. Under the Inada conditions, the optimal budget allocation occurs when the marginal returns from all markets coincide (Loch and Kavadias, 2002). Specifically, the optimal allocation, r_M^* , is the value of r_M that solves the following first order condition for the monopoly's expected return, $E[\pi_M]$:

$$\alpha a (r_M B_M)^{\alpha-1} = p \beta b ((1-r_M) B_M)^{\beta-1}. \quad (5.1)$$

Since the second order condition of the monopoly's expected return, $E''[\pi_M] = (\alpha-1)\alpha a (r_M B_M)^{\alpha-2} + (\beta-1)p \beta b ((1-r_M) B_M)^{\beta-2}$, is less than zero for $\alpha, \beta \in (0,1)$, the value of r_M that solves (5.1) leads to the highest expected return for the monopoly. Clearly, the optimal budget allocation depends on the particular market parameters of $f(\cdot)$ and $g(\cdot)$. However, it is not obvious how the size of the budget influences the share of the budget being invested into the respective markets. We have the following result.

Proposition 5.1: *If the monopoly budget changes from B_M to $\overline{B_M} > B_M$, then the optimal budget allocation changes from r_M^* to $\overline{r_M^*} < r_M^*$ (and vice versa).*

All proofs are provided in Section 5.5.

Proposition 5.1 states that if the monopoly increases (decreases) its budget, it allocates a greater (smaller) share of its budget to the project targeting the emerging market. Indeed, larger investment funds make the emerging market more lucrative compared to the mature market in which large investments experience diminishing returns. However, increasing investment into the emerging market also increases risk exposure since the returns from the emerging market are uncertain. The following proposition characterizes how the market parameters affect the budget allocation decision.

Proposition 5.2: *If $f(x) = ax^\alpha$ and $g(x) = \begin{cases} bx^\beta & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$, the monopoly's optimal budget allocation r_M^* is such that if $\frac{\alpha a}{p\beta b} = B_M^{\beta-\alpha}$*

($>$, $<$), then $r_M^ = \frac{1}{2}$ ($>$, $<$, respectively).*

Proposition 5.2 implies that the market potential parameters a and b make the respective markets more attractive independent of the other market characteristics and the available investment budget. Similarly, the greater the probability that the emerging market will reach its market potential, the more attractive this market becomes. We discuss the effect of market uncertainty in more detail in Section 5.2.2. The marginal productivity of markets, α and β , affect the rate at which the markets offer returns and their impact on the allocation decision thus depends on the budget available to the monopoly. For larger budgets (smaller budgets), either increasing (decreasing) β or decreasing (increasing) α , makes the emerging market more

(less) attractive. Intuitively, an emerging market initially requires more investment than a mature market to achieve a certain return; only for larger investments does the emerging market outperform the mature market. Therefore, if the budget is smaller than a particular threshold, then the mature market is more attractive, and once the budget exceeds this threshold, the emerging market becomes more attractive.

So far, we have focused on the allocation decision of a monopoly wherein the firm needs to account for the parameters of the market return functions and the available investment budget. However, in many instances firm are competing with other firms and project returns further depend on the actions of their competitors. The next section studies the resource allocation problem in a duopoly setting and highlights the effect of competition.

5.2. EFFECT OF COMPETITION – THE DUOPOLY CASE

In this section we study a duopoly setting where both firms are considering investments into projects targeting either the mature or the emerging market. Each Firm n , $n \in \{1,2\}$, decides what share, r_n , of its budget, B_n , to invest into the mature market. As in the monopoly case, we assume that duopoly firms fully invest their budgets. We assume that the products or services of each firm that are targeting a particular market are considered perfect substitutes in that market, i.e., the total market returns depend on the total investment of both firms and follow the previously defined return function $f(\cdot)$ and $g(\cdot)$. The returns achieved by an individual firm from a particular market are proportional to its investment into that market compared to that of its competitor (conceptually, this is similar to Parlar and Weng (2006) where firms' market shares are proportional to the prices they set). At any given investment level, additional investment (including competitive investment) into either market always reduces the average

returns obtained from that market (returns divided by investment) because both markets are associated with diminishing returns. Given the mature market's low rate of marginal productivity, diminishing returns are particularly significant. The proportional allocation of returns implies that firms trade off defending their returns in markets where their competitor is investing heavily versus opportunistically taking market share in markets where their competitor is not investing heavily. Formally, the expected return of Firm n is:

$$E[\pi_n] = \frac{r_n B_n}{r_n B_n + r_{-n} B_{-n}} a(r_n B_n + r_{-n} B_{-n})^\alpha + \frac{(1-r_n) B_n}{(1-r_n) B_n + (1-r_{-n}) B_{-n}} pb((1-r_n) B_n + (1-r_{-n}) B_{-n})^\beta, \quad (5.2)$$

where r_{-n} denotes the share of the budget that the other firm invests into the mature market. To obtain the reaction function of Firm 1's optimal investment given the investment decision of Firm 2, we set the first order condition of (5.2) with respect to r_n to zero:

$$\frac{a(r_n B_n + r_{-n} B_{-n})^{\alpha-2}}{pb(B_n(1-r_n) + B_{-n}(1-r_{-n}))^{\beta-2}} = \frac{((1-r_{-n}) B_n B_{-n} + (1-r_n) \beta (B_n)^2)}{(r_{-n} B_n B_{-n} + r_n \alpha (B_n)^2)}. \quad (5.3)$$

While we were not able to solve (5.3) for r_n in the general case, we can now compare the allocation decision of the monopoly with that of the duopoly firms. To facilitate the comparison of monopoly and duopoly firms, let us assume $B_M = B_1 + B_2$, i.e., the total combined investment budget of the duopoly matches that of the monopoly. Given the functional form of $f(\cdot)$ and $g(\cdot)$, we have the following result:

Theorem 5.1: If $f(x) = ax^\alpha$, $g(x) = \begin{cases} bx^\beta & w.p. \ p \\ 0 & w.p. \ 1-p \end{cases}$, $\alpha < \beta$, $B_M = B_1 + B_2$

and $B_1 > B_2$ ($=$), then $r_1^* B_1 + r_2^* B_2 > r_M^* B_M$ and $r_2^* > r_1^*$ ($=$).

This theorem says that competition alters budget allocation decisions in a clear direction. Under competition, firms overinvest in the mature market. That is, duopoly firms shift a

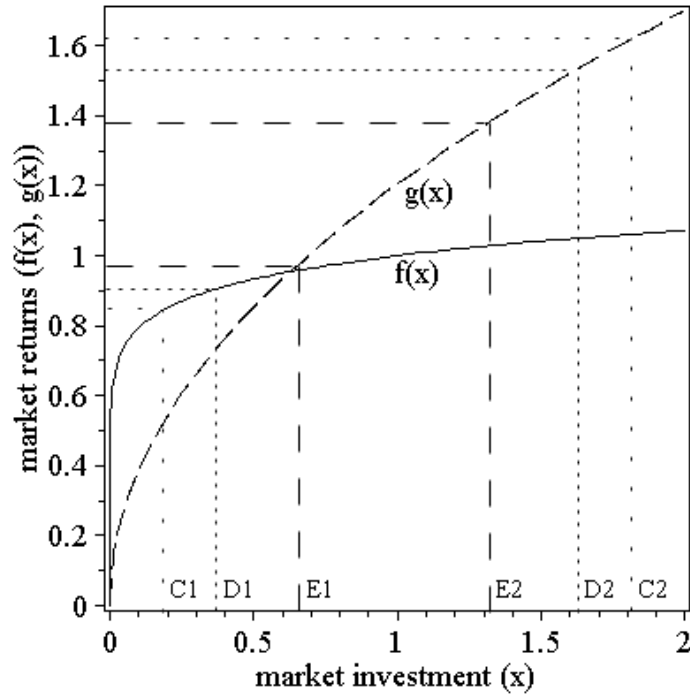
greater share of their budget from the emerging market to the mature market, which offers significant returns for low levels of investment, but with quickly diminishing returns. Given that a monopoly optimizes its resource allocation in a Pareto efficient manner, the strategic interactions between firms induce them to reach decisions that are not Pareto efficient and obtain lower returns.

The intuition behind this result is best understood by recognizing that while a monopoly can exclusively focus on marginal returns from the two markets, duopoly firms also consider average returns (see Proposition 5.A1 in Section 5.5 for a proof of this general result). We illustrate with an example. Consider Figure 5.2 (where $\alpha = 0.1$, $\beta = 0.5$, $a = 1$, $b = 2$, $p = 0.6$, $B_M = 2$, $B_1 = 1$ and $B_2 = 1$), where the monopoly's optimal investment decision, $r_M^* = 0.095$, implies an investment of 0.19 (labeled C1 in Figure 5.2) into the mature market and 1.81 (C2) into the emerging market, leading to equal marginal returns from both markets (i.e., the slopes at these points are identical). However, given the characteristics of the return functions, the mature market provides a greater average return under this allocation. Although this does not affect the monopoly's decision, average returns are important to duopoly firms due to the proportional allocation of returns.

To understand this result, assume first that duopoly firms invest their budget according to the monopoly's budget allocation, $r_1^* = r_2^* = r_M^*$, implying an investment of 0.095 by both Firm 1 and Firm 2. This leads to the highest possible combined duopoly return. However, Firm 1 can increase its return by investing more into the mature market. For example, if Firm 1 triples its mature market investment to $3r_M^*$, as shown in Figure 5.2, there are two key effects: first, the total investment into the mature market increases from 0.19 (C1) to 0.38 (D1), thereby increasing the returns from the mature market, while the total return from the emerging

market decreases. Given the Inada conditions and the fact that a monopoly invests at equal marginal returns, the total loss of returns from the emerging market is greater than the total gain in returns from the mature market. This reduces the total combined returns, but Firm 1's share from the return from the mature market increases from 0.5 to 0.75, while Firm 1's share from the emerging market return shrinks from 0.5 to only 0.44. I.e., by sacrificing a 6% share of the emerging market, Firm 1 has gained an increase of 25% in its share of the mature market. Although Firm 1 has relinquished a part of the market that is providing larger returns (the emerging market), the relative sizes of the lost and gained market shares and the relative sizes of the two markets (and thus the average returns in those markets) lead to higher total returns for Firm 1.

This strategy is anticipated by the other firm which, in turn, decides also to shift a greater share of its own budget into this market. As firms shift more resources into the mature market, both the marginal and average returns from the mature market decline until an equilibrium is reached (E1 and E2 in Figure 5.2). As depicted in Figure 5.2, both firms invest more heavily into the mature market than the monopoly, with $r_1^* = r_2^* = 0.338$, ultimately decreasing the combined returns of both firms. The nature of competition in this setting engages firms in an arms race over the market with greater return per budget unit.



$$\begin{array}{lll}
 C1 = r_M^* B_M & D1 = 3r_M^* B_1 + r_M^* B_2 & E1 = r_1^* B_1 + r_2^* B_2 \\
 C2 = (1 - r_M^*) B_M & D2 = (1 - 3r_M^*) B_1 + (1 - r_M^*) B_2 & E2 = (1 - r_1^*) B_1 + (1 - r_2^*) B_2
 \end{array}$$

Figure 5.2: Resource allocation of monopoly versus duopoly firms;
 $\alpha = 0.1, \beta = 0.5, a = 1, b = 2, p = 0.6, B_M = 2, B_1 = 1$ and $B_2 = 1$

Overinvestment into mature markets has been previously documented (Dankbaar, 1998) and has managerial implications for managers faced with such a project portfolio management problem. Although duopoly firms could achieve their highest possible return if they each invested as a monopoly would, ignoring competitive effects is costly if the other firm acts strategically. Consequently, managers need to recognize that in a competitive setting their firm should invest more heavily into the mature market than a monopoly would, acting in the same market alone. The reason is that duopoly firms must consider both marginal as well as average returns.

5.2.1. BUDGET CONSIDERATIONS

As demonstrated in the previous subsection, the presence of competition affects firms' budget allocation between two markets. In this subsection, we explore how firms' budgets affect their

own as well as their competitor's resource allocation decisions.

To study the impact of the size of firms' budgets on the resource allocation decisions, consider the instance illustrated in Figure 5.3. In this figure, the optimal investment decision of a monopoly (r_M^*) is contrasted with that of two duopoly firms (r_1^* of Firm 1 and r_2^* of Firm 2). We fix Firm 1's budget to 1 and let Firm 2's budget vary from 0 to 3. For comparison, the budget of the monopoly is set to be equal to the combined budget of the two firms in the duopoly case. The case where Firm 2's budget is zero corresponds to the instance where Firm 1 is a monopoly, in which case $r_1^* = 0.1$. The general observations from Figure 5.3 are that a firm's share of budget invested into the mature market is non-increasing in its own budget (as r_2^* is non-increasing in B_2) and that an increase in a firm's budget induces its competitor to invest more into the mature market (as r_1^* is increasing in B_2). This figure further supports our insights from the previous subsection, as one can notice that competition significantly alters the investment decision of Firm 1, even if Firm 2 has a very small budget. Hence, accounting for the presence of competition is critical. Refining this observation, we notice that the share of budget being invested into the mature market is very sensitive to changes in the competitor's budget when it is quite small; however, when the competitor's budget is large, changes to the size of the competitor's budget have a marginal effect on r_1^* .

Intuitively, the greater the budget of Firm 2, the more significant the threat is to Firm 1's return in the mature market and the greater the need for Firm 1 to defend this market by increasing its own investment into that market. But, at the same time, the mature market quickly becomes saturated as investment increases, leading to poor marginal returns. These counterbalancing factors lead to the tempered response by Firm 1.

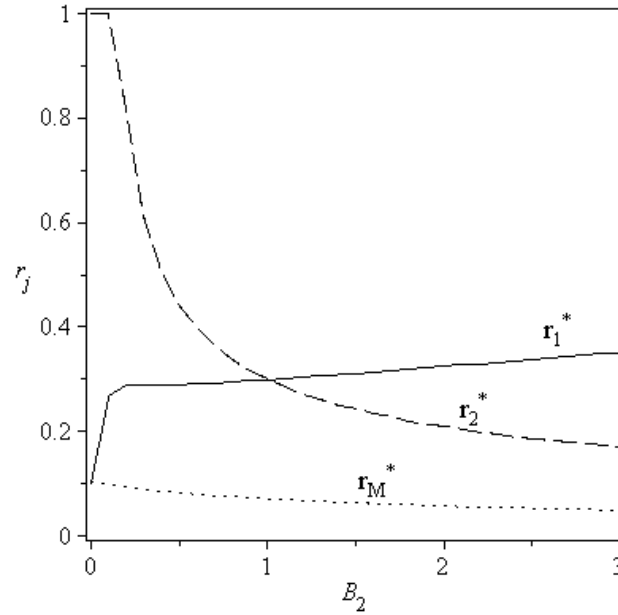


Figure 5.3: Optimal resource allocation into mature market;
 $\alpha = 0.1, \beta = 0.6, a = 0.8, b = 2, p = 0.5, B_1 = 1, B_M = B_1 + B_2$

The insights gained from Figure 5.3 are based on a particular set of market parameters. However, these findings are robust. For example, a reduction in p , the uncertainty associated with the emerging market, or a reduction in a , the market potential of the mature market, decreases the proportion of budget the firm invests into the mature market at any given budget level; however, the relative effects of budget increases still hold.

