

Coasting of the Fueling Trolley System: Dynamic Modeling and Uncertainty Assessment

by

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Author's Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

Structural health monitoring in industrial systems is challenging due to complex component interactions, limited accessibility, and sparse sensing. In such settings, condition-monitoring methods must remain reliable under limited information and significant model-form uncertainty. Bayesian state-space modeling addresses this by representing system interactions stochastically and explicitly quantifying measurement uncertainty. In particular, Kalman filters provide real-time state updates from sensor data while accounting for both process and measurement noise, yielding a practical framework for uncertainty quantification and robust inference.

This thesis develops a physics-based dynamic model of a Fuelling Machine Transport trolley to predict the behavior for condition monitoring purposes. Using the real life coasting data, the unknown model parameters are estimated using the Kalman filter method. Using Kalman filtering with data-driven noise identification, we compare two force-estimation formulations: a simplified 1-DOF model that identifies a net resistive force and a 2-DOF model that separates two coupled force components. Numerical studies show that the 1-DOF formulation is more accurate and robust under the single-sensor setting, whereas the 2-DOF model exhibits multiple convergences and strong prior sensitivity due to structural unobservability.

The coasting data is collected in the form of Gray code signal via the encoder installed on the trolley. Experimental work decodes Gray-code encoder signals and aligns them with event logs to produce position/coasting trajectories that agree with plant records. The single degree of freedom approach is used to find the resistive force in the system. These findings support deploying the simplified estimator with routine consistency checks and route-level trend tracking. Practical recommendations include monitoring $Q_{F_{\text{total}}}$ as a health indicator, periodic re-estimation of Q and R , and optional secondary sensing to improve identifiability. Overall, the thesis demonstrates that carefully chosen models, paired with principled uncertainty quantification, can deliver robust, interpretable estimates for SHM under sparse sensing.

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Dedication

A Dios, que toda la gloria sea para EL, y para mi familia.

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List of Abbreviations

CBC center buffer coupling [16–18](#)

EKF Extended Kalman filter [4](#), [5](#), [8](#)

EM expectation maximization [6](#), [7](#), [68](#), [69](#), [71](#)

FEM finite element method [19](#)

FMT Fueling Machine Transport Trolley [2](#), [10–16](#), [24](#), [28](#), [41](#), [43–45](#), [47–49](#), [57](#), [59](#), [60](#), [71](#), [84](#), [87–91](#), [93](#), [105](#)

KF Kalman filter [2](#), [3](#), [47](#), [49](#), [56](#), [91](#), [96](#), [105](#), [108](#)

LRD Longitudinal Vehicle Railway Dynamics [13](#)

MLE maximum likelihood estimation [6](#), [7](#), [69](#), [105](#), [108](#)

NIS normalized innovation squared [7](#), [8](#), [98](#), [100](#), [102](#), [105](#)

OPG Ontario Power Generation [16](#), [25](#), [28](#), [33](#), [38](#), [44](#), [47](#), [83](#), [88](#), [89](#)

PT Power Track [2](#), [11](#), [12](#), [14–16](#), [19](#), [32](#), [33](#), [35](#), [36](#), [38](#), [42](#), [47](#), [57–59](#), [71](#), [72](#), [78](#)

SHM Structural health monitoring [1](#), [2](#), [4](#), [6](#), [9](#), [48](#), [49](#), [81](#)

SKF Swedish Ball Bearing Factory [24](#)

UKF Unscented Kalman Filter [4](#), [5](#), [8](#)

UQ uncertainty quantification [2](#), [8](#), [9](#)

Chapter 1

Introduction

In many critical industrial systems, ensuring safe and reliable operation depends on effective [Structural health monitoring \(SHM\)](#). However, a persistent challenge arises from the gap between theoretical models and real-world behavior. This gap is particularly pronounced in systems with proprietary or complex components, limited observability, and restricted access to design details, where modeling must rely on incomplete information and simplified assumptions. As a result, the very models used to support [SHM](#) may either fail to capture the true dynamics of the structure or introduce too many parameters to be reliably estimated from sparse measurements. This leads to what is known as model form uncertainty and, in turn, limits the reliability of monitoring and decision-making.

The scope of this thesis is to address this gap in the context of [SHM](#) by estimating the forces acting on a structure from limited measurements. Rather than assuming that a detailed high-fidelity model is available, we aim to obtain reliable force estimates for assessing the health state of the system, even when only simple approximate models are accessible or practically useful. Put differently, this work asks: how can we infer the unknown input forces to a system for [SHM](#) when the available model is structurally incomplete and only sparsely informed by data?

One solution lies in focusing not on the pursuit of ever more complex models, but on extracting value from simple models by properly characterizing their uncertainty. This strategy looks to change the analysis perspective from model accuracy to model utility, acknowledging that in many cases, less precise models can still offer significant insights if their limitations are well understood and accounted for. In this thesis, these simple models are not reduced order representations of an existing high-fidelity model; they are structural simplifications of the physical system, necessitated by the limited observability

in the overall system. Given this model discrepancy, model form uncertainty needs to be accounted and quantified so that it can be propagated across the force estimations.

1.1 Approach

To address the problem of force estimation for [SHM](#) under model form uncertainty, this thesis adopts different model approaches. These models are used not as surrogates for a known high-fidelity model, but as practical approximations of the true dynamics.

We combine these simple models with robust [uncertainty quantification \(UQ\)](#) techniques. The models are formulated in a stochastic state-space form, where uncertainties arising from unmodeled dynamics, parameter errors, and unknown inputs are represented through appropriate noise terms or augmented states. In this way, the models, while structurally basic, are still capable of capturing essential system behavior when paired with methods that quantify confidence in their predictions and explicitly account for model discrepancy.

Bayesian filtering techniques, particularly the [Kalman filter \(KF\)](#) and its variants, provide the core computational framework for this thesis. By treating unknowns—such as states, parameters, and input forces—as probabilistic quantities and updating their distributions as new data become available, these filters enable real-time estimation of both the system response and the forces acting on the structure.

This approach is evaluated on an in-service industrial system, the [Fueling Machine Transport Trolley \(FMT\)](#), where subsystems such as the [Power Track \(PT\)](#) mechanism are difficult to characterize in detail. The is composed by two main components; the trolley and the [PT](#). The trolley is a railway vehicle in charge of handling the exchange of nuclear fuel cell in the reactor, while the [PT](#) is a subsystem that is being drag by the trolley in parallel set of rails. The [PT](#) is in charge of the carrying the cables that supply energy to the trolley and the fuel handling system. By generating a simple model for only the accessible dynamics and estimating the unknown forces and parameters through Bayesian filtering, we obtain force estimates equipped with quantified uncertainty. These estimates can then be used for integration into [SHM](#) workflows, enabling monitoring and decision-making despite structural model incompleteness and limited measurements. While the ultimate goal of this research is to use force estimations and other parameters to extract distinctive features that can give important insights about the health state of the system, the extent of this thesis is to establish a robust estimation of these forces and parameters, that yields to reliable results. This will form the basis for future work involving feature extraction and decision making to asses the health of the system.

1.2 Literature Review

In structural engineering and applied mechanics, estimating internal forces and system parameters from limited external measurements is a central challenge [85, 44]. Real-world systems are complex, noisy, and often only partially observable. Consequently, probabilistic estimation methods are essential for interpreting behavior and identifying governing parameters from indirect observations.

A foundational perspective is the Bayesian filtering framework, which formulates state estimation in dynamic systems as recursive probabilistic inference. System states and measurements are modeled as random variables with specified (or learned) distributions; Bayes' theorem combines prior knowledge with incoming data. While broadly applicable, generic Bayesian filtering can be computationally demanding and may require approximations in real time [10].

The KF is a special case tailored to linear, Gaussian systems. Introduced by Kalman [66], it provides an optimal closed-form estimator via a predict-update cycle, balancing analytical tractability with strong performance. The KF operates in two recursive steps:

Prediction Step

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A} \hat{\mathbf{x}}_{k-1|k-1} \quad (1.1)$$

$$\mathbf{P}_{k|k-1} = \mathbf{A} \mathbf{P}_{k-1|k-1} \mathbf{A}^\top + \mathbf{Q} \quad (1.2)$$

Here, the state estimate $\hat{\mathbf{x}}_{k-1|k-1}$ and its covariance $\mathbf{P}_{k-1|k-1}$ are propagated forward in time using the state-transition matrix $\mathbf{A}$ associated with the chosen model. The process noise covariance $\mathbf{Q}$ accounts for uncertainties due to unmodeled dynamics, parameter errors, and other disturbances acting on the system.

Update Step

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}^\top (\mathbf{H} \mathbf{P}_{k|k-1} \mathbf{H}^\top + \mathbf{R})^{-1} \quad (1.3)$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{y}_k - \mathbf{H} \hat{\mathbf{x}}_{k|k-1}) \quad (1.4)$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}) \mathbf{P}_{k|k-1} \quad (1.5)$$

In the update step, the filter corrects its prediction using the new measurement $\mathbf{y}_k$. The Kalman gain $\mathbf{K}_k$ balances the uncertainty of the prediction, $\mathbf{P}_{k|k-1}$, against the measurement noise covariance $\mathbf{R}$, mapped into the state space by the observation matrix $\mathbf{H}$.

The corrected state estimate $\hat{\mathbf{x}}_{k|k}$ is obtained by adding a residual term proportional to the innovation, i.e. the difference between the actual measurement $\mathbf{y}_k$ and the predicted measurement $\mathbf{H}\hat{\mathbf{x}}_{k|k-1}$. Finally, the updated covariance $\mathbf{P}_{k|k}$ reflects the reduced uncertainty after incorporating the information contained in the measurement. For nonlinear dynamics, the [Extended Kalman filter \(EKF\)](#) linearizes about the current estimate using first-order expansions [53]. Despite its approximations, the [EKF](#) remains widely used in structural dynamics and parameter identification. The [Unscented Kalman Filter \(UKF\)](#) improves nonlinear handling by propagating deterministically chosen sigma points through the nonlinear model [65, 133].

1.2.1 Bayesian Filters for Input-State Estimation

Within [SHM](#), Bayesian filters are extensively used to infer unmeasurable quantities. For example, [UKF](#)-based methods estimate nonlinear structural states from acceleration data, enabling accurate recovery of position and velocity when direct measurements are sparse. In parameter estimation, Bayesian approaches have been used to identify time-invariant or slowly varying properties (e.g., stiffness degradation and damping evolution). Beck and Au [16] pioneered Bayesian damage identification in the early 2000’s. More recently, these filters have been extended not only for parameters but for unknown-input estimations as well [84, 106, 129]. While generally successful (see, e.g., [118]), such extensions can strain filter assumptions and introduce practical identifiability issues, wherein multiple parameter/input combinations explain the data similarly well [78].

In [SHM](#), loads are rarely measured directly, especially for large civil and mechanical structures. As a result, structural response data (accelerations, displacements, strains) must be exploited to infer both the unmeasured input forces and the underlying structural state [82]. This joint input–state (and often parameter) estimation problem underpins key SHM tasks, including damage detection, fatigue assessment, reliability evaluation, and real-time digital twins [129]. The modern unknown-input filtering line is rooted in the unbiased minimum-variance (UMV) framework of Kitanidis[70], who derived conditions under which linear state estimators can remain unbiased in the presence of unknown inputs while minimizing the estimation error covariance. Building on this work, Gillijns and De Moor [49] extended UMV theory to joint input–state estimation, deriving recursive filters for linear discrete-time systems (with direct feedthrough) that estimate both the state and unknown input with minimum variance under unbiasedness constraints.

In parallel, robust variants were proposed. Hsieh [58] introduced robust two-stage Kalman filters for systems with unknown inputs, where an initial stage estimates states

without bias from unknown disturbances and a second stage performs covariance correction for robustness. Yang also proposed an adaptive [EKF](#) for structural damage identification [\[138\]](#). These works form the theoretical backbone for later “Kalman-type” SHM algorithms. A complementary strategy is state augmentation, in which unknown forces are modeled as additional state variables and estimated with a standard Kalman filter. Pan et al. [\[90\]](#) further formalized joint input and state estimation with Kalman filtering, presenting a unified discrete-time framework and showing how various existing estimators (including UMV and augmented KFs) fit into a general class of joint estimators. While this was for linear systems, Yang proposed an alternative approach as an adaptive [EKF](#) for input estimations in nonlinear systems [\[139\]](#).

Lourens et al. [\[76\]](#) proposed such an augmented Kalman filter (AKF) formulation for force identification in structural dynamics, modeling unknown loads as stochastic processes and accounting explicitly for process noise to partially absorb modeling errors. Later Azam and Chatzi [\[12\]](#) proposed a dual Kalman filter in which one filter estimates states and another estimates unknown inputs, using only output-only acceleration measurements, followed by the experimental validation [\[13\]](#). However this method has the limitation that inputs, in order to be estimated, needed to be part of the measurement model. Representative examples include [UKF](#)-based unknown-input estimation [\[115\]](#), modified Kalman filtering with sensor-fault handling [\[31\]](#), and particle filtering for nonlinear damage diagnosis [\[29\]](#).

This introduced another important aspect to the investigation to be considered; partial observability and sparse sensing of the systems states. Lourens was one of the first to address this through joint input state estimation under limited measurement [\[75\]](#). Later to address this challenge, Dermatimanis and Chatzi [\[36\]](#) proposed a Bayesian filtering framework for input–state–parameter estimation from limited output information. By framing the problem in a successive Bayesian filtering and smoothing setting, they showed how parameters and inputs can be jointly estimated in a reliable framework. Later, Tatsis [\[120\]](#) introduced a general substructure-based framework for input–state estimation using limited output measurements, where only a substructure is modeled in detail and the remainder of the structure is treated implicitly. In a similar vein, other studies have expanded on the comparisons of different KF-based estimation approaches for substructured systems [\[121\]](#). This was further explored by Simpson, Chatzi and Chatzis [\[110\]](#), where a sub-structuring approach is presented to overcome model limitations for input–state estimation of offshore wind turbines where only displacement of the system was available. Instead of requiring an accurate global model of the entire turbine–soil–foundation–environment system, they partition the system into a substructure as well.

These approaches demonstrate that, in cases of sparse information, lower-complexity

models can actually be more effective. However, although these approaches alleviate the lack of observability, they introduce a new challenge: the model-form (or process) uncertainty must now be quantified. Consequently, the problem becomes one of joint input–state estimation using substructure models combined uncertainty quantification strategies. These studies demonstrate that the augmented Kalman filtering is a particularly effective strategy for this purpose, especially under sparse sensing conditions. As such, this approach is adopted in this thesis.

Overall, Bayesian filters offer real-time inference with quantified uncertainty, a combination well-matched to SHM needs. Their probabilistic nature supports both optimal estimation (under stated assumptions) and credibility assessment for inferred quantities, enabling risk-informed decision-making in infrastructure management.

1.2.2 Uncertainty Quantification with Kalman Filters

Uncertainty quantification is central to SHM because sensing is sparse, models are approximate, and the system’s conditions are variable. Bayesian state–space methods address both by casting estimation as probabilistic inference on latent states, parameters, and—when needed—unknown inputs [118, 53].

Filter performance hinges on credible process and measurement noise models. When Q and R are not known a priori, several data–driven routes are available, each best suited to different regimes of data availability, computational budget, and model fidelity.

In linear–Gaussian state–space models, the goal is to maximize the likelihood of the data with respect to the noise statistics Q, R . Direct maximum likelihood estimation (MLE) does this by optimizing the innovations likelihood with a numerical optimizer (e.g., quasi–Newton) under a chosen parameterization for Q, R (Cholesky, structured blocks, constraints) [63, 118]. It can converge rapidly and affords fine control over constraints, but the objective is nonconvex and thus sensitive to initialization; without structural constraints, Q or R may also absorb model mismatch and bias uncertainty.

Another route to estimate the noise statistics is the expectation maximization (EM) algorithm, which computes the same MLE solution but does so by alternating an E–step (a Kalman smoother to form expected sufficient statistics) with an M–step that yields closed-form (analytical) updates for Q and R in linear–Gaussian models [35, 107, 47]. Because the M–step is analytical, EM scales more favorably with the number of parameters than generic gradient-based MLE and guarantees a non-decreasing likelihood at each iteration. This makes the estimation typically less sensitive to initialization and missing data, at the cost of slower convergence and a full forward–backward pass per iteration; like direct MLE,

it still converges to a local optimum. In practice, [EM](#) is well suited to batch calibration with longer records or missing samples; its trade-offs are slower convergence and the cost of a full forward–backward pass per iteration, and—like direct [MLE](#)—it converges only to a local optimum. In many applications, [EM](#) is used as a robust first stage to reach a good basin of attraction. However, this approach relies on initial covariance assumptions that can constrain the estimation, causing the [EM](#) approach to be susceptible to initialization conditions [\[15\]](#).

When on–line or lightweight tuning is required, covariance–matching/consistency techniques (e.g., Mehra’s method) adjust Q and R so that the empirical statistics of the innovations agree with those predicted by the filter [\[81\]](#). In this context, the [normalized innovation squared \(NIS\)](#) is a scalar consistency metric formed by weighting each measurement residual by its innovation covariance; under correct noise levels and model, [NIS](#) values follow a chi–square law with degrees of freedom equal to the measurement dimension. A practical check is to average [NIS](#) over a sliding window and compare the result with the expected chi–square interval for that window length and a chosen confidence level (e.g., 95%) [\[14\]](#). When ground truth for the state is available (as in simulation), the normalized estimation error squared (NEES) provides an analogous test by weighting the state–estimation error with the posterior covariance; under a correct filter it is chi–square distributed with degrees of freedom equal to the state dimension. These diagnostics are computationally simple and well suited to in–service retuning to keep [NIS](#)/NEES within expected bounds as operating conditions drift. Their heuristic nature, however, means they can be biased on short windows or under model mismatch, so residual whiteness/independence checks and careful window selection are recommended [\[74\]](#).

If prior engineering knowledge exists—about magnitudes, relative scales, or structure—hierarchical Bayesian formulations provide regularization and uncertainty quantification by placing priors on Q and R (e.g., inverse–Wishart families) and seeking MAP estimates [\[118\]](#). This is advantageous in small–sample settings and when enforcing physically plausible structure (e.g., diagonal or block–diagonal force subblocks) is important. The trade–off is sensitivity to prior/hyperparameter choices, which should be justified or calibrated.

For batch calibration with adequate data, [EM](#) offers a robust starting point; a constrained [MLE](#) pass can then “polish” the solution [\[15\]](#). With limited data or strong domain knowledge, MAP estimation with structured Q/R helps avoid overfitting [\[9\]](#). For in–service adaptation, covariance matching combined with innovation diagnostics ([NIS](#)/NEES and whiteness tests) provides rapid drift correction. Across all cases, one should encode physics in the Q structure (e.g., random–walk forces, small process noise on kinematics) and verify consistency to guard against overconfidence under model mismatch.

Regardless of how Q, R are obtained, innovation-based tests are essential for **UQ**: the **NIS** and normalized estimation error squared (NEES) provide χ^2 -based consistency checks; whiteness and independence can be probed via autocorrelation or Ljung–Box tests [14, 74]. In practice, small posterior covariance does not guarantee accuracy if the model is misspecified or the problem is weakly identifiable—diagnostics should therefore accompany covariance-based confidence.

1.2.3 Partial observability

When loads/excitations are unmeasured, they can be modeled as additional components and inferred jointly with the states via augmented-state **EKF/UKF**, dual (state–parameter) filters, or constrained formulations [118, 115, 76]. However, partial observability often induces identifiability issues, in that multiple input/parameter combinations can produce near indistinguishable outputs over finite windows, especially with a single sensor or weak excitation.

Classical unknown–input observer results provide structural conditions (e.g., rank tests) for the existence of unbiased estimators and highlight the role of invariant zeros and decoupling subspaces [70, 48, 57, 28]. In the linear case, these structural notions are often formalized through the observability matrix, which is built by stacking the output mappings over successive time steps. Full rank of this matrix ensures that distinct initial states (and, in extended formulations, distinct combinations of states and unknown inputs or parameters) produce distinguishable output trajectories in the absence of noise [55]. When stochastic measurement noise is modeled explicitly, the same sensitivity structure appears in the Fisher information matrix (FIM). The FIM is assembled from the sensitivities of the outputs with respect to the unknown states, inputs, or parameters, weighted by the noise statistics [130]. In more general nonlinear or input–augmented settings, the FIM can provide quantitative measures of how well different directions in the state and parameter space are informed by the data. Near–singularity of the FIM signals weakly constrained or practically makes unidentifiable the true combinations of loads and parameters [95].

From a system–identification viewpoint, local/global identifiability further depends on persistence of excitation and experiment design [132]. Building on this concept, recent work has extended observability analysis to nonlinear systems with unmeasured inputs, providing algorithmic tests for state/input/parameter observability and clarifying how sensor layout, input parametrization, and system nonlinearities interact [78, 106]. These tools allow a priori assessment of whether a given configuration renders the joint input–state (and parameter) estimation problem well posed, and they naturally complement sensor–placement

and experiment–design criteria aimed at improving information content [91]. In practice, regularization through the process–noise covariance structure Q (e.g., random–walk priors on slowly varying forces), informative priors (hierarchical Bayes), additional sensing, or targeted excitation design (to increase sensitivity/conditioning) can mitigate ambiguity and improve effective observability.

1.3 Research Interest

1.3.1 Optimal Estimation Under Sparse Sensing

Much of the literature in SHM focuses on refining models to closely replicate real-world systems. While this can be effective in well-understood domains, it becomes problematic under significant model-form uncertainty and sparse sensing conditions. We argue that this fidelity-first focus can be misplaced.

Instead, we ask: between a simplified and a detailed model—neither of which captures the full complexity of the system—which yields more reliable and insightful estimates for the health monitoring of a fueling-machine transport vehicle under a single measurement? Our position is that simple models combined with minimal measurements, when paired with appropriate UQ methods, can deliver meaningful insights without the overhead and fragility of high-fidelity models. They are easier to implement, validate, and adapt, making them particularly attractive in industrial contexts.

Within this agenda, this research goes beyond simply deploying an augmented Kalman filter (AKF) or related estimators. For each data set, I explicitly estimate and tune the uncertainty/noise levels (and more generally, the filter hyperparameters) that make the estimator perform optimally under limited observability. While many authors acknowledge the role of multiple-estimation or trial-and-error tuning, they rarely analyze in depth how noise quantification governs estimator performance under sparse uncertainty and lack of model fidelity.

By framing input and state estimation as a combined problem of model simplification and principled uncertainty quantification, my work shows how uncertainty measures can be used to assess the reliability and behavior of the forces we aim to estimate. This is crucial for decision-making (e.g., maintenance and safety margins), model validation (e.g., detecting model-form bias), and the design of future monitoring systems (e.g., where to place additional sensors or how to design excitation) in sparse-sensing environments.

1.3.2 Methodology

To explore this research question, we take the case of study of an industrial [FMT](#) used in nuclear facilities. This system is an ideal case study in model simplification and uncertainty quantification due to its partially inaccessible and proprietary components, particularly in its power delivery subsystem.

We compare two modeling strategies:

1. A simplified model that treats all the unknown forces as one overall force of interest.
2. A more comprehensive model that attempts to capture the two overall forces of the two main system of the [FMT](#)

These strategies are first evaluated in a controlled numerical simulation designed to replicate key characteristics of the [FMT](#). We then extend the study to an experimental setting, applying the most reliable approach obtained from the numerical study.

1.4 Organization of the Thesis

This thesis is organized as follows.

Chapter 1 introduces the problem context, motivation, and research objectives. It outlines the modeling and uncertainty challenges in industrial systems, reviews relevant literature, and summarizes the proposed approach. **Chapter 2** describes the [FMT](#) system. It identifies the main friction sources and presents the alternative modeling approaches considered. A baseline model is assembled and used to establish the nominal (reference) coasting behavior of the [FMT](#). **Chapter 3** reports a numerical study in which the proposed estimation framework is tested on synthetic coasting data generated by the nominal model. Results are analyzed to assess accuracy, robustness, and identifiability, leading to the selection of the most reliable modeling approach. **Chapter 4** introduces the experimental dataset, details the decoding/preprocessing steps, and applies the selected approach to the measured coasting runs. The chapter presents force estimates with quantified uncertainty and compares them against engineering expectations. **Chapter 5** concludes the thesis by summarizing key findings and contributions. It also provides recommendations for deployment and outlines directions for future work.

Chapter 2

Fueling Machine Transport Trolley System

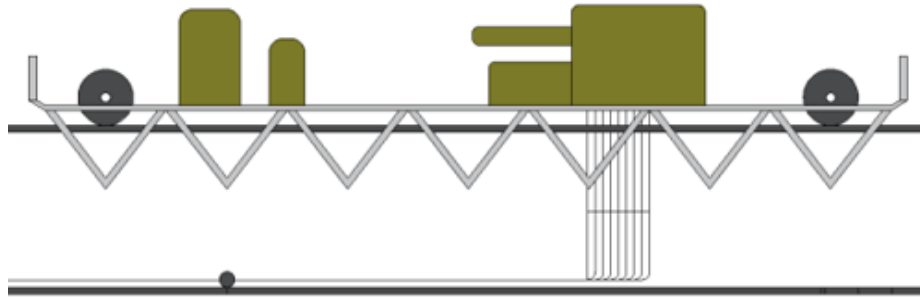


Figure 2.1: FMT sketch.

A FMT system plays a critical role within the infrastructure of a nuclear power station, facilitating the safe and efficient transportation of fuel bundles between the operating units and the fuel transfer mechanisms in the east and west fueling facility auxiliary area. The [FMT](#) is responsible for the transfer of irradiated fuel (IF) bundles from the reactor core to the unirradiated fuel bay.

The [FMT](#) consists of two main structures: the fueling machine trolley and the [PT](#). The trolley is a railway vehicle that carries the fueling machine and other fuel equipment through the power station. The [PT](#), in contrast, is a generally inaccessible power catenary structure situated below the trolley track and carries the cables providing electrical power and control signals to the trolley transfer and fueling machine head systems.. The two

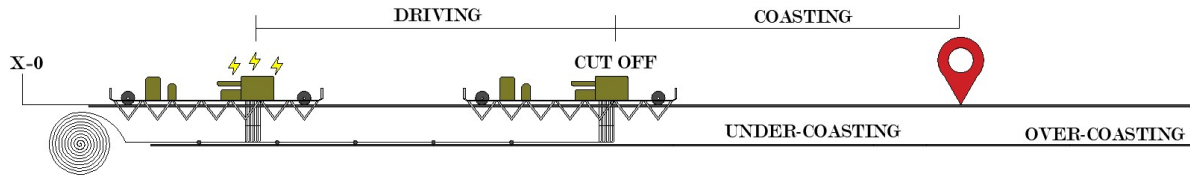


Figure 2.2: FMT coasting representation

systems are coupled by a steel riser frame joining the main interface of the [PT](#) to the first car of the trolley. The [PT](#) is therefore dragged by the trolley along a parallel set of rails located beneath it, allowing it to follow the trolley to all desired destinations, while remaining physically connected to electrical power and control services provided by the station..

The operation of the trolley system is described as follows. The trolley is moved using two electrical motor drives, known as the coarse drive and the fine drive. The coarse drive is used for a fast paced movement with a maximum velocity of 1 m/s. The fine drive is used for a slow pace movement for a more precisely controlled motion of the trolley. In the Fueling Facility Auxiliary Area (FFAA), the fueling machine is picked up from the trolley and connected to the New Fuel Transfer Mechanism (NFTM). The NFTM works cooperatively with the FM head to transfer 16 new fuel bundles from the maintenance storage facility into the FM head magazine for transit to an operating unit. The trolley is moved towards a reactor which requires the new fuel bundles using the coarse drive. In the driving phase, the maximum speed of the trolley is limited to 1 m/s. The trolley destination to a reactor is a precise location, called a set point, at which a carriage system supported by the Reactor Area Bridge (RAB) picks up the fueling machine and attaches it to a fuel channel for fueling purposes.

To stop the trolley at a set point, it is coasted to the set point as close as possible. The coasting is adopted over braking to avoid heavy wear and tear on the brakes from stopping a large moving mass. Coasting is initiated by cutting power to the coarse drive at a fixed point before the reactor set point. Once the trolley stops in the proximity of the reactor set point, the fine drive is used to locate the trolley precisely at the set point.

The [FMT](#) system implements a passive safety feature known as "Trolley Coasting". As the FM trolley transport system approaches its destination, the control system removes power from the coarse drive system, allowing the trolley to passively coast to a stop before its destination. Once the trolley is confirmed to have come to a stop, the fine drive system picks up and drives the trolley to the precise location for head pickup, at very low speeds

using the fine drive system. This passive safety feature (trolley coasting) incorporates a key nuclear safety design feature centered around preventing trolley collisions with station systems, and encroaching into accessed areas of the FM duct while transporting IF onboard. The precise locations where coarse drive power is cut, initiating costing, are referred to as trolley coast setpoints.

The under-coasting is indicative of an increase in the resistance to trolley motion caused by degradation in the condition of power track and the bearings and gear boxes of the trolley system. The trolley over-coasting is an opposite effect due to reduction in friction and drag imposed by the power track due to its maintenance and replacements of worn parts.

The purpose of this chapter is to develop a dynamical model of the [FMT](#) given the available information on the system provided by Ontario Power Generation.

2.1 Longitudinal Railway Vehicle dynamics

[Longitudinal Vehicle Railway Dynamics \(LRD\)](#) describes the movement of a train along its direction of travel [\[32\]](#). By analyzing system motion over the trolley runs, we can then detect variations in these parameters, which will allow us to determine the likelihood of damage or degradation in various components.

The key componets of [LRD](#) models include:

- Structural Assembly: Railway vehicle characteristics, such as the number and type of locomotives and cars, and their distribution along the train's length.
- Resistive Forces: Forces acting on the train during its movement which release energy from the system, such as friction forces on the internal components, rolling resistance, and air resistance.
- Propulsion and Braking Systems: Motion control components which develop acceleration and velocity control, including the motor and braking systems.

The dynamic behaviour of the [FMT](#) can be described by a system of differential equations. As is typical in the discipline, lateral and vertical movement of the train have been assumed negligible in the development of this model [\[33, 45\]](#).

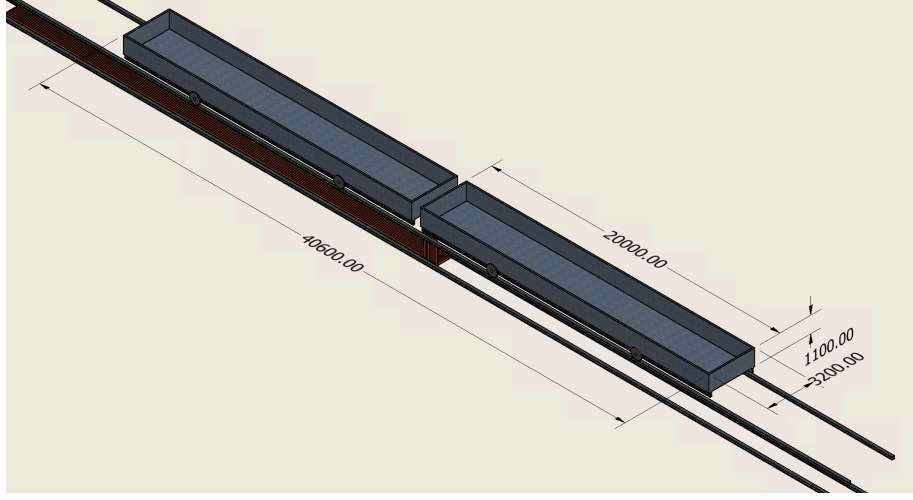


Figure 2.3: Trolley general dimensions (mm)

2.1.1 Trolley Configuration

The trolley, shown in Fig. 2.3, is composed of two cars connected by a coupling link. The dynamic model of the system is presented in Fig. 2.18, where the trolleys are represented as rigid masses connected by a (relatively massless) spring-damper system representative of the coupling link. The leading car of the trolley is also coupled to the power track by a metal frame, composed of STL ASTM A-36 structural steel, that is likewise represented as a spring-damper system. While the resistive forces acting on the trolley and PT will be defined in greater detail in Sections 2.3 and 2.4, they are herein modeled collectively by the lumped terms F_{BC} and F_{FC} for the trolley and F_{PT} for the PT.

Given the structure of the FMT system, the equations of motion for the system can be written in matrix form as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \quad (2.1)$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_{PT} \end{bmatrix}, \quad \dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_{PT} \end{bmatrix}, \quad \ddot{\mathbf{x}} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_{PT} \end{bmatrix}$$

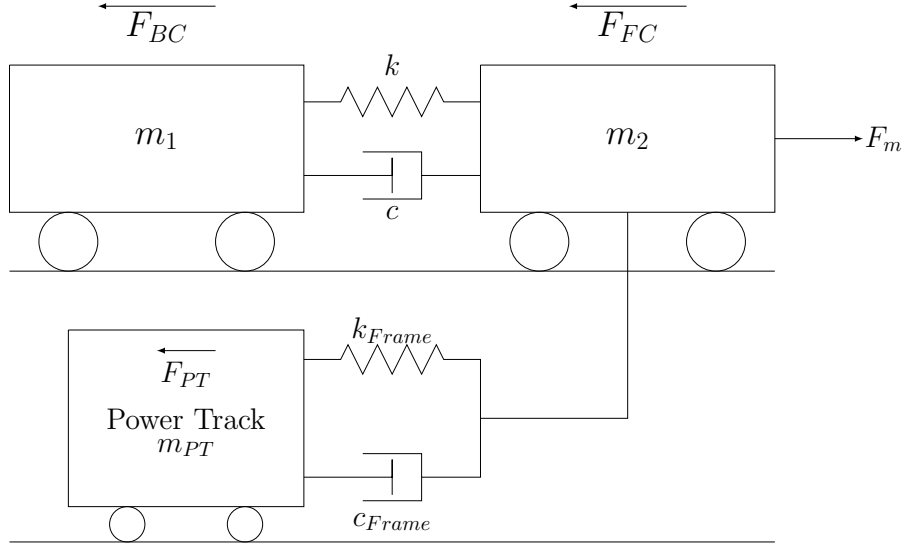


Figure 2.4: Dynamic model of the FMT system. The variables k and c respectively represent the stiffness and damping associated with the trolley coupling pin, while k_{frame} and c_{frame} are the coefficients associated to the frame coupling the PT with the trolley.