These findings can help guide managers as they contemplate their competitive resource allocation decision. We have shown that competition is an important consideration and should lead firms to increase their investment into the mature market. If the competitor is quite small, the actual size of the competitor's budget impacts the allocation decision critically. However, if the competitor's budget is sufficiently large, the actual size has less of an impact. Hence, facing a large competitor, it is sufficient for managers to respond to competition by defending markets with high average returns without having to spend significant resources gaining competitive intelligence regarding the exact size of the competitor's budget. Changes to a firm's own budget have an even more pronounced effect on its investment strategy. With a

very limited budget, the choice is clear: fully invest in the mature market. A gradual increase in a firm's budget leads it to significantly increase its investment into the emerging market (as the mature market becomes saturated). Increasing investment into the emerging market, when faced with budget increases, maximizes the firms' expected returns, but, given the uncertainty associated with the emerging market, this also increases the firms' risk exposure. Consequently, a firm contemplating an increase in its investment budget needs to recognize that an increase in its budget can have two sources of risk: the additional leverage (if borrowing is required to achieve the budget increase) and the additional risk exposure that results from increased investment into the emerging market. The next subsection considers the effect of market uncertainty more closely.

5.2.2. EFFECT OF MARKET UNCERTAINTY

We have characterized the mature market as one that has known investment return projections while the emerging market has uncertain returns. Intuitively, we expect an increase in market uncertainty to reduce investment into the emerging market, and vice versa. Recalling that $1 - p$ is the probability that the emerging market does not provide any returns, we thus expect firms to reduce their investment into the emerging market as this probability increases. Stated differently, we expect r to decrease as p increases. Indeed, Figure 5.4, which depicts the behavior of r for both monopoly and duopoly as a function of p , demonstrates that firms continue to invest a large proportion of their budget into the emerging market even if there is a significant probability that there will be no returns from that market. As the probability of the emerging market achieving its anticipated market potential decreases, firms initially shift resources to the mature market at an increasing rate. However, if this probability is sufficiently small, the duopoly firms shift resources to the mature market at a decreasing rate.

The monopoly, which generally invests a greater share of its budget into the emerging market, is particularly slow to increase investment into the mature market unless the probability of receiving no returns in the emerging market becomes extreme. For example, in the case presented in Figure 5.4, even if $p = 0.1$, it is still optimal for a monopoly to invest 0.7 of its budget into the emerging market.

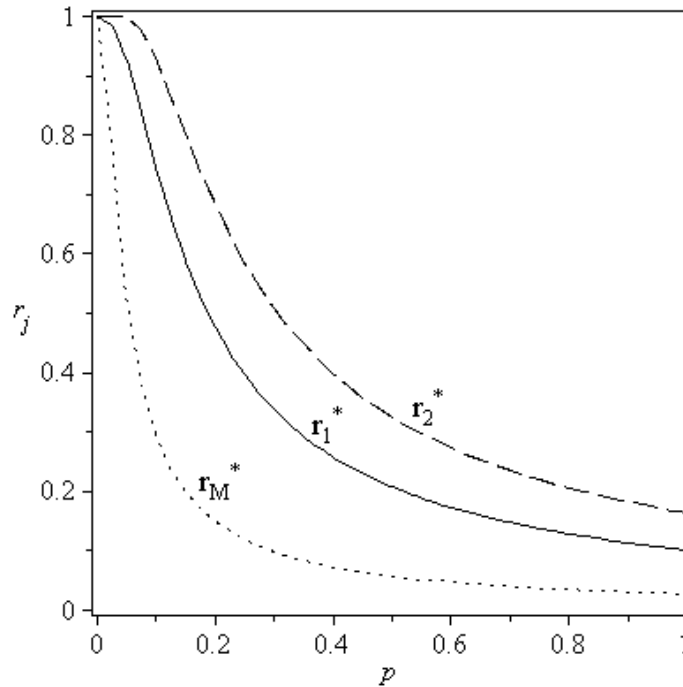


Figure 5.4: Optimal resource allocation into mature market;
 $\alpha = 0.1, \beta = 0.6, a = 0.8, b = 2, B_M = 3, B_I = 2, B_2 = 1$

This result implies that risk neutral decision-makers maintain high investment levels into the emerging market to avoid saturating the mature market even in the presence of substantial market uncertainty. Only if it becomes highly likely that the emerging market will provide no returns, should firms completely avoid the emerging market.

5.2.3. DIFFERENCES IN MARGINAL PRODUCTIVITY

Recall that α and β , $\alpha < \beta$, define the marginal productivity of the mature and emerging markets, respectively (technically, they represent the degree of homogeneity of the return

functions of these markets). A low marginal productivity implies that firms quickly experience diminishing returns for their investments into this market, while a high marginal productivity requires a large investment to obtain the large returns of this market. As the marginal productivity increases, the returns from a market become more linear. As the marginal productivity of the emerging market, β , approaches 1, the emerging market provides linear returns for any amount of investment.

Intuitively, one would expect that firms would take advantage of an increase in β by decreasing r . This is demonstrated in Figure 5.5, which depicts the allocation decisions, r , for both monopoly and symmetric duopoly firms as a function of the marginal productivity of the emerging market. Indeed, when the total of the firms' budgets is (relatively) high—1.25 (implying 0.625 for each duopoly firm)—firms monotonically shift their investment from the mature market into the emerging market as β , the emerging market's marginal productivity, increases. The effect of competition emerges as an important factor—the monopoly responds faster to an increase in marginal productivity in the emerging market and shifts significant resources to the emerging market even when such increase is relatively minor.

However, changing β does not always imply shifting resources from the mature into the emerging market. Since the returns from the emerging market follow the expression $g(x)=bx^\beta$, $\beta < 1$, the size of investment into the emerging market plays an important role. Namely, if the budget dedicated to the emerging market is *high*, then a *high* marginal productivity is preferred, while if the dedicated budget for this market is *low*, then a *low* marginal productivity is preferred. Thus, if firms have small budgets, they may react differently to changes in the marginal productivity. With small budgets, firms may actually shift resources away from the emerging market if the marginal productivity of this market increases. In Figure 5.5, the small

budget duopoly firms ($B_1=B_2=0.025$) monotonically increase investment into the mature market as the marginal productivity of the emerging market increases. The small budget monopoly ($B_M=0.05=B_1+B_2$), initially reduces investment into the mature market as the marginal productivity of the emerging market increases, but, as the returns in the emerging market become more linear, the monopoly also shifts more of its resources to the mature market. This reveals that the firms are more likely to shift resources from the emerging into the mature market as the difference in the two markets' marginal productivities increases, if their budgets are sufficiently small.

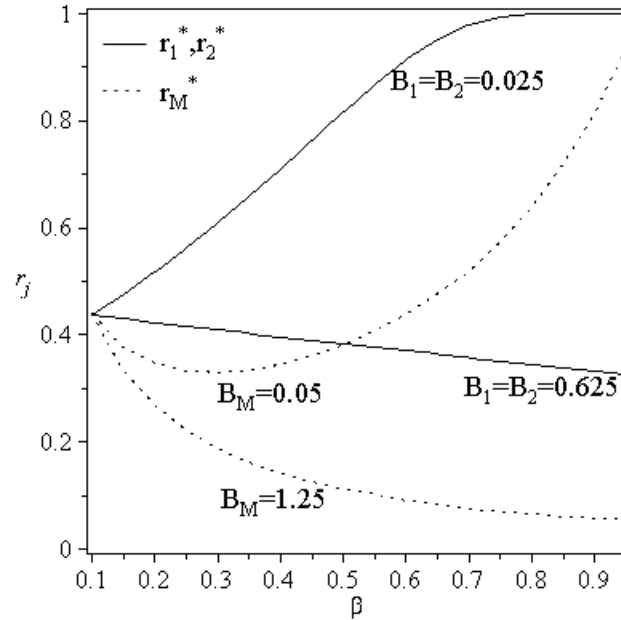


Figure 5.5: Optimal resource allocation into mature market for different budgets and varying β ; $\alpha = 0.1$, $a = 0.8$, $b = 2$, $p = 0.5$

5.3. OLIGOPOLIES

We recognize that, in practice, the number of firms competing in markets may well exceed two. In this section, we study the oligopoly setting with N identical firms in the sense that all face the same investment decision and have the same budget, with $B_1 = B_2 = \dots = B_N \equiv B_O^N$, where the subscript O denotes the oligopoly case and the subscript N denotes the number of

firms participating in the oligopoly. Each Firm n , $n = 1, \dots, N$, is seeking to maximize its return by deciding what share r_i^N of its budget to allocate to the mature market:

$$E[\pi_n] = \frac{r_i^N}{\sum_{i=1}^N r_i^N} a \left(B_O^N \sum_{i \in N} r_i^N \right)^\alpha + \frac{1 - r_i^N}{N - \sum_{i=1}^N r_i^N} pb \left(B_O^N \left(N - \sum_{i \in N} r_i^N \right) \right)^\beta, \quad n = 1, \dots, N, \quad (5.4)$$

where the first term is the return of Firm n from the mature market and the second term is the return of Firm n from the emerging market. Taking the first order conditions of (5.4) and imposing symmetry on the firms' decisions yields:

$$(N - 1 + \alpha) a (N r_O^{N*} B_O^N)^{\alpha-1} = (N - 1 + \beta) pb \left(N (1 - r_O^{N*}) B_O^N \right)^{\beta-1}. \quad (5.5)$$

We have the following result.

Proposition 5.3: If $f(x) = ax^\alpha$ and $g(x) = \begin{cases} bx^\beta & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$, the oligopoly

firms' optimal budget allocation r_O^* is such that if

$$\frac{(N-1+\alpha)a}{(N-1+\beta)pb} = (NB_O^N)^{\beta-\alpha} \quad (>, <), \text{ then } r_O^* = \frac{1}{2} \quad (>, <, \text{ respectively}).$$

This result extends the findings from the monopoly case and subsumes Proposition 5.2 (by setting $N = 1$). The market potential parameters a and b make the respective markets more attractive independent of the marginal productivity of the markets and the available investment budget. As before, for larger budgets either increasing β or decreasing α makes the emerging market more attractive. Conversely, for smaller budgets either increasing β or decreasing α makes the mature market more attractive.

The following theorem characterizes the effect the number of competing firms has on budget allocation decisions.

Theorem 5.2: Let $f(x) = ax^\alpha$, $g(x) = \begin{cases} bx^\beta & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$, and $\alpha < \beta$. As the

number of competing firms increases from $N1$ to $N2$, while the total budget

of all firms is kept constant such that $N1 \cdot B_o^{N1} = N2 \cdot B_o^{N2}$, then $r_o^{N2*} > r_o^{N1*}$.

This result directly extends Theorem 5.1 for the special case where firms have identical budgets: the more firms compete, the more heavily these firms invest into the mature market. Managers need to anticipate that investment into the mature markets will intensify as the number of competitors increases. Consequently, to protect their returns from the mature market, firms should increase their own share of investment into such markets.

Proposition 5.3 characterizes the condition that drives firms to invest a majority of their budget into the mature market. Since competition leads to even higher investment into the mature market, firms may end up investing their budget almost entirely into the mature market. This, of course, is the optimal decision for each firm individually, but detrimental to the combined returns of firms in an oligopoly. Depending on the market parameters, this dynamic may also prevent firms from diversifying their budget over multiple markets, which may contradict some firms' overarching goals of distributing their investments over multiple markets and further building a presence in emerging markets.

5.4. CHAPTER 5 CONCLUSIONS

We have studied and demonstrated how competition affects firms' resource allocation decisions. Our main finding is that while a monopoly optimally considers only the marginal returns of the markets, competition drives firms to account for average returns as well. Therefore, firms invest more heavily into mature markets which have a lower marginal

productivity than emerging markets. This overinvestment occurs even when competing with a firm that is endowed with a small budget. We find it is critical to assess the competing firm's budget, if this firm is small. However, if this firm is large, then the actual size of the competitor's budget does not have a significant impact on a firm's allocation decision. As firms increase their own budget, however, they invest more heavily into the emerging market. We have also shown that increase in the marginal productivity of the emerging market generally leads to increased investment into this market if firms' budgets are sufficiently high. However, firms with small budgets may instead shift resources to the mature market as the marginal productivity of the emerging market increases. Finally, we have extended the framework into an oligopoly setting and have proven that if firms have identical budgets, firms invest into the mature market to a greater degree the greater the number of competitors.

So far, we have focused on settings where firms have project portfolios containing two projects. Through these stylized models, we were able to find tractable results and analyze the key impact of competition on the PPM problem. However, firms often have many projects in their portfolio and compete with a number of other firms in many different markets. In the following chapter, we shift to this more general case where a number of budget-constrained firms are considering projects that target many different markets.

5.5. PROOFS FOR CHAPTER 5

Proof of Proposition 5.1: Substituting $\overline{r_M^*} = \theta r_M^*$ and $\overline{B_M} = \lambda B_M$ into (5.1), we get

$\alpha \alpha (\theta r_M^* \lambda B_M)^{\alpha-1} = p \beta b ((1 - \theta r_M^*) \lambda B_M)^{\beta-1}$, which can be rearranged to:

$$\frac{f'(r_M^* B_M)}{(\theta \lambda)^{1-\alpha}} = - \frac{g'\left(\frac{1}{\theta} B_M - r_M^* B_M\right)}{(\theta \lambda)^{1-\beta}}. \quad (5.P1)$$

We want to show that if $\alpha < \beta$ and $\lambda > 1$, then $\theta < 1$. By contradiction: if we had $\theta \geq 1$, then $\frac{1}{(\theta\lambda)^{1-\alpha}} < \frac{1}{(\theta\lambda)^{1-\beta}}$. Given (5.P1), this implies $f'(r_M^* B_M) > -g'\left(\frac{1}{\theta} B_M - r_M^* B_M\right)$ which for $\theta \geq 1$ cannot hold under (5.1) and the Inada condition $\frac{\partial g^2}{\partial r_M^2} < 0$. Following the same arguments, $\alpha < \beta$ and $\lambda < 1$ necessitate $\theta > 1$. \square

Proof of Proposition 5.2: The monopoly's optimal decision is the allocation r_M^* that solves

$$(5.1), \quad \alpha\alpha(r_M^* B_M)^{\alpha-1} = \beta pb \left((1-r_M^*) B_M\right)^{\beta-1} \text{ or } \frac{\alpha\alpha}{\beta pb} = \frac{\left((1-r_M^*) B_M\right)^{\beta-1}}{\left(r_M^* B_M\right)^{\alpha-1}}. \text{ Thus, if } \frac{\alpha\alpha}{\beta pb} = B_M^{\beta-\alpha},$$

we have $\frac{\left((1-r_M^*) B_M\right)^{\beta-1}}{\left(r_M^* B_M\right)^{\alpha-1}} = B_M^{\beta-\alpha}$ or $r_M^* = \frac{1}{2}$. Similarly, if $\frac{\alpha\alpha}{\beta pb} > B_M^{\beta-\alpha}$ ($<$), we have

$$\frac{\left((1-r_M^*) B_M\right)^{\beta-1}}{\left(r_M^* B_M\right)^{\alpha-1}} > B_M^{\beta-\alpha} \text{ (<, respectively) or } r_M^* > \frac{1}{2} \text{ (<, respectively).} \quad \square$$

Proof of Theorem 5.1: In this proof, we first prove that $B_n > B_{-n} \rightarrow r_n^* < r_{-n}^*$. Next, we show that $B_1 + B_2 = B_M \rightarrow r_1^* B_1 + r_2^* B_2 > r_M^* B_M$. We conclude by proving the result for the special case where both firms have identical budgets.

We first prove that $B_n > B_{-n} \rightarrow r_n^* < r_{-n}^*$. Comparing the response functions of Firm n and Firm $-n$, using (5.3), we get

$$\frac{\left(r_{-n} B_n B_{-n} + r_n \alpha (B_n)^2\right)}{\left((1-r_{-n}) B_n B_{-n} + (1-r_n) \beta (B_n)^2\right)} = \frac{\left(r_n B_n B_{-n} + r_{-n} \alpha (B_{-n})^2\right)}{\left((1-r_n) B_n B_{-n} + (1-r_{-n}) \beta (B_{-n})^2\right)}. \quad (5.P2)$$

Next, we solve (5.P2) for r_n :

$$r_n = -\frac{1}{2(\beta-\alpha)B_n} \left(\begin{array}{l} (\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n \\ \pm \sqrt{\left((\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n\right)^2 - 4(1-\alpha\beta)(\beta-\alpha)B_n B_{-n} r_{-n}} \\ -4(\beta-\alpha)^2 B_{-n}^2 (r_{-n} - r_n^2) \end{array} \right). \quad (5.P3)$$

Since $-4(1-\alpha\beta)(\beta-\alpha)B_n B_{-n} 4r_{-n} - 4(\beta-\alpha)^2 B_{-n}^2 (r_{-n} - r_{-n}^2) < 0$ for all $\alpha < \beta$, we can write (5.P3) as $r_n = -\frac{((\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n \pm [((\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n) - \varepsilon])}{2(\beta-\alpha)B_n}$, where $\varepsilon > 0$. This

leaves two potential solutions, $-\frac{2((\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n) - \varepsilon}{2(\beta-\alpha)B_n}$ and $-\frac{\varepsilon}{2(\beta-\alpha)B_n}$. Since

$-\frac{\varepsilon}{2(\beta-\alpha)B_n} < 0$ for all $\alpha < \beta$, and thus not a feasible solution, the only equilibrium allocation

is:

$$r_n^* = -\frac{1}{2(\beta-\alpha)B_n} \left(\frac{(\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n}{\sqrt{((\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n)^2 - (1-\alpha\beta)(\beta-\alpha)B_n \cdot B_{-n} \cdot 4 \cdot r_{-n}}} + \sqrt{-(\beta-\alpha)^2 \cdot B_{-n}^2 \cdot 4 \cdot (r_{-n} - r_{-n}^2)} \right)$$

To show that $B_n > B_{-n} \rightarrow r_n^* < r_{-n}^*$, we prove that for any given allocation decision by Firm $-n$, the optimal allocation of Firm n into the mature market is always less, i.e.

$$r_n^* - r_{-n} = \frac{1}{2(\beta-\alpha)B_n} \left(\frac{- (\alpha\beta-1)B_{-n} + (\beta-\alpha)B_n - (\beta-\alpha)B_n \cdot 2 \cdot r_{-n}}{\sqrt{((\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n)^2 + (\alpha\beta-1)(\beta-\alpha)B_n \cdot B_{-n} \cdot 4 \cdot r_{-n}}} - \sqrt{-(\beta-\alpha)^2 \cdot B_{-n}^2 \cdot 4 \cdot (r_{-n} - r_{-n}^2)} \right) < 0.$$

This expression holds true if

$$\left[- (\alpha\beta-1)B_{-n} + (\beta-\alpha)B_n - (\beta-\alpha)B_n \cdot 2 \cdot r_{-n} \right]^2 < \left((\alpha\beta-1)B_{-n} - (\beta-\alpha)B_n \right)^2 + (\alpha\beta-1)(\beta-\alpha)B_n \cdot B_{-n} \cdot 4 \cdot r_{-n} - (\beta-\alpha)^2 \cdot B_{-n}^2 \cdot 4 \cdot (r_{-n} - r_{-n}^2)$$

which can be simplified to

$$B_{-n}^2 - B_n^2 < r_{-n} (B_{-n}^2 - B_n^2) \quad (5.P4)$$

If $B_n > B_{-n}$, then (5.P4) holds for all $r_{-n} < 1$. Therefore, $B_n > B_{-n} \rightarrow r_n^* < r_{-n}^*$.

Now, we prove that the combined optimal allocation of the duopoly firms into the mature market is greater than that of the monopoly, i.e., $B_1 + B_2 = B_M \rightarrow r_1^* B_1 + r_2^* B_2 > r_M^* B_M$. The equation describing the optimal allocation of Firm n , (5.3), can be rearranged to

$$\frac{\alpha(r_n B_n + r_{-n} B_{-n})^{\alpha-1}}{b(B_n(1-r_n) + B_{-n}(1-r_{-n}))^{\beta-1}} = \frac{((1-r_{-n})B_{-n} + (1-r_n)\beta B_n)}{B_n(1-r_n) + B_{-n}(1-r_{-n})} \cdot \frac{r_n B_n + r_{-n} B_{-n}}{r_{-n} B_{-n} + r_n \alpha B_n} \quad (5.P5)$$

Similarly, the optimal allocation of the monopoly, (5.1), can be rearranged to

$$\frac{a(r_M B_M)^{\alpha-1}}{b((1-r_M)B_M)^{\beta-1}} = \frac{\beta}{\alpha}. \quad (5.P6)$$

First, we show that

$$\frac{\beta}{\alpha} > \frac{((1-r_{-n})B_{-n} + (1-r_n)\beta B_n)}{B_n(1-r_n) + B_{-n}(1-r_{-n})} \cdot \frac{r_n B_n + r_{-n} B_{-n}}{r_{-n} B_{-n} + r_n \alpha B_n}, \quad (5.P7)$$

which by (5.P5) and (5.P7) implies that $\frac{a(r_M B_M)^{\alpha-1}}{b((1-r_M)B_M)^{\beta-1}} > \frac{a(r_n B_n + r_{-n} B_{-n})^{\alpha-1}}{b(B_n(1-r_n) + B_{-n}(1-r_{-n}))^{\beta-1}}$,

which thus implies $r_M(B_n + B_{-n}) < r_n B_n + r_{-n} B_{-n}$, or $r_1^* B_1 + r_2^* B_2 > r_M^* B_M$.

The functions $\frac{\beta}{\alpha}$ and $\frac{((1-r_{-n})B_{-n} + (1-r_n)\beta B_n)}{B_n(1-r_n) + B_{-n}(1-r_{-n})} \cdot \frac{r_n B_n + r_{-n} B_{-n}}{r_{-n} B_{-n} + r_n \alpha B_n}$ intersect at

$$r_n^{\hat{}} \equiv \frac{(-\beta(B_n + B_{-n}) + (\beta - \alpha)B_{-n}r_{-n} + \alpha B_{-n})}{B_n((\beta - 1)\alpha - (\beta - \alpha)r_{-n})} r_{-n}. \quad \text{For } r_n < r_n^{\hat{}}, (5.P7) \text{ holds and we prove that}$$

for any r_{-n} we have $r_n < r_n^{\hat{}}$, or

$$r_n - r_n^{\hat{}} = \frac{[B_n(\beta - 1)\alpha - (\beta - \alpha)B_n r_{-n} + \beta(B_n + B_{-n}) - (\beta - \alpha)B_{-n}r_{-n} - \alpha B_{-n}]r_{-n}}{B_n((\beta - 1)\alpha - (\beta - \alpha)r_{-n})} < 0. \quad \text{Since the}$$

numerator of this expression is greater than zero and the denominator of this expression is less

than zero for any feasible r_{-n} , we have $r_n < r_n^{\hat{}}$, and thus

$$\frac{\beta}{\alpha} > \frac{((1-r_{-n})B_{-n} + (1-r_n)\beta B_n)}{B_n(1-r_n) + B_{-n}(1-r_{-n})} \cdot \frac{r_n B_n + r_{-n} B_{-n}}{r_{-n} B_{-n} + r_n \alpha B_n}.$$

By (5.P5) and (5.P6), this further implies that we have

$$\frac{a(r_M B_M)^{\alpha-1}}{b((1-r_M)B_M)^{\beta-1}} > \frac{a(r_n B_n + r_{-n} B_{-n})^{\alpha-1}}{b(B_n(1-r_n) + B_{-n}(1-r_{-n}))^{\beta-1}}. \quad \text{And given } \alpha < \beta, \text{ we have}$$

$$r_M(B_n + B_{-n}) < r_n B_n + r_{-n} B_{-n}, \quad (5.P8)$$

i.e., the combined investment of the duopoly firms into the mature market is greater than the investment of the monopoly.

Lastly, in the special case where $B_n = B_{-n} = B_D$ (5.P5) becomes:

$$\frac{a((r_n + r_{-n})B_D)^{\alpha-1}}{b((2 - r_n - r_{-n})B_D)^{\beta-1}} = \frac{(1 - r_{-n}) + (1 - r_n)\beta}{(1 - r_n) + (1 - r_{-n})} \cdot \frac{r_n + r_{-n}}{r_{-n} + r_n\alpha}. \quad (5.P9)$$

This is the response function for Firm n . We insert the response function for Firm $-n$ to get:

$$\frac{((1 - r_n) + (1 - r_{-n})\beta)(r_n + r_{-n})}{((1 - r_n) + (1 - r_{-n}))\alpha} = \frac{((1 - r_{-n}) + (1 - r_n)\beta)(r_{-n} + r_n)}{((1 - r_{-n}) + (1 - r_n))\alpha}$$
 which has two solutions for r_n ,

$r_n = r_{-n}$ and $r_n = \frac{(\alpha\beta - 1)}{\alpha - \beta} + 1 - r_{-n}$. Since we have $r_n + r_{-n} \leq 2$, the second solution can only

hold if $\frac{(\alpha\beta - 1)}{\alpha - \beta} + 1 \leq 2$, which simplifies to $\alpha\beta - \alpha + \beta \geq 1$ (for $\alpha < \beta$). Let $h = \alpha\beta - \alpha + \beta$.

We have $\frac{\partial h}{\partial \alpha} < 0$ and $\frac{\partial h}{\partial \beta} > 0$. Given the feasible region of α and β , $\max(h) < 1$, which

implies that $\alpha\beta - \alpha + \beta < 1$. Consequently, the symmetric solution $r_n = r_{-n} = r_D$ is the only solution if $B_n = B_{-n} = B_D$.

Substituting $r_n = r_{-n} \equiv r_D$ into (5.P9), and rearranging, we get:

$$\frac{a(2r_D B_D)^{\alpha-1}}{b((2 - 2r_D)B_D)^{\beta-1}} = \frac{(1 + \beta)}{(1 + \alpha)}.$$

Next, we rearrange (5.1) to $\frac{a(r_M^* 2B_D)^{\alpha-1}}{pb((1 - r_M^*)2B_D)^{\beta-1}} = \frac{\beta}{\alpha}$ and recognize that if $\alpha < \beta$, then

$$\frac{\beta}{\alpha} > \frac{(1 + \beta)}{(1 + \alpha)}, \text{ and we thus have } \frac{a(r_M^* 2B_D)^{\alpha-1}}{pb((1 - r_M^*)2B_D)^{\beta-1}} > \frac{a(r_D^* 2B_D)^{\alpha-1}}{pb((1 - r_D^*)2B_D)^{\beta-1}} \text{ or}$$

$$\left(\frac{r_M^*}{r_D^*}\right)^{\alpha-1} > \left(\frac{1 - r_M^*}{1 - r_D^*}\right)^{\beta-1}. \text{ Given } \alpha < \beta, \text{ we have } r_D^* > r_M^*, \text{ which equates to}$$

$$r_M(B_n + B_{-n}) < r_n B_n + r_{-n} B_{-n} \text{ with } r_n = r_{-n} \equiv r_D \text{ and}$$

$$B_n + B_{-n} = B_M. \quad \square$$

Proof of Proposition 5.3: This proof follows the same arguments as in the proof of

Proposition 5.1. □

Proof of Theorem 5.2: From (5.5), the optimal budget allocation of firms in an oligopoly of

size $N1$ is the value of r_O^{N1*} that satisfies $\frac{a(N1 \cdot r_O^{N1*} B_O^{N1})^{\alpha-1}}{pb(N1(1-r_O^{N1*})B_O^{N1})^{\beta-1}} = \frac{(N1-1+\beta)}{(N1-1+\alpha)}$ and that of

firm in an oligopoly of size $N2$ is the value of r_O^{N2*} that satisfies

$$\frac{a(N2 \cdot r_O^{N2*} B_O^{N2})^{\alpha-1}}{pb(N2(1-r_O^{N2*})B_O^{N2})^{\beta-1}} = \frac{(N2-1+\beta)}{N2(L-1+\alpha)}. \text{ Since } N2 > N1 \text{ and } \alpha < \beta, \text{ one can show that}$$

$$\frac{(N1-1+\beta)}{(N1-1+\alpha)} > \frac{(N2-1+\beta)}{(N2-1+\alpha)}. \text{ Consequently, } \frac{a(N1 \cdot r_O^{N1*} B_O^{N1})^{\alpha-1}}{pb(N1(1-r_O^{N1*})B_O^{N1})^{\beta-1}} > \frac{a(N2 \cdot r_O^{N2*} B_O^{N2})^{\alpha-1}}{pb(N2(1-r_O^{N2*})B_O^{N2})^{\beta-1}},$$

which simplifies to $\left(\frac{r_O^{N1*}}{r_O^{N2*}}\right)^{\alpha-1} > \left(\frac{1-r_O^{N1*}}{1-r_O^{N2*}}\right)^{\beta-1}$. Since $\alpha < \beta$, we have $r_O^{N2*} > r_O^{N1*}$. \square

The following proposition compares the optimal resource allocation of a monopoly with that of duopoly firms if the two markets offer returns according to the general functions $f(\cdot)$ and $g(\cdot)$, which both follow the Inada conditions.