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_{PT} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c & -c & 0 \\ -c & c + c_{frame} & -c_{frame} \\ 0 & -c_{frame} & c_{frame} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k & -k & 0 \\ -k & k + k_{frame} & -k_{frame} \\ 0 & -k_{frame} & k_{frame} \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} -F_{BC} \\ F_m - F_{FC} \\ -F_{PT} \end{bmatrix}$$

Where k and c represent the stiffness and damping of the trolley coupling, while k_{frame} and c_{frame} describe the corresponding coefficients between the trolley and the PT. The force vector includes the traction force F_m and the resistive forces F_{BC} , F_{FC} , and F_{PT} .

The remainder of this chapter will be devoted to the delineation of the FMT model components, including the structural assembly, the trolley resistive forces, the power track resistive forces, and the forces associated with propulsion and braking.

2.2 Structural Assembly

This section details the derivation of forces associated with the inter-trolley dynamics, specifically the mass of the three [FMT](#) components and the stiffness and damping inherent to the trolley coupling and [PT](#) riser frame.

2.2.1 FMT System Mass

Each trolley car is fabricated from a steel bulkhead structure. As illustrated in Fig. 2.3, the two trolley cars effectively function as large steel enclosures that support and transport the nuclear fuel handling system. According to data provided by [Ontario Power Generation \(OPG\)](#) [4], each trolley car has an approximate mass of 72 metric tons. This total accounts for both the structural mass and the integrated subsystems within the car.

The [PT](#) system comprises 109 intermediate drums, 2 end drums, a flexible cable chain, and a supporting carriage frame onto which all drums are mounted, as shown in Fig. 2.5. The [PT](#) movement is mechanically driven by the motion of the trolley. As the trolley advances, it pulls the movable end of the cable chain, which in turn rotates the end drums. This rotation propels the entire carriage frame along a fixed set of rails that run parallel to the trolley's travel path. In this way, the [PT](#) assembly synchronously follows the trolley's bidirectional traversal.

The individual mass contributions of the primary [PT](#) components are detailed in Table 2.1, based on specifications provided by [OPG](#)'s engineering documentation [1].

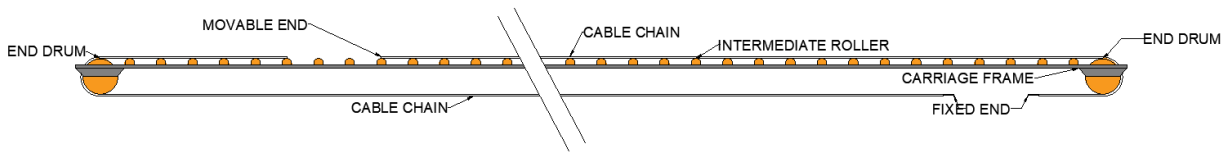


Figure 2.5: Power Track parts

2.2.2 Trolley Coupling

The connection between the trolley cars is modeled as a [center buffer coupling \(CBC\)](#) system, one of the most common used coupling devices. [CBCs](#) are widely employed in freight and heavy-haul railway applications due to their robust mechanical performance,

Table 2.1: Mass Summary of PT Components (3 Significant Figures)

Component	Unit Weight (kg)	Quantity	Total Mass (kg)
Cables	79.0 per m	250 m	1.97×10^4
Cable chain links	5.92 per link	1.81×10^3 links	1.07×10^4
Intermediate drums	87.0 per drum	72 drums	6.26×10^3
End drums	265 per drum	2 drums	5.30×10^2
Carriage frame	7.16×10^3 per frame	1 frame	7.16×10^3
Overall Mass	—	—	4.44×10^4

high load-bearing capacity, and compatibility with standard operating practices [54]. A typical **CBC** assembly consists of a coupler head with a knuckle mechanism, a coupler shank, a draft gear enclosed within the center sill of the carbody, and a vertically inserted coupling pin. The coupling pin locks the knuckles of adjacent coupler heads, providing a secure mechanical connection that resists longitudinal separation while permitting limited rotational motion between vehicles. An sketch of the **CBC** system can be appreciated in Fig. 2.6.

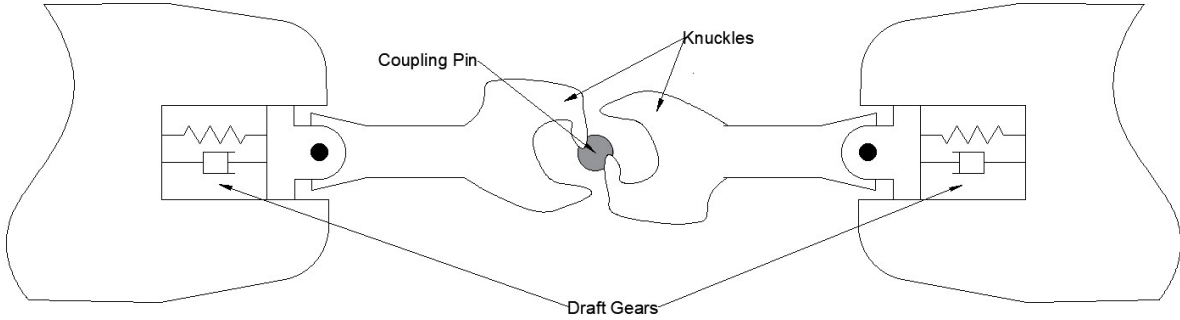


Figure 2.6: CBC Sketch

The draft gear is a critical element of the **CBC** system, responsible for absorbing and dissipating longitudinal forces that arise during traction, braking, and coupling operations. For modeling purposes, the draft gear is approximated as a linear spring-damper system, characterized by stiffness and damping coefficients. Due to the lack of detailed

manufacturer specifications for the specific coupler used in the trolley under analysis, these parameters are estimated based on values reported in the literature.

According to studies in railway vehicle dynamics [71, 116], longitudinal stiffness values for CBC systems typically range from 6 to 20.0 MN/m, with damping coefficients spanning from 10,000 to 100,000 Ns/m. Based on these ranges, this study adopts a nominal stiffness of 10 MN/m and a damping coefficient of 50,000 Ns/m for dynamic simulations. These values represent average characteristics of CBC systems and can modeled as a normally distributed random variable.

$$k \sim \mathcal{N}(\mu = 1.0 \times 10^7 \text{ N/m}, \sigma_k^2)$$

$$c \sim \mathcal{N}(\mu = 5.0 \times 10^4 \text{ Ns/m}, \sigma_c^2)$$

The standard deviations for the stiffness and damping priors were estimated based on the observed ranges in the literature describes in the studies of Knothe and Stichel [71] and Spiriyagin et al [116]. Following the probabilistic modeling approach outlined by Papoulis and Pillai [92], this 95% interval is interpreted as spanning approximately ± 2 standard deviations around the mean. Given a parameter x with literature bounds $[x_L, x_U]$, interpreted as an approximate 95% interval of a normal distribution $x \sim \mathcal{N}(\mu_x, \sigma_x^2)$, we estimate

$$x_U = \mu + 2\sigma, \quad x_L = \mu - 2\sigma$$

$$\mu_x = \frac{x_U + x_L}{2}, \quad \sigma_x \approx \frac{x_U - x_L}{4}.$$

$$\sigma_k = 3.50 \times 10^6 \text{ N/m}$$

$$\sigma_c = 2.25 \times 10^4 \text{ Ns/m}$$

Therefore:

$$k \sim \mathcal{N}(1.00 \times 10^7 \text{ N/m}, (3.50 \times 10^6 \text{ N/m})^2) \quad (2.2)$$

$$c \sim \mathcal{N}(5.00 \times 10^4 \text{ Ns/m}, (2.25 \times 10^4 \text{ Ns/m})^2) \quad (2.3)$$

2.2.3 Power Track Coupling Frame

To accurately represent the dynamic response of the coupling frame that links the trolley to the rail vehicle, a [finite element method \(FEM\)](#) approach was used to estimate its equivalent stiffness and damping properties. The frame geometry was created in SolidWorks CAD software and analyzed using its built-in finite element analysis (FEA) environment to evaluate the structural response, where modal and static simulations were conducted under boundary conditions approximating realistic constraints.

Geometric and Material Assumptions

The frame is constructed using structural steel members with a rectangular hollow section profile of $50.8 \times 50.8 \times 3.05$ mm [127]. The mechanical properties were based on standard values for structural steel as found in engineering design literature [98, 7]. The value ranges used in the analysis are:

Table 2.2: Mechanical Properties of Structural Steel

Property	Value
Young's Modulus (E)	170-220 GPa
Yield Strength	250-415 MPa
Tensile Strength	400-550 MPa
Density	7850 kg/m ³
Estimated Mass	336 kg

Resultant Dynamic Properties

The [FEM](#) simulation displayed in Fig. 2.7 was conducted by applying a longitudinal force of 1 kN at the top connection point of the coupling frame. The bottom region of the frame, where it connects to the [PT](#), was modeled as fully fixed to simulate anchorage to the main vehicle structure. The top portion, which interacts with the trolley, was constrained against lateral displacement to reflect the restricted transverse motion during forward vehicle movement. The entire structure was modeled as a welded steel assembly, assuming rigid connections between all members. To account for uncertainty in material properties, particularly variability in the elastic modulus and fabrication tolerances, the simulation was performed across using the minimum and maximum of plausible input values. Given these assumptions, the resulting range of equivalent longitudinal stiffness values was:

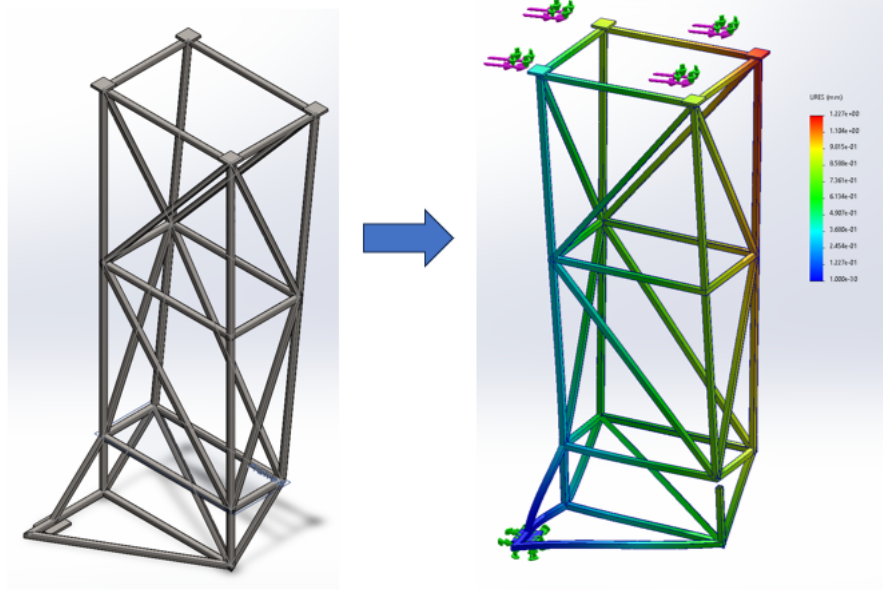


Figure 2.7: Power track - trolley coupling frame

$$k \in [7.12 \times 10^6, 9.18 \times 10^6] \text{ N/m}$$

Based on this range, the mean and standard deviation are estimated as:

$$\mu_k = 8.15 \times 10^6 \text{ N/m}, \quad \sigma_k = 5.15 \times 10^5 \text{ N/m}$$

To estimate the damping coefficient c , a structural damping ratio ζ of 2% was adopted, corresponding to viscous damping. This value is widely accepted as typical for welded steel structures subjected to low-amplitude vibrations and is frequently used in structural dynamics modeling [30]. The damping coefficient is calculated using viscous damping formulation:

$$c = 2\zeta\sqrt{km} \tag{2.4}$$

Applying this formula over the obtained stiffness range results in damping values from approximately 1950 to 2215 Ns/m, resulting in:

$$\mu_c = 2082 \text{ Ns/m}, \quad \sigma_c = 66.22 \text{ Ns/m}$$

Based on the above estimates, we define the following normal prior distributions for the dynamic parameters of the coupling frame:

$$k_{frame} \sim \mathcal{N}(8.15 \times 10^6, (5.15 \times 10^5)^2) \quad (2.5)$$

$$c_{frame} \sim \mathcal{N}(2082, (66.225)^2) \quad (2.6)$$

2.3 Trolley Resistance

2.3.1 Rolling Friction

The complexity in fully understanding rolling friction arises from its dependence on a variety of factors, which can be difficult to isolate in specific scenarios [119]. These factors include adhesion, deformation, elastic hysteresis, abrasion, and the presence of impurities [17]. In the context of railway vehicles, rolling resistance occurs as the train wheels interact with the rails. It is mainly caused by the deformation of both the wheel and rail materials under load, as well as by imperfections in their surfaces. Rolling resistance is influenced by factors such as wheel-rail interface conditions, track quality, and wheel geometry. All these factors contribute to form a combined effect known as the rolling friction coefficient [131].

The rolling friction force can be modeled as proportional to the normal force (N) and directed opposite to the motion of the object [67, 40], such that:

$$F_f = \mu_r N \quad (2.7)$$

For a railway vehicle with a steel wheel–steel rail contact, the coefficient of rolling friction (μ_r) is approximately 2×10^{-4} [61]. Therefore, the friction force per half trolley (weighing 72,000 kg) is calculated as:

$$F_f = \mu_r mg = 0.0002 \times 72000 \times 9.8 = 141.02 \text{ N} \quad (2.8)$$

Although a nominal value of $\mu_r = 2.0 \times 10^{-4}$ is widely cited for clean, dry steel wheel–rail interfaces [61], real-world operating environments often lead to deviations due to surface irregularities, wear, contamination (e.g., dust, rust, or lubricants), and temperature fluctuations. These factors contribute to variability in the rolling friction coefficient and must be accounted for in realistic modeling.

The literature suggests that, under typical but non-extreme service conditions, the coefficient of rolling friction for steel-on-steel contact can vary by approximately $\pm 25\%$ around the nominal value [41, 141]. Based on this observation, a 95% confidence interval of $\mu_r \in [1.5 \times 10^{-4}, 2.5 \times 10^{-4}]$ is selected. This range offers a conservative yet physically grounded estimate of uncertainty, excluding rare or abnormal cases (such as severe weathering or failure modes) while covering expected operational variability.

To formally incorporate this uncertainty into the model, the coefficient of rolling friction is treated as a normally distributed random variable. Thus, the distribution parameters are defined as:

$$\mu_{\mu_r} = 2.0 \times 10^{-4}, \quad \sigma_{\mu_r} = 2.5 \times 10^{-5} \quad (2.9)$$

This normal prior enables the inclusion of epistemic uncertainty in simulation studies and supports sensitivity analysis of the rolling friction’s influence on overall vehicle dynamics.

The prior distribution for μ_r is therefore modeled as:

$$\mu_r \sim \mathcal{N}(2.0 \times 10^{-4}, (2.5 \times 10^{-5})^2) \quad (2.10)$$

2.3.2 Drag Force

Drag is a mechanical force generated by the interaction and contact of a solid body with a fluid (liquid or gas). This phenomenon is essentially the cumulative result of the dynamic fluid acting on the particle’s surface, namely the shear stress and the pressure exerted by the fluid[126]. Considering a uniform flow passing through a defined object the drag force can be define in the following formula [73]:

$$F_d = \frac{\rho C A \dot{x}^2}{2} \quad (2.11)$$

Where ρ is the fluid density, C is the drag coefficient, and A is the cross-sectional area of the body moving through the fluid. In this case, we have assumed an air density of 1.2041 kg/m^3 given a 20 C air temperature [97], a drag coefficient of 6.5 representative of

a cargo railway vehicle [100], and a cross-sectional area of 3.5 m^2 [88]. These parameters are assumed known and constant due to their relatively small impact on the overall forcing contribution to the system dynamics. Therefore, for this model we assert that $F_d = 11.375 \text{ N}$.

2.3.3 Bearing Friction

Bearings friction refers to the resistance encountered when two surfaces, typically the rolling elements and the raceways, come into contact within a bearing assembly. Bearings are mechanical components that facilitate smooth and controlled motion between two or more moving parts. They are commonly used in various machinery, including automotive engines, industrial equipment, and rotating machinery.

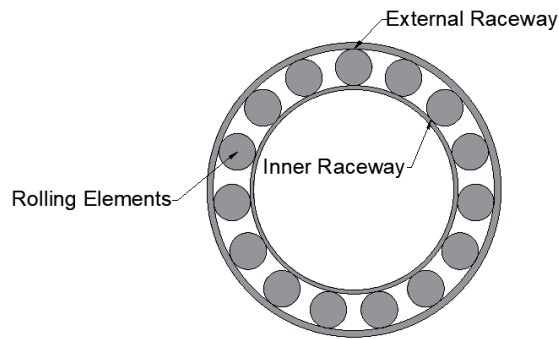


Figure 2.8: Sketch of bearings main components

There are several types of bearings, including ball bearings, roller bearings, and plain bearings, each with its own configuration and operating principles. However, the fundamental concept of bearing friction remains consistent across different types.

Bearings friction arises due to several factors:

- Surface Interaction: As the rolling elements (such as balls or rollers) move along the raceways (the tracks or grooves in which the rolling elements travel), friction occurs at the points of contact between the rolling elements and the raceways.

- Material Properties: The materials used to construct the bearings and the lubricants applied between the rolling elements and the raceways influence friction. Smooth, hard surfaces and high-quality lubricants can help reduce friction and minimize wear.
- Load and Speed: The magnitude of the load placed on the bearing and the speed at which it operates affect friction. Higher loads and speeds typically result in increased friction, which can lead to higher operating temperatures and accelerated wear.
- Lubrication: Proper lubrication is essential for reducing friction and minimizing wear in bearings. Lubricants create a thin film between the rolling elements and the raceways, reducing direct contact and frictional resistance [128].

Researchers have explored different methods for the calculation of mechanical losses due to bearing friction. A preliminary study on the topic was introduced by Palmgren [89], in which the author developed an empirical formula to analyze bearing friction. This method however, did not effectively consider the influence of lubrication factors, and these factors were then included in the work of Jing [64]. A comparison between the two methods shows that the [Swedish Ball Bearing Factory \(SKF\)](#) method has a better correlation with experimental data according [60]. As such, the [SKF](#) method [111] is used to characterize bearing friction in the remainder of this study.

[SKF](#) Frictional Moment Method

Many researchers have adopted the rolling bearing friction model proposed by [SKF](#) as a reference for estimating mechanical losses due to bearing friction [89, 60]. In this model, the total friction torque is decomposed into four main contributions: rolling friction of the rolling elements, sliding friction of the rolling elements, friction in the bearing seals, and friction due to lubricant drag on the rolling elements [111]. Accordingly, the total rolling bearing friction torque is expressed as:

$$M_{TOTAL} = M_{rr} + M_{sl} + M_{seal} + M_{drag} \quad (2.12)$$

Where M_{rr} is the rolling frictional moment, M_{sl} is the sliding frictional moment, M_{seal} is the frictional moment caused by seals in the bearing, and M_{drag} is the frictional moment due to drag by oil baths between the rolling elements. The [SKF](#) approach can only be used once a bearing type and its operational conditions are known. For the case of the [FMT](#), these conditions can be summarized as follows:

- The operating velocity of the trolley is approximately 1 m/s[4].
- The mass of each trolley car is 72,000 kg, per the information provided by OPG.
- The trolley moves in an relatively straight line, and as such no significant axial forces are present in the bearing
- The weight of the trolley is uniformly distributed among the wheels.
- Each trolley contains 4 sets of wheels with 2 wheels per set, where each wheel contains a pair of double roller bearings.

With these operational conditions we are able to determine the radial force to which the bearing is subjected:

$$F_R = \frac{m * g}{N_{bearing}} = 176.4 \text{ kN} \quad (2.13)$$

where $N_{bearing}$ is the number of bearings in the two-car trolley system, which in this case is 8. For the speed of the bearing, n , and the wheel has a diameter of 611 mm.

$$n = \frac{\dot{x}}{0.3055} * \frac{60}{2\pi} \quad (2.14)$$

where r is the wheel radius in meters and $\frac{60}{2\pi}$ represents the transformation to RPM. In this case, each wheel uses a pair of Timken HH221400 cup and cone roller thrust bearings. Specifications are shown in Fig. 2.9 and Table. 2.3. The bearing speed and radial force terms are then used to calculate M_{rr} and M_{sl} within the frictional torque equation.

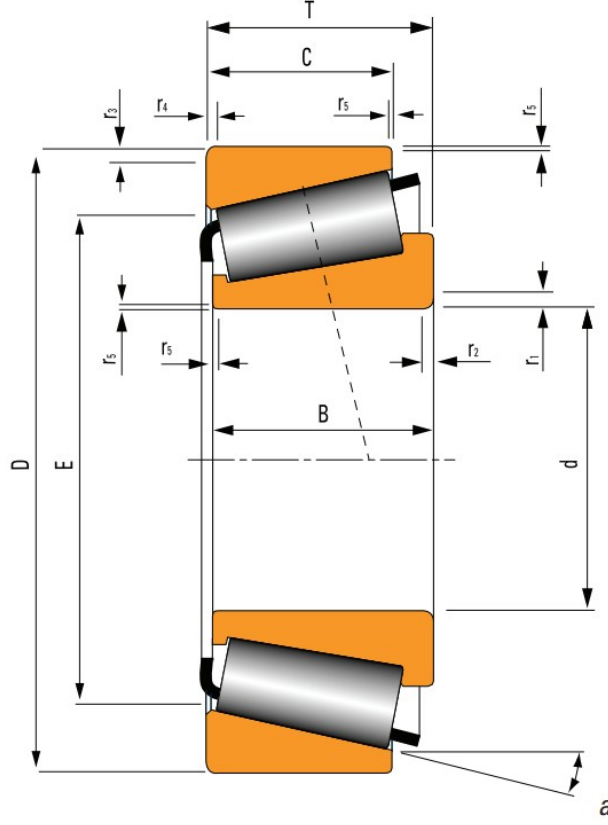


Figure 2.9: Bearing HH221440 sketch [111].

The development of the terms for the bearing frictional torque equation is summarized below. For further details on the assumptions and calculations for each frictional moment coefficient, please reference Appendix A.

- **Rolling Frictional Moment (M_{rr}):** This factor depends on the bearing load and rotational speed, and is given by the equation:

$$M_{rr} = \phi_{ish} \phi_{rs} G_{rr} (vn)^{0.6}$$

$$M_{rr} = 236.06 * 0.9998^n * n^{-0.68} \quad (2.15)$$

Nomenclature	Dimension	Specification
d	95.250 mm	Bore diameter
D	190.5 mm	Outside diameter
B	126.74 mm	Width
d1	116.5 mm	Shoulder diameter inner ring
D1	179.32 mm	Shoulder diameter outer ring
b1	11.91 mm	Width annular lubrication groove outer ring
K	8.5 mm	Diameter of lubrication hole
s	max 11.09 mm	Permissible axial displacement
r1,2	min 1.52 mm	Chamfer Dimension(open bearings)
r3,4	min 1.52 mm	Chamfer Dimension

Table 2.3: Bearing HH221440 list of specification [122]

where the coefficients in Eq. 2.15 are obtained from the calculation of the inlet shear reduction factor (ϕ_{ish}), kinematic replenishment/starvation reduction factor (ϕ_{rs}), rolling frictional variable (G_{rr}), the viscosity (v), and the rotational speed (n) using the bearing's geometric parameters established in Table 2.3.

- **Sliding Frictional Moment (M_{sl}):** This factor depends on the radial and axial load exerted on the bearing, the geometry of the bearing, and its rotational speed.

$$M_{sl} = G_{sl}\mu_{sl}$$

$$M_{sl} = (0.1197n^{-1.4} + (1 - 0.9977n^{-1.4})0.02)37804.725 \quad (2.16)$$

where the coefficients in Eq. 2.16 are obtained from the calculation of the sliding frictional variable (G_{sl}) and the sliding friction coefficient (μ_{sl}). These terms depend on the bearing geometry, the load applied on the bearing, and the rotational speed.

- **Frictional Moment Caused by Seals (M_{seal}):** The selected bearing does not contain a seal. As such, this factor is set to 0.

$$M_{seal} = 0$$

- **Drag Frictional Moment (M_{drag}):** The drag component considers drag force due to the oil bath surrounding the rolling elements during rotation. The selected bearing works with grease instead of oil, so no drag is considered.

$$M_{Drag} = 0$$

The frictional moment therefore ultimately depends on two main factors for the case of the **FMT** bearings, the radial force exerted on the bearing and the rotational speed of the bearing. To transform the frictional moment into a linear force, we establish the following relationship:

$$F_{B-trolley} = \frac{M_{TOTAL}}{r_{wheel}} \quad (2.17)$$

Where r_{wheel} is the radius of the trolley, 305 mm, according to the **OPG** report [88].

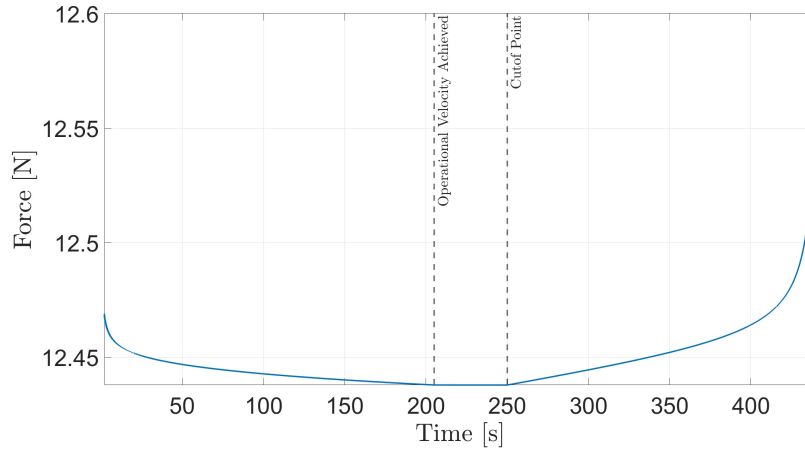


Figure 2.10: Bearing Force response for each trolley bearing

The bearing frictional force ultimately depends on two main factors, the radial force exerted on the bearing and the rotational speed of the bearing. Figure 2.10 shows that the maximum magnitudes are obtained when the trolley has its lowest speeds. However, it should be noted that the variation is relatively minor, with values ranging only between 12.44 N and 12.55 N. Considering the relative magnitude of the force and its comparatively small impact on the longitudinal trolley dynamics, this force is assumed constant in the dynamical model, with an approximate force chosen at 12.5 N per each bearing.

2.3.4 Gearbox Friction

Gearboxes are mechanical devices used to change the speed and torque of rotational motion in machinery by employing gears of different sizes and configurations. The gearbox is

composed of two sets of gear systems. The input motion from the electric motor drives the set of bevel gears shown on the left side of the gearbox as seen on Fig.2.11, which then transmit that motion first to a low speed pinion and subsequently to a low speed gear to develop output movement of the system in the trolley wheels.

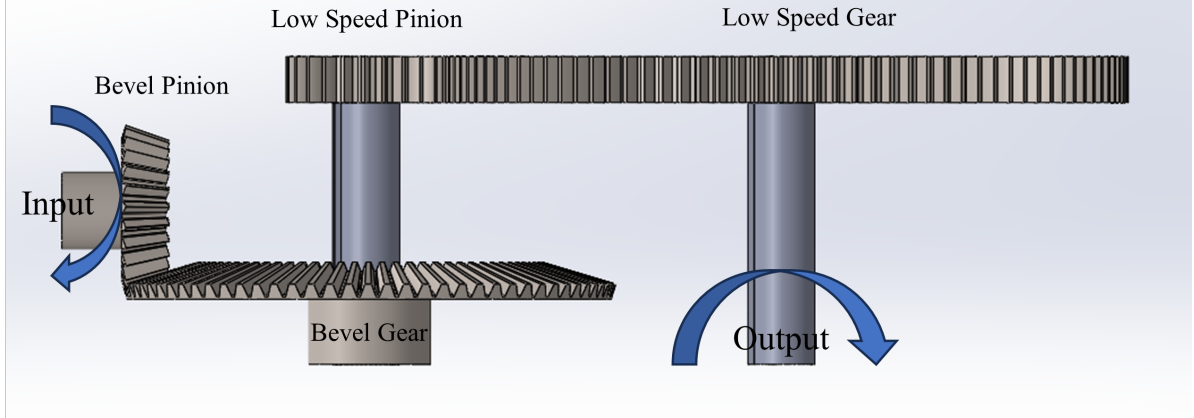


Figure 2.11: Sketch of the gearbox configuration

While the gearbox ratio for the trolley system is known to be 13.85, the exact internal gear dimensions are not available. To address this, a representative gearbox configuration has been proposed, as shown in Table 2.4. This configuration is based on the SEW-EURODRIVE model KK97DRN180L4, which meets the same mechanical torque requirements specified for the trolley system [4]. The KK97 gearbox series is designed for heavy-duty applications, offering high output torque and a compact footprint, making it a suitable reference model. Therefore, the proposed arrangement can be considered a valid approximation of the gearbox likely used in the actual trolley, suitable for simulation and analysis of its nominal performance.

Reference data for the KK97DRN180L4 gearbox, including gear ratio, torque capacity, and dimensional drawings, is available in the official SEW-EURODRIVE product catalog and technical documentation [104, 103].

Gearbox friction is associated with the resistance encountered within a gearbox as it transmits power from the input shaft to the output shaft. Friction in gearboxes is influenced by several factors, including gear meshing, lubrication, and material selection [37]. The mean friction force between a set of gears can be defined by the following relationship [19]:

$$F_G = \mu_{mg} F_{bt} \quad (2.18)$$

Component	Diameter	Gear modulus	Number of teeth
Bevel pinion	100 mm (D1)	10	10
Bevel gear	320 mm (D2)	10	32
Low speed pinion	100 mm (D3)	10	20
Low speed gear	440 mm (D4)	10	44

Table 2.4: Gearbox Specifications

where F_{bt} is the mean normal component of the load applied on the gears teeth and μ_{mg} is the mean sliding friction coefficient in gears. The estimation of the sliding friction coefficient is based on experimental data. The approaches to analyze the mean friction coefficient where made by Buckingham's semi empirical formulas, where important parameters like surface roughness, applied torque and lubrication are neglected [21]. Later work show that Buckingham's approach was incompatible for lubricated conditions [142].

The latest approaches to approximate the mean friction coefficient have been collected in ISO 3663 [3]. One key method of relevance for the trolley gearbox friction calculation considers the micro pitting load carrying capacity [2]. Micropitting in gears is a type of surface fatigue that affects the tooth surfaces of gears. The mean friction coefficient can be calculated using this method as follows:

$$\mu_m = 0.045 \cdot \left(\frac{K_A \cdot K_v \cdot K_{H\beta} \cdot F_{bt} \cdot K_{B\gamma}}{b \cdot \nu_{\Sigma C} \cdot \rho_{n,C}} \right)^{0.2} \cdot (10^3 \cdot \eta_{oil})^{-0.05} \cdot X_R \cdot X_L \quad (2.19)$$

where the factors and associated coefficients used in Eq. 2.19 are defined in Table 2.5. Given a two stage gearbox, each stage will have its own associated friction force due to differences in gear geometries and applied torques. Further definition on the derivation of the coefficients used herein is given in Appendix B.

An important consideration is the lack of available information regarding the actual face widths of the gears in the trolley gearbox. In the absence of this data, a practical estimation range between 50 mm and 150 mm has been established based on standard engineering guidelines, which typically recommend face widths in the range of 8 to 12 times the gear module for industrial gear applications [22, 87]. This approach ensures the gears are capable of transmitting the required torque while maintaining structural integrity. The overall friction force for the gearbox is defined as the summation of the friction forces generated by the two stages of the gearbox. The friction force associated with each stage

Mean Friction Coefficient	Friction Factor	Stage 1	Stage 2
τ_m	Input torque	τ_m	$3.2\tau_m$
K_A	Application factor	2	2
K_v	Dynamic Factor	2	2
$K_{H\beta}$	Face Load factor	1.05	1.05
$K_{B\gamma}$	Helical load factor	1	1
b	Face width	50-150 mm	50-150 mm
$\nu_{\Sigma C}$	Sum of tangential velocities	$0.99 * 2.30 * \dot{x}$	$0.99 * 0.72 * \dot{x}$
$\rho_{n,C}$	Equivalent radius	34.37	53.33
η_{oil}	Dynamic viscosity	17.44	17.44
X_L	Lubricant factor	0.52	0.52
X_R	Roughness factor	1.021	0.09152

Table 2.5: Comparison of friction factors between two stages of the gearbox, where τ_m is the torque generated by the electric motor and $\dot{x}$ is the trolley velocity

is defined in terms of the torque of the electric motor and the velocity of the trolley. The following values have been obtained for a gear with a face width of 100 mm.

$$F_{GBD} = \frac{\tau_m^{1.2}}{\dot{x}^{0.2}} * 3.83 \cdot 10^{-3} \quad (2.20)$$

$$F_{GBD} = \frac{\tau_m^{1.2}}{\dot{x}^{0.2}} * 1.60 \cdot 10^{-2} \quad (2.21)$$

The resulting behavior of the gearbox friction force for a complete trip of the nominal trolley model is shown in Fig. 2.12. As with several of the other resistive forces on the trolley system, the gearbox friction force model shows an increase in friction resistance at low speeds, which is in agreement with the literature [19]. Sudden shifts in the gearbox friction force occur as the input from the motor changes. When the operating velocity is achieved, the gearbox friction decreases due to the fact that the trolley has stopped accelerating. The force applied between the gear teeth shifts to instead maintain a constant velocity. A similar shift in the friction force occurs at the cutoff point, when the input from the motor is removed entirely. The total friction force due to the gearbox is:

$$F_{GB} = F_{GB1} + F_{GB1} = \frac{\tau_m^{1.2}}{\dot{x}^{0.2}} * 1.985 \cdot 10^{-2} \quad (2.22)$$

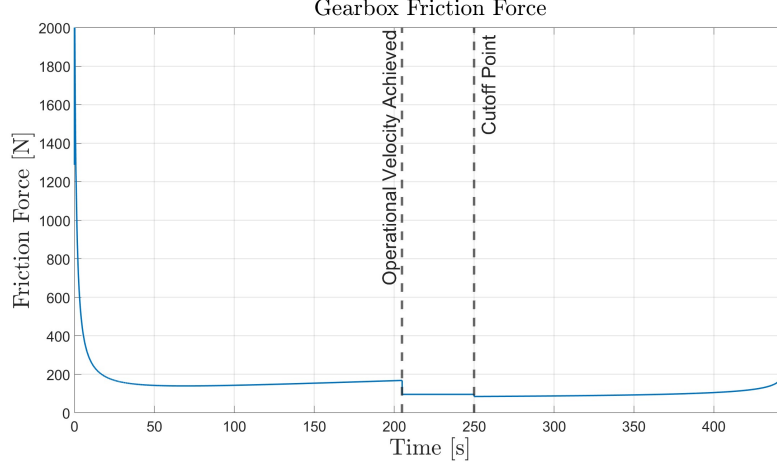


Figure 2.12: Results obtained for the gearbox friction during one trip of the trolley

To define a prior distribution for the gearbox friction coefficient, we focus on the parameter with the greatest uncertainty: the face width of the gear. Since the exact face width is not available from manufacturer specifications, we assume it may range between 50 mm and 150 mm.

The friction force generated by the gearbox is modeled as:

$$F_{GB} = \frac{\tau_m^{1.2}}{\dot{x}^{0.2}} \cdot FT \quad (2.23)$$

As done in previous sections the correspondence bounds have been used to establish a 95% interval for a normal distribution:

$$FT \sim \mathcal{N}(1.97852 \cdot 10^{-2}, (3.648 \cdot 10^{-4})^2) \quad (2.24)$$

2.4 Power Track Resistance

The **PT** provides energy to the system and its accessories, but also adds resistance, which influences the coasting distance. Understanding **PT** resistance is a critical factor in avoiding delays in the fueling operation. To address the challenge of under- or over-coasting, this project aims to determine the dynamic behavior of the **PT** as well as the total resistance

it gives to the trolley.

The main sources of friction of the power track are the interaction between the cable chain and the intermediate drums, the PT rolling friction between its wheels and the rails, and bearings with in the PT drums and wheels. The PT has a overall mass of 44 tons. OPG experimental information has shown that overall resistance force of the PT is 3042.43 N.

2.4.1 Power Track rolling friction

Just like the trolley, the PT rolls on a set of steel wheels and steel rails. We therefore take the same approach for rolling friction in the PT as for rolling friction in the trolley.

$$F_f = \mu_r mg = 84.24N \quad (2.25)$$

Where, similar to the trolley rolling friction, μ_r is modeled as:

$$\mu_r \sim \mathcal{N}(2.0 \times 10^{-4}, (2.5 \times 10^{-5})^2) \quad (2.26)$$

2.4.2 Power Track Bearings friction

In the PT two different types of bearing are used. A pillow block bearing holds the roller while an antifriction bearing is used inside the PT wheel. The location and codes of these bearing are shown in Fig: 2.13

The anti-friction bearings are located in each of the wheels that transport the PT, while the pillow block bearings hold the weight of the cables, power track, and intermediate drums. According to information given by OPG the PT has an overall weight of 44 tons. This weight is held by the wheel bearings, while the pillow block bearings hold the weight of the cable chain, intermediate drums and cables. Given OPG specifications, the corresponding masses of these subsystems are shown in Table 2.8 [1]:

Pillow Block Bearings

Given the information in Table 2.8, it is evident that The pillow block bearing hold an overall mass of 36709 kg. Each drum consists of 2 pillow block bearings, therefore we are considering a total 144 bearings. The applied force on the pillow block bearings is stated as follows:

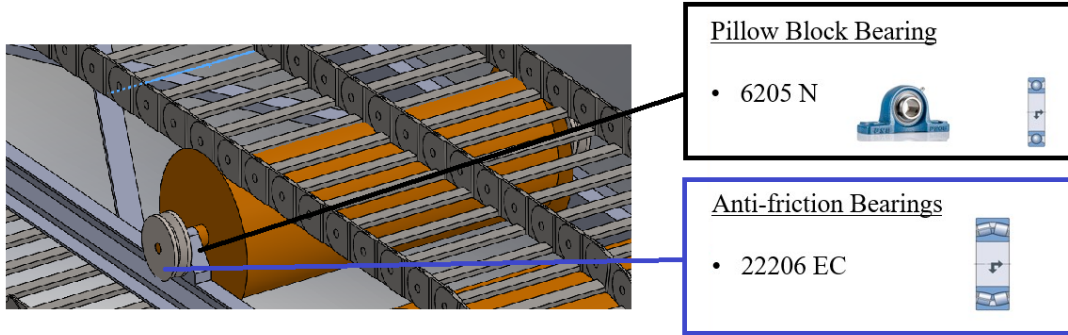


Figure 2.13: PT bearings

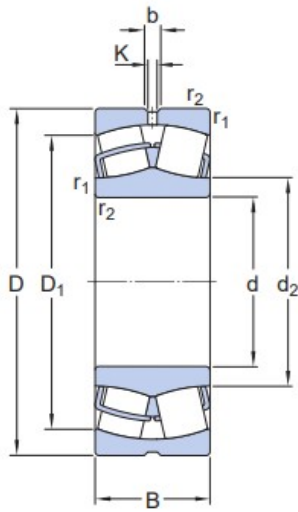


Figure 2.14: Antifriction bearings 22206 EC sketch [112]

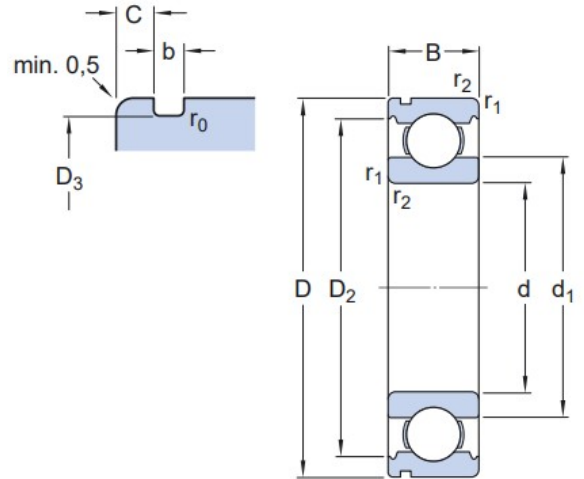


Figure 2.15: 6205 N bearings sketch [113]

$$F_R = \frac{m * g}{n_{bearing}} = \frac{36709 * 9.8}{144} = 2.5 \text{ kN} \quad (2.27)$$

The SKF approach used for the trolley bearings is also appropriate for modeling the

Nomenclature	Dimension	Specification
d	30 mm	Bore diameter
D	62 mm	Outside diameter
B	20 mm	Width
d1	37.6 mm	Shoulder diameter inner ring
D1	53 mm	Shoulder diameter outer ring
b	3.7 mm	Width annular lubrication groove outer ring
K	2 mm	Diameter of lubrication hole
r1,2	min 1 mm	Chamfer Dimension

Table 2.6: Bearing 22206 EC list of specification [112]

Nomenclature	Dimension	Specification
d	25 mm	Bore diameter
D	52 mm	Outside diameter
B	15 mm	Width
d1	34.35 mm	Shoulder diameter
D2	46.21 mm	Recess diameter
b	1.35 mm	Width of nap ring groove
C	2.46 mm	Distance from outer ring to snap ring
r1,2	min 1 mm	Chamfer Dimension(open bearings)

Table 2.7: Bearing 6205 N list of specification [113]

Component	Unit Mass	Quantity / Length
Cables	78.96 kg/m	250 m
Cable chain links	5.917 kg each	1,810 units
Intermediate drums	86.98 kg each	72 units

Table 2.8: Mass specifications of major components [50]

frictional behavior of bearings in the PT.

- **Rolling Frictional Moment (M_{rr}):**

$$M_{rr} = \phi_{ish}\phi_{rs}G_{rr}(vn)^{0.6}$$

$$M_{rr} = 0.159 * 0.9999^n * n^{-0.68} \quad (2.28)$$

where the coefficients in Eq.2.28 are obtained from the calculation of the inlet shear reduction factor (ϕ_{ish}), kinematic replenishment/starvation reduction factor (ϕ_{rs}), rolling frictional variable (G_{rr}), the viscosity (v), and the rotational speed (n) using the bearing's geometric parameters established in Table 2.6.

$$M_{sl} = G_{sl}\mu_{sl}$$

$$M_{sl} = (0.1198n^{-1.4} + (1 - 0.9985n^{-1.4})0.02)144.37 \quad (2.29)$$

where the coefficients in Eq.2.29 are obtained from the calculation of the sliding frictional variable (G_{sl}) and the sliding friction coefficient (μ_{sl}). These terms depend on the bearing geometry, the load applied on the bearing, and the rotational speed.

- **Frictional Moment Caused by Seals (M_{seal}):** The selected bearing does not contain a seal. As such, this factor is set to 0.

$$M_{seal} = 0$$

- **Drag Frictional Moment (M_{drag}):** The drag component considers drag force due to the oil bath surrounding the rolling elements during rotation. The selected bearing works with grease instead of oil, so no drag is considered.

$$M_{Drag} = 0$$

Using the relationship:

$$F_{B-drums} = \frac{M_{TOTAL}}{r_{drums]}$$

where $r_{drums} = \frac{304.8}{2} = 152.4$ mm is the radius of the drum [1].

Anti-Friction Bearings

In the case of the anti-friction bearings mounted in the wheels of the PT, the overall mass of the PT is assumed to be 44 metric tons. The PT is assembled such that each intermediate drum is paired with a corresponding wheel set. As a result, the number of wheel bearings is equal to the number of pillow block bearings.

$$F_R = \frac{m * g}{n_{bearing}} = 3 \text{ kN} \quad (2.30)$$

- **Rolling Frictional Moment (M_{rr}):**

$$M_{rr} = \phi_{ish}\phi_{rs}G_{rr}(vn)^{0.6}$$

$$M_{rr} = 0.59 * 0.9998^n * n^{-0.68} \quad (2.31)$$

where the coefficients in Eq.2.31 are obtained from the calculation of the inlet shear reduction factor (ϕ_{ish}), kinematic replenishment/starvation reduction factor (ϕ_{rs}), rolling frictional variable (G_{rr}), the viscosity (v), and the rotational speed (n) using the bearing's geometric parameters established in Table 2.3. For more details concerning this calculation see Appendix A.

- **Sliding Frictional Moment (M_{sl}):**

$$M_{sl} = G_{sl}\mu_{sl}$$

$$M_{sl} = (0.1197n^{-1.4} + (1 - 0.9981n^{-1.4})0.02)207 \quad (2.32)$$

where the coefficients in Eq.2.32 are obtained from the calculation of the sliding frictional variable (G_{sl}) and the sliding friction coefficient (μ_{sl}). These terms depend on the bearing geometry, the load applied on the bearing, and the rotational speed.

- **Frictional Moment Caused by Seals (M_{seal}):** The selected bearing does not contain a seal. As such, this factor is set to 0.

$$M_{seal} = 0$$

- **Drag Frictional Moment (M_{drag}):** The drag component considers drag force due to the oil bath surrounding the rolling elements during rotation. The selected bearing works with grease instead of oil, so no drag is considered.

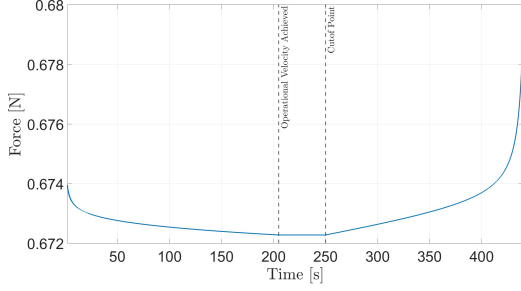
$$M_{Drag} = 0$$

Using the relationship:

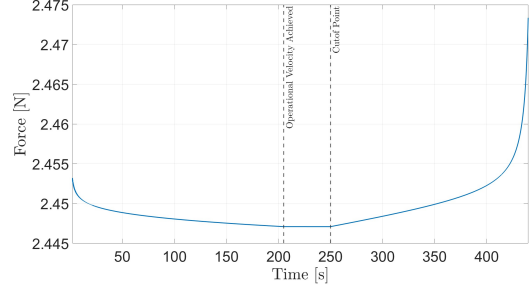
$$F_{B-wheels} = \frac{M_{TOTAL}}{r_{wheel}}$$

where $r_{wheels} = \frac{152.4}{2} = 76.2$ mm is the radius of the drum [1].

As mention for the trolley bearing, given the fact of minor variations in the beaering force, we to model the bearing force as a constant force of 0.673 N and 2.5 N for the drums and wheel bearings respectively.



(a) PT drum bearings



(b) PT wheel bearings

Figure 2.16: Results for each PT bearings

2.4.3 Cable chain friction

As the PT is dragged by the trolley, the energy-supplying cables—carried by a chain—move over a series of drums. The interaction between the cables and these drums contributes significantly to the overall resistance of the power track system. However, due to the uniqueness of this configuration, accurately characterizing this interaction without experimental data is challenging. In particular, there is uncertainty as to whether the chain rolls perfectly over the drums, slides, or exhibits a combination of both behaviors, which directly affects the frictional response. We therefore consider the total resistance force of the PT, as reported by OPG, to be the starting point of these computations. Therefore, the resistance attributable to the cable chain is estimated by subtracting the contributions of bearing friction and rolling resistance from the total measured resistance force.

$$F_{chain} = 3042.46N - F_{B-wheels} - F_{B-drums} - F_f \quad (2.33)$$

$$F_{chain} = 2509.12N \quad (2.34)$$

To estimate the average friction coefficient between the cable chain and the drum, we first approximate the mass of the cable chain and the cables. As stated in Table 2.8, the cables have a mass of 78.96 kg/m, and each cable link weighs 5.917 kg. This results in a total estimated mass of 30.5 tons. Therefore, the average friction coefficient is calculated as:

$$\mu_{chain} \approx 0.0824 \quad (2.35)$$

The computed coefficient of friction, approximately 0.0824, falls between typical values for rolling and sliding friction in steel-on-steel contact. Specifically, the coefficient of rolling resistance for steel on steel is typically around 0.002, while the coefficient of sliding friction ranges from 0.1 to 0.8 depending on surface condition and lubrication [102].

This intermediate value suggests that the interaction between the cable chain and the drum likely involves a combination of rolling and sliding motions. Perfect rolling is unlikely due to surface irregularities, and the flexibility of the chain links. As a result, sliding friction likely contributes significantly to the overall resistance observed in the system.

Assuming a normal prior for the friction coefficient between the cable chain and the drum, based on the estimated value and expected variability, we define the following.

$$\mu_{\text{chain}} \sim \mathcal{N}(0.0824, (0.01)^2) \quad (2.36)$$

2.5 Propulsion and Control

The railway vehicle under study is driven by an electric motor. The traction transmission system consists of an electric motor, a flexible coupling, a gearbox, and a wheel set as presented in Fig. 2.17. During operation, the torque generated by the electric motor is transferred to the gearbox's input pinion via a flexible coupling. Following transmission through the gearbox, the resultant torque is leveraged to drive the wheel set [135].

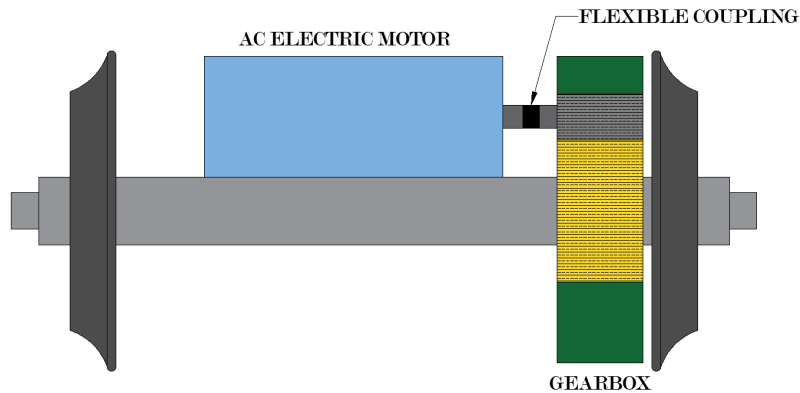


Figure 2.17: General traction transmission system for a railway vehicle

While the internal electronics of the motor influence its interaction with the broader system, a simplified approximation—encompassing soft-start mechanics and general motion control—suffices to effectively represent system dynamics for coasting model calibration. In our system, we incorporate a soft-start model wherein the firing angle of the thyristors is progressively increased, allowing the voltage supplied to the motor to ramp up gradually over a predefined period. This approach mirrors the operational behavior of physical trolley systems, where soft starters manage voltage application to mitigate mechanical and electrical stresses during start-up [5, 136]. The ramp-up period is deliberately brief relative to the overall trolley operation time, encompassing only the initial seconds of runtime. Such modeling aligns with established practices in simulating and calibrating soft-start mechanisms in industrial applications [96].

The force generated by the motor is characterized as the force needed to overcome the collective resistive forces acting on the trolley and drive the trolley to an operating velocity of 1 m/s[4]. The aggregate of all resistive forces encountered will be encapsulated within a single variable denoted as F_{RT} . We can then express the acceleration of the trolley in terms of the difference between the input force from the motor and the resistive forces acting on the trolley.

$$m\ddot{x} = F_{EM} - F_{RT} \quad (2.37)$$

where F_{EM} is the force generated by the electric motor, and F_{RT} is the summation of all the natural resistant forces.

According to the trolley user manual, the system can not exceed an acceleration of 0.1 G [4], which provides a limit on the first stage of trolley motion (acceleration). In the second stage of trolley motion (equilibrium), the sum of all resistant forces will be equal to the force generated by the electric motor, and as such the trolley will move at a constant speed. Currently, it is assumed that this speed is 1 m/s, however this assumption can easily be tuned when encoder data is available from the system. During the final stage of trolley motion (coasting), the action of the resistive forces on the system allows the trolley to coast to a stop. The force of the motor during each of these three stages is defined as follows:

$$F_{EM} = \begin{cases} m\ddot{x} + F_{RT} & \text{where } t \leq t_1; \ddot{x}(t_1) = 0; \dot{x}(t_1) = 1 \\ F_{RT} & \text{where } t_1 < t < t_2 \\ 0 & \text{where } t_2 \leq t \end{cases} \quad (2.38)$$

where t_1 is the time at which the trolley achieves equilibrium and begins to run at a constant velocity and t_2 is the time at which the electric motor is shut down. It is important to note that t_1 will be tuned experimentally based on encoder data described in Chapter 4, leading to the time-dependent equation for the electric motor force shown in equation 2.39. This equation has been set under the assumptions of a slow acceleration emulating a soft start and an acceleration of limit of 0.1 G. This expression will determinate how fast the trolley will achieve its operational velocity. Based on our current approximations of the trolley dynamics and soft start parameters, we attribute force delivered by the motor to the following:

$$F_{EM} = \begin{cases} m * \frac{0.095}{(t+1)^{0.0.922}} + F_{RT} & \text{where } t \leq 205; \ddot{x}(t_1) = 0; \dot{x}(t_1) = 1 \\ F_{RT} & \text{where } 205 < t < t_2 \\ 0 & \text{where } t_2 \leq t \end{cases} \quad (2.39)$$

2.6 Nominal Model of the FMT

In this section the results of modeling the FMT's movement equations are presented with respect to the mean values for the model parameters developed in this chapter. A summary of the findings is displayed in Table 2.9. The frictional forces interacting with the system are delineated in Table 2.10 and their action on the FMT system is shown in Fig. 2.18.

Table 2.9: Nominal Parameters and Mechanical Properties

Parameter	Value
Car mass	72 tonnes
PT mass	44 tons
Trolley bearing force	17.75 N per wheel
Trolley coupling stiffness	1×10^7 N/m
Trolley coupling damping	5×10^5 Ns/m
PT coupling frame stiffness	8.73×10^6 N/m
PT coupling frame damping	2166 Ns/m
Trolley cross-sectional area	3.5 m^2 [4]
Gearbox ratio	13.85 (see Table 2.4)
Drums mass	87 kg
Cable mass	78.96 kg/m
Forces	Value
Trolley rolling friction force	142.02 N
PT chain friction force	2509.12 N
Trolley bearing	12.5 N/bearing
PT drum bearing	0.67 N/bearing
PT wheel bearing	2.5 N/bearing

Table 2.10: Identified resistance forces in the trolley and PT systems

Trolley Resistance Sources	PT Resistance Sources
Rolling friction (F_{fT})	Rolling friction (F_{fPT})
Gearbox friction (F_{GB})	Cable chain friction (F_{chain})
Bearing friction (F_B)	Bearing friction in wheels ($F_{B-wheel}$)
Drag force (F_d)	Bearing friction in drums (F_{B-drum})

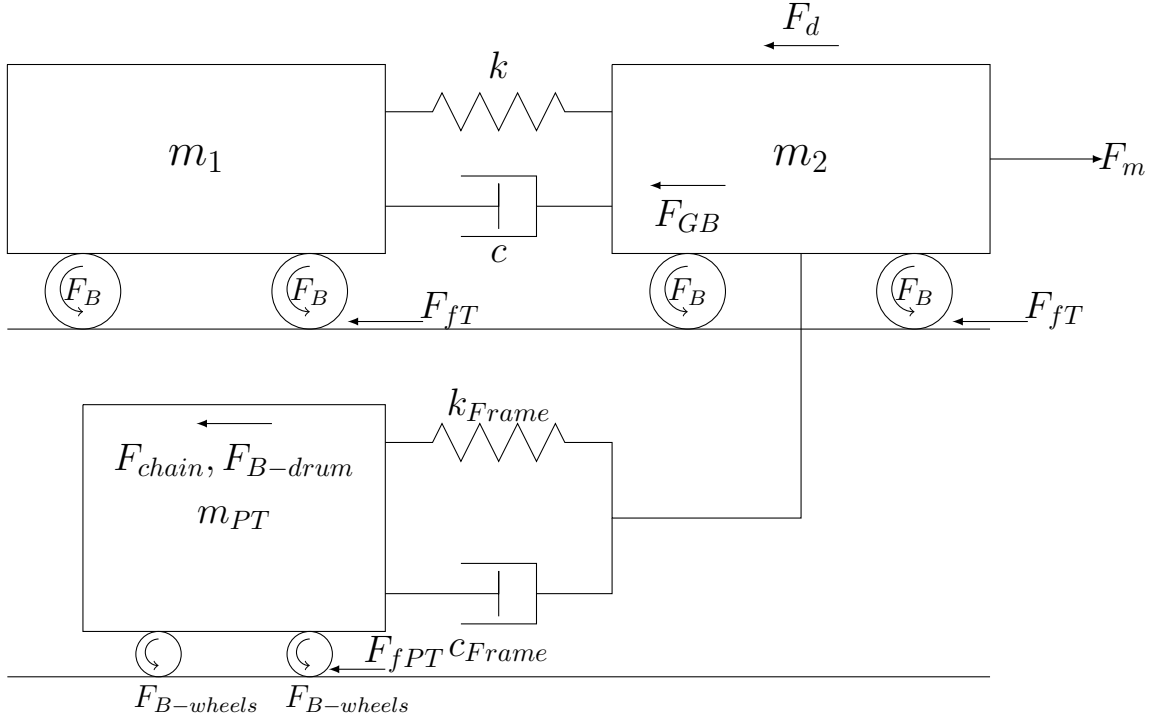


Figure 2.18: FMT dynamic model

Given the development of these parameters and forces, the following system of dynamic equations is obtained to describe the behavior of the [FMT](#):

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \quad (2.40)$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_{PT} \end{bmatrix}, \quad \dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_{PT} \end{bmatrix}, \quad \ddot{\mathbf{x}} = \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_{PT} \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_{PT} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c & -c & 0 \\ -c & c + c_f & -c_f \\ 0 & -c_f & c_f \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k & -k & 0 \\ -k & k + k_f & -k_f \\ 0 & -k_f & k_f \end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix} -F_{fT} - F_B \\ F_m - F_d - F_{fT} - F_B - F_{GB} \\ -F_{fPT} - F_{B-wheels} - F_{chain} - F_{B-drums} \end{bmatrix}$$

2.6.1 System Movement

Given this model, the nominal behavior of the system can be seen in Figure 2.19, where the FMT accelerates for a period of 250 s and achieves equilibrium by moving at constant speed of 1 m/s. At 400 s the electric motor is shut down and the FMT slowly coasts until it reaches a full stop.

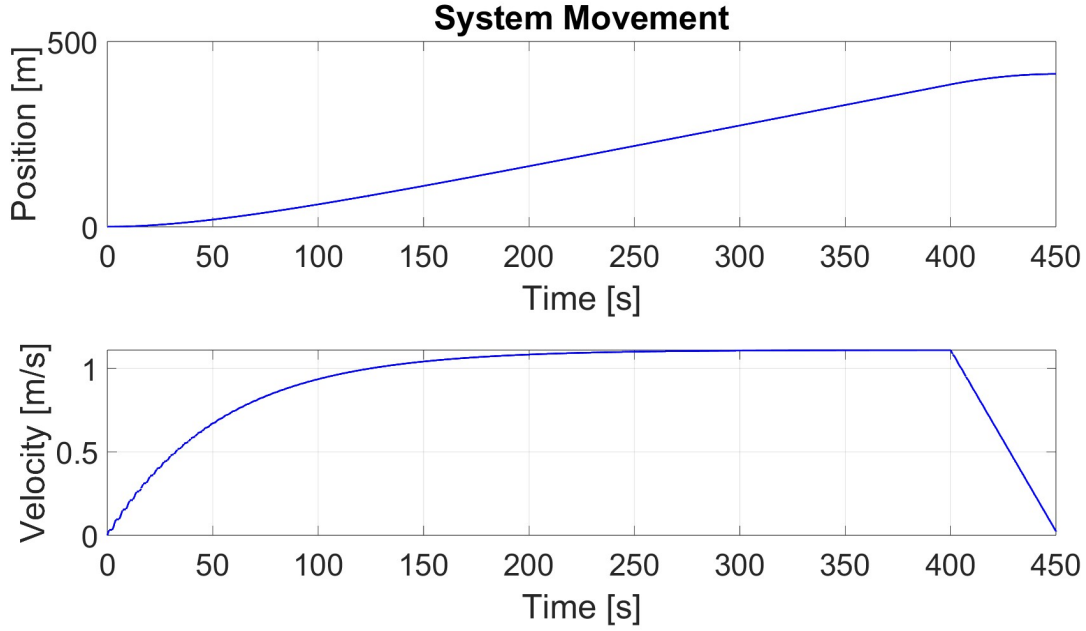


Figure 2.19: FMT Movement

As shown in Figure 2.20 an estimated coasting distance of 24.5 m has been obtained under mean prior conditions. Figure 2.21 reveals that the observed coasting distances in historical trials range from approximately 17 meters to 24 meters, according to reported values by OPG. The model prediction closely matches the upper bound, suggesting that the mean prior assumptions used in the model are consistent with real-world behavior. This comparison provides a preliminary validation of the model's accuracy in replicating

the system’s dynamic response. It also implies that the resistance parameters—such as rolling friction, bearing losses, and cable chain interactions—are reasonably captured by the chosen prior distributions and that model updating with experimental encoder data should yield a sufficient representation of FMT behavior. One key point to note in this analysis is the assumption that the system operates in constant temperature conditions, as the trolley is housed in an underground enclosed environment. As such, we assume negligible impact from temperature variations on the coasting distance of the trolley.

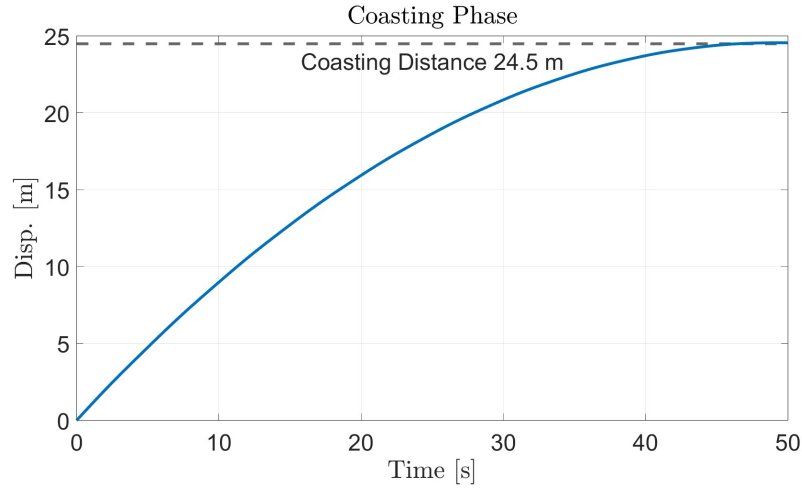


Figure 2.20: FMT model coasting distance

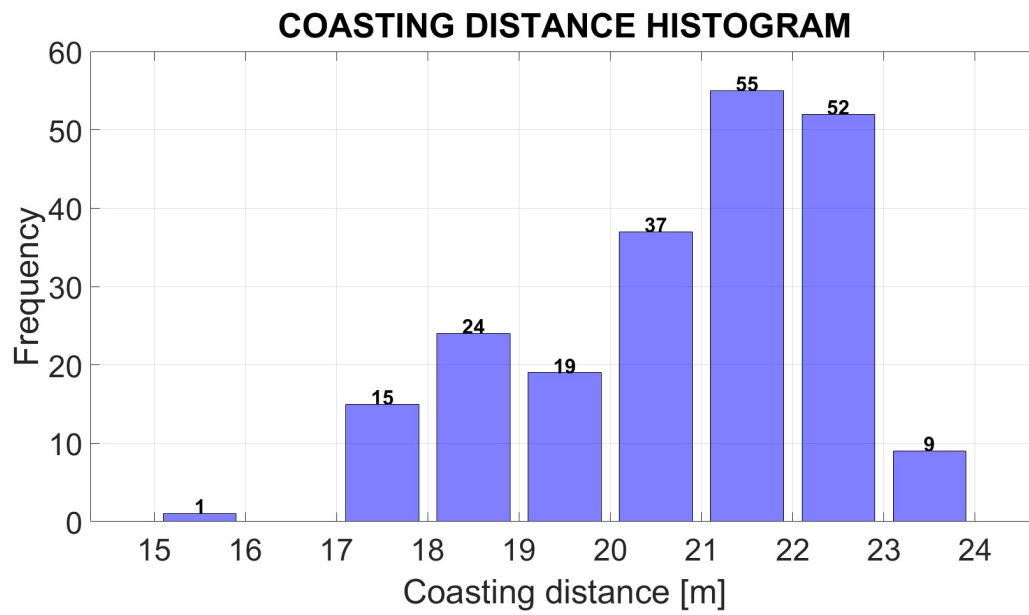


Figure 2.21: Histogram of coasting data given by OPG

2.7 Conclusion

This study developed a dynamic model of the [FMT](#) system by incorporating the major resistive forces affecting motion, including rolling friction, gearbox friction, drag, bearing resistance, and cable chain interaction. Key assumptions included uniform mass distribution, idealized material properties, constant environmental temperature, consistent surface contact conditions like steel on steel, and linear damping behavior. Constraints include limited availability of direct experimental data and uncertainty in certain component behaviors, particularly the interaction between the cable chain and drums.

The model effectively predicted coasting behavior under nominal prior assumptions, with a coasting distance of 24.5 meters—consistent with the empirical data provided by [OPG](#). This indicates the model’s ability to capture the fundamental system dynamics. Future refinements involve integrating real-time sensor data and applying a [KF](#) framework for online health monitoring. This would enable parameter estimation and anomaly detection in operational settings, providing valuable insights for preventive maintenance. [Tab: 2.11](#) show the parameters to be inferred in this subsequent study.

Table 2.11: System Parameters and Mechanical Properties

Parameter	Value
Car mass	72 tonnes
PT mass	44 tons
Trolley bearing	HH221440
Trolley bearing force	17.75 N per wheel
Trolley coupling stiffness	1×10^7 N/m
Trolley coupling damping	5×10^5 Ns/m
PT coupling frame stiffness	8.15×10^6 N/m
PT coupling frame damping	2082 Ns/m
Trolley cross-sectional area	3.5 m ² [4]
Gearbox ratio	13.85 (see Table 2.4)
Rolling friction coefficient	0.0002
PT chain friction coefficient	0.0824
Drums mass	87 kg
Cable mass	78.96 kg/m
PT drum bearing	6205 N
PT wheel bearing	22206 EC

Chapter 3

Estimation Approach

This chapter presents the numerical study conducted to evaluate two distinct modeling approaches for estimating key dynamic parameters of the [FMT](#) system. The overarching goal of this study is to assess the suitability of simplified models to extract reliable information for use in health monitoring and parameter inference applications, particularly under operational conditions characterized by unknown system properties or noisy data.

True-to-life dynamic modeling of any practical system, including the [FMT](#), is unachievable due to limited access to proprietary information, practical time constraints, and unmodeled physical complexities. Moreover, due to the limited sensor information from the [FMT](#), identifying a complete nominal model is impractical in this case. Inferring all parameters involves too many unknowns to resolve reliably. However, it is possible to construct simplified yet informative models that support [SHM](#) and performance analysis. In this work, we deliberately avoid pushing the model toward full physical realism; instead, we contribute a workflow designed to extract useful [SHM](#) information from an intentionally less detailed, surrogate model, thereby testing the boundaries of model simplification for reliable inference.

Two modeling approaches are introduced:

1. A single-degree-of-freedom (1-DOF) model that treats the power track and trolley forces as an aggregated unknown force acting on the system.
2. A two-degree-of-freedom (2-DOF) model that explicitly accounts for the power track subsystem and its dynamic influence.

To infer critical parameters and system states, the [KF](#) is applied to position measurements generated by the coasting simulation model introduced in Chapter 2. This filtering technique enables the simultaneous estimation of both time-varying forces and dynamic parameters, while accounting for measurement noise and process uncertainty.

The remainder of this chapter is organized as follows: Section 3.1 outlines the theoretical estimation framework, initialization strategy, and uncertainty quantification methods. Section 3.2 describes a numerical study performed using a SDOF model with results and analysis. Section 3.3.1 introduces a 2DOF model and discusses the numerical study results applied to this approach. Finally, Section 3.4 summarizes the findings and discusses their implications for practical [SHM](#) applications.