Proposition 5.A1: *When optimizing its resource allocation over two markets with general return functions that follow the Inada conditions, a monopoly considers only marginal returns, whereas duopoly firms also account for the average return per budget allocation unit of the two markets.*

Proof of Proposition 5.A1: Using the general return functions $f(\cdot)$ and $g(\cdot)$, which both follow the Inada conditions, the optimal allocation of a monopoly, r_M^* , is the value of r_M that solves the first order condition of the monopoly's return, $\pi_M = f(r_M B_M) + g((1-r_M)B_M)$:

$$f'(r_M B_M) = -g'((1-r_M)B_M). \quad (5.P10)$$

The return for the duopoly firms is:

$$\pi_n = \frac{r_n B_n}{r_n B_n + r_{-n} B_{-n}} f(r_n B_n + r_{-n} B_{-n}) + \frac{(1-r_n) B_n}{(1-r_n) B_n + (1-r_{-n}) B_{-n}} g((1-r_n) B_n + (1-r_{-n}) B_{-n}). \quad (5.P11)$$

To derive analytical results for the general case, we take the first order condition of (5.P11) for r_n and then impose symmetry such that the firms are identical, i.e., $B_1 = B_2 \equiv B_D$ and $r_1 = r_2 \equiv r_D$:

$$\frac{f(2r_D B_D)}{2r_D} + f'(2r_D B_D) = \frac{g(2(1-r_D) B_D)}{2(1-r_D)} - g'(2(1-r_D) B_D),$$

i.e., the duopoly firms' optimal budget allocation depends on the marginal market return functions, $f'(2r_D B_D)$ and $g'(2(1-r_D) B_D)$, as well as on the average returns per allocation percentage, $\frac{f(2r_D B_D)}{2r_D}$ and $\frac{g(2(1-r_D) B_D)}{2(1-r_D)}$. If $\frac{f(2r_D B_D)}{2r_D} \neq \frac{g(2(1-r_D) B_D)}{2(1-r_D)}$, then we have $f'(2r_D B_D) \neq -g'(2(1-r_D) B_D)$. However, a monopoly with a budget equal to the combined budget of the duopoly firms, $B_M = 2B_D$, invests according to (5.P10),

$$f'(r_M 2B_D) = -g'((1-r_M) 2B_D). \text{ Consequently, if } \frac{f(2r_D B_D)}{2r_D} \neq \frac{g(2(1-r_D) B_D)}{2(1-r_D)},$$

then $r_D \neq r_M$. □

6. CPPM FOR LARGE PORTFOLIOS: BUDGET CONSTRAINED MULTIMARKET COURNOT COMPETITION

Firms often compete against each other with multiple products in a wide range of markets. This is frequently referred to as multimarket contact. Examples include car manufacturers, such as Toyota and GM, that compete in various automotive markets, airlines, such as American Airlines and Delta Air Lines, that offer similar services on a number of key routes, and consumer packaged goods companies, such as Proctor & Gamble and Kimberly-Clark, that compete in a large number of product categories. Since firms' budgets and production capacities are both constrained, they need to make strategic decisions of how to allocate their resources over their products. In this chapter, we address this challenge in a general setting where N firms are competing in S markets.

This chapter is organized as follows. In the next section, we provide the unconstrained multimarket Cournot equilibrium benchmark. In Section 6.2, we introduce the budget-constrained framework, and derive and characterize the equilibrium production decisions. In Section 6.3, we demonstrate the effect of market abstaining—the case where not all firms produce for all markets. Section 6.4 develops a more extensive model that includes differing demand slopes and unit production costs. Finally, Section 6.5 contains conclusions for this chapter.

6.1. UNCONSTRAINED MULTIMARKET COURNOT EQUILIBRIUM BENCHMARK

Consider N firms competing in S independent markets. Each firm n , $n \in N$, determines the quantity output of a product, $q_{s,n}$, to be produced for market s , $s \in S$. The products produced by all firms for a given market are perfectly substitutable. The price of the product in each market s is assumed to be linearly decreasing in the total quantity produced: $p_s = a_s - b_s Q_s$, where $Q_s = \sum_{i=1}^N q_{s,i} \forall s$. Following previous work (Shaked and Sutton, 1990; Brander and Eaton, 1984), we focus on the demand side by assuming that each firm n has the same production cost, c_n , for all products. We further assume $c_n < a_s \forall n, s$, and for now, $b_s = 1 \forall s$. Later, in Section 6.4, we relax the assumption that $b_s = 1 \forall s$. With these assumptions, the profit of

firm n is $\pi_n = \sum_{s=1}^S (a_s - Q_s - c_n) q_{s,n}$.

The well-known Cournot equilibrium production quantities of firm n for market s are:

$$q_{s,n} = \frac{a_s + \sum_{i=1}^N c_i - (N+1)c_n}{N+1}, \quad \forall n, s, \quad (6.1)$$

resulting in the total profit $\pi_n = \sum_{s=1}^S \left(\frac{a_s + \sum_{i=1}^N c_i - (N+1)c_n}{N+1} \right)^2 \forall n$. From (6.1), it is evident that

as the number of competing firms increases, firms will reduce their production quantities across all markets and achieve lower profits. As expected, firms produce more (less) for markets in which their production costs are low (high) and their competitors' costs are high (low).

One implicit assumption in this formulation is that firms have unlimited resources and can produce for any number of markets in any profitable quantity. The total cost of producing the optimal quantity established in (6.1) for all markets is:

$$C_n = \sum_{s=1}^S \left(\frac{a_s + \sum_{i=1}^N c_i - (N+1)c_n}{N+1} c_n \right) \quad (6.2)$$

Therefore, firms will only produce the optimal production quantity derived in (6.1) if their budget is greater than the cost of producing these products, as established in (6.2). An increase in the number of markets or demand growth in those markets results in an increase in the cost of producing at these optimal production levels. Since each firm is not endowed with unlimited resources, a critical decision is how to allocate its resources to maximize profit. The next section presents a model in which firms make the same production decision presented in this section but have a budget constraint.

6.2. BUDGET CONSTRAINED MULTIMARKET COURNOT EQUILIBRIUM

Consider the benchmark model introduced in the previous section. In this section, we impose a budget constraint on each firm. Specifically, each firm n has a budget B_n , where $\sum_{s=1}^S q_{s,n} c_n \leq B_n$. Given this budget constraint, each firm n considers the following profit maximization problem under multimarket Cournot competition:

$$\begin{aligned} \max \pi_n &= \sum_{s=1}^S (a_s - Q_s - c_n) q_{s,n} \\ \text{s.t. } B_n - \sum_{s=1}^S q_{s,n} c_n &\geq 0 \\ q_{s,n} &\geq 0 \quad \forall s. \end{aligned}$$

A key feature of the CPPM problem is that firms are resource constrained. Therefore, we assume that firms' budgets are less than the required total cost of the optimal production in the unconstrained setting (6.2) and assume that each firm invests its entire budget, $B_n = \sum_{s=1}^S q_{s,n} c_n$, producing at full capacity, B_n / c_n . To solve this constrained optimization model with multiple decision makers, we assume that firms release strictly positive quantities of product into each market, $q_{s,n} > 0, \forall s, n$. This assumption is relaxed later in Section 6.3. With these assumptions, we can state the following theorem:

Theorem 6.1: *In a multiproduct Cournot competition involving S markets and N firms, where each firm n is faced with a budget constraint, B_n , the equilibrium production*

quantity of firm n for market s is given by $q_{s,n}^ = \frac{B_n}{c_n S} + \frac{a_s - \frac{1}{S} \sum_{i=1}^S a_i}{(N+1)}$ and firm n 's profit*

at equilibrium is $\pi_n^ = \frac{B_n}{c_n S} \left(\sum_{s=1}^S (a_s - c_n) - \sum_{i=1}^N \frac{B_i}{c_i} \right) + \frac{S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2}{S(N+1)^2}$.*

All proofs are in Section 6.6.

The first term of $q^*, \frac{B_n}{c_n S}$, is the quantity of products produced for each market if the budget was spread evenly across all markets. The second term of $q^*, \frac{a_s - \frac{1}{S} \sum_{i=1}^S a_i}{(N+1)}$, adjusts this average production quantity by considering the demand potential in the particular market compared with the overall average demand potential and the number of competitors. The equilibrium profit π^* , also consists of two terms, where the first term, $\frac{B_n}{c_n S} \left(\sum_{s=1}^S (a_s - c_n) - \sum_{i=1}^N \frac{B_i}{c_i} \right)$, can be interpreted as the “combined market profit per unit” (if all market demands were aggregated

into a single market) multiplied by the production level of the particular firm. This “combined market profit” is then divided by the number of markets. The second term of

$\pi^*, \frac{S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2}{S(N+1)^2}$, is an adjustment factor which is based on the variability in demand

between the markets and the number of competitors. Higher variability in demand leads to higher profits for firms because it reduces the competition effect: in the extreme case, where all markets but one have zero demand, firms have only one market to invest in and competition does not affect the production decision (assuming the budget constraint is binding, as defined in (6.2)).

Generally, firms produce in greater quantities for markets with higher demand potentials, as captured by the second term of q^* . However, there are two key effects which influence the degree to which firms adjust their production quantities to the particular market: variability between market demands has a greater relative impact on production allocation for smaller budgets, which we denote as the “market focus effect”; and the variability between market demands has a lesser impact as the number of competing firms increases, which we denote as the “competition effect”. When all markets have the same demand potential, $a_s = \bar{a} \forall s$, neither effect exists and firms produce an equal amount of product for each market.

To better understand the interplay of these effects when market demands differ, we first derive results for the case of firms with symmetric budgets and costs in Subsection 6.2.1 and then analyze the case of asymmetric budgets and costs in Subsection 6.2.2.

6.2.1. EFFECT OF COMPETITION ON FIRMS WITH SYMMETRIC BUDGETS

In this subsection we explore how competition affects the resource allocation decision of firms

with symmetric budgets and costs. Assuming symmetric budgets, $\bar{B} = B_n \forall n$, and identical production costs, $c_n = \bar{c} \forall n$, an increase in the number of competitors naturally leads to overall higher production capacities. To isolate the effect of competition, we assume that total production capacity across all markets remains the same regardless of the number of competing firms. That is, $\sum_{n=1}^N \frac{B_n}{c_n} = \frac{B}{\bar{c}}$ is fixed for any N . We then have the following result:

Proposition 6.1: *For a fixed total industry budget B and identical unit costs \bar{c} , where firms have identical budgets $\bar{B} = B/N$, increasing N drives firms to produce more (less) for markets with higher (lower) than average demand potential. Namely, $\frac{\partial}{\partial N} \frac{q_{s,n}^*}{\bar{B}} > (<) 0$ if $a_s > (<) \frac{1}{S} \sum_{i=1}^S a_i$. If market demands are not identical, increasing N leads to decreasing profits for individual firms and decreasing industry profits, i.e., $\frac{\partial \pi_n^*}{\partial N} < 0$ and $\frac{\partial \sum_{n=1}^N \pi_n^*}{\partial N} < 0$, respectively.*

Figure 6.1 depicts the change in production allocation for $B = 1$ and $\bar{c} = 1$, where each firm has three target markets with demand potentials $a_1 = 2.7$, $a_2 = 2$ and $a_3 = 1.8$. Since $\bar{a} = 2.2$, an increase in the number of competitors drives firms to produce more in the first market (above average demand potential) and less in the other two markets (below average demand potential). The intuition behind this result lies in understanding how the marginal returns of the markets change with the number of competing firms. A monopoly that contemplates its production allocation between two markets considers the difference in marginal profits it can achieve by shifting units, as well as the resulting changes to the market prices which affects the remaining planned production for those markets. In particular, the price in the market with increased

production decreases, thereby lowering the profits of all previously committed products in that market.

This tradeoff is affected by competition: a firm that shifts its production quantity between two markets under competition can secure the same difference in profits for this shifted units but the effect of the resulting market price changes on the remaining production quantities is shared with its competitors. I.e., the reduction in price experienced in the market with increased production is partly burdened by the competitors' products. In a budget constrained setting, this drives firms to invest more heavily in markets with higher demand potential, and thus higher prices, as shown in Figure 6.1.

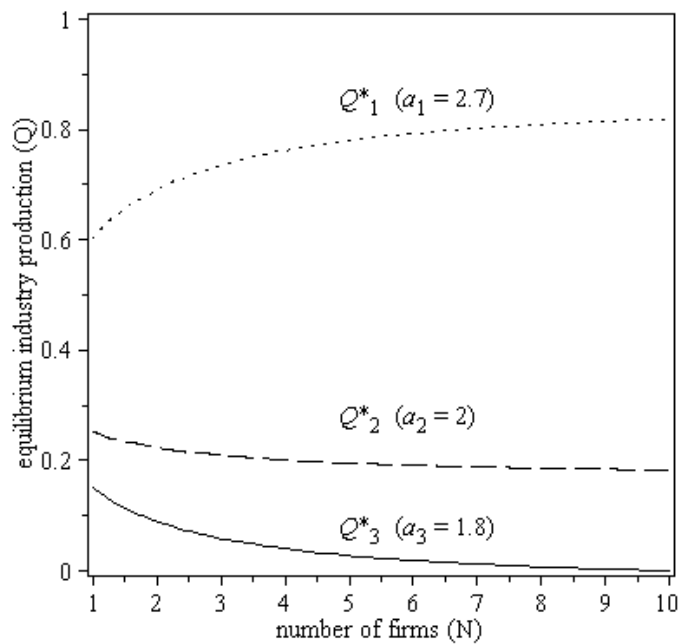


Figure 6.1: Total equilibrium production quantities as a function of N ;
 $\bar{B}=1/N, \bar{c}=1, a_1=2.7, a_2=2, a_3=1.8$

The change in equilibrium allocations caused by competition also changes the individual firms' profits and the industry profits (the sum of all firms' profits). As stated in Theorem 6.1, firms receive higher profits as the variance between the market demands increases; however, the benefit derived from this variance is diminished as the number of firms increases. If there

are differences in demands, this leads to decreasing industry profits as the number of competing firms increases, even if the total production across all markets remains equal. This is consistent with previous findings (Johnson and Myatt, 2006). Figure 6.2 depicts this drop in profits for the same parameters from Figure 6.1, where the lower curve represents the profits of the individual firms and the upper curve represents industry profits.

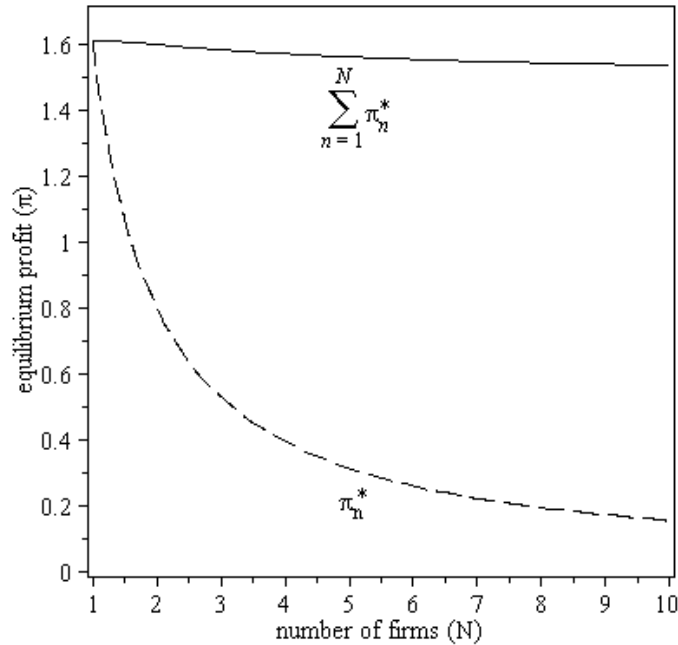


Figure 6.2: Equilibrium profit as a function of N ; $\bar{B}=1/N$, $\bar{c}=1$, $a_1=2.7$, $a_2=2$, $a_3=1.8$

6.2.2. EFFECT OF COMPETITION ON FIRMS WITH ASYMMETRIC BUDGETS

We now turn to the asymmetric setting where firms have different budgets. We show, through an example, that Proposition 6.1 may not hold anymore in the asymmetric case. Figure 6.3 presents an instance where all firms have identical production costs $\bar{c}=1$, but Firm 1 has $B_1=1$ and all other firms have $B_n=1/(N-1)$, $n \geq 2$, so the aggregate capacity is fixed at a level of 2 units if $N \geq 2$. Note that $B_1 > B_n$ for $n \geq 2$, if $N > 2$. All firms are allocating their products over two markets with demand potentials of 6 and 7. Figure 6.3 shows how competition causes the firms with the smaller capacities to produce more heavily for the high demand

market while the firm with the larger capacity produces less for the high demand market and more for the low demand market, contrary to the result of Proposition 6.1. Specifically, as the number of firms increases from 2 to 10, the firm with the large budget reduces its production into the market with the higher demand potential (Market 2) from 0.67 to 0.55, while the firms with the smaller budget increase their total production for Market 2 from 0.67 to 0.91. Similarly, in the market with the lower demand potential (Market 1), the large budget firm increases its productions from 0.33 to 0.45 while the small budget firms decrease their total production from 0.33 to 0.09 as the number of firms increases from 2 to 10. Intuitively, firms with small capacities shift substantial production to high price markets because the burden of the resulting reduction in price experienced in those markets will be carried mostly by the firms with larger capacities. However, as small capacity firms drive down the price in these high demand markets, large capacity firms shift their production to other markets, as seen in Figure 6.3.