3.1 Methodology

3.1.1 Joint Input–State Estimation

In this study, we employ a joint input–state estimation framework based on the [KF](#) to simultaneously estimate the system states and the unknown external force acting on the [FMT](#) system[118]. The state vector is augmented to include both kinematic variables and the input force:

$$\mathbf{x}_k = \begin{bmatrix} p_k \\ v_k \\ F_k \end{bmatrix}, \quad (3.1)$$

where p_k and v_k denote the position and velocity of the system at each time step k , and F_k represents the resistive forces to be identified. Compared to conventional state estimators that only track system dynamics, the joint input–state estimation framework allows simultaneous recovery of unknown external forces, which has been shown to improve robustness in dynamic systems [76, 77]. We adopt this approach because it provides an efficient and theoretically grounded means of identifying resistive forces while maintaining accurate state estimation.

The discrete-time linear stochastic system model can be expressed as:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k \quad (3.2)$$

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \quad (3.3)$$

Here, $\mathbf{A}$ is the augmented state transition matrix incorporating both the physical dynamics of the trolley and the assumed force evolution model. In this work, the external force F_k

is modeled as a random walk, which is a common assumption in input–state estimation to capture slowly varying or unmodeled disturbances [14]. The process noise $\mathbf{w}_k \sim \mathcal{N}(0, \mathbf{Q})$ captures modeling uncertainties and unmodeled dynamics, while the measurement noise $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R})$ accounts for sensor inaccuracies. The output vector $\mathbf{y}_k$ contains the available measurements (position), and the measurement matrix $\mathbf{H}$ maps the augmented state vector to the measurement space.

The KF operates in two recursive steps:

Prediction Step

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A} \hat{\mathbf{x}}_{k-1|k-1} \quad (3.4)$$

$$\mathbf{P}_{k|k-1} = \mathbf{A} \mathbf{P}_{k-1|k-1} \mathbf{A}^\top + \mathbf{Q} \quad (3.5)$$

This step propagates the augmented state estimate and covariance forward in time, predicting both kinematic variables and the input force based only on previously available system.

Update Step

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}^\top (\mathbf{H} \mathbf{P}_{k|k-1} \mathbf{H}^\top + \mathbf{R})^{-1} \quad (3.6)$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{y}_k - \mathbf{H} \hat{\mathbf{x}}_{k|k-1}) \quad (3.7)$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}) \mathbf{P}_{k|k-1} \quad (3.8)$$

The Kalman gain $\mathbf{K}_k$ balances the relative confidence between prediction and measurement. The augmented state update refines both the kinematic estimates and the force estimate, reducing uncertainty in $\mathbf{P}_{k|k}$.

This augmented-state formulation enables direct recovery of the unknown input force in addition to the system’s motion states, providing a complete dynamic picture that supports subsequent structural health monitoring analyses. Importantly, the reliability of this estimation framework is strongly influenced by the correct characterization of the process noise covariance $\mathbf{Q}$ and the measurement noise covariance $\mathbf{R}$. In practice, however, calibrating these covariance matrices is challenging because process and measurement uncertainties are rarely known in advance and may vary with operating conditions and modeling approaches [65, 81]. The identification and calibration of these covariance matrices are therefore essential to the performance of the filter and will be addressed in detail in Sections 3.1.2 and 3.1.3.

3.1.2 Identification of Process Uncertainty

Accurately estimating or tuning the process noise covariance $\mathbf{Q}$ is one of the most challenging tasks in filter design. This difficulty arises because process noise is not directly measurable, often represents unmodeled system dynamics, and can vary significantly with changes in operating conditions [81, 72]. Several methods have been proposed:

- Manual tuning: based on trial-and-error and filter stability [137].
- Innovation-based techniques: which monitor the filter residuals and adapt $\mathbf{Q}$ accordingly [81].
- Maximum Likelihood Estimation (MLE) and Expectation-Maximization (EM) methods: which treat $\mathbf{Q}$ as an unknown to be optimized [108].
- Variance-based methods: which exploit specific segments of measured data to empirically estimate noise characteristics [134].

Among the various methods for characterizing process noise, the EM and MLE algorithms are well-suited for cases in which the state-space model is partially known, but the process noise covariance $\mathbf{Q}$ is unknown or difficult to specify. This makes it an ideal choice for applications involving unmeasured or time-varying system states, such as the force estimation problem addressed in this work.

EM for Process Noise Estimation

The Expectation–Maximization (EM) algorithm is a maximum-likelihood-based iterative method for parameter estimation in models with latent variables [35]. In the context of Kalman filtering, the hidden states $\{\mathbf{x}_k\}$ are treated as latent variables, and the goal is to maximize the likelihood of the observed measurements $\{\mathbf{z}_k\}$ with respect to the unknown parameters—in this case, the process noise covariance $\mathbf{Q}$.

The observed-data log-likelihood involves an intractable integral over the unobserved states. EM circumvents this by alternating between constructing a tight lower bound on the log-likelihood (E-step) and maximizing that bound (M-step). At each iteration, the algorithm guarantees a non-decreasing likelihood value, converging to a local maximum.

E-step (Expectation): Given the current estimate $\mathbf{Q}^{(i)}$, the expectation of the complete-data log-likelihood is computed as follows:

$$\mathcal{Q}(\mathbf{Q} \mid \mathbf{Q}^{(i)}) = \mathbb{E}_{\mathbf{x}_{1:N} \mid \mathbf{z}_{1:N}, \mathbf{Q}^{(i)}} \left[\log p(\mathbf{x}_{1:N}, \mathbf{z}_{1:N} \mid \mathbf{Q}) \right]. \quad (3.9)$$

This requires running a Kalman smoother (e.g., Rauch–Tung–Striebel smoother) to obtain posterior distributions of the hidden trajectory $p(\mathbf{x}_k \mid \mathbf{z}_{1:N})$. From this, the following second moments are obtained:

$$\mathbb{E}[\mathbf{x}_k \mathbf{x}_k^\top \mid \mathbf{z}_{1:N}] = \mathbf{P}_{k|N} + \hat{\mathbf{x}}_{k|N} \hat{\mathbf{x}}_{k|N}^\top, \quad (3.10)$$

$$\mathbb{E}[\mathbf{x}_k \mathbf{x}_{k-1}^\top \mid \mathbf{z}_{1:N}] = \mathbf{P}_{k,k-1|N} + \hat{\mathbf{x}}_{k|N} \hat{\mathbf{x}}_{k-1|N}^\top, \quad (3.11)$$

where $\mathbf{P}_{k|N}$ is the smoothed covariance, and $\mathbf{P}_{k,k-1|N}$ the lag-one smoothed covariance.

M-step (Maximization): Maximize $\mathcal{Q}(\mathbf{Q} \mid \mathbf{Q}^{(i)})$ with respect to $\mathbf{Q}$. For a linear Gaussian system, this leads to the closed-form update:

$$\mathbf{Q}^{(i+1)} = \frac{1}{N-1} \sum_{k=2}^N \mathbb{E}[(\mathbf{x}_k - \mathbf{A}\mathbf{x}_{k-1})(\mathbf{x}_k - \mathbf{A}\mathbf{x}_{k-1})^\top \mid \mathbf{z}_{1:N}]. \quad (3.12)$$

In expanded form, this involves the second moments computed in the E-step:

$$\mathbf{Q}^{(i+1)} = \frac{1}{N-1} \sum_{k=2}^N \left(\mathbb{E}[\mathbf{x}_k \mathbf{x}_k^\top] - \mathbf{A} \mathbb{E}[\mathbf{x}_{k-1} \mathbf{x}_k^\top] - \mathbb{E}[\mathbf{x}_k \mathbf{x}_{k-1}^\top] \mathbf{A}^\top + \mathbf{A} \mathbb{E}[\mathbf{x}_{k-1} \mathbf{x}_{k-1}^\top] \mathbf{A}^\top \right). \quad (3.13)$$

Thus, the EM update for $\mathbf{Q}$ can be interpreted as the average covariance of the smoothed state residuals, where the hidden states are replaced by their posterior expectations.

Convergence and Practical Considerations: In practice, a small number of EM iterations (5–10) is usually sufficient for stable convergence when initialized with reasonable values [107, 47, 42]. The log-likelihood increases monotonically, but convergence is only to a local maximum, since the likelihood surface is generally non-convex [24]. Over-iterating can lead to overfitting if the dataset is short or highly noisy [18].

Although EM has the attractive property of closed-form updates for $\mathbf{Q}$, it can converge slowly or stall in directions where the system is weakly observable. In such cases, direct MLE optimization of the innovations likelihood may be more robust [18].

Maximum Likelihood Estimation for Noise Parameters

The MLE framework offers a systematic approach for identifying the noise characteristics of a Kalman filter [81, 107, 63]. It can estimate the process noise covariance $\mathbf{Q}$ when the measurement noise variance R is known, or jointly estimate both from the same dataset [24]. Here, the method is applied to determine $\mathbf{Q}$ and R for the augmented Kalman filter.

The formulation builds on the discrete-time linear state–space model in Section 3.1.1, where $\mathbf{A}$ and $\mathbf{H}$ are the state transition and measurement matrices, and $\mathbf{Q}$ and R are the unknown process and measurement noise covariances.

For numerical stability, $\mathbf{Q}$ is parameterized by its Cholesky factor:

$$\mathbf{Q} = \mathbf{L}_Q \mathbf{L}_Q^T, \quad (3.14)$$

with $\mathbf{L}_Q$ lower triangular and strictly positive on the diagonal [124]. The measurement noise variance is expressed as:

$$R = r^2, \quad r > 0. \quad (3.15)$$

The following derivation supports two cases: (i) estimating only $\mathbf{Q}$ when R is known; (ii) estimating both $\mathbf{Q}$ and R when no reliable prior exists.

Given $(\mathbf{L}_Q, r)$, the Kalman filter is run over the measurement sequence to compute innovations:

$$\nu_k = z_k - \mathbf{H} \hat{\mathbf{x}}_{k|k-1}, \quad (3.16)$$

with innovation covariance:

$$S_k = \mathbf{H}(\mathbf{A} \mathbf{P}_{k-1|k-1} \mathbf{A}^T + \mathbf{Q})\mathbf{H}^T + r^2. \quad (3.17)$$

Assuming zero mean, Gaussian, the log-likelihood at time k is:

$$\ell_k = -\frac{1}{2} [\log |S_k| + \nu_k^T S_k^{-1} \nu_k + \log(2\pi)], \quad (3.18)$$

and the total log-likelihood is:

$$\mathcal{L}(\mathbf{L}_Q, r) = \sum_{k=1}^N \ell_k. \quad (3.19)$$

The estimation problem is then:

$$(\mathbf{L}_Q^*, r^*) = \arg \max_{\mathbf{L}_Q, r} \mathcal{L}(\mathbf{L}_Q, r), \quad (3.20)$$

subject to $\mathbf{L}_Q$ having positive diagonals and $r > 0$. MATLAB's `fmincon` is used to optimize these parameters until the predicted and measured outputs match in the maximum-likelihood sense.

This approach provides a unified framework for noise identification adaptable to different prior knowledge scenarios, with the estimated covariances then applied in the augmented Kalman filter (Section 3.1) to produce the state and force estimates evaluated in Section 3.2.

3.1.3 Verification of Measurement Noise

In Kalman filtering, the measurement noise covariance matrix $\mathbf{R}$ plays a critical role in determining how much the filter trusts the incoming measurements relative to its model prediction. Therefore, it is necessary to estimate or tune $\mathbf{R}$ based on observed filter performance. A common practical approach to approximate $\mathbf{R}$ is to compute the variance of the measurement signal during a period when no input excitation is applied, or alternatively to relate it to the quantization level of the measurement device [20]. A statistically grounded and widely adopted approach to refining $\mathbf{R}$ is the analysis of innovation statistics [109, 8].

The innovation (or measurement residual) at time k is defined as:

$$\boldsymbol{\nu}_k = \mathbf{z}_k - H\hat{\mathbf{x}}_{k|k-1}, \quad (3.21)$$

where $\mathbf{z}_k$ is the actual measurement, and $\hat{\mathbf{x}}_{k|k-1}$ is the predicted state estimate. The innovation represents the discrepancy between the predicted and measured outputs.

Under the assumptions of linearity, Gaussian noise, and correct model parameters, the innovation is a zero-mean Gaussian variable with covariance:

$$\mathbf{S}_k = H\mathbf{P}_{k|k-1}H^\top + \mathbf{R}, \quad (3.22)$$

where $\mathbf{P}_{k|k-1}$ is the a priori state covariance.

Normalized Innovation Squared (NIS)

To evaluate the statistical consistency of the filter and assess the adequacy of $\mathbf{R}$, we compute the Normalized Innovation Squared (NIS) [14, 53]:

$$\text{NIS}_k = \boldsymbol{\nu}_k^\top \mathbf{S}_k^{-1} \boldsymbol{\nu}_k. \quad (3.23)$$

The NIS follows a chi-squared distribution:

$$\text{NIS}_k \sim \chi_d^2,$$

where d is the dimension of the measurement vector. For scalar measurements, this simplifies to:

$$\text{NIS}_k = \frac{\nu_k^2}{S_k}, \quad \text{with} \quad \mathbb{E}[\text{NIS}_k] = 1. \quad (3.24)$$

Due to the structure of the hidden Markov process assumed in the derivation of the Kalman filter, Evaluating the NIS at each measurement sample over time is equivalent to taking independent samples of the underlying chi-squared distribution. From this resulting distribution of these samples, we can determine whether the filter is statistically consistent [?]:

- $\mathbb{E}[\text{NIS}_k] \gg 1$ indicates an underestimated $\mathbf{R}$ (filter is overconfident).
- $\mathbb{E}[\text{NIS}_k] \ll 1$ suggests an overestimated $\mathbf{R}$ (filter is too cautious).

Therefore, NIS can be used as a quantitative diagnostic for tuning the measurement noise covariance $\mathbf{R}$, especially when ground-truth state information is unavailable.

3.1.4 Workflow

Figure 3.1 shows the complete workflow for joint identification of process and measurement noise parameters and their integration into the augmented Kalman filter.

The identification process begins by collecting the measurement sequence $\{z_k\}$ from the physical system. The state-space model is then defined, including the augmented state vector that incorporates both kinematic states and the unknown forces.

For a given $(\mathbf{L}_Q, r)$, the Kalman filter is executed over the dataset to compute the innovations ν_k and their covariances S_k . These quantities are used to evaluate the log-likelihood $\mathcal{L}(\mathbf{L}_Q, r)$, which serves as the objective function for the numerical optimizer. The constrained optimization (performed in MATLAB's `fmincon`) searches for the parameter values that maximize the likelihood, ensuring $r > 0$ and positive diagonal entries in $\mathbf{L}_Q$.

The resulting $\mathbf{Q}$ and R estimates are then used in the augmented Kalman filter to generate final state and force estimates, along with their associated uncertainty bounds.

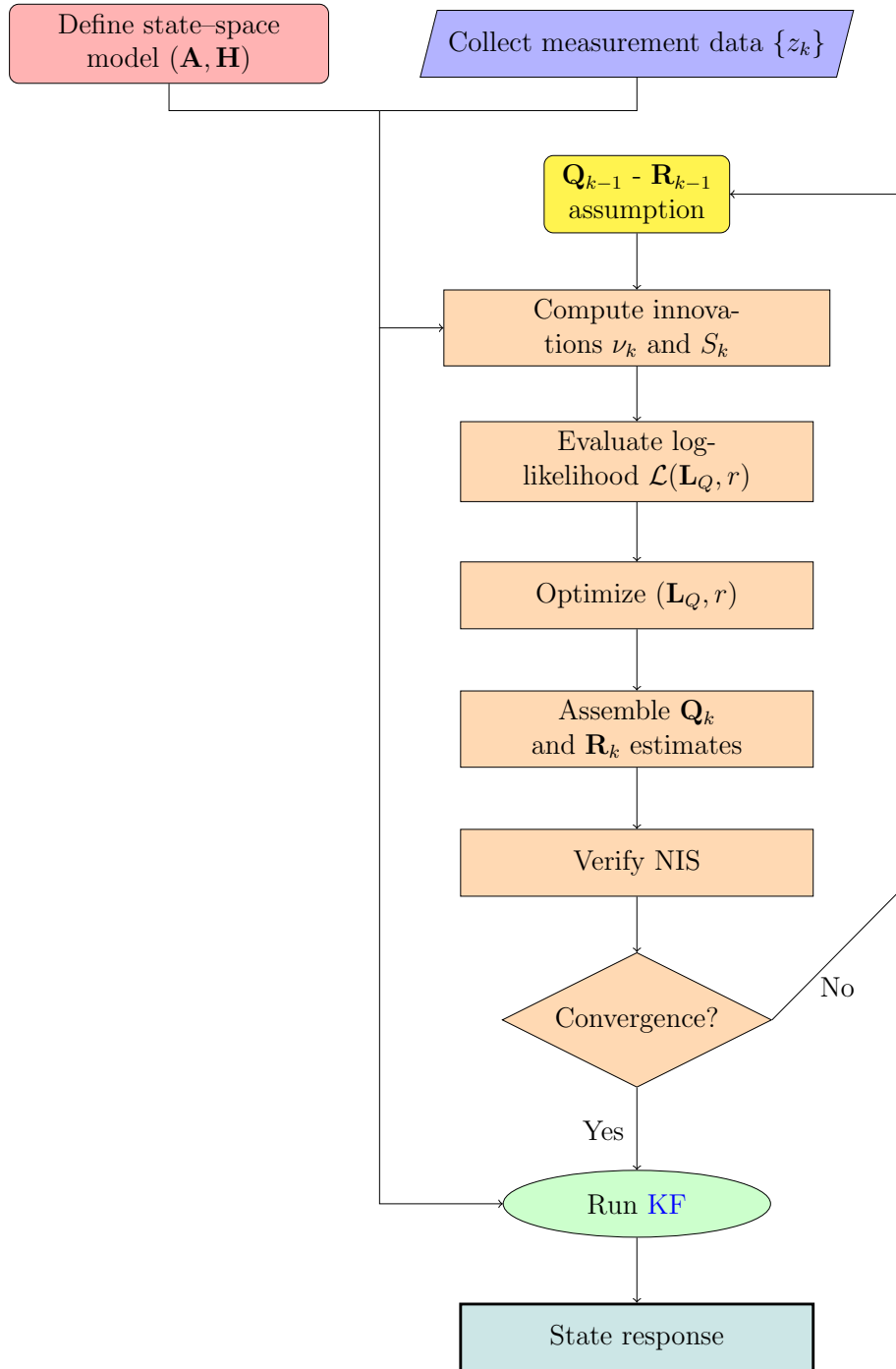


Figure 3.1: Workflow for states and forces estimation through MLE tuning for $\mathbf{Q}$ and $\mathbf{R}$.

3.2 Numerical Study of Force Identification and Uncertainty Quantification in a Simple SDOF Representation of the FMT

The objective of this numerical study is to evaluate how model uncertainty impacts the performance of Kalman filter-based estimation in the context of the [FMT](#). Specifically, the study aims to characterize process noise uncertainty in the simplified model, develop a sufficient Gaussian approximation of this uncertainty (replicating support and first two moments of the distribution), and determine how this modeling choice impacts the performance of the predictive model. This approach allows us to investigate the relationship between model complexity and estimation reliability under conditions of structural and parametric uncertainty.

To assess estimation reliability, we introduce variability into the assertion on the prior of the states and input force. By analyzing filter performance across these variations, we aim to determine how robust the filter will be under field implementation.

To conduct this analysis, we construct and simulate a detailed reference model of the [FMT](#) trolley system, as introduced in Section 2.6, Fig. 2.18. This detailed model captures the majority of the relevant physical interactions—including the effects of rolling resistance, cable chain dynamics, gearbox friction, and bearing forces—resulting in a coupled dynamic system with three degrees of freedom (DOF). For the purposes of this numerical investigation, the detailed model of the [FMT](#) is treated as the “experimental” system from which coasting data are generated. To emulate realistic measurement conditions, Gaussian white noise is superimposed on the simulated displacement signal, consistent with the noise characteristics of the real [FMT](#) system. The displacement is measured by a sensor with a tolerance of ± 78 mm; in order to assess robustness, we additionally consider an inflated tolerance of ± 117 mm (approximately 50% larger). This results in a zero-mean Gaussian noise modeled as $\mathbf{v}_k \sim \mathcal{N}(0, 0.0014^2)$. The resulting noisy measurement signal used in the numerical study is shown in Fig. 3.2.

To evaluate the effect of model-form uncertainty, the modeling approach illustrated in Fig. 3.3. This model reduces the [FMT](#) system to a single-trolley representation, in which the [PT](#) resistance is lumped into a single net force acting on the trolley mass. The intent is to test the filter’s ability to estimate meaningful resistive parameters when the physical representation is highly reduced. The unknown parameter is the total resistance force F_{total} , shown in red.

In this approach, F_{total} is the parameter of interest. It represents the sum of all the the

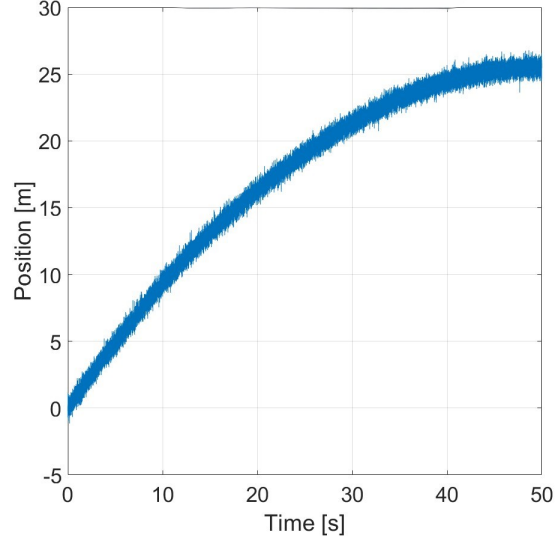


Figure 3.2: Measurement generated for numerica study

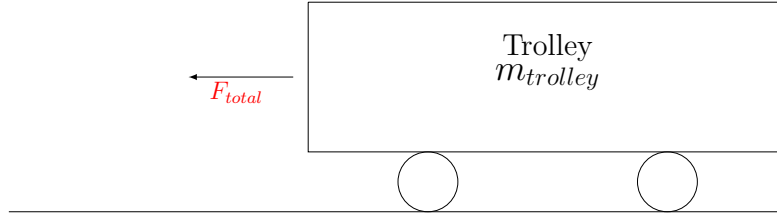


Figure 3.3: In the following approach we look to identify the overall trolley resistance by considering the trolley as one block of mass affected by one overall force.

forces acting on the trolley with the addition of dynamic forces given by the coupling frame that connects the [PT](#) with the trolley. The behavior and range of this force is presented in Fig [3.4a](#). For this numerical study the states and states space equation matrix are described as follows:

$$\mathbf{x}_k = \begin{bmatrix} p_k \\ v_k \\ F_{total} \end{bmatrix}, \quad (3.25)$$

where p_k , v_k , correspond to the position and velocity of the trolley cars and F_{total} is the overall acting on the trolley car. Leading to the following state space equation:

$$\mathbf{x}_{k+1} = \mathbf{A}_{\text{aug}} \mathbf{x}_k, \quad \mathbf{y}_k = \mathbf{H}_{\text{aug}} \mathbf{x}_k$$

$$\mathbf{A} = \mathbf{I}_3 + \Delta t \begin{bmatrix} \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1/2m_t \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

An important aim of this numerical study is to quantify how the choice of prior distributions influences the Kalman filter estimates. To that end, we construct priors that are consistent with the observed coasting behavior of the system. The nominal parameters seen in Fig. 3.14a generate a coasting distance of 25.5 m, which lies in the upper tail of the FMT coasting distance histogram as is referenced in Fig 3.14b. We therefore treat our lower reference values and expand upward to cover the plausible variability.

Uncertainty in the prior of the resistive force will be addressed using the force distributions developed in Chapter 2. Rolling resistance for the trolley and PT depend on the rolling on the rolling friction coefficient, μ_r . The literature reports variability of approximately 25% in μ_r [41]. This is used as a guide to establish the upper bound for the trolley and PT rolling friction as:

$$F_{fT} \in [144, 180] \text{ N}$$

$$F_{fPT} \in [82.2, 102.8] \text{ N}$$

which correspond to normal distributions

$$F_{fT} \sim \mathcal{N}(162, 6^2) \text{ N},$$

$$F_{fPT} \sim \mathcal{N}(92.5, 3.4^2) \text{ N}.$$

To ensure the upper tail of the priors is compatible with the data, we verified that taking the maximum friction values within these ranges and increasing the PT chain-friction force with in reasonable bounds considering a combine behavior of rolling and sliding behavior. This allows the model to reproduce the minimum observed coasting distance of approximately 16m (see Fig. 3.14b). This calibration implies the following working range for the chain friction force:

$$F_{chain} \in [2509, 4715] \text{ N}$$

which we represent as a normal distribution

$$F_{chain} \sim \mathcal{N}(3612, 367.7^2) \text{ N}.$$

The behavior of the force visualized in Fig. 3.4a is given by the interaction between the PT coupling frame and the trolley. This force comprehends the overall sum of all the resistive forces presented in Chapter 2. The mechanical properties of the coupling give the

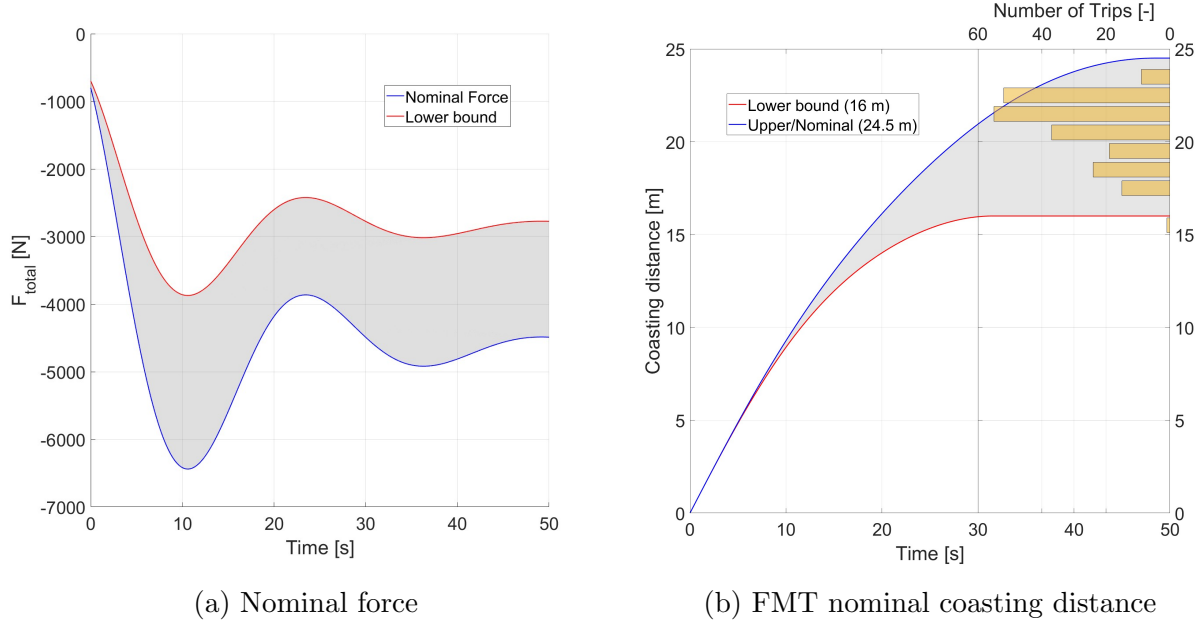


Figure 3.4: Model 3 nominal forces and FMT nominal coasting distance.

sinusoidal behavior of the force. The upper and lower end of the force are obtained by applying the described force ranges into the [FMT](#) model described in [Ch. 2](#).

Given these forces variability the following benchmark of priors is proposed to test the filter prediction capability under uncertain priors:

$$F_{total,0} \sim \mathcal{N}(\mu_{F_{total}}, \sigma_{F,0}^2) \Rightarrow \mu_{F_{Total}} \in \begin{pmatrix} 0 \\ 300 \\ 734 \\ 100 \\ 1400 \end{pmatrix}, \quad \sigma_{F_{total}} \in \begin{pmatrix} 10^0 \\ 10^1 \\ 10^2 \\ 10^3 \\ 10^4 \end{pmatrix}.$$

3.2.1 Analysis Metrics

To assess the robustness of the filter to prior accuracy and sensor noise, we track the following metrics:

Final (steady-state) covariance: We record the terminal value $\mathbf{P}_{T|T}$ and its scalars (trace, eigenvalues). Smaller steady-state values imply higher confidence; comparing across priors reveals how much the prior influences long-run uncertainty.

Distribution of estimates across runs: We summarize the distribution of the final estimates $\hat{\mathbf{x}}_{T|T}$ with sample mean and covariance. Coverage can be checked by reporting the fraction of runs in which the true parameter lies inside the 95% confidence interval defined by $\mathbf{P}_{T|T}$.

Accuracy: root-mean-square (RMS) error: For a scalar component z_k with ground truth $z_k^\star$ and estimate $\hat{z}_{k|k}$,

$$\text{RMS}(z) = \sqrt{\frac{1}{N} \sum_{k=1}^N (\hat{z}_{k|k} - z_k^\star)^2}.$$

We report RMS for the forces of interest and, when relevant, for kinematic states (velocity/position).

Consistency checks (NIS): We monitor the NIS value:

$$\text{NIS}_k = \mathbf{v}_k^\top \mathbf{S}_k^{-1} \mathbf{v}_k,$$

which should be statistically compatible with χ^2 distributions of degrees equal to the measurement and state dimensions, respectively. Persistent under/over-dispersion flags miss-tuned Q or R or overconfident priors.

Online MLE tracking of Q and R : We report $\mathbf{Q}, \mathbf{R}$ across runs. Stability of these estimates across prior choices is evidence of robustness; large drifts suggest mismatch between model and data or over/under-confident priors.

Together, these metrics quantify (i) uncertainty reduction, (ii) steady-state confidence, (iii) accuracy, and (iv) statistical consistency, while the MLE tracking of Q and R provides a principled check on noise-covariance assumptions.

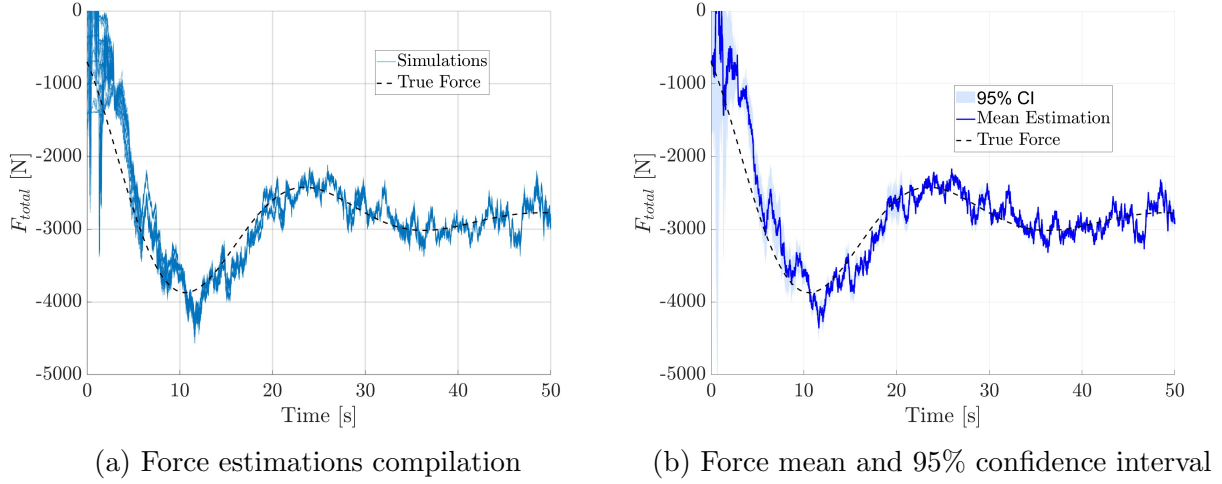


Figure 3.5: Force estimation estimation results

3.2.2 Results and analysis for 1 DOF

Figure 3.5 shows the force estimates across the set of prior choices. A key observation is that trajectories concentrate around similar values rather than diverging into multiple trajectories. This is also evident in Fig. 3.5b, where the across-simulation confidence band is considerably narrow suggesting a consistent convergence among the simulation benchmark. To evaluate the performance of the Kalman filter and verify the adequacy of the process noise covariance obtained through the Maximum Likelihood Estimation (MLE), one representative estimation run was analyzed in detail. The parameters used for this experiment are summarized in Table 3.1, which lists the initial prior for the total force, its associated covariance, and the noise parameters identified through the MLE routine.

Table 3.1: Parameters used for the estimation of residuals shown in Fig. 3.8.

Parameter	Value
Initial force prior, $F_{total,0}$	0 [N]
Initial force covariance, $P_{F_{total},0}$	1×10^5 [N ²]
Measurement noise, R	1.4×10^{-3}
Position process noise, Q_{11}	5.1×10^{-9}
Velocity process noise, Q_{22}	1.1×10^{-8}
Force process noise, Q_{33}	91.7

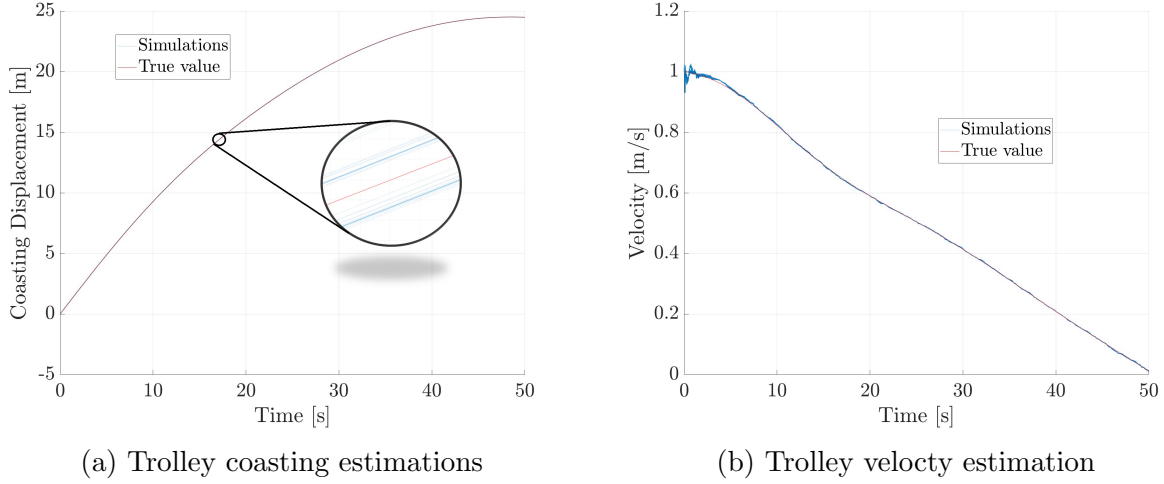


Figure 3.6: Position and velocity estimation for Model 1

Figure 3.7 illustrates the estimated total force $\hat{F}_{total}$ along with its 95% confidence interval, while Figure 3.8 presents the residuals obtained for the position, velocity, and force states. The residuals provide a key statistical diagnostic for assessing whether the process noise covariance Q is properly tuned. In the Kalman filter framework, the innovation or measurement residual is defined as

$$\nu_k = y_k - H \hat{x}_{k|k-1}, \quad (3.26)$$

with an expected covariance

$$S_k = H P_{k|k-1} H^\top + R, \quad (3.27)$$

where $P_{k|k-1}$ is the predicted state covariance and R is the measurement noise variance. The innovations ν_k are expected to be zero-mean, white, and Gaussian distributed if the assumed Q and R correctly represent the system's stochastic characteristics [79, 109].

For the unmeasured force state, the residual analysis was performed using the estimation error

$$e_{F,k} = \hat{F}_{total,k} - F_{true,k}, \quad (3.28)$$

whose empirical variance can be compared with the corresponding diagonal element of the posterior covariance $P_{33,k}$. The posterior covariance represents the filter's predicted uncertainty for the force state:

$$\sigma_{F,k}^2 = P_{33,k}. \quad (3.29)$$

In this case, the residuals of the force state exhibit a standard deviation of approximately 325.9 N, while the posterior covariance from the filter yields an estimated standard deviation of 286.3 N. These two values are in close agreement, differing by about 12%. This proximity indicates that the process noise variance $Q_{33} = 91.73$ obtained through the MLE approach appropriately captures the stochastic behavior of the unmeasured force dynamics. The ratio of the empirical residual variance to the theoretical variance,

$$\rho = \frac{\text{Var}(e_F)}{P_{33}} \approx \frac{(325.9)^2}{(286.3)^2} = 1.29, \quad (3.30)$$

is close to unity, which suggests a consistent and well-calibrated filter. Ratios in the range of $0.5 < \rho < 2$ are generally considered acceptable for well-tuned stochastic filters [20, 53].

If the process noise were underestimated (i.e., Q_{33} too small), the residual variance would be significantly higher than P_{33} , indicating that the filter is overconfident and unable to explain the variability observed in the measurements. Conversely, if Q_{33} were overestimated, the predicted confidence intervals would be excessively wide and the residuals would remain much smaller than expected. The results here demonstrate that neither behavior occurs: the true force trajectory in Fig. 3.7 remains largely within the 95% confidence bounds of the estimated signal, and the residuals in Fig. 3.8 are approximately zero-mean and bounded within the expected range.

Overall, these findings confirm that the process noise covariance estimated via MLE provides a statistically consistent description of the system dynamics. The agreement between the empirical residual variance and the theoretical covariance implies that the stochastic excitation of the force is properly characterized, and that the filter is optimally balancing model fidelity and measurement noise. This behavior is consistent with established criteria for filter consistency and residual analysis [79, 109, 53], validating the use of the MLE-based tuning procedure in this context.

To assess the sensitivity of the MLE-based estimation procedure to the choice of prior conditions, a set of experiments was conducted in which the initial mean and covariance of the total force prior were systematically varied. Figures 3.5 and 3.9 show that the estimated process noise variance for F_{total} exhibits only mild dependence on the prior selection. A consistent trend is observed: as the prior mean approaches zero, the optimizer increases the estimated process noise $Q_{F_{\text{total}}}$. This behavior is expected, since when the prior mean is significantly smaller than the actual (nonzero) force required by the data, the optimizer compensates by enlarging the random-walk variance to allow F_{total} to evolve more rapidly toward the measurement-consistent level. Conversely, when the prior is closer to the true or data-supported magnitude, the mismatch is smaller and the estimated Q converges to lower and more stable values.

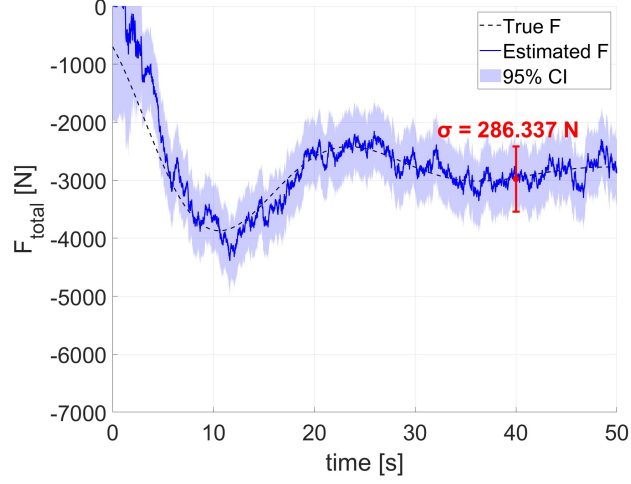


Figure 3.7: Estimated total force $\hat{F}_{total}$ with 95% confidence interval. The shaded region represents $\pm 1.96\sigma$, and the dashed line corresponds to the true applied force.

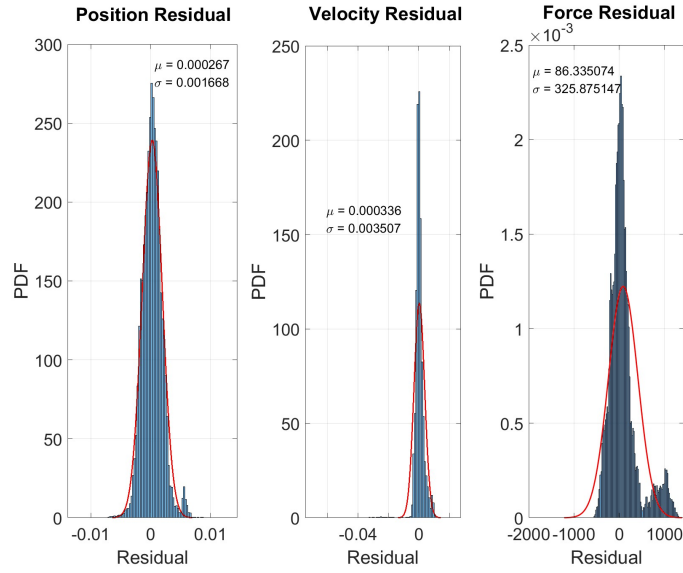


Figure 3.8: Residuals of position, velocity, and force states. The force residuals exhibit a standard deviation of 325.9 N, consistent with the estimated uncertainty of the filter.

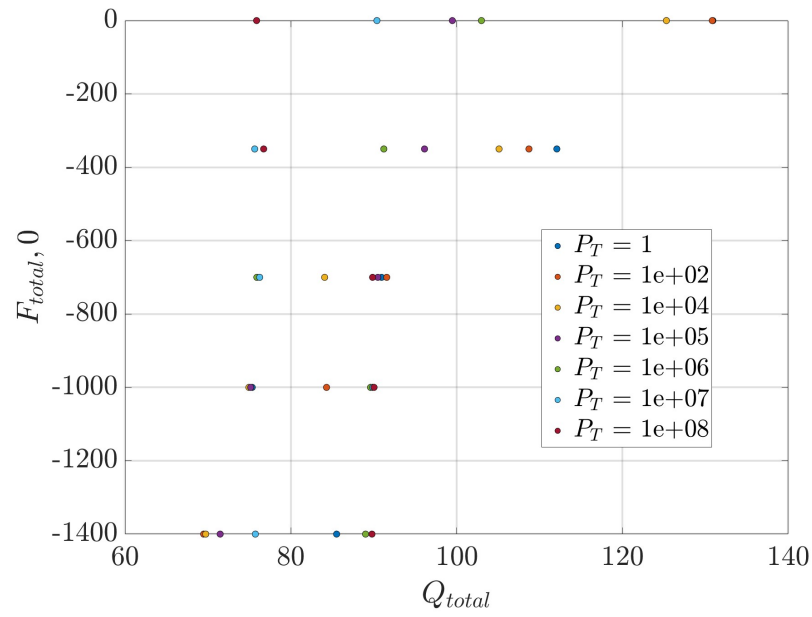


Figure 3.9: Estimated process noise $Q_{F_{total}}$ as a function of the prior mean for the 1 DOF model.

Figure 3.10 compares the root-mean-square error (RMSE) of the final force estimate with the corresponding posterior covariance $P_{33}(T)$. In this 1 DOF configuration, the RMSE of the estimated force remains relatively constant across different prior assumptions, and the final posterior covariance remains within the same order of magnitude. Variations in the covariance are primarily driven by changes in the estimated process noise Q , as shown previously in Fig. 3.9. Since the covariance update equation in the Kalman filter is given by

$$P_{k|k-1} = A P_{k-1|k-1} A^\top + Q, \quad (3.31)$$

any modification in Q directly influences the evolution of the posterior covariance $P_{33}(T)$, which quantifies the uncertainty in the estimated force. Hence, the process noise Q effectively controls the steady-state level of uncertainty encoded in P_{33} .

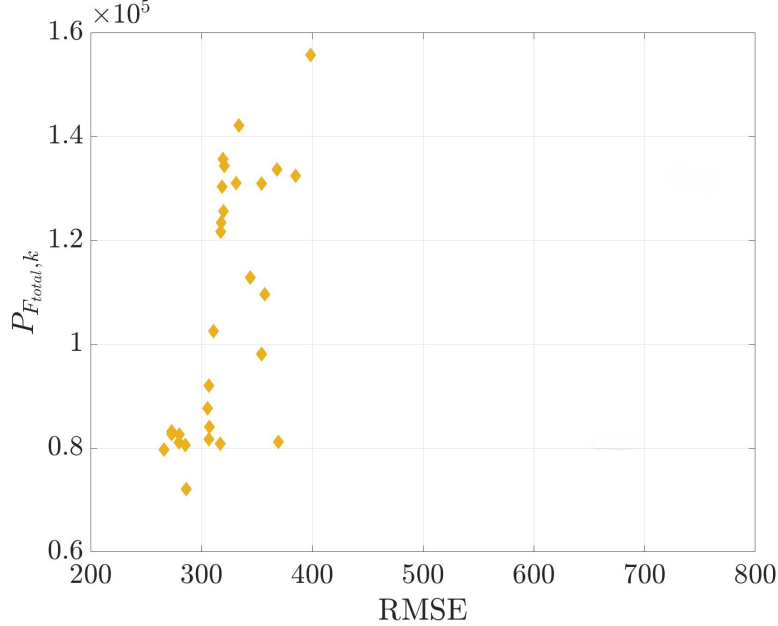


Figure 3.10: Comparison between the final posterior covariance $P_{33}(T)$ and the RMSE of the estimated total force for different prior selections.

Figures 3.10 and 3.9 together demonstrate the internal consistency of the estimation framework. Despite varying the initial prior assumptions, the MLE-based approach converges to statistically coherent solutions, where the posterior covariance P_{33} remains consistent with the observed RMSE of the estimated force. This indicates that the algorithm robustly identifies a process noise level Q that balances model dynamics and measure-

ment information, yielding similar uncertainty characterization regardless of the initial prior. Such robustness to prior selection is a desirable property of maximum-likelihood-based Kalman filtering methods, as also noted in [109, 53, 79], and confirms that the filter effectively adapts its uncertainty representation based on the data.

To further evaluate the statistical consistency of the filter and validate the accuracy of the estimated measurement noise covariance R , the NIS metric was computed for each simulation run.

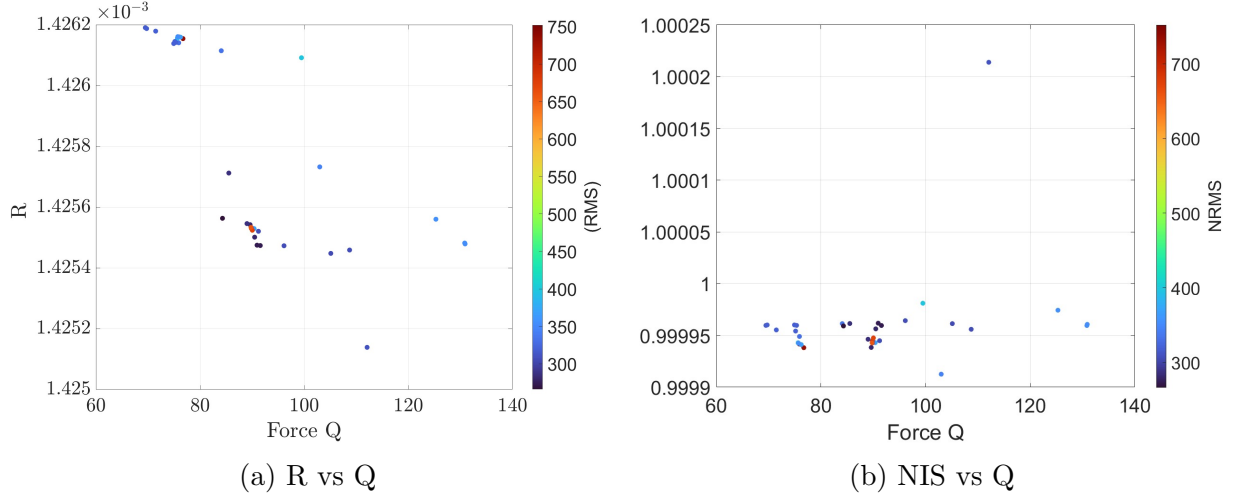


Figure 3.11: 1 DOF mean NIS value

Figure 3.11 presents the mean NIS values obtained across all prior configurations for the 1 DOF model. Regardless of the prior assumptions or the estimation error magnitude, the mean NIS remains close to unity. This behavior indicates that the filter innovations are statistically consistent with the predicted innovation covariance S_k , confirming that the measurement noise covariance R has been correctly estimated through the MLE procedure. The proximity of the mean NIS to its theoretical expectation validates the accuracy of the identified R and demonstrates that the filter is neither overconfident ($\text{NIS} < 1$) nor overly conservative ($\text{NIS} > 1$).

3.2.3 EM vs MLE

There are multiple approaches used by researchers to estimate optimal parameters for uncertain systems, such as the EM algorithm. However, not all approaches are suited for

to all applications. For this application, in which the state representing the resistive force is only weakly observed, the EM algorithm faces near-degeneracy in the E-step posterior that feeds into the M-step and stalls the update of the corresponding element of the process covariance $\mathbf{Q}$. In contrast, direct optimization through the MLE follows the shallow, but nonzero, gradient of the observed filtering likelihood with relative efficiency toward a local optimum in the solution space.

We consider the linear Gaussian state-space model for Model 1 with $\mathbf{A} = \mathbf{I} + \Delta t \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/m_t \\ 0 & 0 & 0 \end{bmatrix}$

and $\mathbf{H} = [1 \ 0 \ 0]$. In our experiments $\Delta t = 9.6 \times 10^{-4}$ and $m_t = 144,000$, so the first nonzero path from x_3 to the measurement is of order $\Delta t^2/m_t \approx 6.4 \times 10^{-12}$, i.e. extremely weak. Therefore, weak observability results poor conditioning. The observability stack $[\mathbf{H}; \mathbf{H}\mathbf{A}; \mathbf{H}\mathbf{A}^2]$ equals

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & \Delta t & 0 \\ 1 & \Delta t & \Delta t^2/m_t \end{bmatrix},$$

whose third column is $\Delta t^2/m_t \approx 6.4 \times 10^{-12}$ in our case.

Let ν_k and S_k be the innovation and innovation variance from the Kalman filter. The observed log-likelihood is

$$\ell(\theta) \equiv \log p_\theta(\mathbf{z}_{1:N}) = -\frac{1}{2} \sum_{k=1}^N (\log |S_k| + \nu_k^\top S_k^{-1} \nu_k + m \log 2\pi), \quad (3.32)$$

where $\theta = \{\mathbf{Q}, \mathbf{R}\}$ (with $\mathbf{A}, \mathbf{H}$ fixed by physics). MLE maximizes (3.32) directly. Gradients propagate through the Riccati recursion, e.g.

$$\frac{\partial \ell}{\partial \mathbf{Q}} = \frac{1}{2} \sum_{k=1}^N \left[(S_k^{-1} \nu_k \nu_k^\top S_k^{-1} - S_k^{-1}) : \frac{\partial S_k}{\partial \mathbf{Q}} \right], \quad \frac{\partial S_k}{\partial \mathbf{Q}} = \mathbf{H} \frac{\partial \mathbf{P}_{k|k-1}}{\partial \mathbf{Q}} \mathbf{H}^\top, \quad (3.33)$$

with $\partial \mathbf{P}_{k|k-1}/\partial \mathbf{Q}$ obtained by sensitivity recursions.

EM constructs the *expected complete-data log-likelihood*

$$\mathcal{Q}(\theta | \theta^{(i)}) = \mathbb{E}_{\mathbf{x}_{1:N} | \mathbf{z}_{1:N}, \theta^{(i)}} [\log p_\theta(\mathbf{x}_{1:N}, \mathbf{z}_{1:N})], \quad (3.34)$$

using the smoothed posterior from a Kalman smoother at $\theta^{(i)}$ (E-step), then maximizes (3.34) in θ (M-step). For $\mathbf{Q}$, the M-step is *closed-form*:

$$\mathbf{Q}^{(i+1)} = \frac{1}{N-1} \sum_{k=2}^N \mathbb{E} \left[(\mathbf{x}_k - \mathbf{A}\mathbf{x}_{k-1})(\mathbf{x}_k - \mathbf{A}\mathbf{x}_{k-1})^\top \middle| \mathbf{z}_{1:N}, \theta^{(i)} \right], \quad (3.35)$$

or, in terms of smoothed moments,

$$\mathbf{Q}^{(i+1)} = \frac{1}{N-1} \sum_{k=2}^N \left(\underbrace{(\mathbf{P}_{k|N} + \hat{\mathbf{x}}_{k|N} \hat{\mathbf{x}}_{k|N}^\top)}_{\mathbb{E}[\mathbf{x}_k \mathbf{x}_k^\top]} - \mathbf{A} \underbrace{(\mathbf{P}_{k-1,k|N} + \hat{\mathbf{x}}_{k-1|N} \hat{\mathbf{x}}_{k|N}^\top)}_{\mathbb{E}[\mathbf{x}_{k-1} \mathbf{x}_k^\top]} \right. \\ \left. - \underbrace{(\mathbf{P}_{k,k-1|N} + \hat{\mathbf{x}}_{k|N} \hat{\mathbf{x}}_{k-1|N}^\top)}_{\mathbb{E}[\mathbf{x}_k \mathbf{x}_{k-1}^\top]} \mathbf{A}^\top + \mathbf{A} \underbrace{(\mathbf{P}_{k-1|N} + \hat{\mathbf{x}}_{k-1|N} \hat{\mathbf{x}}_{k-1|N}^\top)}_{\mathbb{E}[\mathbf{x}_{k-1} \mathbf{x}_{k-1}^\top]} \mathbf{A}^\top \right). \quad (3.36)$$

Equations (3.35)–(3.36) are the standard EM updates for linear dynamical systems [107, 47, 42].

The Fisher information matrix $\mathcal{I}(\theta)$, with $\theta = \mathbf{Q}$, is defined as the negative expected Hessian of the log-likelihood,

$$\mathcal{I}(\theta) = -\mathbb{E} \left[\frac{\partial^2}{\partial \theta^2} \log p_\theta(\mathbf{z}_{1:N}) \right],$$

and is block-structured across all parameters in $\mathbf{Q}$. For Gaussian innovations a useful form is

$$\mathcal{I}_{ij} = \frac{1}{2} \sum_{k=1}^N \text{tr} \left(S_k^{-1} \frac{\partial S_k}{\partial \theta_i} S_k^{-1} \frac{\partial S_k}{\partial \theta_j} \right),$$

where S_k is the innovation covariance and θ_i, θ_j index parameters of $\mathbf{Q}$.

When $\theta_{ii} = Q_{33}$,

$$\frac{\partial S_k}{\partial Q_{33}} = \mathbf{H} \frac{\partial P_{k|k-1}}{\partial Q_{33}} \mathbf{H}^\top,$$

and $\partial P_{k|k-1}/\partial Q_{33}$ excites only the third state component. Since the measurement sees x_3 only through the tiny factor $\Delta t^2/m_t$, every term in the corresponding Fisher block is suppressed by $\mathcal{O}((\Delta t^2/m_t)^2) \approx 10^{-24}$.

Thus the overall Fisher information matrix $\mathcal{I}(\theta)$ is well-conditioned in most directions but nearly singular in the Q_{33} direction. This means the system is technically observable in rank but extremely ill-conditioned, and estimates of Q_{33} will carry huge uncertainty.

The expectation in (3.35) is taken under the current parameter $\theta^{(i)}$. If $Q_{33}^{(i)}$ is small, the smoother posterior variance $(\mathbf{P}_{k|N})_{33}$ becomes small; then the expected residual variance in the (3, 3) entry remains small, so $\mathbf{Q}_{33}^{(i+1)}$ barely increases. EM thus feeds its current uncertainty back into the M-step. Formally, the EM convergence rate is governed by the

fraction of missing information matrix; when the observed Fisher information I_{obs} is tiny relative to the complete-data information I_{comp} , the spectral radius of $I_{\text{miss}}I_{\text{comp}}^{-1}$ approaches 1, and EM becomes very slow [83, 80].

MLE optimizes its gradient and does not use the smoothed posterior at $\theta^{(i)}$ but the observed innovations and their sensitivities. Even if the gradient is small, it is nonzero and an optimizer (with line search or trust region) can take steps that escape the “near-zero” updates that EM produces in (3.36). Empirically, we observe that Q_{total} moves to more realistic values under MLE, while EM remains stuck near its initialization. This behavior is observed in Fig. 3.12b, where we can see the for the EM algorithm the changes in Q_{total} are minimal.

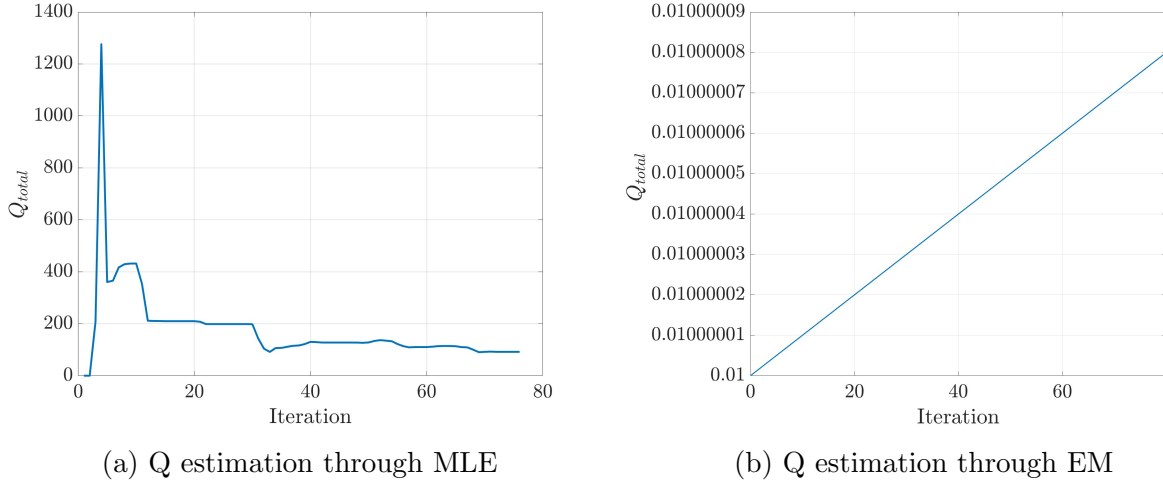


Figure 3.12: Force state uncertainty estimation

3.3 Force Localization Study

While the S-DOF model yields a useful and reliable estimate of the total resistive force acting on the FMT, distinguishing and estimating the individual force components offers distinct advantages for condition and health monitoring. We therefore introduce a 2-DOF model that explicitly represents both the trolley and PT while simplifying the interaction of the two trolley cars to a rigid connection. The purpose is to examine whether the filter can still separate the main resistive contributions for each subsystem with reduced structural

complexity. The forces to be identified are an overall trolley resistance (F_1) and an overall PT resistance (F_2), shown in red in Figure 3.13.

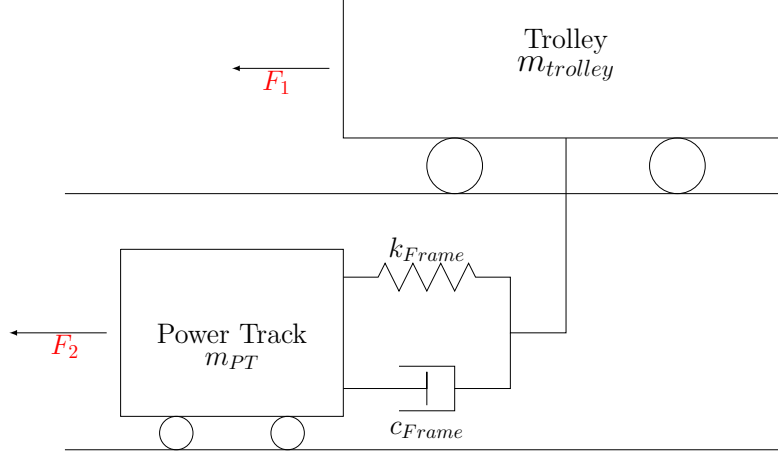


Figure 3.13: A 2-DOF model that includes the rolling friction and power track cable chain friction as parameters to be estimated. This model approximates all dominant physical mechanisms and serves as a surrogate of the full model.

As in Sec. 3.2, the model is going to be tested over a range of difference priors. The forces ranges established for the trolley and PT rolling friction and the PT cable chain friction, we are going to find the ranges for the overall forces acting on the trolley and PT.

$$F_1 \in [534, 565] \text{ N}$$

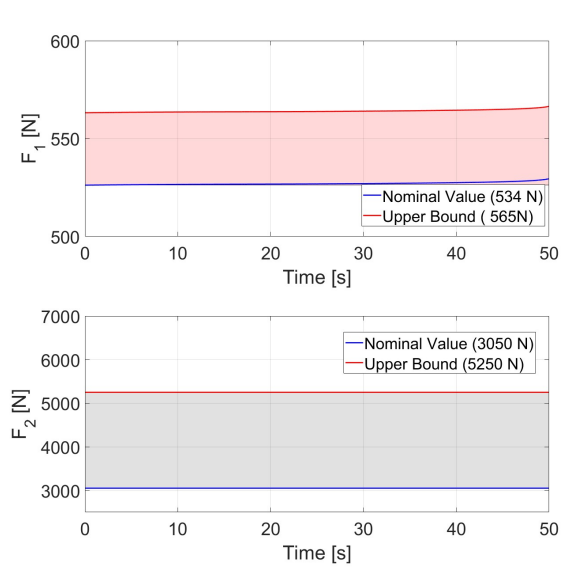
$$F_2 \in [2509, 4715] \text{ N}$$

This can be expressed as a normal distribution for trolley force (F_1) and PT as:

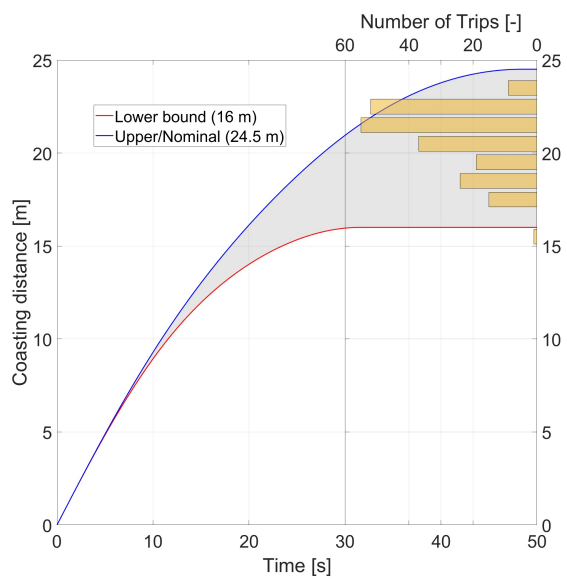
$$\begin{aligned} F_1 &\sim \mathcal{N}(549.5, 5.2^2) \text{ N}, \\ F_2 &\sim \mathcal{N}(4150, 366.7^2) \text{ N}. \end{aligned}$$

For this numerical study the states of are defined as follows:

$$\mathbf{x}_k = \begin{bmatrix} p_k \\ v_k \\ F_1 \\ F_2 \end{bmatrix}, \quad (3.37)$$



(a) Nominal forces



(b) FMT nominal coasting distance

Figure 3.14: Model nominal forces and FMT nominal coasting distance.

where $p_k = \begin{bmatrix} p_{t,k} \\ p_{pt,k} \end{bmatrix}$, $v_k = \begin{bmatrix} v_{t,k} \\ v_{pt,k} \end{bmatrix}$, corresponding to position and velocity of the trolley cars and power track respectively, and the force block is $\begin{bmatrix} F_{1,k} \\ F_{2,k} \end{bmatrix}$. The structural parameters are defined as follows:

$$\mathbf{M} = \begin{bmatrix} 2m_t & 0 \\ 0 & m_{pt} \end{bmatrix}, \quad \mathbf{C}_f = \begin{bmatrix} c_f & -c_f \\ -c_f & c_f \end{bmatrix}, \quad \mathbf{K}_f = \begin{bmatrix} k_f & -k_f \\ -k_f & k_f \end{bmatrix}.$$

Leading to the following state space equation:

$$\mathbf{x}_{k+1} = \mathbf{A}_{\text{aug}} \mathbf{x}_k, \quad \mathbf{y}_k = \mathbf{H}_{\text{aug}} \mathbf{x}_k$$

$$\mathbf{A}_{\text{aug}} = \mathbf{I}_6 + \Delta t \begin{bmatrix} \mathbf{0}_{2 \times 2} & \mathbf{I}_2 & \mathbf{0}_{2 \times 2} \\ -\mathbf{M}^{-1} \mathbf{K}_f & -\mathbf{M}^{-1} \mathbf{C}_f & \mathbf{M}^{-1} \\ \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \end{bmatrix}, \quad \mathbf{H}_{\text{aug}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

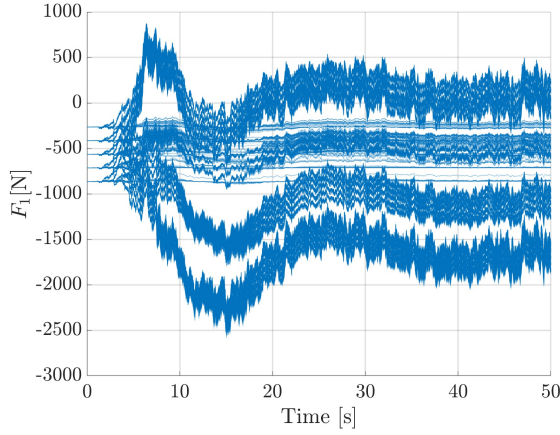
Where the following benchmark of priors are proposed:

$$F_{1,0} \sim \mathcal{N}(\mu_{F_1}, \sigma_{F,0}^2) \Rightarrow \mu_{F_{1T}} \in \begin{pmatrix} 400 \\ 500 \\ 534 \\ 575 \\ 700 \end{pmatrix}, \quad \sigma_{F_1} \in \begin{pmatrix} 10^{-1} \\ 10^0 \\ 10^1 \\ 10^2 \\ 10^3 \end{pmatrix}.$$

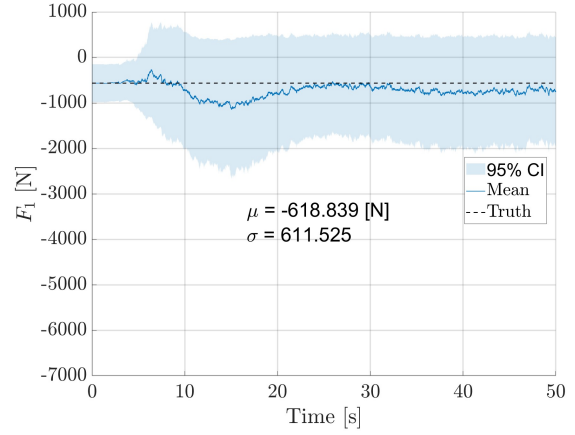
$$F_{2,0} \sim \mathcal{N}(\mu_{F_2}, \sigma_{F,0}^2) \Rightarrow \mu_{F_2} \in \begin{pmatrix} 750 \\ 1700 \\ 3050 \\ 4000 \\ 5200 \end{pmatrix}, \quad \sigma_{F_2} \in \begin{pmatrix} 10^0 \\ 10^1 \\ 10^2 \\ 10^3 \\ 10^4 \end{pmatrix}.$$

3.3.1 Result and analysis - 2 DOF

Results for the trolley force F_1 and the power-track force F_2 are shown in Figs. 3.15 and 3.16. Across the set of prior choices, individual runs of the filter converge to different steady

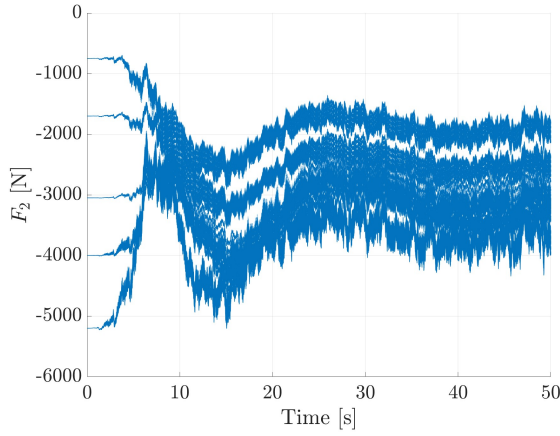


(a) Trolley force estimations compilation

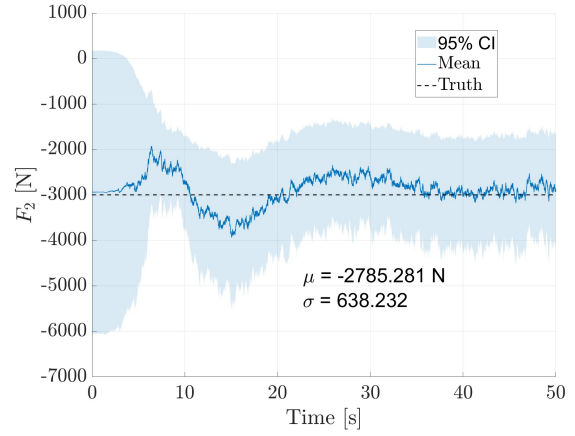


(b) Trolley force 95% confidence interval

Figure 3.15: Trolley Force estimation estimation results



(a) PT force estimations compilation



(b) PT force 95% confidence interval

Figure 3.16: PT force estimation estimation results

values; taken together, the ensemble can be seen to be unidentifiable given the available sensor data. Despite this, the ensemble mean remains close to the ground-truth forces throughout.