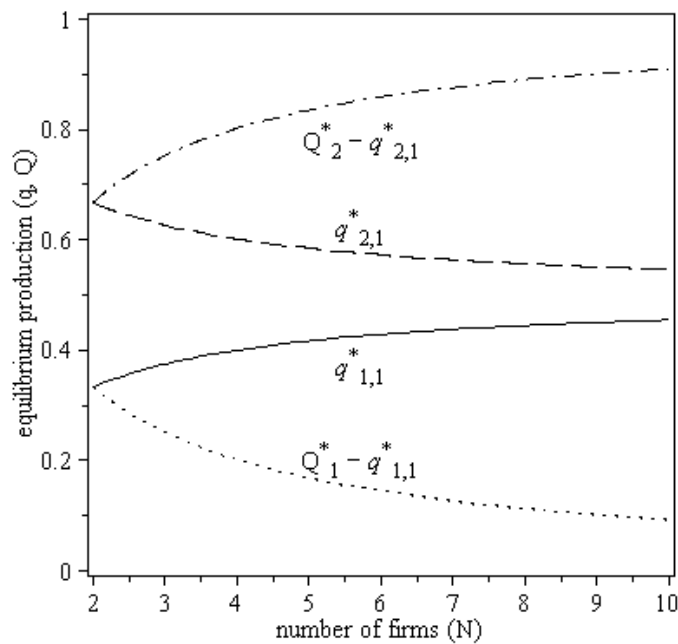


Figure 6.3: Equilibrium production quantities as a function of N ;
 $B_1=1, B_n=1/(N-1)$ for $N>1, \bar{c}=1, a_1=6, a_2=7$

Furthermore, while industry profits decline irrespective of differences in firms' budgets, the individual firms' profits decline particularly fast for firms with large production capacities, while the total profits of all firms with small capacities can actually increase, contrary to Proposition 6.1. Figure 6.4 presents the corresponding profits to the example presented in Figure 6.3. As shown in the figure, total profits of firms with small capacities slightly increases as the number of competing firms increases from 2 to 3. This suggests that it may be profitable for a firm to split its budget into smaller parts: for example, if two firms are competing with budgets of 1 each (i.e., $N = 2$), then a firm that splits its budget into two equal parts (with autonomous decision makers for each part of the budget) can increase its profits according to the curve labeled $\sum_{n=2}^N \pi_n^*$ in Figure 6.4 between $N = 2$ and $N = 3$. However, this only holds if a single firm splits its budget; as stated in Proposition 6.1, if firms have symmetric budgets, total industry profits decreases as the number of firms increases. Therefore, if in the present example, at $N = 2$, both firms were to split their budgets into two equal parts, then both firms would experience reduced profits.

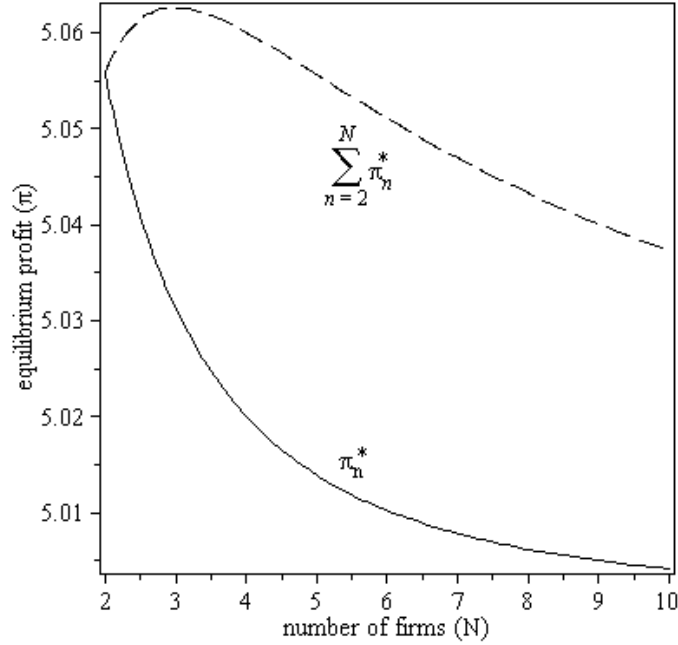


Figure 6.4: Equilibrium profit as a function of N ; $B_1=1, B_n=1/(N-1), \bar{c}=1, a_1=6, a_2=7$

6.2.3. BUDGET ADJUSTMENTS

So far, we have assumed that firms' budgets were fixed. However, it may be reasonable to assume that in certain circumstances firms may be able to adjust their budget (for example, if their planning horizon is sufficiently long). To derive the profitability of increasing the budget, one needs to consider the shadow price associated with the budget. The shadow price of the budget is the marginal profit of increasing the budget by one unit. If the shadow price is positive, a firm can increase its profit by increasing its budget. The following proposition states the value of the budget's shadow price.

Proposition 6.2: *If N firms are competing in S markets, then firm n 's shadow*

$$\text{price, } r_n, \text{ of increasing its budget is } \frac{1}{S} \left(\sum_{t=1}^S a_t - \left(\frac{B_n}{c_n} + \sum_{i=1}^N \frac{B_i}{c_i} \right) \right).$$

The shadow price is determined by the difference of average demand potential and average production capacity, where the production capacity of the particular firm is subtracted twice. Intuitively, the formula in Proposition 6.2 considers the "average market price" gained by

additional production and the reduction of price for the existing products in those markets. An important implication of Proposition 6.2 is that the variance in the demand potentials between the markets does not influence the shadow price of a budget increase. I.e., differences between demand potentials across markets influence a firm's profits (as demonstrated in Theorem 6.1) but does not change the marginal profits a firm can expect from increasing its budget. Instead, the additional profit gained by an incremental increase in budget is based solely on the average demand potentials, and the current production capacities of all firms. Proposition 6.2 also implies that the shadow price of increasing the budget is greater (smaller) the smaller (greater) the current capacity of a firm. Therefore, capacity growth should originate mostly from firms with smaller capacities. Empirical research has broadly shown that small firms grow at a higher rate than large firms (Santarelli et al., 2006).

6.3. MARKET ABSTAINING

The equilibrium production quantities and resulting profits derived in Theorem 6.1 hold only if all firms are producing for all markets. However, given the constraint $q_{s,n} \geq 0 \quad \forall s, n$, the

equilibrium production $q_{s,n}^* = \frac{B_n}{c_n S} + \frac{a_s - \frac{1}{S} \sum_{i=1}^S a_i}{(N+1)}$ is valid only if $q_{s,n}^* > 0$, or:

$$a_s > \frac{\sum_{i=1}^S a_i - (N+1) \frac{B_n}{c_n}}{S} \quad \forall s, n \quad (6.3)$$

We now investigate the case where this condition does not hold for all markets and firms. From (6.3), one can observe that this condition may not be satisfied for every firm for every market, in particular if the difference in demand potentials between markets is high, the number of competitors is small, and firms have small capacities. Intuitively, firms that have small capacities can invest heavily into the markets with high demand potential without

saturation of that market – especially when there are not many competitors in that market. When firms have different capacity levels, some firms may produce for a particular market while other firms do not. In this section, we analyze how these asymmetric production decisions affect the equilibrium resource allocation.

To characterize the solution, we let K_s denote the number of firms not producing for market s . Without loss of generality, we sort all markets by their demand potentials ($a_1 \leq a_2 \leq \dots \leq a_S$), and all firms by their budgets ($B_1 \leq B_2 \leq \dots \leq B_N$). As in Section 6.2, we assume that all firms produce at capacity. We propose the following conjecture for the equilibrium production quantities with market abstaining, $q_{s,n}^\#$. This conjecture can be proved to hold for a number of instances. The case where $S = N = 3$ is provided in Section 6.6.

Conjecture 1: *If N firms are competing in S markets and not all firms produce in all markets, i.e., $K_s > 0$ for some s , then the equilibrium production quantity of firm n for market s is given by*

$$q_{s,n}^\# = \begin{cases} \frac{B_n - \sum_{t=1}^{s-1} c_n q_{t,n}^\#}{c_n S^\#} + \frac{a_s - \frac{1}{S^\#} \sum_{i=s}^S a_i}{N+1} + \frac{\sum_{k=1}^{K_s} \left(\frac{B_k}{c_k S^\#} + \frac{a_s - \frac{1}{S^\#} \sum_{i=s}^S a_i}{N+1} \right)}{N - K_s + 1} & \text{if (6.3) holds} \\ 0 & \text{otherwise,} \end{cases} \quad (6.4)$$

where $S^\# = S - \sum_{i=1}^{s-1} 1$, for $\forall s, n$.

The first two terms of the equilibrium production $q^\#$ are very similar to the previous equilibrium q^* with some modification. The modification adjusts for the budget spent on markets with lower demand potentials than the current market considered. Therefore, for the market with the lowest demand potential, a_1 , the first two terms of $q^\#$ are identical to q^* . For

the other markets, the previously allocated budget, $\sum_{t=1}^{s-1} c_n q_{t,n}$, is removed and only the markets

with higher demand potentials are considered, $S^\# = S - \sum_{i=1}^{s-1} 1$. The third term of $q^\#$ adjusts the

production allocation if other firms decide not to invest in a particular market. Since firms do not produce for markets that violate (6.3), and (6.3) was derived by assuming

$\frac{B_n}{c_n S} + \frac{a_s - \frac{1}{S} \sum_{i=1}^S a_i}{(N+1)} > 0$, we know that this adjustment amount is always negative. Intuitively,

the reduction in competition for a particular market provides firms with an incentive to produce less aggressively for this market and instead compete more heavily in the other markets.

To illustrate the equilibrium production decision proposed by the conjecture, consider the following instance, where Firm 1 (the firm with the lowest budget) produces only for Market 3 (the market with the highest demand potential) and Firm 2 produces only for Markets 2 and 3, and Firm 3 (the firm with the largest budget) produces for all markets. This is clarified in Table 6.1.

	Market 1	Market 2	Market 3
Firm 1	$q_{1,1}^\# = 0$	$q_{2,1}^\# = 0$	$q_{3,1}^\# > 0$
Firm 2	$q_{1,2}^\# = 0$	$q_{2,2}^\# > 0$	$q_{3,2}^\# > 0$
Firm 3	$q_{1,3}^\# > 0$	$q_{2,3}^\# > 0$	$q_{3,3}^\# > 0$

Table 6.1: Equilibrium production summary

We now concentrate on the amounts produced by the firms into the different markets. In this example, since Firm 1 is producing only for Market 3, its entire budget is dedicated to produce the units for this market. Hence, we have that $q_{3,1}^\# = \frac{B_1}{c_1}$. Firm 2 does not produce at

all for Market 1. For Market 2, this firm produces an output equal to

$$q_{2,2}^{\#} = \frac{B_2}{c_2(S-1)} + \frac{a_2 - \frac{1}{S-1} \sum_{i=2}^S a_i}{N+1} + \frac{1}{N} \left(\frac{B_1}{c_1(S-1)} + \frac{a_2 - \frac{1}{S-1} \sum_{i=2}^S a_i}{N+1} \right). \quad \text{The first two terms}$$

correspond to the original q^* but this amount is adjusted by the third term. The third term is the quantity that Firm 1 “would have” produced for Market 2 under the original q^* . The assumption that led to the quantities shown in Table 6.1 is that for each firm n and markets s that violates (6.3), we have $q_{s,n}^{\#} = 0$. Since (6.3) was derived from $q_{s,n}^* > 0$, we know that third term is a negative amount, indicating that Firm 2 is reducing its production allocation for Market 2 because there are fewer competitors in this market than anticipated under the original equilibrium q^* . This term is then divided by N as part of this amount is absorbed by the remaining firms in Market 2, Firms 2 and 3. The conjecture further states that Firm 2 produces

$$q_{3,2}^{\#} = \frac{B_2 - c_2 q_{2,2}^{\#}}{c_2(S-2)} \text{ for Market 3. This means that Firm 2 allocates its remaining budget to the}$$

remaining market.

$$\text{Firm 3 produces } q_{1,3}^{\#} = \frac{B_3}{c_3 S} + \frac{a_1 - \frac{1}{S} \sum_{i=1}^S a_i}{N+1} + \frac{1}{N-1} \left(\frac{B_1}{c_1 S} + \frac{a_1 - \frac{1}{S} \sum_{i=1}^S a_i}{N+1} + \frac{B_2}{c_2 S} + \frac{a_1 - \frac{1}{S} \sum_{i=1}^S a_i}{N+1} \right)$$

into Market 1. Since neither Firm 1 nor Firm 2 are investing into Market 1, part of their anticipated production amount under the original equilibrium q^* is added to Firm 3’s production. Since these amounts are both negative (assuming Table 6.1 was constructed by setting $q_{s,n}^{\#} = 0$ if (6.3) is violated), this implies that Firm 3 reduces production into Market 1 as a monopolist is expected to. However, Firm 3 is not producing the monopoly output for this market as it faces competition in the other markets. Thus, as it becomes the single supplier for Market 1, it adjusts its output downwards, yet, the competitive environments in the other

markets limit its actions. For Market 2, the adjustment is equal to the adjustment made by

$$\text{Firm 2, } q_{2,3}^{\#} = \frac{B_3 - q_{1,3}^{\#}}{c_3(S-1)} + \frac{a_2 - \frac{1}{S-1} \sum_{i=2}^S a_i}{N+1} + \frac{1}{N} \left(\frac{B_1}{c_1(S-1)} + \frac{a_2 - \frac{1}{S-1} \sum_{i=2}^S a_i}{N+1} \right), \text{ and the remaining}$$

$$\text{budget of Firm 3 is used to produce for Market 3, } q_{3,3}^{\#} = \frac{B_3 - c_3 q_{1,3}^{\#} - c_3 q_{2,3}^{\#}}{c_3(S-2)}.$$

In effect, the conjecture says that firms apply the equilibrium production quantities derived in Theorem 6.1 to the markets that are sufficiently profitable. Since the profitability of markets depends on both the demand potentials and the firms' individual budgets, markets that are unattractive to one firm may still attract investment by another firm. Recognizing the opportunity created by the reduction in competition, firms reduce production in markets abandoned by other firms in order to raise prices in those markets.