For F_1 (trolley), the across-simulation dispersion (visualized as a confidence band over the set of runs) increases over time, reflecting the fact that subsets of runs settle at different levels farther from the true value. For F_2 (power-track), a similar multi-modal pattern is present, but the dispersion tends to decrease with time, indicating better agreement among runs. However when analyzing how spread both forces estimation are we can see that the standard deviations are of magnitudes relatively close to each other ($\sigma_{F_1} = 611.5 \text{ N}$ and $\sigma_{F_2} = 638.2 \text{ N}$), showing a relatively same level of uncertainty in the mean estimation across all the estimations. This being a consequence of convergences to multiple combinations of F_1 and F_2 converging to different values.

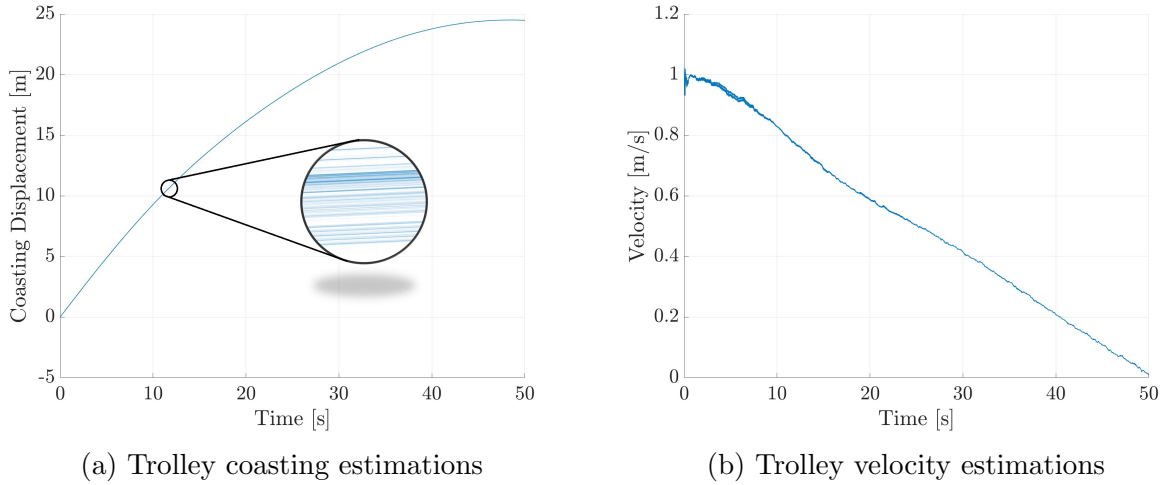


Figure 3.17: Force estimation estimation results

Figures 3.17 and 3.18 illustrate the phenomenon described above: multiple combinations of the forces lead to different final estimates while producing essentially the same measured response. Importantly, these distinct estimates all satisfy the measurement data and are therefore indistinguishable on the basis of the available sensor alone. This reflects an observability/identifiability limitation—individual force contributions cannot be uniquely resolved from a single measurement. To clarify the source of this ambiguity, recall that in the augmented Kalman filter formulation (Model 1) the state vector is defined as:

$$x = [x_1 \ x_2 \ v_1 \ v_2 \ F_1 \ F_2]^\top,$$

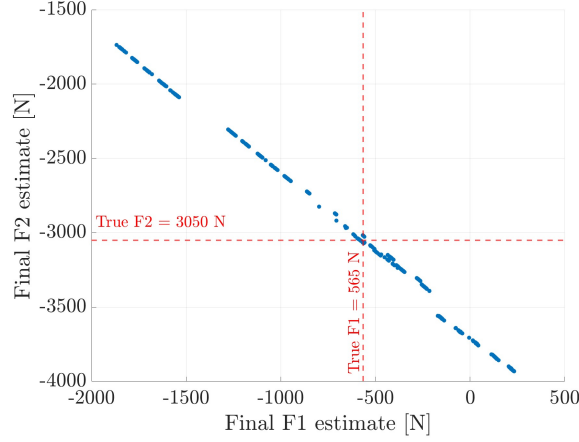


Figure 3.18: F_1 and F_2 converged combinations

where x_1, x_2 are displacements, v_1, v_2 are velocities, and f_1, f_2 are the unknown external forces treated as random-walk states. The measurement equation is

$$y = Hx = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} x,$$

i.e., only the displacement x_1 is measured.

For a discrete-time linear system $x_{k+1} = Ax_k$, $y_k = Hx_k$, the observability matrix is

$$\mathcal{O} = \begin{bmatrix} H \\ HA \\ HA^2 \\ \vdots \\ HA^{n-1} \end{bmatrix},$$

with n the number of states. Full rank of $\mathcal{O}$ is required for complete state observability. Using the augmented system matrices (A, H) , we obtain $\text{rank}(\mathcal{O}) = 4 < 6$. Hence, two dimensions of the state space are unobservable.

The right nullspace of $\mathcal{O}$ reveals the unobservable directions. Numerical computation shows that these are spanned almost entirely by the force states:

$$n_1 \approx \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^\top, \quad n_2 \approx \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^\top.$$

This means that perturbations in F_1 or F_2 alone do not alter the measured output sequence y_k , making the forces individually unobservable. This means that for high magnitude

estimation of one of the forces the other decreases to compensate and keep the equilibrium. However the overall force experienced by the trolley should give a better estimation. We have coupled the estimated trolley force with estimated PT to compute the overall force acting on the trolley. The results are visualized in Figure 3.19. Here we can visualize that as iterations advance the simulations start to converge to same magnitude, however during the first seconds of the simulation the filter struggles to find the correct overall given the fact that during this time the filter is looking to converge to the best estimation of F_1 and F_2 that satisfies the measurement. We can also see the impact of different F_2 force assumptions in the path that the overall force takes, and how high force assumptions make the filter to take more time to converge to the true force. After approximately 25 seconds Figure 3.19a shows a convergence of all the simulation to same force magnitude and starts to follow more accurately the path of the true force acting on the trolley.

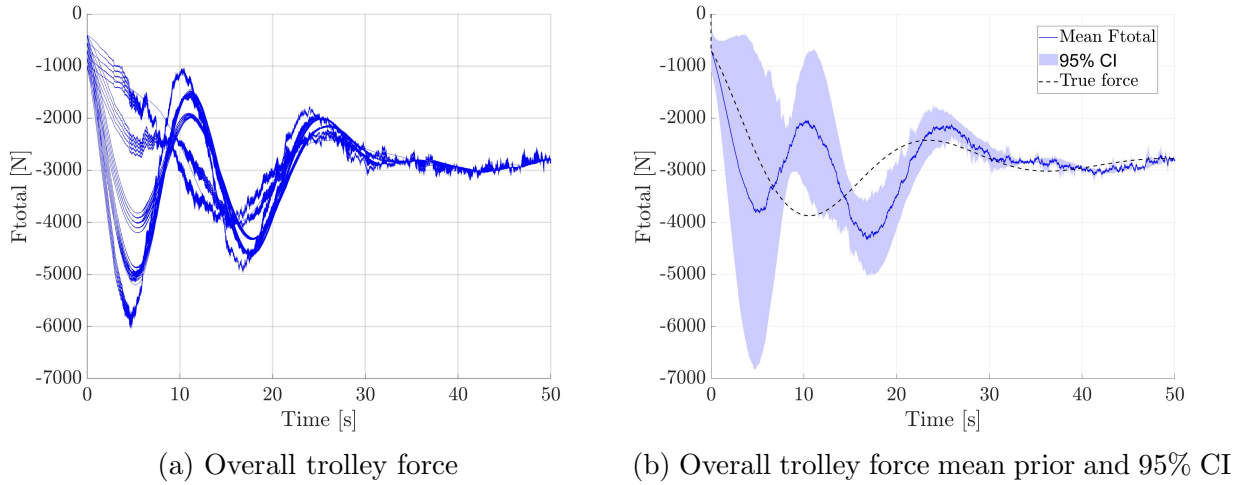


Figure 3.19: Results for the overall force estimation

This study varies the prior means and variances for the forces and examines how they affect the MLE estimates of the process noise Q . Figure 3.20 summarizes the impact of the priors on the estimated Q ; Fig. 3.20a focuses on F_1 and colors points by the prior assumed for F_2 . We take the ground-truth values to be $F_1^* = 565$ N and $F_2^* = 3050$ N.

When the prior for F_2 is close to its true value (e.g., $F_{2,0} \approx -3.0 \times 10^3$ N), the MLE typically assigns a small process noise to F_1 (low Q_{F_1}), and the corresponding estimates of F_1 are tight. As the F_2 prior moves farther from truth, the optimizer compensates by increasing the process noise attributed to F_1 , which manifests as larger Q_{F_1} and a wider spread in the F_1 estimates. This dependence is clearly visible in Fig. 3.20a, where

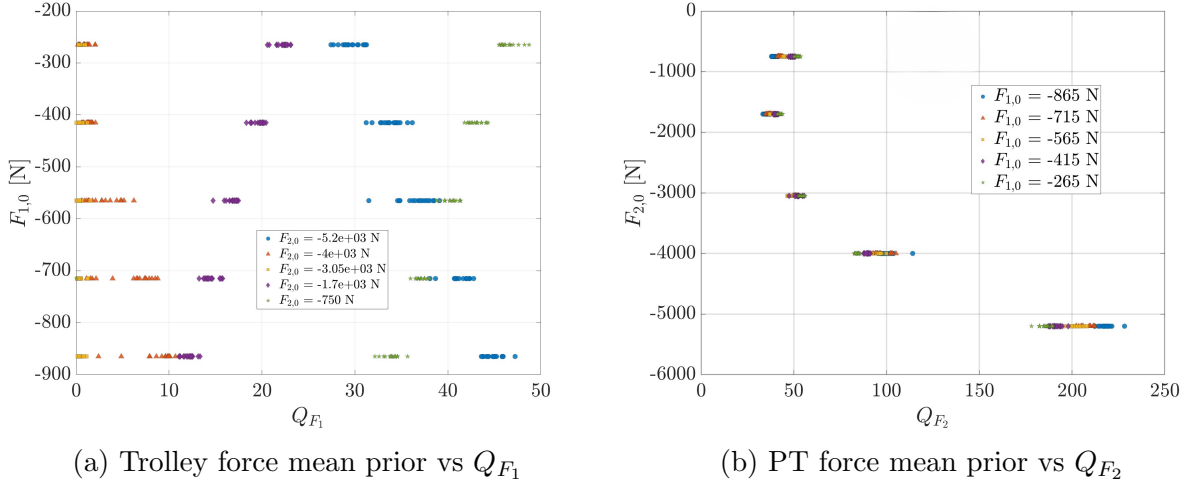


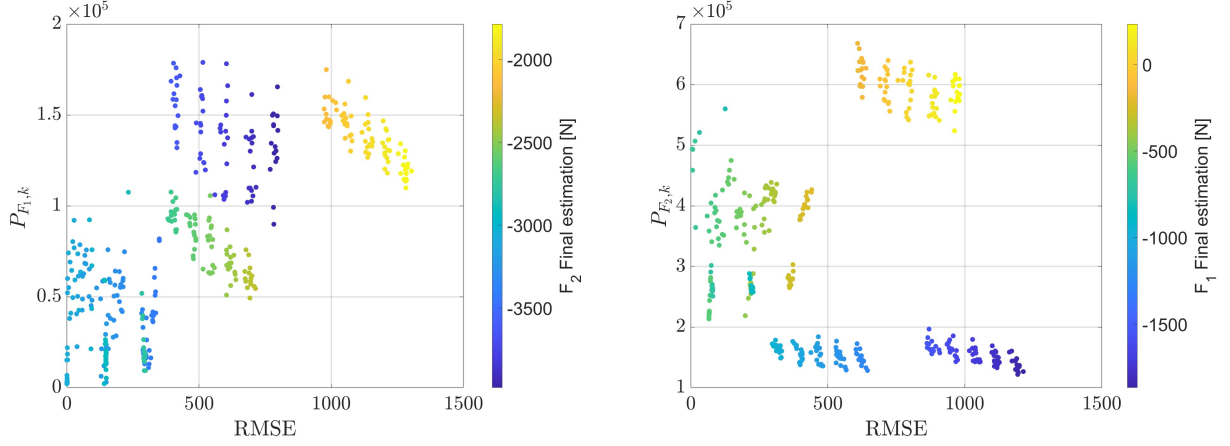
Figure 3.20: 2-DOF Model PT Force estimation Results

classes defined by $F_{2,0}$ separate according to the magnitude of the estimated Q_{F_1} . Different behavior were observe in Fig. 3.20b, the prior assumed for F_1 has only a weak effect on the F_2 noise selected by the MLE. This asymmetry is expected from the model due to the following cases:

- Measurement sensitivity. Only the displacement of the trolley is measured ($H = [1 \ 0 \ 0 \ 0 \ 0]$). The path from the forces to that measurement goes through the second block row of A , where the forces enter as $[F_1 \ F_2]^T$ and then couple via (Kf, Cf) . With $M = \text{diag}(2m_t, m_{pt2})$, the gains $1/(2m_t)$ and $1/m_{pt2}$ are generally unequal. Hence, for comparable force changes, the induced innovations are more sensitive to F_2 than to F_1 .
- Signal magnitude (SNR) The nominal magnitude of F_2 is several times larger than that of F_1 . The combination of a larger input gain and a larger true force yields a higher effective SNR for F_2 in the innovations.

Consequently, misspecifying the F_1 prior produces only a modest bias in the innovations, so the optimizer does not need to inflate Q_{F_2} to explain the data; Q_{F_2} remains relatively stable across F_1 priors. In contrast, misspecifying the F_2 prior strongly perturbs the innovations seen through the measured coordinate; the MLE then compensates by adjusting the other force's process noise (notably Q_{F_2}), which is the dependence highlighted earlier for Fig. 3.20a.

Another useful perspective is to examine the relationship between the final covariance of the estimated forces and their estimation accuracy. The motivation is that, in real-world applications, the true forces are unknown; therefore, the final covariance reported by the filter is often used as a proxy for the reliability of the estimates (see Fig. 3.21).



(a) Trolley force final covariance vs RMSE (F_2 estimation reference) (b) Trolley force final covariance vs RMSE (F_1 estimation reference)

Figure 3.21: 2-DOF Model final covariance estimations

Overall, the results show that the lowest covariance values tend to occur when the estimates of F_1 and F_2 are close to the true forces. However, within this cluster, there are also simulations where the estimates deviate significantly from the truth while the covariance remains small. This suggests that the filter can sometimes be overconfident, producing estimates with low reported uncertainty despite noticeable bias.

This mismatch highlights an important limitation: the final covariance captures internal consistency of the filter but not necessarily external accuracy. In other words, the filter may be “confident but wrong.” Thus, while final covariance is a valuable diagnostic, it is not sufficient on its own as a reliability metric.

3.4 Conclusions

We established a robust methodology for accurate force estimation, thereby providing a trustworthy and reliable framework for subsequent analyses. The 1 DOF model (Model 1)

demonstrated better accuracy and robustness than the 2 DOF model (Model 2), indicating that, for the present problem setup, it is better suited to force estimation. The model structure avoids some of the ill-conditioning that arises in the higher-dimensional formulation and provides more reliable convergence in practice. This makes Model 1 particularly well suited for [SHM](#), where stable and accurate force estimates are essential for detecting abnormal behavior, identifying potential damage at an early stage, and supporting informed maintenance and safety decisions over the service life of the structure.

The structural cause of unobservability in Model 2 can be traced to two facts:

1. Only one displacement x_1 is measured. Without additional measurements of x_2 or of velocities/accelerations, the effects of F_1 and F_2 cannot be disentangled.
2. The stiffness and damping matrices

$$K_f = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}, \quad C_f = \begin{bmatrix} c & -c \\ -c & c \end{bmatrix},$$

are rank-one, coupling only relative motion. This leaves a rigid-body mode of the two masses that is not sensed by the output.

As a result, many combinations of (F_1, F_2) generate identical trajectories of $y(t)$. In maximum-likelihood estimation this appears as a likelihood ridge, where different force allocations achieve the same fit to the observed displacement. The filter therefore converges to different solutions depending on prior choices, reflecting structural unidentifiability.

From the numerical study of Model 1 we find that what is reliably estimated is the combined effect F_{total} acting on the trolley. The estimated process covariance Q captures the variability of this total force: small diagonal entries correspond to regimes where the force residuals are stably identified, while larger entries reflect higher uncertainty or mismatch between the model and data. Comparing Q with the residuals of force/state estimates confirms that Q serves as a proxy for how much unexplained variance the filter attributes to the force input.

A notable finding is that in Model 1 the estimated Q values were more consistent across runs and did not strongly depend on the prior mean of the force. This suggests that the simplified model is not only more robust numerically but also less sensitive to initialization, reflecting a more stable and better-conditioned estimation problem. In contrast, Model 2 exhibited strong dependence on the priors, consistent with the structural unidentifiability discussed above.

In practice, this means that although individual forces F_1 and F_2 cannot be separated in the current single-output setting, the filter can still deliver a robust estimate of their net contribution. The final covariance values therefore provide useful diagnostic information: low covariance coincides with accurate recovery of F_{total} , but low covariance does not guarantee correctness when the model is structurally unidentifiable. Hence, the posterior covariance is informative about confidence, but must always be interpreted in light of the underlying identifiability limitations.

Chapter 4

Experimental data

In this chapter, we present the steps taken to process, decode, and validate the experimental data provided by [OPG](#). The raw data was initially encoded in the form of Gray code, a binary numeral system where two successive values differ by only a single bit. While this encoding is advantageous in minimizing errors during digital transitions, it is not directly suitable for numerical analysis. Therefore, the first step involved decoding the Gray code into standard binary format to render the data interpretable and analyzable. The objective in this stage is to visualize the Gray code as position values corresponding to the trolley at every time step.

Once the data was decoded, we undertook a validation phase to ensure the accuracy of the conversion. This was achieved by cross-referencing key data points with a trolley log file also supplied by [OPG](#). These key points served as reference markers, allowing us to verify the correctness of the decoded data and to identify any anomalies or inconsistencies. This process was critical to ensure that the dataset could be reliably used for further analysis.

Using the decoded data as a measurement, the S-DOF modeling approach is employed to identify the overall resistive force acting on the trolley, thus allowing the methodology proposed in [Chapter 3](#) to be evaluated with real-world data.

In summary, this chapter outlines the critical steps of data decoding, validation against ground truth, and method application.

4.1 Fueling Machine Encoder

The fueling machine encoder is a critical component that enables precise tracking of the fueling machine's position. This device is mechanically coupled to the drive system of the trolley and plays a central role in monitoring its motion. Specifically, it is a shaft encoder, a type of sensor designed to provide digital feedback on the angular position of rotating mechanical components.

The encoder used in the [FMT](#) system employs a 13-bit Gray code encoding scheme. The encoder is composed of two main rotating drums: a high-speed drum and a low-speed drum. These drums are mechanically linked to the motion of the trolley and rotate accordingly. As they rotate, metal brushes mounted on a fixed frame come into contact with conductive segments on the drums. This contact between the brushes and conductive tracks completes electrical circuits, allowing the system to detect binary voltage states.

Each drum is patterned to generate specific Gray code outputs. The pattern of brush contact with conductive and non-conductive segments forms the basis of the Gray code reading. When a brush makes contact with a conductive segment, it registers a binary '1'; when there is no contact, it registers a binary '0'.

The high-speed drum is responsible for the finer resolution of motion and generates 7 bits of the total Gray code. The low-speed drum provides 6 bits and advances more slowly. The design functions similarly to a mechanical odometer: for every full rotation of the high-speed drum, the low-speed drum advances by one bit position. This hierarchical configuration allows the encoder to capture a broad range of motion with high precision. Together, the high-speed and low-speed drums produce a 13-bit Gray code that uniquely represents the position of the trolley at any given moment. For this process the information generated by high speed drum was sufficient to decode the position of the system. The detail of the data are described in [Table 4.1](#).

4.2 Gray Code

The Gray code has the unique property to ensure that transitions between consecutive values result in minimal change, reducing the likelihood of measurement errors in digital systems during state transitions making the Gray code of great utility for tracking angular position in rotative machinery[\[52\]](#).

Gray code sequences can be generated algorithmically or via reflection. For instance, starting with a 1-bit Gray code $\{0, 1\}$, a 2-bit sequence is generated by reflecting and

Table 4.1: Summary of collected data.

Parameter	Value
Sampling rate [Hz]	200
Data length [samples]	427285
Total duration [s]	2136.425
Number of bits	7
Measurement device / sensor	Gray code encoder
Sensor placement	Front trolley car

prefixing the original with ‘0’ and the reflected version with ‘1’, resulting in $\{00, 01, 11, 10\}$.

4.2.1 Gray Code Decoding

In traditional digital systems, Gray code is typically decoded to binary using a bitwise Exclusive Or method. This approach ensures that the single-bit transition property of Gray code is preserved during decoding, minimizing the possibility of errors due to timing mismatches or electrical noise [56]. However, in many mechanical systems—such as absolute rotary encoders—a more practical and widely used approach is adopted: the Gray code is first decoded to binary, then converted to its decimal equivalent, which acts as an index into a pre-calibrated position reference table.

The purpose of decoding the Gray code in our setup is to recover the angular position of rotating machinery from an n -bit absolute encoder. Binary-reflected Gray codes are widely used in rotary/linear encoders because consecutive codewords differ by exactly one bit, which minimizes readout errors during transitions and misalignment [52, 99]. In practice, Gray words are typically converted to ordinary binary and then mapped to an integer position index; this Gray→binary conversion is standard across encoder applications and provides a robust basis for angle estimation. The decoding from Gray code $G = g_{n-1}g_{n-2} \dots g_0$ to binary $B = b_{n-1}b_{n-2} \dots b_0$ follows the recursive rule:

$$b_0 = g_0 \tag{4.1}$$

$$b_i = b_{i-1} \oplus g_i \quad \text{for } i = 1, 2, \dots, n-1 \tag{4.2}$$

Once the binary sequence is obtained, it is converted to its decimal equivalent:

$$D = \sum_{i=0}^{n-1} b_i \cdot 2^i \quad (4.3)$$

Here, D represents the decoded decimal value, which acts as the position index, which then can be related to an angular position of the system.

Table 4.2 illustrates the decoding of 4-bit Gray code values to binary and their corresponding decimal indices.

Table 4.2: Example of 4-bit Gray Code Decoding

Gray Code	Binary (Decoded)	Decimal Index
0000	0000	0
0001	0001	1
0011	0010	2
0010	0011	3
0110	0100	4
0111	0101	5
0101	0110	6
0100	0111	7
1100	1000	8
1101	1001	9
1111	1010	10
1110	1011	11
1010	1100	12
1011	1101	13
1001	1110	14
1000	1111	15

This sequence shows that each Gray code value differs from the previous one by only one bit, fulfilling the unit-distance requirement. After decoding to binary, the resulting decimal index is used to look up or calculate the associated angular displacement.

Such lookup-based decoding strategies are widely documented in industrial automation and instrumentation systems and are considered standard practice in absolute position sensing [59, 43, 52].

In our case, each 13-bit Gray code reading from the fueling machine encoder is treated as a unique index that corresponds to a specific angular position or relative displacement

of the [FMT](#). This approach eliminates the need for complex binary arithmetic and allows for fast, table-based position decoding, which is highly advantageous in real-time control systems.

4.3 Gray code conversion to FMT linear displacement

This section describes the method used to convert the Gray code output of the fueling machine encoder into a linear displacement of the [FMT](#).

The drums are connected via a gear reduction system such that the low-speed drum advances one bit position for every complete revolution of the high-speed drum. This hierarchical design extends the encoder’s measurement range but is not necessary for determining short-range or incremental displacement.

For the purposes of this study, only the output from the high-speed drum is used to track the linear displacement of the [FMT](#). According to the encoder’s datasheet, each increment (i.e., each Gray code step) from the high-speed drum corresponds directly to a fixed linear displacement of the fueling machine of $\Delta x = 78$ mm.

The experimental data received from OPG exhibited a systematic fault in the Gray code values generated by the high-speed drum of the fueling machine encoder. Specifically, the least significant bit (LSB), i.e., bit 0, of the 7-bit Gray code was found to toggle erratically between 0 and 1 at every time step, regardless of the actual mechanical movement of the [FMT](#).

This erroneous behavior was observed at a consistent sampling rate of 0.05 s (i.e., 20 Hz). Table 4.3 demonstrates the faulty toggling of bit 0 over several time steps, while all other bits remained constant:

Table 4.3: Example of Faulty Bit 0 in Gray Code Readings

Time (s)	Gray Code (7-bit)
0.00	1001100
0.05	1001101
0.10	1001100
0.15	1001101
0.20	1001100

To mitigate the effect of this faulty signal, the decision was made to ignore bit 0 entirely

during Gray code decoding. This effectively reduces the number of usable bits from 7 to 6, thereby reducing the total number of distinct positions from $2^7 = 128$ to $2^6 = 64$.

As a consequence, each jump in the 6-bit Gray code now represents a movement that spans two positions in the original 7-bit resolution. Since each 7-bit position corresponds to a displacement of 78 mm, each 6-bit position step now represents:

$$\Delta x = 156 \text{ mm} \quad (4.4)$$

This correction restores the reliability of the displacement measurement at the cost of halving the resolution. Table 4.4 illustrates this mapping, these values map to the same corrected 6-bit Gray code (100110), ensuring that bad data does not result in false displacement readings. The position is only updated when a more significant bit changes, corresponding to actual motion of the [FMT](#).

Table 4.4: Validation of Bit 0 Removal

7-bit Gray Code	No Bit 0 (6-bit)	$\Delta x \rightarrow 78 \text{ mm}$	$\Delta x \rightarrow 156 \text{ mm}$
1001100	100110	0 mm	0 mm
1001101	100110	78 mm	0 mm
1001111	100111	156 mm	156 mm
1001110	100111	234 mm	156 mm
1001010	100101	312 mm	312 mm
1001011	100101	390 mm	312 mm
1001001	100100	468 mm	468 mm
1001000	100100	546 mm	468 mm
1000000	100000	624 mm	624 mm
1000001	100000	702 mm	624 mm
1010001	101000	780 mm	780 mm

4.3.1 OPG Data Set

The following results were obtained by applying the corrected Gray code decoding method to the dataset provided by [OPG](#). As illustrated in Figure 4.1, the [FMT](#) begins its journey at the west fuel handling and auxiliary area, denoted as W-FFAA. From there, it travels sequentially to each reactor, beginning with Reactor 1 (R1), where it coasts and comes to a full stop for operational procedures.

After completing activities at R1, the operator drives the [FMT](#) forward to R2, then to R3, R4, and ultimately arrives at the east fuel handling and auxiliary area, labeled as E-FFAA. The return trip mirrors this route in reverse order, stopping again at each reactor face before finally returning to the W-FFAA. Information regarding the collected data is detailed in [Tab:4.1](#).

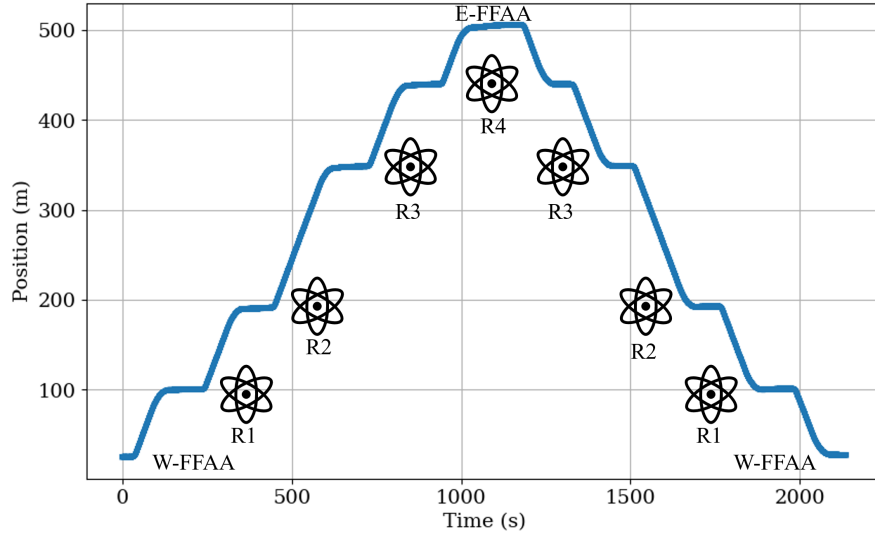


Figure 4.1: FMT decoded trajectory.

4.4 Data Validation

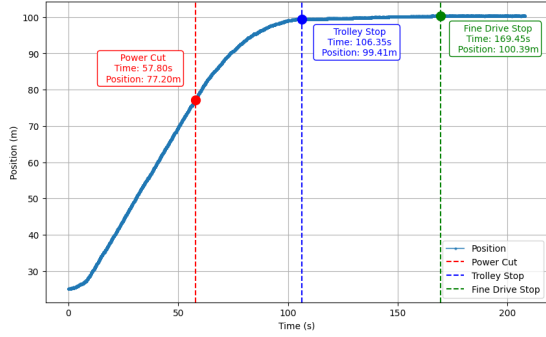
The [OPG](#) log file contains operational information for each [FMT](#) trip, and as such can be used to validate the encoder outputs. Data from the log files associated with the encoder measured trips are shown in [Table 4.5](#). Each trip represents a segment of the [FMT](#)'s journey across the reactor faces and fuel handling stations. The dataset includes positional coordinates, coasting zones, and distances traveled.

[Table 4.5](#) presents the starting location on the track where the [FMT](#)'s coarse drive begins propelling the system. The coasting start point indicates where the coarse drive motor is shut down, followed by the end point, where the [FMT](#) comes to a complete stop. After this, the system is finely adjusted using the fine drive to reach the exact operational location.

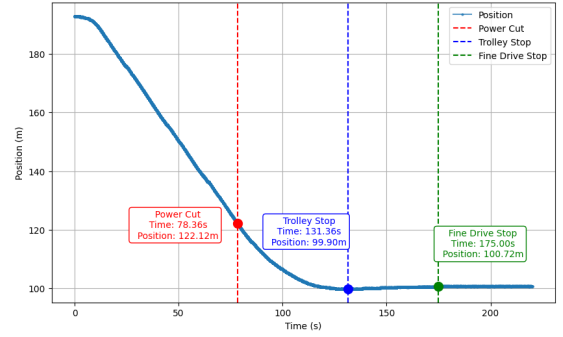
Table 4.5: OPG log file data

Destination	Starting Loc (m)	Coasting Start (m)	End Point (m)	Coast Dist (m)	Direction
R1	24.95	77.2	99.42	22.22	West–East
R2	100.39	169.92	189.66	20.74	
R3	190.92	324.01	346.30	22.29	
R4	347.01	415.09	438.15	23.06	
E-FFAA	439.25	478.88	503.87	24.13	
R4	505.83	460.47	439.48	20.99	East–West
R3	439.33	369.24	348.63	20.61	
R2	348.09	212.65	192.21	20.44	
R1	192.85	122.12	99.60	22.52	
W-FFAA	100.44	49.99	27.58	22.41	

Using this information, we segmented the data into individual trips performed by the FMT. This allowed us to clearly identify and mark the key reference points listed in Table 4.5. The results in Fig. 4.2 show the estimated position trace obtained from the decoded



(a) FMT transit from W-FFAA to R1



(b) FMT transit from R2 to R1

Figure 4.2: Arrivals to R1

coasting data. To validate the coasting distance against the event log, we identified the coasting start and end points by aligning the decoded encoder signal with the coasting time interval reported in the log. This procedure enables a direct comparison between the distance computed from the Gray-code decoder and the coasting distance reported by the log.

From Fig. 4.2, the decoded coasting distance is 22.21 m for W-FFAA→R1, whereas the log reports 22.22 m. The absolute difference is therefore 0.01 m, corresponding to a relative error of approximately 0.045% with respect to the log value. This close agreement indicates that the Gray-code decoding and time alignment are consistent with the logged coasting interval. The error results for all the trips are presented in Tab 4.6.

Table 4.6: OPG log file data for FMT data validation

Trip	Log file coast (m)	Gray code coast (m)	Error (%)
E-FFAA→R1	22.22	22.21	0.05
R1→R2	20.74	20.01	3.52
R2→R3	21.29	21.54	1.17
R3→R4	23.06	23.30	1.04
R4→E-FFAA	24.99	24.13	3.44
E-FFAA→R4	20.99	20.44	2.62
R4→R3	20.69	20.06	3.04
R3→R2	20.44	20.42	0.10
R2→R1	22.52	22.22	1.33
R1→W-FFAA	22.41	22.58	0.76

To analyze the velocity and acceleration of the [FMT](#), a numerical approximation of these values for each trip was determined using a finite difference approach. The objective was to evaluate whether the key points provided in the log file—such as the start of coasting or complete stoppage—correspond accurately with the dynamic behavior of the system as derived from the decoded position data. This method is commonly used in motion analysis due to its simplicity and effectiveness in handling time-series data [\[25\]](#).

The results in [Fig 4.3](#) were obtained for the arrivals at the reactor positions, showing how the computed velocity and acceleration align with the expected operational transitions defined in the log file. In the plots, the yellow-shaded regions represent the coasting phase, during which the coarse drive motor is shut down and the system continues moving due to inertia. The green-shaded regions correspond to the fine drive phase, where precise adjustments are made to accurately position the [FMT](#) at the designated stop point.

In addition to the successful validation of the encoder data, it is important to note that the [FMT](#) exhibits a relatively constant velocity of approximately 1 m s^{-1} across all transit segments. This consistent behavior confirms the stable operation of the coarse drive system during the initial propulsion phase showing an operational velocity of 1 m/s, this confirms our assumption that the system is moving at this speed before coasting.

4.5 Method Application

In this section, we apply the [KF](#) estimation workflow process described in [Chapter 3](#), this time using experimental measurements obtained from the encoder of the physical setup.

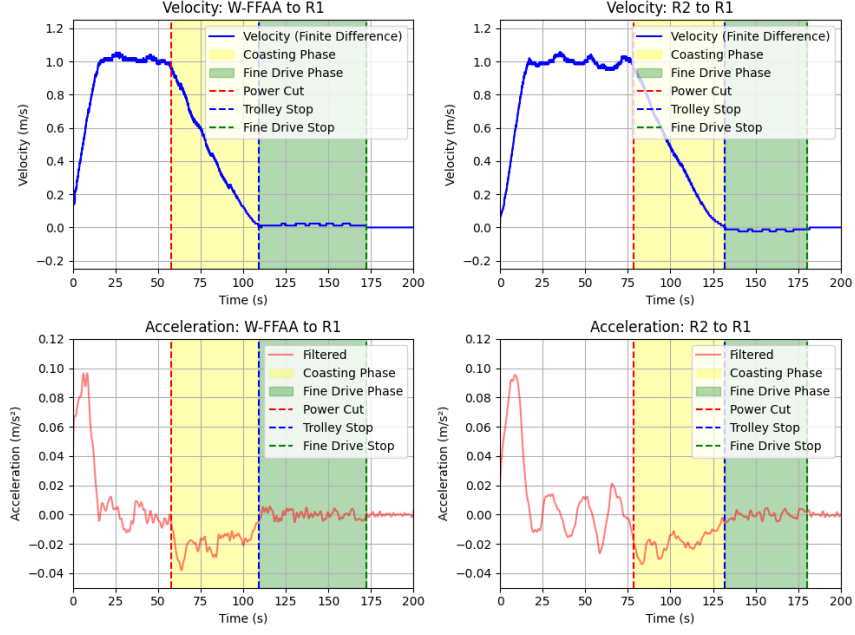


Figure 4.3: velocity and acceleration for R1 arrivals

The purpose of this section is to validate the proposed estimation framework under realistic measurement conditions and assess its performance when confronted with sensor noise and unmodeled dynamics.

From the numerical study, it was established that the most effective configuration was the S-DOF model, shown in Fig. 3.3. This simplified representation was found to provide stable and accurate estimates of the total resistive force acting on the trolley, denoted as F_{total} . We recall that the state vector for this model is defined as:

$$\mathbf{x}_k = \begin{bmatrix} p_k \\ v_k \\ F_{\text{total}} \end{bmatrix}$$

Leading to the following state space equation:

$$\mathbf{x}_{k+1} = \mathbf{A}_{\text{aug}} \mathbf{x}_k, \quad \mathbf{y}_k = \mathbf{H}_{\text{aug}} \mathbf{x}_k$$

$$\mathbf{A} = \mathbf{I}_3 + \Delta t \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/2m_t \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Before applying the Kalman filter we ask whether the measurement noise can be modeled as zero-mean, white, and approximately Gaussian. From finite data we cannot *prove* normality; instead we gather evidence via residual analysis [66, 53]. We exploit a short equilibrium segment where the trolley travels at a known constant speed $v^* = 1$ m/s. Let $\{z_k\}$ be the measured velocities at times $\{t_k\}$. We form noise-only residuals by subtracting the constant reference,

$$e_k = z_k - v^*,$$

and examine their distribution within that window (Fig. 4.4). The histogram plot of $\{e_k\}$ is visually close to normal. It is reasonable in this operating regime to model the measurement noise as zero-mean, white, and approximately Gaussian—the standard assumption used by the Kalman filter.

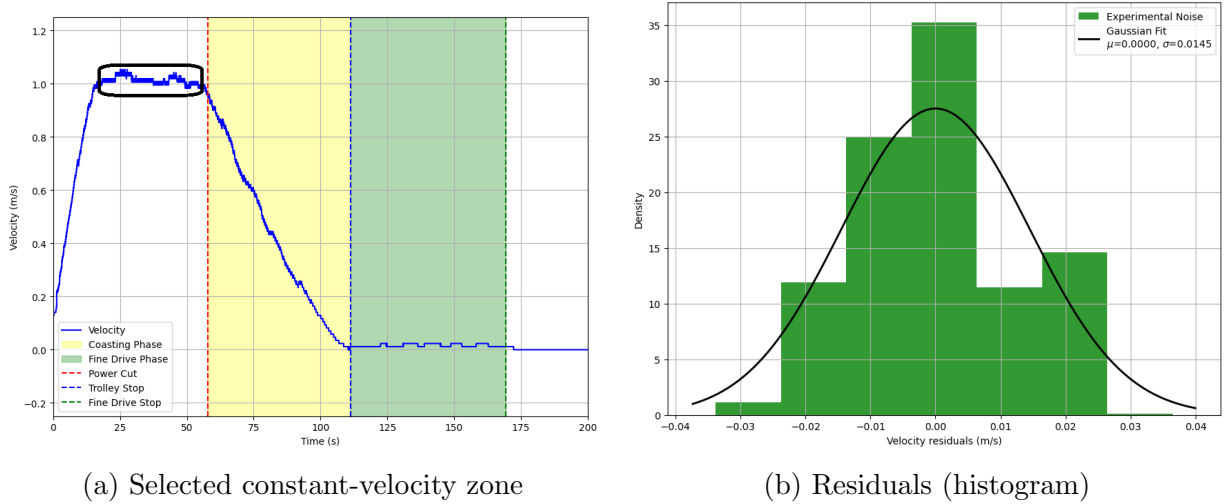
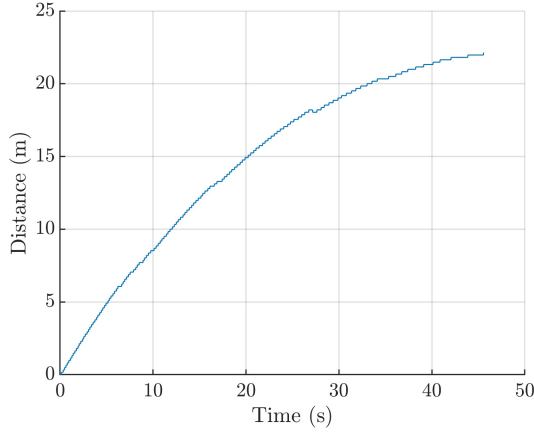


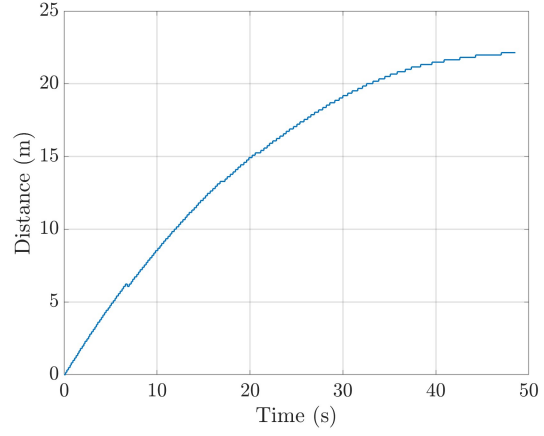
Figure 4.4: Velocity residuals $\{e_k\}$ during the equilibrium segment.

4.5.1 R1 arrival

In this section we propose to analyze the estimation approach applied to arrivals at R1, displayed in Fig. 4.5. To evaluate and verify the estimation method under real conditions, we repeat the same numerical study described previously, but now using experimental data. In this case we select the coasting distance shown in Fig. 4.5a we analyze the coasting phase of the FMT during its arrival from the West Maintenance Station and R2 to the R1 terminal. The information corresponding to experimental analysis in detailed Tab:4.7.



(a) WFFAA→R1



(b) R2→R1

Figure 4.5: FMT coasting motion for R1 arrivals

Table 4.7: Summary of collected data for R1 arrival.

Parameter	WFFAA→R1	R2→R1
Sampling rate [Hz]	200	200
Data length [samples]	9111	9712
Total duration [s]	45.550	48.555

To assess the stability and sensitivity of the filter under experimental conditions, we employ the same prior benchmarks used in the numerical study. The initial total force is modeled as a Gaussian random variable:

$$F_{\text{total},0} \sim \mathcal{N}(\mu_{F_{\text{total}}}, \sigma_{F,0}^2),$$

where different mean and variance combinations are explored to evaluate the filter's robustness to initialization. The priors are defined as:

$$\mu_{F_{\text{total}}} \in \begin{pmatrix} 0 \\ 300 \\ 734 \\ 1000 \\ 1400 \end{pmatrix}, \quad \sigma_{F_{\text{total}}} \in \begin{pmatrix} 10^0 \\ 10^1 \\ 10^2 \\ 10^3 \\ 10^4 \end{pmatrix}.$$

Results

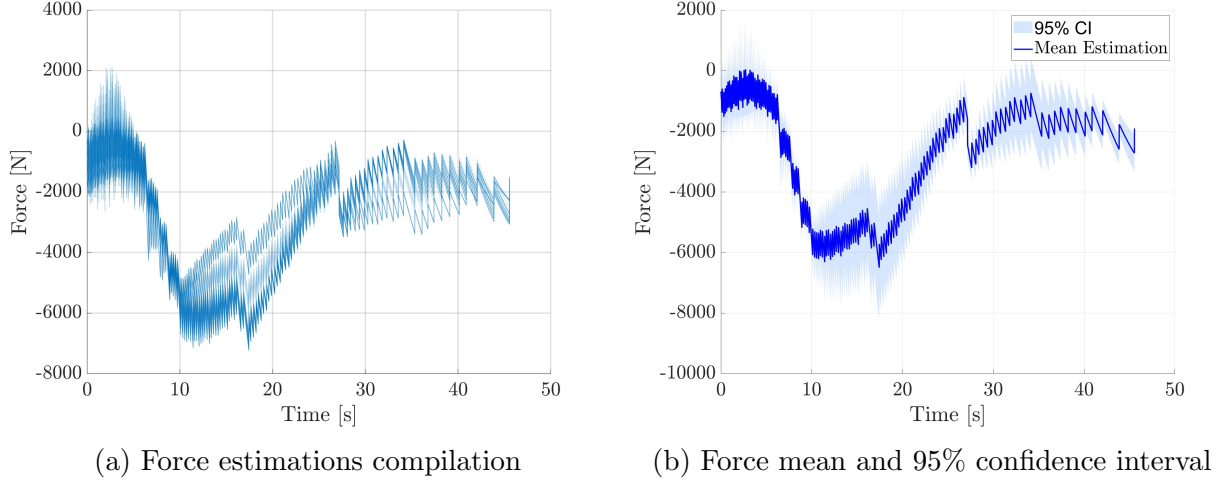


Figure 4.6: Force estimation results WFFAA→R1

Figures 4.6 and 4.7 show the force estimates across the set of prior choices for the two arrivals at R1. Just like in the numerical study describe in Sec. 3.2 the trajectories concentrate around similar values, showing consistent convergence among the simulation benchmark, showing again robustness to the different prior assumptions.

To evaluate the performance of the Kalman filter and verify the adequacy of the process noise covariance obtained through MLE, one representative estimation run for each arrival was analyzed in detail. The parameters used for this experiment are summarized in Table 4.8, which lists the initial prior for the total force, its associated covariance, and the noise parameters identified through the MLE procedure.

Table 4.8: Parameters used for the estimation shown in Fig. 4.8.

Parameter	Value	Value
Initial force prior, $F_{total,0}$	0 [N]	0 [N]
Initial force covariance, $P_{F_{total},0}$	1×10^6 [N ²]	1×10^6 [N ²]
Measurement noise, R	7.8×10^{-6}	7.3×10^{-6}
Position process noise, Q_{11}	3.8×10^{-4}	3.8×10^{-4}
Velocity process noise, Q_{22}	1.5×10^{-4}	3.2×10^{-4}
Force process noise, Q_{33}	1.2×10^4	5.9×10^4

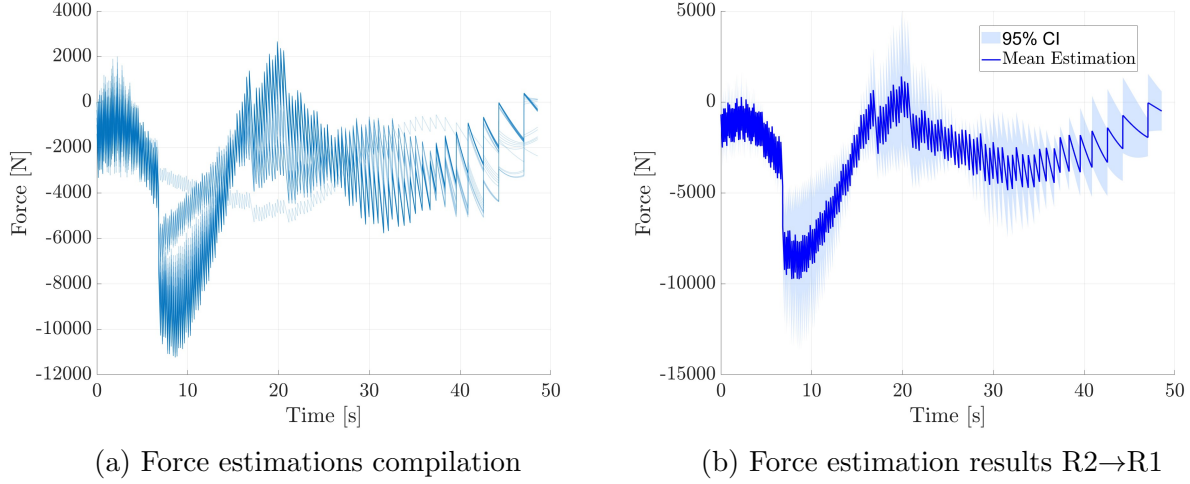


Figure 4.7: Force estimation estimation results

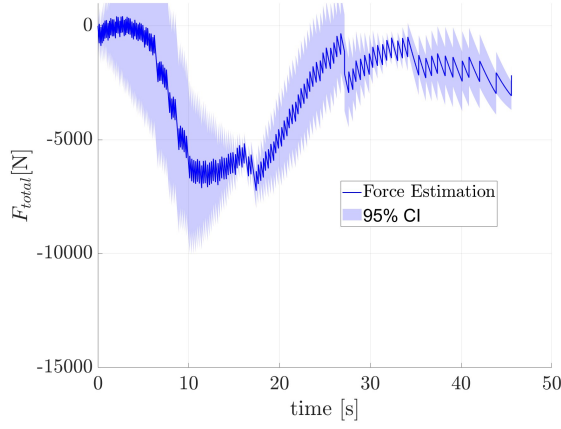
The estimated total force is presented in Fig. 4.8, along with its 95% confidence interval for each arrival. In Fig. 4.8a, the estimation exhibits physically consistent evolution, with uncertainty bounds that adapt over time as more measurements are incorporated. This indicates that the selected Q and R values effectively capture the underlying system and measurement variability. Fig. 4.8b shows a less consistence behavior of the uncertainty bounds is observed. This uncertainty increments could be due to several factors. Among the most common, changes in the resistive forces behavior or encoder lecture errors. This leads to the KF to increase the uncertainty during these time steps of the estimation.

To interpret if the estimation for Q and R are correctly estimated we use the residuals as guidance:

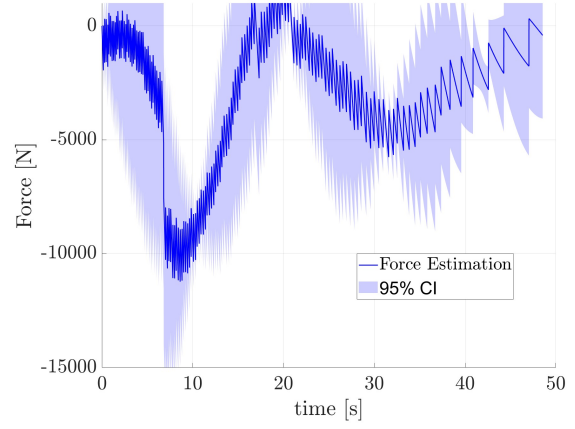
$$\nu_k = y_k - H \hat{x}_{k|k-1}, \quad (4.5)$$

Where ν_k are expected to be zero-mean, white, and Gaussian distributed if the assumed Q and R correctly represent the system's stochastic characteristics. Figure 4.9 shows the empirical probability density function (PDF) of the residuals obtained from the experimental data. The residuals exhibit a near-zero mean of 1×10^{-6} and a standard deviation of 3.9×10^{-4} and 3.7×10^{-4} , indicating that the innovations are centered around zero and consistent with the Gaussian assumption. This behavior suggests that the filter is statistically consistent and that the identified noise covariances (Q, R) are appropriate for the observed data.

To assess the sensitivity of the MLE-based estimation procedure to the choice of prior conditions, we explore the impact of prior assumptions on Q estimation. In figure 4.10, we

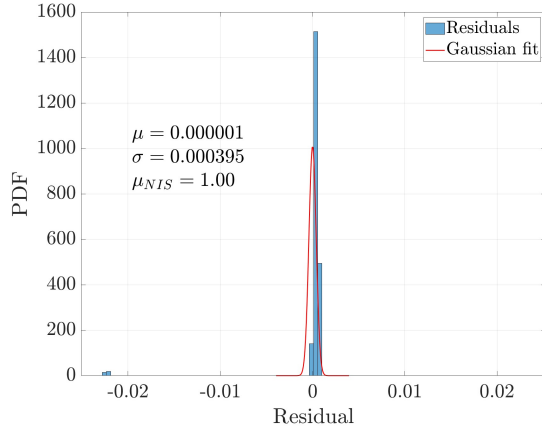


(a) WFFAA→R1

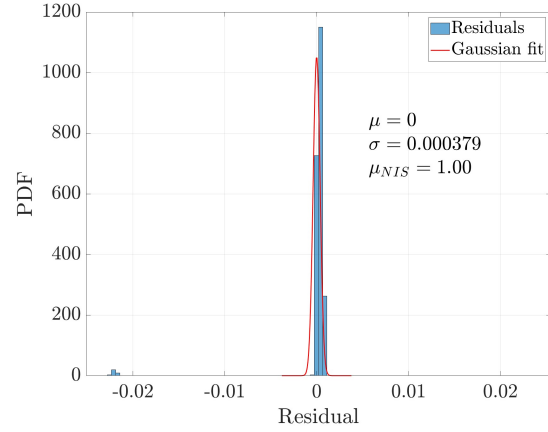


(b) R2→R1

Figure 4.8: Estimated total force $\hat{F}_{total}$ with 95% CI



(a) WFFAA→R1



(b) R2→R1

Figure 4.9: Measurement residual distribution

see a similar behavior as the numerical study in Sec.3.2 for the two arrival measurements, where as the prior mean approaches zero, the optimizer increases the estimated process noise. This behavior shows that priors close to zero are relatively far from the true value as the optimizer is trying to compensate by increasing the force variance to allow the force to evolve more rapidly toward the measurement-consistent level. Importantly, the estimate of Q is coupled with the assumed initial covariance of the estimation (P). This two factors are working together to compute an overall uncertainty.

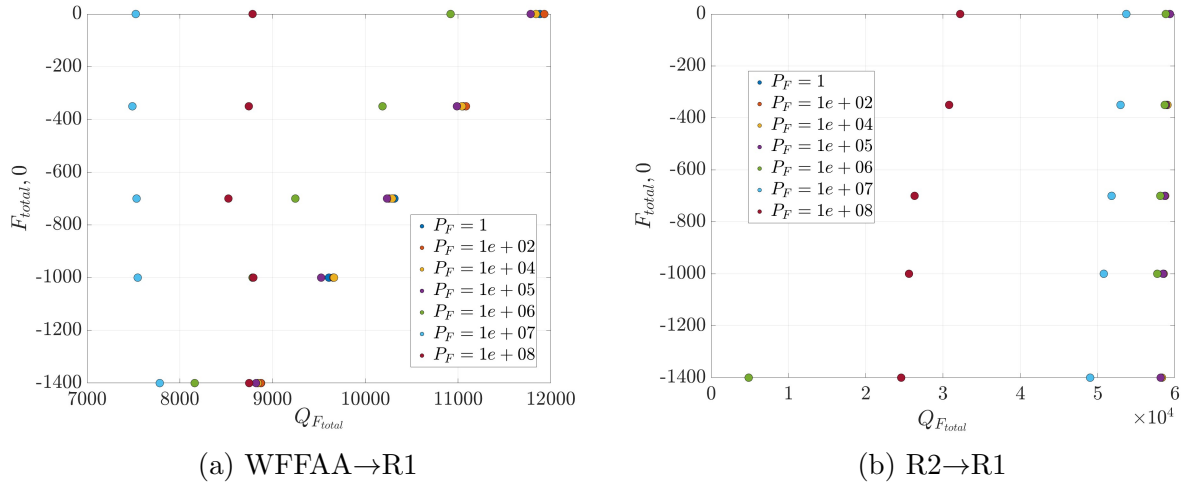
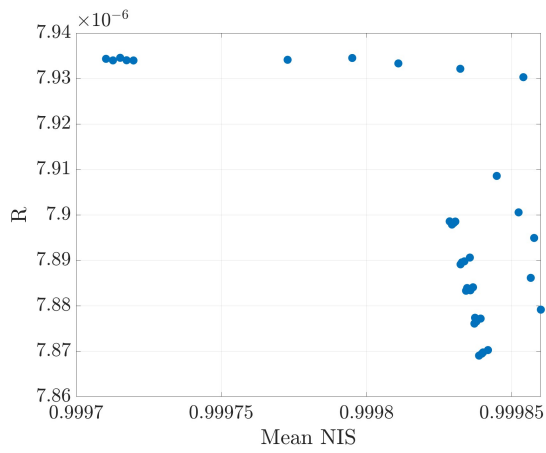
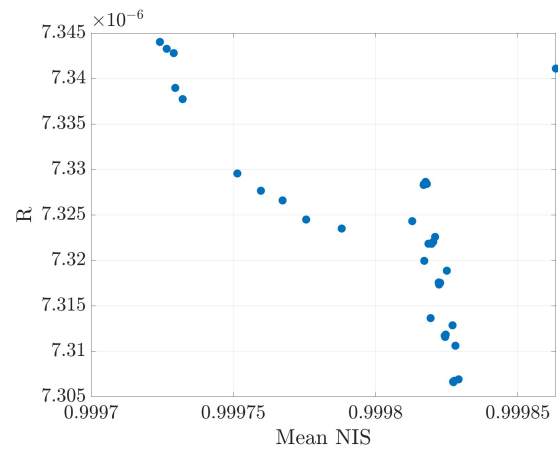


Figure 4.10: Estimated process noise $Q_{F_{total}}$ vs the prior force mean

An additional consistency check is to verify that the measurement-noise assumptions yield a mean NIS close to unity (i.e., $\overline{NIS} \approx 1$). Figure 4.11 plots the estimated R values against the mean NIS across simulations. For all runs, the mean NIS is approximately 1, indicating that the MLE-identified R is consistent with the innovation statistics and that the filter is neither over- nor under-confident on average. We also note that the two arrival cases do not produce identical R estimates, although the values are close. This discrepancy is expected and can be due to encoder errors (e.g., encoder sliding) leading to the MLE to adapt optimal R to each dataset accordingly.



(a) WFFAA→R1



(b) R2→R1

Figure 4.11: R estimations with respect their NIS value

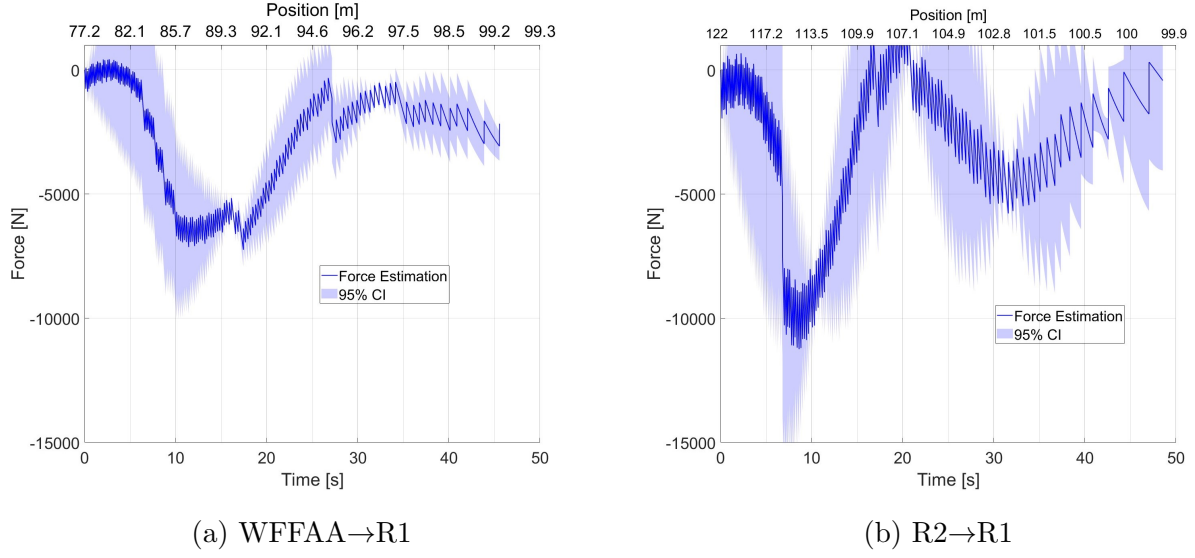


Figure 4.12: R1 arrivals

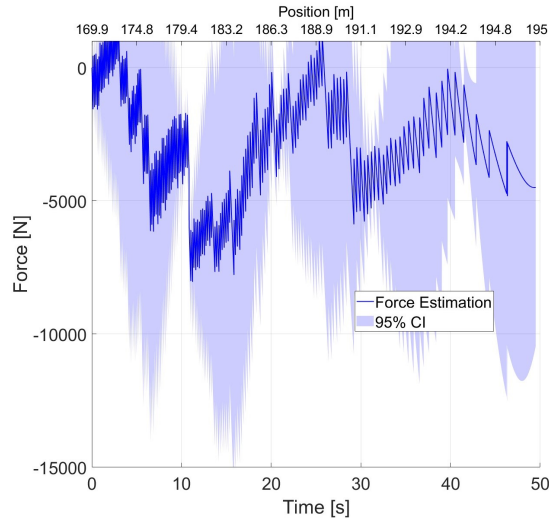
4.5.2 FMT arrivals

Herein we present the result of the method applied to all the trips in the data set. We are looking to compare the behavior of the forces arriving to same destination but coming from opposite direction. In order to estimate the force, the same prior assumptions for all the estimations have been established (mean prior and covariance). In the previous section its shown that for this approach the initial conditions will not affect the estimation of the force behavior.

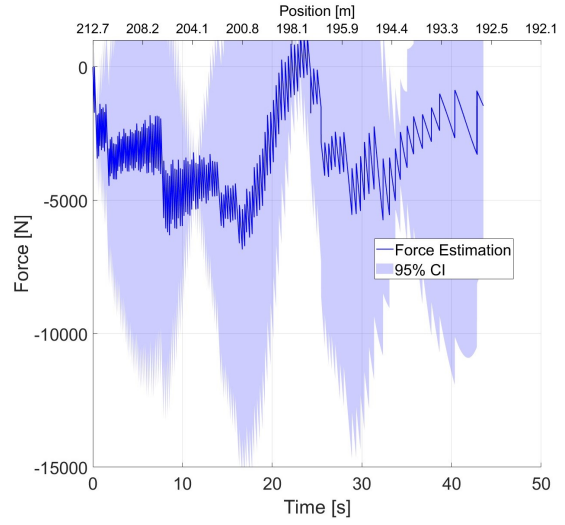
The force estimates across the different arrivals allow a direct comparison of how the system load evolves during coasting. In Figs. 4.12 - 4.15, the inferred overall force exhibits a common profile: an initial peak at the start of coasting followed by a decay toward a quasi-steady level. This motif is consistent with the numerical study in Sec. 3.2 using the 1-DOF formulation based on the nominal model of Chapter 2.

Table 4.9 summarizes the estimated process-noise variances for position and velocity (Q_{pos} , Q_{vel}), the force random-walk variance ($Q_{F_{\text{total}}}$), the measurement noise (R), and the mean NIS for each arrival. Two points stand out:

1. Stable kinematic and measurement noise. The estimates for Q_{pos} , Q_{vel} , and R are of similar order across datasets, indicating a consistent characterization of kinematic uncertainty and sensor noise. The mean NIS is ≈ 1 for all runs, which supports the

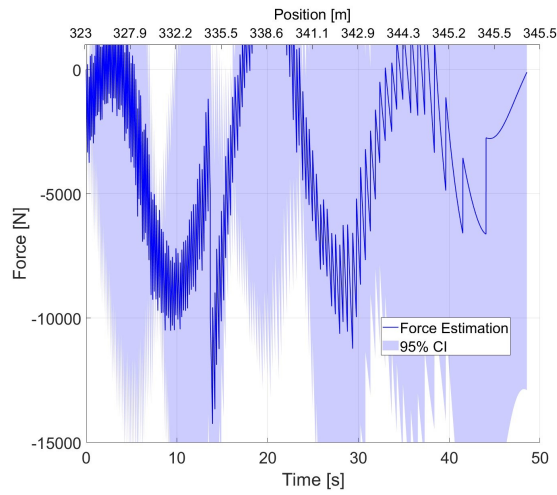


(a) R1→R2

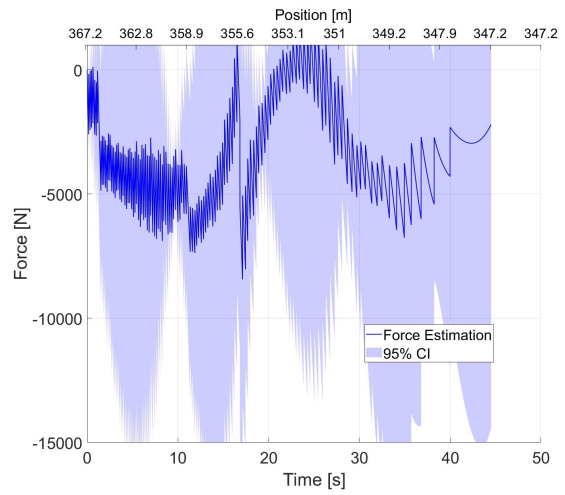


(b) R3→R2

Figure 4.13: R2 arrivals

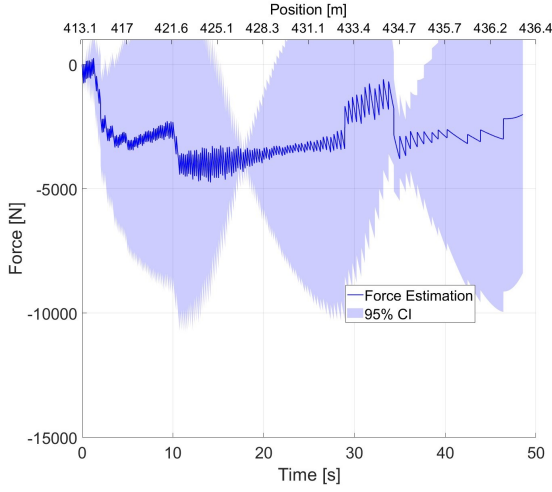


(a) R2→R3

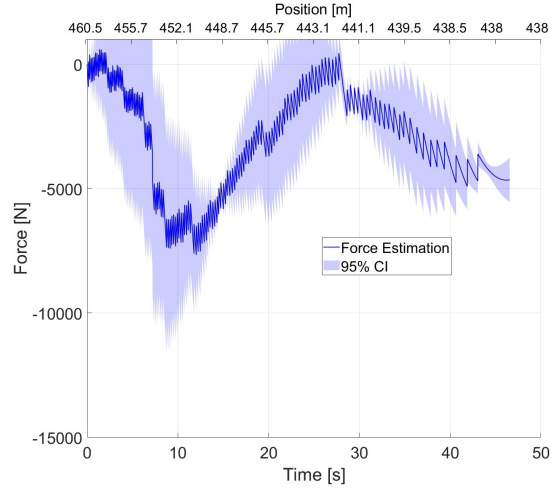


(b) R4→R3

Figure 4.14: R3 arrivals



(a) R3→R4



(b) E-FFAA→R4

Figure 4.15: R4 arrivals

Table 4.9: Data set estimation parameter results

Trip	Q_{pos}	Q_{vel}	$Q_{F_{total}}$	R	Mean NIS
E-FFAA→R1	3.8×10^{-4}	1.5×10^{-4}	1.2×10^4	7.8×10^{-6}	0.99
R2→R1	3.9×10^{-4}	1.1×10^{-4}	5.9×10^4	7.3×10^{-6}	0.99
R1→R2	3.9×10^{-4}	1.1×10^{-4}	6.9×10^4	1.1×10^{-6}	0.99
R3→R2	3.8×10^{-4}	2.7×10^{-4}	4.6×10^4	7.6×10^{-6}	0.99
R2→R3	3.6×10^{-4}	5.6×10^{-4}	1.9×10^5	7.9×10^{-6}	0.99
R4→R3	3.5×10^{-4}	2.5×10^{-4}	6.4×10^5	7.0×10^{-6}	0.99
R3→R4	3.7×10^{-4}	4.2×10^{-4}	1.6×10^4	7.7×10^{-6}	0.99
E-FFAA→R4	3.8×10^{-4}	$2, 2 \times 10^{-4}$	7.0×10^3	7.94×10^{-6}	0.99

conclusion that the combined noise model (Q , R) is statistically consistent with the innovations and that the filter is neither systematically over- nor under-confident.

2. Trajectory-dependent force uncertainty. In contrast, $Q_{F_{total}}$ varies appreciably across arrivals, implying that the effective variability of the resistive force is route-dependent. This change in uncertainty could be the result of more complex interactions in the resistive force that can be sign of wear or system malfunction. To test this hypothesis more data sets of the same arrival will be needed for comparison and behavior validation. Larger values of $Q_{F_{total}}$ correspond to force profiles with greater temporal

variability and wider confidence intervals.