6.4. BUDGET CONSTRAINED MULTIMARKET EQUILIBRIUM WITH VARYING COSTS AND DEMAND SLOPES

In this section, we provide additional insights using a more general model. Thus far, we have restricted the demand slope to be 1 and that we assumed that each firm has the same production cost for all different products. Now, we relax these two assumptions and allow the demand slope b_s to take any positive value and the unit production cost to differ from product to product. We define $c_{s,n}$ as the unit production cost of firm n for the product targeting market s . Consequently, firm n 's optimization problem is given by:

$$\begin{aligned} \max \pi_n &= \sum_{s=1}^S (a_s - b_s Q_s - c_{s,n}) q_{s,n} \\ \text{s.t. } B_n - \sum_{s=1}^S c_{s,n} q_{s,n} &\geq 0 \\ q_{s,n} &\geq 0 \quad \forall s \end{aligned}$$

According to Laye and Laye (2008), this extended problem setup has a single solution (which can be found with common solvers), but no closed form result has been found. The general case of the budget constrained multimarket Cournot equilibrium adds further complexity and counter-intuitive results. We demonstrate by presenting two numerical illustrations.

6.4.1. INCREASING PRODUCTION COSTS MAY INCREASE PROFITS

Firms generally aim to reduce unit production costs in order to increase their profits. However, in a budget constrained multimarket Cournot equilibrium, we find that in some instances firms can increase their profits by *increasing* their production costs. This is a particularly interesting result because strategically increasing unit costs is typically easier to accomplish than reducing unit costs.

Consider three firms with budgets $B_1 = 0.3$, $B_2 = 0.4$, $B_3 = 0.6$ that are competing in three markets with demand potentials $a_1 = 3$, $a_2 = 4$, $a_3 = 3.5$ and demand slopes $b_1 = b_2 = b_3 = 1$. The unit production costs of all firms for all markets is 0.2, leading to the equilibrium production quantities and resulting profits displayed in Table 6.2.

	Firm 1	Firm 2	Firm 3
$q_{1,n}$	0.37	0.54	0.88
$q_{2,n}$	0.63	0.79	1.12
$q_{3,n}$	0.50	0.67	1.00
π_n	1.73	2.30	3.43

Table 6.2: Equilibrium production quantities and resulting profits; $B_1=0.3$, $B_2=0.4$, $B_3=0.6$, $a_1=3$, $a_2=4$, $a_3=3.5$, $b_1=b_2=b_3=1$, $c_{s,n}=0.2 \quad \forall s, n$

As presented in Table 6.2, firms produce for Market 3 proportional to their budgets. This occurs since the demand potential of this market, a_3 , coincides with the average demand potential of all three markets; e.g., Firm 3 has twice the budget of Firm 1 and produces twice the amount of product of Firm 1 for Market 3. As was demonstrated in Figure 6.3, the firms with larger budgets (e.g. Firm 3) invest proportionally more into markets with lower demand potentials (Market 1) and vice versa. However, their larger budgets, and thus higher production levels, do lead to higher profits.

We now assume an increase in unit production costs for each firm in all markets, as illustrated in Table 6.3.

	$c_{s,1}$	$c_{s,2}$	$c_{s,3}$
Market 1	0.25	1.0	1.0
Market 2	1.0	0.25	0.25
Market 3	1.0	0.25	0.25

Table 6.3: New unit production cost parameters

With costs increasing, we expect profits to decrease, *ceteris paribus*. However, in this particular instance, the profits of all firms increase. Table 6.4 shows the new production quantities and resulting profits.

	Firm 1	Firm 2	Firm 3
$q_{1,n}$	1.2	-	-
$q_{2,n}$	-	0.88	1.28
$q_{3,n}$	-	0.72	1.12
π_n	1.86	2.41	3.61

Table 6.4: Equilibrium production quantities and resulting profits with higher production costs; $B_1=0.3$, $B_2=0.4$, $B_3=0.6$, $a_1=3$, $a_2=4$, $a_3=3.5$, $b_1=b_2=b_3=1$, costs as in Table 6.3

The key to understanding this counter-intuitive result lies in the way firms adapt their production decision to competition. As previously discussed, firms produce more for markets with higher demand potentials (and vice versa) but reduce the degree to which they shift their production between these markets as the number of competing firms increases. This

consequently leads to lower returns for all firms as was depicted in Figure 6.2. However, if the cost structure of firms is such that it provides certain firms with cost advantages for some of the markets, then firms will produce more heavily in those markets. Depending on the market parameters and the firms' budgets, this can lead to an overall more efficient production allocation and thus higher profits for all firms. The results presented in Table 6.4 demonstrates how the change in production costs has led Firm 1 to produce exclusively on Market 1, while Firm 2 and 3 focus on the other two markets.

While many counter-examples can be constructed that demonstrate how cost increases generally lead to profit reductions, this section highlights the opportunity that exists in some instances to increase profits for all firms by strategically increasing production costs. In particular, such opportunities exist if increases in costs lead firms to specialize in certain markets, thus reducing the competition effect that drives firms away from Pareto optimal solutions. This result provides another explanation to why firms are often found to focus on certain markets. While increasing market returns of markets is one explanation (Selove, 2010), our work demonstrates that dividing up markets between firms can have advantages for all firms, even with decreasing market returns.

6.4.2. BENEFICIARIES FROM CHANGES IN DEMAND SLOPES

Typically, a firm that is focusing on producing for a particular market is expected to benefit from an increase in demand in that market to a greater degree than firms that are less focused on this market or do not produce any product for that market. However, in this example we illustrate that the opposite may occur in some instances. Consider again three firms that compete in three separate markets. Let us assume that their budgets vary from each other such that $B_1 = 0.03$, $B_2 = 0.21$, $B_3 = 0.52$. Assume the unit production of each firm are identical

across the three markets such that $c_{s,1} = 0.1$, $c_{s,2} = 0.3$, $c_{s,3} = 0.2$, for $\forall s$. Lastly, the three markets have different demand potentials $a_1 = 3$, $a_2 = 4$, $a_3 = 3.5$ but share identical demand slopes $b_1 = b_2 = b_3 = 1$. The equilibrium production quantities and resulting profits are presented in Table 6.5.

	Firm 1	Firm 2	Firm 3
$q_{1,n}$	-	-	0.45
$q_{2,n}$	0.21	0.41	1.14
$q_{3,n}$	0.09	0.29	1.01
π_n	0.63	1.32	4.86

Table 6.5: Equilibrium production quantities and resulting profits; $B_1=0.03$, $B_2=0.21$, $B_3=0.52$, $a_1=3$, $a_2=4$, $a_3=3.5$, $b_1=b_2=b_3=1$, $c_{1,1}=c_{2,1}=c_{3,1}=0.1$, $c_{1,2}=c_{2,2}=c_{3,2}=0.3$, $c_{1,3}=c_{2,3}=c_{3,3}=0.2$

As in the previous example in Subsection 6.4.1, the firm with the largest budget (Firm 3) is using a larger share of its capacity for the product targeting the market with the lowest demand potential (Market 1) than firms with smaller budgets. In fact, since Firms 1 and 2 violate (6.3) for Market 1, they do not produce any amount for Market 1.

Assume that the market demand slope of b_1 decreases to 0.2, while the slopes of b_2 and b_3 both increase to 1.5 and 1.4, respectively. Since a decrease in the demand slope leads to higher prices (at equal production levels), we generally expect a decrease in demand slopes to lead to higher profits. Conversely, demand slope increases generally lead to lower prices and thus lower profits in those markets (at equal production levels). As presented in Table 6.5, Firm 3 is the only firm to produce for the market experiencing a decrease in its demand slope while Firms 1 and 2 are exclusively producing for the markets that are experiencing an increase in demand slopes. This may lead one to expect this change in demands to benefit Firm 3 more than Firms 1 and 2. However, Table 6.6 demonstrates that Firm 3's profit decreases while the profits of Firms 1 and 2 increase.

	Firm 1	Firm 2	Firm 3
$q_{1,n}$	-	-	1.49
$q_{2,n}$	0.19	0.38	0.58
$q_{3,n}$	0.11	0.32	0.53
π_n	0.64	1.34	4.48

Table 6.6: Equilibrium production quantities and resulting profits with differing demand slopes; $B_1=0.03$, $B_2=0.21$, $B_3=0.52$, $a_1=3$, $a_2=4$, $a_3=3.5$, $b_1=0.2$, $b_2=1.5$, $b_3=1.4$, $c_{1,1}=c_{2,1}=c_{3,1}=0.1$, $c_{1,2}=c_{2,2}=c_{3,2}=0.3$, $c_{1,3}=c_{2,3}=c_{3,3}=0.2$

A comparison between the production quantities of Tables 6.5 and 6.6 reveal that the change in demand slopes drives Firm 3 to produce even more heavily in Market 1, thereby reducing its production for the markets with the higher demand potentials. As a result, Firms 1 and 2 have less competition in Markets 2 and 3 and thus receive higher profits.

Bulow, Geanakoplos and Klemperer (1985) have found a similarly counter-intuitive instance in a reduced setting where one firm is a monopolist in one market and is competing with another firm in a second market. They showed that a demand shock that raises prices in the single-firm market could lower the profit of the firm with access to both markets while raising the profit of the firm that only has access to the second market with unchanged demand. They found that the key to this result was whether the products of the two firms were strategic substitutes or strategic complements. Our results highlight the counter-intuitive demand effects that can occur even without explicitly modeling substitution and complementarity effects.

6.5. CHAPTER 6 CONCLUSIONS

In this chapter, we studied the budget constrained production decision of firms in a competitive setting. Our key contribution is a closed-form solution of the equilibrium production quantities and resulting profits for any given number of firms and markets that considers the firms' individual budgets and production costs as well as differences in demand potentials between

markets. The provided solutions clearly demonstrate that competition has a significant impact on equilibrium production decisions, in a way that was not *a priori* obvious. In particular, competition generally drives firms to produce more for markets with larger than average demand potentials, thereby reducing industry-wide profits. In asymmetric settings, large budget firms produce in an opposite manner to this general trend and focus on markets with low demand potential, thereby benefitting smaller firms. Our results can help explain counter-intuitive numerical instances where, for example, an increase in firms' unit costs leads to an increase in profits for all firms.

6.6. PROOFS FOR CHAPTER 6

Proof of Theorem 6.1: We first derive the Karush–Kuhn–Tucker (KKT) conditions, where the KKT multiplier for the budget constraint is defined as r_n :

$$-\left(a_s - q_{s,n} - \sum_{i=1}^N q_{s,i} - c_n\right) + r_n \geq 0 \perp q_{s,n} \geq 0 \quad \forall s \in S, n \in N,$$

$$B_n - \sum_{s=1}^S q_{s,n} c_n \geq 0 \perp r_n \geq 0 \quad \forall n \in N.$$

Assuming that firms produce a positive amount into each market, $q_{s,n} > 0$, and invest their entire budget, the KKT conditions imply:

$$a_s - q_{s,n} - \sum_{i=1}^N q_{s,i} - c_n = r_n, \tag{6.P1}$$

$$B_n - \sum_{s=1}^S q_{s,n} c_n = 0. \tag{6.P2}$$

We subtract an instance of (6.P1), with $n = 1$, summed over S from an instance of (6.P1), with $s = 1$ and $n = 1$, multiplied by S :

$$\begin{aligned}
& - \frac{S \left(a_1 - q_{1,1} - \sum_{i=1}^N q_{1,i} - c_1 = r_1 \right)}{\sum_{s=1}^S \left(a_s - q_{s,1} - \sum_{i=1}^N q_{s,i} - c_1 = r_1 \right)} \\
& \quad Sa_1 - Sq_{1,1} - S \sum_{i=1}^N q_{1,i} - Sc_1 - \sum_{s=1}^S \left(a_s - q_{s,1} - \sum_{i=1}^N q_{s,i} - c_1 \right) = Sr_1 - Sr_1 \\
& \quad \Rightarrow Sa_1 - Sq_{1,1} - S \sum_{i=1}^N q_{1,i} - \sum_{s=1}^S a_s + \sum_{s=1}^S q_{s,1} + \sum_{s=1}^S \sum_{i=1}^N q_{s,i} = 0
\end{aligned}$$

and then separate the element $n = 1$ out of the summations over the set N :

$$\begin{aligned}
& Sa_1 - Sq_{1,1} - S \sum_{i=2}^N q_{1,i} - Sq_{1,1} - \sum_{s=1}^S a_s + \sum_{s=1}^S q_{s,1} + \sum_{s=1}^S \sum_{i=2}^N q_{s,i} + \sum_{s=1}^S q_{s,1} = 0 \\
& Sa_1 - \sum_{s=1}^S a_s - 2Sq_{1,1} + 2 \sum_{s=1}^S q_{s,1} - S \sum_{i=2}^N q_{1,i} + \sum_{s=1}^S \sum_{i=2}^N q_{s,i} = 0
\end{aligned}$$

By (6.P2) we have $\sum_{s=1}^S q_{s,n} = \frac{B_n}{c_n}$, so:

$$Sa_1 - \sum_{s=1}^S a_s - 2Sq_{1,1} + 2 \frac{B_1}{c_1} - S \sum_{i=2}^N q_{1,i} + \sum_{s=1}^S \sum_{i=2}^N q_{s,i} = 0 \tag{6.P3}$$

Next, we subtract an instance of (6.P1) with $n = 1$ from (6.P1):

$$\begin{aligned}
& a_s - q_{s,n} - \sum_{i=1}^N q_{s,i} - c_n = r_n \\
& - \left(a_s - q_{s,1} - \sum_{i=1}^N q_{s,i} - c_1 = r_1 \right) \\
& \quad \frac{-q_{s,n} + q_{s,1} - c_n + c_1 = r_n - r_1.
\end{aligned}$$

This can be expressed as:

$$q_{s,n} = r_1 - r_n + c_1 - c_n + q_{s,1} \tag{6.P4}$$

Finally, we substitute the value of $q_{1,i}$ and $q_{s,i}$ from (6.P4) into (6.P3):

$$Sa_1 - \sum_{s=1}^S a_s - 2Sq_{1,1} + 2 \frac{B_1}{c_1} - S \sum_{i=2}^N (r_1 - r_i + c_1 - c_n + q_{1,1}) + \sum_{s=1}^S \sum_{i=2}^N (r_1 - r_i + c_1 - c_n + q_{s,1}) = 0.$$

Rearranging:

$$Sa_1 - \sum_{s=1}^S a_s - 2Sq_{1,1} + 2\frac{B_1}{c_1} - S\sum_{i=2}^N (r_1 - r_i + c_1 - c_n) - S(N-1)q_{1,1} \\ + S\sum_{i=2}^N (r_1 - r_i + c_1 - c_n) + (N-1)\sum_{s=1}^S q_{s,1} = 0,$$

cancelling terms and using (6.P2):

$$Sa_1 - \sum_{s=1}^S a_s - 2Sq_{1,1} + 2\frac{B_1}{c_1} - S(N-1)q_{1,1} + (N-1)\frac{B_1}{c_1} = 0,$$

we have:

$$Sa_1 - \sum_{s=1}^S (a_s) + (N+1)\frac{B_1}{c_1} = S((N-1)+2)q_{1,1} \\ \Rightarrow q_{1,1} = \frac{B_1}{c_1 S} + \frac{a_1 - \frac{1}{S}\sum_{s=1}^S (a_s)}{N+1}$$

This result holds for any instance of s and n ; therefore, $q_{s,n}^* = \frac{B_n}{c_n S} + \frac{\left(a_s - \frac{1}{S}\sum_{t=1}^S a_t\right)}{(N+1)}$ is the

unique Nash equilibrium production quantity for $\forall s \in S, n \in N$.

Substituting this equilibrium production quantity into the profit expression

$\pi_n = \sum_{s=1}^S \left[\left(a_s - \sum_{i=1}^N (q_{s,i}) - c_n \right) q_{s,n} \right]$, we have:

$$\pi_n = \sum_{s=1}^S \left[\left(a_s - \sum_{i=1}^N \left(\frac{B_i}{c_i S} + \frac{\left(a_s - \frac{1}{S}\sum_{t=1}^S a_t \right)}{(N+1)} \right) - c_n \right) \left(\frac{B_n}{c_n S} + \frac{\left(a_s - \frac{1}{S}\sum_{t=1}^S a_t \right)}{(N+1)} \right) \right]$$

$$\begin{aligned}
&= \sum_{s=1}^S \left[\frac{B_n}{c_n S} a_s - \frac{B_n}{c_n S} \sum_{i=1}^N \left(\frac{B_i}{c_i S} \right) - \frac{B_n}{c_n S} \frac{N \left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} - \frac{B_n}{c_n S} c_n + \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} a_s \right. \\
&\quad \left. - \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} \sum_{i=1}^N \left(\frac{B_i}{c_i S} \right) - \frac{N \left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)^2}{(N+1)^2} - \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} c_n \right] \\
&= \sum_{s=1}^S \left[\frac{B_n}{c_n S} \left(a_s - \sum_{i=1}^N \left(\frac{B_i}{c_i S} \right) - c_n \right) \right] - \sum_{s=1}^S \left[\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right) \left(\frac{B_n}{c_n S} \frac{N}{(N+1)} + \frac{1}{(N+1)} \sum_{i=1}^N \left(\frac{B_i}{c_i S} \right) \right) \right] \\
&\quad + \frac{(N+1) \sum_{s=1}^S (a_s^2) - \frac{(N+1)}{S} \sum_{s=1}^S \left(a_s \sum_{t=1}^S a_t \right) - N \sum_{s=1}^S (a_s^2) + \frac{2N}{S} \sum_{s=1}^S \left(a_s \sum_{t=1}^S a_t \right) - \frac{N}{S} \left(\sum_{t=1}^S a_t \right)^2}{(N+1)^2} \\
&\quad - \sum_{s=1}^S \left[\frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} c_n \right]
\end{aligned}$$

Since $\sum_{s=1}^S \left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right) = 0$ and $\sum_{s=1}^S \left(a_s \sum_{t=1}^S a_t \right) = \left(\sum_{t=1}^S a_t \right)^2$, we have:

$$\pi_n = \frac{B_n}{c_n S} \left(\sum_{s=1}^S (a_s - c_n) - \sum_{i=1}^N \frac{B_i}{c_i} \right) + \frac{S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2}{S(N+1)^2} \quad \forall n \in N. \quad \square$$

Proof of Proposition 6.1: By Theorem 6.1, we have $\frac{\partial q_{s,n}^*/\bar{B}}{\partial N} = \frac{\partial}{\partial N} \left(\frac{1}{c_n S N} + \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1) \frac{B}{N}} \right)$,

or:

$$\frac{\partial q_{s,n}^*/\bar{B}}{\partial N} = \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{\left(1 + \frac{1}{N} \right)^2 B N^2}.$$

Since $\left(1 + \frac{1}{N} \right)^2 B N^2 > 0$ for all feasible B and N , we have $\frac{\partial q_{s,n}^*/\bar{B}}{\partial N} > (<) 0$ if $a_s > (<) \frac{1}{S} \sum_{i=1}^S a_i$.

By Theorem 6.1, we also have $\frac{\partial \pi_n^*}{\partial N} = \frac{\partial}{\partial N} \left(\frac{B}{N\bar{c}S} \left(\sum_{s=1}^S (a_s - \bar{c}) - \frac{B}{\bar{c}} \right) + \frac{S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2}{S(N+1)^2} \right)$, or:

$$\frac{\partial \pi_n^*}{\partial N} = -\frac{B}{N^2 \bar{c} S} \left(\sum_{s=1}^S (a_s - \bar{c}) - \frac{B}{\bar{c}} \right) - 2 \frac{S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2}{S(N+1)^3}. \quad (6.P5)$$

We have assumed that budgets are below the total production cost of the unconstrained equilibrium, defined in (6.2). Given the symmetry assumptions of this proposition, (6.2) represents the maximum budget of each firm:

$$\begin{aligned} \bar{B}_{\max} &= \sum_{s=1}^S \left(\frac{a_s + \sum_{i=1}^N \bar{c} - (N+1)\bar{c}}{N+1} \bar{c} \right) \\ &= \sum_{s=1}^S \left(\frac{a_s - \bar{c}}{N+1} \bar{c} \right) \end{aligned} \quad (6.P6)$$

By substituting $N \cdot (6.P6)$ (the maximum total budget of N firms) into $\sum_{s=1}^S (a_s - \bar{c}) - \frac{B}{\bar{c}}$ (from

6.P5), we can show that $\sum_{s=1}^S (a_s - \bar{c}) - \frac{N \cdot \bar{B}_{\max}}{\bar{c}} > 0$, or $\sum_{s=1}^S (a_s - \bar{c}) - \frac{N}{N+1} \sum_{s=1}^S (a_s - \bar{c}) > 0$, for all

feasible $\bar{c} < a_s$. Furthermore, $S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2$ is a measure of variance and cannot be

negative and we have $\frac{B}{N^2 \bar{c} S} > 0$ and $\frac{2}{S(N+1)^3} > 0$ for are all feasible B, N, S and \bar{c} .

Therefore, $\frac{\partial \pi_n^*}{\partial N} < 0$.

By Theorem 6.1 we also have $\frac{\partial \sum_{n=1}^N \pi_n^*}{\partial N} = \frac{\partial}{\partial N} \sum_{n=1}^N \left[\frac{B}{NcS} \left(\sum_{s=1}^S (a_s - \bar{c}) - \frac{B}{c} \right) + \frac{S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2}{S(N+1)^2} \right],$

or:

$$\frac{\partial \sum_{n=1}^N \pi_n^*}{\partial N} = -(N-1) \frac{S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2}{S(N+1)^3}.$$

Since $S \sum_{s=1}^S (a_s^2) - \left(\sum_{t=1}^S a_t \right)^2 > 0$ and $\frac{N-1}{S(N+1)^3} > 0$ for all feasible N and S , we have $\frac{\partial \sum_{n=1}^N \pi_n^*}{\partial N} < 0$. \square

Proof of Proposition 6.2: The shadow price r_n is the KKT multiplier for the budget constraint from Theorem 6.1. Specifically, (6.P1) defines the shadow price of firm n 's budget constraint.

Substituting the equilibrium quantity, $q_{s,n}^* = \frac{B_n}{c_n S} + \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)}$, into (6.P1) we have:

$$\begin{aligned} r_n &= a_s - \left(\frac{B_n}{c_n S} + \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} \right) - \sum_{i=1}^N \left(\frac{B_i}{c_i S} + \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} \right) \\ &= a_s - \frac{B_n}{c_n S} - \frac{\left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} - \sum_{i=1}^N \left(\frac{B_i}{c_i S} \right) - \frac{N \left(a_s - \frac{1}{S} \sum_{t=1}^S a_t \right)}{(N+1)} \\ &= a_s - \frac{B_n}{c_n S} - \sum_{i=1}^N \left(\frac{B_i}{c_i S} \right) - a_s + \frac{1}{S} \sum_{t=1}^S a_t \\ &= \frac{1}{S} \left(\sum_{t=1}^S a_t - \left(\frac{B_n}{c_n} + \sum_{i=1}^N \left(\frac{B_i}{c_i} \right) \right) \right) \quad \forall n \in N. \quad \square \end{aligned}$$

Proof of Conjecture 1 for one instance: We prove the special case defined in Table 6.1.

Without loss of generality, we sort all markets by demand potential such that $a_1 < a_2 < a_3$ and

all firms by budgets such that $B_1 < B_2 < B_3$. Assuming Conjecture 1 holds, then by (6.4) we have:

$$q_{1,1} = q_{2,1} = 0, \quad q_{3,1} = \frac{B_1}{c_1},$$

$$q_{1,2} = 0$$

$$q_{2,2} = \frac{B_2}{2c_2} + \frac{B_1}{6c_1} + \frac{a_2 - \frac{1}{2}(a_2 + a_3)}{3} \quad (6.P7)$$

$$q_{s1,n3} = \frac{B_3}{3c_3} + \frac{B_1}{6c_1} + \frac{B_2}{6c_2} + \frac{a_1 - \frac{1}{3}(a_1 + a_2 + a_3)}{2} \quad (6.P8)$$

As before, we have the following KKT conditions:

$$-\left(a_s - q_{s,n} - \sum_{i=1}^N q_{s,i} - c_n\right) + r_n \geq 0 \perp q_{s,n} \geq 0 \quad \forall s \in S, n \in N$$

$$B_n - \sum_{s=1}^S q_{s,n} c_n \geq 0 \perp r_n \geq 0 \quad \forall n \in N$$

Assuming that firms produce at capacity, we have:

$$B_n - \sum_{s=1}^S q_{s,n} c_n = 0 \quad \forall n \quad (6.P9)$$

We define T_n as the number of markets that firm n is not producing for and have:

$$a_s - q_{s,n} - \sum_{i=1}^N q_{s,i} - c_n = a_t - q_{t,n} - \sum_{i=1}^N q_{t,i} - c_n = r_n \quad s, t = \{T_n + 1, \dots, S\}, n \in N \quad (6.P10)$$

Using (6.P10), we can solve for any $q_{t,n}$:

$$q_{t,n} = a_t - \sum_{i=1}^N q_{t,i} - a_s + q_{s,n} + \sum_{i=1}^N q_{s,i} \quad s, t = \{T_n + 1, \dots, S\}, n \in N \quad (6.P11)$$

Substituting (6.P11) into (6.P9), we get:

$$B_n - \sum_{t=T_n+1}^S \left(a_t - a_s + q_{s,n} + \sum_{i=1}^N q_{s,i} - \sum_{i=1}^N q_{t,i} \right) c_n = 0 \quad s = \{T_n + 1, \dots, S\}, n \in N$$

and by expanding the terms:

$$\frac{B_n}{c_n} + (S - T_n) \cdot a_s - \sum_{T_n+1}^S (a_t) - (S - T_n) \cdot q_{s,n} - (S - T_n) \sum_{i=1}^N q_{s,i} + \sum_{T_n+1}^S \left(\sum_{i=1}^N q_{t,i} \right) = 0$$

$$s = \{T_n + 1, \dots, S\}, n \in N$$

which leads to:

$$a_s - \frac{1}{S - T_n} \sum_{t=T_n+1}^S (a_s) = \left(q_{s,n} - \frac{1}{S - T_n} \frac{B_n}{c_n} \right) + \left(\sum_{i=1}^N q_{s,i} - \frac{1}{S - T_n} \sum_{t=T_n+1}^S \sum_{i=1}^N q_{t,i} \right)$$

and by (6.P9):

$$\begin{aligned} & a_s - \frac{1}{S - T_n} \sum_{t=T_n+1}^S (a_s) \\ &= \left(q_{s,n} - \frac{1}{S - T_n} \frac{B_n}{c_n} \right) + \left(\sum_{i=1}^N q_{s,i} - \frac{1}{S - T_n} \sum_{i=1}^N \frac{B_i}{c_i} + \sum_{t=1}^{T_n} \sum_{i=n+1}^N q_{t,i} \right) \end{aligned} \quad s = \{T_n + 1, \dots, S\}, n \in N \quad (6.P12)$$

Apart from the trivial case, $q_{3,1}$, we prove the Conjecture only for $q_{2,2}$. The other cases can be shown using similar logic and the full proof can be obtained upon request from the authors.

If $q_{1,1} = q_{2,1} = 0$, then (6.P8) implies $q_{3,1} = B_1$.

Substituting the proposed solutions (6.P7) and (6.P8) into (6.P12) for $n = 2$ and $s = 2$, we have:

$$\begin{aligned}
a_2 - \frac{1}{3-1} \sum_{t=2}^3 (a_t) &= \left(q_{2,2} - \frac{1}{S-1} \frac{B_2}{c_2} \right) + \left(\sum_{i=1}^N q_{2,i} - \frac{1}{S-1} \sum_{i=1}^N \frac{B_i}{c_i} + \frac{1}{S-1} q_{1,3} \right) \\
a_2 - \frac{1}{3-1} \sum_{t=2}^3 (a_t) &= \left(\frac{B_2}{2c_2} + \frac{B_1}{6c_1} + \frac{a_2 - \frac{1}{2}(a_2 + a_3)}{3} - \frac{1}{3-1} \frac{B_2}{c_2} \right) \\
&+ \left(0 + \frac{B_2}{2c_2} + \frac{B_1}{6c_1} + \frac{a_2 - \frac{1}{2}(a_2 + a_3)}{3} + \frac{B_3}{3c_3} + \frac{B_1}{12c_1} - \frac{B_2}{12c_2} - \frac{a_1 - \frac{1}{3}(a_1 + a_2 + a_3)}{4} \right. \\
&+ \left. \frac{a_2 - \frac{1}{2}(a_2 + a_3)}{3} - \frac{1}{3-1} \sum_{i=1}^N \frac{B_i}{c_i} + \frac{1}{3-1} \left(\frac{B_3}{3c_3} + \frac{B_1}{6c_1} + \frac{B_2}{6c_2} + \frac{a_1 - \frac{1}{3}(a_1 + a_2 + a_3)}{2} \right) \right) \\
a_2 - \frac{1}{3-1} \sum_{t=2}^3 (a_t) &= \frac{B_1}{2c_1} + \frac{B_2}{2c_2} + \frac{B_3}{2c_3} - \frac{1}{2} \sum_{i=1}^N \frac{B_i}{c_i} + a_2 - \frac{1}{2}(a_2 + a_3)
\end{aligned}$$

which always holds true. □

7. CONCLUSIONS AND FUTURE DIRECTIONS

In this thesis we explore analytical models of the project portfolio management problem under competition. We demonstrate that competition can significantly affect firms' resource allocation decisions, sometimes in a counter-intuitive manner.

In Chapter 3, we develop a CPPM model in which two firms make binary investment decisions to either invest into a mature market or into an emerging market. We demonstrate that even in this simple model, there are many possible investment dynamics between the two firms that affect the optimal timing of the decision, the long term considerations and the communication strategy. If firms have asymmetric budgets, the number of possible interactions increases even further and we show numerically the counter-intuitive result that that a firm might be better off when the competitor's budget increases.

Chapter 4 adds further complexity to the initial model. Returns from the emerging market are assumed to be uncertain and firms decide whether to invest early, thereby subjecting themselves to that uncertainty, or to delay investment until the uncertainty is resolved. Furthermore, firms are restricted to invest in one market only, but can decide how much to invest into their target market. We are able to prove that the existence of two investment projects in the portfolio can lead firms to strategically delay investment, even if there is limited uncertainty in the market. Contrary to the single project setting, which is well explored in the real option literature, our setting provides firms with the flexibility to wait without conceding the leadership position to its competitor. However, we also prove that if diffusion effects are high, firms may need to make decisions in a Prisoner's Dilemma type situation which can lead

them to invest too early. Finally, we develop a straight-forward ration that can allow managers to make decision in a timely and effective manner.