The arrivals to R3 (Fig. 4.14) are the least similar to the other routes: the inferred force shows more erratic fluctuations and wider uncertainty bands. This behavior aligns with the parameters in Table 4.9, where the R3-related trips exhibit the largest $Q_{F_{\text{total}}}$ values. The joint evidence (irregular force traces plus inflated force process noise) suggests that the system experiences different or more variable conditions on the R3 approaches—for example, changes in rolling resistance, local grade or alignment, intermittent drivetrain interactions, or minor sensor timing effects that manifest as effective force variability. Importantly, the mean NIS remaining near unity indicates that the estimator has appropriately adjusted its uncertainty to the data rather than becoming overconfident.

Route-dependent $Q_{F_{\text{total}}}$ has direct diagnostic value in that it flags segments where the effective resistive environment is more susceptible to abrupt changes and warrants closer inspection. For R3, practical follow-ups include (i) checking for systematic speed differences at coast-entry, (ii) inspecting track condition and alignment near the R3 corridor, (iii) comparing encoder health (dropouts/quantization patterns) across routes, and (iv) repeating the analysis on adjacent trips to assess persistence.

4.6 Remarks

This chapter presented the acquisition, decoding, and analysis of experimental coasting data and applied the proposed model, uncertainty-aware estimation framework. The Gray-code decoding and time alignment produced coasting distances in close agreement with the plant log (typical absolute errors of order centimeters and $< 1\%$ relative error). This supports the fidelity of the position input used by the estimator and the soundness of the event-based segmentation.

Batch identification of the measurement noise R (via MLE/EM) yielded mean NIS values near unity across all runs, indicating statistical consistency of the innovation process and absence of persistent over/under-confidence. Estimated kinematic process noises (Q_{pos} , Q_{vel}) were stable across datasets, suggesting repeatable encoder/kinematic characteristics under nominal operation.

In contrast, the force random-walk variance $Q_{F_{\text{total}}}$ varied with the arrival trajectory. Arrivals to R3 exhibited the largest $Q_{F_{\text{total}}}$ and the most irregular force traces, pointing to genuinely different or more variable operating conditions on that corridor (e.g., local track/grade effects, drivetrain interactions, or sensor timing artifacts). The estimator

responded appropriately by inflating uncertainty rather than producing overconfident but biased estimates.

The 1-DOF formulation remained well-behaved on experimental data, reproducing the expected “initial peak-then-settle” force pattern observed in simulation. Given the single position measurement and partial observability, the simplified model offers superior robustness to priors and avoids the multi-modality seen when attempting to separate multiple force components with insufficient sensing.

Recommendations for monitoring.

- Track $Q_{F_{\text{total}}}$ as a route-specific health indicator to flag changes in the resistive environment.
- Maintain an innovation-based watch (e.g., windowed NIS bounds and residual whiteness tests) to ensure online consistency.
- Where feasible, add a secondary sensor (e.g., an acceleration proxy) or apply occasional excitation to strengthen identifiability on problematic routes (notably R3).
- Periodically re-estimate Q and R to accommodate seasonal effects, maintenance actions, or other shifts in operating conditions.

These findings motivate a deployment strategy centered on the simplified estimator with routine consistency checks and route-level trend tracking.

Chapter 5

Conclusion

This thesis developed and evaluated an uncertainty-aware workflow for estimating health-indicative forces in a [FMT](#) under sparse sensing and model-form uncertainty. The main contributions are:

- Physics-based model (Chap. 2): A dynamic model incorporating major resistive mechanisms (rolling/gearbox friction, aerodynamic drag, bearing losses, cable-chain interaction) was assembled with clear assumptions and constraints. The model reproduced nominal coasting behavior consistent with plant data (coasting distance ~ 24.5 m), establishing a sound baseline for estimation.
- [KF](#) workflow for system identification under high model form uncertainty (Chap. 3): A workflow for states and parameters estimation and uncertainty quantification is proposed. The [MLE](#) algorithm is used to estimate the process and measurement uncertainty, followed by a validation using the [NIS](#) mean value. This develop a strong approach for robust estimation.
- Model selection via numerical study (Chap. 3): A systematic comparison of a simplified 1-DOF model (net force) and a more detailed 2-DOF model (separate forces) showed that the 1-DOF formulation is more accurate and robust under the present single-measurement setting. The higher-dimensional model exhibited sensitivity to priors and multi-modality arising from structural unobservability.
- Experimental validation and UQ (Chap. 4): A Gray-code decoding and alignment pipeline produced position traces in close agreement with plant logs ($< 1\%$ error). Batch identification of noise statistics ([MLE](#)) yielded mean [NIS](#) near unity across

runs, indicating statistical consistency. The force process variance $Q_{F_{\text{total}}}$ varied by route, highlighting operational differences (notably on R3).

5.1 Key Findings

- Efficacy of model simplification. With a single displacement measurement, the simplified 1-DOF formulation delivers stable, physically plausible estimates and avoids the multi-modality encountered when attempting to separate multiple force components. The resulting estimation problem is better conditioned and less sensitive to initialization.
- Structural unidentifiability of the detailed model. In the 2-DOF case, measuring only x_1 together with rank-one stiffness/damping couplings leaves a rigid-body mode unsensed, making distinct (F_1, F_2) pairs observationally equivalent. This manifests as a likelihood ridge and prior-dependent convergence, consistent as the numerical results showed.
- Noise identification and consistency. EM/MLE-identified R values produced innovations consistent with Gaussian assumptions (mean NIS ≈ 1) and stable kinematic process noises $(Q_{\text{pos}}, Q_{\text{vel}})$, supporting the credibility of the learned noise model. Posterior covariance is informative about *within-run* confidence but must be interpreted alongside identifiability and innovation diagnostics.
- Route-dependent force variability. Elevated $Q_{F_{\text{total}}}$ and more irregular force traces on R3 suggest genuinely different operating conditions (e.g., local track/grade, drivetrain interactions, or sensor timing artifacts). Monitoring this parameter provides actionable route-level health insight.

5.2 Practical Implications

The results support deploying the simplified estimator for on-line monitoring with routine consistency checks. The following suggestions are made to enhance the monitoring process in practice:

1. Track $Q_{F_{\text{total}}}$ as a route-specific indicator of resistive environment changes;

2. Use windowed NIS and residual-whiteness tests for on-line consistency and drift detection;
3. When possible, augment sensing (e.g., acceleration proxy) or apply occasional excitation to strengthen identifiability on problematic routes (notably R3);
4. Periodically re-estimate Q and R to accommodate seasonal or maintenance-induced shifts.

5.3 Limitations and Future Work

This work relies on a single position measurement and modeling assumptions (uniform mass distribution, linear damping, idealized contact, and a partially specified cable-chain behavior). While the estimator adapts uncertainty (via Q) to the data, structural unidentifiability persists in models that separate forces without additional sensing. Moreover, residual non-Gaussianities (e.g., quantization and intermittent dropouts) remain and are only partially captured by Gaussian noise models.

Future efforts include:

1. Augment sensing for observability. Incorporate secondary measurements (e.g., an acceleration proxy or a second displacement) to mitigate the observability limitations that arise from single-channel sensing and reduce structural unidentifiability.
2. Route-aware anomaly detection. Investigate anomaly-detection thresholds based on route-specific trends in $Q_{F_{\text{total}}}$ and corresponding force behavior to distinguish nominal variability from emergent faults.
3. Damage-mechanism characterization. Model distinct forms of damage in the system and identify characteristic shapes in the resistive force, enabling more specific diagnostics.
4. Cable-chain physics refinement. Refine the physical model of the cable-chain interaction as additional data become available, relaxing idealizations used in the model formulation.
5. Robust noise modeling. Address residual non-Gaussianities by evaluating robust or heavy-tailed noise models and missing-data handling to better capture quantization and dropout effects.

5.4 Closing Remarks

Under sparse sensing and model–form uncertainty, the pursuit of ever–higher fidelity is not always the most effective path. This thesis shows that a carefully chosen model, coupled with principled uncertainty quantification (KF with MLE noise learning and innovation diagnostics), can deliver robust, interpretable, and deployable estimates. The approach provides a practical foundation for condition monitoring on the FMT and similar industrial assets, and establishes a blueprint for data–informed model selection and uncertainty management in constrained sensing environments.

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APPENDICES

Appendix A

Bearing Resistance

The SKF model considers different aspects of the rolling bearing friction torque:

$$M_{TOTAL} = M_{rr} + M_{sl} + M_{seal} + M_{drag} \quad (A.1)$$

For M_{rr} , the rolling frictional moment can be calculated as following:

$$M_{rr} = \phi_{ish} \phi_{rs} G_{rr} (vn)^{0.6} \quad (A.2)$$

Where v is the viscosity of the lubricant which in this case is $110 \text{ mm}^2/\text{s}$, n is the angular velocity of the bearing that we previously calculated as 31.25 RPM . The other components of the equation require more calculation that will be detailed as follows:

ϕ_{ish} is the inlet shear reduction factor that is obtained as following:

$$\phi_{ish} = \frac{1}{1 + 1.84 * 10^{-9} (nd_m)^{1.28} v^{0.64}} \quad (A.3)$$

where d_m is the mean diameter of the bearing in mm ($0.5 * (D + d)$), which is 142.875 mm .

$$\phi_{ish} = \frac{1}{1 + 1.84 * 10^{-9} (31.25 * 142.875)^{1.28} 110^{0.64}} = 0.9982 \quad (A.4)$$

ϕ_{rs} is the Kinematic replenishment/starvation reduction factor. This term correct the displace of lubricant away from the race ways during over-rolling of the moving elements in the bearing. ϕ_{rs} can be estimated using the following equation:

$$\phi_{rs} = \frac{1}{e^{\left[K_{rs} vn (d+D) \sqrt{\frac{K_z}{2(D+d)}} \right]}} \quad (A.5)$$

where K_{rs} is known as the replenishment/starvation constant and for grease and oil-air lubrication is established as $6 * 10^{-8}$. K_z is the bearing type related constant and for a cylindrical roller bearing is 6.2. By replacing the values, the following is obtained:

$$\phi_{rs} = 0.9938 \quad (\text{A.6})$$

G_{rr} is a rolling frictional variable that depends on the bearing type, radial force, mean bearing diameter, and axial load. For cylindrical roller bearings we establish G_{rr} as following:

$$G_{rr} = 2.13 * 10^{-6} d_m^{2.41} F_r^{0.31} \quad (\text{A.7})$$

$$G_{rr} = 14.068 \quad (\text{A.8})$$

Replacing on the rolling frictional moment equation is obtained the following:

$$M_{rr} = 1847.11 [Nmm] \quad (\text{A.9})$$

The next term the total moment equation M_{sl} represents the the sliding frictional moment in the bearing, which is calculated as follows:

$$M_{sl} = G_{sl} \mu_{sl} \quad (\text{A.10})$$

where G_{sl} is a sliding frictional variable that depends on the bearing type, radial force, mean bearing diameter, and axial load. For cylindrical roller bearings we establish G_{rr} as following:

$$G_{sl} = 0.16 d_m^{0.9} F_a + 0.0015 d_m F_r \quad (\text{A.11})$$

where F_a is the axial force on the bearing, but lets recall that in our assumptions we established that there is no axial force in our system, therefore the first term in the equation is equal to zero. Replacing we obtain:

$$G_{sl} = 37804.725 [Nmm] \quad (\text{A.12})$$

μ_{sl} is the sliding friction coefficient that can be calculated using the following equation:

$$\mu_{sl} = 0.12 \phi_{bl} + (1 - \phi_{bl}) 0.02 \quad (\text{A.13})$$

where ϕ_{bl} is the weighting factor for the sliding friction coefficient that can be calculated using the following equation:

$$\phi_{bl} = \frac{1}{e^{[2.6x10^{-8}(nv)^{1.4}d_m]}} \quad (\text{A.14})$$

Replacing we obtain the following:

$$\phi_{bl} = 0.7177 \quad (\text{A.15})$$

Now we can replace in the sliding frictional coefficient equation to obtain the following:

$$\mu_{sl} = 0.0917 \quad (\text{A.16})$$

Finally we replace on the sliding frictional moment to obtain the following:

$$M_{sl} = 3464.50 [Nmm] \quad (\text{A.17})$$

The next term on the total frictional moment equation M_{seal} represents the frictional moment caused by the seals of the bearing. However, according to SKF [111] this type of bearing does not requires a seal therefore this term is zero. The same happens to M_{drag} which is the frictional moment associated to the drag caused by the oil bath the rolling elements experiment in the bearing. In this situation the bearings are never under a oil bath due to the fact that we are using grease as lubricant, and this element is not covering the elements in bath. The grease distributes along the bearing and stays in the element surface to lubricate contacts. Therefore we establish the the total frictional moment:

$$M_{TOTAL} = 1847.11 + 3464.50 + 0 + 0 = 5316.61 [Nmm] \quad (\text{A.18})$$

To calculate the friction force associated to the bearing we divide by the wheel radius, that is given by OPG (305 mm):

$$F_B = \frac{M_{TOTAL}}{305} \quad (\text{A.19})$$

$$F_B = \frac{5316.61}{305} = 17.43 \text{ N per bearing} \quad (\text{A.20})$$

Therefore the total bearing friction force applied to each car will be 48.84, assuming each car has 4 bearings. It should be noted that the previous calculation for the bearing resistance has been made for the maximum speed that the trolley will faced of 1 m/s, this parameter depends on the trolley's velocity.

Appendix B

Gearbox Resistance

The parameters for the average friction coefficient in gears are calculated as follows:

Application Factor K_A

ISO Standard 6336 [3] proposes a procedure to calculate the application factor. This factor is defined as the ratio between the equivalent torque and nominal torque.

$$K_A = \frac{T_{eq}}{T_n} \quad (\text{B.1})$$

While the equivalent torque is calculated using the following equation, obtained from different sets of experiments under the same conditions, where aim is to make the gear fail and record the live in cycles of the gear:

$$T_{eq} = \left(\frac{n_1 T_1^p + n_2 T_2^p + \dots + n_i T_i^p}{n_1 + n_2 + \dots + n_i} \right)^{\frac{1}{p}} \quad (\text{B.2})$$

where $i = 1, 2, \dots, n$ is the number of the test, n_i is the number of the cycles, T_i is the torque at which failure occurred, and p is the slope in the Wöhler fatigue curves. Wöhler fatigue curves, also known as S-N curves, are fundamental in understanding the fatigue life of materials. These curves graphically represent the relationship between the stress applied to a material (S) and the number of cycles (N) it can withstand before failure [?].

Due to the fact that value are obtained experimentally we are going to used as a guide the numbers obtained by [34], where the K_a is calculated under different loads and different

velocities in Straight cylindrical gear systems following the ISO standard parameters [3]. After trials in two different sets of gear, he was able to calculate values of $K_a = 1.19$ and $K_a = 1.222$. For our gear box model we have established the following application factor due to the fact that we are using cylindrical gears of the same width as the one perform in the experiments by Cura [34]:

$$K_A = 1.20 \quad (\text{B.3})$$

Dynamic Factor K_v

According to ISO Standard the dynamic factor should be set at 2.00 if the gear is not working outside of resonance conditions. In gears, resonance conditions refer to scenarios where the natural frequencies of the gear system align with the frequencies of external forces or operational speeds [39]. This alignment can lead to excessive vibrations, potentially causing damage or failure of the gear components. Analyzing and understanding these resonance conditions is crucial for designing reliable and durable gear systems, especially in critical applications. The main reason for a gear to reach resonance conditions is to be working under speeds and loads that are out of its capability [94], therefore:

$$K_v = 2.00 \quad (\text{B.4})$$

Face Load Factor $K_{H\beta}$

The face load factor $K_{H\beta}$, represents the load distribution over the common face width in meshing gears. In the international standard for cylindrical gear rating, ISO 6336-1, that following parameters in Table B.1 are set to considered to select the appropriate value for the face load factor based on the working conditions. This factor consider face load interaction due misalignment in gears shafts.

For our model we are going to assume the gear haven't gone through any modification and for practical purposes the shafts cannot be considered aligned however for the purpose of our model we are going to assume they are, therefor we can say according to Table B.1:

$$K_{H\beta} = 1.05 \quad (\text{B.5})$$

Transverse load in plane of action in Newtons F_{bt}

The transverse load in plane of action F_{bt} in Newtons is the normal force acting against the face of the gears as they interact with each other.

Modification	Supposed deviation of the shaft	Deviation of the shaft (maximum) mm (in)	$K_{H\beta}$	Life time (hours)
Without modification	Worst case	0.2 (0.008)	2.53	499
Without modification	Reasonable case	0.1 (0.004)	1.05–2.53	1200
Without modification	No deviation	0.0	1.05	280000
With modification	Worst case	0.2 (0.008)	1.77	6750
With modification	Reasonable case	0.1 (0.004)	1.28–1.77	14500
With modification	No deviation	0.0	1.28	113000

Table B.1: Face Load factor established by ISO Standard [69]

To have a better perspective lets put the force in terms of the torque of the electric motor:

$$F_{bt} = \frac{\tau_M \rho}{\cos(\alpha)} \quad (\text{B.6})$$

Where we know that α is 20 degrees established by standards AGMA 3-6 [6], and ρ is the pitch radius of the impulsing gear. The pitch radius is the point where the two gears make contact with each other. Once the electric motor is shut down the normal force of the gears will be given by the remaining inertia of the system that will be give by $m\ddot{x}$.

This approach is valid while the electric motor is giving power to the NFT. After the electric motor shuts down the remaining inertia of the system gives the motion of the system. Therefore the the load acting on the gears is expressed in terms of the mass and acceleration of the system:

$$F_{bt} = \frac{m\ddot{x}\rho}{\cos(\alpha)} \quad (\text{B.7})$$

Helical load factor for scuffing $K_{B\gamma}$

The helical factor is a result of considering the circumference in the helical gears and how this affect the the mean friction coefficient. Due to the fact the we are assuming that we are assuming our gearbox is working with straight gears, no helical factor is considered.

$$K_{B\gamma} = 1.00 \quad (\text{B.8})$$

Face width in mm b

The face width of the gear is described in Fig. B.1, as 100 mm.

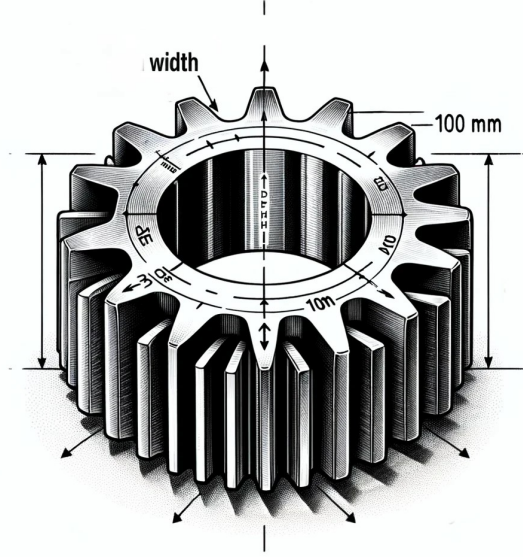


Figure B.1: Face width

Sum of tangential velocities at the pitch point $\nu_{\Sigma C}$

Ideally the tangential velocity between the set of two interacting gears should be exactly the same. Nevertheless the process has minor losses. According the American Gear Manufacturers Association [6], the efficiency of transmission between straight cylindrical and bevel gears can be in the range of 0.99-0.98 [93]. Considering this we can derive the following expression for the $\nu_{\Sigma C}$.

$$\nu_{\Sigma C} = 0.99v_T + v_T \quad (\text{B.9})$$

Where v_T is the tangential velocity, and the factor of 0.99 is assuming an efficiency of 99% of movement transition to the impulsed gear.

Equivalent radius of curvature at the pitch diameter in mm $\rho_{n,C}$

The equivalent radius a concept related to the curvature of the gear teeth at the pitch circle. The pitch circle is an imaginary circle used in gear design to simplify the analysis of gear teeth and their interaction.

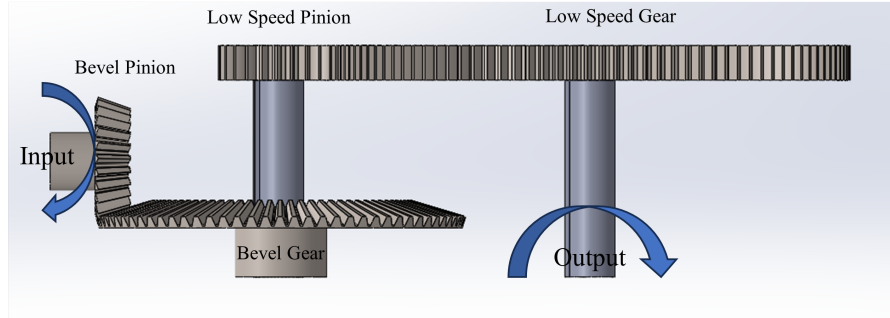


Figure B.2: Sketch of the gearbox configuration

For two mating gears, the equivalent radius of curvature ($\rho_{n,C}$) at the pitch point can be calculated from the radii of curvature ρ_1 and ρ_2 of the two contacting gear teeth at their pitch points:

$$\rho_{n,C} = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \quad (\text{B.10})$$

To apply this concept to the gearbox of interest, two stages of gears must be considered. The gearbox configuration is shown in Fig B.2, where a set of bevel gears and cylindrical gears are presented for the first and second stage respectively. For the first stage of gears the values for pitch radius are established using the software Solid works to model the gears: 50mm for the impulsing and 110 mm for the impulsed gear. For the second stage its established 70mm for the impulsing and 224 mm. obtaining the following values for $\rho_{n,C}$:

$$\rho_{n,C1} = 34.375 \quad (\text{B.11})$$

$$\rho_{n,C2} = 53.33 \quad (\text{B.12})$$

Dynamic viscosity of the lubricant in mPa's η_{oil}

For the oil selection it has been decided to use the Shell Omala, considered a high quality lubricant for heavy duty gears, recommended for high performance. Assuming a high quality lubricant will make the gears perform under optimal conditions, which is aligned to the assumptions that are being made.. According to the data sheet we have that [105]:

$$\eta_{oil} = 17.44 \text{ mPa s} \quad (\text{B.13})$$

Lubricant Factor X_L

The lubricant factor is given by the following expression:

$$X_L = 0.6 \dots 1.5 * \eta_{oil}^{-0.05} \quad (\text{B.14})$$

Assuming we are working with a new lubricant that the factor will be 0.6, this factor is associated to operating conditions of the lubricant, for high quality lubricants we obtain the following [3]:

$$X_L = 0.52 \quad (\text{B.15})$$

Roughness Factor X_R

The surface roughness factor can be calculated using the following equation:

$$X_R = 2.2 \cdot \left(\frac{Ra}{\rho_{n,C}} \right)^{0.25} \quad (\text{B.16})$$

where Ra is the arithmetic mean roughness between the interacting gears.

$$R_a = \frac{R_{a1} + R_{a2}}{2} \quad (\text{B.17})$$

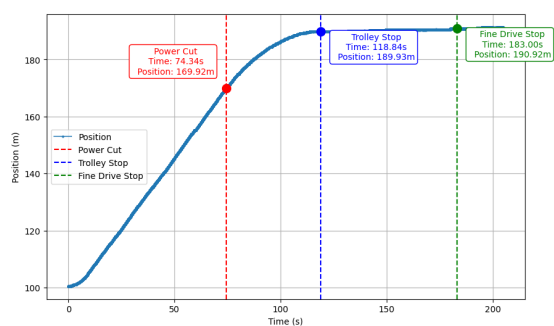
According to AGMA 3-6 [6] the recommended roughness for general purpose gears is of $1.6\mu m$. Now we assume the both interacting gears have the same surface roughness, therefore $R_a = 1.6$. Considering that, we obtain the following for the roughness factor [6]:

$$X_{R1} = 1.021 \quad (\text{B.18})$$

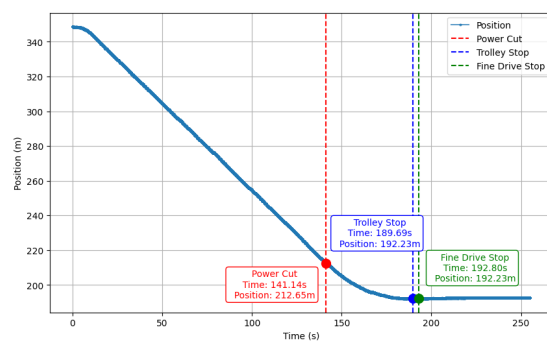
$$X_{R2} = 0.9156 \quad (\text{B.19})$$

Appendix C

Experimental Data

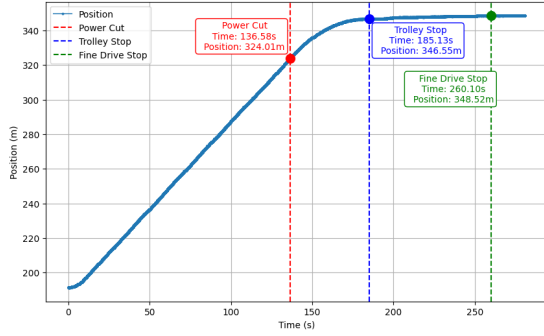


(a) FMT transit from R1 to R2

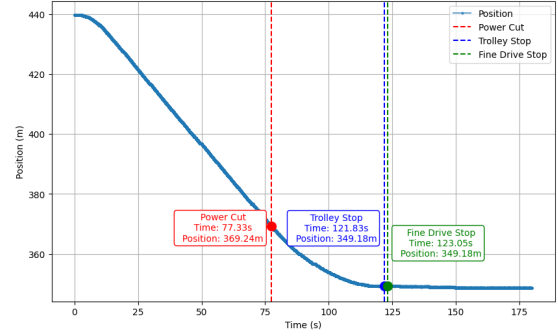


(b) FMT transit from R3 to R2

Figure C.1: Arrivals to R2

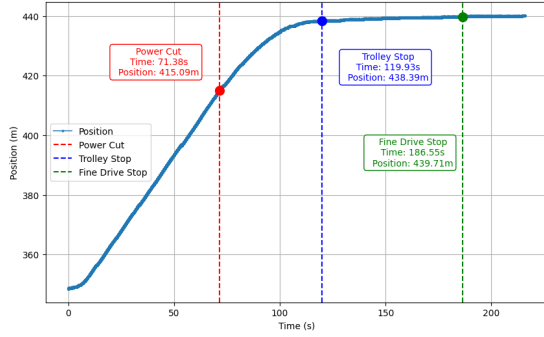


(a) FMT transit from R2 to R3

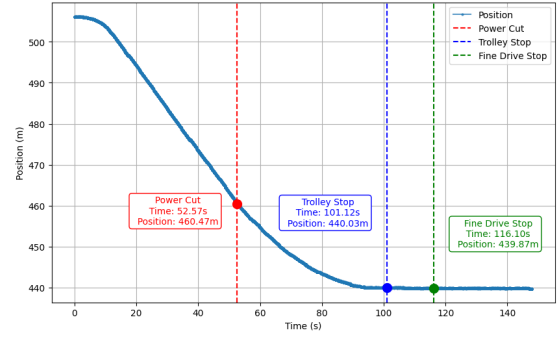


(b) FMT transit from R4 to R3

Figure C.2: Arrivals to R3

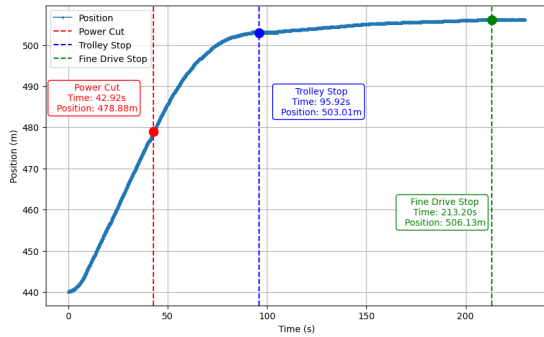


(a) FMT transit from R3 to R4

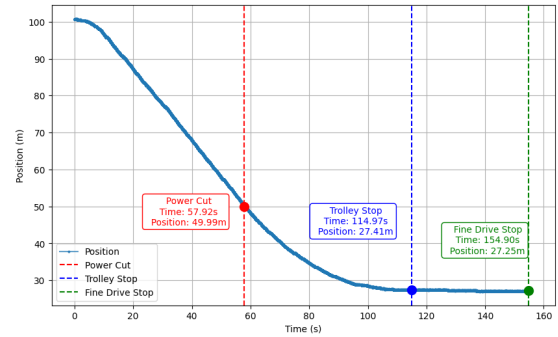


(b) FMT transit from E-FFAA to R4

Figure C.3: Arrivals to R4



(a) FMT transit from R4 to E-FFAA



(b) FMT transit from R1 to W-FFAA

Figure C.4: Arrivals to FFAA

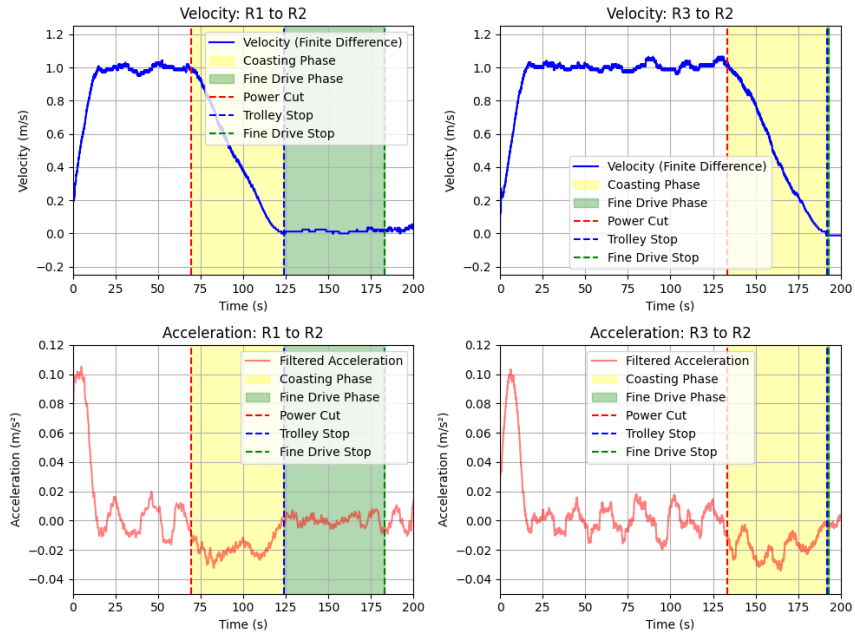


Figure C.5: velocity and acceleration for R2 arrivals

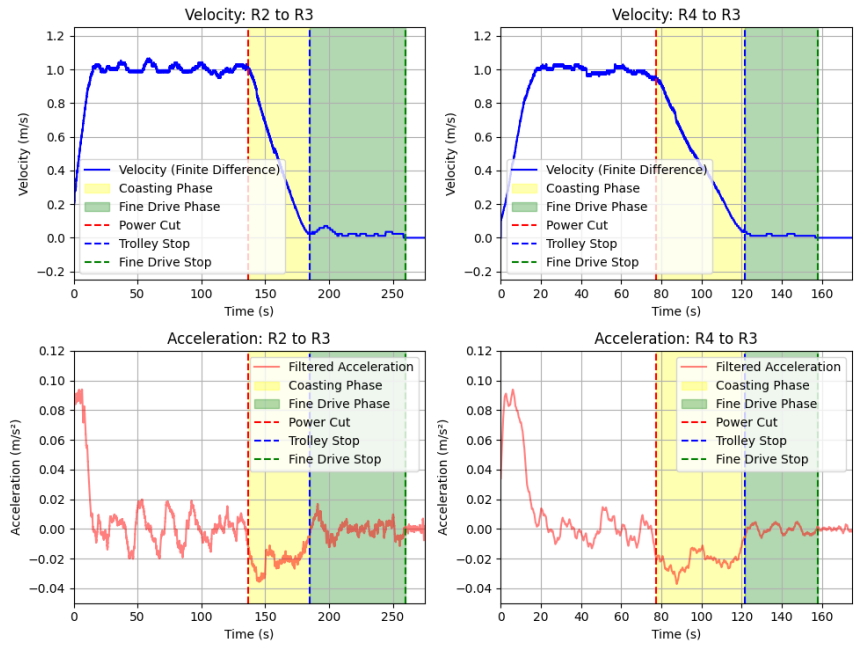


Figure C.6: velocity and acceleration for R3 arrivals

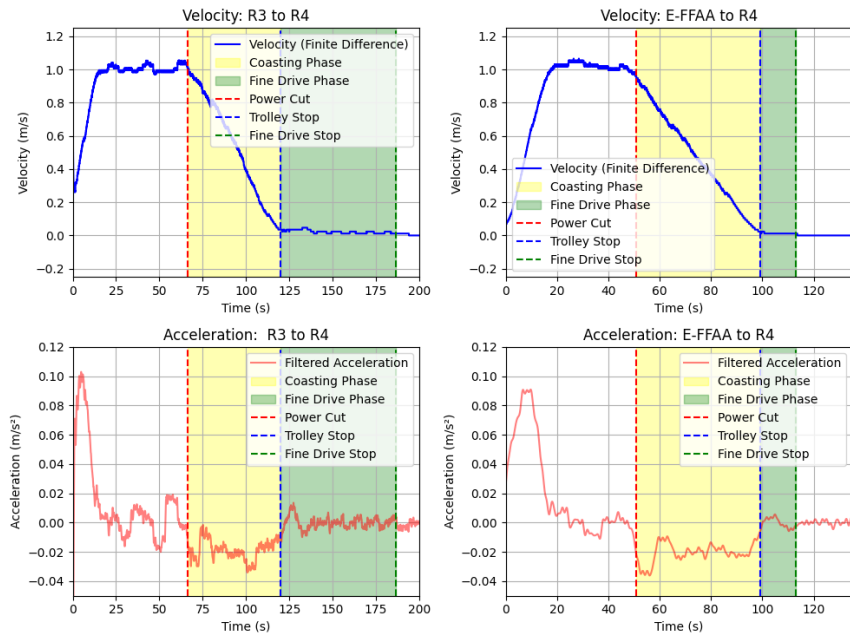


Figure C.7: velocity and acceleration for R4 arrivals