In Chapter 5, we proceed to continuous investment decisions where firms decide how to allocate their limited budget over the available projects. We prove that while a monopoly optimally only considers the marginal returns of the markets, competition drives firms to account for both marginal and average returns. Therefore, firms invest more heavily into mature markets which have a lower marginal productivity than emerging markets. This overinvestment occurs even when competing with a firm that is endowed with a small budget. We find it is critical to assess the competing firm's budget, if this firm is small. However, if this firm is large, then the actual size of the competitor's budget does not have a significant impact on a firm's allocation decision. As firms increase their own budget, however, they invest more heavily into the emerging market. For the special case where firms have identical budgets, we show that further increases in competition lead firms to invest even more heavily into mature markets.

Finally, Chapter 6 explores the most general setting where N firms compete in S markets. We prove a closed-form solution of the equilibrium production quantities and resulting profits for any given number of firms and markets. The solution considers the firms' individual budgets and production costs as well as differences in demand potentials between markets. Using this result, we are able to clearly demonstrate the effect of competition on the allocation decision and prove that competition between firms with identical budgets drives them to produce more for markets with larger than average demand potentials, thereby reducing industry-wide profits. If firms' budgets differ, large budget firms produce in an opposite manner to this general trend and focus on markets with low demand potential, thereby

benefitting smaller firms. Our results can help explain counter-intuitive numerical instances where, for example, an increase in firms' unit costs leads to an increase in profits for all firms.

While this thesis provides a number of insights into the CPPM problem, several aspects remain open for further research. For example, some markets may provide increasing returns, which could lead to different investment strategies. In addition, firms may not only differ with respect to the size of their budgets but their investment into a market may also influence market demands in different ways. For example, a firm such as Apple may increase market demand to a greater degree than another firm in an emerging technology market such as the tablet market. Furthermore, years of research in PPM have revealed a number of other factors that can be important to an investment decision: investment projects may be interrelated in terms of their costs or expected returns, projects may be targeting short-term vs. long-term objectives, and projects may differ with respect to how well they fit into the overarching goal of a firm. Deriving insights from a model that incorporates these additional complexities into a CPPM model would be highly useful to decision makers who are currently forced to make intuitive decisions due to the lack of appropriate analytical models.

APPENDIX

APPENDIX A: MAPLE CODE

CPPM: BINARY INVESTMENT DECISIONS – CODE FOR FIGURE 3.3

```
> Prison := 1/(2*ln(k)/b+1/x);
> Chicken := 1/(-ln(1/(2*k))/b+1/(2*x));
> Stag := 1/(-2*ln(2/k)/b+2/x);
> Hero := 1/(ln(k)/b+1/x);
> Mature := b/(2*ln(2));
> Emerging := 1*b/(2*ln(2));
> plot(eval([Prison, Chicken, Stag, Hero, Emerging, [Mature, x, x = 0 .. 8]], {b = 1, k = 4, l =
  10}), x = 0 .. 8, y = 0 .. 8, labeldirections = [horizontal, vertical], labels = ["P , investment
  into M", "P , investment into E"], axes = "framed", font = [times, 16], axesfont = [times, 16],
  thickness = [2, 2, 2, 2, 0, 0], legend = ["eq. (1)", "eq. (2)", "eq. (3)", "eq. (4)", "", ""], linestyle =
  [solid, dot, dash, dashdot, solid, solid], color = black)
```

CPPM WITH UNCERTAIN RETURNS: WHEN, WHERE AND HOW MUCH TO INVEST – CODE LEADING TO FIGURE 4.3

```
> MM := (u[M]-c)^2;
> ME := (u[M]-c)^2;
> EM := (v+u[E]-c)^2;
> EMS := s[E];
> EE := (2*v+u[E]-c)^2;
> EES := s[E];
> MMEML := -u[E]+c+(1/2)*sqrt(2)*u[M]-(1/2)*sqrt(2)*c;
> MMEMD := -v^2-2*v*u[E]+2*v*c-u[E]^2+2*u[E]*c-(5/9)*c^2+(4/9)*u[M]^2-
  (8/9)*u[M]*c;
> MMEMI := (1/3)*c-u[E]+(2/3)*u[M];
> MMEEL := (1/2)*u[M]-(1/2)*u[E];
> MEEML := u[M]-u[E];
```

```

> MEEEEL := -(1/2)*u[E]+(1/2)*c+(1/2)*sqrt(2)*u[M]-(1/2)*sqrt(2)*c;
> MEEEEI := -(1/2)*u[E]-(1/4)*c+(3/4)*u[M];
> MEEED := (5/4)*c^2+(9/4)*u[M]^2-(9/2)*u[M]*c-4*v^2-4*v*u[E]+4*v*c-
  u[E]^2+2*u[E]*c;
> EMEEEL := (1/2)*sqrt(2)*(-u[E]+c);
> EMEEI := u[E]-c;
> EMEEED := (7/5)*v^2-(2/5)*v*u[E]+(2/5)*v*c-u[E]^2+2*u[E]*c-c^2;
> MMEMF := -(3/4)*c^2+(1/4)*u[M]^2-(1/2)*u[M]*c-v^2-2*v*u[E]+2*v*c-
  u[E]^2+2*u[E]*c;
> MMEEF := (1/4)*u[M]^2-(1/2)*u[M]*c-v^2-v*u[E]+v*c-(1/4)*u[E]^2+(1/2)*u[E]*c;
> MEEMF := u[M]^2-2*u[M]*c-v^2-2*v*u[E]+2*v*c-u[E]^2+2*u[E]*c;
> MEEEF := (3/4)*c^2+u[M]^2-2*u[M]*c-v^2-v*u[E]+v*c-(1/4)*u[E]^2+(1/2)*u[E]*c;
> EMEEF := -(3/4)*u[E]+(3/4)*c;
> MMEEDF := MMEEF;
> MEEMDF := MEEMF;
> MMEEIL := MMEEL;
> MEEMIL := MEEML;
> EECCEF := (7/36)*(-2*v-u[E]+c)^2;
> EELEEF := (1/4)*(-2*v-u[E]+c)^2;
> EELEED := (1/8)*(-2*v-u[E]+c)^2;
> EECMM := (7/9)*v^2-(2/9)*v*u[E]+(2/9)*v*c-(5/9)*u[E]^2+(10/9)*u[E]*c-(5/9)*c^2;
> EMLEED := -(7/4)*v^2+(1/2)*v*u[E]-(1/2)*v*c+(5/4)*u[E]^2-(5/2)*u[E]*c+(5/4)*c^2;
> vu[M] := 1.35;
> vu[E] := 1;
> vc := .5;
> plot([eval(MEEMDF, {c = vc, u[E] = vu[E], u[M] = vu[M]}), [eval(MEEMIL, {u[E] =
  vu[E], u[M] = vu[M]}), v, v = 0 .. 50], eval(MEEEF, {c = vc, u[E] = vu[E], u[M] = vu[M]}),
  eval(MMEMD, {c = vc, u[E] = vu[E], u[M] = vu[M]}), eval(MEEED, {c = vc, u[E] = vu[E],
  u[M] = vu[M]}), [eval(MEEEEI, {c = vc, u[E] = vu[E], u[M] = vu[M]}), v, v = 0 .. 50],
  eval(EMLEED, {c = vc, u[E] = vu[E]}), eval(EELEED, {c = vc, u[E] = vu[E]}), v = 0 ..
  .75, s[E] = 0 .. 1.5, legend = ['MEEMDF', 'MEEMIL', 'MEEEF', 'MMEMD', 'MEEED',

```

'MEEEI', 'EMLEED', 'EELEED'], color = [yellow, gold, turquoise, red, blue, navy, green, plum], legendstyle = [font = [Times, Roman, 8]], axes = boxed);

CPPM: CONTINUOUS INVESTMENT DECISIONS - CODE FOR FIGURE 5.3

```

> with(plots);
> Eq5b := (r2*B1*B2+r1*alp*B1^2)*a*(r1*B1+r2*B2)^(alp-2)-((1-r2)*B1*B2+(1-
  r1)*bet*B1^2)*b*((1-r1)*B1+(1-r2)*B2)^(bet-2);
> Eq5 := eval(Eq5b, {B1 = B2, B2 = B1, r1 = r2, r2 = r1});
> Eq5M := alp*a*(r3*B3)^(alp-1)-bet*b*((1-r3)*B3)^(bet-1);
> xlistbet := Array(1 .. 50);
> ylistbet := Array(1 .. 50);
> ylistr1 := Array(1 .. 50);
> ylistr2 := Array(1 .. 50);
> ylistr3 := Array(1 .. 50);
> ylistrD1 := Array(1 .. 50);
> ylistrD2 := Array(1 .. 50);
> ylistrM1 := Array(1 .. 50);
> ylistrM2 := Array(1 .. 50);
> ylistrD1x := Array(1 .. 50);
> ylistrD2x := Array(1 .. 50);
> ylistrM1x := Array(1 .. 50);
> ylistrM2x := Array(1 .. 50);
> i := 2;
> for B2 from .2 by .1 to 3 do
  i := i+1;
  xlistbet[i] := B2;
  ylistr3[i] := fsolve(eval(Eq5M, {B3 = 1+B2, a = .8, alp = .1, b = 1, bet = .6}) = 0, r3);
  ylistbet[i] := fsolve(eval({Eq5, Eq5b}, {B1 = 1, a = .8, alp = .1, b = 1, bet = .6}), {r1 =
    0 .. 1, r2 = 0 .. 1}); temp := ylistbet[i];
  ylistr1[i] := solve(temp[1]-r1, r1);
  ylistr2[i] := solve(temp[2]-r2, r2);
  ylistrM1x[i] := ylistr3[i]*(1+B2);

```

```

ylistrM2x[i] := (1-ylistr3[i])*(1+B2);
ylistrD1x[i] := ylistr1[i]+ylistr2[i]*B2;
ylistrD2x[i] := 1-ylistr1[i]+(1-ylistr2[i])*B2;
ylistrM1[i] := eval(a*x^alp, {a = .8, alp = .1, x = ylistrM1x[i]});
ylistrM2[i] := eval(b*x^bet, {b = 1, bet = .6, x = ylistrM2x[i]});
ylistrD1[i] := eval(a*x^alp, {a = .8, alp = .1, x = ylistrD1x[i]});
ylistrD2[i] := eval(b*x^bet, {b = 1, bet = .6, x = ylistrD2x[i]})
> end do;
> unassign('B2');
> ylistr1[1] := .1016302723;
> ylistr1[2] := .2674162666;
> ylistr2[1] := 1;
> ylistr2[2] := 1;
> ylistr3[1] := fsolve(eval(Eq5M, {B3 = 1+0, a = .8, alp = .1, b = 1, bet = .6}) = 0, r3);
> ylistr3[2] := fsolve(eval(Eq5M, {B3 = 1+.1, a = .8, alp = .1, b = 1, bet = .6}) = 0, r3);
> xlistbet[1] := 0;
> xlistbet[2] := .1;
> display(plot([seq([xlistbet[i], ylistr3[i]], i = 1 .. 31)], B[2] = .2 .. 3, r[j] = 0 .. 1, axes = boxed,
color = black, linestyle = ["dot"]), plot([seq([xlistbet[i], ylistr1[i]], i = 1 .. 31)], B[-n] = .2 .. 3,
r[j] = 0 .. .3, axes = boxed, color = black), plot([seq([xlistbet[i], ylistr2[i]], i = 1 .. 31)], axes =
boxed, color = black, linestyle = ["dash"]), font = [family, style, 13], labelfont = [family, style,
13], caption = "Figure 3: Optimal resource allocation into mature market; &alpha; = 0.1,
&beta; = 0.6, a = 0.8, b = 2, p = 0.5, B = 1, B = B +B ");

```

**CPPM: BUDGET CONSTRAINED MULTIMARKET COURNOT COMPETITION – CODE FOR
FIGURE 6.1 AND FIGURE 6.2**

```

> q2 := B/(3*N)+(a[s]-(a[M]+a[E]+a[G])*(1/3))/(N+1);
> plot(eval([(eval(q2, a[s] = a[M]))*N, (eval(q2, a[s] = a[E]))*N, (eval(q2, a[s] = a[G]))*N],
{B = 1, a[E] = 2, a[G] = 2.7, a[M] = 1.8}), N = 1 .. 10, Q = 0 .. 1, labels = ["number of firms
(N)", "equilibrium industry production (Q)"], labeldirections = [horizontal, vertical], color =

```

```

black, axes = boxed, linestyle = ["solid", "dash", "dot"], font = [times, 12], captionfont =
[times, 14]);
> profit2 := B*(a[M]+a[E]+a[G]-c-N*(B/(N*c)))/(N*c*S)+(S*(a[M]^2+a[E]^2+a[G]^2)-
(a[M]+a[E]+a[G])^2)/(S*(N+1)^2);
> plot([eval(profit2*N, {B = 1, S = 3, c = 1, a[E] = 2, a[G] = 2.7, a[M] = 1.8}), eval(profit2, {B
= 1, S = 3, c = 1, a[E] = 2, a[G] = 2.7, a[M] = 1.8})], N = 1 .. 10, Pi = 0 .. 1.7, axes = boxed,
labels = ["number of firms (N)", "equilibrium profit (&pi;)"], labeldirections = [horizontal,
vertical], color = black, linestyle = ["solid", "dash", "solid", "dash"], font = [times, 12],
captionfont = [times, 14]);

```

APPENDIX B: LIST OF NOTATIONS

ALL CHAPTERS

N	number of firms
n	subscript for firm n
S	number of markets
s	subscript for market s
M	mature market
E	emerging market
π_n	profit of firm n
B_n	budget of firm n

CPPM: BINARY INVESTMENT DECISIONS

$P_{n,s}$	project investment opportunity of firm n into market s
z_s	total investment by all firms into market s
a_s	maximum return potential of market s
b_s	twice the inflection point of market s
$v_{n,s}$	binary decision variable of whether firm n invests into market s ($v=1$) or not ($v=0$)
α	relative difference in return potentials (a_E/a_M)
β	relative difference in inflection points (b_E/b_M)
r	proportional share of investment made by Firm 1 vs. Firm 2 ($B_1/[B_1+B_2]$)

CPPM WITH UNCERTAIN RETURNS: WHEN, WHERE AND HOW MUCH TO INVEST

t_n	timing decision of firm n
s_n	market decision of firm n
I	invest early
D	delay investment
k_n	capacity expansion of firm n
c_n	unit cost of firm n
p_s	linear price function for market s
A_s	demand intercept for market s
μ_s	mean market demand of market s

σ	variance of emerging market demand
x	number of firms investing in emerging market (early or delayed)
v	diffusion effect of each firm in emerging market
Δ_s	mean demand minus unit cost of market s

CPPM: CONTINUOUS INVESTMENT DECISIONS

$f(x)$	market returns from mature market given total investment x by both firms
$g(x)$	market returns from emerging market given total investment x by both firms
a	market potential of mature market
b	market potential of emerging market
α	marginal productivity of mature market
β	marginal productivity of emerging market
p	probability that emerging market will reach its market potential
r_n	share of firm n 's budget that it invests into mature market

CPPM: BUDGET CONSTRAINED MULTIMARKET COURNOT COMPETITION

$q_{s,n}$	production quantity of firm n into market s
Q_s	total production quantity of all firms into market s
a_s	demand intercept of market s
c_n	unit production cost of firm n
C_n	total cost of production in unconstrained setting
r_n	shadow price of budget constraint
K_s	number of firms not producing for market s
$c_{s,n}$	unit production cost of firm n for market s
b_s	demand slope of market s

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